Different types of Instruments

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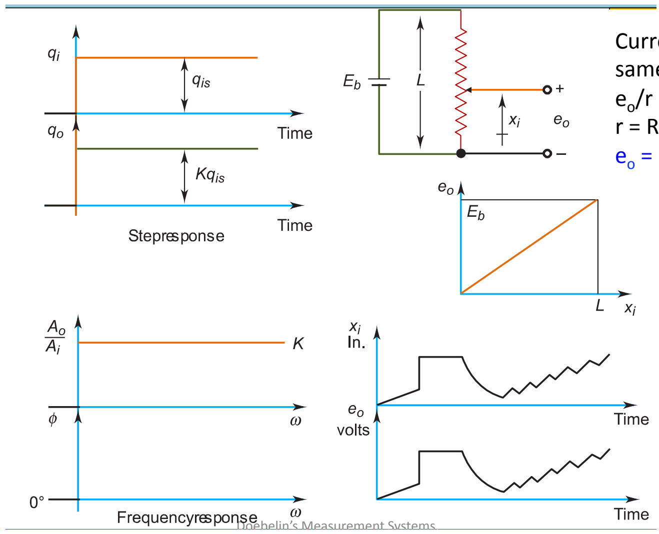
Zeroth order instrument



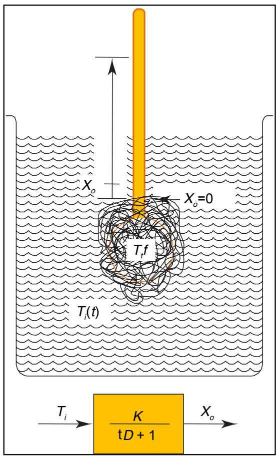
Current is same:

$e_o/r = E_b/R$

$r = R*x_i/L$

$e_o = E_b/L * x_i$

First order Instrument



$$x_o = \frac{K_{ex} V_b}{A_c}(T_{tf} - T_{ref})$$

Recall: K_{ex} = 1/V dV/dT

x_o: displacement from reference mark, m (x_0 = 0 when $T_{tf} = T_{ref}$), m
T_{tf}: temperature of fluid in bulb, °C
K_{ex}: coefficient of thermal expansion of thermometer fluid, $m^3/(m^3\text{-}^oC)$
(more precisely differential expansion coefficient of thermometer fluid and glass bulb)
A_c: cross-sectional area of capillary tube

Thermometer as a 1st order instrument

- Energy in – energy out = energy stored

$$UA_b(T_i - T_{tf})dt - 0 = \rho V_b C dT_{tf}$$

- U: overall heat transfer coefficient across bulb-wall, $W/m^2\text{-}^oC$
- A_b: heat transfer area of bulb wall, m^2
- ρ: mass density of thermometer fluid, kg/m^3
- C: specific heat of thermometer fluid, J/(kg-K)

Thermometer as a 1st order instrument

- Therefore, we get

$$\rho V_b C \frac{dT_{tf}}{dt} + UA_b T_{tf} = UA_b T_i$$

- Taking derivative of the first equation with respect to t, we get

$$\frac{d}{dt} x_o = \frac{K_{ex} V_b}{A_c} \left(\frac{d}{dt} T_{tf} - 0 \right)$$

Thermometer as a 1st order instrument

- Substituting expressions for T_{tf} and dT_{tf}/dt, we get

$$\rho V_b C \frac{A_c}{K_{ex} V_b} \frac{dx_o}{dt} + UA_b \left(\frac{A_c}{K_{ex} V_b} x_o + T_{ref} \right) = UA_b T_i$$

$$\tau \frac{dq_o}{dt} + q_o = K q_i$$

(Standard form, where t is time lag, K is static sensitivity)

- Dividing by: $UA_b \frac{A_c}{K_{ex} V_b}$

to obtain expressions for tau and K $\tau = \frac{\rho C V_b}{UA_b}; K = \frac{K_{ex} V_b}{A_c}$

Trade-offs in designing a thermometer

- Want large K: For which need large V_b and/or small A_c
- Want small tau: For which need large A_b, large U, small rho, small C, small V_b

- Decreasing A_c is an option – but mostly we find that there is a trade-off
- That is, increasing sensitivity leads to a slower response, and vice-versa