MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 5

(February 16, 2022)

(1) Let L_0, L_1, L_2 and L_3 be the Lagrange polynomials for distinct nodes x_0, x_1, x_2 and x_3. Find all $j \geqslant 0$ such that

$$x_0^j L_0(x) + x_1^j L_1(x) + x_2^j L_2(x) + x_3^j L_3(x) = x^j.$$

(2) Let $x_0, \ldots, x_k$ be distinct nodes and define $g(x) := f[x_0, x_1, \ldots, x_k, x]$. Prove that $g[y_0, \ldots, y_n] = f[x_0, \ldots, x_k, y_0, \ldots, y_n]$.

(3) If $f(x) = g(x)h(x)$ then find a formula for the divided differences for f in terms of those of g and h.

(4) Construct a Hermite polynomial $H_3(x)$ for the following data for $(x, f(x), f'(x))$:

$$(8.3, 17.56492, 3.116256), \quad (8.6, 18.50515, 3.151762).$$

If the function here is $f(x) = x\ln(x)$ then compute $f(8.4)$ and the errors.

(5) Construct a Hermite polynomial $H_3(x)$ for the following data for $(x, f(x), f'(x))$:

$$(0.8, 0.22363362, 2.1691753), \quad (1, 0.65809197, 2.0466965).$$

If the function here is $f(x) = \sin(e^x - 2)$ then compute $f(0.9)$ and the errors.

(6) Use 5-digit rounding arithmetic and compute the table for the values of $\sin(x)$ and its derivative, $\cos(x)$, at 0.30, 0.32 and 0.35. Obtain the corresponding Hermite polynomial H and compute $H(0.34)$. Compare the actual error and the one predicted by the error formula.

(7) Compute the natural cubic spline for the following data:

x	-0.5	-0.25	0
$f(x)$	-0.0247500	0.3349375	1.1010000

(8) Compute the natural cubic spline for the following data:

x	0.1	0.2	0.3	0.4
$f(x)$	-0.62049958	-0.28398668	0.00660095	0.24842440

(9) Compute the clamped cubic spline for the data in the above problem and $f'(0.1) = 3.58502082$ and $f'(0.4) = 2.16529366$.