MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 1

(January 12, 2022)

(1) Find the 4-th Taylor $P_4(x)$ polynomial for the function $f(x) = xe^{x^2}$ at $x = 0$.

(2) Let $f(x) = (1-x)^{-1}$. Find the n-th Taylor polynomial $P_n(x)$ for $f(x)$ about $x = 0$.

(3) For $f(x)$ and $P_n(x)$ as in the above problem, find a value of n such that $P_n(x)$ approximates $f(x)$ to within 10^{-6} on $[0, 0.5]$.

(4) If we use k digits and the chopping method to approximate a real number $y \neq 0$ then prove that the relative error is $\leqslant 10^{-k+1}$.

(5) If we use k digits and the rounding method to approximate a real number $y \neq 0$ then prove that the relative error is $\leqslant 0.5 \times 10^{-k+1}$.

(6) Suppose $x = \frac{5}{7}$ and $y = \frac{1}{3}$. Use five-digit chopping to compute $x \oplus y$, $x \ominus y$, $x \otimes y$ and $x \oplus y$. Compute the absolute and the relative errors in the above 4 operations.

(7) Let $p = 0.546217$ and $q = 0.546201$. Use five-digit arithmetic to compute $p \ominus q$ and determine the absolute and the relative errors using the methods of chopping and rounding. Compute the number of significant digits in both these methods for the result.

(8) Consider the quadratic equation $x^2 + 62.10x + 1 = 0$ whose roots are (approximately) $x_1 = -0.01610723$ and $x_2 = -62.08390$.

Use the four-digit rounding arithmetic to compute the roots using the formula

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Compute the absolute and the relative errors.

(9) Evaluate $f(x) = x^3 - 6.1x^2 + 3.2x + 1.5$ at $x = 4.71$ using three-digit arithmetic in both the chopping and the rounding methods. Compute the absolute and the relative errors.