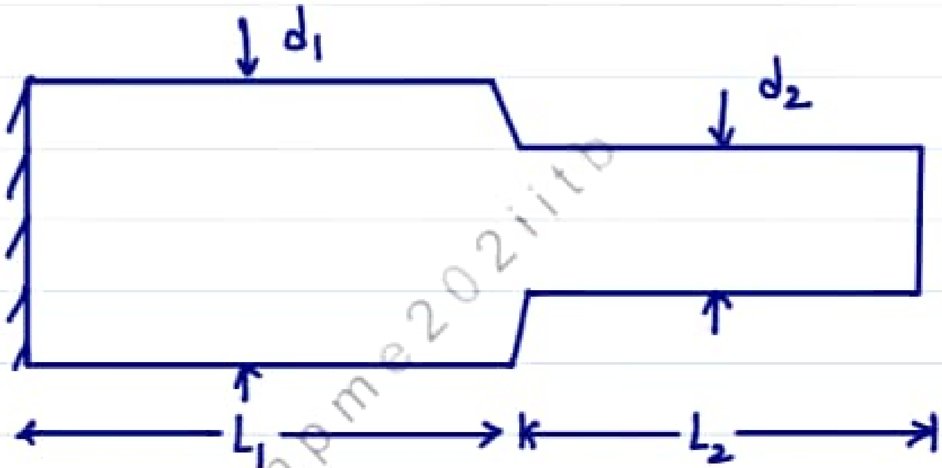
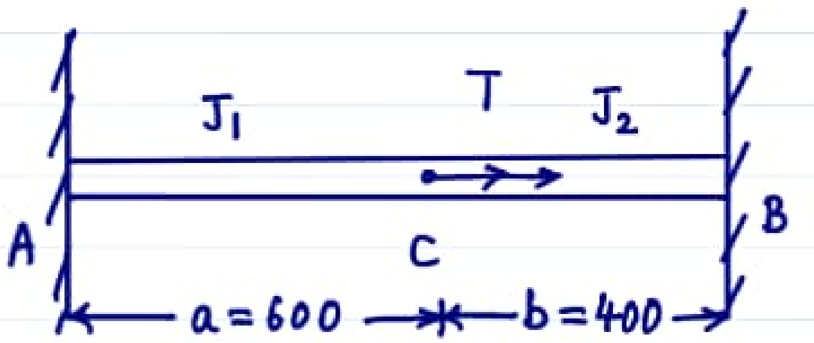
ME 202
LECTURE 5
TUTORIAL 1
11 JAN 2022

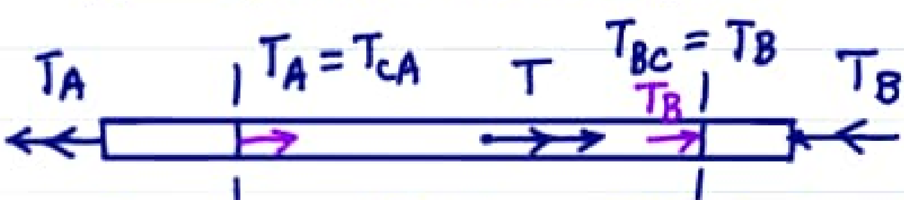
1.



2.



d = 50mm



Torque equilibrium $T - T_B - T_A = 0$

2 unknowns 1 static eqm eqn

Statically indeterminate

Angle of twist @ B wrt A = 0 $\qquad \theta = \dfrac{TL}{GJ}$

$$\theta_B = \theta_{CA} + \theta_{BC}$$
$$= \frac{T_A a}{GJ} - \frac{T_B b}{GJ} = 0$$

$T_A a = T_B b$, $T_A = \dfrac{T b}{a+b}$, $T_B = \dfrac{T a}{a+b}$

$T = 400P$ N-mm

$T_A = 0.16P$ N-m, $T_B = 0.24P$ N-m

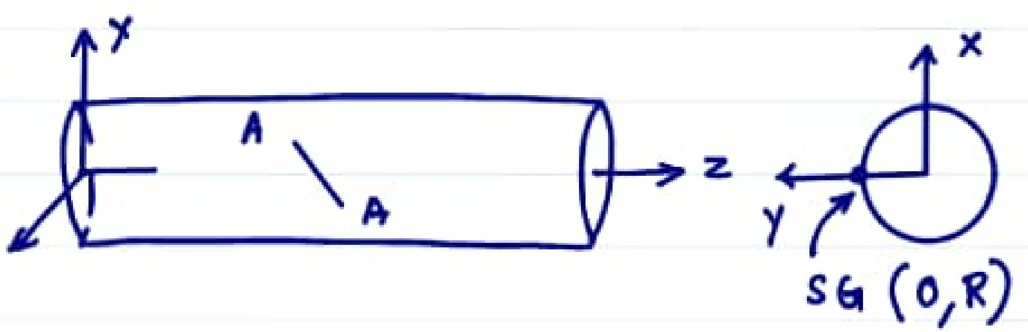
T_{AC} $\qquad$ T_{BC}

$$\tau = \frac{16T}{\pi d^3} = \tau_{max} = \frac{T_{max}(d/2)}{J} = \underbrace{45 \times 10^6 \text{ N/m}^2}_{\text{given}}$$

$$\frac{0.24P \times 0.05/2}{\frac{\pi}{32} \times (0.05)^4} = 45 \times 10^6$$

$$\Rightarrow P = 4602 \text{ N}$$

3.



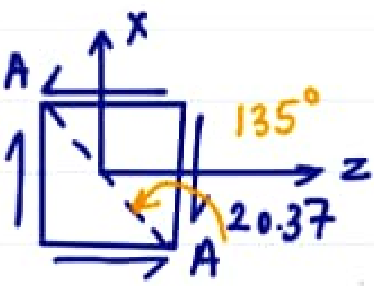
At SG, $\sigma_{zx} = -G\alpha y = -\dfrac{TR}{J}$

$\sigma_{zy} = +G\alpha x = 0$

$$\sigma_{zx} = -\frac{500 \times 1000 \times 25}{\pi/32 \times 50^4} = -20.37 \ \frac{N}{mm^2}$$

$$= -20.37 \ MPa$$

At SG,



$\sigma_{xx} = 0, \quad \sigma_{zz} = 0, \quad \sigma_{zx} = -20.37$

$$\sigma_{zx} = 2G\epsilon_{zx} \Rightarrow \epsilon_{zx} = \frac{-20.37}{2G}$$

$\epsilon_{AA} = 339 \times 10^{-6}$ Given

$\theta \neq \cancel{\alpha}$ twist !

θ wrt z axis

Use strain transform rules,

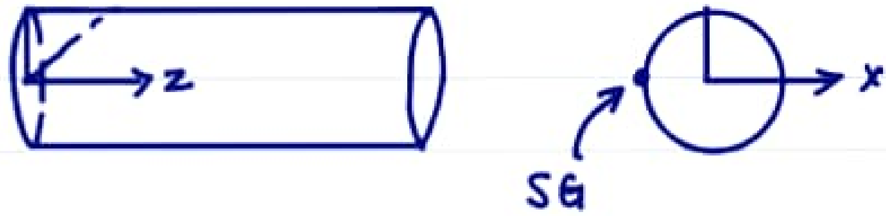
$$\epsilon_{AA} = \epsilon_{zz}\cos^2\theta + \epsilon_{xx}\sin^2\theta + 2\epsilon_{xz}\sin\theta\cos\theta$$

$$339 \times 10^{-6} = 0 + 0 + 2\left(\frac{-20.37}{2G}\right)\sin 135^\circ \cos 135^\circ$$

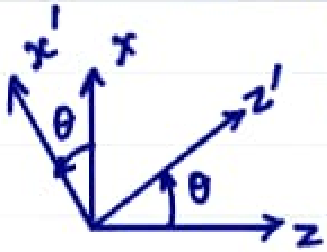
$G = 30044$ MPa

$= 30.044$ GPa

Y ↑ ↗ x ↑ y

HW: 

Strain transf rule



$$\underset{\sim}{\epsilon}' = \underset{\sim}{Q}\,\underset{\sim}{\epsilon}\,\underset{\sim}{Q}^T$$

$$\underset{\sim}{Q} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

4.

Axial Impact

$M = \rho A L$

Conservation of energy

$$\cancel{PE}_1 + KE_1 = PE_2 + \cancel{KE}_2$$

before impact   after impact

$$\frac{1}{2} M v^2 = \frac{1}{2} K x^2$$

$$\rho A L v^2 = \left(\frac{AE}{L}\right) x^2$$

x = compression

$$x = \sqrt{\frac{\rho}{E}}\, L v$$

Max compression

Torsional Impact

$$\cancel{PE_1} + KE_1 = PE_2 + \cancel{KE_2}$$

before locking   after locking

$$\frac{1}{2} I\omega^2 = \frac{1}{2} MK^2 \left(\frac{2\pi N}{60}\right)^2 = \frac{1}{2} k_T \theta^2$$

$$= \frac{1}{2}\left(\frac{GJ}{L}\right)\alpha^2 L^2$$

Solve for α max angle of twist / length

Max shear stress $= G\alpha \dfrac{d}{2}$

Alternate method:

After bearing jam,

$$I\ddot{\theta} + k_T\theta = 0 \qquad (T = I\alpha)$$

$$MK^2\ddot{\theta} + \frac{GJ}{L}\theta = 0$$

$$\theta(t) = A\cos\sqrt{\frac{GJ}{MK^2L}}\,t + B\sin\sqrt{\frac{GJ}{MK^2L}}\,t$$

ang. def. after bearing jam.

ICs: $\theta(0) = 0$, $\dot{\theta}(0) = \omega = \dfrac{2\pi N}{60}$

$$A = 0, \quad B = \omega \sqrt{\frac{MK^2 L}{GJ}}$$

$$\theta_{max} = B = \omega \sqrt{\frac{MK^2 L}{GJ}}$$

$$\alpha_{max} = \sqrt{\frac{MK^2}{GJL}\,\omega^2}$$

Note: $I\ddot{\theta} + k_T\,\theta = 0$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k_T \theta^2\right) = 0$$

$$\Rightarrow \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k_T \theta^2 = \text{constant of motion}$$

$$\Rightarrow \frac{1}{2} I \dot{\theta}_1^2 + \frac{1}{2} k_T \theta_1^2 = \frac{1}{2} I \dot{\theta}_2^2 + \frac{1}{2} k_T \theta_2^2$$

same as earlier approach