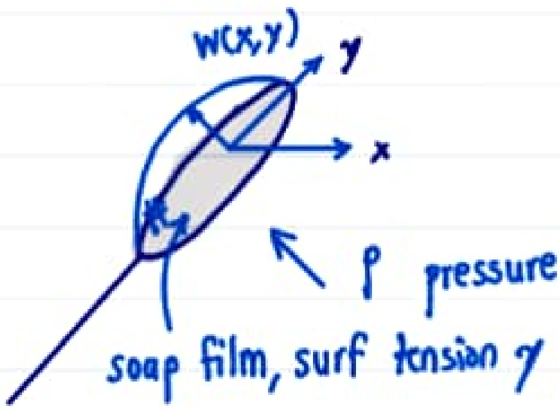
ME 202

LECTURE 9

THU 20 JAN 2022

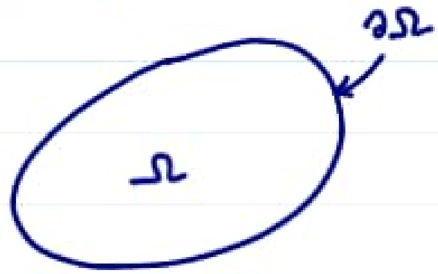
## SOAP BUBBLES ☺

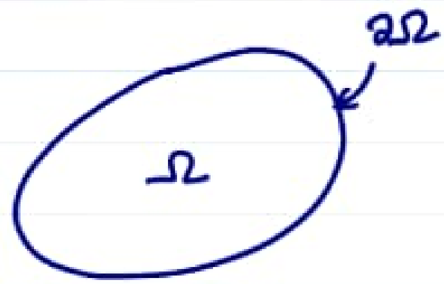




$w(x,y)$ shape of soap bubble

<u>Assume small displacements</u>

| TORSION EQN | MEMBRANE EQN |
| --- | --- |
| $\varphi(x,y)$ Prandtl stress func. | $w(x,y)$ z-disp of film/membrane |
| $\nabla^2\varphi = -2G\alpha$ in $\Omega$ | $\nabla^2 w = -p/\gamma$ in $\Omega$ |
| $G$ shear modulus $N/m^2$ | $p$ pressure $N/m^2$ |
| $\alpha$ intensity of twist rad/m | $\gamma$ surf tension $N/m \equiv J/m^2$ |
| $\varphi = 0$ on $\partial\Omega$ | $w = 0$ on $\partial\Omega$ |
| $T = 2\int_\Omega \varphi \, dx\,dy$ | $V = \int_\Omega w \, dx\,dy$ |
| $T$ torque N-m | $V$ enclosed volume |

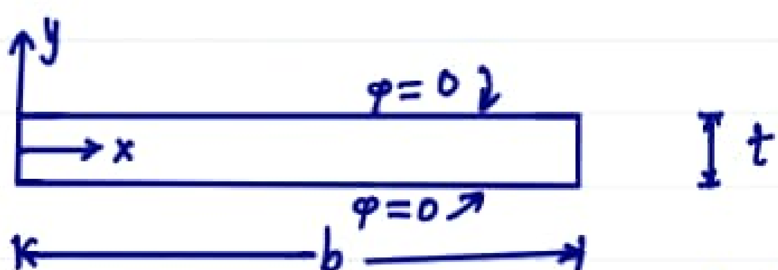



Use this analogy to visualize $\varphi$.

<u>Utility</u>

thin rect c/s of shaft




$b \gg t$

$\nabla^2\varphi = \partial^2\varphi + \partial^2\varphi = -2G\alpha$ , $\varphi$

$\partial x^2 \quad \partial y^2$

ignored

$$\frac{\partial \varphi}{\partial y} = -2G\alpha y + C_1, \quad \varphi = -2G\alpha\frac{y^2}{2} + C_1 y + C_2$$

$\varphi = 0$ when $y = \pm t/2$

$$\varphi = G\alpha\left(\frac{t^2}{4} - y^2\right)$$

$$\text{Torque} = 2\int_\Omega \varphi \, dx\,dy$$

$$= 2G\alpha \int_0^b dx \int_{-t/2}^{t/2} \left(\frac{t^2}{4} - y^2\right) dy$$

$$T = \frac{G\alpha b t^3}{3} \qquad \frac{N}{m^2}\,\frac{rad}{m}\,mm^3 \equiv Nm \checkmark$$

$$K_t = \frac{Gbt^3}{3}$$

Expt

$b/t$

1

5

10

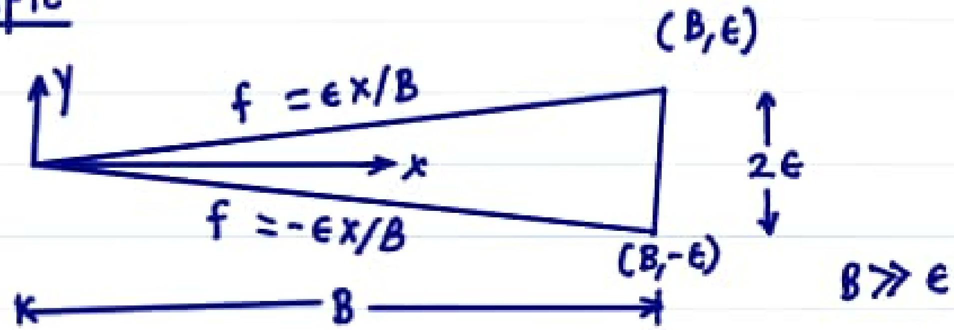
...

$$\tau_{xz} = \frac{\partial \varphi}{\partial y} = -2G\alpha y$$

$$\tau_{yz} = -\frac{\partial \varphi}{\partial x} = 0$$

$$\tau_{max} = \sqrt{\tau_{xz}^2 + \tau_{yz}^2} = G\alpha t = \frac{3T}{bt^2}$$

## Example




$$\underbrace{\cancel{\frac{\partial^2\varphi}{\partial x^2}}}_{\approx 0} + \frac{\partial^2\varphi}{\partial y^2} = -2G\alpha , \quad \varphi = 0 \text{ on } f = \pm\frac{\epsilon x}{B}$$

$$\varphi = G\alpha(f^2 - y^2)$$

$$T = -2G\alpha \int_0^B dx \int_{-f}^{f} dy \, (y^2 - f^2)$$

$$T = \frac{2}{3} G\alpha\epsilon^3 B$$

$$\varphi(x,y) = K\left(y^2 - \frac{\epsilon^2 x^2}{B^2}\right) \cancel{(x-B)}$$

$$\frac{\partial\varphi}{\partial y} = K 2y, \quad \frac{\partial^2\varphi}{\partial y^2} = 2K$$

$$\frac{\partial\varphi}{\partial x} = -\frac{K\epsilon^2 2x}{B^2}, \quad \frac{\partial^2\varphi}{\partial x^2} = -\frac{2K\epsilon^2}{B^2}$$

$$\left|\frac{\partial^2\varphi}{\partial x^2}\right| << \left|\frac{\partial^2\varphi}{\partial y^2}\right| \qquad \frac{\epsilon^2}{B^2} << 1$$

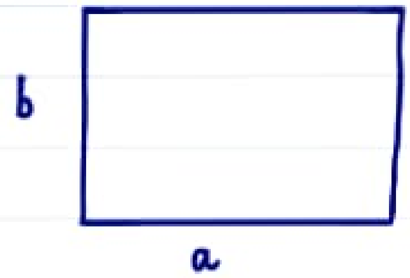
$$\nabla^2\varphi \approx \frac{\partial^2\varphi}{\partial y^2} = 2K = -2G\alpha$$

$$\Rightarrow K = -G\alpha$$

$$\varphi = -G\alpha\left(y^2 - \frac{\epsilon^2 x^2}{B^2}\right)$$

For sections

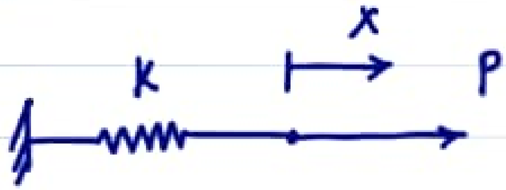
need approx $K_t$




$b \not\gg a$

- ☐ Fourier series
- ☐ Principal of Min Potential Energy PMPE

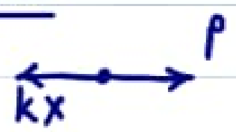
<u>PMPE</u>




Find x

<u>Force Balance</u>

$\Sigma F = 0$




$$P - kx = 0$$

$$\Rightarrow x = \frac{P}{k}$$

<u>PMPE</u>

$$\Pi = \Pi_s + \Pi_P$$

$$\Pi(x) = \frac{1}{2}kx^2 + (-Px)$$

Find x that minimizes $\Pi$.

$$\frac{d\Pi}{dx} = 0 \Rightarrow kx - P = 0$$

$$\Rightarrow x = P/k.$$

# Force and Potential Energy


$$F = ma = mv\frac{dv}{dx}$$

$$\int_1^2 F\,dx = \int_1^2 mv\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= KE_2 - KE_1$$

want

$$KE_1 + PE_1 = KE_2 + PE_2$$

$\Pi \equiv PE$/ Potential

F should take this form $F = -\frac{d\Pi}{dx}$

"conservative"

| $\Pi$ | Force |
|---|---|
| $-Px$ | $P$ |
| $\frac{1}{2}kx^2$ | $-kx$ |
| $mgx$ | $-mg$ |