

Formulae Sheet

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Linear combination of random variables X and Y and fixed numbers a and b :

$$E[aX + bY] = aE[X] + bE[Y]$$

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) \text{ if } X \text{ and } Y \text{ are independent.}$$

Discrete Random Variables

Let X be a discrete random variable with a pmf of $f(x)$ then

$$E[X] = \sum_x x f(x)$$

$$Var(X) = E[(X - E[X])^2] = \sum_x (x - E[X])^2 f(x) = E[X^2] - (E[X])^2$$

| pmf | $E(X)$ | $Var(X)$ |
|--------------------------------------|---------------------|-----------------------|
| $\pi^x (1 - \pi)^{1-x}$ | π | $\pi(1 - \pi)$ |
| $\binom{n}{x} \pi^x (1 - \pi)^{n-x}$ | $n\pi$ | $n\pi(1 - \pi)$ |
| $(1 - \pi)^x \pi$ | $\frac{1-\pi}{\pi}$ | $\frac{1-\pi}{\pi^2}$ |
| $\frac{\lambda^x}{x!} e^{-\lambda}$ | λ | λ |

Continuous Random Variables

Let X be a continuous random variable with a pdf of $f(x)$ then

$$E[X] = \int_{x \in \Omega_x} x f(x) dx$$

$$Var(X) = E[(X - E[X])^2] = \int_{x \in \Omega_x} (x - E[X])^2 f(x) dx = E[X^2] - (E[X])^2$$

| pdf | $E(X)$ | $Var(X)$ |
|--------------------------|---------------------|-----------------------|
| $\lambda e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| $\frac{1}{b-a}$ | $\frac{b+a}{2}$ | $\frac{(b-a)^2}{12}$ |

| point estimate | critical value | standard error |
|-------------------------|----------------|--|
| p | z^* | $\sqrt{\frac{p(1-p)}{n}}$ |
| $p_1 - p_2$ | z^* | $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ |
| \bar{x} | t_{df}^* | $\sqrt{\frac{s^2}{n}}$ |
| $\bar{x}_1 - \bar{x}_2$ | t_{df}^* | $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ |

$$p_{pooled} = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}$$

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$$