An one-shot approximation procedure for linear quadratic feedback synthesis ★

Siddhartha, Manav Debasish Chatterjee, Ravi Banavar [1]

Systems and Control Engineering, Indian Institute of Technology, Bombay, India

Abstract: This article introduces an offline approximate feedback synthesis algorithm for continuous-time unconstrained linear quadratic (LQ) optimal control problems. The driving engine behind the approximation algorithm is a special kind of *quasi-interpolation* technique based on the theory of *approximate approximations*. The explicit feedback control is approximated via the quasi-interpolation scheme on a uniform-cardinal grid over the phase-space. This method theoretically provides guarantees for uniform i.e. $\mathbb{L}^\infty$ convergence of the approximated feedback law to the actual optimal feedback, upto some saturation error, which can be reduces to an arbitrary number. We provide a simple numerical example to illustrate our results.

Keywords: optimal control, linear quadratic regulator, approximation algorithm.

1. INTRODUCTION

This article focuses on a class of continuous-time unconstrained optimal control problems where the system dynamics is linear and the cost function is quadratic in state and action variables. These well-known class of problems in the control-theory literature are referred to as the *linear quadratic* (LQ) optimal control problems. More specifically, our article concerns with a special class of unconstrained LQ problems known as *linear quadratic regulator* (LQR) problems, where the time horizon is infinite. For linear time-invariant systems with constant state and actuation matrices and time-invariant quadratic costs with constant weighting matrices, it is a well known fact that the optimal feedback policy $x \longmapsto \varphi^*(x)$ is an affine function of system states Liberzon (2012) which guarantees stability (exponential) (Liberzon, 2012, Chapter 6, Theorem 6.1) of the closed loop system.

The aim of this article is to introduce a quasi-interpolation-based approximation technique to furnish *one-shot* approximate unconstrained LQ feedback maps. The quasi-interpolation technique we employ here is a grid-based technique, which constructs the approximate feedback map based on the available values of the optimal feedback map on a uniform grid. The most prominent feature of the technique is the ability to furnish approximate feedback maps with *uniform* approximation guarantees on potentially unbounded sets. The synthesis procedure follows a ***challenge-answer*** maxim: given a prespecified level of accuracy say, $\varepsilon > 0$ with respect to the uniform metric, exhibit if possible, a "small" parameter h (the step size) and a "large" parameter $\mathcal{D}$ (the shape parameter), such that the approximate feedback respects the prespecified level of accuracy. The method, does *not* involve convergence in a conventional sense. However, since the preset level of accuracy is *arbitrary* and the positive parameters h and $\mathcal{D}$ can always be picked to achieve that prespecified level of accuracy, our technique is perfectly suited for engineering applications. To this end we show *pseudo-convergence* of the approximate feedback map to the optimal one.

The article unravels as follows: The basic unconstrained LQ problem has been set up in §2. The main results of this article along with a brief overview of the approximate approximations technique is presented in §3. Finally we provide a numerical example based of the developed technique for illustration in §4.

1.1 Notation

We employ standard notation in this article. We let $\mathbb{N}^* := \{1, 2, \ldots\}$ denote the set of positive integers, $\mathbb{N} := \mathbb{N}^* \cup \{0\}$, and $\mathbb{Z}$ denote the integers. The vector space $\mathbb{R}^d$ is equipped with standard inner product, $\langle x, y\rangle := \sum_{j=1}^d x_j y_j'$ for every $x, y \in \mathbb{R}^d$. By $(\mathbb{R}^d)^\star$ we mean the dual space of $\mathbb{R}^d$ which is of course isomorphic to the primal vector space $\mathbb{R}^d$ in view of the Riesz representation theorem. By $\alpha \in (\mathbb{N}^* \cup \{0\})^d$ we denote a multi-index of length $|\alpha| := \alpha_1 + \ldots + \alpha_d$, and we set $z^\alpha := z_1^{\alpha_1} \cdots z_d^{\alpha_d}$ for $z \in \mathbb{R}^d$. The usual α order derivative of a function f is denoted by

$$\partial^\alpha f(x) := \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots x_d^{\alpha_d}} f(x),$$

and thus we denote $\nabla_k f := (\partial^\alpha f)_{|\alpha|=k}$, the vector of partial derivatives.

2. PROBLEM SETUP

Let us consider a continuous-time linear time-invariant dynamical system modeled by the ordinary differential equation

$$\dot{x}(t) = Ax(t) + Bu(t) \quad \text{for } t \geq 0, \tag{2.1}$$

with the following data:

((2.1)-a) $x(t) \in \mathbb{R}^d$ is the vector of states;

((2.1)-b) $u(t) \in \mathbb{R}^m$ is the control action at time t;

((2.1)-c) $A \in \mathbb{R}^{d\times d}$ is the system matrix, and $B \in \mathbb{R}^{d\times m}$ is the control/actuation matrix.

★ Siddhartha Ganguly is supported by the PMRF grant RSPMRF0262, from the Ministry of Human Resource Development, Govt. of India. The authors are with Systems and Control Engineering, IIT Bombay.

[1] `Email addresses: {sganguly, dchatter, banavar}@iitb.ac.in.`

Consider the following infinite horizon unconstrained LQR problem for the system (2.1), given by:

$$\begin{aligned}
&\underset{u}{\text{minimize}} \quad \mathbb{J}(x,u) := \int_0^{+\infty} \Big(\langle x(t),\, Qx(t)\rangle + \langle u(t),\, Ru(t)\rangle \Big)\,\mathrm{d}t \\
&\text{subject to} \quad \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) & \text{for } t \geq 0, \\ x(0) = x_0, \end{cases}
\end{aligned} \tag{2.2}$$

where $Q = Q^\top \in \mathbb{R}^{d\times d}$ is a given positive semi-definite state-weighting matrix and $R = R^\top \in \mathbb{R}^{m\times m}$ is a given positive definite matrix. The solution to the LQR problem (2.2) can be obtained by using Bellman's dynamic programming (DP) principle. We want to find the optimal cost-to-go function $\mathbb{R}^d \ni x \longmapsto \mathbb{J}^*(x) \in \mathbb{R}$, which satisfies:

$$\min_{u\in\mathbb{R}^m} \Bigg(\langle x(t),\, Qx(t)\rangle + \langle u(t),\, Ru(t)\rangle + \bigg\langle \frac{\partial \mathbb{J}^*}{\partial x},\, \big(Ax(t)+Bu(t)\big)\bigg\rangle \Bigg) = 0, \tag{2.3}$$

for all $x \in \mathbb{R}^d$. The solution of (2.3) is given by:

$$\begin{cases} P = P^\top \succcurlyeq 0, \\ Q + A^\top P + PA - PBR^{-1}B^\top P = 0, \\ K = -R^{-1}B^\top P, \\ u^*(t) = Kx(t). \end{cases} \tag{2.4}$$

Note that the feedback gain K in (2.4) is independent of the state information and thus the map $x \longmapsto u^*(x)$ is a static linear state feedback. Also it is worth noting that the algebraic Riccati equation (ARE)

$$Q + A^\top P + PA - PBR^{-1}B^\top P = 0, \tag{2.5}$$

is quadratic in P and thus the solution is non-trivial. However it is a well known that under some stabilizability conditions, (2.5) has a unique, positive definite solution, to wit:

Theorem 1. (Bitmead and Gevers, 1991, Theorem 10.9). Consider the algebraic Riccati equation (2.5) associated with the problem (2.2). Assume that

(1) the pair $\big(A, B\big)$ is stabilizable;
(2) the pair $\big(A, Q^{1/2}\big)$ has no unobservable modes on the imaginary axis;
(3) the weighting matrices $Q \succcurlyeq 0$, and $R \succ 0$.

Then the Riccati equation (2.5) has a solution P, which is unique, maximal and positive definite symmetric.

Remark 2. We say that P is a stabilizing solution of the ARE if the closed loop system matrix $F := \big(A + BK\big)$ is Hurwitz, i.e. its eigen values has negative real part.

3. MAIN RESULT

Our main result — the approximate feedback synthesis algorithm — is established in this section. We begin with a brief presentation of the chief technical tool driving our algorithm.

3.1 Approximate approximations

Approximation of multivariable functions via quasi-interpolation with data sites $(mh)_{m\in\mathbb{Z}^d}$ on uniform grids has been studied extensively in the past. *Approximate approximations* as an approximate quasi-interpolation scheme was first introduced in Maz'ya (1991). We provide a brief summary and a few relevant results on approximate approximations pertinent to our work. For simplicity, we restrict ourselves in the realm of data distributions with regular center Maz'ya and Schmidt (1996). However, the theory is also well developed to deal with irregularly centered data sites Maz'ya and Schmidt (2001). Let a function $u : \mathbb{R}^d \longrightarrow \mathbb{R}$ be the object of approximation, i.e., the approximand. The idea behind the quasi-interpolation scheme is to represent the approximand $u(\cdot)$ as the sum of the scaled basis functions shifted by d-dimensional data distributions. For a fixed step size $h > 0$ and a shape parameter $\mathcal{D} \in \,]0, +\infty[$, such an approximant is

$$\widehat{u}(x) := \mathcal{D}^{-d/2} \sum_{m\in\mathbb{Z}^d} u(x_m)\, \psi\left(\frac{x - x_m}{h\sqrt{\mathcal{D}}}\right) \quad x \in \mathbb{R}^d, \tag{3.6}$$

where the data points/sites $x_m := mh$ are specified on a uniform grid of dimension d. The generating function $\psi(\cdot)$ belongs to a certain class of *nice* functions with enough smoothness attributes, and the linear combination of dilated shifts of $\psi(\cdot)$ forms an *approximate partition of unity* i.e. $\mathbb{R} \ni \xi \longmapsto \mathcal{D}^{-d/2} \sum_{m\in\mathbb{Z}^d} \psi\left(\frac{\xi - m}{\sqrt{\mathcal{D}}}\right) \approx 1$. Notice that this is a point of departure from the plain quasi-interpolants, where the generating function forms an exact partition of unity. The shape parameter $\mathcal{D}$ results in *peaked/narrowed* and *flat* generating function for its high and low values. As the scheme (3.6) is a quasi-interpolation procedure, the generating functions $\psi\left(\frac{\cdot - x_j}{h\sqrt{\mathcal{D}}}\right)$ do not satisfy the stringent cardinality property that every conventional interpolation schemes has to satisfy: $\psi\left(\frac{x_i - x_j}{h\sqrt{\mathcal{D}}}\right) = \delta_{\mathrm{ij}}$, where δ_{ij} is the Kronecker-delta function. We now provide the main theorem that we are going to use to arrive at our main. We shall see that the error estimate consists of two terms – one of which decays with some order depending upon the smoothness attributes of the function, and the other depends on the scaling parameter $\mathcal{D}$ of the generating functions.

Theorem 3. (Maz'ya and Schmidt, 2007, Chapter 2) Consider a function $u(\cdot)$ belonging to the class of Lipschitz continuous functions i.e., u satisfies the inequality $\|u(x+y) - u(x)\| \leq L_0\|y\|$ with the Lipschitz rank L_0 for all $x \in \mathbb{R}^d$. Let $\{mh : m \in \mathbb{Z}^d\} \subset \mathbb{R}^d$ is the set of data sites. In addition, suppose that the generating function ψ satisfy:

- *the continuous moment condition* of order N:
$$\int_{\mathbb{R}^d} \psi(y)\,\mathrm{d}y = 1, \ \int_{\mathbb{R}^d} y^\alpha \psi(y)\,\mathrm{d}y = 0, \ \text{for all } \alpha,\ 1 \leq |\alpha| < N; \tag{3.7}$$
- *the decay condition* with exponent K i.e.
$$|\partial^\beta \psi(x)|\,(1 + \|x\|)^K < +\infty, \tag{3.8}$$
for $x \in \mathbb{R}^d$, $K > d, 0 \leq |\beta| \leq \mu$, where β is a multi-index and μ is the smallest integer greater that $d/2$, and $\mathcal{F}\psi(0) = 1$, $\mathcal{F}$ being the Fourier transform operator.

Then

$$|\widehat{u}(x) - u(x)| \leq C_\gamma L_0 h\sqrt{\mathcal{D}} + \Delta_0(\psi, \mathcal{D}), \tag{3.9}$$

where $\Delta_0(\psi, \mathcal{D}) := \mathcal{E}_0(\psi, \mathcal{D})\|u\|_\infty$ is the saturation error, $C_\gamma := N \cdot \Gamma(N+1)/\Gamma(N+2)$ is a constant. The term $\mathcal{E}_0(\psi, \mathcal{D})$ is given by

$$\mathcal{E}_0(\psi, \mathcal{D}) := \sum_{\nu\in\mathbb{Z}^d\setminus\{0\}} \mathcal{F}\psi(\sqrt{D}\,\nu)\, e^{2\pi i\langle x,\,\nu\rangle}. \tag{3.10}$$

The assumptions on $\psi(\cdot)$ guarantees that for any preassigned $\varepsilon > 0$, we can choose a $\mathcal{D}_{\min} > 0$ such that for any $\mathcal{D} > \mathcal{D}_{\min}$, $\|\mathcal{E}_0(\psi, \mathcal{D})\|_\infty \leq \varepsilon$; see (Maz'ya and Schmidt, 2007, Chapter 2, Corollary 2.13).

Remark 4. Notice that the approximate approximation formula (3.6) involves an infinite sum over a d-dimensional integer lattice to approximate the function $u(\cdot)$ at a point $x \in \mathbb{R}^d$. Thus naturally an infinite number of summands plays a part in constructing the approximant $\widehat{u}(\cdot)$. However, this sum can be truncated in applications due the decay properties that the generating function enjoys. To this end, we can define a truncated approximant Maz'ya and Schmidt (2007) which we continue to call $\widehat{u}(\cdot)$ by abuse of notation:

$$\widehat{u}(x) := \mathcal{D}^{-d/2} \sum_{m \in \mathbb{F}} u(mh)\, \psi\left(\frac{x - mh}{h\sqrt{\mathcal{D}}}\right), \quad x \in \mathbb{R}^d, \tag{3.11}$$

where $\mathsf{B}(x, \Pi) := \{x_0 \in \mathbb{R}^d \mid |x - x_0| \le \Pi\}$ and $\mathbb{F} := \{mh, m \in \mathbb{Z}^d \mid mh \in \mathsf{B}(x, \Pi)\}$ is a finite set over which the summation (3.11) is performed. Then fixing the parameter $\Pi > 0$, we can obtain a *uniform estimate* similar to the preceding one where the saturation error can be controlled by choosing $\mathcal{D}$ appropriately. We refer the reader to (Maz'ya and Schmidt, 2007, Chapter 2, Section 2.3.2) for more details.

We now summarize the key features of approximate approximations:

- For any $u(\cdot)$ in an appropriate class of functions, $\widehat{u}(\cdot)$ approximates $u(\cdot)$ in the uniform norm within some *preassigned* error-margin, say, $\varepsilon > 0$, by means of *pseudo convergence* in two steps:
  - On the right hand side of (3.9), the second term $\Delta_0(\psi, \mathcal{D})$ — the *saturation error* — depends on the shape parameter $\mathcal{D}$, and can be lowered within $\frac{\varepsilon}{2}$ by increasing $\mathcal{D}$ after fixing a small h, say 0.1.
  - The first term on the right hand side of (3.9) converges to zero as $h \longrightarrow 0$. Thus after fixing h and reducing $\Delta_0(\psi, \mathcal{D})$ below $\frac{\varepsilon}{2}$, reduce h (below 0.1) such that the first term stays below $\frac{\varepsilon}{2}$. The total error, consequently, stays within the preassigned bound ε.
- An upper bound on h and a lower bound on $\mathcal{D}$ to match the preassigned saturation error can always be obtained (in some cases precise formulas can be given — see Maz'ya and Schmidt (2001)); see (Maz'ya and Schmidt, 2007, Chapter 2) and (Maz'ya and Schmidt, 2007, Chapter 3, Table 1) for precise error estimates and relevant data.
- While the approximant $\widehat{u}(\cdot)$ of $u(\cdot)$ in (3.6) employs an infinite sum to perform the approximation, in practice the summation can be truncated since the generating functions can be chosen to be in the Schwarz class (the class of rapidly decreasing functions); see Remark 4 and (Maz'ya and Schmidt, 2007, Chapter 2, Section 2.3.2) for more details.

Here is our main result:

Theorem 5. Consider the linear quadratic optimal control problem (2.2) and let $\mathcal{G} \ni x \longmapsto \varphi^*(x) := -R^{-1}B^\top Px$ be the optimal feedback function, where $\mathcal{G} \subset \mathbb{R}^d$ is a bounded set. Let $x \longmapsto \widehat{\varphi}(x)$ be the approximate feedback policy to $\varphi^*(\cdot)$ obtained via the quasi-interpolation engine (3.6). Fix a preassigned uniform error margin $\varepsilon > 0$. Then there exists a pair $(h, \mathcal{D}) \in\,]0, +\infty[^2$ such that

$$\|\varphi^* - \widehat{\varphi}\|_\infty \le \varepsilon. \tag{3.12}$$

Remark 6. Theorem 5 is a qualitative statement asserting, for each $\varepsilon > 0$, the existence of a pair $(h, \mathcal{D})$ of strictly positive numbers that ensure that the approximated feedback function is *uniformly* within ε of the optimal feedback function. Our proof of Theorem 5 (given below) illustrates a general mechanism of quantitatively determining the pair $(h, \mathcal{D})$ of positive parameters needed to answer the challenge posed by a given uniform error margin $\varepsilon > 0$; a specific illustration will be given in the numerical example in §4.

Proof. The feedback function $\mathcal{G} \ni x \longmapsto \varphi^*(x)$ is a linear/affine function of the state variables and thus it is Lipschitz continuous with the Lipschitz rank say L_0. Then we can find an approximate map $\widehat{\varphi}(\cdot)$ (see Theorem 3) which has the representation

$$\widehat{\varphi}(x) = \sum_{m \in \mathbb{F}} \varphi^*(mh)\, \psi\left(\frac{x - mh}{h\sqrt{\mathcal{D}}}\right), \tag{3.13}$$

for all $x \in \mathcal{G}$ and the following estimate holds

$$|\varphi^*(x) - \widehat{\varphi}(x)| \le C_\gamma L_0 h \sqrt{\mathcal{D}} + \Delta_0(\psi, \mathcal{D}). \tag{3.14}$$

In (3.14) the various quantities are as follows:

- $\Delta_0(\psi, \mathcal{D}) := \mathcal{E}_0(\psi, \mathcal{D}) \|\varphi^*\|_\infty$,
- $\mathcal{E}_0$ is as in (3.10),
- the constant $C_\gamma := N \cdot \Gamma(N+1)/\Gamma(N+2)$, where N is the order of moment condition that $\psi(\cdot)$ satisfies as defined in (3.7), $\Gamma(\cdot)$ is the gamma function, and
- L_0 is the Lipschitz constant of $\varphi^*(\cdot)$.

Fix $h = h_0 > 0$, from (Maz'ya and Schmidt, 2007, Chapter 2, Corollary 2.13) it follows that for the preassigned $\varepsilon > 0$, we can find $\mathcal{D}_{\min} > 0$ such that when $\mathcal{D} \ge \mathcal{D}_{\min}$,

$$\|\mathcal{E}_0(\psi, \mathcal{D})\|_\infty \le \frac{\varepsilon}{2\|\varphi^*\|_\infty},$$

thus it follows that

$$\|\Delta_0(\psi, \mathcal{D})\|_\infty \le \frac{\varepsilon}{2}. \tag{3.15}$$

Once $\Delta_0(\psi, \mathcal{D})$ has been reduced to an arbitrary number, we can fix

$$h := \frac{\varepsilon}{2 C_\gamma L_0 \sqrt{\mathcal{D}}}. \tag{3.16}$$

Combining the estimate (3.15) with (3.16), we observe that

$$\|\varphi^* - \widehat{\varphi}\|_\infty \le \varepsilon, \tag{3.17}$$

thus given a prespecified $\varepsilon > 0$ we can pick $(h, \mathcal{D}) \in\,]0, +\infty[$ such that the estimate (3.17) always holds. The proof is complete.

4. NUMERICAL EXPERIMENTS

Consider the infinite time linear quadratic (LQ) problem :

$$\begin{aligned}
&\underset{u}{\text{minimize}} && \mathbb{J}(x, u) := \int_0^{+\infty} \big(\langle x(t),\, Qx(t)\rangle + \langle u(t),\, Ru(t)\rangle\big)\, dt \\
&\text{subject to} && \begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -2x_1(t) + x_2(t) + u(t), \\ x_1(0) = 2, \text{ and } x_2(0) = -3, \end{cases}
\end{aligned} \tag{4.18}$$

where the weighting matrices are defined as

$$Q := \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, \; R := \frac{1}{4}.$$

The feedback gain matrix K and the solution of the Riccati equation i.e. the symmetric positive definite matrix P corresponding to the optimal control problem (4.18) can be readily calculated. In fact we have

$$P = \begin{pmatrix} 1.732 & 0.3660 \\ 0.3660 & 1.4729 \end{pmatrix}, \; K = \begin{pmatrix} 1.4641 & 5.8916 \end{pmatrix}.$$

Thus, the optimal LQ feedback policy is given by

$$\mathbb{R}^2 \ni (x_1, x_2) \longmapsto \varphi(x_1, x_2) := 1.464x_1 + 5.90x_2. \qquad (4.19)$$

Note that, for practical application the feedback synthesis procedure is preferred to be carried out over a compact set instead of the whole $\mathbb{R}^2$. To this end, consider the sublevel set of the quadratic function $\mathbb{R}^d \ni z \longmapsto V(z) := \langle z, Pz \rangle$ associated with the closed loop system $\dot{x} = (A + BK)x$, i.e. the ellipsoid

$$\mathcal{E}(P; a) := \{ z \in \mathbb{R}^d : z^\top P z \leq a \} \qquad (4.20)$$

where $P \in \mathbb{R}^{d \times d}$ is the solution of the ARE, and $a \in [0, +\infty[$. It is easy to see that $\mathcal{E}(\cdot, \cdot)$ is a convex, compact subset of $\mathbb{R}^d$. Furthermore, for any $x(0) \in \mathcal{E}$, $x(t) \in \mathcal{E}$, for all $t \geq 0$ i.e $\mathcal{E}$ is an positively invariant set under the optimal LQ feedback law. For numerical purposes, we have defined a square grid on $x_1 - x_2$ i.e $\mathcal{G} := [-1, 1] \times [-1, 1] \subset \mathbb{R}^2$. The region $\mathcal{G}$ contains the positively invariant ellipsoid $\mathcal{E}(P; 1)$

Check again!

. For the quasi-interpolation we picked our basis function

$$x \longmapsto \psi(x) := \frac{1}{\sqrt{\pi}} \mathrm{e}^{-\|x\|^2}. \qquad (4.21)$$

For illustration, let us fix a error tolerance $\varepsilon = 0.005$; fix the shape parameter $\mathcal{D} = 2$ and choose the parameter $h = \frac{\varepsilon}{2C_\gamma L_0 \sqrt{\mathcal{D}}} = 0.0026$. We picked a conservative choice of the Lipschitz constant $L_0 = 2$ and the multi-dimensional Gaussian kernel (4.21) satisfies the moment condition (3.7) with $M = 2$ and thus $C_\gamma = 0.33$. With these ingredients, we employed the quasi-interpolant

$$\widehat{\varphi}(x) := \frac{1}{\pi \mathcal{D}} \sum_{m \in \mathbb{F}} \varphi^*(mh)\, \mathrm{e}^{-\frac{\|x - mh\|^2}{\mathcal{D}h^2}}, \qquad (4.22)$$

and it is guaranteed that for the given $\varepsilon = 0.005$, our one-shot synthesis produces the pair $(h, \mathcal{D}) = (0.0026, 2)$ such that $\|\varphi^*(x) - \widehat{\varphi}(x)\| \leq 0.005$; the Figure 2 numerically verifies this fact. The Table 1 shows a list of user-defined tolerance values and the corresponding h and $\mathcal{D}$ needed to achieve it.

Table 1.

Threshold (ε)	h	$\mathcal{D}$
10×10^{-3}	5.353×10^{-3}	2
5×10^{-3}	2.676×10^{-3}	2
1×10^{-3}	0.437×10^{-3}	3

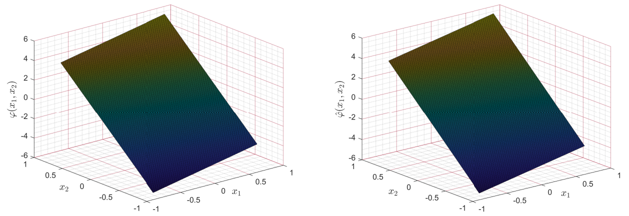

Fig. 1. The optimal and the approximated LQ feedback policies

The figure 1 shows the optimal and the approximated feedback policies. The approximation is quality is quite good, summing only a few terms of the series (4.22) over $\mathbb{F}$.

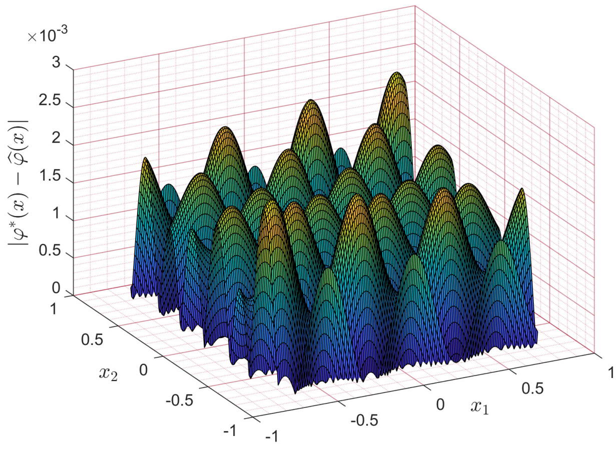

Fig. 2. Error between the optimal and the approximate feedback polices

5. CONCLUSION

This article introduced an approximate feedback synthesis technique based on a quasi-interpolation scheme. We showed that the approximation engine is capable to furnish a feedback function which can be made arbitrarily close to the smooth optimal feedback by choosing the pair $(h, \mathcal{D})$, uniformly over the whole state-space. The numerical result illustrates the effectiveness of the proposed algorithm.

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