Metal Casting - 1

Outline

- Casting basics
- Patterns and molds
- Melting and pouring analysis
- Solidification analysis
- Casting defects and remedies

Casting Basics

- A casting is a metal object obtained by *pouring molten* metal into a *mold* and allowing it to *solidify*.

Gearbox casting

Magnesium casting

Aluminum manifold

Cast wheel

Casting: Brief History

- 3200 B.C. – Copper part (a frog!) cast in Mesopotamia. Oldest known casting in existence
- 233 B.C. – Cast iron plowshares (in China)
- 500 A.D. – Cast crucible steel (in India)
- 1642 A.D. – First American iron casting at Saugus Iron Works, Lynn, MA
- 1818 A.D. – First cast steel made in U.S. using crucible process
- 1919 A.D. – First electric arc furnace used in the U.S.
- Early 1970's – Semi-solid metalworking process developed at MIT
- 1996 – Cast metal matrix composites first used in brake rotors of production automobile

Complex, 3-D shapes

- Near net shape → Mass conserving process
- Low scrap →
- Relatively quick process →
- Intricate shapes → Pattern = Product!
- Large hollow shapes —
- No limit to size
- Reasonable to good surface finish

Capabilities

- Dimensions
  - sand casting - as large as you like
  - small - 1 mm or so
- Tolerances
  - 0.005 in to 0.1 in
- Surface finish
  - die casting 8-16 micro-inches (1-3 μm)
  - sand casting - 500 micro-inches (2.5-25 μm)

Processes

- Sand
- Shell
- Plaster
- Ceramic
- Investment
- Lost foam
- Pressure
- Vacuum

- Die
- Centrifugal
- Squeeze
- Semi-solid
- Single crystal
- Directional solidification
- Slush
- Continuous

Metals processed by casting

- Sand casting – 60%
- Investment casting – 7%
- Die casting – 9%
- Permanent mold casting – 11%
- Centrifugal casting – 7%
- Shell mold casting – 6%

why this is most common?

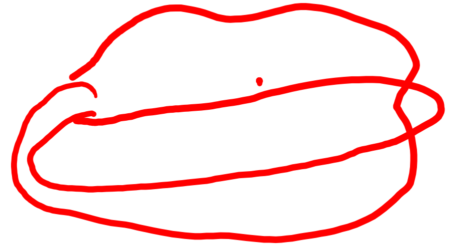
(1) Availability

(2) Cost

(3) Flexible size, material

Aluminium Lower

Casting: Basic Steps

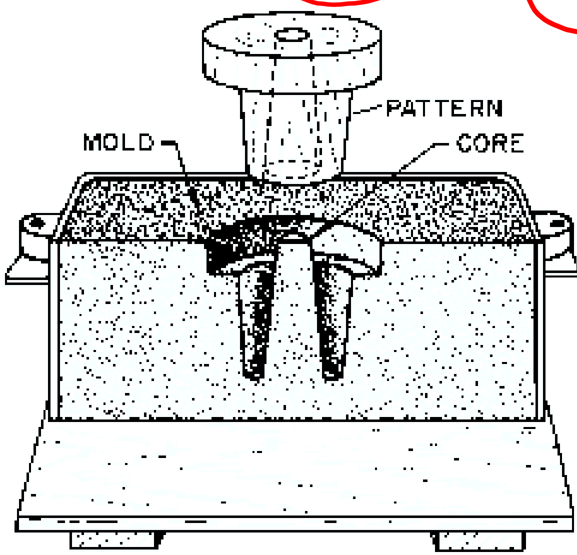


- Basic steps in casting are:
  - Preparation of pattern(s), core(s) and mold(s)
  - Melting and pouring of liquefied metal
  - Solidification and cooling to room temperature
  - Removal of casting - shakeout
  - Inspection (for possible defects)

Solid material for hollow shape

Pattern Making

- Pattern is a replica of the exterior surface of part to be cast – used to create the mold cavity
- Pattern materials – wood, metal, plaster



Pattern Making

- Pattern usually larger than cast part

Allowances made for:

- ***Shrinkage***: to compensate for metal shrinkage during cooling from freezing to room temp

$$\text{Shrinkage allowance} = \alpha L(T_f - T_0)$$

$L\alpha\Delta T$

expressed as *per unit length* for a given material

α = coeff. of thermal expansion, T_f = freezing temp

T_0= room temp

e.g. Cast iron allowance = 1/96 in./ft

aluminum allowance = 3/192 in./ft

Pattern Making

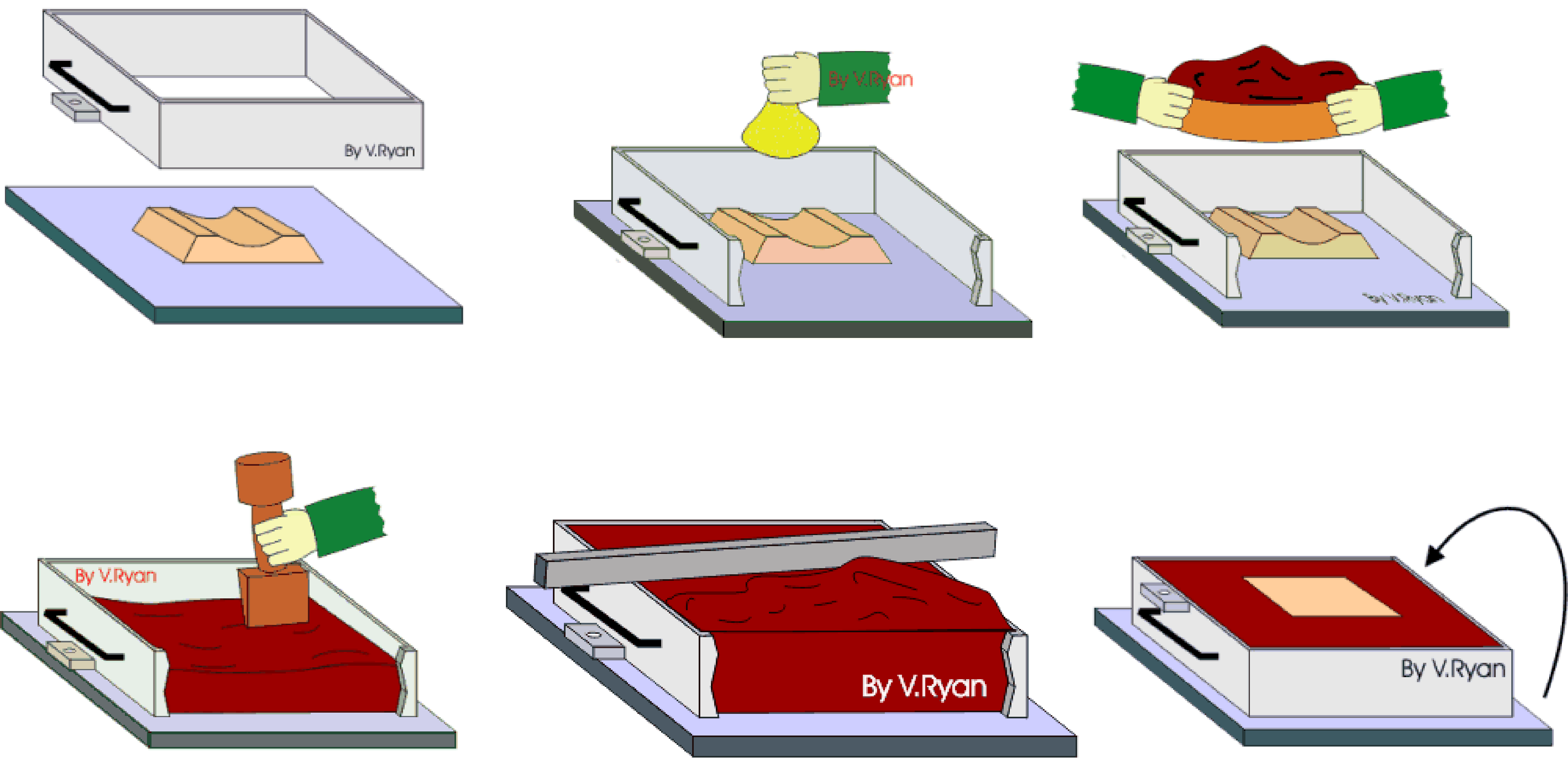
- Pattern allowances made for:
  - ***Machining***: excess dimension that is removed by machining; depends on part dimension and material to be cast

    Functional + Aesthetics

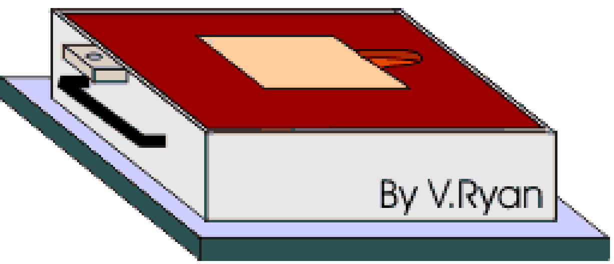
    e.g. cast iron, dimension 0-30 cm, allowance = 2.5 mm; aluminum, allowance = 1.5 mm

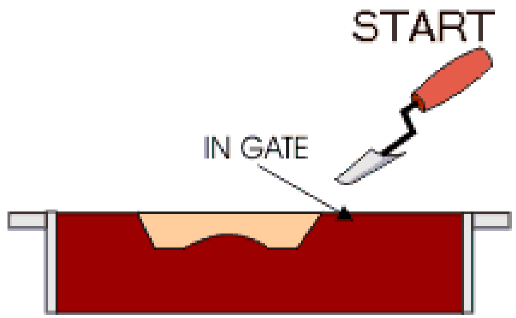
  - ***Draft***: taper on side of pattern parallel to direction of extraction from mold; for ease of pattern extraction; typically 0.5~2 degrees

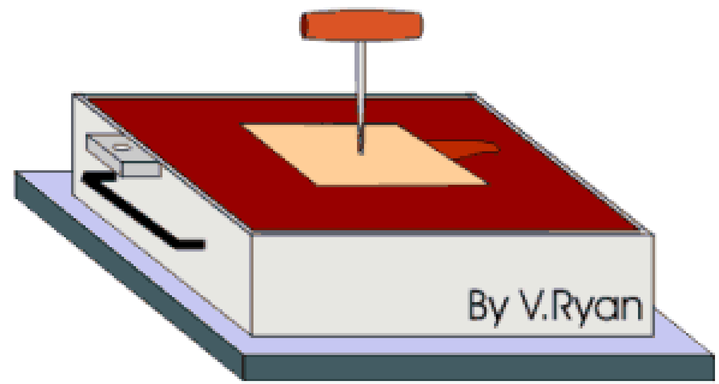
Mold Making: Sand Casting

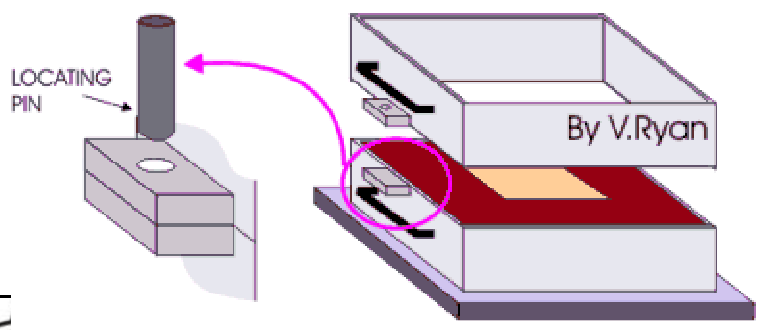


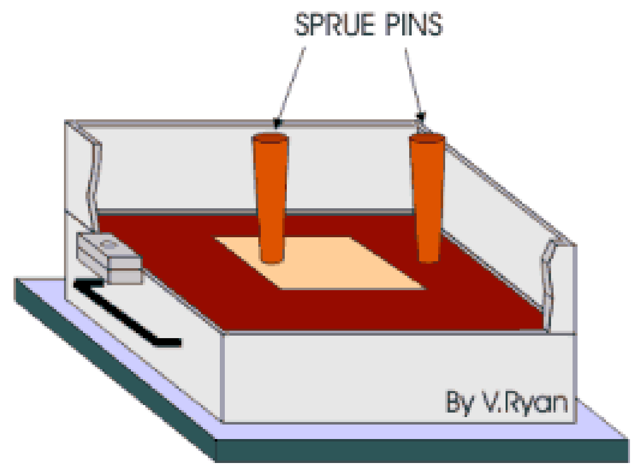
Mold Making: Sand Casting



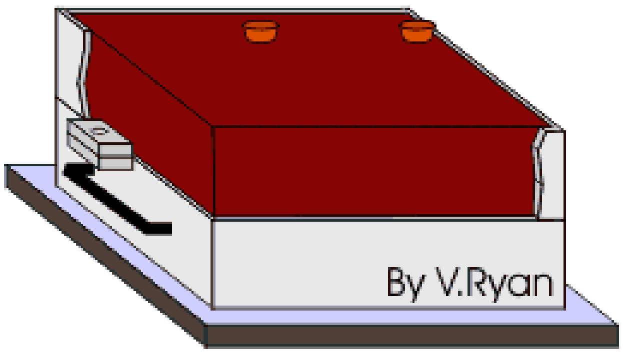


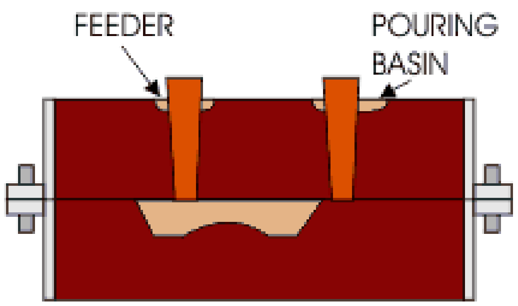


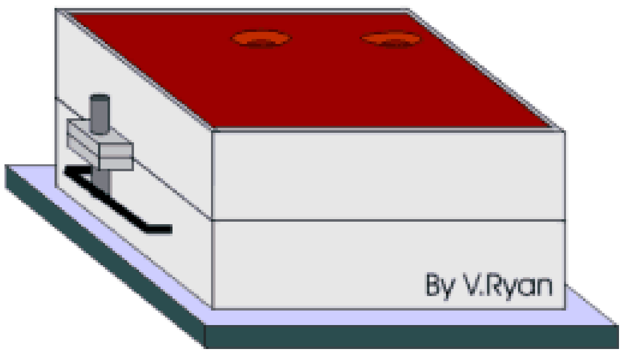


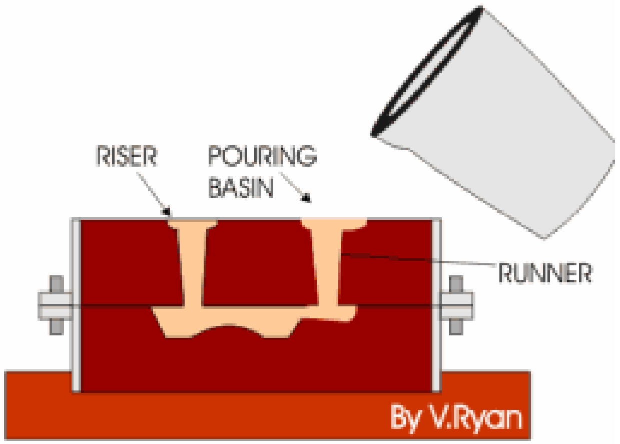


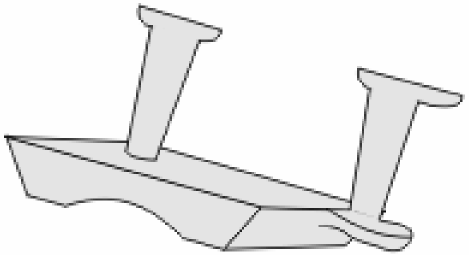
Mold Making: Sand Casting











Casting video: http://www.designinsite.dk/htmsider/pb0211wmv.htm

Sand Casting

- Green sand mold:
  sand + clay + water + additives
- Typical composition (by wt.):
  - 70-85% sand, 10-20% clay, 3-6% water, 1-6% additives
- Important properties of molding sand:
  - Strength
  - Permeability ??
  - Deformation
  - Flowability
  - Refractoriness

Melting

Heat - I need to give to my material

- For a pure metal:

  total heat energy required, H =

  energy to raise temp of metal to melting point, T_m + heat of fusion, H_f + energy to raise temp of liquid metal to pouring temp, T_p

m

$$H = \rho V \left[c_s(T_m - T_0) + H_f + c_l(T_p - T_m)\right]$$

- Heat required for alloys more complex
- Gas fired, electric arc and induction furnaces used to melt metal

Melting

- Solubility of gases (hydrogen and nitrogen) in molten metal an issue
- Solubility of H_2, *S*:

Tp iovg

$$S = Ce^{-\frac{E_s}{k\theta}}$$

$$S = C \exp\left[-E_s/(k\theta)\right]$$

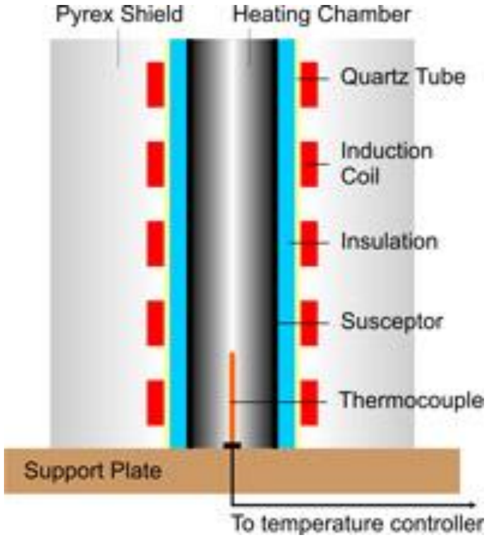
E_s = heat of solution of 1 mol of H_2

θ = absolute temp, *C* and *k* are constants

e.g. 1 atm pressure, liquid solubility of H_2 in iron = 270 cc/kg; in aluminum = 7 cc/kg

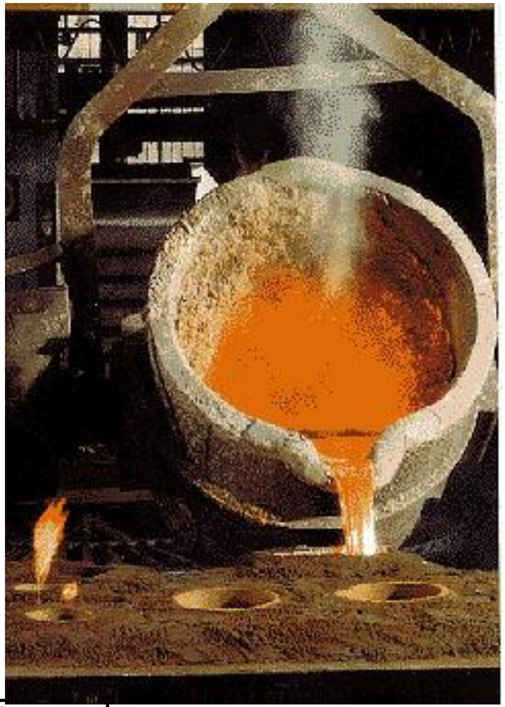
Melting Furnaces

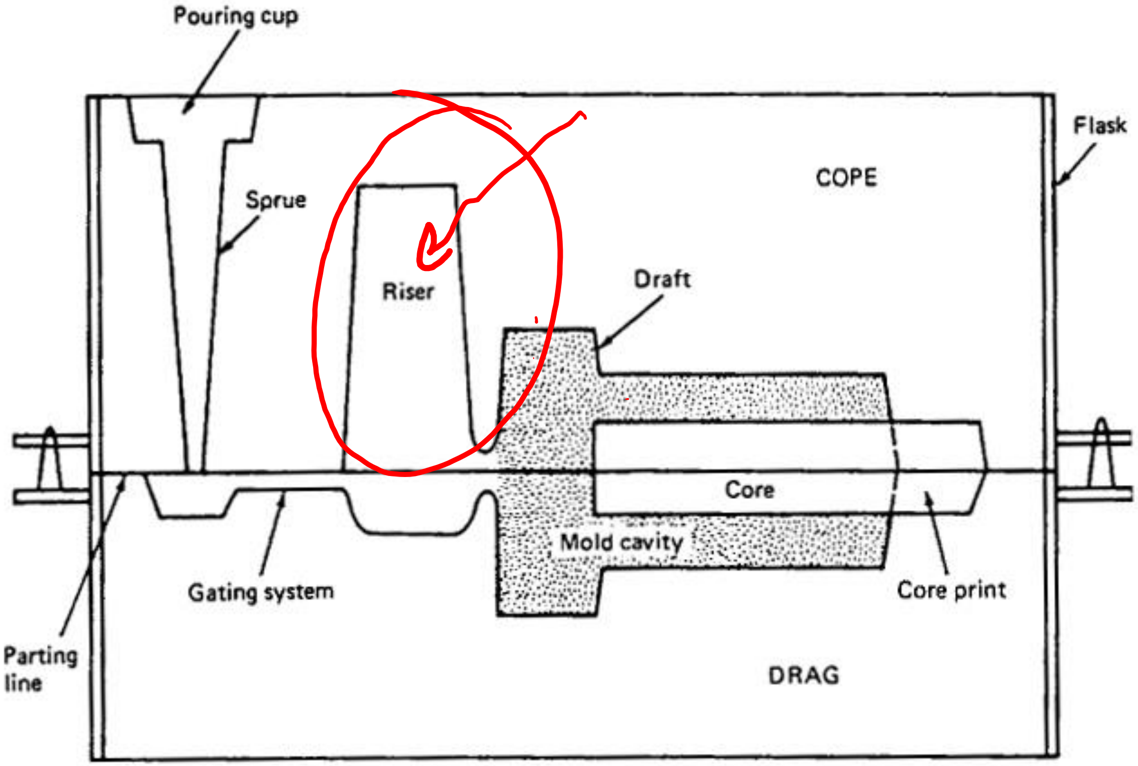
Induction Heating



Pouring

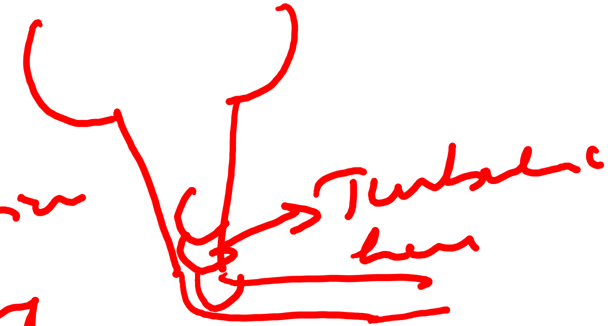
- An important step in casting since it impacts mold filling ability and casting defects





Pouring

Aspiration sucking



- Key aspects of pouring
  - Pouring rate
    - Too slow → metal freezes before complete mold filling
    - Too fast → inclusion of slag, aspiration of gas, etc.
    - Reynolds number: Laminar versus turbulent flow

(1) Aspir Pouring

(2) Mold erosion

$$\mathrm{Re} = \frac{\rho V D}{\eta}$$

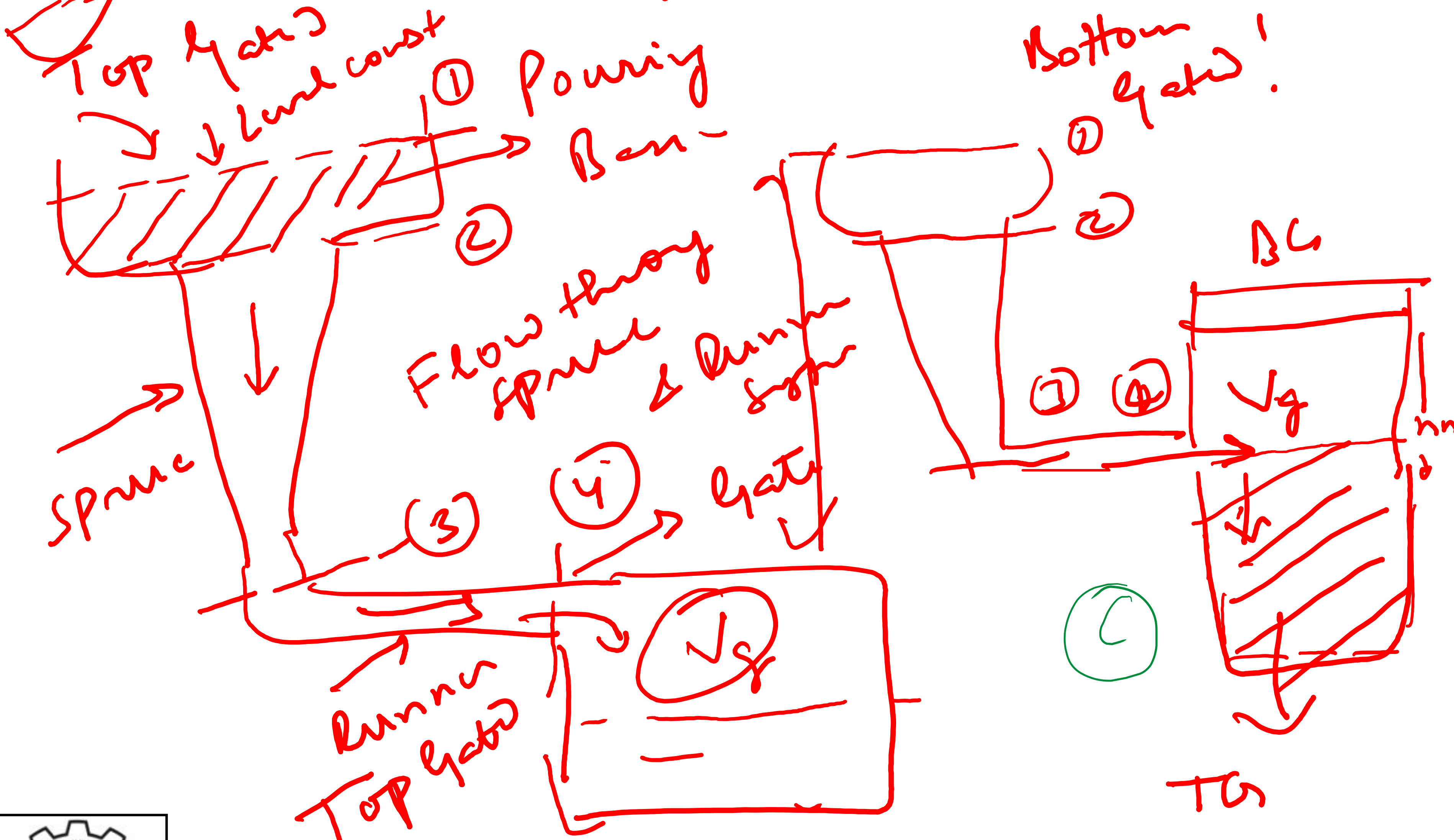
ρ = density of liquid, V = mean flow velocity, D = tube diameter, η = dynamic viscosity of liquid

Re → Es

e kθρ

- Most steels reach mildly turbulent flow conditions easily (Re > 3500)
  - Superheat ~ $(T_p - T_m)$; T_p = pouring temp
    - Too high → increased gas solubility → porosity problems

Basic Molding systems!



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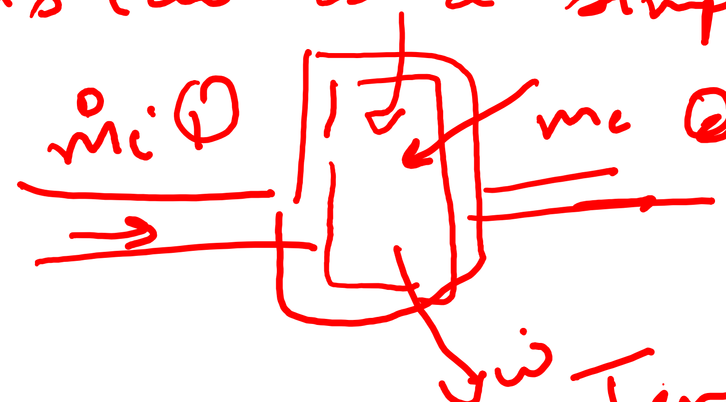
First law of Thermodynamics of open system

$$\cancel{\frac{dE_{cv}}{dt}}^{0} = \cancel{\dot{Q}}^{0} - \cancel{\dot{W}}^{0} + \sum \dot{m}_i \left(h_i + \frac{v_i^2}{2} + g z_i\right) - \sum \dot{m}_e \left(h_e + \frac{v_e^2}{2} + g z_e\right)$$

Bernoulli's law is a simplification



$h = \cancel{u} + pv$

① Steady state

② $\dot{q} = 0$

③ $\dot{w} = 0$

④ Temp const.

⑤ incomp

Temp is const

$$\dot{m}_1 \left(p v_1 + \frac{v_1^2}{2} + g H_1\right) = \dot{m}_2$$

Pouring Analysis (Sprue/Gating Design)

- Fluid flow in sprue/gating/mold can be analyzed using energy balance i.e. Bernoulli's theorem

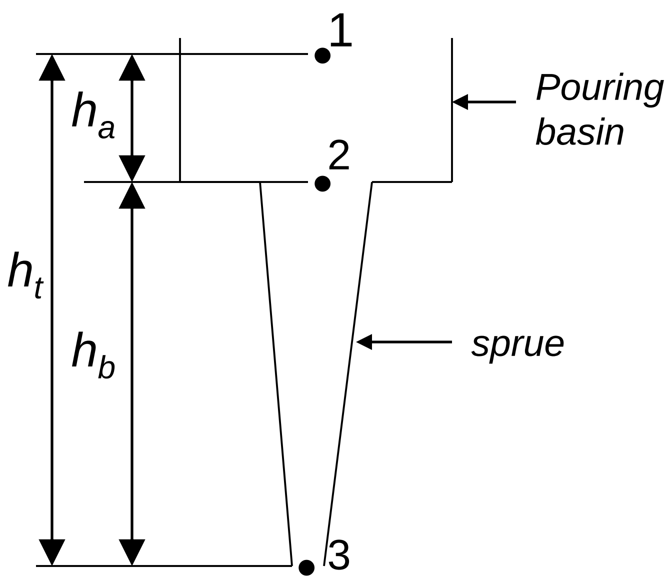
$$h_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + F = const.$$

$P + \rho g H + \frac{1}{2}\rho v^2 = const.$

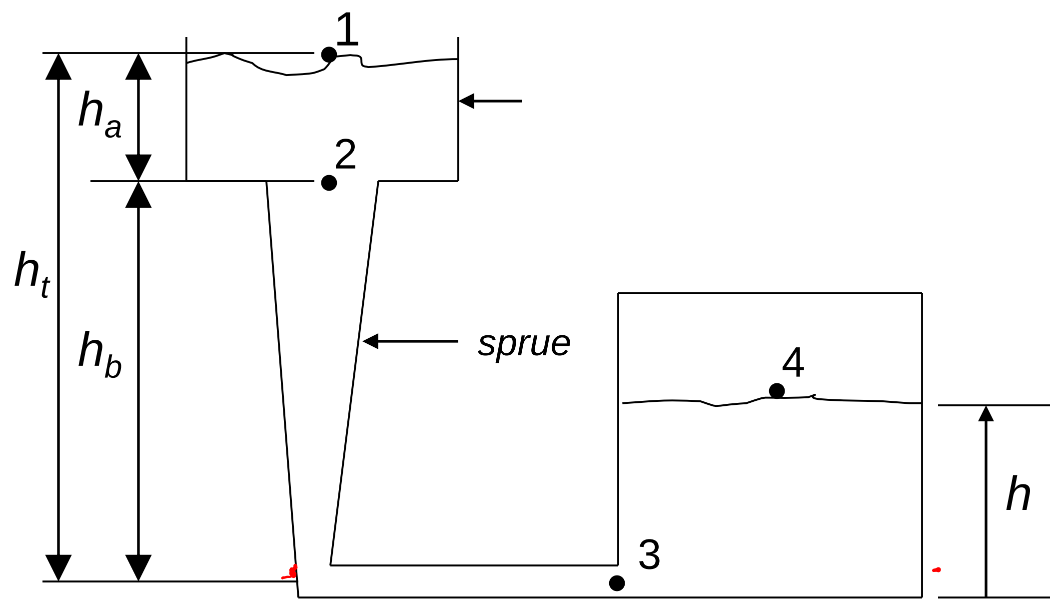
Bernoulli

- Assumptions of analysis
  - Incompressible fluid
  - Negligible frictional losses
  - Entire mold is at atmospheric pressure

Pouring Analysis (Sprue/Gating Design)



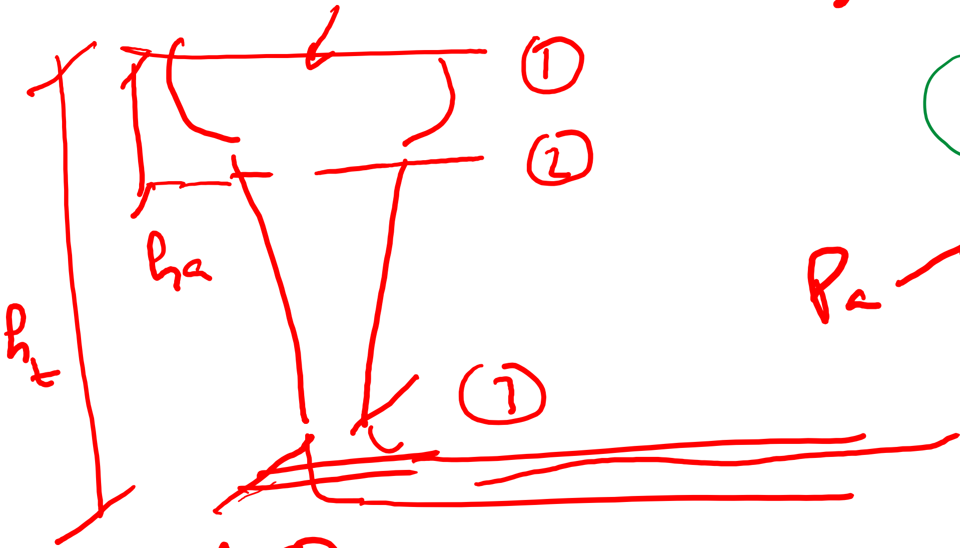
Top Gated Mold



Bottom Gated Mold

- Design of sprue and gating system (runners + gates) based on Bernoulli's theorem

Let us consider top gated design first,



1 & 2

$$\underset{p_a}{\cancel{p_1}} + \rho g h_1 + \frac{1}{2}\rho \underset{0}{\cancel{v_1^2}}$$

$$= \underset{p_a}{\cancel{p_3}} + \underset{0}{\cancel{\rho g h_3}} + \frac{1}{2}\rho v_3^2$$

1 & 3

$$\sqrt{2gh_t} = v_3$$

$$\frac{1}{2}\rho v_3^2 = \rho g h_t$$

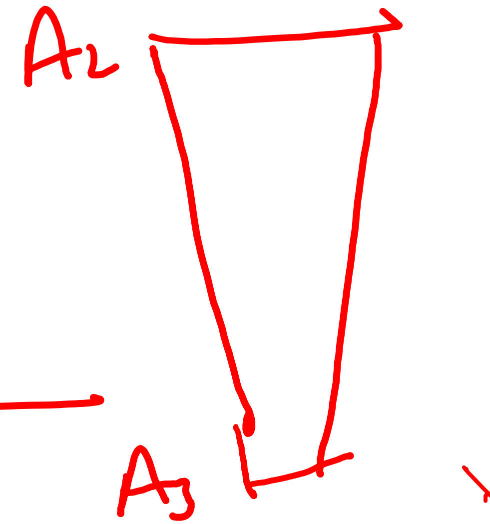
$$v_3 = \sqrt{2gh_t}$$

$$v_2 = \sqrt{2gh_a}$$

If $A_2 = A_3$

$v_2 = \sqrt{2 g h_c}$

$v_3 = \sqrt{2 g h_t}$



Continuity

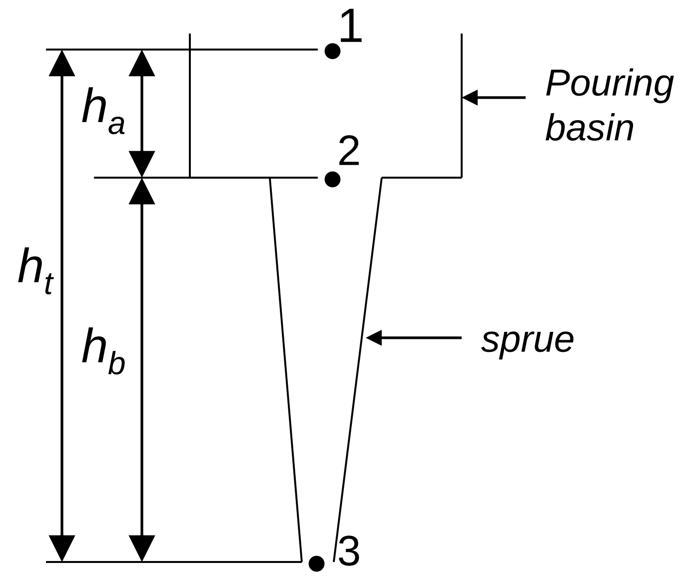
$$A_2 v_2 = A_3 v_3$$

$$A_2 / A_3 = \frac{v_3}{v_2} = \sqrt{\frac{h_t}{h_c}}$$

Pouring Analysis (Sprue/Gating Design)

- Applying energy balance between points 1 and 3

$$h_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = h_3 + \frac{V_3^2}{2g} + \frac{P_3}{\rho g}$$



Top Gated Mold

Assuming entire mold is at atmospheric pressure and velocity of melt at point 1 ~ 0

$$V_3 \approx \sqrt{2gh_t}$$

Pouring Analysis (Sprue Design)

- Consider the geometry of freely falling liquid from the pouring basin; also assume permeable walls (e.g. sand mold)

$$V_2 \approx \sqrt{2gh_a} \qquad V_3 \approx \sqrt{2gh_t}$$

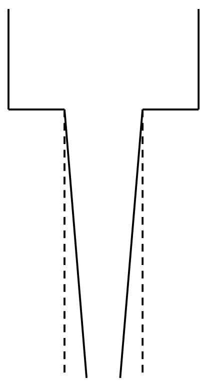
- Assuming continuity of fluid flow, flow rate at point 2 = flow rate at point 3:

$$A_2V_2 = A_3V_3 \Rightarrow \boxed{\frac{A_3}{A_2} = \frac{V_2}{V_3} = \sqrt{\frac{h_a}{h_t}}}$$

Pouring Analysis (Sprue Design)

- Result suggests a parabolic shape for sprue

$$\frac{A_3}{A_2} = \sqrt{\frac{h_a}{h_t}}$$



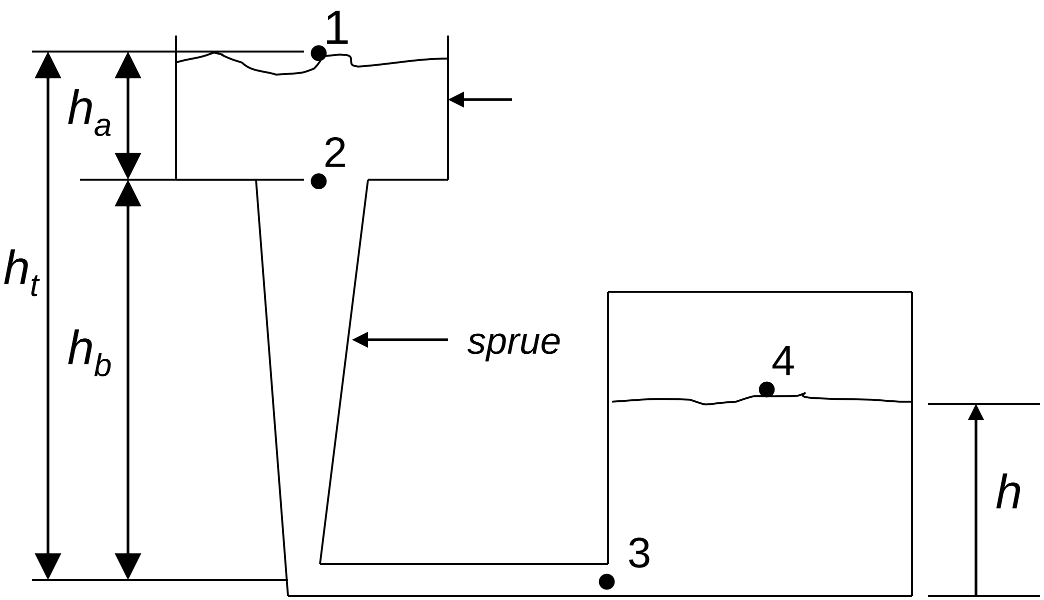
- A straight sprue can lead to aspiration of gases from the mold (for a permeable mold) into the molten metal → porous castings

Pouring Analysis (Sprue/Gating Design)

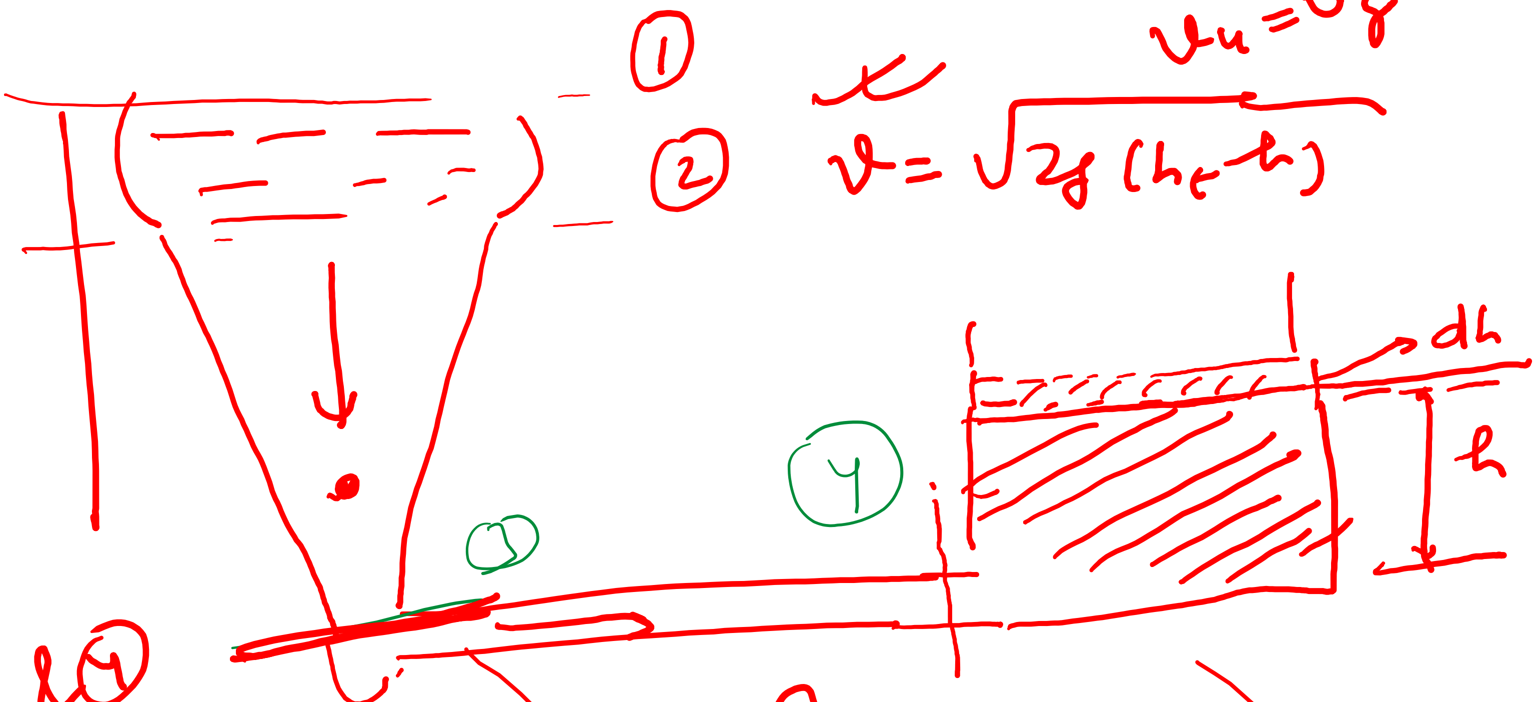
- As metal is poured into mold, the effective "head" decreases

- Velocity of metal at point 3:

$$V_3 \approx \sqrt{2g(h_t - h)}$$



Bottom Gated Mold



$$v_4 = v_g$$

$$v = \sqrt{2g\,(h_t - h)}$$

① & ④

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho \cancel{v_1^2} \to 0 = P_4 + \cancel{\rho g h_4}^{\,0} + \tfrac{1}{2}\rho v_4^2$$

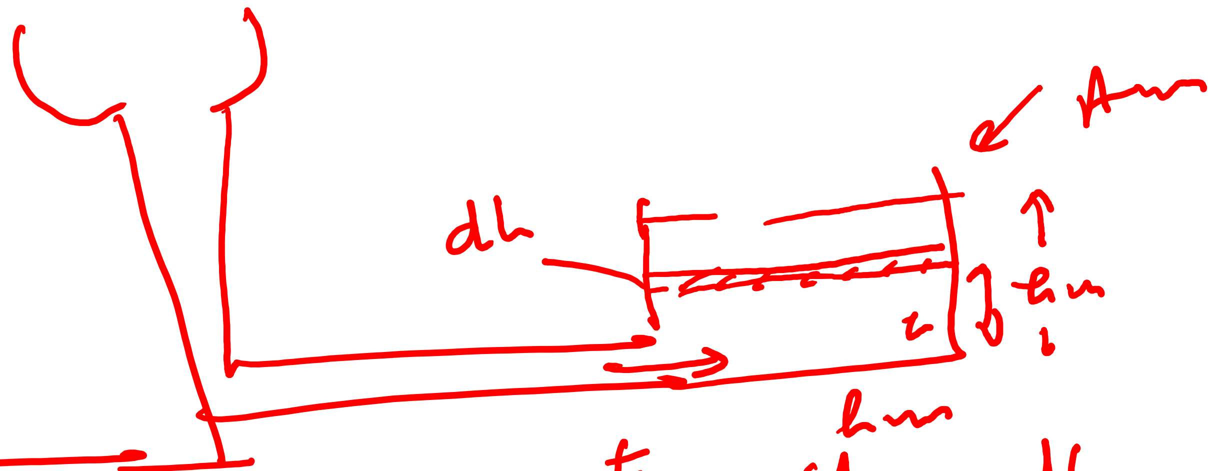
P_a $\quad h_t$

$$\rho g\,(h_t - h) = \tfrac{1}{2}\rho v_4^2$$

$$v_g = \sqrt{2g\,(h_t - h)}$$

Filling time

B.C.



$$v_g = \sqrt{2g(h_t - h)}$$

A_g

$$A_g \cdot v_g = A_m \cdot \frac{dh}{dt}$$

$$A_g \cdot \sqrt{2g(h_t - h)} = A_m \cdot \frac{dh}{dt}$$

$$\int_0^t dt = \int_0^{h_m} \frac{A_m}{A_g} \cdot \frac{dh}{\sqrt{2g(h_t - h)}} \rightarrow \text{B.C.}$$

$$v_3 = v_g = \sqrt{2g h_t}$$

$$T_f = \frac{V_m}{A_g \cdot \sqrt{2g h_t}}$$

Mold Filling Analysis

- Bottom gated mold: In time *dt* increase in volume of metal in mold = $A_m dh$, where A_m = cross-section of the mold cavity
- Volumetric flow rate of metal delivered to mold at point 3 (gate) = $A_3 V_3$
- Volume balance at point 3:

$$A_m dh = A_3 \sqrt{2g(h_t - h)}dt$$

- Mold filling time, t_f

$$\frac{1}{\sqrt{2g}} \int_0^{h_m} \frac{dh}{\sqrt{h_t - h}} = \frac{A_3}{A_m} \int_0^{t_f} dt \Rightarrow \boxed{t_f = \frac{2A_m}{A_3\sqrt{2g}}\left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)}$$

Mold Filling Analysis

- Mold filling time, t_f

$$\frac{1}{\sqrt{2g}}\int_0^{h_m}\frac{dh}{\sqrt{h_t - h}} = \frac{A_3}{A_m}\int_0^{t_f} dt \Rightarrow \boxed{t_f = \frac{2A_m}{A_3\sqrt{2g}}\left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)}$$

- Mold filling time for top gated mold

$$t_f = \frac{Mold\ \ Volume}{Flow\ Rate} = \frac{A_m h_m}{A_g V_g}$$

- Above calculations represent the minimum time necessary

Example Problem 1

Given a top gated mold with the following:

Sprue height, h_t = 20 cm

Cross-section of sprue base, A_3 = 2.5 cm^2

Volume of mold cavity, V = 1560 cm^3

Find:

a) Flow velocity at sprue base

b) Flow rate of metal into mold cavity

c) Mold filling time

Example Problem 1 (contd)

Solution:

a) Velocity at sprue base

$$V_3 = \sqrt{2gh_t} = \sqrt{2(981)(20)} = 198.1 \text{ cm/s}$$

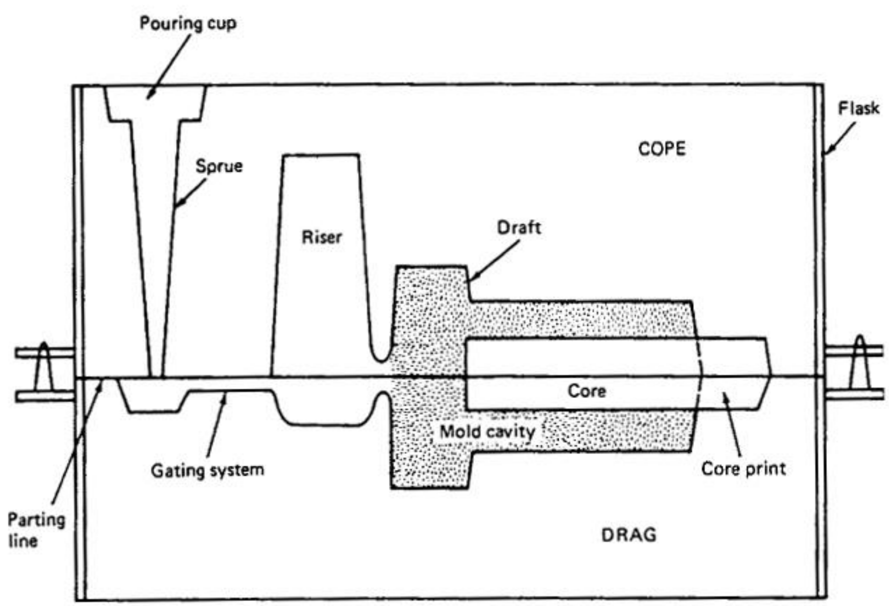
b) Flow rate, $Q = A_3V_3 =$ $(2.5)(198.1) = 495 \text{ cm}^3\text{/s}$

c) Mold filling time

$$t_f = \frac{1560}{495} = 3.2 \text{ s}$$

Example Problem 2

- Consider the sand mold shown below. You wish to pour molten iron so that the flow into the mold cavity is not very turbulent. Determine the diameter of the gate for the given problem data.



Example 2 (contd.)

Data for problem:

- Iron data:

  - density = 7860 kg/m^3

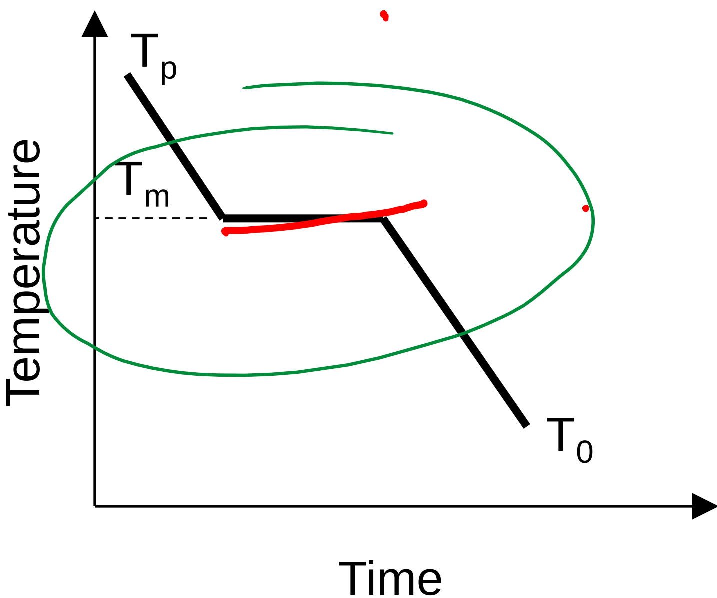
  - viscosity at pouring temp. = 2.25×10^{-3} $N.s/m^2$

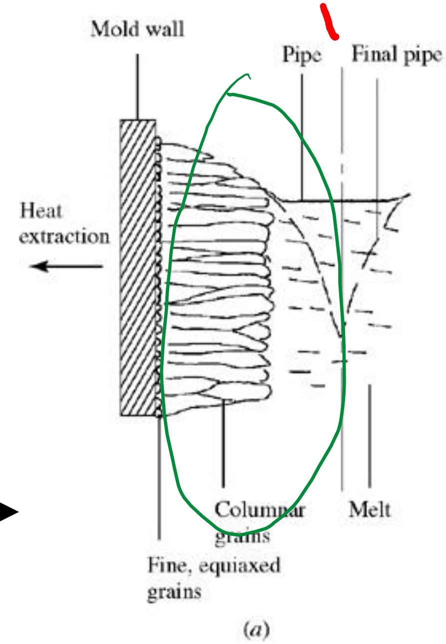
- Sprue height (including pouring cup) = 2 in. or 0.051 m
- Assume that the runners and gates are uniform in cross-section; ignore riser

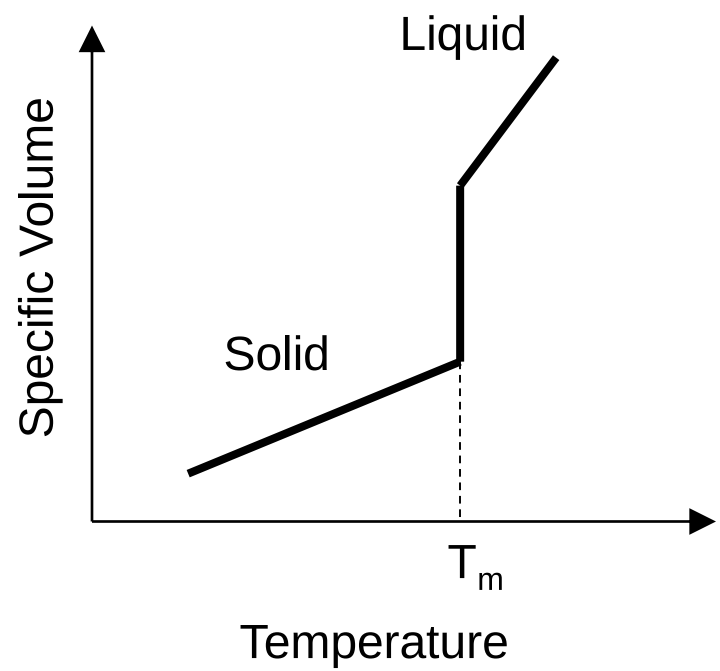
Solidification

- Pure metals
  - Solidify at approx. constant temperature

    when phase change is happening
  - Initiation of solidification requires undercooling







Solidification

Liquidus

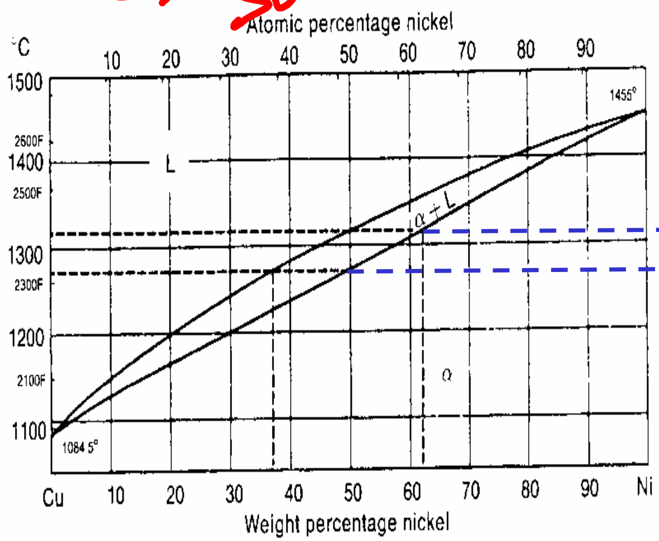
T_L

T_S

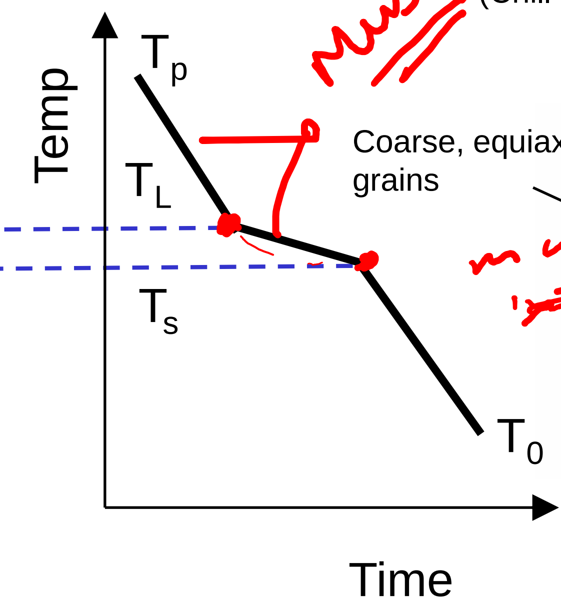
Solidus

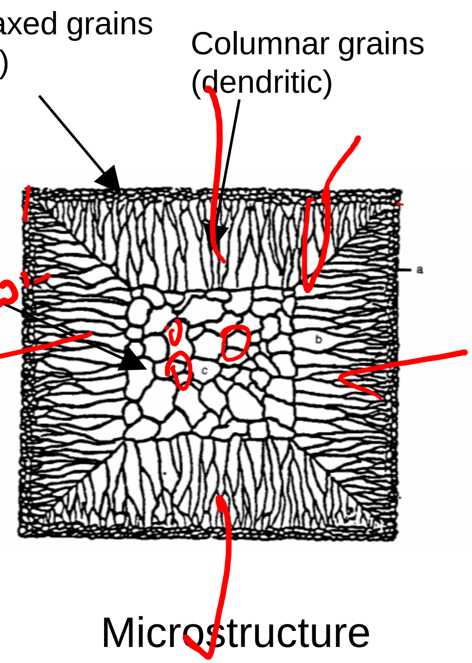
Some other charts

- Alloys
  - Solidify over a temperature range
  - Composition and microstructure determined by phase diagram of alloy



Binary Phase Diagram

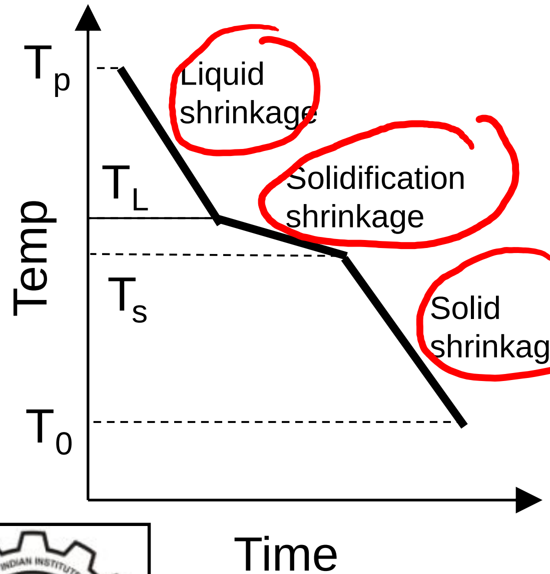




Microstructure

Shrinkage

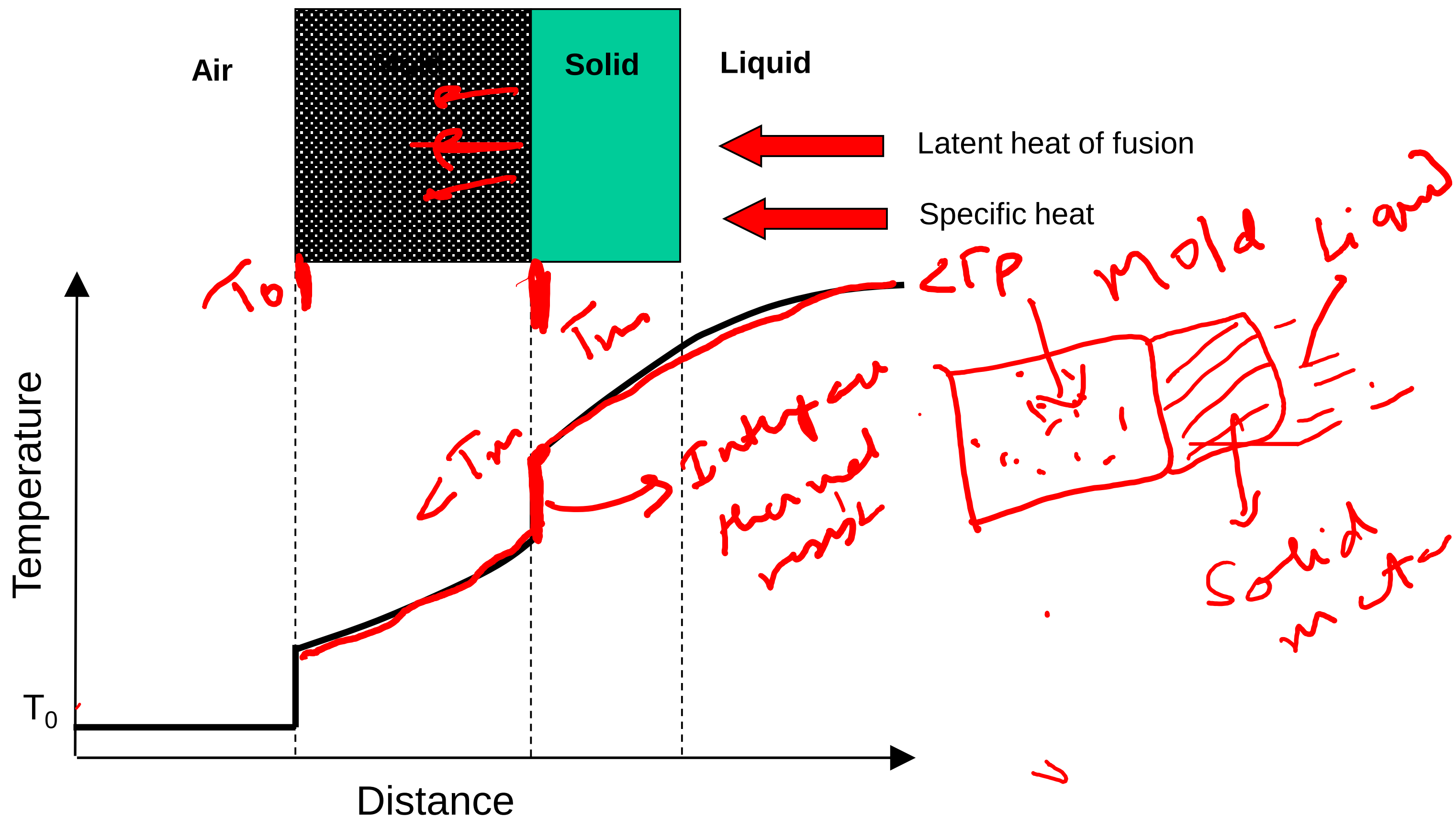
- Shrinkage: most metals shrink when cooled from the liquid state
  - Liquid shrinkage
  - Solidification shrinkage
  - Solid shrinkage

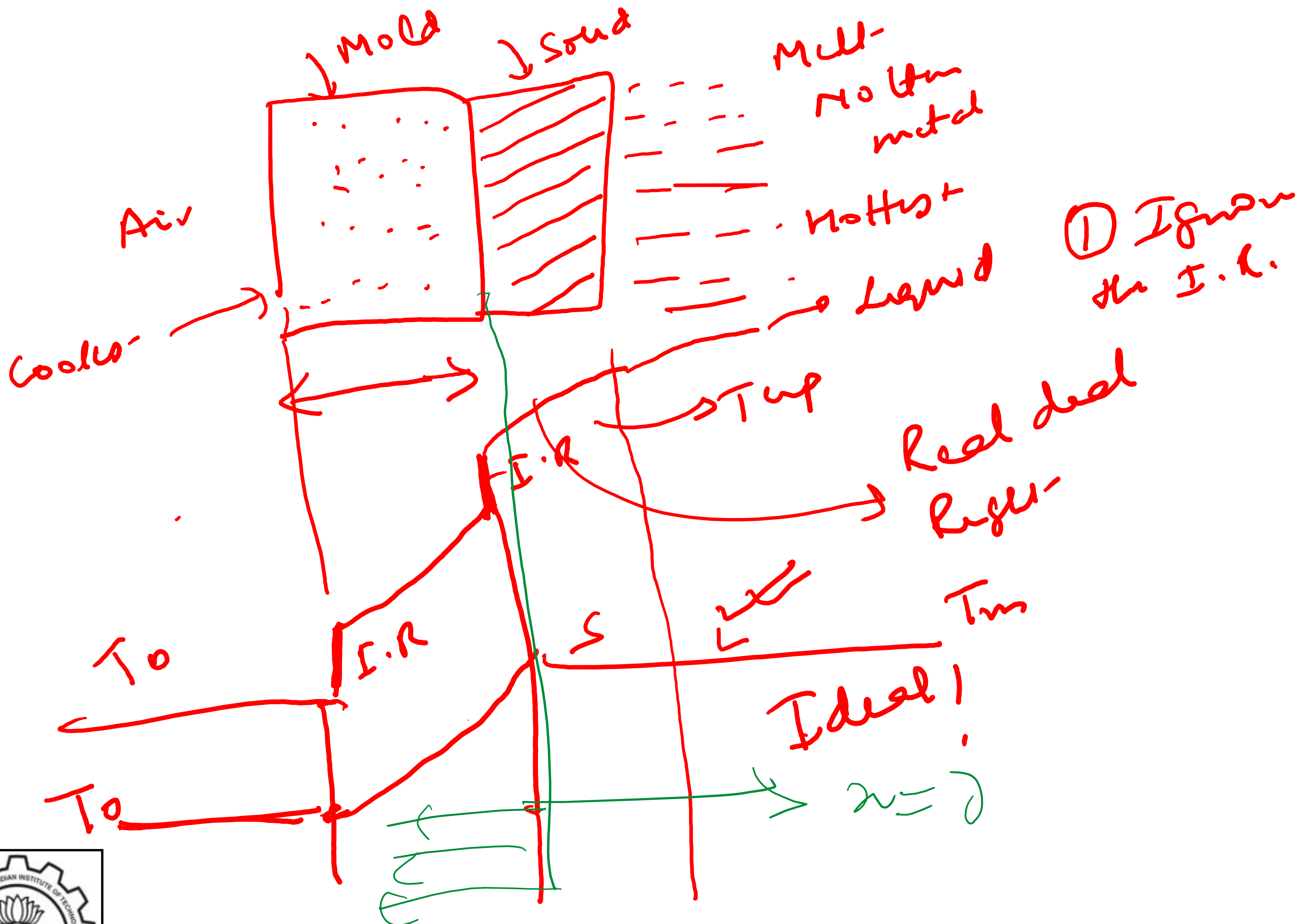


Metal	Solidification Shrinkage (%)	Coeff. Thermal Exp.	Solid Shrinkage (%)
Aluminum alloys	7	25×10^{-6}	6.7
Cast iron	1.8	13×10^{-6}	4
Steel	3	14×10^{-6}	7.2
Copper alloys	5.5	17×10^{-6}	6

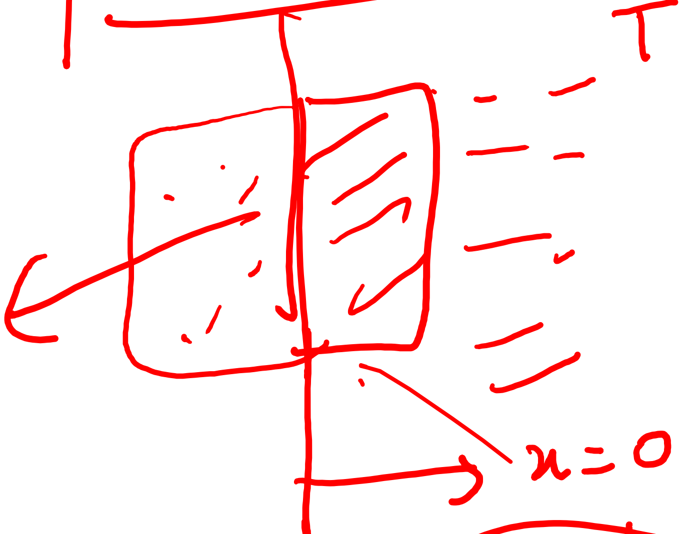
Heat Transfer During Solidification

- Casting: non-steady state heat flow
- Consider the 1-d solidification of a pure metal



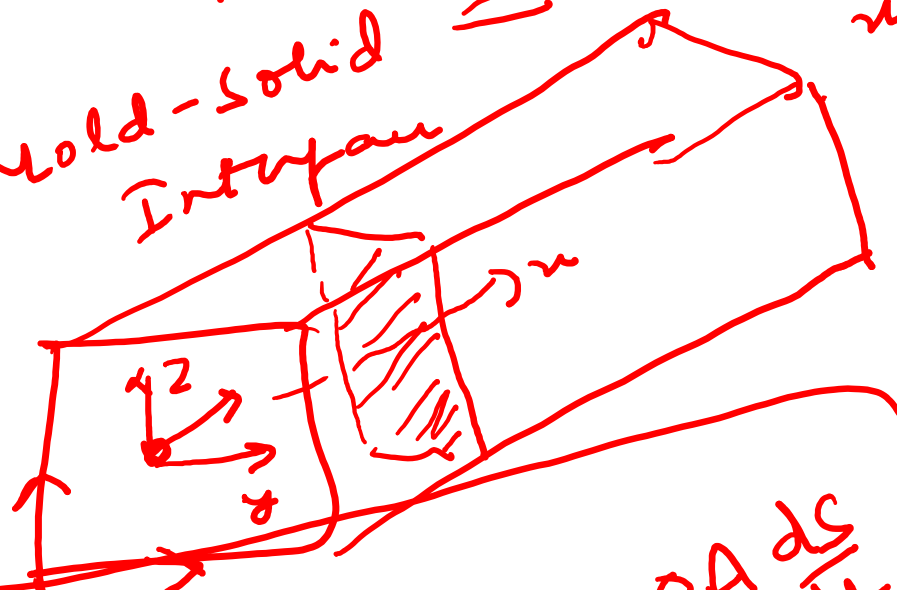

Mold
Solid
Melt
Molten metal
Air
Hottest
Liquid
Cooler
① Ignore the I.R.
Tmp
I.R
Real deal Region
To
I.R
S
L
Tm
Ideal!
To
x=0

uc $$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$ → One dimen.

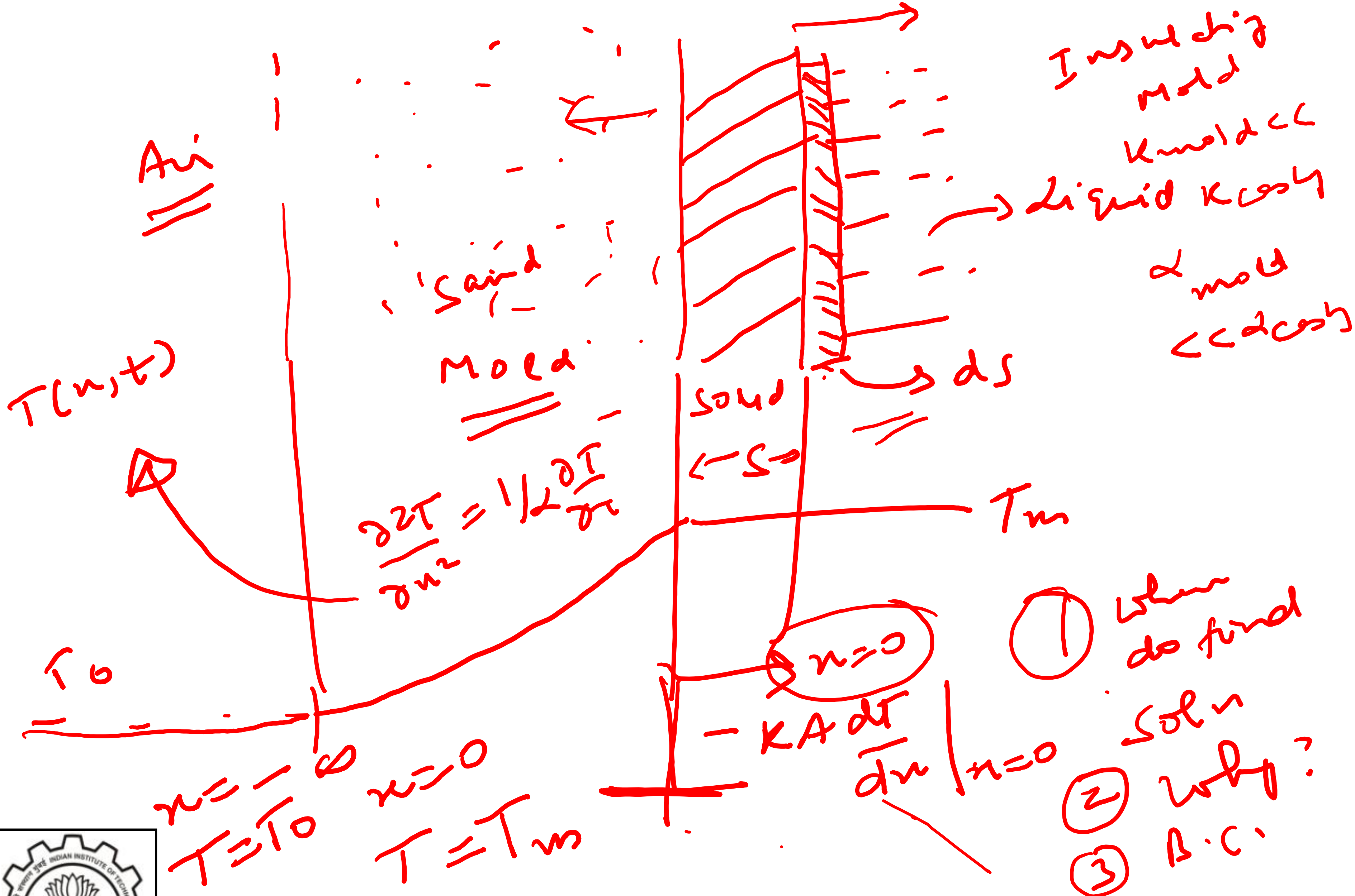


T(x) 1-D

Mold-solid Interface



$$\dot{q} = \boxed{-KA\frac{\partial T}{\partial x}\Big|_{x=0} = \dot{m}\,\Delta H_f = \rho \frac{dV}{dt}\Delta H_f = \rho A \frac{ds}{dt}}$$



Air
Sand Mold
Insulating Mold
$K_{mold} << K_{liquid}$ $K_{casting}$
$\alpha_{mold} << \alpha_{casting}$
ds
solid
S
T_m
$T(x,t)$
T_0
$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
$x = -\infty$
$T = T_0$
$x = 0$
$T = T_m$
$x=0$
$-KA\frac{dT}{dx}\Big|_{x=0}$
① when do find soln
② why?
③ B.C.

Heat Transfer Analysis: Insulating Molds

- Insulating mold example: sand mold
- Solidification rate for such molds depends primarily on the thermal properties of the mold
- Assumptions of analysis:
  - One dimensional heat transfer
  - Uniform thickness of solidified metal
  - No thermal resistance at mold-metal interface
  - No temperature gradient within solid and liquid metal
  - Mold is semi-infinite in size
  - Mold thermal properties are uniform
  - Zero superheat
  - Pure metal

k_{mold} $k_{casting}$

(1) $k_{mold} \ll k_{casting}$

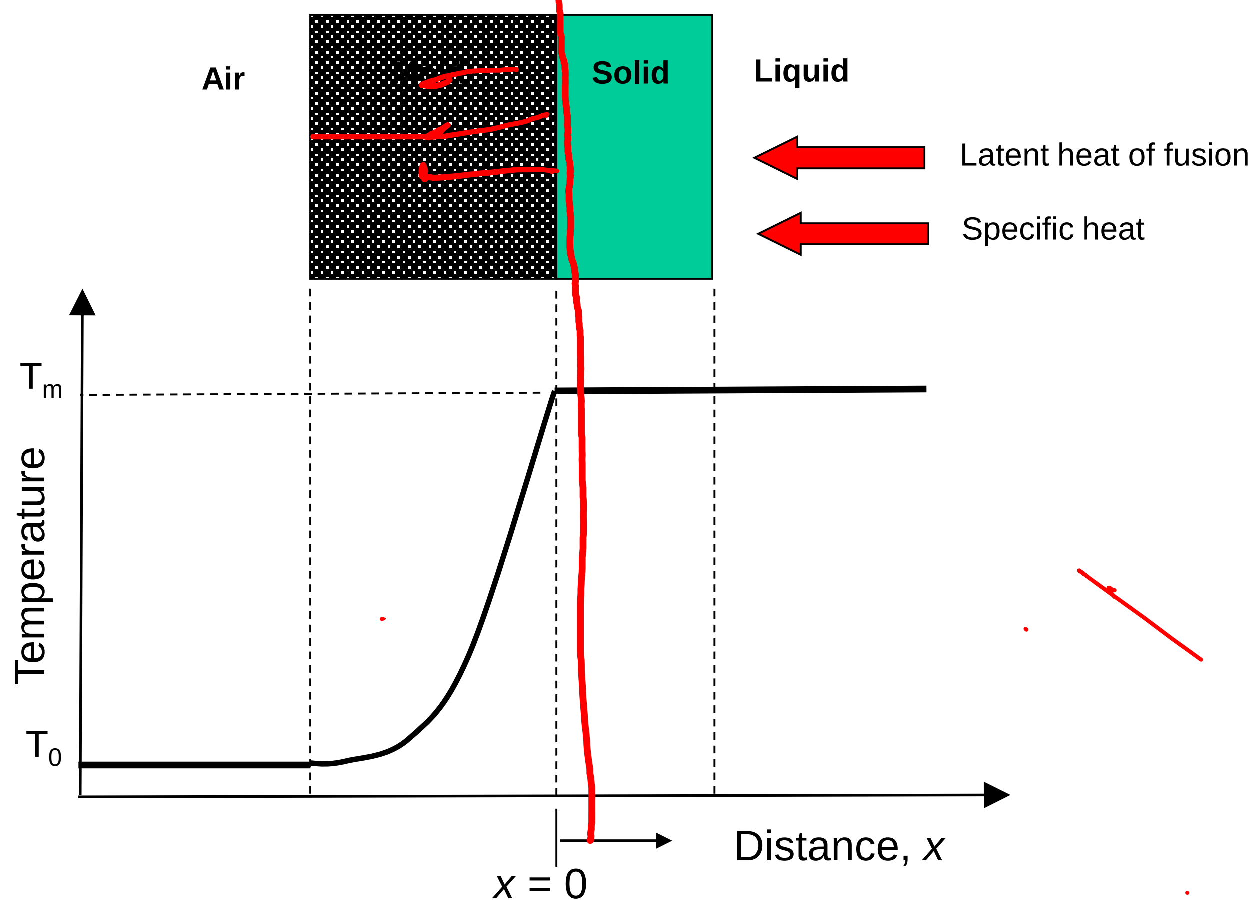
$\alpha = k/\rho c$

$\alpha_{mold} \ll \alpha_{casting}$

k, ρ, c

Heat Transfer Analysis: Insulating Molds

- Implications of assumptions on instantaneous temperature distribution across mold

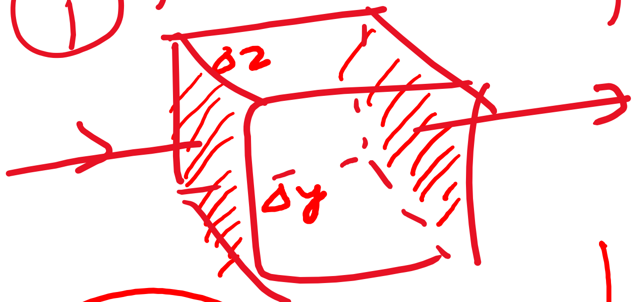


$$\boxed{-KA\frac{\partial T}{\partial x}}$$

cube, $\Delta x, \Delta y, \Delta z$

$$K\frac{\partial^2 T}{\partial x^2} + \cancel{\dot{q}} = \rho C_p \frac{\partial T}{\partial t}$$

① f ② $f+\Delta f$



$$-KA\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-KA\frac{\partial T}{\partial x}\right)\cdot \Delta x$$

$\Delta x \cdot A$

Δx

$-KA\frac{\partial T}{\partial x}$ ①

$= f$

$$\boxed{df = \frac{\partial f}{\partial x}\cdot dx}$$

$$\Delta f = \frac{\partial f}{\partial x}\cdot \Delta x$$

$$K\frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

②

$$\cancel{-\left(KA\frac{\partial T}{\partial x}\right)} + \frac{\partial}{\partial x}\left(-KA\frac{\partial T}{\partial x}\right)\cdot \Delta x - \cancel{\left(-KA\frac{\partial T}{\partial x}\right)} = \boxed{\rho C_p \frac{\partial T}{\partial t}\cdot \Delta x\, A}$$

① $T(x,t) = u(x) \cdot v(t)$

② Laplace

③ Green's function

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k} \frac{\partial T}{\partial t}$$

Conduction

Heat Accumulation

$\alpha = k/\rho c$

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}}$$

→ Laplace Eqn

Heat Transfer Analysis: Insulating Molds

- Governing equation of transient heat transfer:

$$\boxed{\frac{\partial T}{\partial t} = \alpha_m \frac{\partial^2 T}{\partial x^2}} \quad (1)$$

α_m = thermal diffusivity of mold = $k_m/(\rho_m c_m)$

k_m = thermal conductivity of mold

c_m = specific heat of mold

ρ_m = density of mold

- For the assumed boundary conditions, the general solution to (1) is:

$$\boxed{\frac{T - T_m}{T_0 - T_m} = erf\left(\frac{-x}{2\sqrt{\alpha_m t}}\right)} \quad (2)$$

where *erf*() = Gaussian error function

- Note that Eq. (2) can be differentiated to obtain temperature gradient within the mold

Insulating mold

Solution to 1D heat conduction eqn

$$T(x,t) = T_m + (T_o - T_m) \cdot erf\left(\frac{-x}{2\sqrt{\alpha_m t}}\right) \qquad (3)$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x exp(-z^2) \cdot dz = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3 \cdot 1!} + \frac{x^5}{5 \cdot 2!} + \frac{x^7}{7 \cdot 3!} + \ldots \right)$$

differentiating to obtain the temperature gradient

$$\frac{\partial T}{\partial x} = \frac{T_m - T_0}{\sqrt{\pi \alpha_m t}} exp\left(\frac{-x^2}{4\alpha_m t}\right) \qquad (4)$$

$$T(x,t) = T_m + (T_0 - T_m)\,\mathrm{erf}\left(\frac{-x}{2\sqrt{\alpha_m t}}\right)$$

$$\frac{\partial T}{\partial x} = \frac{-e^{\cancelto{1}{-\frac{x^2}{4t\alpha_m}}}\,(T_0 - T_m)}{\sqrt{\pi t \alpha}}$$

$$-kA\left.\frac{\partial T}{\partial x}\right|_{x=0} = \dot{m}\,\Delta H_f = \rho\,\frac{dV}{dt}\cdot\Delta H_f$$

$$= \rho\cdot A\cdot\frac{ds}{dt}\cdot\Delta H_f$$

$$-k\cancel{A}\frac{\partial T}{\partial x}\Big|_{x=0} = \rho_{cas}\cdot\cancel{A}\cdot\frac{ds}{dt}\cdot\Delta H_f$$

$$\frac{k\,(T_m-T_0)}{\sqrt{\pi t \alpha_m}} = \rho_{cas}\cdot\frac{ds}{dt}\cdot\Delta H_f$$

$$\int_0^t \frac{k_m\,(T_m-T_0)}{\Delta H_f\,\rho_{cas}\sqrt{\pi t \alpha_m}}\cdot dt = \int_0^s ds$$

$$S = \frac{2\sqrt{k_m}\sqrt{t}\,(T_m - T_0)}{\Delta H_f \sqrt{\pi}\, \rho_{cast} \sqrt{\alpha_m}}$$

$$\alpha_m = \frac{k_m}{\rho_m c_m}$$

$$S = \frac{2\sqrt{k_m \rho_m c_m}\,\sqrt{t}\,(T_m - T_0)}{\Delta H_f \sqrt{\pi} \cdot \rho_{cast}}$$

$$t_s \propto \left(\frac{V}{A}\right)^2$$

$$\sqrt{t_s} = \frac{S \cdot \rho_{cas} \cdot \Delta H_f \sqrt{\pi}}{2\sqrt{k_m \rho_m c_m}\,(T_m - T_0)}$$

$(V/A)^2$

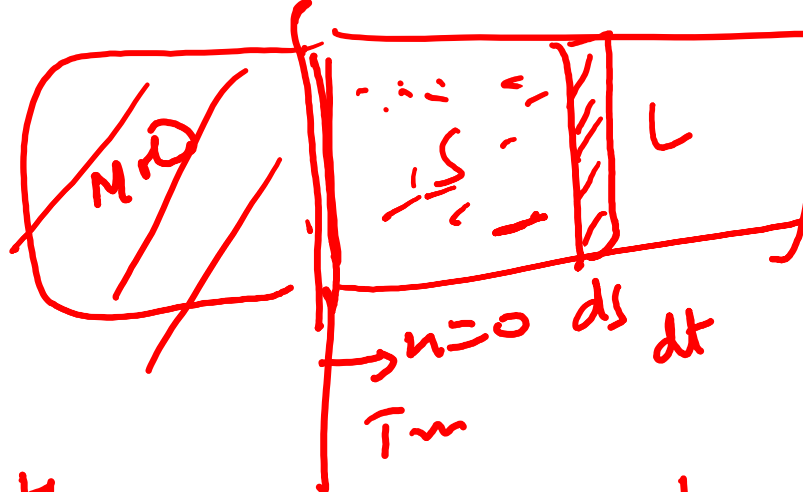
$$t_s = \frac{S^2 \pi}{4 k_m \rho_m c_m} \left(\frac{\rho_{cas} \Delta H_f}{(T_m - T_0)}\right)^2$$

Rate Steady State

J/kg

$\dot{m} H_f$

J/s



$$KA \left.\frac{dT}{dx}\right|_{x=0} = \dot{m} H_f$$

$$\left.\frac{dT}{dx}\right|_{x=0}$$

$$KA \left.\frac{dT}{dx}\right|_{x=0} = \rho \cdot \frac{dV}{dt} H_f$$

$$KA \frac{T_m - T_o}{\sqrt{\pi \alpha_m t}} = \rho \frac{dV}{dt} H_f = \rho A \frac{ds}{dt} H_f$$

$$K_m \frac{T_m - T_o}{\sqrt{\pi \alpha_m t}} = \rho_{cast} \cdot H_f \frac{ds}{dt}$$

sand/mold

$$\frac{K_m (T_m - T_o) \cdot dt}{H_f \cdot \rho_{cast} \sqrt{\pi \alpha_m t}} = ds$$

$$\alpha_m = \frac{k_m}{\rho_m C_m}$$

$$\frac{1}{H_f \rho_{cast}} \int \sqrt{\frac{K_m \rho_m C_m}{\pi}} (T_m - T_o) = \int_0^s ds$$

$$\sqrt{t} = \frac{S \cdot H_f \cdot \rho_{cast} \sqrt{\pi}}{2 (T_m - T_o) \sqrt{K_m \rho_m C_m}}$$

$$\sqrt{t} = \frac{S H_f \, \rho \cos\theta \sqrt{\pi}}{2(T_m - T_0)\sqrt{k_m \rho_m c_m}}$$

$$t = \left[\frac{S H_f \quad \rho \cos\theta}{2(T_m - T_0)}\right]^2 \frac{}{k_m \rho_m c_m}$$

$$\boxed{t = \frac{\pi}{4}\left(\frac{H_f \, \rho \cos\theta}{(T_m - T_0)}\right)^2 \left(\frac{V}{A}\right)^2 \cdot \frac{1}{k_m \rho_m c_m}}$$

Insulating Mold

$$t = \frac{\pi}{4}\left[\frac{\rho_{casting} H_f}{(T_m - T_0)}\right]^2 \left(\frac{V}{A}\right)^2 \frac{1}{k_m \rho_m c_m}$$

$K_m << K_{casting}$

K, ρ, c

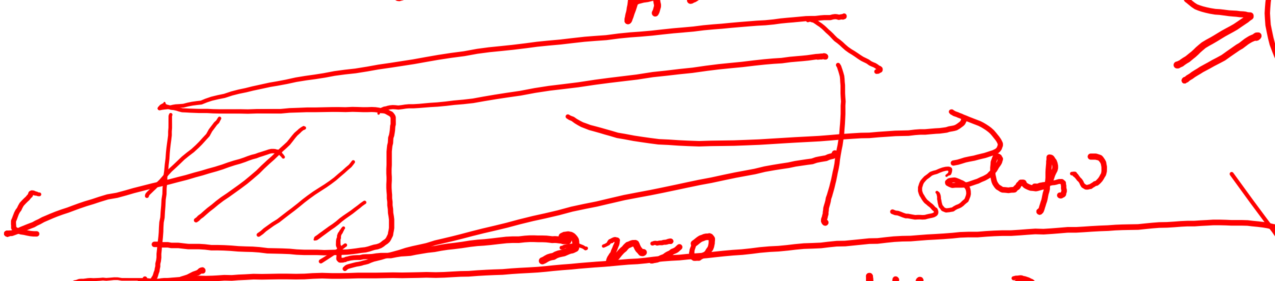
$\alpha_m << \alpha_{casting}$

What affects solidification?

reduce $\left(\frac{V}{A}\right)$

$t \propto \left(\frac{V}{A}\right)^2$

$\left(\frac{V}{A}\right)^2_{riser} > \left(\frac{V}{A}\right)^2_{casting}$



$V = A \frac{ds}{dt}$

$$KA \frac{dT}{dx}\Big|_{x=0} = H_f \cdot \frac{dV}{dt} \rho_{casting}$$

Heat Transfer Analysis: Insulating Molds

- Quantity of practical interest: solidification time
- Can be obtained from energy balance at mold-solid interface
- Rate of heat flow into mold at mold-solid interface:

$$\dot{Q}\Big|_{x=0} = -k_m A \frac{\partial T}{\partial x}\Big|_{x=0} \qquad (5)$$

A = area of mold-metal interface

$$\dot{Q}\Big|_{x=0} = -A\sqrt{\frac{k_m \rho_m c_m}{\pi t}}\left(T_m - T_0\right) \qquad (6)$$

Insulating mold

if the metal is cast at its melting temperature, then the heat entering the mold is the latent heat of fusion

$$\frac{dQ}{dt} = \rho_{casting} \Delta H_f \frac{dV}{dt} = \rho_{casting} \Delta H_f A \frac{dS}{dt} \quad (7)$$

- ΔH_f = latent heat of fusion
- V = volume of solidified metal
- A = area of mold-metal interface
- S = thickness of solidified metal (x)

Insulating mold

- Corresponding heat flux

$$\left(\frac{q}{A}\right)_{casting} = \rho_{casting} \Delta H_f \frac{dS}{dt} \qquad (8)$$

Insulating mold

- heat flux away from mold-metal interface = heat flux to mold-metal interface due to solidification

- so $$\left(\frac{q}{A}\right)_{mold_{x=0}} = \left(\frac{q}{A}\right)_{casting} \qquad (9)$$

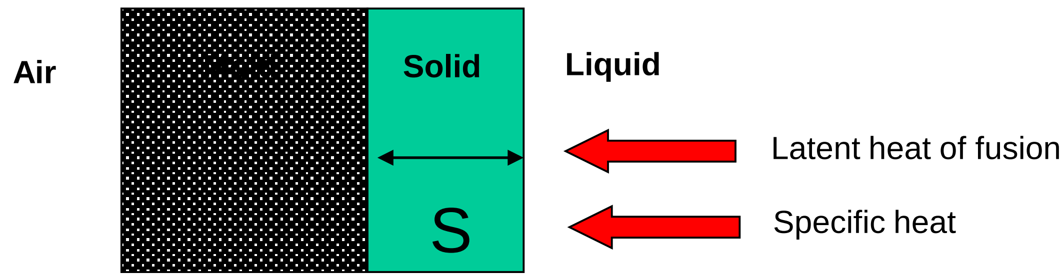
$$\sqrt{\frac{k_m \rho_m c_m}{\pi t}}\left(T_m - T_0\right) = \rho_{casting} \Delta H_f \frac{dS}{dt} \qquad (10)$$

Heat Transfer Analysis: Insulating Molds

Integrating from S = 0 and t = 0 to S = S and t = t

$$S = \frac{2}{\sqrt{\pi}}\left(\frac{T_m - T_0}{\rho_{casting}\Delta H_f}\right)\sqrt{k_m \rho_m c_m t} \qquad (11)$$

Also, S = V/A



Solidification time

$$t = \left[\frac{\pi}{4} \left(\frac{\rho_{casting} \Delta H_f}{T_m - T_0} \right)^2 \frac{1}{k_m \rho_m c_m} \right] \left(\frac{V}{A} \right)^2 \qquad (12)$$

mold

one-dim

- Subscripts

  m = mold
- ΔH_f = latent heat of solidification
- T_m = metal melting temperature
- T_0 = initial mold temperature

Solidification time – Ex. 5-1

- You are sand casting a magnesium part with dimensions of 10 cm by 10 cm by 2.5 cm. The environment temperature is 25^{o}C.
- Determine the time for the part to solidify if the metal is poured at its melting point.
- Determine the time for the part to solidify if the metal is poured at 50^{o}C above its melting point, so as to alleviate the potential problem of short shots.

Solidification time – Ex. 5-2

Material	Specific heat (kJ/kg-^oC)	Density (kg/m^3)	Thermal conductivity (W/m-K)
Sand (solid)	1.16	1500	0.6
Magnesium (solid)	1.07	1700	154
	Melting point (^oC)	Latent heat of solidification (kJ/kg)	Specific heat (kJ/kg-K)
Magnesium (liquid)	650	384	1.38

Solidification time – Ex. 5-3

- N.B. solidification is a phase change that occurs at the melting point
- Insulating mold:
  - $k_{mold} = 0.6 << k_{casting} = 154$ W/m-K
  - $\alpha_{mold} = 3.4 \times 10^{-7} << \alpha_{casting} = 6.6 \times 10^{-5}$ m^2/s
- Solidification time:

$$t = \left[\frac{\pi}{4} \left(\frac{\rho_c \Delta H_f}{T_m - T_0} \right)^2 \frac{1}{k_m \rho_m c_m} \right] \left(\frac{V}{A} \right)^2$$

Solidification time – Ex. 5-4

- ΔH_f = 384 kJ/kg
- ρ_c = 1700 kg/m^3
- T_m = 650^{o}C
- T_o = 25^{o}C
- k_m = 0.6 x 10^{-3} kW/m-K
- ρ_m = 1500 kg/m^3
- c_m = 1.16 kJ/kg-K

Solidification time – Ex. 5-5

- $V = 0.1 \times 0.1 \times 0.025 = 2.5 \times 10^{-4}\ m^3$
- $A = 2 \times (0.1 \times 0.1) + 4 \times (0.1 \times 0.025) = 0.03\ m^2$
- $(V/A)^2 = 6.94 \times 10^{-5}\ m^2$

Solidification time – Ex. 5-6

- So

$$t = \left[\frac{\pi}{4} \left(\frac{1700 \times 384}{650 - 25} \right)^2 \frac{1}{0.6 \times 10^{-3} \times 1500 \times 1.16} \right] \left(6.94 \times 10^{-5} \right)$$

- t = 57 s

Solidification time – Ex. 5-7

- Now, we have to take into account cooling the liquid from (650 + 50)^{o}C to 650^{o}C
- So, the latent heat of solidification (ΔH_f) will be increased by $c_p \Delta T$

Solidification time – Ex. 5-8

- For liquid magnesium
  - $c_p = 1.38$ kJ/kg-K
  - $\Delta T = 50^oC$
- So

$$\Delta H_f = H_f + c_p \Delta T$$

$$= 384 + 1.38 \times 50 = 453 \text{ kJ/kg}$$

Solidification time – Ex. 5-9

- So

$$t = \left[\frac{\pi}{4} \left(\frac{1700 \times 453}{650 - 25} \right)^2 \frac{1}{0.6 \times 10^{-3} \times 1500 \times 1.16} \right] \left(6.94 \times 10^{-5} \right)$$

t = 79 s (a bit slower)

Heat Transfer Analysis: Insulating Molds

- Replacing *S* with (*V/A*), where *V* = volume of metal solidified at time *t* and re-arranging

$$t = \left[\frac{\pi}{4} \left(\frac{\rho_{casting} \Delta H_f}{T_m - T_0} \right)^2 \frac{1}{k_m \rho_m c_m} \right] \left(\frac{V}{A} \right)^2 \qquad (13)$$

$$\boxed{t = C_m \left(\frac{V}{A} \right)^2} \qquad (14) \Rightarrow \text{Chvorinov's Rule}$$

- Chvorinov's rule can be used for more complex castings to provide a first approximation of solidification time

Riser Design

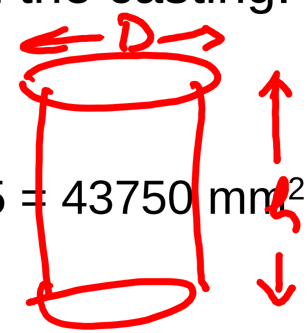
- Riser design is based on Chvorinov's rule
- Function of riser: to feed the mold cavity with molten metal in order to compensate for shrinkage

$$\boxed{t_{riser} \geq t_{mold}} \Rightarrow \left(\frac{V_{riser}}{A_{riser}}\right)^2 \geq \left(\frac{V_{mold}}{A_{mold}}\right)^2 \Rightarrow \left(\frac{V_{riser}}{A_{riser}}\right) \geq \left(\frac{V_{mold}}{A_{mold}}\right)$$

- Design and place risers so that solidification begins in the casting and ends in the riser → shrinkage defects such as voids, porosity are limited to the riser

Riser Design Example

An open cylindrical riser must be designed for a sand mold. The part to be cast is a 125 mm x 125 mm x 25 mm plate. The foundryman knows from past experience that the total solidification time for casting this part is 2 min. It is required that the height-to-diameter ratio of the riser be 1. Find the dimensions of the riser so that its total solidification time is 30% longer than the casting.



Volume of casting, $V_c = 125 \times 125 \times 25 = 390625\ mm^3$

Surface area of casting, $A_c = 2 \times 125 \times 125 + 4 \times 125 \times 25 = 43750\ mm^2$

For defect-free casting it is required that $t_{riser} \geq t_{casting}$

Volume of riser, $V_r = \pi D^2 h/4 = \pi D^3/4$

Surface area of riser across which heat transfer takes place, $A_r = \pi D h + \pi D^2/4$

(Note: no heat transfer across riser-casting boundary)

Problem states that $t_{riser} = 1.3\ t_{casting}$

$(V_r/A_r)^2 = 1.3(V_c/A_c)^2 \rightarrow \boxed{\mathbf{D = h = 50.74\ mm}}$

$t_{s\ riser} >$

$1.3\, t_{s\ casting}$

$l_c = 125$

$b_c = 125$

$h_c = 25$

$V_c = l_c b_c h_c$

$A_c = 2(l_c b_c + h_c b_c + l_h b_h)$

$\frac{h_r}{d_r} = 1$

$t_s = 2\,\text{min}$

Cylindrical riser

$V_r = \frac{\pi d_r^2}{4} \cdot h_r$

$A_r = \frac{\pi d_r^2}{4}$

If top riser

$l_c b_c h_c$

Top riser

Side riser

$t_r\ 2$ $\quad$ $t_c\ 2$

$$\left(\frac{V_r}{A_r}\right) \geq 1.3\left(\frac{V_c}{A_c}\right)$$

$$V_c = l_c b_c h_c$$

$$A_c = 2(l_c b_c + b_c h_c + l_c h_c)$$

casting

Riser

$$V_r = \frac{\pi d_r^2}{4} h_r$$

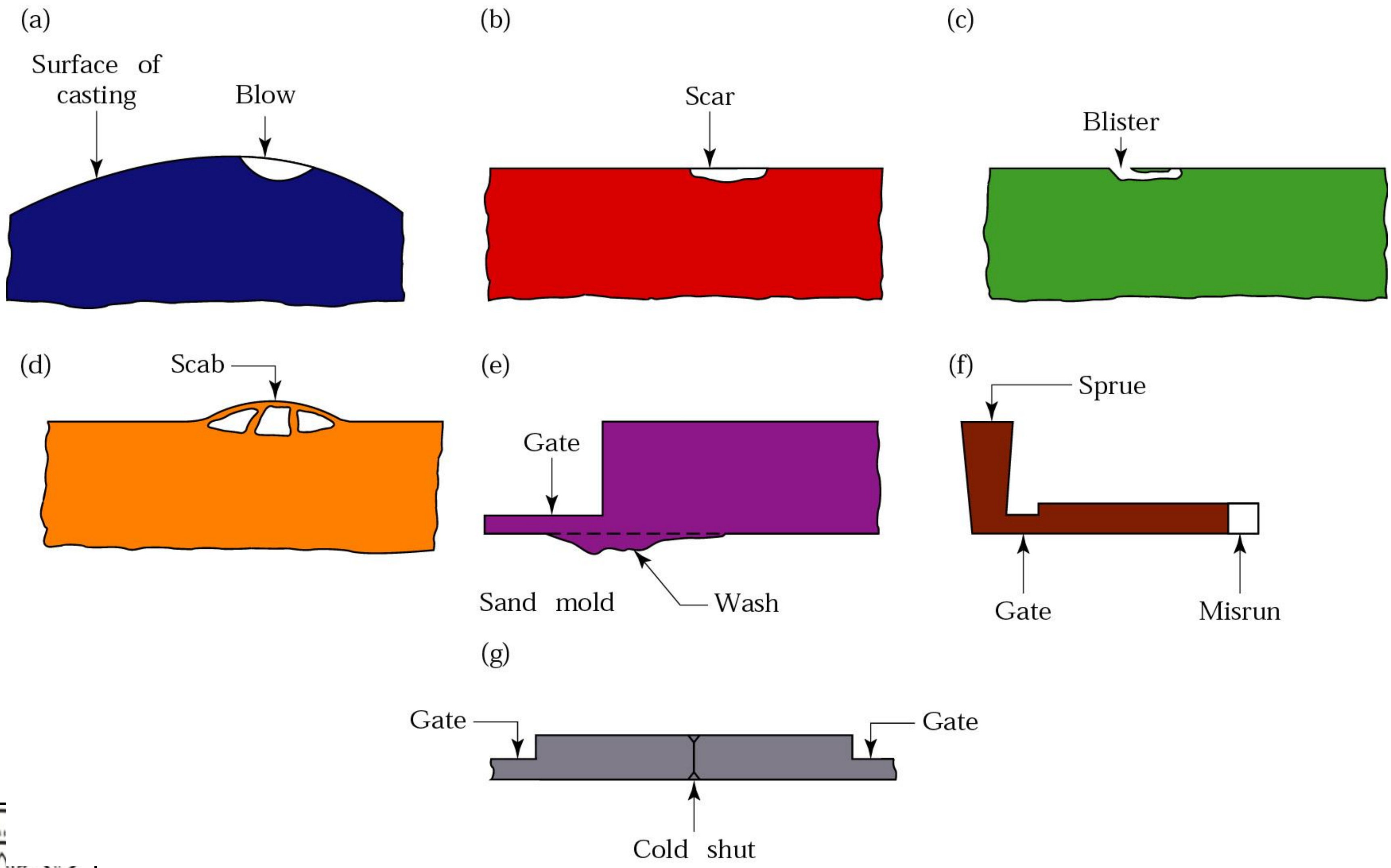
$$A_r = \pi d_r \cdot h_r + \frac{\pi}{4} d_r^2$$

$$h_r / d_r = 1$$

$$1.3\left(\frac{V_c}{A_c}\right)^2 = \left(\frac{V_r}{A_r}\right)^2 = \frac{\pi/4 \, d_r^3}{\pi \cdot d_r^2 + \pi/4 \cdot d_r^2}$$

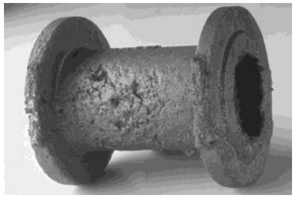
d_r ?

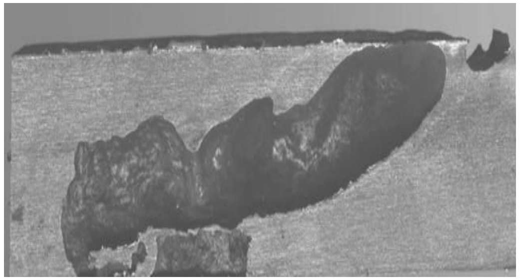
Casting Defects



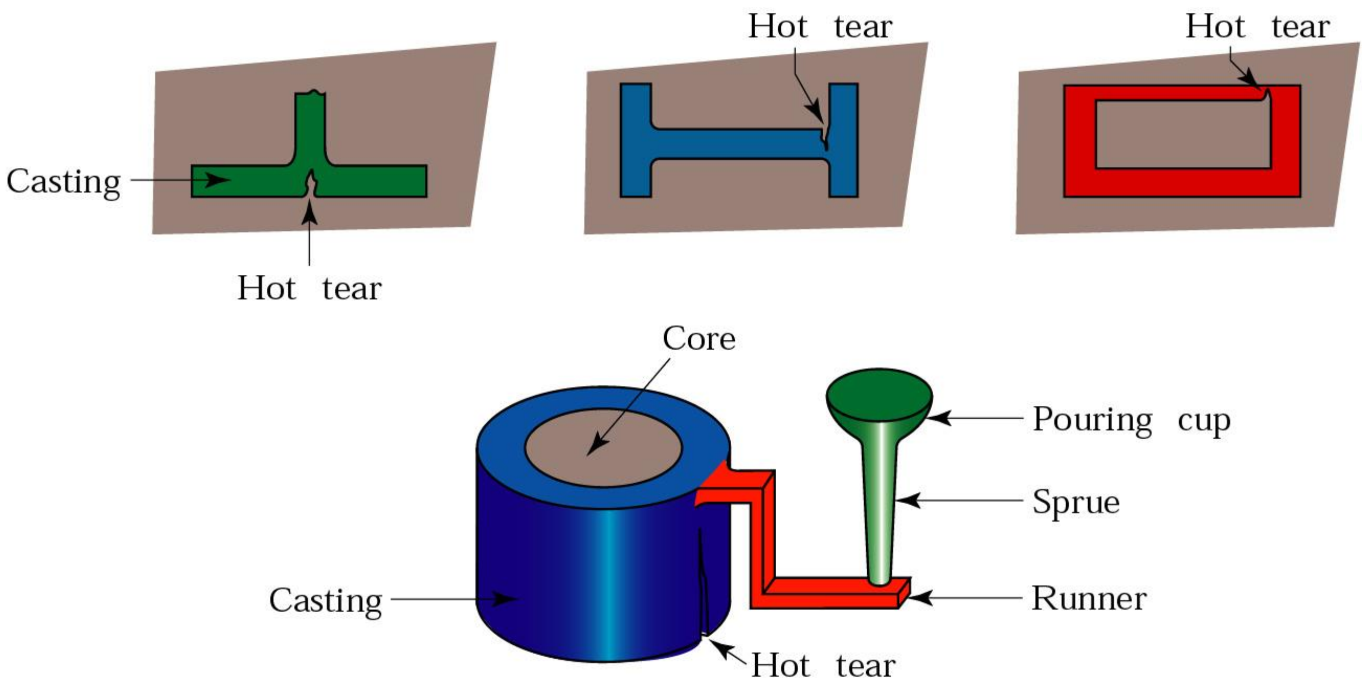
Casting Defects

- Metallic projections
  - Flash: excess metal solidified outside mold cavity
    - *Causes*: insufficient clamping force, improper parting line
- Cavities (voids)
  - Shrinkage cavities: voids inside casting
    - *Causes*: contraction during solidification
    - *Remedies*: use similar (V/A) ratios, use gradually increasing section modulus toward riser, proper gating/riser design, use of chills
  - Blowholes: void on surface of casting
    - *Causes*: excessive gas entrapment, lack of adequate venting
    - *Remedies*: de-gas melt, add vents



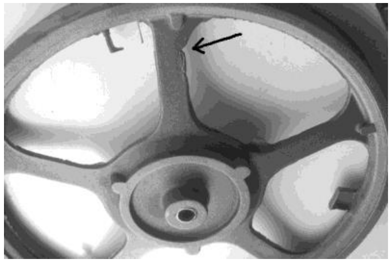


Defects - Hot Tears



Casting Defects

- Discontinuities
  - Hot tears: intercrystalline failure in casting that occurs at a high temperature within mold; usually forms in sections that solidify last and where geometrical constraints are present
    - *Causes*: large differences in section thickness, abrupt changes in section thickness, too many branching/connected sections, mold has high hot strength and stiffness
    - *Remedies*: through casting and mold redesign
  - Cold shut: incomplete fusion of two molten metal flows that meet inside the mold from opposite directions
    - *Causes*: insufficient superheat, inadequate risers
    - *Remedies*: increase superheat, add additional risers



Casting Defects

- Defective surface
  - Scabs: thin layer of molten metal that enters gaps in mold and solidifies
    - *Causes*: improper mold design
- Incomplete castings
  - Misrun: incomplete casting
    - *Cause*: insufficient superheat
    - *Remedy*: increase superheat
- Inclusions
    - *Remedy*: clean melt before pouring, improve strength of mold