ME 202

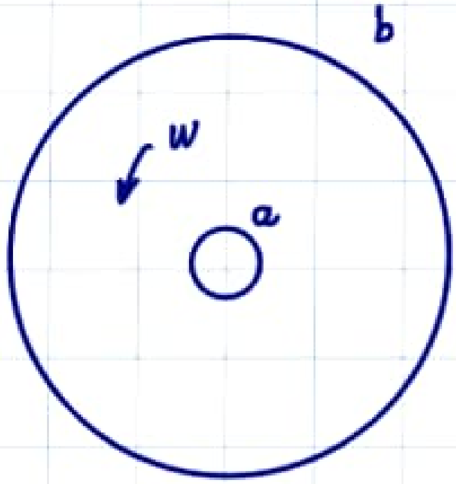
LECTURE 28

TUE 22 MAR 2022

Dnyanesh Pawaskar

Problem

Polycarbonate



$a = 10\,mm$
$b = 60\,mm$
$E = 2\ GPa$
$\nu = 0.35$
$\rho = 1200\ kg/m^3$

Failure when radial stress = 65 MPa

Eqm $\quad \dfrac{d\sigma_{rr}}{dr} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \rho r \omega^2 = 0$

Stress Approach

$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\sigma_{\theta\theta}\right) = \frac{du}{dr}$$

$$\epsilon_{\theta\theta} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\sigma_{rr}\right) = \frac{u}{r}$$

Equate du/dr,

$$r\frac{d\sigma_{\theta\theta}}{dr} - \nu r\frac{d\sigma_{rr}}{dr} + (1+\nu)(\sigma_{\theta\theta} - \sigma_{rr}) = 0$$

Hooke's Law + SDR

From eqm, $\sigma_{\theta\theta} = r\dfrac{d\sigma_{rr}}{dr} + \sigma_{rr} + \rho\omega^2 r^2$

$$r\frac{d^2\sigma_{rr}}{dr^2} + 3\frac{d\sigma_{rr}}{dr} + (3+\nu)\rho\omega^2 r = 0$$

soln $\sigma_{rr} = \underbrace{C_1 + \dfrac{C_2}{r^2}}_{CF} \underbrace{- \left(\dfrac{3+\nu}{8}\right)\rho\omega^2 r^2}_{PI}$

$$\sigma_{\theta\theta} = C_1 - \frac{C_2}{r^2} - \frac{(1+3\nu)}{8}\rho\omega^2 r^2$$

Apply BCs $\sigma_{rr}(a) = 0$, $\sigma_{rr}(b) = 0$

$$\sigma_{rr} = \frac{3+\nu}{8}\rho\omega^2\left[a^2 + b^2 - \left(\frac{ab}{r}\right)^2 - r^2\right]$$

$$\sigma_{\theta\theta} = \frac{3+\nu}{8}\rho\omega^2\left[a^2 + b^2 + \left(\frac{ab}{r}\right)^2 - \frac{1+3\nu}{3+\nu}r^2\right]$$

Radial shess max $\dfrac{d\sigma_{rr}}{dr} = 0$

$$-(ab)^2(-2)r^{-3} - 2r = 0 \Rightarrow r = \sqrt{ab}$$

$$\sigma_{rr}^{max} = \frac{3+\nu}{8}\rho\omega^2(b-a)^2 = \sigma_c$$

failure

$$\omega_c = \sqrt{\frac{8\sigma_c}{(3+\nu)\rho(b-a)^2}}$$

$$= 7193.14 \text{ rad/s}$$

$$= 68,689.45 \text{ RPM}$$

$$\omega = \frac{2\pi N}{60}$$

If, max tangential stress $= \sigma_c$

$$\frac{d\sigma_{\theta\theta}}{dr} = 0$$

$$(ab)^2(2r^{-3}) + \left(\frac{1+3\nu}{3+\nu}\right)2r = 0$$

$\sigma_{\theta\theta}(a) > \sigma_{\theta\theta}(b)$ check

$$\sigma_{\theta\theta}(a) = \sigma_c \Rightarrow \omega_c = \sqrt{\frac{4\sigma_c}{\rho\left[(3+\nu)b^2 + (1-\nu)a^2\right]}}$$

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Displacement Approach

check $$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + (1-\nu^2)\frac{\rho\omega^2 r}{E} = 0$$

$$u = \underbrace{C_1 r + \frac{C_2}{r^2}}_{CF} \underbrace{- \frac{(1-\nu^2)\rho\omega^2 r^3}{8E}}_{PI}$$

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left[\underset{\frac{du}{dr}}{\underset{\parallel}{\epsilon_{rr}}} + \nu \underset{\frac{u}{r}}{\underset{\parallel}{\epsilon_{\theta\theta}}}\right]$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}\left[\epsilon_{\theta\theta} + \nu\epsilon_{rr}\right]$$

Apply TBCs $\sigma_{rr}(a) = 0$, $\sigma_{rr}(b) = 0$

same as before.

$\sigma_{rr} =$

$\sigma_{\theta\theta} =$

$$u = \frac{(3+\nu)(1-\nu)}{8E}\left(a^2+b^2 - \frac{1+\nu}{3+\nu}r^2 + \frac{1+\nu}{1-\nu}\frac{a^2b^2}{r^2}\right)\rho\omega^2 r$$

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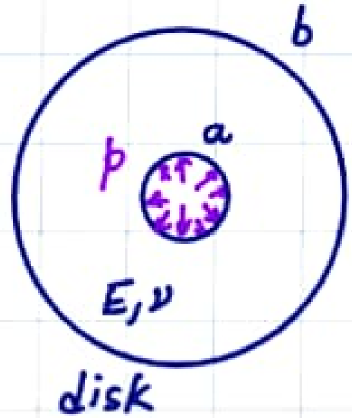
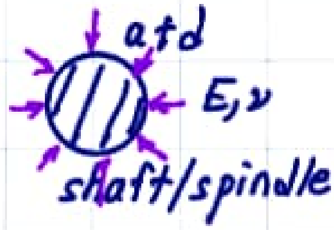
Problem

$\omega = 0$

Everything is plane stress





$d \ll a < b$

{interference fit/ press fit

calculate contact pressure

For shaft, solid disk

finite stresses

$$\sigma_{rr} = C_1 + \cancel{\frac{C_2}{r^2}}^{\,0} = C_1 = -p$$

$$\sigma_{\theta\theta} = C_1 - \cancel{\frac{C_2}{r^2}}^{\,0} = C_1 = -p$$

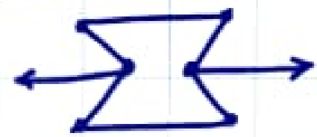
BC $\sigma_{rr}(a) = -p \Rightarrow C_1 = -p$

$$\epsilon_{\theta\theta} = \frac{u}{r} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\sigma_{rr}\right)$$

$$u = \frac{-rp}{E}(1-\nu) = u_s$$

$$u_s(a) = \frac{-ap}{E}(1-\nu)$$

Aside



For disk,

$$\sigma_{rr} = C_1 + \frac{C_2}{r^2}, \quad \sigma_{\theta\theta} = C_1 - \frac{C_2}{r^2}$$

BCs $\sigma_{rr}(a) = -p$, $\sigma_{rr}(b) = 0$

$$\sigma_{rr} = \frac{a^2 p}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\sigma_{\theta\theta} = \frac{a^2 p}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

$$u = \frac{r}{E}(\sigma_{\theta\theta} - \nu\sigma_{rr}) = u_d(r)$$

$$u_d(a) = \frac{a p}{E(b^2 - a^2)}(b^2\nu - a^2\nu + a^2 + b^2)$$

Press fit,

$$u_d(a) = u_s(a+d) + d$$

$$= u_s(a) + d \qquad \text{solve for } p$$

$$p = \frac{Ed(b^2 - a^2)}{2ab^2}$$

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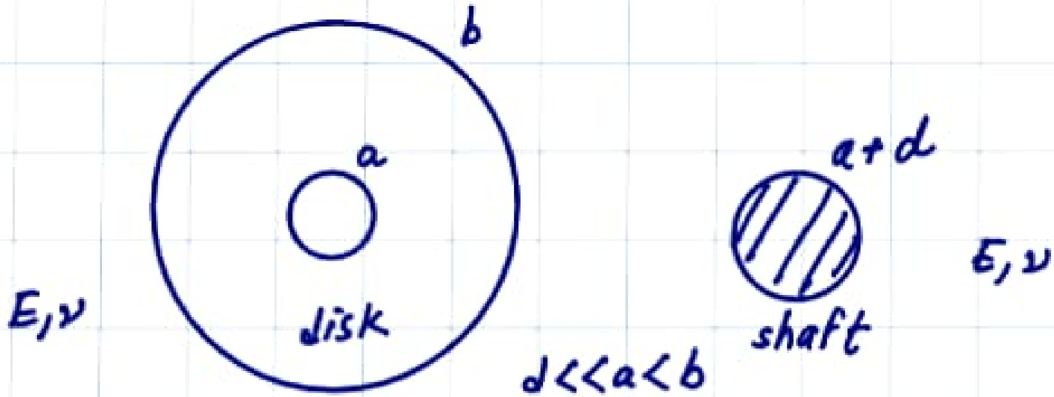
Problem



Calculate ω at which disk comes loose off the shaft.

Approach 1 Find ω at which $p=0$

Approach 2 Disks spin independently

$$\cancel{a}+d+u_s(a+d)=\cancel{a}+u_d(a)$$

Approach 1

Disk $\sigma_{rr} = C_1 + \dfrac{C_2}{r^2} - \dfrac{3+\nu}{8}\rho\omega^2 r^2$

BC $\sigma_{rr}(b)=0 \Rightarrow C_1 = \dfrac{3+\nu}{8}\rho\omega^2 b^2 - \dfrac{C_2}{b^2}$

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$$\sigma_{rr}(a) = \frac{3+\nu}{8}\rho\omega^2 b^2 - \frac{C_2}{b^2} + \frac{C_2}{a^2} - \frac{3+\nu}{8}\rho\omega^2 a^2$$

At $\omega = 0$, $\sigma_{rr}(a) = -p$ at zero rot speed

$$\Rightarrow C_2 = \frac{-p}{\frac{1}{a^2} - \frac{1}{b^2}}$$

$$\sigma_{rr}(a) = -p + \left(\frac{3+\nu}{8}\right)(b^2 - a^2)\rho\omega^2$$

Disk loose when $\sigma_{rr}(a) = 0$

$$\omega^2 = \frac{8p}{(3+\nu)\rho(b^2 - a^2)}$$

$$\omega^2 = \frac{4E\delta}{(3+\nu)\rho a b^2}$$

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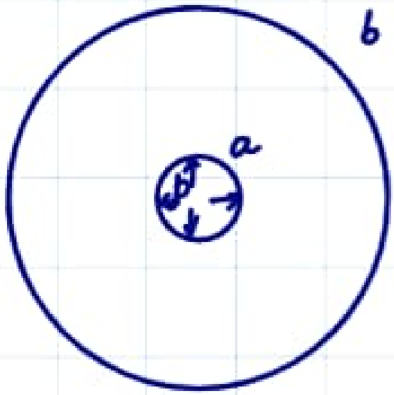

$$\underset{\sim}{t} = \underset{\approx}{\sigma}\,\underset{\sim}{n} \quad @\, r=a$$

$$\begin{pmatrix} p \\ 0 \end{pmatrix} = \begin{pmatrix} \sigma_{rr} & \cancel{\sigma_{r\theta}} \\ \cancel{\sigma_{r\theta}} & \sigma_{\theta\theta} \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \; p = \sigma_{rr}(-1) \quad @\, r=a$$

$$\Rightarrow \; \sigma_{rr}(a) = -p$$