ME 202

LECTURE 18

TUE 15 FEB 2022

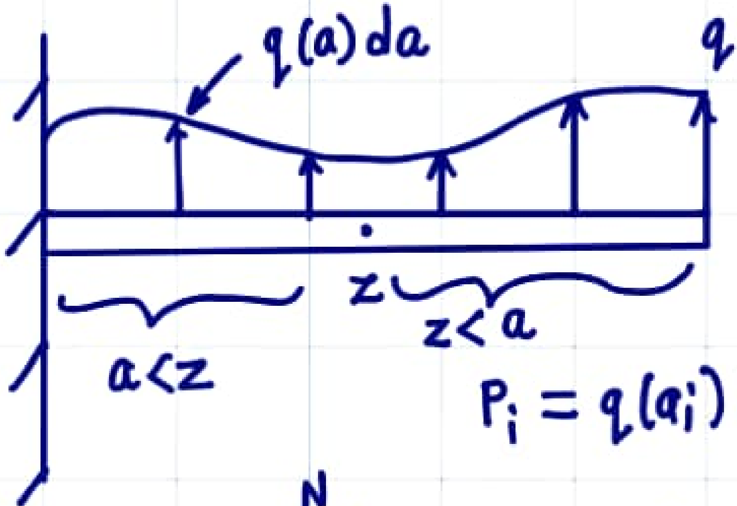
D N PAWASKAR

Recall, impulse response func.

$G(z,a)$ for cantilever

Deflection @ z due to unit point force @ a

$$G(z,a) = G(a,z)$$



$$P_i = q(a_i)\,\Delta a_i$$

$$u(z) = \sum_{i=1}^{N} P_i \, G(z,a_i)$$

$$u(z) = \int_0^L q(a)\,da \; G(z,a)$$

note

Std trick in linear systems.

$$u(z) = \int_0^z q(a)\frac{1}{EI}\left(\frac{za^2}{2} - \frac{a^3}{6}\right)da$$

$$+ \int_z^L \frac{q(a)}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right)da$$

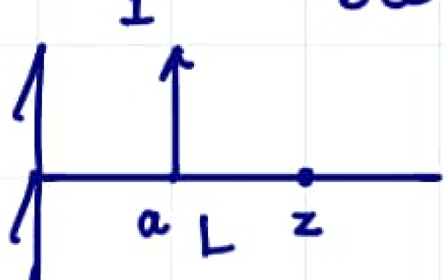
Example 1 $\quad q = q_0$

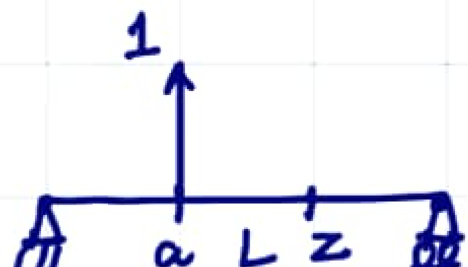
check

$$u(z) = \frac{q_0 z^2}{24EI}\left(6L^2 - 4zL + z^2\right)$$

Note: 1. Use correct $G(z,a)$ in each integration

2. $G(z,a)$ will depend on BCs

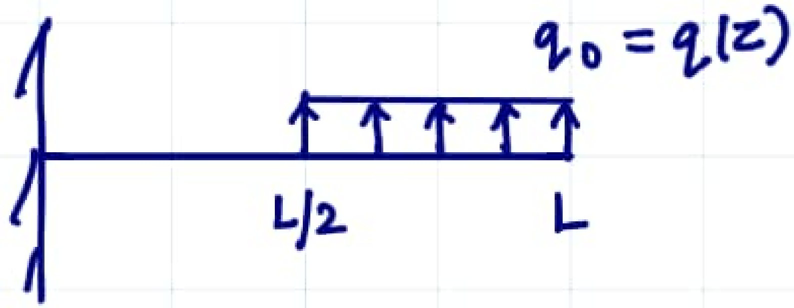




$$G(z,a) \neq G(z,a)$$

3. Use $q(a)$ in integration.

Example 2



$u(L)$ want

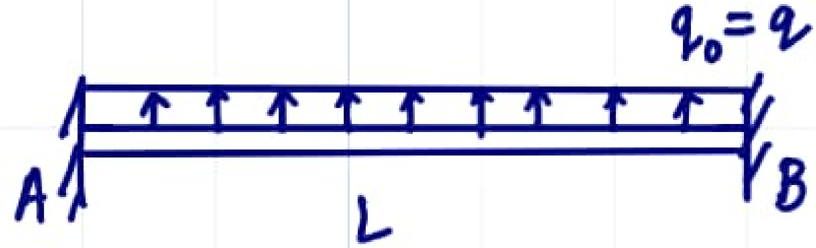
$$u(L) = \int_0^{L/2} G(L,a)\,\cancelto{0}{q(a)}\,da + \int_{L/2}^{L} G(L,a)\,q(a)\,da$$

$$= \int_{L/2}^{L} \frac{1}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) q_0 \, da \qquad a \leq L$$

$$= \frac{41}{384} \frac{q_0 L^4}{EI}$$

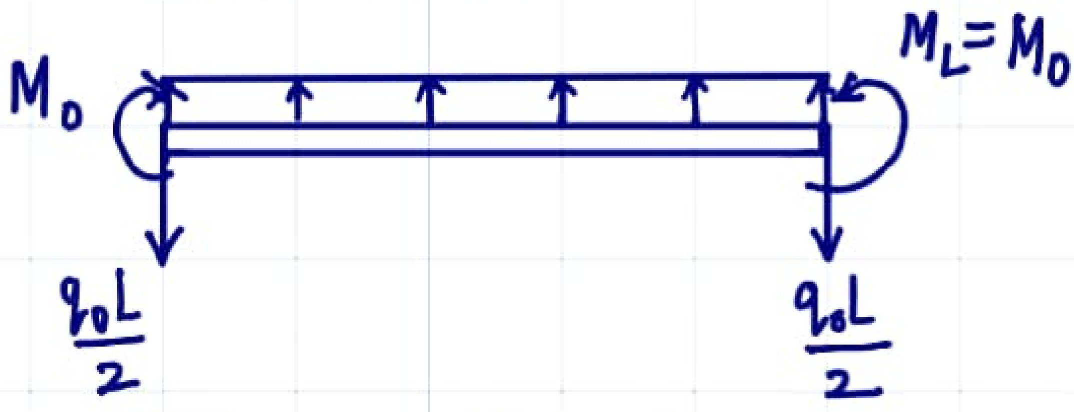
Fixed-fixed Beam

Consider this,



Goal $u(z)$, $u(L/2)$

$$M(z) = EI\,u''$$

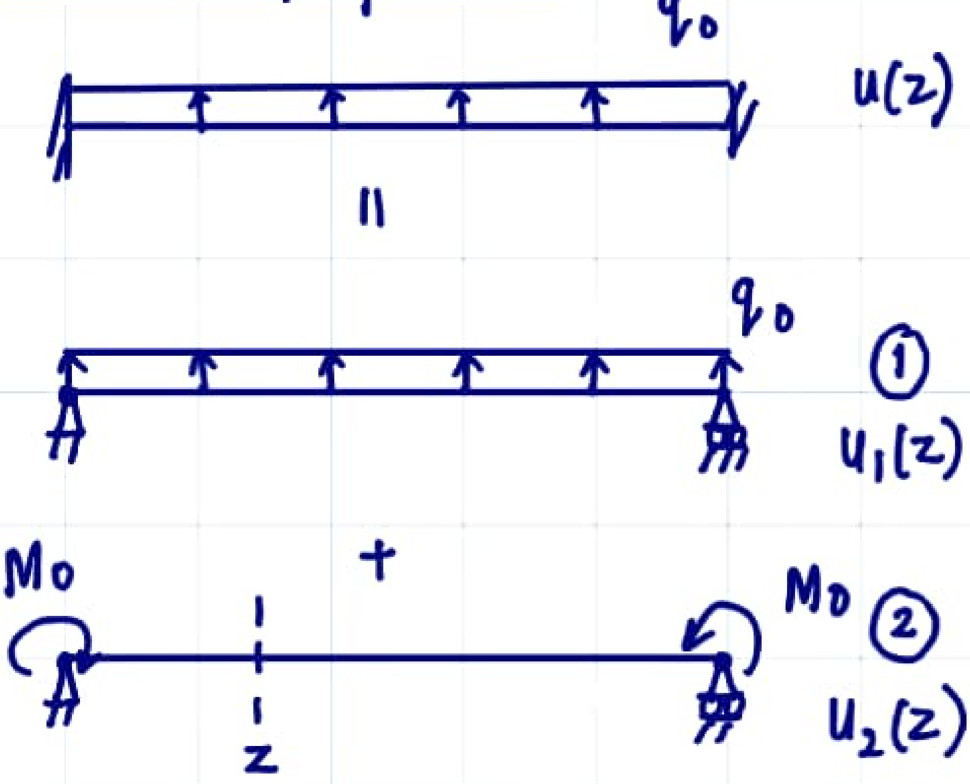


Take moments @ A

$$-M_0 + M_L - \frac{q_0 L}{2} L + \frac{q_0 L}{2} L = 0$$

Statically indeterminate.

Linear Superposition



① solved earlier $u_1(z)$ ✓

In ②, $M(z) = M_0$

$$EI\, u_2'' = M_0$$

$$u_2 = \frac{M_0}{EI}\left(\frac{z^2}{2} + c_1 z + \cancelto{0}{c_2}\right)$$

$$u_2(0) = 0, \quad u_2(L) = 0$$

$$u_2(z) = \frac{M_0}{2EI}\, z(z-L)$$

$$u(z) = u_1(z) + u_2(z)$$

M_0 as yet unknown

Use BCs $u'(0) = 0$

$$\frac{q_0}{2EI}\left(\frac{L^3}{12} - \frac{M_0 L}{q_0}\right) = 0$$

$$M_0 = \frac{q_0 L^2}{12}$$

check $u'(L) = 0$ identically

Take M_0, plug into $u_2(z)$

$$u(z) = u_1(z) + u_2(z)$$

$$u(z) = \frac{q_0}{24EI} z^2 (z-L)^2$$

$$u_{max} = u\left(\frac{L}{2}\right) = \frac{1}{384} \frac{q_0 L^4}{EI}$$

$$M_{max} =$$