ME 202 S3
Tutorial 10

Thu 07 Apr 2022

Problem 1

- Consider a simply supported beam of length L and flexural rigidity EI being buckled by an axial load P. Your goal is to double the first buckling load to $2\pi^2 EI/L^2$ by introducing two rotational springs one at each end. Find the minimum value of the rotational spring stiffness to achieve the stated goal.

A. Use the second order differential equation approach to solve this problem.

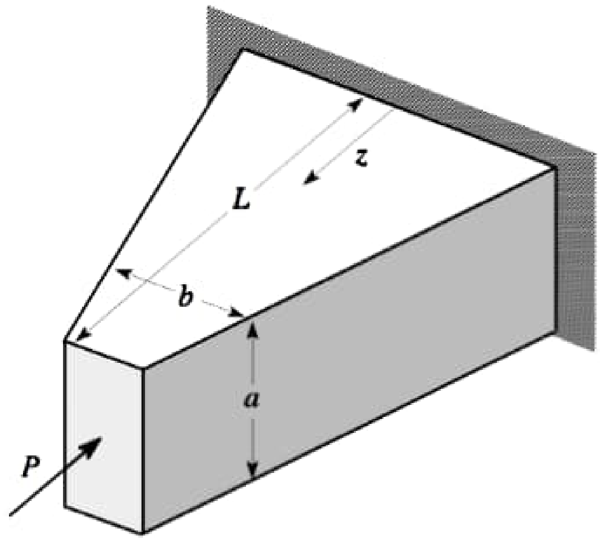
B. Use the potential energy approach to solve this problem using a single DOF approximation for the deflected shape.

Problem 2

- Consider a simply supported beam of length L and flexural rigidity EI being buckled by an eccentrically applied axial load P. Use the second order differential equation approach to show that as P approaches the critical buckling load, the deflection of the beam becomes uncontrollably large.

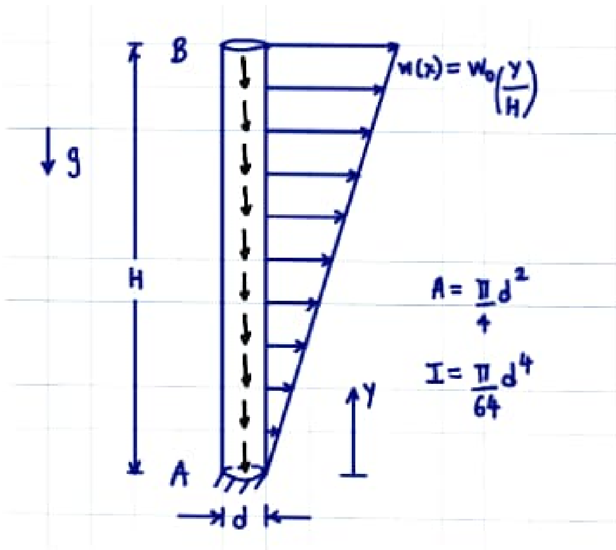
Problem 3

- Consider the column of length $L, L >> a, b$ of variable cross-section with $b = a(3/2 - z/L)$. Use the potential energy method to calculate the approximate first buckling load of the column. Also comment on about which axis the column is likely to buckle first.



Problem 4

- A solid cylindrical tower AB of height H and diameter d (d << H) is shown. The properties of the material are: elastic modulus E, mass density rho. The gravitational acceleration g acts in the direction shown. A linearly distributed wind load per unit length with maximum intensity w_0 acts transversely as shown. Find the approximate maximum height H_max of the tower that can be built without causing the tower to buckle under its own weight i.e. find the height H_max for which the tower becomes unstable.
- Assume that the tower is fixed into the ground at A. Make and justify physically reasonable assumptions about the deflected shape of the tower. Note that the self-weight of the tower acts in a distributed manner throughout the axis of the tower as indicated by the black arrows.



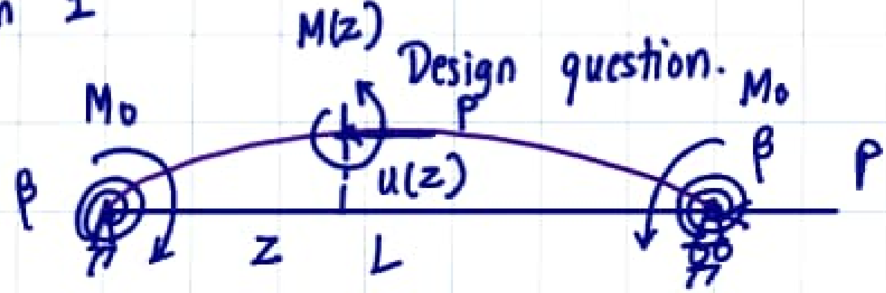
ME 202

LECTURE 34 TUTORIAL 10

THU 07 APR 2022

Problem 1

Design question.



Find β s.t. $P^* = \dfrac{2\pi^2 EI}{L^2}$

$$M_0 = \beta u'(0), \quad M_0 = -\beta u'(L)$$

M_0 resistive moment due to spring

$$M(z) + Pu - M_0 = 0$$

$$EIu'' + Pu = M_0$$

$$u = \frac{M_0}{P} + A\cos\lambda z + B\sin\lambda z, \quad \lambda^2 = \frac{P}{EI}$$

$$u(0) = 0 \Rightarrow \frac{M_0}{P} + A = 0, \quad A = -\frac{M_0}{P}$$

$$u(L) = 0 \Rightarrow \frac{M_0}{P} + A\cos\lambda L + B\sin\lambda L = 0$$

$$u'(0) = \frac{M_0}{\beta},$$

$$B = -\frac{M_0}{P}\frac{(1-\cos\lambda L)}{\sin\lambda L} = \frac{M_0}{\lambda\beta}$$

want $P = \dfrac{2\pi^2 EI}{L^2}$

$$\Rightarrow \lambda L = \pi\sqrt{2} = 4.4429$$

$$\frac{\beta L}{EI} = 3.3820$$

$$\beta = 3.3820\,\frac{EI}{L}.$$

Alt,

$$-A\lambda\sin\lambda L + B\lambda\cos\lambda L = -\frac{M_0}{\beta}$$

$$u'(L) = -\frac{M_0}{\beta}$$

Elastic Instability, L.S. Srinath Adv. Mech. of Solids

Energy approach,

$$\Pi = \int_0^L \left(\frac{EI}{2} u''^2 - \frac{P}{2} u'^2\right) dz + \frac{1}{2}\beta(u'(0))^2 + \frac{1}{2}\beta(u'(L))^2$$

Assume $u = a \sin\frac{\pi z}{L}$

$$\Pi(a) = \frac{a^2\pi^2}{4L^3}\left(-PL^2 + EI\pi^2\right) + \frac{\beta a^2 \pi^2}{L^2}$$

$\frac{d\Pi}{da} = 0, \quad a \neq 0$

$$P = \frac{EI\pi^2 + 4\beta L}{L^2} \underset{\text{given}}{=} \frac{2\pi^2 EI}{L^2}$$

$$\beta = \frac{\pi^2 EI}{L^2}$$

$$= 2.4674 \frac{EI}{L^2}$$

2.

$$u = A\cos\lambda z + B\sin\lambda z + C + D$$

$u(0)=0, \quad u(L)=0, \quad u''(0)=0, \; u''(L)=\dfrac{Pe}{EI}$

ASymmetry $\Rightarrow$ Deflection determinate.

$$A = \frac{-Pe}{EI\lambda^2 \sin\lambda L}, \quad B=0, \; D=0$$

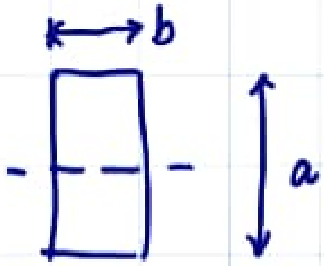
$$C = \frac{Pe}{\lambda^2 L EI}$$

$$u(z) = e\left(\frac{z}{L} - \frac{\sin\lambda z}{\sin\lambda L}\right)$$

As $\lambda L \to \pi$, $u \to \infty$

$$P \to \frac{\pi^2 EI}{L^2}$$

3. $I = \dfrac{ba^3}{12}$ case 1



$$I = \frac{a^4}{12}\left(\frac{3}{2} - \frac{z}{L}\right)$$

$$\Pi = \int_0^L \left(\frac{EI}{2} u''^2 - \frac{P}{2} u'^2\right) dz$$

Assume $u = Cz^2$

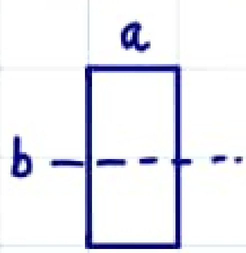
$$\frac{Ea^4 c^2 L}{6} - \frac{2}{3} Pc^2 L^3 = \Pi$$

$$\frac{d\Pi}{dc} = 0, \qquad P_1^* = \frac{Ea^4}{4L^2}$$

Case 2 $\quad I = \dfrac{ab^3}{12} = \dfrac{a^4}{12}\left(\dfrac{3}{2} - \dfrac{z}{L}\right)^3$

Assume $u = Cz^2$

$$\frac{d\Pi}{dc} = 0 \Rightarrow \quad P_2^* = \frac{5Ea^4}{16L^2}$$



$P_1^* < P_2^*$

4. $H \to H_{max}$, $u \to \infty$ asymmetric load

$$\Pi = \int_0^H \left(\frac{EI}{2} u''^2 - wu\right) dz - \int_0^H \frac{P}{2} u'^2 dz$$

$$m = \rho A$$

$$P = mg(H-z)$$

Assume $u = Cz^2$

$$\Pi = 2C^2 EI\,H - w_0 \frac{CH^3}{4} - \frac{C^2 H^4 mg}{6}$$

$$\frac{d\Pi}{dC} = 0$$

$$\left(4EIH - \frac{H^4 mg}{3}\right) C = \frac{w_0 H^3}{4}$$

$$H \to H_{max} = \left(\frac{12EI}{\rho A g}\right)^{1/3} = \left(\frac{3d^2 E}{4\rho g}\right)^{1/3}$$

Alt,

$$\int_0^H \left(\frac{EI}{2} u''^2 - wu\right) dz - \int_0^H mg\,dz \int_0^z \frac{1}{2} u'^2 \, d\xi \,, \quad u = C\xi^2$$

$$= \Pi$$