ME 202

LECTURE 23 TUTORIAL 7

THU 10 MAR 2022

## Problem 1


Conductive Heat Transfer 101 3D $\nabla^2 T = 0$

1D $\frac{d^2T}{dx^2} = 0$

$$T(x) = \frac{T_1\left(x+\frac{h}{2}\right) - T_2\left(x-\frac{h}{2}\right)}{h}$$

$$\sigma(z,x) = \left(-x\,u''(z) - \alpha T(x)\right)E$$



Next page

1 of 7

$$M(z) = 0 = -\int_{\Omega} \sigma x \, dx\, dy$$

$$\Rightarrow \quad u'' E \int_{\Omega} x^2 \, dx\, dy + \alpha E \int_{\Omega} T(x)\, x \, dx\, dy = 0$$

$$\Rightarrow \quad u'' E \frac{bh^3}{12} + \alpha E \frac{bh^2}{12} (T_1 - T_2) = 0$$

$$\Rightarrow \quad u'' = \frac{\alpha}{h} (T_2 - T_1)$$

$$\Rightarrow \quad u = \frac{\alpha}{h} (T_2 - T_1) \frac{z^2}{2} + \underbrace{c_1 z + c_2}_{\substack{u(0)=0 \\ u(L)=0}}$$

$$\Rightarrow \quad u(z) = \frac{\alpha}{h} (T_2 - T_1) \left( \frac{z^2}{2} - \frac{Lz}{2} \right)$$

# Problem 2


$$@\ z = L, \quad \frac{\alpha T L^2}{h} - \frac{R L^3}{3EI} = 0$$

$$\Rightarrow R = \frac{3EI}{L}\frac{\alpha T}{h}$$

$$u(z) = \frac{\alpha T z^2}{h} - \frac{R}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$= \frac{\alpha T}{h}\left[z^2 - \frac{3}{L}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)\right]$$

$$u'' = \frac{T\alpha}{h}\left(\frac{3z}{L} - 1\right)$$

$$u''(0) = -\frac{T\alpha}{h}, \quad u''(L) = \frac{2T\alpha}{h}$$

$$\sigma = (-x u'' - \alpha T) E$$

$$\sigma_{max} = -2\alpha T E \quad @ \ z = L$$

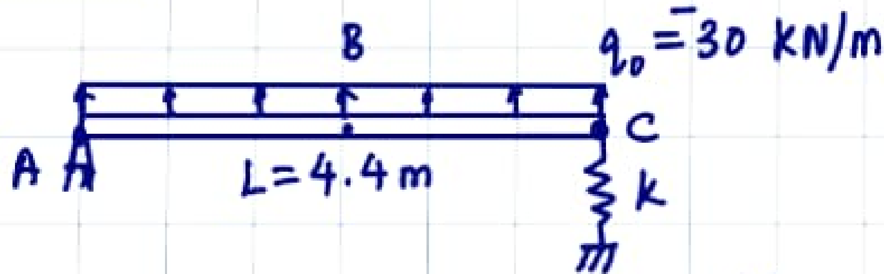
$$-S_c = -2\alpha T E \Rightarrow T_{max} = \frac{S_c}{2\alpha E}$$

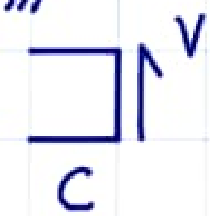
Max operating temp

FOS factor of safety.

$$T_{max} := \frac{T_{max}}{2} \leftarrow \text{FOS}$$

## Problem 3




### Method 1

$$EI \frac{d^4u}{dz^4} = q_0$$

BCs: $u(0) = 0$, $EI u''(0) = 0$, $EI u''(L) = 0$

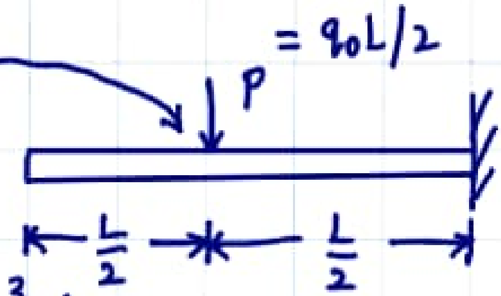
$$-EI u'''(L) = -k\,u(L)$$

ODE $u = \frac{1}{EI}\left(\frac{q_0 z^4}{24} + \frac{c_1 z^3}{6} + \frac{c_2 z^2}{2} + c_3 z + c_4\right)$

$$u = \frac{q_0 z}{24EIk}\left(kL^3 - 2kLz^2 + kz^3 + 12EI\right)$$

$$u(L) = \frac{q_0 L}{2k}$$

$$u\left(\frac{L}{2}\right) = \frac{q_0 L}{384EIk}\left(5kL^3 + 96EI\right)$$

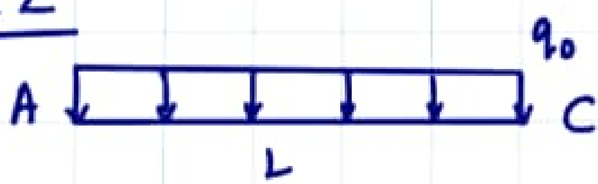



$$\delta = P\left(\frac{L}{2}\right)^3 \frac{1}{3EI}$$

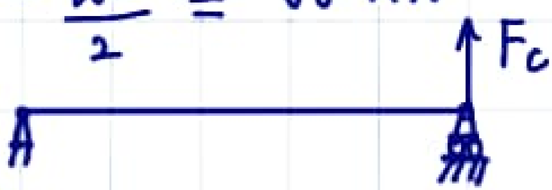
$$= \frac{PL^3}{24EI}, \quad k = \frac{24EI}{L^3}$$

$$u\left(\frac{L}{2}\right) = \frac{3q_0L^4}{128EI}$$

## Method 2




$$F_c = \frac{q_0L}{2} = 66 \text{ kN}$$

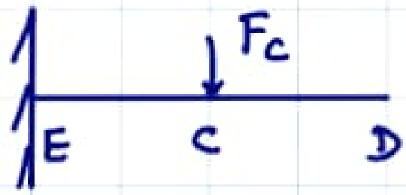



Def @ B as if DE is rigid ($E \to \infty$)

$$\delta_B' = \frac{5q_0L^4}{384EI} = 0.366 \text{ m}$$

$\delta_D =$

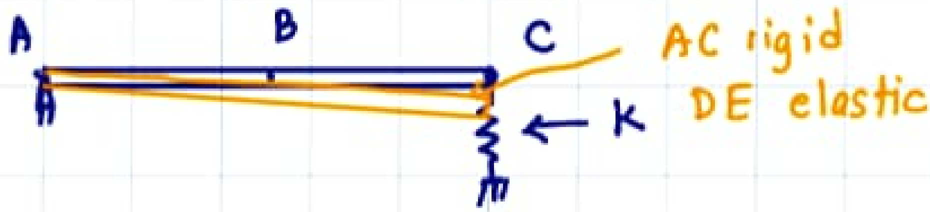
$$\frac{F_c}{EI}\left(\frac{L}{2}\left(\frac{L}{2}\right)^2 - \frac{1}{6}\left(\frac{L}{2}\right)^3\right)$$




$$= 1.4641 \text{ m}$$

$$\delta_c = \frac{F_c}{3EI}\left(\frac{L}{2}\right)^3 = 0.58564 \text{ m}$$

$$= \frac{q_0 L}{2k}$$




$$\delta_B = \delta_B' + \frac{1}{2}\delta_c$$

$$= 0.366 + \frac{1}{2} 0.58564$$

$$= 0.658845 \text{ m}$$