ME 202

LECTURE 20

28 FEB 2022

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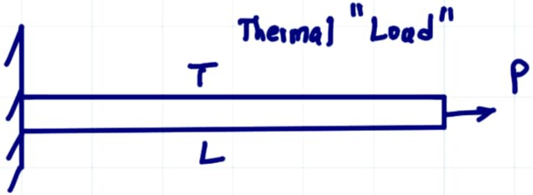
THERMAL STRESSES/ DEFLECTION IN BEAMS DUE TO TEMPERATURE



$\Delta T = T$ temp difference

E, α CTE $/^{\circ}K$ $mm/mm^{\circ}K$

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δ = total axial deflection

$$\delta = \frac{PL}{AE} + \alpha T L$$

$$\frac{\delta}{L} = \frac{P}{AE} + \alpha T$$

$$\epsilon = \epsilon_e + \epsilon_t$$

total strain = elastic strain + thermal strain

anelastic strain

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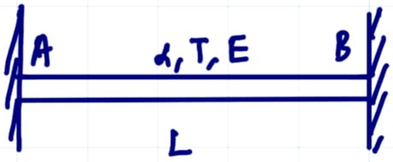
Extend this,

$$\epsilon = \frac{\sigma}{E} + \alpha T = \frac{dw}{dz}$$

$$\sigma = E(\epsilon - \alpha T)$$ Mod Hooke's Law

Thermal Stresses

$$' \equiv \frac{d}{dz}$$



Method 1

$$\sigma = E\,(w' - \alpha T)\,, \qquad \sigma' + \cancel{b} = 0$$

$$w'' = 0 \Rightarrow w(z) = c_1 z + c_2$$

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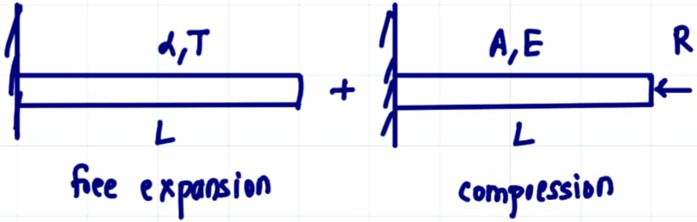
BCs $\Rightarrow$ $c_2 = 0$ $\Rightarrow$ $w = 0$

$\sigma = -E\alpha T$ $\quad$ $E\alpha T$ compressive

Method 2



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Deflection at B = 0

$$0 = \alpha T L - \frac{RL}{AE} \Rightarrow R = AE\alpha T$$

$$\text{Compressive stress} = 0 + \frac{R}{A} = E\alpha T$$

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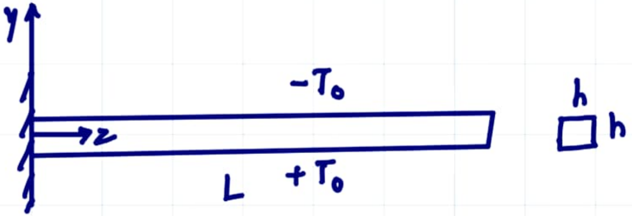
Beam Bending Due to Temp



Apply temp gradient across thickness.

$$T(y) = \frac{-T_0 y}{h/2} = \frac{-2T_0 y}{h} \quad \text{Given}$$

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$$T\left(\pm \frac{h}{2}\right) = \mp T_0$$

Goal: $u(z)$ Vertical deflection

$$\epsilon_{zz} = -y \frac{d^2 u}{dz^2} = \frac{\sigma}{E} + \alpha T$$

$$\Rightarrow \sigma = (-y u'' - \alpha T) E$$

$$M = -\int_\Omega \sigma y \underbrace{dz\,dy}_{da} = 0$$

$$= \int_\Omega u'' E y^2 \, da + \int_\Omega E \alpha T y \, da$$

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$$0 = EIu'' + E\alpha \int_{\Omega} \frac{-T_0 y}{h/2}\, y\, da$$

$$0 = \cancel{E}\cancel{I}u'' - \cancel{E}\alpha T_0 \frac{2}{h} \cancel{I}$$

$$u'' = \frac{2\alpha T_0}{h} \qquad \text{BCs} \quad u(0) = 0, \quad u'(0) = 0$$

$$u = \frac{\alpha T_0 z^2}{h}$$

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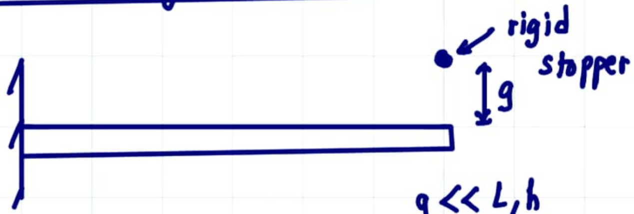
Beam Bending with Gap



$g << L, h$

g initial gap

1. Compute T_1 at which beam hits the gap
2. Find reaction at gap when $T = 2T_1$

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To close the gap,

$$u(L) = g \Rightarrow \frac{\alpha T L^2}{h} = g$$

$$\Rightarrow T_1 = \frac{g h}{\alpha L^2}$$

Method 1 Incremental

After gap is closed, available thermal load $= T - T_1$ $\quad T > T_1$

$T = 2T_1$

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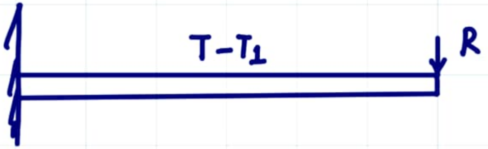

New ref config

$$\frac{\alpha(T-T_1)L^2}{h} - \frac{RL^3}{3EI} = 0 \qquad T_1 = \frac{gh}{\alpha L^2}$$

$$R = \left(\frac{\alpha T L^2}{h} - g\right)\frac{3EI}{L^3}$$

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Method 2

Direct application of full thermal load $T > T_1$

Total def @ free end

$$\frac{\alpha T L^2}{h} - \frac{R L^3}{3EI} = g$$

$$R = \left(\frac{\alpha T L^2}{h} - g\right)\frac{3EI}{L^3} \quad T > T_1$$

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$$T = 2T_1, \quad I = \frac{h^4}{12}$$

$$R = \frac{Egh^4}{4L^3}$$

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