ME 202 S3
Tutorial 9

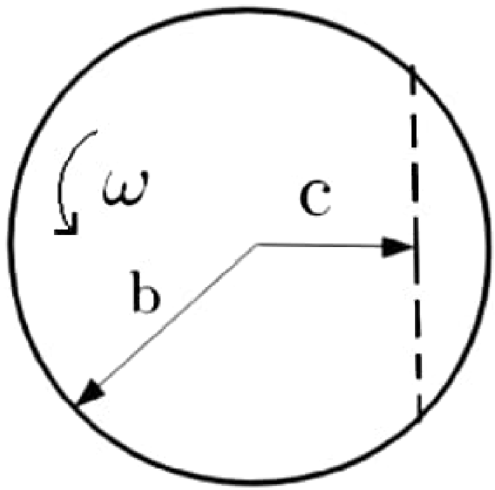
Thu 24 Mar 2022

Problem 1

- Recall the problem discussed in the previous class of a hollow disk (inner radius a, outer radius b) press fitted onto a solid disk of diameter $a + d$ where d is the interference $d \ll a$. Both disks are made of the same material. Find the rotational speed at which the outer disk comes loose of the inner disk using the displacement method.

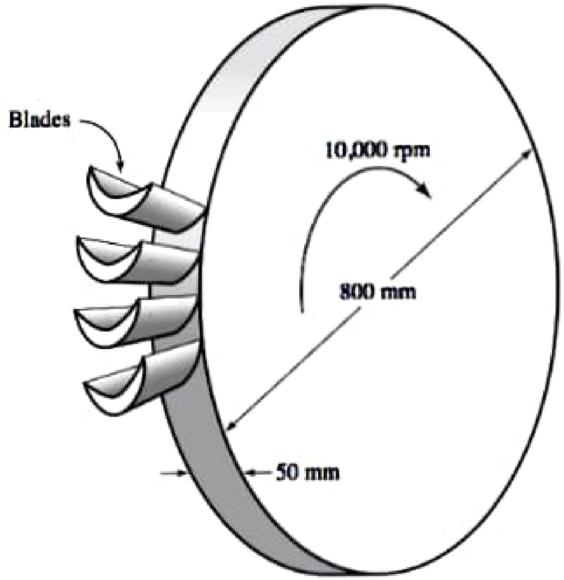
Problem 2

- Consider a solid disk with outer radius b rotating in its plane. There exists a weak plane in the disk on the chord of the circle at a distance c = b/2 from the center of the disk as shown. The weak plane will fail when the maximum normal stress across the plane reaches a critical value N. Find the maximum safe rotational speed of the disk.



Problem 3

- 50 turbine blades each of mass 400g are mounted on the outer periphery of a solid disk and the entire assembly is rotated at 10,000 rpm. Find the maximum tensile stress in the disk. Assume E = 210 GPa, v = 0.3, rho = 7700 kg/m^3 for the disk. Assume that the blades are rigid.



Problem 4

- Consider a stationary hollow disk (inner radius a, outer radius b) under internal pressure p. The material is known to fail when the Tresca (maximum shear stress) criterion is satisfied. It is further given that the material is elastic perfectly plastic i.e. yield stress of the material as measured in a uniaxial tension test is constant Y. Find the pressure at which plastic failure is just initiated in the disk. Also find the pressure at which the entire disk undergoes plastic failure.

Problem 5

- Consider a solid disk made of an elastic perfectly plastic material with uniaxial yield stress Y. Find the rotational speed at which plastic failure (yield) is just initiated in the disk assuming the Tresca criterion is applicable. Also find the rotational speed at which the entire disk fails plastically.

ME 202

LECTURE 29 TUT 9

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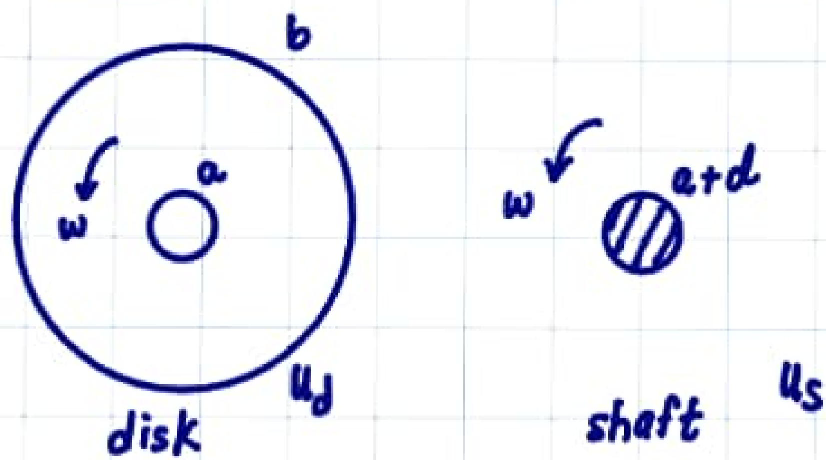
Problem 1



After fit/deformation,

$$\cancel{a} + u_d(a) = \cancel{a} + d + u_s(a + \cancel{d}^{\,0})$$

$$\frac{(3+\nu)(1-\nu)}{8E}\left[a^2+b^2-\frac{1+\nu}{3+\nu}a^2+\frac{1+\nu}{1-\nu}\frac{a^2b^2}{a^2}\right]\rho\omega^2 a$$

$$= d+\frac{(3+\nu)(1-\nu)}{8E}\left[a^2+0^2-\frac{1+\nu}{3+\nu}a^2 + \frac{1+\nu}{1-\nu}\cdot 0\right]\rho\omega^2 a$$

$$\Rightarrow \omega^2 = \frac{4Ed}{(3+\nu)\rho a b^2}$$ as before

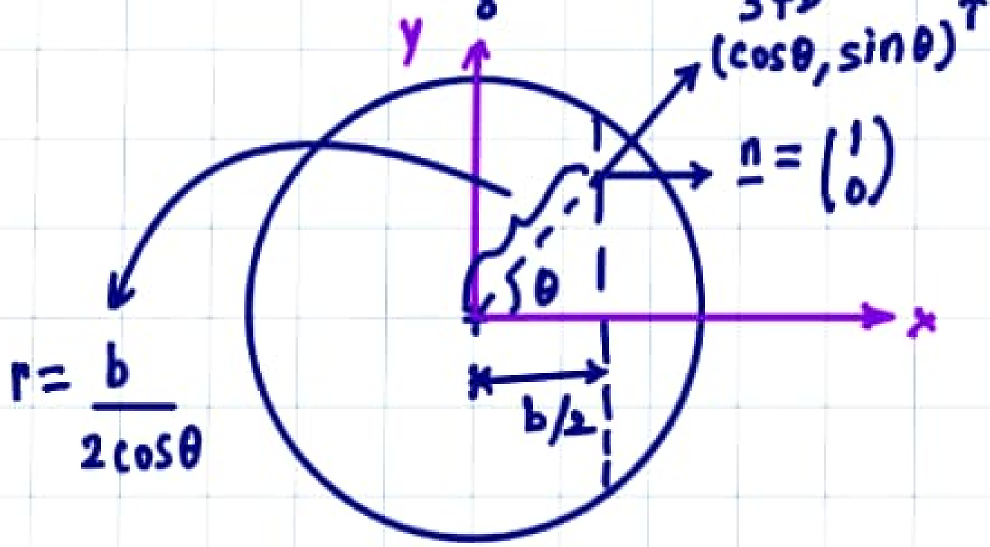
$\rho \equiv kg/m^3 \equiv Ns^2/m^4$

Problem 2

Solid disk $a = 0$

$$\sigma_{rr} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(b^2 - r^2\right)$$

$$\sigma_{\theta\theta} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(b^2 - \frac{1+3\nu}{3+\nu}r^2\right)$$



$$\underset{\text{polar coords}}{\underline{n}} = \underset{\sim}{Q}\,\underline{n} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \begin{pmatrix}\cos\theta\\ -\sin\theta\end{pmatrix}$$

$$\underline{t} = \underset{\sim}{\sigma}\,\underline{n} = \begin{pmatrix} \sigma_{rr} & 0 \\ 0 & \sigma_{\theta\theta} \end{pmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

$$\underline{t} = \sigma_{rr}\cos\theta - \sigma_{\theta\theta}\sin\theta$$

$$\sigma_n(\theta,\omega) = \underline{t}\cdot\underline{n} = \sigma_{rr}\cos^2\theta + \sigma_{\theta\theta}\sin^2\theta$$

normal cpt of traction

put

$$\sigma_n(\theta,\omega) = \frac{3+\nu}{8}\rho\omega^2 b^2\left(1 - \frac{1}{4\cos^2\theta}\right)\cos^2\theta$$

$$+ \frac{3+\nu}{8}\rho\omega^2 b^2\left(1 - \frac{1+3\nu}{3+\nu}\frac{1}{4\cos^2\theta}\right)\sin^2\theta$$

$$\sigma_n = \frac{3+\nu}{8}\rho\omega^2 b^2\left(1 - \frac{1}{4} - \frac{1+3\nu}{3+\nu}\frac{\sin^2\theta}{4\cos^2\theta}\right)$$

$$\frac{\partial\sigma_n}{\partial\theta} = 0 \Rightarrow \theta = 0^\circ$$

$$\sigma_r^{max} = \frac{3+\nu}{8} \rho \omega^2 b^2 \left(1 - \frac{1}{4}\right) = N$$

$$\omega_{max} = \sqrt{\frac{32 N}{3(3+\nu)\rho b^2}}$$

Problem 3

Force exerted by each blade

$$= MR\omega^2$$

$$= 0.4 \times 0.43 \times \left(\frac{10000 \times 2\pi}{60}\right)^2$$

$$= 189 \text{ kN}$$

σ_{rr} at $r=b$

$$= \frac{50 \times 189 \times 10^3}{\pi \times 800 \times 50} = 75 \text{ MPa}$$

$$\sigma_{rr} = C_1 + \cancel{\frac{C_2}{r^2}} - \left(\frac{3+\nu}{8}\right)\rho\omega^2 r^2$$

$$\sigma_{\theta\theta} = C_1 - \cancel{\frac{C_2}{r^2}} - \left(\frac{1+3\nu}{8}\right)\rho\omega^2 r^2$$

Finite stresses as $r \to 0 \Rightarrow C_2 = 0$

$$\frac{\rho\omega^2}{8} = 1.055 \times 10^9 \text{ N/m}^4$$

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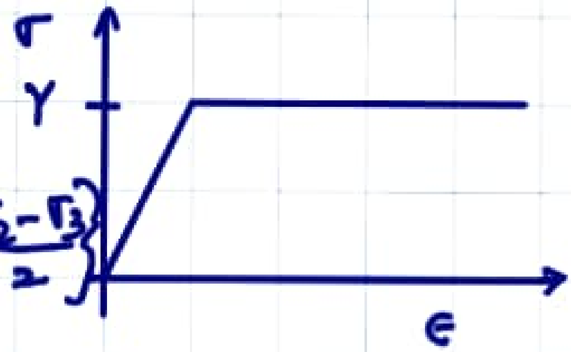
$\sigma_{rr}(0.4) = 75 \text{ MPa}$

$\Rightarrow \quad C_1 = 632 \text{ MPa}$

$\sigma_{max} = 632 \text{ MPa} = C_1$

Problem 4

$$\tau_{max} = \max\left\{ \frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_2-\sigma_3|}{2}, \frac{|\sigma_2-\sigma_3|}{2} \right\}$$

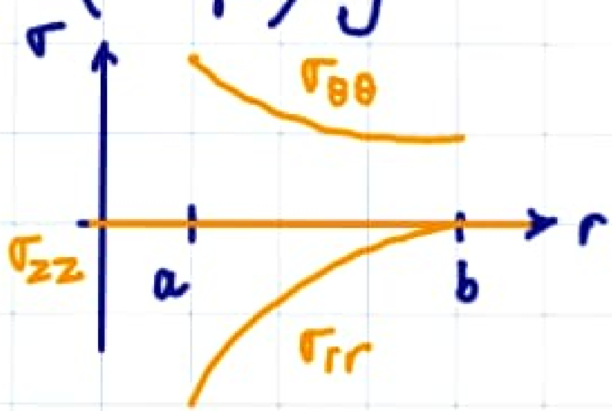


$$\begin{pmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \underset{\substack{\parallel \\ \sigma_{zz}}}{0} \end{pmatrix}$$

$\sigma_1 = \sigma_{rr}$, $\sigma_2 = \sigma_{\theta\theta}$,
$\sigma_3 = 0$

Internal pressure p

$$\left.\begin{aligned} \sigma_{rr} &= \frac{pa^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right) \\ \sigma_{\theta\theta} &= \frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right) \end{aligned}\right\} \text{Elastic solutions.}$$



$$\frac{\sigma_{\theta\theta}-\sigma_{rr}}{2} = \frac{Y}{2}$$

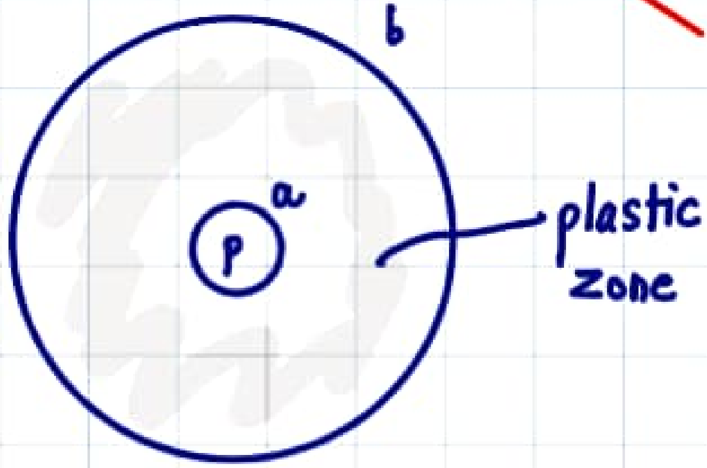
$$\frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{a^2}-1+\frac{b^2}{a^2}\right)=Y$$

$$P_0=\frac{Y(b^2-a^2)}{2b^2}$$ onset of plastic def.

Entire disk plastically yielded,

$r=b$ $$\frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{b^2}-1+\frac{b^2}{b^2}\right)=Y$$

$$P_L=\frac{Y(b^2-a^2)}{2a^2}$$ limit load/ pressure



Inside plastic zone,

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad \text{eqm}$$

$$\sigma_{\theta\theta} - \sigma_{rr} = Y$$

$$\frac{d\sigma_{rr}}{dr} = \frac{Y}{r} \Rightarrow \sigma_{rr} = Y \ln r + c$$

$$\sigma_{rr}(b) = 0 \Rightarrow c = -Y \ln b$$

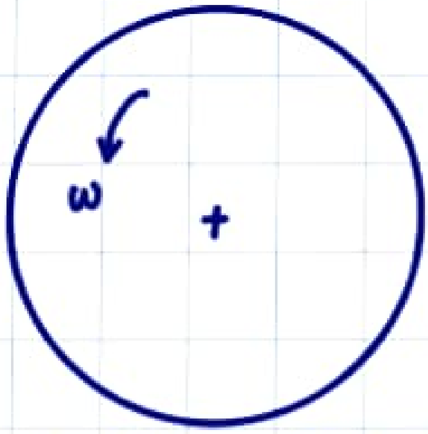
$$\sigma_{rr}(a) = -P_L$$

$$-P_L = Y \ln a - Y \ln b$$

$$\Rightarrow P_L = Y \ln \frac{b}{a}$$

limit pressure for full plastic deformation.

Problem 5



ω_0 speed at which plastic def. just starts

ω_L ——————"—— entire disk is plastically deformed

$$\omega_0 = \sqrt{\frac{8Y}{(3+\nu)\rho b^2}}$$

$$\omega_L = \sqrt{\frac{3Y}{\rho b^2}}$$

Homework

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