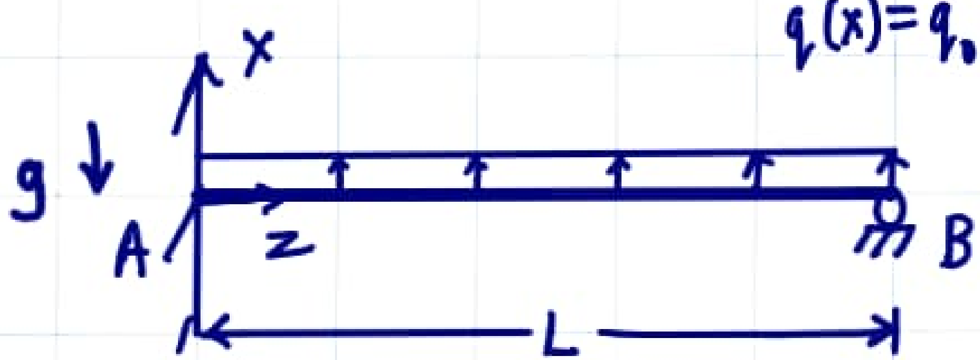
ME 202

LECTURE 19

TUTORIAL 6

THU 17 FEB 2022

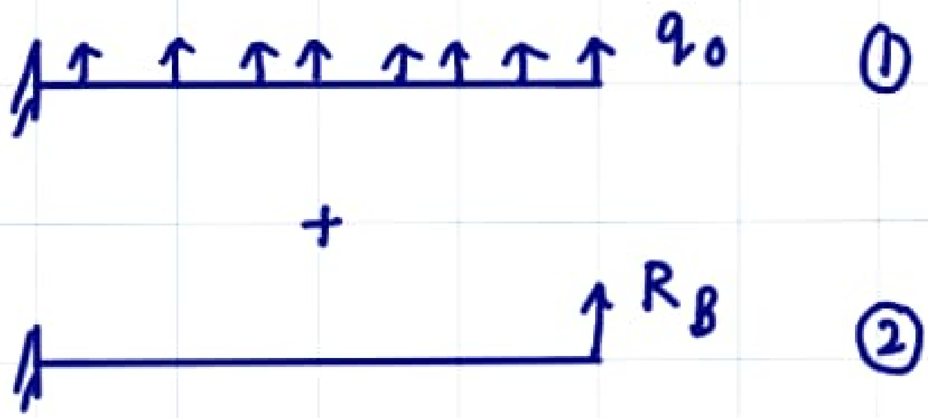
Problem 1



$$q_0 = -\rho A g \equiv \frac{kg}{m^3}\, m^2\, \frac{N}{kg} \equiv \frac{N}{m}$$

In statics, $g \equiv N/kg$

$$\left.\begin{array}{l} \text{3 reactions} \\ \text{2 equations } \; \Sigma F_x = 0,\ \Sigma M_y = 0 \end{array}\right\} \text{stat. indet.}$$



$$u(z) = u_1(z) + u_2(z)$$

$$u(z) = \frac{q_0 z^2}{24EI}\left(6L^2 - 4Lz + z^2\right) + \frac{R_B}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$u(L) = 0 \Rightarrow$$

$$\frac{q_0 L^2}{24EI}\left(6L^2 - 4L^2 + L^2\right) + \frac{R_B L^3}{3EI} = 0$$

$$R_B = -\frac{3}{8} q_0 L$$

$$R_A + R_B + q_0 L = 0$$

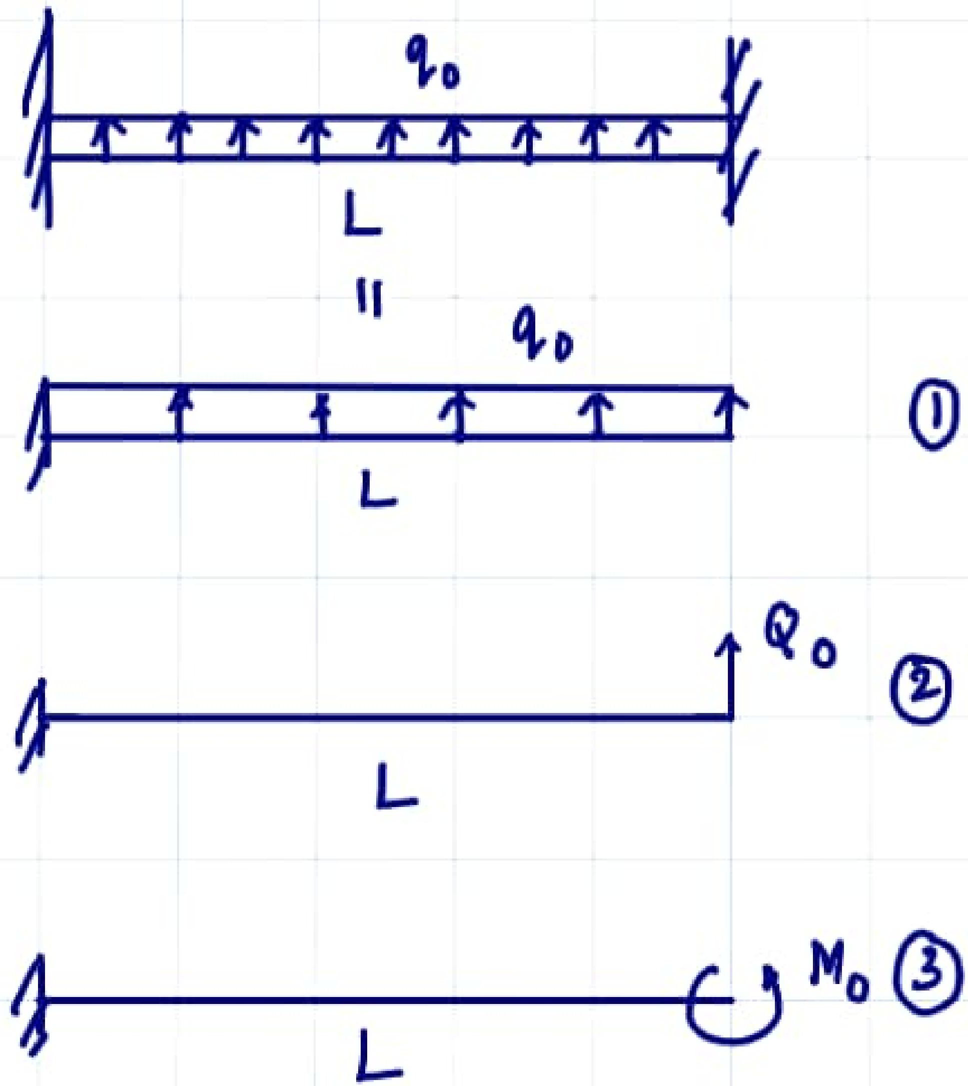
$$R_A = -\frac{5}{8} q_0 L$$

$$M_A + q_0 L \frac{L}{2} + \left(-\frac{3}{8} q_0 L\right) L = 0$$

$$M_A = -\frac{q_0 L^2}{8}$$

$$u(z) = \frac{q_0}{EI} \left(\frac{L^2 z^2}{16} - \frac{L z^3}{24} + \frac{z^4}{24} \right)$$

Problem 2



$$u(z) = u_1(z) + u_2(z) + u_3(z)$$

$$u(z) = \frac{q_0 z^2}{24EI}\left(z^2 + 6L^2 - 4Lz\right) + \frac{Q_0}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right) + \frac{M_0 z^2}{2EI}$$

$$u'(z) = \frac{q_0}{24EI}\left(4z^3 + 12L^2 z - 12Lz\right) + \frac{Q_0}{EI}\left(Lz - \frac{z^2}{2}\right) + \frac{M_0 z}{EI}$$

$u(L) = 0, \quad u'(L) = 0$ Fixed-fixed

$$\frac{1}{EI}\left(\frac{q_0 L^4}{8} + \frac{Q_0 L^3}{3} + \frac{M_0 L^2}{2}\right) = 0$$

$$\frac{1}{EI}\left(\frac{q_0 L^3}{6} + \frac{Q_0 L^2}{2} + M_0 L\right) = 0$$

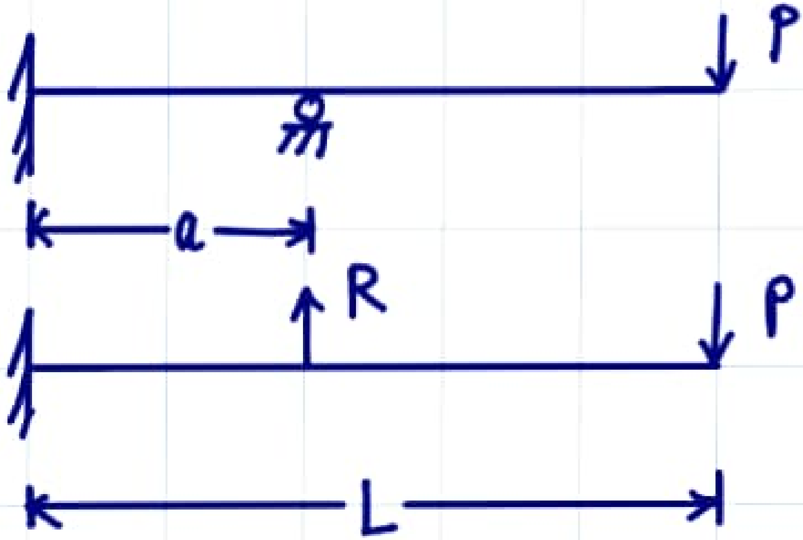
Solve for $Q_0 = -\frac{q_0 L}{2}$

$M_0 = +\frac{q_0 L^2}{12}$

Plug back to get

$$u(z) = \frac{q_0 z^2}{24EI}(z-L)^2$$

Problem 3



Given $u(a) = 0$

$$u(a) = -\frac{P}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) + \frac{R}{EI}\left(\frac{a^3}{2} - \frac{a^3}{6}\right)$$

$$= 0$$

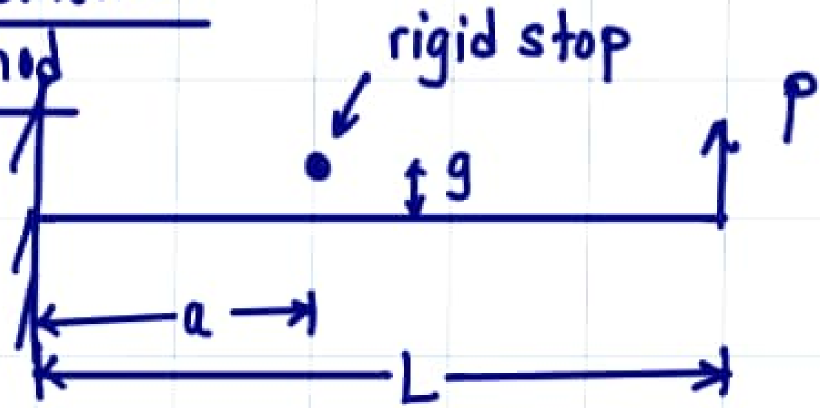
$$R = \frac{3}{2}\frac{PL}{a} - \frac{P}{2}$$

$$u(L) = \frac{-P}{EI}\,\frac{L^3}{3} + \frac{Ra^2L}{2EI} - \frac{Ra^3}{6EI}$$

Problem 4

Contact Mechanics 101

Incremental Method



$g =$ initial gap

Find force P_0 to close the gap.

$$u(a) = \frac{P}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = g$$

stat. det.

$$P_0 = \frac{gEI}{\frac{La^2}{2} - \frac{a^3}{6}}$$

Total force $P_1 > P_0$.

Used P_0 to close the gap.

Available force $= P_1 - P_0$

Contact made/ Gap closed.*

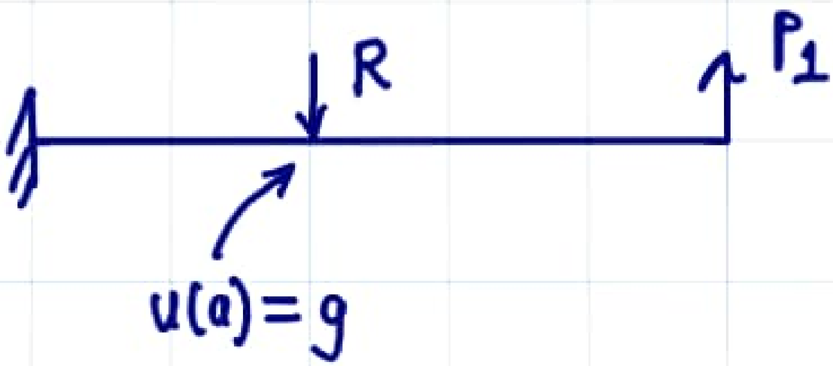
At $z = a$,

$$-\frac{Ra^3}{3EI} + \frac{P_1 - P_0}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = 0$$

$$R = \frac{3P_1}{a^3}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) - \frac{3EIg}{a^3}$$

* stat. indet.

Full / Lumpsum application of force P_1 $\qquad P_1 > P_0$



$$\frac{-Ra^3}{3EI} + \frac{P_1}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = g$$

$$R = \frac{3P_1}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) - \frac{3EIg}{a^3}$$

$L = 0.7\ \text{m}, \quad E = 200\ \text{GPa}$

$a = 0.5\ \text{m}, \quad g = 0.5\ \text{mm}$

$$I = \frac{1}{12}\left(60 \times 10^{-3}\right)^4 = 1.08 \times 10^{-6}\ \text{m}^4$$

$$EI = 2.16 \times 10^{5}\ \text{Nm}^2$$

$$\frac{P}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = g$$

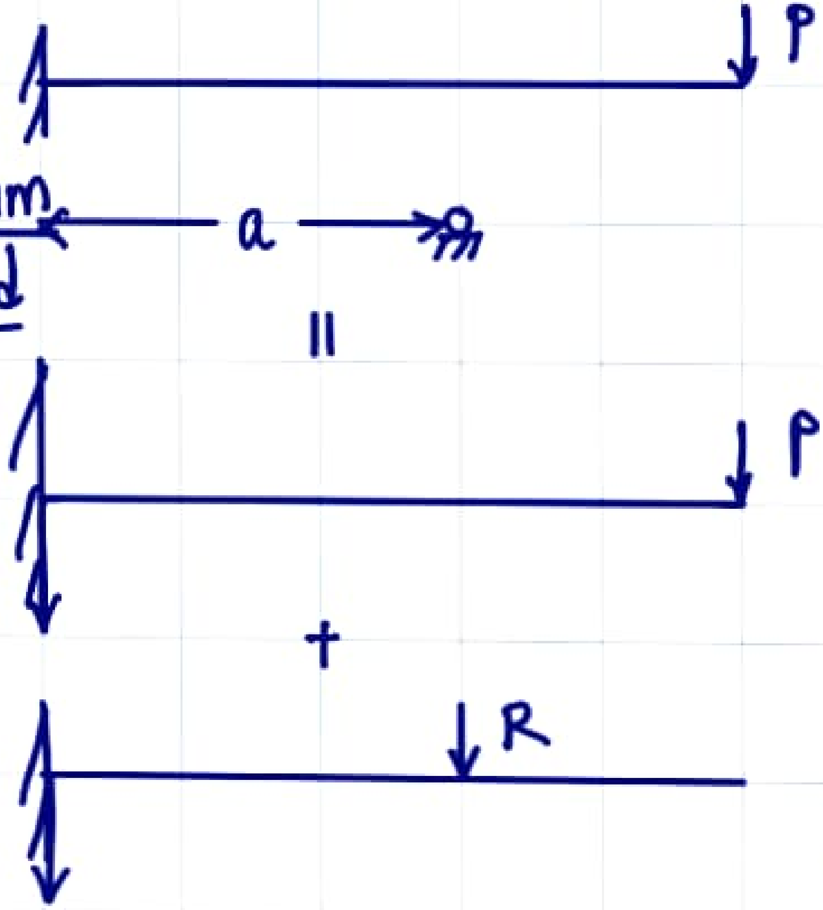
$$P_0 = 1.62\ \text{kN}$$

End deflection at C $= \dfrac{P_0 L^3}{3EI}$

$= 0.8575\ \text{mm}$

$P > P_0$

Lumpsum Method



Given

$$u(L) = \frac{P}{EI}\frac{L^3}{3} + \frac{R}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = \delta_L = 1 \text{ mm}$$

Given

$$u(a) = \frac{P}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) + \frac{R}{EI}\frac{a^3}{3} = \delta_a = 0.5 \text{ mm}$$

$\delta_a = g$

$$\begin{pmatrix} L^3/3 & \frac{La^2}{2}-\frac{a^3}{6} \\ \frac{La^2}{2}-\frac{a^3}{6} & \frac{a^3}{3} \end{pmatrix} \begin{pmatrix} P \\ R \end{pmatrix} = \begin{pmatrix} \delta_L EI \\ \delta_a EI \end{pmatrix}$$

$P = 5.634$ kN

$R = -6.423$ kN