ME 202

LECTURE 16 TUTORIAL 5

THU 10 FEB 2022

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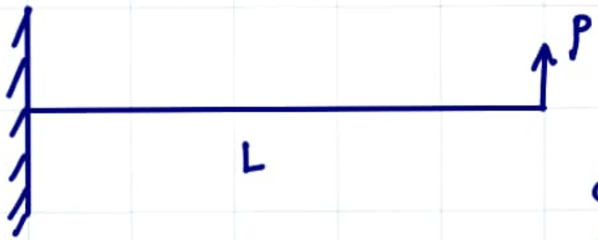
Axial vs Bending Deflections



c/s square $a \times a$

$a = L/10$

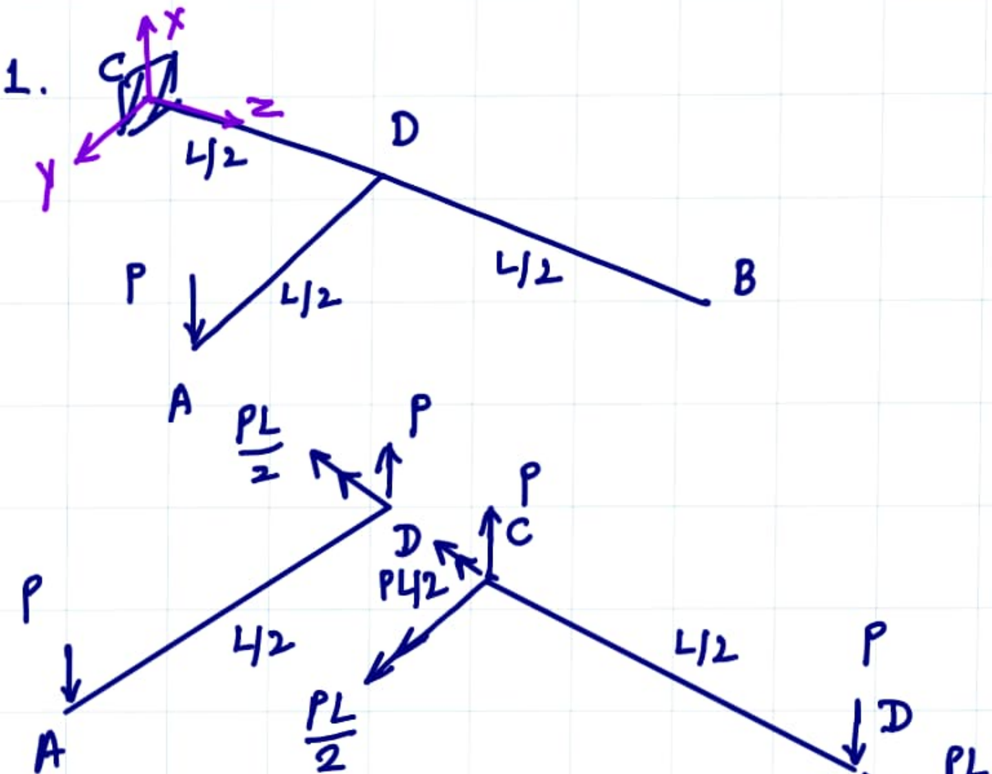
$$A = a^2 = L^2/100, \quad I = a^4/12 = L^4/120{,}000$$

$$\delta_{axial} = \frac{PL}{AE} = 100P/EL$$

$$\delta_{bending} = \frac{PL^3}{3EI} = 40{,}000\,P/EL$$

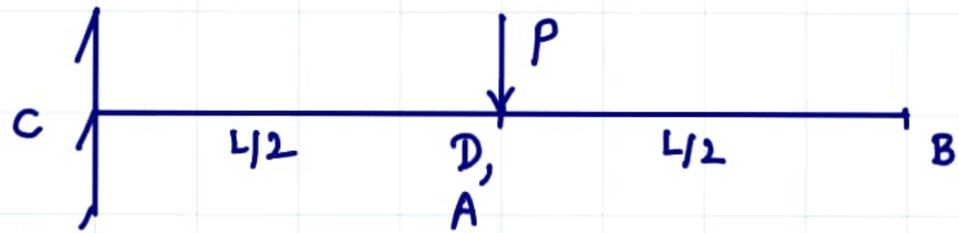
400 times more

1.



$$\delta_A = \underbrace{\frac{P(L/2)^3}{3EI}}_{\text{bending of AD}} + \underbrace{\frac{P(L/2)^3}{3EI}}_{\text{bending of CD}} + \underbrace{\frac{(PL/2)(L/2)(L/2)}{GJ}}_{\text{twist of CD}}$$

$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)$$



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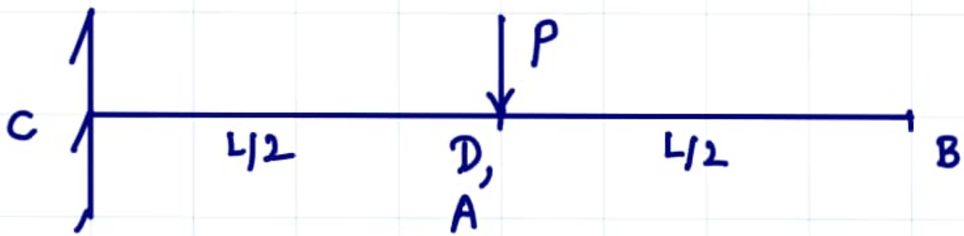


$$\delta_B = \frac{P}{EI}\left(\frac{a^2 z}{2} - \frac{a^3}{6}\right), \quad z = L, \quad a = \frac{L}{2} \qquad \text{see notes}$$

$$= \frac{P}{EI} L^3 \frac{5}{48}$$

In CD, $T_{max} = \frac{PL}{2}$, $M_{max} = \frac{PL}{2}$

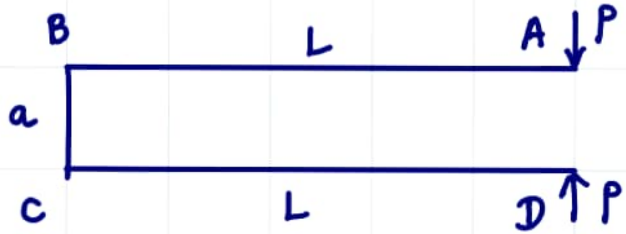
$$\sigma_{zz} = \frac{32M}{\pi D^3} = \frac{16PL}{\pi D^3}, \quad \sigma_{zx} = \frac{16T}{\pi D^3}$$

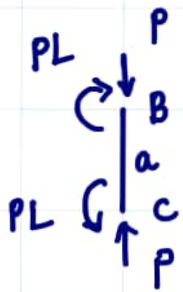
$$= \frac{8PL}{\pi D^3}$$

$$\tau_{max} = \left(\left(\frac{\sigma_{zz} - \sigma_{xx}}{2} \right)^2 + \sigma_{zx}^2 \right)^{1/2}$$

$$= 8\sqrt{2}\, \frac{PL}{\pi D^3}$$

2.





$$\theta_B = \frac{PLa}{2EI}$$

$$\theta_C = \frac{-PLa}{2EI}$$

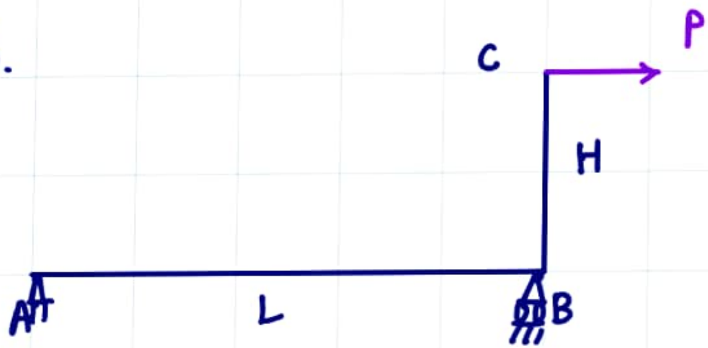
$$\delta_A = \underbrace{\frac{PL^3}{3EI}}_{\text{bending of AB}} + \underbrace{\theta_B L}_{\text{rotation @ B due to bending of BC}}$$

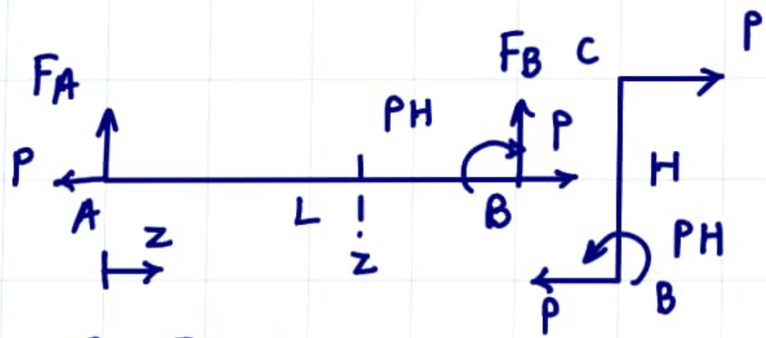
$$= \frac{PL^2}{6EI}(2L+3a)$$

$$AD' = AD - 2\delta_A = a - \frac{PL^2}{3EI}(2L+3a)$$

$$M_{max} = PL$$

3.

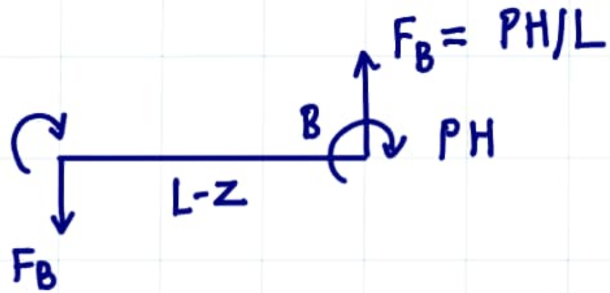




$$F_A + F_B = 0$$

$$F_B L - PH = 0 \Rightarrow F_B = \frac{PH}{L}$$

$M(z) = F_B(L-z) - PH$



$$M(z) = -\frac{PHz}{L} \qquad \text{check } M(0) = 0 \checkmark$$

$$= EIu''$$

$$u'' = -\frac{PH}{EIL} z$$

$$u' = \frac{-PH}{LEI}\left(\frac{z^2}{2} + C_1\right)$$

$$u = \frac{-PH}{LEI}\left(\frac{z^3}{6} + C_1 z + C_2\right)$$

BCs $\quad u(0) = 0, \quad u(L) = 0$

$\Rightarrow \quad C_2 = 0 \quad , \quad C_1 = -L^2/6$

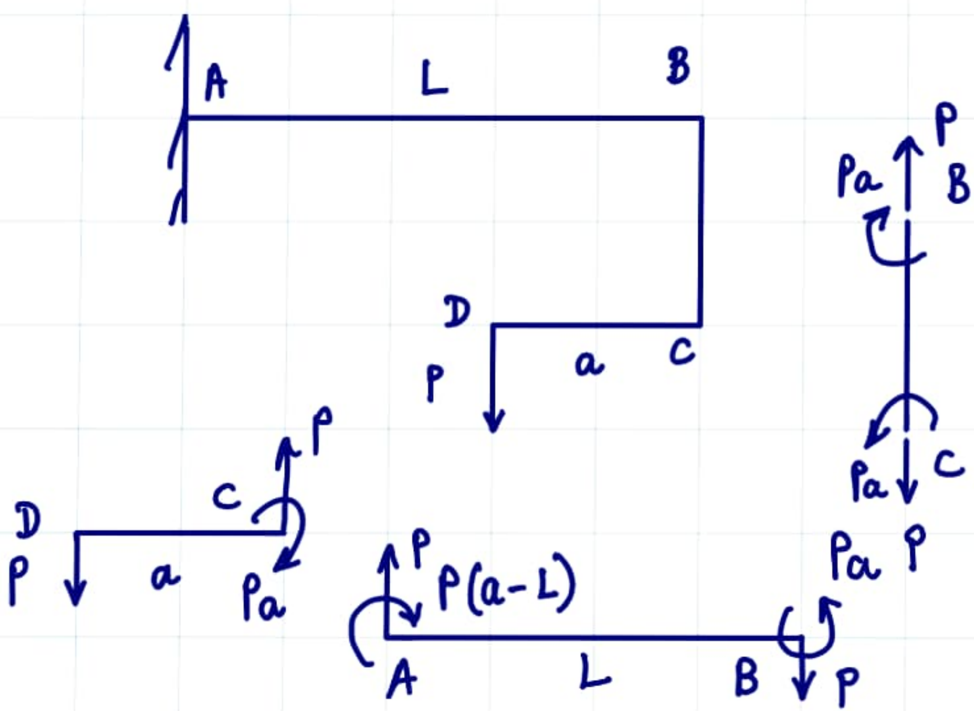
want angle at B, $\quad u'(L) = \dfrac{-PH}{LEI}\left(\dfrac{L^2}{2} - \dfrac{L^2}{6}\right)$

$$= \frac{-PHL}{3EI}$$

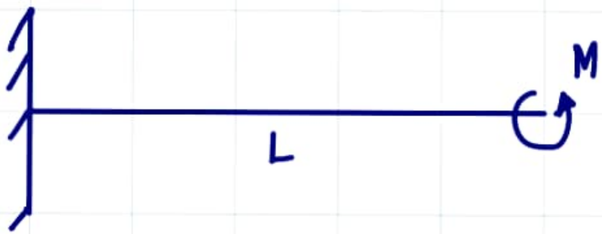
$$\delta_c = \underbrace{\frac{PH^3}{3EI}}_{\text{bending of BC}} + \underbrace{\frac{PHL}{3EI} . H}_{\substack{\text{rotation at B} \\ \text{from bending of AB}}}$$

$$= 30.95 \text{ mm}$$

4.



Useful Result for Angle Calculations in Bent Beam Problems



$$u = \frac{Mz^2}{2EI}, \quad u' = \theta = \frac{Mz}{EI}$$

$$\theta_L - \theta_0 = \frac{ML}{EI} - 0 = \frac{ML}{EI}$$

$$u = -\frac{Mz(L-z)}{2EI} = \frac{M(z^2-Lz)}{2EI}$$

$$u' = \frac{M}{2EI}(2z-L) = \theta$$

$$\theta_L - \theta_0 = \frac{ML}{2EI} - \left(\frac{-ML}{2EI}\right) = \frac{ML}{EI}$$

Both are consistent as both beams have
same applied moment at ends.
This has been used in Prob #2 of
this tutorial.