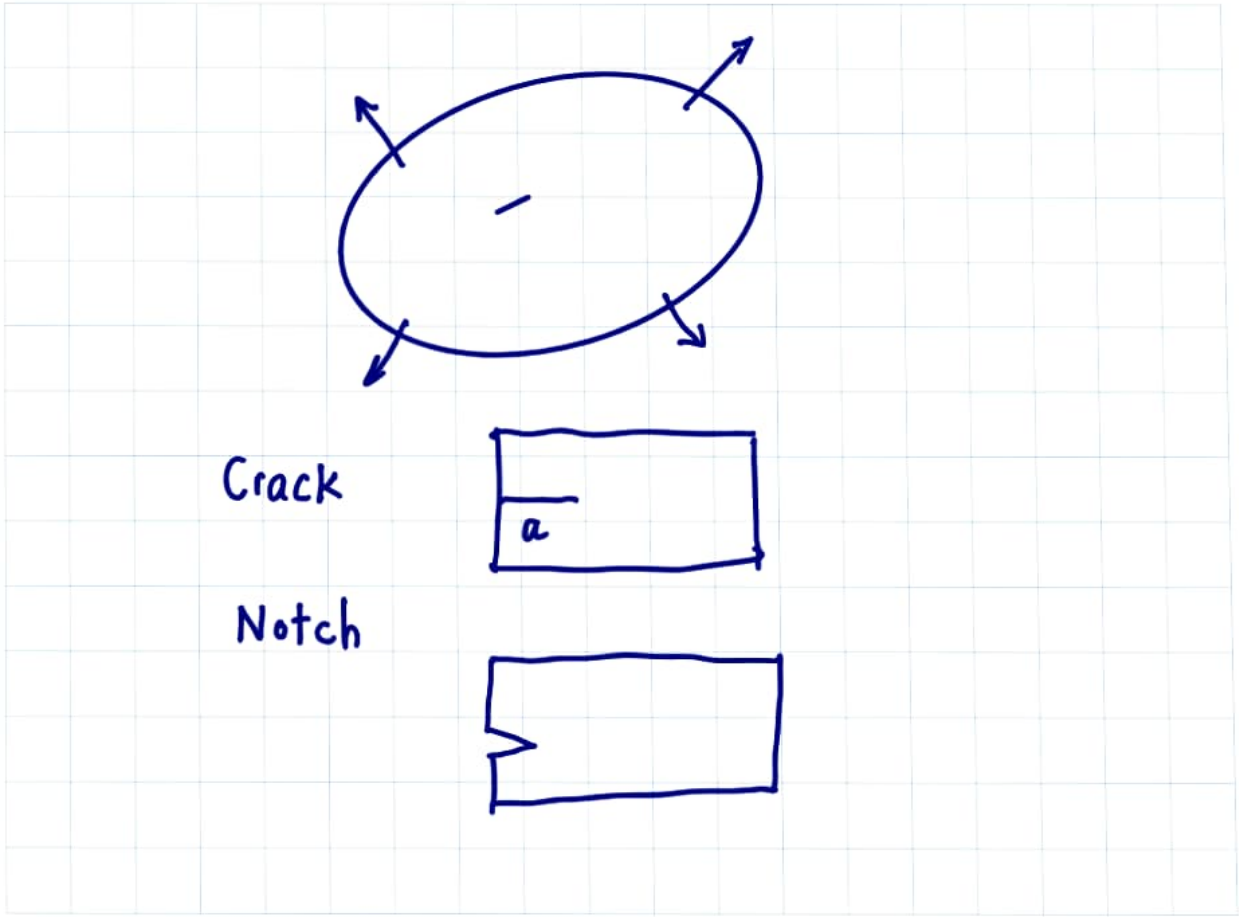
ME 202
LECTURE 25
TUE 15 MAR 2022

PMPE Fracture Mechanics

Assume pre-existing crack.

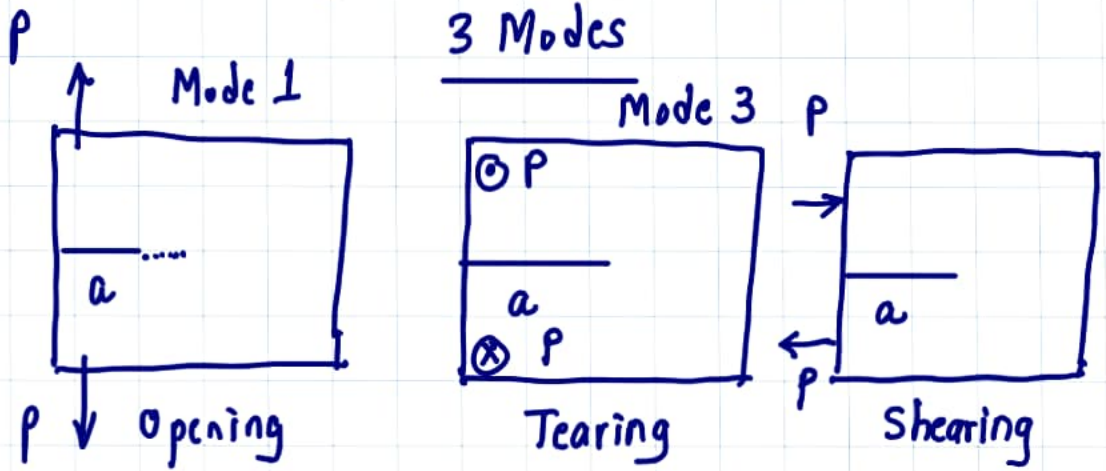
DNYANESH PAWASKAR


Crack
a
Notch

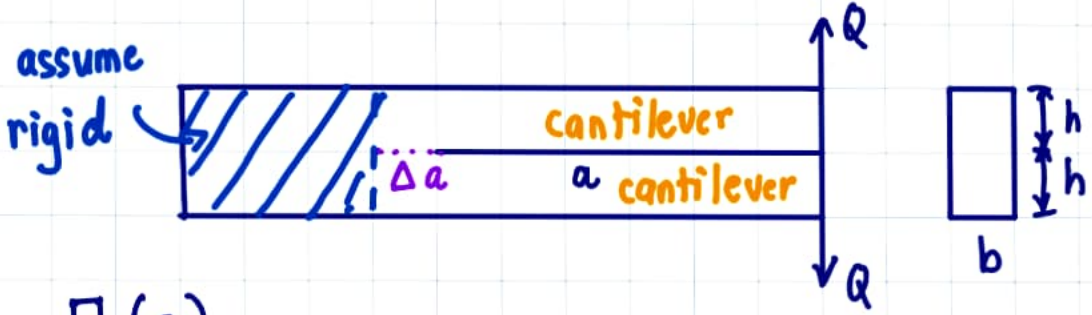
SE → Surface Energy

Linear Elastic Fracture Mechanics

γ J/m^2



Goal : Calculate load at which crack of length a grows.



$\Pi(a)$

Treat a like a configuration parameter.

if $\Pi(a+\Delta a) < \Pi(a)$ system "choose" the new crack length $a+\Delta a$.

PE of cantilever with end force

$$\Pi = \int_0^a \frac{EI}{2} u''^2 \, dz - Q\,u(a)$$

$$u(z) = \frac{Q}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right), \quad u(a) = \frac{Qa^3}{3EI}$$

$$= -\frac{1}{2} Q\,u(a) = \frac{1}{2}\frac{Q^2}{EI}\frac{a^3}{3} - Q\,u(a)$$

$$\Pi = \frac{1}{2}ku^2 - Pu = -\frac{1}{2}Pu$$

In cracked beam,

$$\Pi(a) = -\frac{1}{2}Q \cdot \frac{Qa^3}{3EI} \cdot 2 + 2\gamma ab$$

$$\Pi(a+\Delta a) = -\frac{1}{2}Q \cdot \frac{Q}{3EI}(a+\Delta a)^3 2 + 2\gamma(a+\Delta a)b$$

Crack will grow if

$\Pi(a+\Delta a) \leq \Pi(a)$ for some Δa

PMPE

no matter how small that Δa is.

$\Delta a \to 0$

$$\Pi(a) + \Delta a\, \Pi'(a) + \underbrace{0(\Delta a^2)}_{\to 0} \leq \Pi(a)$$

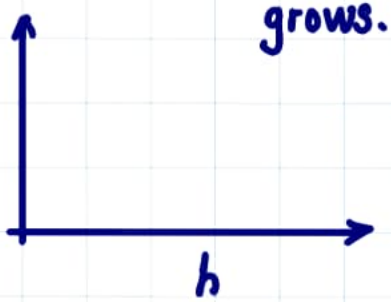
$$\Pi'(a) = 0 \quad \text{or} \quad \frac{d\Pi}{da} = 0$$

$$\frac{d\Pi}{da} = \frac{-Q^2 3a^2}{3E\, bh^3/12} + 2\gamma b = 0$$

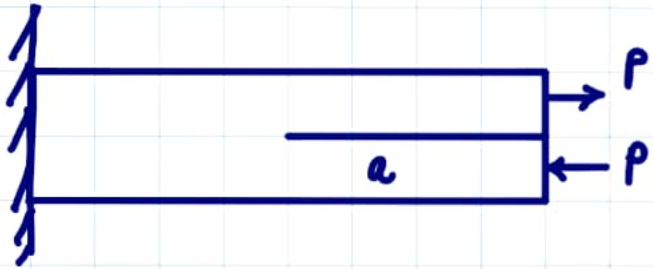
$$Q = \sqrt{\frac{E\gamma b^2 h^3}{6a^2}}$$

Load at which crack grows.

$$Q \propto \frac{bh^{3/2}}{a}$$



Mode 2



$$\Pi(a) = \left(-\frac{1}{2} P \frac{Pa}{AE} + \gamma ab\right) 2 \qquad A = bh$$

$$\frac{d\Pi}{da} = 0 \Rightarrow P = \sqrt{2E\gamma hb^2}$$

Mode 3