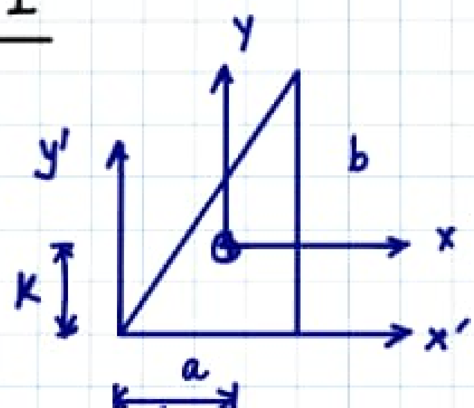
ME 202

LECTURE 36

TUE 12 APR 2022

Problem 1



$$I_{xx}' = \int_{\Omega} {y'}^2 \, dx' \, dy'$$

$$= \int_0^a dx' \int_0^{bx'/a} {y'}^2 \, dy'$$

$$= \frac{ab^3}{12}$$

$$I_{yy}' = \frac{ba^3}{4}$$

$$I_{xy}' = -\int_{\Omega} x'y' \, dx' \, dy'$$

$$= \frac{-b^2a^2}{8}$$

$$I_{xx}' = I_{xx} + Ak^2$$

$$I_{yy}' = I_{yy} + Ah^2$$

$$I_{xy}' = I_{xy} - Ahk$$

$$\frac{ab^3}{12} = I_{xx} + \frac{ab}{2}\left(\frac{b}{3}\right)^2 \Rightarrow I_{xx} = \frac{ab^3}{36}$$

$$I_{yy} = \frac{ba^3}{36}, \quad I_{xy} = \frac{-a^2b^2}{72}$$

$$A = \frac{-M_y I_{xx} + M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = -E\frac{d^2u}{dz^2}$$

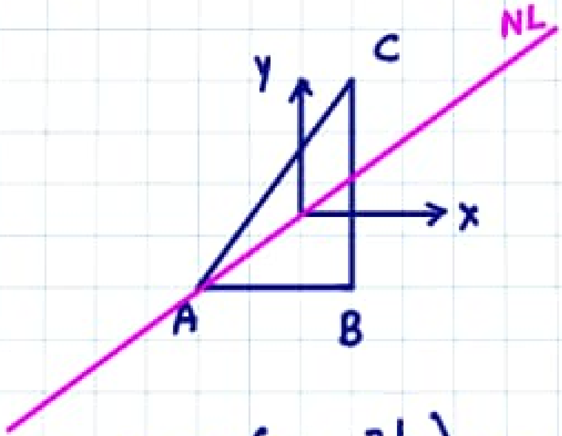
$$B = \frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = -E\frac{d^2v}{dz^2}$$

$$A = -6.006 \times 10^9 \ \frac{Nm}{m^4}$$

$$B = +6.006 \times 10^9 \ \frac{Nm}{m^4}$$

Eqn of neutral line $Ax + By = 0$

$$y = x$$



$$\sigma_{zz} = Ax + By$$

$$\sigma_{zz} \text{ @ } C \left(\frac{a}{3}, \frac{2b}{3}\right) = 60.06 \text{ MPa}$$

tensile

$$\sigma_{zz} \text{ @ } B \left(\frac{-a}{3}, \frac{-b}{3}\right) = -60.06 \text{ MPa}$$

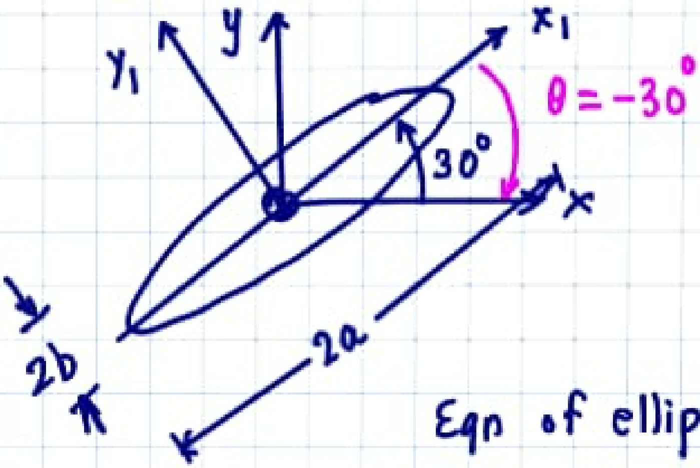
$$A = -E \frac{d^2u}{dz^2} \Rightarrow u = -\frac{A}{E}\frac{z^2}{2} + \cancel{c_1 z} + \cancel{c_2}$$

$$u_{max} = -\frac{A}{E}\frac{L^2}{2} = +30.03 \text{ mm}$$

$$B = -E \frac{d^2v}{dz^2}, \quad v = -\frac{B}{E}\frac{z^2}{2} + \cancel{d_1 z} + \cancel{d_2}$$

$$v_{max} = \frac{-BL^2}{2E} = -30.03 \text{ mm}$$

Example 2



$M_x = M$

$M_y = 0$

Eqn of NL = Goal

Eqn of ellipse $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$

$x_1 = ar\cos\varphi$

$y_1 = br\sin\varphi$

$dx_1 dy_1 = J\, dr d\varphi$

$= abr\, dr d\varphi$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial r} & \frac{\partial x_1}{\partial \varphi} \\ \frac{\partial y_1}{\partial r} & \frac{\partial y_1}{\partial \varphi} \end{vmatrix}$$

$$I_{x_1x_1} = I^1_{xx} = \int_\Omega y_1^2\, dx_1 dy_1$$

$$= \int_0^{2\pi} \int_{r=0}^{1} (br\sin\theta)^2\, abr\, dr d\varphi$$

$$= \frac{\pi}{4} a b^3$$

$$I'_{yy} = \frac{\pi}{4} a^3 b$$

$$I'_{xy} = 0$$

Use transf. rules $\theta = -30^\circ$, $\quad c = \cos\theta$, $s = \sin\theta$

$$I_{xx} = I'_{xx} c^2 + I'_{yy} s^2 + 2 I'_{xy} sc$$
$$= \frac{\pi}{16} (3ab^3 + ba^3)$$

$$I_{yy} = I'_{xx} s^2 + I'_{yy} c^2 - 2 I'_{xy} sc$$
$$= \frac{\pi}{16} (3a^3 b + ab^3)$$

$$I_{xy} = \left(I'_{yy} - I'_{xx}\right) sc + I'_{xy} (c^2 - s^2)$$
$$= \sqrt{3}\, \frac{\pi}{16} (ab^3 - ba^3)$$

$$\Delta = \det(\underset{\sim}{I}) = I'_{xx} I'_{yy} - (I'_{xy})^2$$

invariant of $\underset{\sim}{I}$

$$= I_{xx} I_{yy} - I_{xy}^2$$
$$= \frac{\pi^2}{16} a^4 b^4$$

$$A = \frac{M\sqrt{3}}{\pi}\left(\frac{1}{a^3 b} - \frac{1}{ab^3}\right)$$

$$B = \frac{M}{\pi}\left(\frac{3}{ab^3} + \frac{1}{a^3 b}\right)$$

$$\sigma_{zz} = Ax + By$$

NL $\quad Ax + By = 0$

$$y = \sqrt{3}\left(\frac{a^3 b - ab^3}{3a^3 b + ab^3}\right)x$$

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