ME 311: Microprocessors and Automatic Control

Basics of Boolean Logic
K-map fundamentals

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Identities of Boolean algebra

Operations with 0 and 1

$X+0 = X$, $X+1=1$, $X\cdot 1=X$, $X\cdot 0=0$

Indempotent theorem

$X+X = X$ $\quad$ $X\cdot X = X$

Involution theorem

$(X')' = X$

Complementary

$X+Y = Y+X$, $\quad$ $X\cdot Y = Y\cdot X$

Identities of Boolean algebra

- Associative

$(X+Y)+Z = X+(Y+Z) = X+Y+Z$

- Distributive

$X \cdot (Y+Z) = X \cdot Y + X \cdot Z$

$X+(Y \cdot Z) = (X+Y) \cdot (X+Z)$

- Simplification

$X \cdot Y+X \cdot Y'=X$ $\quad X+X \cdot Y= X$ $\quad (X+Y') \cdot Y =X \cdot Y$

$(X+Y) \cdot (X+Y')=X$ $\quad X \cdot (X+Y) = X$

$(X \cdot Y')+Y = X+Y$

Identities of Boolean algebra

- DeMorgan's Law

$(X+Y+Z+\ldots)' = X'Y'Z'\ldots$

$(XYZ\ldots)' = (X'+Y'+Z'+\ldots)$

- Factoring

$(X+Y)(X'+Z) = XZ + X'Y$

$XY+YZ+X'Z=XY+X'Z$

Karnaugh Maps (K-map)

- Templates for K maps

B \ A	0	1
0	0	2
1	1	3

				A	A
	C \ AB	00	01	11	10
	0	0	2	6	4
C	1	1	3	7	5
			B	B	

				P	P
	Q \ PZ	00	01	11	10
	0	0	0	1	0
Q	1	0	1	1	1
			Z	Z	

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Karnaugh Maps (K-map)

- Example

Given truth table ➔ Draw K map ➔ Write expression by circling Terms with adjacent 1s.

B \ A	0	1
0	0	2
1	1	3

A	B	R
0	0	1
0	1	1
1	0	0
1	1	0

B \ A	0	1
0	1	0
1	1	0

Basically think of sum of products Method:
For circled 1s see that B is Having value 0 and 1. Thus B will not show up in expression Corresponding to this pair. What Will show up? See A = 0 for this Pair so A′ will come up.
There are no other 1s so R = A′

Sum of products way
R = A′B′+A′B
= A′(B′+B) = A′

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Karnaugh Maps (K-map)

- Examples 3 input variables

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

Q	P	Z	R
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Given truth table ➔ Draw K map ➔ Write expression by circling Terms with adjacent 1s.

Q \ PZ	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Expression: Watch each pairs of 1s
R = PZ + QZ + QP

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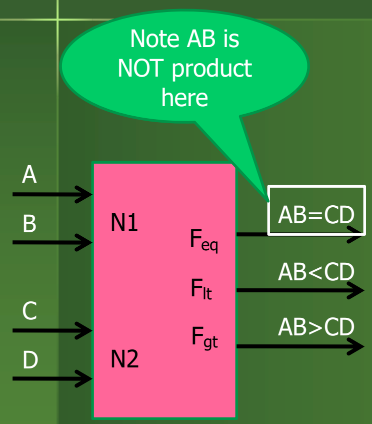
Comparator

- Two bit comparator



A	B	C	D	Feq	Flt	Fgt
0	0	0	0	1	0	0
		0	1	0	1	0
		1	0	0	1	0
		1	1	0	1	0
0	1	0	0	0	0	1
		0	1	1	0	0
		1	0	0	1	0
		1	1	0	1	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	1	0	0
		1	1	0	1	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	1	0	0

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Comparator

- K maps: expressions

K map for Feq

CD \ AB	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	0	1

K map for Flt

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	1
10	1	1	0	0

K map for Fgt

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

$$F_{eq} = A'B'C'D' + A'BC'D + AB'CD' + ABCD$$

$$F_{lt} = A'B'D + B'CD + A'C$$

$$F_{gt} = AC' + ABD' + BC'D'$$

Diagonal entries: XOR gates

Example K-map

- Full adder truth table 2.39 (Katz book)
- Corresponding k-map fig 2.45 (pg 72 katz book)

Example K-map

- Full adder truth table 2.40 ()
- Corresponding k-map fig 2.46 (pg 72 katz)

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Example K-map

- Another K map fig 2.47

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Example K-map

- Another K map function of four variables
- Pg 73

Addition vs Binary OR

- In binary OR say A + B we represent OR by + symbol (remember A and B are binary numbers 0 or 1)
- Addition of two binary numbers say 0010 + 1010 we may use addition + symbol again however, its very different from “bitwise OR” of these two numbers
- Q: how? Why we got to follow different rules for addition? And why Boolean identities (1+1 = 1) is not valid in addition operation.

Addition vs Binary OR

- Q: What is fundamental definition of addition
- Just look at fundamentals of representation of multidigit numbers in decimal or any system and see rules of addition
- Example