

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Discrete Random Variables

Let X be a discrete random variable with a pmf of $p(x)$ then

$$E[X] = \sum_x xp(x)$$

Let X be a random variable, then

$$Var(X) = E[X^2] - E[X]^2$$

Linear combination of random variables X and Y and fixed numbers a and b :

$$E[aX + bY] = aE[X] + bE[Y]$$

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y)$$

Distribution	pmf	$E(X)$	$Var(X)$
Binomial	$\binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$
Geometric	$(1-p)^{n-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Discrete Uniform	$\frac{1}{n}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$
Poisson	$\frac{\lambda^k}{k!}e^{-\lambda}$	λ	λ

Continuous Random Variables

$$E[X] = \int_{x \in \Omega_x} xf(x) dx$$

Distribution	pmf	$E(X)$	$Var(X)$
Continuous Uniform	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2

Central Limit Theorem

Under certain conditions:

$$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$$

$$(\bar{x}_1 - \bar{x}_2) \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$$

$$\hat{p} \sim N(p, \frac{p(1-p)}{n})$$

$$(\hat{p}_1 - \hat{p}_2) \sim N((p_1 - p_2), \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2})$$

Also note that

$$\hat{p}_{pooled} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$$

Linear Regression

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \epsilon$$

$$b_1 = \frac{s_y}{s_x} R \quad b_0 = \bar{y} - b_1 \bar{x}$$

Maximum Likelihood

The likelihood function

$$L(\bar{\theta}) = f_1(x_1, \bar{\theta}) \times \dots \times f_n(x_n, \bar{\theta})$$

MLE estimator of $\bar{\theta}$ is

$$\hat{\bar{\theta}} = \operatorname{argmax}_{\bar{\theta}} L(\bar{\theta}) = \operatorname{argmax}_{\bar{\theta}} \ln L(\bar{\theta})$$

```
dnorm(0.025)
```

```
## [1] 0.3988176
```

```
dnorm(0.05)
```

```
## [1] 0.3984439
```

```
dnorm(0.10)
```

```
## [1] 0.3969525
```

```
dnorm(0.80)
```

```
## [1] 0.2896916
```

```
dnorm(0.90)
```

```
## [1] 0.2660852
```

```
pnorm(0.025)
```

```
## [1] 0.5099725
```

```
pnorm(0.05)
```

```
## [1] 0.5199388
```

```
pnorm(0.10)
```

```
## [1] 0.5398278
```

```
pnorm(0.80)
```

```
## [1] 0.7881446
```

```
pnorm(0.90)
```

```
## [1] 0.8159399
```

```
qnorm(0.025)
```

```
## [1] -1.959964
```

```
qnorm(0.05)
```

```
## [1] -1.644854
```

```
qnorm(0.10)
```

```
## [1] -1.281552
```

```
qnorm(0.80)
```

```
## [1] 0.8416212
```

```
qnorm(0.90)
```

```
## [1] 1.281552
```