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Lecture : 04.03.2021

- Fundamental Standards

Fundamental quantities for which independant standards are defined: length, time, mass & temperature.

Some of the thing we take care of while choosing the standards :

i) Should not be rare and should be ~~easily reproduc~~ easy to reproduce. Labs all over the world need to have ~~tha~~ access to them.

ii) Should be insensitive to modifying and interfering inputs.

iii) Should be well known and should be accepted throughout the world. Shouldn't be controversial. Should give the same result every time it's reproduced.

Hierarchy in fundamental standards : For traceability

- Global Reference

i) National Reference standards

ii) Working standards

iii) Interlaboratory standards

- Vernier Caliper

A common length measuring instrument. It gives additional accuracy by implementing two scales: The main scale and the vernier scale. Unequal divisions in these scales give way to extra accuracy.

Let, a = Size of main scale division

p = No. of main scale divisions } for same interval

q = No. of vernier scale divisions

$$\text{Scale readability} = \frac{a(q-p)}{q}$$

- Screw Gauge

Also used to measure length. It has a main scale and a circular scale. We define pitch as the distance moved on the main scale for one screw rotation.

Let, n = No. of divisions on the circular scale

p = Pitch

$$\text{Scale readability} = \frac{p}{n}$$

Lecture : 08.03.2021

- Gauge blocks:

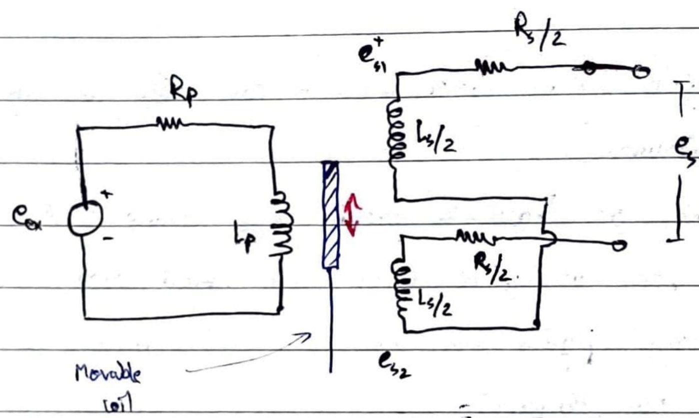
  - Physical objects whose dimensions we can rely on for checking various instruments.
  - Small blocks of hard & dimensionally stable steel.
  - Made in sets of wide range and small steps, such that any arbitrary figure can be reproduced using the limited blocks at our disposal.
  - Extremely important to control all interfering and modifying inputs for greater accuracy.

  Similarly, we have angle blocks.
  - Angular displacement is essentially based on length measurements so a fundamental standard isn't necessary.
  - We still have working standards for ease.
  - Again, angle blocks have a wide range & several steps.

- Magnetoresistive displacement transducer : Refer the slides

Lecture : 09.03.2021

- Variable inductance pickup → Differential transformer



$$e_{ex} = i_p R_p + L_p \frac{di_p}{dt} \quad \text{— ①}$$

$$e_{s1} = M_1 \frac{di_p}{dt}$$

$$e_{s2} = M_2 \frac{di_p}{dt}$$

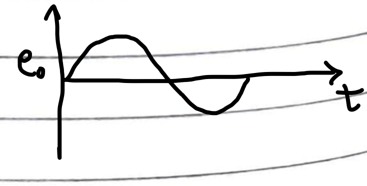
M_1 & M_2 are mutual inductances.

$$e_s = e_{s1} - e_{s2} = (M_1 - M_2)\frac{di_p}{dt} \quad \text{— ②}$$

For symmetric configuration,

$M_1 = M_2$

$e_s = 0$

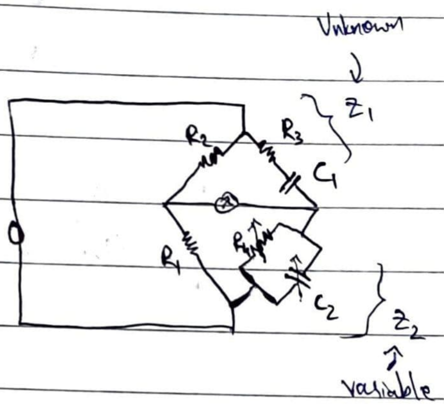


Nearly linear near the null point.

- AC bridges

i) Wien bridge.

Used to measure an unknown capacitance.



When the bridge is balanced:

$$\frac{R_2}{R_1} = \frac{Z_1}{Z_2}$$

Impedance $Z_1 = R_3 + \frac{1}{j\omega C_1} = R_3 - \frac{j}{\omega C_1}$

$$Z_2 = \frac{\frac{1}{j\omega C_2} \times R_4}{R_4 + \frac{1}{j\omega C_2}} = \frac{R_4}{1 + j\omega C_2 R_4}$$

Substituting,

$$R_2 \left(\frac{R_4}{1 + j\omega C_2 R_4} \right) = R_1 \left(R_3 - \frac{j}{\omega C_1} \right)$$

$$\omega C_1 R_2 R_4 = (R_1 R_3 + j\omega C_2 R_1 R_3 R_4)\omega C_1 - jR_1 + \omega C_2 R_1 R_4$$

Comparing complex & real parts,

$$\omega^2 = \frac{1}{R_3 R_4 C_1 C_2} \quad \text{—— (1)}$$

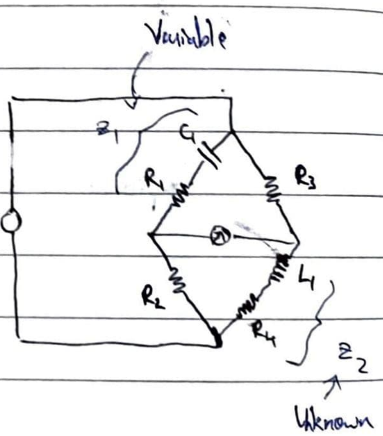
$$wC_1 R_2 R_4 = wR_1 (C_1 R_3 + C_2 R_4)$$

$$\frac{R_2}{R_1} = \frac{C_2}{C_1} + \frac{R_3}{R_4} \quad \text{——— (2)}$$

By changing the variable elements R_4 & C_2, we can find the value of R_3 & C_1.

ii) Hay bridge

Used to measure an unknown inductance & resistance.



$$\frac{Z_1}{R_2} = \frac{R_3}{Z_2}$$

$$\frac{R_1 - j/wC_1}{R_2} = \frac{R_3}{R_4 + jwL_1}$$

$$R_1 R_4 + jwL_1 R_1 - \frac{jR_4}{wC_1} + \frac{L_1}{C_1} = R_3 R_2$$

Comparing complex & real parts,

$$w^2 = \frac{R_4}{L_1 R_1 C_1} \quad \text{——— (1)}$$

And, $R_1 R_4 + \frac{L_1}{C_1} = R_3 R_2$

Finally, the eqn for unknowns are:

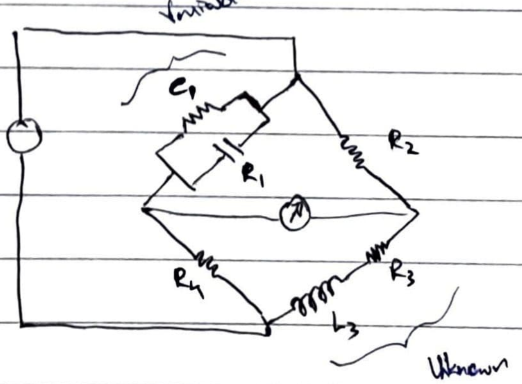
$$L_1 = \frac{R_2 R_3 C_1}{1 + w^2 R_1^2 C_1^2}$$

$$R_4 = \frac{w^2 C_1^2 R_2 R_3 R_1}{1 + w^2 R_1^2 C_1^2}$$

Where R_1 & C_1 is set to balance the bridge.

iii) Maxwell - Wien's bridge

Used to measure unknown resistance & inductance.



Using the same analysis, We finally get:

$$L_3 = C_1 R_2 R_4$$

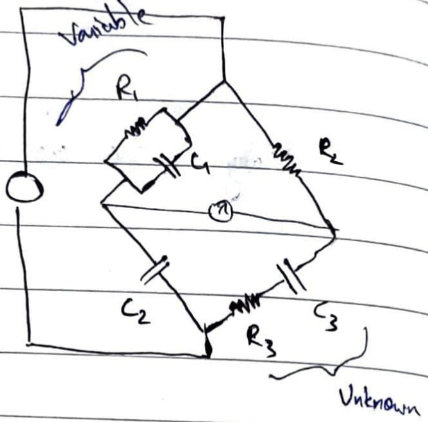
$$R_3 = \frac{R_2 R_4}{R_1}$$

For a balanced bridge

Note: The unknowns are independant of frequency w.

iv) Schering bridge :

Used to measure unknown capacitance & resistance.



Using the same analysis, we get :

$$C_3 = \frac{R_1 C_2}{R_2}$$

$$R_3 = \frac{C_1 R_2}{C_2}$$

For a balanced bridge

Note : The unknowns are independant of frequency ω.

Lecture : 11.03.2021

- Capacitance pick-up

Concept : By changing the overlapping area of the two plates of the capacitor, we change the capacitance which is picked up.
We have seen several bridges that can be used to pick up this capacitance.

Similarly, instead of ~~ex~~ changing the overlapping area, we can also change the distance between the plates.

We can also use these concepts to measure the angular displacements. Like rotation to change overlap area.

Sensitivity is adjusted based on ~~x~~ whether it's used for long range or short range measurements.

8.854×10^{-12} F/m

$$C = \frac{\varepsilon_0 \varepsilon_r A}{x}$$

Non linearity $\eta = \frac{\Delta h}{h+\Delta h}$

Sensitivity

i) When area is changed, i.e, the plate is moved parallel to itself

$$\frac{\Delta C}{\Delta A} = \frac{\varepsilon_0 \varepsilon_r}{x} \qquad \text{where, } \Delta A = w \Delta l$$

ii) When the distance b/t the plates ~~...~~ is changed

$$\frac{\Delta C}{\Delta x} = \frac{\varepsilon_0 \varepsilon_r A}{x^2}$$ (mod for +ve value)

- Piezoelectric Transducers.

- Generating electric charge within them upon deformation. This is called piezoelectric effect.
- The effect is linear & reversible.
- So it is direction sensitive. Polarity switches for compression and tension. It helps us detect the direction.
- Used for measuring force, acceleration & pressure.
- Low response time.

Refer the slides for more.

- Integration route or the differentiation route?

When we have a system with a lot of error, differentiation tends to amplify this noise, while integration averages out the noise and actually give near to perfect solutions.

lecture : 15.03.2021 & 16.03.2021

- Strain gauge

Concept: Change in resistance with elongation or compression of the strain gauge. This change is picked up using the wheatstone bridge.

We know, $R = \frac{\rho L}{A}$

$$\frac{dR}{R} = \left[\frac{L}{A}d\rho + \frac{\rho}{A}dL - \frac{\rho L}{A^2}dA\right] \times \frac{A}{\rho L}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Also, $\frac{A+dA}{A} = (1-\varepsilon\nu)^2 \approx 1-2\varepsilon\nu$

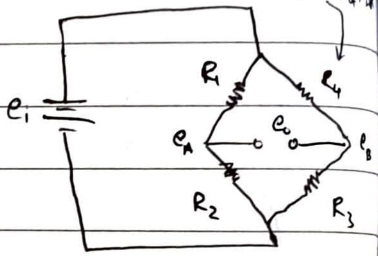
$$\frac{dA}{A} = -2\varepsilon\nu = -2\nu\frac{dL}{L}$$

$$\frac{dR}{A} = \underbrace{\frac{d\rho}{\rho}}_{\text{Change in resistance due to piezoelectric effect}} + \underbrace{(1+2\nu)\frac{dL}{L}}_{\text{Change in resistance due to the change in length}}$$

Change in mechanical property (Resistivity) due to mechanical stress/Force : Piezo electric effect

$$\boxed{\text{Gage factor} = \frac{dR/R}{dL/L} = \frac{d\rho/\rho}{dL/L} + (1+2\nu)}$$

After stress
$R_4 = R_4 + \Delta R$



$$e_A - e_B = e_o$$

$$e_o = \left[\frac{R_1}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right] e_i$$

Let $R_1 = R_2$

$R_3 = R_4 = R_G$

$$e_o = e_i\left[\frac{1}{2} - \frac{R_G + \Delta R}{R_G + R_G + \Delta R}\right]$$

Let $S_g = \dfrac{\Delta R / R_G}{\Delta L / L} \Rightarrow \Delta R = S_g \varepsilon R_G$

$$e_o = e_i\left[\frac{1}{2} - \frac{R_G + S_g\varepsilon R_G}{2R_G + S_g\varepsilon R_G}\right]$$

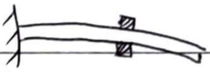
$$= e_i\left[\frac{1}{2} - \frac{1+\varepsilon S_g}{2+\varepsilon S_g}\right]$$

$$e_o = \frac{e_i(-\varepsilon S_g)}{2(2+\varepsilon S_g)}$$ } Non linear

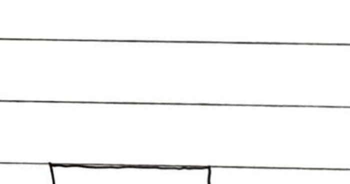
Here,, input = Strain (ε)
output = Voltage (e_o)

How to make the input output relation linear ?

Case I Use two strain gauges



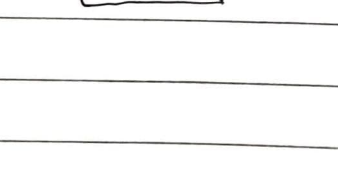
$$e_o = e_i\left[\frac{1}{2} - \frac{R_G+\Delta R}{2R_G}\right]$$



$$= e_i\left(\frac{-\Delta R}{2R_G}\right)$$

$$= -e_i S_g \varepsilon/2$$

Case II Use four strain gauges



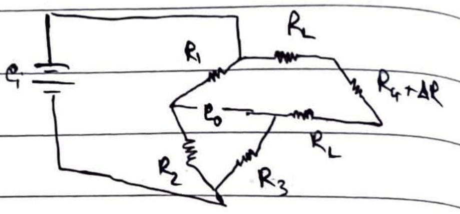
$$e_o = e_i\left[\frac{R_G-\Delta R}{2R_G} - \frac{R_G+\Delta R}{2R_G}\right]$$

$$= \frac{e_i(-2\Delta R)}{2R_G}$$

$$= -e_i\varepsilon S_g$$

$$\frac{e_o}{e_i} = \left[\frac{R_1}{R_1+R_2} - \frac{2R_L+R_4+\Delta R}{R_3+2R_L+R_4+\Delta R}\right]$$

$$= \left[\frac{1}{2} - \frac{2R_L/R_4 + 1 + \Delta R/R_4}{\frac{R_3}{R_4} + 2R_L/R_4 + 1 + \frac{\Delta R}{R_L}}\right]$$



Lead wire resistance

Lecture : 18.03.2021

★ Second order systems

General eqⁿ : $a_2 \dfrac{d^2 q_o}{dt^2} + a_1 \dfrac{dq_o}{dt} + a_0 q_o = b_o q_i$

Let's define :

$k = \dfrac{b_o}{a_o}$ — Static sensitivity, Unitless

$w_n = \sqrt{\dfrac{a_o}{a_2}}$ — Undamped natural frequency, Unit : Hz

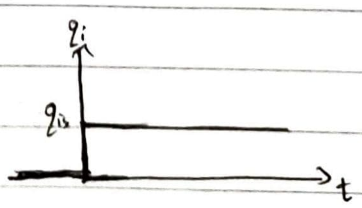
$\xi = \dfrac{a_1}{2\sqrt{a_o a_2}}$ — Damping ratio, Unitless

Equivalent eqⁿ : $\dfrac{1}{w_n^2} \dfrac{d^2 q_o}{dt^2} + \dfrac{2\xi}{w_n} \dfrac{dq_o}{dt} + q_o = kq_i$

I Step response to a 2^{nd} order system

IC : $q_o(0) = 0$

$q_o'(0) = 0$



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$$\frac{1}{\omega_n^2} q_0'' + \frac{2\xi}{\omega_n} q_0' + q_0 = k q_{is}$$

Taking laplace transform :

$$\frac{1}{\omega_n^2}\left[s^2 Q_0 - \cancel{s q_0(0)} - \cancel{q_0'(0)}\right] + \frac{2\xi}{\omega_n}\left[s Q_0 - \cancel{q_0(0)}\right] + Q_0 = \frac{k q_{is}}{s}$$

$$\left[\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} + 1\right] Q_0 = \frac{k q_{is}}{s}$$

$$Q_0 = \frac{k q_{is} \omega_n^2}{s(s^2 + 2\xi \omega_n + \omega_n^2)}$$

$$= \frac{k q_{is} \omega_n^2}{[(s + \xi\omega_n)^2 + (\omega_n^2 - \xi^2\omega_n^2)]s}$$

$$= \frac{k q_{is} \omega_n}{\sqrt{1-\xi^2}} \times \frac{1}{s} \times \frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + (\sqrt{\omega_n^2 - \xi\omega_n^2})^2}$$

Now proceed using the convolution theorem.

Final solution :

Case 1 $\xi = 0$

$$q_0(t) = [1 - \cos(\omega_n t)] k q_{is}$$

Case 2 : $0 < \xi < 1$

$$q_o(t) = \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\sqrt{1-\xi^2}\, \omega_n t + \phi\right) \right] k q_{i3}$$

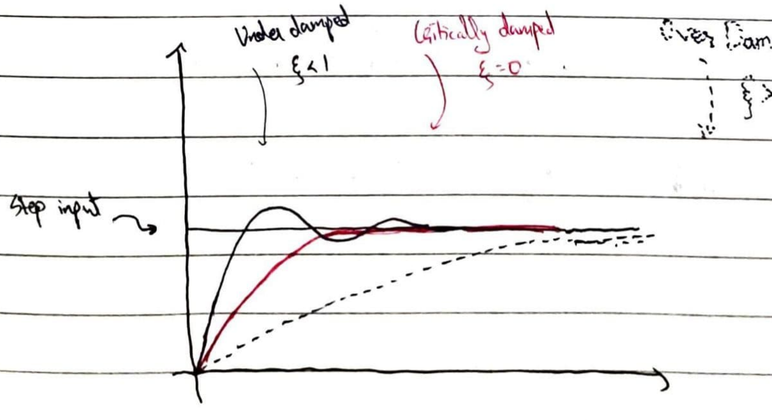
Where, $\theta = \sin^{-1}\left(\sqrt{1-\xi^2}\right)$

Case 3 : $\xi = 1$

$$q_o(t) = k q_{i3} \left(1 - e^{-\omega_n t} - \omega_n t\, e^{-\omega_n t}\right)$$

Case 4 : $\xi > 1$

$$\frac{q_o(t)}{k q_{i3}} = 1 - \left(\frac{\xi + \sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\right) \exp\left[\left(-\xi + \sqrt{\xi^2-1}\right)\omega_n t\right] + \left(\frac{\xi - \sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\right) \exp\left[\left(-\xi - \sqrt{\xi^2-1}\right)\omega_n t\right]$$

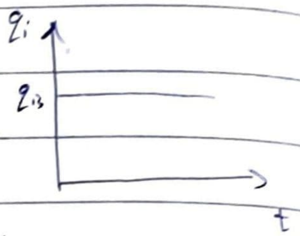


Output shifts to the right as $\xi \uparrow$

Output frequency $\uparrow$ with $\omega_n \uparrow$

Homework : ME226 190100011

Q1. Step response :



Initial condition : $q_o(0) = 0$

$\left.\frac{dq_o}{dt} = 0\right|_{t=0}$

$$\frac{1}{w_n^2}\frac{d^2q_o}{dt^2} + \frac{2\xi}{w_n}\frac{dq_o}{dt} + q_o = kq_{is}$$

laplace transform

$$\frac{1}{w_n^2}\left[s^2Q_o - s\cancel{q_o(0)}^{0} - \cancel{\dot{q}_o(0)}^{0}\right] + \frac{2\xi}{w_n}\left[sQ_o - \cancel{q_o(0)}^{0}\right] + Q_o = \frac{kq_{is}}{s}$$

$$Q_o = \frac{kq_{is}\,w_n^2}{s(s^2 + 2\xi w_n s + w_n^2)}$$

$$= \frac{kq_{is}\,w_n}{\sqrt{1-\xi^2}} \times \underbrace{\frac{1}{s}}_{F(s)} \times \underbrace{\frac{\sqrt{1-\xi^2}\,w_n}{s^2 + 2\xi w_n s + w_n^2}}_{G(s)}$$

$$\frac{b}{(s-a)^2 + b^2} \Rightarrow b = w_n\sqrt{1-\xi^2}$$

$$a = -w_n\xi$$

Inverse laplace using convolution theorem.

$$q_o = \frac{kq_{is}\,w_n}{\sqrt{1-\xi^2}} \times \int H(\tau)\, e^{-\xi w_n(t-\tau)} \sin\left[\sqrt{1-\xi^2}\, w_n (t-\tau)\right] d\tau$$

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Look at the denominator : $s^2 + 2\xi w_n s + w_n^2$

Case I : $\xi > 1$ $\Rightarrow$ ~~Real~~ 2 real roots

$$s_1, s_2 = -w_n \xi \pm w_n \sqrt{\xi^2 - 1}$$

$$Q_o = kq_{i3} \times \frac{s_1 s_2}{s(s-s_1)(s-s_2)} = kq_{i3} \underbrace{\frac{1}{s}}_{F(s)} \times \underbrace{\frac{s_1 s_2}{(s-s_1)(s-s_2)}}_{G(s)}$$

$$\cancel{q_o = kq_{i3} \left[1 - \left(\frac{s_2 e^{s_1 t} - s_1 e^{s_2 t}}{s_2 - s_1} \right) \right]}$$

$$q_o = kq_{i3} \int_0^t H(t-\tau) \left(\frac{e^{s_1 \tau} - e^{s_2 \tau}}{s_2 - s_1} \right) d\tau$$

$$= kq_{i3} \left[\frac{s_1 e^{s_1 t} - s_2 e^{s_2 t}}{s_2 - s_1} \right]_0^t$$

$$= kq_{i3} \left[\frac{s_1 e^{s_1 t} - s_2 e^{s_2 t}}{s_2 - s_1} + 1 \right]$$

$$= kq_{i3} \left[\frac{w_n(-\xi + \sqrt{\xi^2 - 1}) e^{w_n t(-\xi + \sqrt{\xi^2-1})} - w_n(-\xi - \sqrt{\xi^2-1}) e^{w_n t(-\xi - \sqrt{\xi^2 - 1})}}{-2 w_n \sqrt{\xi^2 - 1}} + 1 \right]$$

Case II : $\xi = 1$ $\Rightarrow$ $s_1 = s_2 = -w_n \xi = -w_n$

$$Q_o = kq_{i3} w_n^2 \times \frac{1}{\underbrace{s}_{F(s)} \underbrace{(s + w_n)^2}_{G(s)}}$$

$$q_o = k q_{i3}\, \omega_n^2 \int_0^t H(t-\tau) \times \tau e^{-\omega_n \tau}\, d\tau$$

$$= k q_{i3}\, \omega_n^2 \left[\tau (-1/\omega_n) e^{-\omega_n \tau} - 1/\omega_n^2 e^{-\omega_n \tau} \right]_0^t$$

$$= k q_{i3} \left[-t \omega_n e^{-\omega_n t} - e^{-\omega_n t} - 1 \right]$$

Case III : $\xi < 1$

$$Q_o = \frac{k q_{i3}\, \omega_n}{\sqrt{1-\xi^2}} \times \underbrace{\frac{1}{s}}_{F(s)} \times \underbrace{\frac{\sqrt{1-\xi^2}\, \omega_n}{s^2 + 2\xi \omega_n + \omega_n^2}}_{G(s)}$$

$$q_o = \frac{k q_{i3}\, \omega_n}{\sqrt{1-\xi^2}} \int_0^t H(\tau)\, e^{-\xi \omega_n (\tau)} \sin\left(\sqrt{1-\xi^2}\, \omega_n (\tau)\right) d\tau$$

$$= \frac{k q_{i3}\, \omega_n}{\sqrt{1-\xi^2}} \int_0^t e^{-\xi \omega_n \tau} \sin\left(\sqrt{1-\xi^2}\, \omega_n \tau\right) d\tau$$

$$= \frac{k q_{i3}\, \omega_n}{\sqrt{1-\xi^2}} \int_0^t e^{a\tau} \sin(b\tau)\, d\tau$$

$$a = -\xi \omega_n \quad , \quad b = \sqrt{1-\xi^2}\, \omega_n$$

$$q_o = \frac{k q_{i3}\, \omega_n}{\sqrt{1-\xi^2}} \left[\frac{e^{a\tau}}{a^2+b^2} (a \sin b\tau - b \cos b\tau) \right]_0^t$$

$$\frac{q_o}{k q_{i3}} = 1 - \frac{e^{\xi \omega_n t}}{\sqrt{1-\xi^2}} \left(\xi \sin(\sqrt{1-\xi^2}\, \omega_n t) - \sqrt{1-\xi^2} \cos(\omega_n t \sqrt{1-\xi^2}) \right)$$

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Homework : ME 226 190100011

Q2.

Partial fractions :

$$Q_o = \frac{k q_{i3} w_n^2}{s(\underbrace{s^2 + 2\xi w_n s + w_n^2}))}$$

Roots for this quadratic are :

$$s_1, s_2 = \frac{-2\xi w_n \pm \sqrt{4\xi^2 w_n^2 - 4w_n^2}}{2}$$

$$= -\xi w_n \pm w_n\sqrt{\xi^2 - 1}$$

$$= w_n \{ \xi \pm \sqrt{\xi^2 - 1} \}$$

So,

$$s_1 = w_n (\xi + \sqrt{\xi^2 - 1})$$

$$s_2 = w_n (\xi - \sqrt{\xi^2 - 1})$$

Case I : $\xi > 1$

$$\frac{1}{s(s^2 + 2\xi w_n s + w_n^2)} = \frac{A}{s} + \frac{B}{s - s_1} + \frac{C}{s - s_2}$$

$$\Rightarrow A(s-s_1)(s-s_2) + Bs(s-s_2) + Cs(s-s_1) = 0$$

$$A + B + C = 0 \longrightarrow (1)$$

$$A(s_1 + s_2) + B s_2 + C s_1 = 0 \longrightarrow (2)$$

$$A s_1 s_2 = 1 \longrightarrow (3)$$

$$A = \frac{1}{s_1 s_2} = \frac{1}{W_n^2}$$

$$Eq\,[(2) - s_1 \times (1)] \Rightarrow A s_2 + B(s_2 - s_1) = 0$$

$$B = \frac{-A s_2}{s_2 - s_1} = \frac{1}{s_1(s_1 - s_2)}$$

$$B = \frac{1}{s_1(s_1 - s_2)} = \frac{1}{W_n(\xi + \sqrt{\xi^2 - 1})} \times \frac{1}{2W_n\sqrt{\xi^2-1}}$$

$$= \frac{1}{2W_n^2(\xi + \sqrt{\xi^2-1})\sqrt{\xi^2-1}} = \frac{-1}{2W_n^2}\frac{(\xi + \sqrt{\xi^2-1})}{\sqrt{\xi^2-1}}$$

$$C = -A - B$$

$$= \frac{1}{2W_n^2}\frac{(\xi - \sqrt{\xi^2-1})}{\sqrt{\xi^2-1}}$$

So, $Q_o = k q_{i} W_n^2 \left[\frac{A}{s} + \frac{B}{s - s_1} + \frac{C}{s - s_2} \right]$

Inverse laplace

$$q_o = k q_{i} W_n^2 \left(A + B e^{s_1 t} + C e^{s_2 t} \right)$$

$$q_o = k q_{is} \left[1 - \frac{(\xi + \sqrt{\xi^2-1})}{2\sqrt{\xi^2-1}} e^{\omega_n t(-\xi + \sqrt{\xi^2-1})} + \frac{\xi - \sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}} e^{\omega_n t(-\xi - \sqrt{\xi^2-1})} \right]$$

Case II : $\xi = 1$

Equal roots

~~Product of roots~~

$$s_1 = s_2 = -\omega_n$$

$$Q_o = \frac{k q_{is} \omega_n^2}{s(s+\omega_n)^2}$$

$$\frac{1}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$$A(s+\omega_n)^2 + Bs(s+\omega_n) + Cs = 1$$

$$A + B = 0$$

$$A = 1/\omega_n^2$$

$$B = -1/\omega_n^2$$

$$2A\omega_n s + B\omega_n s + Cs = 0$$

$$C = -1/\omega_n$$

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$$Q_o = k q_{i} \omega_n^2 \left[\frac{1}{\omega_n^2 s} - \frac{1}{\omega_n^2 (s+\omega_n)} - \frac{1}{\omega_n (s+\omega_n)^2} \right]$$

$$= k q_{i} \left[1 - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2} \right]$$

Inverse laplace :

$$q_o = k q_{i} \left[1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t} \right]$$

Case III : $\xi < 1$

Imaginary roots :

$$Q_o = \frac{k q_{i} \omega_n^2}{s(s^2 + 2\xi \omega_n s + \omega_n^2)}$$

$$\frac{1}{s(s^2 + 2\omega_n \xi s + \omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{(s^2 + 2\omega_n \xi s + \omega_n^2)}$$

$$A(s^2 + 2\omega_n \xi s + \omega_n^2) + (Bs + C)s = 1$$

$$A = 1/\omega_n^2$$

$$B = -A = -1/\omega_n^2$$

$$C = -2A\xi\omega_n = -\frac{2\xi}{\omega_n}$$

$$Q_o = k q_{i3} \left[\frac{1}{s} - \frac{(s + 2\xi w_n)}{s^2 + 2\xi w_n s + w_n^2} \right]$$

$$= k q_{i3} \left[\frac{1}{s} - \frac{(s + 2\xi w_n)}{(s + w_n \xi)^2 + w_n^2 (1 - \xi^2)} \right]$$

$$= k q_{i3} \left[\frac{1}{s} - \frac{(s+\xi w_n)}{(s + w_n \xi)^2 + w_n^2 (1+\xi^2)} - \frac{\xi w_n}{(s+w_n\xi)^2 + w_n^2 (1+\xi^2)} \right]$$

Let $\Rightarrow$ $w_d = w_n \sqrt{1-\xi^2}$

$$Q_o = k q_{i3} \left[\frac{1}{s} - \underbrace{\frac{(s+\xi w_n)}{(s+w_n\xi)^2 + w_d^2}} - \frac{\xi w_d}{(s+w_n\xi)^2 + w_d^2} \times \left(\frac{1}{\sqrt{1-\xi^2}} \right) \right]$$

Inverse laplace

$$\frac{s-a}{(s-a)^2 + b^2} \qquad \frac{a}{(s-a)^2 + b^2}$$

$$q_o = k q_{i3} \left[1 - e^{-\xi w_n t} \cos(w_d t) - \frac{\xi e^{-\xi w_n t}}{\sqrt{1-\xi^2}} \sin(w_d t) \right]$$

Let, $\phi = \sin^{-1}\left(\sqrt{1-\xi^2}\right)$

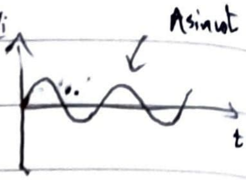
$\sin\phi = 1 - \xi^2$

$$\sqrt{1-\xi^2} \cos(w_d t) + \xi \sin(w_d t) = \sin(w_d t + \phi)$$

$$q_o = k q_{i3} \left[1 - \frac{e^{-\xi w_n t}}{\sqrt{1-\xi^2}} \times \sin\left(\sqrt{1-\xi^2}\, w_n t + \phi\right) \right]$$

Lecture : 23.03.2021

- Frequency response of a 2^{nd} order system



$$\frac{1}{w_n^2}\frac{d^2q_o}{dt^2} + \frac{2\xi}{w_n}\frac{dq_o}{dt} + q_o = kA\sin wt$$

- For the homogeneous solⁿ :

$$\frac{1}{w_n^2}\frac{d^2q_{oh}}{dt^2} + \frac{2\xi}{w_n}\frac{dq_{oh}}{dt} + q_{oh} = 0$$

Let us assume, $q_{oh} = e^{\lambda t}$, Substituting

$$\frac{\lambda^2}{w_n^2} + \frac{2\xi\lambda}{w_n} + 1 = 0$$

$$\lambda^2 + 2\xi w_n\lambda + w_n^2 = 0$$

So, $\lambda_1, \lambda_2 = -\xi w_n \pm w_n\sqrt{\xi^2-1}$

And, $q_{oh} = c_1 e^{\lambda_1} + c_2 e^{\lambda_2}$

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- For the particular solⁿ : $q_{op} = B\cos\omega t + C\sin\omega t$

$$-\frac{\omega^2}{\omega_n^2}(B\cos\omega t + C\sin\omega t) + \frac{2\xi\omega}{\omega_n}(-B\sin\omega t + C\cos\omega t) + (B\cos\omega t + C\sin\omega t) = kA\sin\omega t$$

Solve for B & C in this eqⁿ.

Finally, we get

$$q_o = q_{oh} + q_{op}$$

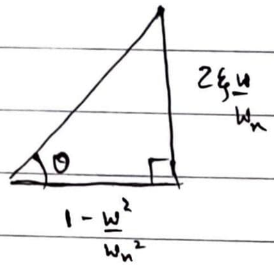
$$q_o = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + B\cos\omega t + C\sin\omega t$$

$$\boxed{\begin{aligned} q_o = {} & \underbrace{c_1 e^{\omega_n t(-\xi+\sqrt{\xi^2-1})} + c_2 e^{-\omega_n t(\xi+\sqrt{\xi^2-1})}}_{\text{Transient : Gets affected by initial cond}^n} \\ & \underbrace{+ \frac{2\xi(\omega/\omega_n)\,kA}{(1-\omega^2/\omega_n^2)^2 + 4\xi^2\omega^2/\omega_n^2}\cos\omega t + \frac{(1-\omega^2/\omega_n^2)\,kA}{(1-\omega^2/\omega_n^2)^2 + 4\xi^2\omega^2/\omega_n^2}\sin\omega t}_{\text{Steady state}} \end{aligned}}$$

So, the steady state output will always be the same irrespective of the initial condⁿ.

For steady state,

$$\frac{q_0|_{steady}}{kA} = \frac{2\xi\,(\omega/\omega_n)\cos\omega t + \left(1-\omega^2/\omega_n^2\right)\sin\omega t}{\left(1-\omega^2/\omega_n^2\right)^2 + 4\xi^2\,\omega^2/\omega_n^2}$$



$$\sin\theta = \frac{2\xi\,\omega/\omega_n}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2}}}$$

$$\frac{q_0}{kA} = \frac{\sin\theta\cos\omega t + \cos\theta\sin\omega t}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2}}}$$

$$= \frac{\sin(\omega t+\theta)}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2}}}$$

- Frequency Response (Without transient state)

$$\frac{1}{\omega_n^2}\frac{d^2q_0}{dt^2} + \frac{2\xi}{\omega_n}\frac{dq_0}{dt} + q_0 = Kq_i$$

Let $D = \frac{d}{dt}$

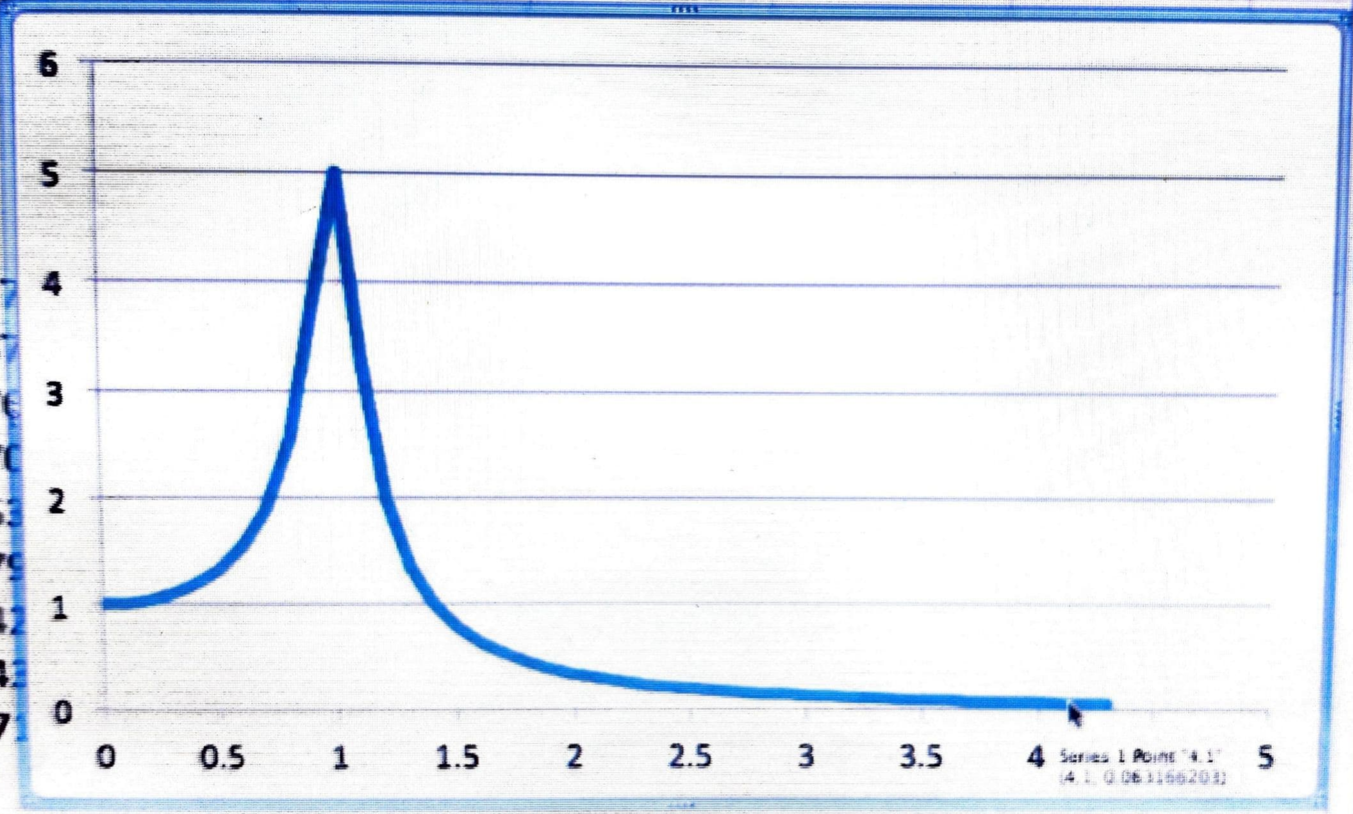
$$\left(\frac{D^2}{w_n^2} + \frac{2\xi}{w_n} D + 1\right) q_o = k q_{i3}$$

$$\frac{q_o}{k q_{i3}} = \frac{1}{\left(\frac{D^2}{w_n^2} + \frac{2\xi D}{w_n} + 1\right)}$$

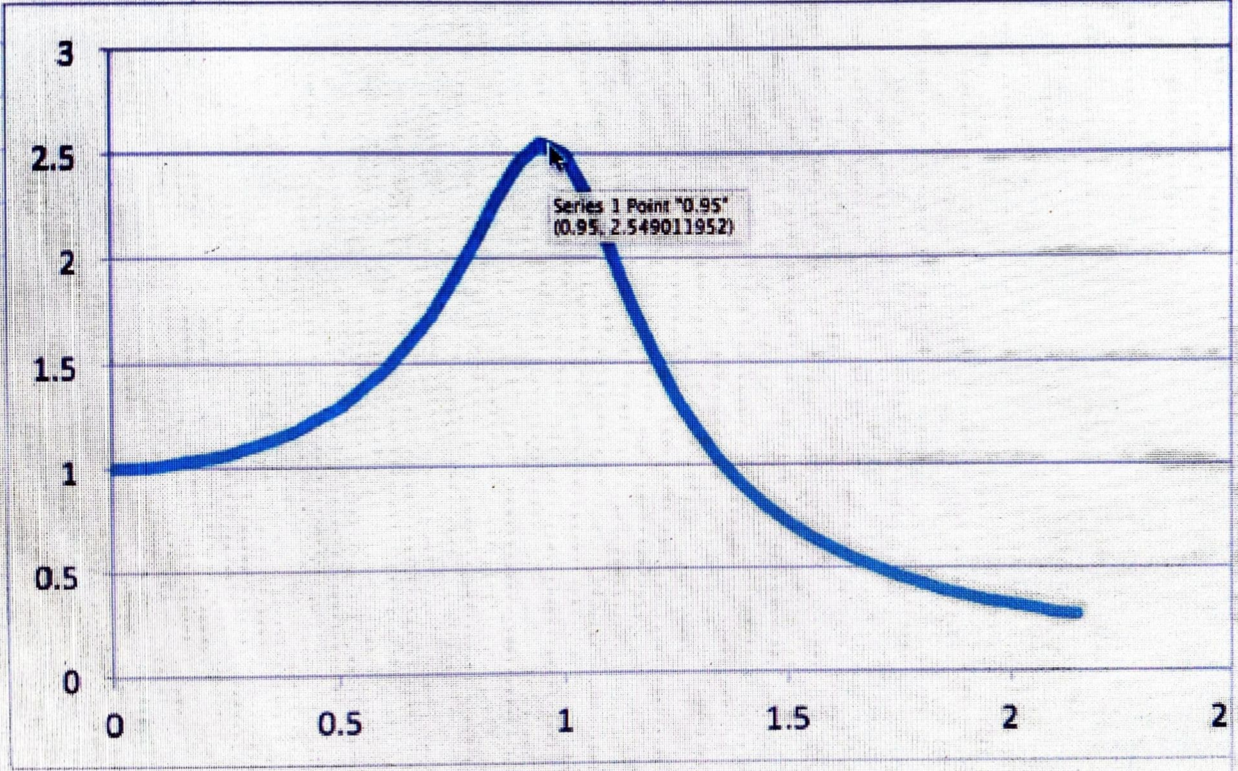
For $D = jw$,

$$\frac{q_o}{k q_{i3}} = \frac{1}{\left[\frac{j^2 w^2}{w_n^2} + \frac{2\xi jw}{w_n} + 1\right]}$$

0.1	
q_is)	phase
1	
89495	0.0201992
76362	0.0416425
52023	0.0658387
11366	0.0949517
16372	0.1325515
73779	0.1853479
083558	0.2679104
336541	0.418224
208036	0.758377
5	#DIV/0!
797975	-0.80864979
521721	-0.49934672



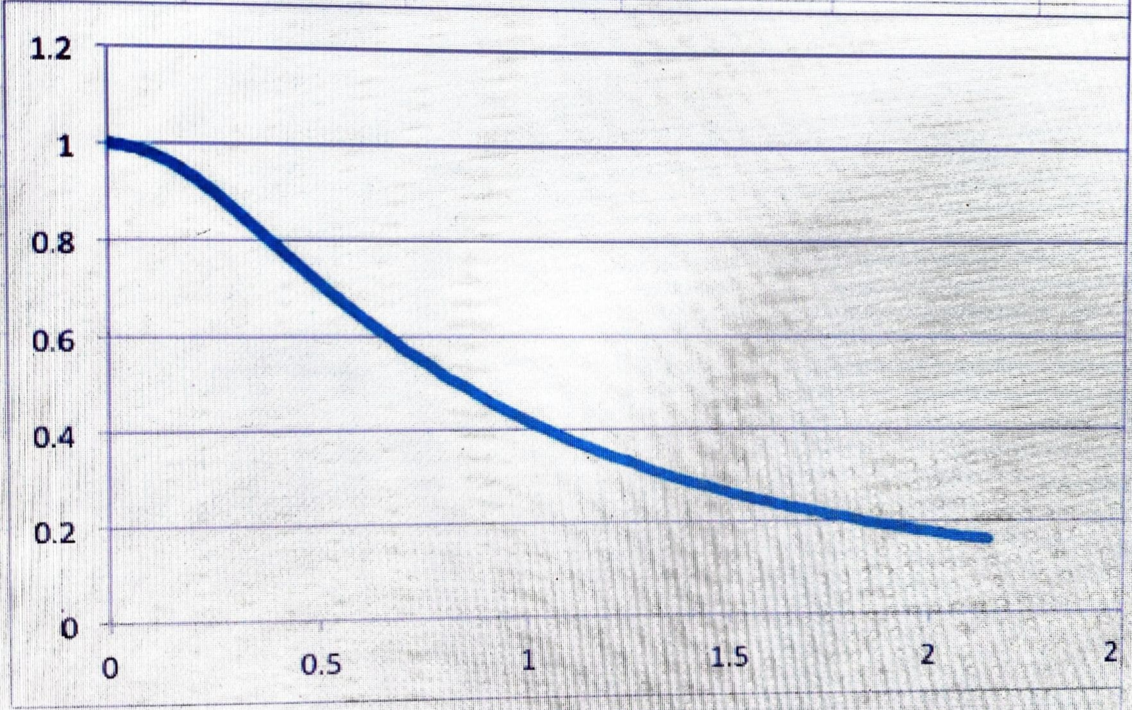
0.2	
_is)	phase
1	0
0482	0.020047439
7753	0.040382076
0615	0.061304161
0685	0.083141232
4981	0.106264863
5942	0.131111648
5839	0.158210768
5067	0.188221505
5008	0.221985683
1325	0.260602392
9166	0.305535759
1434	0.35877067



Select Switch Plot

q_0/(K q_is)

F	G
1.2	
_0/(K q_is)	phase
1	0
0.9953298	0.119725398
0.98166677	0.237835927
0.95998372	0.352871862
0.93169499	0.463647609
0.89842283	0.569313191
0.86178081	0.669357414
0.82321781	0.763567571
0.78393406	0.851966327
0.74485829	0.93474387
0.70666525	1.012197011
0.66981381	1.084680589



II Impulse response to a 2nd order system : $q_i = A\delta$

Case 1 : $\xi = 0$

$$\frac{q_o(t)}{KA} = \omega_n \sin(\omega_n t)$$

Case 2 : $0 < \xi < 1$

$$\frac{q_o(t)}{kA} = \frac{\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t)$$

$$= \frac{\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n t\sqrt{1-\xi^2}\right)$$

Case 3 : $\xi = 1$

$$\frac{q_o(t)}{kA} = \omega_n^2 t e^{-\omega_n t}$$

Case 4 : $\xi > 1$

$$\frac{q_o(t)}{kA} = \frac{\omega_n}{2\sqrt{\xi^2-1}} \left[e^{-\omega_n t(\xi-\sqrt{\xi^2-1})} - e^{-\omega_n t(\xi+\sqrt{\xi^2-1})} \right]$$

ME 226

Homework

19010001

IC

$q_o(0) = 0$

$q_o'(0) = 0$

Q1. Impulse function : $A\delta$

$$\frac{1}{w_n^2}\frac{d^2q_o}{dt^2} + \frac{2\xi}{w_n}\frac{dq_o}{dt} + q_o = kA\delta$$

Taking laplace,

$$\frac{1}{w_n^2}\left[s^2Q_o - s\,\cancel{q_o(0)}^{\,0} - \cancel{q_o'(0)}^{\,0}\right] + \frac{2\xi}{w_n}\left[sQ_o - \cancel{q_o(0)}^{\,0}\right] + Q_o = kA$$

$$Q_o\left[\frac{s^2}{w_n^2} + \frac{2\xi s}{w_n} + 1\right] = kA$$

$$Q_o = \frac{kAw_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

$$= \frac{kAw_n^2}{(s+w_n\xi)^2 + [w_n^2 - \xi^2 w_n^2]}$$

Case 1 : $w_n^2 - \xi^2 w_n^2 > 0$

$$Q_o = \frac{kAw_n^2}{\sqrt{w_n^2 - \xi^2 w_n^2}}\ \frac{\sqrt{w_n^2 - \xi^2 w_n^2}}{(s+w_n\xi)^2 + \left(\sqrt{w_n^2 - \xi^2 w_n^2}\right)^2}$$

Taking inverse laplace,

$$q_o(t) = \frac{kAw_n^2}{\sqrt{w_n^2 - \xi^2 w_n^2}}\ e^{-w_n\xi t}\sin\left(w_n t\sqrt{1-\xi^2}\right)$$

Case 2 : $w_n^2 - \xi^2 w_n^2 = 0$

$$Q_o = \frac{kAw_n^2}{(s+w_n\xi)^2}$$

Taking inverse= laplace,

$$q_o(t) = kAw_n^2 \times te^{-w_n t}$$

Case 3 : $w_n^2 - \xi w_n^2 < 0$

$$Q_o = \frac{kAw_n^2}{(s+w_n\xi)^2 - (\xi^2 w_n^2 - w_n^2)}$$

$$Q_0 = \frac{kA w_n^2}{\sqrt{\xi^2 w_n^2 - w_n^2}} \times \frac{\sqrt{\xi^2 w_n^2 - w_n^2}}{(s+\xi w_n)^2 - \left(\sqrt{\xi^2 w_n^2 - w_n^2}\right)^2}$$

Taking inverse laplace:

$$q_o(t) = \frac{kA w_n^2}{\sqrt{\xi^2 w_n^2 - w_n^2}} \times e^{-w_n \xi t} \left[\frac{e^{w_n t \sqrt{\xi^2-1}} + e^{-w_n t \sqrt{\xi^2-1}}}{2} \right]$$

$$= \frac{kA w_n}{2\sqrt{\xi^2-1}} e^{-w_n \xi t} \left(e^{w_n t \sqrt{\xi^2-1}} + e^{-w_n t \sqrt{\xi^2-1}} \right)$$

$\boxed{Q2.}$ Ramp ~~impulse~~ response: $q_i t$

$$\frac{1}{w_n^2}\frac{d^2 q_o}{dt^2} + \frac{2\xi}{w_n}\frac{dq_o}{dt} + q_o = k q_i t$$

Solving for homogeneous eq^n: $q_o = e^{\lambda t}$

$$\frac{1}{w_n^2}\lambda^2 e^{\lambda t} + \frac{2\xi}{w_n}\lambda e^{\lambda t} + e^{\lambda t} = 0$$

$$\frac{\lambda^2}{w_n^2} + \frac{2\xi\lambda}{w_n} + 1 = 0$$

$$\lambda^2 + 2\xi w_n \lambda + w_n^2 = 0$$

$$\lambda = -\xi w_n \pm w_n \sqrt{\xi^2-1}$$

$$\therefore \quad q_{o,h} = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Where, $\lambda_1 = \left(-\xi + \sqrt{\xi^2-1}\right) w_n$

$\lambda_2 = -\left(\xi + \sqrt{\xi^2-1}\right) w_n$

Solving for particular sol^n: $q_{o,p} = At + B$

$$0 + \frac{2\xi A}{w_n} + At + B = k q_i t$$

$A = k q_i$

$$B = -\frac{2\xi A}{w_n} = -\frac{2\xi k q_i}{w_n}$$

$$q_0(t) = q_{0,h} + q_{0,p}$$

$$= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + kq_i t - \frac{2\xi_s}{w_n} kq_i$$

Imposing initial condition :

$$q_0(0) = c_1 + c_2 - \frac{2\xi_s}{w_n} kq_i = 0$$

$$q_0'(0) = \lambda_1 c_1 + \lambda_2 c_2 + kq_i = 0$$

We get,

$$c_1 = \frac{\lambda_2 \left(-\frac{2\xi_s}{w_n} kq_i\right) - kq_i}{\lambda - \lambda_2} = \frac{kq_i}{2w_n}\left[2\xi_s + \frac{2\xi_s^2 - 1}{\sqrt{\xi_s^2 - 1}}\right]$$

$$c_2 = \frac{\lambda_1 \left(-\frac{2\xi_s}{w_n} kq_i\right) - kq_i}{\lambda_2 - \lambda_1} = \frac{kq_i}{2w_n}\left[2\xi_s - \frac{2\xi_s^2 - 1}{\sqrt{\xi_s^2 - 1}}\right]$$

So finally, we get

$$q_0(t) = \frac{kq_i}{2w_n}\left[2\xi_s + \frac{2\xi_s^2 - 1}{\sqrt{\xi_s^2 - 1}}\right] e^{w_n t\left(-\xi_s + \sqrt{\xi_s^2 - 1}\right)} + \frac{kq_i}{2w_n}\left[2\xi_s - \frac{2\xi_s^2 - 1}{\sqrt{\xi_s^2 - 1}}\right] e^{-w_n t\left(\xi_s + \sqrt{\xi_s^2 - 1}\right)}$$

$$+ kq_i\left(t - \frac{2\xi_s}{w_n}\right)$$

Case 2 : $\xi_s = 1$

$$\lambda_1 = \lambda_2 = -\xi w_n$$

$$q_{0,h} = c_1 e^{\lambda t} + c_2 t e^{\lambda t}$$

$$q_0(t) = c_1 e^{\lambda t} + c_2 t e^{\lambda t} + kq_i\left(t - \frac{2\xi_s}{w_n}\right)$$

Imposing IC :

$$q_0(0) = c_1 - \frac{2kq_i\xi_s}{w_n} = 0 \quad \rightarrow \quad c_1 = kq_i \times \frac{2\xi_s}{w_n} = \frac{2kq_i}{w_n}$$

$$q_0'(0) = \lambda c_1 + c_2 + kq_i = 0$$

$$-w_n\xi_s \times \frac{2kq_i}{w_n} + c_2 \quad kq_i = 0$$

$$c_2 = kq_i$$

$$q_o(t) = \left(\frac{2kq_i}{w_n} + t\, kq_i\right) e^{-w_n t} + kq_i\left(t - \frac{2}{w_n}\right)$$

<u>Case 3</u> : $\xi < 1$

$$\lambda = -\xi w \pm i w\sqrt{1-\xi^2}$$

$$q_o(t) = e^{-\xi w_n t}\left[c_1 \cos(w_n t\sqrt{1-\xi^2}) + c_2 \sin(w_n t\sqrt{1-\xi^2})\right] + kq_i\left(t - \frac{2\xi}{w_n}\right)$$

Imposing IC :

$$q_o(0) = c_1 - kq_i \times \frac{2\xi}{w_n} = 0 \quad \Rightarrow \quad c_1 = kq_i \frac{2\xi}{w_n}$$

$$q_o'(0) = -\xi w_n c_1 + c_2 w_n\sqrt{1-\xi^2} + kq_i = 0 \quad \Rightarrow \quad c_2 = \frac{(2\xi^2 - 1)\, kq_i}{w_n\sqrt{1-\xi^2}}$$

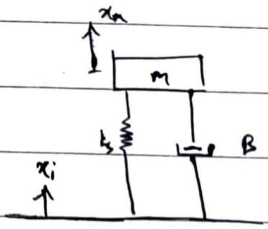
So finally,

$$\frac{q_o(t)}{kq_i} = e^{-\xi w_n t}\left[\frac{2\xi}{w_n}\cos(w_n t\sqrt{1-\xi^2}) + \frac{(2\xi^2-1)}{w_n\sqrt{1-\xi^2}}\sin(w_n t\sqrt{1-\xi^2})\right] + t - \frac{2\xi}{w_n}$$

Lecture 30.03.2021

- Seismic Pick-up Transducer

A second order instrument to detect earthquakes.



$$k_s(x_i - x_m) + B(\dot{x}_i - \dot{x}_m) + M\ddot{x}_i = M\ddot{x}_m + M\ddot{x}_i$$

$$M(\ddot{x}_i - \ddot{x}_m) + B(\dot{x}_i - \dot{x}_m) + k_s(x_i - x_m) = M\ddot{x}_i$$

$$M\ddot{x}_o + B\dot{x}_o + k_s x_o = M\ddot{x}_i$$

$$\frac{M}{k_s}\ddot{x}_o + \frac{B}{k_s}\dot{x}_o + x_o = \frac{M}{k_s}\ddot{x}_i$$

$$\left(\frac{D^2}{w_n^2} + \frac{2\xi}{w_n} + 1\right)x_o = \frac{D^2}{w_n^2}x_i$$

Note : Assuming that the input sinusoidal &

$$\frac{M}{k_s} = \frac{1}{w_n^2} = \frac{1}{(2\pi f_n)^2} \; ; \quad \frac{B}{k_s} = \frac{2\xi}{w_n}$$

$w_n = 2\pi f_n$

Also, $x_o = x_i - x_m$

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$$\frac{x_o}{x_i} = \frac{D^2/w_n^2}{\left[\frac{D^2}{w_n^2} + \frac{2\xi}{w_n}D + 1\right]}$$

$D = jw$

$$\frac{x_o}{x_i} = \frac{-w^2/w_n^2}{-w^2/w_n^2 + \frac{2\xi}{w_n}(jw) + 1}$$

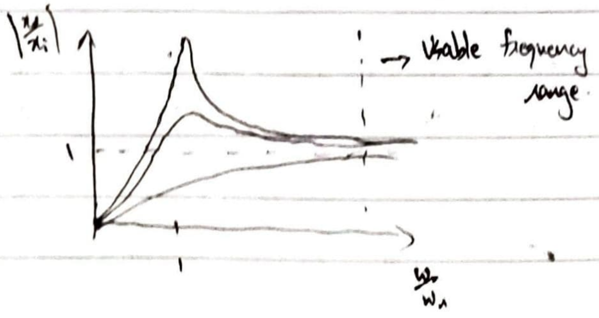
$$= \frac{-w^2/w_n^2}{\left(1 - w^2/w_n^2\right) + \frac{2\xi w}{w_n} j}$$

$$\left|\frac{x_o}{x_i}\right| = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \qquad \left(r = \frac{w}{w_n}\right)$$

Analysing, when $r \to 0$, $\left|\frac{x_o}{x_i}\right| \to 0$

$r \to 1$, $\left|\frac{x_o}{x_i}\right| \gg 1$

$r \to \infty$, $\left|\frac{x_o}{x_i}\right| = 1$



For large frequency, $\Omega \to \infty$

So we compress the spring, but by the time it reacts, we elongate it.

High frequency doesn't allow the spring to react & so $x_d = 0$.

Since seismic transducers are usable for high Ω values, we can design the system to choose the natural frequency ω_n.

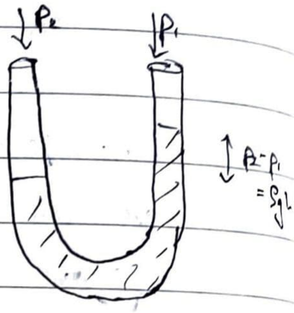
$$\omega_n = \sqrt{\frac{k_s}{m}}$$

For example, to measure $\omega = 30\ Hz$

We can ~~set~~ set $\omega_n = 5\ Hz$

Lecture : 01.04.2021

- Pressure manometer



- Manometer is a deflection type gauge.
- Continuous output
- Pressure ↑ → Height ↑

Refinements / Corrections required

For high accuracy :

- Thermal expansion of scale needs to be considered
- Variation of manometer fluid density with temperature
- Local value of g should be determined.

Additional error sources?

- Non verticality of tubes
- Difficulty in reading h due to meniscus formed by cappilarity.

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Lecture : 05.04.2021

- McLeod Gauge

Refer the slides for setup :

Used for measuring very small pressures p_i.
We entrap gas in one of the arms to produce
pressure $p > p_i$.

So now, $p - p_i = \rho g h$

Using Boyle's law, $PV = \text{constant}$

$$p_i \forall = p h A_t$$

$$p = \frac{p_i \forall}{h A_t}$$

So,
$$p - p_i = \rho g h$$
$$\frac{p_i \forall}{h A_t} - p_i = \rho g h$$

$$p_i = \frac{\rho g h^2 A_t}{\forall - h A_t}$$

If $h A_t \ll \forall$,

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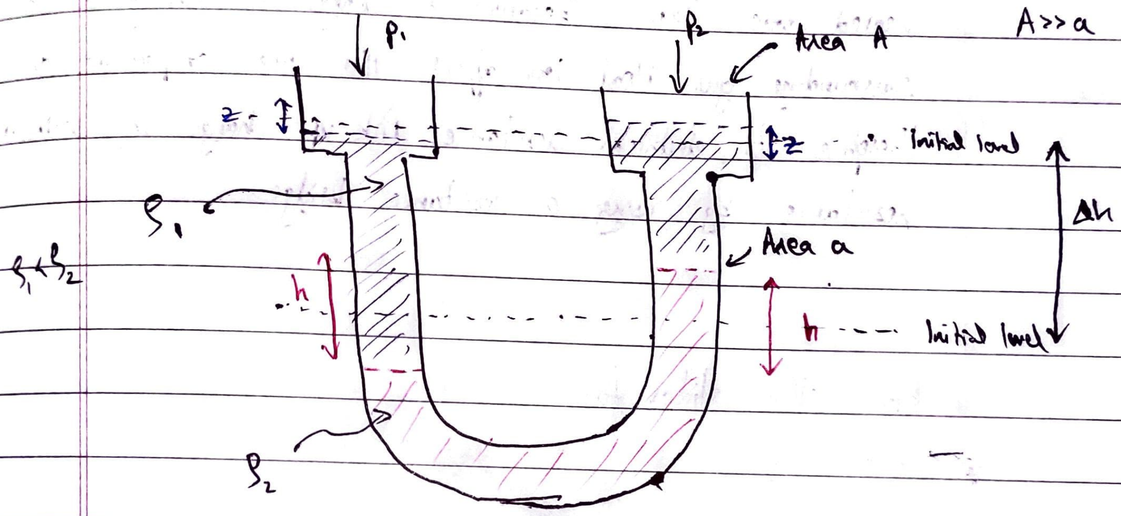
$$\boxed{P_i = \frac{\rho g h^2 A_t}{\forall}}$$

- Micromanometer



Substituting volume, $A\frac{z}{2} = a\frac{h}{2}$

$Az = ah$

$$P_1 + \rho_1 g\left(\Delta h - \frac{z}{2} + \frac{h}{2}\right) = P_2 + \rho_1 g\left(\Delta h + \frac{z}{2} - \frac{h}{2}\right) + \rho_2 g h$$

$$P_1 - P_2 = \rho_1 g(z-h) + \rho_2 g h \approx (\rho_2 - \rho_1) g h$$

Since $A >> a$, $z << h$

Lecture : 08.04.2021

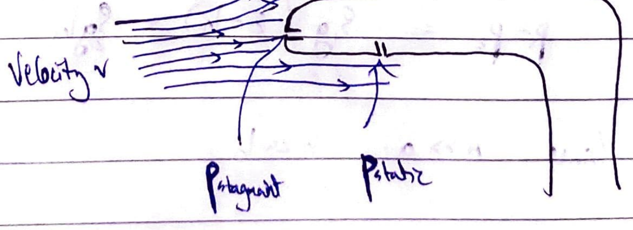
- Pisani Gauge

Change in gas pressure will affect heat loss from a heated wire. This is because of change in density of the surrounding gas. Heat loss affects the wire temperature and therefore its electrical resistance. Pick up change in electrical resistance by using a resistance bridge.

Read the slides for:
- Pisani Gauge
- Ionization gauges
- Diaphragm based pressure gauges

* Flow Measurement

- Velocity measurement using Pitot Static Tube



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$$P_{stat} + \frac{1}{2}\rho v^2 = P_{stag}$$

$$\boxed{v = \sqrt{\frac{2(P_{stag} - P_{stat})}{\rho}}}$$

Used in aircrafts and missiles to determine the speed and altitude.

Sources of error in P_{stat}

i Misalignment of tube axis: This exposes the tube to a component of velocity.

ii Non zero tube diameter: Flow accelerates around the tube. This causes a decrease in static pressure.

iii Influence of tube support: Flow stagnates on tube support leading edge. Pressure increases at the stagnation point. This effect propogates upstream.

So, non zero tube diameter accelerates flow.
Influence of tube support de accelerates flow.

These errors are made to cancel out in Prandtl pitot tube
Refer slides for set up.

Sources of error in P_{stag} :

i Misalignment : Prevents formation of a true stagnation point, at the measuring hole.

ii 2D & 3D velocity fields : Due to finite probe size and non uniform velocity field, ~~there~~ there is actually a range of velocity at the tube tip.

iii Effect of viscosity : Due to finite viscosity fluid, correction in $(P_{stag} - P_{stat})$ is required

$$P_{stag} - P_{stat} = \frac{C \rho V^2}{2}$$

$$C = 1 + \frac{4}{Re}$$

Where, $10 < Re \left(= \frac{\rho V R}{\mu} \right) < 100$

R is the outer radius of the tube

- Variation of pressure with altitude in our atmosphere

$$P = 101325 \left(1 - 2.25577 \times 10^{-5} h \right)^{5.25588}$$

Air pressure

Altitude above sea level

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Lecture : 15.04.2021

* Tracer particles & properties : PIV

PIV works on the principle that the tracer particles are located at two point of time and the distance travelled by them is recorded.
This info gives us the flow velocity.

Synchronizer controlls the flow of time of these event.
The two points of time can be long if the flow velocity is low.
Otherwise, the interval should be short if the flow velocity is high.
We can say that the distance travelled by the tracer particle could be almost constant.

Now, what particles could be used for tracing?

* Mechanical constraints :
- Sp. gravity should be near to that of water.
Shouldn't be too high that it settles down nor too low.

Stokes no. : $St \ll 1$
A particle with low stokes number follows fluid streamlines.

$$St = \frac{\tau_p}{\tau_f} \qquad \text{where} \qquad \tau_p = \frac{\rho_p d_p^2}{18\mu}$$

$\therefore$ Particle size should be low.

★ Optical constraint :

Should be able to scatter and reflect light.

Larger particle size required to scatter light.

Trade off.

★ Chemical constraint

Should not react with fluid.

★ Magnetic flowmeter

- Uses Faraday's law of Electromagnetic induction.
- Magnetic field is generated and channeled into the fluid flowing through the pipe.
- Flow of conductive fluid through the magnetic field will produce emf.
- Voltage signals sensed by electrodes located on the flow tube walls.
- Faster the flow, more the voltage generated. Proportional

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Major advantage : Linear relation

Constraint : Flow has to be uniform.
Should have sufficient length upstream for a uniform fully developed flow.
Minimum 3-5 diameter length upstream.

Flow meter does not obstruct flow. Can be applied to any kind of dirty or abrasive liquids.

Liquid should be conductive. Constraint : Hydrocarbons & gases cannot be measured.

* Ultrasonic Flowmeter

Uses sound waves to determine the velocity of a fluid flowing in a pipe.

Under flow condition, the frequency of the reflected wave is different due to the doppler effect.

When the fluid flow is faster, frequency shift increases linearly.

Do not obstruct flow & can be applied to dirty & abrasive liquids.

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Advantages :

- Handles high pressure
- High turndown
- Handles extreme temperature
- Low maintenance
- Highly reliable
- Clamped outside the pipe without penetration

Disadvantages :

- High cost
- Sensitive to high stray vibrations
- Problems with pipe diameter change due to build up

Lecture : 19.04.2021

★ Hot wire Anemometry

Principle : Amount of cooling experienced by a heated wire can be related to the local flow velocity.

Current is passed through the wire to maintain a certain temperature.
More the flow, faster the cooling and more is the current required to maintain the temperature.

We need a high response rate. We need a quick response.
For this we need a low thermal inertia.
For that, we need to wire to be small and mass to be less.

$$\frac{dQ}{dt} = P - F$$

Q is the internal energy of sensor
P is the electrical input power to sensor
F is the total rate of heat transferred from the sensor

$$F = q_c + q_f + q_r + q_s$$

Convection from sensor to flow
Conduction to supporting prongs
Radiation from sensor to surrounding
transfer to quartz substrate

- q_1 (Heat loss to prong supports) can be neglected if the ratio l/d of the wire is large.

- q_r (Radiation loss) can be neglected.

Free convection is without an air flow but the air around the heated body getting hot and rising. Replaced by colder air.
Forced convection is due to blown air over the surface to cool it.

$$P = I^2 R_s \quad \text{where} \quad R_s = R_f [1 + \alpha (T_s - T_f)]$$

$$F = h \times Area \times \Delta T = [C_0 + C_1 \sqrt{U}] \times Area \times \Delta T$$

$$\frac{dQ}{dt} = P(I, T_s) - F(U, T_s)$$

$$\frac{m C_p \, dT_s}{dt} = P(I, T_s) - F(U, T_s)$$

We want to find velocity U.
So, we can keep I or T_s constant and relate U to the variable term.

Constant temperature mode is preferred so that the thermal inertia ($mC_p dT_s/dt$ term) does not come in the picture.

Lecture : 20.04.2021

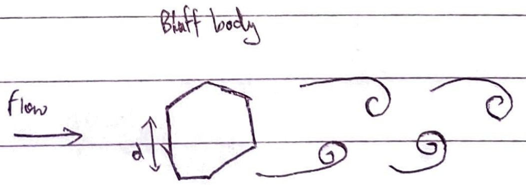
* Uncertainty

Let, $y = f(x_1, x_2, x_3 \dots)$

$$\Delta y = \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \dots$$

$$\Delta y = \sqrt{\left[\frac{\partial y}{\partial x_1}\Delta x_1\right]^2 + \left[\frac{\partial y}{\partial x_2}\Delta x_2\right]^2 + \dots}$$

* Vortex Flowmeter



Strouhal number (St) : $\dfrac{d \times f}{V_{av}}$ (Dimensionless)

d - Diameter of bluff body

f - Frequency of vortex

V_{av} - Average flow velocity

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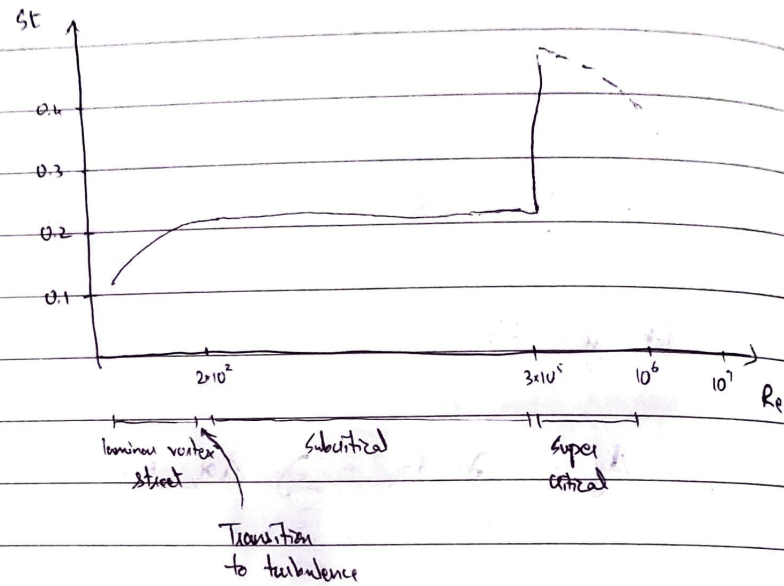
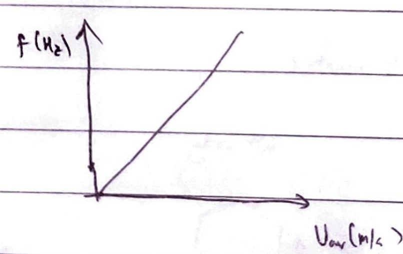


We can see that St no. is constant for a range of Reynolds no.

For a given bluff body, we can say that : $f \propto V_{av}$



Advantages :

- Low pressure loss
- Large turn down ratio
- Low cost : compared to Coriolis & Ultrasonic flowmeter
- Wide operating temp & pressure range.
- Suitable for various fluids : Air, water, ~~stea~~ steam.
- Linear output : $f \propto V_{avg}$

Disadvantage :

- Sensitive to upstream flow conditions.

Concept :

- A bluff body intercepts the flow, generating wakes behind it.
- We define strouhal number and operate in the range where it is constant. So we have : $f \propto V_{avg}$
- To find the frequency, we use some sensors, generally piezoelectric, to detect fall & rise in pressure.
- We can optimize the location of the sensor for accurate readings and to minimize permanent pressure loss.