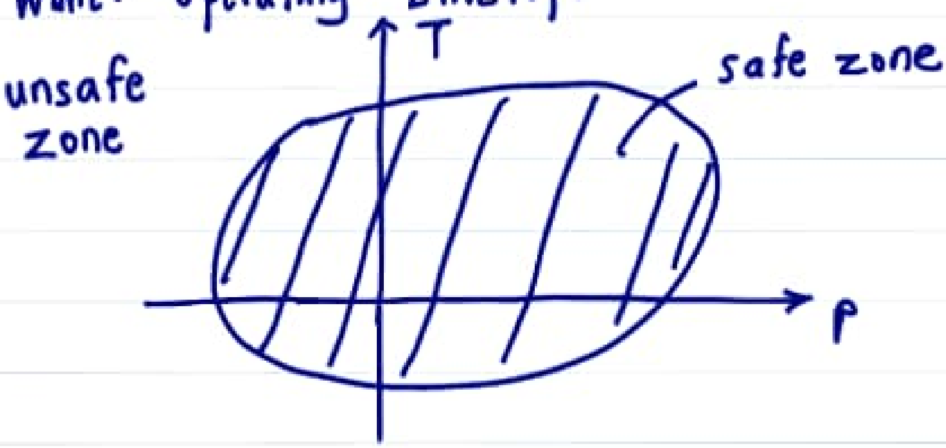
ME 202
LECTURE 6
13 JAN 2022

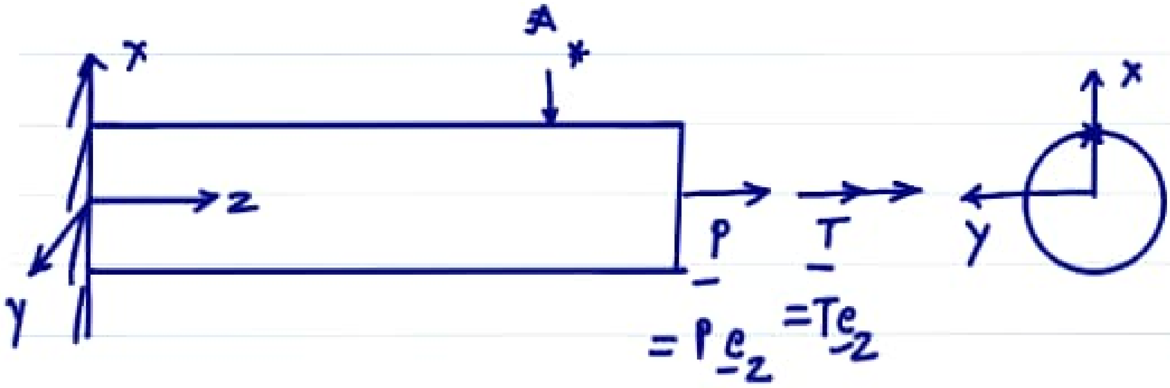
Torsion / Twist + Axial Force
T P

Drill Bit

Power Drill, boschtools.com

Want: Operating Envelope





$$\sigma_{zz} = \frac{P}{A} = \frac{4P}{\pi D^2}$$

$* =$ point of interest $\left(\frac{D}{2}, 0\right)$

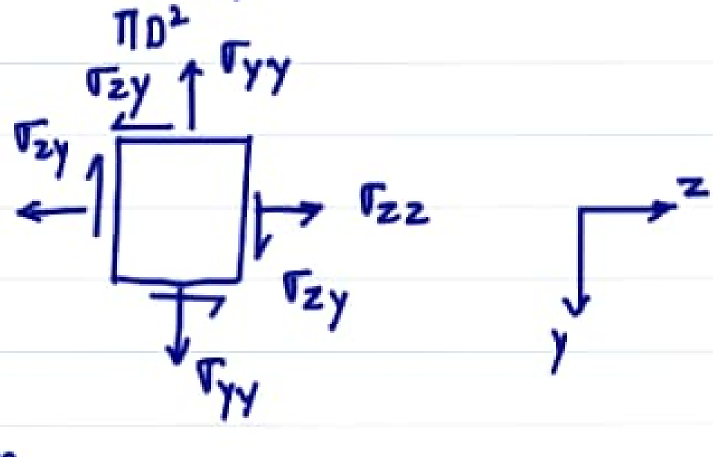
$$\sigma_{zy} = +G\alpha x = \frac{16T}{\pi D^3}$$

$$\sigma_{zx} = -G\alpha y = 0$$

$$\underset{\sim}{\sigma} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 16T/\pi D^3 \\ 0 & \frac{16T}{\pi D^3} & \frac{4P}{\pi D^2} \end{pmatrix}$$ 2D stress @ *

Stress element

Top of *



Theory of Failure.

Soy, material brittle. ⇒ fail by fracture.
Max Normal Stress Theory.

nucleation + growth

Use formulas from Mohr's circle

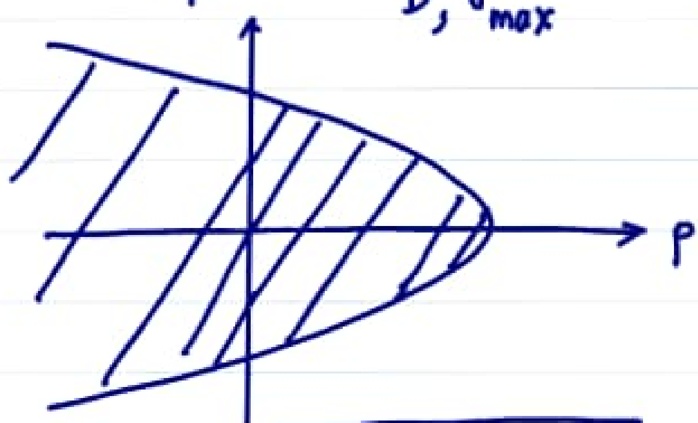
$$\sigma_{max} = \underbrace{c}_{\text{center}} + \underbrace{R}_{\text{radius}}$$

$$= \left(\frac{\sigma_{zz} + \cancel{\sigma_{yy}}}{2}\right) + \sqrt{\left(\frac{\sigma_{zz} - \cancel{\sigma_{yy}}}{2}\right)^2 + \sigma_{zy}^2}$$

$$\sigma_{max} = \frac{4P}{2\pi D^2} + \sqrt{\frac{4P^2}{\pi^2 D^4} + \frac{256 T^2}{\pi^2 D^6}}$$

known exptly.

Used either for design or analysis.



$$2P + \sqrt{4P^2 + 256T^2} = N$$

Material ductile $\Rightarrow$ fails by plastic deformation
Movement of dislocations.
$\Downarrow$
slip

Max shear stress theory.

From Mohr's circle,

$$R = \sqrt{\left(\frac{\sigma_{zz} - \sigma_{yy}}{2}\right)^2 + \sigma_{zy}^2}$$

$$\sqrt{\left(\frac{4P}{2\pi D^2}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2} = \tau_{max}$$

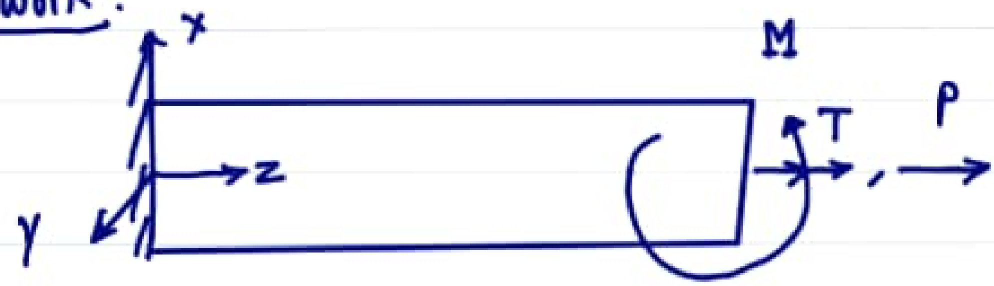
$\uparrow$ From expts.

$$P^2 D^2 + 64T^2 = \left(\frac{\sigma_Y \pi D^3}{4}\right)^2$$

$= \frac{\sigma_Y}{2} \leftarrow$ yield stress from tensile test

Ellipse in P-T space.

<u>Homework</u>:



At free end, $M \underline{e}_y + T \underline{e}_z$

$P \underline{e}_z$

$$\sigma_{zz} = \frac{P}{A} + \left(-\frac{M}{I}x\right)$$

Euler-Bernoulli beam theory.

$$\sigma_{zy} = \frac{16T}{\pi D^3}$$

will review in later class.

Find operating envelope in P, M, T space for safe operation of shaft, max shear stress theory τ_{max} given from expts

Torsion of Non-circular cross-sections

- Angle of Twist
- T_{max} for safe operation

T from aerodynamics.

Airfoil

c/s Wing / Turbine blade

Theory based on circular c/s, at some z.

centroid

origin here for now.

$OP = OP'$

$\theta = \alpha z$, α unit angle of twist

small angles.

$u = -\alpha y z$

$v = +\alpha x z$

$w = w(x,y)$ warping.

From expt observations

or prove formally $\omega \neq 0$ for non-circ c/s
later.

$$\underset{\alpha}{\text{Disp}} \longrightarrow \text{Strains} \longrightarrow \text{Stresses} \longrightarrow \underset{T}{\text{Torque}}$$

Goal: $T = K_t \alpha$

$\uparrow$ Torsional stiffness (G, geometry)