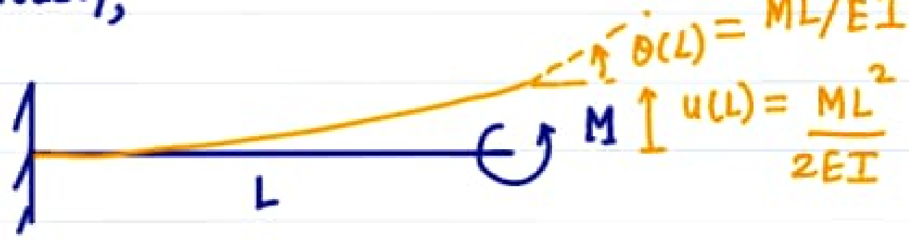
ME 202

LECTURE 14

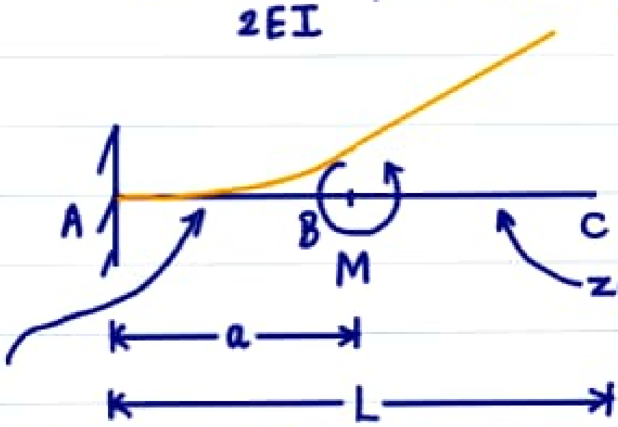
TUTORIAL 4

TUE 01 FEB 2022

1. Previously,



$$u(z) = \frac{Mz^2}{2EI}, \quad u'(z) = \frac{Mz}{EI} = \theta(z)$$



$M = EIu''$

zero BM $\Rightarrow u'' = 0$

$u = Az + B$

$$\underbrace{u = \frac{Mz^2}{2EI}, \; u'(z) = \frac{Mz}{EI}}_{AB}, \; u(a) = \frac{Ma^2}{2EI}, \; u'(a) = \frac{Ma}{EI}$$

Over BC, $u(z) = Az + B$

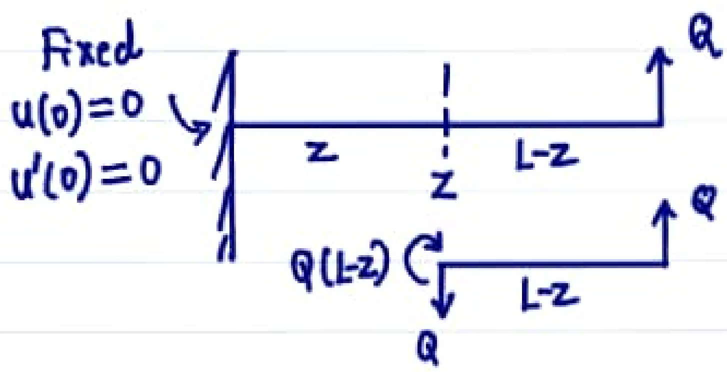
$$\underset{BC}{u(a)} = Aa + B = \frac{Ma^2}{2EI} = \underset{AB}{u(a)}$$

$$\underset{BC}{u'(a)} = A = \frac{Ma}{EI} = \underset{AB}{u'(a)}$$

Solve for A, B

$$\Rightarrow \left.\begin{aligned} u(z) &= \frac{Ma}{EI}\left(z - \frac{a}{2}\right) \\ u'(z) &= \frac{Ma}{EI} \end{aligned}\right\} BC$$

Apply EB theory (for pure bending) to cases with SF also.



$M = EI u''$, $Q(L-z) = EI u''$

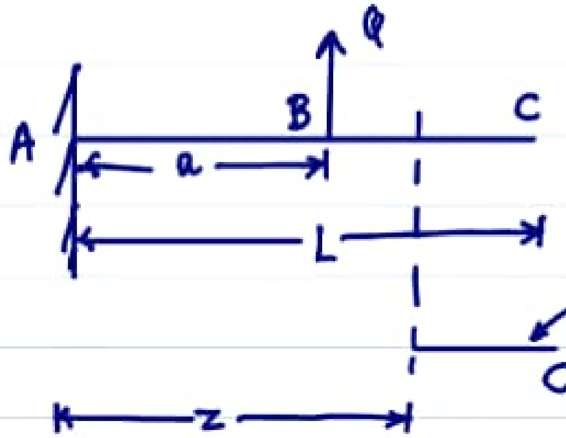
$u'' = \frac{Q}{EI}(L-z)$, $u' = \frac{Q}{EI}\left(Lz - \frac{z^2}{2} + C_1\right)$

$$u = \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6} + C_1 z + C_2\right)$$

BCs @ $z=0 \Rightarrow c_1=0,\ c_2=0$

$$u = \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$u(L) = \frac{QL^3}{3EI}, \quad u'(L) = \theta(L) = \frac{QL^2}{2EI}$$



nothing happening here

$M=0=EIu''$

$u = Az+B$ over BC

Over AB, $M(z) = Q(a-z) = EI\,u''$

integrate twice + BCs

$$\left. u(z) = \frac{Q}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right) \right\} \begin{array}{l} AB \\ 0<z<a \end{array}$$

$$u(a) = \frac{Qa^3}{3EI}, \quad u'(a) = \frac{Qa^2}{2EI}$$

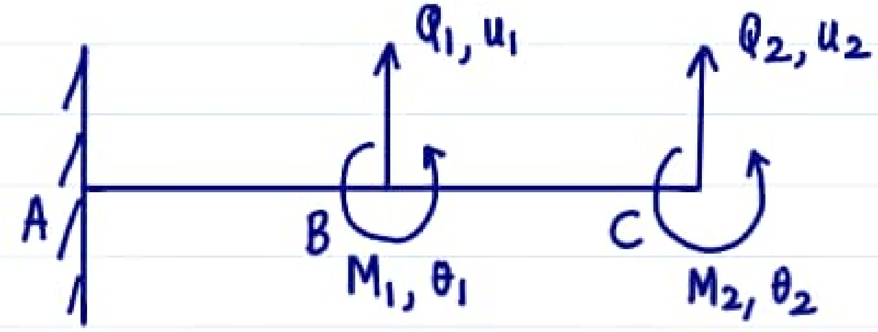
Match slope + displacement @ B,

$$A = \frac{Qa^2}{2EI}, \quad B = -\frac{Qa^3}{6EI}$$

$$\left. u(z) = \frac{Qa^2}{2EI}z - \frac{Qa^3}{6EI} \right\} \begin{array}{l} BC \\ a<z<L \end{array}$$

$$u(L) = \frac{Qa^2L}{2EI} - \frac{Qa^3}{6EI}$$

$$u'(L) = \frac{Qa^2}{2EI}$$



Expect

Linear Superposition

$$\begin{pmatrix} u_1 \\ u_2 \\ \theta_1 \\ \theta_2 \end{pmatrix} = \underbrace{\begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ & & & \\ & & & \\ & & & C_{44} \end{pmatrix}}_{\text{Compliance Matrix}} \begin{pmatrix} Q_1 \\ Q_2 \\ M_1 \\ M_2 \end{pmatrix}$$

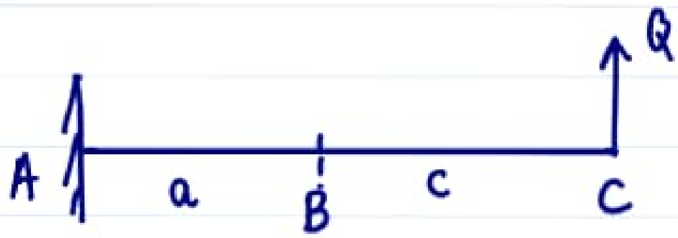
$$u_2 = \frac{Q_2L^3}{3EI} + \frac{M_2L^2}{2EI} + \frac{Q_1}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) + \frac{M_1}{EI}\left(La - \frac{a^2}{2}\right)$$

$$u_1 = \frac{Q_2}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) + \frac{Q_1a^3}{3EI} + \frac{M_1a^2}{2EI} + \frac{M_2a^2}{2EI}$$

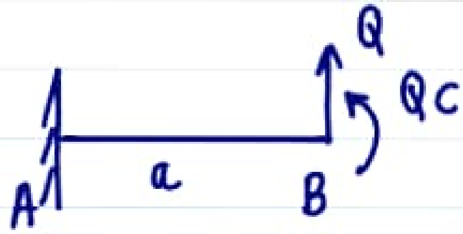
$$\theta_1 = \frac{Q_1 a^2}{2EI} + \frac{M_1 a}{EI} + \frac{M_2 a}{EI} + \frac{Q_2}{EI}\left(La - \frac{a^2}{2}\right)$$

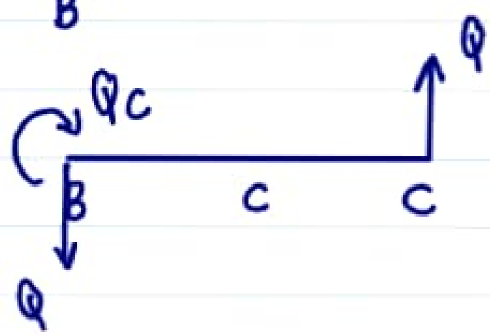
$$\theta_2 = \frac{Q_1 a^2}{2EI} + \frac{Q_2}{EI}\left(La - \frac{a^2}{2}\right) + \frac{M_2 L}{EI} + \frac{M_1 a}{EI}$$

<u>Note:</u>



$$u(a+c) = \frac{Q}{3EI}(a+c)^3$$





Total
Deflection of C

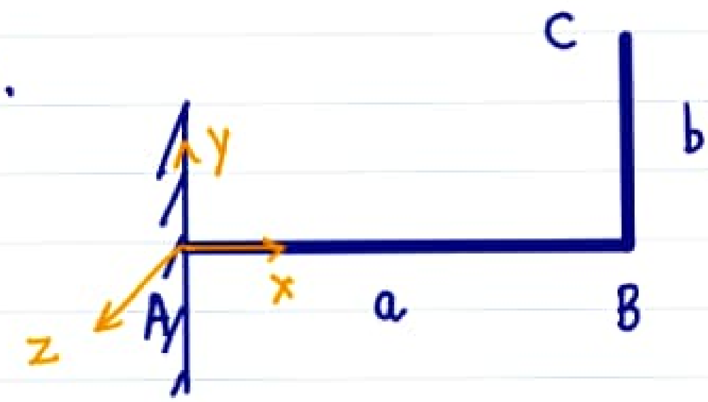
$$= \frac{Qc^3}{3EI} + \frac{Qa^3}{3EI} + \frac{Qc}{2EI}a^2$$

$$+ \underbrace{\frac{Qa^2}{2EI}}_{\theta \text{ @ } B \text{ due to } Q \text{ acting @ } B} \cdot c$$

$$+ \underbrace{\frac{Qca}{EI}}_{\theta \text{ @ } B \text{ due to } Qc \text{ acting @ } B} \cdot c$$

$$= \frac{Q}{3EI}\underbrace{(a^3 + c^3 + 3a^2c + 3ac^2)}_{(a+c)^3} \text{ as expected.}$$

2.



TBC