ME 202

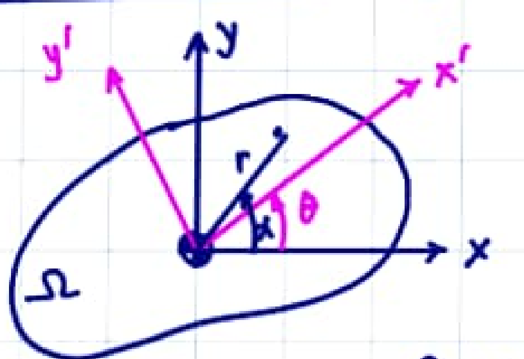
LECTURE 35

MON 11 APR 2022

Unsymmetric Bending

Review on moment of inertia/second moment of area

$\rho = 1$



Ω 2D planar c/s

● centroid

$$I_{xx} = \int_\Omega y^2\,da,$$

$$I_{yy} = \int_\Omega x^2\,da, \quad I_{xy} = \overset{*}{-}\int_\Omega xy\,da$$

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

$$I_{x'x'} = I'_{xx} = \int_\Omega y'^2\, da'$$

$$da' = J\,da \qquad J = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{vmatrix}$$

$$dx'dy' = J\,dx\,dy$$

$$dx' = \cos\theta\, dx + \sin\theta\, dy \qquad J = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$
$$dy' = -\sin\theta\, dx + \cos\theta\, dy$$

$$= 1$$

$$I'_{xx} = \int_\Omega (-x\sin\theta + y\cos\theta)^2\, dx\,dy$$
$$= \sin^2\theta \int_\Omega x^2\, da + \cos^2\theta \int_\Omega y^2 da - 2\sin\theta\cos\theta \int_\Omega xy\, da$$

$$I_{xx}' = I_{xx}\cos^2\theta + I_{yy}\sin^2\theta + 2\sin\theta\cos\theta\, I_{xy}$$

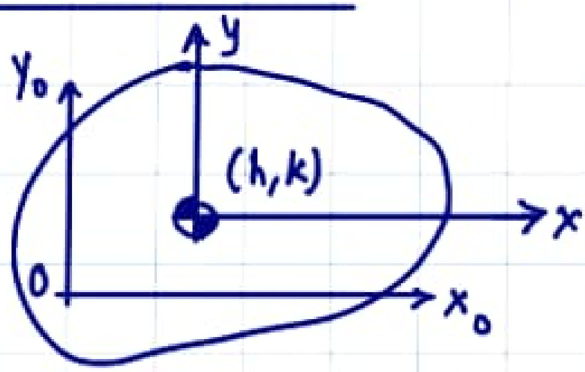
$$I_{yy}' = I_{xx}\sin^2\theta + I_{yy}\cos^2\theta - 2\sin\theta\cos\theta\, I_{xy}$$

$$I_{xy}' = (I_{yy} - I_{xx})\sin\theta\cos\theta + I_{xy}(\cos^2\theta - \sin^2\theta)$$

same as stress transf. rules

$$\underline{L} = \underset{\sim}{I}\,\underline{\omega}\ , \quad \underline{T} = \underset{\sim}{I}\,\underline{\alpha}\ , \text{ etc.}$$

Parallel Axis Thm

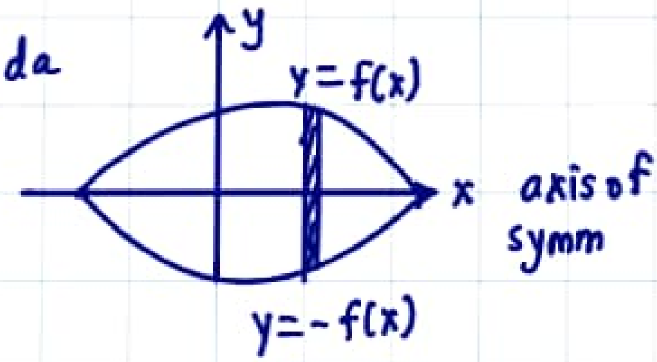


$$x = x_0 - h, \quad y = y_0 - k$$

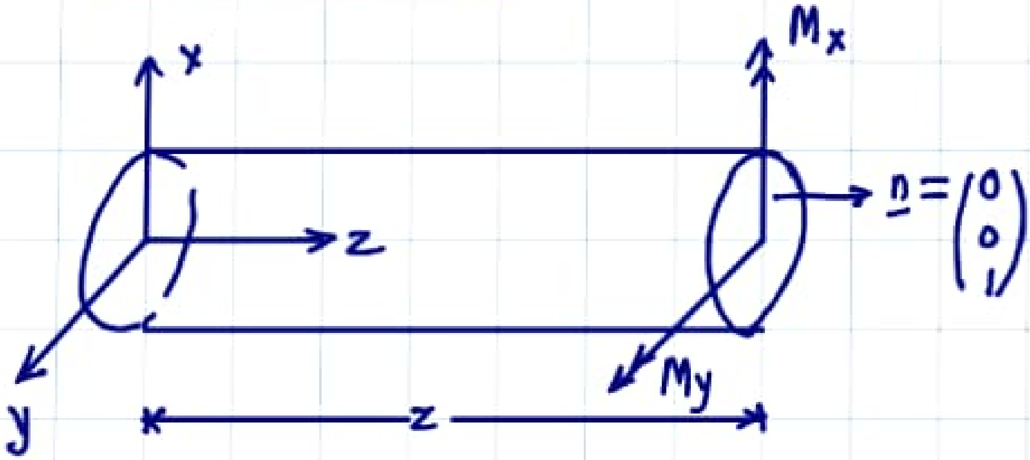
$$I_{xx}^{o} = I_{xx} + Ak^2, \quad I_{yy}^{o} = I_{yy} + Ah^2$$

$$I_{xy}^{o} = I_{xy} - Ahk$$

$$I_{xy} = -\int_{\Omega} xy \, da$$



Return to Bending



$$\sigma_{zz} = (Ax + By) = E\,\epsilon_{zz}$$ uniaxial

$$\epsilon_{zz} = -x \frac{d^2u}{dz^2} - y \frac{d^2v}{dz^2}$$

u disp along x, v disp along y

Eqm on c/s,

Force Eqm $\int_{\Omega} \underline{t} \, da = \underline{0}$ $\qquad \underline{t} = \begin{pmatrix} \cancel{\sigma_{xz}}^{\,0} \\ \cancel{\sigma_{yz}}^{\,0} \\ \sigma_{zz} \end{pmatrix}$

$$\int_{\Omega} \sigma_{zz} \, da = 0$$

$$A \int_{\Omega} x \, da + B \int_{\Omega} y \, da = 0$$

$\forall A, B$

for all

$\Rightarrow$ x-axis, y-axis $\Big\}$ centroidal axes

Moment Eqm on c/s

$$\int_{\Omega} \underbrace{(\underline{x} \times \underline{t})} \, da = M_x \underline{e}_x + M_y \underline{e}_y$$

$$\underline{e}_x (y \sigma_{zz} - z \cancel{\sigma_{zy}}) - \underline{e}_y (x \sigma_{zz} - z \cancel{\sigma_{zx}}) + \underline{e}_z (x \cancel{\sigma_{zy}} - y \cancel{\sigma_{zx}})$$

$$M_x = \int_\Omega y\, \sigma_{zz}\, dx\,dy$$

✓ $$M_y = -\int_\Omega x\, \sigma_{zz}\, dx\,dy$$

$$-\int_\Omega x\,(Ax + By)\, dx\,dy = M_y$$

known/given

$$-A\, I_{yy} + B\, I_{xy} = M_y$$

$$-A\, I_{xy} + B\, I_{xx} = M_x$$

calculated from geom

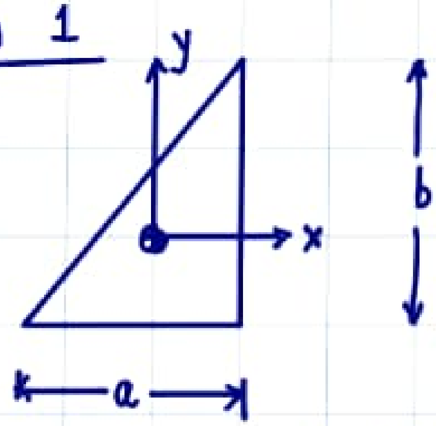
$$A = -\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = -E\frac{d^2 u}{dz^2}$$

$$B = -\frac{-M_x I_{yy} + M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = -E\frac{d^2 v}{dz^2}$$

Symm bending earlier

$I_{xy} = 0$, $M_y \neq 0$, $M_x = 0$

Problem 1



c/s of a cantilever, $L = 1$ m

$$\underline{M} = \begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} 10 \text{ Nm} \\ 0 \end{pmatrix} \quad @ \ z = L$$

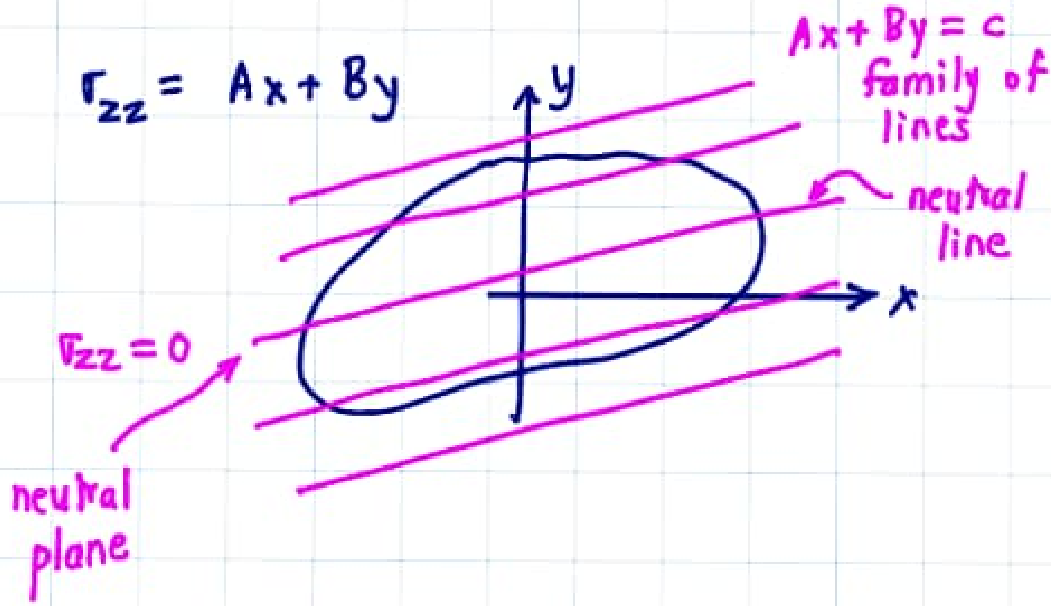
Calculate max horiz and vert disp

u_{max}, v_{max}

$E = 100$ GPa, $a = 10$mm, $b = 20$mm

Obtain eqn of neutral line

Points on the triangle with max, min σ_{zz}.


σzz = Ax + By
Ax + By = c
family of lines
neutral line
y
x
σzz = 0
neutral plane