Formulae Sheet

$$\begin{split} \bar{x} &= \frac{\sum_{i=1}^{n} x_i}{n} \\ P(A^c) &= 1 - P(A) \end{split} \qquad \qquad s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \end{split}$$

Linear combination of random variables X and Y and fixed numbers a and b:

$$E[aX+bY]=aE[X]+bE[Y] \\ Var(aX+bY)=a^2Var(X)+b^2Var(Y)$$
 if X and Y are independent.

Discrete Random Variables

Let X be a discrete random variable with a pmf of f(x) then

$$E[X] = \sum_x x f(x)$$

$$Var(X) = E[(X - E[X])^2] = \sum_x (x - E[X])^2 f(x) = E[X^2] - (E[X])^2$$

| pmf | E(X) | Var(X) |
|------------------------------------|---------------------|-----------------------|
| $\pi^x (1-\pi)^{1-x}$ | π | $\pi(1-\pi)$ |
| $\binom{n}{x}\pi^x(1-\pi)^{n-x}$ | $n\pi$ | $n\pi(1-\pi)$ |
| $(1-\pi)^x\pi$ | $\frac{1-\pi}{\pi}$ | $\frac{1-\pi}{\pi^2}$ |
| $\frac{\lambda^x}{x!}e^{-\lambda}$ | λ | λ |

Continuous Random Variables

Let X be a continuous random variable with a pdf of f(x) then

$$E[X] = \int_{x \in \Omega_x} x f(x) \, dx$$

$$Var(X) = E[(X - E[X])^2] = \int_{x \in \Omega_x} (x - E[X])^2 f(x) \, dx = E[X^2] - (E[X])^2$$

| pdf | E(X) | Var(X) |
|--------------------------|---------------------|-----------------------|
| $\lambda e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| 1 | $\frac{b+a}{2}$ | $\frac{(b-a)^2}{12}$ |