Bulk Deformation - 1

ME 206

Manufacturing Processes 1

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Outline

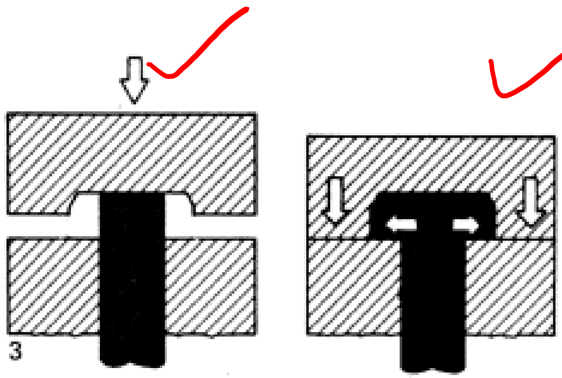
- What is bulk deformation?
- Cold vs hot working
- Forging introduction
- Forging analysis
- Forging defects

What is bulk deformation?

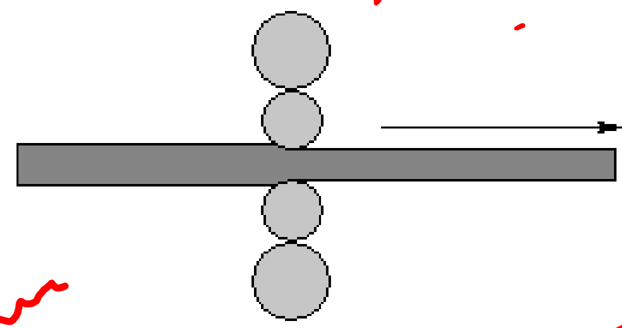
- Bulk deformation (or forming): processes characterized by large amount of plastic deformation (large strains) carried out at elevated or room temperature
- Bulk plastic flow of material under uniaxial or multi-axial stresses dominated by compression
- Mass conserving processes → volume is constant

Bulk Deformation Processes

- Examples include: forging, rolling, extrusion, rod drawing etc.



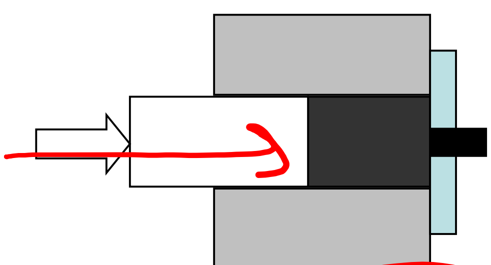
Forging (heading)



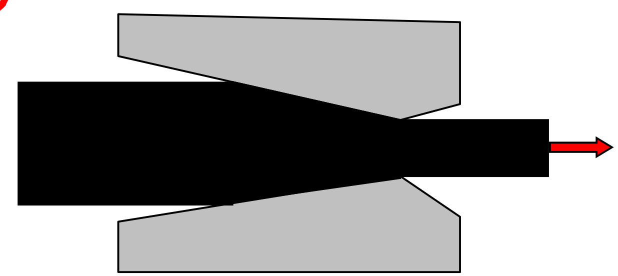
Rolling

Primary compression

Shear driven failure?



Extrusion



Rod Drawing

Cold vs Hot Working

- Many bulk deformation processes carried out at elevated temperatures

- Cold working: $T < 0.3T_m$ Strain hard
  - Usually a finishing step

- Warm working: $T = 0.3\text{~}0.5T_m$
  - Intermediate or final step

Microstructural
Residual
→ H/T Process?

- Hot working: $T > 0.5T_m$
  - Initial step

Cold Working

Advantages

- Better dimensional control
- Superior surface finish
- Strain hardening of surface layer can be beneficial

Limitations

- Greater material strength → higher forces
- Stronger tooling required
- Lower ductility → small deformations
- Lower malleability → harder to shape material
- Anisotropic surface properties

Hot Working

Advantages

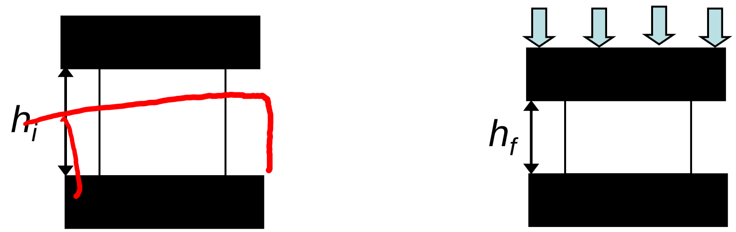
- Lower yield strength → lower forces
- High ductility → larger strains possible
- Higher malleability → easy to shape metal

Limitations

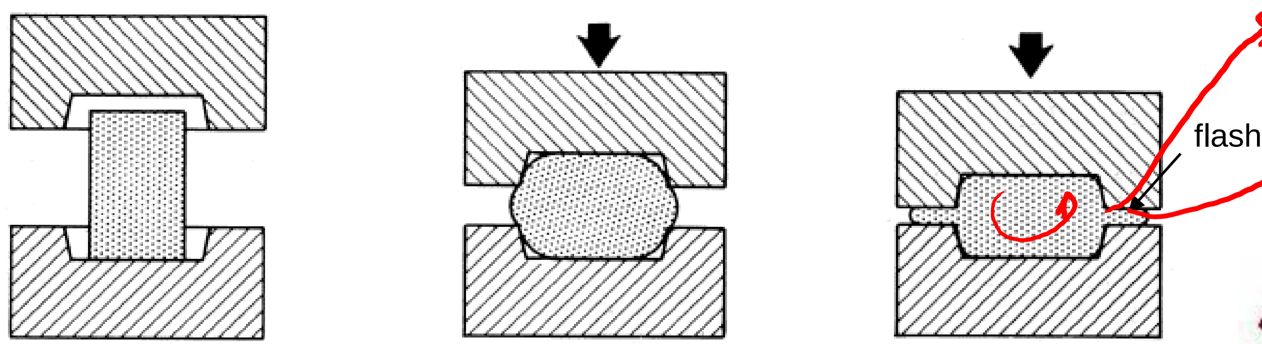
- Easily forms oxide layers (scales) on surface → poor surface finish
- Harder to control dimensions

Forging

- Shape change via compressive forces
- Types of forging processes
  - Open die forging (upsetting)

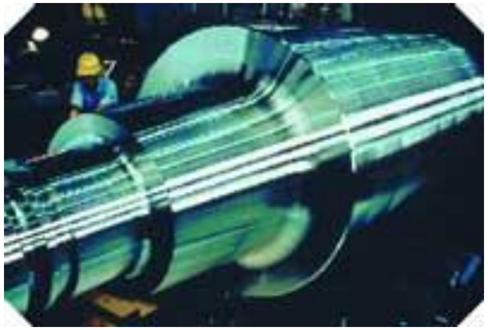


  - Closed or impression die forging



Open Die Forging

- Carried out between flat dies
- Parts weighing few lbs ~ 150 tons e.g. solid shafts, spindles/rotors, rings, etc.
- Often carried out in steps
- Wide range of ferrous and non-ferrous metals



Stepped shaft

Spindles

Source: http://www.forging.org

Closed Die Forging

- Carried out between shaped dies
- Parts weighing few ounces ~ 25 tons e.g. crankshafts,connecting rods, aircraft parts etc.
- Most engineering metals: carbon steels, stainless steels, aluminum, bronze, etc.

Aircraft bulkhead

Connecting rods

Source: http://www.forging.org

Forgings

- Coins
- Landing gear
- Crank shafts
- Turbine shafts

Forge!

Forging presses

- Large machines
  - hold dies
  - form parts

Press types

- Servo-hydraulic presses
- Servo-electrical presses
- Mechanical presses
- Screw presses
- Hammers
  – gravity drop
  – power drop
  – counter blow (two rams)
  – high pressure gas

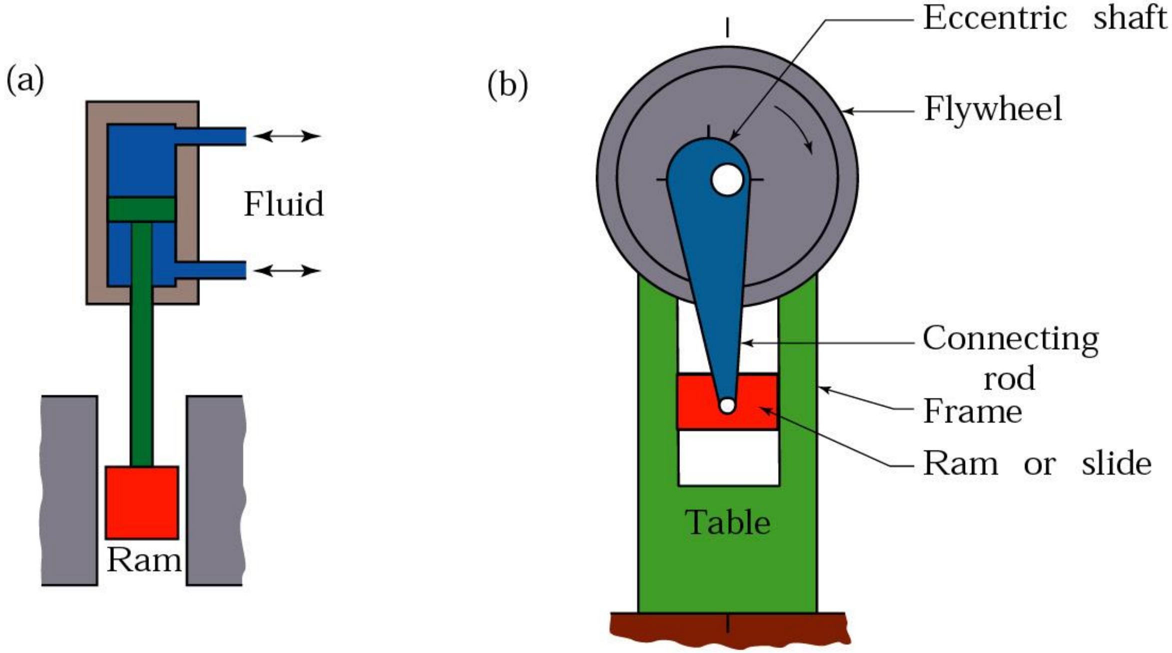
Encoder

Servo Control

Rigid Ball Screw drives Motor!

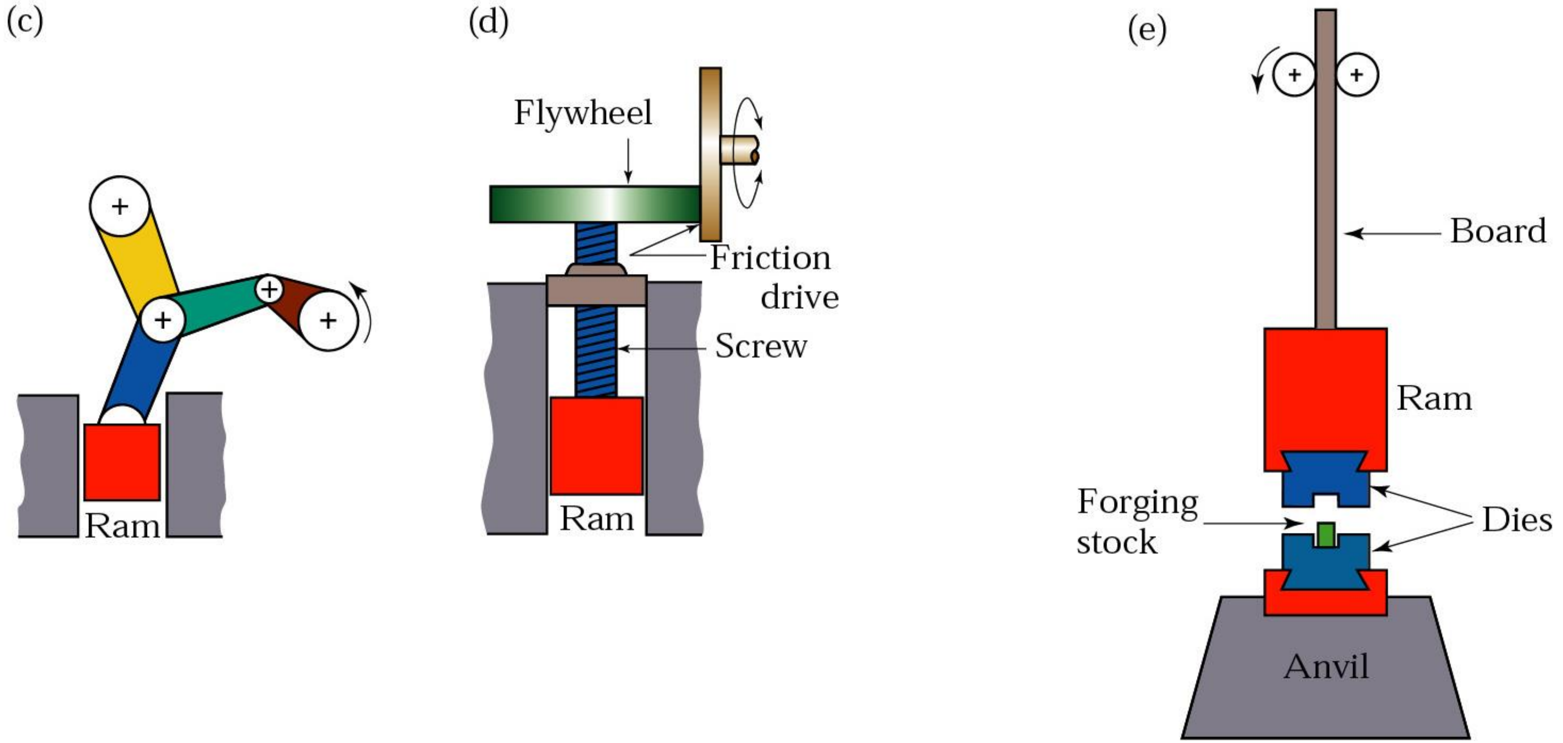
50,000 ton press

Forges



Schematic illustration of the principles of various forging machines. (a) Hydraulic press. (b) Mechanical press with an eccentric drive; the eccentric shaft can be replaced by a crankshaft to give the up-and-down motion to the ram. (continued)

Forges



Schematic illustration of the principles of various forging machines. (c) Knuckle-joint press. (d) Screw press. (e) Gravity drop hammer.

Forging steps

- Prepare slug
  - saw
  - flame cut
  - shear
- Clean slug surfaces
  - shot blast
  - flame

Forging steps

- For hot forging
  - heat up and descale forging
  - make sure press is hot
- Lubricate
  - oil
  - soap
  - MoS_2
  - glass
  - graphite

Lubrication purposes

- Reduce friction
- Reduce die wear
- Thermally insulate part
  - – to keep it warm

① Sliding Friction

$\tau_{fr} = \mu p$

$\mu p \leq \tau_{flow}$

② Sticking Friction

$\mu p \geq \tau_{flow}$

$\boxed{\tau_f = \tau_{flow}}$

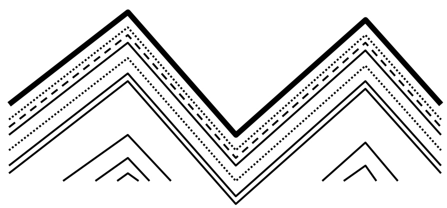
Sticking Friction

Forging steps

- Forge
- Remove flash
  - trim
  - machine
- Check dimensions
- Post processing, if necessary
  - heat treat
  - machine

Effect on grain structure

- Large grains are broken up.
- Grains can be made to flow.



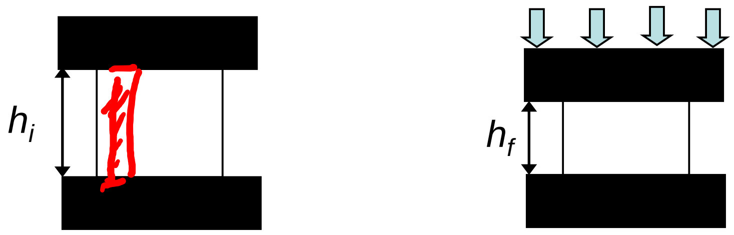
grain fragmentation due to strain!

Dies

- Final part shape determined by die accuracy
- Multiple parts can be made in one die
- Progressive shaping can be done in one die set
- Need to be stronger than highest forging stress

Forging Analysis

- Simple stress analysis possible for open die forging process



$\epsilon_{zz} = 0$
$\tau_{xz} = 0$
$\tau_{yz} = 0$

Plane stress / Plane strain

- Analysis method called “slab” analysis
- Applicable to plane strain compression with low sliding friction

Source: http://www.forging.org

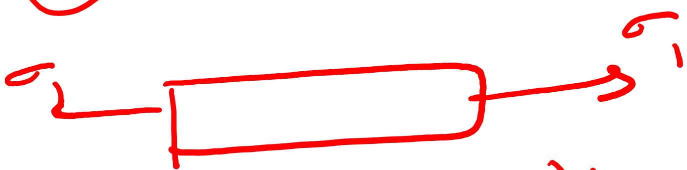
Slab analysis assumptions

- Entire forging is plastic
  - no elasticity
- Material is perfectly plastic
  - strain hardening and strain rate effects later
- Friction coefficient (μ) is constant $\tau_f = \mu p$
  - all sliding, to start
- Plane strain
  - no z-direction deformation
- In any thin slab, stresses are uniform
- Three conditions exist: All sliding; Sliding-sticking transition and Fully sticking

Two basic concepts

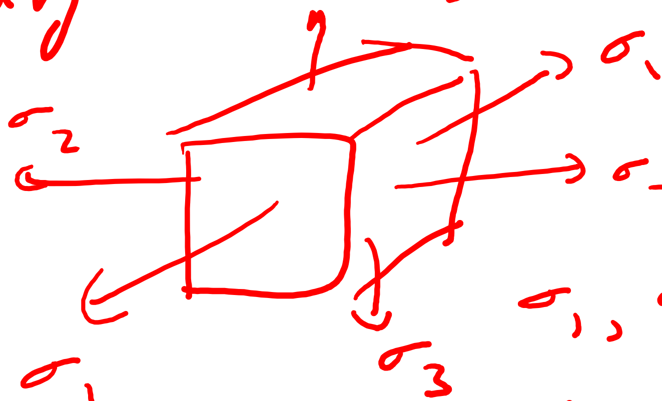
Tresca criteria

1 when does yielding occur



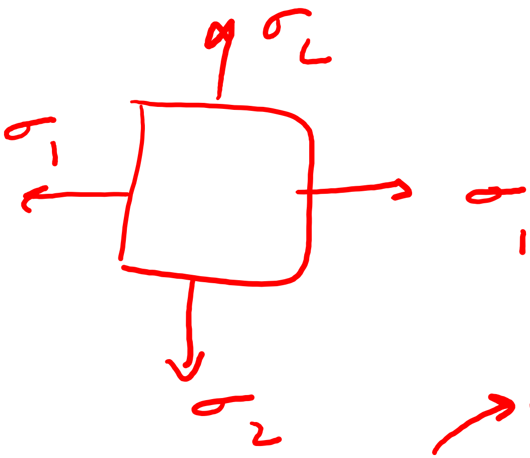
σ_{flow} or Y_f

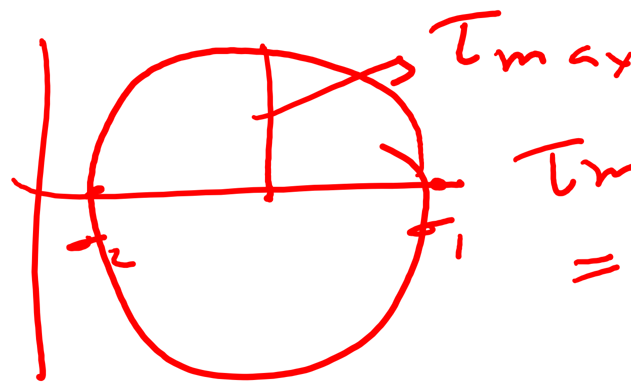
$\sigma_1 =$



$\sigma_1, \sigma_2, \sigma_3$ are principal stresses

$\tau = \frac{\sigma_1 - \sigma_3}{2}$





$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$2\tau = \sigma_1 - \sigma_2 \rightarrow 2K$

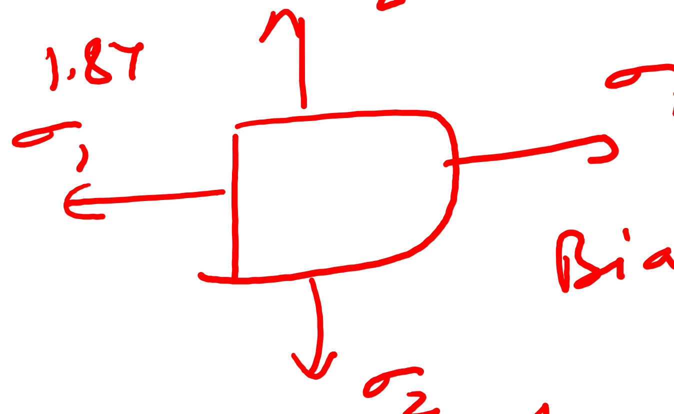
$\boxed{2K = \sigma_{flow} = Y_f}$

Yielding!

Concept of Y ② (A)

Y

Uniaxial

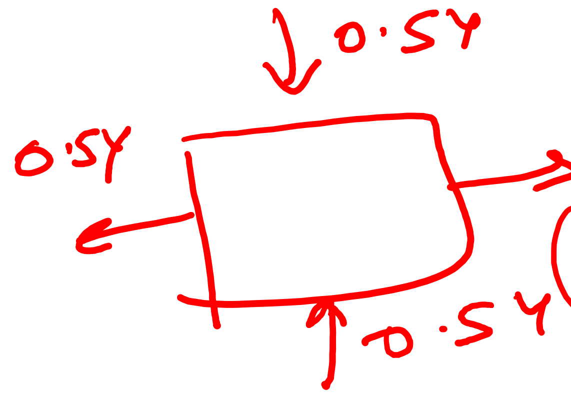


Biaxial

or

Triaxial Loading!

Yielding stress!



(B)

$\sigma_1 = Y$

$$\frac{\sigma_1 - \sigma_3}{2} = \tau_{max}$$

$$\boxed{\sigma_1 - \sigma_3 = 2K = Y}$$

$$\boxed{\sigma_{flow} = 2k = Y_f}$$ For plane strain condn

Tresca criteria for rigid con

$$\epsilon_3 = 0$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_1 + \sigma_2)$$

$$\frac{\sigma_3}{E} = \frac{\sigma_1 + \sigma_2}{2}$$

Using von Mises

$$\boxed{2Y_f^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$2Y_f^2 = (\sigma_1 - \sigma_2)^2 + \left(\sigma_2 - \left(\frac{\sigma_1 + \sigma_2}{2}\right)\right)^2 + \left(\frac{\sigma_1 + \sigma_2}{2} - \sigma_1\right)^2$$

$$2Y_f^2 = (\sigma_1 - \sigma_2)^2 + \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

$$\boxed{2Y_f^2 = \frac{3}{2}(\sigma_1 - \cancel{\sigma_2})^2}$$

$\epsilon_3 = 0$

$$\boxed{\epsilon_x + \epsilon_y + \epsilon_z = 0}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_1)$$

$$\frac{2}{\sqrt{3}} Y_f = \sigma_1$$

$$\frac{\sigma_3}{E} = \frac{\cancel{\nu}}{E}(\sigma_1 + \sigma_2)$$

Plane strain

$$\boxed{2K = \; = \frac{2}{\sqrt{3}} \sigma_{f100} = 1.15\, \sigma_{f100}}$$

$$\frac{\sigma_3}{\cancel{E}} = \frac{(\sigma_1 + \sigma_2)}{2\cancel{E}}$$

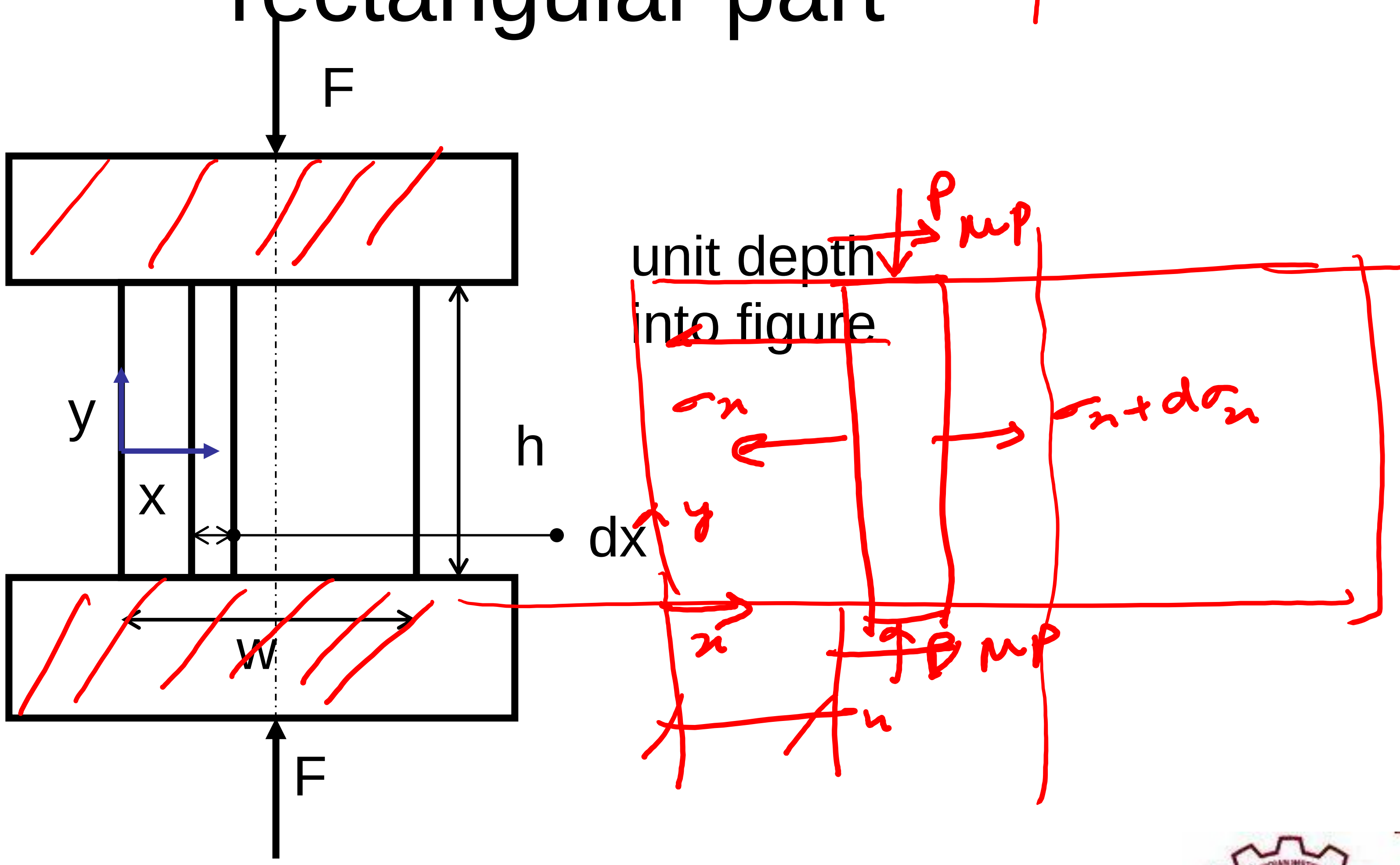
$$2Y_f^2 = (\sigma_1 - \cancel{\sigma_2})^2 + (\cancel{\sigma_2} - \cancel{\sigma_3})^2 + (\cancel{\sigma_3} - \sigma_1)^2$$

Uniaxial $\sigma_2 = 0 \quad \sigma_3 = 0$

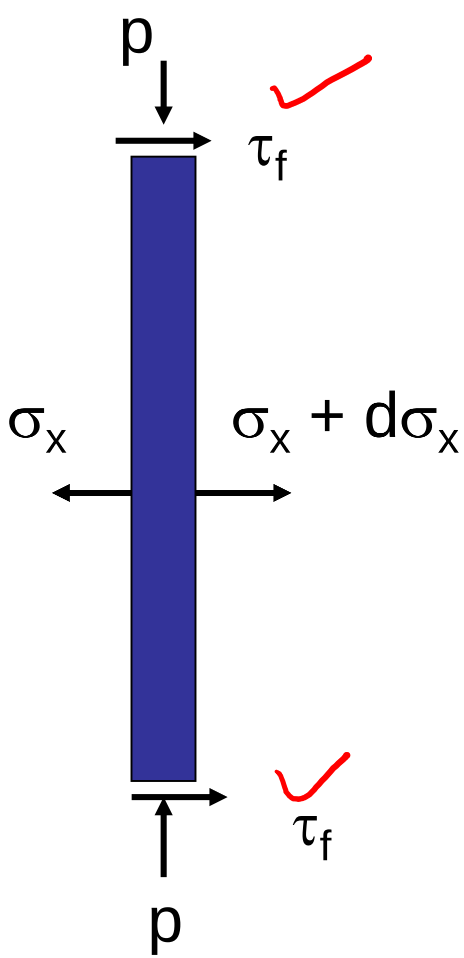
$$2Y_f^2 = 2\sigma_1^2$$

$$Y_f' = 1.15\, Y_f$$

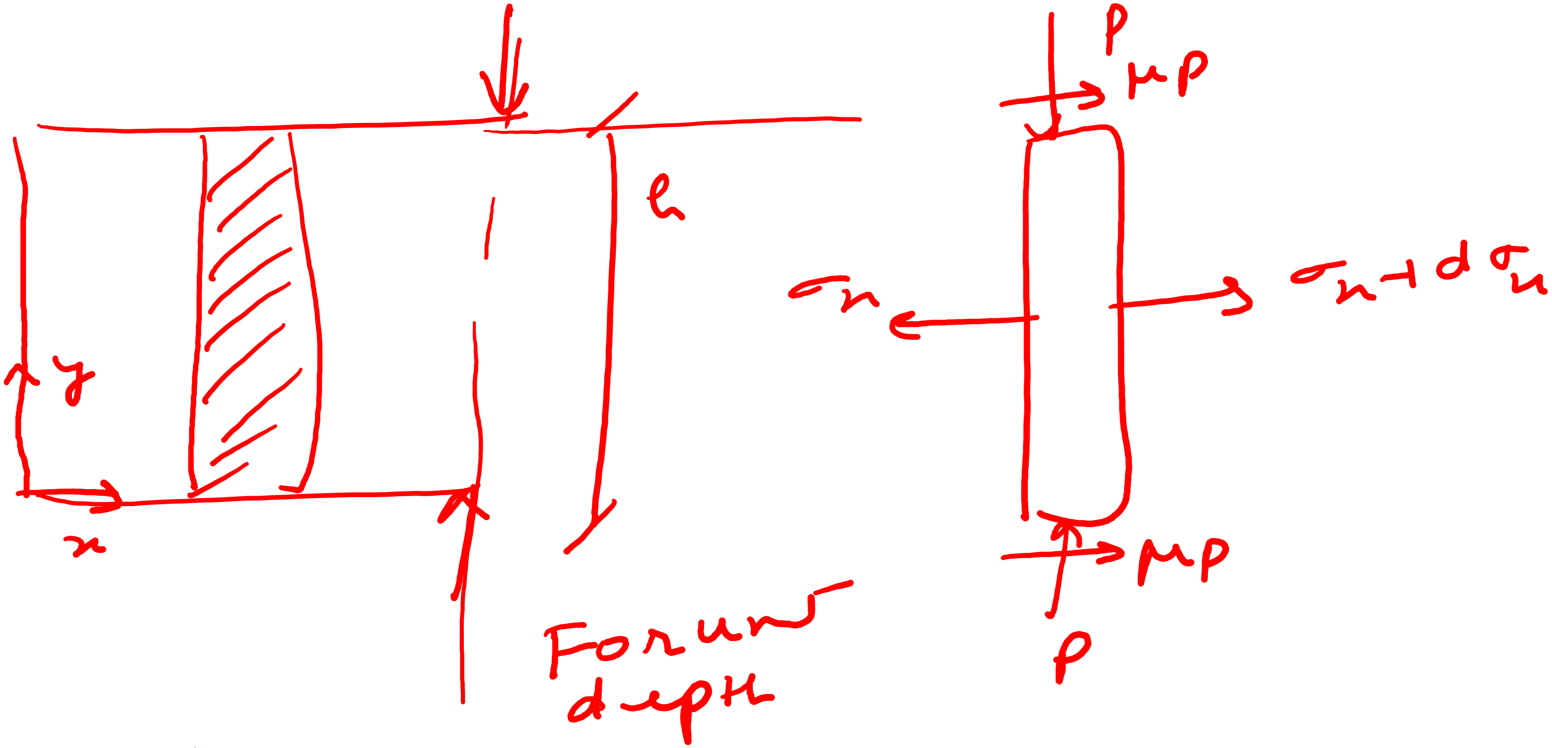
Open die forging analysis – rectangular part



Expanding the dx slice on LHS



- p = die pressure
- σ_x, $d\sigma_x$ from material on side
- $\tau_{friction}$ = friction force = μp

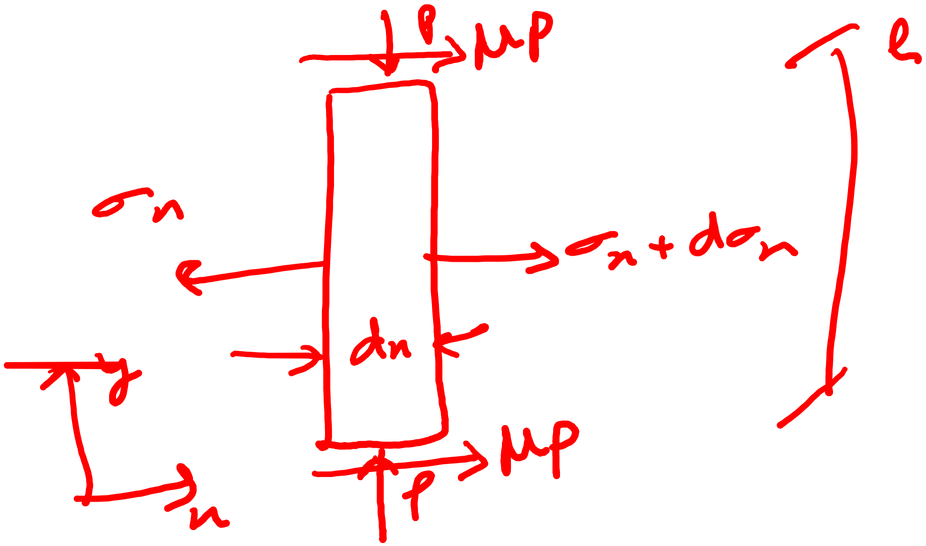


$$\Sigma F_x = 0$$

$$-\sigma_x h + (\sigma_x + d\sigma_x)\cdot h + 2\mu p \cdot dx = 0$$

$$d\sigma_x \cdot h = -2\mu p \cdot dx$$

$$d\sigma_x = \frac{-2\mu p \cdot dx}{h} \quad \text{—— (i)}$$

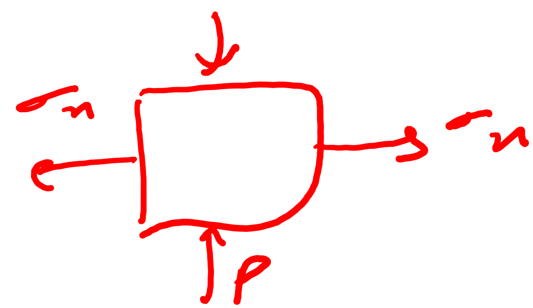


unit depth

$$d\sigma_x \cdot h = -2\mu p\, dx$$

$$\sum F_x = 2\mu p \cdot dx + (\sigma_x - d\sigma_x)\cdot h - \sigma_x \cdot h = 0$$

$$\boxed{d\sigma_x \cdot h + 2\mu p \cdot dx} = 0 \quad \text{— (1)}$$



Tresca

$$(\sigma_x - (-p)) = 2k \quad \text{— (2)}$$

$$\sigma_x + p = 2k$$

The yielding condition, $\sigma_1 - \sigma_3 = 2K$

$$\sigma_x - (-p) = 2K$$

$$\sigma_x + p = 2K$$

$$\frac{d\sigma_x}{dx} = -\frac{dp}{dx} \simeq \frac{\Delta\sigma_x}{\Delta x} = -\frac{\Delta p}{\Delta x} \quad \text{So,}$$

$$\Delta\sigma_x = -\Delta p$$

$$\boxed{d\sigma_x = -dp}$$

$$-dp = -\frac{2\mu p}{h} \cdot dx$$

$$dp = \frac{2\mu p}{h} \cdot dx$$

K

$$dp = \frac{2\tau_f}{h} \cdot dx$$

K

$$\int \frac{dp}{p} = \int \frac{2\mu}{h} \cdot dx$$

friction in stick dp

$$\boxed{\sigma_x + p = 2k}$$

$$\frac{d\sigma_x}{dx} + \frac{dp}{dx} = 0 \qquad \frac{\Delta\sigma_x}{\Delta x} + \frac{\Delta p}{\Delta x} = 0$$

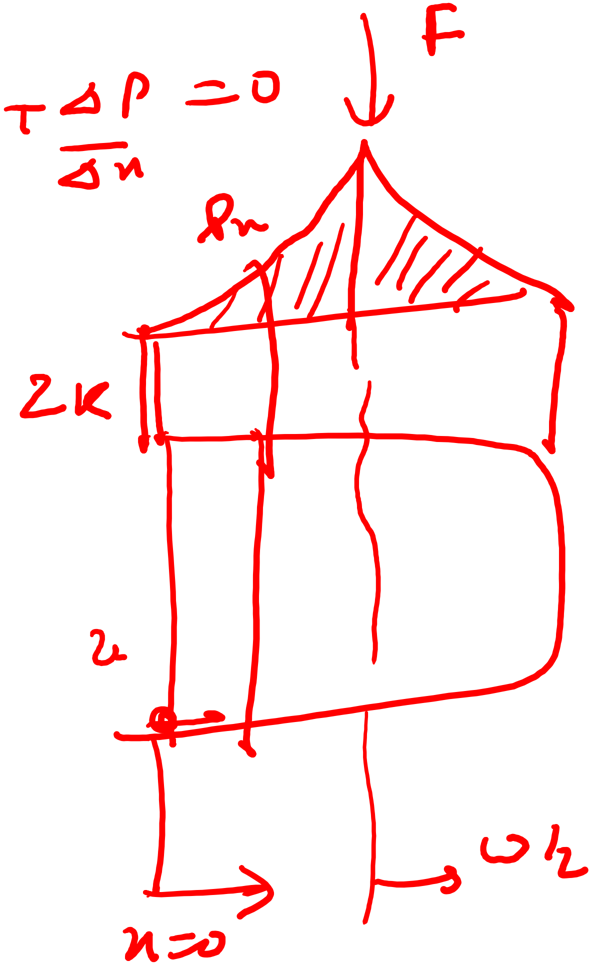
$$\boxed{d\sigma_x = -dp} \quad \text{— (2)}$$

$$d\sigma_x \cdot h = -2\mu p \cdot dx$$

$$d\sigma_x = \frac{-2\mu p}{h} \cdot dx$$

$$-dp = \frac{-2\mu p}{h} \cdot dx$$

$$\boxed{\int_{2k}^{p_x} \frac{dp}{p} = \int_0^x \frac{2\mu}{h} \cdot dx}$$



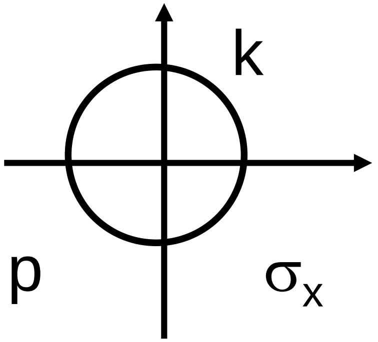
$x = 0$
$p = 2k$

Force balance in x-direction

$$h d\sigma_x + 2\tau_{friction} dx = 0$$

$$d\sigma_x = -\frac{2\tau_{friction}}{h} dx$$

Mohr's circle



$$\sigma_x + p = 2k = \frac{2}{\sqrt{3}}\sigma_{flow} = 1.15 \cdot \sigma_{flow}$$

(distortion energy (von Mises) criterion, plane strain)

N.B. all done on a per unit depth basis

Force balance

Differentiating, and substituting into Mohr's circle equation

$$d(2k) = d(\sigma_x + p) \quad \therefore\ dp = -d\sigma_x$$

$$d\sigma_x = -\frac{2\tau_{friction}}{h}dx \qquad \therefore\ dp = \left(\frac{2\tau_{friction}}{h}\right)dx$$

noting: $\tau_{friction} = \mu p$

$$dp = \frac{2\mu}{h}p\,dx \longrightarrow \frac{dp}{p} = \frac{2\mu}{h}dx$$

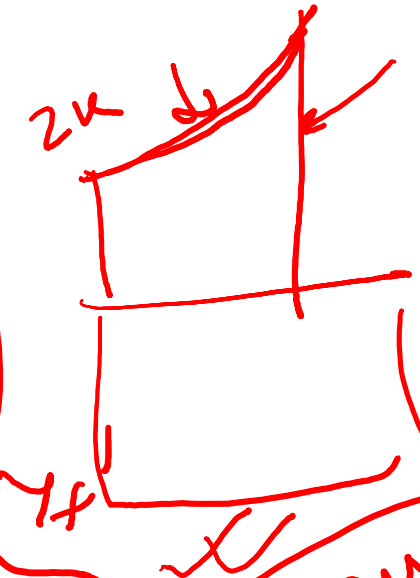
Sliding region

$$\boxed{p_{max} = 2k\, e^{\mu w/h}}$$

$$p_x = 2k\, e^{2\mu x/h}$$

$$p_x = 1.15\,\sigma_f\, e^{2\mu x/h}$$

$$\int_{2k}^{p_x} \frac{dp}{p} = \int_0^x \frac{2\mu}{h} dx$$



- Noting: @ x = 0, σ_x = 2k = 1.15 σ_{flow}

$$\ln p \Big|_{2k}^{p_x} = \frac{2\mu x}{h}$$

$$\ln\left(\frac{p_x}{2k}\right) = \frac{2\mu x}{h}$$

$$p_x/2k = e^{2\mu x/h}$$

$$p_x = 2K\, e^{2\mu x/h}$$

Forging Analysis

- Simplest condition in plane strain forging is "All Sliding" which means that the frictional shear stress is: $\tau_{friction} = \mu p$
  - There is an exact solution for this where pressure increases exponentially
  - An approximate solution which is arrived at by using Taylor's series approximation

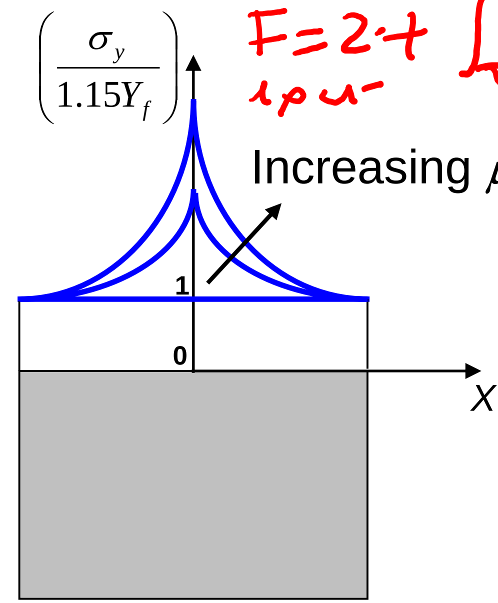
Forging pressure – sliding region

$F_{H} = 2\int_0^{w/2} p(x)\cdot dx$

$F = 2\cdot t\int_0^{w/2} p(x)\cdot dx$

1 p u

$$\ln p_x - \ln(2k) = 2\mu \frac{x}{h}$$



Sliding region result ($0 < x < x_k$)

$$\frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right)$$

$$p_x = 1.15\cdot\sigma_{flow}\cdot\exp\left(\frac{2\mu x}{h}\right)$$

N.B done on a per unit depth basis

Forging pressure – approximation

Exact $p_x = 1.15\sigma_f\, e^{\frac{2\mu x}{h}}$

- Taking the first two terms of a Taylor's series expansion for the exponential about 0, for $|x| \leq 1$

$$\exp(x) = 1 + x + \cancelto{0}{\frac{x^2}{2!}} + \cancelto{0}{\frac{x^3}{3!}} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$$

yields

$$\frac{p_x}{2k} = \left(1 + \frac{2\mu x}{h}\right) \qquad p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{2\mu x}{h}\right)$$

Average forging pressure – all sliding approximation

- using the Taylor's series approximation

$$\frac{p_{ave}}{2k} = \frac{\displaystyle\int_0^{\frac{w}{2}} \frac{p_x}{2k}\,dx}{\frac{w}{2}} = \frac{\displaystyle\int_0^{\frac{w}{2}} \left(1 + \frac{2\mu x}{h}\right) dx}{\frac{w}{2}} = \frac{\left.\left(x + \dfrac{2\mu x^2}{2h}\right)\right|_0^{\frac{w}{2}}}{\frac{w}{2}}$$

$$\boxed{\frac{p_{ave}}{2k} = \left(1 + \frac{\mu w}{2h}\right)}$$

$\Rightarrow$ Force from load, $P_{ave} \times w \times t$

$$p_{ave} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right)$$

N.B done on a per unit depth basis

Forging force – all sliding approximation

$$F_{forging} = p_{ave} \cdot width \cdot depth$$

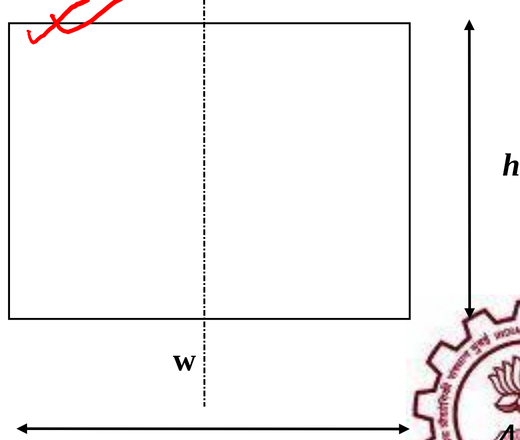
Textbooks! Forging Force estimate!

$$\boxed{F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right) \cdot w \cdot depth}$$

Approx.

Example Problem

A rectangular workpiece has the following original dimensions: w = 100 mm, *h* = 25 mm, and depth d= 20 mm and is being ~~open die~~ forged in plane strain. The true stress-true strain curve of the metal is given by $\sigma_t = 400\varepsilon_t^{0.5}$ MPa and the coefficient of friction is $\mu = 0.1$. Calculate the forging force required to reduce the height by 20%. Use both the "exact" and the average pressure methods.



$w_i = 100$ mm

$h_i = 25$ mm

20% reduction

$d = 20$ mm

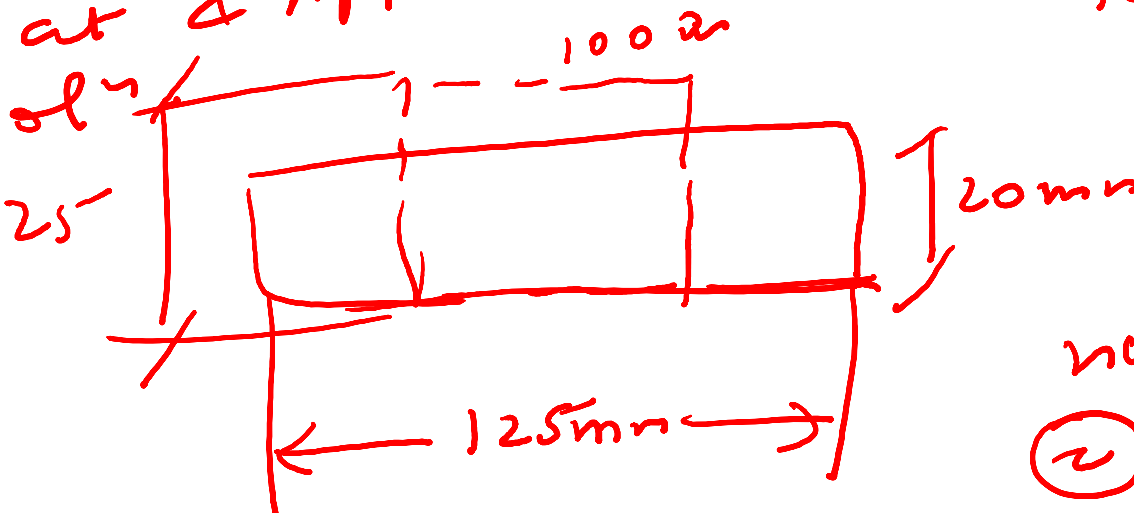
It is a plane strain condn.

$\sigma = 400\,\epsilon^{0.5}$

$\sigma = K\epsilon^n$ Flow stress a function of strain

$\mu = 0.1$

Exact & Approximate Soln



$100 \times 25 = 20 \times w$

(1) Find out the new dimension

(2)

② Find σ_{flow}

$$\sigma_{flow} = 400\,\epsilon^{0.5}$$

$$\epsilon = \ln\left(\frac{h_i}{h_f}\right) = \ln\left(\frac{25}{20}\right) = 0.2231$$

$$\sigma_{flow} = 400\,(0.2231)^{0.5} = 188.95\ MPa$$

③ <u>Exact sol^n</u>

$$F_{exact} = 1.15\,\sigma_f \cdot 2 \cdot d \int_0^{w/2} e^{\frac{2\mu x}{h}} \cdot dx$$

$$= 1.15\,\sigma_f \times 2d \int_0^{125/2} e^{\frac{2\times 0.1 \cdot x}{20}} \cdot dx$$

$$= 1.15 \times 188.95 \times 2 \times 20 \times 86.824$$

$$= 754.6\ \underline{\underline{KN}}$$

(4) $P_{ave} = 1.15\,\sigma_{flow}\left(1+\frac{\mu D}{2h}\right)$

$F_{ave} = 1.15\,\sigma_{flow}\left(1+\frac{\mu D}{2h}\right) \times D \times d$

$F_{ave} = 1.15 \times 188.95 \times \left(1+\frac{0.1 \times 125}{2 \times 20}\right)$

125×20

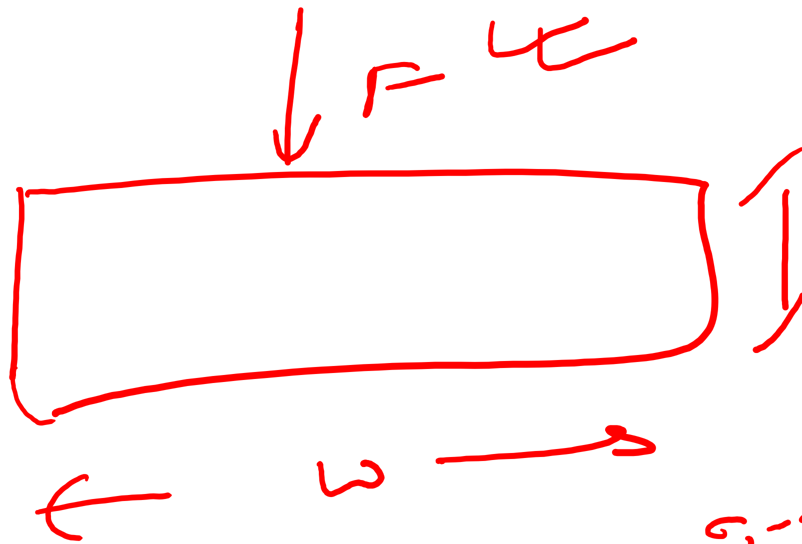
$P_{max} =$

$\mu P_{max} =$

$F_{approx} = \underline{\underline{311\ kN}}$

Is the assumption of all sliding correct?

$\sigma_n + p =$



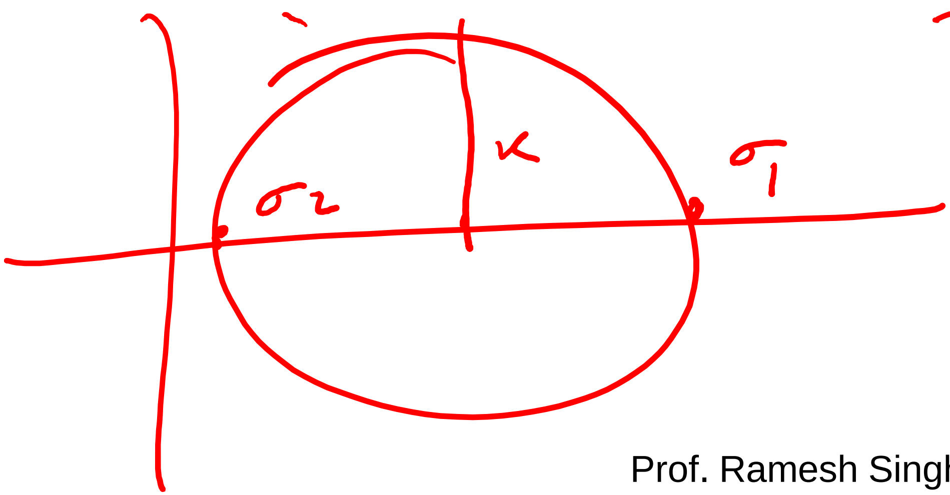
20%

40%

60%

80%

$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

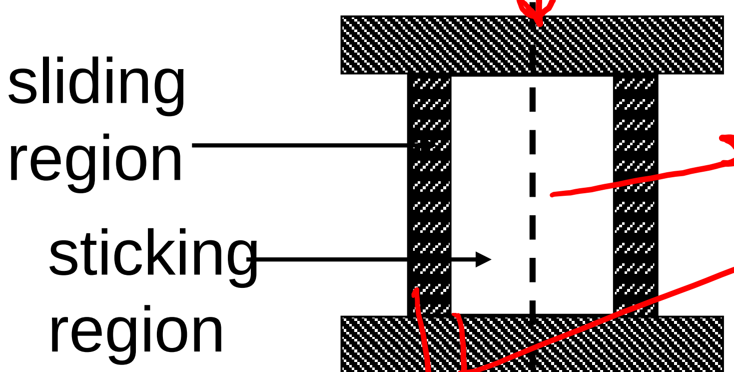


$\sigma_1 - \sigma_2 = 2k$ Tresca

$\sigma_n + p = 2k$

Slab - die interface

- Sliding if $\tau_f < \tau_{flow}$
- Sticking if $\tau_f \geq \tau_{flow}$
  - can't have a frictional stress on a material greater than its flow (yield) stress
  - deformation occurs in a sub-layer just within the material with stress τ_{flow}



Transition

Sliding / sticking transition

- Transition will occur at x_k
- using $k = \mu p$, in:

$$\frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right) \qquad \frac{k}{2\mu k} = \exp\left(\frac{2\mu x_k}{h}\right)$$

- hence:

$$\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}$$

Let the transition occur at x_k

$$\frac{P_x}{2K} = e^{\frac{2\mu x}{h}}$$

@ x_k $\mu P = K$

$\therefore$ $P_{x_k} = K/\mu$

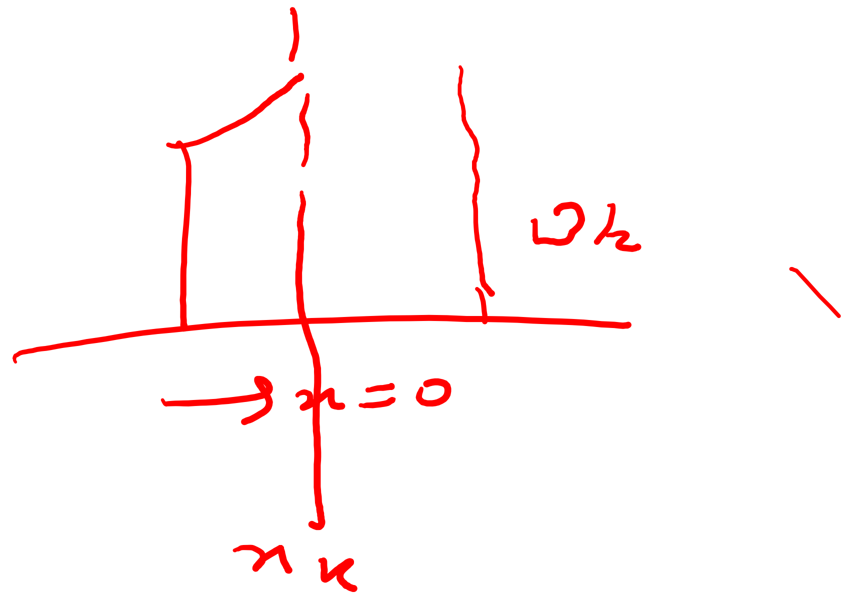
$$\frac{K}{2K\mu} = e^{\frac{2\mu x_k}{h}}$$

$$\boxed{\frac{P_x}{2k} = e^{\frac{2\mu x}{h}}} \quad -(1)$$

$2k = 1.15\,\tau_{f100}$
$= 2\,\tau_{f100}$

$\mu p = k$

@ x_k $\quad p = k/\mu$



$$\frac{k}{\mu \times 2k} = e^{\frac{2\mu x_k}{h}}$$

$$\frac{1}{2\mu} = e^{\frac{2\mu x_k}{h}} \qquad \log\left(\frac{1}{2\mu}\right) = \frac{2\mu x_k}{h}$$

$$\boxed{\frac{x_k}{h} = \frac{1}{2\mu}\log\left(\frac{1}{2\mu}\right)}$$

1 Sliding region

2 Sticking region

something else happens

$\frac{x_k}{h} = \frac{1}{2\mu}\ln\left(\frac{1}{2\mu}\right)$

$x_k = \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$

sliding

$0 \le x \le x_k$

$p_x = 2k\, e^{2\mu x/h}$

$P_{av} = \int_0^{x_k} 2k\, e^{2\mu x/h} \cdot dx$

x_k

$P_{av} \times 2x_k$

Force slid + force sticking

$F_{ext-sliding} = 2d \int_0^{x_k} 2k\, e^{2\mu x/h} \cdot dx$

Prof. Ramesh Singh, Notes by Dr. Singh/ Dr. Colton

Approximation

$$\boxed{\frac{P_x}{2k} = e^{\frac{2\mu x}{h}}}$$

Approx.

$$\boxed{\frac{P_x}{2k} = \left(1 + \frac{2\mu x}{h}\right)}$$

$$P_x = 2k\left(1 + \frac{2\mu x}{h}\right)$$

$$P_{av} = 2k \int_0^{x_k} \left(1 + \frac{2\mu x}{h}\right) . dx$$

x_k

$$\frac{P_x}{2k} = e^{2\mu x / h} \rightarrow \text{Exact}$$

Sliding → A

Sticking → B

All Sliding

All sticking!

Prof. Ramesh Singh, Notes by Dr. Singh/ Dr. Colton

In the sliding region

$0 \le x \le x_k$

$$\frac{p_x}{2k} = e^{\frac{2\mu x}{h}}$$

Exact & Approximate

✓ $p_x = 2k\, e^{\frac{2\mu x}{h}}$

✓ $p_{approx} = 2k\left(1 + \frac{2\mu x}{h}\right)$

Sticking region

$$dp = \frac{2\mu}{h} p dx$$

- Using p = k/ μ

$$dp = \frac{2\mu}{h} \frac{k}{\mu} dx$$

$$\int_{p_{x_k}}^{p_x} dp = \int_{x_k}^{x} \frac{2k}{h} dx$$

$$p_x - p_{x_k} = \frac{2k}{h}\left(x - x_k\right)$$

Sticking region

$$\boxed{dp = \frac{2\tau_f}{h} . dx}$$

$$\boxed{dp = \frac{2\mu p}{h} . dx}$$

$\tau_f = K$ or $\mu p \Rightarrow K$

$$dp = \frac{2K}{h} . dx$$

$$\int_{p_{xk}}^{p_x} dp = \frac{2K}{h}(x - x_K)$$

$$p_x - p_{xk} = \frac{2K}{h}(x - x_K)$$

$$p_x = p_{xk} + \frac{2K}{h}(x - x_K)$$

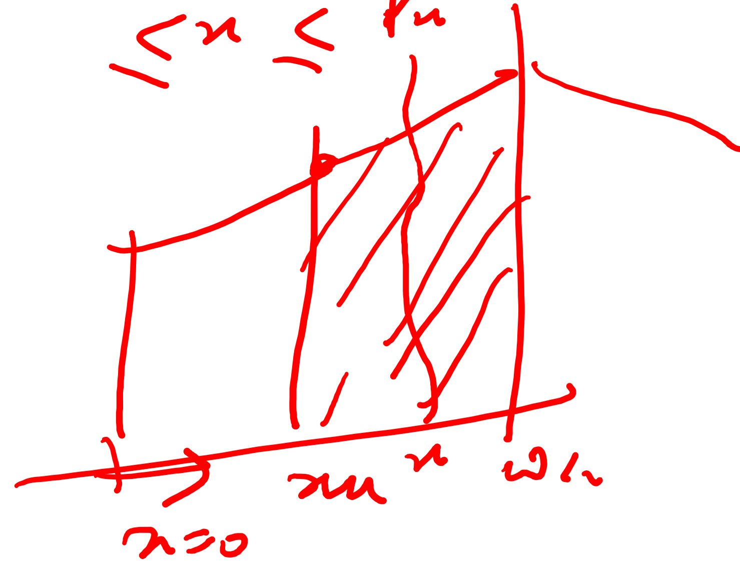
$$\boxed{dp = \frac{2\mu p}{h} . dx}$$

$\mu p \rightarrow \tau_f$

$$dp = \frac{2k}{h} . dx$$

$$\boxed{dp = \frac{2\tau_f}{h} . dx}$$

$\leq x \leq p_x$



$$dp = \frac{2k}{h} . dx$$

$$\int_{p_{xk}}^{p_x} dp = \frac{2k}{h} \int_{x_k}^{x} . dx$$

$$\boxed{\int_{p_{xk}}^{p_x} dp = \frac{2k}{h} (x - x_k)}$$

Sticking region

We know that

- at x = x_k, $p_{x_k} = k/\mu$
- and $$\frac{x_k}{h} = \frac{1}{2\mu}\ln\frac{1}{2\mu}$$

$\mu p = k$

$p_{x_k} = k/\mu$

$\frac{x_k}{h} = \frac{1}{2\mu}\ln\left(\frac{1}{2\mu}\right)$

$p_x - p_{x_k} = \frac{2k}{h}(x - x_k)$

Forging pressure - sticking region

Combining (for $x_k < x < w/2$)

$$p_x = \frac{k}{\mu} + 2k\left(\frac{x}{h} - \frac{1}{2\mu}\ln\left(\frac{1}{2\mu}\right)\right)$$

which is the nature of eqn

$$\int_{x_k}^{w/2} \frac{p_x}{2k} = \frac{1}{2\mu}\left(1-\ln\left(\frac{1}{2\mu}\right)\right)+\frac{x}{h}$$

$$p_x = 1.15\cdot\sigma_{flow}\cdot\left[\frac{1}{2\mu}\left(1-\ln\left(\frac{1}{2\mu}\right)\right)+\frac{x}{h}\right]$$

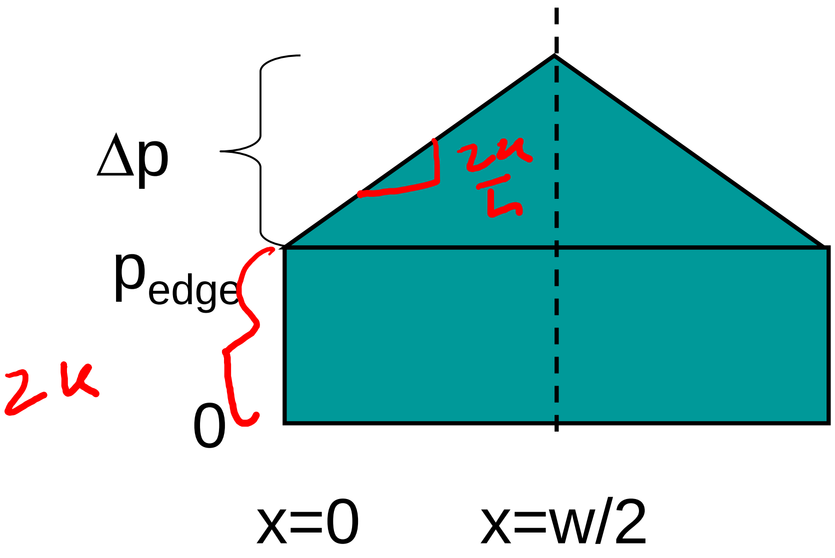
$$p_{av} = \int_{x_k}^{w/2} p_x \cdot dx \Big/ (w/2 - x_k)$$

Forging pressure – all sticking approximation

- If $x_k << w$, we can assume all sticking, and approximate the total forging force per unit depth (into the figure) by:

$dp = \frac{2k}{h} \cdot dx$

slope



Forging pressure – all sticking approximation

$$p_{edge} = 2k$$

$$\frac{\int_0^{w/2} p_x \cdot dx}{w/2}$$

$$\int_{2k}^{p_x} dp = \int_0^x \frac{2k}{h} dx \qquad p_x - 2k = \frac{2k}{h}(x)$$

$$\therefore \frac{p_x}{2k} = \left(1 + \frac{x}{h}\right)$$

$$p_{av} = \frac{\int_0^{w/2} 2k\left(1+\frac{x}{h}\right) \cdot dx}{w/2}$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{x}{h}\right)$$

Average forging pressure – all sticking approximation

$$\frac{p_{ave}}{2k} = \frac{\int_0^{\frac{w}{2}} \frac{p_x}{2k} dx}{w/2} = \frac{\int_0^{\frac{w}{2}} \left(1 + \frac{x}{h}\right) dx}{w/2} = \frac{\left(x + \frac{x^2}{2h}\right)\Big|_0^{\frac{w}{2}}}{w/2}$$

$$\frac{p_{ave}}{2k} = \left(1 + \frac{w}{4h}\right)$$

$$P_{ave} = 2k\left(1 + \frac{w}{4h}\right)$$

$$P_{ave\ all\ stick} = 2K\left(1 + \frac{\mu w}{2h}\right)$$

$$p_{ave} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{w}{4h}\right)$$

Forging force – all sticking approximation

$$F_{forging} = p_{ave} \cdot width \cdot depth$$

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{w}{4h}\right) \cdot w \cdot depth$$

Sticking and sliding

- If you have both sticking and sliding, and you can't approximate by one or the other,
- Then you need to include both in your pressure and average pressure calculations.

$$F_{forging} = F_{sliding} + F_{sticking}$$

$$F_{forging} = \left(p_{ave} \cdot A\right)_{sliding} + \left(p_{ave} \cdot A\right)_{sticking}$$

Material Models

Strain hardening (cold – below recrystallization point)

$$\sigma_{flow} = Y = \underline{\underline{K\varepsilon^n}} \rightarrow$$

$\underline{\underline{K\varepsilon^n}}$ Strain hardening!

Strain rate effect (hot – above recrystallization point)

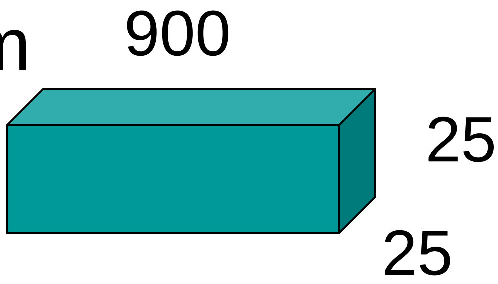
$$\sigma_{flow} = Y = \underline{\underline{C\left(\dot{\varepsilon}\right)^m}}$$

But hot thermally softens!

$$\dot{\varepsilon} = \frac{1}{h}\frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}}$$

Forging - Ex. 1-1

- Lead 25 mm x 25 mm x 900 mm
- σ_y = 6.89 MPa
- h_f = 6.25 mm, μ = 0.25
- Show effect of friction on total forging force.
- Use the slab method.
- Assume it doesn't get wider in 900 mm direction.
- Assume cold forging.



$w = 25$

$w_f = 100$

$h_f = 6.25$

(1) x_k

$\frac{x_k}{h} = \frac{1}{2\mu} \ln\left(\frac{1}{2\mu}\right)$

Forging - Ex. 1-2

- At the end of forging:

h_f = 6.25 mm, w_f = 100 (conservation of mass)

- Sliding / sticking transition

$$\frac{x_k}{h} = \frac{1}{2\cancel{\alpha}} \ln \frac{1}{2\cancel{\alpha}} \quad \mu$$

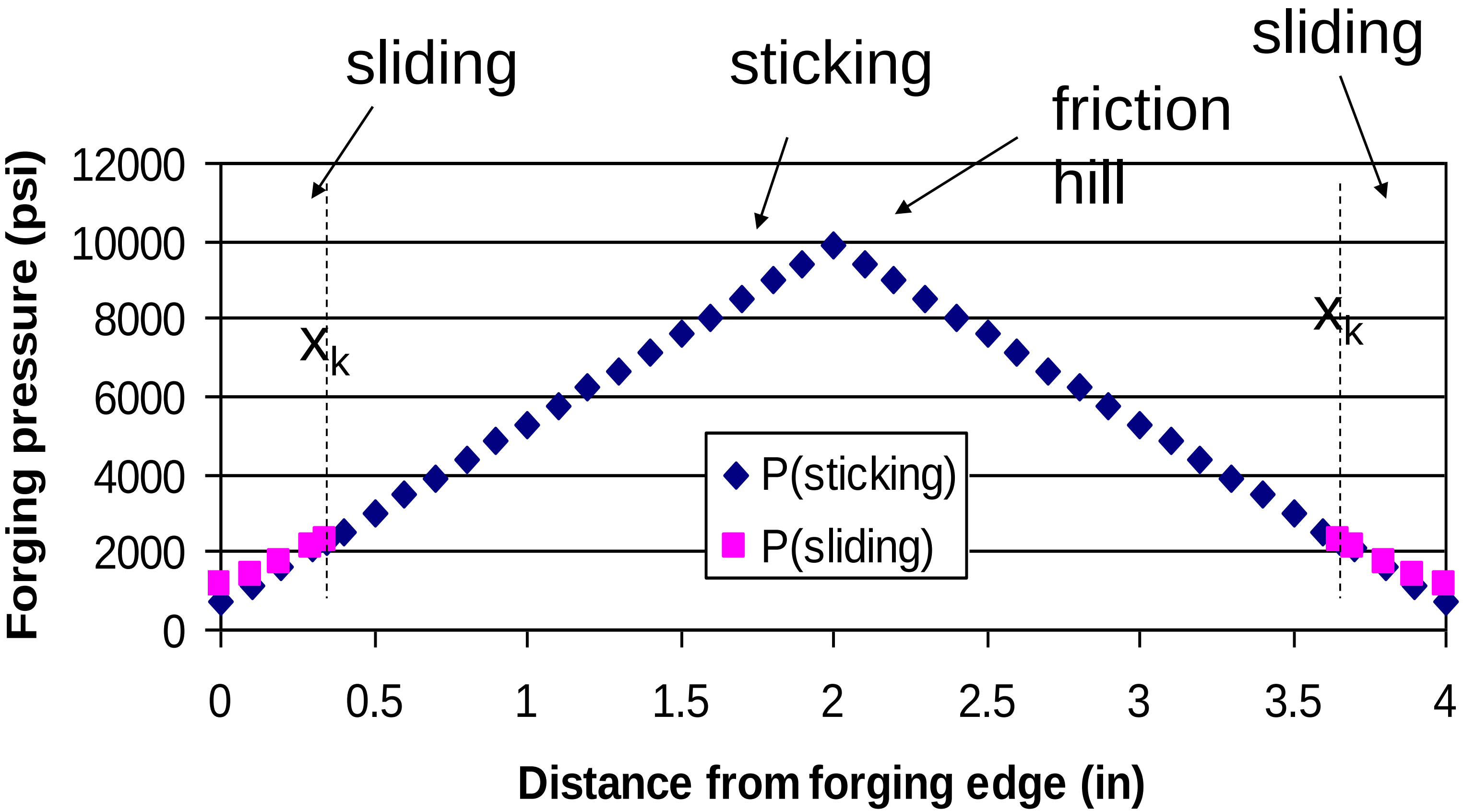
μ

Forging - Ex. 1-3

- Sliding region:

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \exp\left(\frac{2\propto x}{h_f}\right)$$

Forging - Ex. 1-5



Forging - Ex. 1-6

- Friction hill
  - forging pressure must be large (8.7x) near the center of the forging to “push” the outer material away against friction

Forging - Ex. 1-7

- Determine the forging force from:

$$Force = \iint p \cdot dA$$

- since we have plane strain

$$\frac{F}{unit\ depth} = \int_0^x p_x dx$$

Forging - Ex. 1-8

- We must solve separately for the sliding and sticking regions

$$F_{forging} = 2\left(\left(\int_{0}^{x_k} p_x dx\right).depth\right)_{sliding} + 2\left(\left(\int_{x_k}^{w/2} p_x dx\right).depth\right)_{sticking}$$

Approx!

Forging - Ex. 1-16

or since the part is 36” deep:
F(both) = 337 Tonnes

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{w}{4h}\right) \cdot w \cdot depth$$

F(all sticking) = 363 tonnes

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right) \cdot w \cdot depth$$

F(all sliding) = 496,800 lbs = 218 tons
Can we use exact solution for this???
All sticking over-estimates actual value.

Forging – Effect of friction

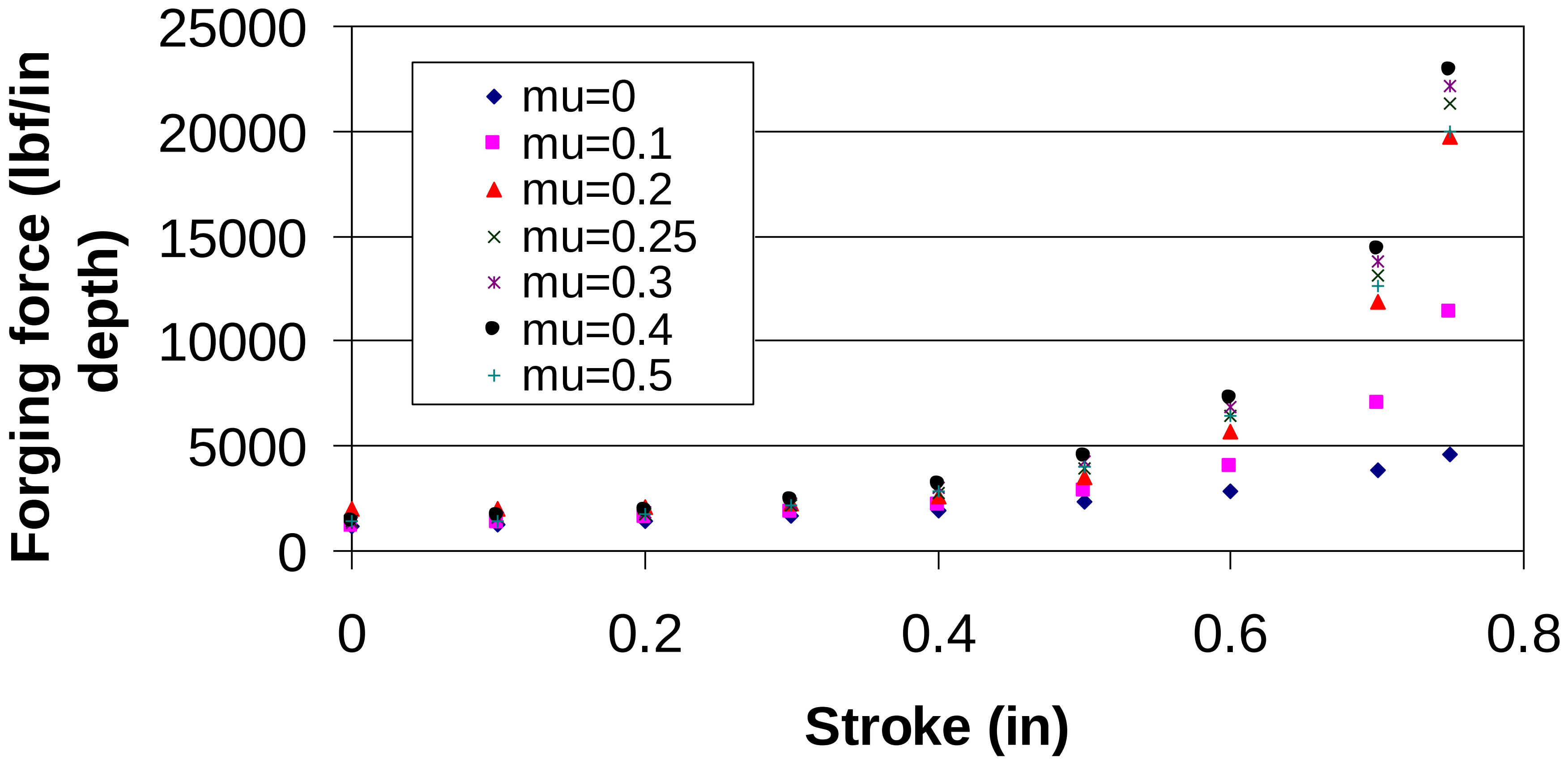
- Effect of friction coefficient (μ) – all sticking

Friction coefficient	Fmax (lbf/in depth)	xk	Stick/slide
0	4600	2	slide
0.1	11365	2	slide
0.2	19735	0.573	both
0.25	21331	0.347	both
0.3	22182	0.213	both
0.4	22868	0.070	both
0.5	23000	0	stick

- Friction is <u>very</u> important

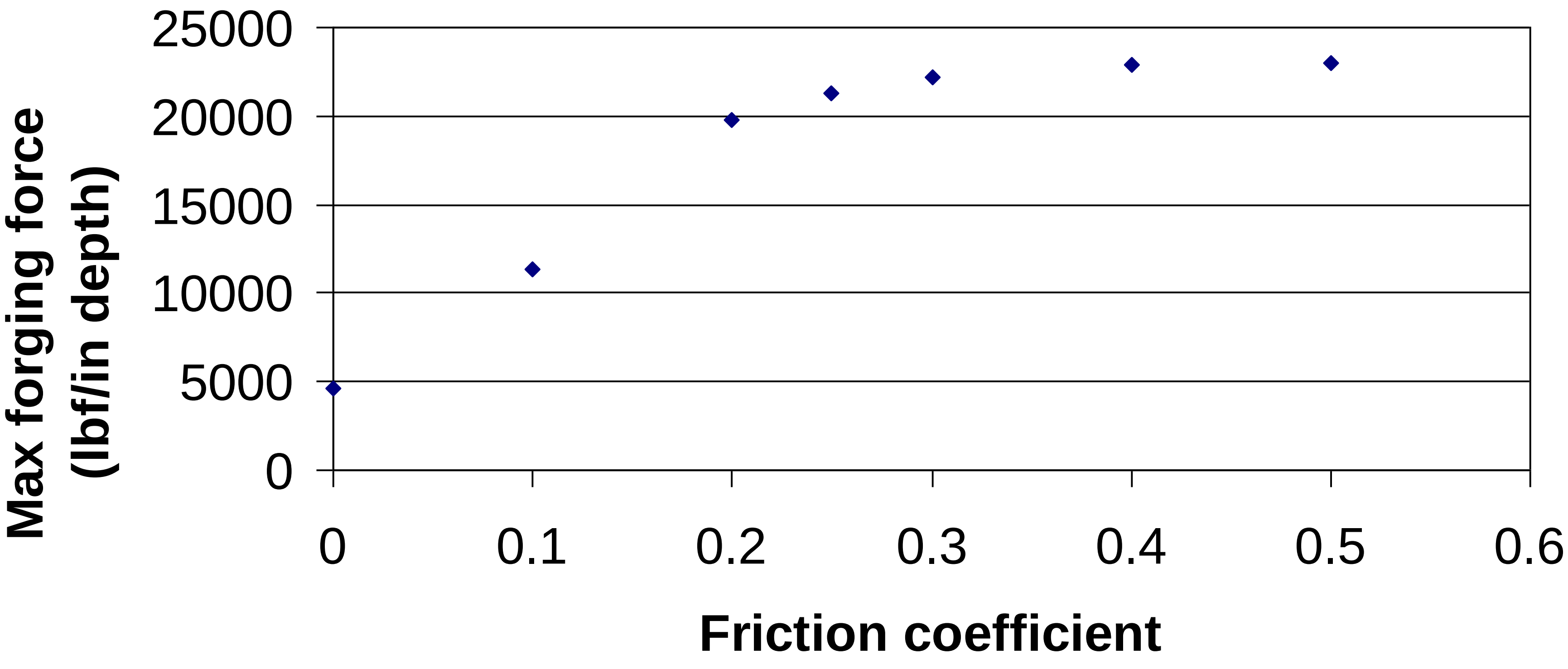
Forging - Ex. 1-17

Forging force vs. stroke – all sticking



Forging - Ex. 1-19

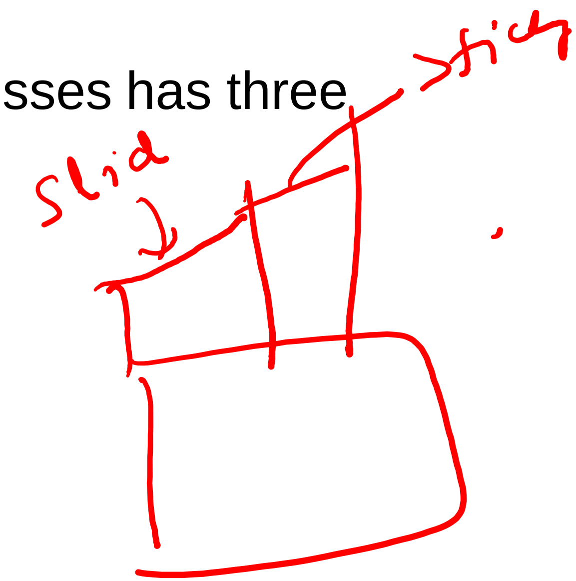
Maximum forging force vs. friction coefficient (μ) all sticking



Deformation Work

In general, work done in bulk deformation processes has three components

Total work, $W = W_{ideal} + W_{friction} + W_{redundant}$



Work of ideal plastic deformation, W_{ideal}

= (area under true stress-true strain curve)(volume)

$$= (\text{volume})\left(\int_0^{\varepsilon_t} \sigma_t \, d\varepsilon_t\right)$$

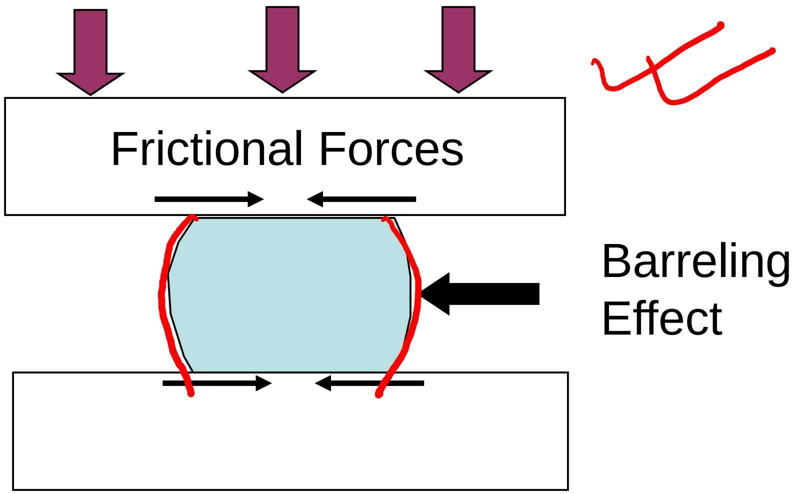
For a true stress-true strain curve $\sigma_t = K\varepsilon_t^n$:

$$\boxed{W_{ideal} = (\text{volume})\left(\frac{K\varepsilon_t^{n+1}}{n+1}\right) = (\text{volume})\bar{Y}_f \varepsilon_t}$$

$\bar{Y}_f = \text{Avg. flow stress}$

Deformation Work

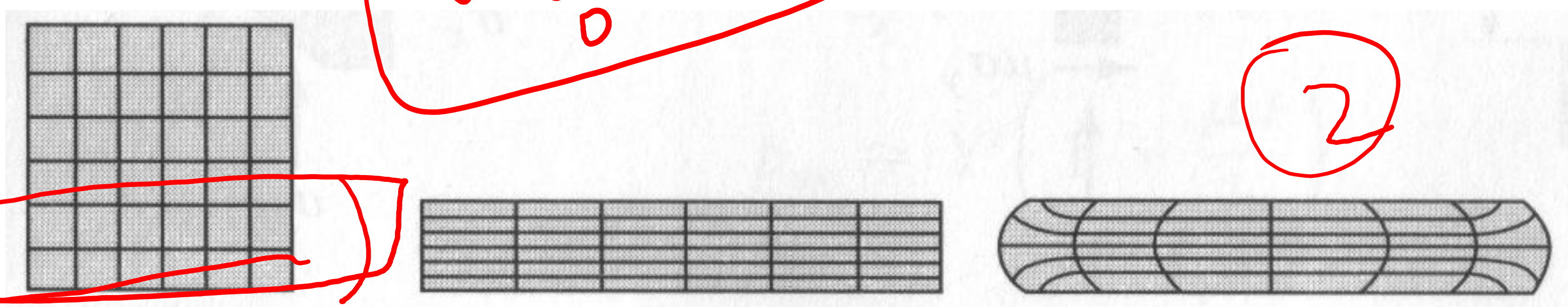
Friction between dies and workpiece causes inhomogeneous (non-uniform) deformation called barreling



Deformation Work

Internal shearing of material requires redundant work to be expended

$V \int_0^{\epsilon} \sigma \cdot d\epsilon$ = Lower Bound Energy

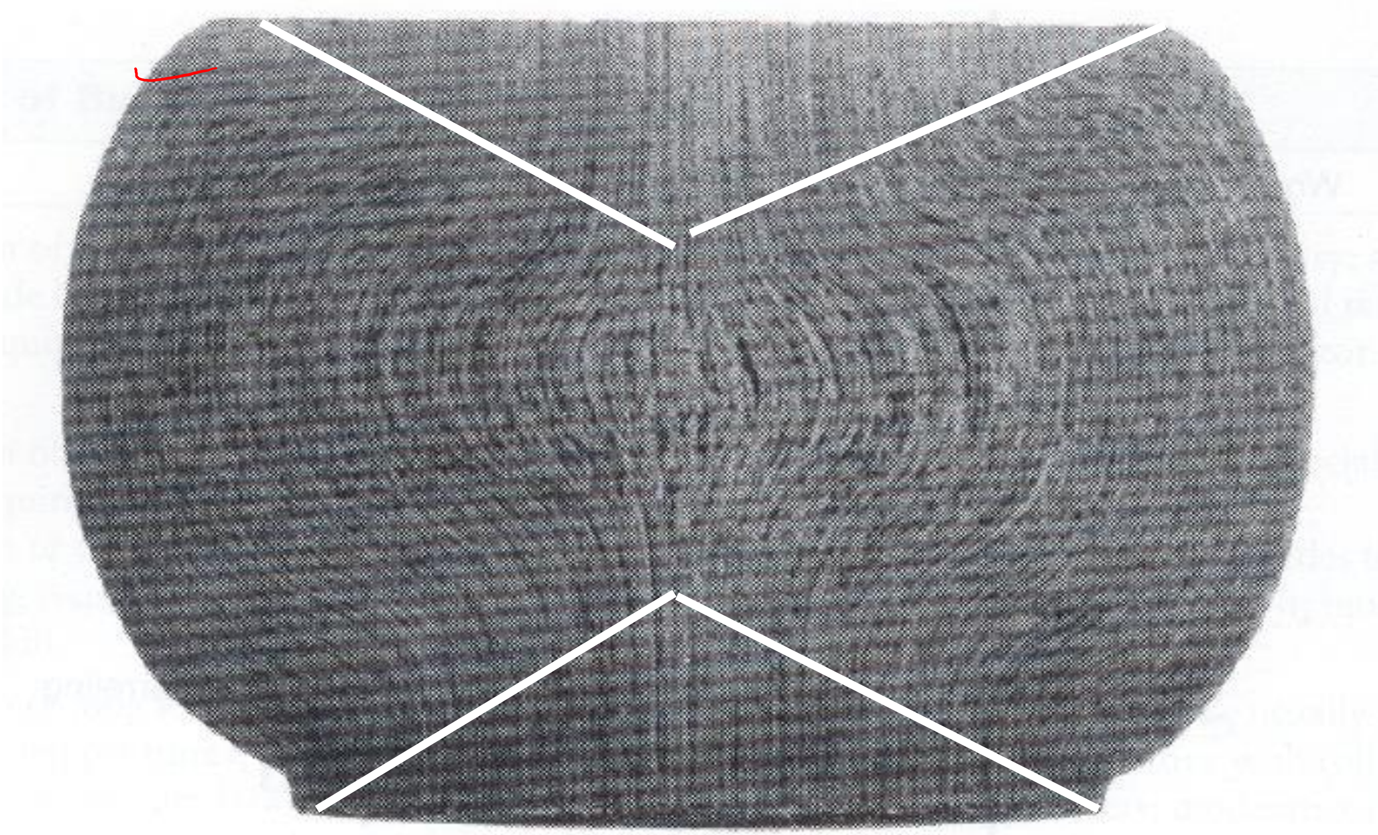


2

Ideal Deformation

Redundant Deformation

Redundant Zone



Closed/Impression Die Forging

- Analysis more complex due to large variation in strains in different parts of workpiece

- Approximate approaches

  – Divide forging into simple part shapes e.g. cylinders, slabs etc. that can be analyzed separately

  – Consider entire forging as a simplified shape

Closed/Impression Die Forging

Steps in latter analysis approach

- ***Step 1***: calculate average height from volume *V* and total projected area A_t of part (including flash area)

$$h_{avg} = \frac{V}{A_t} = \frac{V}{Lw}$$

h avg

- ***Step 2***:

$$\varepsilon_{avg} = \text{avg. strain} = \ln\left(\frac{h_i}{h_{avg}}\right)$$

$$\dot{\varepsilon}_{avg} = \text{avg. strain rate} = \frac{v}{h_{avg}}$$

Closed/Impression Die Forging

- ***Step 3***: calculate flow stress of material Y_f for cold/hot working

$\sigma_f(\epsilon)$ Cold $\sigma_f(\dot{\epsilon})$ Hot

- ***Step 4***:

$$\text{Avg. forging load} = F_{avg} = K_p Y_f A_t$$

K_p = pressure multiplying factor

= 3~5 for simple shapes without flash

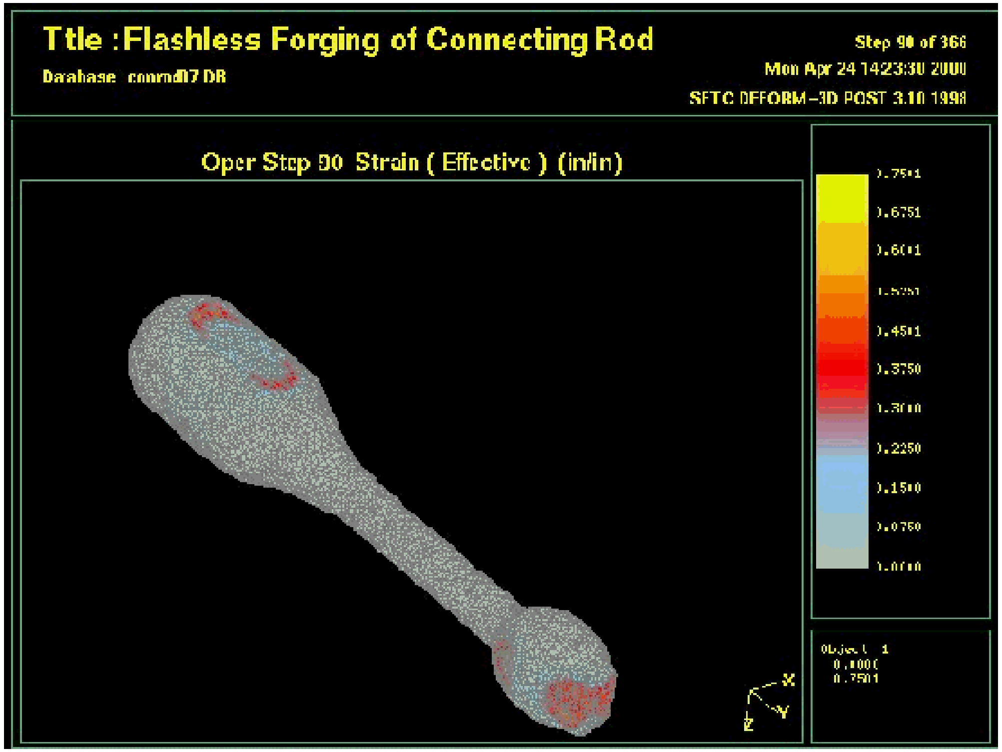
= 5~8 for simple shapes with flash

= 8~12 for complex shapes with flash

Empirical

Other Analysis Methods

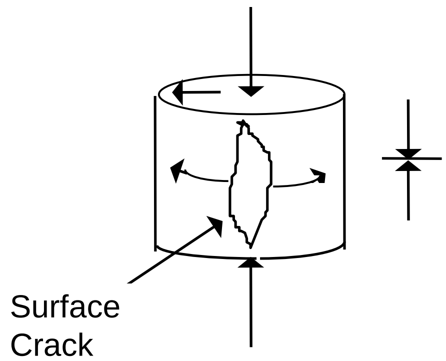
- Complex closed die forging simulated using finite element software



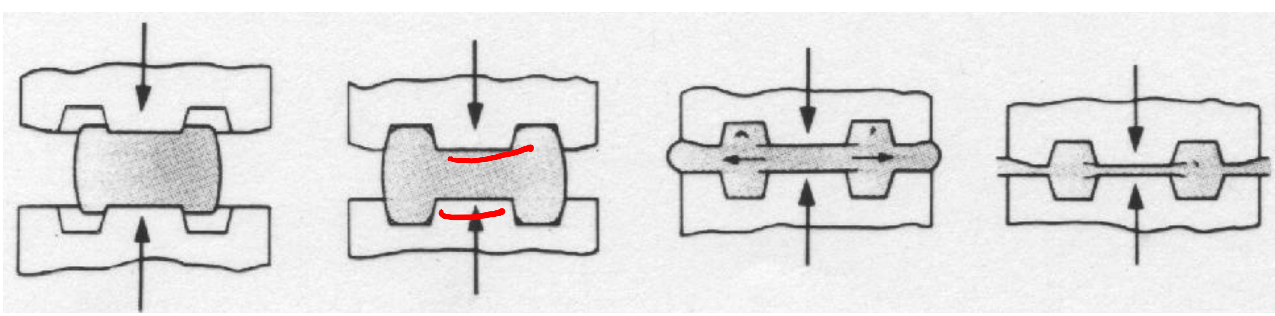
Source: http://nsmwww.eng.ohio-state.edu/html/f-flashlessforg.html

Forging Defects

- Surface cracking due to tensile stresses

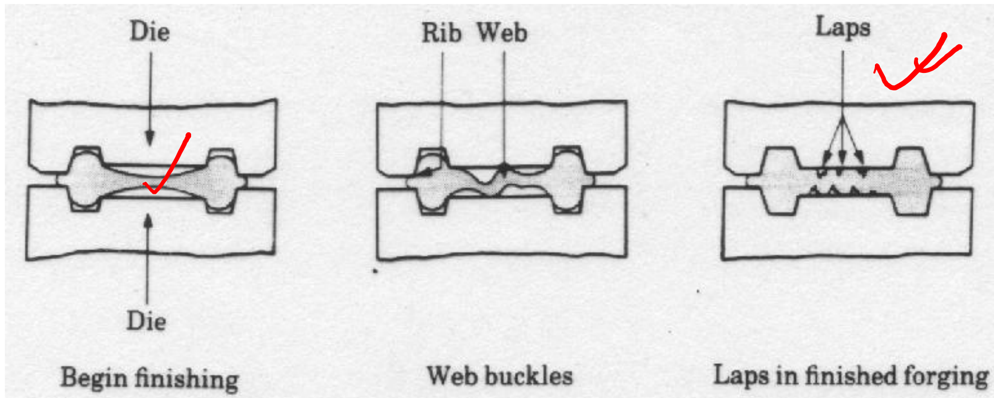


- Internal cracking in thick webs

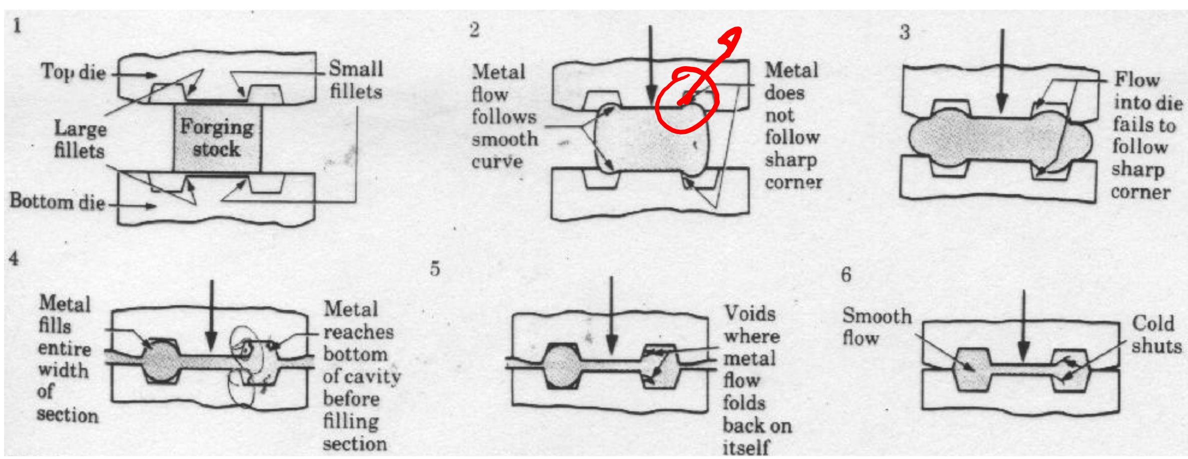


Forging Defects

- Laps due to buckling of thin webs



- Cold shuts due to small radii fillets in die



1 P.S. Forging
2 Ideal Preform
3 Approx. predict flow.
4 Defects

Summary

- What is bulk deformation?
- Cold vs hot working
- Forging analysis
  - Slab analysis
- Forging defects