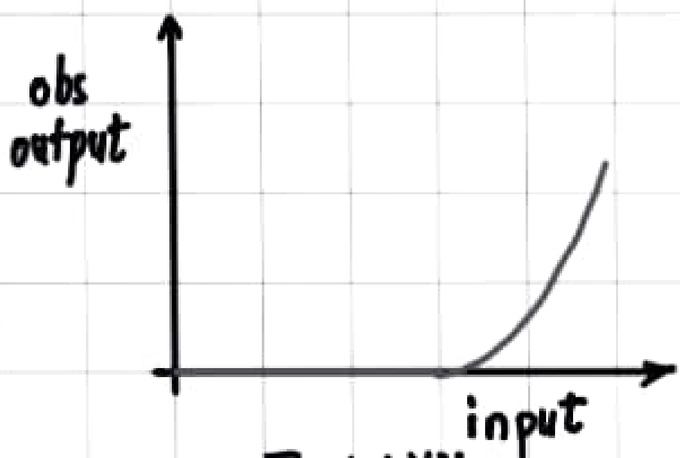
ME 202

LECTURE 30

MON 28 MAR 2022

BUCKLING



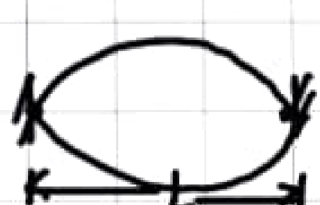
Instability

Columns

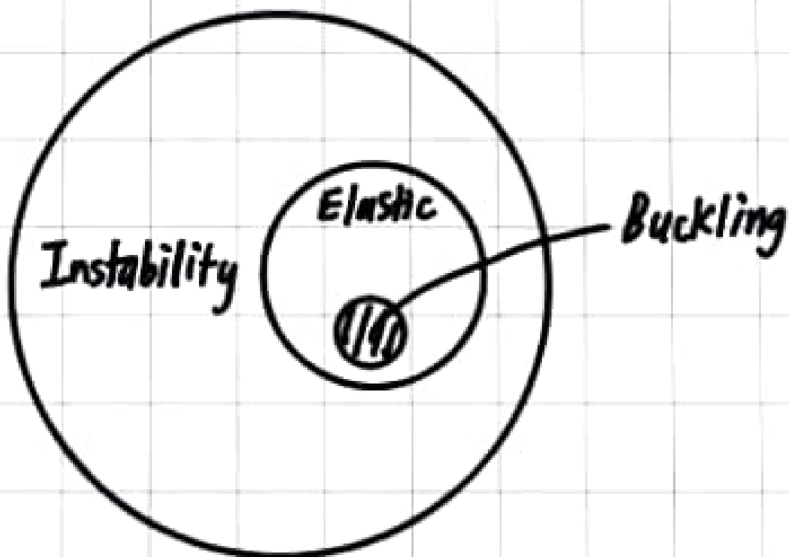
Tower

Pull-in instability

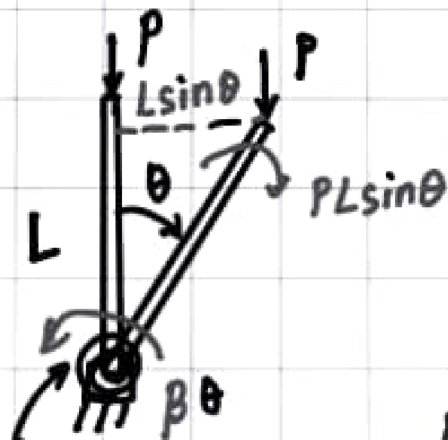
Snap through.



Whirling



1 DOF problem



rigid rod

torsion spring β

Nm/rad

$\theta = 0$ is an eqm configuration.

$$\Sigma M = 0$$

$$\beta\theta = PL\sin\theta$$

$$p = \frac{PL}{\beta}$$

$$\theta = p\sin\theta$$

$$\frac{\theta}{p}, \sin\theta$$

For small angles, $\sin\theta \approx \theta$

DNYANESH PAWASKAR

$\theta = p\,\theta \qquad \theta = 0$

θ indeterminate $\Leftarrow$ $p=1$ $\Rightarrow$ another eqm position exists

$P = \dfrac{\beta}{L}$ critical/buckling load.

Indeterminate angular disp is an artefact of the linearization.

$\theta = p\,\theta, \qquad \dfrac{\beta}{L}\,\theta = P\,\theta$

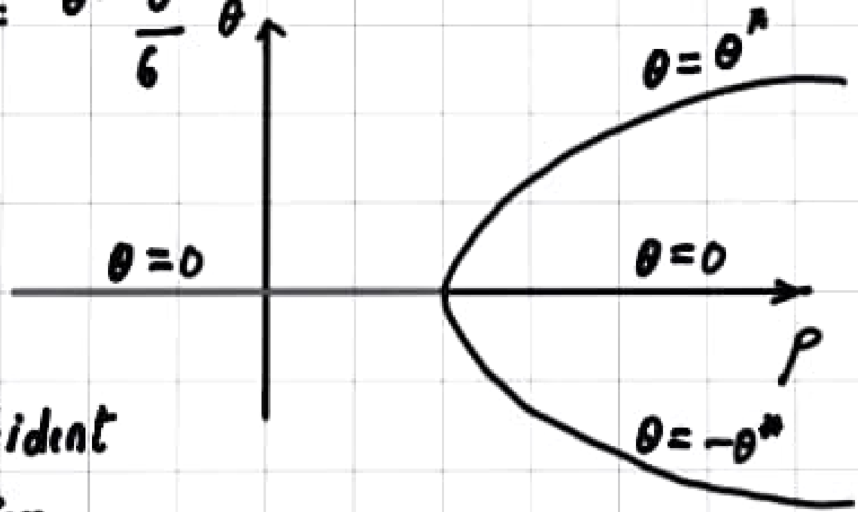
1D eigenvalue problem. $\underset{\sim}{A}\,\underline{v} = \lambda\,\underline{v}$

Buckling load is the critical load at which a system in stat. eqm has non-trivial solutions (in addition to the trivial/zero solution).

$$\sin\theta \approx \theta - \frac{\theta^3}{6}$$

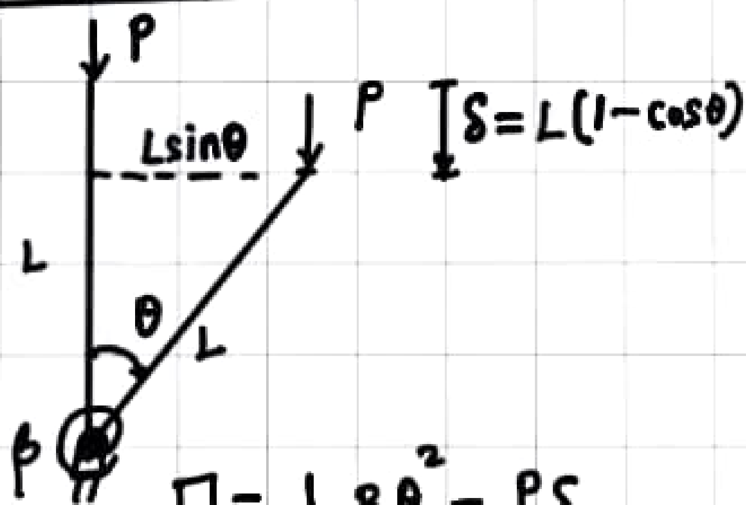
$$\frac{\theta}{P} = \theta - \frac{\theta^3}{6}$$

Bifurcation Diagram



Pitchfork/Trident Bifurcation.

Potential Energy Approach



$$\Pi = \frac{1}{2}\beta\theta^2 - P\delta$$

$$\Pi(\theta) = \frac{1}{2}\beta\theta^2 - PL(1-\cos\theta)$$

$$= \frac{1}{2}\beta\theta^2 - p\frac{\beta}{\cancel{L}}\cancel{L}(1-\cos\theta)$$

$$= \beta\left[\frac{1}{2}\theta^2 - p(1-\cos\theta)\right]$$

$p<1$ stable eqm at $\theta=0$

$p=1$ critical transition

$p>1$ unstable eqm at $\theta=0$

+

2 stable equilibria

$$\frac{\partial \Pi}{\partial \theta} = \theta - p\sin\theta$$

$$\frac{\partial^2 \Pi}{\partial \theta^2} = 1 - p\cos\theta$$

$$\left.\frac{\partial^2 \Pi}{\partial \theta^2}\right|_{\theta=0} = 1-p$$

DNYANESH PAWASKAR

Buckling load is the critical load at which equilibrium changes from stable to unstable.