ME338 – Manufacturing Process II

Lecture 15 – Geometrical Tolerance*

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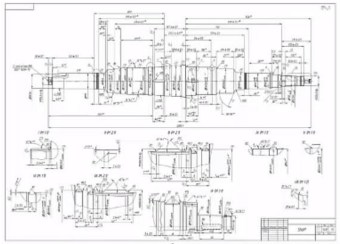
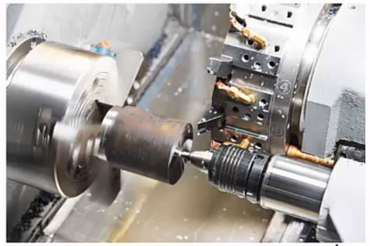
Several slides taken from Prof. Joshi and Prof. Pande's earlier lectures on engineering metrology

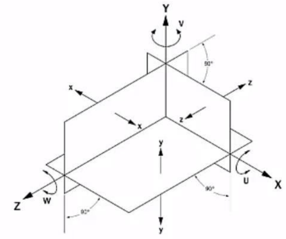
Important steps in mass production



Blueprint/Drawing Interpretation

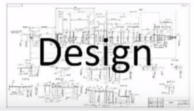


Engineering process / how a part is made

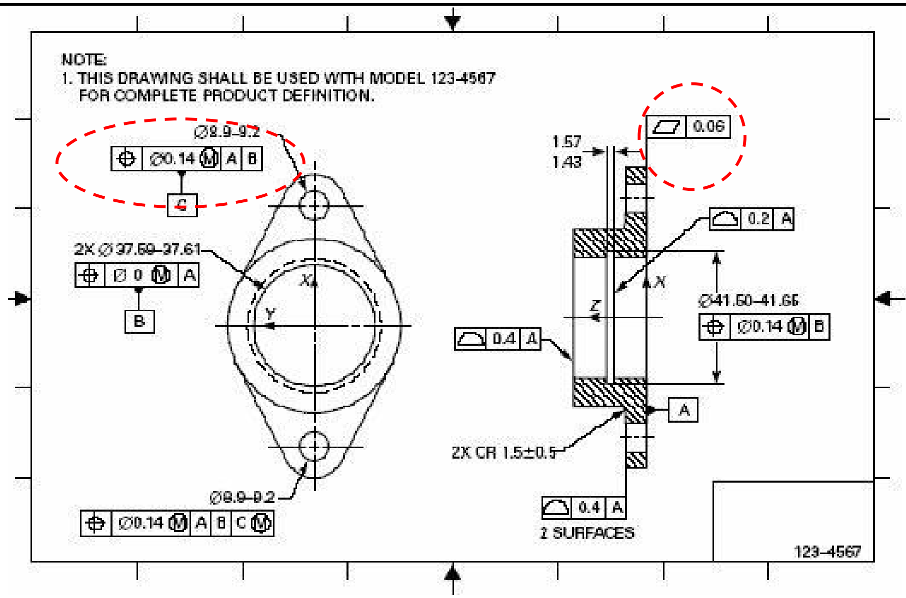


GD&T ⌖

These three concepts would give you the greatest impact in mechanical engineering.

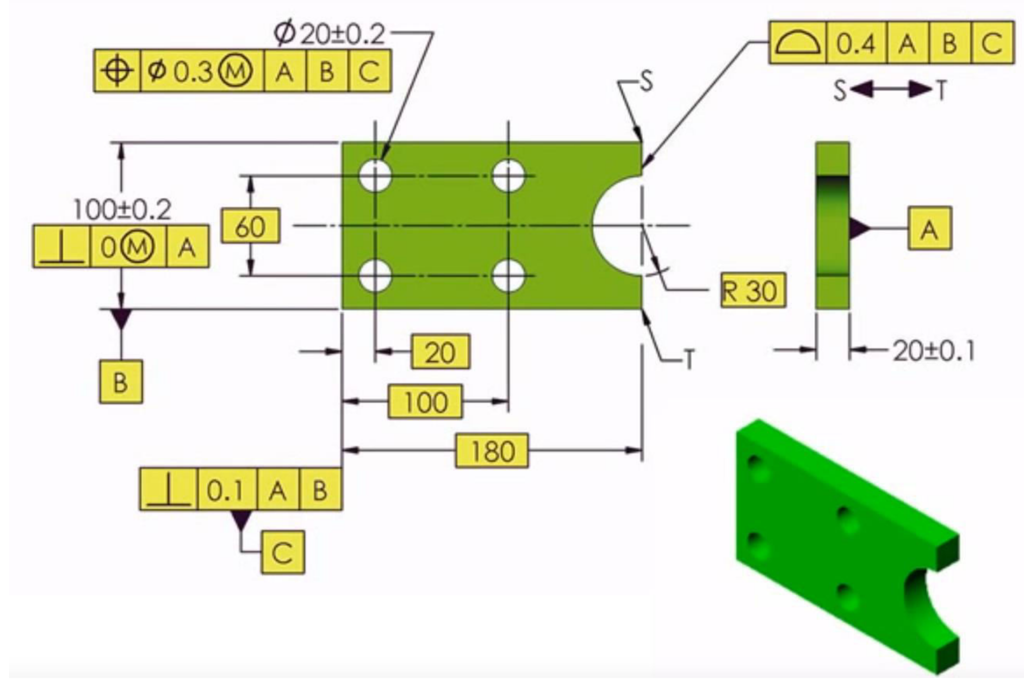


Sample engineering drawing



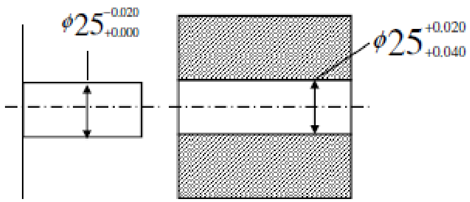
Besides dimensional tolerances (DT), geometrical tolerances (GT) are also required, which need to be mentioned in the drawing sheet.
Else, there will be serious problems during the part assembly

Sample engineering drawing (2)

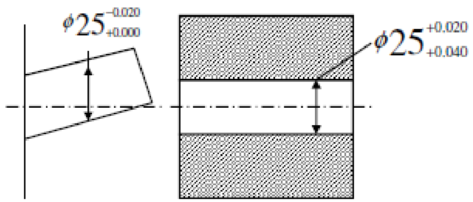


Tolerance of Size and Geometry

- The tolerance on 'Size' is also called as 'Dimensional' tolerance.
- Dimensional tolerances may not be sufficient to manufacture the desired fit perfectly.
- Additional tolerances, called 'Geometrical' Tolerances are required:

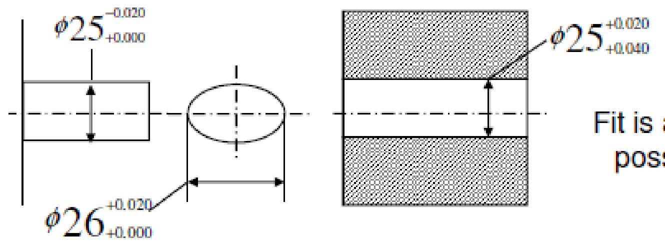


Fit is acceptable, assembly possible
Parts in Correct **Orientation**

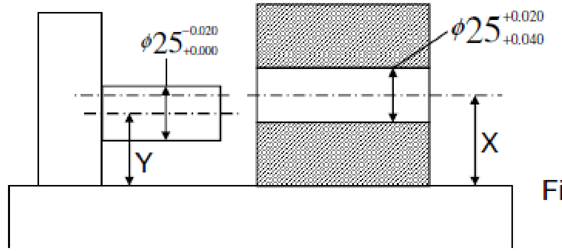


Fit is acceptable, assembly Not possible
Parts not in correct **Orientation**

Tolerance of Size and Geometry



Fit is acceptable, assembly Not possible Parts not in correct **Shape**



Fit is acceptable, assembly Not possible
Parts not in correct **Position**

Tolerance of Size and Geometry

- For proper selection of FIT, it is also important that we specify additional tolerances so that the desired FIT is achieved.
- These types of tolerances which help specify the functional requirements more clearly, are called as 'Geometrical Tolerances'.
- These are of following three types –
  - Tolerances on **Shape or Form**
  - Tolerances on **Orientation**
  - Tolerances on **Position**
- Geometric tolerance can be expressed in the following manner:

Symbol	Tolerance Value	Reference datum

Geometric Tolerance Symbols

	Type	Characteristics	Symbol
For *individual* features (Peak-to-Valley measure)	Form	Straightness	⏤
		Flatness	▱
		Circularity (roundness)	○
		Cylindricity	⌭
For *individual* or *related* features	Profile	Profile of a line	⌒
		Profile of a surface	⌓
For *related* features	Orientation	Angularity	∠
		Perpendicularity	⊥
		Parallelism	//
	Location	Position	⌖
		Concentricity	◎
		Symmetry	⌯
	Runout	Runout	↗
		Total runout	⌰

GD&T characteristics

Type	Application	Characteristic	Symbol	Datums	Shape of tolerance zone
Form	Single Feature	Straightness	⏤	Datums not allowed	Parallel lines or planes, cylinder
		Flatness	⏥		Parallel planes
		Circularity	○		Concentric circles
		Cylindricity	⌭		Concentric cylinders
Profile	Single or Related Feature	Profile of line	⌒	Datums required*	2D uniform boundary
		Profile of surface	⌓		3D uniform boundary
Location	Related Feature	Position	⌖		Parallel planes, cylinder, sphere, cone
		Concentricity	◎	Datums required	Cylinder
		Symmetry	⌯		Parallel planes
Orientation		Parallelism	//		Parallel planes, cylinder
		Perpendicularity	⊥		
		Angularity	∠		
Run-Out		Circular Runout	↗		Concentric circles, parallel circles
		Total Runout	⌰		Concentric cylinders, parallel planes

* There are some exceptions when profile and position may not require a datum.

Various types of Geometric Tolerances

Form /Shape tolerance

Sr. No.	Characteristics	Symbol
1.	Straightness	⏤
2.	Flatness	⏥
3.	Circularity	○
4.	Cylindricity	⌭
5.	Profile of a Line	⌒
6.	Profile of a Surface	⌓

Orientation tolerance

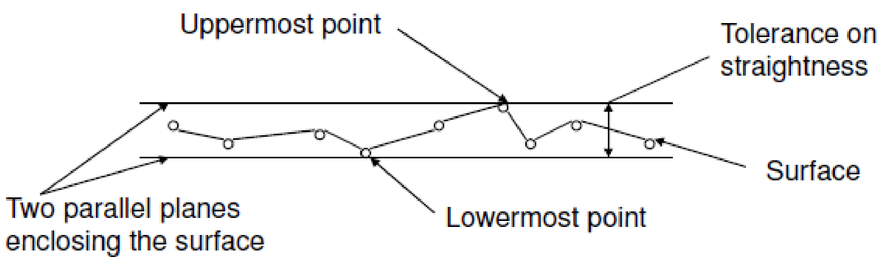
S. N.	Characteristics	Symbol
1.	Parallelism	//
2.	Perpendicularity	⊥
3.	Angularity	∠

Position tolerance

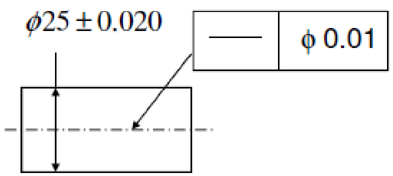
S.N.	Characteristics	Symbol
1.	Position	⌖
2.	Concentricity/Co-axiality	◎
3.	Symmetry	⌯
4.	Run out	↗

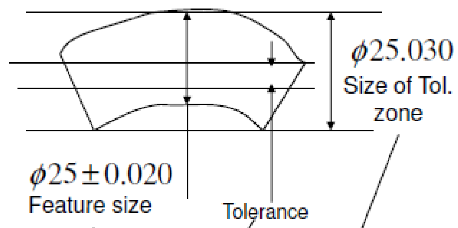
Geometric Tolerances: Straightness

- Straightness:
  - It is the characteristic of a line where all the elements of a line are colinear.
  - In general, there could be two lines, within which, all the points on a line lie.

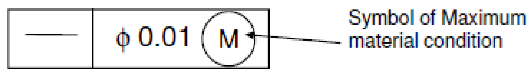


Geometric Tolerances: Straightness





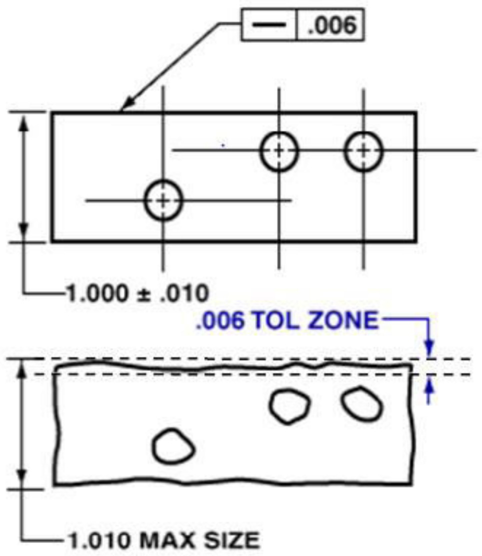
Concept of Maximum Material Condition



As the feature size reduces below its MMC, the tolerance goes on increasing as can be seen from the adjoining table.

Feature Size	Tolerance	Size of zone
25.020	ϕ 0.01	25.030
25.010	ϕ 0.02	25.030
25.000	ϕ 0.03	25.030
24.990	ϕ 0.04	25.030

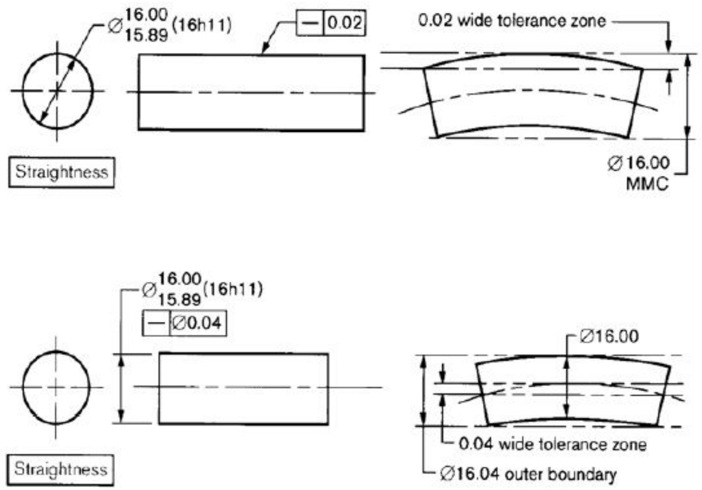
Straightness tolerance



Straightness Tolerance – Line / Axis

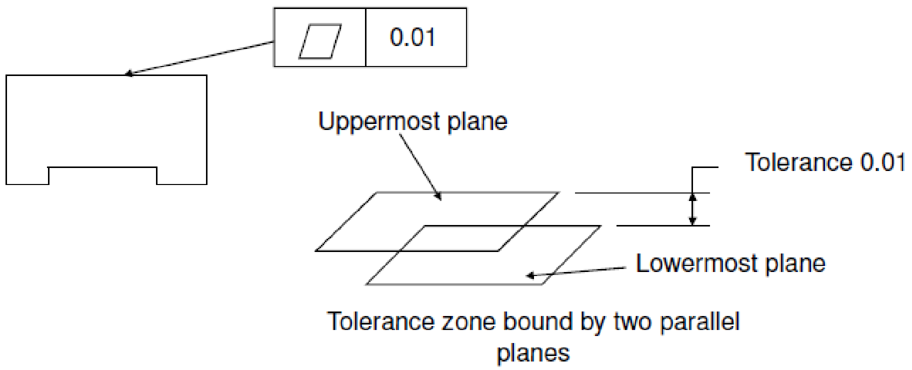
On the Drawing

Interpretation

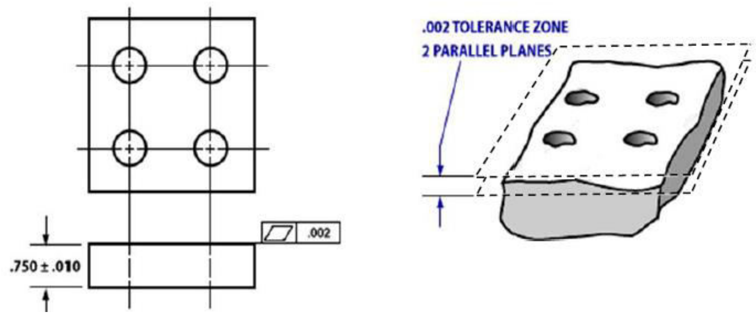


Geometric Tolerances: Flatness

- Flatness:
  - It is defined as minimum distance between two planes within which all the points on a surface lie.
  - A surface along which all the points lie along single plane is called as perfectly flat surface.
  - The maximum material condition is not applicable to Flatness

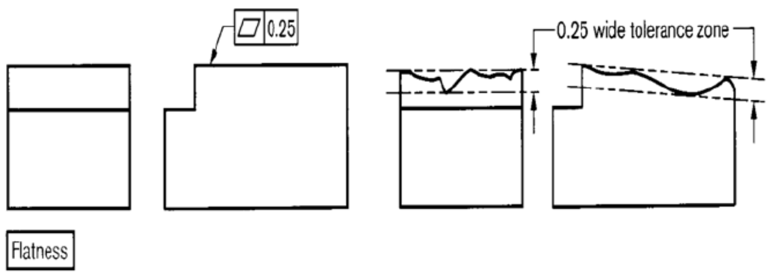


Flatness tolerance



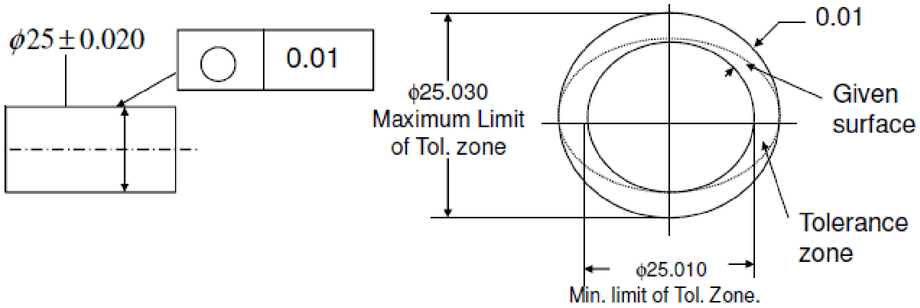
On the Drawing

Interpretation



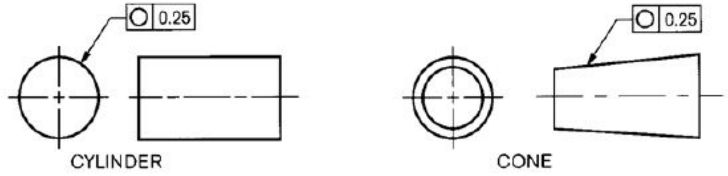
Geometric Tolerances: Circularity

- Circularity:
  - It is defined for a cylindrical or conical surfaces. It defines the distance between the surface and its axis.
  - Ideally, all points on a surface (at a cross-section), should be equidistant from the axis for the cross-section to be perfectly circular.
  - The tolerance on circularity is defined by two concentric circles within which a surface can lie.
  - The distance between two the concentric circles is called tolerance.

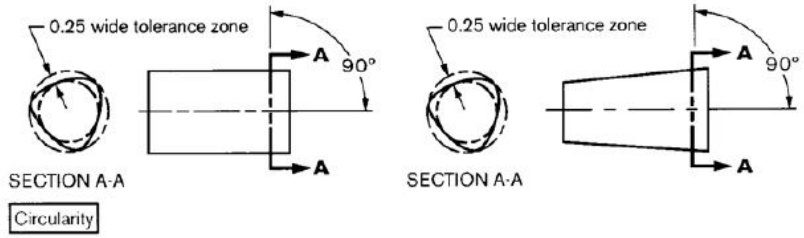


Circularity (Roundness) Tolerance

On the Drawing

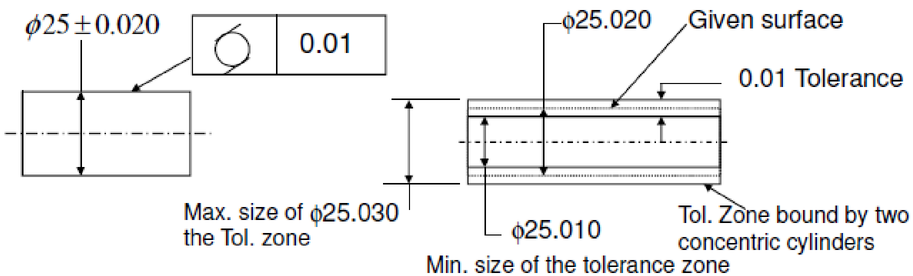


Interpretation

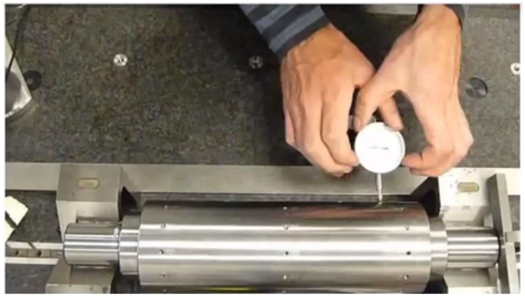


Geometric Tolerances: Cylindricity

- Cylindricity:
  - It is defined for a surface of revolution. It defines the distance between the surface and its axis.
  - Ideally, all points on a surface (of revolution), should be equidistant from the axis for the cross-section to be perfectly cylindrical.
  - The tolerance on cylindricity is defined by two concentric cylinders within which a surface can lie.
  - The distance between the two concentric cylinders is called tolerance.

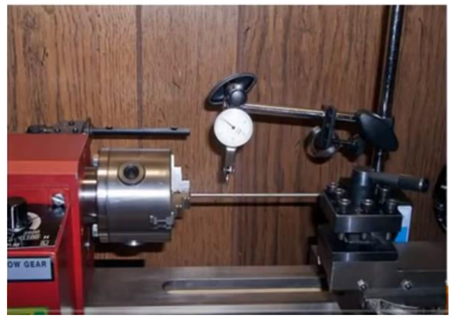


Measurement of geometrical tolerances



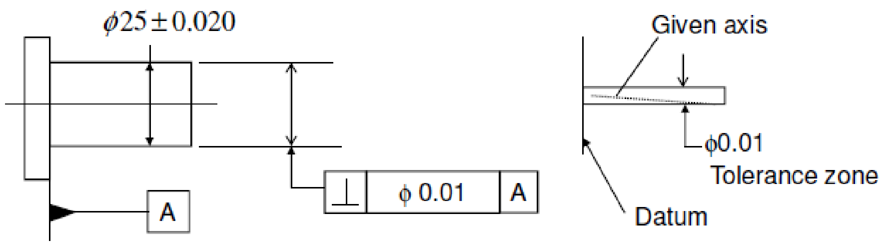
Cylindricity measurement

Flatness measurement in disk brake



Geometric Tolerances: Perpendicularity

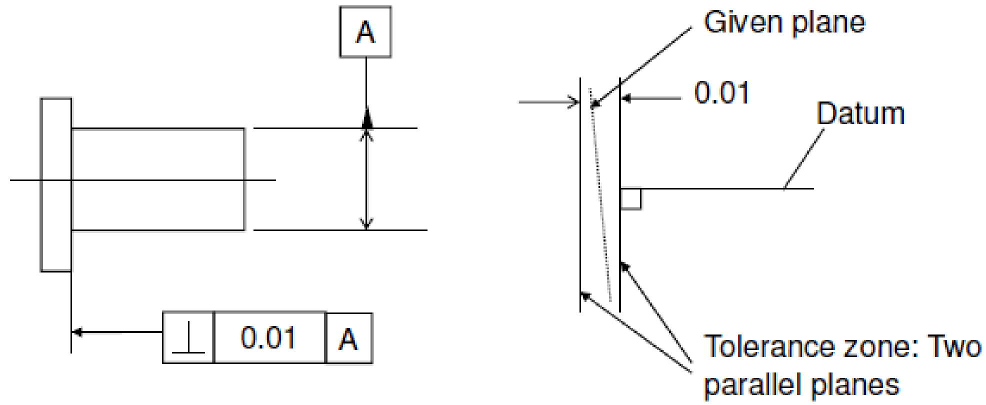
- Perpendicularity:
  - It is defined for a feature (like surface or line) with reference to another feature called reference.
  - It defines the distance between two lines or surfaces that are parallel to each other and perpendicular to the datum surface and encompass the line or surface in question.



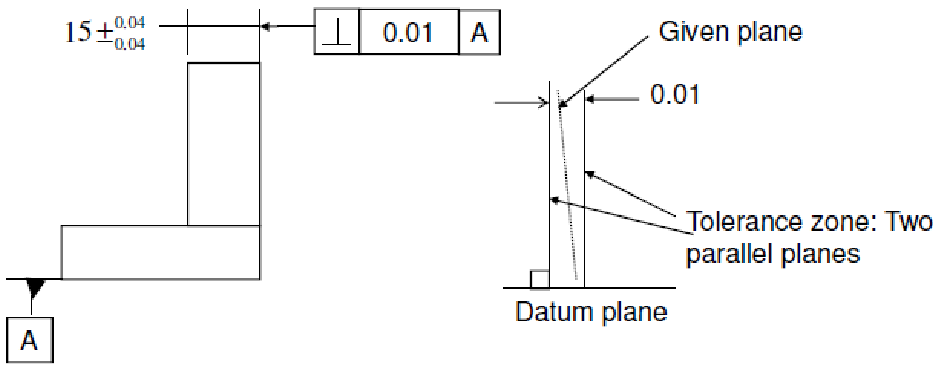
Maximum size of the part = Maximum size permitted by the dimensional tolerance (25.020) + geometrical tolerance (0.01) = 25.03 mm

Minimum size of the part = Minimum size permitted by the dimensional tolerance (24.98) - Geometrical tolerance (0.01) = 24.97 mm

Perpendicularity of a Surface with Line as a datum



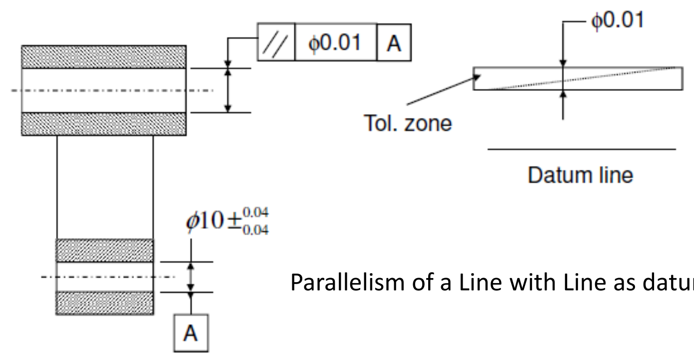
Perpendicularity of a Surface with Surface as a datum



- Maximum size of the part = Maximum size permitted by the dimensional tolerance (15.040) + geometrical tolerance (0.01) = 15.05 mm
- Minimum size of the part = Minimum size permitted by the dimensional tolerance (14.96) - Geometrical tolerance (0.01) = 14.95 mm

Geometric Tolerances: Parallelism

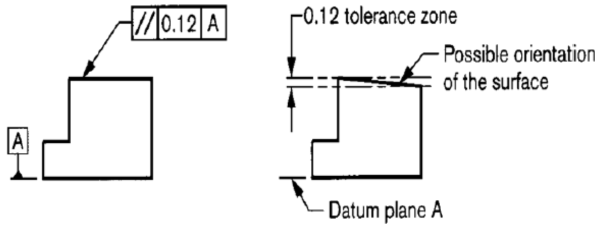
- Perpendicularity:
  - It is defined for a feature (like surface or line) with reference to another feature called reference.
  - It defines the distance between two lines or surfaces that are parallel to each other and parallel to the datum surface and encompass the line or surface in question.



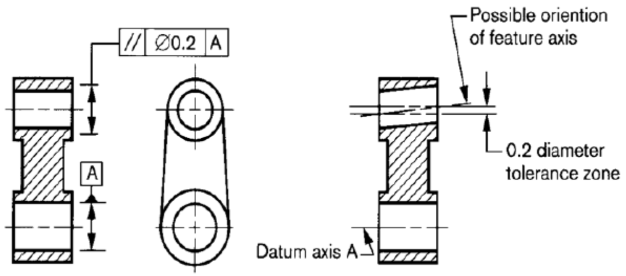
Parallelism of a Line with Line as datum

Parallelism Tolerance

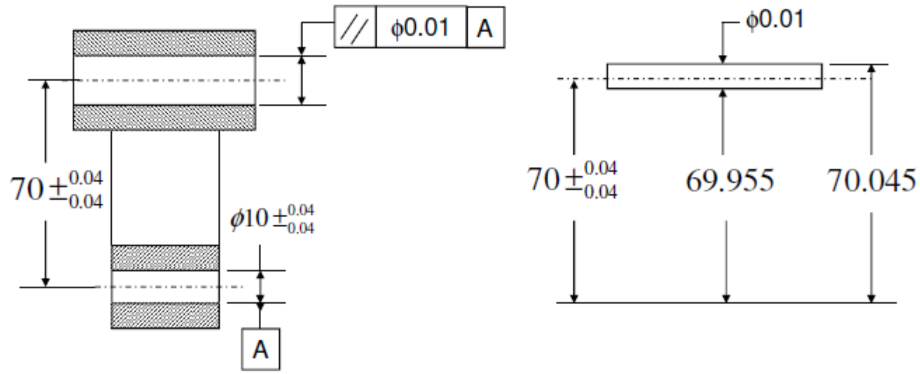
On the Drawing   Interpretation



On the Drawing   Interpretation

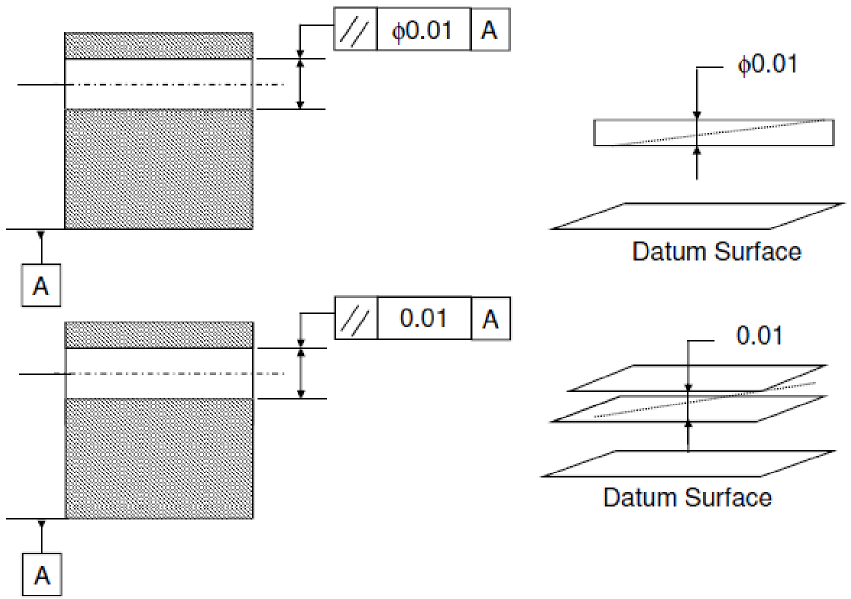


Parallelism of a Line with Line as a datum



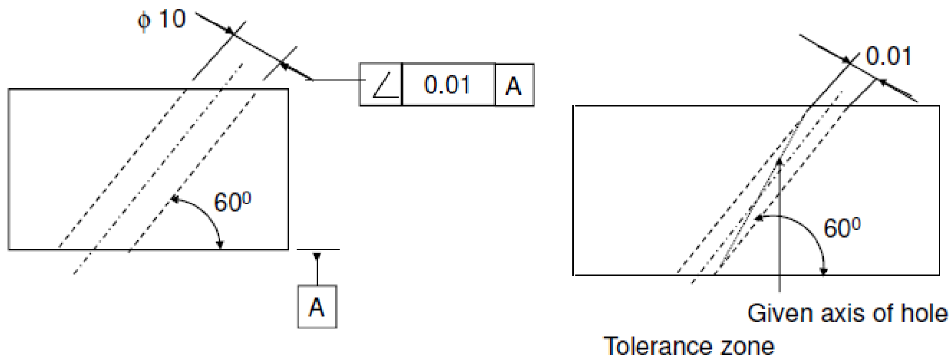
- Max. distance between holes = Max distance permitted by the dimensional tolerance (70.040) + (1/2) Geometrical tolerance (0.005) = 70.045 mm
- Min. distance between holes = Min. distance permitted by the dimensional tolerance (69.96) – (1/2) Geometrical tolerance (0.005) = 69.955 mm

Parallelism of a Line with Surface as a datum



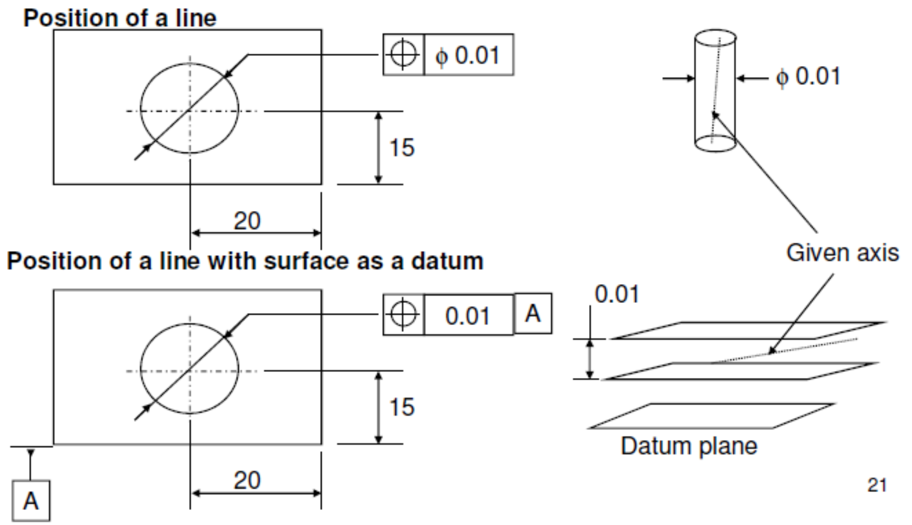
Geometric Tolerances: Angularity

- Angularity:
  - It is defined for a feature (like surface or line) with reference to another feature called reference.
  - It defines the distance between two lines or surfaces that are at an angle to the datum surface and encompass the line or surface in question.



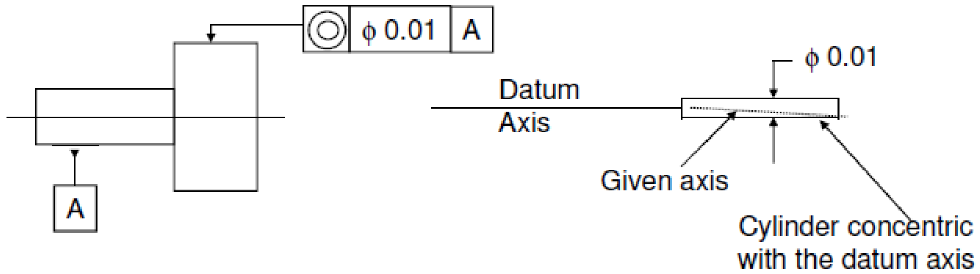
Geometric Tolerances: Position

- Position:
  - It defines the perfect (exact) location of a point, line or a surface in relation to the other datum



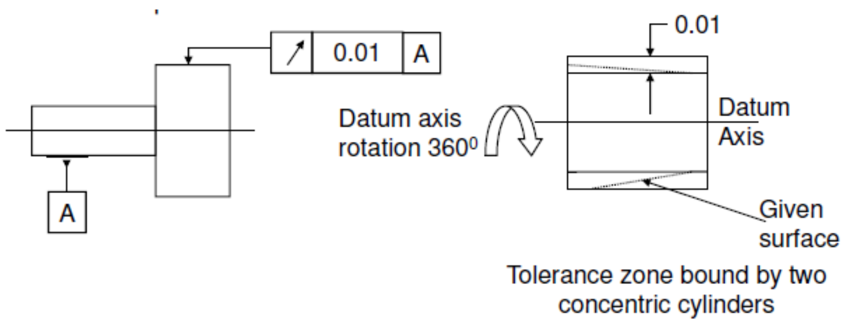
Geometric Tolerances: Concentricity

- Concentricity:
  - It defines the position of an axis in relation to the other datum axis. It defines a cylinder which coincides with the datum axis and of diameter given by the geometrical tolerance.



Geometric Tolerances: Run out

- Run out:
  - It defines the deviation from the desired form and orientation during one full rotation of the part on the datum axis.



Material Condition Modifiers

- Maximum material condition (MMC) is the condition in which a feature of size contains the maximum amount of material within the stated limits of size
- Least material condition (LMC) is the condition in which a feature of size contains the least amount of material within the stated limits of size.
- Maximum Material Condition (MMC)
  - Largest (size) shaft
  - Smallest (size) Hole
- Symbol : M
- Regardless of Feature size (RFS)
- ISO recommends
  - Form of a feature is Perfect (Ideal) at MMC
  - If not specified, treat Feature Tolerance as RFS.

Modifier Symbols

- Each geometric tolerance for a feature of size applies in one of the following three contexts
  - Regardless of Feature Size (RFS)
  - modified to Maximum Material Condition (MMC)
  - modified to Least Material Condition (LMC)

Tolerances	Applicable modifiers
⌖ ⊥ // ∠	MMC, RFS, LMC
—	MMC, RFS
↗ ⌰ ◎	RFS

ME338 – Manufacturing Process II

Lecture 16 – Surface Texture & Roughness

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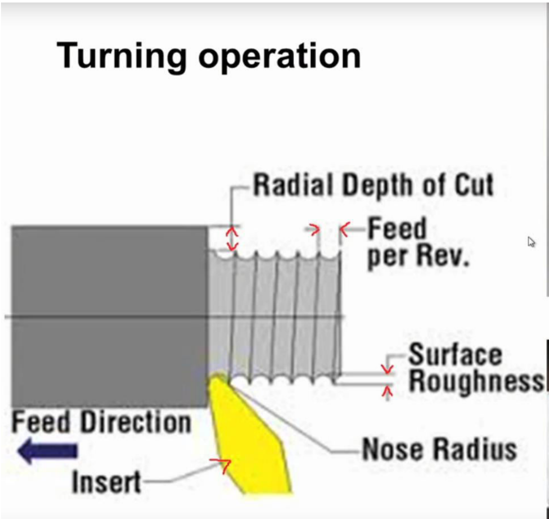
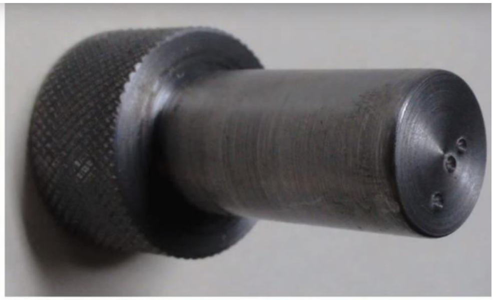
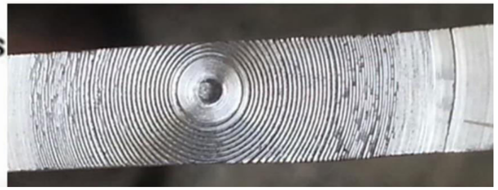
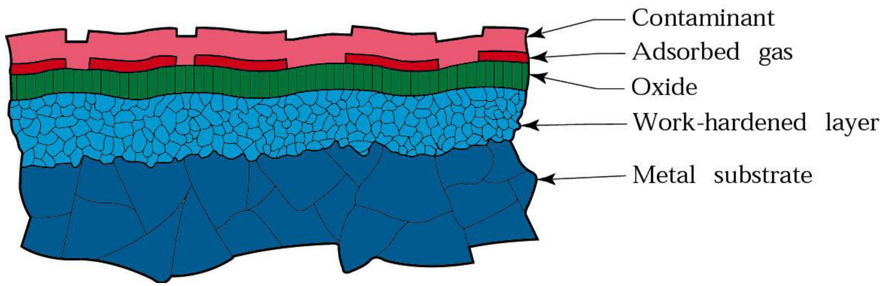
Surface texture in turning process







Surface structure

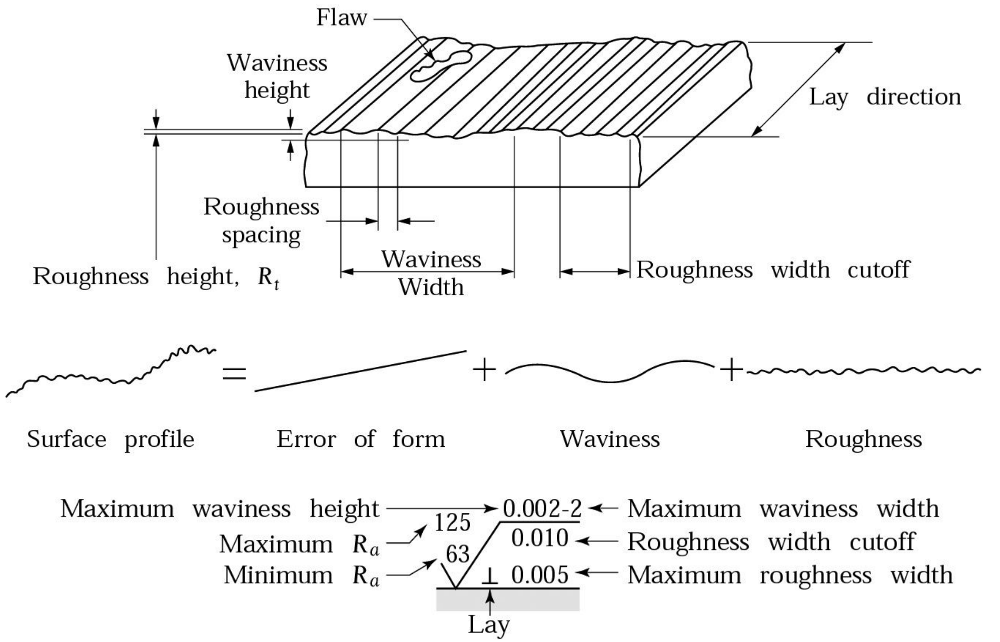


- Depth and properties of work-hardened layer is called surface structure
  - Depends upon the processing methods
  - Machining with dull tool, poor cutting conditions, thickness of work hardened layer will be more.
- Residual stress is generated in the work-hardened layer
- Thermal and electrical conductivity of contacting body will be decided by their respective surfaces
  - Rough surfaces have high electrical and thermal resistance because of fewer contact points, also have more chances of corrosion, shorter fatigue life
- Surface treatments (annealing, coatings) can influence the properties of surfaces

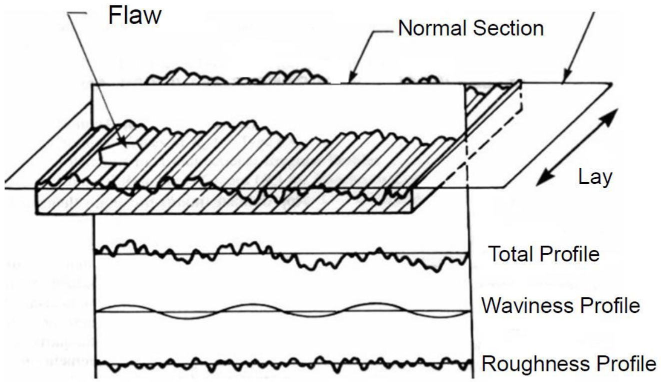
Surface texture and roughness

- All surfaces have their own set of characteristics, known as surface texture.
- Surface texture is expressed in terms of following quantities:
  - Flaws or defects: random, scratches, cracks, cavities etc
  - Lay or directionality: direction of predominant surface pattern, visible through the naked eye
  - Waviness: recurrent deviation from a flat surface, like waves on the surface of water
    - Waviness width and waviness height
    - Caused by vibration/deflection of tools, uneven lubrication, temperature variation across the workpiece
  - Roughness: closely spaced irregular deviations of a scale smaller than that for waviness.
    - Expressed in terms of its height

Surface texture and roughness



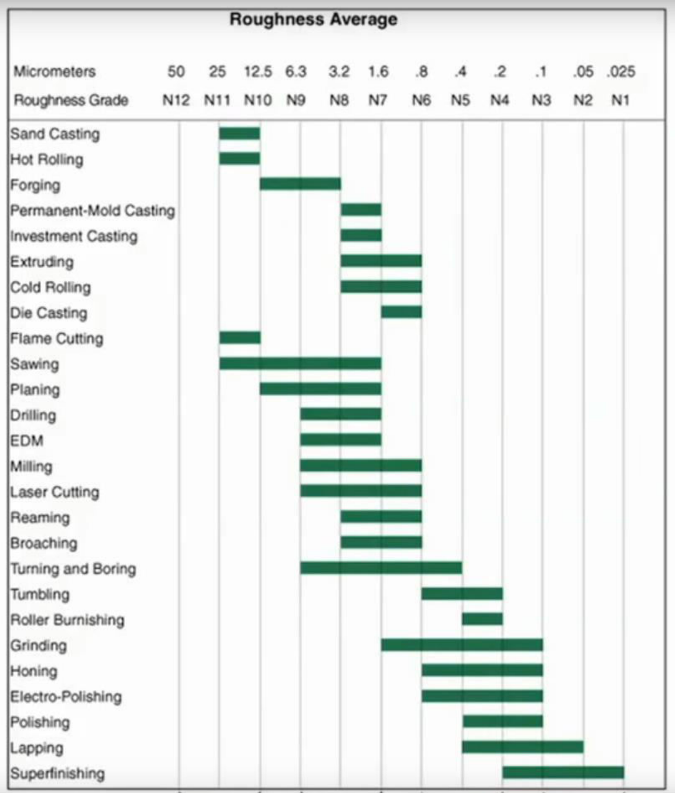
Surface Topography



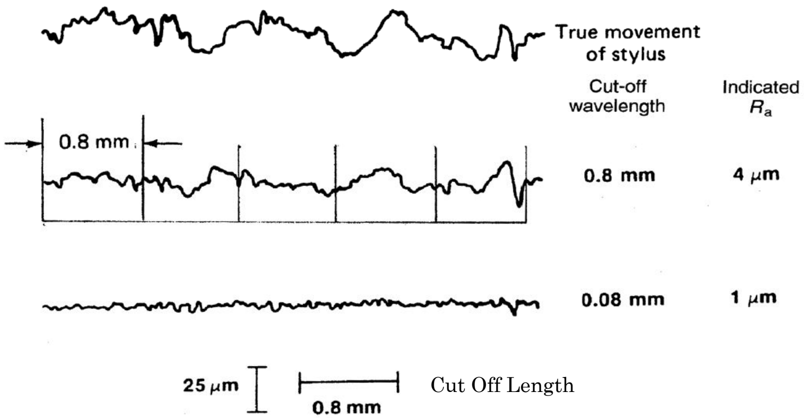
- Lay – machining pattern
- Waviness – Long wave profile
  - Axis errors, vibrations, run out
- Roughness – Short wave profile
  - Tool Feed marks, Cutting phenomena

Surface texture

- Roughness:
  - Finer irregularities in the surface texture
  - Roughness caused by tool chatter
  - Roughness is measured perpendicular to the lay direction
- Waviness:
  - More widely spaced component of surface texture
  - Waviness is an error in form due to incorrect geometry of the tool producing the surface
- Lay:
  - direction of the predominant surface pattern, ordinarily determined by the production method employed.
- Surface texture:
  - Repetitive or random deviations from the nominal surface that form the pattern of the surface.
  - Includes roughness, waviness, lay and flaws(defects)



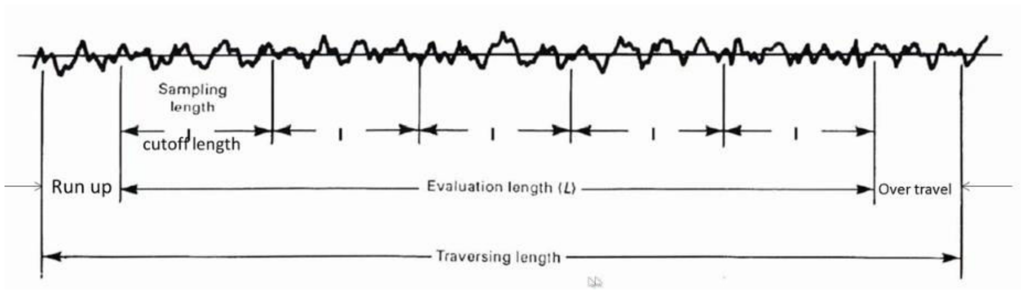
Assessment of Surface Roughness



- Standards : ISO, BS, ANSI, DIN
- 2D Parameters
  - Amplitude based
- 3D Parameters
  - Area based , Hybrid

Surface roughness value will be different for different measurement length

Cut off length



- The area selected for assessment and evaluation of the roughness parameters having the cutoff wavelength.
- Any surface irregularities spaced farther apart than the sampling length are considered waviness.
- Cutoff length depends upon the manufacturing process
- Evaluation length is normally 5x more than the cut off length

Selection of Cut off length

Machining Process	Sampling length, mm
Milling, Boring	0.8, 2.5, 8, 10
Turning	0.8, 2.5
Grinding	0.25, 0.8, 2.5
Planing	2.5, 8, 10, 25

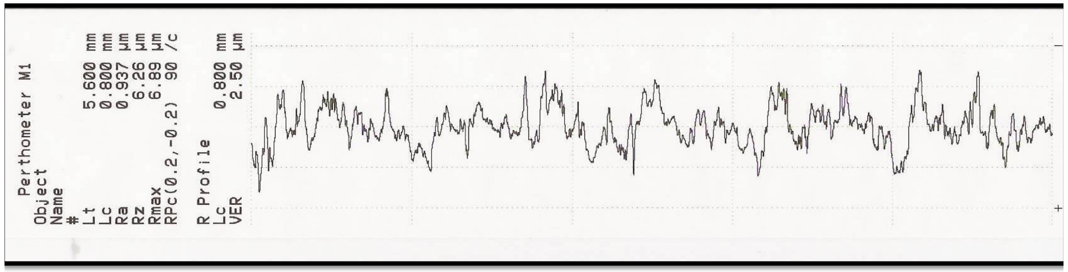
Milling, Ra = 0.8 – 7 µm
Grinding, Ra = 0.025 – 1.6 µm

Periodic Profiles	Non-Periodic Profiles		Cut-off	Sampling Length/ Evaluation Length
Spacing Distance RSm (mm)	Rz (µm)	Ra (µm)	λc (mm)	λc (mm)/L
>0.013-0.04	To 0.1	To 0.02	0.08	0.08/0.4
>0.04-0.13	>0.1-0.5	>0.02-0.1	0.25	0.25/1.25
>0.13-0.4	>0.5-10	>0.1-2	0.8	0.8/4
>0.4-1.3	>10-50	>2-10	2.5	2.5/12.5
>1.3-4.0	>50	>10	8	8/40

Surface Roughness: Measurement

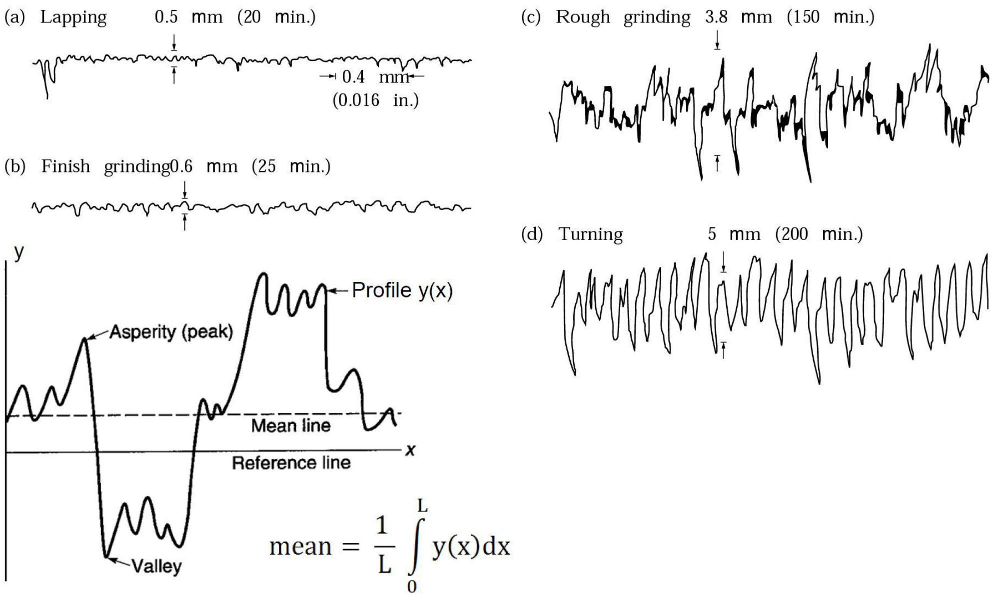
- Perthometer :
  - Used to measure surface roughness (line surface roughness)
  - Contact based profilometer

Example: surface roughness



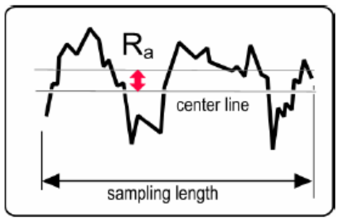
Parameter	Value
Total length(L_t)	5.6 mm
Cut-off length (L_c)	0.8mm
Measured Length (L_m)	5.6-(2*0.8)=4mm
Average Roughness (R_a)	0.937µm
Average Roughness Depth (R_z)	6.26 µm
Maximum peak-to-valley (R_{max})	6.89 µm
RPc (0.2,-0.2)	90/c
Vertical Scale Division	2.5 µm

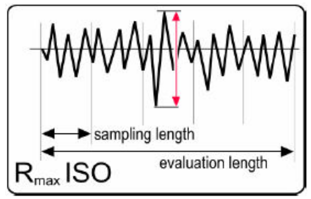
Surface roughness Profiles

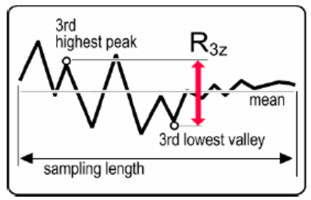


Surface Roughness Parameters

- Amplitude Based Parameters (several methods)
  - R_a – Arithmetic Average (CLA)
  - R_q – Root Mean Square (RMS)
  - R_T – total Peak-to-Valley (also known as R_{max})
  - R_z – Ten-point height
  - R_{3z} – height difference between 3rd highest peak and 3rd valley
  - R_v (Valley): Lowest valley
  - R_p (Peak): Highest peak

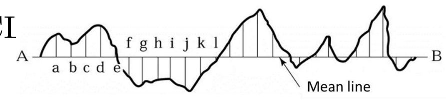




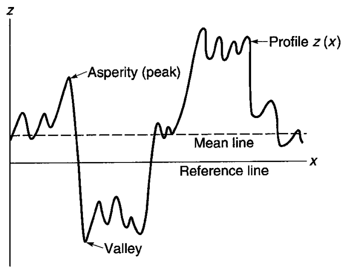


Surface roughness: measurement methods

- Arithmetic mean value (R_a): arithmetic average of the absolute values of the roughness profile ordinates from mean line
  - Also termed as Center line average (CI
  - $R_a = \frac{y_a + y_b + y_c + y_d \ldots\ldots\ldots + y_n}{n} = \frac{1}{n}\sum_{i=1}^{n} y_i$
  - $y_a, y_b, y_c \ldots y_n$ are the absolute values from the "mean line"



- Mean line is defined such that the area between the profile and the mean line above the line is equal to the below the mean line
- Ra, CLA, or Arithmetic average is the arithmetic mean of the absolute values of the vertical deviation from the "mean line" through the profile



$$R_a = CLA = AA = \frac{1}{L}\int_0^L |z - m|\, dx$$

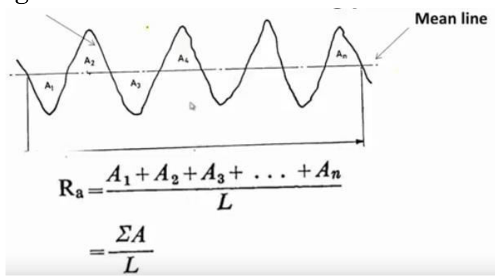
$$m = \frac{1}{L}\int_0^L z\, dx \qquad \text{m: Mean line}$$

where L is the sampling length of the profile (profile length).

Surface roughness: measurement methods

- Ra can also be determined by measuring the area under the curve and dividing it by the sampling length

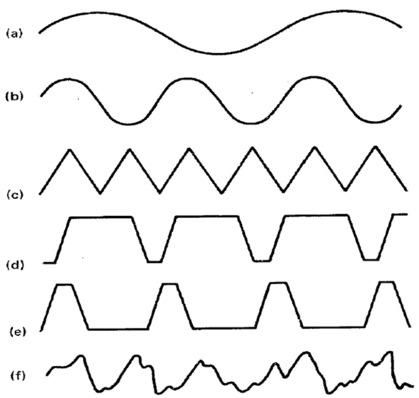
$$R_a = \frac{A_1 + A_2 + A_3 + A_4 \ldots\ldots\ldots\ldots + A_n}{L} = \frac{1}{L}\sum_{n=1}^{n} A_i$$

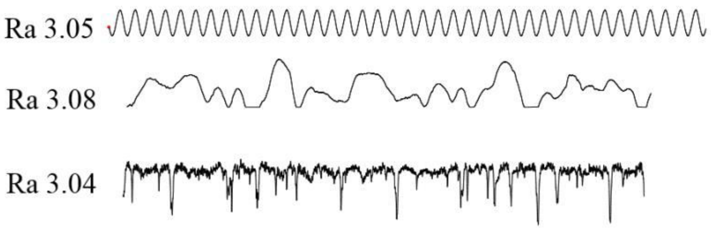


- In some cases, area is measured by planimeter and used to determine the average surface roughness
- Ra value will not differentiate between different types of surfaces
- No distinction between peak and value

Limitation of Ra

- Ra is used primarily to monitor production processes where gradual change in the surface finish due to wear can occur
- Ra is an average, defects in the surface do not generally influence the results, therefore it is not useful in detecting defects
- Ra focuses only on the profile amplitude
- Ra does not differentiate between peak and valleys, Profiles can be completely different for the same Ra value

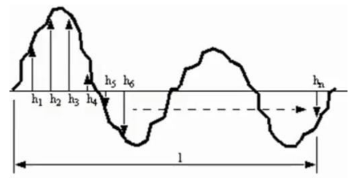


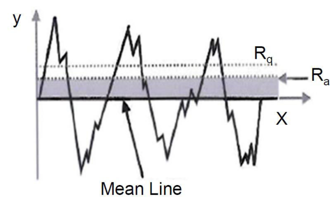


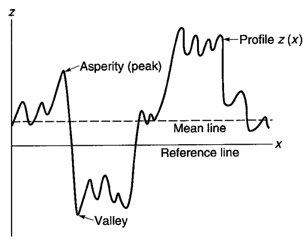
Surface roughness: measurement methods

- Root mean square roughness (R_q):
  - Rq is the square root of the arithmetic mean of the square of the vertical deviation from the mean line
  - Rq is more sensitive to peaks and valleys than Ra as the amplitudes are squared
  - Rq is typically 11% more than Ra (more preferred)
  - Used to control very fine surfaces in academic measurements
  - Sum of the areas above the line is equal to the sum of areas below center line.
  - $R_q = \sqrt{\frac{y_a^2 + y_b^2 + y_c^2 + y_d^2 \ldots\ldots\ldots + y_n^2}{n}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}$

$$R_q^2 = \frac{1}{L}\int_0^L (z - m)^2\, dx \qquad \text{m – mean line}$$







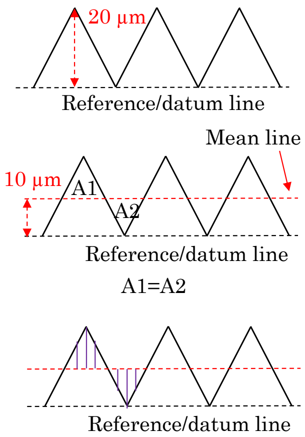
If mean line is not provided, you need to find the mean line first

Example

- Peak-to-valley height given: 20 µm, Find Ra?
- Steps: Both Ra and Rq require the mean line
  - Find mean line (mean line is that line, above and below which area is same)
  - Find the deviation of the points above and below the mean line
  - Take the absolute values of those points and divide them by number of points

$$R_a = \frac{y_a + y_b + y_c + y_d \dots\dots\dots\dots\dots + y_n}{n}$$

$$R_a = \frac{A_1 + A_2 + A_3 + A_4 \dots\dots\dots\dots\dots + A_n}{L}$$



Ra = 5 µm

Example - Roughness

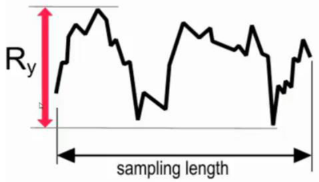
- In the measurement of surface roughness, heights of successive 10 peaks and troughs were measured from a datum and were as follows:

No.	1	2	3	4	5	6	7	8	9	10
h_t(microns)	33	25	30	19	22	18	27	29	20	32

- If the measurements were obtained on 10 mm length, determine the Ra and RMS values of surface roughness.

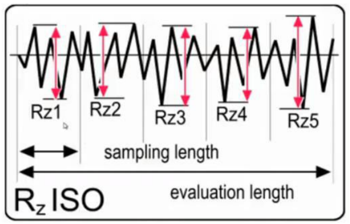
Other surface roughness parameters

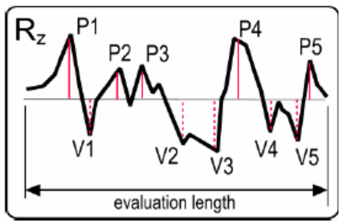
- Maximum roughness height (R_y):
  - Vertical distance from the deepest valley to the height peak.
  - Represents the amount of maximum material to be removed in order to obtain a smooth surface



- Rmax ISO: Maximum peak-to-valley profile height. The greatest peak-to-valley distance within any one sampling length
- R_z Ten-point height. The difference between the average absolute value of the five highest peaks and the five lowest valleys over the evaluation length

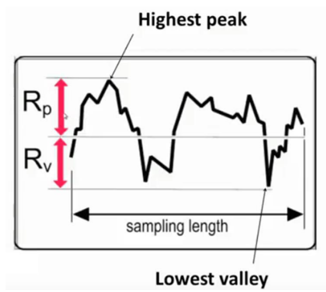
$$R_z = \frac{(P_1 + P_2 + \dots + p_5) - (V_1 + V_2 + \dots + V_5)}{5}$$





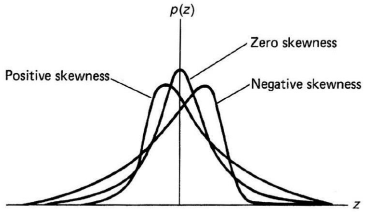
Other surface roughness

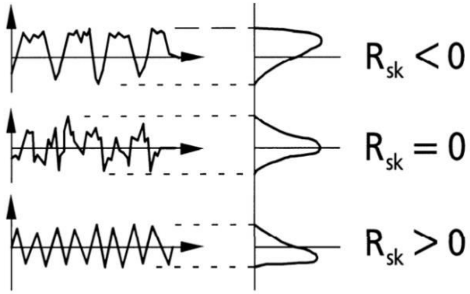
- Rp (Peak): Highest peak.
  - Maximum distance between the mean line and the highest point within the sample.
- Rv (Valley): Lowest valley.
  - Maximum distance between the mean line and the lowest point within the sample.
- Rt (P-V): Maximum peak-to-valley height. The absolute value between the highest and lowest peaks.
- Rz: Ten-point height.
  - The average absolute value of the five highest peaks and the five lowest valleys over the evaluation length.



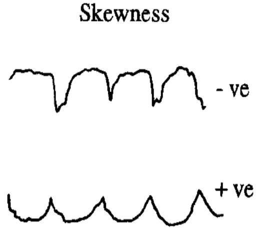
Classification of Surface Profiles: Skewness (R_{sk})

- Shape of distribution of heights
- The skew direction indicates whether the bulk of the material is above or below the mean line
- Negative skewness indicates a predominantly valley-based surface





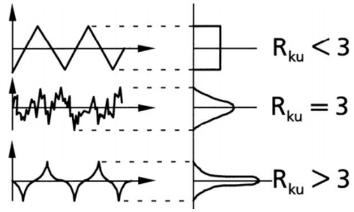
$$R_{sk} = \frac{1}{R_q^{\,3}}\left[\frac{1}{l_r}\int_0^{l_r} Z^3(x)\,dx\right]$$



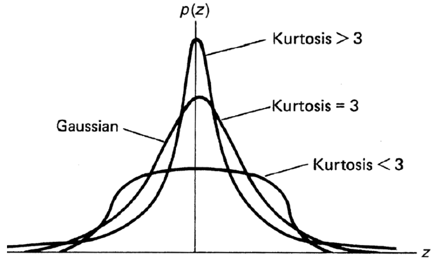
Skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean

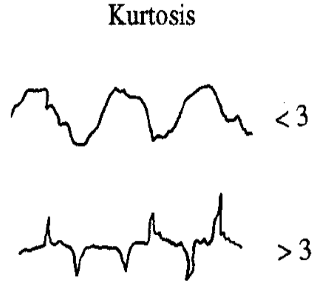
Classification of Surface Profiles : Kurtosis (R_{ku})

- Measure of the sharpness of the profile
- Unlike R_{sk} parameter, kurtosis can not only detect whether the profile spikes are evenly distributed but also provides a measure of the spikiness of the area



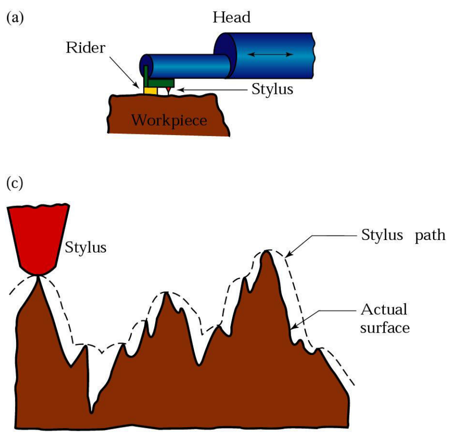
$$R_{ku} = \frac{1}{R_q^{\,4}}\left[\frac{1}{l_r}\int_0^{l_r} Z^4(x)\,dx\right]$$



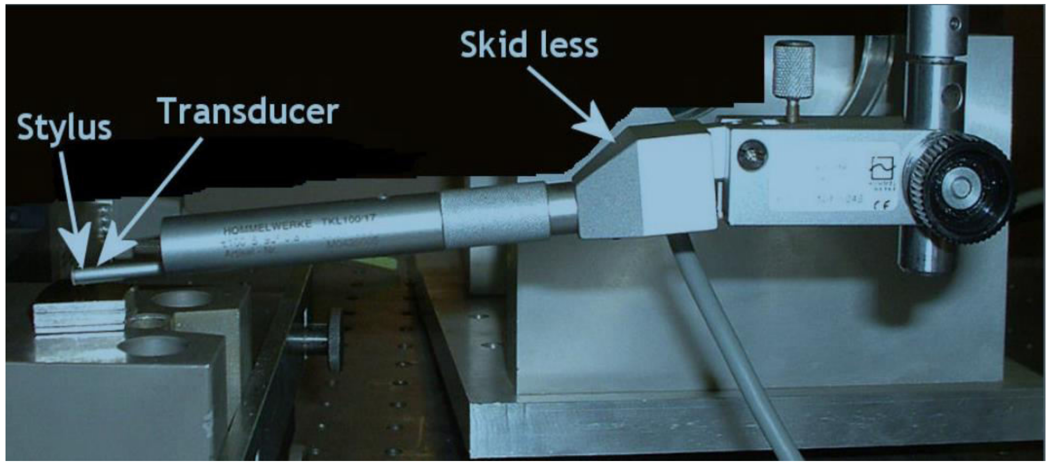


3D Surface Roughness Measurement

- Mechanical stylus based measurement
- Optical methods
- Capacitance based method
- Air flow measurement

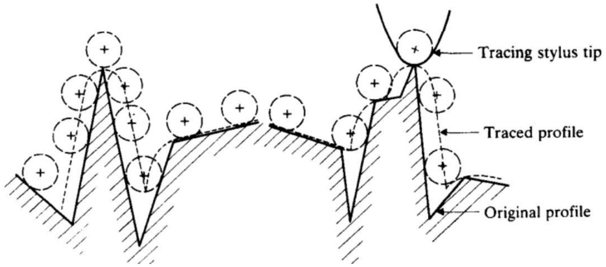


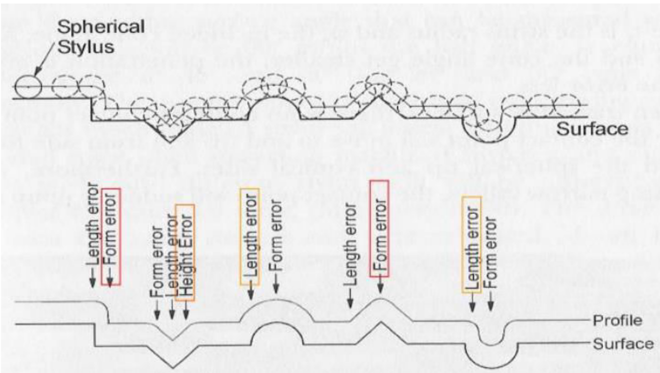
Stylus Instrument Probe



Stylus based measurements: Errors

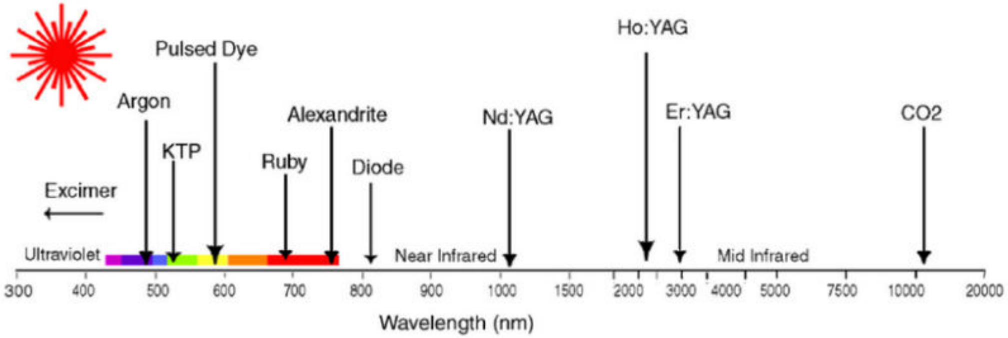
- The curvature tends to round off peaks;
- Reduce the depths of valleys
- The peak height is unaffected.





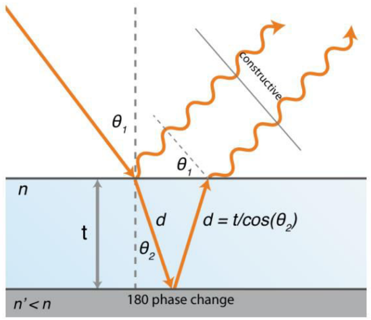
Precision Measurement using Light

- Use of light for measurement of length, displacement, angles and associated parameters
- This is mainly due to compact, portable laser systems with narrow bandwidth and high coherence

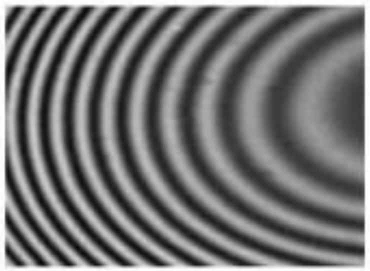


Interference

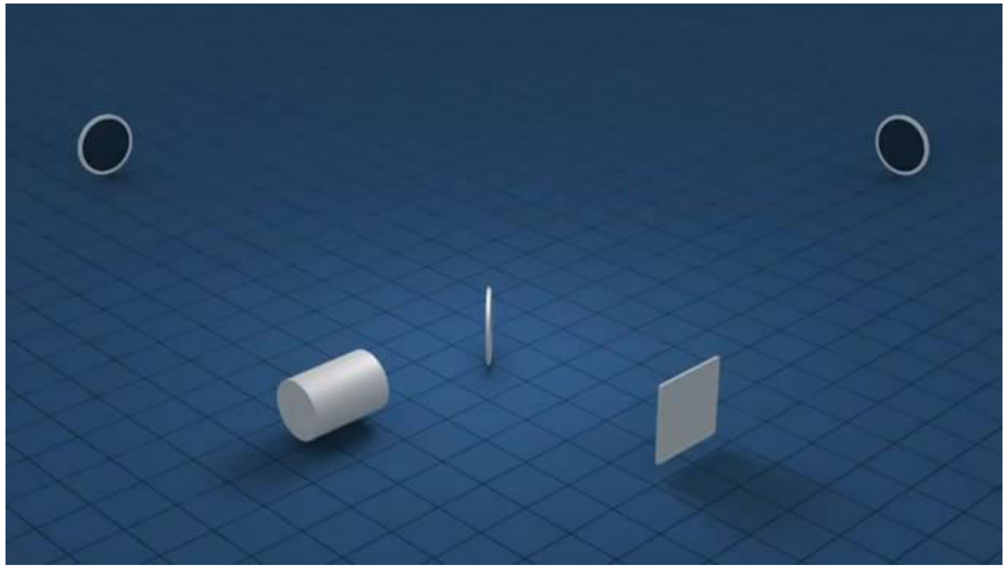
- When two or more light beams allow to cross each other, there may be a modification to the intensity obtained by their superposition
  - Monochromatic and Coherent Source
- If the modified intensity gets maximum, its constructive interference.
  - Bright spots/ring
- If the resultant or modified intensity is zero or minimum, it is destructive interference
  - Dark spots/ring
- From metrology viewpoint, it is a series of non-contact techniques using the interference of light waves to determine surface shape and transmission properties.



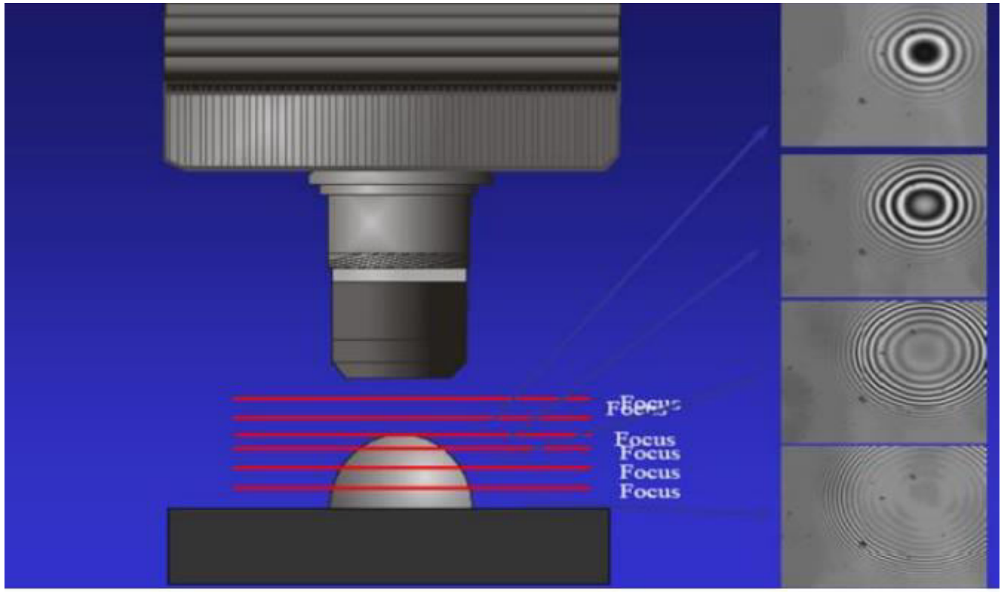
Thin Film Interference



Optical interference

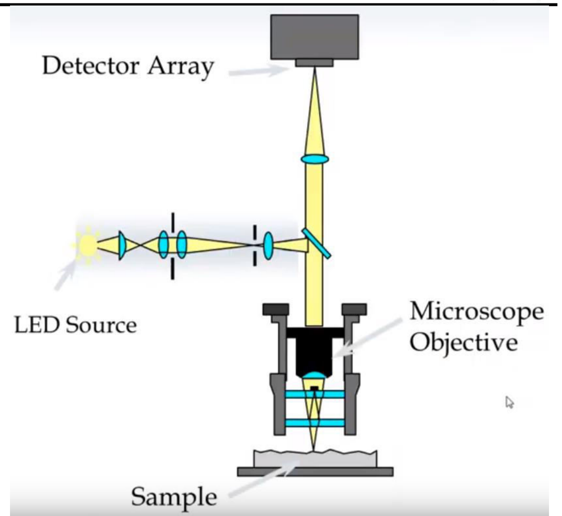


Optical Interferometers

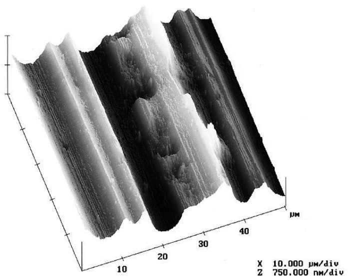


White Light Interferometer

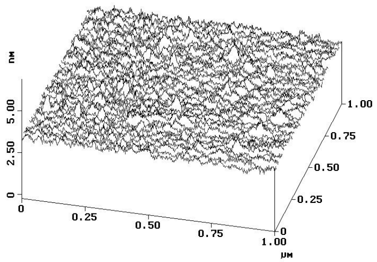
- Interference microscope
- combines an interferometer and microscope into one instrument.
- It is used for measuring engineering surfaces that demand testing with high resolving power.



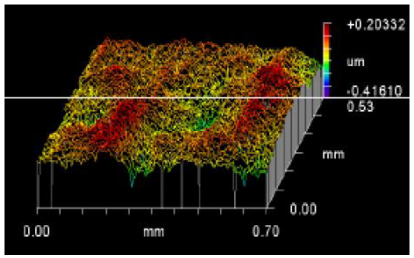
Three-Dimensional Surface Measurement



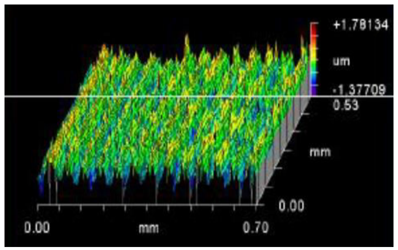
Surface of rolled aluminum



highly polished silicon surface measured in an atomic force microscope. The surface roughness is Rq = 0.134 nm.



Isotropic - Random



Turned surface - Periodic

ME338 – Manufacturing Process II

Lecture 16 – Statistical Quality control (SQC)

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Quality Improvement and Variations

- All processes, however good, are characterized by a certain amount of variation if measured using an instrument of sufficient resolution
  - Quality is inversely proportional to variability
  - More variation in the process, more issues in quality
- Quality Improvement means reduction of variations
- Variation can be due to random causes or assignable causes
- Random causes of variation:
  - human variability within each operation cycle, variations in raw materials, machine vibration, and so on.
  - Difficult to identify and control
- Assignable causes of variation:
  - Set up error, operator mistakes, defective raw materials, tool failures , machine wear/malfunctions , and so on
  - Causes can be identified and eliminated
- When the variability is confined to the Random Causes only, the process is said to be in statistical control

Traditional Statistical Tools

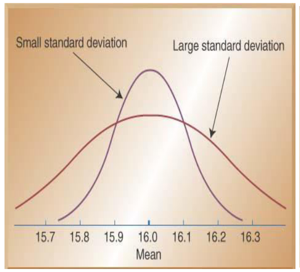
- The Mean:
  - Measure of central tendency
  - Measures of variability of data around the mean

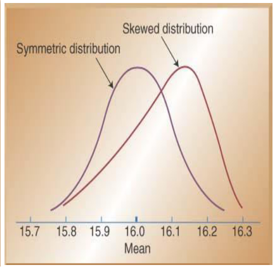
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- The Range:
  - Difference between largest/smallest observation in a set of data
  - $(R = x_{max} - x_{min})$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

- Standard Deviation measures the amount of data dispersion around mean
- Distribution of Data shape
  - bell shaped distribution or
  - Skewed distribution

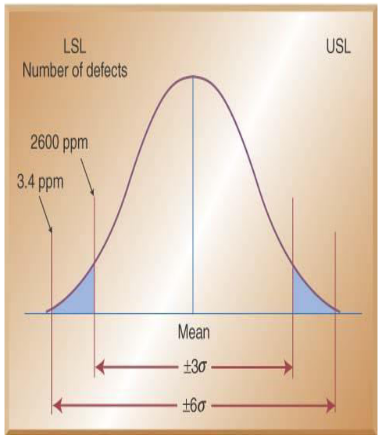




±6 Sigma versus ± 3 Sigma

- Motorola coined "six-sigma" to describe their higher quality efforts back in 1980's
- Ordinary quality standard requiring mean ±3σ to be within tolerances implies that 99.74% of production is between LSL and USL
- Six sigma is much stricter: mean ±6σ must be within tolerances implying that 99.99966% production between LSL and USL
- same proportions apply to control limits in control charts
- Six-sigma quality standard is now a benchmark in many industries

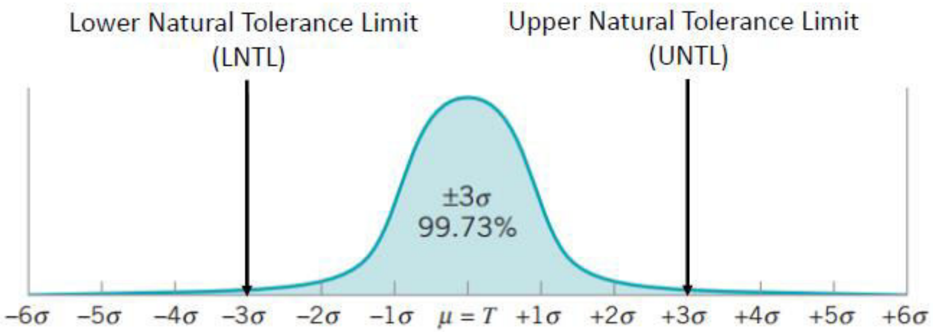
PPM Defective for ±3σ versus ±6σ quality



Process Capability

- Process capability relates to the normal variations in the output when the process is in statistical control
  - It shows manufacturer's capability to produce a product within the required tolerances
- For a process under statistical control:
  - Normally distributed
  - Process capability (PC) equals ±3σ standard deviations about the mean output value
    - $PC = \mu \pm 3\sigma$
    - $\mu = mean$
    - $\sigma = \ standard\ deviation$

Process Capability



Spec. Limit	Percent Inside Specs	ppm Defective
±1 Sigma	68.27	317300
±2 Sigma	95.45	45500
±3 Sigma	99.73	2700
±4 Sigma	99.9937	63
±5 Sigma	99.999943	0.57
±6 Sigma	99.9999998	0.002

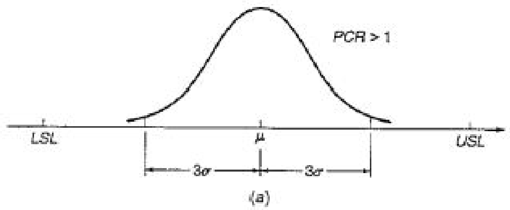
Process Capability (C_p)

- Tolerances are specified based on the allowable variability that will achieve required function and performance
- The process capability (Cp):

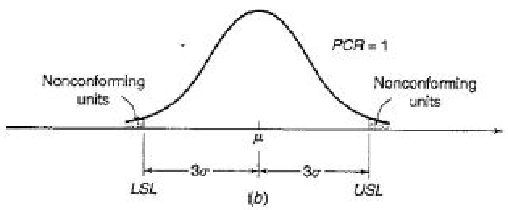
$$C_p = \frac{Tolerance\ range}{6\sigma_{sample}} = \frac{USL - LSL}{6\sigma_{sample}}$$

- USL/LSL : Upper / lower specification limit (customer defines it)
- If the process is running off the center, its actual capability will be less than the C_p obtained as above
- The automotive industry requires C_p to be 1.33
  - Tolerance range is 33% more than ±3σ

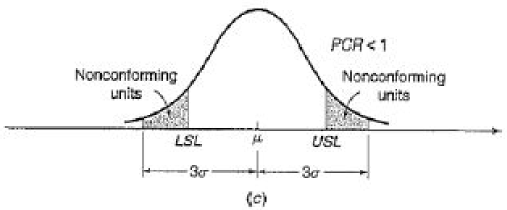
Process Capability (C_p)



Cp > 1



Cp = 1



Cp < 1

Process Capability Index (C_p and C_{pk})

- C_{pk} takes process centering into account
- $C_{pk} = \dfrac{Separation\ between\ the\ process\ mean\ and\ nearest\ tolerance\ limit}{6\sigma_{sample}}$

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma_s}, \frac{\mu - LSL}{3\sigma_s}\right)$$

- If the process is centered at the nominal dimensions, then $C_p = C_{pk}$
- C_p is the simplest indicator of a process capability while C_{pk} gives a better picture.
- C_p will give a description of form while C_{pk} provides both form and location.
- C_p index does not take into consideration the placement of process with respect to the given limits or the specification width while C_{pk} considers the centering of the process distribution.
- C_p is also known as the "process potential index" while C_{pk} is known as "process capability index" or "process performance index".

Example: C_p and C_{pk}

- Three bottling machines (A, B, C) are being evaluated for possible use. The machines must be capable of meeting the design specification of 15.8-16.2 oz. with at least a process capability index of 1.0 (Cp≥1)
- The table below shows the information gathered from production runs on each machine. **Are they all acceptable?**

Machine	σ	USL-LSL	6σ
A	.05	.4	.3
B	.1	.4	.6
C	.2	.4	1.2

- Solution:
  - Machine A

$$\mathbf{Cp}\,\frac{\mathbf{USL-LSL}}{\mathbf{6\sigma}} = \frac{\mathbf{.4}}{\mathbf{6(.05)}} = \mathbf{1.33}$$

  - Machine B

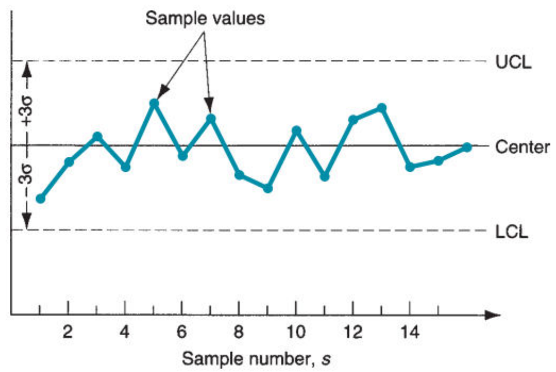
$$Cp\frac{USL-LSL}{6\sigma} = \frac{.4}{6(.1)} = 0.67$$

  - Machine C

$$Cp\frac{USL-LSL}{6\sigma} = \frac{.4}{6(.2)} = 0.33$$

Statistical Process Control

- Use of various statistical methods to assess and analyze variations in a process
- A control chart is a graphical technique in which statistics computed from measured values of a certain process characteristic are plotted over time to determine if the process remains in statistical control



UCL: Upper Control Limit

LCL: Lower Control Limit

Control charts: Types and importance

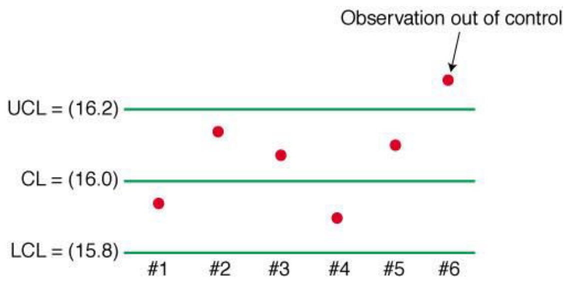
- Variables Control Charts
  - These charts are applied to data that follow a continuous distribution.
  - X-bar and R-chart
- Attributes Control Charts
  - These charts are applied to data that follow a discrete distribution.
  - P and C-chart
- Popularity of control charts
  - Control charts are a proven technique for improving productivity.
  - Control charts are effective in defect prevention.
  - Control charts prevent unnecessary process adjustment.
  - Control charts provide diagnostic information.
  - Control charts provide information about process capability.

Statistical Process Control: X-Bar and R-chart

- Control charts for variables X bar and R charts)
  - Quality characteristics are measurements or enumeration.
  - E.g. diameter, surface finish etc.
- Control charts for attributes (p and c charts)
  - Whether a part is defective or how many defects are in the sample
- The x bar chart is used to plot the average measured value of a certain quality characteristic for each of a series of samples taken from the production process.
  - It indicates how the process mean changes over time.
- The R chart plots the range of each sample,
  - To monitor the variability of the process and indicating whether it changes over time.

Control Charts

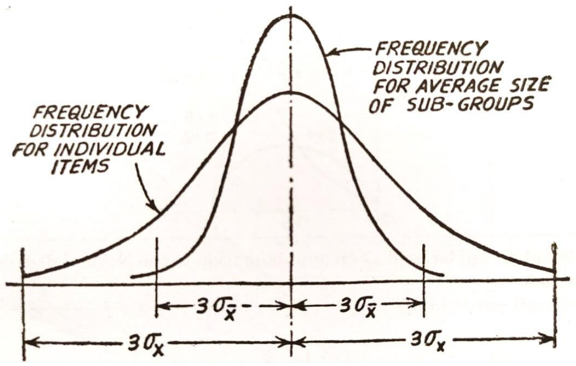
- Variable measurements: Use X- Bar and R charts
  - Something that is measured
  - Length, diameter, weight, volume
- Using Attributes: Use P and C charts
  - Something that is counted
  - Yes/No kind of measurement
  - No of defects, Number of broken cookies in a pack, rotten apples in a carton, No of customer complaints
- Control Charts show sample data plotted on a graph with CL, UCL, and LCL



Control Charts for Variables: X-bar and R-chart

- A quality characteristic is normally distributed with mean μ and standard deviation σ
- For a sample of size n:
  - Mean of sample means = $\bar{\bar{x}}$
  - Sample Deviation (σ_{sample}) = $\frac{\sigma}{\sqrt{n}}$
- Control limits
  - Upper control limit (UCL) :$\bar{\bar{x}} + 3\sigma_{sample} = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}}$
  - Lower control limit (UCL):$\bar{\bar{x}} - 3\sigma_{sample} = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}}$
- Use x-bar charts to monitor the changes in the mean of a process (central tendencies)
- Use R-bar charts to monitor the dispersion or variability of the process

σ_{sample} Vs σ



$$\sigma_{sample} = \frac{\sigma}{\sqrt{n}}$$

- Standard deviation of sample size (σ_{sample}) is smaller than standard deviation of individual samples (σ)
- However, the mean will remain same

Control Charts for Variables: X-bar and R-chart

- Number of Samples / Sample size (observations per sample)
- A series of samples (e.g., k = 20 or more is generally recommended) each of small size (e.g., n = 3, 4 , 5, or 6 parts per sample) are collected and the characteristic of interest is measured for each part
- The sample mean ($\bar{x}$)and mean of sample means ($\bar{\bar{x}}$)is:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i; \text{ and } \bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k} \bar{x}_i$$

- $\bar{\bar{x}}$ serves as an estimator of process mean μ
- The center line (CL) of X-bar chart is defined by $\bar{\bar{x}}$
- Similarly, the center line (CL) of R chart is $\bar{R}$,mean of range R obtained for all samples
- An estimator of σ is given by $\bar{\sigma}$
- $\bar{\sigma} = \frac{\bar{R}}{d_2}$where d_2 is an adjustment factor

Control Charts for Variables: X-bar and R-chart

- Control limits for X-bar chart are defined as:
  - $LCL = \bar{\bar{x}} - 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$
  - $UCL = \bar{\bar{x}} + 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$
- Control limits for R chart are defined as:
  - $LCL = D_3\bar{R}$
  - $UCL = D_4\bar{R}$

Table of d_2, A_2, D_3, and D_4 values as a Function of the subgroup sample size, n

n	d_2	A_2	D_3	D_4
2	1.128	1.880	0	3.267
3	1.693	1.023	0	2.574
4	2.059	0.729	0	2.282
5	2.326	0.577	0	2.114
6	2.534	0.483	0	2.004
7	2.704	0.419	0.076	1.924

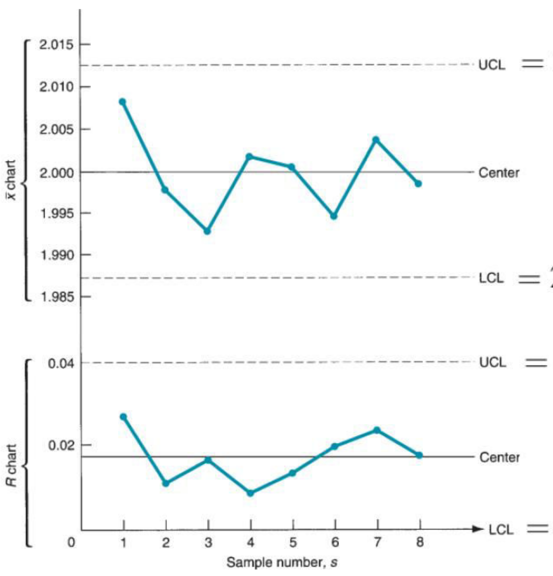
Example: X-bar and R-chart

- Eight samples of size 4 have been collected from a manufacturing process that is in statistical control, and the dimension of interest has been measured for each part .
- The calculated x bar values (units are cm) for the eight samples are: 2.008, 1.998, 1.993, 2.002 , 2.001 , 1.995, 2.004, and 1.999
- The calculated R values (cm) are 0.027, 0.011 , 0.017 , 0.009, 0.014, 0.020, 0.024, and 0.018
- Number of samples, k = 8 and sample size, n = 4
- From Table, $A_2 = 0.729$; $D_3 = 0$ and $D_4 = 2.282$

$$\bar{\bar{x}} = (2.008 + 1.998 + \cdots + 1.999)/8 = 2.000$$

$$\bar{R} = (0.027 + 0.011 + \cdots + 0.018)/8 = 0.0175$$

Example: X-bar and R-chart



UCL $= 2.000 + 0.729(0.0175) = 2.0128$

LCL $= 2.000 - 0.729(0.0175) = 1.9872$

UCL $= 2.282(0.0175) = 0.0399$

LCL $= 0(0.0175) = 0$

-

Example: Constructing a X-bar Chart

- A quality control inspector at the soft drink company has taken three samples with four observations each of the volume of bottles filled. If the σ of the bottling operation is .2 ounces, use the below data to develop control charts with limits of 3σ for the 16 oz. bottling operation

	observ 1	observ 2	observ 3	observ 4	mean	range
samp 1	15.8	16	15.8	15.9	15.88	0.2
samp 2	16.1	16	15.8	15.9	15.95	0.3
samp 3	16	15.9	15.9	15.8	15.90	0.2

- K=3, n=4 (sample size)
- Center line and control limit formulas

$$\bar{\bar{x}} = \frac{x_1 + x_2 + \ldots x_n}{k}, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where (k) is the # of sample means and (n) is the # of observations w/in each sample

$$UCL_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

Example: Constructing a X-bar Chart

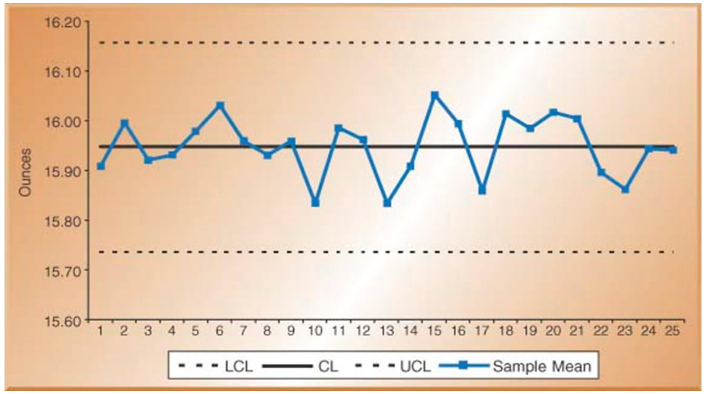
Center line ($\bar{\bar{x}}$):

$$\bar{\bar{x}} = \frac{15.875 + 15.975 + 15.9}{3} = 15.92$$

Control limits for±3σ limits:

$$UCL_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 15.92 + 3\left(\frac{.2}{\sqrt{4}}\right) = 16.22$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 15.92 - 3\left(\frac{.2}{\sqrt{4}}\right) = 15.62$$



Process is within control limits

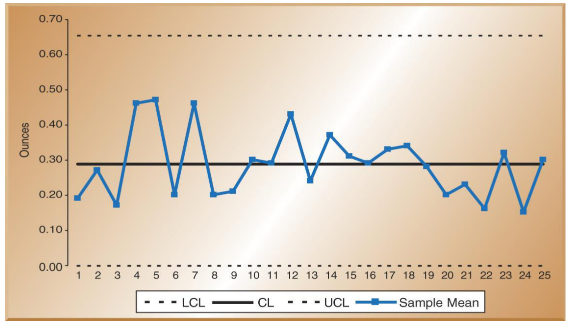
Control Chart for Range (R)

- Center Line and Control Limit formulas:

$$\bar{R} = \frac{0.2 + 0.3 + 0.2}{3} = .233$$

$$UCL_R = D_4\bar{R} = 2.28(.233) = .53$$

$$LCL_R = D_3\bar{R} = 0.0(.233) = 0.0$$



Process is within control limits

Factors for three sigma control limits

Sample Size (n)	Factor for x-Chart	Factors for R-Chart	
	A2	D3	D4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

X-Bar and R charts

- Samples are drawn from a Production Process and Inspected
- Let K = No. of samples, n = No. of observations per sample
- X- Bar Chart Monitors mean of the process
- Sample mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Mean of Samples

$$\bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k} \bar{x}_i$$

- Upper control Limit

$$UCL = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}}$$

- lower control Limit

$$LCL = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}}$$

R Control Limits

- The x-chart provides information about the variation around the average value for each subgroup. It is also important to know whether the range of values is stable from group to group.
- If some observations exhibit very large ranges and others very small ranges, you might conclude that the sprayer is not functioning consistently over time.
- To test this, you can create a control chart of the average subgroup ranges, called a range chart.
- Monitors dispersion or Variability of Process
- Factors : D4, D3 (From Table)

$$\text{Centerline} = \bar{R}$$

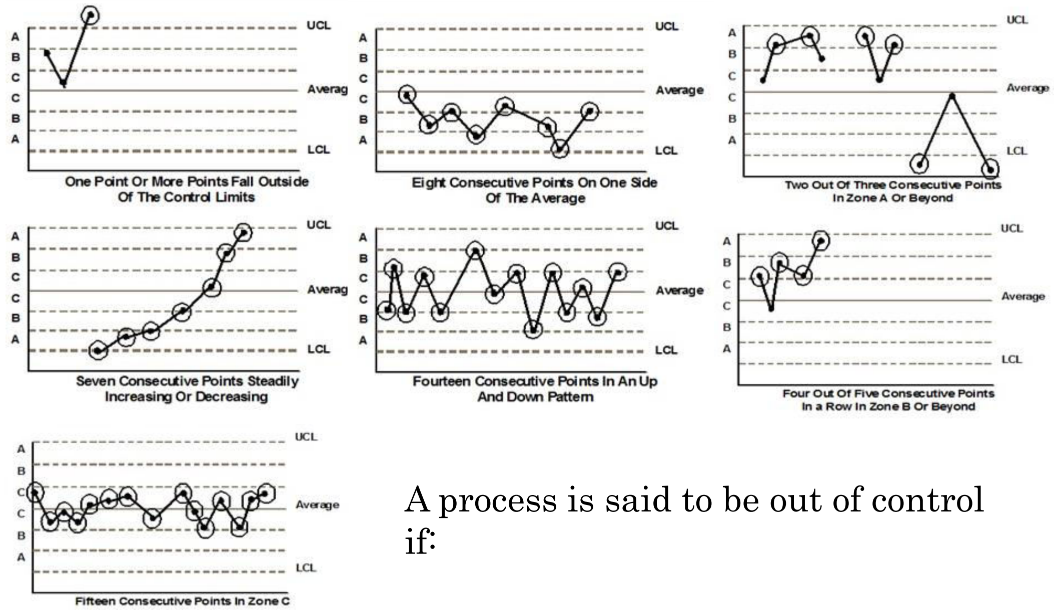
$$UCL = D_4 \cdot \bar{R}$$

$$LCL = D_3 \cdot \bar{R}$$

Factors for X Bar and R Charts

Sample Size n	Factor for $\bar{x}$-Chart A_2	Factors for R-Chart D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72

Example: Out of control – ±3σ limits

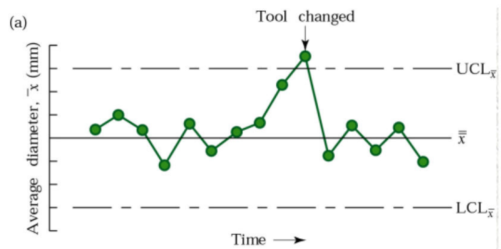


A process is said to be out of control if:

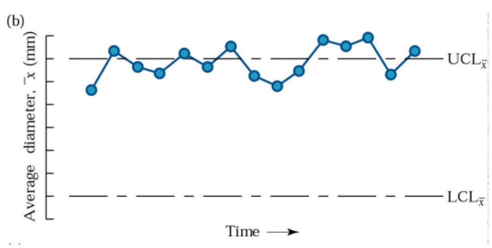
Chart Interpretation

- When control charts are used to monitor production quality, random samples are drawn from the process of the same size n used to construct the charts
- R chart is examined before x bar to ensure whether variability is under control
- Look for out of control points and find out if any assignable cause
- Other patterns
  - Trends or cyclical patterns in the data, which may mean wear or other factors that occur as a function of time;
  - Sudden changes in average level of the data; and
  - Points consistently near the upper or lower limits.

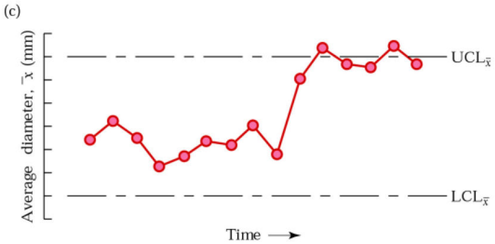
Chart Interpretation



Process begins to become out of control because of such factors as tool wear (drift).
The tool is changed and the process is then in statistical control.



Process parameters are not set properly; thus all parts are around the upper control limit (shift in mean)



Process becomes out of control because of factors such as a change in the properties of the incoming material (shift in mean)

Example

The data shown are the deviations from nominal diameter for holes drilled in a carbon-fiber composite material used in aerospace manufacturing.

The values reported are deviations from nominal in ten-thousandths of an inch.

1. Set up X-bar and R charts on the process. Is the process in statistical control?
2. Estimate the process standard deviation using the range method.
3. If specifications are at nominal ±100, what can you say about the capability of this process?

Sample Number	x_1	x_2	x_3	x_4	x_5
1	−30	+50	−20	+10	+30
2	0	+50	−60	−20	+30
3	−50	+10	+20	+30	+20
4	−10	−10	+30	−20	+50
5	+20	−40	+50	+20	+10
6	0	0	+40	−40	+20
7	0	0	+20	−20	−10
8	+70	−30	+30	−10	0
9	0	0	+20	−20	+10
10	+10	+20	+30	+10	+50
11	+40	0	+20	0	+20
12	+30	+20	+30	+10	+40
13	+30	−30	0	+10	+10
14	+30	−10	+50	−10	−30
15	+10	−10	+50	+40	0
16	0	0	+30	−10	0
17	+20	+20	+30	+30	−20
18	+10	−20	+50	+30	+10
19	+50	−10	+40	+20	0
20	+50	0	0	+30	+10

Example

(1)

X-bar chart:

CL = 10.9, UCL = 50.35, LCL = −28.55

***R* chart:**

CL = 54.1, UCL = 123.5, LCL = 0

Process is in statistical control.

(2)

Process SD is estimated as $\dfrac{\overline{R}}{d_2} = 23.3$

(3) $C_{pk} = 200 / (6 \times 23.3) = 1.43$

Sample Number	x_1	x_2	x_3	x_4	x_5
1	−30	+50	−20	+10	+30
2	0	+50	−60	−20	+30
3	−50	+10	+20	+30	+20
4	−10	−10	+30	−20	+50
5	+20	−40	+50	+20	+10
6	0	0	+40	−40	+20
7	0	0	+20	−20	−10
8	+70	−30	+30	−10	0
9	0	0	+20	−20	+10
10	+10	+20	+30	+10	+50
11	+40	0	+20	0	+20
12	+30	+20	+30	+10	+40
13	+30	−30	0	+10	+10
14	+30	−10	+50	−10	−30
15	+10	−10	+50	+40	0
16	0	0	+30	−10	0
17	+20	+20	+30	+30	−20
18	+10	−20	+50	+30	+10
19	+50	−10	+40	+20	0
20	+50	0	0	+30	+10

Control Chart for Attributes: P-chart

- In the p chart , the quality characteristic of interest is the proportion of nonconforming or defective units.
- P-chart follows binomial distribution
- For k samples, each of size n , and D_i is number of defectives in i^{th} sample
- Average proportion defective in a sample is $\bar{p} = \dfrac{\sum_{i=1}^{k} D_i}{kn}$
- The centreline is: $\bar{p} = \dfrac{\sum_{i=1}^{k} D_i}{kn}$ $\qquad \sigma_p = \sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$
- The control limits are:

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

n= no of observations in a sample

P-Chart Example:

- A Production manager for a tire company has inspected the number of defective tires in five random samples with 20 tires in each sample. The table below shows the number of defective tires in each sample of 20 tires. Calculate the control limits.

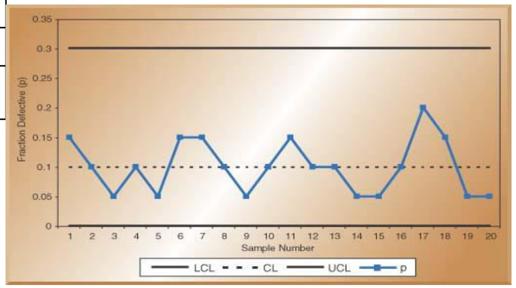
Sample	Number of Defective Tires	Number of Tires in each Sample	Proportion Defective
1	3	20	.15
2	2	20	.10
3	1	20	.05
4	2	20	.10
5	1	20	.05
Total	9	100	.09

$$CL = \bar{p} = \frac{\#\,\text{Defectives}}{\text{Total Inspected}} = \frac{9}{100} = .09$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{(.09)(.91)}{20}} = 0.64$$

$$UCL_p = \bar{p} + z(\sigma_p) = .09 + 3(0.64) = .282$$

$$LCL_p = \bar{p} - z(\sigma_p) = .09 - 3(0.64) = -.102 = 0$$



P-chart: example

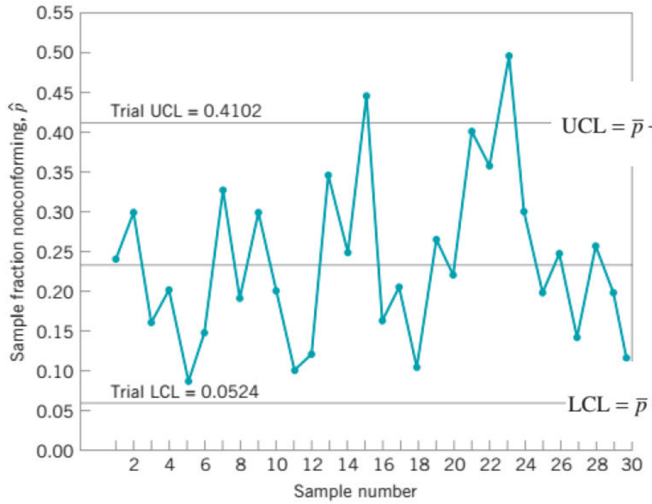
- Frozen orange juice concentrate is packed in 6 oz cardboard cans.
- These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel
- By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel.
- Set up a control chart to improve the fraction of non conforming cans produced by this machine.
- Total 30 samples each having 50 number of juice cans were inspected.

P-chart: example

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

P-chart: example



$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$

Example

- In 12 samples of size n = 7, the average value of the sample means is 6.860 cm for the dimension of interest, and the mean of the ranges of the samples is 0.027 cm. Determine (a) lower and upper control limits for the x chart and (b) lower and upper control limits for the R chart. (c) What is your best estimate of the standard deviation of the process?
  - Ans : x bar chart: LCL = 6.8487 cm and UCL = 6.8713 cm; R chart: LCL = 0.0205 cm and UCL = 0.0519 cm ; estimate of SD = 0.00998 cm)
- A p chart is to be constructed. Six samples of 25 parts each have been collected, and the average number of defects per sample was 2.75. Determine the center , LCL and UCL for the p chart .
  - Ans : p bar = CL = 0.11, LCL = 0 and UCL = 0.298)
- The upper and lower control limits for a p chart are: LCL = 0.19 and UCL = 0.24. Determine the sample size n that is used with this control chart. Ans : p bar = 0.17, n = 259)

P-chart

- P-chart (Proportion chart) is closely related to the C-chart (count chart). It depicts the proportion of items with a particular attribute, such as defects.
- The P-chart is often used to analyze the proportion of defects in each subgroup.
- **P Control Chart**
  - Chart constructed in terms of
  - p = Average Proportion Defective in a sample
- Use a P-Chart for yes/no or good/bad decisions (discrete items) in which defective items are clearly identified in terms of proportions
  - How many good decisions out of total decisions
- Use a C-Chart for more general counting when there can be more than one defect per unit
  - Number of flaws or stains in a carpet sample cut from a production run
  - Number of complaints per customer at a hotel

Example: Inspection Data – No of Defects

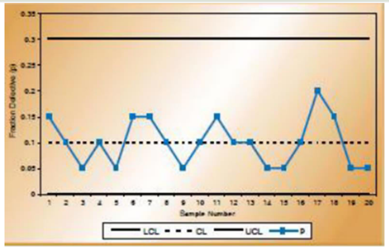
- In a Production Process, data on number of defective parts has been collected. 20 samples have been drawn each with 20 observations.
- Construct P chart for the inspection process.

Sample size (k) : 20
Observations per sample (n) : 20
Total Observations : 400
Total number of defects observed : 40
Fraction defective $\overline{p}$ = 0.1
σ_p = 0.067

Chart limits

UCL = 0.301
LCL = -0.1 -- > 0.0

Sample Number	Number of Defective Tires	Number of Observations Sampled	Fraction Defective
1	3	20	.15
2	2	20	.10
3	1	20	.05
4	2	20	.10
5	1	20	.05
6	3	20	.15
7	3	20	.15
8	2	20	.10
9	1	20	.05
10	2	20	.10
11	3	20	.15
12	2	20	.10
13	2	20	.10
14	1	20	.05
15	1	20	.05
16	2	20	.10
17	4	20	.20
18	3	20	.15
19	1	20	.05
20	1	20	.05
Total	40	400	



C-Chart Example

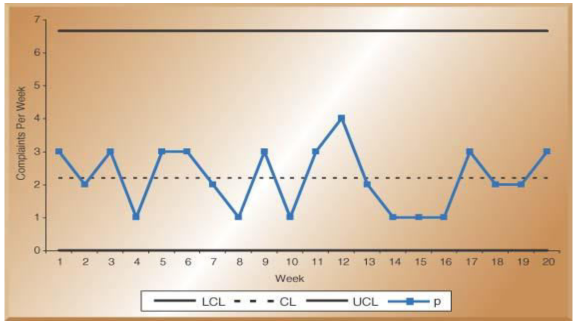
- C-chart (count chart) follows Poisson's distribution
- The number of weekly customer complaints are monitored in a large hotel using a c-chart. Develop three sigma control limits using the data table below.

Week	Number of Complaints
1	3
2	2
3	3
4	1
5	3
6	3
7	2
8	1
9	3
10	1
Total	22

$$CL = \overline{c} = \frac{\#\text{complaints}}{\#\text{ of samples}} = \frac{22}{10} = 2.2$$

$$UCL_c = \overline{c} + z\sqrt{\overline{c}} = 2.2 + 3\sqrt{2.2} = 6.65$$

$$LCL_c = \overline{c} - z\sqrt{\overline{c}} = 2.2 - 3\sqrt{2.2} = -2.25 = 0$$



Confidence level

Suppose $X_1, X_2, \ldots, X_n$ are random sample from N(μ, σ²) where μ is unknown and σ is known.

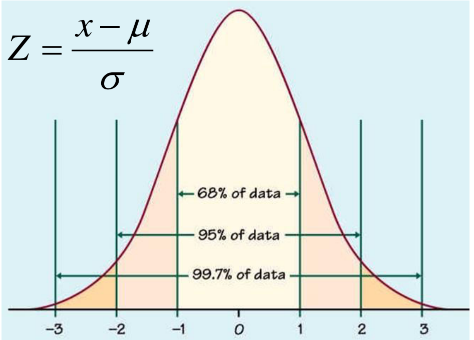
A 100(1-α)% confidence interval for μ is: $\bar{x} \pm z_{\frac{\alpha}{2}} . \frac{\sigma}{\sqrt{n}}$

- A confidence level is a measure of the degree of reliability of a confidence interval. It is denoted as 100(1-α)%.
  - A confidence level of 100(1-α)% implies that 100(1-α)% of all samples would include the true value of the parameter estimated.
  - Most frequently used confidence levels are 90%, 95% and 99%.
- The higher the confidence level, higher are chances that true value of the parameter being estimated lies within the interval.

Confidence level: using Z-table

- A confidence level of 100(1-α)% implies that 100(1-α)% of all samples would include the true value of the parameter estimated.
- Z-table is used when population standard deviation (σ) is known but mean is not known. Sample mean is available.

Confidence level	Z value
90%	1.65
95%	1.96
99%	2.58
99,9%	3.291



- 68% of cases in a Normal distribution fall within 1 std. deviation of the mean
- 95% within 2 std. dev. (actually 1.96)
- 99.7% within 3 std. dev.

Example: confidence level

Randomly sampling 100 students for their CPI, you get a sample mean of 8.0 and a population standard deviation of 1.2. What is the 95% confidence interval?

Solution steps:

- First find the standard deviation for sample size (n): $\sigma_{sample}: \frac{\sigma}{\sqrt{n}} = 0.12$
- Calculate the lower confidence boundary: 8.0 – (1.96* 0.12) = 7.77
- Calculate the upper confidence boundary: 8.0 + (1.96* 0.12) = 8.24
- You are 95% confident that the interval 8.0 +/- 0.12 or 7.77 to 8.24 contains the true student population mean GPA.