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ME 226

Endsem

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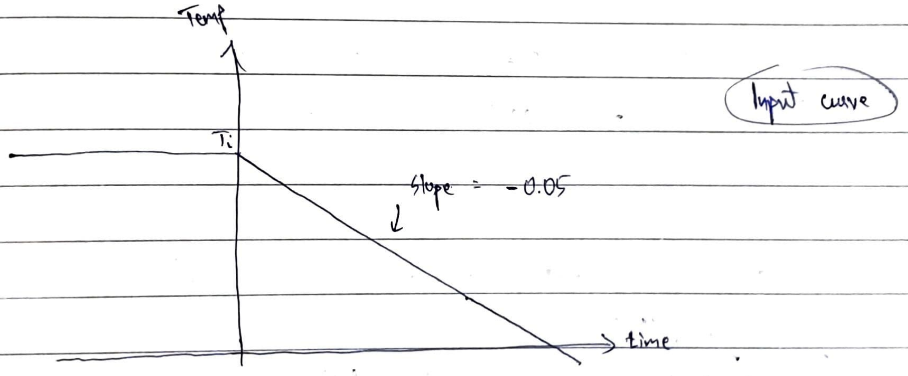
Q1.

a) Given, response time τ = 12 s

Speed of balloon = 5 m/s

Rate of temp drop = $\frac{0.2}{20}$ °C/m × 5 m/s = 0.05 °C/s

Lets say, the initial temperature at the ground was T_i.



This graph is a superposition of a step and a ramp input.

Input = $T_i - 0.05t$

The output will also be a superposition of the output for a step & ramp input.

Output of step input $= kq_i \left[1 - e^{-t/\tau}\right] + q_o(0)\, e^{-t/\tau}$

$= T_i \left[1 - e^{-t/12}\right] + T_i\, e^{-t/12}$

$= T_i$

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Output of ramp input $= k q_{in} [(t-\tau) + \tau e^{-t/\tau}]$
$= -0.05 [~~(t-15)~~ (t-12) + 12 e^{-t/12}]$

$\therefore$ Total output $= T_i - 0.05 [t-12 + 12 e^{-t/12}]$

Now given, thermometer shows an output of 2°C at altitude 3000 m.

Time taken to reach altitude 3000 m $= \frac{3000}{5} = 600$ s

$\therefore$ $2 = T_i - 0.05 [600 - 12 + 12 e^{-600/12}]$

$T_i = 2 + 0.05 [600 - 12 + 12 e^{-600/12}]$
$= 31.4$°C

$\therefore$ Output $=$ ~~31.4~~ $31.4 - 0.05 [t-12 + 12 e^{-t/12}]$

~~Note that for steady state ($t > 5\tau$), we get a steady error.~~

~~Error =~~

So now, we know that the initial temperature $= 31.4$°C

$\therefore$ Actual temperature $= T_i - 0.05 t$
$0 = 31.4 - 0.05 t$
$t = 628$ s

Altitude $= t \times 5 = 628 \times 5 = \boxed{3140 \text{ m}}$

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b) Assumptions :

- Thermometer is a first order instrument
- There is no time delay when the balloon sends the temperature output that the thermometer records.
- Temperature drop rate and velocity of balloon is constant

Q2.

a) Reynolds no. $$Re = \frac{\rho v d}{\eta}$$

$$= \frac{1000 \times 1 \times 0.25}{8.6 \times 10^{-4}}$$

$$= 290697.67$$

Uncertainty :

$$\Delta Re = \sqrt{\left(\frac{\partial Re}{\partial \rho}\Delta\rho\right)^2 + \left(\frac{\partial Re}{\partial v}\Delta v\right)^2 + \left(\frac{\partial Re}{\partial d}\Delta d\right)^2 + \left(\frac{\partial Re}{\partial \eta}\Delta\eta\right)^2}$$

$$= \sqrt{\left(\frac{vd}{\eta}\Delta\rho\right)^2 + \left(\frac{\rho d}{\eta}\Delta v\right)^2 + \left(\frac{\rho v}{\eta}\Delta d\right)^2 + \left(\frac{-\rho v d}{\eta^2}\Delta\eta\right)^2}$$

$$= \sqrt{\left(\frac{1 \times 0.25}{8.6 \times 10^{-4}} \times 10\right)^2 + \left(\frac{10^3 \times 0.25}{8.6 \times 10^{-4}} \times \frac{1 \times 5}{100}\right)^2 + \left(\frac{10^3 \times 1}{8.6 \times 10^{-3}} \times 1 \times 10^{-3}\right)^2 + \left(\frac{-10^3 \times 1 \times 0.25}{(8.6 \times 10^{-4})^2} \times \frac{8.6 \times 10^{-4} \times 4}{100}\right)^2}$$

$$= \sqrt{8.45 \times 10^6 + 2.11 \times 10^8 + 1.35 \times 10^6 + 1.35 \times 10^8}$$

$$= 18862.66$$

$\therefore$ % Uncertainty $= \frac{\Delta Re}{Re} \times 100 = \frac{18862.66}{290697.67} \times 100 = 6.49\%$

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b) We should measure velocity more precisely since its term is the largest in the calculation of ~~and~~ uncertainity.

$$\left(\frac{\rho d}{\eta}\Delta v\right)^2$$ is larger than the other terms.

Measuring ~~the~~ the velocity precisely reduces the Δv term and reduces the total uncertainity.

c) Suggestion 1 : Ultrasonic flowmeter
- It can measure flow for any fluid, even if its dirty or abrasive.
- It is non intrusive
- High turndown ratio
- Can withstand high temperatures & pressures.

Suggestion 2 : Vortex flowmeter
- It is cheap
- Has no moving parts, hence durable.
- Linear output ($\because$ Frequency $\propto V_{av}$)
- Wide operating range.

Q3.

a) We make the fluid flow over a heated wire. The amount of cooling experienced by the hot wire is related to the local velocity of the flowing fluid.

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b)

Balancing energy :

$$\frac{dQ}{dt} = P - F$$

$$\frac{dQ}{dt} = P - (q_c + q_p + q_r + q_z)$$

Q = Internal energy
P = Power given by circuit
F = Rate of heat transfered from the wire
q_c = Heat transfer rate due to convection
q_p = Heat transfer rate due to conduction to prongs
q_r = " " " " " radiation
q_z = " " " to quartz substrate

q_r is negligible. We increase (l/d) ratio of prong to reduce q_p and neglect it. q_z is also small.

The principle primarily wants $F = q_c$. The flowing fluid results in forced convection.

Due to Nusselt number, we derive heat transfer coefficient

$$h = C_0 + C_1\sqrt{U}$$

where, C_0 & C_1 are constants
U is the flow velocity.

$\therefore$ $q_c = mh\Delta T$ ~~F~~ $\rightarrow F(U,T)$.

wire surface temperature

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$$P = iR^2 = iR^2(1+\alpha\Delta T)^2$$

$\therefore$ $$dQ = mC_p dT$$

$$\frac{dQ}{dt} = P - F$$

$$\boxed{mC_p \frac{dT}{dt} = P(I, T) - F(U, T)}$$

Where, T is wire surface temp.
C_p is sp. heat capacity of wire
m is wire mass

c) In constant current mode :

$$mC_p \frac{dT}{dt} = P(T) - F(U,T)$$

We find velocity with respect to temperature & its derivative

In constant temp mode :

$$0 = P(I) - F(U)$$

We find velocity for the current value required to maintain the temperature as constant.
This method is preferred as we get rid of $mC_p\, dT/dt$ term.

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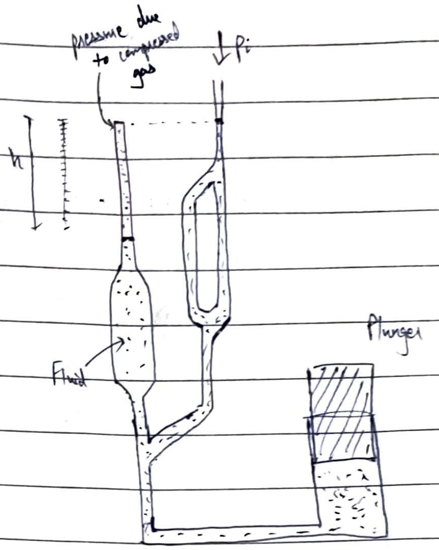
1Q4)

a)

Principle :
On pushing the plunger, we push the liquid up against pressure.
p_i is the pressure input.
After the junction, the right arm fluid rises up against p_i
& the left arm fluid rises up against pressure of the compress gas. (pressure p)



$p > p_i$ and we push the plunger till the height of fluid in the right arm is at level with the top of the left arm.

p can be found by boyle's law.
$\Delta p = p - p_i = \rho g h$.

So we solve for p_i.

Mcloed gauge is a modified manometer.

b)

For pressure p : According to Boyle's law

$pV = \text{constant}$

Let the area of the left arm capillary be : A
Let the volume of left arm above the junction be : V

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$$\therefore \quad p \cdot hA = p_i \times V$$

$$p = \frac{p_i V}{hA}$$

Now, we can say by bernoulli,

$$p_i + \rho g h = p$$

$$p_i + \rho g h = \frac{p_i V}{hA}$$

$$\rho g h = p_i \left(\frac{V}{hA} - 1 \right)$$

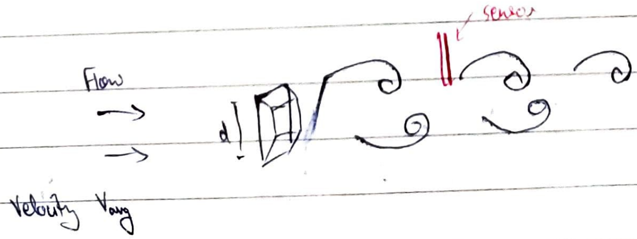
$$\boxed{p_i = \frac{\rho g h^2 A}{V - hA}}$$

Further, if $V \gg hA$,

$$\boxed{p_i = \frac{\rho g h^2 A}{V}}$$

Q5.

Inclusion of a bluff body in flow results in wakes behind it. Wakes result in alternating high and low pressure.



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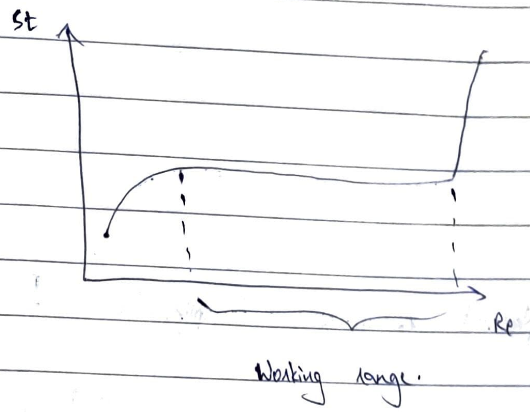
Now we define a non dimensional quantity, Strouhal number (St)

$$St = \frac{d \times f}{V_{avg}}$$

d : Bluff diameter
f = Wake frequency or vortex frequency

We observe that for a range of reynolds no, St is almost constant. This is our working range.



$$St = \text{constant}$$

$$\frac{d \times f}{V_{avg}} = \text{constant}$$

$$\frac{f}{V_{avg}} = \text{constant}$$

We get, $V_{avg} \propto f$

A linear relationship between V_{avg} & f.

Pressure sensors or piezo sensors can be installed behind the bluff body who's signals can be processed to find out the frequency. Auto correlation is used to reduce noise.
We can optimize the location of sensor to assure good accuracy and least pressure loss.

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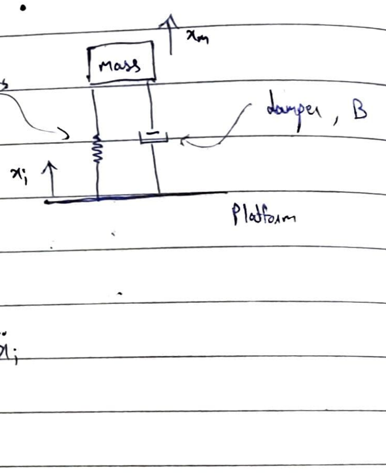
- It is cost effective compared to other flow measuring instruments
- No moving parts
- Linear relationship
- High turndown ratio
- Wide operating range.

Note that : The instrument is sensitive to upstream flow conditions

Q6.

a)

Relative displacement : $x_o = x_i - x_m$



Using force balance,

$$k_s x_o + B\dot{x}_o = M\ddot{x}_m$$

$$k_s x_o + B\dot{x}_o + M\ddot{x}_i = M\ddot{x}_m + M\ddot{x}_i$$

$$k_s x_o + B\dot{x}_o + M(\ddot{x}_i - \ddot{x}_m) = M\ddot{x}_i$$

$$k_s x_o + B\dot{x}_o + M\ddot{x}_o = M\ddot{x}_i$$

$$\frac{M}{k_s}\ddot{x}_o + \frac{B}{k_s}\dot{x}_o + x_o = \frac{M}{k_s}\ddot{x}_i$$ $\Rightarrow$ Governing eq^n

Comparing with our general second order equation,

$\frac{1}{W_n^2} = \frac{M}{k_s}$; x_i = Input ; $K = \frac{M}{k_s}$

$\frac{B}{k_s} = \frac{2\xi}{W_n}$; x_o = Output ;

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Rewriting,
$$\frac{1}{W_n^2}\ddot{x}_o + \frac{2\xi}{W_n}\dot{x}_o + x_o = k\ddot{x}_i$$

$$\left(\frac{D^2}{W_n^2} + \frac{2\xi}{W_n}D + 1\right)x_o = kD^2 x_i$$

$$\frac{x_o}{x_i} = \frac{kD^2 \times W_n^2}{D^2 + 2\xi W_n D + W_n^2} \qquad (k \times W_n^2 = 1)$$

$$\frac{x_o}{x_i} = \frac{D^2}{D^2 + 2\xi W_n D + W_n^2} \qquad (D = jw)$$

$$\frac{x_o}{x_i} = \frac{-W^2}{-W^2 + 2\xi W_n \times jW + W_n^2}$$

$$\frac{x_o}{x_i} = \frac{-W^2/W_n^2}{1 - W^2/W_n^2 + 2\xi W/W_n\, j}$$

$$\left|\frac{x_o}{x_i}\right| = \frac{W^2/W_n^2}{\sqrt{\left(1-\frac{W^2}{W_n^2}\right)^2 + \left(2\xi\frac{W}{W_n}\right)^2}} \qquad (W/W_n = r)$$

$$\left|\frac{x_o}{x_i}\right| = \frac{r^2}{\sqrt{(1-r^2)^2 + 4\xi^2 r^2}}$$

Note : Here, we assume the input to be sinusoidal

b) For : $r \to 0 \Rightarrow \left|\frac{x_o}{x_i}\right| \to 0$

$r \to 1 \Rightarrow \left|\frac{x_o}{x_i}\right| >> 1$ (For small ξ)

$r \to \infty \Rightarrow \left|\frac{x_o}{x_i}\right| = 1$

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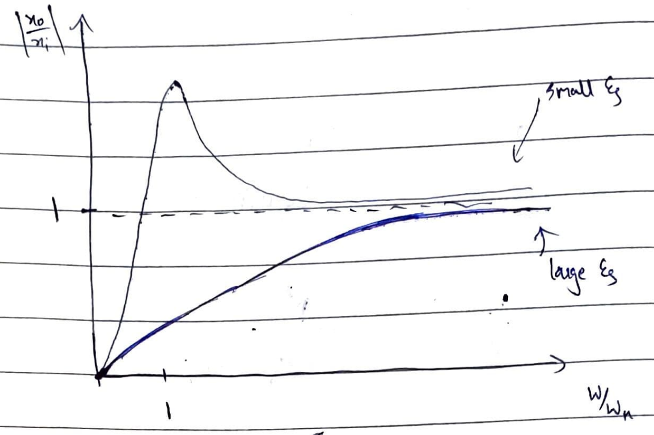
For this instrument, we want $|x_i/x_o| = 1$

That is, $x_m = 0$

We want it to be insensitive to certain vibrations



This peak at $\delta = 1$ tells us that the instrument will vibrate vigorously when the input frequency is equal to the natural frequency of the instrument.

$|x_o/x_i| = 1$ for large frequencies tell us that the instrument cannot cope up with the high frequency and fails to give any displacement at all.

Q7.

a) Given, pulse width = 6 ns

Time between shots = Time delay = 6 ns

Laser repetition = 1/frequency = 1 second

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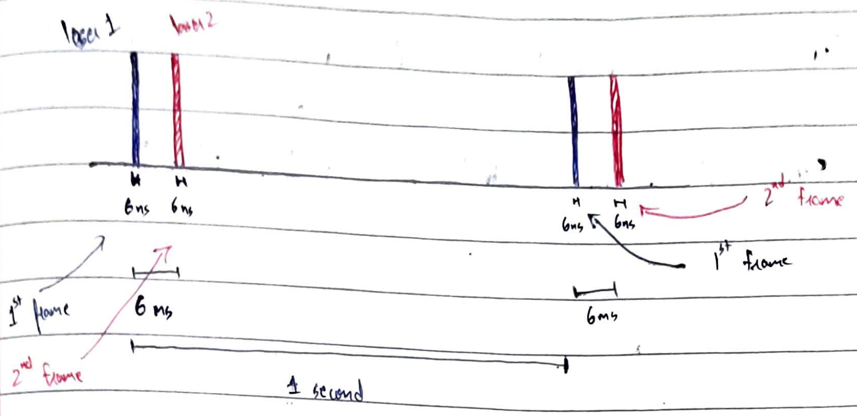
Timing diagram



b)

As seen, the laser is realized for only 6 ns during which the camera captures the picture. This is a really short duration and can be assumed that it is captured for a frozen time.

We can also prove this quantitatively.
The displacement that the flow goes during this time interval :

$$d = 6\,ns \times 40\,mm/s$$
$$= 6 \times 10^{-9} \times 40 \times 10^{-3}$$
$$= 240 \times 10^{-12}\,m$$
$$\approx 0 \text{ meters}$$

c)

If the particles are not stationary, the image captured would be distorted due to the particle motion.
Such frames cannot be analyzed by cross correlation or particle tracking method.
We require a stationary image of the tracer particles so we need the flow to be frozen.

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d)

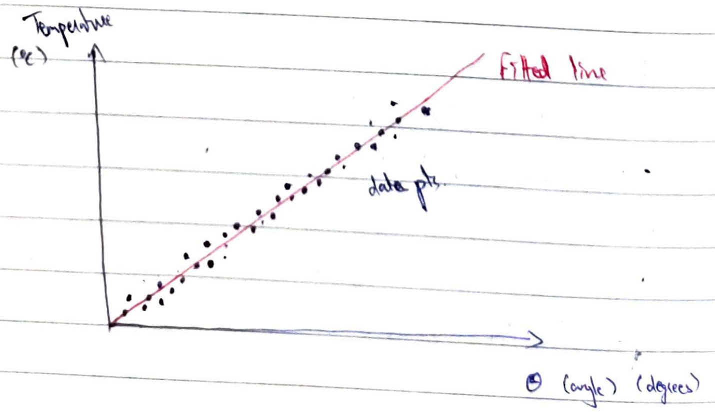
PIV is a non intrusive technique.
The instrument uses lazer to capture particle position and thus no object needs to be inserted to disturb the fluid flow.
Example of an intrusive instrument is the pitot tube.

Q8.

Let us take the example of a pressure thermometer.
This instrument takes temperature as input at the bulb and gives motion as the final output via linkage.
In particular, the motion is an ~~angle~~ angular deflection.

To callibrate an instrument, we give it a known input.
This could be done by exposing the pressure thermometer to a temperature controlled environment or to an environment where we simultaneously measure the temperature using a calibrated thermometer.

We plot an input output relation of our pressure thermometer.
It will be a Temperature vs θ curve. While plotting, temperature is known, instrument gives the angle.
We prefer a linear input output relation for our instrument, so we fit a linear line to the curve we just plotted.



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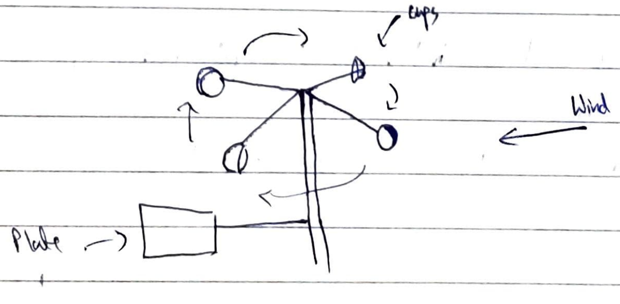
We also find the uncertainity in our readings with respect to our actual readings.

Now we can use this calibrated curve to use this instrument. When the instrument gives a certain angular deflection for a temperature, we use to curve to find the corresponding temperature.
We can also mark the scale in the instrument itself using this curve.

Q9.

In Archery, it is very important to know the wind speed and the wind direction.
To test archery skills in olympics, different wind speeds and direction are often the parameters.

We can accurately measure the wind speed and direction using a cup anemometer.



It has cups which in the presence of wind experience a force and start revolving about the instrument axis.
A flat plate also experience a force, causing it to deflect and positioning itself in the wind direction. This helps us get the wind direction.

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The number of cup revolutions can be counted using a digital counter which we studied in class.
Using the conservation of momentum, we can get a linear relationship between the no. of revolutions and the wind speed.

Note that this gives us the speed and direction parameters in only one plane, the horizontal plane.
This instrument would ~~give~~ also get affected by the wind speed in vertical direction. ~~basis, because of the~~

The instrument can be accurately used when the wind velocity in the vertical component is much lesser.
The cup revolution should not have any resistance at it's joints, but this is not the ideal case. This will give some error too.

In archery, we ~~also~~ should have the knowledge of wind in all the three coordinates, so we can use three such instrument in the three planes (xy, yz & zx) and take the equivalent speed and direction using vector addition.

For stronger winds, instruments like ~~pt~~ Prandtl pitot tube can also be use.
Again, we need three pitot tube along three axis.