Dynamic response of first-order instruments

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Frequency response of first-order instruments

- Governing equation: $\tau \frac{dq_o}{dt} + q_o = Kq_i$
- Given: $q_i = A\cos(\omega t)$
- Given (for simplicity only): $q_o = 0$ at t = 0
- Easy to find $q_{oc} = Be^{-t/\tau}$
- Also find $q_{op} = C_1 \cos(\omega t) + C_2 \sin(\omega t)$
- No duplication. Putting in the governing equation to find C₁ and C₂

 $\tau[-C_1\omega\sin(\omega t) + C_2\omega\cos(\omega t)] + [C_1\cos(\omega t) + C_2\sin(\omega t)] = KA\cos(\omega t)$

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Frequency response

Solving to get C₁ and C₂ as

$$C_1 = \frac{KA}{1 + \tau^2 \omega^2} \qquad C_2 = \frac{\tau \omega KA}{1 + \tau^2 \omega^2}$$

• Therefore, the complete solution is:

$$q_o = Be^{-t/\tau} + \frac{KA}{1 + \tau^2 \omega^2} [\cos(\omega t) + (\omega \tau) \sin(\omega t)]$$

- Use the given initial condition to find $B = -\frac{KA}{1 + \tau^2 \omega^2}$
- Can we solve this problem using Laplace transform?

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Step response of first-order instruments

- Governing equation: $\tau \frac{dq_o}{dt} + q_o = Kq_i$
- Given q_i is a step input of magnitude q_{is}
- Given (for simplicity only): $q_o = 0$ at t = 0
- Taking Laplace transform: $L\left[\tau \frac{dq_o}{dt} + q_o\right] = L[Kq_i]$ Get: $\tau[sQ q_o(0)] + Q = \frac{Kq_{is}}{s}$ Or $Kq_o = \tau q_o(0)$
- Or, $Q = \frac{Kq_{is}}{s(\tau s + 1)} + \frac{\tau q_0(0)}{(\tau s + 1)}$

$$q_0 = L^{-1} \left[\frac{Kq_{is}}{s(\tau s + 1)} \right] + L^{-1} \left[\frac{\tau q_0(0)}{(\tau s + 1)} \right]$$

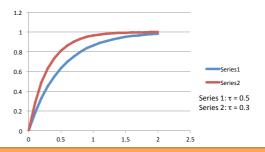
Step response

• Final solution:

$$q_o = Kq_{is}(1 - e^{-t/\tau}) + q_0(0)e^{-t/\tau}$$

- Speed of response depends on the value of $\boldsymbol{\tau}$
- Faster response for a smaller value of $\boldsymbol{\tau}$
- For $q_0(0) = 0$, get

$$\frac{q_o}{Kq_{is}} = (1 - e^{-t/\tau})$$



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