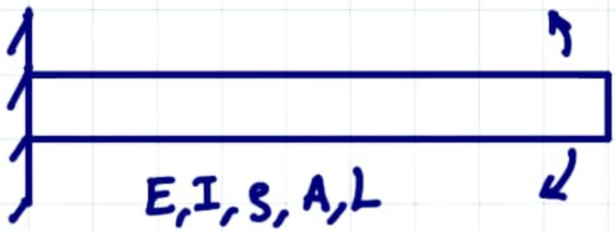
ME 202

LECTURE 21

THU 03 MAR 2022

Dnyanesh Pawaskar

Recall,



Vibrating beam

$$k \propto \frac{EI}{L^3}, \quad m = \rho A L$$

$$\omega \propto \sqrt{\frac{k}{m}} \Rightarrow \omega \propto \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$$

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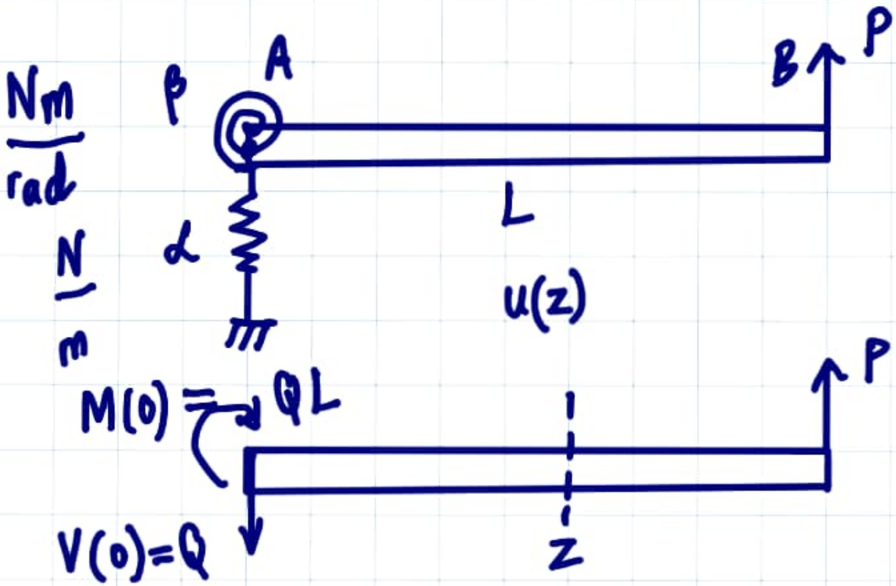
BEAMS WITH ELASTIC SUPPORTS



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$$P = V(0) = \alpha\, u(0)$$

$$P L = M(0) = \beta\, u'(0) = \beta\, \theta(0)$$

$$M(z) = P(L-z) = EI\, u''(z)$$

New BCs $\quad u(0) = \dfrac{P}{\alpha}, \quad u'(0) = \dfrac{PL}{\beta}$

$$u(z) = \frac{P}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right) + \frac{PLz}{\beta} + \frac{P}{\alpha}$$

$$u(L) = \frac{PL^3}{3EI} + \frac{PL^2}{\beta} + \frac{P}{\alpha}$$

$$= \frac{PL^3}{3EI}\left(1 + \frac{3EI}{\beta L} + \frac{3EI}{\alpha L^3}\right)$$

if $\frac{3EI}{\beta L}$, $\frac{3EI}{\alpha L^3}$ v small

$\Rightarrow$ rigid wall
elastic/compliant beam

v large

$\Rightarrow$ rigid beam
compliant wall

So far,

$$EI\,u'' = M(z) \quad 2^{nd} \text{ order ODE}$$

4^{th} order beam eqn ODE

Recall, $V(z) = -M'(z)$, $q(z) = -V'(z)$

$$\Rightarrow q(z) = M''(z) = \frac{d^2M}{dz^2}$$

$$\frac{d^2}{dz^2}\left(EI\,\frac{d^2u}{dz^2}\right) = \frac{d^2M}{dz^2} = q(z)$$

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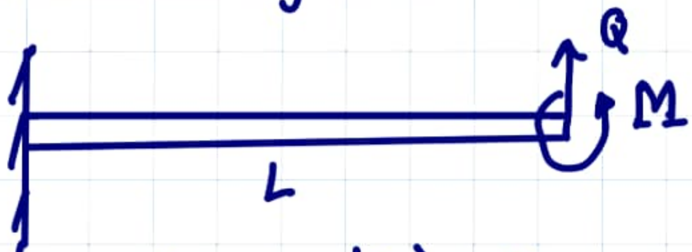
$$EI\,u'''' = q(z)$$ 4^{th} order ODE

Need 4 integration constants.

Example 1



BCs $u(0)=0,\ u'(0)=0$

$EI\,u''(L) = M,\ -EI\,u'''(L) = Q$

ODE $EI\,u'''' = 0$

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$$u = A + Bz + Cz^2 + Dz^3$$

$$u' = B + 2Cz + 3Dz^2$$

$$u'' = 2C + 6Dz$$

$$u''' = 6D$$

$$A = 0$$

$$B = 0$$

$$C = \frac{1}{2}\frac{M+QL}{EI}$$

$$D = \frac{-Q}{6EI}$$

$$u = \frac{Mz^2}{2EI} + \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right) \text{ as before}$$

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Example 2

$$EI\,u'''' = q_0$$

$$u(0)=0,\quad u'(0)=0,\quad u'(L)=0,\quad u(L)=0$$

$$u = \frac{q_0}{EI}\left(\frac{z^4}{24} + c_1\frac{z^3}{6} + c_2\frac{z^2}{2} + c_3 z + c_4\right)$$

$$u = \frac{q_0 z^2}{24EI}(z-L)^2$$

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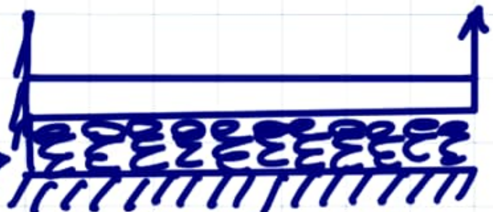
$$u_{max} = u(L/2) = \frac{q_0 L^4}{384\,EI}$$

Uniformly Distributed Stiffness

$$k = \frac{N}{m^2}$$



Beams on elastic foundation

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$$EI\,u'''' = -ku$$ resistive force / length

$$EI\,u'''' + ku = 0$$

$$u = e^{rz}$$

$$u = A\cos\lambda z + B\sin\lambda z + C\cosh\lambda z + D\sinh\lambda z$$

$$\frac{k}{EI} = \lambda^4$$

$$u(0) = 0, \quad u'(0) = 0$$

$$EI\,u''(L) = 0$$

$$-EI\,u'''(L) = P$$

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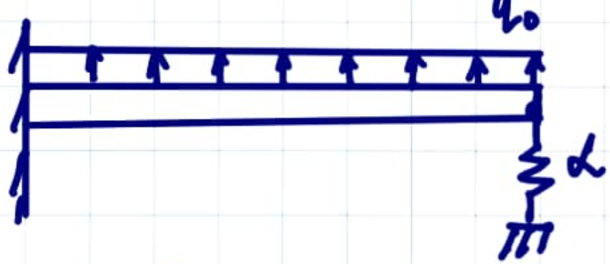
Problem



Method 1 4^{th} order ODE

$$EI\,u'''' = q_0$$

$$u = \frac{q_0}{EI}\left(\frac{z^4}{24} + c_1\frac{z^3}{6} + c_2\frac{z^2}{2} + c_3 z + c_4\right)$$

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4 BCs $u(0) = 0$

$$u'(0) = 0$$

$$EI\,u''(L) = 0$$

$$+EI\,u'''(L) = +\alpha\, u(L)$$

$$c_1 = \frac{-1}{8}\,\frac{5L^4\alpha + 24EIL}{3EI + L^3\alpha}$$

$$c_2 = \frac{-L^2}{2} - c_1 L \qquad c_3 = 0,\ c_4 = 0$$

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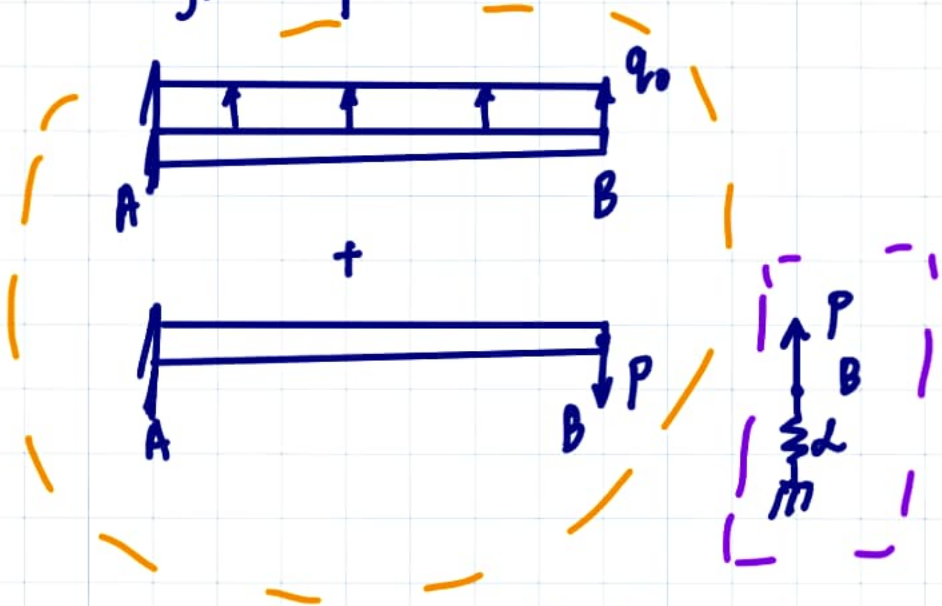
Method 2 Linear superposition

original problem =



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Def of B from circled prob = Def of B from boxed problem

Displacement compatibility / continuous

$$\frac{q_0 L^4}{8EI} - \frac{PL^3}{3EI} = \frac{P}{\alpha}$$

$$P = \frac{q_0 L^4 / 8EI}{L^3/3EI + 1/\alpha}$$

$$u(z) = \frac{q_0}{EI}\left(\frac{z^4}{24} - \frac{Lz^3}{6} + \frac{L^2z^2}{4}\right) - \frac{P}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$