Experiment 6: Strains in a Ring under Combined Bending and Extension

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1 Objective

The aim of this experiment is

- To measure strains using bonded foil strain gauges in combination with a Wheatstone Bridge.
- Compare with linear elastic solution in a "proving" ring (circular beam with rectangular cross-section) subjected to combined extension and bending with strains measured from experiment.

2 Experimental Setup

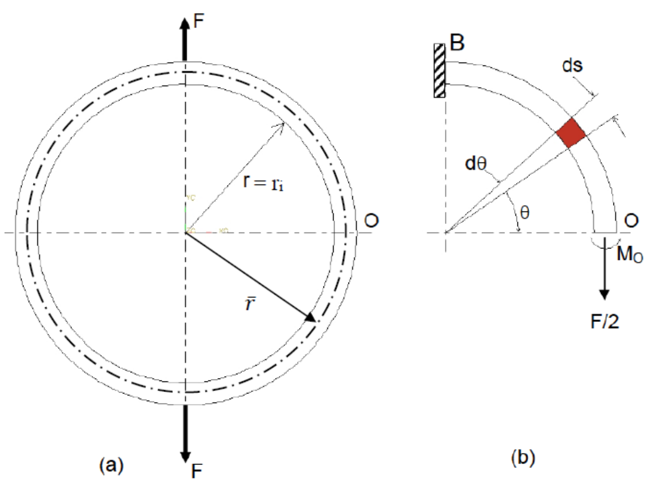
The instruments used in the experiment are:

1. **Wheatstone bridge module with Strain Gauges**: The module has the three fixed resistors of the Wheatstone bridge set up and we install the strain gauge as the fourth variable resistor. The strain gauges used in the experiment have a resistance of 120 ohms.
2. **Strain Indicator**: This is a variable conversion machine which converts in the voltage value of the bridge directly into strain.
3. **Circular ring**: The curved beam specimen made of Aluminium which undergoes loading. It is loaded at the bottom-most point of the circle using weights.

3 Theory

A thin ring of internal radius r_i subjected to a diametrical pull F is shown in the figure (a) below. Using symmetry the free body diagram of a quarter ring is shown in figure (b). At $\theta = 0$, the shear force is zero and hence only an axial load of F/2 and bending moment M_0, which is indeterminate are acting on the cross section.

We measure the strains at 4 locations:
Outer and innermost point at $\theta = 180°$ by Strain gauge 1 and Strain gauge 2 respectively
Inner and outermost point at $\theta = 0°$ by Strain gauge 3 and Strain gauge 4 respectively



We gather the experiment data by loading the weight hanger with weights to provide the diametric pull on the circular ring. We take readings of the strain values from the strain indicator. Before loading, we set the strain indicator to zero so that the strain readings for zero loading is zero. We take strain readings for 1 kg, 2 kg, 3 kg, 4 kg and 5 kg weights.

To find the theoretic values of strains, we start with using the energy method to find the value of M_0 at $\theta = 0°$,

$$M_0 = \frac{Fr}{2}\left(1 - \frac{2}{\pi}\right)$$

And, the bending stress due to moment is:

$$\sigma_b(r) = \frac{M(R-r)}{Ar(\bar{r}-R)} = \frac{F\left(\frac{1}{2}-\frac{1}{\pi}\right)(R-r)}{A(\bar{r}-R)} = \frac{F\left(\frac{1}{2}-\frac{1}{\pi}\right)(R-r)}{Ae}$$

Total stress on the cross section due to bending moment and axial stress:

$$\sigma(r) = \frac{F}{2A} + \sigma_b(r) = \frac{F}{2A} + \frac{F\left(\frac{1}{2}-\frac{1}{\pi}\right)(R-r)}{Ae}$$

Where E is the Young's Modulus, the strain is :

$$\epsilon(r) = \frac{\sigma(r)}{E} = \frac{F}{2AE} + \frac{F\left(\frac{1}{2}-\frac{1}{\pi}\right)(R-r)}{AEe}$$

4 Results

Given,
$D_o = 131$ mm & $D_i = 108.6$ mm,
The Young's modulus of the aluminium specimen, E = 69 GPa

$$R = \frac{r_o - r_i}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{65.5 - 51.08}{\ln\left(\frac{65.5}{51.08}\right)} = 59.725 \text{ mm}$$

$$e = \bar{r} - R = \frac{r_o + r_i}{2} - R = 0.1749 \text{ mm}$$

$$A = (r_o - r_i)\,t = (65.5 - 54.3) * 9.7 = 108.64 \text{ mm}^2$$

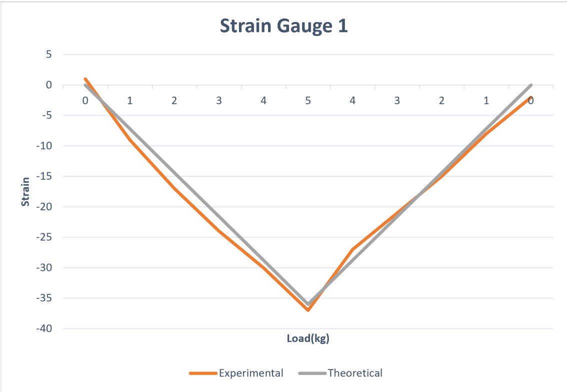
With all these values calculated, we can plug them along with the load value (F) and the location (r) where we want to find the strain in the formula to get the theoretical strain. Let us look at one example of the calculation for theoretical strain and error : For strain gauge 1 (r=r_o) and 1 kg weight (F=9.8 N)

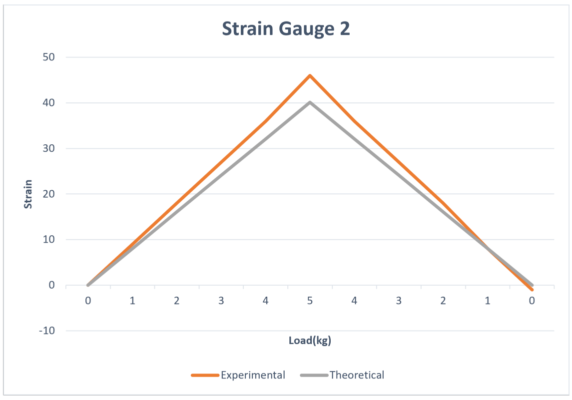
$$\epsilon\left(r_o\right) = \frac{F}{2AE} + \frac{F\left(\frac{1}{2}-\frac{1}{\pi}\right)(R-r_o)}{AEe} = \frac{9.81}{2 * 108.64 * 10^{-6} * 69 * 10^{9}} + \frac{9.81\left(\frac{1}{2}-\frac{1}{\pi}\right)(59.725-65.5) * 10^{-3}}{108.64 * 10^{-6} * 69 * 10^{9} * 0.1749 * 10^{-3}}$$
$$= -7.197067918 * 10^{-6}$$

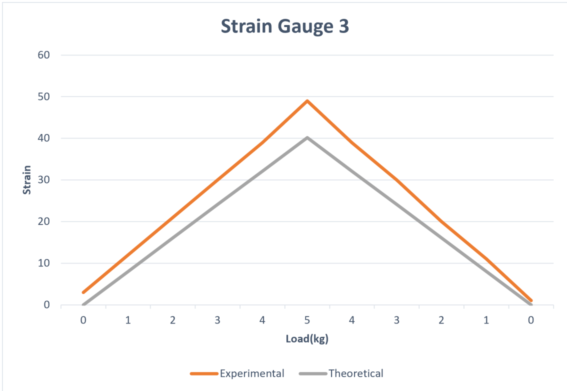
$$\text{Error } = \frac{\left(-9 * 10^{-6}\right) - \left(-7.197067918 * 10^{-6}\right)}{-7.197067918 * 10^{-6}} = 25.05\%$$

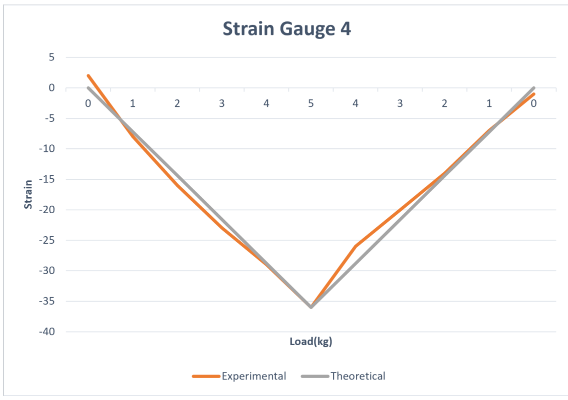
Similarly, all the results for the experiment can be compiled in the table below.
Note: The experimental and theoretical strain values are of the order of microns (10^{-6})

Load (in Kg)	S1 (×10^(-6))	Theoritical S1(10^-6)	Error %	S2 (×10^(-6))	Theoritical S2(10^-6)	Error %	S3 (×10^(-6))	Theoritical S3(10^-6)	Error %	S4 (×10^(-6))	Theoritical S4(10^-6)	Error %
0	1	0	0	0	0	0	3	0	0	2	0	0
1	-9	-7.197067918	25.05092495	9	8.029895354	12.08116174	12	8.029895354	49.441549	-8	-7.197067918	11.1563777
2	-17	-14.39413584	18.10365134	18	16.05979071	12.08116174	21	16.05979071	30.7613554	-16	-14.39413584	11.1563777
3	-24	-21.59120375	11.15637773	27	24.08968606	12.08116174	30	24.08968606	24.5346242	-23	-21.59120375	6.52486199
4	-30	-28.78827167	4.209104122	36	32.11958142	12.08116174	39	32.11958142	21.4212586	-29	-28.78827167	0.73546732
5	-37	-35.98533959	2.8196494	46	40.14947677	14.57185423	49	40.14947677	22.0439317	-36	-35.98533959	0.04073996
5	-37	-35.98533959	2.8196494	46	40.14947677	14.57185423	49	40.14947677	22.0439317	-36	-35.98533959	0.04073996
4	-27	-28.78827167	6.211806291	36	32.11958142	12.08116174	39	32.11958142	21.4212586	-26	-28.78827167	9.68544309
3	-21	-21.59120375	2.738169486	27	24.08968606	12.08116174	30	24.08968606	24.5346242	-20	-21.59120375	7.36968523
2	-15	-14.39413584	4.209104122	18	16.05979071	12.08116174	20	16.05979071	24.5346242	-14	-14.39413584	2.73816949
1	-8	-7.197067918	11.15637773	8	8.029895354	0.372300673	11	8.029895354	36.9880866	-7	-7.197067918	2.73816949
0	-2	0	0	-1	0	0	1	0	0	-1	0	0









5 Observations

- The outer circumference of the circular ring undergoes compression due to the effect of bending moment, so we get a negative strain value.
- Similarly, the inner circumference of the circular ring undergoes tension due to the effect of bending moment, so we get a positive strain value.
- Strain increases with the increase in load and is directly proportional theoretically. A similar pattern can be observed in the experimental values too.
- Strain values recorded on Gauge 1 and Gauge 4 are the same, and so are the values recorded on Gauge 2 and Gauge 3. This tells us that the force is indeed equally distributed in a symmetric specimen.
- We observe the same strain values during loading and unloading to which we can comment that the specimen is within it's elastic range.
- We observe that the magnitude of strain is smaller on the outer circumference than the inner circumference. Note that the bending stress when defined in terms of force is directly proportion to the radial distance of the point on the specimen from the neutral axis or (R-r). We can see that the neutral axis is closer to the inner circumference than the outer circumference and we can also observe that the magnitude of bending stress is larger for the point on the outer circumference.
  However, the tensile force results in a tensile stress which opposes the compressive bending stress on the outer circumference and adds to the tensile bending stress in the inner circumference.
  So finally, we get that the net magnitude of stress is smaller on the outer circumference. than the inner circumference, and so is strain.

6 References

1. Lab Manual
2. Wikipedia