Mathematica solution and additional explanation for Q1

Q1.

Following information is given in Q1. The top of the riser is insulated which means that the riser will not lose heat from the top. It also says that the riser is located at the top of the casting which means there will be no heat loss from the bottom as well. This the riser volume Vr=$\pi r^2 h$ and the surface area through which heat is lost is Ar=2πrh. Vc/Ac =30 as given in the problem. Chvorinov's rule states $\left(\frac{V}{A}\right)^2$ of riser ⩾ $\left(\frac{V}{A}\right)^2$ of casting, Under limiting conditions, they can be equal [5 marks]

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In[89]:= Vr = π r² h
         Ar = 2 π r h
Out[89]= h π r²
Out[90]= 2 h π r
In[91]:= Solve[(Vr / Ar) == 30, r]
Out[91]= {{r → 60}}
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The radius of the riser is 60 mm and diameter is 120 mm.

If the riser is insulated further it means that the riser's conductivity will be lower than that of sand now, the solidification time of riser will increase by a factor of 3. A detailed explanation is attached as a separate file. Now the new relationship is 3×$\left(\frac{V}{A}\right)^2$of riser⩾ $\left(\frac{V}{A}\right)^2$ of casting. Under limiting conditions [7 marks],

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In[92]:= Solve[3 (Vr / Ar)² == 30², r]
Out[92]= {{r → -20 √3}, {r → 20 √3}}
In[93]:= N[{{r → -20 √3}, {r → 20 √3}}]
Out[93]= {{r → -34.641}, {r → 34.641}}
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The acceptable radius is 34.641 mm and diameter 69.28 mm. This reduction can be attributed to an improvement in the insulation which can lead to a reduction in the volume of riser required. Better insulation of riser is known to reduce the wastage of material in riser by feeding the material longer [3].

The solidification time is given by

$$t = \left[\frac{\pi}{4}\left(\frac{\rho_c \Delta H_f}{T_m - T_o}\right)^2 \frac{1}{K_m \rho_m C_m}\right]\left(\frac{V}{A}\right)^2$$

It is clear that solidification time is a function of Geometry (V/A) and thermal properties.

Ordinarily, the thermal properties of sand is same for riser and casting. To ensure feeding from riser,

$$t_{riser} \geq t_{casting}$$

$$(V/A)^2_{riser} \geq (V/A)^2_{casting}$$

We know that now the riser is insulated which means $K_{m\,riser} = \frac{K_m}{3}$

$$t_{s\,riser} = \left[\frac{\pi}{4}\left(\frac{\rho_c \Delta H_f}{T_m - T_0}\right)^2 \frac{1}{\left(\frac{K_m}{3}\right)\cdot \rho_m C_m}\right]\left(\frac{V}{A}\right)^2_{riser} \quad -(2)$$

The conductivity of riser is 1/3 conductivity of sand mold. The solidification time for casting is given by,

$$t_{s\,casting} = \left[\frac{\pi}{4}\left(\frac{\rho_c \Delta H_f}{T_m - T_0}\right)^2 \frac{1}{K_m \rho_m C_m}\right]\left(\frac{V}{A}\right)^2_{cast} \quad -(3)$$

The solidification time of riser should be equal to or greater, under limiting conditions,

$$t_{s\,riser} \geq t_{s\,casting}$$

From Eq (2) and (3),

$$3\left(V/A\right)^2_{riser} \geq \left(V/A\right)^2_{casting}$$

If riser loses heat at 1/3 the rate of casting.