Math/CS 466/666: Programming Project 1

This project explores the Cholesky factorization of a symmetric positive definite matrix. For this project the class has not been randomly grouped into teams. Instead, please form your own teams on WebCampus consisting of 3 to 4 students. Please welcome any student who wishes to join your group so no people are left out. If you prefer to work independently that is also fine.

Each team should present their work in the form of a typed report using clear and properly punctuated English. Pencil and paper calculations may be typed or hand written. Where appropriate include full program listings and output. Every team member should participate in the work and be prepared to independently answer questions concerning the material. One report per team must be submitted through WebCampus as a single pdf file to complete this project.

1a. Write a program to find the Cholesky factorization $A = U^T U$ of a symmetric positive definite 2×2 matrix A. Note that if your programming language of choice has a built-in Cholesky factorization subroutine, do not use it for this exercise. Your program should report an error if the input matrix is not symmetric or not positive definite; otherwise it should report the factor U.

b. Test your program using the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}.$$

Which ones are symmetric positive definite? For those that are, what is the factor U?

c. The Julia programming language has a Cholesky factorization routine included with the linear algebra library. The command

```
julia> using LinearAlgebra
```

loads this library into your workspace. After loading the linear algebra library type

```
julia> A=rand(3,3)
julia> B=A'*A
julia> z=cholesky(B)
julia> propertynames(z)
```

to randomly create a positive definite matrix B, find its Cholesky decomposition and finally display what fields are available for use.

d. Compute the residual error of the factorization found in the previous problem using

```
julia> z.L*z.U-B
julia> norm(z.L*z.U-B)
```

Comment on the accuracy of the factorization.

e. The file `prog1c.dat` contains square matrices of varying sizes separated by blank lines. The description of each matrix consists of an integer specifying the dimension of the matrix followed by the entries of the matrix in row-major order. Use the Julia command `cholesky` to check which of these matrices are positive definite and to find the Cholesky decomposition for each one that is.