ME 202

LECTURE

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Energy Methods

- Minimum Potential Energy
    - Approximate solutions

      Automated algos which improve the solution

  "There are no solutions. Only tradeoffs"

  " Don't let the perfect be the enemy of the good"
    - Basis of FEM (Finite Element Method)

- Minimum Complementary Potential Energy

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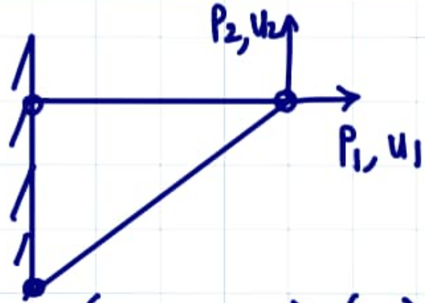
Principle of Minimum Potential Energy

Recall,

Bar structure

$$\Pi = \frac{1}{2} k u^2 + (-Pu)$$



$$\underset{\sim}{K}\,\underline{u} = \underline{P}$$

π

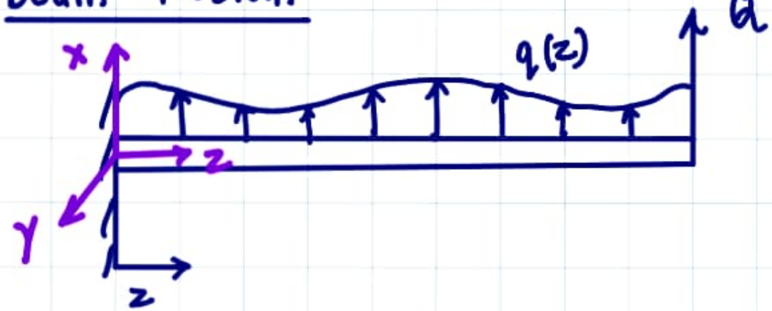
$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

$\Pi(u_1, u_2)$

Linear algebraic system

Π = Stored Elastic Energy + Pot. of Ext. Forces

Beam Problem



strain energy density

$$SED = \frac{1}{2}\sigma_{zz}\epsilon_{zz} \quad J/m^3$$

$$= \frac{1}{2}E\epsilon_{zz}^2 = \frac{1}{2}E x^2 u''^2$$

$$\epsilon_{zz} = -x u''$$

$$\sigma_{zz} = E\epsilon_{zz}$$

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Stored Elastic Energy $= \int_0^L dz \int_\Omega \frac{1}{2} E x^2 u''^2 \, da$

$$= \int_0^L dz \, \frac{1}{2} E u''^2 I$$

$$= \int_0^L \frac{1}{2} EI \left(\frac{d^2u}{dz^2}\right)^2 dz$$

$$\Pi = \int_0^L \frac{1}{2} E I u''^2 dz + \left(-\int_0^L q(z)\, u \, dz - Q\, u(L)\right)$$

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Note: Π is a function of $u(z)$.

Acc to PMPE, $u(z)$ which minimizes Π

is the one that keeps the beam in equilibrium.

$\Downarrow$

satisfies the $2^{nd}/4^{th}$ order beam equation.

Approximations

Come up with a linear algebraic system.

① choose a $u(z)$ with unknown coefficients but which will obey the kinematic boundary conditions.

BCs

Kinematic BCs   KBCs
slope / deflection

Natural BCs   NBCs
shear force / bending moment

KBCs: $u(0) = 0, \quad u'(0) = 0$

NBCs: $-EI\,u'''(L) = Q_0$

$EI\,u''(L) = M_0$