

CW (CNN, Konwolucja)

Konwolucja w CNN (Convolutional Neural Network)

Input (konwolucji) $\mathbf{u} = (u(i, j))_{i=1, j=1}^{N, N}$, $u(i, j) \in \mathbb{R} \quad (1 \leq i, j \leq N)$

Output (konwolucji) $\mathbf{x} = (x(i, j))_{i=1, j=1}^{N, N}$, $x(i, j) \in \mathbb{R} \quad (1 \leq i, j \leq N)$
($N = 5$)

Jądro (*ang.* kernel) $\mathbf{w} = (w(i', j'))_{i'=-H, j'=-H}^{H, H}$
 $w(i', j') \in \mathbb{R} \quad (-H \leq i', j' \leq H)$
($H = 1$)

Konwolucja (splot) $\mathbf{u} \mapsto \mathbf{x} = \mathbf{w} * \mathbf{u}$

Konwolucja = ekstrakcja cech obrazów (feature extraction)

$$x(i, j) = f\left(\left\{\sum_{i'=-H}^H \sum_{j'=-H}^H w(i', j') u(i + i', j + j')\right\} - \theta\right)$$

f funkcja progowa, θ próg

Uwaga. $(i + i', j + j') \notin \{1, \dots, N\} \times \{1, \dots, N\} \Rightarrow u(i + i', j + j') = 0$
(Jak to implementować w Pythonie?)

Dane dla zadań

$$\mathbf{u}_i = \begin{pmatrix} u(1, 1) & u(1, 2) & u(1, 3) & u(1, 4) & u(1, 5) \\ u(2, 1) & u(2, 2) & u(2, 3) & u(2, 4) & u(2, 5) \\ u(3, 1) & u(3, 2) & u(3, 3) & u(3, 4) & u(3, 5) \\ u(4, 1) & u(4, 2) & u(4, 3) & u(4, 4) & u(4, 5) \\ u(5, 1) & u(5, 2) & u(5, 3) & u(5, 4) & u(5, 5) \end{pmatrix} \quad (1 \leq i \leq 5)$$

■ = 1.0 $\in \mathbb{R}$ (float)

0 $\in \mathbb{R}$ (float)

$$\begin{aligned} \mathbf{u}_1 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 \\ 0 & \blacksquare & 0 & \blacksquare & 0 \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \mathbf{u}_2 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 \\ \blacksquare & 0 & \blacksquare & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 \end{pmatrix} \\ \mathbf{u}_3 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \end{pmatrix} & \mathbf{u}_4 &= \begin{pmatrix} 0 & 0 & \blacksquare & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \mathbf{u}_5 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \blacksquare & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$$\mathbf{w}_j = \begin{pmatrix} w(-1, -1) & w(-1, 0) & w(-1, 1) \\ w(0, -1) & w(0, 0) & w(0, 1) \\ w(1, -1) & w(1, 0) & w(1, 1) \end{pmatrix} \quad (1 \leq j \leq 4)$$

$$\begin{array}{cccc}
\mathbf{w}_1 = \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & 0 & 0 \\ \blacksquare & 0 & 0 \end{array} &
\mathbf{w}_2 = \begin{array}{ccc} 0 & 0 & \blacksquare \\ 0 & 0 & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{array} &
\mathbf{w}_3 = \begin{array}{ccc} \blacksquare & \blacksquare & 0 \\ 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \end{array} &
\mathbf{w}_4 = \begin{array}{ccc} 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \end{array}
\end{array}$$

Zadania. (Konwolucja)

Niech funkcja $\mathbf{u} \mapsto \mathbf{x} = (x(i, j))_{i=1, j=1}^{5,5} = \mathbf{w} * \mathbf{u}$ będzie zdefiniowana wzorem

$$x(i, j) = f\left(\left\{\sum_{i'=-1}^1 \sum_{j'=-1}^1 w(i', j')u(i + i', j + j')\right\} - \theta\right)$$

z funkcją progową

$$f(x) = \begin{cases} 0 & \text{gdy } x < 0 \\ 1 & \text{gdy } x \geq 0 \end{cases}$$

i progiem $\theta = 2.5$. Obliczyć i wyświetlić jako obraz $\mathbf{w}_j * \mathbf{u}_i$ ($1 \leq i \leq 5, 1 \leq j \leq 4$).

Uwaga. Konwolucja przeprowadza ekstrakcję cech obrazów (feature extraction).