MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 4

(February 09, 2022)

(1) Let $f : [0,1] \to \mathbb{R}$ be continuously differentiable and define
$$B_n(x) = \sum_{k=0}^{n} \binom{n}{k} f\left(\frac{k}{n}\right) x^k (1-x)^{n-k}.$$
Prove that $\lim_{n\to\infty} B_n(x) = f(x)$ for each $x \in [0,1]$.

(2) If $f(x) = x^2$ then show that $B_n(x) = \left(\frac{n-1}{n}\right) x^2 + \frac{1}{n}x$.

(3) Use the above $B_n(x)$ to determine n such that $|B_n(x) - x^2| < 10^{-2}$ for all $x \in [0,1]$.

(4) Use Neville's method to approximate $\sqrt{3}$ with $f(x) = 3^x$ and $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$ and $x_4 = 2$. Find the absolute and relative errors.

(5) Use Neville's method to approximate $\sqrt{3}$ with $f(x) = \sqrt{x}$ and $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$ and $x_4 = 5$. Find the absolute and relative errors.

(6) If $P_3(x)$ is the interpolating polynomial for the following data then use Neville's method to find y if $P_3(1.5) = 0$.

x	0	0.5	1	2
$f(x)$	0	y	3	2

(7) Use the forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data and approximate $f(-\frac{1}{3})$.

x	-0.75	-0.5	-0.25	0
$f(x)$	-0.07181250	-0.02475000	0.33493750	1.10100000

(8) Use the backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data and approximate $f(0.25)$.

x	0.1	0.2	0.3	0.4
$f(x)$	-0.62049958	-0.28398668	0.00660095	0.24842440

(9) A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$,$\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.