ME 202

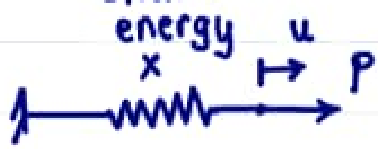
LECTURE 10

TUTORIAL 3

MON 24 JAN 2022

Principle of Minimum Potential Energy

$$\Pi_{total} = \Pi_{\substack{\text{stored}\\\text{elastic/}\\\text{strain}\\\text{energy}}} + \Pi_{\substack{\text{externally}\\\text{applied}\\\text{forces}}}$$

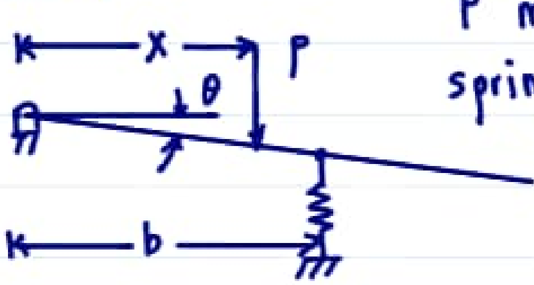


$$\Pi(u) = \frac{1}{2}kx^2 - Pu \quad , \quad u = x$$

$$\Pi(x) = \frac{1}{2}kx^2 - Px$$

disp in direction of P @ point of application of P

1. Rigid pointer.



P moves thru $x\theta$

spring compression $b\theta$

$\sin\theta \approx \theta$

$$\Pi(\theta) = \frac{1}{2} k\, b^2 \theta^2 - P x \theta$$

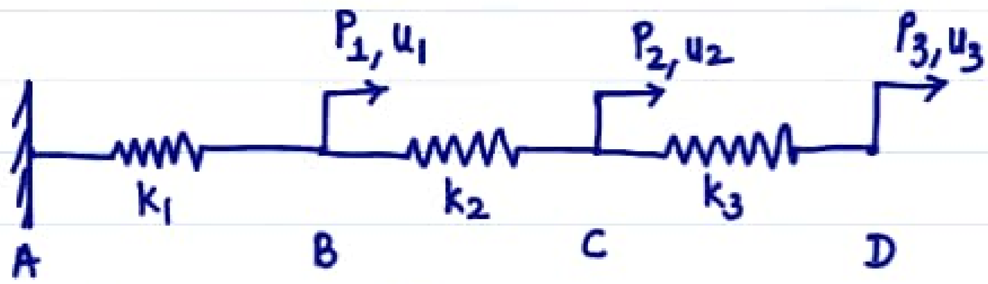
$$\text{PMPE} \Rightarrow \quad \Pi'(\theta) = 0 \quad \Rightarrow \quad \frac{1}{2} k\, b^2 2\theta - Px = 0$$

$$x = \frac{b^2 k \theta}{P}$$

$$= 150^2 \times \frac{800}{1000} \times \frac{\pi}{60} \times \frac{1}{8}$$

$$= 117.81 \text{ mm}$$

2.



$$k_1 = \frac{AE}{a}, \quad k_2 = \frac{AE}{b}, \quad k_3 = \frac{AE}{c}$$

$$\Pi(u_1, u_2, u_3) = \frac{1}{2} k_1 (u_1 - 0)^2 + \frac{1}{2} k_2 (u_2 - u_1)^2$$

$$+ \frac{1}{2} k_3 (u_3 - u_2)^2 + (-P_1 u_1 - P_2 u_2 - P_3 u_3)$$

$$\text{PMPE} \Rightarrow \frac{\partial \Pi}{\partial u_1} = 0 \Rightarrow \quad k_1 u_1 + k_2 (u_1 - u_2) = P_1$$

$$\frac{\partial \Pi}{\partial u_2} = 0 \Rightarrow \quad k_2 (u_2 - u_1) + k_3 (u_2 - u_3) = P_2$$

$$\frac{\partial \Pi}{\partial u_3} = 0 \Rightarrow \quad k_3 (u_3 - u_2) = P_3$$

$$\underbrace{\begin{pmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{pmatrix}}_{\text{stiffness matrix}} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}}_{\text{disp. vector}} = \underbrace{\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}}_{\text{load vector}}$$

symmetric & pos. def.

all e-vals +ve

$$\underset{\sim}{K}\,\underline{u} = \underline{P}, \qquad \underset{\sim}{K}^{-1} = \underset{\sim}{C}$$

compliance matrix

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ \text{symm} & & C_{33} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

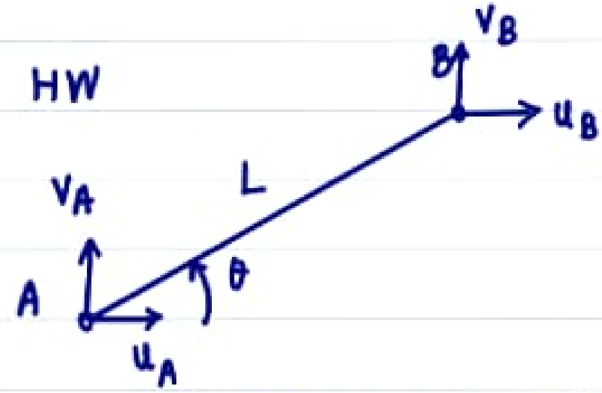
$$u_3 = C_{31}P_1 + C_{32}P_2 + C_{33}P_3 = \underset{\text{given}}{0}$$

$$0.03 \times 2.7 + 0.042 \times 1.8 + 0.05 \times P_3 = 0$$

$$P_3 = -3.132 \text{ kN}$$

$P_3 = 3.132$ kN in the dir given in prob.

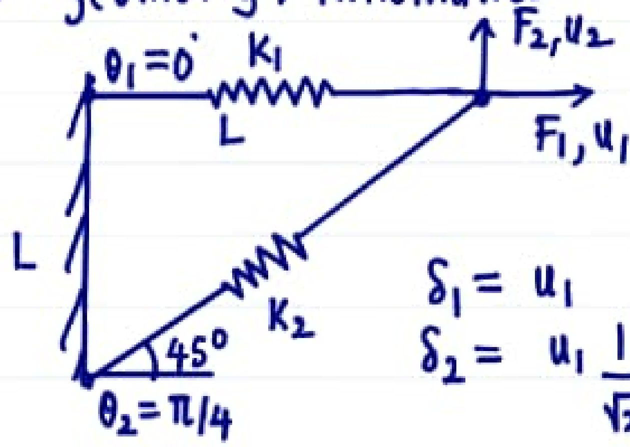
3. Result HW



$$\underset{\text{extension}}{\delta} = (u_B - u_A)\cos\theta + (v_B - v_A)\sin\theta$$

approx for $|u_A|, |u_B|, |v_A|, |v_B| \ll L$

From geometry / kinematics



$$k_1 = \frac{AE}{L} \qquad \delta_1 = u_1$$

$$k_2 = \frac{AE}{L\sqrt{2}} \qquad \delta_2 = u_1 \frac{1}{\sqrt{2}} + u_2 \frac{1}{\sqrt{2}}$$

$$\Pi(u_1, u_2) = \frac{1}{2} k_1 \delta_1^2 + \frac{1}{2} k_2 \delta_2^2 + (-F_1 u_1 - F_2 u_2)$$

$$= \frac{1}{2} k_1 u_1^2 + \frac{1}{2} k_2 \frac{1}{2}(u_1 + u_2)^2 - F_1 u_1 - F_2 u_2$$

PMPE

$$\frac{\partial \Pi}{\partial u_1} = 0 \Rightarrow k_1 u_1 + \frac{k_2}{2}(u_1 + u_2) = F_1$$

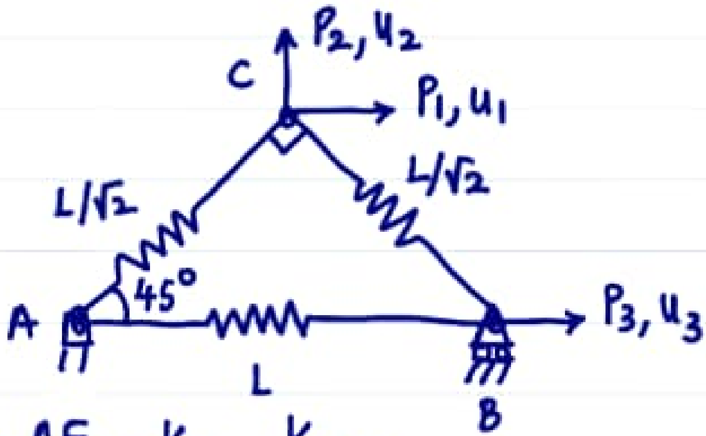
$$\frac{\partial \Pi}{\partial u_2} = 0 \Rightarrow \frac{k_2}{2}(u_1 + u_2) = F_2$$

$$\frac{AE}{L}\begin{pmatrix} 1+\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

$$u_1 = \frac{L}{AE}(F_1 - F_2) = -3.0315 \times 10^{-6} \text{ m}$$

$$u_2 = \frac{L}{AE}(-F_2 + 3.82843 F_2) = 2.018 \times 10^{-5} \text{ m}$$

4.



$P_1 = P$
$P_2 = -P$
$P_3 = 0$

$$k_{AB} = \frac{AE}{L} = k, \quad k_{AC} = k_{BC} = \frac{AE}{L/\sqrt{2}} = k\sqrt{2}$$

Bar	Stiffness	Angle	Extension
AB	k	0	u_3
AC	$k\sqrt{2}$	45°	$\frac{u_1+u_2}{\sqrt{2}}$
CB	$k\sqrt{2}$	−45°	$(u_3-u_1)\cos(-45°) + (0-u_2)\sin(-45°) = (u_3-u_1+u_2)/\sqrt{2}$

$$\Pi = \frac{1}{2}ku_3^2 + \frac{1}{2}k\sqrt{2}\,\frac{1}{2}(u_1+u_2)^2 + \frac{1}{2}k\sqrt{2}\,\frac{1}{2}(u_3-u_1+u_2)^2$$

$$-P_1u_1 - P_2u_2 - P_3u_3$$

PMPE $\Rightarrow$ $\frac{\partial \Pi}{\partial u_1} = 0,\ \frac{\partial \Pi}{\partial u_2} = 0,\ \frac{\partial \Pi}{\partial u_3} = 0$

$\underset{\sim}{\nabla}_{u}\Pi = \underline{0}$ gradient of Π wrt $\underline{u}$'s

$$\frac{k}{\sqrt{2}}(u_1+u_2) - \frac{k}{\sqrt{2}}(u_3-u_1+u_2) = P_1$$

$$\frac{k}{\sqrt{2}}(u_1+u_2) + \frac{k}{\sqrt{2}}(u_3-u_1+u_2) = P_2$$

$$ku_3 + \frac{k}{\sqrt{2}}(u_3-u_1+u_2) = P_3$$

$$\begin{pmatrix} \frac{2k}{\sqrt{2}} & 0 & \frac{-k}{\sqrt{2}} \\ 0 & \frac{2k}{\sqrt{2}} & \frac{k}{\sqrt{2}} \\ \frac{-k}{\sqrt{2}} & \frac{k}{\sqrt{2}} & k\left(1+\frac{1}{\sqrt{2}}\right) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$k = \frac{AE}{L} = 260$ kN/mm $\qquad \underset{\sim}{K}\,\underline{u} = \underline{P}$

$$\underline{u} = \underset{\sim}{K}^{-1}\,\underline{P} \qquad P_1 = +650\ \text{kN}$$

$$= \underset{\sim}{C}\,\underline{P} \qquad P_2 = -650\ \text{kN}$$

$$P_3 = 0$$

$$u_3 = 2.5\ \text{mm}$$

$$u_3 = C_{31}P_1 + C_{32}P_2 = C_{31}P - C_{32}P$$

$$1.5 = 1.92311\times 10^{-3}\,P + 1.92311\times 10^{-3}\,P$$

$$\underline{P_{max} = 390\ \text{kN}}$$