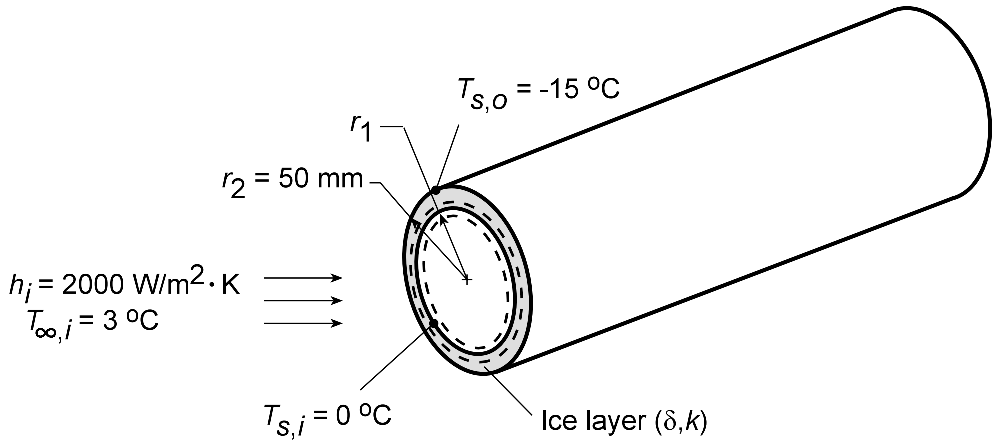
PROBLEM 3.51

KNOWN: Pipe wall temperature and convection conditions associated with water flow through the pipe and ice layer formation on the inner surface.

FIND: Ice layer thickness δ.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Negligible pipe wall thermal resistance, (3) negligible ice/wall contact resistance, (4) Constant k.

PROPERTIES: *Table A.3*, Ice (T = 265 K): k ≈ 1.94 W/m·K.

ANALYSIS: Performing an energy balance for a control surface about the ice/water interface, it follows that, for a unit length of pipe,

$$q'_{conv} = q'_{cond}$$

$$h_i\left(2\pi r_1\right)\left(T_{\infty,i} - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\ln\left(r_2/r_1\right)/2\pi k}$$

Dividing both sides of the equation by r_2,

$$\frac{\ln\left(r_2/r_1\right)}{\left(r_2/r_1\right)} = \frac{k}{h_i r_2} \times \frac{T_{s,i} - T_{s,o}}{T_{\infty,i} - T_{s,i}} = \frac{1.94\,\mathrm{W/m\cdot K}}{\left(2000\,\mathrm{W/m^2\cdot K}\right)\left(0.05\,\mathrm{m}\right)} \times \frac{15^\circ\mathrm{C}}{3^\circ\mathrm{C}} = 0.097$$

The equation is satisfied by $r_2/r_1 = 1.114$, in which case $r_1 = 0.050\ \mathrm{m}/1.114 = 0.045\ \mathrm{m}$, and the ice layer thickness is

$$\delta = r_2 - r_1 = 0.005\,\mathrm{m} = 5\,\mathrm{mm}$$ <

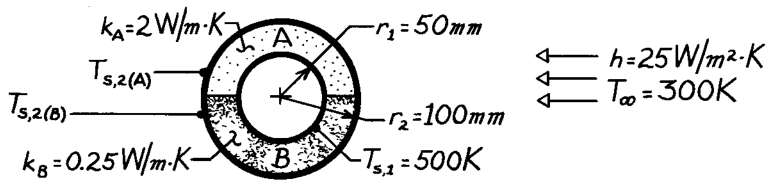
COMMENTS: With no flow, $h_i \to 0$, in which case $r_1 \to 0$ and complete blockage could occur. The pipe should be insulated.

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



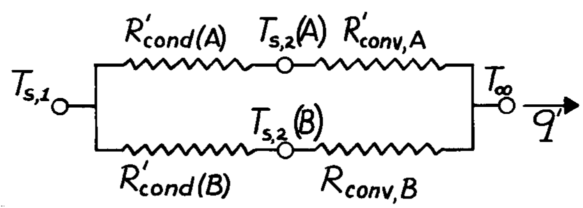
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$$R'_{conv,A} = R'_{conv,B} = 1/\pi\, r_2 h$$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\, k_A}$$

$$R'_{cond(\mathbf{B})} = \frac{\ln(r_2/r_i)}{\pi\, k_B}$$



<

The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.1273\ \text{m}\cdot\text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \qquad R'_{cond(B)} = 8\ R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500-300)\text{K}}{(0.1103+0.1273)\text{m}\cdot\text{K/W}} + \frac{(500-300)\text{K}}{(0.8825+0.1273)\text{m}\cdot\text{K/W}} = (842 + 198)\ \text{W/m} = 1040\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 407\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 325\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = (T_{s,1} - T_\infty)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m}\cdot\text{K/W}.$

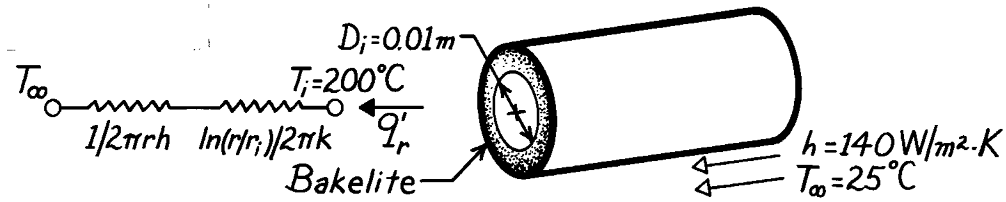
Hence $q' = (500-300)\text{K}/0.1923\ \text{m}\cdot\text{K/W} = 1040\ \text{W/m}.$

PROBLEM 3.53

KNOWN: Surface temperature of a circular rod coated with bakelite and adjoining fluid conditions.

FIND: (a) Critical insulation radius, (b) Heat transfer per unit length for bare rod and for insulation at critical radius, (c) Insulation thickness needed for 25% heat rate reduction.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation and contact resistance.

PROPERTIES: *Table A-3,* Bakelite (300K): k = 1.4 W/m·K.

ANALYSIS: (a) From Example 3.4, the critical radius is

$$r_{cr} = \frac{k}{h} = \frac{1.4\ \text{W/m}\cdot\text{K}}{140\ \text{W/m}^2\cdot\text{K}} = 0.01\text{m}. \qquad <$$

(b) For the bare rod,

$$q' = h\left(\pi\ D_i\right)\ \left(T_i - T_\infty\right)$$

$$q' = 140\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\pi\times 0.01\text{m}\right)\ \left(200-25\right)^\circ\text{C} = 770\ \text{W/m} \qquad <$$

For the critical insulation thickness,

$$q' = \frac{T_i - T_\infty}{\dfrac{1}{2\pi\ r_{cr}h} + \dfrac{\ln\left(r_{cr}/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\dfrac{1}{2\pi\times\left(0.01\text{m}\right)\times 140\ \text{W/m}^2\cdot\text{K}} + \dfrac{\ln\left(0.01\text{m}/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}}$$

$$q' = \frac{175^\circ\text{C}}{\left(0.1137+0.0788\right)\ \text{m}\cdot\text{K/W}} = 909\ \text{W/m} \qquad <$$

(c) The insulation thickness needed to reduce the heat rate to 577 W/m is obtained from

$$q' = \frac{T_i - T_\infty}{\dfrac{1}{2\pi\ rh} + \dfrac{\ln\left(r/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\dfrac{1}{2\pi\left(r\right)140\ \text{W/m}^2\cdot\text{K}} + \dfrac{\ln\left(r/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}} = 577\frac{\text{W}}{\text{m}}$$

From a trial-and-error solution, find

$$r \approx 0.06\ \text{m}.$$

The desired insulation thickness is then

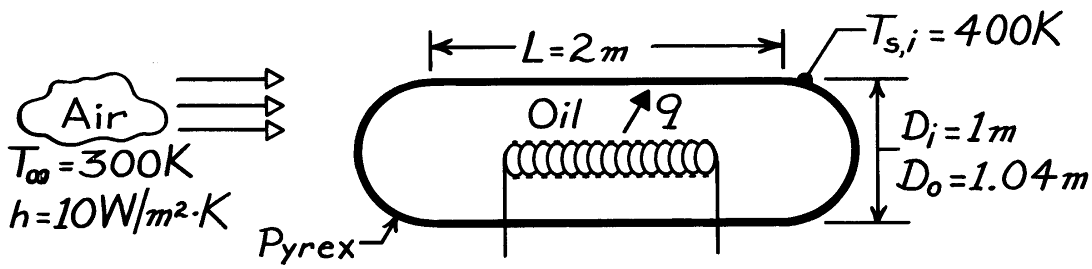
$$\delta = \left(r - r_i\right) \approx \left(0.06 - 0.005\right)\text{m} = 55\ \text{mm}. \qquad <$$

PROBLEM 3.54

KNOWN: Geometry of an oil storage tank. Temperature of stored oil and environmental conditions.

FIND: Heater power required to maintain a prescribed inner surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial direction, (3) Constant properties, (4) Negligible radiation.

PROPERTIES: *Table A-3*, Pyrex (300K): k = 1.4 W/m·K.

ANALYSIS: The rate at which heat must be supplied is equal to the loss through the cylindrical and hemispherical sections. Hence,

$$q = q_{cyl} + 2q_{hemi} = q_{cyl} + q_{spher}$$

or, from Eqs. 3.28 and 3.36,

$$q = \frac{T_{s,i} - T_\infty}{\dfrac{\ln(D_o/D_i)}{2\pi\ Lk} + \dfrac{1}{\pi\ D_o Lh}} + \frac{T_{s,i} - T_\infty}{\dfrac{1}{2\pi\ k}\left[\dfrac{1}{D_i} - \dfrac{1}{D_o}\right] + \dfrac{1}{\pi D_o^2 h}}$$

$$q = \frac{(400-300)\text{K}}{\dfrac{\ln 1.04}{2\pi(2\text{m})1.4\ \text{W/m}\cdot\text{K}} + \dfrac{1}{\pi(1.04\text{m})2\text{m}\left(10\ \text{W/m}^2\cdot\text{K}\right)}}$$

$$+ \frac{(400-300)\text{K}}{\dfrac{1}{2\pi(1.4\ \text{W/m}\cdot\text{K})}(1-0.962)\text{m}^{-1} + \dfrac{1}{\pi(1.04\text{m})^2 10\ \text{W/m}^2\cdot\text{K}}}$$

$$q = \frac{100\text{K}}{2.23\times10^{-3}\ \text{K/W} + 15.30\times10^{-3}\ \text{K/W}} + \frac{100\text{K}}{4.32\times10^{-3}\ \text{K/W} + 29.43\times10^{-3}}$$

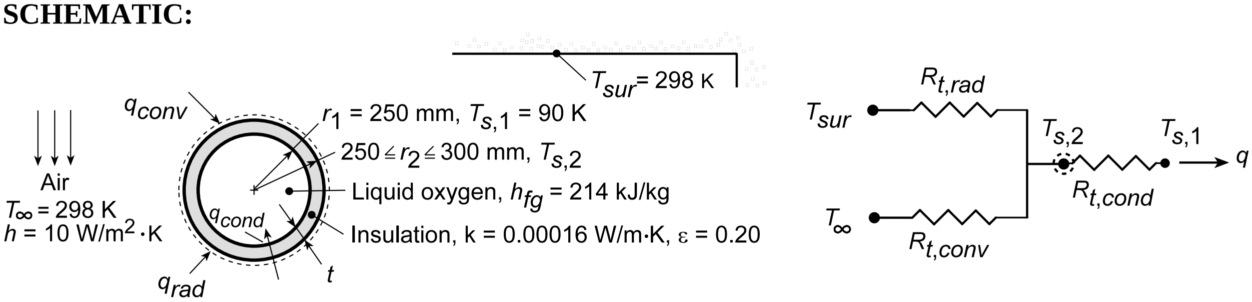
$$q = 5705\text{W} + 2963\text{W} = 8668\text{W}.$$ <

PROBLEM 3.55

KNOWN: Diameter of a spherical container used to store liquid oxygen and properties of insulating material. Environmental conditions.

FIND: (a) Reduction in evaporative oxygen loss associated with a prescribed insulation thickness, (b) Effect of insulation thickness on evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Negligible conduction resistance of container wall and contact resistance between wall and insulation, (3) Container wall at boiling point of liquid oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the insulation, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that $q_{conv} + q_{rad} = q_{cond} = q$. Hence,

$$\frac{T_\infty - T_{s,2}}{R_{t,conv}} + \frac{T_{sur} - T_{s,2}}{R_{t,rad}} = \frac{T_{s,2} - T_{s,1}}{R_{t,cond}} = q \qquad (1)$$

where $R_{t,conv} = \left(4\pi r_2^2 h\right)^{-1}$, $R_{t,rad} = \left(4\pi r_2^2 h_r\right)^{-1}$, $R_{t,cond} = (1/4\pi k)\left[(1/r_1) - (1/r_2)\right]$, and, from Eq. 1.9, the radiation coefficient is $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. With t = 10 mm ($r_2$ = 260 mm), ε = 0.2 and $T_\infty = T_{sur}$ = 298 K, an iterative solution of the energy balance equation yields $T_{s,2} \approx 297.7$ K, where $R_{t,conv}$ = 0.118 K/W, $R_{t,rad}$ = 0.982 K/W and $R_{t,cond}$ = 76.5 K/W. With the insulation, it follows that the heat gain is

$q_w \approx 2.72$ W

Without the insulation, the heat gain is

$$q_{wo} = \frac{T_\infty - T_{s,1}}{R_{t,conv}} + \frac{T_{sur} - T_{s,1}}{R_{t,rad}}$$

where, with $r_2 = r_1$, $T_{s,1}$ = 90 K, $R_{t,conv}$ = 0.127 K/W and $R_{t,rad}$ = 3.14 K/W. Hence,

q_{wo} = 1702 W

With the oxygen mass evaporation rate given by $\dot{m} = q/h_{fg}$, the percent reduction in evaporated oxygen is

$$\%\ \text{Reduction} = \frac{\dot{m}_{wo} - \dot{m}_w}{\dot{m}_{wo}} \times 100\% = \frac{q_{wo} - q_w}{q_{wo}} \times 100\%$$

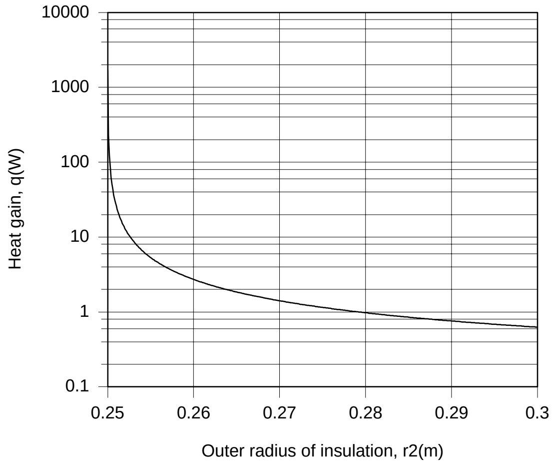
Hence,

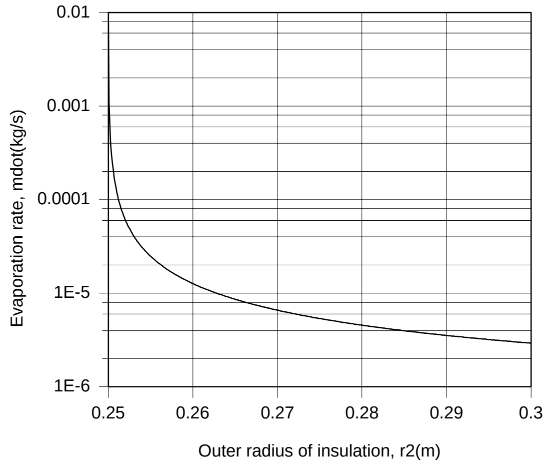
$$\%\ \text{Reduction} = \frac{(1702 - 2.7)\,\text{W}}{1702\,\text{W}} \times 100\% = 99.8\% \qquad <$$

Continued...

PROBLEM 3.55 (Cont.)

(b) Using Equation (1) to compute $T_{s,2}$ and q as a function of r_2, the corresponding evaporation rate, $\dot{m} = q/h_{fg}$, may be determined. Variations of q and $\dot{m}$ with r_2 are plotted as follows.





Because of its extremely low thermal conductivity, significant benefits are associated with using even a thin layer of insulation. Nearly three-order magnitude reductions in q and $\dot{m}$ are achieved with $r_2 = 0.26$ m. With increasing r_2, q and $\dot{m}$ decrease from values of 1702 W and 8×10^{-3} kg/s at $r_2 = 0.25$ m to 0.627 W and 2.9×10^{-6} kg/s at $r_2 = 0.30$ m.

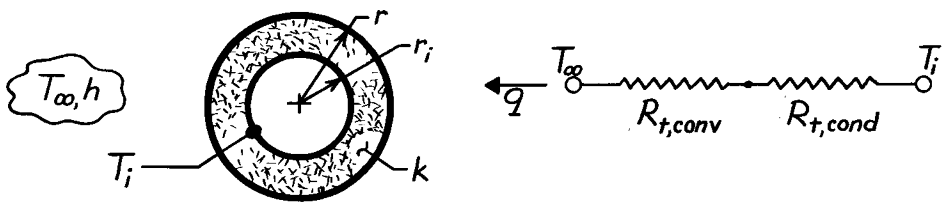
COMMENTS: Laminated metallic-foil/glass-mat insulations are extremely effective and corresponding conduction resistances are typically much larger than those normally associated with surface convection and radiation.

PROBLEM 3.56

KNOWN: Sphere of radius r_i, covered with insulation whose outer surface is exposed to a convection process.

FIND: Critical insulation radius, r_{cr}.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (spherical) conduction, (3) Constant properties, (4) Negligible radiation at surface.

ANALYSIS: The heat rate follows from the thermal circuit shown in the schematic,

$$q = (T_i - T_\infty)/R_{tot}$$

where $R_{tot} = R_{t,conv} + R_{t,cond}$ and

$$R_{t,conv} = \frac{1}{hA_s} = \frac{1}{4\pi\ hr^2} \qquad (3.9)$$

$$R_{t,cond} = \frac{1}{4\pi\ k}\left[\frac{1}{r_i} - \frac{1}{r}\right] \qquad (3.36)$$

If q is a maximum or minimum, we need to find the condition for which

$$\frac{d\,R_{tot}}{dr} = 0.$$

It follows that

$$\frac{d}{dr}\left[\frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right] + \frac{1}{4\pi\ hr^2}\right] = \left[+\frac{1}{4\pi\ k}\frac{1}{r^2} - \frac{1}{2\pi\ h}\frac{1}{r^3}\right] = 0$$

giving

$$r_{cr} = 2\frac{k}{h}$$

The second derivative, evaluated at $r = r_{cr}$, is

$$\frac{d}{dr}\left[\frac{dR_{tot}}{dr}\right] = -\frac{1}{2\pi\ k}\ \frac{1}{r^3} + \frac{3}{2\pi\ h}\ \frac{1}{r^4}\Bigg]_{r=r_{cr}}$$

$$= \frac{1}{(2k/h)^3}\left\{-\frac{1}{2\pi\ k} + \frac{3}{2\pi\ h}\frac{1}{2k/h}\right\} = \frac{1}{(2k/h)^3}\frac{1}{2\pi\ k}\left\{-1 + \frac{3}{2}\right\} > 0$$

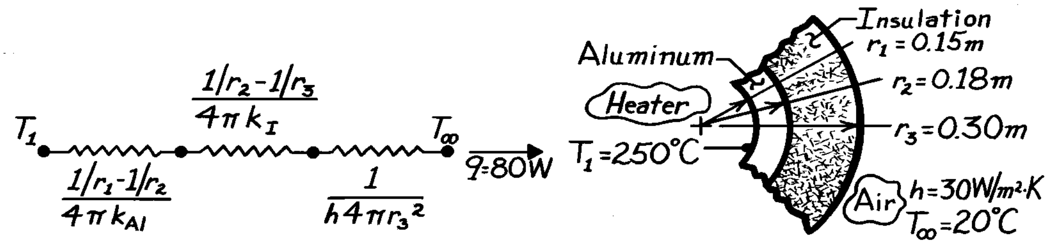
Hence, it follows no optimum R_{tot} exists. We refer to this condition as the critical insulation radius. See Example 3.4 which considers this situation for a cylindrical system.

PROBLEM 3.57

KNOWN: Thickness of hollow aluminum sphere and insulation layer. Heat rate and inner surface temperature. Ambient air temperature and convection coefficient.

FIND: Thermal conductivity of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation exchange at outer surface.

PROPERTIES: *Table A-1*, Aluminum (523K): k ≈ 230 W/m·K.

ANALYSIS: From the thermal circuit,

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \frac{T_1 - T_\infty}{\dfrac{1/r_1 - 1/r_2}{4\pi\, k_{A1}} + \dfrac{1/r_2 - 1/r_3}{4\pi\, k_I} + \dfrac{1}{h 4\pi\, r_3^2}}$$

$$q = \frac{(250-20)^\circ\,C}{\left[\dfrac{1/0.15 - 1/0.18}{4\pi(230)} + \dfrac{1/0.18 - 1/0.30}{4\pi\, k_I} + \dfrac{1}{30(4\pi)(0.3)^2}\right]\dfrac{K}{W}} = 80\ W$$

or

$$3.84\times10^{-4} + \frac{0.177}{k_I} + 0.029 = \frac{230}{80} = 2.875.$$

Solving for the unknown thermal conductivity, find

$$k_I = 0.062\ W/m\cdot K.$$ <

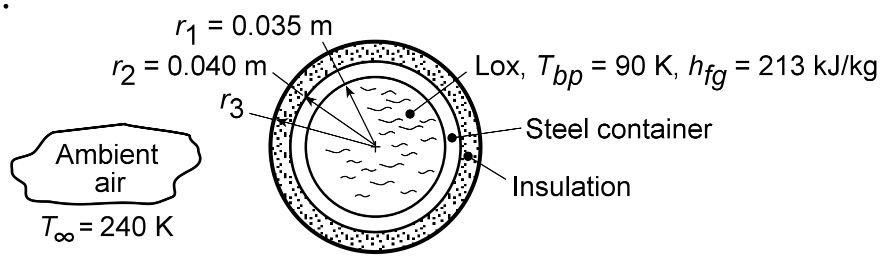
COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation. Hence uncertainties in knowledge of h or k_{A1} have a negligible effect on the accuracy of the k_I measurement.

PROBLEM 3.58

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface, (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K.

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m}h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^5\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

Since the conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.35\,\text{m}} - \frac{1}{0.40\,\text{m}}\right) = 2.4\times10^{-3}\,\text{K/W}$$

it is clear that exclusive reliance must be placed on the insulation and that a special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.40\,\text{m}} - \frac{1}{r_3}\right)$$

which yields r_3 = 0.4021 m. The minimum insulation thickness is therefore $\delta = (r_3 - r_2)$ = 2.1 mm.

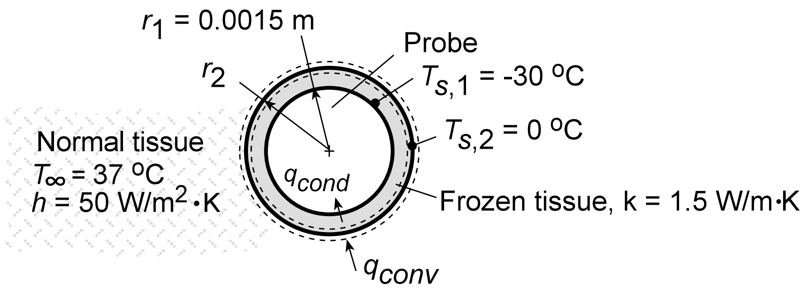
COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.

PROBLEM 3.59

KNOWN: Diameter and surface temperature of a spherical cryoprobe. Temperature of surrounding tissue and effective convection coefficient at interface between frozen and normal tissue.

FIND: Thickness of frozen tissue layer.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible contact resistance between probe and frozen tissue, (3) Constant properties.

ANALYSIS: Performing an energy balance for a control surface about the phase front, it follows that

$$q_{conv} - q_{cond} = 0$$

Hence,

$$h\left(4\pi r_2^2\right)\left(T_\infty - T_{s,2}\right) = \frac{T_{s,2} - T_{s,1}}{\left[\left(1/r_1\right) - \left(1/r_2\right)\right]/4\pi k}$$

$$r_2^2\left[\left(1/r_1\right) - \left(1/r_2\right)\right] = \frac{k}{h}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)}$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = \frac{k}{hr_1}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)} = \frac{1.5\,\text{W/m}\cdot\text{K}}{\left(50\,\text{W/m}^2\cdot\text{K}\right)\left(0.0015\,\text{m}\right)}\left(\frac{30}{37}\right)$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = 16.2$$

$$\left(r_2/r_1\right) = 4.56$$

It follows that r_2 = 6.84 mm and the thickness of the frozen tissue is

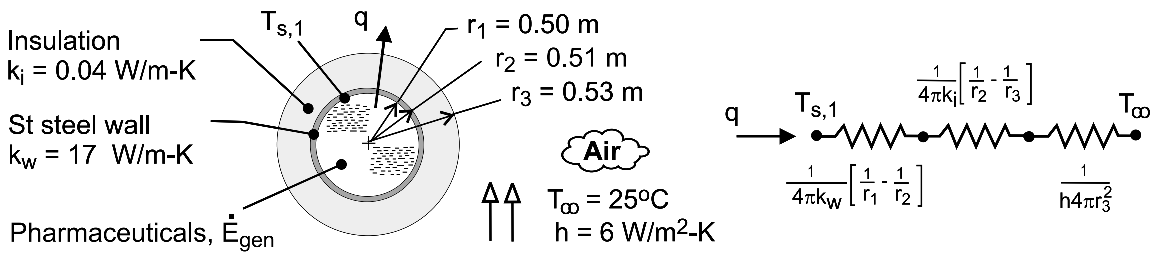
$$\delta = r_2 - r_1 = 5.34\,\text{mm}$$ <

PROBLEM 3.60

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}} = \frac{(50 - 25)^\circ\text{C}}{\frac{1}{4\pi\,(17\,\text{W/m}\cdot\text{K})}\left(\frac{1}{0.50\text{m}} - \frac{1}{0.51\text{m}}\right) + \frac{1}{4\pi\,(0.51\text{m})^2\, 6\,\text{W/m}^2\cdot\text{K}}}$$

$$\dot{E}_g = q = \frac{25^\circ\text{C}}{\left(1.84\times10^{-4} + 5.10\times10^{-2}\right)\text{K/W}} = 489\,\text{W} \qquad <$$

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 25^\circ\text{C} + 489\,\text{W}\left[1.84\times10^{-4} + \frac{1}{4\pi\,(0.04)}\left(\frac{1}{0.51} - \frac{1}{0.53}\right) + \frac{1}{4\pi\,(0.53)^2\, 6}\right]\frac{\text{K}}{\text{W}}$$

$$T_{s,1} = 25^\circ\text{C} + 489\,\text{W}\left[1.84\times10^{-4} + 0.147 + 0.047\right]\frac{\text{K}}{\text{W}} = 120^\circ\text{C} \qquad <$$

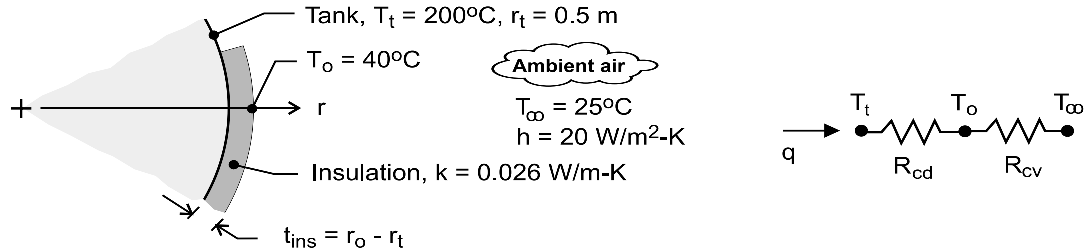
COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection.

PROBLEM 3.61

KNOWN: Spherical tank of 1-m diameter containing an exothermic reaction and is at 200°C when the ambient air is at 25°C. Convection coefficient on outer surface is 20 $W/m^2 \cdot K$.

FIND: Determine the thickness of urethane foam required to reduce the exterior temperature to 40°C. Determine the percentage reduction in the heat rate achieved using the insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, (3) Convection coefficient is the same for bare and insulated exterior surface, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

PROPERTIES: *Table A-3,* urethane, rigid foam (300 K): k = 0.026 W/m·K.

ANALYSIS: (a) The heat transfer situation for the heat rate from the tank can be represented by the thermal circuit shown above. The heat rate from the tank is

$$q = \frac{T_t - T_\infty}{R_{cd} + R_{cv}}$$

where the thermal resistances associated with conduction within the insulation (Eq. 3.35) and convection for the exterior surface, respectively, are

$$R_{cd} = \frac{\left(1/r_t - 1/r_o\right)}{4\pi k} = \frac{\left(1/0.5 - 1/r_o\right)}{4\pi \times 0.026\ \text{W/m}\cdot\text{K}} = \frac{\left(1/0.5 - 1/r_o\right)}{0.3267}\ \text{K/W}$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{4\pi h r_o^2} = \frac{1}{4\pi \times 20\ \text{W/m}^2\cdot\text{K} \times r_o^2} = 3.979 \times 10^{-3} r_o^{-2}\ \text{K/W}$$

To determine the required insulation thickness so that T_o = 40°C, perform an energy balance on the o-node.

$$\frac{T_t - T_o}{R_{cd}} + \frac{T_\infty - T_o}{R_{cv}} = 0$$

$$\frac{(200-40)\text{K}}{\left(1/0.5 - 1/r_o\right)/0.3267\ \text{K/W}} + \frac{(25-40)\text{K}}{3.979 \times 10^{-3} r_o^2\ \text{K/W}} = 0$$

$$r_o = 0.5135\ \text{m} \qquad t = r_o - r_i = (0.5135 - 0.5000)\ \text{m} = 13.5\ \text{mm}$$ <

From the rate equation, for the bare and insulated surfaces, respectively,

$$q_o = \frac{T_t - T_\infty}{1/4\pi h r_t^2} = \frac{(200-25)\text{K}}{0.01592\ \text{K/W}} = 10.99\ \text{kW}$$

$$q_{ins} = \frac{T_t - T_\infty}{R_{cd} + R_{cv}} = \frac{(200-25)}{(0.161 + 0.01592)\text{K/W}} = 0.994\ \text{kW}$$

Hence, the percentage reduction in heat loss achieved with the insulation is,

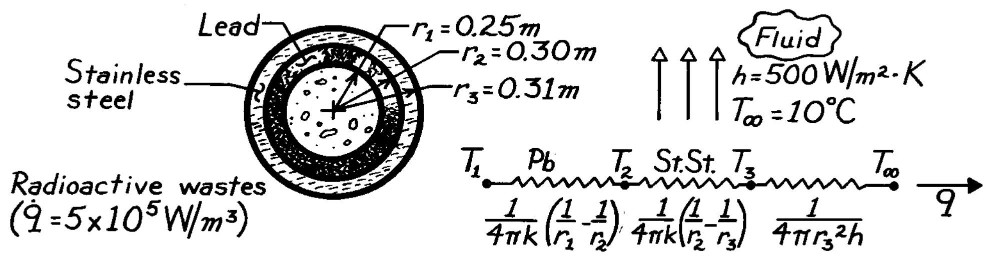
$$\frac{q_{ins} - q_o}{q_o} \times 100 = -\frac{0.994 - 10.99}{10.99} \times 100 = 91\%$$ <

PROBLEM 3.62

KNOWN: Dimensions and materials used for composite spherical shell. Heat generation associated with stored material.

FIND: Inner surface temperature, T_1, of lead (proposal is flawed if this temperature exceeds the melting point).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, MP = 601K; St.St.: 15.1 W/m·K.

ANALYSIS: From the thermal circuit, it follows that

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \dot{q}\left[\frac{4}{3}\pi\ r_1^3\right]$$

Evaluate the thermal resistances,

$$R_{Pb} = \left[1/\left(4\pi \times 35.3\ \text{W/m}\cdot\text{K}\right)\right]\ \left[\frac{1}{0.25\text{m}} - \frac{1}{0.30\text{m}}\right] = 0.00150\ \text{K/W}$$

$$R_{St.St.} = \left[1/\left(4\pi \times 15.1\ \text{W/m}\cdot\text{K}\right)\right]\ \left[\frac{1}{0.30\text{m}} - \frac{1}{0.31\text{m}}\right] = 0.000567\ \text{K/W}$$

$$R_{conv} = \left[1/\left(4\pi \times 0.31^2\ \text{m}^2 \times 500\ \text{W/m}^2\cdot\text{K}\right)\right] = 0.00166\ \text{K/W}$$

$$R_{tot} = 0.00372\ \text{K/W}.$$

The heat rate is $q = 5\times10^5\ \text{W/m}^3\left(4\pi/3\right)\left(0.25\text{m}\right)^3 = 32,725\ \text{W}$. The inner surface temperature is

$$T_1 = T_\infty + R_{tot}\ q = 283\text{K} + 0.00372\text{K/W}\left(32{,}725\ \text{W}\right)$$

$$T_1 = 405\ \text{K} < \text{MP} = 601\text{K}.$$ <

Hence, from the thermal standpoint, the proposal is adequate.

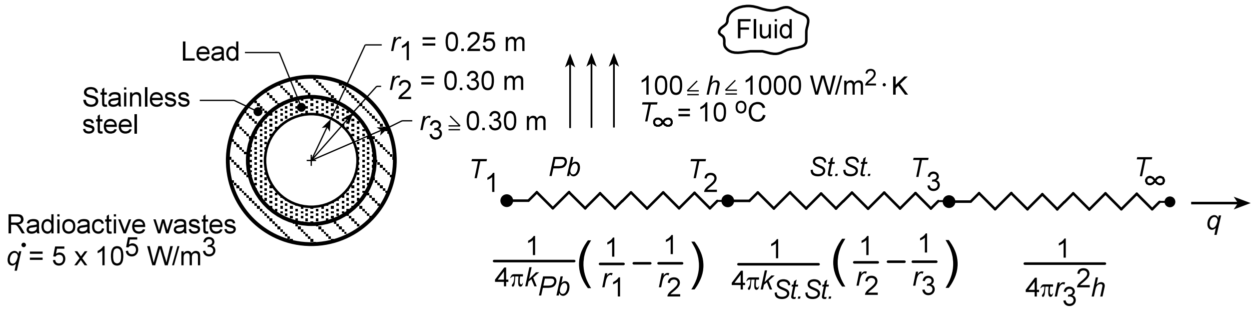
COMMENTS: In fabrication, attention should be given to maintaining a good thermal contact. A protective outer coating should be applied to prevent long term corrosion of the stainless steel.

PROBLEM 3.63

KNOWN: Dimensions and materials of composite (lead and stainless steel) spherical shell used to store radioactive wastes with constant heat generation. Range of convection coefficients h available for cooling.

FIND: (a) Variation of maximum lead temperature with h. Minimum allowable value of h to maintain maximum lead temperature at or below 500 K. (b) Effect of outer radius of stainless steel shell on maximum lead temperature for h = 300, 500 and 1000 $W/m^2{\cdot}K$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300 K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, St. St.: 15.1 W/m·K.

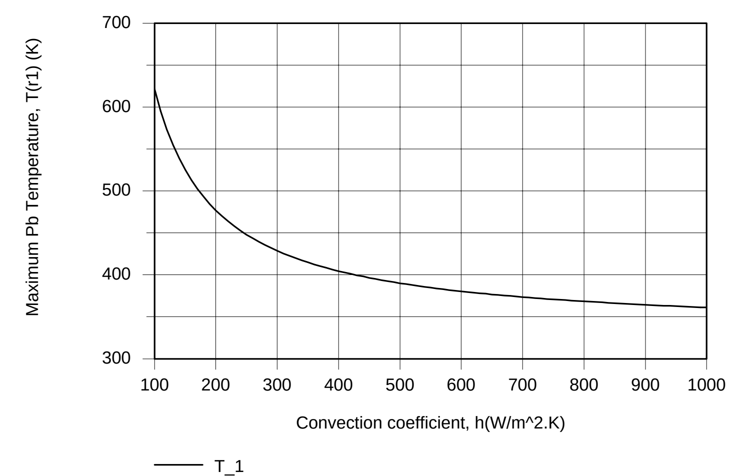
ANALYSIS: (a) From the schematic, the maximum lead temperature T_1 corresponds to $r = r_1$, and from the thermal circuit, it may be expressed as

$$T_1 = T_\infty + R_{tot} q$$

where q = $\dot{q}(4/3)\pi r_1^3 = 5 \times 10^5\ W/m^3\,(4\pi/3)(0.25\,m)^3 = 32,725\,W$. The total thermal resistance is

$$R_{tot} = R_{cond,Pb} + R_{cond,St.St} + R_{conv}$$

where expressions for the component resistances are provided in the schematic. Using the *Resistance Network* model and *Thermal Resistance* tool pad of IHT, the following result is obtained for the variation of T_1 with h.



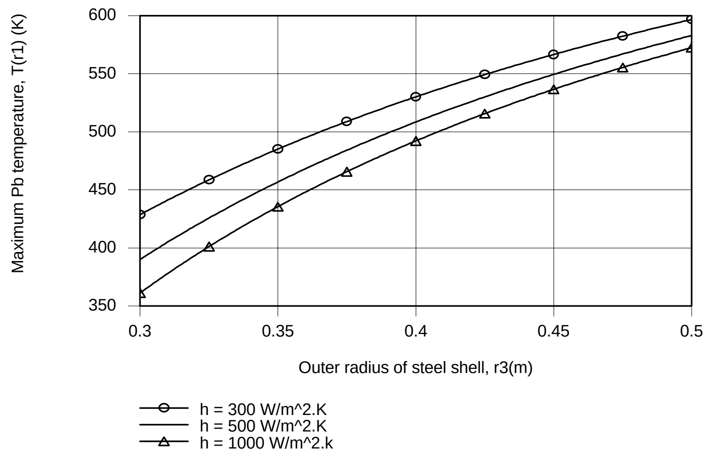
Continued...

PROBLEM 3.63 (Cont.)

To maintain T_1 below 500 K, the convection coefficient must be maintained at

$$h \geq 181\ \text{W/m}^2\cdot\text{K}$$ <

(b) The effect of varying the outer shell radius over the range $0.3 \leq r_3 \leq 0.5$ m is shown below.



For h = 300, 500 and 1000 $\text{W/m}^2\cdot\text{K}$, the maximum allowable values of the outer radius are $r_3 = 0.365$, 0.391 and 0.408 m, respectively.

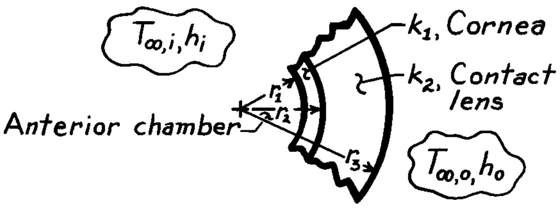
COMMENTS: For a maximum allowable value of $T_1 = 500$ K, the maximum allowable value of the total thermal resistance is $R_{tot} = (T_1 - T_\infty)/q$, or $R_{tot} = (500 - 283)\text{K}/32{,}725\ \text{W} = 0.00663$ K/W. Hence, any increase in $R_{cond,St.St}$ due to increasing r_3 must be accompanied by an equivalent reduction in R_{conv}.

PROBLEM 3.64

KNOWN: Representation of the eye with a contact lens as a composite spherical system subjected to convection processes at the boundaries.

FIND: (a) Thermal circuits with and without contact lens in place, (b) Heat loss from anterior chamber for both cases, and (c) Implications of the heat loss calculations.

SCHEMATIC:

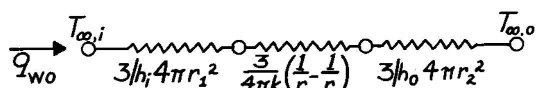


r_1=10.2mm $\quad k_1$=0.35 W/m·K
r_2=12.7mm $\quad k_2$=0.80 W/m·K
r_3=16.5mm
$T_{\infty,i}$=37°C $\quad h_i$=12 W/m^2·K
$T_{\infty,o}$=21°C $\quad h_o$=6 W/m^2·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Eye is represented as 1/3 sphere, (3) Convection coefficient, h_o, unchanged with or without lens present, (4) Negligible contact resistance.

ANALYSIS: (a) Using Eqs. 3.9 and 3.36 to express the resistance terms, the thermal circuits are:

Without lens:



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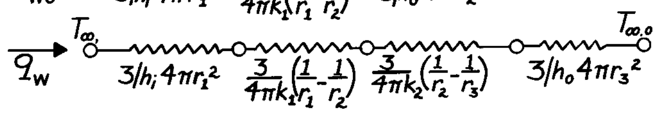
With lens:



<

(b) The heat losses for both cases can be determined as $q = (T_{\infty,i} - T_{\infty,o})/R_t$, where R_t is the thermal resistance from the above circuits.

Without lens:

$$R_{t,wo} = \frac{3}{12\,\text{W/m}^2\cdot\text{K}\,4\pi\left(10.2\times10^{-3}\text{m}\right)^2} + \frac{3}{4\pi\times0.35\text{ W/m}\cdot\text{K}}\left[\frac{1}{10.2}-\frac{1}{12.7}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{ W/m}^2\cdot\text{K}\,4\pi\left(12.7\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+246.7\text{ K/W}=451.1\text{ K/W}$$

With lens:

$$R_{t,w} = 191.2\text{ K/W}+13.2\text{ K/W}+\frac{3}{4\pi\times0.80\text{ W/m}\cdot\text{K}}\left[\frac{1}{12.7}-\frac{1}{16.5}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\,\text{W/m}^2\cdot\text{K}\,4\pi\left(16.5\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+5.41\text{ K/W}+146.2\text{ K/W}=356.0\text{ K/W}$$

Hence the heat loss rates from the anterior chamber are

Without lens: $q_{wo} = (37-21)^\circ\text{C}/451.1\text{ K/W}=35.5\text{mW}$ <

With lens: $q_w = (37-21)^\circ\text{C}/356.0\text{ K/W}=44.9\text{mW}$ <

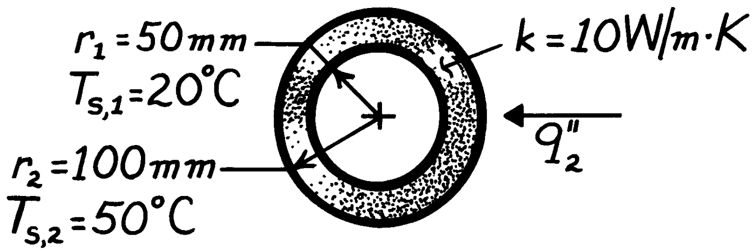
(c) The heat loss from the anterior chamber increases by approximately 20% when the contact lens is in place, implying that the outer radius, r_3, is less than the critical radius.

PROBLEM 3.65

KNOWN: Thermal conductivity and inner and outer radii of a hollow sphere subjected to a uniform heat flux at its outer surface and maintained at a uniform temperature on the inner surface.

FIND: (a) Expression for radial temperature distribution, (b) Heat flux required to maintain prescribed surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No generation, (4) Constant properties.

ANALYSIS: (a) For the assumptions, the temperature distribution may be obtained by integrating Fourier's law, Eq. 3.33. That is,

$$\frac{q_r}{4\pi}\int_{r_1}^{r}\frac{dr}{r^2} = -k\int_{T_{s,1}}^{T} dT \quad \text{or} \quad -\frac{q_r}{4\pi}\left.\frac{1}{r}\right|_{r_1}^{r} = -k\left(T - T_{s,1}\right).$$

Hence,

$$T(r) = T_{s,1} + \frac{q_r}{4\pi\, k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

or, with $q''_2 \equiv q_r / 4\pi\, r_2^2$,

$$T(r) = T_{s,1} + \frac{q''_2 r_2^2}{k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$ <

(b) Applying the above result at r_2,

$$q''_2 = \frac{k\left(T_{s,2} - T_{s,1}\right)}{r_2^2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]} = \frac{10\ \text{W/m}\cdot\text{K}\ \left(50-20\right)^{\circ}\text{C}}{\left(0.1\text{m}\right)^2\left[\frac{1}{0.1} - \frac{1}{0.05}\right]\frac{1}{\text{m}}} = -3000\ \text{W/m}^2.$$ <

COMMENTS: (1) The desired temperature distribution could also be obtained by solving the appropriate form of the heat equation,

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = 0$$

and applying the boundary conditions $T(r_1) = T_{s,1}$ and $-k\left.\frac{dT}{dr}\right]_{r_2} = q''_2$.

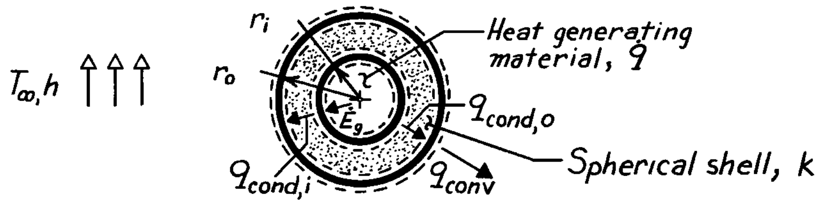
(2) The negative sign on q''_2 implies heat transfer in the negative r direction.

PROBLEM 3.66

KNOWN: Volumetric heat generation occurring within the cavity of a spherical shell of prescribed dimensions. Convection conditions at outer surface.

FIND: Expression for steady-state temperature distribution in shell.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Steady-state conditions, (3) Constant properties, (4) Uniform generation within the shell cavity, (5) Negligible radiation.

ANALYSIS: For the prescribed conditions, the appropriate form of the heat equation is

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=0$$

Integrate twice to obtain,

$$r^2\frac{dT}{dr}=C_1 \qquad \text{and} \qquad T=-\frac{C_1}{r}+C_2. \tag{1,2}$$

The boundary conditions may be obtained from energy balances at the inner and outer surfaces. At the inner surface (r_i),

$$\dot{E}_g=\dot{q}\left(4/3\pi\ r_i^3\right)=q_{cond,i}=-k\left(4\pi\ r_i^2\right)dT/dr)_{r_i} \qquad dT/dr)_{r_i}=-\dot{q}r_i/3k. \tag{3}$$

At the outer surface (r_o),

$$q_{cond,o}=-k4\pi\ r_o^2\ dT/dr)_{r_o}=q_{conv}=h4\pi\ r_o^2\left[T\left(r_o\right)-T_\infty\right]$$

$$dT/dr)_{r_o}=-\left(h/k\right)\ \left[T\left(r_o\right)-T_\infty\right]. \tag{4}$$

From Eqs. (1) and (3), $C_1=-\dot{q}r_i^3/3k$. From Eqs. (1), (2) and (4)

$$-\frac{\dot{q}r_i^3}{3kr_o^2}=-\left[\frac{h}{k}\right]\left[\frac{\dot{q}r_i^3}{3r_ok}+C_2-T_\infty\right]$$

$$C_2=\frac{\dot{q}r_i^3}{3hr_o^2}-\frac{\dot{q}r_i^3}{3r_ok}+T_\infty.$$

Hence, the temperature distribution is

$$T=\frac{\dot{q}r_i^3}{3k}\ \left[\frac{1}{r}-\frac{1}{r_o}\right]+\frac{\dot{q}r_i^3}{3hr_o^2}+T_\infty. \qquad <$$

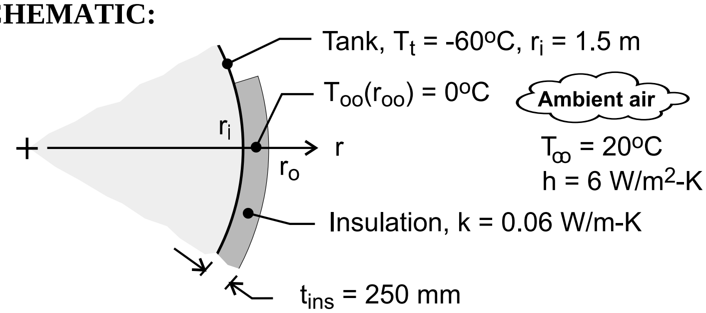
COMMENTS: Note that $\dot{E}_g=q_{cond,i}=q_{cond,o}=q_{conv}$.

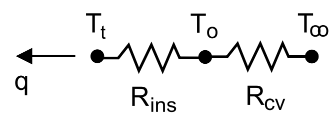
PROBLEM 3.67

KNOWN: Spherical tank of 3-m diameter containing LP gas at -60°C with 250 mm thickness of insulation having thermal conductivity of 0.06 W/m·K. Ambient air temperature and convection coefficient on the outer surface are 20°C and 6 W/m^2·K, respectively.

FIND: (a) Determine the radial position in the insulation at which the temperature is 0°C and (b) If the insulation is pervious to moisture, what conclusions can be reached about ice formation? What effect will ice formation have on the heat gain? How can this situation be avoided?

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

ANALYSIS: (a) The heat transfer situation can be represented by the thermal circuit shown above. The heat gain to the tank is

$$q = \frac{T_\infty - T_t}{R_{ins} + R_{cv}} = \frac{\left[20-(-60)\right]\text{K}}{\left(0.1263 + 4.33\times10^{-3}\right)\text{K/W}} = 612.4\ \text{W}$$

where the thermal resistances for the insulation (see Table 3.3) and the convection process on the outer surface are, respectively,

$$R_{ins} = \frac{1/r_i - 1/r_o}{4\pi k} = \frac{(1/1.50 - 1/1.75)\,\text{m}^{-1}}{4\pi\times 0.06\ \text{W/m}\cdot\text{K}} = 0.1263\ \text{K/W}$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{h4\pi r_o^2} = \frac{1}{6\ \text{W/m}^2\cdot\text{K}\times 4\pi(1.75\ \text{m})^2} = 4.33\times10^{-3}\ \text{K/W}$$

To determine the location within the insulation where T_{oo} (r_{oo}) = 0°C, use the conduction rate equation, Eq. 3.35,

$$q = \frac{4\pi k\left(T_{oo} - T_t\right)}{\left(1/r_i - 1/r_{oo}\right)} \qquad\qquad r_{oo} = \left[\frac{1}{r_i} - \frac{4\pi k\left(T_{oo} - T_t\right)}{q}\right]^{-1}$$

and substituting numerical values, find

$$r_{oo} = \left[\frac{1}{1.5\ \text{m}} - \frac{4\pi\times 0.06\ \text{W/m}\cdot\text{K}\left(0-(-60)\right)\text{K}}{612.4\ \text{W}}\right]^{-1} = 1.687\ \text{m}$$ <

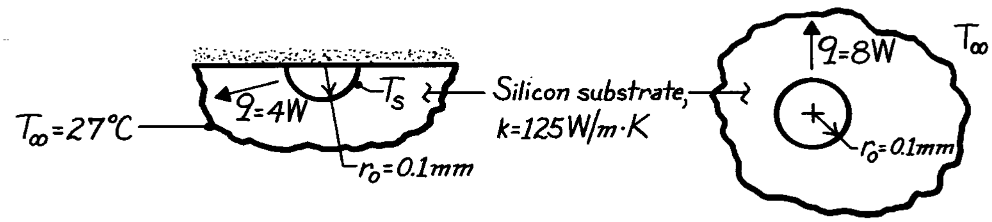
(b) With r_{oo} = 1.687 m, we'd expect the region of the insulation $r_i \le r \le r_{oo}$ to be filled with ice formations if the insulation is pervious to water vapor. The effect of the ice formation is to substantially increase the heat gain since k_{ice} is nearly twice that of k_{ins}, and the ice region is of thickness (1.687 – 1.50)m = 187 mm. To avoid ice formation, a vapor barrier should be installed at a radius larger than r_{oo}.

PROBLEM 3.68

KNOWN: Radius and heat dissipation of a hemispherical source embedded in a substrate of prescribed thermal conductivity. Source and substrate boundary conditions.

FIND: Substrate temperature distribution and surface temperature of heat source.

SCHEMATIC:



ASSUMPTIONS: (1) Top surface is adiabatic. Hence, hemispherical source in semi-infinite medium is equivalent to spherical source in infinite medium (with q = 8 W) and heat transfer is one-dimensional in the radial direction, (2) Steady-state conditions, (3) Constant properties, (4) No generation.

ANALYSIS: Heat equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad r^2 dT/dr=C_1$$

$$T(r)=-C_1/r+C_2.$$

Boundary conditions:

$$T(\infty)=T_\infty \qquad T(r_o)=T_s$$

Hence, $C_2 = T_\infty$ and

$$T_s=-C_1/r_o+T_\infty \quad \text{and} \quad C_1=r_o\left(T_\infty-T_s\right).$$

The temperature distribution has the form

$$T(r)=T_\infty+\left(T_s-T_\infty\right)r_o/r \qquad <$$

and the heat rate is

$$q=-kAdT/dr=-k2\pi\ r^2\left[-\left(T_s-T_\infty\right)r_o/r^2\right]=k2\pi\ r_o\left(T_s-T_\infty\right)$$

It follows that

$$T_s-T_\infty=\frac{q}{k2\pi\ r_o}=\frac{4\ \text{W}}{125\ \text{W/m}\cdot\text{K}\ 2\pi\left(10^{-4}\ \text{m}\right)}=50.9^\circ\text{C}$$

$$T_s=77.9^\circ\text{C}. \qquad <$$

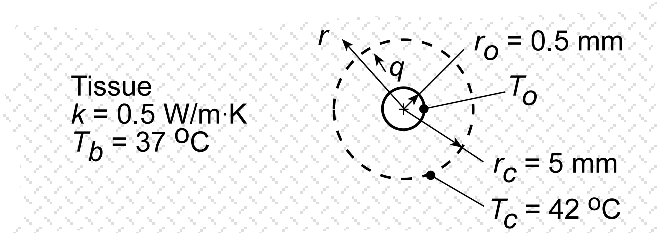
COMMENTS: For the semi-infinite (or infinite) medium approximation to be valid, the substrate dimensions must be much larger than those of the transistor.

PROBLEM 3.69

KNOWN: Critical and normal tissue temperatures. Radius of spherical heat source and radius of tissue to be maintained above the critical temperature. Tissue thermal conductivity.

FIND: General expression for radial temperature distribution in tissue. Heat rate required to maintain prescribed thermal conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant k.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

Integrating twice,

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

Since $T \to T_b$ as $r \to \infty$, $C_2 = T_b$. At $r = r_o$, $q = -k\left(4\pi r_o^2\right)dT/dr\big|_{r_o} = -4\pi k r_o^2\, C_1 / r_o^2 = -4\pi k C_1$.

Hence, $C_1 = -q/4\pi k$ and the temperature distribution is

$$T(r)=\frac{q}{4\pi kr}+T_b$$ <

It follows that

$$q=4\pi kr\left[T(r)-T_b\right]$$

Applying this result at $r = r_c$,

$$q=4\pi\left(0.5\,W/m\cdot K\right)\left(0.005\,m\right)\left(42-37\right)^{\circ}C=0.157\,W$$ <

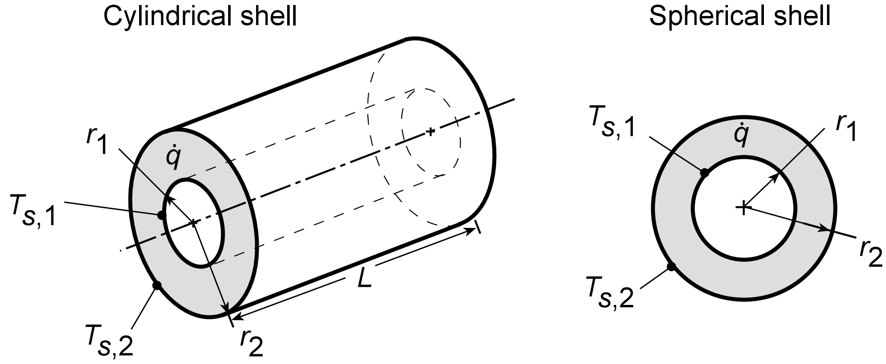
COMMENTS: At $r_o = 0.0005$ m, $T(r_o) = \left[q/\left(4\pi k r_o\right)\right]+T_b = 92°C$. Proximity of this temperature to the boiling point of water suggests the need to operate at a lower power dissipation level.

PROBLEM 3.70

KNOWN: Cylindrical and spherical shells with uniform heat generation and surface temperatures.

FIND: Radial distributions of temperature, heat flux and heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Uniform heat generation, (3) Constant k.

ANALYSIS: (a) For the *cylindrical shell*, the appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

The general solution is

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2$$

Applying the boundary conditions, it follows that

$$T(r_1)=T_{s,1}=-\frac{\dot{q}}{4k}r_1^2+C_1\ln r_1+C_2$$

$$T(r_2)=T_{s,2}=-\frac{\dot{q}}{4k}r_2^2+C_1\ln r_2+C_2$$

which may be solved for

$$C_1=\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\Big/\ln(r_2/r_1)$$

$$C_2=T_{s,2}+(\dot{q}/4k)r_2^2-C_1\ln r_2$$

Hence,

$$T(r)=T_{s,2}+(\dot{q}/4k)\left(r_2^2-r^2\right)+\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\frac{\ln(r/r_2)}{\ln(r_2/r_1)}$$ <

With $q''=-k\,dT/dr$, the heat flux distribution is

$$q''(r)=\frac{\dot{q}}{2}r-\frac{k\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]}{r\ln(r_2/r_1)}$$ <

Continued...

Similarly, with q = q'' A(r) = q'' (2πrL), the heat rate distribution is

$$q(r) = \pi L \dot{q} r^2 - \frac{2\pi Lk\left[(\dot{q}/4k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{\ln(r_2/r_1)}$$ <

(b) For the *spherical shell*, the heat equation and general solution are

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$T(r) = -(\dot{q}/6k)r^2 - C_1/r + C_2$$

Applying the boundary conditions, it follows that

$$T(r_1) = T_{s,1} = -(\dot{q}/6k)r_1^2 - C_1/r_1 + C_2$$

$$T(r_2) = T_{s,2} = -(\dot{q}/6k)r_2^2 - C_1/r_2 + C_2$$

Hence,

$$C_1 = \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right] \Big/ \left[(1/r_1) - (1/r_2)\right]$$

$$C_2 = T_{s,2} + (\dot{q}/6k)r_2^2 + C_1/r_2$$

and

$$T(r) = T_{s,2} + (\dot{q}/6k)\left(r_2^2 - r^2\right) - \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)}$$ <

With q''(r) = - k dT/dr, the heat flux distribution is

$$q''(r) = \frac{\dot{q}}{3}r - \frac{\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)}\frac{1}{r^2}$$ <

and, with q = $q''\left(4\pi r^2\right)$, the heat rate distribution is

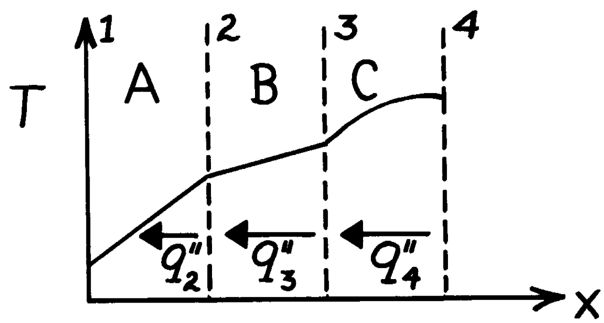
$$q(r) = \frac{4\pi\dot{q}}{3}r^3 - \frac{4\pi\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)}$$ <

PROBLEM 3.71

KNOWN: Temperature distribution in a composite wall.

FIND: (a) Relative magnitudes of interfacial heat fluxes, (b) Relative magnitudes of thermal conductivities, and (c) Heat flux as a function of distance x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) For the prescribed conditions (one-dimensional, steady-state, constant k), the parabolic temperature distribution in C implies the existence of heat generation. Hence, since dT/dx *increases* with *decreasing* x, the heat flux in C *increases* with *decreasing* x. Hence,

$$q''_3 > q''_4$$

However, the linear temperature distributions in A and B indicate no generation, in which case

$$q''_2 = q''_3$$

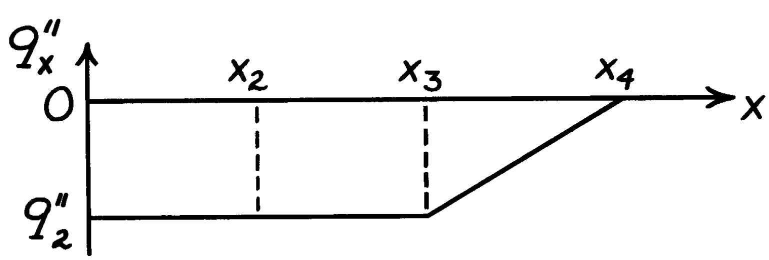
(b) Since conservation of energy requires that $q''_{3,B} = q''_{3,C}$ and $dT/dx)_B < dT/dx)_C$, it follows from Fourier's law that

$$k_B > k_C.$$

Similarly, since $q''_{2,A} = q''_{2,B}$ and $dT/dx)_A > dT/dx)_B$, it follows that

$$k_A < k_B.$$

(c) It follows that the flux distribution appears as shown below.



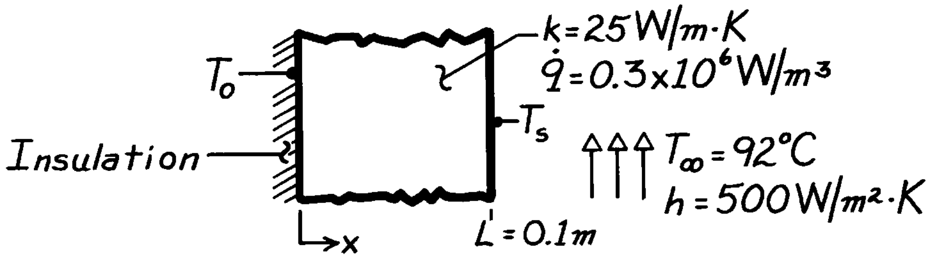
COMMENTS: Note that, with $dT/dx)_{4,C} = 0$, the interface at 4 is adiabatic.

PROBLEM 3.72

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: From Eq. 3.42, the temperature at the inner surface is given by Eq. 3.43 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.46,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ\text{C} + 0.3\times10^6 \frac{\text{W}}{\text{m}^3}\times 0.1\text{m}/500\text{W/m}^2\cdot\text{K} = 92^\circ\text{C} + 60^\circ\text{C} = 152^\circ\text{C}.$$

It follows that

$$T_o = 0.3\times10^6\,\text{W/m}^3\times(0.1\text{m})^2/2\times25\text{W/m}\cdot\text{K} + 152^\circ\text{C}$$

$$T_o = 60^\circ\text{C} + 152^\circ\text{C} = 212^\circ\text{C}. \qquad <$$

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500\text{W/m}^2\cdot\text{K}(152-92)^\circ\text{C} = 30\text{kW/m}^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

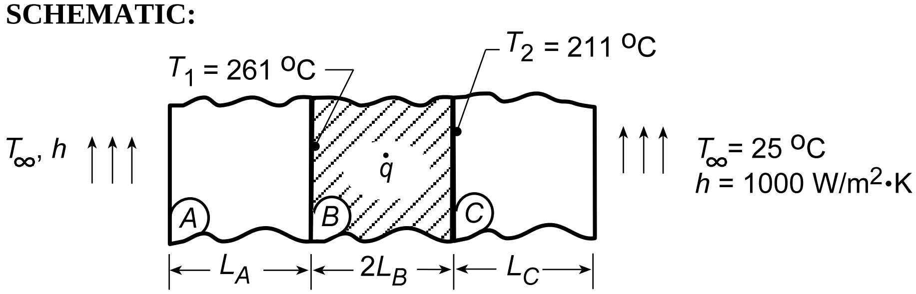
$$q''_{conv} = \dot{q}L = 0.3\times10^6\text{W/m}^3\times0.1\text{m} = 30\text{kW/m}^2.$$

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

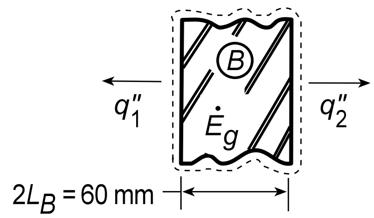
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

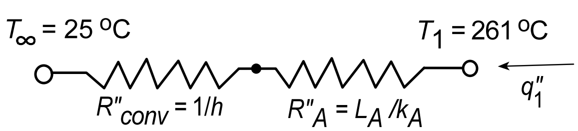
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

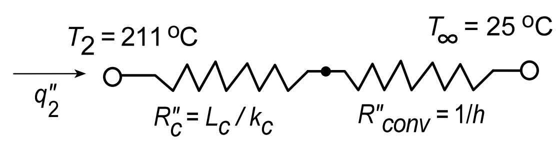
$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \ .$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:





$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right)$$

$$q_1'' = \left(261 - 25\right)^\circ C \bigg/ \left(\frac{1}{1000\,W/m^2\cdot K} + \frac{0.030\,m}{25\,W/m\cdot K}\right)$$

$$q_1'' = 236^\circ C/\left(0.001 + 0.0012\right)m^2\cdot K/W$$

$$q_1'' = 107,273\,W/m^2$$

$$q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_2'' = \left(211 - 25\right)^\circ C \bigg/ \left(\frac{0.020\,m}{50\,W/m\cdot K} + \frac{1}{1000\,W/m^2\cdot K}\right)$$

$$q_2'' = 186^\circ C/\left(0.0004 + 0.001\right)m^2\cdot K/W$$

$$q_2'' = 132,857\,W/m^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\,W/m^2\right)\Big/2\times 0.030\,m = 4.00\times 10^6\,W/m^3 \ .$$ <

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273 \text{ W/m}^2 \qquad (5)$$

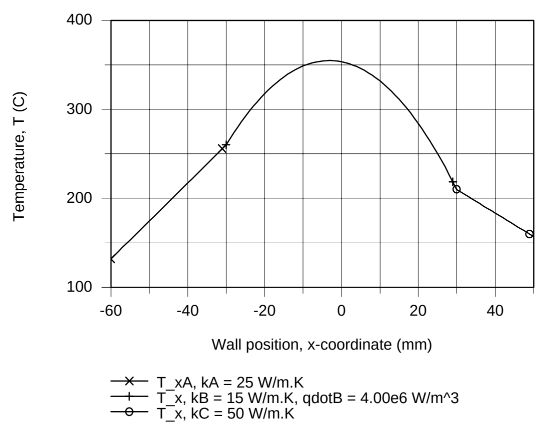
Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

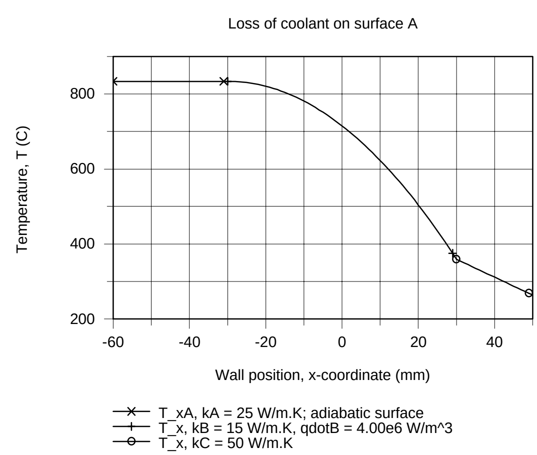
$$k_B = 15.3 \text{ W/m} \cdot \text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = -L_B and +L_B, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

T_1 = 835°C $\qquad$ T_2 = 360°C <



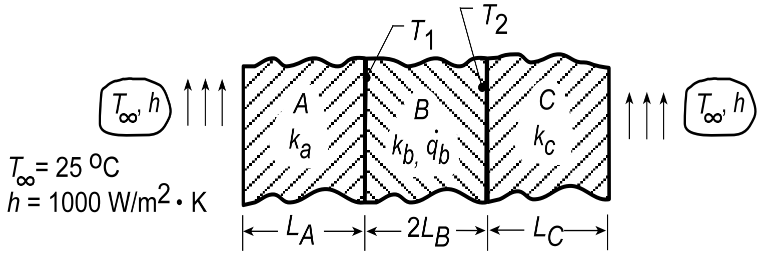


PROBLEM 3.74

KNOWN: Composite wall exposed to convection process; inside wall experiences a uniform heat generation.

FIND: (a) Neglecting interfacial thermal resistances, determine T_1 and T_2, as well as the heat fluxes through walls A and C, and (b) Determine the same parameters, but consider the interfacial contact resistances. Plot temperature distributions.

SCHEMATIC:



$k_A = 25\,W/m\cdot K \qquad L_A = 30\,mm$

$k_B = 15\,W/m\cdot K \qquad L_B = 30\,mm$

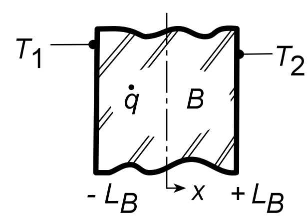
$k_C = 50\,W/m\cdot K \qquad L_C = 20\,mm$

$\dot{q}_B = 4\times10^6\,W/m^3$

ASSUMPTIONS: (1) One-dimensional, steady-state heat flow, (2) Negligible contact resistance between walls, part (a), (3) Uniform heat generation in B, zero in A and C, (4) Uniform properties, (5) Negligible radiation at outer surfaces.

ANALYSIS: (a) The temperature distribution in wall B follows from Eq. 3.41,

$$T(x)=\frac{\dot{q}_B L_B^2}{2k_B}\left(1-\frac{x^2}{L_B^2}\right)+\frac{T_2-T_1}{2}\frac{x}{L_B}+\frac{T_1-T_2}{2}. \qquad (1)$$

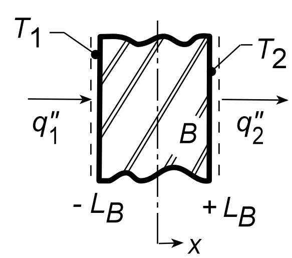


The heat fluxes to the neighboring walls are found using Fourier's law,

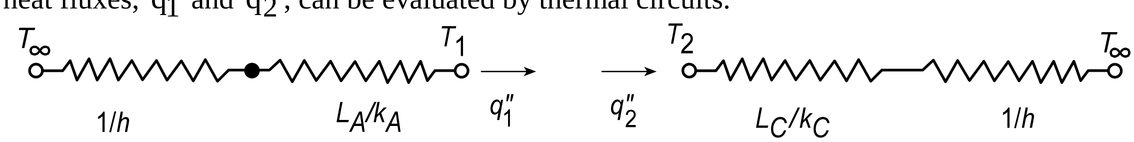
$$q''_x=-k\frac{dT}{dx}.$$

At $x=-L_B$: $\quad q''_x(-L_B)-k_B\left[+\frac{\dot{q}_B}{k_B}(L_B)+\frac{T_2-T_1}{2L_B}\right]=q''_1 \quad (2)$

At $x=+L_B$: $\quad q''_x(L_B)-k_B\left[-\frac{\dot{q}_B}{k_B}(L_B)+\frac{T_2-T_1}{2L_B}\right]=q''_2 \quad (3)$



The heat fluxes, q''_1 and q''_2, can be evaluated by thermal circuits.



Substituting numerical values, find

$$q''_1=(T_\infty-T_1)^\circ C\Big/(1/h+L_A/k_A)=(25-T_1)^\circ C\Big/\left(1/1000\,W/m^2\cdot K+0.03\,m/25\,W/m\cdot K\right)$$

$$q''_1=(25-T_1)^\circ C\Big/(0.001+0.0012)K/W=454.6(25-T_1) \qquad (4)$$

$$q''_2=(T_2-T_\infty)^\circ C\Big/(1/h+L_C/k_C)=(T_2-25)^\circ C\Big/\left(1/1000\,W/m^2\cdot K+0.02\,m/50\,W/m\cdot K\right)$$

$$q''_2=(T_2-25)^\circ C\Big/(0.001+0.0004)K/W=714.3(T_2-25). \qquad (5)$$

Continued...

Substituting the expressions for the heat fluxes, Eqs. (4) and (5), into Eqs. (2) and (3), a system of two equations with two unknowns is obtained.

Eq. (2): $$-4\times10^{6}\,\mathrm{W/m^3}\times0.03\,\mathrm{m}+15\,\mathrm{W/m\cdot K}\,\frac{T_2-T_1}{2\times0.03\,\mathrm{m}}=q_1''$$

$$-1.2\times10^{5}\,\mathrm{W/m^2}-2.5\times10^{2}\left(T_2-T_1\right)\mathrm{W/m^2}=454.6\left(25-T_1\right)$$

$$704.6\,T_1-250\,T_2=131{,}365 \qquad (6)$$

Eq. (3): $$+4\times10^{6}\,\mathrm{W/m^3}\times0.03\,\mathrm{m}-15\,\mathrm{W/m\cdot K}\,\frac{T_2-T_1}{2\times0.03\,\mathrm{m}}=q_2''$$

$$+1.2\times10^{5}\,\mathrm{W/m^2}-2.5\times10^{2}\left(T_2-T_1\right)\mathrm{W/m^2}=714.3\left(T_2-25\right)$$

$$250\,T_1-964\,T_2=-137{,}857 \qquad (7)$$

Solving Eqs. (6) and (7) simultaneously, find

T_1 = 260.9°C $\qquad$ T_2 = 210.0°C $\qquad$ <

From Eqs. (4) and (5), the heat fluxes at the interfaces and through walls A and C are, respectively,

$$q_1''=454.6\left(25-260.9\right)=-107{,}240\,\mathrm{W/m^2}$$ <

$$q_2''=714.3\left(210-25\right)=+132{,}146\,\mathrm{W/m^2}\,.$$ <

Note directions of the heat fluxes.

(b) Considering interfacial contact resistances, we will use a different approach. The general solution for the temperature and heat flux distributions in each of the materials is

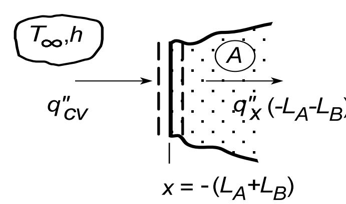
$$T_A\left(x\right)=C_1x+C_2 \qquad q_x''=-k_AC_1 \qquad -\left(L_A+L_B\right)\le x\le -L_B \qquad (1,2)$$

$$T_B\left(x\right)=-\frac{\dot{q}_B}{2k_B}x^2+C_3x+C_4 \qquad q_x''=-\frac{\dot{q}_B}{k_B}x+C_3 \qquad -L_B\le x\le L_B \qquad (3,4)$$

$$T_C\left(x\right)=C_5x+C_6 \qquad q_x''=-k_CC_5 \qquad +L_B\le x\le\left(L_B+L_C\right) \qquad (5,6)$$

To determine C_1 ... C_6 and the distributions, we need to identify boundary conditions using surface energy balances.

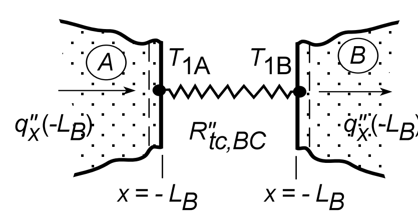
At $x = -(L_A + L_B)$:



$$-q_x''\left(-L_A-L_B\right)+q_{cv}''=0 \qquad (7)$$

$$-\left(-k_AC_1\right)+h\left[T_\infty-T_A\left(-L_A-L_B\right)\right] \qquad (8)$$

At $x = -L_B$: The heat flux must be continuous, but the temperature will be discontinuous across the contact resistance.



$$q_{x,A}''\left(-L_B\right)=q_{x,B}''\left(-L_B\right) \qquad (9)$$

$$q_{x,A}''\left(-L_B\right)=\left[T_{1A}\left(-L_B\right)-T_{1B}\left(-L_B\right)\right]\Big/R_{tc,AB}'' \qquad (10)$$

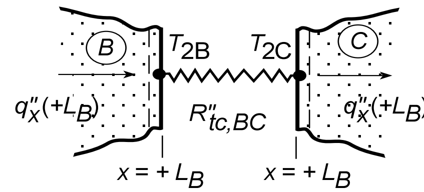
Continued...

PROBLEM 3.74 (Cont.)

At x = + L_B: The same conditions apply as for x = -L_B,

$$q''_{x,B}\left(+L_B\right) = q''_{x,C}\left(+L_B\right) \qquad (11)$$

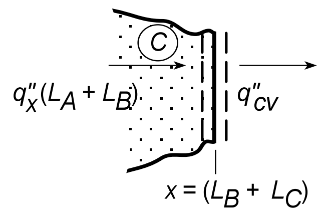
$$q''_{x,B}\left(+L_B\right) = \left[T_{2B}\left(+L_B\right) - T_{2C}\left(+L_B\right)\right] \Big/ R''_{tc,BC} \qquad (12)$$



At x = +(L_B + L_C):

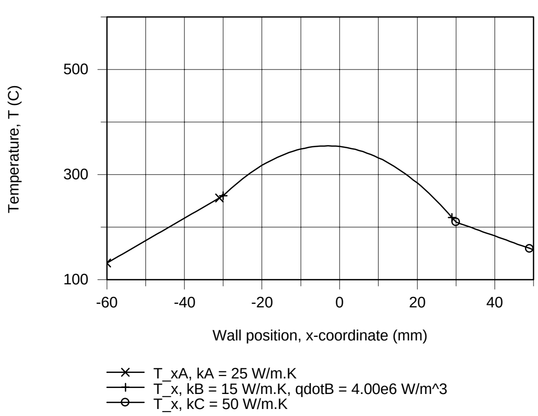
$$-q_{x,C}\left(L_B + L_C\right) - q''_{cv} = 0 \qquad (13)$$

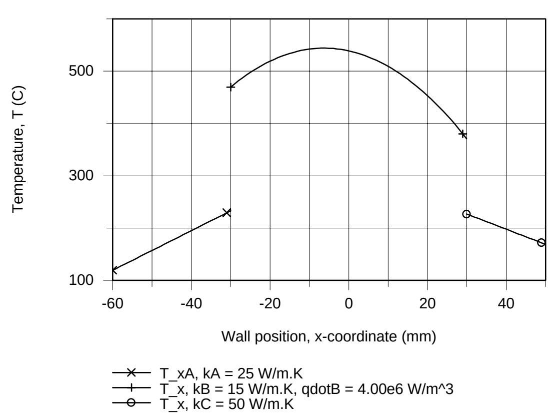
$$-\left(-k_C C_5\right) - h\left[T_C\left(L_B + L_C\right) - T_\infty\right] = 0 \qquad (14)$$



Following the method of analysis in IHT Example 3.6, User-Defined Functions, we solve the system of equations above for the constants C_1 ... C_6 for conditions with negligible and prescribed values for the interfacial constant resistances. The results are tabulated and plotted below; q''_1 and q''_2 represent heat fluxes leaving surfaces A and C, respectively.

Conditions	T_{1A} (°C)	T_{1B} (°C)	T_{2B} (°C)	T_{2C} (°C)	q''_1 (kW/m^2)	q''_2 (kW/m^2)
$R''_{tc} = 0$	260	260	210	210	106.8	132.0
$R''_{tc} \neq 0$	233	470	371	227	94.6	144.2

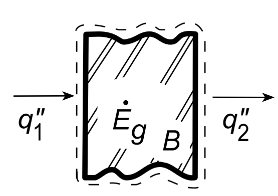




COMMENTS: (1) The results for part (a) can be checked using an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} = -\dot{E}_g$$

$$q''_1 - q''_2 = -\dot{q}_B \times 2L_B$$



where

$$q''_1 - q''_2 = -107,240 - 132,146 = 239,386\ \mathrm{W/m^2}$$

$$-\dot{q}_B L_B = -4\times 10^6\ \mathrm{W/m^3} \times 2\left(0.03\ \mathrm{m}\right) = -240,000\ \mathrm{W/m^2}\ .$$

Hence, we have confirmed proper solution of Eqs. (6) and (7).

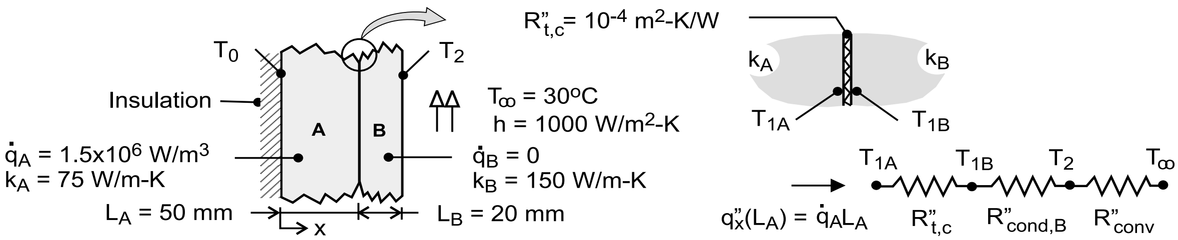
(2) Note that the effect of the interfacial contact resistance is to increase the temperature at all locations. The total heat flux leaving the composite wall (q_1 + q_2) will of course be the same for both cases.

PROBLEM 3.75

KNOWN: Composite wall of materials A and B. Wall of material A has uniform generation, while wall B has no generation. The inner wall of material A is insulated, while the outer surface of material B experiences convection cooling. Thermal contact resistance between the materials is $R''_{t,c} = 10^{-4} m^2 \cdot K/W$. See Ex. 3.6 that considers the case without contact resistance.

FIND: Compute and plot the temperature distribution in the composite wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties, and (3) Inner surface of material A is adiabatic.

ANALYSIS: From the analysis of Ex. 3.6, we know the temperature distribution in material A is parabolic with zero slope at the inner boundary, and that the distribution in material B is linear. At the interface between the two materials, $x = L_A$, the temperature distribution will show a discontinuity.

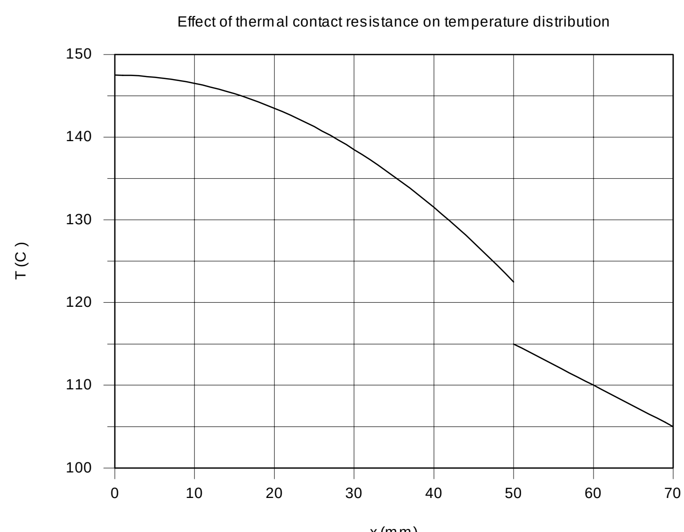
$$T_A(x) = \frac{\dot{q} L_A^2}{2k_A}\left(1 - \frac{x^2}{L_A^2}\right) + T_{1A} \qquad 0 \le x \le L_A$$

$$T_B(x) = T_{1B} - (T_{1B} - T_2)\frac{x - L_A}{L_B} \qquad L_A \le x \le L_A + L_B$$

Considering the thermal circuit above (see also Ex. 3.6) including the thermal contact resistance,

$$q'' = \dot{q} L_A = \frac{T_{1A} - T_\infty}{R''_{tot}} = \frac{T_{1B} - T_\infty}{R''_{cond,B} + R''_{conv}} = \frac{T_2 - T_\infty}{R''_{conv}}$$

find $T_A(0) = 147.5°C$, $T_{1A} = 122.5°C$, $T_{1B} = 115°C$, and $T_2 = 105°C$. Using the foregoing equations in IHT, the temperature distributions for each of the materials can be calculated and are plotted on the graph below.



COMMENTS: (1) The effect of the thermal contact resistance between the materials is to increase the maximum temperature of the system.

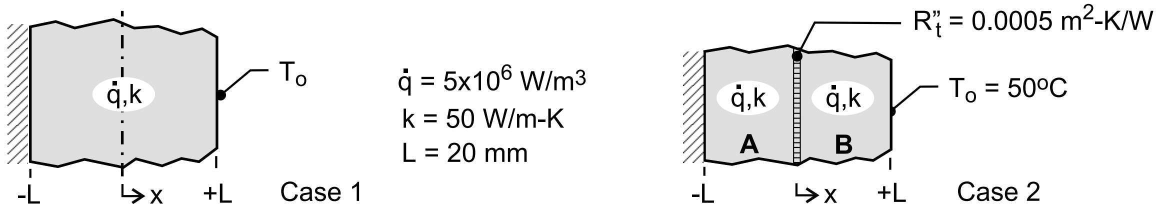
(2) Can you explain why the temperature distribution in the material B is not affected by the presence of the thermal contact resistance at the materials' interface?

PROBLEM 3.76

KNOWN: Plane wall of thickness 2L, thermal conductivity k with uniform energy generation $\dot{q}$. For case 1, boundary at x = -L is perfectly insulated, while boundary at x = +L is maintained at T_o = 50°C. For case 2, the boundary conditions are the same, but a thin dielectric strip with thermal resistance $R''_t = 0.0005\ m^2 \cdot K/W$ is inserted at the mid-plane.

FIND: (a) Sketch the temperature distribution for case 1 on T-x coordinates and describe key features; identify and calculate the maximum temperature in the wall, (b) Sketch the temperature distribution for case 2 on the same T-x coordinates and describe the key features; (c) What is the temperature difference between the two walls at x = 0 for case 2? And (d) What is the location of the maximum temperature of the composite wall in case 2; calculate this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the plane and composite walls, and (3) Constant properties.

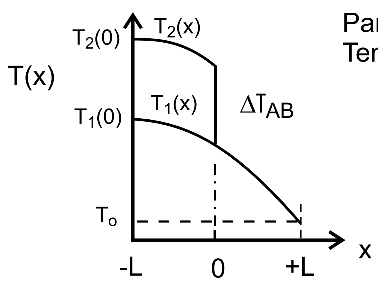
ANALYSIS: (a) For case 1, the temperature distribution, $T_1(x)$ vs. x, is parabolic as shown in the schematic below and the gradient is zero at the insulated boundary, x = -L. From Eq. 3.43,

$$T_1(-L) - T_1(+L) = \frac{\dot{q}(2L)^2}{2k} = \frac{5\times10^6\ W/m^3\,(2\times0.020\ m)^2}{2\times50\ W/m\cdot K} = 80°C$$

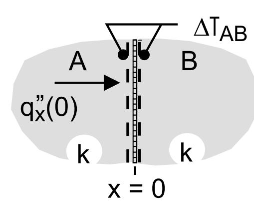
and since $T_1(+L) = T_o$ = 50°C, the maximum temperature occurs at x = -L,

$$T_1(-L) = T_1(+L) + 80°C = 130°C$$

(b) For case 2, the temperature distribution, $T_2(x)$ vs. x, is piece-wise parabolic, with zero gradient at x = -L and a drop across the dielectric strip, ΔT_{AB}. The temperature gradients at either side of the dielectric strip are equal.



Parts (a,b)
Temperature distributions



Part (d) Surface energy balance

(c) For case 2, the temperature drop across the thin dielectric strip follows from the surface energy balance shown above.

$$q''_x(0) = \Delta T_{AB}/R''_t \qquad\qquad q''_x(0) = \dot{q}L$$

$$\Delta T_{AB} = R''_t\,\dot{q}L = 0.0005\ m^2\cdot K/W\times5\times10^6\ W/m^3\times0.020\ m = 50°C.$$

(d) For case 2, the maximum temperature in the composite wall occurs at x = -L, with the value,

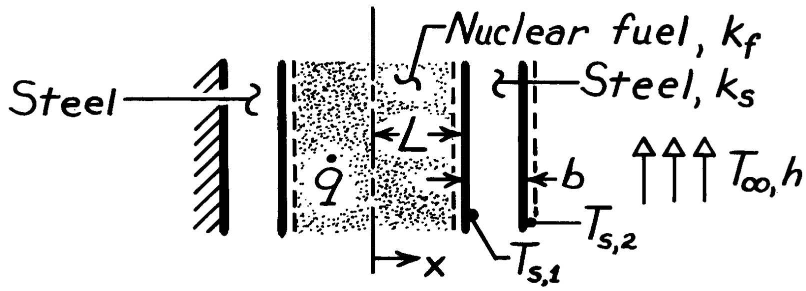
$$T_2(-L) = T_1(-L) + \Delta T_{AB} = 130°C + 50°C = 180°C$$ <

PROBLEM 3.77

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k_f}=0 \qquad (-L \le x \le +L)$$

is $$T=-\frac{\dot{q}}{2k_f}x^2+C_1x+C_2.$$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in}=\dot{E}_{out}=0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L}=-\frac{\dot{q}}{k_f}(-L)+C_1=0 \qquad \text{or} \qquad C_1=-\frac{\dot{q}L}{k_f}$$

$$T=-\frac{\dot{q}}{2k_f}x^2-\frac{\dot{q}L}{k_f}x+C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g=q_{cond}=q_{conv}$, or on a unit area basis.

$$\dot{q}(2L)=\frac{k_s}{b}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right).$$

Hence,

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+T_{s,2} \qquad \text{where} \qquad T_{s,2}=\frac{\dot{q}(2L)}{h}+T_\infty$$

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+\frac{\dot{q}(2L)}{h}+T_\infty.$$

Continued …..

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\;\frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

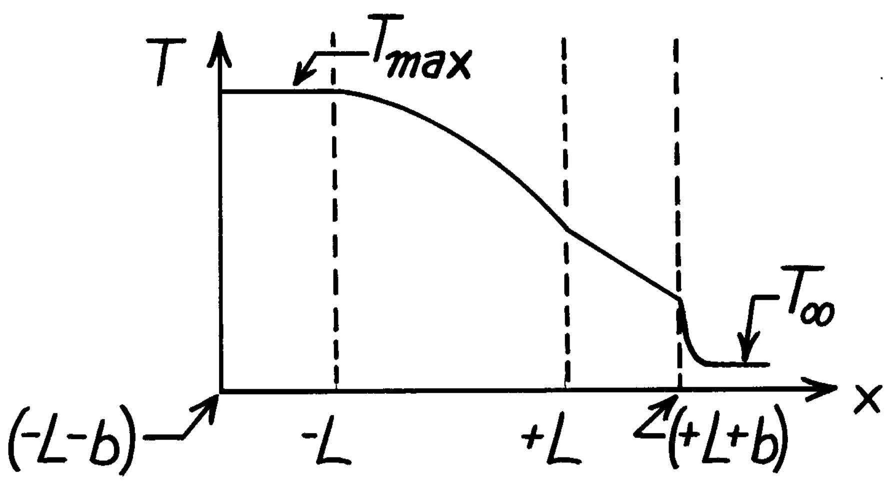
$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right] + T_\infty$$ <

(b) For the temperature distribution shown below,

$(-L-b) \le x \le -L$: $\quad dT/dx=0,\ T=T_{max}$
$-L \le x \le +L$: $\quad |dT/dx| \uparrow$ with $\uparrow$ x
$+L \le x \le L+b$: $\quad (dT/dx)$ is const.

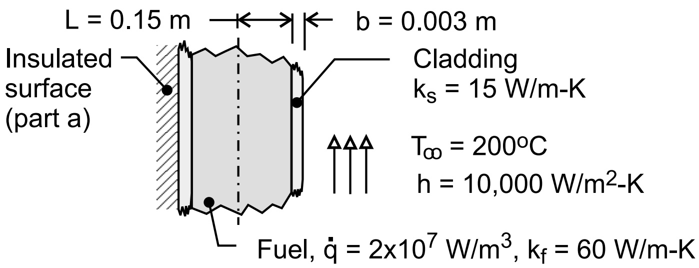


PROBLEM 3.78

KNOWN: Thermal conductivity, heat generation and thickness of fuel element. Thickness and thermal conductivity of cladding. Surface convection conditions.

FIND: (a) Temperature distribution in fuel element with one surface insulated and the other cooled by convection. Largest and smallest temperatures and corresponding locations. (b) Same as part (a) but with equivalent convection conditions at both surfaces, (c) Plot of temperature distributions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance.

ANALYSIS: (a) From Eq. C.1,

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{1}$$

With an insulated surface at x = -L, Eq. C.10 yields

$$T_{s,1} - T_{s,2} = \frac{2\dot{q}L^2}{k_f} \tag{2}$$

and with convection at x = L + b, Eq. C.13 yields

$$U\left(T_{s,2} - T_\infty\right) = \dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{3}$$

Substracting Eq. (2) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{4\dot{q}L^2}{k_f}$$

$$T_{s,2} = T_\infty + \frac{2\dot{q}L}{U} \tag{4}$$

Continued …..

and substituting into Eq. (2)

$$T_{s,1} = T_\infty + 2\dot{q}L\left(\frac{L}{k_f} + \frac{1}{U}\right) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1),

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2}{U} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty$$

or, with $U^{-1} = h^{-1} + b/k_s$,

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty \tag{6}$$ <

The maximum temperature occurs at x = - L and is

$$T(-L) = 2\dot{q}L\left(\frac{b}{k_s} + \frac{1}{h} + \frac{L}{k_f}\right) + T_\infty$$

$$T(-L) = 2\times 2\times 10^7\ \mathrm{W/m^3} \times 0.015\,\mathrm{m}\left(\frac{0.003\mathrm{m}}{15\,\mathrm{W/m\cdot K}} + \frac{1}{10,000\,\mathrm{W/m^2\cdot K}} + \frac{0.015\,\mathrm{m}}{60\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 530°\mathrm{C}$$ <

The lowest temperature is at x = + L and is

$$T(+L) = -\frac{3}{2}\frac{\dot{q}L^2}{k_f} + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty = 380°\mathrm{C}$$ <

(b) If a convection condition is maintained at x = - L, Eq. C.12 reduces to

$$U\left(T_\infty - T_{s,1}\right) = -\dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,1} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{7}$$

Subtracting Eq. (7) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty - T_{s,1} + T_\infty\right) \qquad \text{or} \qquad T_{s,1} = T_{s,2}$$

Hence, from Eq. (7)

Continued …..

PROBLEM 3.78 (Cont.)

$$T_{s,1} = T_{s,2} = \frac{\dot{q}L}{U} + T_\infty = \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (8)$$

Substituting into Eq. (1), the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (9)$$ $<$

The maximum temperature is at x = 0 and is

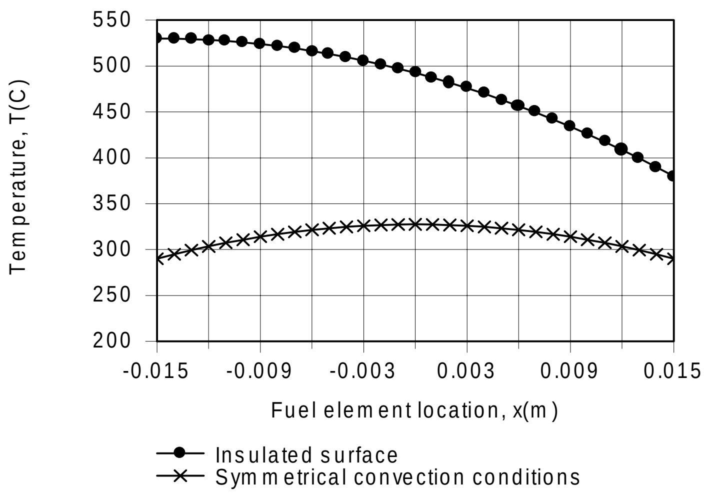
$$T(0) = \frac{2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})^2}{2\times 60\,\mathrm{W/m\cdot K}} + 2\times10^7\ \mathrm{W/m^3}\times 0.015\,\mathrm{m}\left(\frac{1}{10,000\,\mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C}$$

$$T(0) = 37.5°\mathrm{C} + 90°\mathrm{C} + 200°\mathrm{C} = 327.5°\mathrm{C}$$ $<$

The minimum temperature at x = ± L is

$$T_{s,1} = T_{s,2} = 2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})\left(\frac{1}{10,000\,\mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 290°\mathrm{C}$$ $<$

(c) The temperature distributions are as shown.



The amount of heat generation is the same for both cases, but the ability to transfer heat from both surfaces for case (b) results in lower temperatures throughout the fuel element.

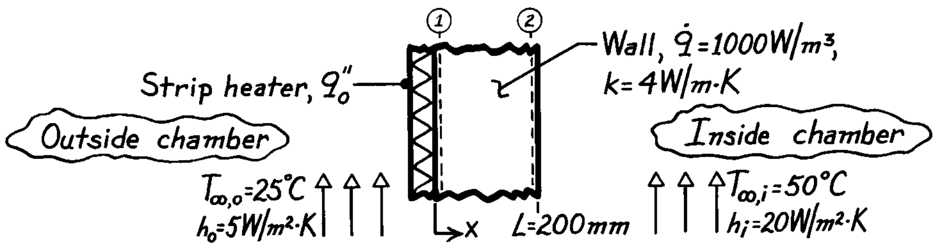
COMMENTS: Note that for case (a), the temperature in the insulated cladding is constant and equivalent to $T_{s,1}$ = 530°C.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

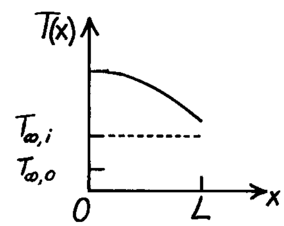
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \tag{1}$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \tag{2}$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow$ $T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \tag{3}$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \tag{4}$$

Continued …..

and from Eq. (3) with x = L and $T(L) = T_2$,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \tag{5,6}$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$
<

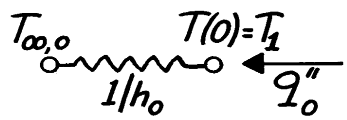
$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2 \times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$
<

Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

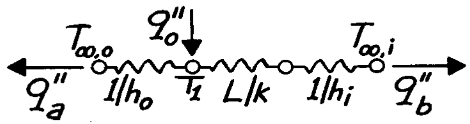


(c) The value of q''_o when $T(0) = T_1 = 65°C$ follows from the circuit

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\,\text{C} = 200\ \text{W/m}^2.$$
<

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

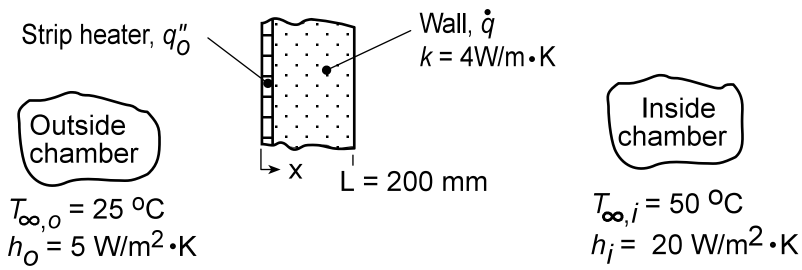
$$T_1 = 55^\circ\text{C}.$$
<

PROBLEM 3.80

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation and strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions ($T_{\infty,i}$, h_i, $T_{\infty,o}$, h_o). Strip heater acts to guard against heat losses from the wall to the outside.

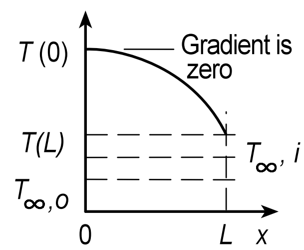
FIND: Compute and plot q''_o and T(0) as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i}$ = 30, 50 and 70°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: If no heat generated within the wall will be lost to the outside of the chamber, the gradient at the position x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution must be parabolic as shown in the sketch.



To determine the required heater flux q''_o as a function of the operation conditions $\dot{q}$ and $T_{\infty,i}$, the analysis begins by considering the temperature distribution in the wall and then surface energy balances at the two wall surfaces. The analysis is organized for easy treatment with equation-solving software.

Temperature distribution in the wall, T(x): The general solution for the temperature distribution in the wall is, Eq. 3.40,

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

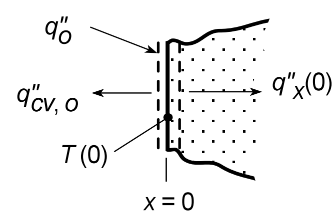
and the guard condition at the outer wall, x = 0, requires that the conduction heat flux be zero. Using Fourier's law,

$$q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0} = -kC_1 = 0 \qquad (C_1 = 0) \tag{1}$$

At the outer wall, x = 0,

$$T(0) = C_2 \tag{2}$$

Surface energy balance, x = 0:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

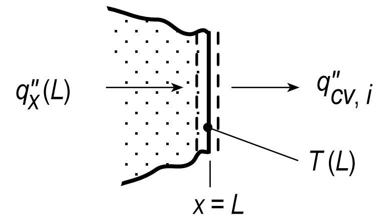
$$q''_o - q''_{cv,o} - q''_x(0) = 0 \tag{3}$$

$$q''_{cv,o} = h\left(T(0) - T_{\infty,o}\right), q''_x(0) = 0 \tag{4a,b}$$

Continued...

PROBLEM 3.80 (Cont.)

Surface energy balance, x = L:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

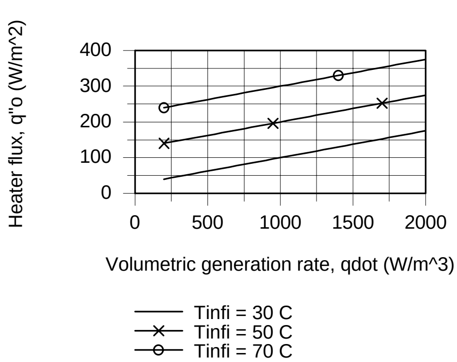
$$q''_x(L) - q''_{cv,i} = 0 \qquad (5)$$

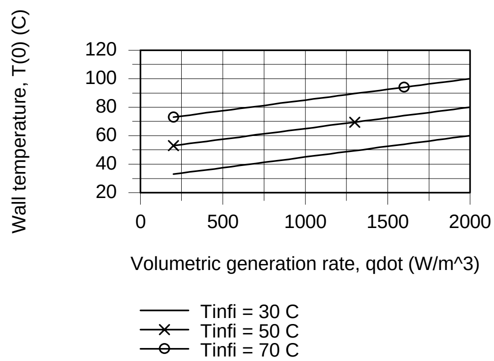
$$q''_x(L) = -k \left.\frac{dT}{dx}\right)_{x=L} = +\dot{q}L \qquad (6)$$

$$q''_{cv,i} = h\left[T(L) - T_{\infty,i}\right]$$

$$q''_{cv,i} = h\left[-\frac{\dot{q}}{2k}L^2 + T(0) - T_{\infty,i}\right] \qquad (7)$$

Solving Eqs. (1) through (7) simultaneously with appropriate numerical values and performing the parametric analysis, the results are plotted below.





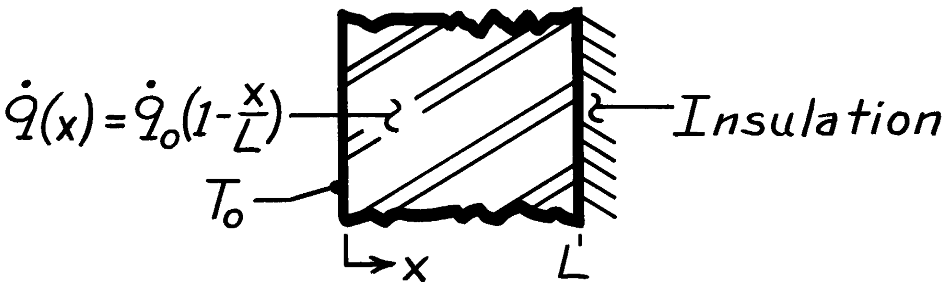
From the first plot, the heater flux q''_o is a linear function of the volumetric generation rate $\dot{q}$. As expected, the higher $\dot{q}$ and $T_{\infty,i}$, the higher the heat flux required to maintain the guard condition ($q''_x(0) = 0$). Notice that for any $\dot{q}$ condition, equal changes in $T_{\infty,i}$ result in equal changes in the required q''_o. The outer wall temperature T(0) is also linearly dependent upon $\dot{q}$. From our knowledge of the temperature distribution, it follows that for any $\dot{q}$ condition, the outer wall temperature T(0) will track changes in $T_{\infty,i}$.

PROBLEM 3.81

KNOWN: Plane wall with prescribed nonuniform volumetric generation having one boundary insulated and the other isothermal.

FIND: Temperature distribution, T(x), in terms of x, L, k, $\dot{q}_o$ and T_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) Constant properties.

ANALYSIS: The appropriate form the heat diffusion equation is

$$\frac{d}{dx}\left[\frac{dT}{dx}\right]+\frac{\dot{q}}{k}=0.$$

Noting that $\dot{q}=\dot{q}(x)=\dot{q}_o\ (1-x/L)$, substitute for $\dot{q}(x)$ into the above equation, separate variables and then integrate,

$$d\left[\frac{dT}{dx}\right]=-\frac{\dot{q}_o}{k}\left[1-\frac{x}{L}\right]dx \qquad \frac{dT}{dx}=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]+C_1.$$

Separate variables and integrate again to obtain the general form of the temperature distribution in the wall,

$$dT=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]dx+C_1dx \qquad T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+C_1x+C_2.$$

Identify the boundary conditions at x = 0 and x = L to evaluate C_1 and C_2. At x = 0,

$$T(0)=T_o=-\frac{\dot{q}_o}{k}(0-0)+C_1\cdot 0+C_2 \qquad \text{hence, } C_2=T_o$$

At x = L,

$$\left.\frac{dT}{dx}\right]_{x=L}=0=-\frac{\dot{q}_o}{k}\left[L-\frac{L^2}{2L}\right]+C_1 \qquad \text{hence, } C_1=\frac{\dot{q}_oL}{2k}$$

The temperature distribution is

$$T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+\frac{\dot{q}_oL}{2k}x+T_o.$$ <

COMMENTS: It is good practice to test the final result for satisfying BCs. The heat flux at x = 0 can be found using Fourier's law or from an overall energy balance

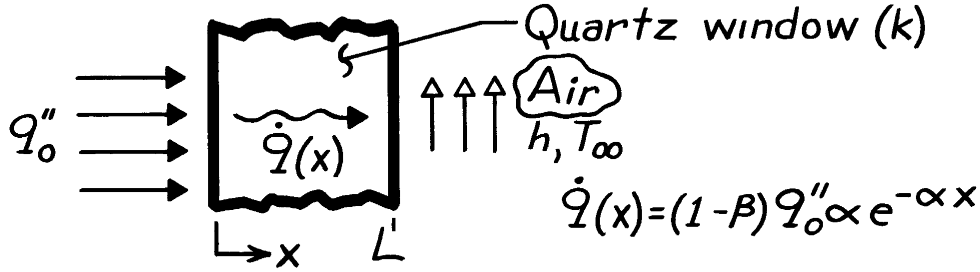
$$\dot{E}_{out}=\dot{E}_g=\int_0^L \dot{q}dV \qquad \text{to obtain} \qquad q''_{out}=\dot{q}_oL/2.$$

PROBLEM 3.82

KNOWN: Distribution of volumetric heating and surface conditions associated with a quartz window.

FIND: Temperature distribution in the quartz.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation emission and convection at inner surface (x = 0) and negligible emission from outer surface, (4) Constant properties.

ANALYSIS: The appropriate form of the heat equation for the quartz is obtained by substituting the prescribed form of $\dot{q}$ into Eq. 3.39.

$$\frac{d^2T}{dx^2}+\frac{\alpha(1-\beta)q_o''}{k}e^{-\alpha x}=0$$

Integrating,

$$\frac{dT}{dx}=+\frac{(1-\beta)q_o''}{k}e^{-\alpha x}+C_1 \qquad T=-\frac{(1-\beta)}{k\alpha}q_o''e^{-\alpha x}+C_1x+C_2$$

Boundary Conditions:

$$-k\ dT/dx)_{x=o}=\beta q_o''$$
$$-k\ dT/dx)_{x=L}=h\left[T(L)-T_\infty\right]$$

Hence, at x = 0:

$$-k\left[\frac{(1-\beta)}{k}q_o''+C_1\right]=\beta q_o''$$
$$C_1=-q_o''/k$$

At x = L:

$$-k\left[\frac{(1-\beta)}{k}q_o''e^{-\alpha L}+C_1\right]=h\left[-\frac{(1-\beta)}{k\alpha}q_o''e^{-\alpha L}+C_1L+C_2-T_\infty\right]$$

Substituting for C_1 and solving for C_2,

$$C_2=\frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+\frac{q_o''}{k}+\frac{q_o''(1-\beta)}{k\alpha}e^{-\alpha L}+T_\infty.$$

Hence,

$$T(x)=\frac{(1-\beta)q_o''}{k\alpha}\left[e^{-\alpha L}-e^{-\alpha x}\right]+\frac{q_o''}{k}(L-x)+\frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+T_\infty. \quad <$$

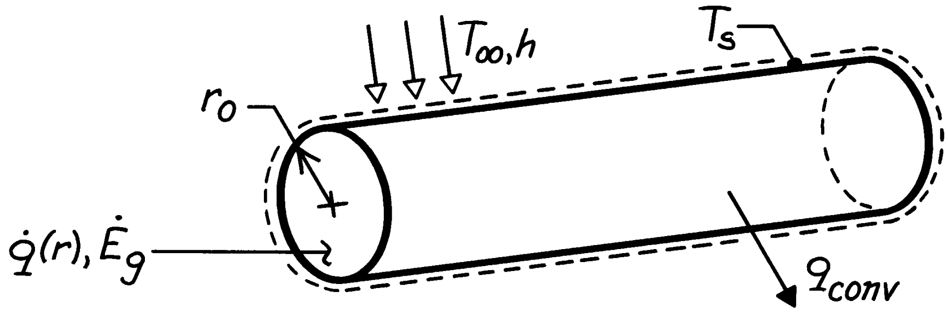
COMMENTS: The temperature distribution depends strongly on the radiative coefficients, α and β. For $\alpha \to \infty$ or $\beta = 1$, the heating occurs entirely at x = 0 (no volumetric heating).

PROBLEM 3.83

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left(1-\frac{r^2}{r_o^2}\right)$$

$$r\frac{dT}{dr} = -\frac{\dot{q}_o r^2}{2k} + \frac{\dot{q} r^4}{4kr_o^2} + C_1 \qquad T = -\frac{\dot{q}_o r^2}{4k} + \frac{\dot{q}_o r^4}{16kr_o^2} + C_1 \ln r + C_2.$$

From the boundary conditions,

$$\frac{dT}{dr}\Big|_{r=0} = 0 \rightarrow C_1 = 0 \qquad -k\frac{dT}{dr}\Big|_{r=r_o} = h\left[T(r_o) - T_\infty\right]$$

$$+\frac{\dot{q}_o r_o}{2} - \frac{\dot{q}_o r_o}{4} = h\left[-\frac{\dot{q}_o r_o^2}{4k} + \frac{\dot{q}_o r_o^2}{16k} + C_2 - T_\infty\right]$$

$$C_2 = \frac{\dot{q}_o r_o}{4h} + \frac{3\dot{q}_o r_o^2}{16k} + T_\infty.$$

Hence

$$T(r) = T_\infty + \frac{\dot{q}_o r_o}{4h} + \frac{\dot{q}_o r_o^2}{k}\left[\frac{3}{16} - \frac{1}{4}\left(\frac{r}{r_o}\right)^2 + \frac{1}{16}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (\dot{q}_o r_o)/4h$$

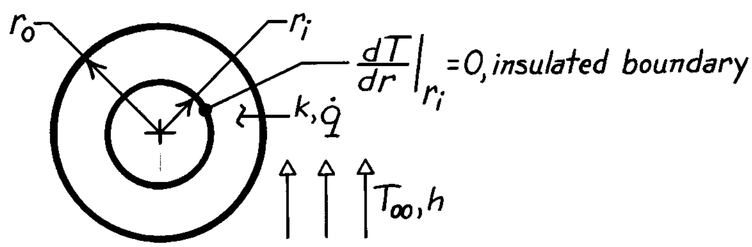
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of $r_i,\ r_o,\ \dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $$\left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k} r_i + C_1 \frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k} r_i^2$$

at $r = r_o$: $$-k\left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad \text{surface energy balance}$$

$$k\left[-\frac{\dot{q}}{2k} r_o + \left(\frac{\dot{q}}{2k} r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k} r_o^2 + \left(\frac{\dot{q}}{2k} r_i^2\right)\ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q} r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q} r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q} r_i^2}{2k} \ln\left(\frac{r}{r_o}\right) - \frac{\dot{q} r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

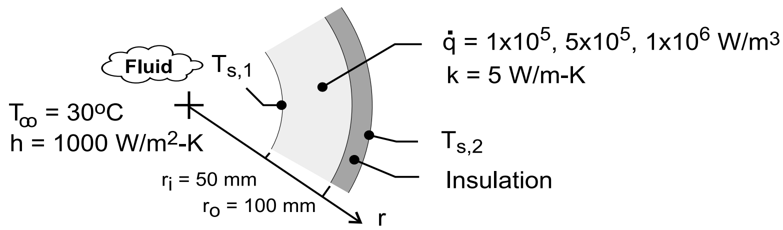
$$q_r'(r) = -k(2\pi\ r_o)\left.\frac{dT}{dr}\right)_{r_o} = -2\pi\ k r_o\left[-\frac{\dot{q}}{2k} r_o + \frac{\dot{q} r_i^2}{2k}\ \frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.85

KNOWN: The solid tube of Example 3.7 with inner and outer radii, 50 and 100 mm, and a thermal conductivity of 5 W/m·K. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 $W/m^2 \cdot K$.

FIND: Calculate and plot the temperature distributions for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. Use Eq. (7) with Eq. (10) of the Example 3.7 in the *IHT Workspace*.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties and (4) Uniform volumetric generation.

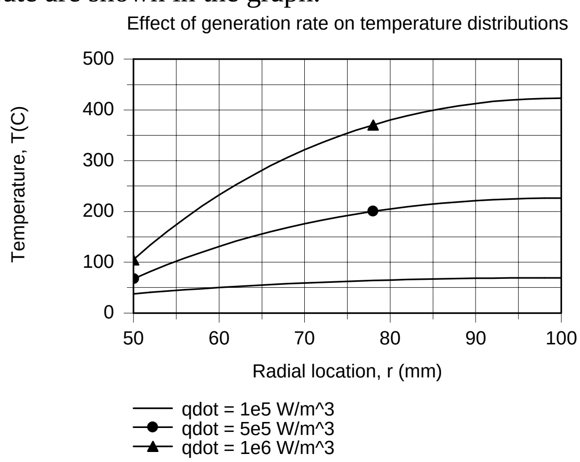
ANALYSIS: From Example 3.7, the temperature distribution in the tube is given by Eq. (7),

$$T(r) = T_{s,2} + \frac{\dot{q}}{4k}\left(r_2^2 - r^2\right) - \frac{\dot{q}}{2k} r_2^2 \ell n\left(\frac{r_2}{r}\right) \qquad r_1 \le r \le r_2 \qquad (1)$$

The temperature at the inner boundary, $T_{s,1}$, follows from the surface energy balance, Eq. (10),

$$\pi \dot{q}\left(r_2^2 - r_1^2\right) = h 2\pi r_1\left(T_{s,1} - T_\infty\right) \qquad (2)$$

For the conditions prescribed in the schematic with $\dot{q} = 1 \times 10^5 W/m^3$, Eqs. (1) and (2), with r = r_1 and T(r) = $T_{s,1}$, are solved simultaneously to find $T_{s,2}$ = 69.3°C. Eq. (1), with $T_{s,2}$ now a known parameter, can be used to determine the temperature distribution, T(r). The results for different values of the generation rate are shown in the graph.



COMMENTS: (1) The temperature distributions are parabolic with a zero gradient at the insulated outer boundary, r = r_2. The effect of increasing $\dot{q}$ is to increase the maximum temperature in the tube, which always occurs at the outer boundary.

(2) The equations used to generate the graphical result in the *IHT Workspace* are shown below.

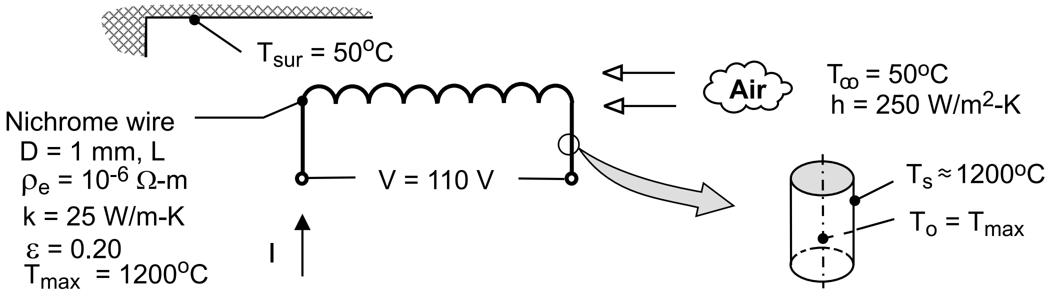
```
// The temperature distribution, from Eq. 7, Example 3.7
T_r = Ts2 + qdot/(4*k) * (r2^2 – r^2) – qgot / (2*k) * r2^2*ln (r2/r)
// The temperature at the inner surface, from Eq. 7
Ts1 = Ts2 + qdot / (4*k) * (r2^2 – r1^2) – qdot / (2*k) * r2^2 * ln (r2/r1)
// The energy balance on the surface, from Eq. 10
pi * qdot * (r2^2 – r1^2) = h * 2 * pi * r1 * (Ts1 – Tinf)
```

PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.55), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(1473+323)\,K\left(1473^2+323\right)^2 K^2 = 46.3\,W/m^2\cdot K$$

$$h_t = (250+46.3)\,W/m^2\cdot K = 296.3\,W/m^2\cdot K$$

$$\dot{q}_{max} = \frac{2h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.3\,W/m^2\cdot K\right)}{0.0005m}(1150°C) = 1.36\times10^9\,W/m^3$$

Hence, with $\quad \dot{q} = \dfrac{I^2 R_e}{\forall} = \dfrac{I^2\left(\rho_e L/A_c\right)}{LA_c} = \dfrac{I^2\rho_e}{A_c^2} = \dfrac{I^2\rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2}\frac{\pi D^2}{4} = \left(\frac{1.36\times10^9\,W/m^3}{10^{-6}\,\Omega\cdot m}\right)^{1/2}\frac{\pi\,(0.001m)^2}{4} = 29.0\,A \qquad <$$

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E\cdot A_c}{I_{max}\,\rho_e} = \frac{110\,V\left[\pi\,(0.001m)^2/4\right]}{29.0\,A\left(10^{-6}\,\Omega\cdot m\right)} = 2.98m \qquad <$$

and the power rating is

$$P_{elec} = \Delta E\cdot I_{max} = 110\,V\,(29\,A) = 3190\,W = 3.19\,kW \qquad <$$

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.53) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \dfrac{\dot{q}\,r_o^2}{4k} + T_s = \dfrac{1.36\times10^9\,W/m^3\,(0.0005m)^2}{4\,(25\,W/m\cdot K)} + 1200°C = 1203°C$. With only a

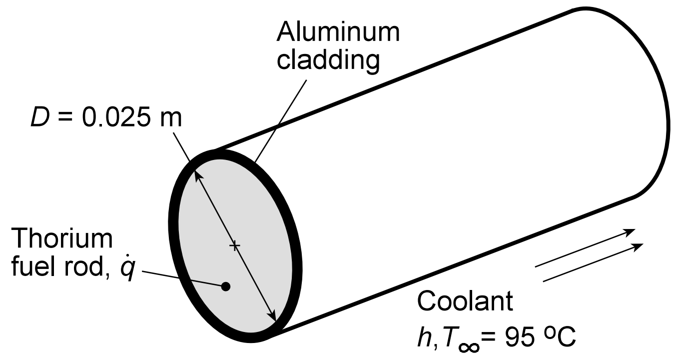
3°C temperature difference between the centerline and surface of the wire, the assumption is *excellent*.

PROBLEM 3.87

KNOWN: Energy generation in an aluminum-clad, thorium fuel rod under specified operating conditions.

FIND: (a) Whether prescribed operating conditions are acceptable, (b) Effect of $\dot{q}$ and h on acceptable operating conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r-direction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible temperature gradients in aluminum and contact resistance between aluminum and thorium.

PROPERTIES: *Table A-1*, Aluminum, pure: M.P. = 933 K; *Table A-1*, Thorium: M.P. = 2023 K, k ≈ 60 W/m·K.

ANALYSIS: (a) System failure would occur if the melting point of either the thorium or the aluminum were exceeded. From Eq. 3.53, the maximum thorium temperature, which exists at r = 0, is

$$T(0) = \frac{\dot{q} r_o^2}{4k} + T_s = T_{Th,max}$$

where, from the energy balance equation, Eq. 3.55, the surface temperature, which is also the aluminum temperature, is

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} = T_{Al}$$

Hence,

$$T_{Al} = T_s = 95^\circ C + \frac{7\times10^8\ W/m^3 \times 0.0125\,m}{14,000\ W/m^2\cdot K} = 720^\circ C = 993\,K$$

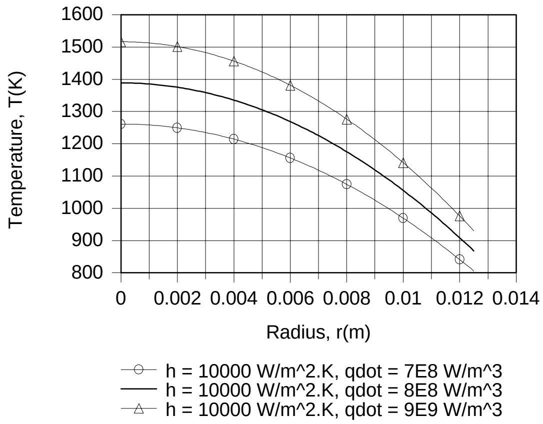
$$T_{Th,max} = \frac{7\times10^8\ W/m^3\,(0.0125m)^2}{4\times60\ W/m\cdot K} + 993\,K = 1449\,K \qquad <$$

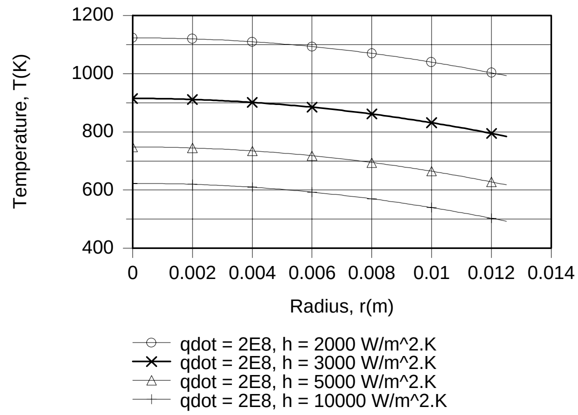
Although $T_{Th,max}$ < M.P.$_{Th}$ and the thorium would not melt, T_{al} > M.P.$_{Al}$ and the cladding would melt under the proposed operating conditions. The problem could be eliminated by *decreasing* $\dot{q}$, *increasing* h or using a cladding material with a higher melting point.

(b) Using the one-dimensional, steady-state conduction model (solid cylinder) of the IHT software, the following radial temperature distributions were obtained for parametric variations in $\dot{q}$ and h.

Continued...

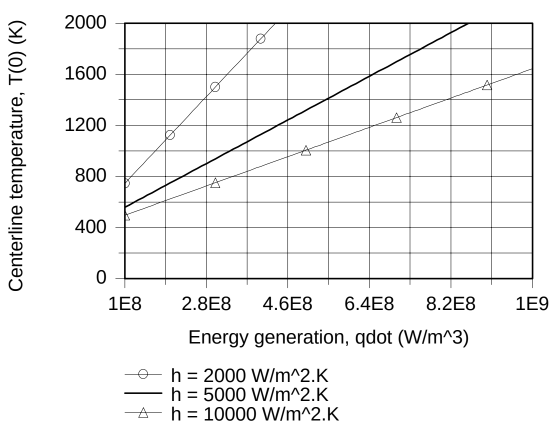
PROBLEM 3.87 (Cont.)

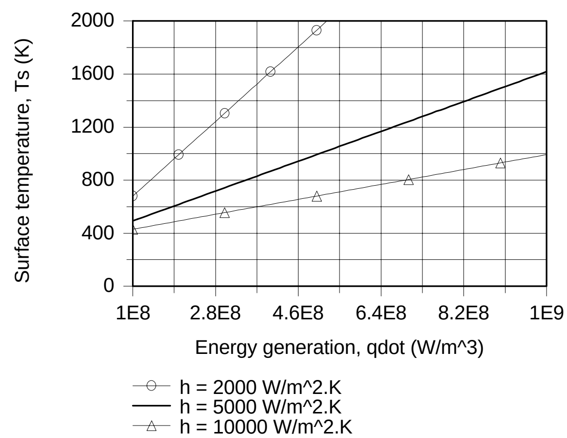




For h = 10,000 W/m$^2\cdot$K, which represents a reasonable upper limit with water cooling, the temperature of the aluminum would be well below its melting point for $\dot{q} = 7 \times 10^8$ W/m^3, but would be close to the melting point for $\dot{q} = 8 \times 10^8$ W/m^3 and would exceed it for $\dot{q} = 9 \times 10^8$ W/m^3. Hence, under the best of conditions, $\dot{q} \approx 7 \times 10^8$ W/m^3 corresponds to the maximum allowable energy generation. However, if coolant flow conditions are constrained to provide values of h < 10,000 W/m$^2\cdot$K, volumetric heating would have to be reduced. Even for $\dot{q}$ as low as 2×10^8 W/m^3, operation could not be sustained for h = 2000 W/m$^2\cdot$K.

The effects of $\dot{q}$ and h on the centerline and surface temperatures are shown below.





For h = 2000 and 5000 W/m$^2\cdot$K, the melting point of thorium would be approached for $\dot{q} \approx 4.4 \times 10^8$ and 8.5×10^8 W/m^3, respectively. For h = 2000, 5000 and 10,000 W/m$^2\cdot$K, the melting point of aluminum would be approached for $\dot{q} \approx 1.6 \times 10^8$, 4.3×10^8 and 8.7×10^8 W/m^3. Hence, the envelope of acceptable operating conditions must call for a reduction in $\dot{q}$ with decreasing h, from a maximum of $\dot{q} \approx 7 \times 10^8$ W/m^3 for h = 10,000 W/m$^2\cdot$K.

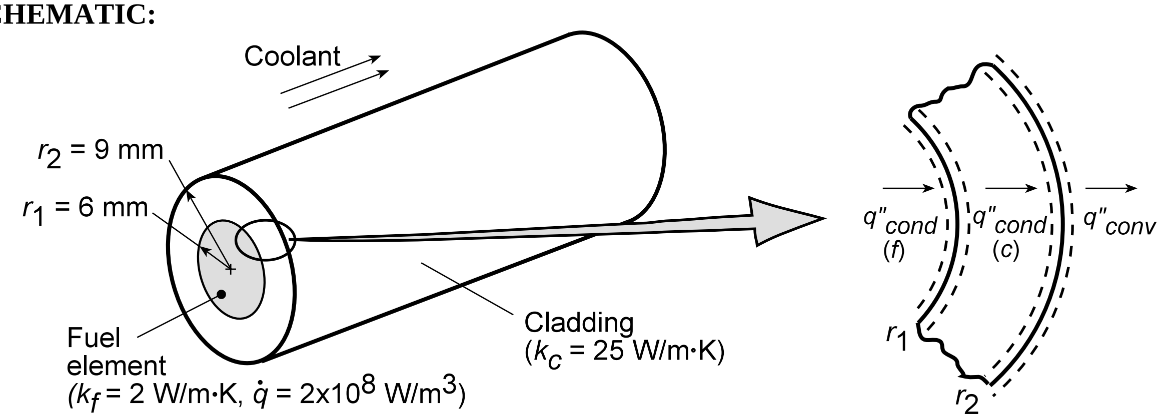
COMMENTS: Note the problem which would arise in the event of a *loss of coolant*, for which case h would *decrease* drastically.

PROBLEM 3.88

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.49 and 3.23, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(\frac{dT_f}{dr}\right)=-\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr}=-\frac{\dot{q}r}{2k_f}+\frac{C_1}{k_f r} \qquad T_f=-\frac{\dot{q}r^2}{4k_f}+\frac{C_1}{k_f}\ln r+C_2 \tag{1,2}$$

$$\frac{dT_c}{dr}=\frac{C_3}{k_c r} \qquad T_c=\frac{C_3}{k_c}\ln r+C_4 \tag{3,4}$$

The corresponding boundary conditions are:

$$dT_f/dr\big)_{r=0}=0 \qquad T_f(r_1)=T_c(r_1) \tag{5,6}$$

$$-k_f\frac{dT_f}{dr}\bigg)_{r=r_1}=-k_c\frac{dT_c}{dr}\bigg)_{r=r_1} \qquad -k_c\frac{dT_c}{dr}\bigg)_{r=r_2}=h\left[T_c(r_2)-T_\infty\right] \tag{7,8}$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f=-\frac{\dot{q}r^2}{4k_f}+C_2 \tag{9}$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f}+C_2=\frac{C_3\ln r_1}{k_c}+C_4 \tag{10}$$

Continued...

PROBLEM 3.88 (Cont.)

Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h}T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty \tag{14}$$ <

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty\,. \tag{15}$$ <

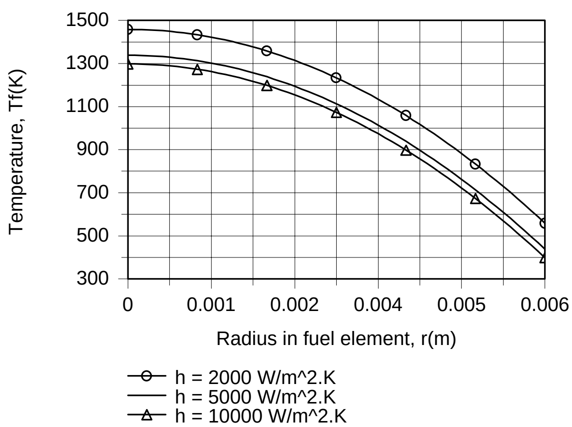
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 $W/m^2 \cdot K$ is

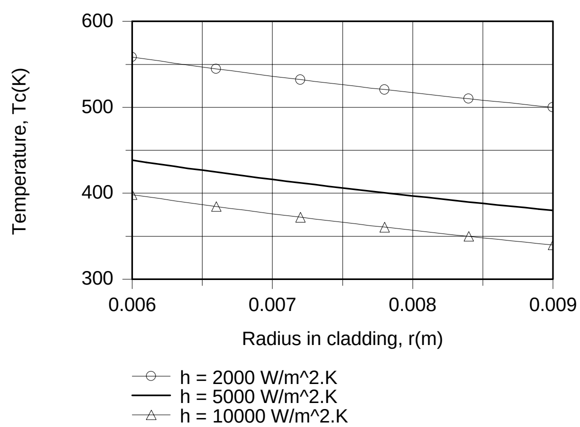
$$T_f(0) = \frac{2\times10^8\,W/m^3 \times (0.006\,m)^2}{4\times 2\,W/m\cdot K} + \frac{2\times10^8\,W/m^3 \times (0.006\,m)^2}{2\times 25\,W/m\cdot K}\ln\frac{0.009\,m}{0.006\,m}$$

$$+\frac{2\times10^8\;W/m^3\,(0.006\,m)^2}{2\times(0.09\,m)\,2000\,W/m^2\cdot K} + 300\,K$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,K = 1458\,K\,.$$ <

(c) Temperature distributions for the prescribed values of h are as follows:





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Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \rightarrow \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

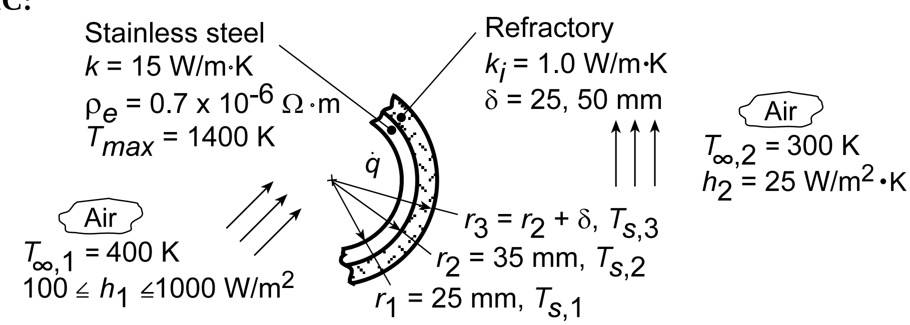
COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2/4k_f = (2\times10^8\ W/m^3)(0.006\ m)^3/8\ W/m\cdot K = 900\ K$, in which case, with no cladding and $h \rightarrow \infty$, $T_f(0) = 1200$ K. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.

PROBLEM 3.89

KNOWN: Dimensions and properties of tubular heater and external insulation. Internal and external convection conditions. Maximum allowable tube temperature.

FIND: (a) Maximum allowable heater current for adiabatic outer surface, (3) Effect of internal convection coefficient on heater temperature distribution, (c) Extent of heat loss at outer surface.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform heat generation, (4) Negligible radiation at outer surface, (5) Negligible contact resistance.

ANALYSIS: (a) From Eqs. 7 and 10, respectively, of Example 3.7, we know that

$$T_{s,2} - T_{s,1} = \frac{\dot{q}}{2k} r_2^2 \ln\frac{r_2}{r_1} - \frac{\dot{q}}{4k}\left(r_2^2 - r_1^2\right) \tag{1}$$

and

$$T_{s,1} = T_{\infty,1} + \frac{\dot{q}\left(r_2^2 - r_1^2\right)}{2h_1 r_1} \tag{2}$$

Hence, eliminating $T_{s,1}$, we obtain

$$T_{s,2} - T_{\infty,1} = \frac{\dot{q}r_2^2}{2k}\left[\ln\frac{r_2}{r_1} - \frac{1}{2}\left(1 - r_1^2\big/r_2^2\right) + \frac{k}{h_1 r_1}\left(1 - r_1^2\big/r_2^2\right)\right]$$

Substituting the prescribed conditions (h_1 = 100 W/m^2·K),

$$T_{s,2} - T_{\infty,1} = 1.237\times10^{-4}\left(m^3\cdot K\big/W\right)\dot{q}\left(W\big/m^3\right)$$

Hence, with T_{max} corresponding to $T_{s,2}$, the maximum allowable value of $\dot{q}$ is

$$\dot{q}_{max} = \frac{1400-400}{1.237\times10^{-4}} = 8.084\times10^6\ W\big/m^3$$

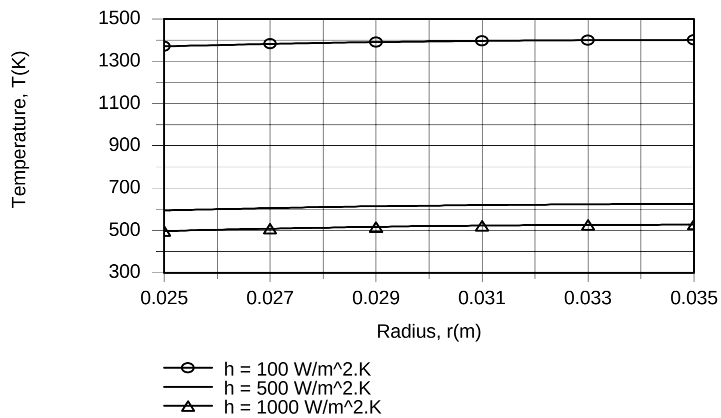
with

$$\dot{q} = \frac{I^2\,Re}{\forall} = \frac{I^2 \rho_e\, L/A_c}{LA_c} = \frac{\rho_e I^2}{\left[\pi\left(r_2^2 - r_1\right)\right]^2}$$

$$I_{max} = \pi\left(r_2^2 - r_1^2\right)\left(\frac{\dot{q}}{\rho_e}\right)^{1/2} = \pi\left(0.035^2 - 0.025^2\right)m^2\left(\frac{8.084\times10^6\ W\big/m^3}{0.7\times10^{-6}\,\Omega\cdot m}\right)^{1/2} = 6406\,A \quad <$$

Continued …..

(b) Using the one-dimensional, steady-state conduction model of IHT (hollow cylinder; convection at inner surface and adiabatic outer surface), the following temperature distributions were obtained.



The results are consistent with key implications of Eqs. (1) and (2), namely that the value of h_1 has no effect on the temperature drop across the tube ($T_{s,2} - T_{s,1} = 30$ K, irrespective of h_1), while $T_{s,1}$ decreases with increasing h_1. For h_1 = 100, 500 and 1000 W/m$^2\cdot$K, respectively, the ratio of the temperature drop between the inner surface and the air to the temperature drop across the tube, $(T_{s,1} - T_{\infty,1})/(T_{s,2} - T_{s,1})$, decreases from 970/30 = 32.3 to 194/30 = 6.5 and 97/30 = 3.2. Because the outer surface is insulated, the heat rate to the airflow is fixed by the value of $\dot{q}$ and, irrespective of h_1,

$$q'(r_1) = \pi\left(r_2^2 - r_1^2\right)\dot{q} = -15,240\,\text{W} \qquad <$$

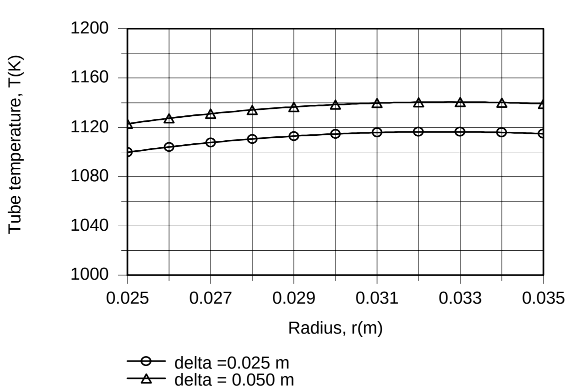
(c) Heat loss from the outer surface of the tube to the surroundings depends on the total thermal resistance

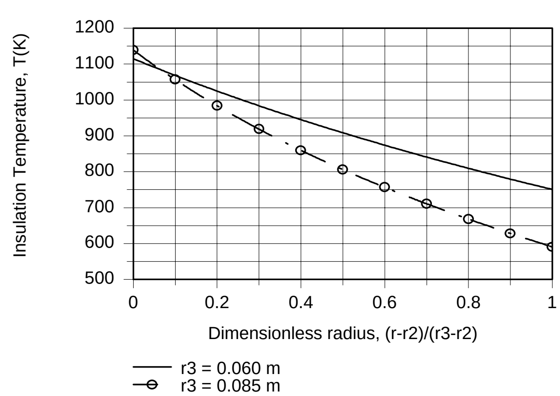
$$R_{tot} = \frac{\ln(r_3/r_2)}{2\pi L k_i} + \frac{1}{2\pi r_3 L h_2}$$

or, for a unit area on surface 2,

$$R''_{tot,2} = (2\pi r_2 L) R_{tot} = \frac{r_2 \ln(r_3/r_2)}{k_i} + \frac{r_2}{r_3 h_2}$$

Again using the capabilities of IHT (hollow cylinder; convection at inner surface and heat transfer from outer surface through $R''_{tot,2}$), the following temperature distributions were determined for the tube and insulation.





Continued...

PROBLEM 3.89 (Cont.)

Heat losses through the insulation, $q'(r_2)$, are 4250 and 3890 W/m for δ = 25 and 50 mm, respectively, with corresponding values of $q'(r_1)$ equal to -10,990 and -11,350 W/m. Comparing the tube temperature distributions with those predicted for an adiabatic outer surface, it is evident that the losses reduce tube wall temperatures predicted for the adiabatic surface and also shift the maximum temperature from r = 0.035 m to r ≈ 0.033 m. Although the tube outer and insulation inner surface temperatures, $T_{s,2} = T(r_2)$, increase with increasing insulation thickness, Fig. (c), the insulation outer surface temperature decreases.

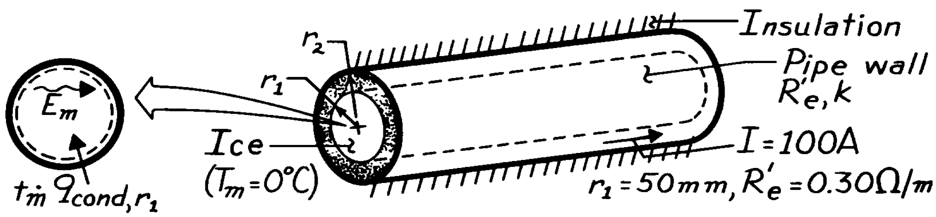
COMMENTS: If the intent is to maximize heat transfer to the airflow, heat losses to the ambient should be reduced by selecting an insulation material with a significantly smaller thermal conductivity.

PROBLEM 3.90

KNOWN: Electric current I is passed through a pipe of resistance R'_e to melt ice under steady-state conditions.

FIND: (a) Temperature distribution in the pipe wall, (b) Time to completely melt the ice.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation in the pipe wall, (5) Outer surface of the pipe is adiabatic, (6) Inner surface is at a constant temperature, T_m.

PROPERTIES: *Table A-3,* Ice (273K): ρ = 920 kg/m^3; *Handbook Chem. & Physics,* Ice: Latent heat of fusion, $h_{sf} = 3.34\times10^5$ J/kg.

ANALYSIS: (a) The appropriate form of the heat equation is Eq. 3.49, and the general solution, Eq. 3.51 is

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1\ln r + C_2$$

where

$$\dot{q} = \frac{I^2 R'_e}{\pi\left(r_2^2 - r_1^2\right)}.$$

Applying the boundary condition $(dT/dr)_{r_2} = 0,$ it follows that

$$0 = \frac{\dot{q}r_2}{2k} + \frac{C_1}{r_2}$$

Hence $\qquad C_1 = \dfrac{\dot{q}r_2^2}{2k}$

and $\qquad T(r) = -\dfrac{\dot{q}}{4k}r^2 + \dfrac{\dot{q}r_2^2}{2k}\ln r + C_2.$

Continued …..

PROBLEM 3.90 (Cont.)

Applying the second boundary condition, $T(r_1) = T_m$, it follows that

$$T_m = -\frac{\dot{q}}{4k} r_1^2 + \frac{\dot{q} r_2^2}{2k} \ln r_1 + C_2.$$

Solving for C_2 and substituting into the expression for T(r), find

$$T(r) = T_m + \frac{\dot{q} r_2^2}{2k} \ln \frac{r}{r_1} - \frac{\dot{q}}{4k}\left(r^2 - r_1^2\right). \qquad <$$

(b) Conservation of energy dictates that the energy required to completely melt the ice, E_m, must equal the energy which reaches the inner surface of the pipe by conduction through the wall during the melt period. Hence from Eq. 1.11b

$$\Delta E_{st} = E_{in} - E_{out} + E_{gen}$$

$$\Delta E_{st} = E_m = t_m \cdot q_{cond,r_1}$$

or, for a unit length of pipe,

$$\rho\left(\pi\, r_1^2\right) h_{sf} = t_m\left[-k\left(2\pi\, r_1\right)\left[\frac{dT}{dr}\right]_{r_1}\right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -2\pi r_1 k t_m \left[\frac{\dot{q} r_2^2}{2k r_1} - \frac{\dot{q} r_1}{2k}\right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -t_m \dot{q} \pi \left(r_2^2 - r_1^2\right).$$

Dropping the minus sign, which simply results from the fact that conduction is in the negative r direction, it follows that

$$t_m = \frac{\rho h_{sf} r_1^2}{\dot{q}\left(r_2^2 - r_1^2\right)} = \frac{\rho h_{sf} \pi\, r_1^2}{I^2 R_e'}.$$

With r_1 = 0.05m, I = 100 A and $R_e' = 0.30$ Ω/m, it follows that

$$t_m = \frac{920\,\text{kg/m}^3 \times 3.34 \times 10^5\,\text{J/kg} \times \pi \times (0.05\,\text{m})^2}{(100\,\text{A})^2 \times 0.30\,\Omega/\text{m}}$$

or $\qquad t_m = 804\,\text{s}. \qquad <$

COMMENTS: The foregoing expression for t_m could also be obtained by recognizing that all of the energy which is generated by electrical heating in the pipe wall must be transferred to the ice. Hence,

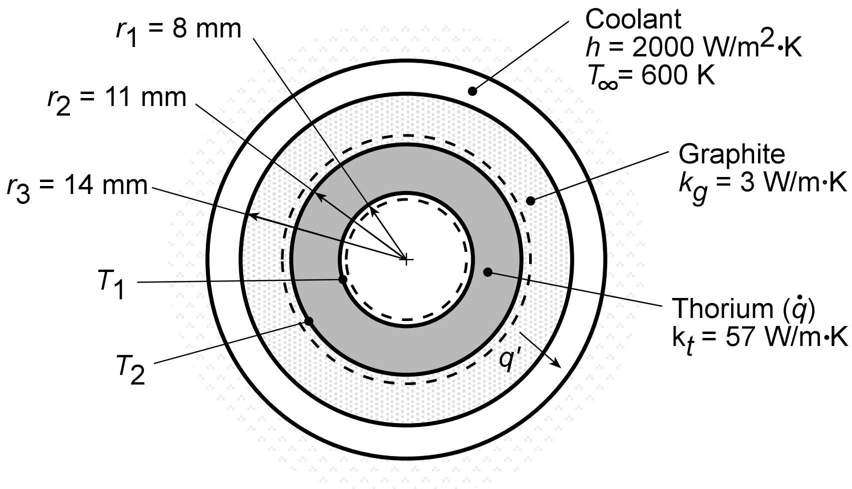
$$I^2 R_e' t_m = \rho h_{sf} \pi\, r_1^2.$$

PROBLEM 3.91

KNOWN: Materials, dimensions, properties and operating conditions of a gas-cooled nuclear reactor.

FIND: (a) Inner and outer surface temperatures of fuel element, (b) Temperature distributions for different heat generation rates and maximum allowable generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Thoriun: $T_{mp} \approx 2000$ K; *Table A.2*, Graphite: $T_{mp} \approx 2300$ K.

ANALYSIS: (a) The outer surface temperature of the fuel, T_2, may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{tot}}$$

where

$$R'_{tot} = \frac{\ln(r_3/r_2)}{2\pi\, k_g} + \frac{1}{2\pi r_3 h} = \frac{\ln(14/11)}{2\pi\,(3\,\mathrm{W/m\cdot K})} + \frac{1}{2\pi\,(0.014\,\mathrm{m})\left(2000\,\mathrm{W/m^2\cdot K}\right)} = 0.0185\,\mathrm{m\cdot K/W}$$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 10^8\,\mathrm{W/m^3} \times \pi\left(0.011^2 - 0.008^2\right)\mathrm{m}^2 = 17{,}907\,\mathrm{W/m}$$

Hence,

$$T_2 = q'R'_{tot} + T_\infty = 17{,}907\,\mathrm{W/m}\,(0.0185\,\mathrm{m\cdot K/W}) + 600\,\mathrm{K} = 931\,\mathrm{K}$$ <

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right)$$

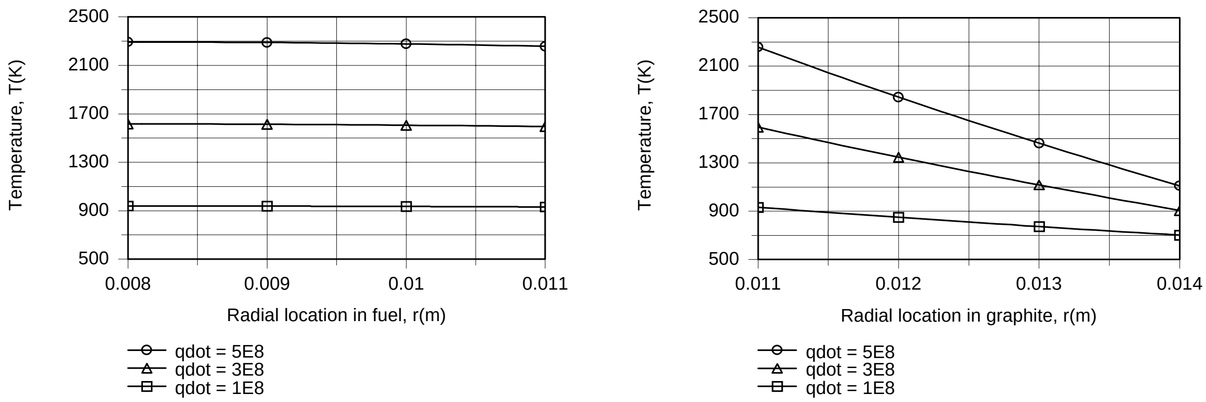
$$T_1 = 931\,\mathrm{K} + \frac{10^8\,\mathrm{W/m^3}\,(0.011\,\mathrm{m})^2}{4\times 57\,\mathrm{W/m\cdot K}}\left[1 - \left(\frac{0.008}{0.011}\right)^2\right] - \frac{10^8\,\mathrm{W/m^3}\,(0.008\,\mathrm{m})^2}{2\times 57\,\mathrm{W/m\cdot K}}\ln\left(\frac{0.011}{0.008}\right)$$

Continued...

PROBLEM 3.91 (Cont.)

$$T_1 = 931\,K + 25\,K - 18\,K = 938\,K$$ <

(b) The temperature distributions may be obtained by using the IHT model for one-dimensional, steady-state conduction in a hollow tube. For the fuel element ($\dot{q} > 0$), an adiabatic surface condition is prescribed at r_1, while heat transfer from the outer surface at r_2 to the coolant is governed by the thermal resistance $R''_{tot,2} = 2\pi r_2 R'_{tot} = 2\pi(0.011\text{ m})0.0185\text{ m}\cdot\text{K/W} = 0.00128\text{ m}^2\cdot\text{K/W}$. For the graphite ($\dot{q}$ = 0), the value of T_2 obtained from the foregoing solution is prescribed as an inner boundary condition at r_2, while a convection condition is prescribed at the outer surface (r_3). For $1 \times 10^8 \le \dot{q} \le 5 \times 10^8\text{ W/m}^3$, the following distributions are obtained.



The comparatively large value of k_t yields small temperature variations across the fuel element, while the small value of k_g results in large temperature variations across the graphite. Operation at $\dot{q} = 5 \times 10^8\text{ W/m}^3$ is clearly unacceptable, since the melting points of thorium and graphite are exceeded and approached, respectively. To prevent softening of the materials, which would occur below their melting points, the reactor should not be operated much above $\dot{q} = 3 \times 10^8\text{ W/m}^3$.

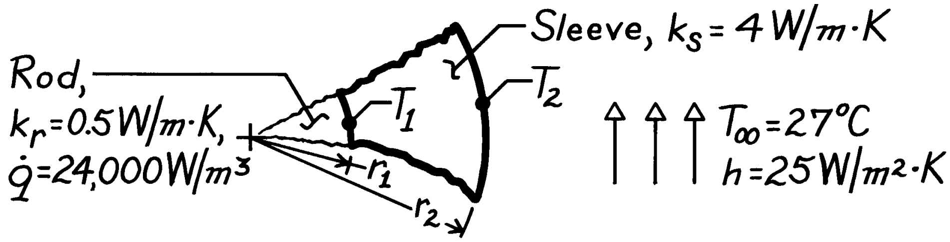
COMMENTS: A contact resistance at the thorium/graphite interface would increase temperatures in the fuel element, thereby reducing the maximum allowable value of $\dot{q}$.

PROBLEM 3.92

KNOWN: Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection.

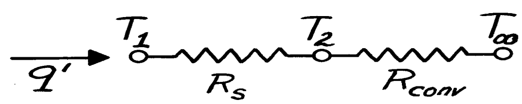
FIND: (a) Temperature at the interface between rod and sleeve and on the outer surface, (b) Temperature at center of rod.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in rod and sleeve, (2) Steady-state conditions, (3) Uniform volumetric generation in rod, (4) Negligible contact resistance between rod and sleeve.

ANALYSIS: (a) Construct a thermal circuit for the sleeve,



where

$$q' = \dot{E}'_{gen} = \dot{q}\pi\, D_1^2/4 = 24{,}000\ \text{W/m}^3 \times \pi \times (0.20\ \text{m})^2/4 = 754.0\ \text{W/m}$$

$$R'_s = \frac{\ln(r_2/r_1)}{2\pi\, k_s} = \frac{\ln(400/200)}{2\pi \times 4\ \text{W/m}\cdot\text{K}} = 2.758 \times 10^{-2}\,\text{m}\cdot\ \text{K/W}$$

$$R_{conv} = \frac{1}{h\pi\, D_2} = \frac{1}{25\ \text{W/m}^2\cdot\text{K} \times \pi \times 0.400\ \text{m}} = 3.183 \times 10^{-2}\,\text{m}\cdot\ \text{K/W}$$

The rate equation can be written as

$$q' = \frac{T_1 - T_\infty}{R'_s + R'_{conv}} = \frac{T_2 - T_\infty}{R'_{conv}}$$

$$T_1 = T_\infty + q'\left(R'_s + R'_{conv}\right) = 27^\circ\text{C} + 754\ \text{W/m}\left(2.758 \times 10^{-2} + 3.183 \times 10^{-2}\right)\text{K/W}\cdot\text{m} = 71.8^\circ\text{C} \quad <$$

$$T_2 = T_\infty + q' R'_{conv} = 27^\circ\text{C} + 754\ \text{W/m} \times 3.183 \times 10^{-2}\text{m}\cdot\ \text{K/W} = 51.0^\circ\text{C}. \quad <$$

(b) The temperature at the center of the rod is

$$T(0) = T_o = \frac{\dot{q} r_1^2}{4k_r} + T_1 = \frac{24{,}000\ \text{W/m}^3 (0.100\ \text{m})^2}{4 \times 0.5\ \text{W/m}\cdot\text{K}} + 71.8^\circ\text{C} = 192^\circ\text{C}. \quad <$$

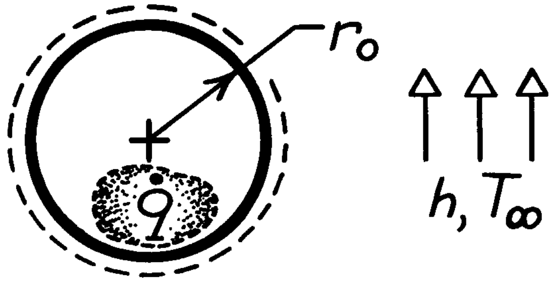
COMMENTS: The thermal resistances due to conduction in the sleeve and convection are comparable. Will increasing the sleeve outer diameter cause the surface temperature T_2 to increase or decrease?

PROBLEM 3.93

KNOWN: Radius, thermal conductivity, heat generation and convection conditions associated with a solid sphere.

FIND: Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation.

ANALYSIS: Integrating the appropriate form of the heat diffusion equation,

$$\frac{1}{r^2}\frac{d}{dr}\left[kr^2\frac{dT}{dr}\right]+\dot{q}=0 \qquad \text{or} \qquad \frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=-\frac{\dot{q}r^2}{k}$$

$$r^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3k}+C_1 \qquad \frac{dT}{dr}=-\frac{\dot{q}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2.$$

The boundary conditions are: $\left.\frac{dT}{dr}\right]_{r=0}=0$ hence $C_1=0,$ and

$$-k\left.\frac{dT}{dr}\right]_{r_o}=h\left[T(r_o)-T_\infty\right].$$

Substituting into the second boundary condition ($r = r_o$), find

$$\frac{\dot{q}r_o}{3}=h\left[-\frac{\dot{q}r_o^2}{6k}+C_2-T_\infty\right] \qquad C_2=\frac{\dot{q}r_o}{3h}+\frac{\dot{q}r_o^2}{6k}+T_\infty.$$

The temperature distribution has the form

$$T(r)=\frac{\dot{q}}{6k}\left(r_o^2-r^2\right)+\frac{\dot{q}r_o}{3h}+T_\infty. \qquad <$$

COMMENTS: To verify the above result, obtain $T(r_o) = T_s$,

$$T_s=\frac{\dot{q}r_o}{3h}+T_\infty$$

Applying energy balance to the control volume about the sphere,

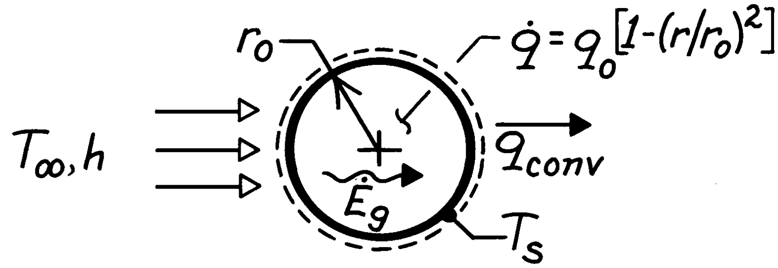
$$\dot{q}\left[\frac{4}{3}\pi\, r_o^3\right]=h4\pi\, r_o^2\left(T_s-T_\infty\right) \qquad \text{find} \qquad T_s=\frac{\dot{q}r_o}{3h}+T_\infty.$$

PROBLEM 3.94

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence
$$r^2\frac{dT}{dr} = -\frac{\dot{q}_o}{k}\left(\frac{r^3}{3}-\frac{r^5}{5r_o^2}\right)+C_1$$

$$T = -\frac{\dot{q}_o}{k}\left(\frac{r^2}{6}-\frac{r^4}{20r_o^2}\right)-\frac{C_1}{r}+C_2.$$

From the boundary conditions,

$$dT/dr\,|_{r=0} = 0 \quad \text{and} \quad -k\,dT/dr\,|_{r=r_o} = h\left[T(r_o)-T_\infty\right]$$

it follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3}-\frac{r_o}{5}\right) = h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6}-\frac{r_o^2}{20}\right)+C_2-T_\infty\right]$$

$$C_2 = \frac{2r_o\dot{q}_o}{15h}+\frac{7\dot{q}_o r_o^2}{60k}+T_\infty.$$

Hence
$$T(r) = T_\infty+\frac{2r_o\dot{q}_o}{15h}+\frac{\dot{q}r_o^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_o}\right)^2+\frac{1}{20}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + \left(2r_o\dot{q}_o/15h\right).$$

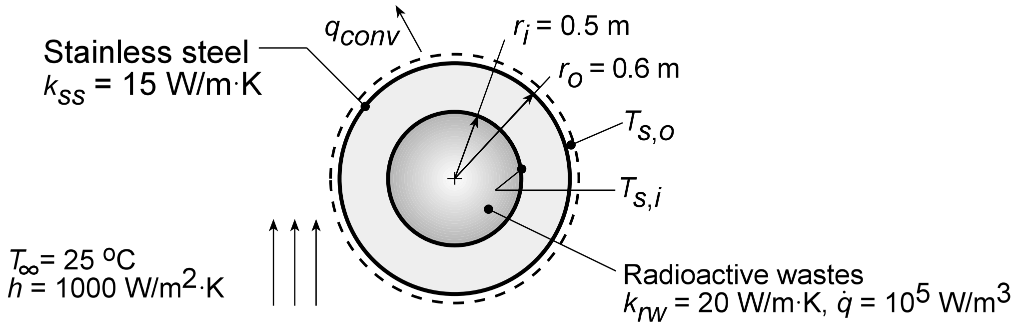
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.95

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m^3,

$$T_{s,o} = T_\infty + \frac{\dot{q}r_i^3}{3hr_o^2} = 25^\circ C + \frac{10^5 \text{ W/m}^2 (0.5\text{ m})^3}{3000\text{ W/m}^2\cdot K(0.6\text{ m})^2} = 36.6^\circ C. \qquad <$$

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$. Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right)r_o\left(T_{s,o} - T_\infty\right) = 36.6^\circ C + \frac{1000\text{ W/m}^2\cdot K}{15\text{ W/m}\cdot K}(0.2)0.6\text{ m}\left(11.6^\circ C\right) = 129.4^\circ C. \qquad <$$

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right) \quad <$$

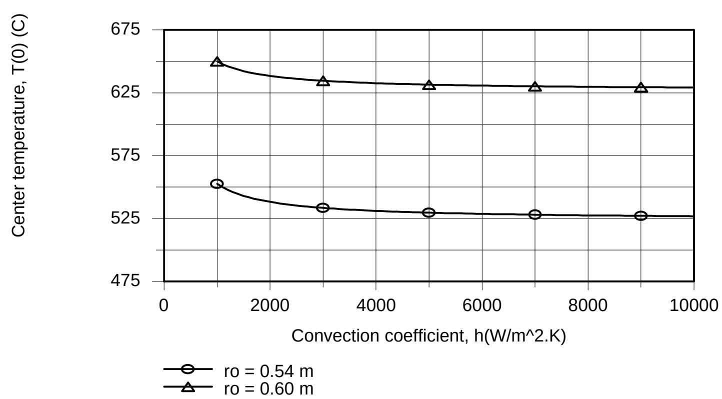
At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 129.4^\circ\text{C} + \frac{10^5\ \text{W/m}^3\,(0.5\,\text{m})^2}{6(20\,\text{W/m}\cdot\text{K})} = 337.7^\circ\text{C} \quad <$$

(d) The feasibility assessment may be performed by using the IHT model for one-dimensional, steady-state conduction in a solid sphere, with the surface boundary condition prescribed in terms of the total thermal resistance

$$R''_{tot,i} = \left(4\pi r_i^2\right)R_{tot} = R''_{cnd,i} + R''_{cnv,i} = \frac{r_i^2\left[(1/r_i) - (1/r_o)\right]}{k_{ss}} + \frac{1}{h}\left(\frac{r_i}{r_o}\right)^2$$

where, for r_o = 0.6 m and h = 1000 W/m^2·K, $R''_{cnd,i}$ = 5.56 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 6.94 × 10^{-4} m^2·K/W, and $R''_{tot,i}$ = 6.25 × 10^{-3} m^2·K/W. Results for the center temperature are shown below.



Clearly, even with r_o = 0.54 m = $r_{o,min}$ and h = 10,000 W/m^2·K (a practical upper limit), T(0) > 475°C and the desired condition can not be met. The corresponding resistances are $R''_{cnd,i}$ = 2.47 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 8.57 × 10^{-5} m^2·K/W, and $R''_{tot,i}$ = 2.56 × 10^{-3} m^2·K/W. The conduction resistance remains dominant, and the effect of reducing $R''_{cnv,i}$ by increasing h is small. *The proposed extension is not feasible.*

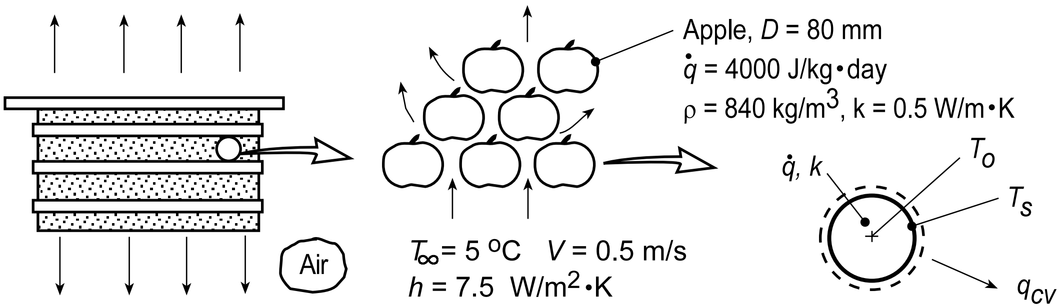
COMMENTS: A value of $\dot{q}$ = 1.79 × 10^5 W/m^3 would allow for operation at T(0) = 475°C with r_o = 0.54 m and h = 10,000 W/m^2·K.

PROBLEM 3.96

KNOWN: Carton of apples, modeled as 80-mm diameter spheres, ventilated with air at 5°C and experiencing internal volumetric heat generation at a rate of 4000 J/kg·day.

FIND: (a) The apple center and surface temperatures when the convection coefficient is 7.5 $W/m^2 \cdot K$, and (b) Compute and plot the apple temperatures as a function of air velocity, V, for the range $0.1 \le V \le 1$ m/s, when the convection coefficient has the form $h = C_1 V^{0.425}$, where $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$.

SCHEMATIC:



ASSUMPTIONS: (1) Apples can be modeled as spheres, (2) Each apple experiences flow of ventilation air at $T_\infty = 5°C$, (3) One-dimensional radial conduction, (4) Constant properties and (5) Uniform heat generation.

ANALYSIS: (a) From Eq. C.24, the temperature distribution in a solid sphere (apple) with uniform generation is

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (1)$$

To determine T_s, perform an energy balance on the apple as shown in the sketch above, with volume V = $4/3\pi r_o^3$,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad -q_{cv} + \dot{q}\forall = 0$$

$$-h\left(4\pi r_o^2\right)\left(T_s - T_\infty\right) + \dot{q}\left(4/3\pi r_o^3\right) = 0 \qquad (2)$$

$$-7.5\,W/m^2 \cdot K\left(4\pi \times 0.040^2 m^2\right)\left(T_s - 5^\circ C\right) + 38.9\,W/m^3\left(4/3\pi \times 0.040^3 m^3\right) = 0$$

where the volumetric generation rate is

$$\dot{q} = 4000\,J/kg \cdot day$$

$$\dot{q} = 4000\,J/kg \cdot day \times 840\,kg/m^3 \times (1\,day/24\,hr) \times (1\,hr/3600\,s)$$

$$\dot{q} = 38.9\,W/m^3$$

and solving for T_s, find

$$T_s = 5.14^\circ C \qquad <$$

From Eq. (1), at r = 0, with T_s, find

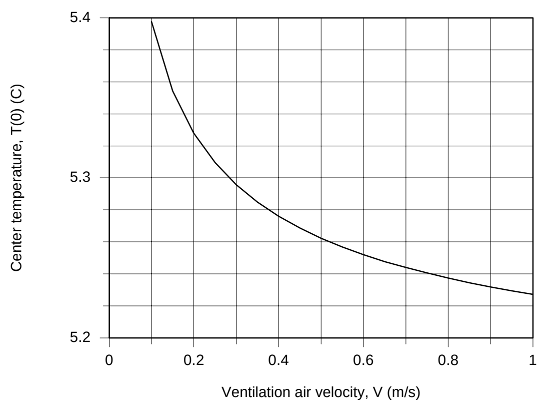
$$T(0) = \frac{38.9\,W/m^3 \times 0.040^2 m^2}{6 \times 0.5\,W/m \cdot K} + 5.14^\circ C = 0.12^\circ C + 5.14^\circ C = 5.26^\circ C \qquad <$$

Continued...

(b) With the convection coefficient depending upon velocity,

$$h = C_1 V^{0.425}$$

with C_1 = 10.1 W/m^2·K·(m/s)$^{0.425}$, and using the energy balance of Eq. (2), calculate and plot T_s as a function of ventilation air velocity V. With very low velocities, the center temperature is nearly 0.5°C higher than the air. From our earlier calculation we know that T(0) - T_s = 0.12°C and is independent of V.



COMMENTS: (1) While the temperature within the apple is nearly isothermal, the center temperature will track the ventilation air temperature which will increase as it passes through stacks of cartons.

(2) The IHT Workspace used to determine T_s for the base condition and generate the above plot is shown below.

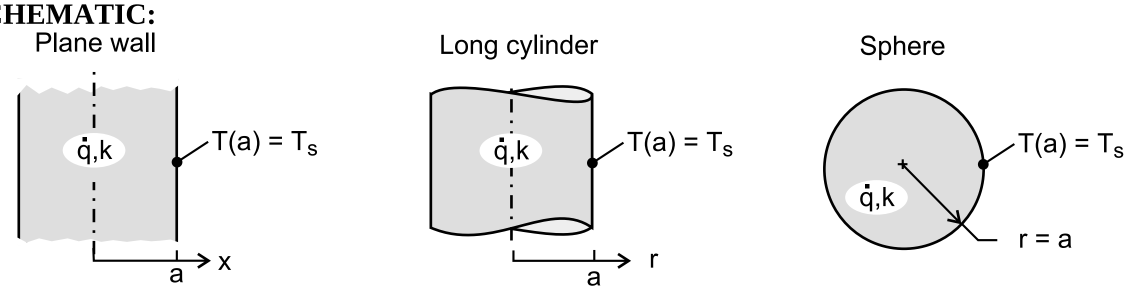
```
// The temperature distribution, Eq (1),
T_r = qdot * ro^2 / (4 * k) * ( 1- r^2/ro^2 )  +  Ts

// Energy balance on the apple, Eq (2)
- qcv + qdot * Vol = 0
Vol = 4 / 3 * pi *  ro ^3

// Convection rate equation:
qcv = h* As * ( Ts - Tinf )
As = 4 * pi * ro^2

// Generation rate:
qdot = qdotm * (1/24) * (1/3600) * rho          // Generation rate, W/m^3; Conversions: days/h and h/sec

// Assigned variables:
ro = 0.080               // Radius of apple, m
k = 0.5                  // Thermal conductivity, W/m.K
qdotm = 4000             // Generation rate, J/kg.K
rho = 840                // Specific heat, J/kg.K
r = 0                    // Center, m; location for T(0)
h =  7.5                 // Convection coefficient, W/m^2.K; base case, V = 0.5 m/s
//h = C1 * V^0.425       // Correlation
//C1 = 10.1
//V = 0.5                // Air velocity, m/s; range 0.1 to 1 m/s
Tinf = 5                 // Air temperature, C
```

PROBLEM 3.97

KNOWN: Plane wall, long cylinder and sphere, each with characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.

FIND: (a) On the same graph, plot the dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[\dot{q}a^2/2k]$, *vs.* the dimensionless characteristic length, x/a or r/a, for each shape; (b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area; and (c) Which shape would be preferred for use as a nuclear fuel element? Explain why?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties and (4) Uniform volumetric generation.

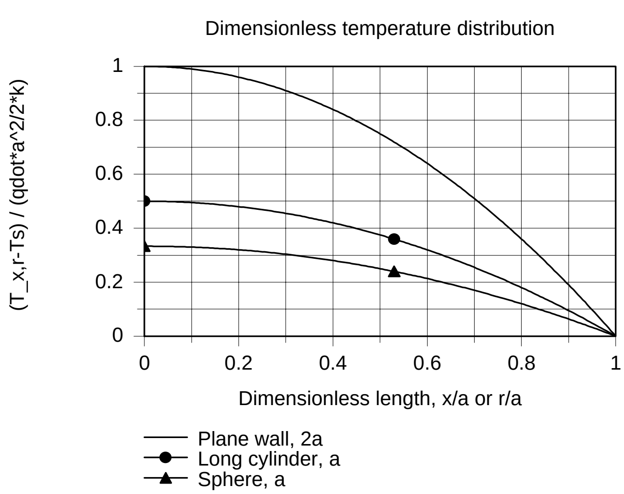
ANALYSIS: (a) For each of the shapes, with T(a) = T_s, the dimensionless temperature distributions can be written by inspection from results in Appendix C.3.

Plane wall, Eq. C.22
$$\frac{T(x)-T_s}{\dot{q}a^2/2k} = 1-\left(\frac{x}{a}\right)^2$$

Long cylinder, Eq. C.23
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1-\left(\frac{r}{a}\right)^2\right]$$

Sphere, Eq. C.24
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1-\left(\frac{r}{a}\right)^2\right]$$

The dimensionless temperature distributions using the foregoing expressions are shown in the graph below.



Continued …..

(b) The sphere shape has the smallest temperature difference between the center and surface, T(0) – T(a). The ratio of volume-to-surface-area, $\forall/A_s$, for each of the shapes is

Plane wall

$$\frac{\forall}{A_s} = \frac{a(1\times1)}{(1\times1)} = a$$

Long cylinder

$$\frac{\forall}{A_s} = \frac{\pi a^2 \times 1}{2\pi a \times 1} = \frac{a}{2}$$

Sphere

$$\frac{\forall}{A_s} = \frac{4\pi a^3/3}{4\pi a^2} = \frac{a}{3}$$

The smaller the $\forall/A_s$ ratio, the smaller the temperature difference, T(0) – T(a).

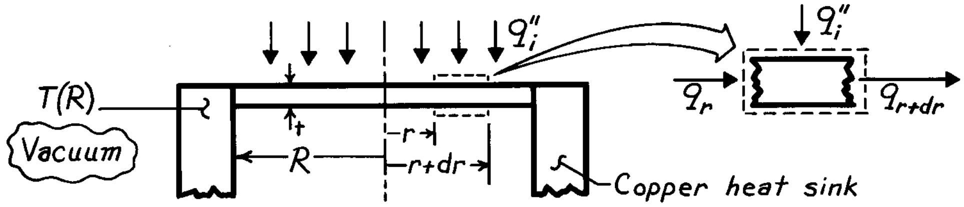
(c) The sphere would be the preferred element shape since, for a given $\forall/A_s$ ratio, which controls the generation and transfer rates, the sphere will operate at the lowest temperature.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T. \qquad <$$

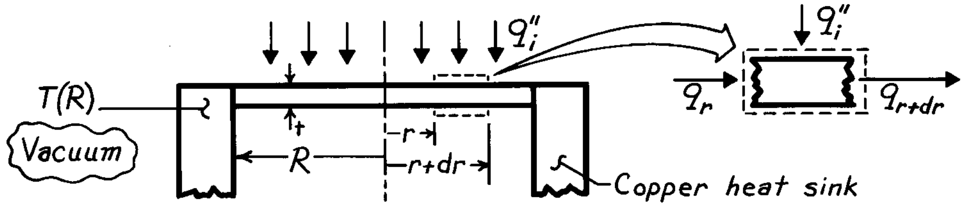
COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T.$$ <

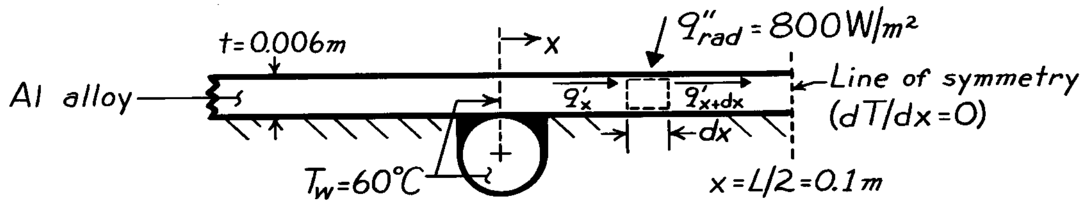
COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.99

KNOWN: Net radiative flux to absorber plate.

FIND: (a) Maximum absorber plate temperature, (b) Rate of energy collected per tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (x) conduction along absorber plate, (3) Uniform radiation absorption at plate surface, (4) Negligible losses by conduction through insulation, (5) Negligible losses by convection at absorber plate surface, (6) Temperature of absorber plate at x = 0 is approximately that of the water.

PROPERTIES: *Table A-1,* Aluminum alloy (2024-T6): k ≈ 180 W/m·K.

ANALYSIS: The absorber plate acts as an extended surface (a conduction-radiation system), and a differential equation which governs its temperature distribution may be obtained by applying Eq.1.11a to a differential control volume. For a unit length of tube

$$q'_x + q''_{rad}(dx) - q'_{x+dx} = 0.$$

With $$q'_{x+dx} = q'_x + \frac{dq'_x}{dx}dx$$

and $$q'_x = -kt\frac{dT}{dx}$$

it follows that,

$$q''_{rad} - \frac{d}{dx}\left[-kt\frac{dT}{dx}\right] = 0$$

$$\frac{d^2T}{dx^2} + \frac{q''_{rad}}{kt} = 0$$

Integrating twice it follows that, the general solution for the temperature distribution has the form,

$$T(x) = -\frac{q''_{rad}}{2kt}x^2 + C_1x + C_2.$$

Continued …..

The boundary conditions are:

$$T(0) = T_w \qquad\qquad C_2 = T_w$$

$$\left.\frac{dT}{dx}\right]_{x=L/2} = 0 \qquad C_1 = \frac{q''_{rad}L}{2kt}$$

Hence,

$$T(x) = \frac{q''_{rad}}{2kt}\, x(L-x) + T_w.$$

The maximum absorber plate temperature, which is at x = L/2, is therefore

$$T_{max} = T(L/2) = \frac{q''_{rad}L^2}{8kt} + T_w.$$

The rate of energy collection per tube may be obtained by applying Fourier's law at x = 0. That is, energy is transferred to the tubes via conduction through the absorber plate. Hence,

$$q' = 2\left[-k\,t\left.\frac{dT}{dx}\right]_{x=0}\right]$$

where the factor of two arises due to heat transfer from both sides of the tube. Hence,

$$q' = -Lq''_{rad}.$$

Hence $\qquad T_{max} = \dfrac{800\dfrac{W}{m^2}(0.2m)^2}{8\left[180\dfrac{W}{m\cdot K}\right](0.006m)} + 60^\circ C$

or $\qquad T_{max} = 63.7^\circ C$ $\qquad$ <

and $\qquad q' = -0.2m \times 800\ W/m^2$

or $\qquad q' = -160\ W/m.$ $\qquad$ <

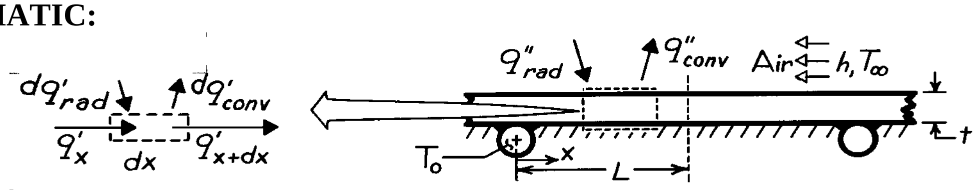
COMMENTS: Convection losses in the typical flat plate collector, which is not evacuated, would reduce the value of q'.

PROBLEM 3.100

KNOWN: Surface conditions and thickness of a solar collector absorber plate. Temperature of working fluid.

FIND: (a) Differential equation which governs plate temperature distribution, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Adiabatic bottom surface, (4) Uniform radiation flux and convection coefficient at top, (5) Temperature of absorber plate at x = 0 corresponds to that of working fluid.

ANALYSIS: (a) Performing an energy balance on the differential control volume,

$$q'_x + dq'_{rad} = q'_{x+dx} + dq'_{conv}$$

where
$$q'_{x+dx} = q'_x + \left(dq'_x / dx\right)dx$$
$$dq'_{rad} = q''_{rad} \cdot dx$$
$$dq'_{conv} = h\left(T - T_\infty\right) \cdot dx$$

Hence, $$q''_{rad}dx = \left(dq'_x / dx\right)dx + h\left(T - T_\infty\right)dx.$$

From Fourier's law, the conduction heat rate per unit width is

$$q'_x = -k\ t\ dT/dx \qquad \frac{d^2T}{dx^2} - \frac{h}{kT}\left(T - T_\infty\right) + \frac{q''_{rad}}{kt} = 0.$$ <

(b) Defining $\theta = T - T_\infty, d^2T/dx^2 = d^2\theta / dx^2$ and the differential equation becomes,

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_{rad}}{kt} = 0.$$

It is a second-order, differential equation with constant coefficients and a source term, and its general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$$

where $\lambda = \left(h/kt\right)^{1/2}$, $\quad S = q''_{rad} / kt.$

Appropriate boundary conditions are:

$$\theta\left(0\right) = T_o - T_\infty \equiv \theta_0, \qquad d\theta/dx)_{x=L} = 0.$$

Hence, $$\theta_o = C_1 + C_2 + S/\lambda^2$$

$$d\theta/dx)_{x=L} = C_1\ \lambda e^{+\lambda L} - C_2\ \lambda e^{-\lambda L} = 0 \qquad C_2 = C_1\ e^{2\lambda L}$$

Hence, $$C_1 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{2\lambda L}\right) \qquad C_2 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{-2\lambda L}\right)$$

$$\theta = \left(\theta_0 - S/\lambda^2\right)\left[\frac{e^{\lambda x}}{1+e^{2\lambda L}} + \frac{e^{-\lambda x}}{1+e^{-2\lambda L}}\right] + S/\lambda^2.$$ <