

# ME 311: Microprocessors and Automatic Control



## Concept of feedback in Mechatronics systems



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# Important Concepts



- What is feedback?
- Open Loop Vs Closed Loop system
- How to process the feedback variable to achieve the desired goal? OR
- When goal is given, how to decide what should be the control algorithm? Is it unique?

We will answer these questions in the next part of course

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# Input and output



- It's important to know what are input and output variables given any system for control
- The relationship between input and output variables is obtained by the way of mathematical modeling we saw: applying standard physics law (Newton's, Kirchoff's etc.)
- From control perspective several representations could be useful: ODEs, State space, transfer function etc.

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## Concept of Feedback: Example



- Goal: to place mass at a desired position

Current pos      Desired pos

- Identify: quantity of interest (output) and quantity you can dictate (input)
- What is differential equation connecting input and output?

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# Concept of Feedback: Example



- Goal: to place mass at a desired position
- Control input: Force F
- Output: displacement x

Current pos

Desired pos

Equation

$$F = m\ddot{x}$$



Q: To take mass to desired position  $x_d$  can you think what F should be applied?

- One way is to define/ plan trajectory from current position  $x_0$  to  $x_d$  and get corresponding  $\ddot{x}_d(t)$  and apply  $F = m\ddot{x}_d$  and hope that with this the desired position would be reached



Q: Can you see what is the problem with such solution

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# Concept of Feedback: Example



- Goal: to place mass at a desired position
- Input: Force F
- Output: displacement x

Equation  $F = m\ddot{x}$

- A: no feedback hence we would not know if we have really achieved the task, especially in the presence of external disturbances such as friction.
- Hence we need to sense the current position and make use of the same in developing F to be applied.



Q: what function  $F=f(x)$  (notice NOT  $f(t)$ ) should F be, so as to achieve the desired goal?? Can you think of?

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# Concept of Feedback: Example



- Goal: to place mass at a desired position
- Input: Force F
- Output: displacement x
- Equation  $F = m\ddot{x}$

- Q: How to come up with such function?
- One idea is to see error between current x and  $x_d$  and make F to be some function of this error such that  $F = 0$  when error is 0.
- Another idea is to consider virtual springs and dampers in the system such that equilibrium position of such spring mass damper system coincides with  $x_d$ .  $F = -k_p(x - x_d) = -k_p e$

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# Concept of Feedback: Example



- Goal: to place mass at a desired position
- Input: Force F
- Output: displacement x
- Equation  $F = m\ddot{x}$



Q: How do we know that such F would work towards achieving our goal?

- A: carry out analysis by substituting this F in equation of dynamics. Writing everything in e variable, thus

$$F = -k_p(x - x_d) = -k_p e = m\ddot{x} = m\ddot{e}$$

$$\therefore m\ddot{e} + k_p e = 0$$

- This is equation of harmonic system without damping and for initial error  $e_0$  the final error will keep oscillating and never be zero as desired  $\otimes$  so how to take care of this!

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# Concept of Feedback: Example



- In effect what we have done is ‘attached’ a virtual spring to the mass whose equilibrium position is at  $x_d$  which actually is leading to indefinite oscillations.
- A: Look at the final error equation and see what form we would desire for this equation. The form we would desire is

$$m\ddot{e} + k_d\dot{e} + k_p e = 0$$

- In effect we would like to have additional virtual damper in the system which would reduce oscillations finally to zero meaning  $e = 0$  which means  $x = x_d$  ☺



Q: can you see what expression for  $F$  would give the above equation of “closed loop dynamics” ??

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# Concept of Feedback: Example



- Back substituting we would see that to get the desired equation of error dynamics  $m\ddot{e} + k_d\dot{e} + k_p e = 0$  we would need

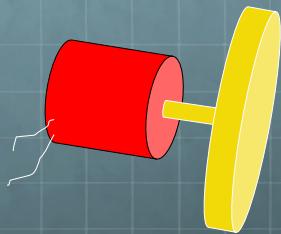
$$\begin{aligned} F &= -k_p e - k_d \dot{e} \\ &= -k_p(x - x_d) - k_d \dot{x} \end{aligned}$$

Notice both  
position and  
velocity are  
FED BACK.

- Every time doing such analysis is difficult especially for more complex systems hence we would need additional tools to analyze and synthesize control input ( $F$  in this case) to achieve the desired goal in terms of desired output
- For linear systems these would be transfer function or state space analysis. However the most fundamental is ode equations

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# Concept of Feedback: Example



Goal: to place disc at a desired angle

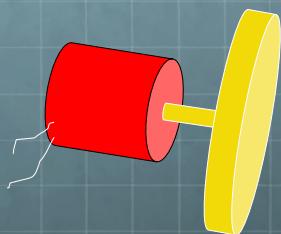
Feedback??? Theta

- Identify: quantity of interest (output) : Theta and quantity you can dictate (input) : Torque
- Some may claim input can be voltage given to motor as well!! Possible but we then need motor dynamics to be considered in analysis as well
- The system is mathematically exactly same as previous system equation being  $\tau = J\ddot{\theta}$
- HW: work out the control laws and analysis similar to the previous example!!

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# Concept of Feedback: Example



Goal: to place disc at a desired angle

Feedback??? Theta

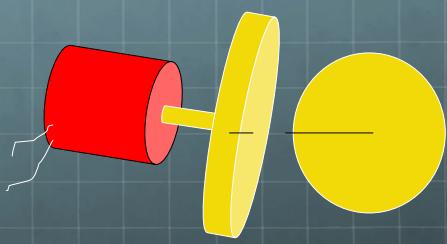
- Q: what would happen in the presence of friction? Would your proposed PD control work?
- Can you think again based on error dynamics in the presence of friction, what would be additional term in control input that would help overcome friction or minimize bad effects of friction?
- Q: what if I would like to move disc along some desired function of theta in time??

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## Feedback: Example



- Goal: to rotate disc with desired speed
- Feedback???
- Can the angle act as feedback?
- Desired output? Control input?
- What will be equation of dynamics in terms of speed  $\omega$ ?

- Can you develop control law that can achieve this goal?



Q: Do you require  $kd$  term in this case?

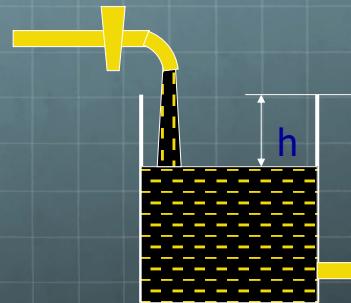
- A: NO. we do not because since the system in the new variable is of first order there will be no overshoot and error response would be exponential!!

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## Feedback: Example



- Goal: to maintain height of liquid constant
- Feedback???

- Identify: quantity of interest (output) and quantity you can dictate (input)

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## Example : Feedback

- Driving car / riding bicycle
- Dynamics is more complex! What is feedback for such case?

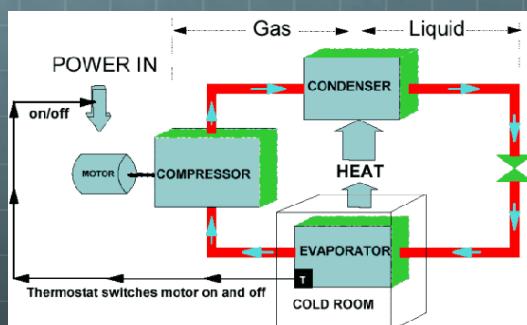


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## Feedback: Example

Air conditioning system



Goal: to maintain temp constant at  $22^{\circ}\text{C}$

Feedback???

- Identify: quantity of interest (output) and quantity you can dictate (input)

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# Feedback: Example



- Goal: to maintain focus on object no matter what the distance is
- Feedback???

- Identify: quantity of interest (output) and quantity you can dictate (input)

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# Thank You

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