CONDUCTION 3

HEAT TRANSFER FROM EXTENDED SURFACES

Objectives

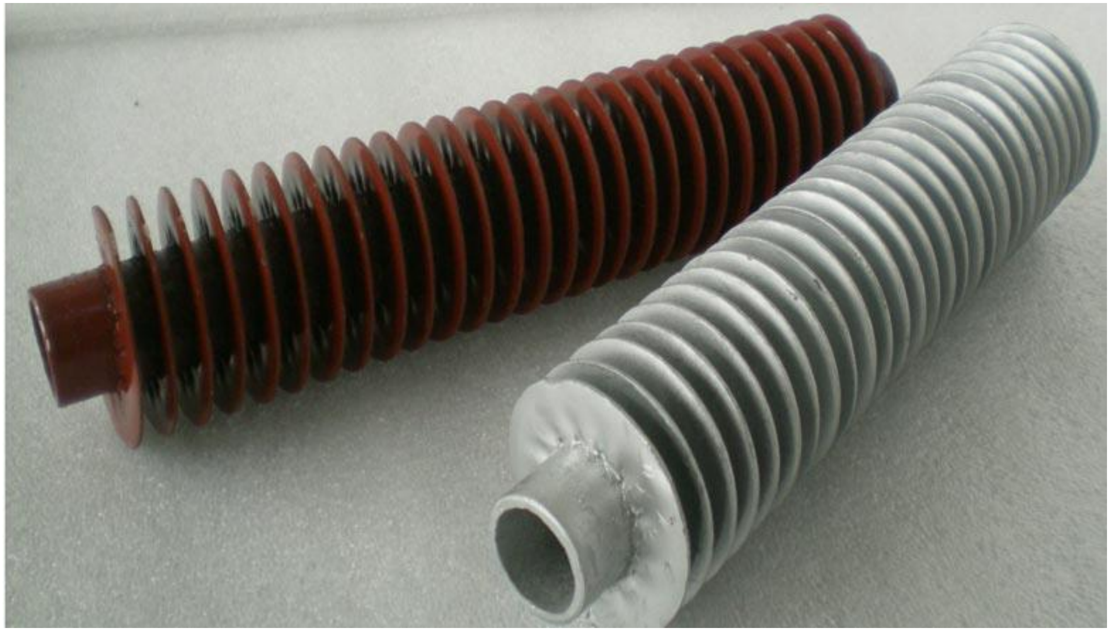
The significance of enhancing the heat transfer by using fins or extended surfaces is presented initially.

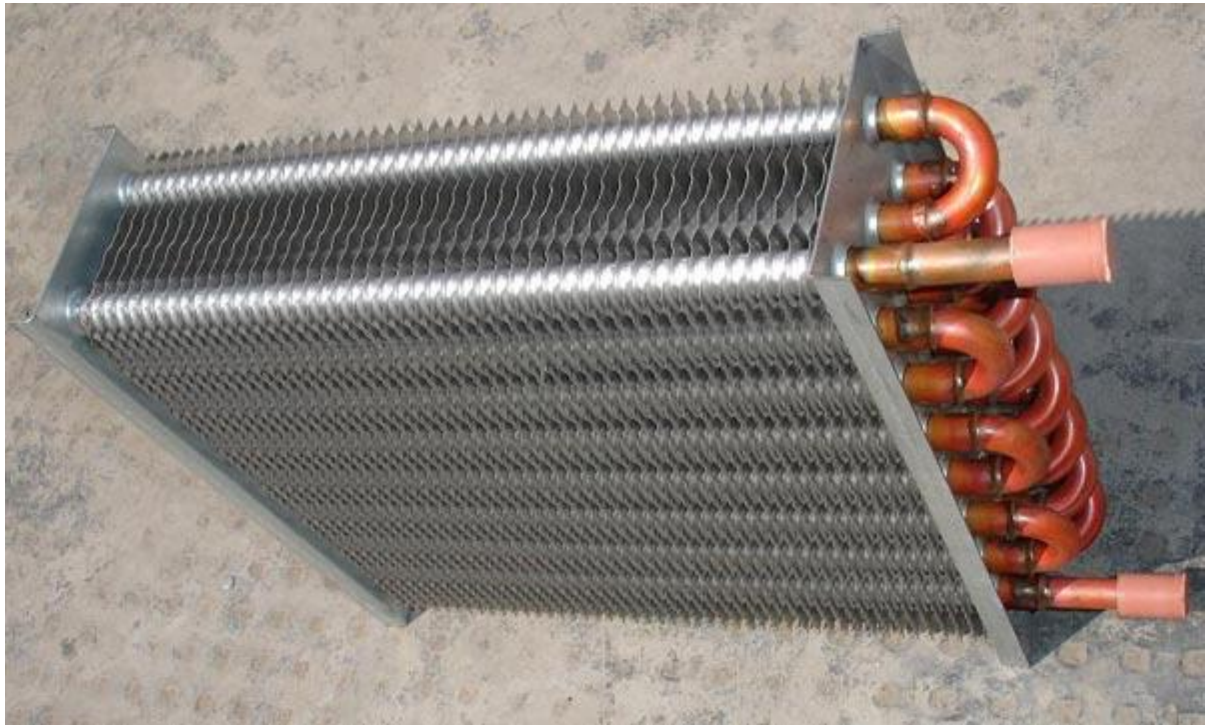
A general form of the energy equation for one dimensional conditions in an extended surface is derived.

APPLICATIONS OF FINS

Fins on engine cylinder

Fins on engine cylinder

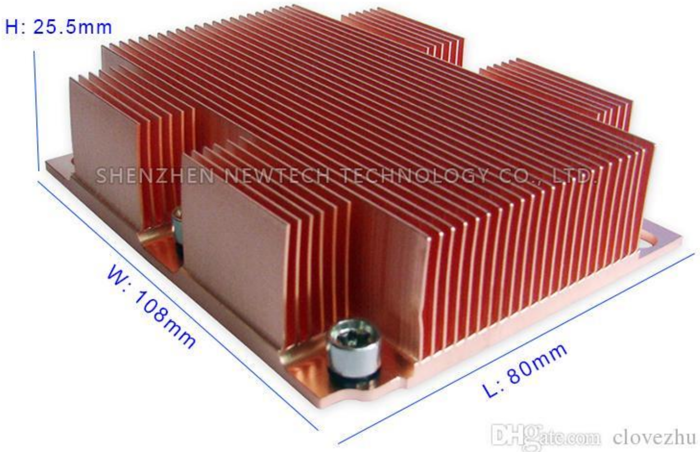




Finned tubes

Finned U tube
aircooled heat exchanger

COMPUTER COOLING



INTRODUCTION

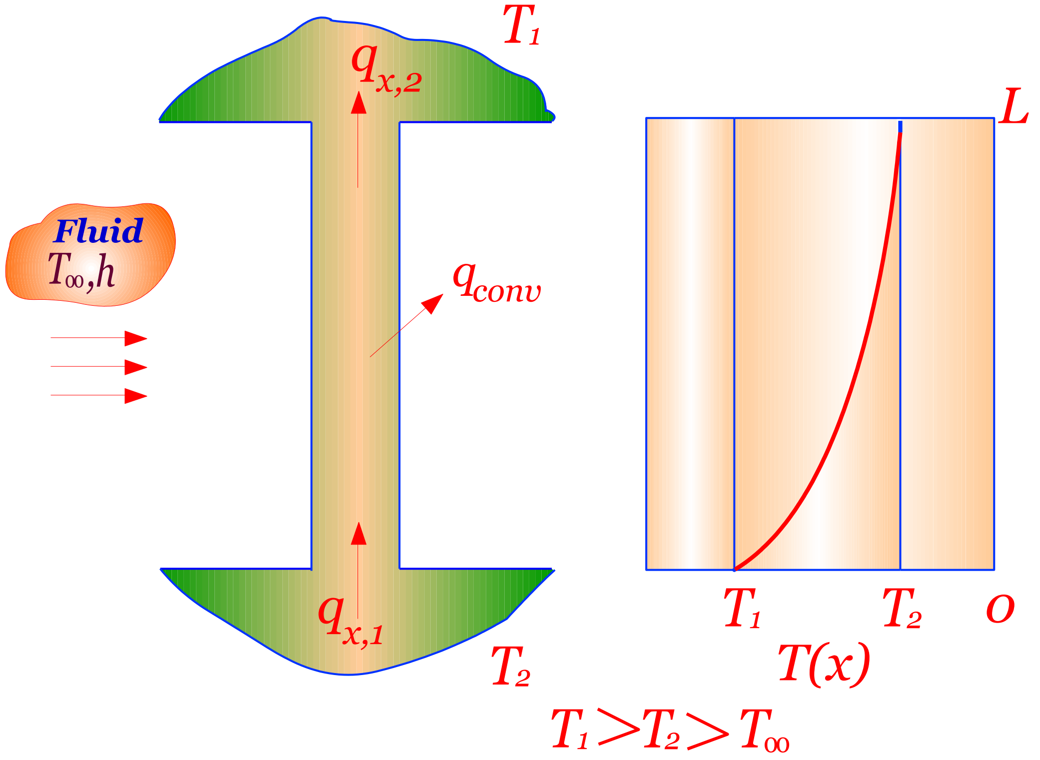
The term extended surface is commonly used in reference to a solid that experiences energy transfer by conduction within its boundaries, as well as energy transfer by convection and/or radiation between its boundaries and the surroundings.

A strut is used to provide mechanical support to two walls that are at different temperatures.

A temperature gradient in the x-direction sustains heat transfer by conduction internally, at the same time there is energy transfer by convection from the surface.

The most frequent application is one in which an extended surface is used specifically to enhance the heat transfer rate between a solid and an adjoining fluid.

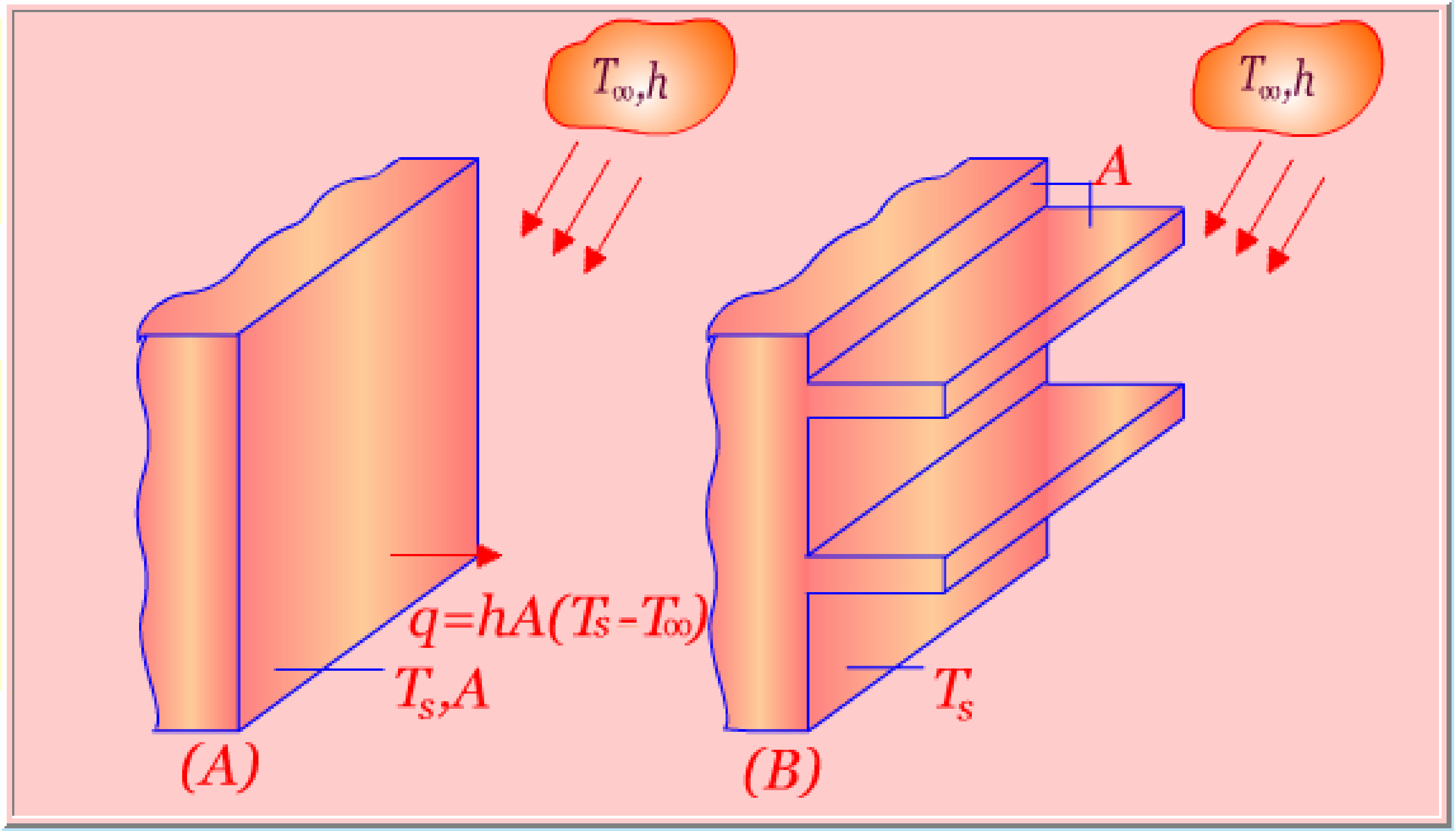
Such an extended surface is termed a FIN.



Consider a plane wall
The rate of heat transfer is given by Newton's law of cooling as

$$\dot{Q}_{conv} = hA(T_s - T_\infty)$$

T_s - the surface temperature
T_∞ - Temperature of the surrounding medium
h - Heat transfer coefficient
A – Heat Transfer area



Use Of Fins To Enhance Heat Transfer From A Plane Wall
(A) Bare Surface (B) Finned Surface

If T_s is fixed, there are two ways in which the heat transfer rate may be increased.
The convection coefficient h could be increased by

- Increasing the fluid velocity
- The fluid temperature T_∞ could be reduced

LIMITATIONS:

Many situations would be encountered in which increasing heat transfer coefficient to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitively high.

Such costs are comprised of the blower or pump power requirements needed to increase heat transfer coefficient through increased fluid motion.

Moreover, the second option of reducing temperature of the surrounding medium is often impractical.

There exists a third option, that is, the heat transfer rate may be increased by increasing the surface area across which the convection occurs.

This may be done by providing fins that extend from the wall into the surrounding fluid.

The thermal conductivity of the fin material has a strong effect on the temperature distribution along the fin and therefore influences the degree to which the heat transfer rate is enhanced.

Ideally, the fin material should have a large thermal conductivity to minimize temperature variations from its base to its tip.

In the limit of infinite thermal conductivity, the entire fin would be at the temperature of the base surface, thereby providing the maximum possible heat transfer enhancement.

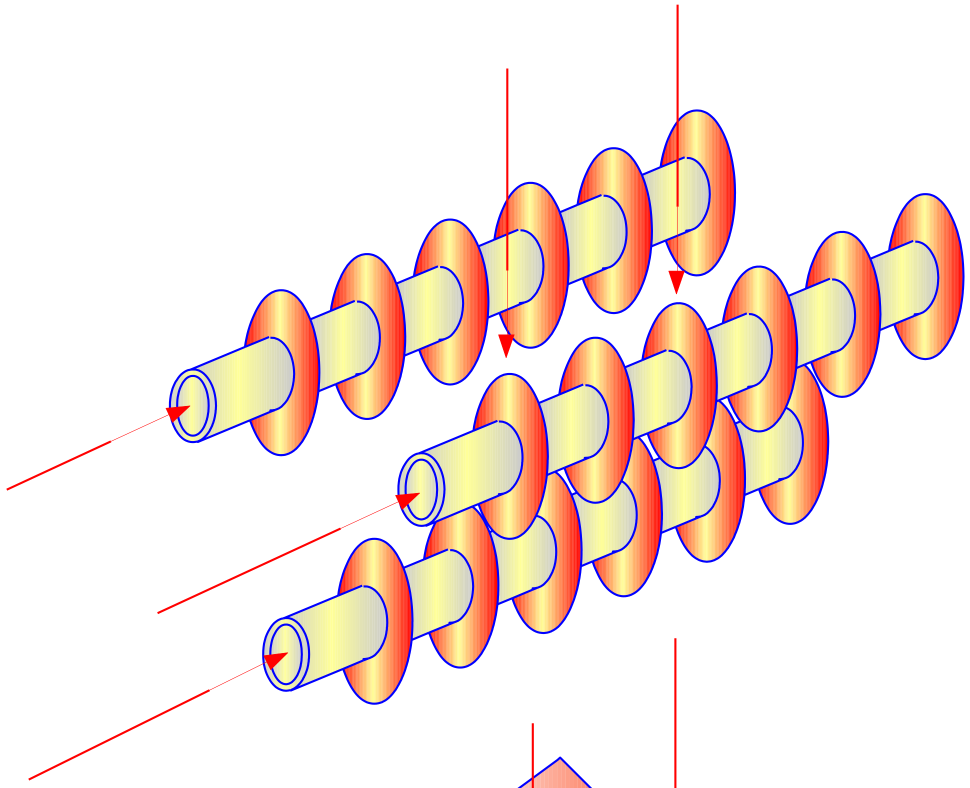
APPLICATIONS:

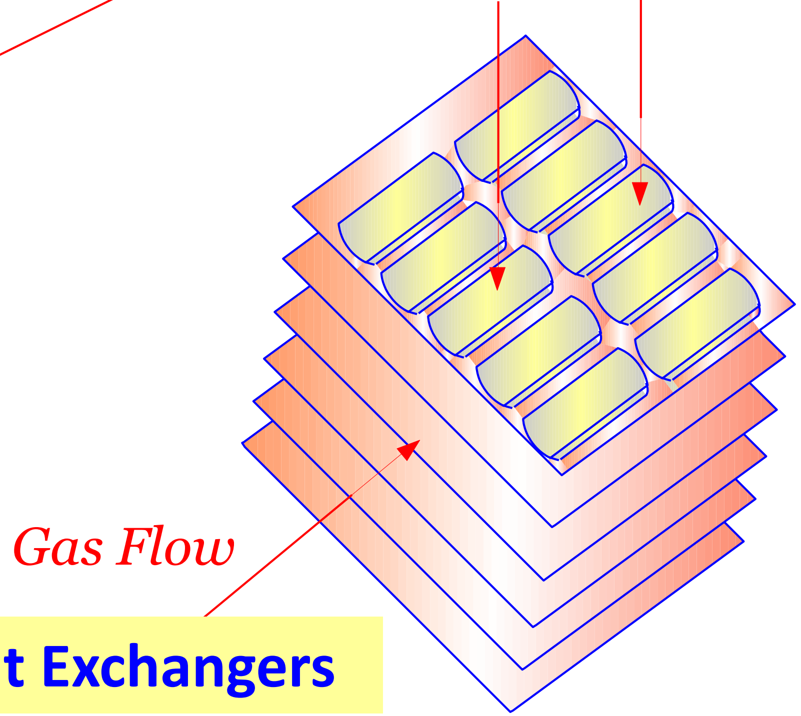
There are several fin applications,

- the arrangement for cooling engine heads on motorcycles and lawn-mowers or
- for cooling electric power transformers
- the tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner.

Two common finned tube arrangements are shown

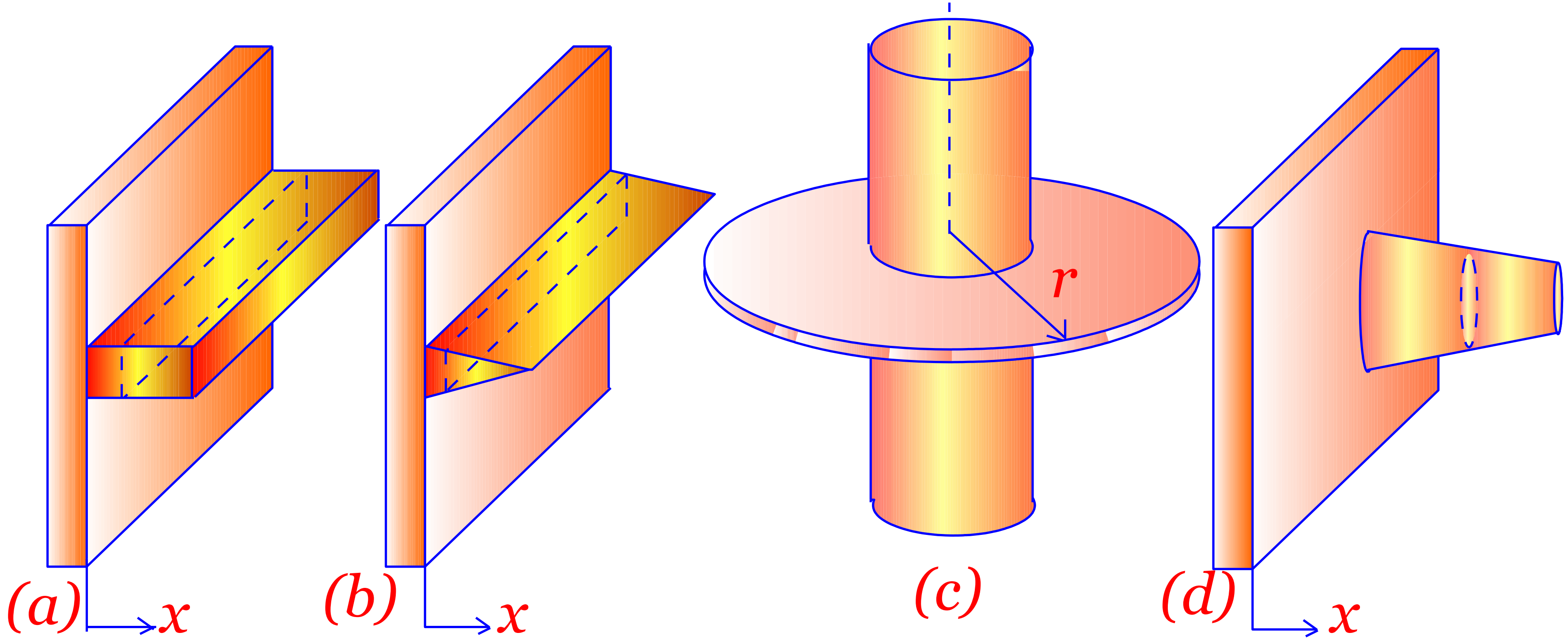
In any application, selection of a particular fin configuration may depend on space, weight, manufacturing and cost considerations, the extent to which the fins reduce the surface convection coefficient and increase in the pressure drop associated with flow over the fins.





Schematic Of Typical Finned Tube Heat Exchangers

DIFFERENT FIN CONFIGURATIONS



- A straight fin is any extended surface that is attached to a plane wall. It may be of uniform cross sectional area, or its cross sectional area may vary with the distance *x* from the wall.
- An annular fin is one that is circumferentially attached to a cylinder, and its cross section varies with radius from the centerline of the cylinder.
- The foregoing fin types have rectangular cross sections, whose area may be expressed as a product of the fin thickness *t* and the width *w* for straight fins or the circumference for annular fins.
- In contrast a pin fin, or spine, is an extended surface of circular cross section.
- Pin fins may also be of uniform or non-uniform cross section.

A GENERAL CONDUCTION ANALYSIS OF EXTENDED SURFACES

To determine the heat transfer rate associated with a fin, we must first obtain the temperature distribution along the fin.
The analysis is simplified if certain assumptions are made.

Heat transfer is assumed to be in only one dimensional i.e., in the longitudinal (x) direction, even though conduction within the fin is actually two dimensional.

The rate at which the energy is convected to the fluid from any point on the fin surface must be balanced by the rate at which the energy reaches that point due to conduction in the transverse (y, z) direction. However, in practice the fin is thin and temperature changes in the longitudinal direction are much larger than those in the transverse direction.

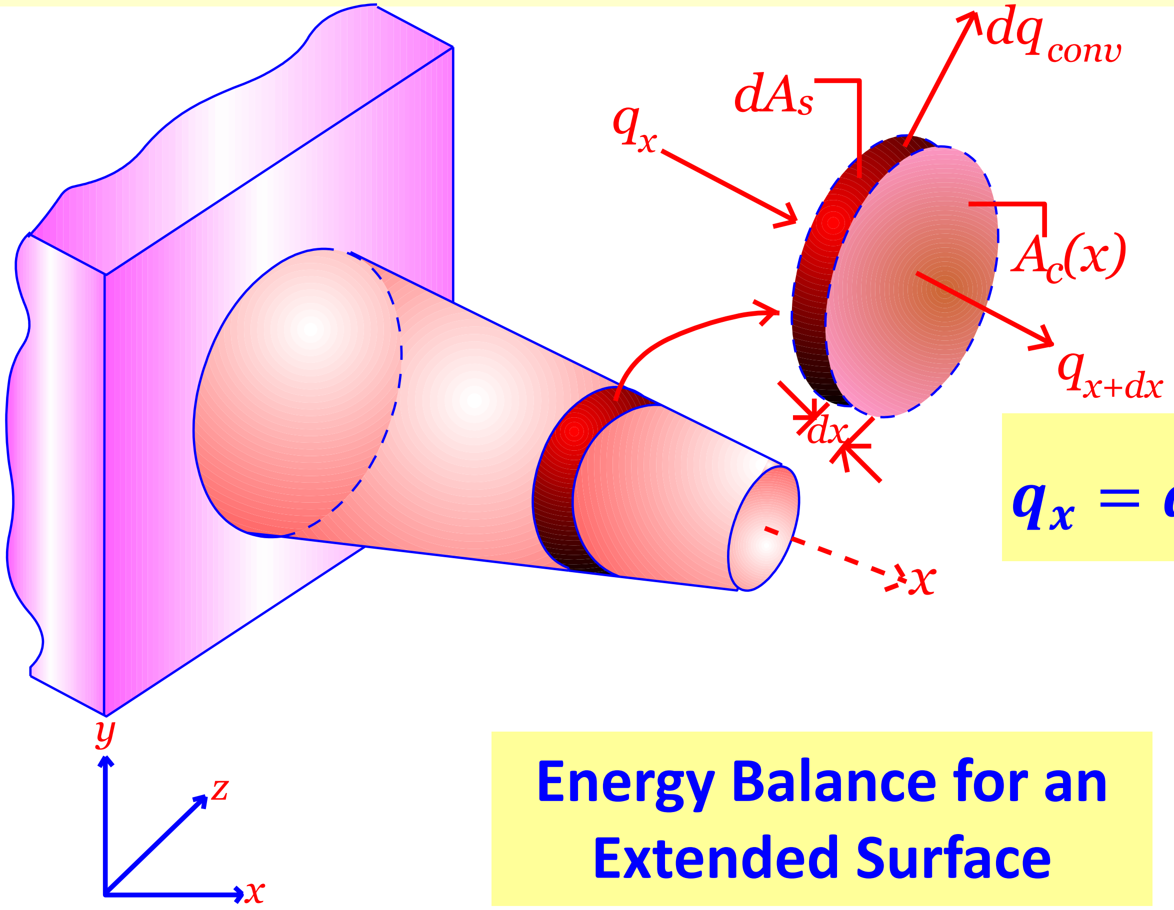
Steady state conditions are assumed.

Thermal conductivity is assumed to be constant .

Radiation from the surface is assumed to be negligible .

Convection heat transfer coefficient is assumed to be uniform over the surface

We begin our analysis by performing an energy balance on an appropriate differential element. Consider the extended surface of Figure.



Applying the conservation of energy requirement to the differential element of Fig.

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = q_x + \frac{dq_x}{dx}dx + dq_{conv}$$

$$\frac{dq_x}{dx}dx + dq_{conv} = 0$$

$$q_x = -kA_c\frac{dT}{dx}$$

$$dq_{conv} = h\,dA_s(T - T_\infty)$$

$$\frac{d}{dx}\left(-kA_c\frac{dT}{dx}\right)dx + h\,dA_s(T - T_\infty) = 0$$

Energy Balance for an Extended Surface

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

This result provides a general from of the energy equation for one dimensional condition in an extended surface. Its solution for appropriate boundary conditions would provide the temperature distribution, which could then be used with Fourier law to calculate the conduction rate at any distance x

TEMPERATURE DISTRIBUTION FOR RECTANGULAR FIN AND PIN FIN

The temperature distribution for rectangular fin and pin fin with various boundary conditions is obtained from the general form of the energy equation for an extended surface.

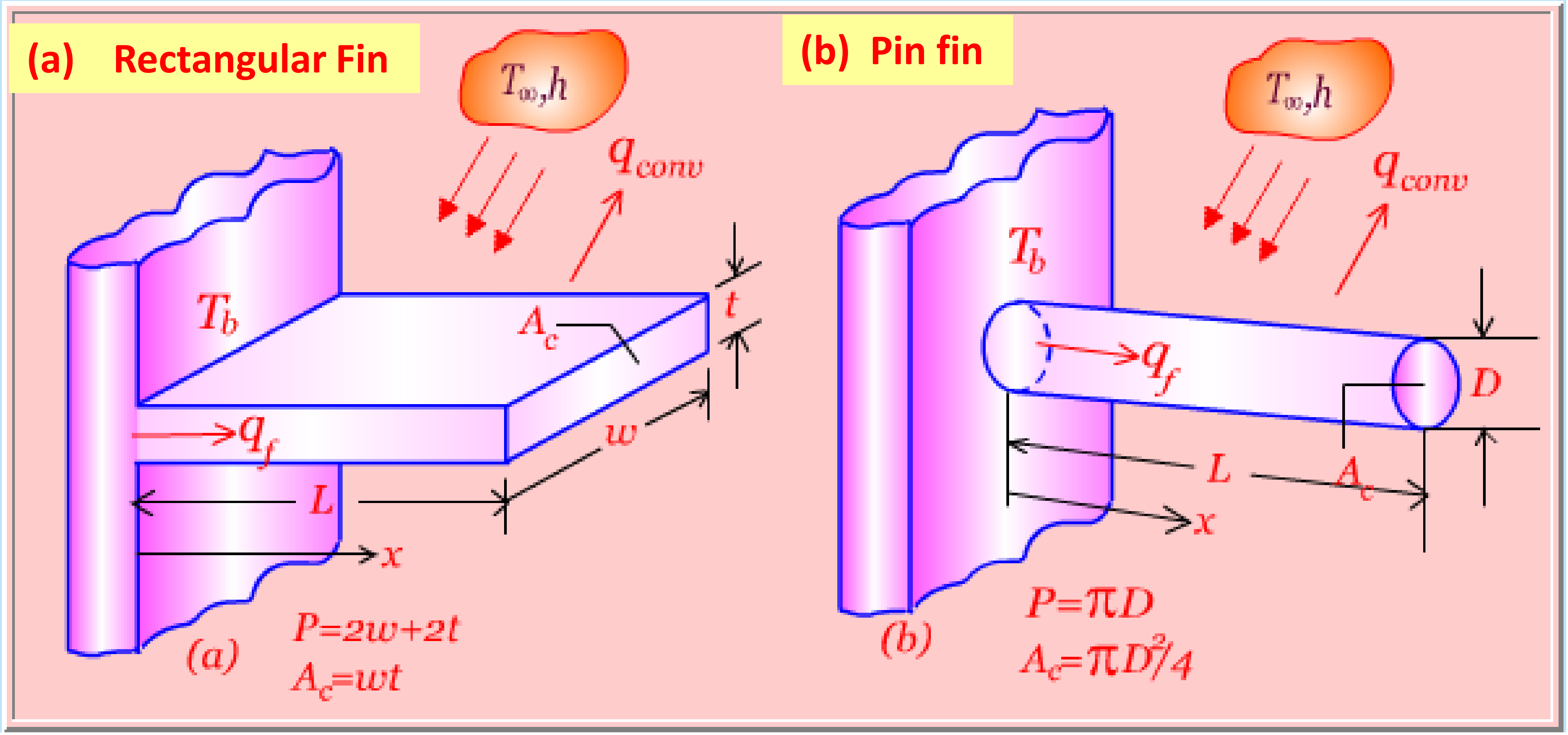
Straight rectangular and pin fins of uniform cross section are considered

Each fin is attached to a base surface of temperature $T(0) = T_b$ and extends into a fluid of temperature T_∞.

Figure Fins of Uniform Cross Section

(a) Rectangular Fin

(b) Pin Fin



$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

For the prescribed fins, A_c is a constant and $A_s = Px$ A_s is the surface area measured from the base to x and P is the fin perimeter.

$$\frac{dA_c}{dx} = 0 \qquad A_s = Px \qquad \frac{dA_s}{dx} = P$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\theta(x) = T(x) - T_\infty \qquad m^2 = \frac{hP}{kA_c}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Equation is linear, homogenous, second order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1e^{mx} + C_2e^{-mx}$$

To evaluate the constant C_1 and C_2 of this equation, it is necessary to specify appropriate boundary conditions.

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One such condition may be specified in terms of the temperature at the base of the fin ($x = 0$)

$$\theta(x) = T(x) - T_\infty \qquad \theta(0) = T_b - T_\infty = \theta_b$$

The second condition, specified at the fin tip ($x = L$), may correspond to any one of the four different physical conditions.

- Convection heat transfer from the fin tip
- Adiabatic condition at the fin tip
- Prescribed temperature maintained at the fin tip
- Infinite fin (very long fin)

CASE B, ADIABATIC CONDITION AT THE FIN TIP

The assumption that the convective heat loss from the fin tip is negligible reduces to the condition that the tip may be treated as adiabatic and we obtain

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta(x) = T(x) - T_\infty$$

Base of the fin is at constant temperature $\theta(0) = \theta_b$ and the fin tip is adiabatic $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

$$\theta(0) = T_b - T_\infty = \theta_b$$

$$\theta_b = C_1 + C_2$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = mC_1 e^{mL} - mC_2 e^{-mL} = 0$$

$$C_2 = C_1 e^{2mL}$$

$$\theta_b = C_1 + C_2$$

$$\theta_b = C_1 + C_1 e^{2mL}$$

$$C_1 = \frac{\theta_b}{1 + e^{2mL}}$$

$$C_2 = \frac{\theta_b}{1 + e^{2mL}} e^{2mL}$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta(x) = \frac{\theta_b}{1 + e^{2mL}} e^{mx} + \frac{\theta_b}{1 + e^{2mL}} e^{2mL} e^{-mx}$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{mx}}{1 + e^{2mL}} + \frac{e^{-mx}}{1 + e^{2mL}} e^{2mL}$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mx}}{1+e^{2mL}} e^{2mL}$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{-mL}}{e^{-mL}} \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mL}}{e^{-mL}} \frac{e^{-mx}}{1+e^{2mL}} e^{2mL}$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{-m(L-x)}}{e^{-mL}+e^{mL}} + \frac{e^{m(L-x)}}{e^{-mL}+e^{mL}}$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{-m(L-x)}+e^{m(L-x)}}{e^{-mL}+e^{mL}}$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$\frac{\theta(x)}{\theta_b} = \frac{\cosh[m(L-x)]}{\cosh mL}$$

$$\theta(x) = T(x) - T_\infty$$

$$m^2 = \frac{hP}{kA_c}$$

$$\frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh mL}$$

Heat Transfer Rate

$$Q = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$\theta(x) = \frac{\theta_b}{1+e^{2mL}} e^{mx} + \frac{\theta_b}{1+e^{2mL}} e^{2mL} e^{-mx}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = \frac{\theta_b}{1+e^{2mL}} m - \frac{\theta_b}{1+e^{2mL}} e^{2mL} m$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = m\theta_b \left[\frac{1}{1+e^{2mL}} - \frac{e^{2mL}}{1+e^{2mL}}\right]$$

$$Q = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$Q = -kA_c m\theta_b \left[\frac{1}{1+e^{2mL}} - \frac{e^{2mL}}{1+e^{2mL}}\right]$$

$$Q = -kA_c m\theta_b \frac{e^{-mL}}{e^{-mL}} \left[\frac{1}{1+e^{2mL}} - \frac{e^{2mL}}{1+e^{2mL}}\right]$$

$$Q = -kA_c m\theta_b \left[\frac{e^{-mL}}{e^{-mL}+e^{mL}} - \frac{e^{mL}}{e^{-mL}+e^{mL}}\right]$$

$$Q = -kA_c m\theta_b \left[\frac{e^{-mL}-e^{mL}}{e^{-mL}+e^{mL}}\right]$$

$$Q = kA_c m\theta_b \left[\frac{e^{mL}-e^{-mL}}{e^{-mL}+e^{mL}}\right]$$

$$m^2 = \frac{hP}{kA_c}$$

$$Q = kA_c\theta_b \sqrt{\frac{hP}{kA_c}} \left[\frac{e^{mL}-e^{-mL}}{e^{-mL}+e^{mL}}\right]$$

$$Q = \theta_b \sqrt{hPkA_c} Tanh mL$$

CASE D, INFINITE FIN (VERY LONG FIN)

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$x = 0, \theta = \theta_b = T_b - T_\infty$$

$$\theta(0) = \theta_b = C_1 + C_2$$

$$\theta_b = C_1 + C_2$$

$$\theta_b = C_2$$

$$x \to \infty, \theta = T_\infty - T_\infty = 0$$

$$\theta(\infty) = 0 = C_1 e^{\infty} + C_2 e^{-\infty}$$

$$0 = C_1 e^{\infty} + C_2(0)$$

$$0 = C_1 e^{\infty} \qquad C_1 = 0$$

$$\theta(x) = \theta_b e^{-mx}$$

$$T(x) - T_\infty = (T_b - T_\infty) e^{-mx}$$

$$m^2 = \frac{hP}{kA_c}$$

$$q_f = -kA_c \left.\frac{dT}{dx}\right|_{x=0} == -kA_c \left.\frac{d\theta}{dx}\right|_{x=0} = -kA_c \left(-\theta_b m e^{-mx}\right)\Big|_{x=0}$$

$$q_f = mkA_c\theta_b = \sqrt{\frac{hP}{kA_c}}(kA_c\theta_b) = \theta_b\sqrt{hPkA_c}$$

$$q_f = \theta_b\sqrt{hPkA_c}$$

SUMMARY OF TEMPERATURE DISTRIBUTION AND HEAT TRANSFER RATE

Case	Tip condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection heat transfer $h A_c \left[T(L) - T_\infty\right] = -kA_c \left.\frac{dT}{dx}\right\|_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$\theta_b \sqrt{hPk A_c}\, \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$\sqrt{h P k A_c}\,\theta_b \operatorname{Tanh} mL$
C	Prescribed temperature $\theta(L) = \theta_L$	$\frac{\left(\theta_L/\theta_b\right)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$\sqrt{h P k A_c}\,\theta_b \frac{\cosh mL - \left(\theta_L/\theta_b\right)}{\sinh mL}$
D	Infinite fin ($L \to \infty$)	e^{-mx}	$\theta_b \sqrt{h P k A_c}$

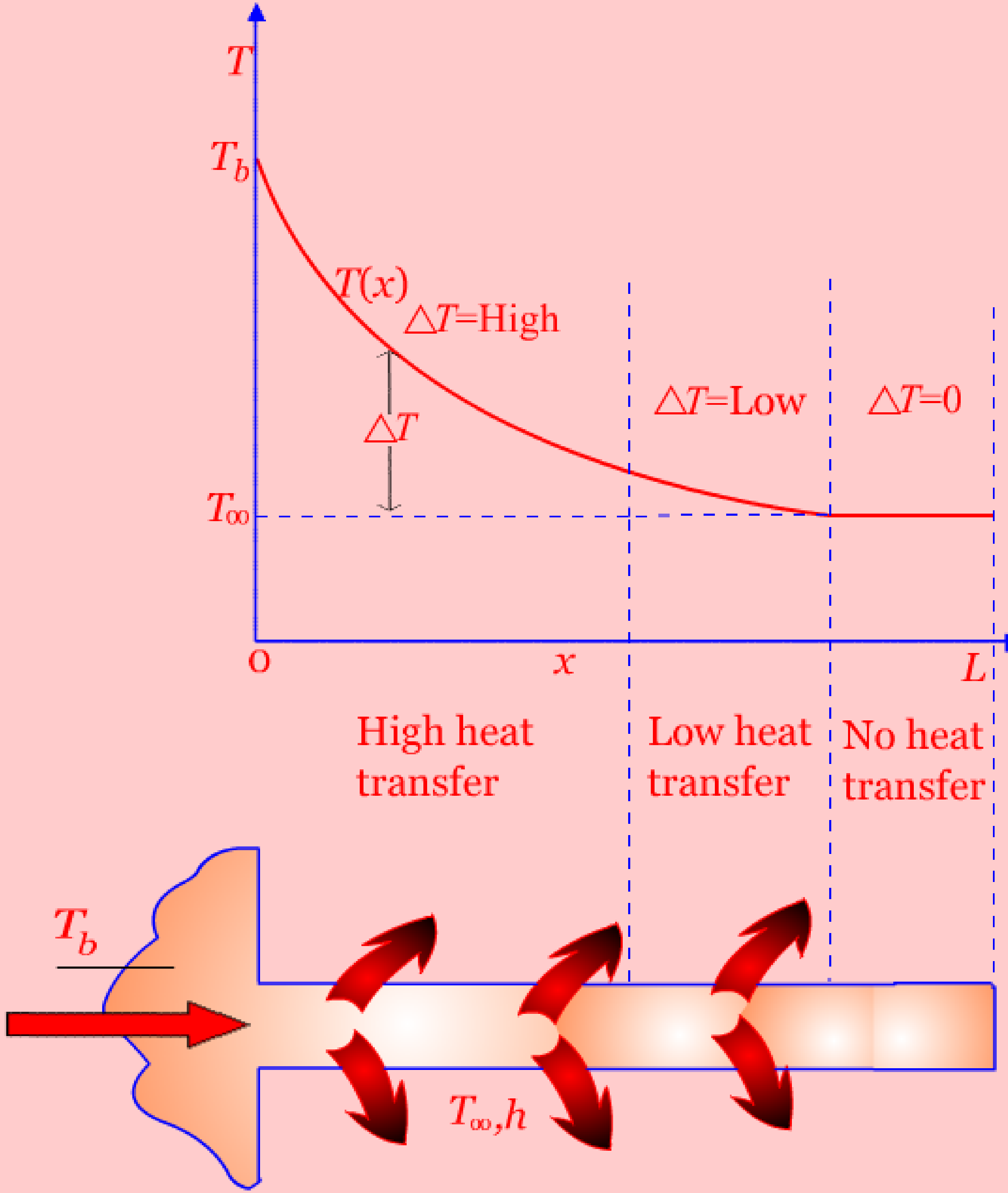
PROPER LENGTH OF THE FIN

An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified.

You may be tempted to think that the longer the fin, the larger the surface area and thus the higher the rate of heat transfer. Therefore, for maximum heat transfer, the fin should be infinitely long.

However, the temperature drops along the fin exponentially and reaches the environment temperature at some length.

The part of the fin beyond this length does not contribute to heat transfer since it is at the temperature of the environment

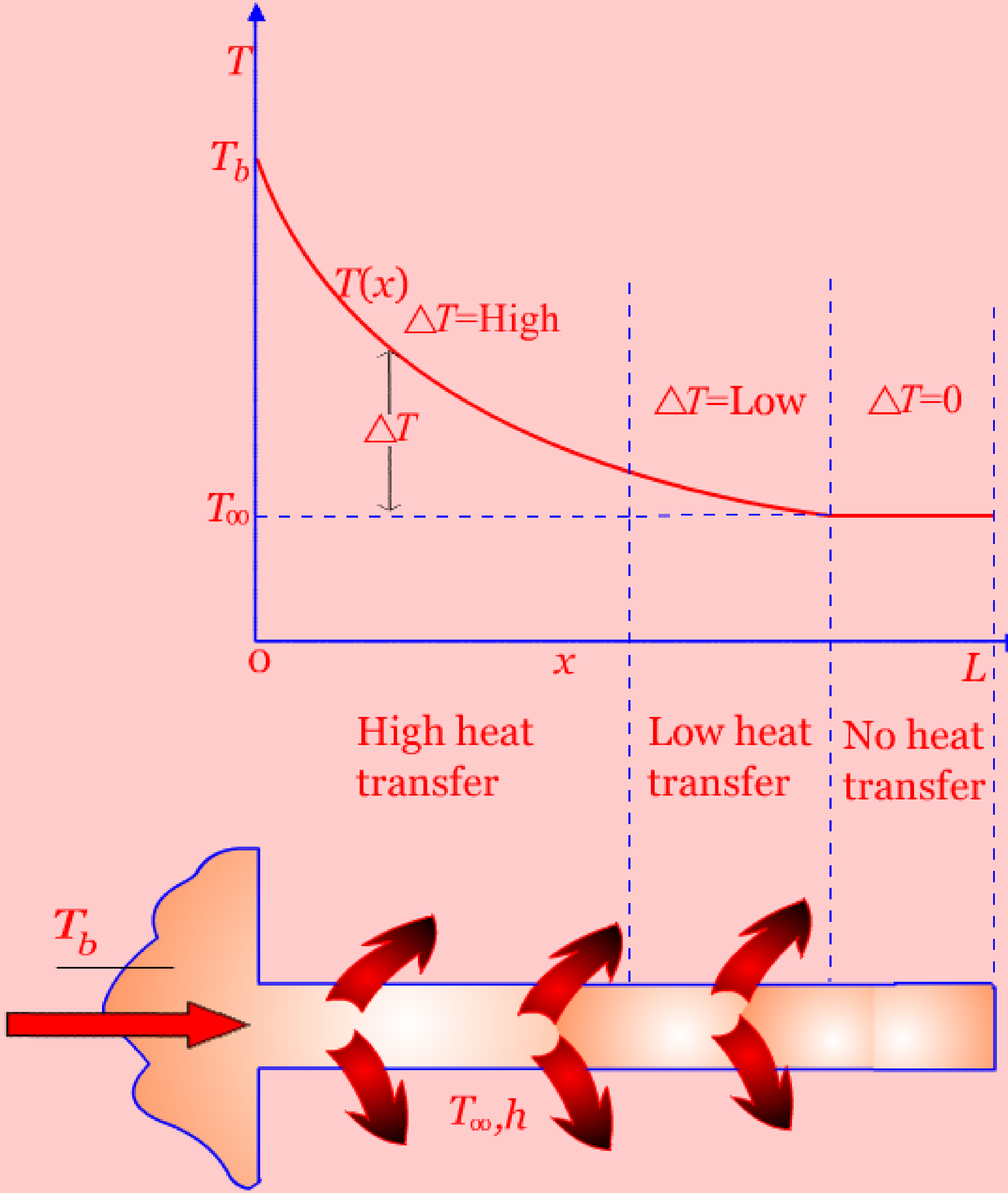


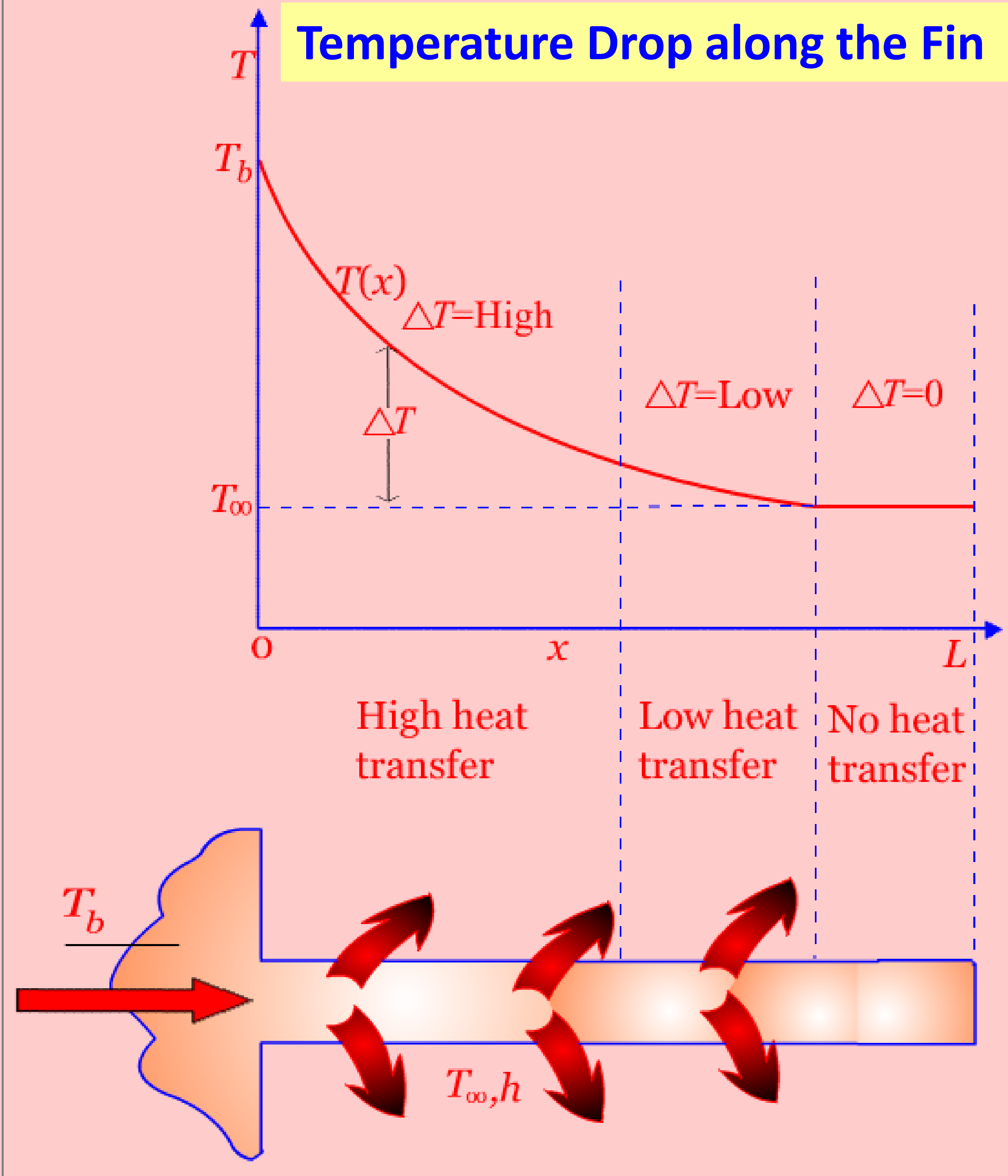
PROPER LENGTH OF THE FIN

Therefore, designing such an "extra long" fin is out of question since it results in material waste, excessive weight, and increased size and thus increased cost with no benefit in return

In fact, such a long fin will hurt performance since it will suppress fluid motion and thus reduce the convection heat transfer coefficient.

Therefore, fins that are so long that the temperature approaches the environment temperature cannot be recommended





To get a sense of proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fins under the same conditions.

The ratio of these two heat transfers is heat transfer ratio

$$\frac{Q_{fin\ with\ adiabatic\ tip}}{Q_{fin\ infinitely\ long}} = \frac{\theta_b\sqrt{hPkA_c}(TanhmL)}{\theta_b\sqrt{hPkA_c}}$$

$$\frac{Q_{fin\ with\ adiabatic\ tip}}{Q_{fin\ infinitely\ long}} = TanhmL$$

The variation of heat transfer from a fin relative to that from an infinitely long fin

Heat transfer from a fin increases with mL almost linearly at first, but the curve reaches a plateau later and reaches a value for the infinitely long fin at about $mL = 5$.

Therefore, a fin whose length is $L = 5/m$ can be considered to be an infinitely long fin.

Reducing the fin length by half in that case (from $mL = 5$ to $mL = 2.5$) causes a drop of just 1 percent in heat transfer.

We certainly would not hesitate sacrificing 1 percent in heat transfer performance in return for 50 percent reduction in the size and possibly the cost of the fin.

In practice, a fin length that corresponds to about $mL = 1$ will transfer 76.2 percent of the heat that can be transferred by an infinitely long fin, and thus it should offer a good compromise between heat transfer performance and the fin size

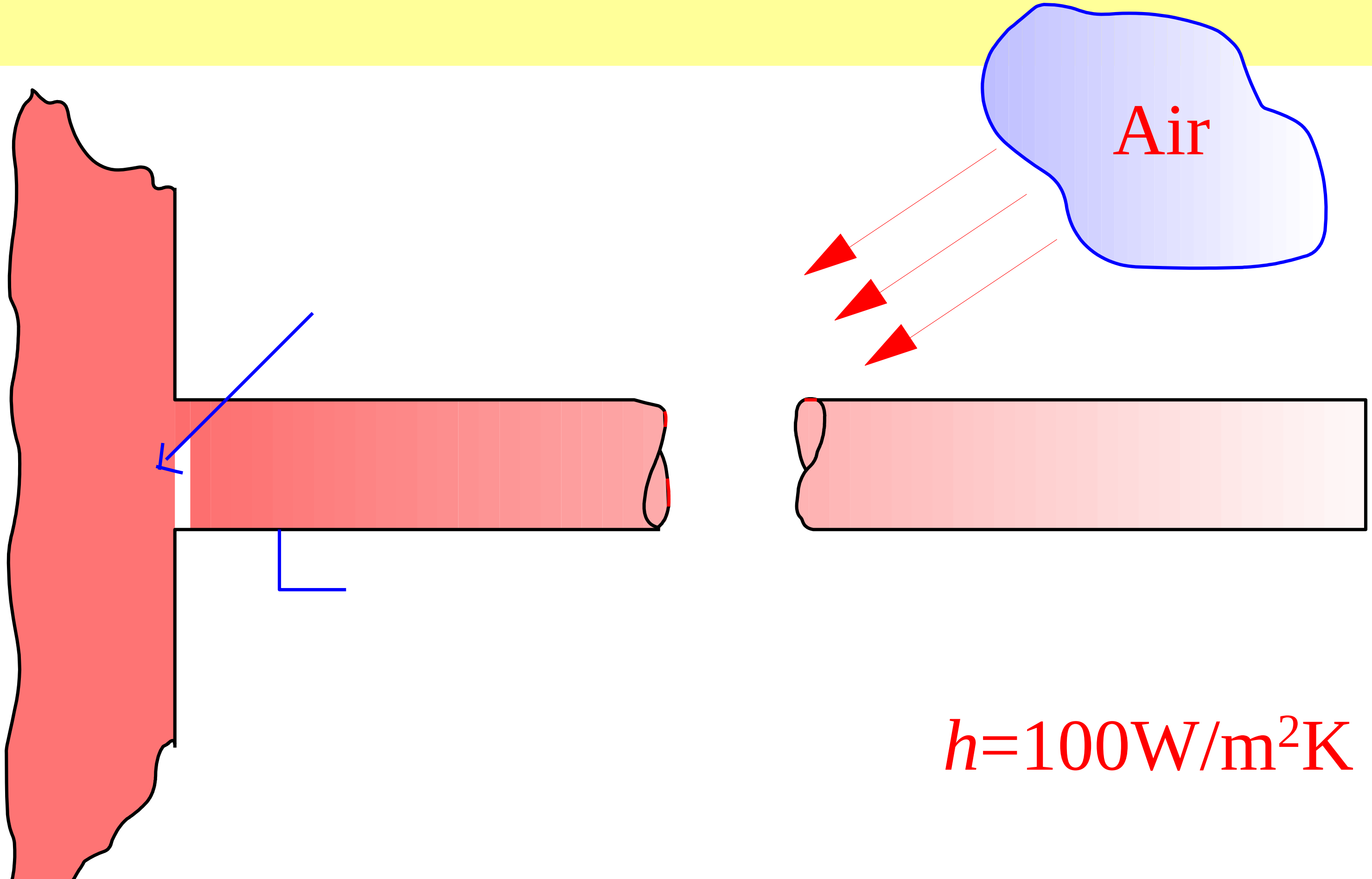
mL	$TanhmL$
0.1	0.1
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

$$\frac{Q_{fin\ with\ adiabatic\ tip}}{Q_{fin\ infinitely\ long}} = TanhmL$$

A very long rod 5 mm in diameter has one end maintained at 100℃. The surface of the rod is exposed to ambient air at 25℃ with a convection heat transfer coefficient of 100 W/m^2K.

- Determine the temperature distributions along rods constructed from pure copper, 2024 aluminium alloy and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
- Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss

Known : A long circular rod exposed to ambient air.



Find :

Temperature distribution and heat loss when rod is fabricated from copper, an aluminum alloy, or stainless steel.

How long rods must be to assume infinite length.

Assumptions:

- Steady state conditions
- One dimensional conduction along the rod
- Constant properties
- Negligible radiation exchange with surroundings
- Uniform heat transfer coefficient

Properties : At $[T = T_b + T_\infty/2 = 100 + 25/2 = 62.5℃ = 335\ K]$

Copper: $k = 398\ W/m.K$; Aluminium: $k = 180\ W/m.K$; Stainless steel, $k = 14\ W/m.K$

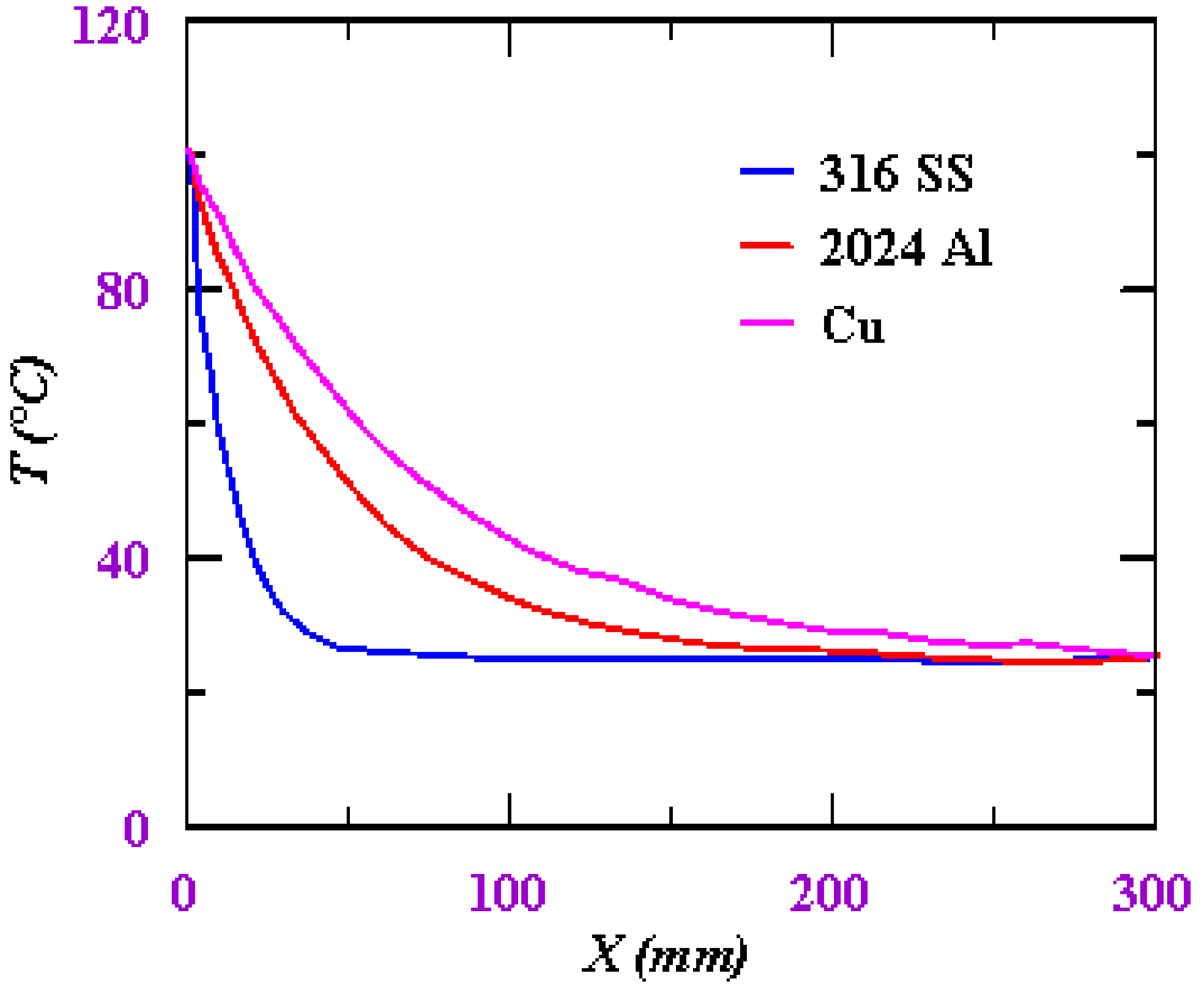
Analysis:

1. Subject to the assumption of an infinitely long fin, the temperature distributions are determined from equation,

$$T(x) - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$m^2 = \frac{hP}{kA_c}$$

$$q_f = \theta_b\sqrt{hPkA_c}$$



$$m^2 = \frac{hP}{kA_c} = \frac{h(\pi d)}{k\frac{\pi d^2}{4}} = \frac{4h}{kd}$$

$$m = \sqrt{\frac{4h}{kd}}$$

$$d = 5 \times 10^{-3}\ m$$

$$h = 100\ \frac{W}{m^2K}$$

Copper: $k = 398\ W/m.K$;
Aluminium: $k = 180\ W/m.K$;
Stainless steel, $k = 14\ W/m.K$

Copper: $m = 14.2\ m^{-1}$
Aluminium: $m = 21.2 m^{-1}$
Stainless steel, $m = 75.6 m^{-1}$

There is little additional heat transfer associated with extending the length of the rod much beyond 50, 200 and 300 mm, respectively, for stainless steel, aluminium alloy and copper.

$$q_f = \theta_b\sqrt{hPkA_c}$$

$$q_f = (T_b - T_\infty)\sqrt{hPkA_c}$$

$$q_{f,copper} = (100 - 25)\sqrt{100 \times \pi(5 \times 10^{-3})(398)\frac{\pi(5 \times 10^{-3})^2}{4}}$$

$d = 5 \times 10^{-3}\ m$

$h = 100\ \dfrac{W}{m^2K}$

$$q_{f,copper} = 8.3W$$
$$q_{f,aluminium} = 5.6\ W$$
$$q_{f,stainlesssteel} = 1.6\ W$$

$$m = \sqrt{\frac{4h}{kd}}$$

Copper: $k = 398\ W/m.K$;
Aluminium: $k = 180\ W/m.K$;
Stainless steel, $k = 14\ W/m.K$

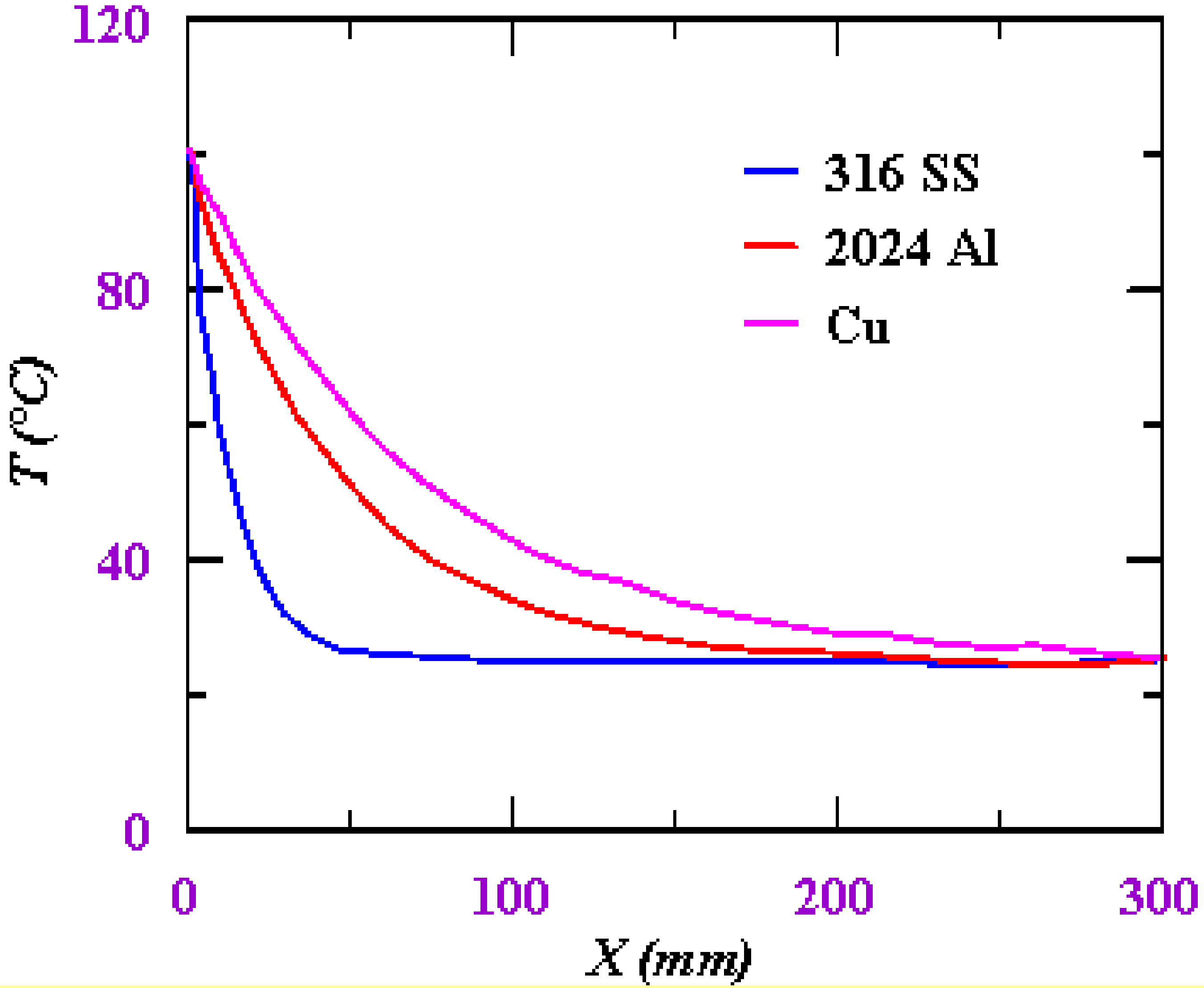
Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of the approximation may be made by comparing Equations $q_f = \theta_b\sqrt{hPkA_c}$ (infinitely long rod) and $q_f = \theta_b\sqrt{hPkA_c}TanhmL$ (adiabatic fin tip) to a satisfactory approximation, the expressions provide equivalent results if $mL \geq 2.65$. Hence, a rod may be assumed to be infinitely long if

$$L \geq L_\infty = \frac{2.65}{m} = \frac{2.65}{\sqrt{\frac{4h}{kd}}} = 2.65\sqrt{\frac{kd}{4h}}$$

$$L_{\infty,copper} = 2.65\sqrt{\frac{398(5 \times 10^{-3})}{4 \times 100}}$$

$L_{\infty,copper} = 0.19\ m$ $\quad L_{\infty,aluminium} = 0.13\ m$

$L_{\infty,stainless\ steel} = 0.04\ m$



Copper: $m = 14.2\ m^{-1}$
Aluminium: $m = 21.2 m^{-1}$
Stainless steel, $m = 75.6 m^{-1}$

$q_{f,copper} = 8.3 W$
$q_{f,aluminium} = 5.6\ W$
$q_{f,stainlesssteel} = 1.6\ W$

$L_{\infty,copper} = 0.19\ m$
$L_{\infty,aluminium} = 0.13\ m$
$L_{\infty,stainless\ steel} = 0.04\ m$

Fin heat transfer rate may accurately be predicted from the infinite fin approximation if $mL \geq 2.65$

However, if the infinite fin approximation is to accurately predict the temperature distribution $T(x)$, a larger value of mL would be required. This value may be inferred from $T(x) - T_\infty = (T_b - T_\infty)e^{-mx}$ and the requirement that the tip temperature be very close to the fluid temperature. Hence, if we require that $T(L) = T_\infty$, it follows that $mL \geq 2.65$, in which case L_∞ is 0.33,0.23 and 0.07 m for the copper aluminium alloy, and stainless steel, respectively.

FIN EFFICIENCY

Consider the surface of the plane wall at temperature T_b exposed to a medium at temperature T_∞.

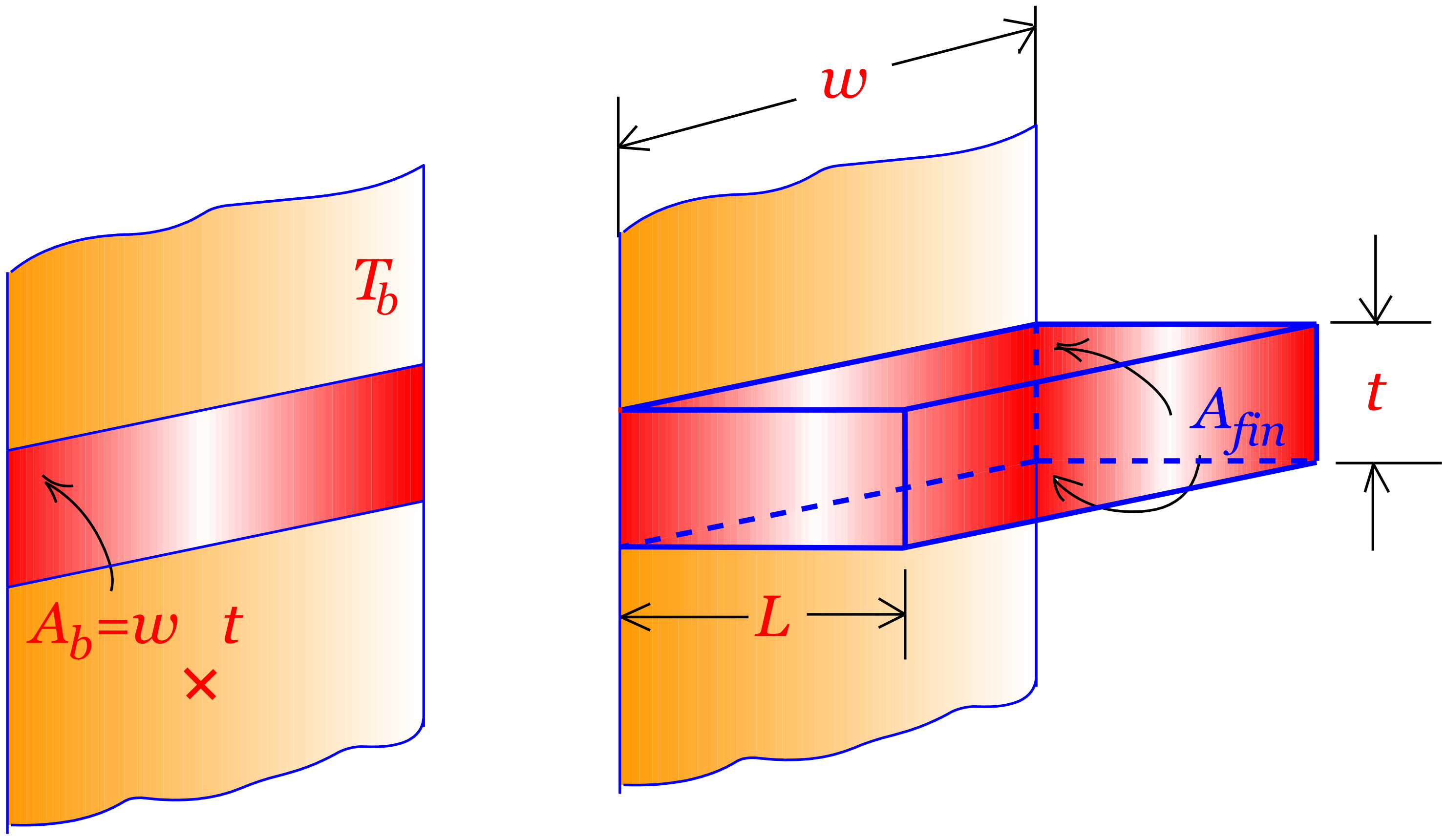
Heat is lost from the surface to the surrounding medium by convection with a heat transfer coefficient of h.

Neglecting radiation, heat transfer from a surface area A is expressed as

$$Q_{conv} = hA(T_b - T_\infty)$$

Now let us consider a fin of constant cross sectional area $A_c = A_b$ and length L that is attached to the surface with a perfect contact.

This time heat will flow from the surface to the fin by conduction. From the fin to the surrounding medium by convection with the same heat transfer coefficient h.



(a)Surface without fins

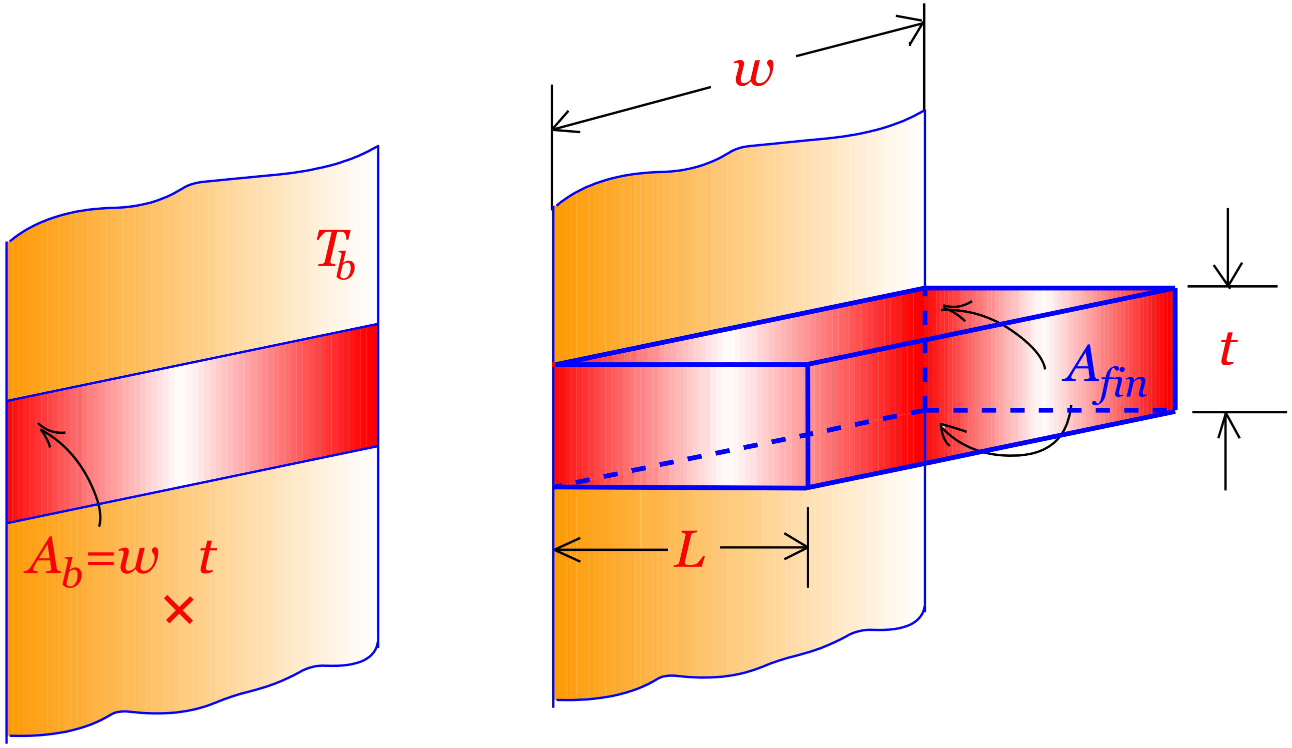
(b) Surface with a fin

$A_{fin} = 2 \times w \times L + w \times t$
$\cong 2 \times w \times L$

Fins Enhance Heat Transfer from a Surface by Enhancing Surface

FIN EFFICIENCY

- The temperature of the fin will be T_b at the fin base and gradually decrease toward the fin tip.
- Convection from the fin surface causes the temperature at any cross section to drop somewhat from the midsection toward the outer surfaces.
- However, the cross sectional area of the fins is usually very small, and thus the temperature at any cross section can be considered to be uniform.
- Also, the fin tip can be assumed for convenience and simplicity to be insulated by using the corrected length for the fin instead of the actual length.
- In the limiting case of zero thermal resistance or infinite thermal conductivity $(k \rightarrow \infty)$, the temperature of the fin will be uniform at the base value of T_b.



(a)Surface without fins

(b) Surface with a fin

$A_{fin} = 2 \times w \times L + w \times t$
$\cong 2 \times w \times L$

- The heat transfer from the fin will be maximum in this case and can be expressed as

$$Q_{conv} = hA_{fin}(T_b - T_\infty)$$

FIN EFFICIENCY

In the limiting case of zero thermal resistance or infinite thermal conductivity $(k \to \infty)$, the temperature of the fin will be uniform at the base value of T_b.

In reality, however, the temperature of the fin will drop along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference $T(x)$ toward the fin tip, as shown in Figure.

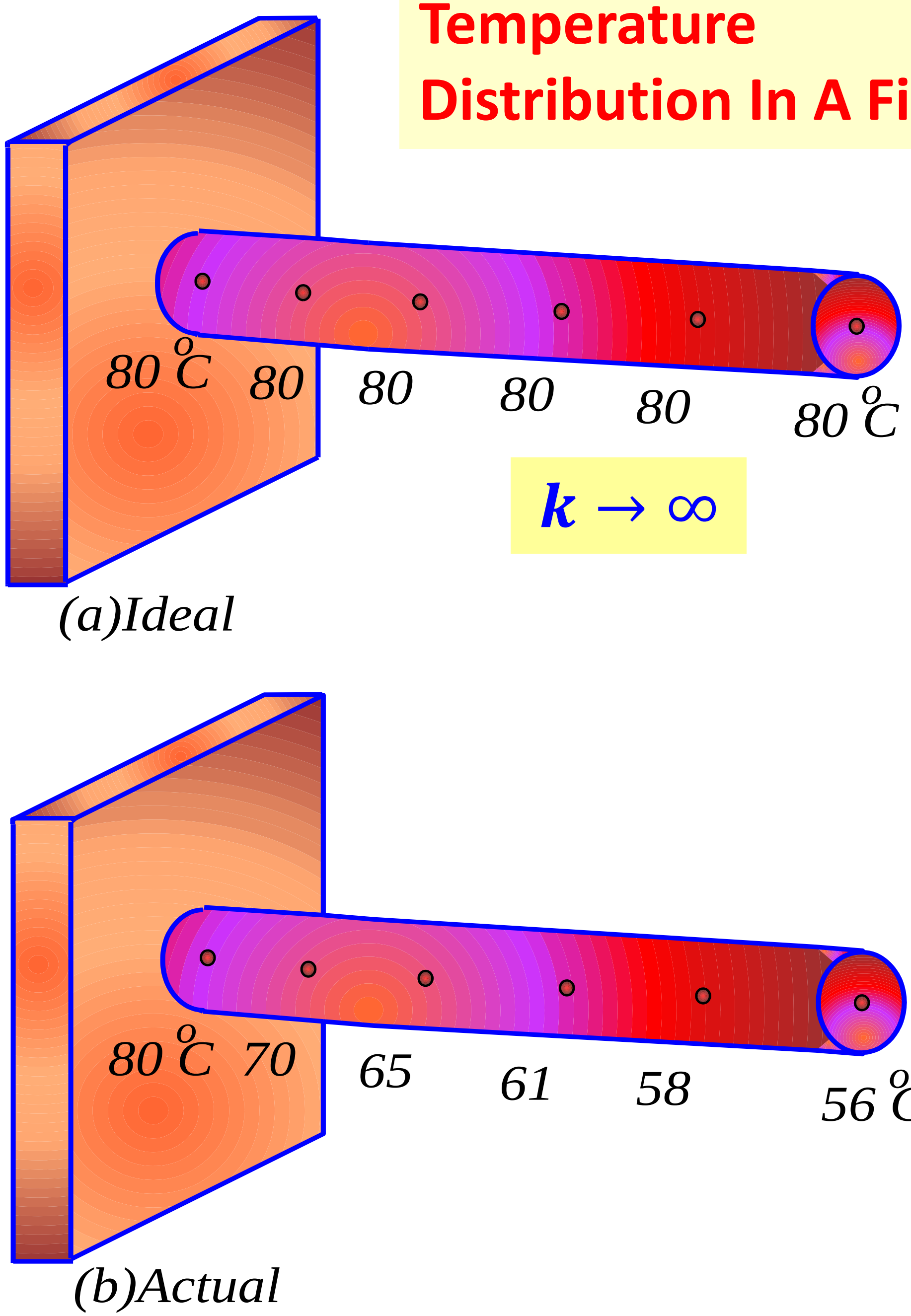
To account for the effect of this decrease in temperature on heat transfer, we define fin efficiency

$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}}$$

q_{fin} - Actual heat transfer rate from the fin

$q_{fin,max}$ - Ideal heat transfer rate from the fin if the entire fin were at the base temperature

Ideal And Actual Temperature Distribution In A Fin



$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}}$$

q_{fin} - Actual heat transfer rate from the fin

$q_{fin,max}$ - Ideal heat transfer rate from the fin if the entire fin were at the base temperature

This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of very long fins and fins with insulated tips, the fin efficiency can be expressed as

$$\eta_{long,fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{(T_b - T_\infty)\sqrt{hPkA_c}}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}}{hA_{fin}} = \frac{\sqrt{hPkA_c}}{hPL} = \frac{1}{L}\sqrt{\frac{kA_c}{hP}}$$

$$\eta_{long,fin} = \frac{1}{mL}$$

$$\eta_{insulated} = \frac{q_{fin}}{q_{fin,max}} = \frac{\theta_b\sqrt{hPkA_c}TanhmL}{hA_{fin}\theta_b} = \frac{\sqrt{hPkA_c}TanhmL}{hA_{fin}} = \frac{\sqrt{hPkA_c}TanhmL}{hPL} = \frac{TanhmL}{L}\sqrt{\frac{kA_c}{hP}}$$

$$\eta_{insulated} = \frac{TanhmL}{mL}$$

- An important consideration in the design of finned surfaces is the selection of the proper fin length *L.*
- Normally the longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.
- But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction.
- Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided.
- The efficiency of most fins used in practice is above 90 percent.

$$\eta_{long,fin} = \frac{1}{mL} \qquad m^2 = \frac{hP}{kA_c}$$

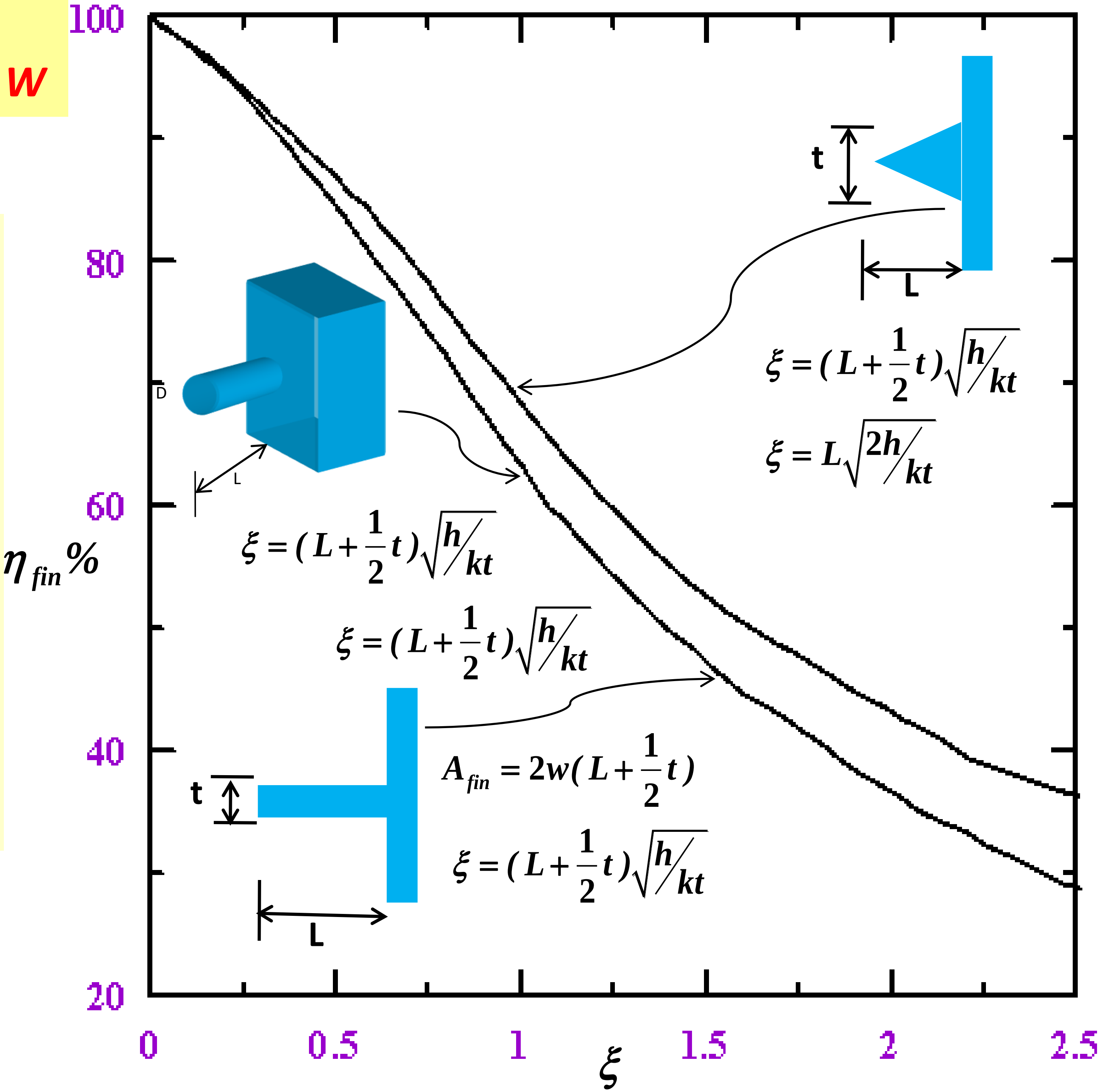
$$\eta_{insulated} = \frac{TanhmL}{mL}$$

mL	$TanhmL$
0.1	0.1
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

Efficiency Of Circular, Rectangular and Triangular Fins On A Plain Surface Of Width *W*

For most fins of constant thickness encountered in practice, the fin thickness t is too small relative to the fin length L , and thus the fin tip area is negligible.

Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

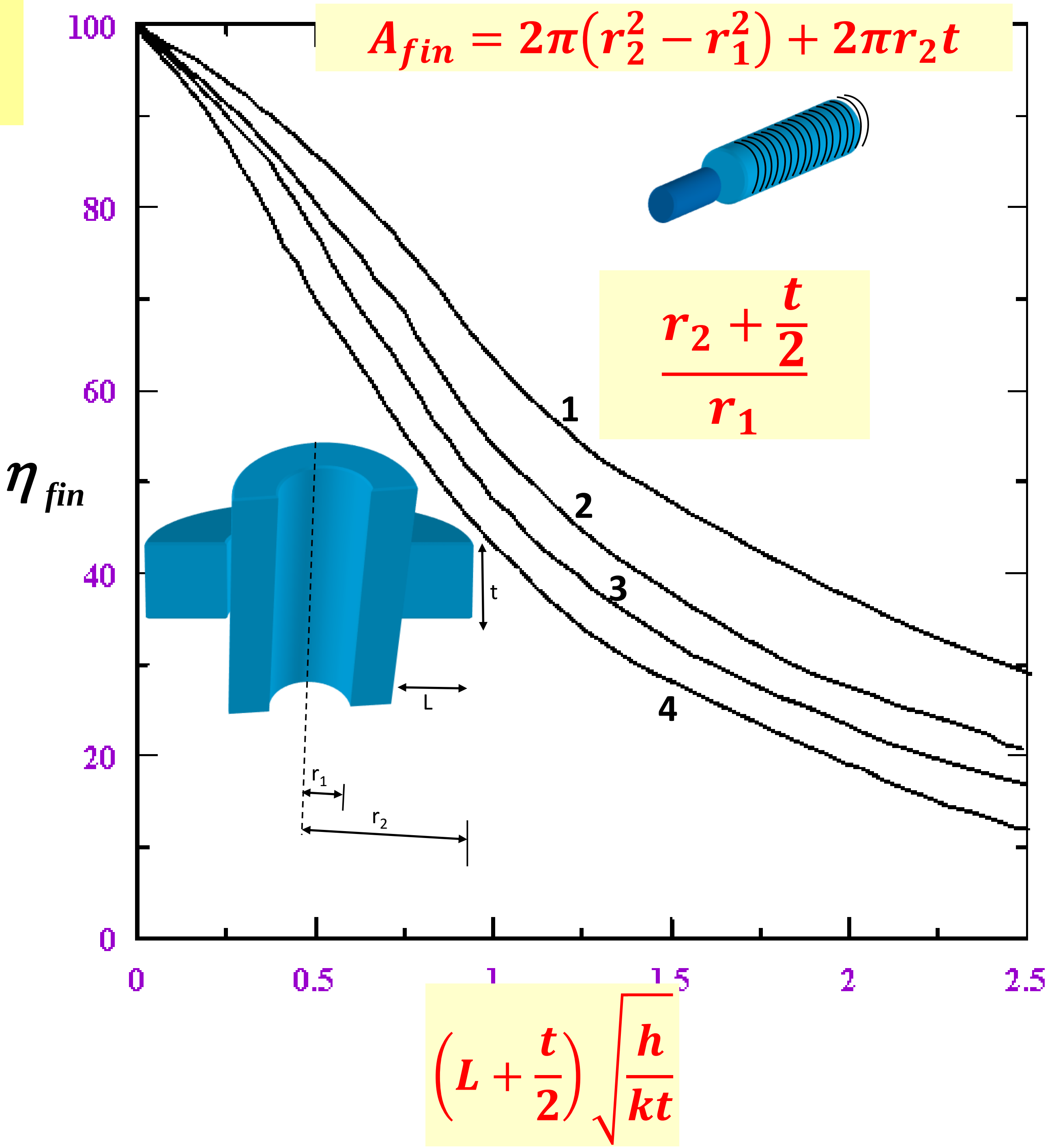


Efficiency Of Circular Fins Of Length *L* And Constant Thickness *T*

$$A_{fin} = 2\pi\left(r_2^2 - r_1^2\right) + 2\pi r_2 t$$

$$\left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}}$$

$$\frac{r_2 + \frac{t}{2}}{r_1}$$



FIN EFFECTIVENESS

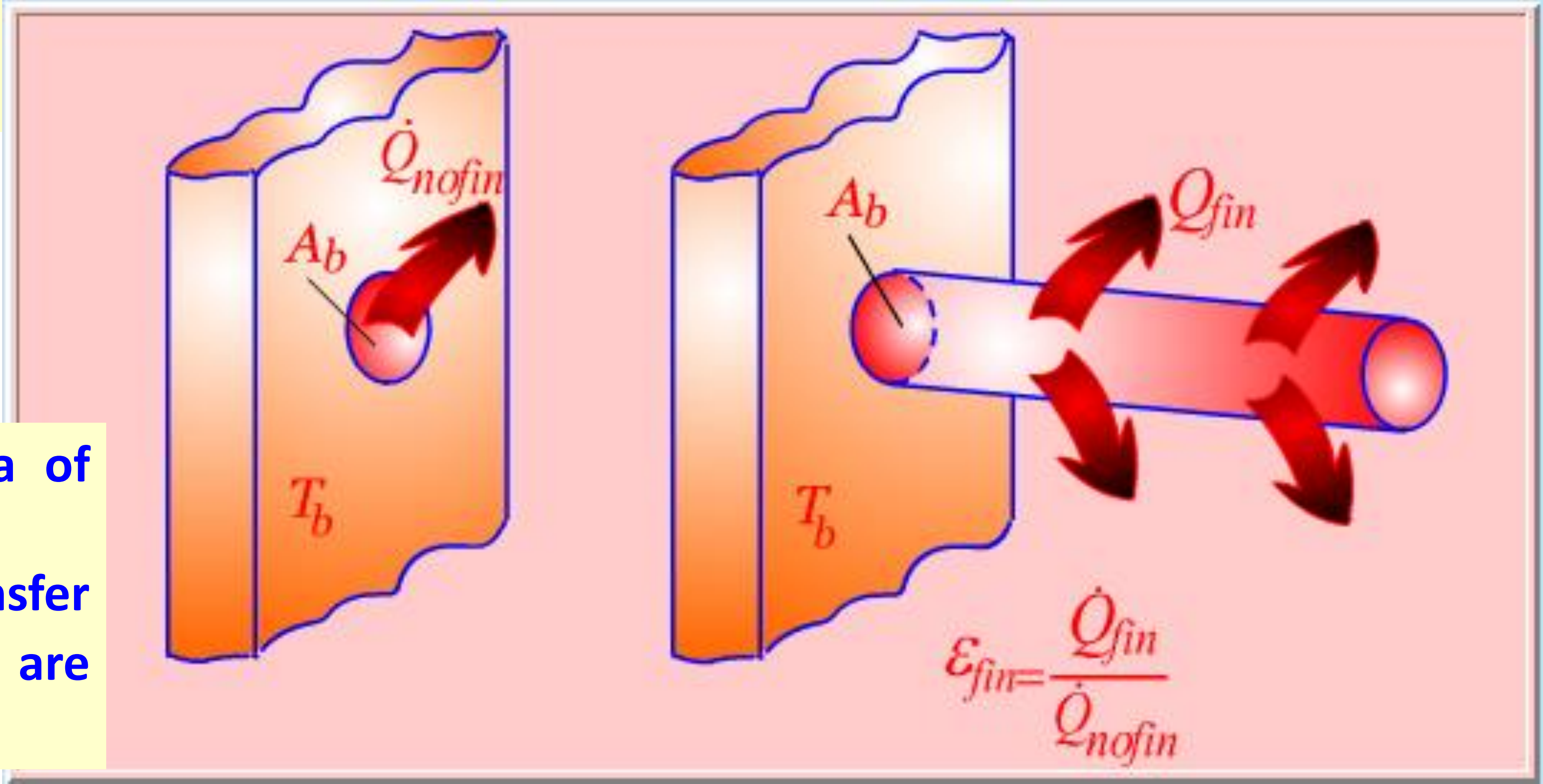
Fins are used to enhance heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins.

In fact, there is no assurance that adding fins on a surface will enhance heat transfer.

The performance of the fins is judged on the basis of enhancement of heat transfer relative to the no fin case.

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\ fin}}$$

$$\varepsilon_{fin} = \frac{q_{fin}}{hA_b(T_b - T_\infty)}$$

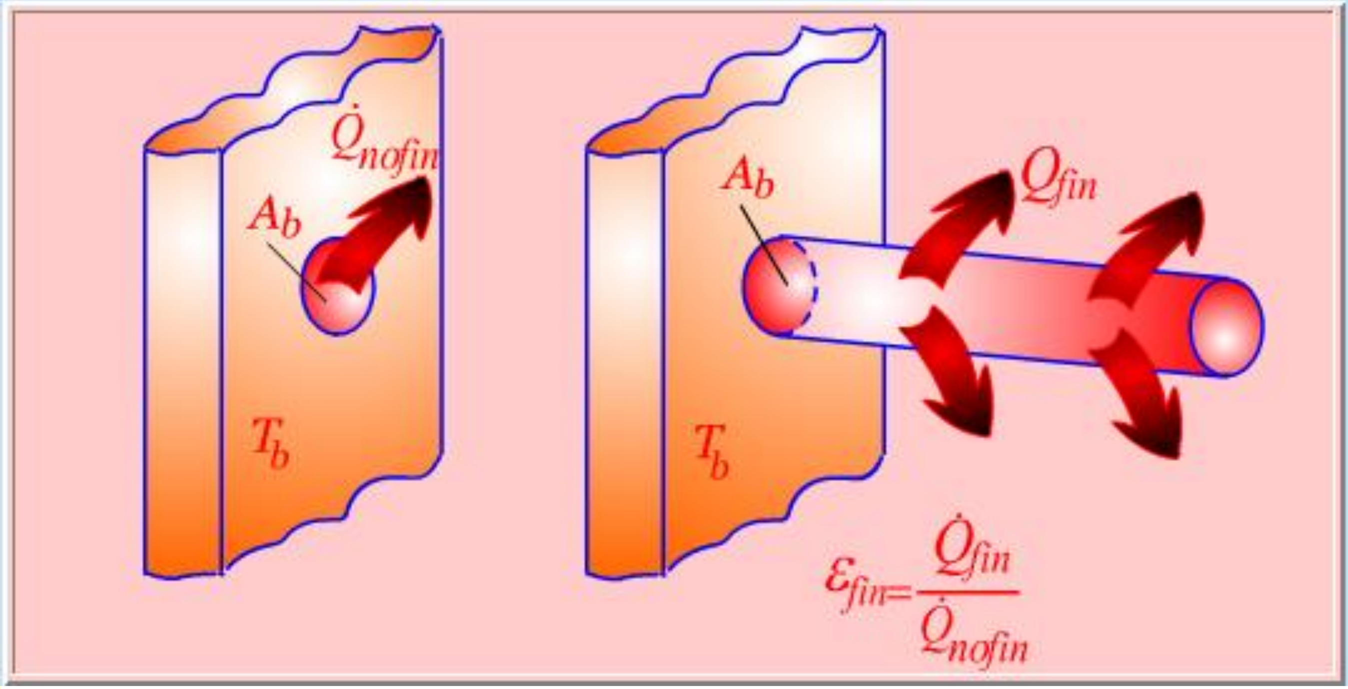


A_b - Cross sectional area of the fin at the base

$q_{no\ fin}$ - Rate of heat transfer from this area if no fins are attached to the surface.

The physical significance of effectiveness of fin can be summarized below

- An effectiveness of $\varepsilon_{fin} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium



- An effectiveness of $\varepsilon_{fin} < 1$ indicates that the fin actually acts as insulation, slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used.

- An effectiveness of $\varepsilon_{fin} > 1$ indicates that the fins are enhancing heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of maximizing effectiveness of a specified cost or minimizing cost for a desired effectiveness.

RELATION BETWEEN FIN EFFICIENCY AND FIN EFFECTIVENESS

The fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\,fin}} = \frac{q_{fin}}{hA_b(T_b - T_\infty)}$$

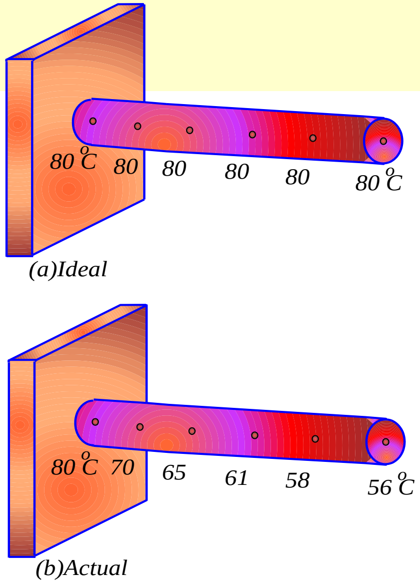
$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{q_{fin}}{hA_{fin}(T_b - T_\infty)}$$

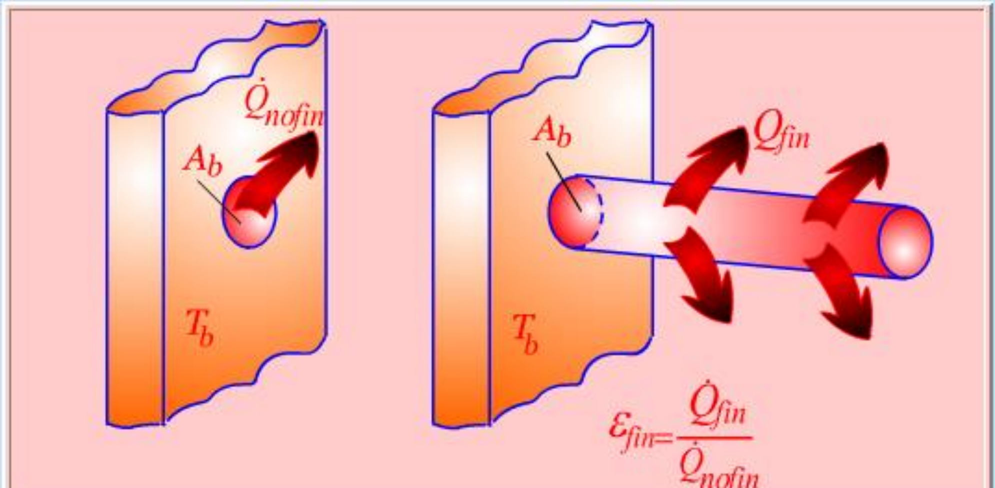
$$q_{fin} = \eta_{fin} h A_{fin}(T_b - T_\infty)$$

$$\varepsilon_{fin} = \frac{\eta_{fin} h A_{fin}(T_b - T_\infty)}{hA_b(T_b - T_\infty)}$$

$$\varepsilon_{fin} = \frac{\eta_{fin} A_{fin}}{A_b}$$

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.





FIN EFFECTIVENESS OF A SUFFICIENTLY LONG FIN WITH A UNIFORM CROSS SECTION

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\ fin}} = \frac{(T_b - T_\infty)\sqrt{hPkA_c}}{hA_b(T_b - T_\infty)}$$

$$\varepsilon_{fin} = \sqrt{\frac{kP}{hA_c}}$$

$$A_c = A_b$$

The thermal conductivity k of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminium, and iron being the most common ones. Perhaps the most widely used fins are made of aluminium because of its low cost and weight and its resistance to corrosion.

The ratio of the perimeter to the cross sectional area of the fin $\frac{P}{A_c}$ should be as high as possible. This criterion is satisfied by thin plate fins or slender pin fins

The use of fins is most effective in applications involving low convection heat transfer coefficient. Thus, the use of fins is more easily justified when the medium is a gas instead of a liquid and the heat transfer is by natural convection instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the gas side.

DETERMINATION OF THE RATE OF HEAT TRANSFER FROM A FINNED SURFACE

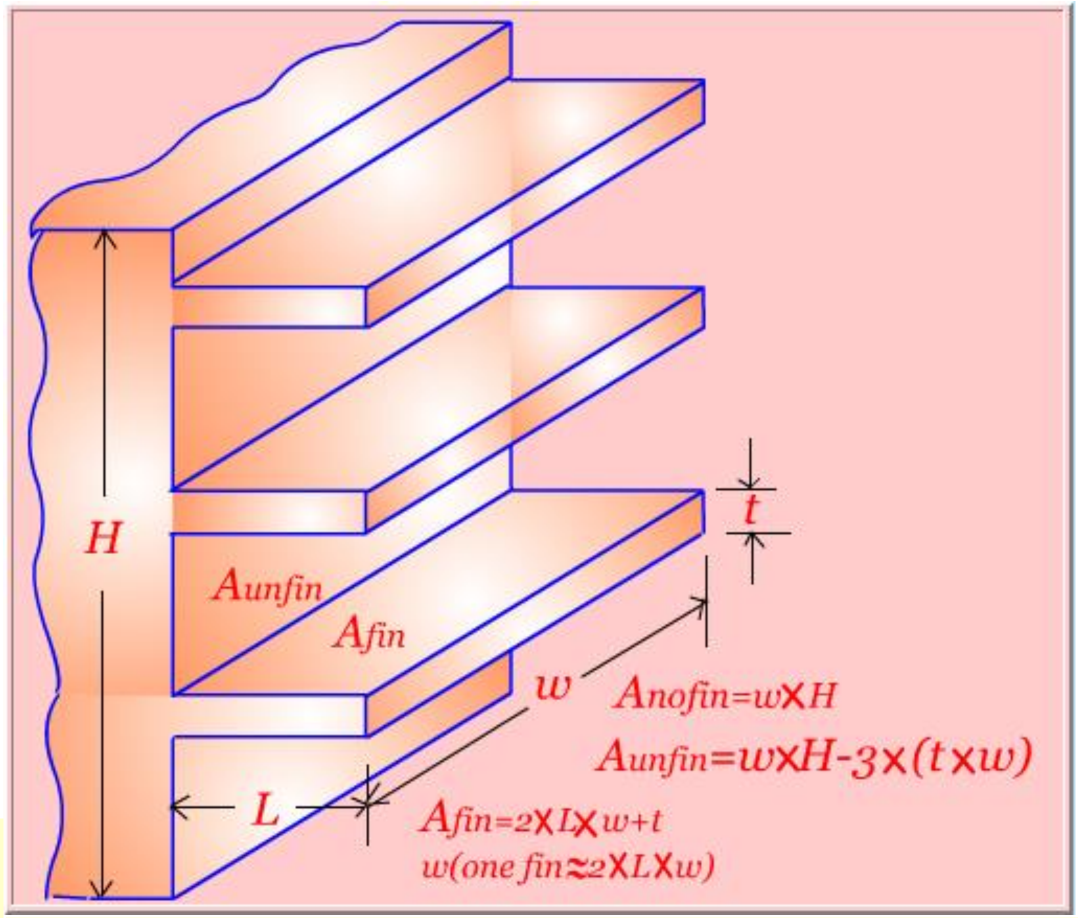
When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$q_{total,fin} = q_{unfin} + q_{fin}$$

$$q_{total,fin} = hA_{unfin}(T_b - T_\infty) + \eta_{fin}A_{fin}(T_b - T_\infty)$$

$$q_{total,fin} = h\left(A_{unfin} + \eta_{fin}A_{fin}\right)(T_b - T_\infty)$$

A_{fin} is the total surface area of all the fins on the surface, and
A_{unfin} is the area of the unfinned portion of the surface



Various Surface Areas Associated With A Rectangular Surface With Three Fins

OVERALL EFFECTIVENESS FOR A FINNED SURFACE

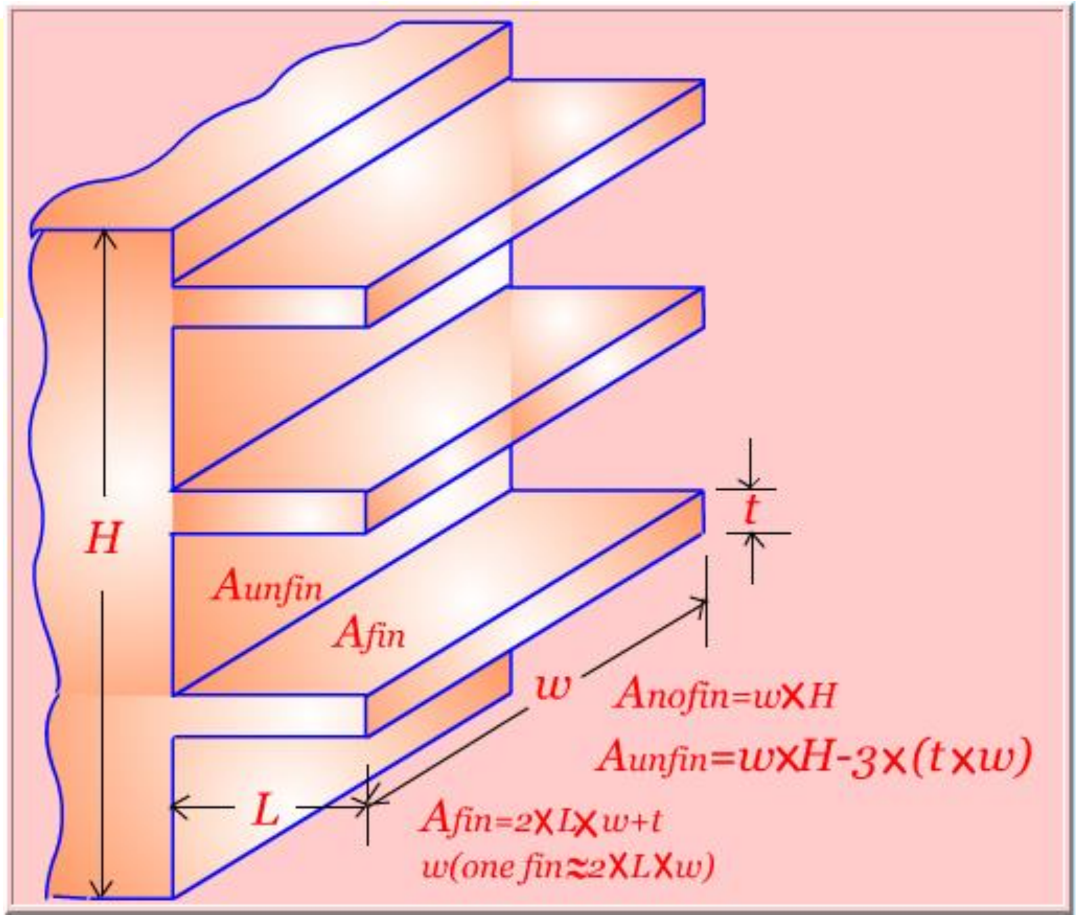
Overall effectiveness for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\,fin}} = \frac{h\left(A_{unfin} + \eta_{fin} A_{fin}\right)(T_b - T_\infty)}{h A_{nofin}(T_b - T_\infty)}$$

A_{nofin} is the area of the surface when there are no fins

A_{fin} is the total surface area of all the fins on the surface, and

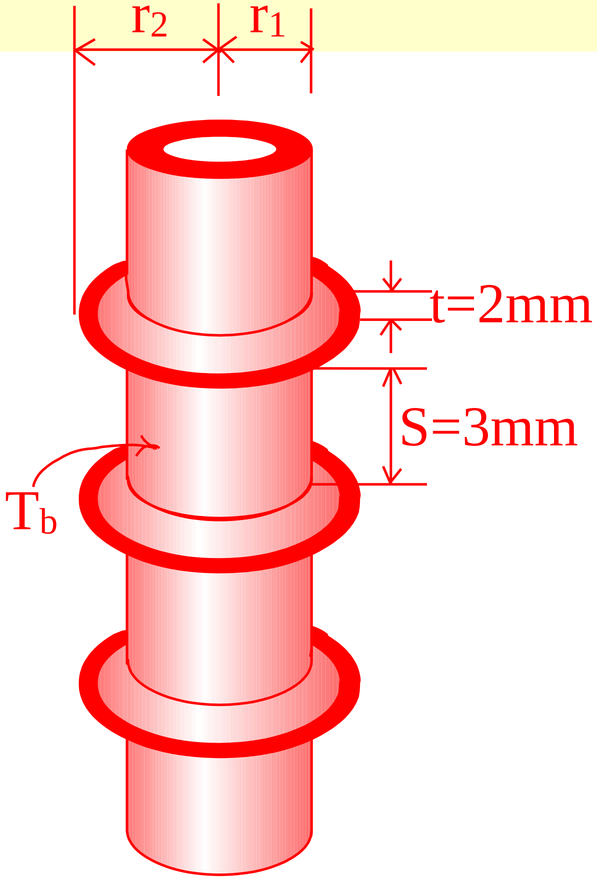
A_{unfin} is the area of the unfinned portion of the surface



Various Surface Areas Associated With A Rectangular Surface With Three Fins

Note that the overall fin effectiveness depends on the fin density (i.e. number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.

Steam in a heating system flows through tubes whose outer diameter is D_1= 3 cm and whose walls are maintained at a temperature of 125°C. Circular aluminium fins (k = 180 W/m°C) of outer diameter D_2= 6 cm and constant thickness t = 2mm are attached to the tube, as shown in the Figure. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at 27°C, with a combined heat transfer coefficient of h = 60 W/m^2 °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.



Known: Properties of the fin, ambient conditions, heat transfer coefficient, dimensions of the fin.

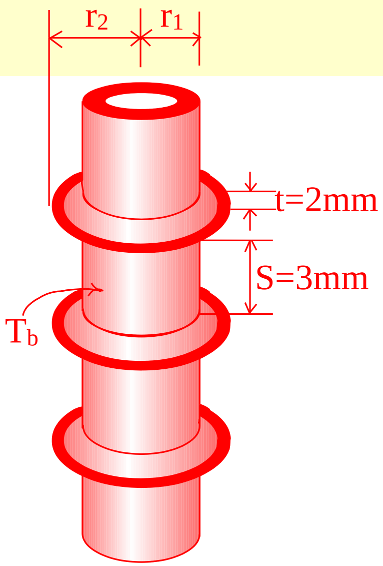
Find: To find the increase in heat transfer from the tube per meter of its length as a result of adding fins.

Assumptions:

1. Steady operating conditions exist.
2. The heat transfer coefficient is uniform over the entire fin surfaces.
3. Thermal conductivity is constant.
4. Heat transfer by radiation is negligible.

Analysis:

In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be,



$$A_{nofin} = \pi D_1 L = \pi(0.03)(1) = 0.0942\ m^2$$

$$Q_{nofin} = hA_{nofin}(T_b - T_\infty) = 60 \times 0.0942 \times (125 - 27)$$

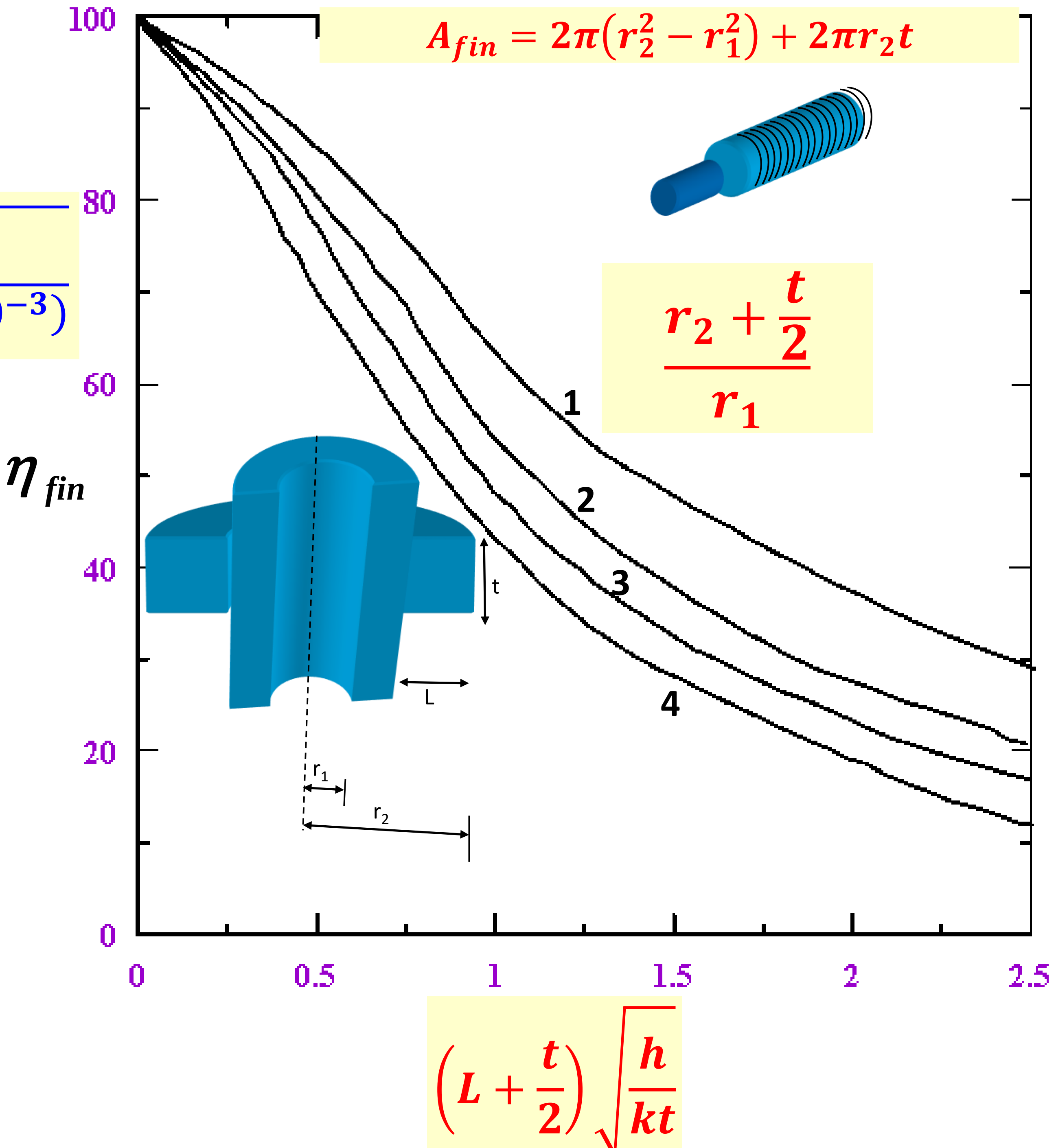
$$Q_{nofin} = 554\ W$$

$$\frac{r_2 + \frac{t}{2}}{r_1} = \frac{30 + \frac{2}{2}}{15} = 2.07$$

$$\left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.015 + \frac{2 \times 10^{-3}}{2}\right)\sqrt{\frac{60}{180(2 \times 10^{-3})}}$$

$$\left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = 0.21 \qquad \frac{r_2 + \frac{t}{2}}{r_1} = 2.07$$

From the graph, $\eta_{fin} = 0.95$



$$q_{total,fin} = h(A_{unfin} + \eta_{fin}A_{fin})(T_b - T_\infty)$$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$

$$\eta_{fin} = 0.95$$

$$A_{fin} = 2\pi\left((30\times10^{-3})^2 - (15\times10^{-3})^2\right) + 2\pi(30\times10^{-3})2\times10^{-3}$$

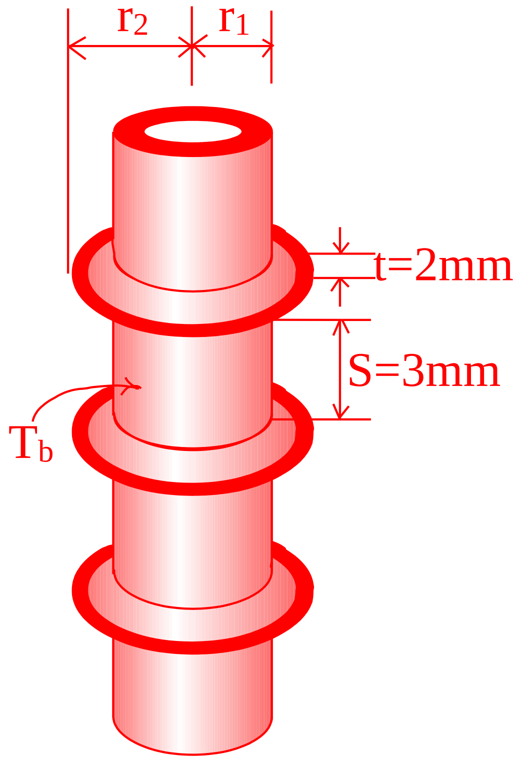
$$A_{fin} = 0.00462\ m^2$$

$$A_{unfin} = \pi D_1 S = \pi(30\times10^{-3})(3\times10^{-3}) = 0.000283m^2$$

$$A_{unfin} = 0.000283m^2$$

$$q_{total,fin} = 60\left(0.000283 + 0.95\,(0.00462)\right)(125 - 27)$$

$$q_{total,fin} = 27.46\ W$$



Noting that there are 200 fins and thus 200 inter-fin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$Q_{total,fin} = 27.46\times200 = 5492\ W$$

$$Q_{nofin} = 554\ W$$

Therefore, the increase in the heat transfer from the tube per meter of its length as a result of the addition of fins is

$$Q_{increase} = Q_{total,fin} - Q_{no,fin} = 5492\ - 554 = 4938\ W$$

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\ fin}} = \frac{5492}{554} = 9.91$$

$$Q_{total,fin} = 5492\ W$$

$$Q_{nofin} = 554\ W$$

Comments:

The efficiency of the finned tube is 95 %

The overall effectiveness of the finned tube is 9.91

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of the finned surface.