MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 3

(January 26, 2022)

The accuracy in problems (1) – (4) is expected within 10^{-2}.

(1) Use the Newton-Raphson method with $p_0 = -1.5$ to solve
$$\cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) = 0.$$

(2) Use the Newton-Raphson method with $p_0 = -0.5$ to solve
$$e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3 = 0.$$

(3) Use the modified Newton-Raphson method in problem (1) above.

(4) Use the modified Newton-Raphson method in problem (2) above.

(5) For $p_0 = 0.5$ and $p_n = \dfrac{2 - e^{p_{n-1}} + p_{n-1}^2}{3}$, generate first five terms of the sequence $\{\hat{p}_n\}$ using the Aitken's Δ^2-method.

(6) Find appropriate polynomials of degree at most one and at most two interpolating $f(x) = \cos x$ on $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$ to approximate $\cos(0.45)$. Find the absolute errors.

(7) Repeat the above problem for $f(x) = \sqrt{1 + x}$.

(8) Use appropriate Lagrange polynomials of degrees one, two and three to find $f(8.4)$ with the following data:
$$\begin{aligned} f(8.1) &= 16.94410 \\ f(8.3) &= 17.56492 \\ f(8.6) &= 18.50515 \\ f(8.7) &= 18.82091 \end{aligned}$$

(9) Use appropriate Lagrange polynomials of degrees one, two and three to find $f(0.25)$ with the following data:
$$\begin{aligned} f(0.1) &= -0.29004986 \\ f(0.2) &= -0.56079734 \\ f(0.3) &= -0.81401972 \\ f(0.4) &= -1.0526302 \end{aligned}$$