ME 202

LECTURE 12

THU 27 JAN 2022

BENDING / FLEXURE

Applications: Structures
Machine components
Furniture
Bio-mechanics

L >> c/s dimensions.

BEAM

Q1: Deflected shape?

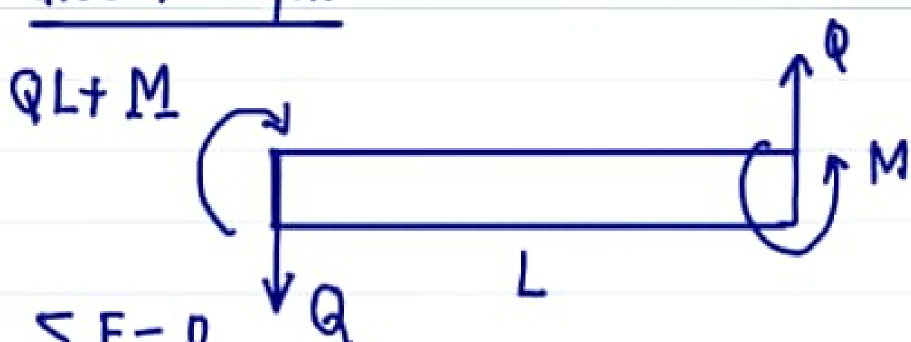
Q'2: What is M_{max}, Q_{max}?

Equilibrium

Global Equilibrium: Applied to entire beam

Local Eqm : Applied piecewise.

<u>Global Eqm</u>

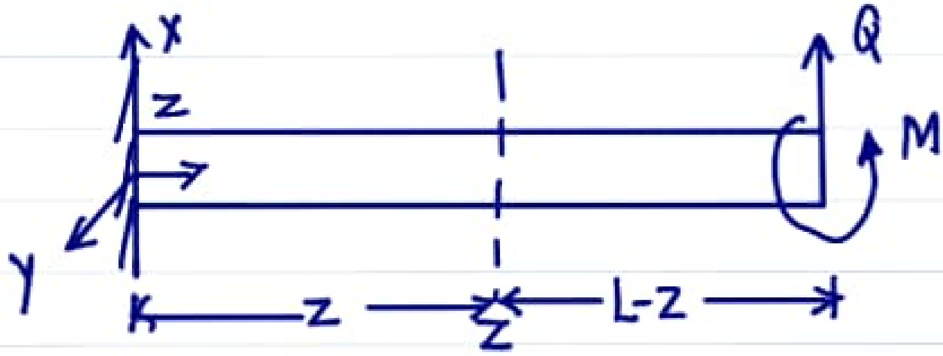


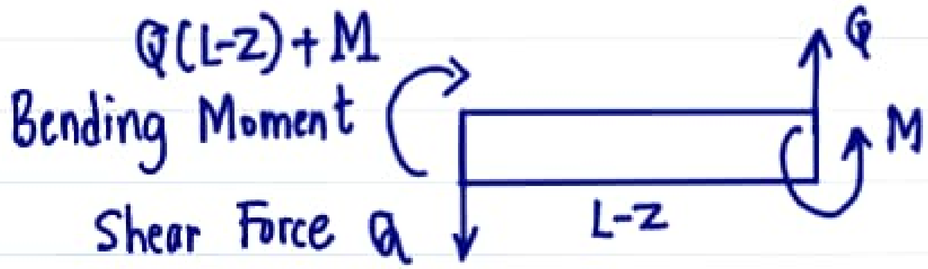
$\Sigma \underline{F} = \underline{0}$

$\Sigma \underline{M} = \underline{0}$ support reactions

Wall prevents rigid body motion.
by constraining vert motion &
ang displacement.

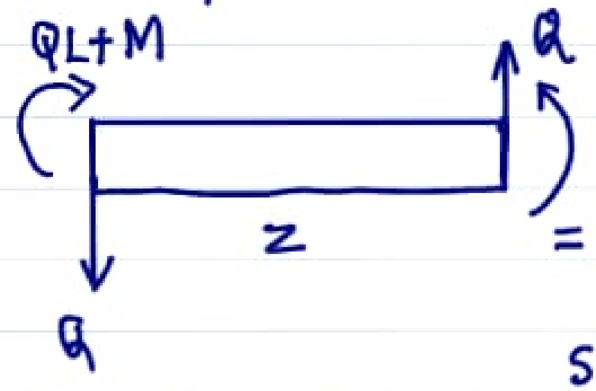
<u>LOCAL EQM</u> : Any internal section
is in eqm





$\Sigma F_x = 0, \ \Sigma M_y = 0$

OR



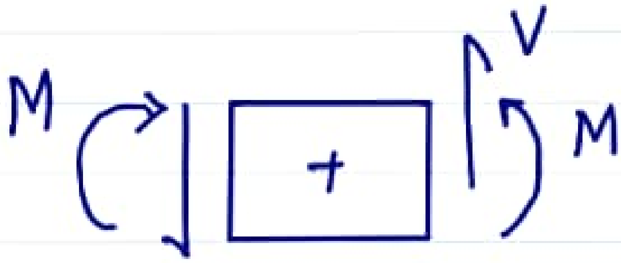
$M + QL - Qz$

$= M + Q(L-z)$

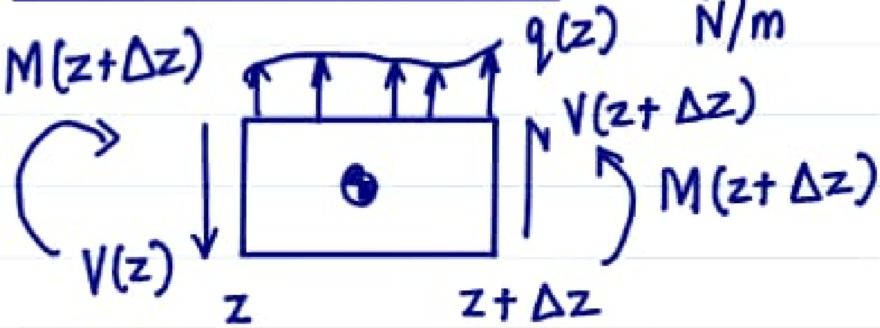
SF BM

Sign Convention for $V(z)$, $M(z)$

SF/BM	Area, Normal $\pm e_z$	Sign
+	+	+
−	−	+
−	+	−
+	−	−



Local Equilibrium Applied to $\Delta z \to 0$.



$$\sum F_x = 0 \Rightarrow V(z+\Delta z) - V(z) + q(z)\Delta z = 0$$

Used: $$\int_a^{a+\epsilon} f(x)\,dx \approx f(a)\epsilon \quad \text{as } \epsilon \to 0$$

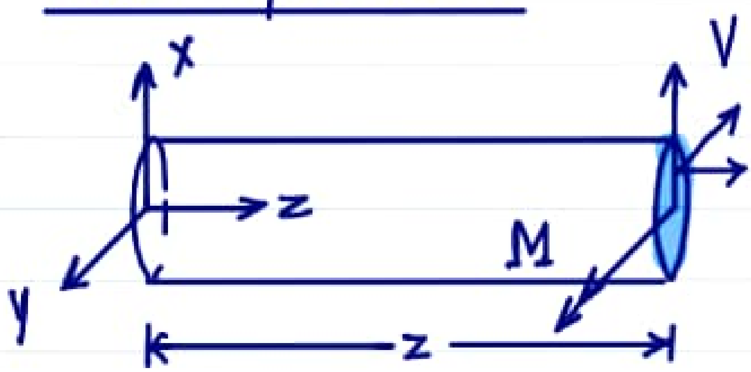
As $\Delta z \to 0$, $$\frac{dV}{dz} + q = 0$$

$$\sum M_y = 0 \Rightarrow M(z+\Delta z) + V(z+\Delta z)\frac{\Delta z}{2} + V(z)\frac{\Delta z}{2} - M(z) = 0$$

$$\frac{dM}{dz} + V(z) = 0 \quad \text{as } \Delta z \to 0$$

Local Equilibrium

$\underline{t} = \underset{\sim}{\sigma}\,\underline{n}$



$\underline{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

At z,
$$\begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ M \\ 0 \end{pmatrix}$$

$$\underset{\sim}{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}, \quad \underline{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \underline{t} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$\underline{t} = \underset{\sim}{\sigma}\,\underline{n} \Rightarrow t_x = \sigma_{xz},\ t_y = \sigma_{yz},\ t_z = \sigma_{zz}$$

$$\int_\Omega \underline{t}\, da = V \underline{e}_x, \quad \int_\Omega (\underline{x} \times \underline{t})\, da = M \underline{e}_y$$

$$\int_\Omega (\sigma_{xz}\, \underline{e}_x + \sigma_{yz}\, \underline{e}_y + \sigma_{zz}\, \underline{e}_z)\, da = V \underline{e}_x$$

$$\int_{\Omega} \sigma_{xz}\, da = \bar{V}, \quad \int_{\Omega} \sigma_{yz}\, da = 0, \quad \int_{\Omega} \sigma_{zz}\, da = 0$$

$$\underline{x} \times \underline{t} = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ x & y & z \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix}$$

$$= \underline{e}_x \left(y\sigma_{zz} - z\sigma_{zy} \right) - \underline{e}_y \left(x\sigma_{zz} - z\sigma_{zx} \right)$$

$$+ \underline{e}_z \left(x\sigma_{zy} - y\sigma_{zx} \right)$$