DCFoil Documentation

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Summary

DCFoil is a program for the dynamic analysis and design optimization of composite hydrofoils.

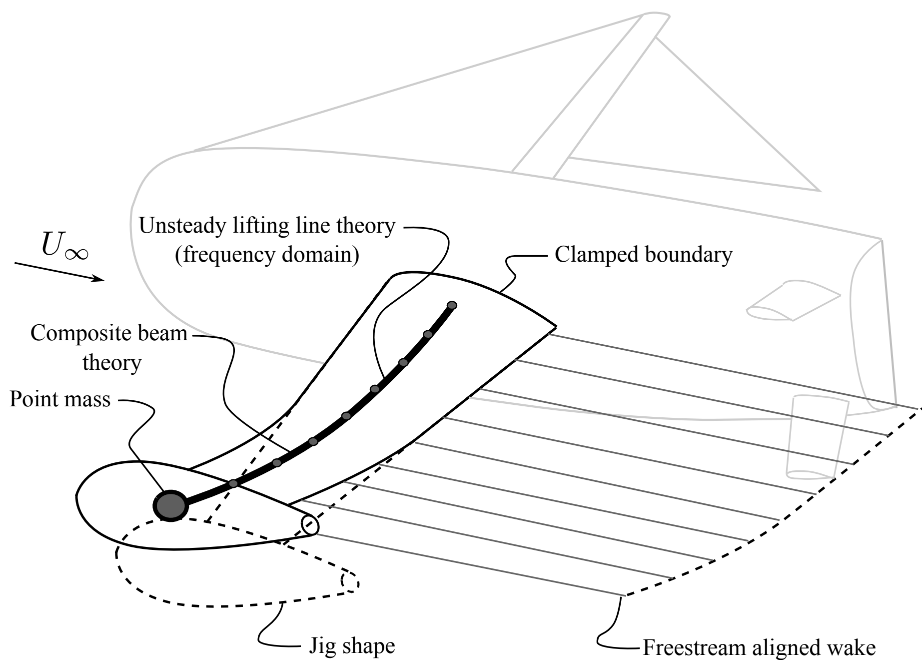

Figure 1: DCFoil modeling approach

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2.2.1 Steady lifting line

The lifting line model derives from Glauert [1, Ch. XI] and works for arbitrary chord. Specifically, we are after sectional lift slopes ($c_{\ell_\alpha} = dc_\ell/d\alpha = a_0$). We assume

- the chord is small compared to the span,
- the wing is symmetric about the centerline,
- span is straight and orthogonal to the freestream
- trailing vortices are shed from the trailing edge and align with the freestream

The wing is represented by superimposing "horseshoe" systems of vortex lines (analagous to a wire with electrical current). This is because the circulation across a wing is not constant. The free vortex system is a sheet of trailing vortices springing from the trailing edge. The induced velocity of an element of the line (ds) at point P from one vortex line of constant strength Γ is

$$dq = \frac{\Gamma}{4\pi r^2}\sin(\theta)ds \tag{1}$$

but in practice, one would solve this is an integral over the entire vortex line, so we will build up to the full wing.

To begin solution, we first assume the circulation is the Fourier series[1]

$$\Gamma(y) = 2U_\infty s\sum_{n=1}^{\infty} a_n \sin(n\theta) \quad \text{where } y = -\frac{s}{2}\cos(\theta) \quad \text{and } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]. \tag{2}$$

The difficulty is now determining the Fourier coefficients a_n so we need some relations for $\Gamma(y)$ to solve it.

One relation is the equation for the normal induced velocity (downwash velocity) at a point along the span

$$w(y) = \frac{1}{4\pi}\int_{-s/2}^{s/2}\frac{\frac{d\Gamma}{d\eta}}{y-\eta}d\eta = \boxed{-U_\infty\sum_{n=1}^{\infty}\frac{na_n\sin(n\theta)}{\sin(\theta)}} \tag{3}$$

where η is the spanwise coordinate and s is total span. We skipped a few steps in the derivation [2, Sec. 3.7].

The second relation is from sectional lift as a function of circulation. Recall that the circulation at a section (derived from Kutta-Joukowski lift theorem) is

$$\Gamma(y) = \frac{1}{2}c_\ell c U_\infty = \frac{1}{2}a_0 c\left(U_\infty\alpha - w(y)\right) \tag{4}$$

where we made use of $c_\ell = a_0\alpha_{\text{eff}} = a_0(\alpha - w/U_\infty)$. After substitution of the Fourier series form and combining Equations (3) and (4), we end up with

$$\sum_{n=1}^{\infty} a_n\sin(n\theta)\left(n\mu + \sin(\theta)\right) = \mu\alpha\sin(\theta) \quad \text{where } \mu(\theta) = \frac{a_0 c(\theta)}{4s} \tag{5}$$

[1] Kerwin and Hadler [2] use $\tilde{y}$ as θ

Here's the digestion of the Julia code which does the numerical solution of Equation (5) but symmetrically about the centerline.

$$
\begin{aligned}
&\tilde{y} = \left[0, \frac{\pi}{2}\right] \quad \text{of size nNodes} \\
&\mathbf{n} = [1:2:2\times \text{nNodes}] \\
&\mathbf{c} = c\sin(\tilde{y}) \quad \text{(parametrized vector leading to elliptical planform)} \\
&\mathbf{b} = \frac{\pi}{4}\frac{\mathbf{c}}{s/2}\alpha\sin(\tilde{y}) \quad \text{(RHS of Equation (5) in vector form)} \\
&\tilde{y}n = \tilde{y} \otimes \mathbf{n} \quad \text{(outer product)} \\
&\mathbf{A_0} = \begin{bmatrix} | & | & \\ \sin(\tilde{y}) & \sin(\tilde{y}) & \cdots \\ | & | & \end{bmatrix} \quad \text{(square matrix of } \sin(\tilde{y})) \\
&\mathbf{A_1} = \frac{\pi}{4}\frac{\mathbf{c}}{s/2} \otimes \mathbf{n} \quad \text{(outer product representing } n\mu \text{ on LHS)} \\
&\mathbf{A} = \sin(\tilde{y}n) \odot (\mathbf{A_0} + \mathbf{A_1}) \\
&\mathbf{Ax} = \mathbf{b} \quad \text{(solve linear system for } \mathbf{x} = a_n) \\
&\Gamma(y) = 4U_\infty s/2 \left(\underbrace{\sin(\tilde{y}n)\mathbf{x}}_{\text{mat-vec product}} \right) \\
&c_\ell = \frac{2\Gamma(y)}{U_\infty c} \\
&c_{\ell_\alpha} = \frac{c_\ell}{\alpha}
\end{aligned}
$$

2.2.2 Extension to unsteady frequency domain

We are interested in the sectional lift and moments for a harmonically oscillating body. Theodorsen [3] came up with the Theodorsen function $C(k)$ to account for the lag and deficit in forces. It is applied as a transfer function to the static hydrodynamics.

$$
\begin{Bmatrix} F_z \\ M_y \end{Bmatrix}_i = -\left(\left[\mathbf{m_f}\begin{Bmatrix} \ddot{w} \\ \ddot{\psi} \end{Bmatrix}\right]_i + \left[\mathbf{c_f}\begin{Bmatrix} \dot{w} \\ \dot{\psi} \end{Bmatrix}\right]_i + \left[\mathbf{k_f}\begin{Bmatrix} w \\ \psi + \alpha_0 \end{Bmatrix}\right]_i + \left[\hat{\mathbf{c}}_\mathbf{f}\begin{Bmatrix} \dot{w}' \\ \dot{\psi}' \end{Bmatrix}\right]_i + \left[\hat{\mathbf{k}}_\mathbf{f}\begin{Bmatrix} w' \\ \psi' \end{Bmatrix}\right]_i \right)\Delta y_i \tag{6}
$$

where Δy_i is the strip width at node i, which we assume to be equal to element length.

$$
\mathbf{m_f} = \pi\rho_f b^2 \begin{bmatrix} 1 & ab \\ ab & b^2\left(\frac{1}{8}+a^2\right) \end{bmatrix} \tag{7}
$$

$$
\mathbf{c_f}(k) = \frac{1}{2}\rho_f b U_0 \left(\cos(\Lambda) \begin{bmatrix} c_{\ell_\alpha} 2C(k) & -b\left[2\pi + c_{\ell_\alpha}(1-2a)C(k)\right] \\ c_{\ell_\alpha} eb2C(k) & \frac{b}{2}(1-2a)(2\pi b - c_{\ell_\alpha} 2ebC(k)) \end{bmatrix} \right) \tag{8}
$$

$$
\mathbf{k_f}(k) = \frac{1}{2}\rho_f U_0^2 \cos(\Lambda) \left(\cos(\Lambda) \begin{bmatrix} 0 & -C(k)2bc_{\ell_\alpha} \\ 0 & -2eb^2 c_{\ell_\alpha} C(k) \end{bmatrix} \right) \tag{9}
$$

$$
\hat{\mathbf{c}}_\mathbf{f}(k) = \frac{1}{2}\rho_f b U_0 \sin(\Lambda) \left(\begin{bmatrix} 2\pi b & 2\pi ab^2 \\ 2\pi ab^2 & 2\pi b^3\left(\frac{1}{8}+a^2\right) \end{bmatrix} \right) \tag{10}
$$

$$
\hat{\mathbf{k}}_\mathbf{f}(k) = \frac{1}{2}\rho_f b U_0 \sin(\Lambda) \left(U_0\cos(\Lambda) \begin{bmatrix} c_{\ell_\alpha} 2C(k) & -c_{\ell_\alpha} b(1-2a)C(k) \\ 2ebc_{\ell_\alpha} C(k) & \pi b^2 - c_{\ell_\alpha} eb^2(1-2a)C(k) \end{bmatrix} \right) \tag{11}
$$

The extra $\hat{\square}$ matrices account for sweep effects on the quasi-steady (damping and stiffness) aerodynamics and are lumped into their respective global matrices if they are in phase with velocity or displacements.

3 Static solution

4 Forced vibration solution

5 Flutter solution

5.0.1 Mode space reduction

To reduce the problem size, we use mode space reduction to a reduced set of N_r generalized coordinates. The displacement field is approximated by

$$\mathbf{u} \approx \mathbf{Q}_r(y)\mathbf{q}(t) \tag{12}$$

where $\mathbf{q} \in \mathbb{R}^{N_r}$ is a vector of retained generalized coordinates and $\mathbf{Q}_r \in \mathbb{R}^{N_s \times N_r}$ is a matrix with columns corresponding to eigenvectors. This is typically called the normal mode method. One then solves the eigenvalue problem

$$\left(\mathbf{K_s} - \omega_i^2 \mathbf{M_s}\right) \bar{\mathbf{u}}_i = 0 \tag{13}$$

where ω_i is the natural frequency. We compute $\bar{\mathbf{u}}_i$ and collect them in the matrix

$$\mathbf{Q}_r = \begin{bmatrix} | & | & & | \\ \bar{\mathbf{u}}_1 & \bar{\mathbf{u}}_2 & \cdots & \bar{\mathbf{u}}_{N_r} \\ | & | & & | \end{bmatrix}. \tag{14}$$

Now the reduced stiffness and mass matrices are

$$\mathbf{M_s}_r = \mathbf{Q}_r^T \mathbf{M_s} \mathbf{Q}_r = \mathbf{I}_r \in \mathbb{R}^{N_r \times N_r} \tag{15}$$

$$\mathbf{K_s}_r = \mathbf{Q}_r^T \mathbf{K_s} \mathbf{Q}_r = \operatorname{diag}\left[\omega_i^2\right] \tag{16}$$

and the governing equation reduces to

$$\mathbf{M_s}_r \ddot{\mathbf{q}} + \mathbf{C_s}_r \ddot{\mathbf{q}} + \mathbf{K_s}_r \mathbf{q} - \mathbf{Q}_r^T \mathbf{f}_{\text{hydro}} = \mathbf{0} \tag{17}$$

To apply this to the hydrodynamic loads, we obtain from Equation (**??**)

$$\mathbf{f}_{\text{hydro},r} = -\left(\mathbf{M_f}_r \ddot{\mathbf{u}} + \mathbf{C_f}_r \dot{\mathbf{u}} + \mathbf{K_f}_r \mathbf{u}\right) \tag{18}$$

where the matrices are

$$\mathbf{M_f}_r = \mathbf{Q}_r^T \mathbf{M_f} \mathbf{Q}_r \tag{19}$$

$$\mathbf{C_f}_r = \mathbf{Q}_r^T \mathbf{C_f} \mathbf{Q}_r \tag{20}$$

$$\mathbf{K_f}_r = \mathbf{Q}_r^T \mathbf{K_f} \mathbf{Q}_r. \tag{21}$$

Note, since cavitating flow leads to non-symmetric matrices, we cannot do all the simplifications Jonsson et al. [4] uses.

References

[1] H. Glauert. *The Elements of Aerofoil and Airscrew Theory*. Cambridge Science Classics. Cambridge University Press. doi: 10.1017/CBO9780511574481.

[2] Justin E Kerwin and Jacques B Hadler. Principles of naval architecture series: Propulsion. *The Society of Naval Architects and Marine Engineers (SNAME)*, pages 18–30, 2010.

[3] T. Theodorsen. General theory of aerodynamic instability and the mechanism of flutter. Technical Report Rept. 496, NACA, May 1934.

[4] Eirikur Jonsson, Gaetan K. W. Kenway, Graeme J. Kennedy, and Joaquim R. R. A. Martins. Development of flutter constraints for high-fidelity aerostructural optimization. In *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Denver, CO, June 2017. AIAA 2017-4455.