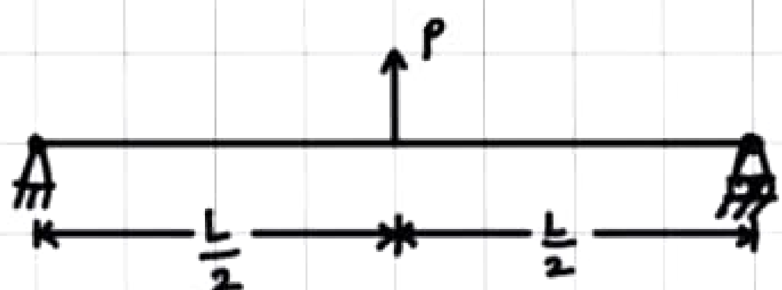
ME 202

LECTURE 26 TUTORIAL 8

THU 17 MAR 2022

1.




$$\Pi(u) = \int_0^L \frac{EI}{2} u''^2 \, dz - P\, u\left(\frac{L}{2}\right)$$

$$u(z) = a_1 \sin \frac{\pi z}{L}$$

$$u''(z) = -a_1 \left(\frac{\pi}{L}\right)^2 \sin \frac{\pi z}{L}$$

$$\Pi(a_1) = \frac{EIL}{4} a_1^2 \left(\frac{\pi}{L}\right)^4 - P a_1$$

$$\Pi'(a_1) = 0 \Rightarrow a_1 = \frac{2PL^3}{\pi^4 EI}$$

$$u(z) = \frac{2PL^3}{\pi^4 EI} \sin\frac{\pi z}{L} \qquad \text{approx.}$$

$$u(z) = \frac{Pz}{48EI}(3L^2 - 4z^2) \qquad \text{exact}$$

$$0 \leq z \leq \frac{L}{2}$$

Approx $\quad u\left(\frac{L}{2}\right) = 0.02053 \dfrac{PL^3}{EI}$

Exact $\quad 0.02083 \dfrac{PL^3}{EI}$

Approx $\quad u''\left(\frac{L}{2}\right) = 0.2026 \; PL$

Exact $\quad 0.25 \; PL$

$$u(z) = a_1 \sin\frac{\pi z}{L} + a_3 \sin\frac{3\pi z}{L} + a_5 \sin\frac{5\pi z}{L}$$

$N = 3$

Orthogonality of sines $\displaystyle\int_0^L \sin\frac{m\pi z}{L}\sin\frac{n\pi z}{L}\,dz$

$$= \frac{L}{2}\,\delta_{mn}$$

$$\Pi = \frac{EIL}{4}\left[a_1^2\left(\frac{\pi}{L}\right)^4 + a_3^2\left(\frac{3\pi}{L}\right)^4 + a_5^2\left(\frac{5\pi}{L}\right)^4\right]$$

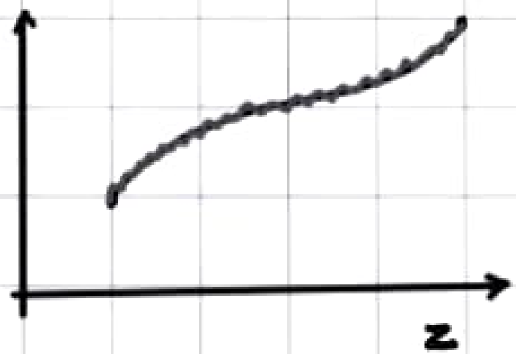
$$-P(a_1 - a_3 + a_5)$$

$$\frac{\partial\Pi}{\partial a_1} = 0,\quad \frac{\partial\Pi}{\partial a_3} = 0,\quad \frac{\partial\Pi}{\partial a_5} = 0$$

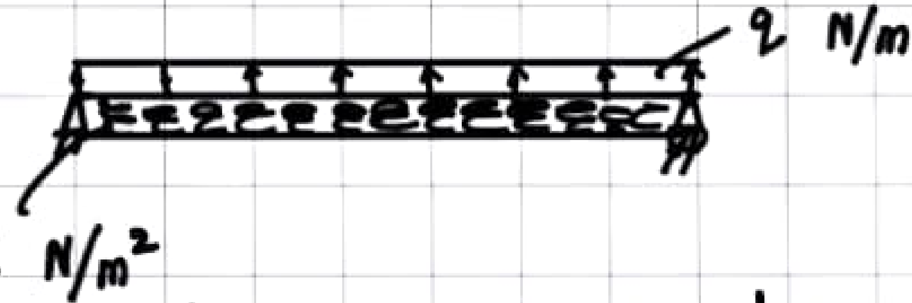
$$a_1 = \frac{2PL^3}{\pi^4 EI},\quad a_3 = \frac{-2PL^3}{81\pi^4 EI},\quad a_5 = \frac{2PL^3}{625\pi^4 EI}$$

| approx → | $u_{max}$ | $M_{max}$ |
|---|---|---|
| 1 term | $0.02055\ PL^3/EI$ | $0.2026\ PL$ |
| 2 terms | $0.02081\ PL^3/EI$ | $0.2252\ PL$ |
| 3 terms | $0.02083\ PL^3/EI$ | $0.2333\ PL$ |

DNYANESH PAWASKAR




2.



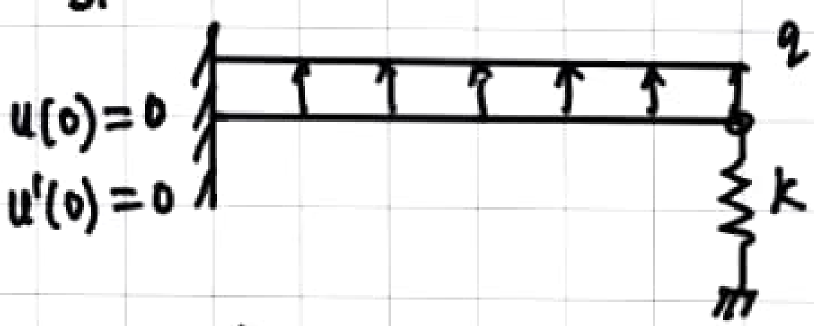

$$\Pi[u] = \int_0^L \frac{EI}{2} u''^2 dz - \int_0^L q u \, dz + \int_0^L \frac{1}{2} k u^2 dz$$

$$u(z) = \sum_{n=1,3,5\ldots}^{\infty} a_n \sin \frac{n\pi z}{L}$$

$$\Pi(a_1, a_3, \ldots) = \sum_{n=1,3,5}^{\infty} \left\{ a_n^2 \left[ \left(\frac{EI}{2}\right)\left(\frac{L}{2}\right)\left(\frac{n\pi}{L}\right)^4 + \frac{kL}{4} \right] - a_n \frac{2qL}{n\pi} \right\}$$

$$\frac{\partial \Pi}{\partial a_n} = 0 \Rightarrow a_n = \frac{4qL^4/(n^5\pi^5 EI)}{1 + kL^4/(n^4\pi^4 EI)}$$

3.




$$\Pi = \int_0^L \left(\frac{EI}{2} u'' - qu\right) dz + \frac{1}{2} k u^2(L)$$

$$u = \cancel{a_0} + \cancel{a_1} z + a_2 z^2 + a_3 z^3$$

physically ok.

$$\Pi(a_2, a_3) = 2a_2^2 EIL + 6EI a_3^2 L^3 + 6EIa_2 a_3 L^2$$

$$+\frac{1}{2}k\left(a_2 L^2 + a_3 L^3\right)^2 - \frac{1}{3}a_2 L^3 q - \frac{1}{4}a_3 L^4 q$$

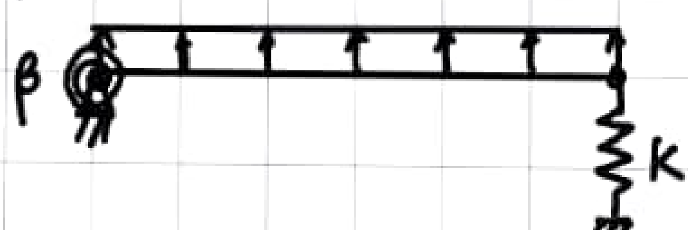
PMPE $\dfrac{\partial \Pi}{\partial a_2} = 0, \quad \dfrac{\partial \Pi}{\partial a_3} = 0$

$\Rightarrow$

$$a_2 = \frac{30EIL^2 q + kL^5 q}{48EI(3EI + kL^3)}$$

$$a_3 = \frac{-L(12EIq + kL^3 q)}{48EI(3EI + kL^3)}$$

If,

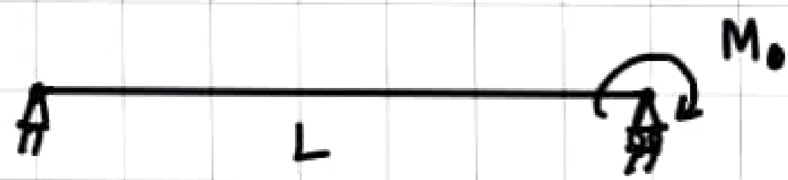



$$u = \cancel{a_0}^{\,0} + a_1 z + a_2 z^2 + a_3 z^3$$

$$u'(0) = a_1 \neq 0$$

$$\Pi = \frac{1}{2}\beta\left(u'(0)\right)^2 + \frac{1}{2}k\left(u(L)\right)^2 + \int_0^L \left(\frac{EI}{2}u''^2 - qu\right)dz$$

4.




KBCS $\quad u(0)=0, \quad u(L)=0$

+

NBC $\quad u''(0) = 0$

$$u = c_0 + c_1 z + c_2 z^2 + c_3 z^3$$

Apply BCs $\qquad c_0 = 0, \; c_2 = 0$

$$c_1 L + c_3 L^3 = 0$$

$$u(z) = c_1 z - \frac{c_1}{L^2} z^3 \qquad N = 1$$

$$\Pi = \int_0^L \frac{EI}{2} u''^2 \, dz + (+M_0 u'(L))$$

$$= \frac{6EIc_1^2}{L} + M_0 c_1 (-2)$$

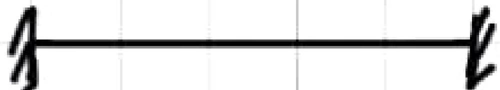
$$\frac{\partial \Pi}{\partial c_1} = 0 \Rightarrow \quad c_1 = \frac{M_0 L}{6EI}$$

$$u(z) = \frac{M_0 L}{6EI}\left(z - \frac{z^3}{L^2}\right)$$

Max $\quad u' = 0 \Rightarrow 3z^2 = L^2 \Rightarrow z = \frac{L}{\sqrt{3}}$

$$u_{max} = \frac{M_0 L^2}{9\sqrt{3}\, EI}$$

5.



$u=0$ $u'=0$ $\qquad\qquad$ $\left.\begin{matrix} u=0 \\ u'=0 \end{matrix}\right\}$ same for $v(z)$

$u(z)$ exact

$v(z)$ approx $\qquad v(z) = u(z) + \varphi(z)$

BCs $\quad \varphi(0)=0, \ \varphi'(0)=0, \ \varphi(L)=0, \ \varphi'(L)=0$

$$\Pi[u] < \Pi[v]$$

$$\Pi[v] = \int_0^L \frac{EI}{2}(u''+\varphi'')^2 dz - \int_0^L q(u+\varphi)\,dz$$

$$= \int_0^L \frac{EI}{2}(u''^2 + 2u''\varphi'' + \varphi''^2)dz - \int_0^L qu\,dz - \int_0^L q\varphi\,dz$$

$$\Pi[v] = \Pi[u] + \underbrace{\int_0^L (EIu''\varphi''dz - q\varphi)}_{I_1} + \underbrace{\int_0^L \frac{EI}{2}\varphi''^2 dz}_{+ve}$$

$u$ exact $\quad EI\,u'''' = q(u)$

$v$ approx $\quad EI\,v'''' \neq q(v)$

$$I_1 = -\int_0^L EI\,u'''\,\varphi'\,dz + \underbrace{\left[EI\,u''\,\varphi'\right]_0^L}_{\substack{0 \\ \text{BCs on } \varphi'}} - \int_0^L q\,\varphi\,dz$$

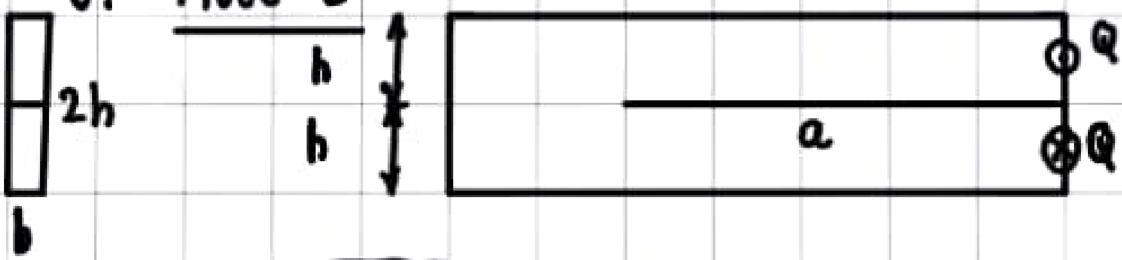
$$\int u\,dv = [uv] - \int v\,du$$

$$= \int_0^L EI\,u''''\,\varphi\,dz + \underbrace{\left[-EI\,u'''\,\varphi\right]_0^L}_{\substack{0 \\ \text{BCs on } \varphi}} - \int_0^L q\,\varphi\,dz$$

$$= \int_0^L (\underbrace{EI\,u'''' - q}_{\substack{0 \quad u \text{ exact}}})\,\varphi\,dz$$

$$\Pi[u] < \Pi[v]$$

6. Mode 3




$$Q = \sqrt{\frac{E\gamma h b^4}{6a^2}}$$

load to open this crack in Mode 3.