1 Discrete Random Variables Review

1.1 pmf and cdf

Let X be a random variable that represents total number of heads in 3 coin flips. Draw a plot of the cdf of X.

1.2 Expected Value and Variance

You are playing a die game where you roll a six sided die. You pay \$2 at each round to play. If you roll 1 or 2 or 3 then you win \$2.50. If you roll 4 or 5 then you lose \$6 (in addition to the \$2 you paid to play the game). If you roll 6 then you win \$10. What is the expected value and variance of your pay in this game?

1.3 Review of Expected Value and Variance in R

```
x <- c(0.50, -8, 8)
f_x <- c(1/2, 1/3, 1/6)

x*f_x
```

```
## [1]  0.250000 -2.666667  1.333333
```

```
mu <- sum(x*f_x)

mu
```

```
## [1] -1.083333
```

```
x-mu
```

```
## [1]  1.583333 -6.916667  9.083333
```

```
(x-mu)^2
```

```
## [1]  2.506944 47.840278 82.506944
```

```
(x-mu)^2*f_x
```

```
## [1]  1.253472 15.946759 13.751157
```

```
sigma2 <- sum((x-mu)^2*f_x)

sigma2
```

```
## [1] 30.95139
```

```
sigma <- sqrt(sigma2)
sigma
```

```
## [1] 5.563397
```

2 Combinations

3 Bernoulli Trials

4 Binomial Distribution

Conditions

- The trials have to be independent from each other.
- The probability of success has to be the same for each trial.
- The number of trials is fixed.
- Each trial outcome is either a success or a failure.

Probability mass function

Example

A vet has been assigned to work at a farm where 70% of the animals have been infected by avian influenza. The vet selects 10 animals at random. What is the probability that 1 of the selected animals are infected? What is the probability that 2 of the selected animals are infected? What is the probability that 3 of the selected animals are infected?

4.1 Using R for binomial distribution

Probability mass function

```
dbinom(x = 1, size = 10, prob = 0.70)
```

```
## [1] 0.000137781
```

If you know order of your arguments you do not have to specify them.

```
dbinom(1, 10, 0.70)
```

```
## [1] 0.000137781
```

But it is good practice to write the arguments.

```
dbinom(x = 2, size = 10, prob = 0.70)
```

```
## [1] 0.001446701
```

```
dbinom(x = 3, size = 10, prob = 0.70)
```

```
## [1] 0.009001692
```

```
dbinom(x = 1, size = 10, prob = 0.70) +
  dbinom(x = 2, size = 10, prob = 0.70) +
  dbinom(x = 3, size = 10, prob = 0.70)
```

```
## [1] 0.01058617
```

Cumulative Distribution Function

```
pbinom(q = 3, size = 10, prob = 0.70)
```

```
## [1] 0.01059208
```

Quantile function

```
qbinom(p = 0.000137781, size = 10, prob = 0.70)
```

```
## [1] 1
```

```
qbinom(p = 0.003, size = 10, prob = 0.70)
```

```
## [1] 3
```

4.2 Expected Value

4.3 Variance

Example: You can tell the difference between tap water and bottled water with a probability of 0.95. You are given 20 glasses of water without knowing whether they are bottled or from the tap. Calculate the expected value and variance of correct responses.

```
dbinom(x = 0:20, size = 20, prob = 0.95)
```

```
##  [1] 9.536743e-27 3.623962e-24 6.541252e-22 7.457027e-20 6.021550e-18
##  [6] 3.661102e-16 1.739024e-14 6.608289e-13 2.040309e-11 5.168784e-10
## [11] 1.080276e-08 1.865931e-07 2.658952e-06 3.108928e-05 2.953482e-04
## [16] 2.244646e-03 1.332759e-02 5.958215e-02 1.886768e-01 3.773536e-01
## [21] 3.584859e-01
```

5 Geometric Distribution

5.1 Probability mass function

5.2 Expected Value

5.3 Variance

Example

According to a Gallup survey the latest trend shows that 21% of people think abortion should be illegal in all circumstances. A journalist is writing an article and wants to include different opinions. What is the probability that the first person to think that abortion should be illegal in all circumstances is the third person the journalist approaches.

```r
dgeom(x = 2, prob = 0.21)
```

```
## [1] 0.131061
```

Note that the argument `x` refers to the number of failures not n.

The journalist starts their search again. What is the expected number of people they will approach before they find a person who thinks abortion should be illegal in all circumstances? What is the standard deviation?