ME 202 Help Session

17 Feb 2022

9-10pm

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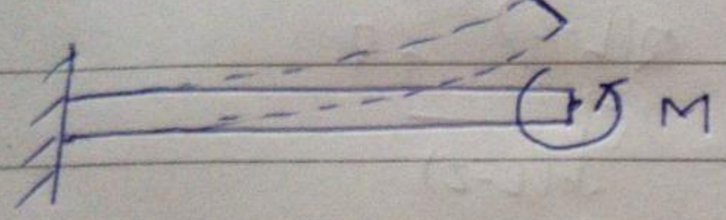
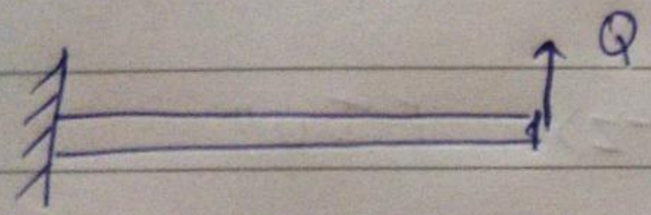
Problems we’ll solve in this session:

- Displacement problem with multiple beams
- Application of PMPE to Torsion of non circular C/S
- Distributed Loads

Some basics:

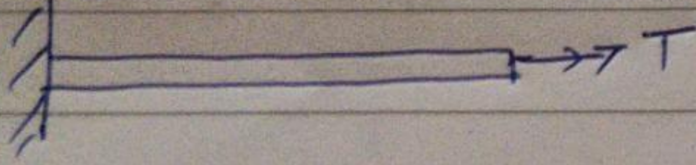


$$u(z) = \frac{Mz^2}{2EI} \qquad U'(z) = \frac{Mz}{EI}$$



$$U(z) = \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

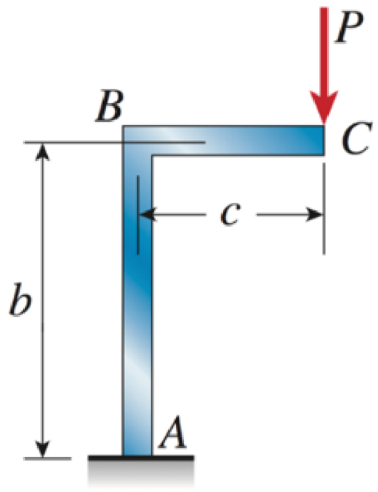
$$U'(z) = \frac{Q}{EI}\left(Lz - \frac{z^2}{2}\right)$$

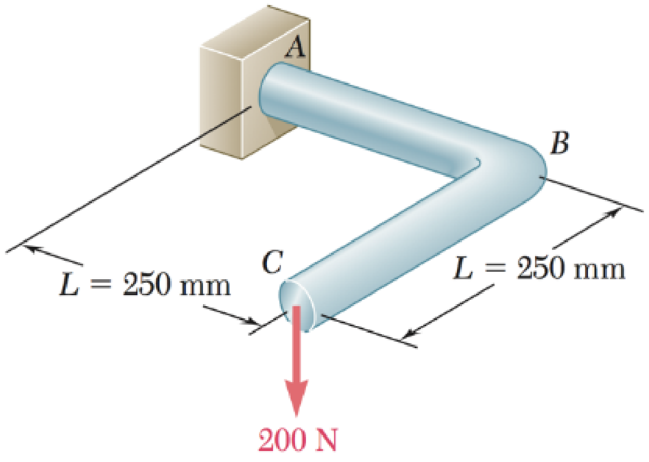


$$\alpha = \frac{T}{GJ} \qquad \theta = \alpha x$$

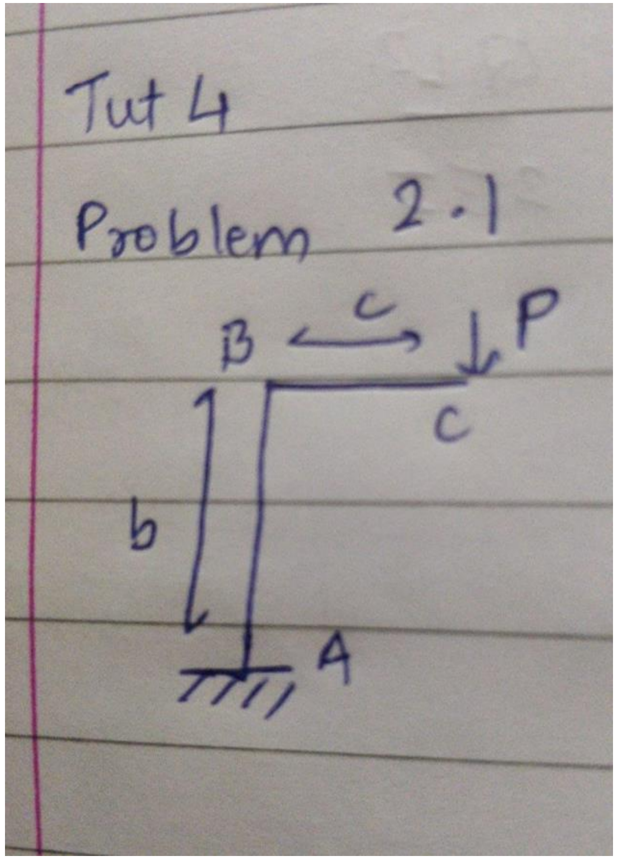
Problem 2 of Tutorial 4

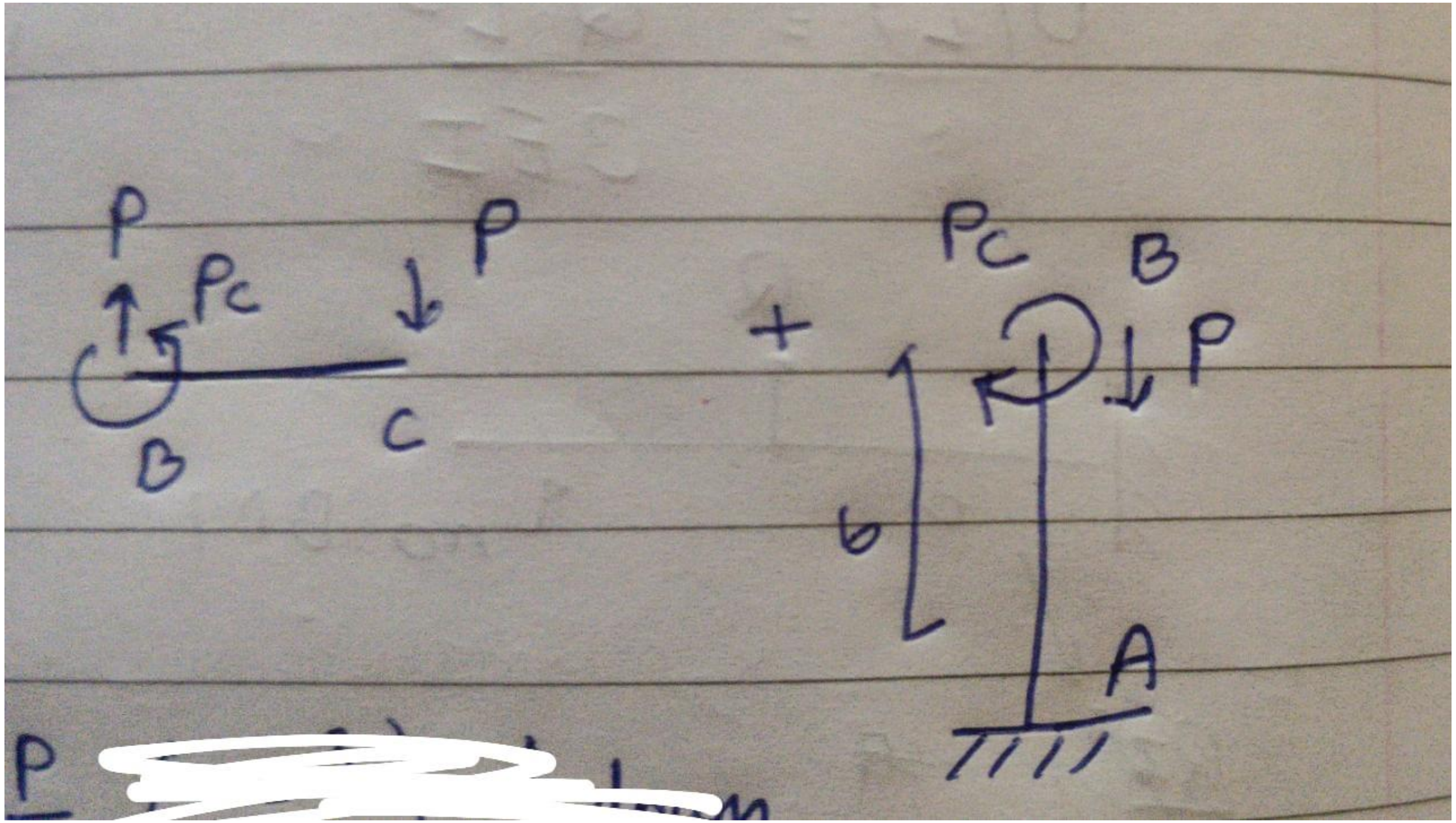
2. Consider the bent beams shown in the following figures. Use the results of problem 1 (and the torque-twist formula if necessary) to obtain the vertical deflection of the point C in each case. Assume that the joint B is rigid i.e the angle ABC remains 90 deg even after deformation. In each case, express your result in terms of the given dimensions (*b*, *c*, L as applicable) and the material properties *E*, *G* and geometric properties *A*, *I*, *J*.



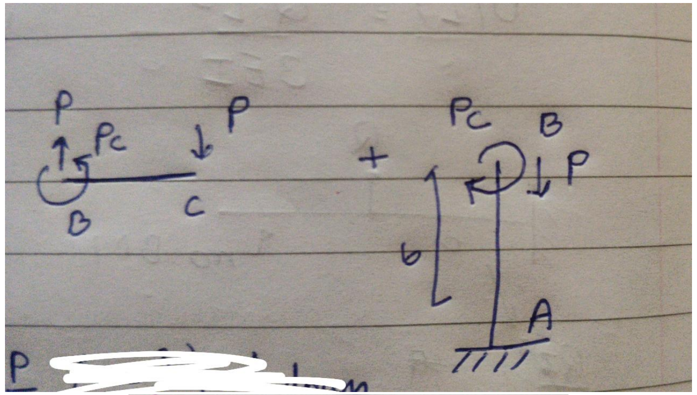


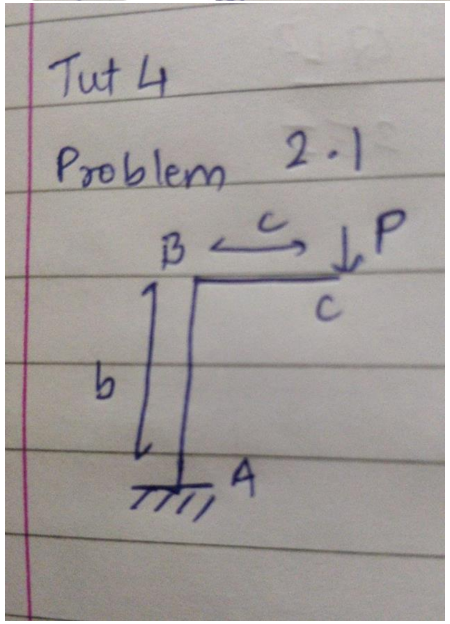
Tut 4 Problem 2.1

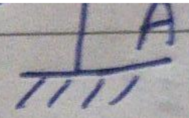




Tut 4 Problem 2.1







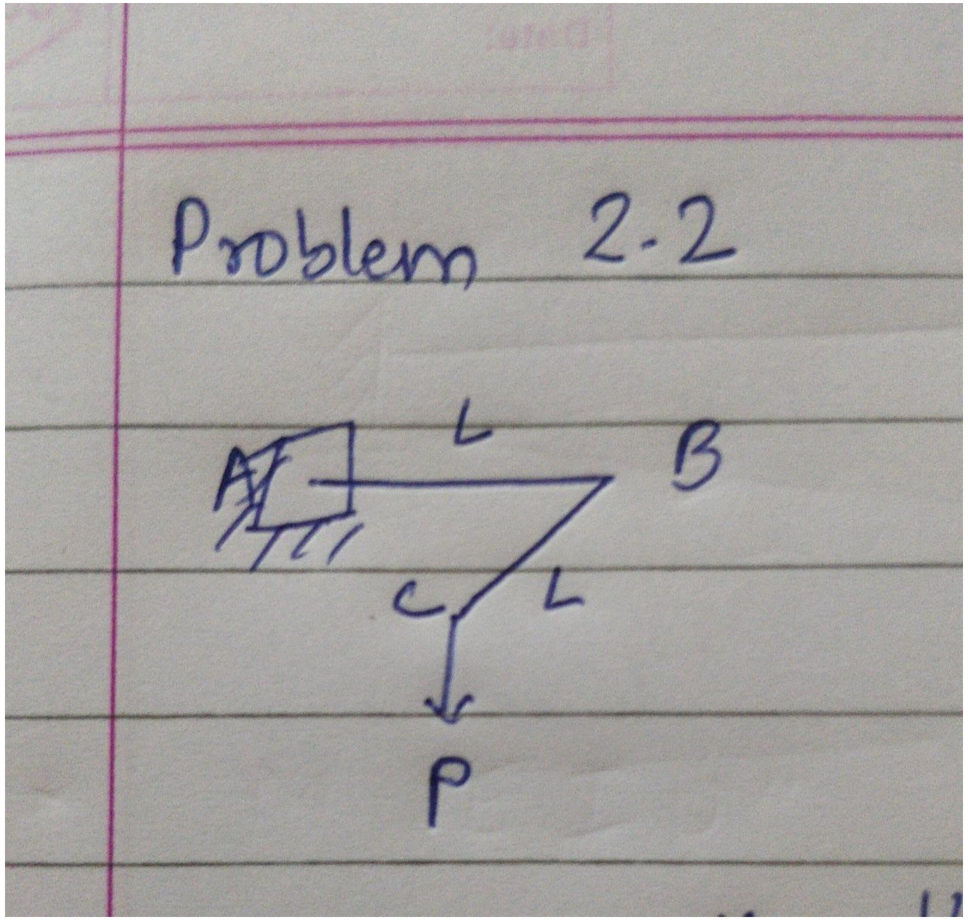
$$u_{C/B} = \frac{P}{EI}\left(\frac{c^3}{3}\right) \downarrow \text{down}$$

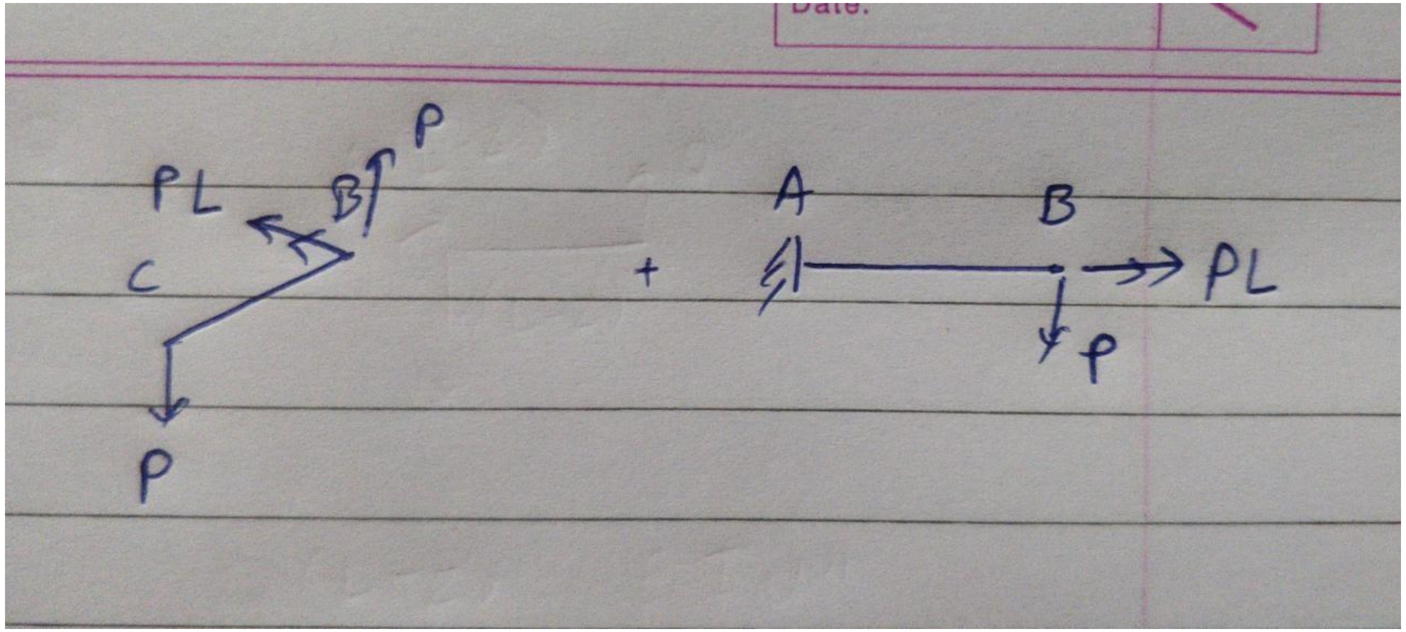
$$u_{B/A} = \frac{\cancel{PL}\,Pb}{AE} \downarrow + \theta_B\, c \downarrow$$

$$= \frac{Pb}{AE} + \frac{(Pc)\,b}{EI}\, c$$

$$\therefore\ u_C = \frac{Pc^3}{3EI} + \frac{Pb}{AE} + \frac{Pbc^2}{EI}$$

Tut 4 Problem 2.2

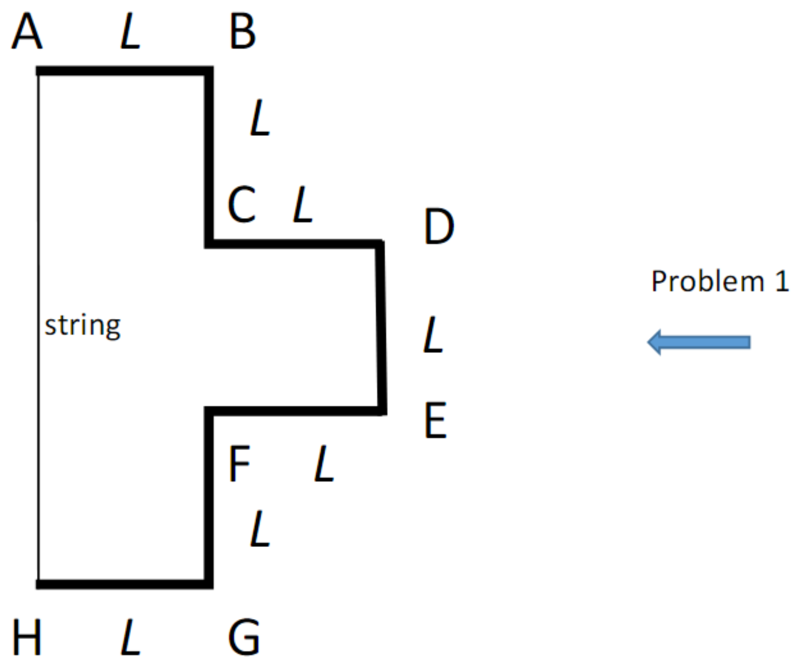




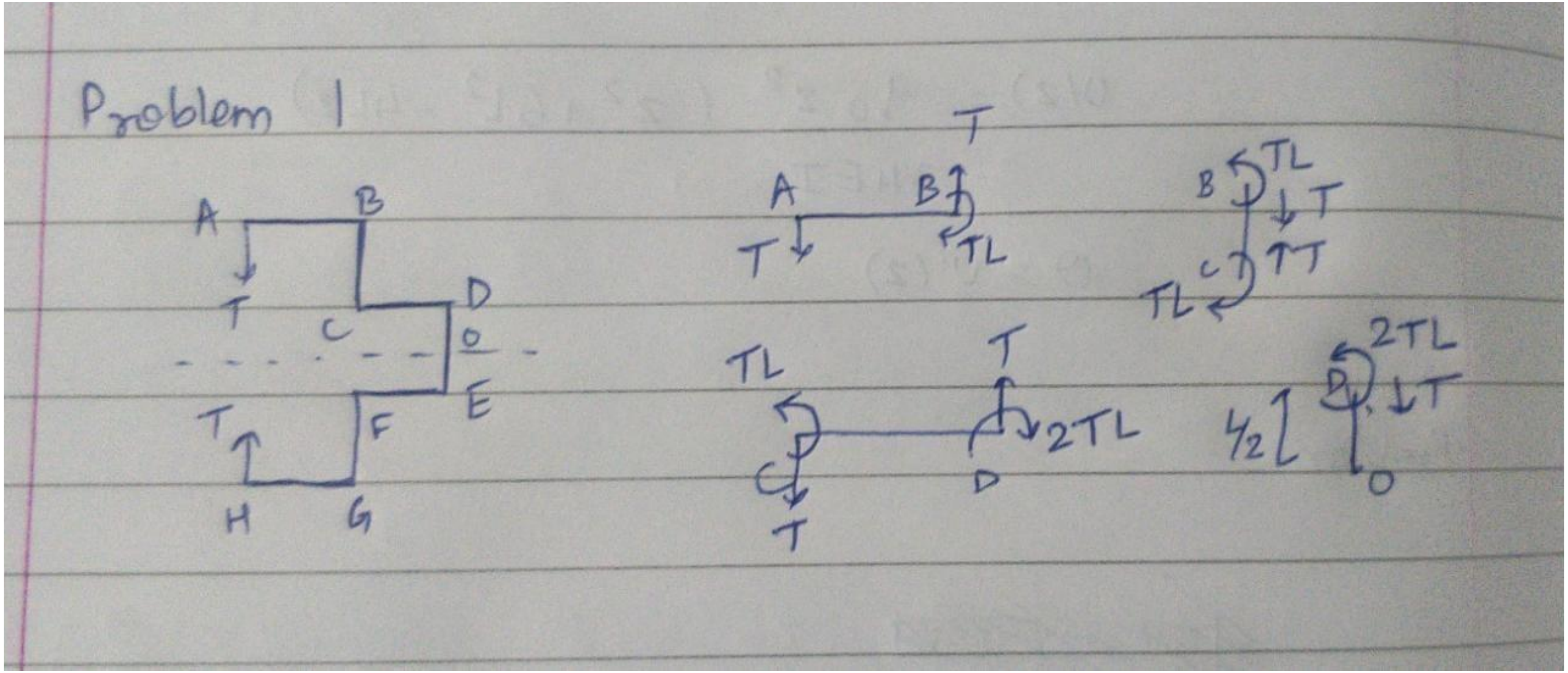
What are components of $U_c = \frac{PL^3}{3EI} + (PL) * \frac{L}{GJ} * L + \frac{PL^3}{3EI}$

Old Midsem Problem 1

1. (10 points) Consider the planar bent beam ABCDEFGH (each segment AB, BC, CD, DE, EF, FG, GH has length L , and flexural rigidity is EI, and $\angle ABC = \angle BCD = \angle CDE = \angle DEF = \angle EFG = \angle FGH = \pi/2$). A string is attached between A and H and is slowly tightened to tension T. The initial distance between A and H is $3L$. Calculate (a) the new distance between A and H and, (b) the location and magnitude of the maximum bending moment in the beam. Ignore axial deformations in the beam and the elasticity of the string. Assume both the beam and string are weightless.



Old Midsem Problem 1



$$U_A = \frac{TL^3}{3EI} + \frac{TL^2}{EI} * L + \frac{TL^3}{3EI} + \frac{TL^3}{2EI} + \left(\frac{TL^2}{2EI} + \frac{TL^2}{EI}\right) * L + 2TL * \frac{L}{2} * \frac{1}{EI} * 2L$$

Old Midsem Problem 1

$$U_{AB} = \frac{TL^3}{3EI} \qquad U_{AC} = \theta_{BC} * L = \frac{TL^2}{EI} * L$$

$$U_{CD} = \frac{TL^3}{3EI} + \frac{TL^3}{2EI} + \left(\frac{TL^2}{2EI} + \frac{TL^2}{EI}\right) * L$$

$$U_{DO} = \frac{2TL\,(L/2)}{EI} * \underline{\underline{(2L)}}$$

Application of PMPE to Torsion of non circular c/s

2 Application of PMPE to Torsion of non ○ c/s.

We find ϕ using PMPE and use it to find k_T

Step 1 : ϕ such that $\phi = 0$ on $\partial \Omega$

Step 2 : $$\Pi = \frac{1}{2G} \int_{\Omega} \left(\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 \right) dx\,dy - 2\alpha \int_{\Omega} \phi \, dx\,dy$$

Step 3 : $\frac{\partial \Pi}{\partial K} = 0$

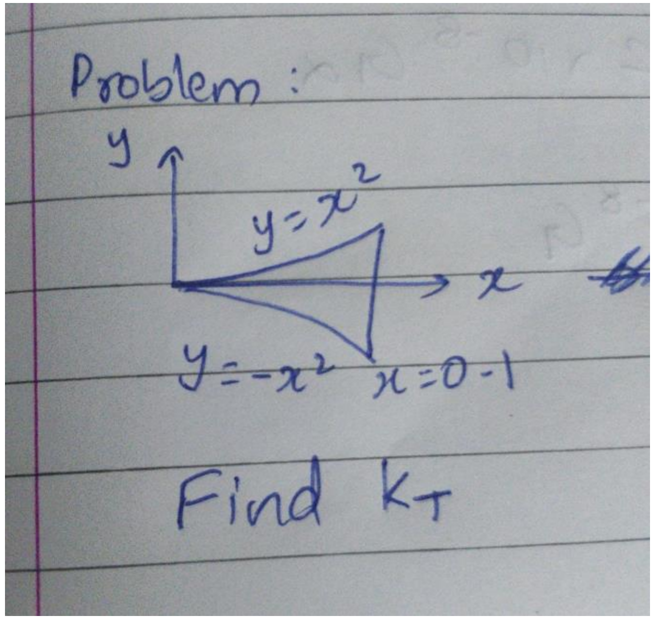
↑ variable in ϕ

Step 4 : $T = 2\int \phi \, dx\,dy$

Step 5 : $k_t = T/\alpha$

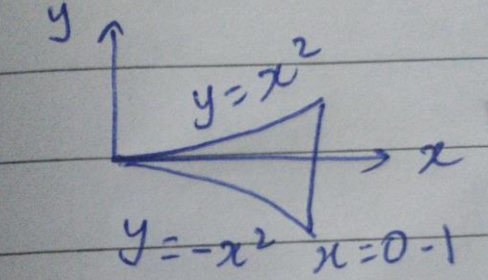
Old Midsem Problem 2:

3. (10 points) The profile of the solid cross-section of a long (length >> 1) shaft is enclosed within the curves $y = x^2, y = -x^2, x = 0.1$ as shown in the figure in the $x - y$ plane. An axial torque T directed along with z –axis axis on the shaft. Use the principle of minimum potential energy to determine the approximate torsional stiffness of this shaft. The shear modulus of the shaft material is G. Assume the shaft is weightless.



Problem:

Problem :



Find K_T

Step ① $\phi(x,y) = K(y^2 - x^4)(x - 0.1)$

$\phi = 0$ on surface ✓

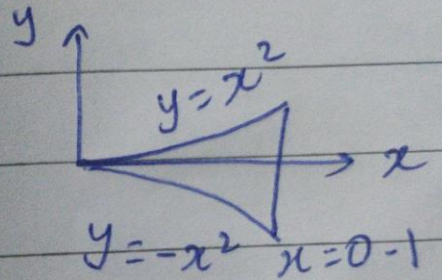
Step ② Finding T

$$\frac{\partial \phi}{\partial x} = K(y^2 - x^4) - 4K x^3 (x - 0.1)$$

$$\frac{\partial \phi}{\partial y} = 2Ky(x - 0.1)$$

Problem:

Problem :



Find K_T

$$\frac{\partial \phi}{\partial y} = 2ky(x-0.1)$$

$$\Pi = \frac{1}{2G}\int_{\Omega}\left\{\left(\frac{\partial \phi}{\partial x}\right)^2+\left(\frac{\partial \phi}{\partial y}\right)^2\right\}dx\,dy - 2\alpha\int_{\Omega}\phi\,dx\,dy$$

$$= \frac{56\times10^{-12}}{G}K^2 - \overset{4.7}{\cancel{8}}\times10^{-10}\,K\alpha$$

Step 3 $\quad \frac{\partial \pi}{\partial k} = 0 \Rightarrow K = 43\,G\alpha$

Step 4 $\quad \phi = 43G\alpha(y^2-x^4)(x-0.1)$

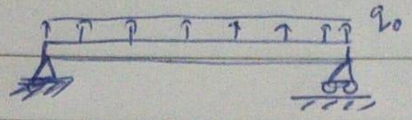
$$T = 2\int_{\Omega}\phi\,dx\,dy = 2\times10^{-8}\,G\alpha$$

Step 5 $\quad K_t = \frac{T}{\alpha} = 2\times10^{-8}\,G$

Distributed Loads

Help Session - 17/2/22

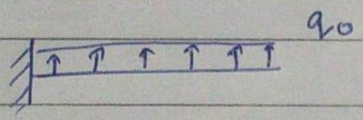
Pinned - Pinned



$$v(z) = \frac{q_0}{2EI}\left(\frac{z^4}{12} - \frac{Lz^3}{6} + \frac{L^3 z}{12}\right)$$

$$\theta = v'(z)$$

Cantilever

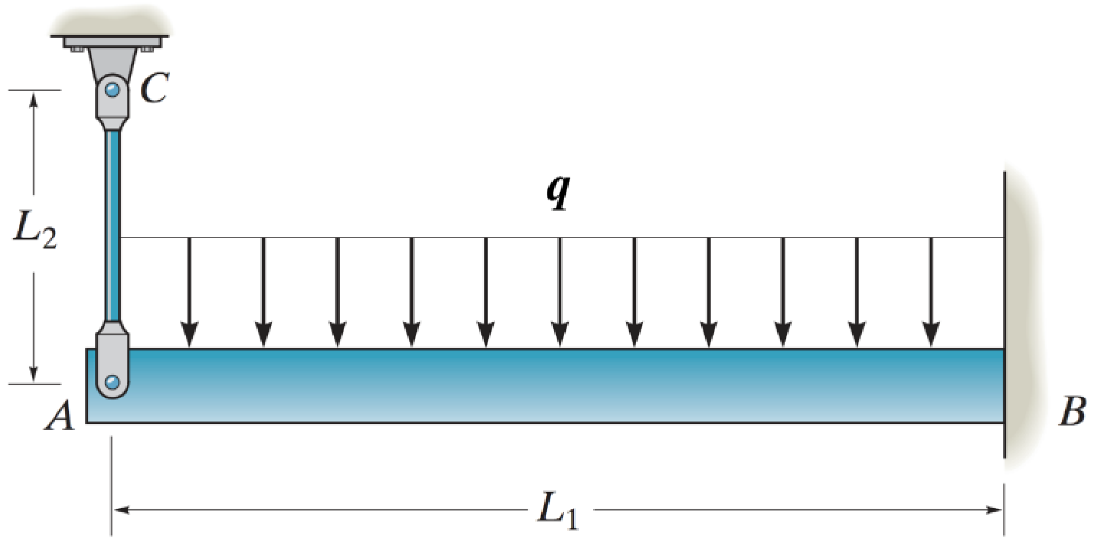


$$v(z) = \frac{q_0 z^2}{24EI}\left(z^2 + 6L^2 - 4Lz\right)$$

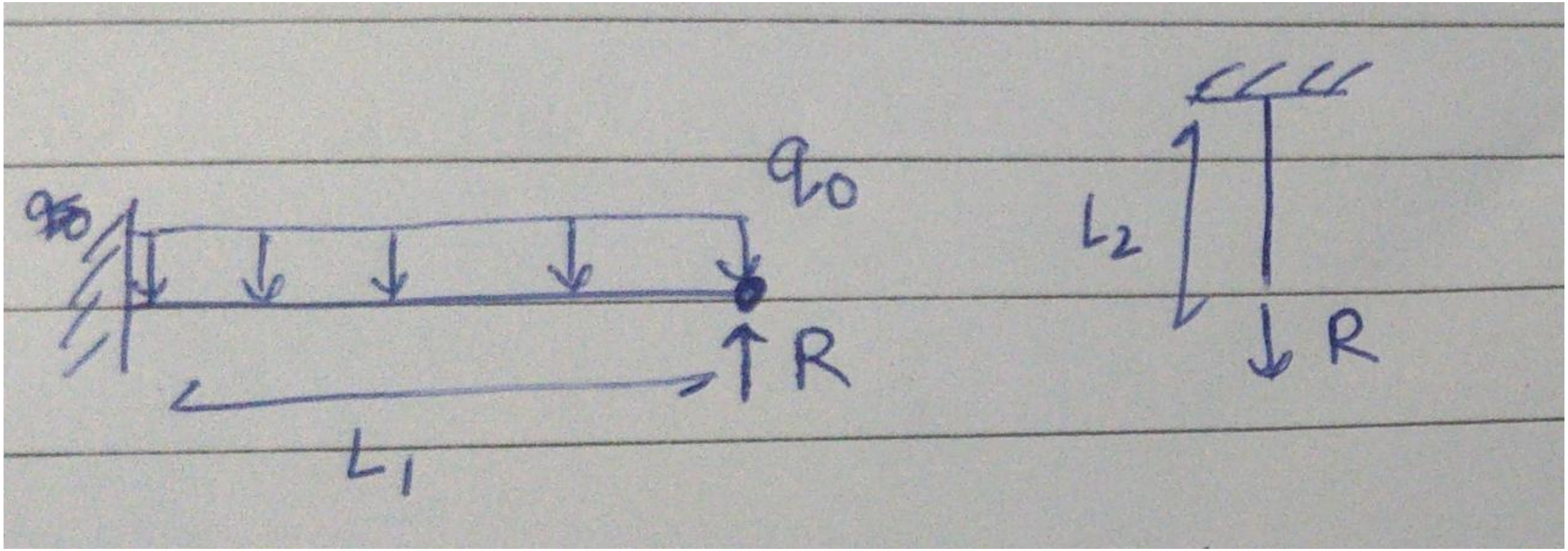
$$\theta = v'(z)$$

Old Midsem Problem 3

2. (10 points) The elastic beam AB of length L_1 and flexural rigidity EI is fixed into a wall at B ($z = L_1$) and connected to an elastic bar AC at A ($z = 0$) via a pinned connection. The bar has length L_2, elastic modulus E, and cross-sectional area A. If the beam is subjected to a uniformly distributed load per unit length of intensity q as shown, find (a) the tensile force in the bar and (b) the maximum deflection of the beam (c) the maximum bending moment in the beam. Assume both the beam and bar are weightless.

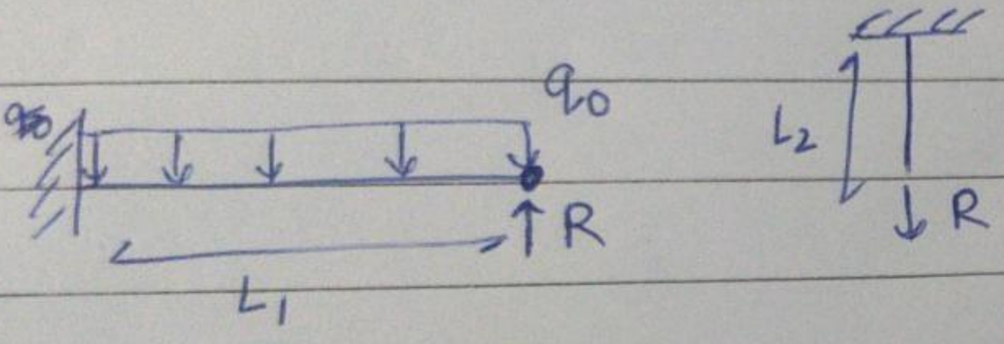


Old Midsem Problem 3



$$U(A) = \frac{3q_0L_1^4}{24EI}\ down - \frac{RL_1^3}{3EI}(up) = \frac{RL_2}{AE}$$

Old Midsem Problem 3



$$v(\text{end point}) = \frac{q_0 L_1^2 (3L_1^2)}{24EI}\Big\downarrow_{\text{down}} - \frac{R}{EI}\frac{L_1^3}{3}\Big\uparrow_{\text{up}} = \frac{RL_2}{AE}\Big\downarrow_{\text{down}}$$

R ✓

Old Midsem Problem 3

$\therefore$ Max deflection v

$v'(z) = 0$	boundary
check v at $v'(z) = 0$	check ends

$\therefore$ Maximum M

$$M(z) = EIv'' = Pz - \frac{qz^2}{2}$$

$M'(z) = 0$ for M_{max} ✓✓

Thanks for attending
All the best!