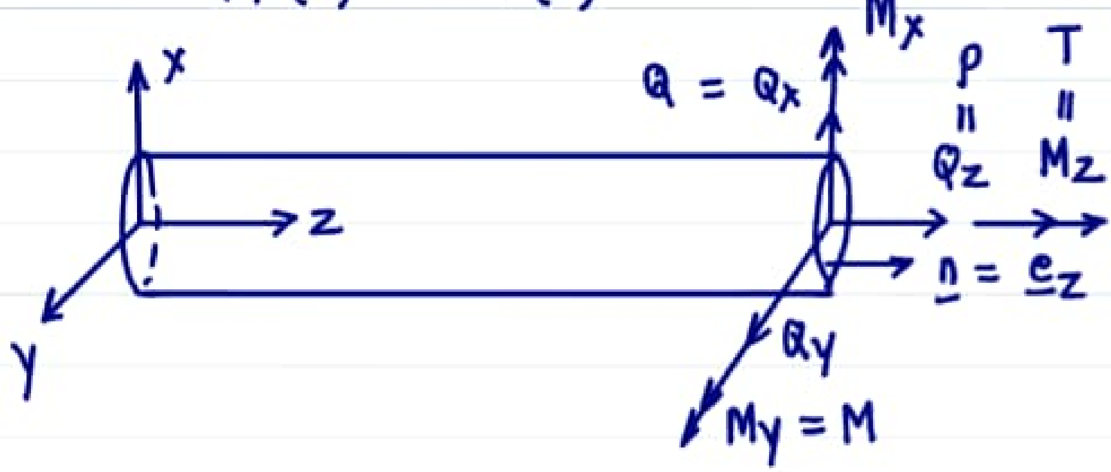
ME 202

LECTURE 13

MON 31 JAN 2022

Previously,

$$M'(z) = -V(z)$$



$$\int_\Omega \underline{t}\, da = Q_x \underline{e}_x + Q_y \underline{e}_y + Q_z \underline{e}_z$$

$$\int_\Omega \underline{x} \times \underline{t}\, da = M_x \underline{e}_x + M_y \underline{e}_y + M_z \underline{e}_z$$

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$$\underline{e}_x(y\sigma_{zz}-z\sigma_{zy})-\underline{e}_y(x\sigma_{zz}-z\sigma_{zx})$$

$$+\underline{e}_z(x\sigma_{zy}-y\sigma_{zx}) = M_x\underline{e}_x + M_y\underline{e}_y + M_z\underline{e}_z$$

$$Q_x = \int_\Omega \sigma_{xz}\,dx\,dy$$

$$Q_y = \int_\Omega \sigma_{yz}\,dx\,dy$$

$$Q_z = \int_\Omega \sigma_{zz}\,dx\,dy$$

$$M_x = \int_\Omega (y\sigma_{zz}-z\sigma_{zy})\,dx\,dy$$

$$M_y = \int_\Omega (-x\sigma_{zz}+z\sigma_{zx})\,dx\,dy$$

$$M_z = \int_\Omega (x\sigma_{zy}-y\sigma_{zx})\,dx\,dy$$

<u>Pure Bending</u>

$Q_x = 0,\ Q_y = 0,\ Q_z = 0$

$M_x = 0,\ M_y = M,\ M_z = 0$

Proposal
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$\sigma_{zz} = Ax$$

$$\int_\Omega Ax\,dx\,dy = 0 = Q_z$$

$$\int_\Omega x\,dx\,dy = 0$$ origin is at centroid

$$0 = M_x = \int_\Omega y\,Ax\,dx\,dy$$

$$\Rightarrow \int xy\,dx\,dy = 0$$ x-y symmetry

of c/s

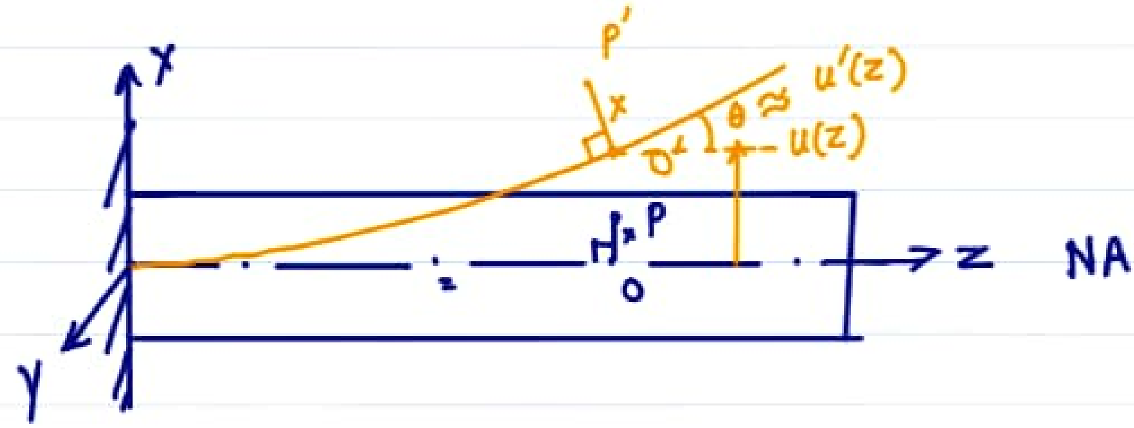
$$M_y = M = \int_\Omega -x\,A x \, dx\, dy \qquad \text{2}^{\text{nd}} \text{ moment of area @ y-axis}$$

$$M(z) = -A \int_\Omega x^2 \, dx\, dy = -A\, I_{yy}$$

$$\int_\Omega x^0 \, dx\, dy = \text{Area},$$

$$\Rightarrow \quad A = -\frac{M(z)}{I_{yy}}, \qquad \sigma_{zz}(x,y,z) = -\frac{M(z)}{I_{yy}}\, x$$

Kinematics/Strain-Disp Relationship



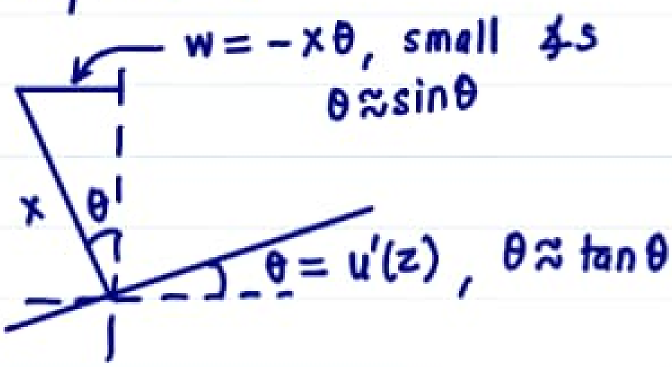
$P(z,x)$ orig location of a point.

Let $u(z)$ be x-disp / vertical disp of the neutral axis.

$u = u(z)$ x-disp of P

$v = 0$ No y-disp of P

$$w = -x\frac{du}{dz}$$



Kinematic Assumptions of Euler-Bernoulli beam theory

$$\epsilon_{zz} = \frac{\partial w}{dz} = -x\frac{d^2u}{dz^2}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = 0, \quad \epsilon_{yy} = \frac{\partial v}{\partial y} = 0$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0, \quad \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0$$

$$\epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0$$

Only one strain $\epsilon_{zz} = -x\dfrac{d^2u}{dz^2}$

small angles, $\underbrace{\dfrac{d^2u}{dz^2} \approx \kappa}_{\text{curvature}} = \dfrac{u''}{(1+u'^2)^{3/2}}$

Hooke's Law 1D $\quad \sigma_{zz} = E\,\epsilon_{zz}$

$$Ax = -\frac{M}{I}x = E\left(-x\frac{d^2u}{dz^2}\right)$$

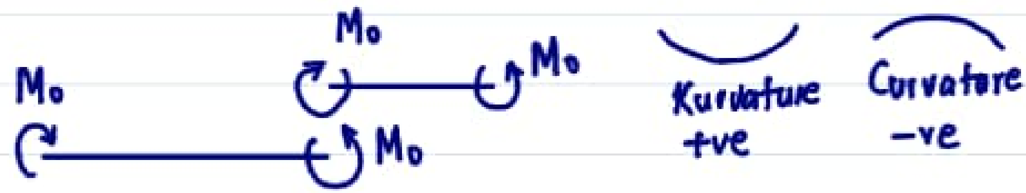
$$\boxed{M(z) = EI\,u''} \qquad ' \equiv \frac{d}{dz}$$

2^{nd} order beam equation

Moment - curvature relationship

Used to get deflection curve $u(z)$.

Deflection of cantilever



$$M(z) = + M_0 = EI\, u''$$

$$u' = \frac{M_0}{EI} z + C_1$$

$$u(z) = \frac{M_0 z^2}{2EI} + C_1 z + C_2$$

Get C_1, C_2 from end/boundary conditions

$$u(0) = 0, \quad u'(0) = 0 \Rightarrow C_1 = 0, \; C_2 = 0$$

$$u(z) = \frac{M_0 z^2}{2EI}$$