ME 202

LECTURE 17

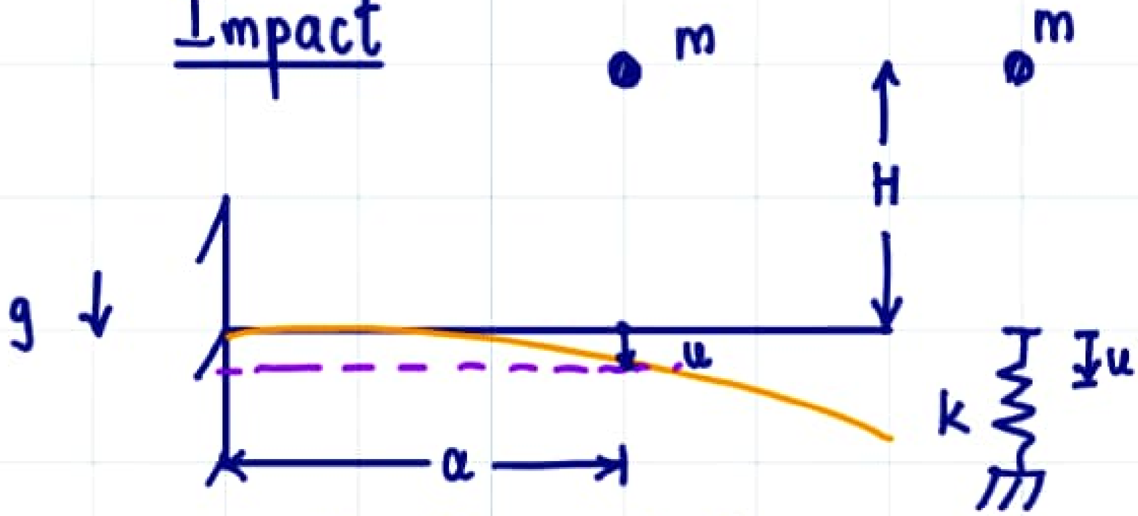
MON 14 FEB 2022

"Dynamics"

Getting dynamical insight / scaling laws from static solutions.

Real dynamics PDEs.

Impact



mg statically applied

$$u_s = \frac{Pa^3}{3EI} = \frac{mga^3}{3EI} \quad \downarrow$$

static

$$\text{local stiffness} = \frac{P}{u_s} = \frac{3EI}{a^3} = k$$

mg applied dynamically.

u dynamic displacement

Max displacement of beam

$$PE_1 + \cancel{KE_1} = PE_2 + \cancel{KE_2}$$

$$mg(H+u) = \frac{1}{2} k u^2$$

$$\frac{mg}{K}(H+u) = \frac{1}{2} u^2$$

$$u_s = \frac{mg}{K} \quad \text{known}$$

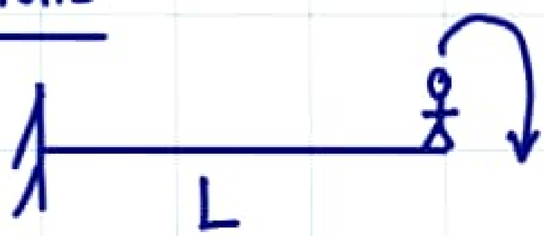
Quadratic eqn for u

$$u^2 - 2 u u_s - 2 u_s H = 0$$

Suddenly applied load $H = 0$

$$u = 2 u_s$$

Vibrations



How does ω (nat freq of vibes) depend on L?

Vibrations
- mass m
- stiffness k

mass x acceleration + stiffness x deflection = 0

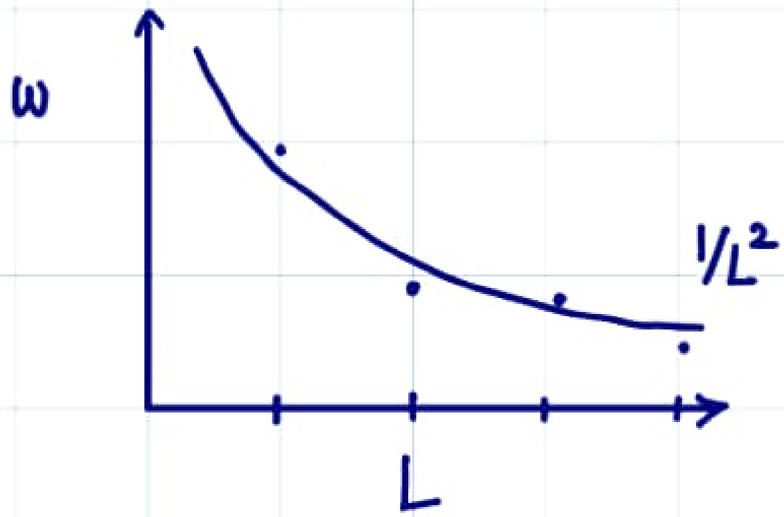
$$\omega = \sqrt{\frac{k}{m}}$$

Mass $= \rho A L$

Stiffness $\propto \dfrac{EI}{L^3}$

$$\omega \propto \sqrt{\frac{EI}{L^3}\frac{1}{\rho A L}}$$

$$\omega \propto \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$$

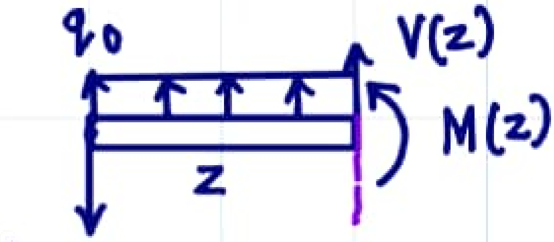


Comprised accuracy but obtained scaling law.

Deflection of Beams

Want $u(z)$ vert displacement.

$$EI\,u'' = M(z)$$



$$\Sigma M_y = 0$$

$$\Rightarrow M(z) + \frac{q_0 L z}{2} - \frac{q_0 z^2}{2} = 0$$

$$M(z) = \frac{q_0}{2}\left(z^2 - Lz\right)$$

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Recall, $V' = -q, \quad M' = -V$

$\Rightarrow \quad M'' = q = q_0$

$$M(z) = \frac{q_0 z^2}{2} + \tilde{c}_1 z + \tilde{c}_2$$

Pinned-pinned $\quad M(0) = 0, \; M(L) = 0$

$$u'' = \frac{q_0}{2EI}\left(z^2 - Lz\right)$$

$$u = \frac{q_0}{2EI}\left(\frac{z^4}{12} - \frac{Lz^3}{6} + c_1 z + c_2\right)$$

$u(0) = 0, \; u(L) = 0 \quad$ BCs for simply supported

$\Rightarrow c_2 = 0, \quad c_1 = \frac{L^3}{12}$

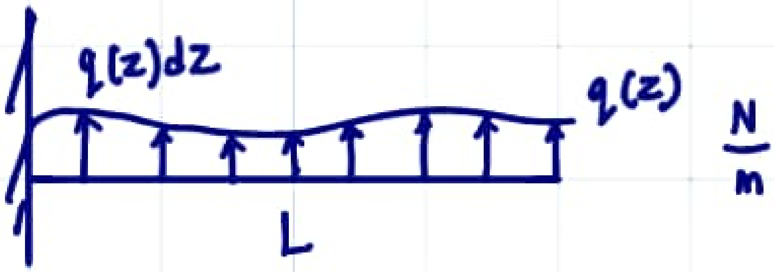
$$u(z) = \frac{q_0}{24EI}\left(L^3 z - 2Lz^3 + z^4\right)$$

$$u_{max} = u\left(\frac{L}{2}\right) = \frac{5q_0L^4}{384EI}$$

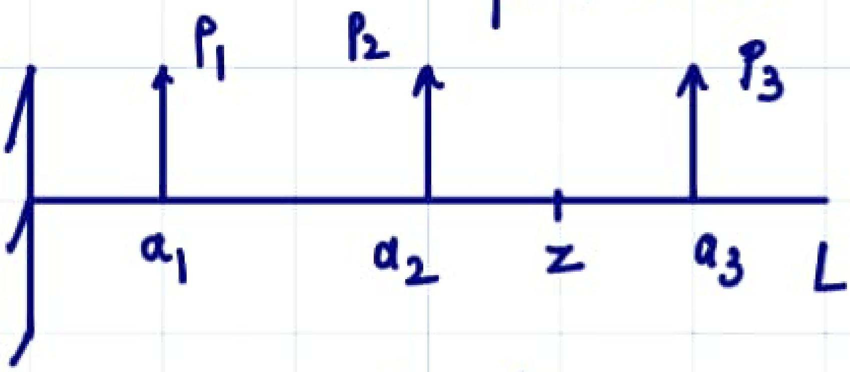
$$\theta = u'(z)$$

$$\theta_{max} = u'(0) = \frac{q_0L^3}{24EI}$$

Cantilever



Distributed load — collection of "infinite" point loads



$$u(z) = \frac{P_1}{EI}\left(\frac{z a_1^2}{2} - \frac{a_1^3}{6}\right)$$

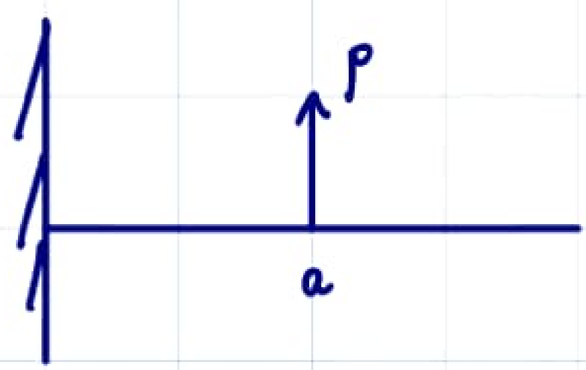
$$+ \frac{P_2}{EI}\left(\frac{z a_2^2}{2} - \frac{a_2^3}{6}\right) + \frac{P_3}{EI}\left(\frac{a_3 z^2}{2} - \frac{z^3}{6}\right)$$

$$= \sum_{i=1}^{N} P_i \underbrace{G(z, a_i)}_{\text{appropriately chosen}}$$

$$G(z,a) = \begin{cases} \dfrac{1}{EI}\left(\dfrac{z a^2}{2} - \dfrac{a^3}{6}\right) & a \leq z \\[2ex] \dfrac{1}{EI}\left(\dfrac{a z^2}{2} - \dfrac{z^3}{6}\right) & z \leq a \end{cases}$$

$G(z,a) =$ deflection @ z due to

unit point load @ a



$$G(z,a) = G(a,z)$$

$G(z,a)$ Green's function for cantilever.

Linear superposition.