MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 2

(January 19, 2022)

(1) Use the Bisection method to find the approximations $p_1, \ldots, p_5$ for the root of $f(x) = \sqrt{x} - \cos x = 0$ on $[0, 1]$.

(2) Use the Bisection method to find solutions accurate to within 10^{-2} for the root of $f(x) = x^3 - 7x^2 + 14x - 6 = 0$ on $[1, 3.2]$.

(3) Let $f(x) = (x + 2)(x + 1)^2 x(x - 1)^3(x - 2)$. To which zero of f does the Bisection method converge when applied on the interval $[-1.5, 2.5]$?

(4) Show that the fixed point theorem does not ensure a unique fixed point of $f(x) = 3^{-x}$ on the interval $[0, 1]$, even though a unique fixed point on this interval does exist.

(5) The following two methods are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$:

$$p_n = p_{n-1} - \frac{p_{n-1}^3 - 21}{3p_{n-1}^2}, \qquad p_n = \left(\frac{21}{p_{n-1}}\right)^{1/2}.$$

(6) Let $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_3.

(7) Let $f(x) = x^2 - 6$. Use the Secant method to find p_4 with $p_0 = 3$ and $p_1 = 2$.

(8) Let $f(x) = x^2 - 6$. Use the method of false position to find p_4 with $p_0 = 3$ and $p_1 = 2$.

(9) An object falling vertically through the air is faces viscous resistance as well as the gravitational force. Assume that an object with mass a is dropped from a height d_0 and that the height of the object after t seconds is

$$d_t = d_0 - \frac{ag}{k}t + \frac{a^2 g}{k^2}(1 - e^{-kt/a})$$

where $g = 9.80665\ m/s^2$ and $k = 0.24\ kg/m$ represents the coefficient of air resistance. If an object of mass $0.25\ kg$ is dropped from a height of 300 meters then find, to within 0.01 error, the time it takes this object to hit the ground.