MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 9

(April 06, 2022)

(1) Use the Modified Euler method with $h = 0.25$ to approximate the solution to the initial-value problem: $y' = 1 + (t - y)^2$, $2 \leqslant t \leqslant 3$, $y(2) = 1$, and compare the results to the actual values with $y(t) = t + \frac{1}{1-t}$.

(2) Repeat the above problem using the midpoint method.

(3) Repeat the above problem using the Runge-Kutta method of order four.

(4) Given the linear system:

$$\begin{aligned} 2x_1 - 6\alpha x_2 &= 3 \\ 3\alpha x_1 - x_2 &= \frac{3}{2} \end{aligned}$$

(a) find the values of α for which the system has no solution,
(b) find the values of α for which there are infinitely many solutions,
(c) assuming that a unique solution exists for a given α, find the solution in terms of α.

(5) Use Gaußian elimination and three-digit chopping arithmetic to solve the linear system:

$$\begin{aligned} 0.03x_1 + 58.9x_2 &= 59.2 \\ 5.31x_1 - 6.10x_2 &= 47.0 \end{aligned}$$

and compare the approximations to the actual solution $(10, 1)$.

(6) Solve the above linear system using Gaußian elimination with partial pivoting and three-digit rounding arithmetic.

(7) For the matrix A given below, find a permutation matrix P such that PA has an LU factorization and find the LU factorization of PA:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

(8) For the matrix A given below, find a permutation matrix P such that PA has an LU factorization and find the LU factorization of PA:

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & 1 \end{pmatrix}.$$

(9) For the matrix A given below, find a permutation matrix P such that PA has an LU factorization and find the LU factorization of PA:

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}.$$