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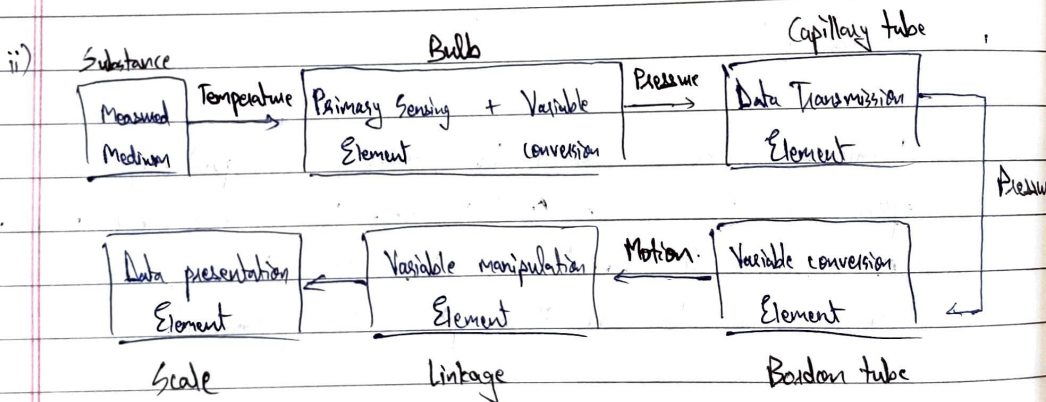
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Q1.

- i) The instrument is vapor pressure thermometer. It is used to measure temperature of the required substance. The instrument consists of a bulb, a capillary tube, borden tube and a linkage pointing to a scale. The bulb is placed in contact with the substance whose temperature needs to be measured. The volatile fluid in the bulb evaporates due to the rise in temperature (or ~~can~~ volatile vapour condenses due to fall in temp resulting in an increase in pressure which is transmitted to the borden tube which converts it into motion of the linkage to display the temperature on the scale.



- iii) Calibration of an instrument takes place with a standard. We could use an existing calibrated thermometer to calibrate our instrument. We can take, say ~~water~~ ice, measure its temperature with the calibrated thermometer. The linkage position in the pressure thermometer would represent the same temperature. Now we can start heating ice, keep recording its actual temperature with the calibrated thermometer.

and label the deviation of the linkage as that temperature. We can also find a linear relation between the angular deviation and temperature and fit a linear line to represent the instrument's behaviour.

Q2.

- Interfering input: Instruments are sometimes unintentionally sensitive to some condition or quantities. These are called interfering input.
- Modifying input: Quantities that bring about a change in the input output relation of the instrument. Modifying input can affect both desired and interfering input.
- ~~Interfering input~~ Modifying input:
  - Input is temperature and the output is motion, measured on a scale. This scale can expand or contract with varying temperature. This might affect the input output relation.
  - The ambient temperature might affect the pressure of transmission element. This can also modify the input output relation and should be taken into account.
- Interfering input:
  - High temperature of the substance being measured could expand the bulb. This is unintentional and not intended. This can reduce the pressure as volume increases.
  - If the thermometer has some acceleration, it might develop extra pressure giving inaccurate readings.

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Page \_\_\_\_\_**Q3.**

Assuming the instrument be of the first order.  
The input is a step function.

$$\tau \frac{dq_o}{dt} + q_o = A \quad \forall t > 0$$

Laplace,

$$\tau [Q_o - q_o(0)] + Q_o = \frac{A}{s}$$

$$Q_o = \left[ \frac{A}{s} + \tau q_o(0) \right] \frac{1}{(1 + \tau s)}$$

$$= \frac{A}{s(1 + \tau s)} + \frac{\tau q_o(0)}{1 + \tau s}$$

Inverse Laplace,

$$q_o(t) = A(1 - e^{-t/\tau}) + q_o(0)e^{-t/\tau}$$



Q3.

Assuming that temperature probe is a first order instrument.

- So, for time  $t : 0 \leq t \leq 7 \text{ sec}$ , for which  $\tau = 30 \text{ sec}$

$$\begin{aligned} q_o(t) &= 40(1 - e^{-t/\tau}) + 25e^{-t/\tau} \\ &= 40(1 - e^{-t/30}) + 25e^{-t/30} \end{aligned} \quad \text{--- (1)}$$

$$\begin{aligned} \text{At } t = 7 \text{ s, } q_o(t) &= 40(1 - e^{-7/30}) + 25e^{-7/30} \\ &= 28.12^\circ\text{C} \end{aligned}$$

- So, for  $7 \leq t \leq 8$ , for which  $\tau = 6 \text{ sec}$

$$\begin{aligned} q_o(t) &= 80(1 - e^{-(t-7)/\tau}) + 28.12e^{-(t-7)/\tau} \\ &= \cancel{80(1 - e^{-t/6})} + 80(1 - e^{-(t-7)/6}) + 28.12e^{-(t-7)/6} \end{aligned} \quad \text{--- (2)}$$

$$\begin{aligned} \text{At } t = 15 \text{ s, } q_o(15) &= 80(1 - e^{-8/6}) + 28.12e^{-8/6} \\ &= 66.32^\circ\text{C} \end{aligned}$$

- So, for  $15 \leq t$ , for which  $\tau = 20$ ,

$$\begin{aligned} q_o(t) &= 25(1 - e^{-(t-15)/\tau}) + 66.32e^{-(t-15)/\tau} \\ &= 25(1 - e^{-(t-15)/20}) + 66.32e^{-(t-15)/20} \end{aligned} \quad \text{--- (3)}$$

$$\begin{aligned} \text{At } t = 30 \text{ s, } q_o(30) &= 25(1 - e^{-15/20}) + 66.32e^{-15/20} \\ &= 44.52^\circ\text{C} \end{aligned}$$

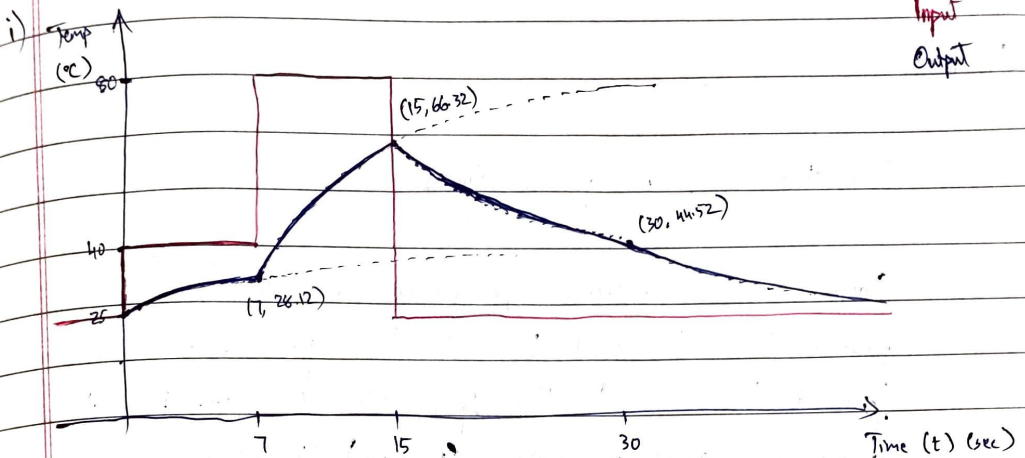
Note: Used the general expression of step response of first order:

$$q_o(t) = kq_s(1 - e^{-t/\tau}) + q_o(0)e^{-t/\tau}$$

Where  $k = 1$ .

Also further used continuity to find initial condition every time the set up is changed. Shifted time to origin for every set up for

Simplicity.



ii) As calculated at the beginning,

$$q_0(7) = 28.12^\circ\text{C}$$

$$q_0(15) = 66.32^\circ\text{C}$$

$$q_0(30) = 44.52^\circ\text{C}$$

- iii). Time constant represents the response time of the instrument. For a thermometer, the response time would be faster if the conductivity of the substance being, whose temperature is being measured, is higher. Since the same thermometer is used in every case, the conductivity of the thermometer liquid would be the same, so it won't be a factor. It is the same like a wooden and steel chair at same temperatures, but the wooden chair won't feel that hot due to slow conductivity. Faster the conductivity, faster is the response and lower is the  $\tau$  value.

Since water conducts faster than air,  $\tau_{\text{water}} < \tau_{\text{air}}$ . For the third case, a wet probe would conduct the temperature faster as water on the probe would heat up faster than mercury and then it stays on the bulb to conduct the heat. This is still not as effective as the medium

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being water itself. So,  $T_{\text{water}} < T_{\text{wet probe}} < T_{\text{dry}}$

Q4.

First, let us solve for a first order instrument where,

$$q_i = A \sin \omega t ; \quad q_o(0) = 0 ; \quad k = 1$$

$$\therefore \tau \frac{dq_o}{dt} + q_o = k q_i = A \sin \omega t$$

Laplace,

$$\tau [Q_o \cdot \overset{0}{s}] + Q_o = A \cdot \left( \frac{\omega}{s^2 + \omega^2} \right)$$

$$Q_o = A \left( \frac{\omega}{s^2 + \omega^2} \right) \left( \frac{1}{1 + \tau s} \right)$$

$$= A \omega \left[ \frac{Xs + Y}{s^2 + \omega^2} + \frac{Z}{1 + \tau s} \right]$$

$$(Xs + Y)(1 + \tau s) + Z(s^2 + \omega^2) = 1$$

$$Xs + X\tau s^2 + Y + Y\tau s + Zs^2 + Z\omega^2 = 1$$

$$X\tau + Z = 0 ; \quad Y\tau + X = 0 ; \quad Y + Z\omega^2 = 1$$

$$Z = -X\tau$$

$$X = -Y\tau$$

$$\therefore Z = Y\tau^2$$

$$Y + Y\tau^2\omega^2 = 1$$

$$Y = \frac{1}{1 + \tau^2\omega^2}$$

$$Z = \frac{\tau^2}{1 + \tau^2\omega^2} \quad \& \quad X = -\frac{\tau}{1 + \tau^2\omega^2}$$



$$Q_0 = A\omega \left[ \frac{Xs}{s^2 + \omega^2} + \frac{Y}{s^2 + \omega^2} + \frac{Z}{1 + \tau s} \right]$$

$$= A\omega X \left( \frac{s}{s^2 + \omega^2} \right) + A\omega Y \left( \frac{\omega}{s^2 + \omega^2} \right) + A\omega Z \left( \frac{1}{1 + \tau s} \right)$$

Inverse Laplace,

$$q_0(t) = A\omega X \cos \omega t + A\omega Y \sin \omega t + \frac{A\omega Z}{\tau} e^{-t/\tau}$$

$$= \frac{-A\omega Z}{1 + \tau^2 \omega^2} \cos \omega t + \frac{A}{1 + \tau^2 \omega^2} \sin \omega t + \frac{A\omega Z}{1 + \tau^2 \omega^2} e^{-t/\tau}$$

Now given,  $q_i = \sin 2t + 0.3 \sin 20t$

Superimposing  $\Rightarrow q_1 = \sin 2t$  &  $q_2 = 0.3 \sin 20t$

We get,

$$A=1, \omega=2$$

$$A=0.3, \omega=20$$

$$q_0(t) = \frac{-2(0.2)}{1 + (0.2 \times 2)^2} \cos 2t + \frac{1}{1 + (0.2 \times 2)^2} \sin 2t + \frac{2 \times 0.2}{1 + (0.2 \times 2)^2} e^{-t/0.2}$$

$$- \frac{0.3 \times 20 \times 0.2}{1 + (20 \times 0.2)^2} \cos 20t + \frac{0.3}{1 + (0.2 \times 20)^2} \sin 20t + \frac{0.3 \times 20 \times 0.2}{1 + (0.2 \times 20)^2} e^{-t/0.2}$$

$$q_0(t) = -0.345 \cos 2t + 0.862 \sin 2t + 0.345 e^{-t/0.2}$$

$$- 0.07 \cos 20t + 0.018 \sin 20t + 0.07 e^{-t/0.2}$$

For steady state,  $t \rightarrow \infty$

$$q_0 = -0.345 \cos 2t + 0.862 \sin 2t - 0.07 \cos 20t + 0.018 \sin 20t$$



being water itself.  $\therefore$ ,  $T_{\text{water}} < T_{\text{water}} < T_{\text{dry}}$

**Q5.**

i) The two gauges are pressure gauges.

Gauge 1 is a deflection type pressure gauge. In this instrument, the pressure from the fluid hits the lower piston, generating force. This force is transmitted to the spring via the rod. Force to the spring will cause a motion. ( $F = -kx$ ) This motion will give a rotation to the linkage ~~with~~ which will point to a calibrated scale to give the pressure value.

Gauge 2 is a null type pressure gauge. In this instrument, the pressure from the fluid also hits the lower piston, generating force that is transmitted by a rod. Now to counter this force, we apply standard weights on a platform, giving a downward opposing force. We try different combination of weights to balance the force, or stop the motion (acceleration = 0). Now that we know the force, we can divide it with the area of the lower ~~piston~~ piston to find pressure. ( $P = F/A$ )

ii) I would prefer the null <sup>Gauge 2</sup> ~~deflection~~ type. It is less prone to errors. We only use standard weight and there could not be any mistakes. While in deflection type (Gauge 1), the spring may wear away or the linkage may get loose. The more the parts of the instrument, more the data gets manipulated or corrupted which adds errors at every step.

Q6.

Parameters that need to be measured to understand the transmission of the virus are:

1. How long do the infected droplet stay and on what surfaces.
2. What are the size of the droplets exhaled by an infected person.
3. What temperatures are favourable for spread of virus.
4. How does the population density affect the spread of infection.
5. Distribution of infected exhaled air which mix with the atmosphere.
6. Effect of wind in spreading infection.
7. How much protection can wearing a mask give to people.
8. For how long can an infected person infect other people around him.
9. Amount of air inhaled and exhaled by a person in one cycle.

Strategy for measuring:

- Point no. 1 : We can try placing droplets which carry infection on different surfaces under different conditions and observe how long the virus can survive.
- Point no. 4 : We can gather the data from different cities of varied population density and observe the rate of growth of infection and try to model a mathematical equation taking into consideration the commute in the city to understand the spread.
- Point no. 8 : We can test an infected person at regular intervals to record how long he/she can be a carrier of infection.