ME 202

LECTURE 8

TUTORIAL 2

MON 18 JAN 2022

Problem 1

Torsional pendulum

Gere & Goodno 7th ed.

$$\text{Torque} = \frac{GJ}{L}\theta$$

$$I\ddot{\theta} + \frac{GJ}{L}\theta = 0$$

$$\ddot{\theta} + \omega_t^2 \theta = 0, \quad \omega_t = \sqrt{\frac{GJ}{LI}}$$

GJ ← wire

LI ← mass moment of inertia

Max safe angle of twist under torsion only.

$$\tau_{max} = \frac{16T}{\pi D^3} = \frac{16}{\pi D^3}\frac{G}{L}\frac{\pi D^4}{32}\theta_{max}$$

τ_{max}: from expt given

$$\theta_{max} = \frac{2L\tau_{max}}{GD}$$

However, wire is under both torsion & tension.

$$\sigma_{zz} = \frac{P}{A} = \frac{4Mg}{\pi D^2}, \quad \sigma_{xz} = \frac{16T}{\pi D^3}$$

$$\text{Max shear stress} = \left(\left(\frac{\sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2\right)^{1/2} = \tau_{max}$$

$$\sqrt{\left(\frac{2Mg}{\pi D^2}\right)^2 + \left(\frac{GD\theta_{max}}{2L}\right)^2} = \tau_{max}$$

Problem 2

$\omega = 2\pi f$

↑ rad/s ↑ Hz

$\text{Power} = T\omega = T\,2\pi f$

$$\text{Torque } T = \frac{1800 \times 10^3}{2\pi \times 4}$$

$$= 71.62 \times 10^6 \text{ N mm}$$

$$P = -540 \times 10^3 \text{ N}$$

$d_2 = 300\ mm, \quad d_1 = 250\ mm$

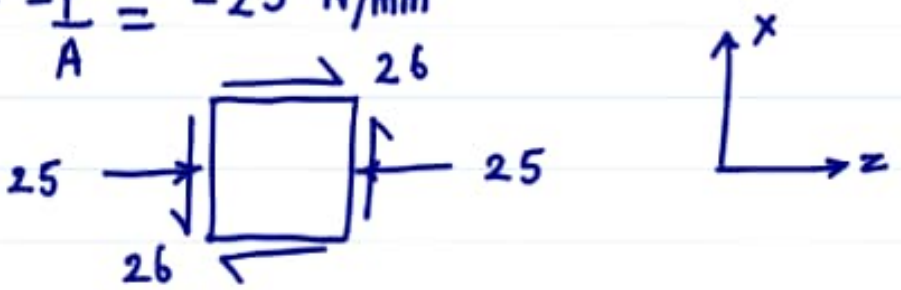
$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 21{,}598\ mm^2$$

$$J = \frac{\pi}{32}(d_2^4 - d_1^4) = 411{,}720{,}443\ mm^4$$

$$\tau_{yz} = +G\alpha x = 0$$

$$\tau_{xz} = -G\alpha y = \frac{T}{J}\frac{d_2}{2} = 26\ N/mm^2 = 26\ MPa$$

$$\tau_{zz} = -\frac{P}{A} = -25\ N/mm^2$$



Principal Stresses

$$\tau_1, \tau_2 = \frac{\tau_{zz} + \tau_{xx}}{2} \pm \sqrt{\left(\frac{\tau_{zz} - \tau_{xx}}{2}\right)^2 + \tau_{zx}^2}$$

$$\tau_1 = 16\ MPa, \quad \tau_2 = -41\ MPa$$

Max Tensile Stress = 16 MPa

Max Comp Stress = 41 MPa

Max Shear Stress = $\frac{\sigma_1 - \sigma_2}{2}$ = 28.5 MPa

Problem 3

$$\sigma_{zz} = \sigma_{zz}^{\text{bending}} = \frac{32\,M}{\pi D^3}$$

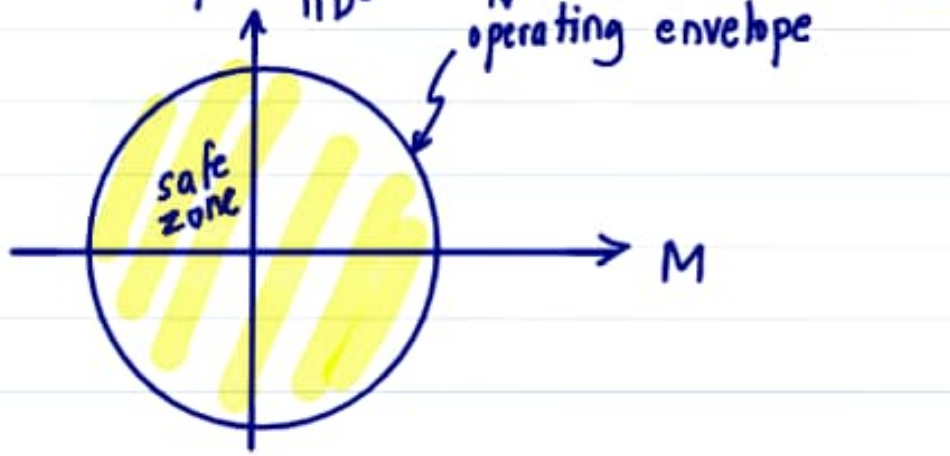
$$\sigma_{zx} = \frac{16\,T}{\pi D^3}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + \sigma_{zx}^2} = S \quad \text{(given)}$$

$$\sqrt{\left(\frac{16\,M}{\pi D^3}\right)^2 + \left(\frac{16\,T}{\pi D^3}\right)^2} = S$$

$$\left(M^2 + T^2\right)^{1/2} \frac{16}{\pi D^3} = \frac{S}{N}$$

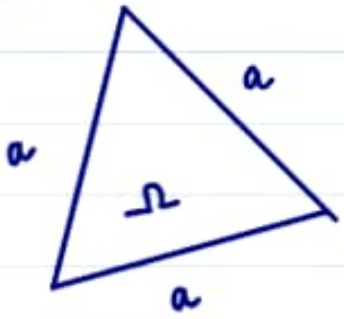
factor of safety > 1



HW: P, M, T space

Problem 4

4.1



Goal $T = K_t \alpha$

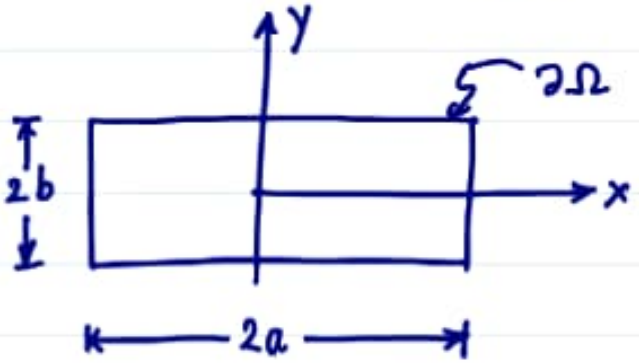
Find $\varphi(x,y)$ s.t. $\nabla^2 \varphi = -2G\alpha$ inside Ω

$\varphi = 0$ on $\partial\Omega$

$\varphi = K\, F(x,y)$

$T = 2\int_\Omega \varphi \, da$

4.2



Try Same trick $\varphi(x,y) = K(x-a)(x+a)(y-b)(y+b)$

$= K(x^2-a^2)(y^2-b^2)$

$\varphi = 0$ on $\partial\Omega$

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = K(y^2 - b^2)2 + K(x^2 - a^2)2$$

$$\neq -2G\alpha \quad \forall\ x, y \in \Omega$$

↑ for all

Work: Fourier series

Consider, thin rectangle $a \gg b$