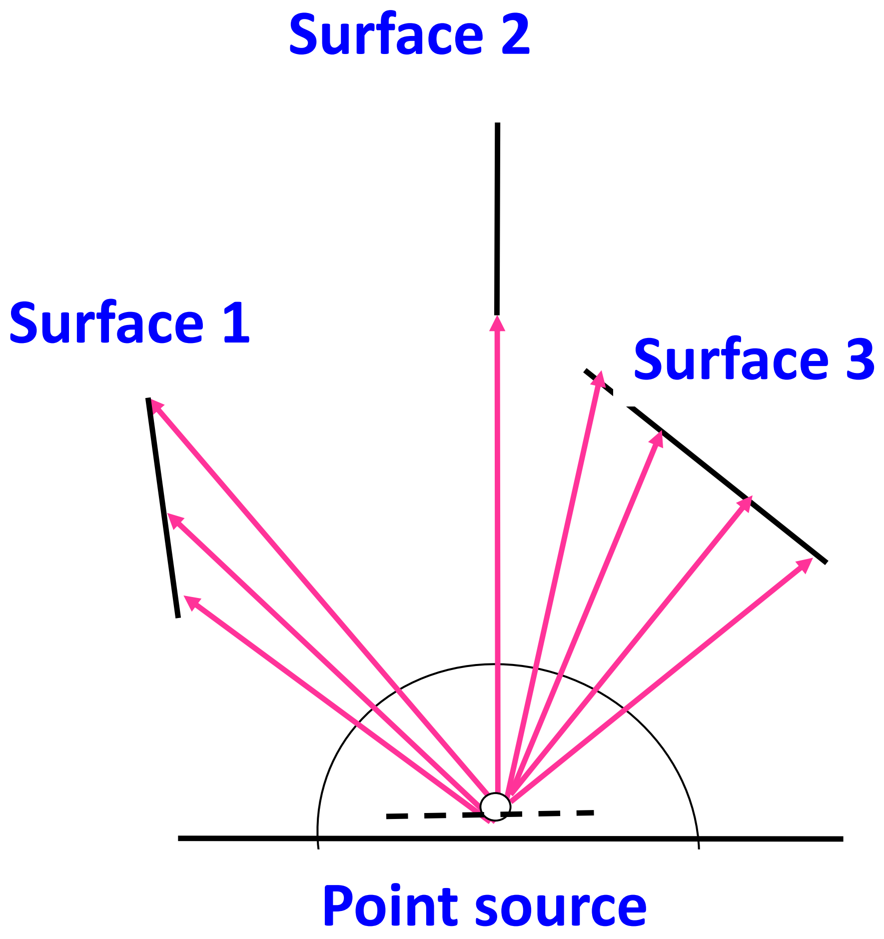
RADIATION EXCHANGE BETEWEEN SURFACES

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures.

This dependence on the orientation is accounted for by the *view factor*

By facing the fire from front or back – maximum radiation

By facing the fire from the side – minimum radiation



VIEW FACTOR (Shape Factor, Configuration Factor And Angle Factor)

F_{ij} – the fraction of the radiation leaving surface i that strikes the surface j directly

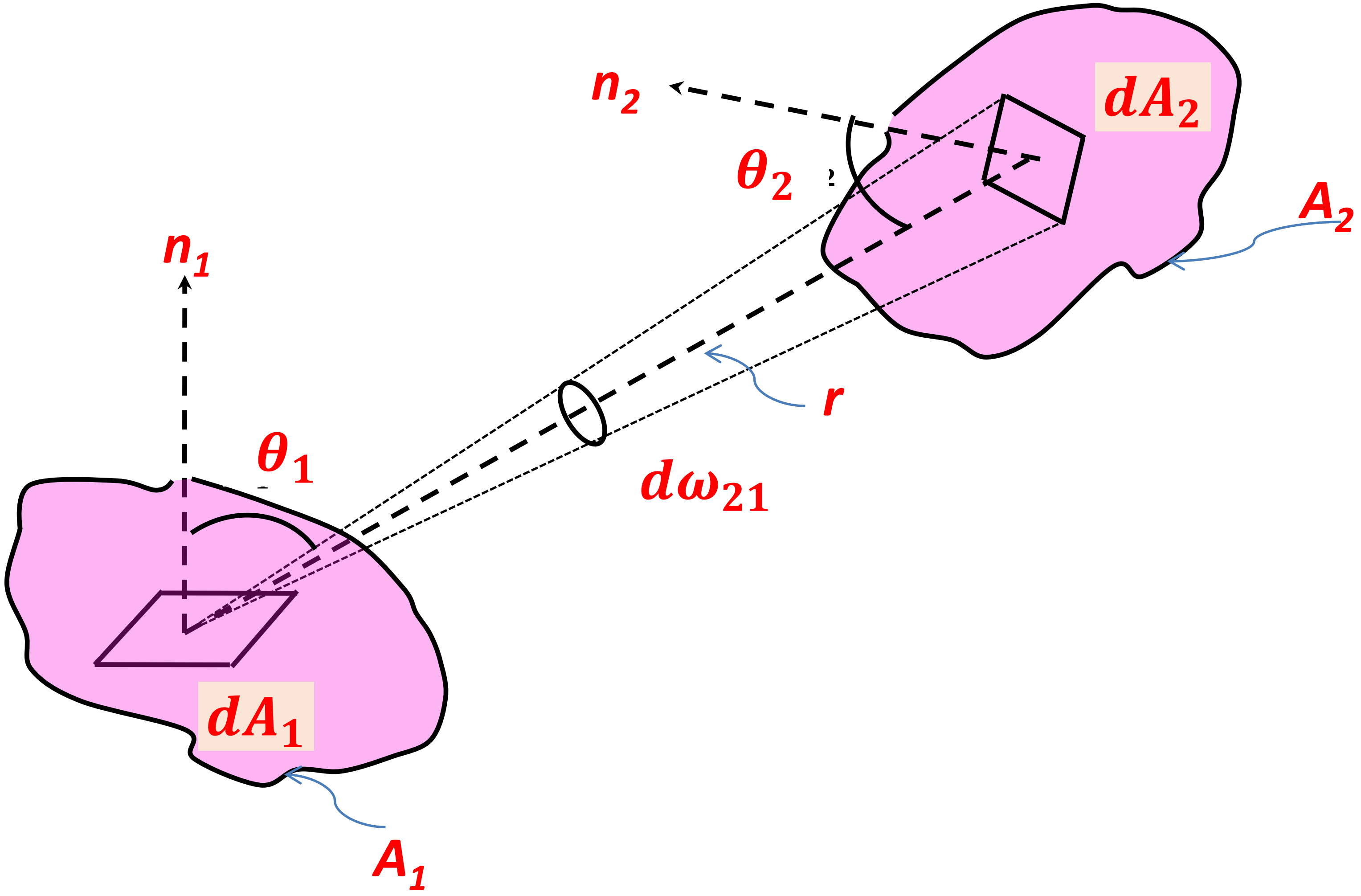
F_{12} – the fraction of the radiation leaving surface 1 that strikes the surface 2 directly

F_{21} – the fraction of the radiation leaving surface 2 that strikes the surface 1 directly

Consider two differential surfaces dA_1 and dA_2 on two arbitrarily oriented surfaces A_1 and A_2

Distance between dA_1 and dA_2 is r

Angle between the normals of the surfaces and the line that connects dA_1 and dA_2 are θ_1 and θ_2



Geometry for the determination of the view factor

VIEW FACTOR (Shape Factor, Configuration Factor And Angle Factor)

Surface 1 emits and reflects radiation diffusely in all directions with a constant intensity of I_1

Solid angle subtended by dA_2 when viewed by dA_1 is $d\omega_{21}$

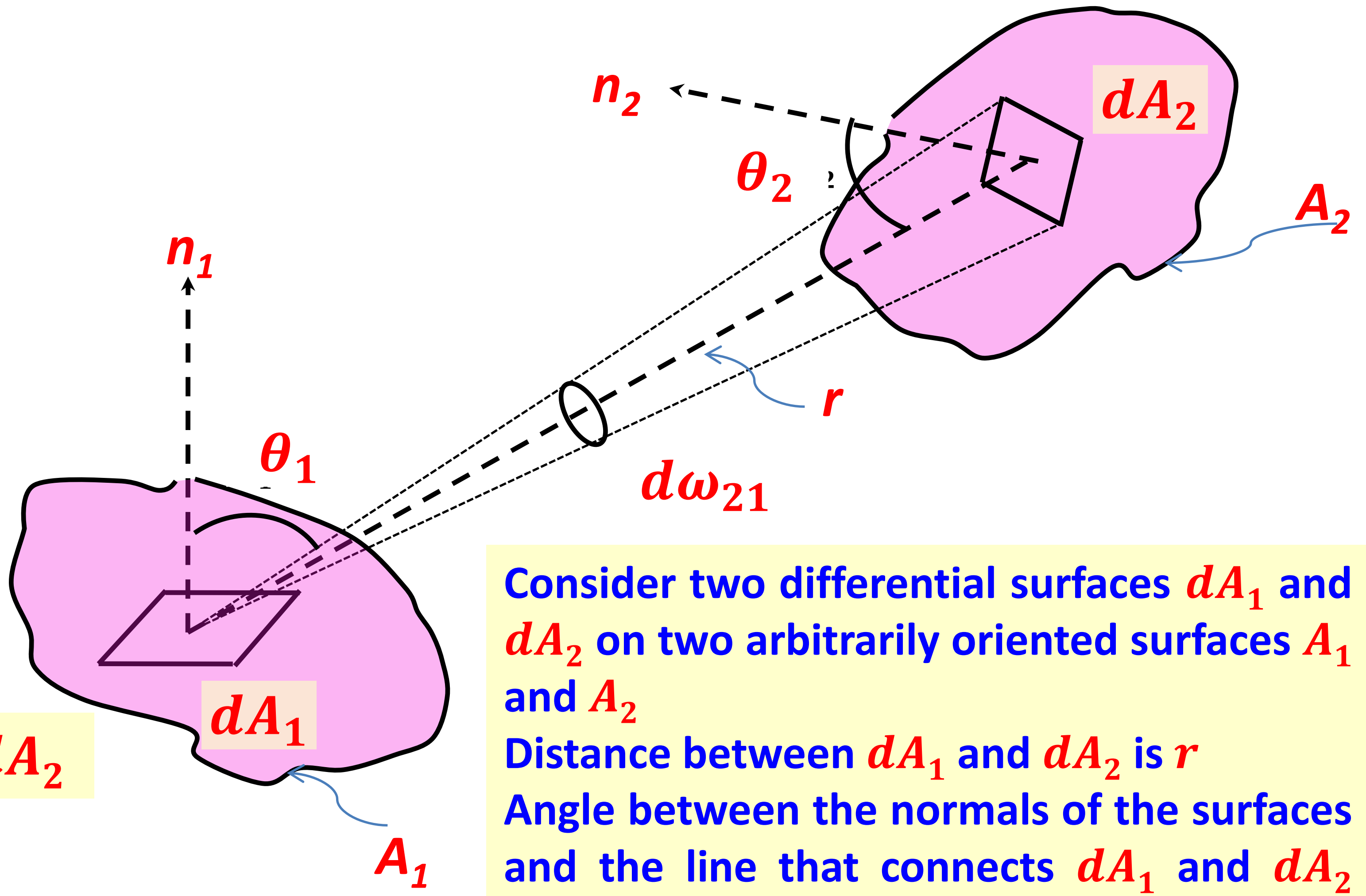
Rate at which the radiation leaves the dA$_1$ in the direction of θ_1 = $I_1 cos\theta_1\ c$

$$d\omega_{21} = \frac{dA_2 cos\theta_2}{r^2}$$

Portion of the radiation that strikes dA_2

$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 cos\theta_1 dA_1 d\omega_{21}$$

$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 cos\theta_1 dA_1 \frac{dA_2 cos\theta_2}{r^2}$$



Consider two differential surfaces dA_1 and dA_2 on two arbitrarily oriented surfaces A_1 and A_2

Distance between dA_1 and dA_2 is r

Angle between the normals of the surfaces and the line that connects dA_1 and dA_2 are θ_1 and θ_2

Geometry for the determination of the view factor

The total rate at which radiation leaves dA_1 (via emission and reflection) in all directions is radiosity (which is $J_1 = \pi I_1$) times the surface area

$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1$$

Differential view factor $dF_{dA_1 \to dA_2}$ which is the fraction of radiation leaving dA_1 that strikes dA_2 directly

$$\dot{Q}_{dA_1 \to dA_2} = I_1 cos\theta_1 dA_1 \frac{dA_2 cos\theta_2}{r^2}$$

$$dF_{dA_1 \to dA_2} = \frac{\dot{Q}_{dA_1 \to dA_2}}{\dot{Q}_{dA_1}} = \frac{I_1 cos\theta_1 dA_1 \frac{dA_2 cos\theta_2}{r^2}}{\pi I_1 dA_1}$$

$$dF_{dA_1 \to dA_2} = \frac{cos\theta_1 cos\theta_2}{\pi r^2} dA_2$$

The view factor from a differential area dA_1 to a finite area A_2 can be determined from the fact that the fraction of radiation leaving dA_1 that strikes A_2 is the sum of the fractions of radition striking the differential areas dA_2.

Therefore, the view factor $F_{dA_1 \to A_2}$ is determined by integrating $dF_{dA_1 \to dA_2}$ over A_2

$$F_{dA_1 \to dA_2} = \int_{A_2} \frac{cos\theta_1 cos\theta_2}{\pi r^2} dA_2$$

The total rate at which radiation leaves the entire A_1 (via emission and reflection) in all directions is

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1$$

The portion of this radiation that strikes dA_2 is determined by considering the radiation that leaves dA_1 and strikes dA_2 and integrating it over A_1

$$\dot{Q}_{A_1 \to dA_2} = \int_{A_1} \dot{Q}_{dA_1 \to dA_2} = \int_{A_1} \frac{I_1 cos\theta_1 cos\theta_2}{r^2} dA_1 dA_2$$

Integration of this relation over A_2 gives the radiation that strikes the entire area A_2

$$\dot{Q}_{A_1 \to A_2} = \int_{A_2} \dot{Q}_{A_1 \to dA_2} = \int_{A_1} \int_{A_2} \frac{I_1 cos\theta_1 cos\theta_2}{r^2} dA_1 dA_2$$

$$\dot{Q}_{A_1\to A_2} = \int_{A_2} \dot{Q}_{A_1\to dA_2} = \int_{A_1}\int_{A_2} \frac{I_1 cos\theta_1 cos\theta_2}{r^2} dA_1 dA_2$$

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1$$

$$F_{12} = F_{A_1\to A_2} = \frac{\dot{Q}_{A_1\to A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{cos\theta_1 cos\theta_2}{\pi r^2} dA_1 dA_2$$

These relations confirm that the view factors between two surfaces depends on their relative orientations and the distance between them

Rules to compute view factors

- **Reciprocity rule**
- **Summation rule**
- **Superposition rule**
- **Symmetry rule**

Reciprocity Rule

$$F_{12} = F_{A_1 \to A_2} = \frac{\dot{Q}_{A_1 \to A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

$$F_{21} = F_{A_2 \to A_1} = \frac{\dot{Q}_{A_2 \to A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

These relations confirm that the view factors between two surfaces depends on their relative orientations and the distance between them.

Comparing the above two equations, we get

$$A_1 F_{12} = A_2 F_{21}$$

The view factor relations developed above are applicable to any two surfaces *i and j* provided that the surfaces are diffuse emitters and diffuse reflectors (so that the assumption of constant intensity is valid)

Reciprocity Rule

$$A_i F_{i \to j} = A_j F_{j \to i}$$

When determining the pair of view factors, easier one is evaluated first and then the more difficult one by applying the reciprocity relation

SUMMATION RULE

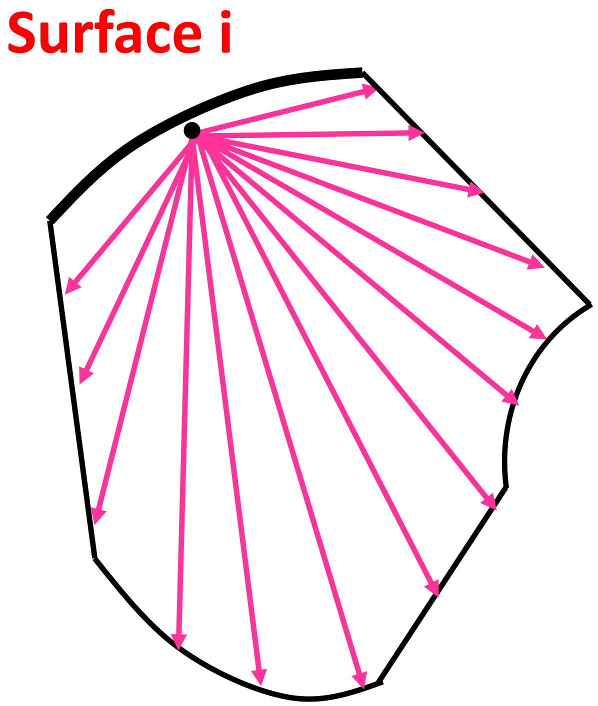
The conservation of energy principle requires that the entire radiation leaving any surface *i* of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.*

$$\sum_{j=1}^{N} F_{i \to j} = 1$$

N **- number of the surfaces of an enclosure**

For an enclosure with three surfaces

$$\sum_{j=1}^{3} F_{1 \to j} = F_{1 \to 1} + F_{1 \to 2} + +F_{1 \to 3} = 1$$



Radiation leaving any surface i of an enclosure must be intercepted completely by the surfaces of the enclosure.

Therefore, the sum of the view factors from surface i to each one of the surfaces of the enclosure must be unity.

To calculate radiation exchange in an enclosure of N surfaces, a total of N^2 view factors is needed.

$$\begin{vmatrix} F_{11} & F_{12} & F_{13} & .. & .. & F_{1N} \\ F_{21} & F_{22} & F_{23} & .. & .. & F_{2N} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ F_{N1} & F_{N2} & F_{N3} & .. & .. & F_{NN} \end{vmatrix}$$

Summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors

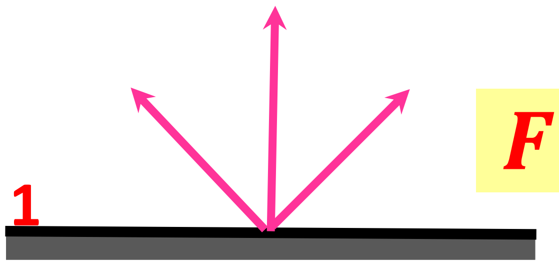
Reciprocity rule gives $\frac{1}{2}N(N-1)$ additional relations

The total number of view factors that need to be evaluated directly for an N surface enclosure becomes

$$N^2 - \left[N - \frac{1}{2}N(N-1)\right] = \frac{1}{2}N(N-1)$$

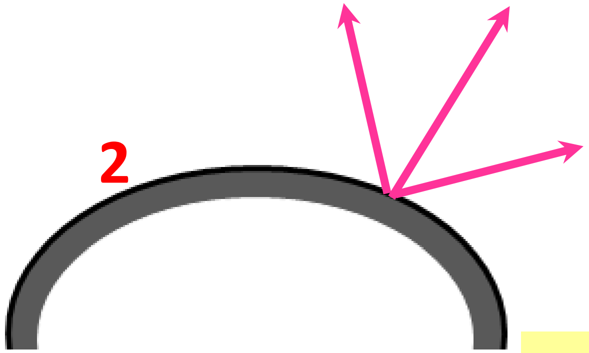
$N = 3$ Total number of view factors = $\frac{1}{2} \times 3(3-1) = 3$

Few view typical view factor scenarios



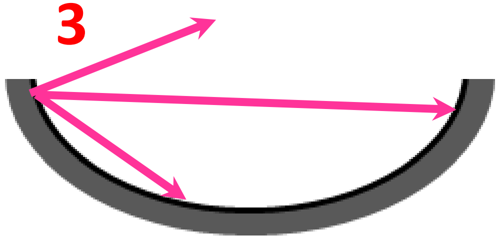
$$F_{1\to1} = 0$$

a) **Plane surface**



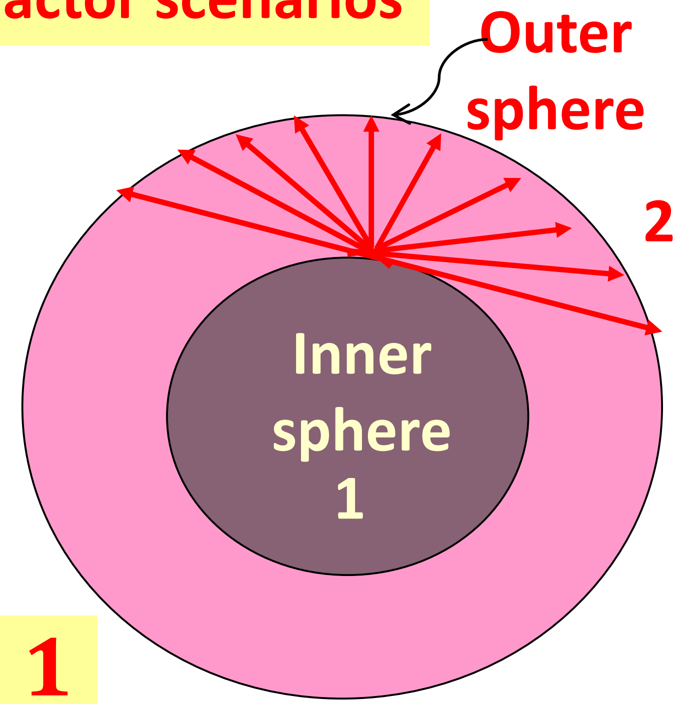
b) **Convex surface**

$$F_{2\to2} = 0$$



c) **Concave surface**

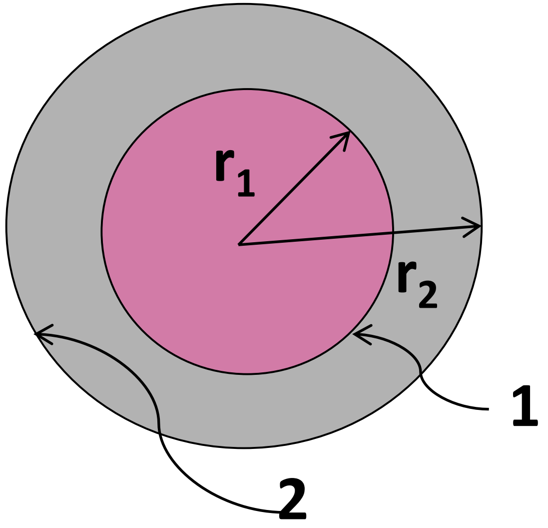
$$F_{3\to3} \neq 0$$



$$F_{1\to2} = 1$$

In a geometry that consists of two concentric spheres, the view factor since the entire radiation leaving the surface of the smaller sphere will be intercepted by the larger sphere

Problem: Determine the view factor of two concentric spheres



Solution: The view factors associated with two concentric spheres are to be determined.

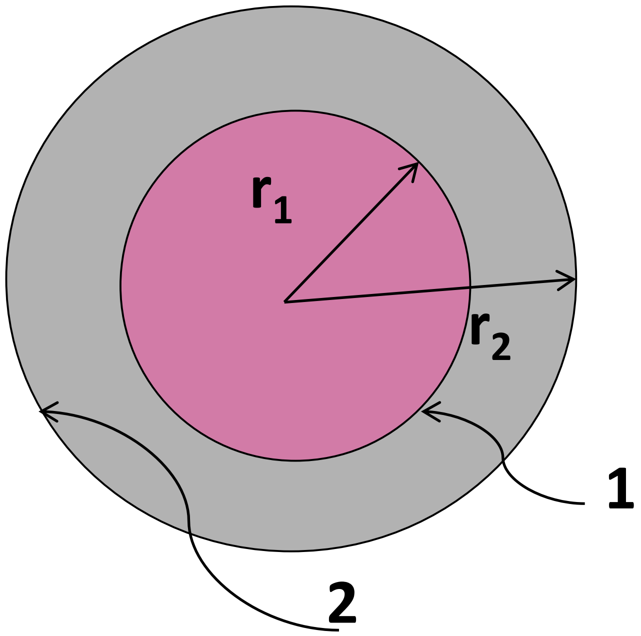
Assumptions: The surfaces are diffuse emitters and reflectors.

Analysis: The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two surface enclosure. Therefore, $N = 2$ and this enclosure involves $N^2 = 22 = 4$ view factors, which are F_{11}, F_{12}, F_{21} and F_{22}. In this two surface enclosure, we need to determine only

$$\frac{1}{2}N(N-1) = \frac{1}{2} \times 2(2-1) = 1$$

view factor directly. The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but two view factors directly in this case by a simple inspection.

$F_{11} = 0$, since no radiation leaving surface 1 strikes itself
$F_{12} = 1$, since all radiation leaving surface 1 strikes surface 2



SUMMATION RULE

$$\sum_{j=1}^{N} F_{i\to j} = 1$$

$$F_{11} + F_{12} = 1 \quad F_{11} = 0$$

$$F_{12} = 1$$

Reciprocity Rule

$$A_i F_{i\to j} = A_j F_{j\to i}$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} F_{12} = \frac{r_1^2}{r_2^2}(1)$$

$$F_{21} = \frac{r_1^2}{r_2^2}$$

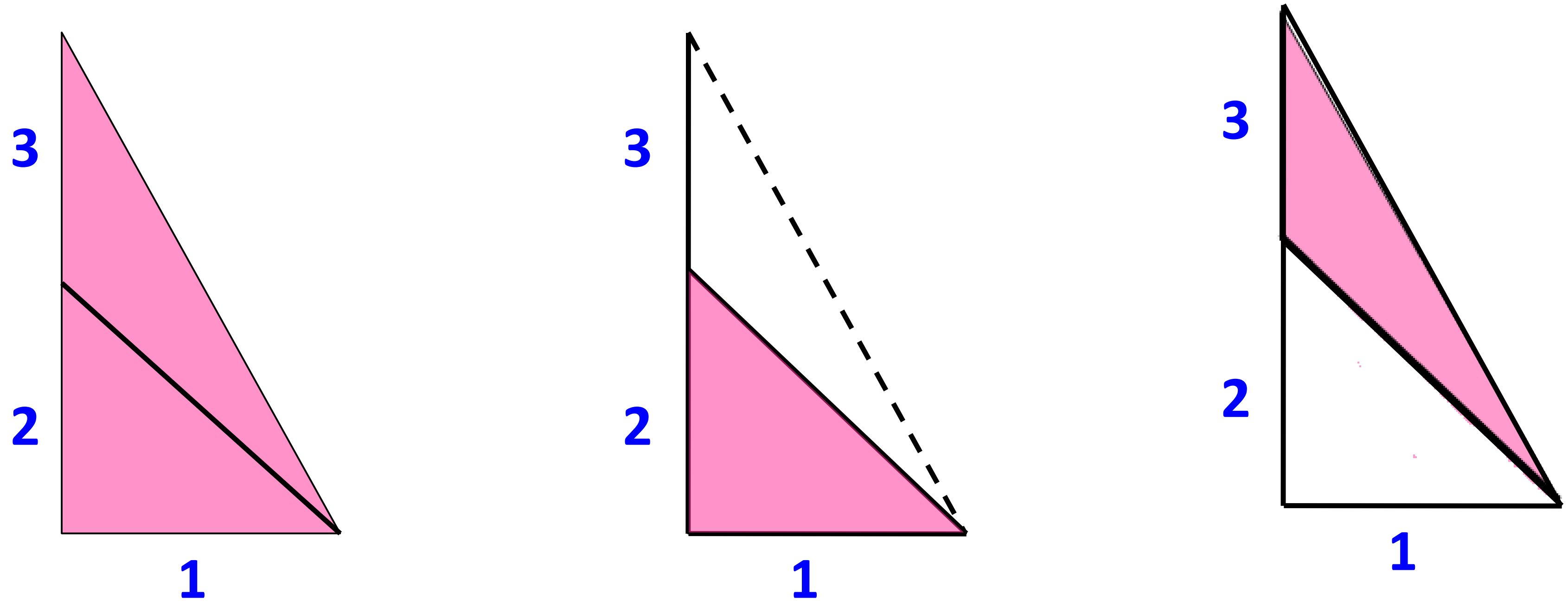
SUMMATION RULE

$$\sum_{j=1}^{N} F_{i\to j} = 1$$

$$F_{21} + F_{22} = 1 \quad F_{22} = 1 - F_{21}$$

$$F_{22} = 1 - \frac{r_1^2}{r_2^2}$$

SUPERPOSITION RULE



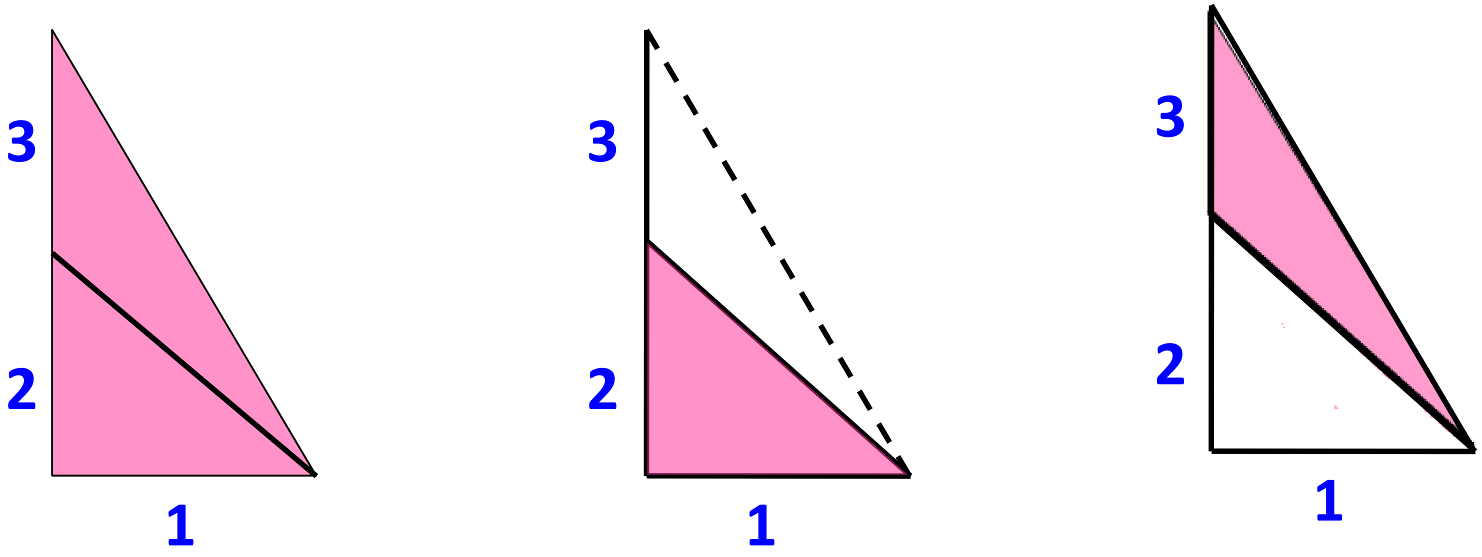
$$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$$

Sometimes, the view factor associated with a given geometry is not available in standard tables and charts. In such cases, it is desirable to express the given geometry as the sum or difference of some geometries with known view factors

The view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j

Note that the reverse of this is not true. That is, the view factor from a surface j to a surface i is not equal to the sum of the view factors from the parts of surface j to surface i

To obtain a relation $F_{(2\text{-}3)\rightarrow 1}$



$$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$$

$$A_1F_{1\rightarrow(2,3)} = A_1F_{1\rightarrow 2} + A_1F_{1\rightarrow 3}$$

By reciprocity rule applied to the individual terms of the RHS

$$A_1F_{1\rightarrow(2,3)} = A_2F_{2\rightarrow 1} + A_3F_{3\rightarrow 1}$$

By reciprocity rule

$$A_1F_{1\rightarrow(2,3)} = (A_2 + A_3)F_{(2,3)\rightarrow 1}$$

$$(A_2 + A_3)F_{(2,3)\rightarrow 1} = A_2F_{2\rightarrow 1} + A_3F_{3\rightarrow 1}$$

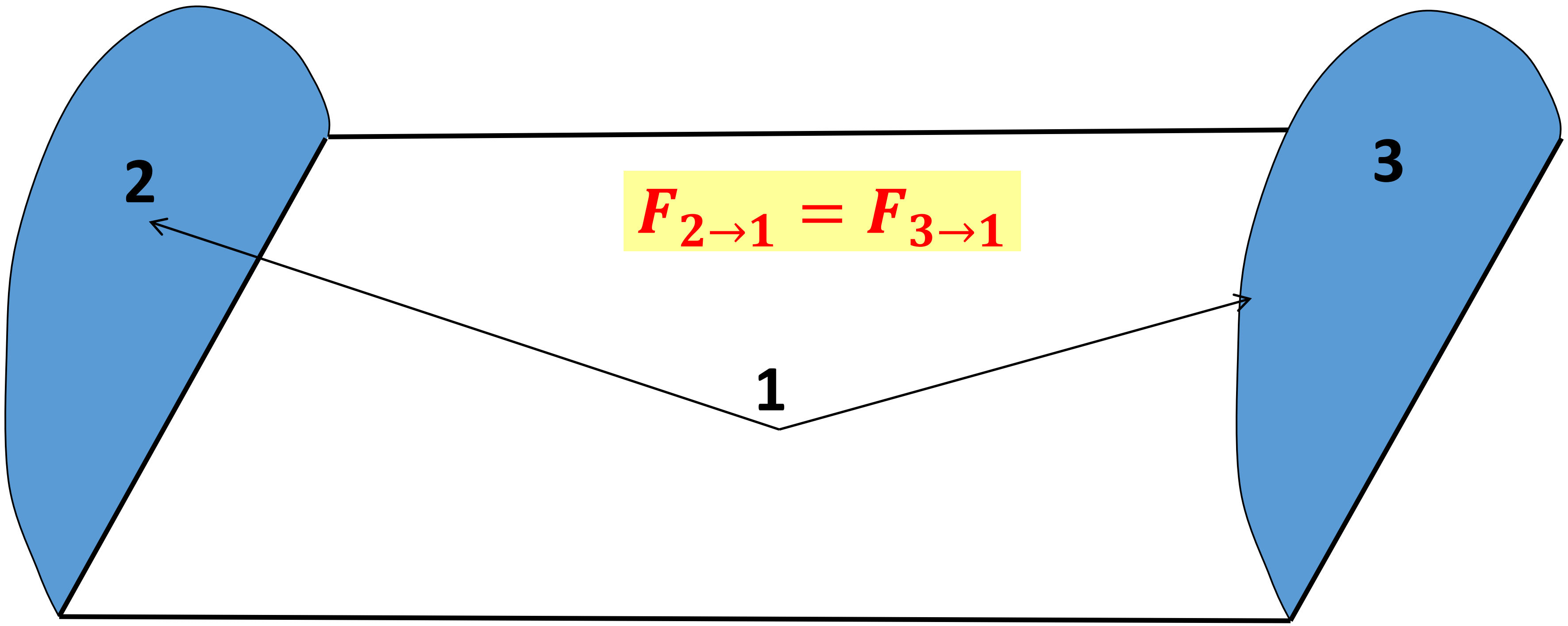
$$F_{(2,3)\rightarrow 1} = \frac{A_2F_{2\rightarrow 1} + A_3F_{3\rightarrow 1}}{(A_2 + A_3)}$$

$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$ **is possible**

$F_{(2,3)\rightarrow 1} = F_{2\rightarrow 1} + F_{3\rightarrow 1}$ **is not possible**

The view factor from a surface *i* to a surface *j* is equal to the sum of the view factors from surface *i* to the parts of surface *j*

SYMMETRY RULE



$$F_{2\to1} = F_{3\to1}$$

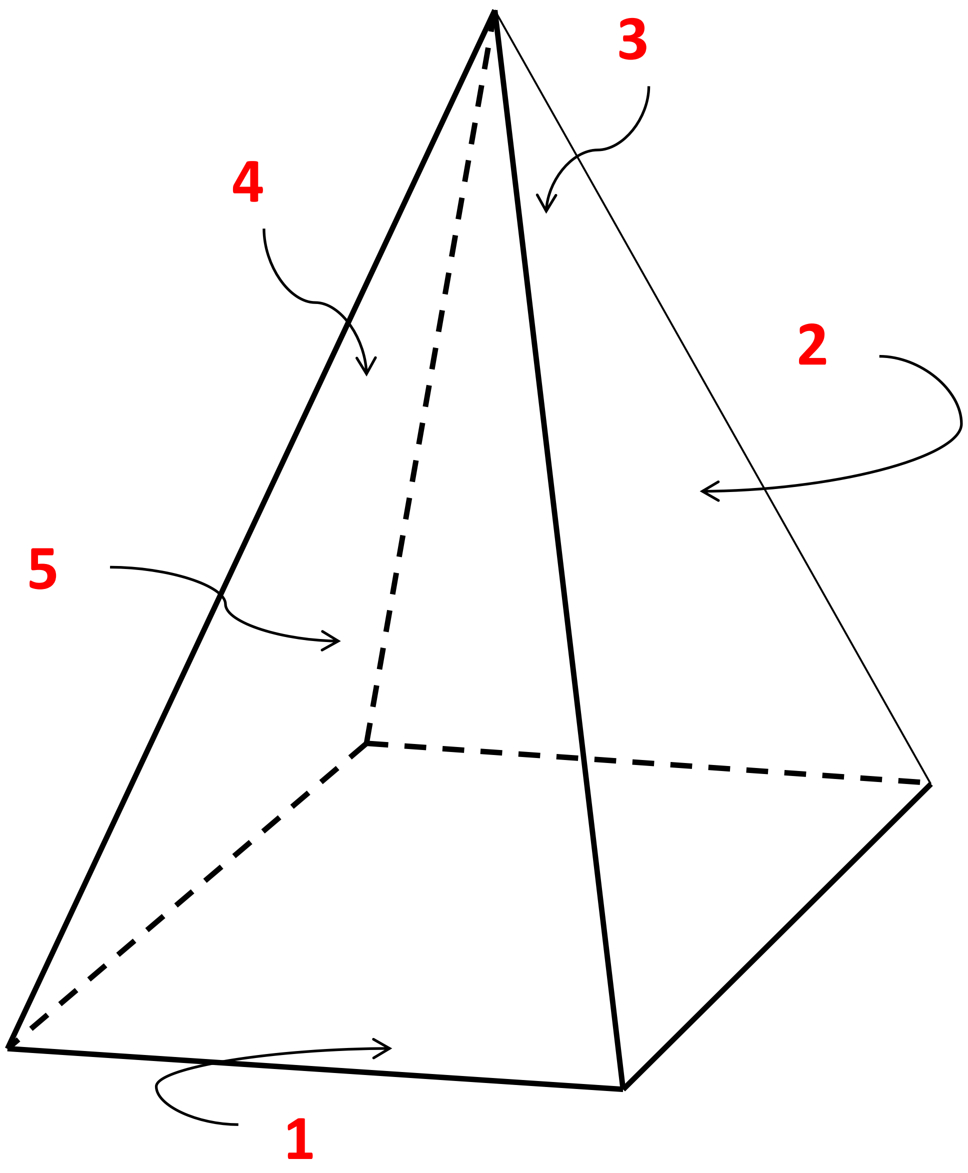
Two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface

$$F_{j\to i} = F_{k\to i}$$

By reciprocity rule

$$F_{i\to j} = F_{i\to k}$$

Problem: Determine the view factors from the base of the pyramid to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

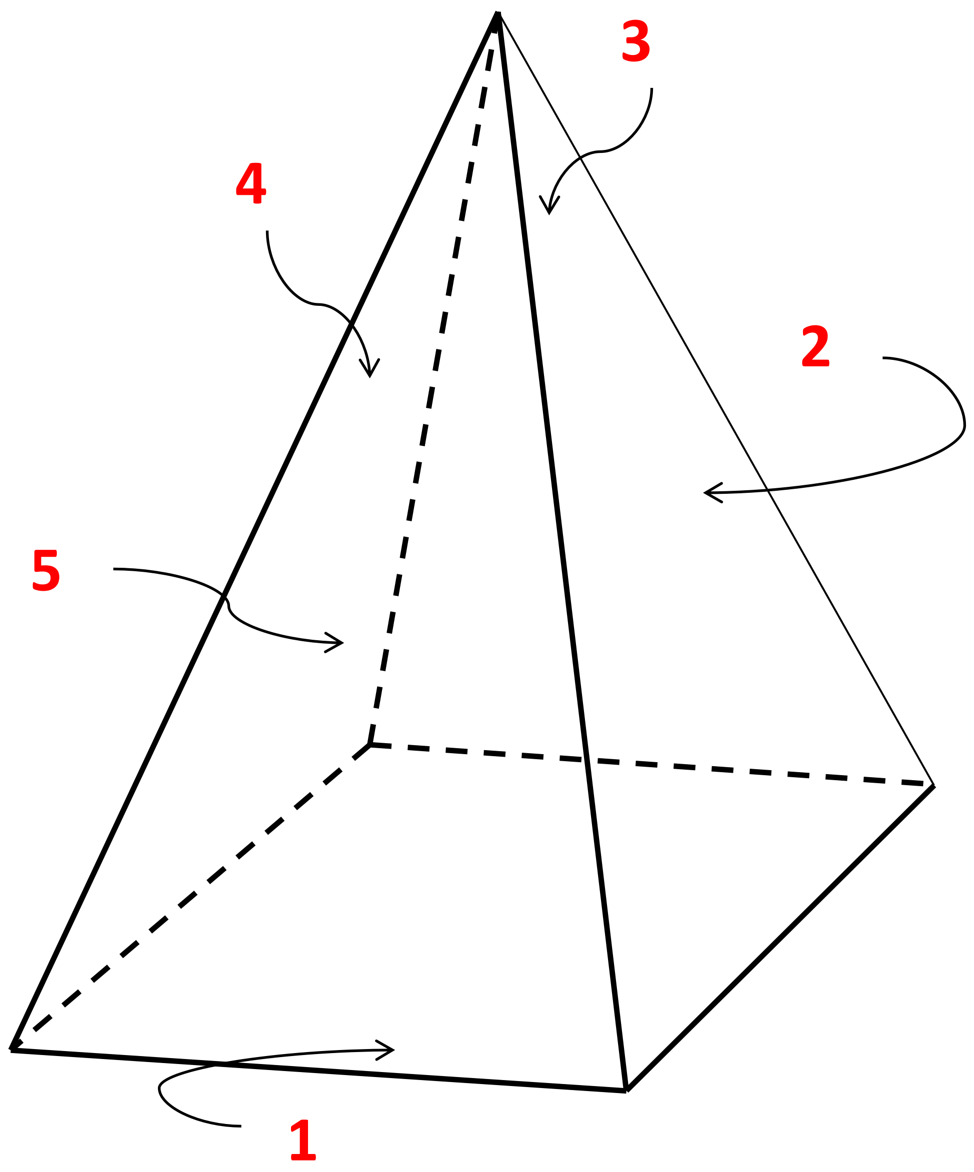


Solution: The view factors from the base of a pyramid to each of its four side surfaces for the case of a square base are to be determined.

Assumptions: the surfaces are diffuse emitters and reflectors

Analysis: the base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4 and 5) form a five surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the symmetry rule, we have

$$F_{12} = F_{13} = F_{14} = F_{15}$$



Symmetry rule

$$F_{12} = F_{13} = F_{14} = F_{15}$$

SUMMATION RULE

$$\sum_{j=1}^{N} F_{i\rightarrow j} = 1$$

$$F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

$$F_{11} = 0$$

$$F_{12} = F_{13} = F_{14} = F_{15} = 0.25$$

Discussion:

Note that each of the four side surfaces of the pyramid receive one-fourth of the entire radiation leaving the base surface, as expected. Also note that the presence of symmetry greatly simplified the determination of the view factors.

Problem: Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Fig.

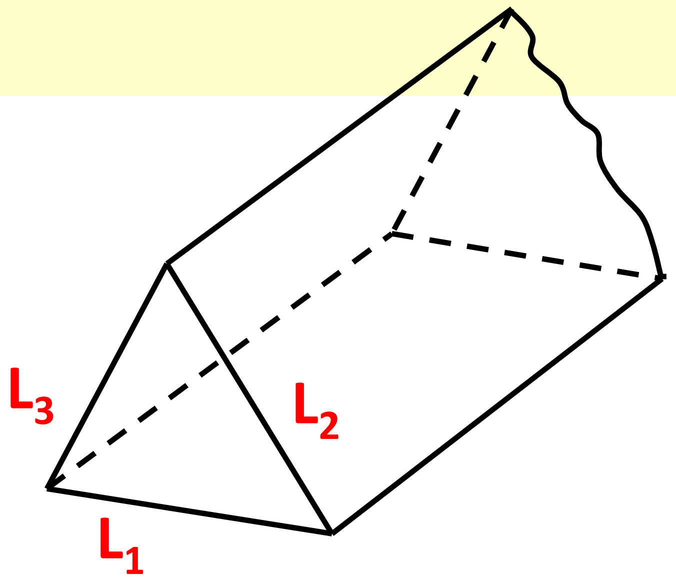
Solution: The view factors associated with an infinitely long triangular duct are to be determined.

Assumptions: The surfaces are diffuse emitters and reflectors

Analysis: The widths of the sides of the triangular cross section of the duct are L_1, L_2, and L_3 and the surface areas corresponding to them are A_1, A_2 and A_3 respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three surface enclosure, $N = 3$.

This enclosure involves $N^2 = 32 = 9$ view factors, and we need to determine the view factors only (directly).

$$\frac{1}{2}N(N-1) = \frac{1}{2} \times 3(3-1) = 3$$



$$F_{11} = F_{22} = F_{33} = 0$$

Since all the three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

By applying the summation rule to each of the three surfaces gives

SUMMATION RULE

$$\sum_{j=1}^{N} F_{i \to j} = 1$$

$$F_{11} + F_{12} + F_{13} = 1 \qquad A_1 F_{12} + A_1 F_{13} = A_1$$

$$F_{21} + F_{22} + F_{23} = 1 \qquad A_2 F_{21} + A_2 F_{23} = A_2$$

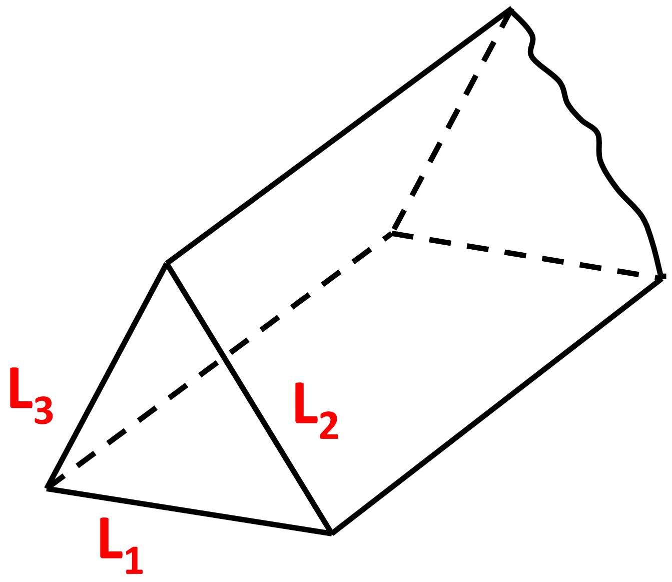
$$F_{31} + F_{32} + F_{33} = 1 \qquad A_3 F_{31} + A_3 F_{32} = A_3$$

By applying the reciprocity rule

$$A_1 F_{12} = A_2 F_{21} \qquad A_1 F_{12} + A_1 F_{13} = A_1$$

$$A_1 F_{13} = A_3 F_{31} \qquad A_1 F_{12} + A_2 F_{23} = A_2$$

$$A_2 F_{23} = A_3 F_{32} \qquad A_3 F_{13} + A_2 F_{23} = A_3$$



Solving these three algebraic equations, we get F_{12}, F_{13}, F_{23}

$$A_1F_{12} = A_2F_{21} \qquad A_1F_{12} + A_1F_{13} = A_1$$

$$A_1F_{13} = A_3F_{31} \qquad A_1F_{12} + A_2F_{23} = A_2$$

$$A_2F_{23} = A_3F_{32} \qquad A_3F_{13} + A_2F_{23} = A_3$$

Solving these three algebraic equations, we get F_{12}, F_{13}, F_{23}

$$F_{12} = \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1}$$

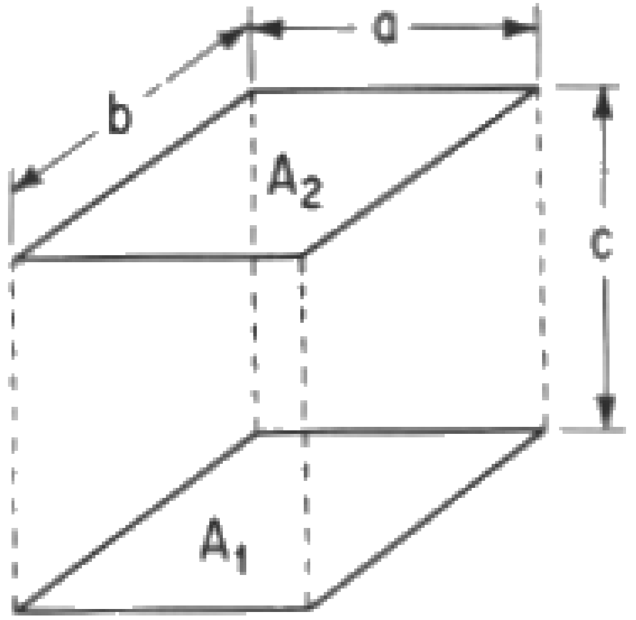
$$F_{23} = \frac{A_2 + A_3 - A_1}{2A_1} = \frac{L_2 + L_3 - L_1}{2L_1}$$

$$F_{13} = \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1}$$

View factor expressed for some geometries of finite size (3D)

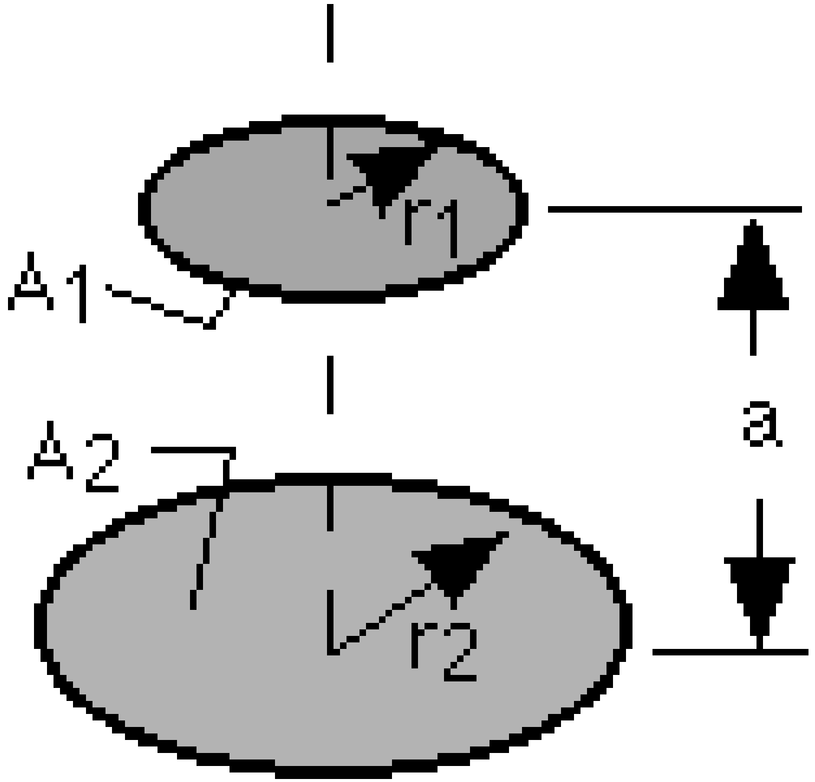
Aligned parallel rectangles

$$X = \frac{a}{c}$$
$$Y = \frac{b}{c}$$



$$F_{1-2} = \frac{2}{\pi XY}\left\{ \ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}} - X\tan^{-1}X - Y\tan^{-1}Y \right\}$$

Disk to parallel coaxial disk of unequal radius



$$R = \frac{r}{a}$$

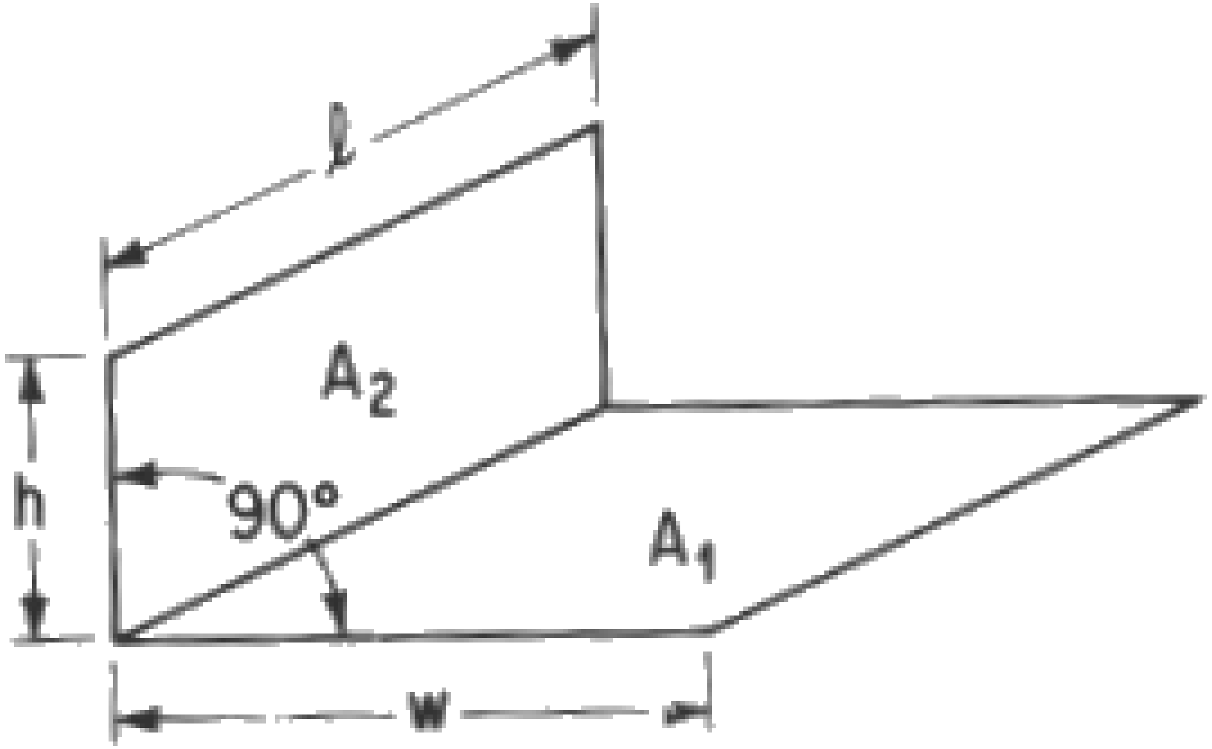
$$X = 1 + \frac{(1 + R_2^2)}{R_1^2}$$

$$F_{1-2} = \frac{1}{2}\left\{ X - \left[X^2 - 4\left(\frac{R_2}{R_1}\right)^2 \right]^{1/2} \right\}$$

Two finite rectangles of same length, having one common edge, and at an angle of 90º to each other

$$H = \frac{h}{l}$$

$$W = \frac{w}{l}$$

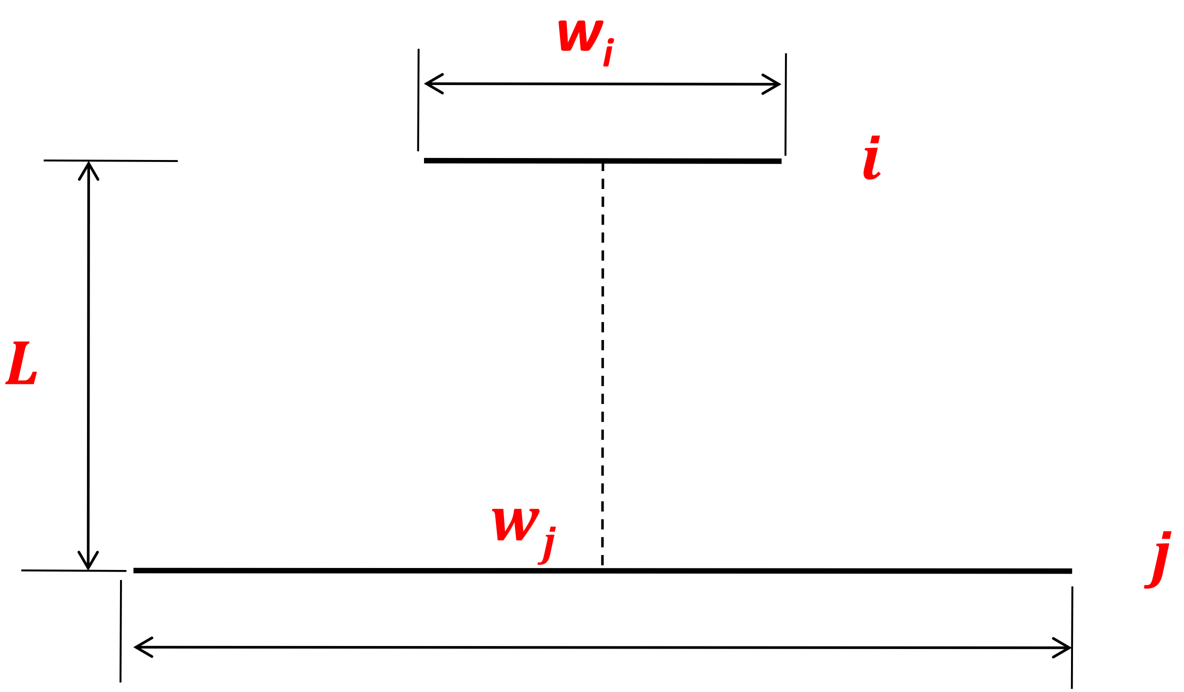


$$F_{1-2} = \frac{1}{W\pi}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - \sqrt{H^2+W^2}\tan^{-1}\sqrt{\frac{1}{H^2+W^2}}\right.$$

$$\left. + \frac{1}{4}\ln\left\{\frac{\left(1+W^2\right)\left(1+H^2\right)}{1+W^2+H^2}\left[\frac{W^2\left(1+W^2+H^2\right)}{\left(1+W^2\right)\left(W^2+H^2\right)}\right]^{W^2}\left[\frac{H^2\left(1+H^2+W^2\right)}{\left(1+H^2\right)\left(H^2+W^2\right)}\right]^{H^2}\right\}\right)$$

View factor expression for some infinitely long (2D) geometries

Parallel plates with midlines connected by perpendicular line

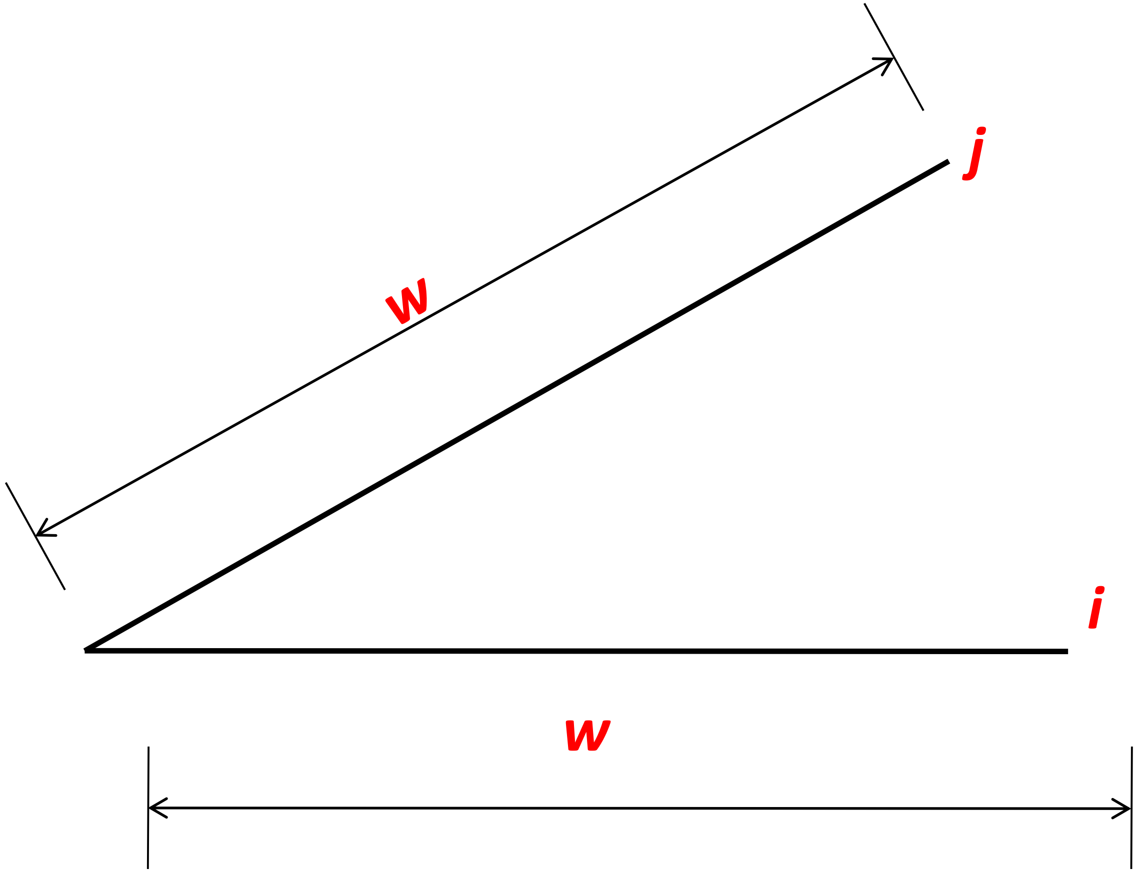


$$W_i = \frac{w_i}{L} \quad \& \quad W_j = \frac{w_j}{L}$$

$$F_{i\to j} = \frac{\left[\left(W_i + W_j\right)^2 + 4\right]^{1/2} - \left[\left(W_j - W_i\right)^2 + 4\right]^{1/2}}{2W_i}$$

View factor expression for some infinitely long (2D) geometries

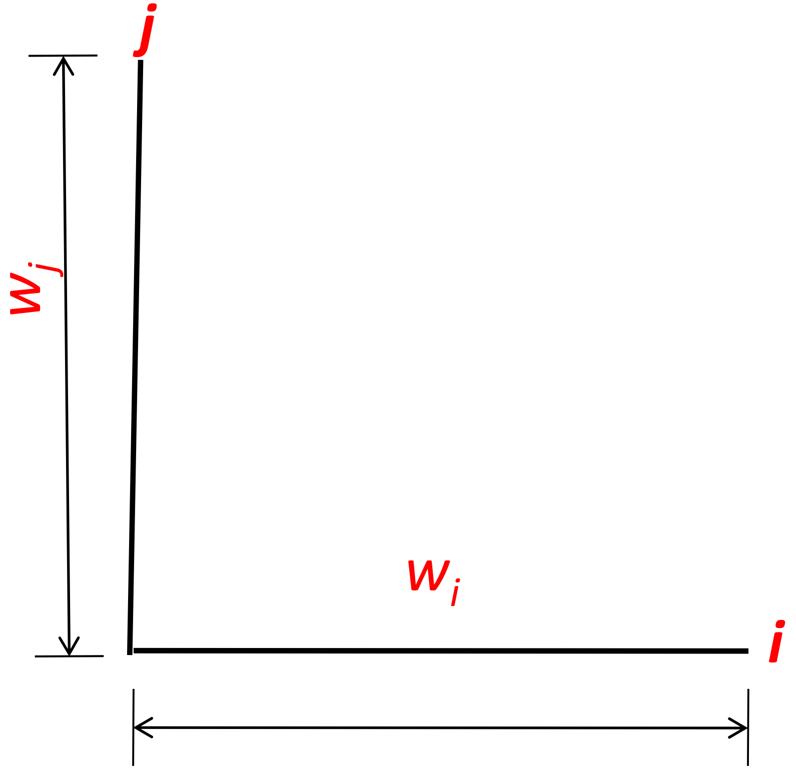
Inclined plate of equal width and with a common edge



$$F_{i \to j} = 1 - \sin \frac{1}{2} \alpha$$

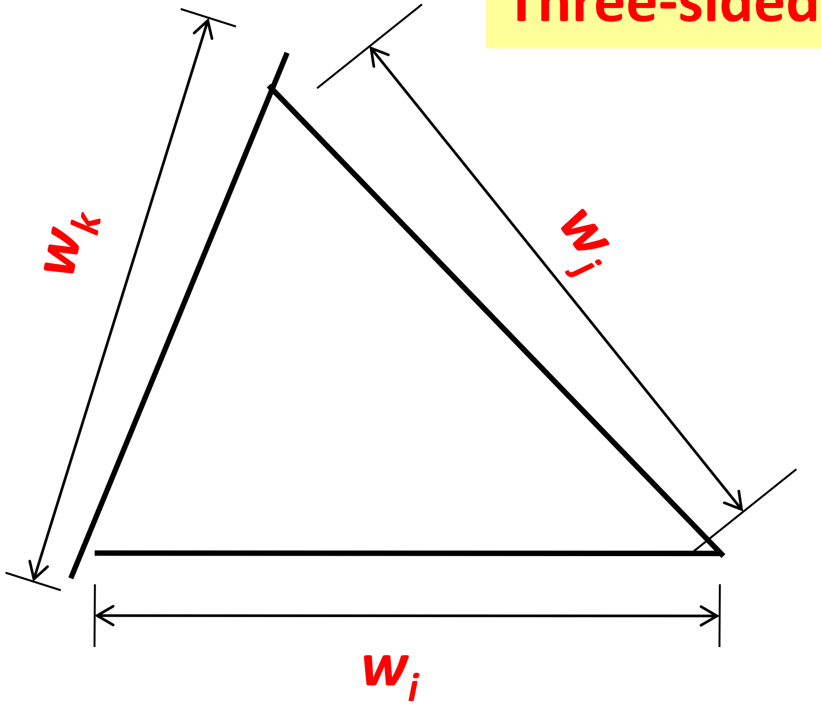
View factor expression for some infinitely long (2D) geometries

Perpendicular plates with common edge



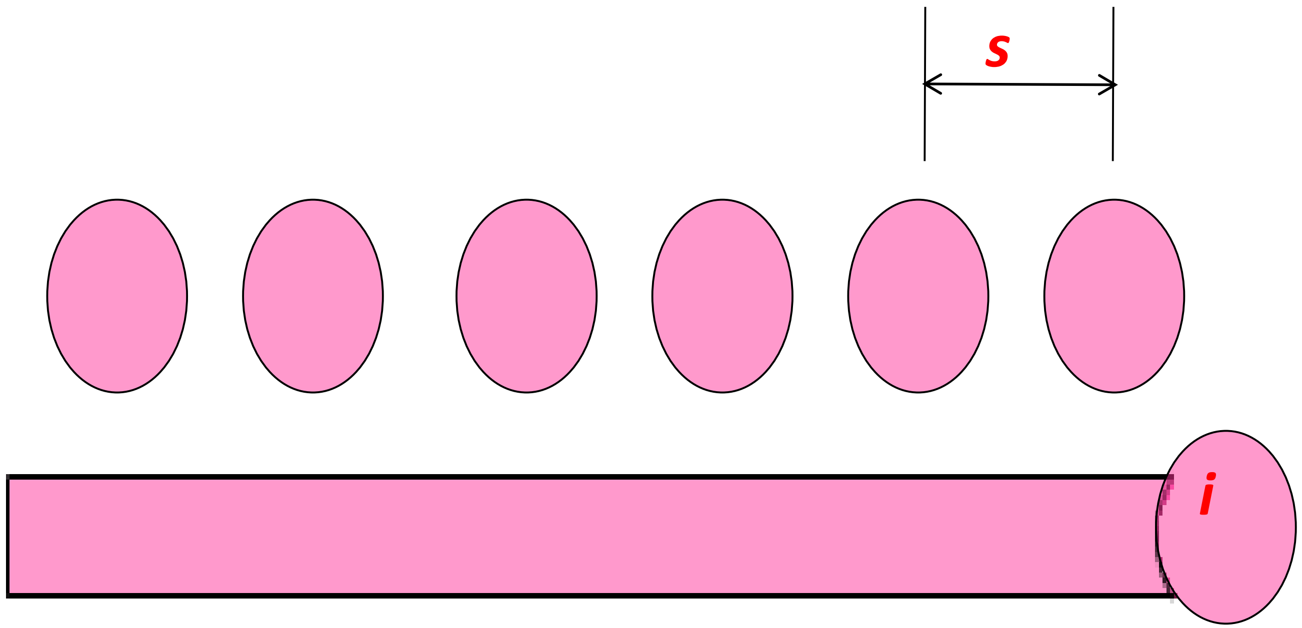
$$F_{i\to j}=\frac{1}{2}\left\{1+\frac{w_j}{w_i}-\left[1+\left(\frac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$$

Three-sided enclosure



$$F_{i\to j}=\frac{w_i+w_j-w_k}{2w_i}$$

View factor expression for some infinitely long (2D) geometries



$$F_{i\to j} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

A CATALOG OF RADIATION HEAT TRANSFER CONFIGURATION FACTORS

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https://web.engr.uky.edu/rtl/Catalog/tablecon.html

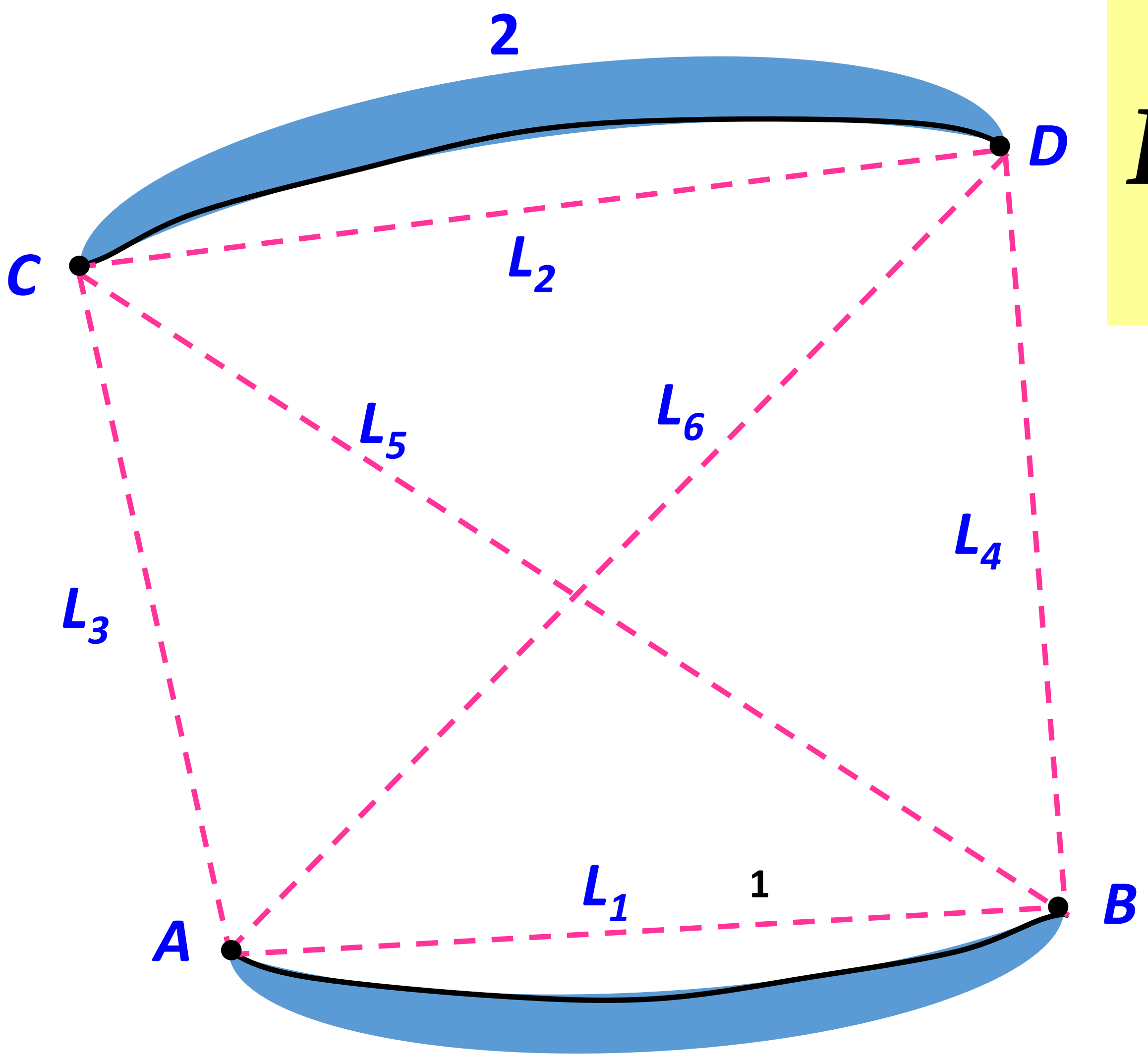
View factors between infinitely long surfaces: The cross-string method

Many problems encountered in practice involve geometries of constant cross section such as channels and ducts that are very long in one direction relative to the other direction.

Such geometries can conveniently be considered to be two-dimensional, since any radiation interaction through their end surfaces is negligible.

These geometries can subsequently be modeled as being infinitely long, and the view factor between their surfaces can be determined by the amazingly simple cross-strings method developed by H.C.Hottel in the 1950s.

The surfaces of the geometry need not be flat; they can be convex, concave or any irregular shape



$$F_{i\to j}=\frac{(L_5+L_6)-(L_3+L_4)}{2L_1}$$

$$F_{i\to j}=\frac{\sum(\textit{Crossed strings})-\sum(\textit{Uncrossed strings})}{2\times(\textit{String on surface i})}$$

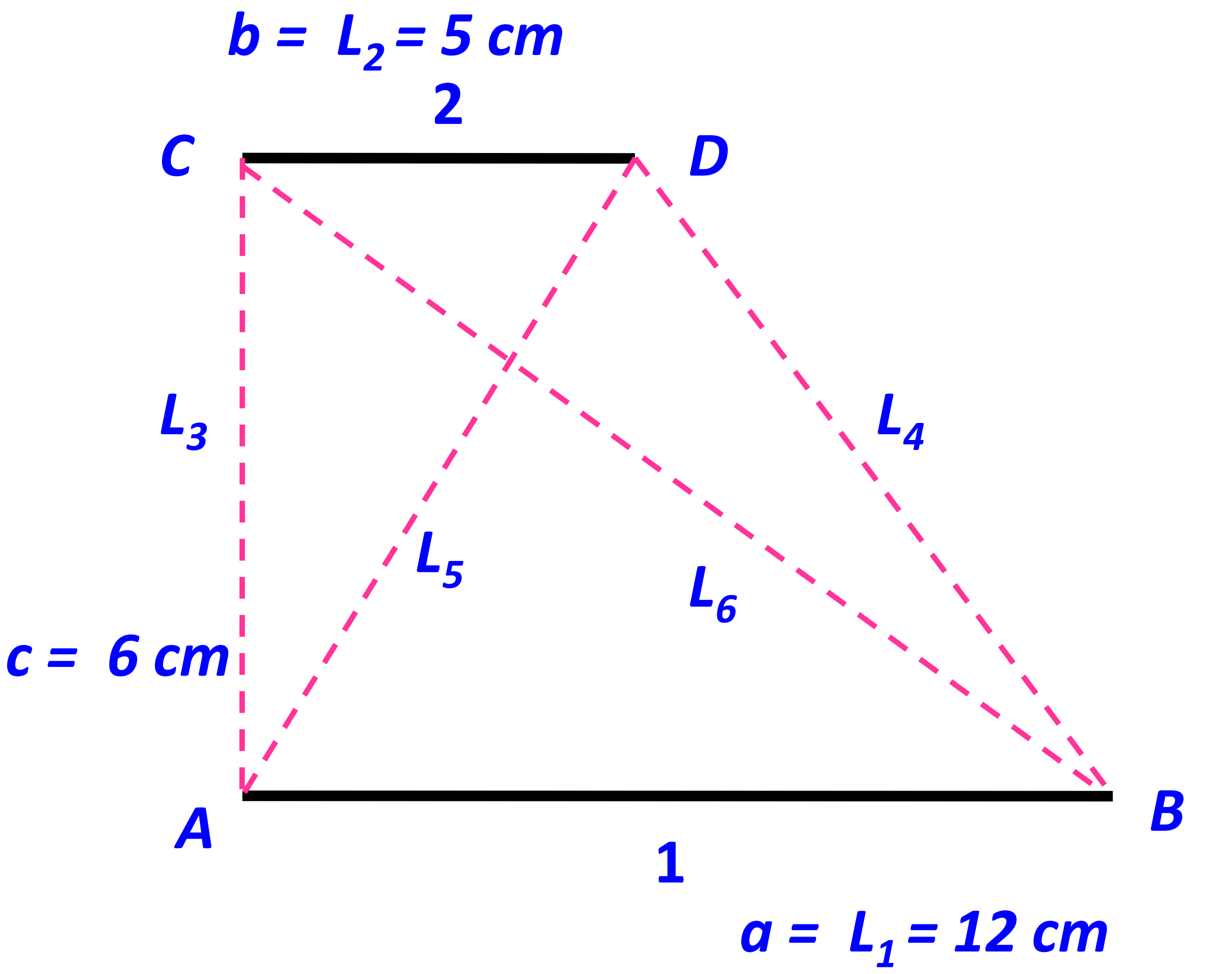
The crossed-strings method is applicable even when the two surfaces considered share a common edge, as in a triangle.

In such cases, the common edge can be treated as an imaginary string of zero length.

The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.

Problem: Two infinitely long parallel plates of widths a = 12 cm and b = 5 cm are located a distance c =6 cm apart, as in fig. Determine the view factor F1-2 from surface 1 to surface 2 by using the crossed strings method.

Derive the crossed strings formula by forming triangles on the given geometry and using earlier equations for view factors between the sides of triangles.



Solution: The view factors between two infinitely long parallel plates are to be determined using the crossed string method, and the formula for the view factor is to be derived.

Assumptions: The surfaces are diffuse emitters and reflectors.

Analysis: a. First we label the endpoints of both surfaces and draw straight dashed lines between the endpoints as in fig. Then, we identify the crossed and uncrossed strings and apply the crossed strings method.

$$F_{1\to 2} = \frac{\sum(Crossed\ strings) - \sum(Uncrossed\ strings)}{2\times(String\ on\ surface\ 1)} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$L_1 = a = 12\ cm \qquad L_4 = \sqrt{7^2 + 6^2} = 9.22\ cm$$

$$L_2 = b = 5\ cm \qquad L_5 = \sqrt{5^2 + 6^2} = 7.81\ cm$$

$$L_3 = c = 6\ cm \qquad L_6 = \sqrt{12^2 + 6^2} = 13.42\ cm$$

$$F_{1\to 2} = \frac{(7.81 + 13.42) - (6 + 9.22)}{2\times 12} = 0.25$$

(b) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four surface enclosure. Then applying the summation rule to surface 1 yields

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

$$F_{11} = 0 \qquad F_{12} = 1 - F_{13} - F_{14}$$

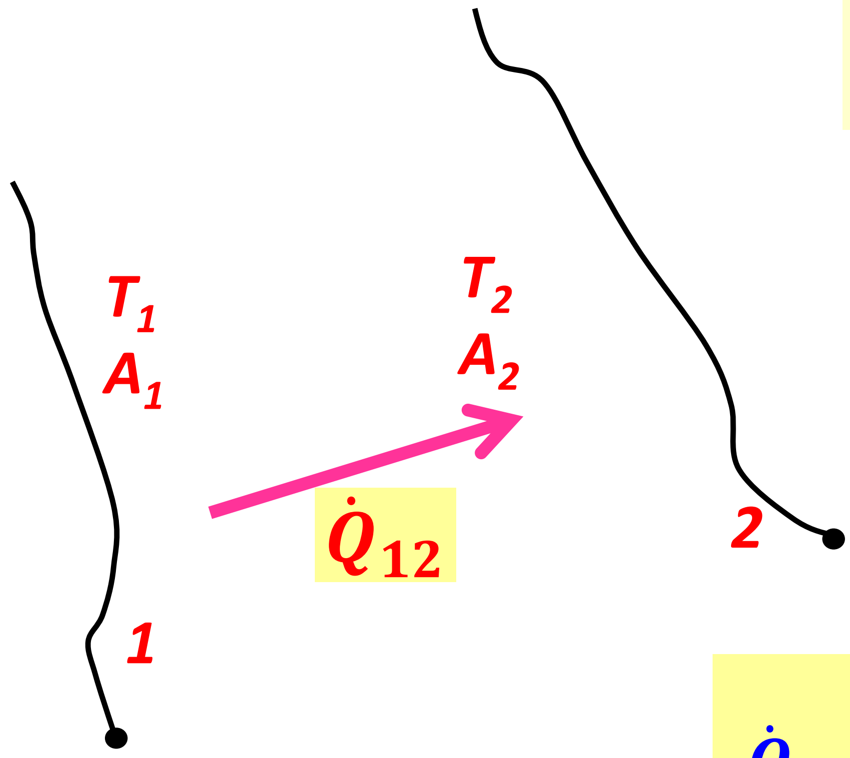
Where the view factors F_{13} and F_{14} can be determined by considering the triangles ABC and ABD respectively

$$F_{13} = \frac{L_2 + L_3 - L_6}{2L_1} ; F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

$$F_{12} = 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1}$$

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

Radiation heat transfer between black surfaces



Two general black surfaces maintained at uniform temperature T_1 and T_2

By reciprocity relation, $A_1 F_{1\rightarrow 2} = A_2 F_{2\rightarrow 1}$

$$\dot{Q}_{12} = \begin{pmatrix} \text{Radiation leaving the} \\ \text{entire surface 1} \\ \text{that strikes surface 2} \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving the} \\ \text{entire surface 2} \\ \text{that strikes surface 1} \end{pmatrix}$$

$$\dot{Q}_1 = \dot{Q}_{1\rightarrow 2} = -\dot{Q}_2$$

Negative value of $\dot{Q}_{1\rightarrow 2}$ indicates that the net radiation heat transfer is from surface 2 to 1

Enclosure with N black surfaces

Consider an enclosure consisting of N black surfaces maintained at specified temperatures

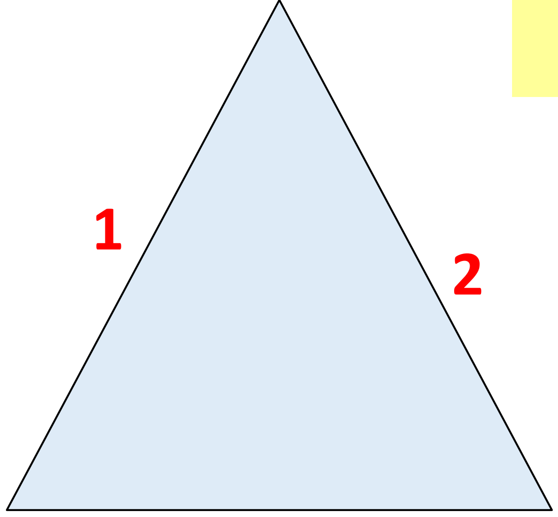
The net radiation heat transfer from any surface i of this enclosure is determined by adding up the net radiation heat transfers from surface i to each of the surfaces of the enclosure

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\, \sigma\left(T_i^4 - T_j^4\right)$$

Negative value of heat transfer indicates that the net radiation heat transfer from surface i (ie., surface i gains radiation energy instead of losing).

Also, the net heat transfer from a surface to itself to zero, regardless of the shape of the surface.

Enclosure with N black surfaces



$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\,\sigma\left(T_i^4 - T_j^4\right)$$

The net heat transfer from a surface to itself to zero, regardless of the shape of the surface.

$$\dot{Q}_{1\to 1} = \dot{Q}_{2\to 2} = \dot{Q}_{3\to 3} = 0$$

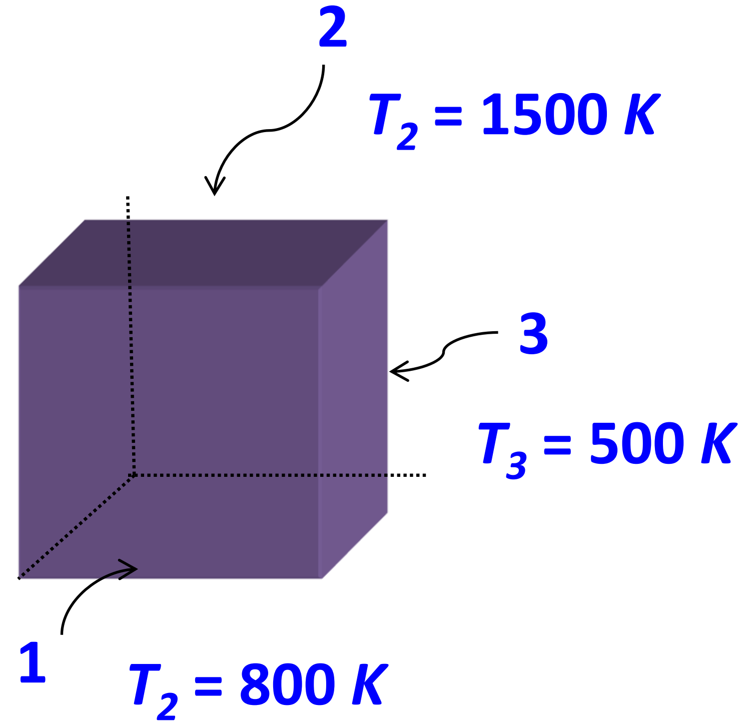
$$\dot{Q}_1 = \sum_{j=1}^{3} \dot{Q}_{1\to j} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{A_1 F_{1\to 2}}} + \frac{\sigma\left(T_1^4 - T_3^4\right)}{\frac{1}{A_1 F_{1\to 3}}}$$

$$\dot{Q}_2 = \sum_{j=1}^{3} \dot{Q}_{2\to j} = \frac{\sigma\left(T_2^4 - T_1^4\right)}{\frac{1}{A_2 F_{2\to 1}}} + \frac{\sigma\left(T_2^4 - T_3^4\right)}{\frac{1}{A_2 F_{2\to 3}}}$$

$$\dot{Q}_3 = \sum_{j=1}^{3} \dot{Q}_{3\to j} = \frac{\sigma\left(T_3^4 - T_1^4\right)}{\frac{1}{A_3 F_{3\to 1}}} + \frac{\sigma\left(T_3^4 - T_2^4\right)}{\frac{1}{A_3 F_{3\to 2}}}$$

RADIATION HEAT TRANFER IN A BLACK FURNACE

Problem: Consider the 5 m× 5 m× 5 m cubical furance shown in figure, whose surfaces closely approximate black surfaces. The base, top and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (a) the net rate of radiation heat transfer between the base and the side surfaces, (b) the net rate of radiation heat transfer between the base and the top surface, (c)the net radiation heat transfer from the base surface.



SOLUTION: The surfaces of a cubical furnace are black and are maintained at uniform temperatures. The net rate of radiation heat transfer between the base and side surfaces, between the base and the top surface, from the base surface are to be determined.

ASSUMPTIONS: The surfaces are black and isothermal.

ANALYSIS: (a) considering that the geometry involves six surfaces, we may be tempted at first to treat the furnace as a six surface enclosure. However, the four side surfaces possess the same properties, and thus we can treat them as a single side surface in radiation analysis. We consider the base surface to be surface 1, the top surface to be surface 2, and the side surfaces to be surface 3.

Then, the problem reduces to determining $\dot{Q}_{1\to3}, \dot{Q}_{1\to2}$ and $\dot{Q}_1$

$$\dot{Q}_{1\to3} = A_1 F_{1\to3}\sigma\left(T_1^4 - T_3^4\right)$$

$$F_{1\to1} + F_{1\to2} + F_{1\to3} = 1$$

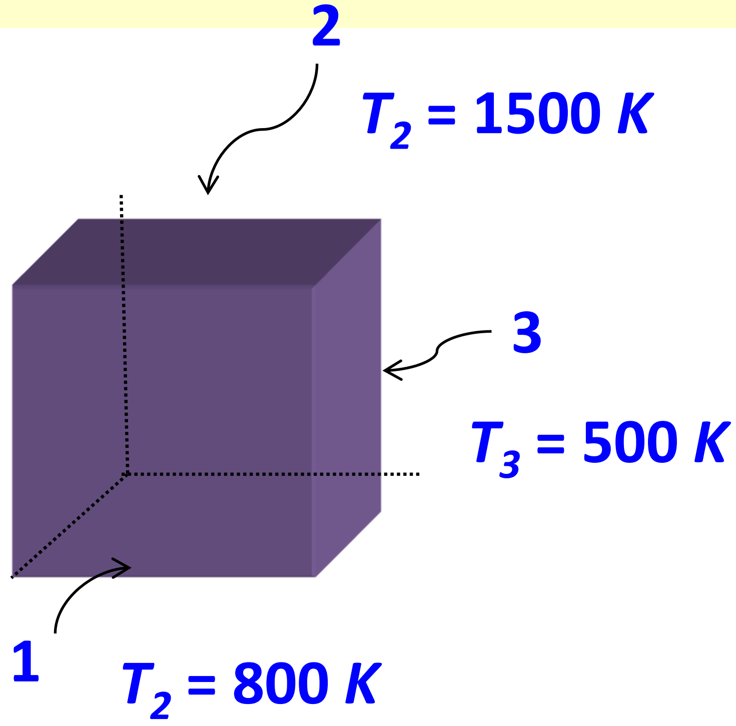
$$F_{1\to1} = 0 \qquad F_{1\to2} = 0.2$$

Details about calculations of $F_{1\to2}$ are in the next slide

$$0 + 0.2 + F_{1\to3} = 1 \qquad F_{1\to3} = 0.8$$

$$\dot{Q}_{1\to3} = (25)(0.8)\left(5.67\times10^{-8}\right)\left[(800)^4 - (500)^4\right]$$

$$\dot{Q}_{1\to3} = 394\ kW$$



$$F_{i\to j}=\frac{2}{\pi\bar{X}\bar{Y}}\left\{\begin{array}{l} ln\left[\frac{\left(1+\bar{X}^2\right)\left(1+\bar{Y}^2\right)}{1+\bar{X}^2+\bar{Y}^2}\right]^{\frac{1}{2}}+\bar{X}\left(1+\bar{Y}^2\right)^{\frac{1}{2}}Tan^{-1}\left(\frac{\bar{X}}{\left(1+\bar{Y}^2\right)^{\frac{1}{2}}}\right) \\ +\bar{Y}\left(1+\bar{X}^2\right)^{\frac{1}{2}}Tan^{-1}\left(\frac{\bar{Y}}{\left(1+\bar{X}^2\right)^{\frac{1}{2}}}\right)-\bar{X}\,Tan^{-1}\left(\bar{X}\right)-\bar{Y}\,Tan^{-1}\left(\bar{Y}\right)\end{array}\right\}$$

$$\bar{X}=\frac{X}{L}=1;\bar{Y}=\frac{Y}{L}=1\Rightarrow F_{1-2}=0.2$$

$$F_{1\to 2}=\frac{2}{\pi}\left\{\begin{array}{l} ln\left[\frac{(1+1)(1+1)}{1+1+1}\right]^{\frac{1}{2}}+1(1+1)^{\frac{1}{2}}Tan^{-1}\left(\frac{1}{(1+1)^{\frac{1}{2}}}\right) \\ +1(1+1)^{\frac{1}{2}}Tan^{-1}\left(\frac{1}{(1+1)^{\frac{1}{2}}}\right)-1Tan^{-1}(1)-1Tan^{-1}(1)\end{array}\right\}$$

$$F_{1\to 2}=\frac{2}{\pi}\{0.14384+0.87042+0.87042-0.7853-0.7853\}$$

$$F_{1\to 2}=0.2$$

The net rate of radiation from surface 1 to surface 2 is given by

$$\dot{Q}_{1\to2} = A_1 F_{1\to2} \sigma\left(T_1^4 - T_2^4\right)$$

$$F_{1\to2} = 0.2$$

$$\dot{Q}_{1\to2} = (25)(0.2)\left(5.67\times10^{-8}\right)\left[(800)^4 - (1500)^4\right]$$

$$\dot{Q}_{1\to2} = -1319\ kW$$

The negative sign indicates that net radiation heat transfer is from surface 2 to surface 1

The net rate of radiation from surface 1

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\, \sigma\left(T_i^4 - T_j^4\right)$$

$$\dot{Q}_1 = \sum_{j=1}^{3} \dot{Q}_{1\to3} = \dot{Q}_{1\to1} + \dot{Q}_{1\to2} + \dot{Q}_{1\to3}$$

$$\dot{Q}_1 = 0 + (-1319) + 394$$

$$\dot{Q}_1 = -925\ kW$$

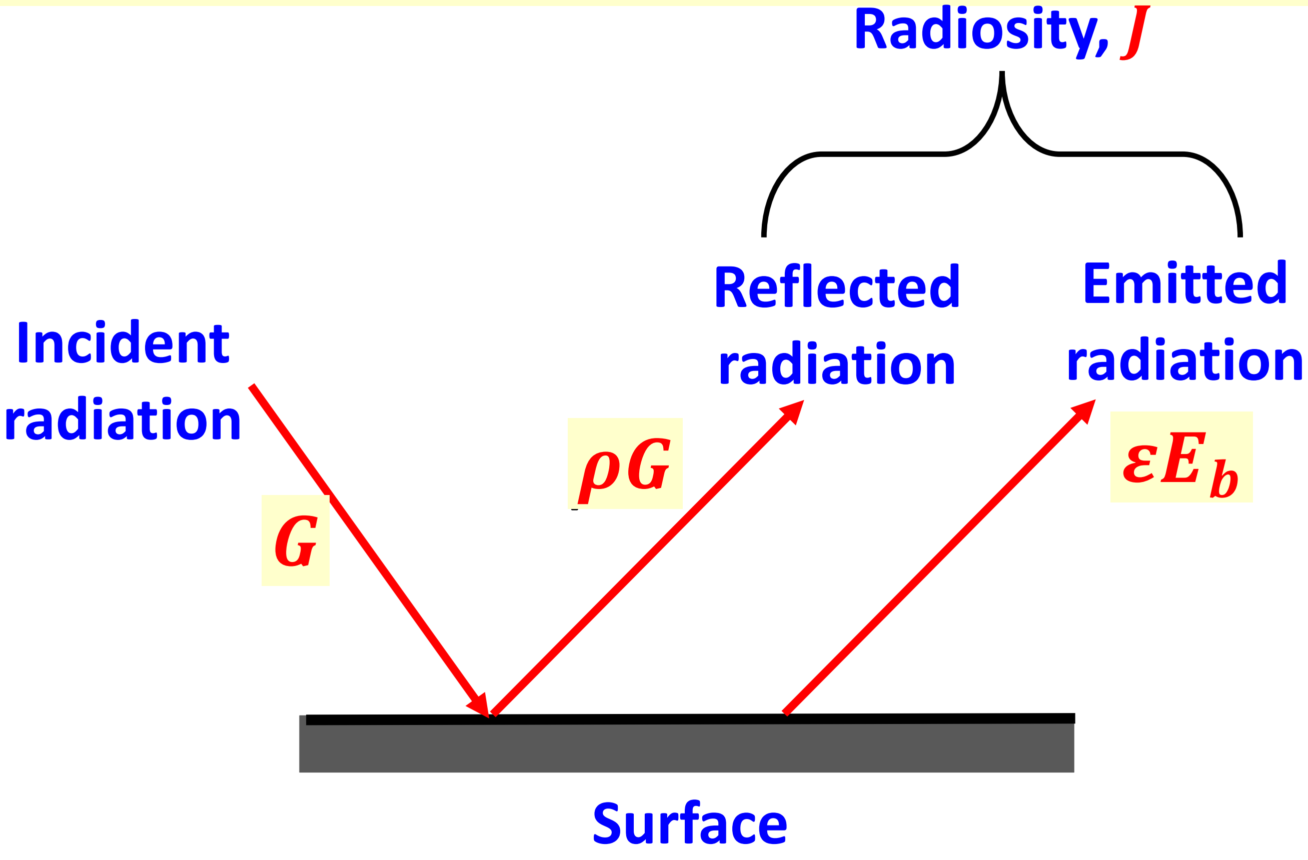
The negative sign indicates that net radiation heat transfer is to surface 1. that is, the base of the furnace is gaining net radiation at a rate of about 925 kW.

Radiation heat transfer: Diffuse, Gray surfaces

Assume the surfaces of an enclosure to be opaque, diffuse and gray

The surfaces are non-transparent, they are diffuse emitters and reflectors and their radiation properties are independent of wavelength

Each surface of the enclosure is isothermal, and both the incoming and outgoing radiation are uniform over each surface



Radiosity is the total radiation energy leaving a surface per unit time and per unit area

$$J_i = \varepsilon_i E_{bi} + \rho_i G_i$$

$$\alpha_i + \rho_i + \tau_i = 1$$

$$\tau_i = 0 \qquad \rho_i = 1 - \alpha_i$$

$$\alpha_i = \varepsilon_i \qquad \rho_i = 1 - \varepsilon_i$$

$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i$$

$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i$$

$E_{bi} = \sigma T_i^4$ is the blackbody emissive power of surface i

G_i is the irradiation (ie., the radiation energy incident on surface *i* per unit time per unit area)

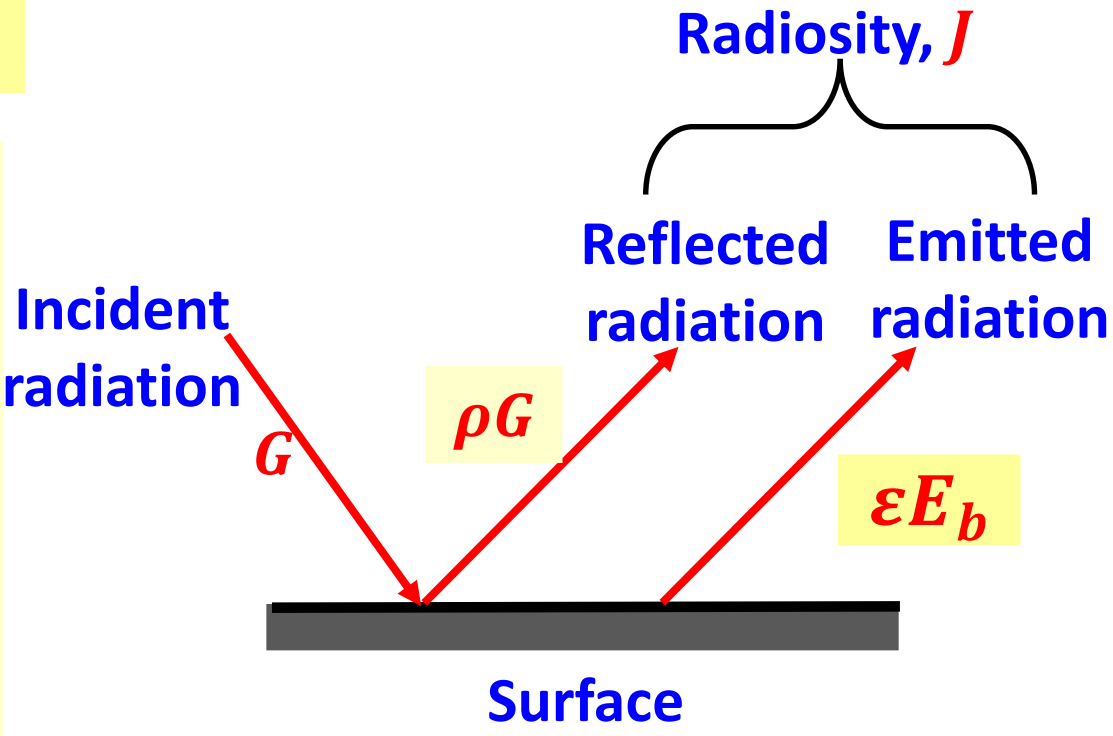
For a surface that can be approximated as a blackbody $(\varepsilon_i = 1)$, the radiosity relation reduces to

$$J_i = E_{bi} = \sigma T_i^4$$

The radiosity of a blackbody is equal to its emissive power.

This is expected, since a blackbody does not reflect any radiation

Thus radiation coming from a blackbody is due to emission only.



NET RADIATION HEAT TRANSFER TO OR FROM THE SURFACE

During a radiation interaction, a surface loses energy by emitting radiation and gains energy by absorbing radiating energy emitted by other surfaces

A surface experiences a net gain or a net loss of energy, depending on which quantity is larger.

The net rate of radiation heat transfer from a surface i of surface area A_i is denoted by $\dot{Q}_i$

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving the} \\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident on the} \\ \text{entire surface } i\end{pmatrix}$$

$$\dot{Q}_i = A_i(J_i - G_i)$$

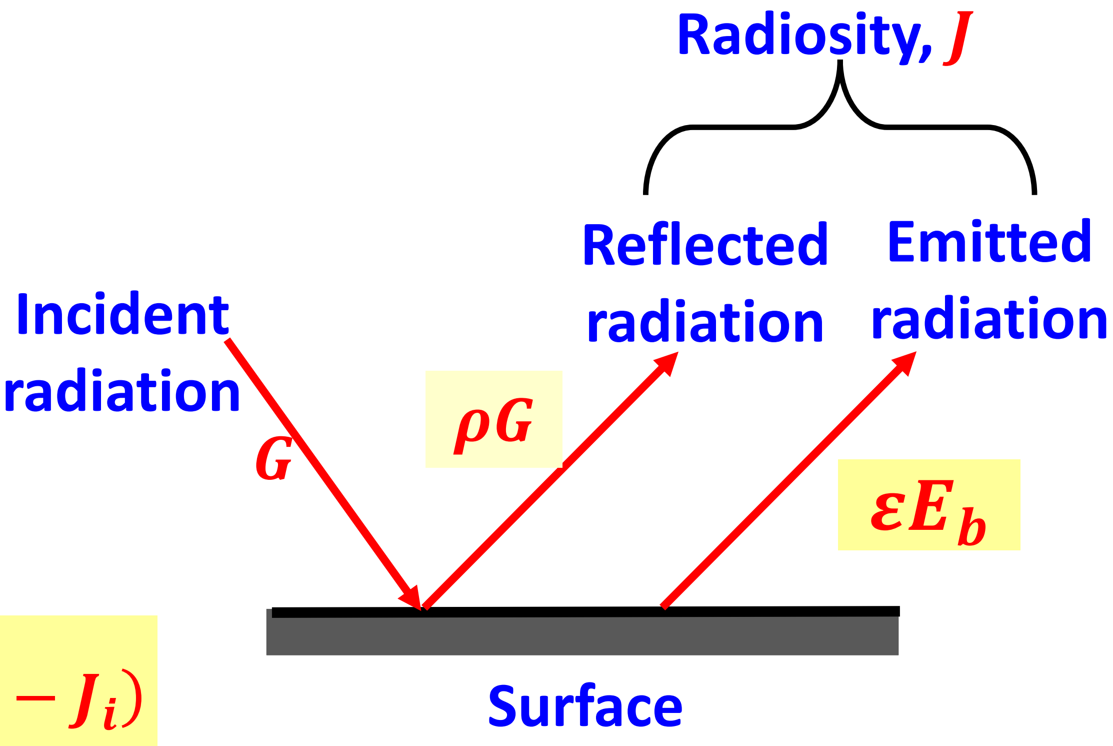
$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i$$

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{(1 - \varepsilon_i)}$$

$$\dot{Q}_i = A_i(J_i - G_i) = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{(1 - \varepsilon_i)}\right)$$

$$\dot{Q}_i = A_i\left(\frac{J_i - \varepsilon_i J_i - J_i + \varepsilon_i E_{bi}}{(1 - \varepsilon_i)}\right)$$

$$\dot{Q}_i = \frac{A_i \varepsilon_i}{1 - \varepsilon_i}(E_{bi} - J_i)$$

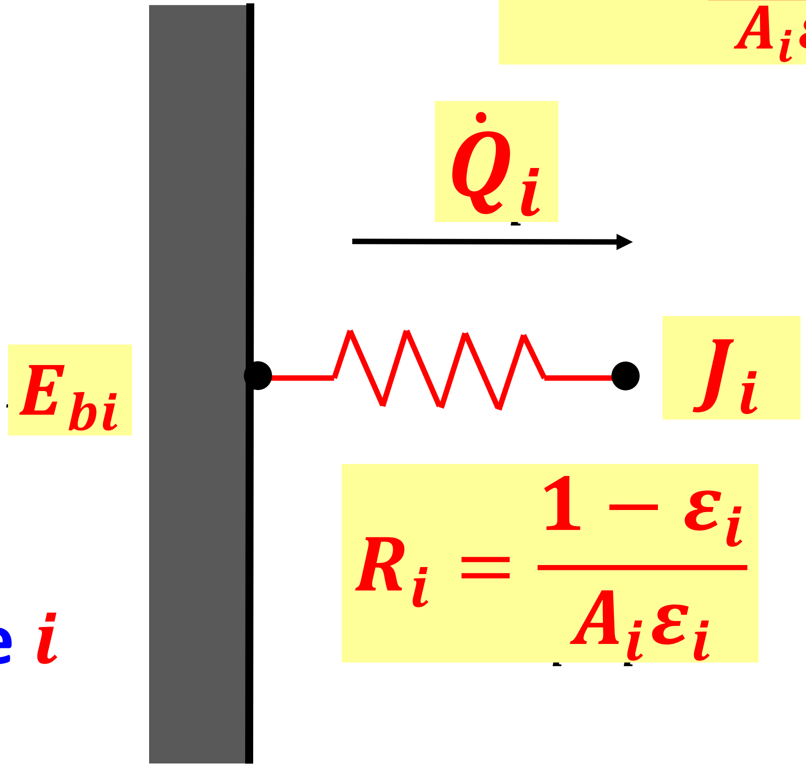


$$\dot{Q}_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}(E_{bi} - J_i)$$

$$\dot{Q}_i = \frac{(E_{bi} - J_i)}{\dfrac{1-\varepsilon_i}{A_i \varepsilon_i}}$$

$$\dot{Q}_i = \frac{(E_{bi} - J_i)}{R_i}$$

$$R_i = \frac{1-\varepsilon_i}{A_i \varepsilon_i}$$



$$R_i = \frac{1-\varepsilon_i}{A_i \varepsilon_i}$$

Surface i

$(E_{bi} - J_i)$ is the potential difference

R_i is the surface Resistance

$\dot{Q}_i$ Net rate of radiation heat transfer corresponds to the current in the electrical analogy

The direction of the net radiation heat transfer depends on the relative magnitude of J_i (the radiosity) and E_{bi} (the emissive power of a blackbody at the temperature of the surface). It will be from the surface if $E_{bi} > J_i$ and to the surface if $J_i > E_{bi}$.

A negative value for $\dot{Q}_i$ indicates that heat transfer is to the surface. All of this radiation energy gained must be removed from the other side of the surface through some mechanism if the surface temperature is to remain constant

The surface resistance to radiation for blackbody is zero since $1-\varepsilon_i$ and $J_i = E_{bi}$. The net rate of radiation heat transfer in this case is determined directly from $\dot{Q}_i = A_i(J_i - G_i)$

RE-RADIATING SURFACE

Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic since their back sides are well insulated and the net heat transfer through them is zero.

When the convection effects on the front (heat transfer) side of such a surface is negligible and steady state conditions are reached, the surface must lose as much radiation energy as it gains, and thus $\dot{Q}_i = 0$

In such cases, the surface is said to reradiate all the radiation energy it receives, and such a surface is called a RERADIATING SURFACE

$$\dot{Q}_i = \frac{(E_{bi} - J_i)}{\dfrac{1 - \varepsilon_i}{A_i \varepsilon_i}} = \frac{(E_{bi} - J_i)}{R_i} \Longrightarrow E_{bi} = J_i$$

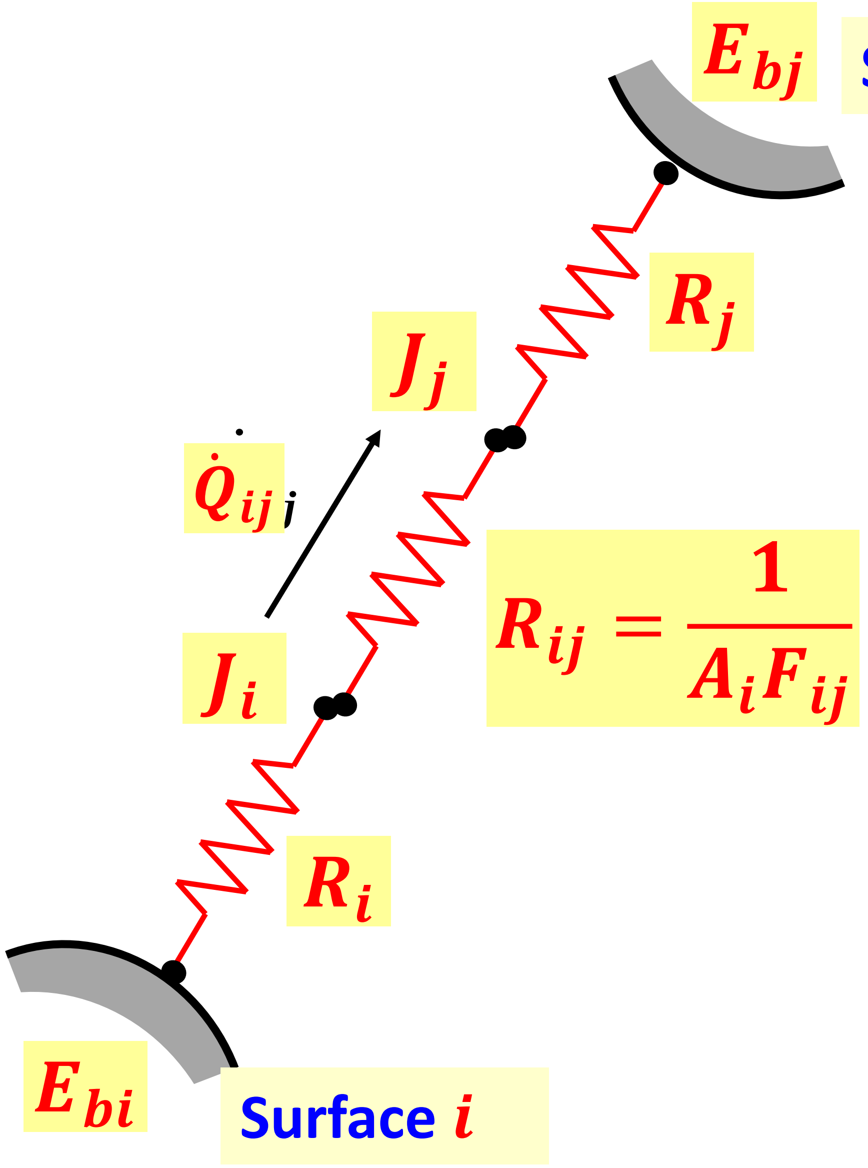
The temperature of a reradiating surface under steady conditions can easily be determined from the equation above once its radiosity is known.

Note that the temperature of a reradiating surface is independent of its emissivity

In radiation analysis, the surface resistance of a reradiating surface is disregarded since there is no heat transfer through it .

This is like the fact that there is no need to consider a resistance in an electrical network if no current is flowing through it

Net radiation heat transfer between any two diffuse, gray and opaque surfaces



Consider two diffuse, gray and opaque surfaces of arbitrary shape maintained at uniform temperatures

J – radiosity – rate of radiation leaving a surface per unit surface area

F_{ij} – view factor – fraction of radiation leaving surface i that strikes surface j

Net rate of radiation heat transfer from surface i to surface j

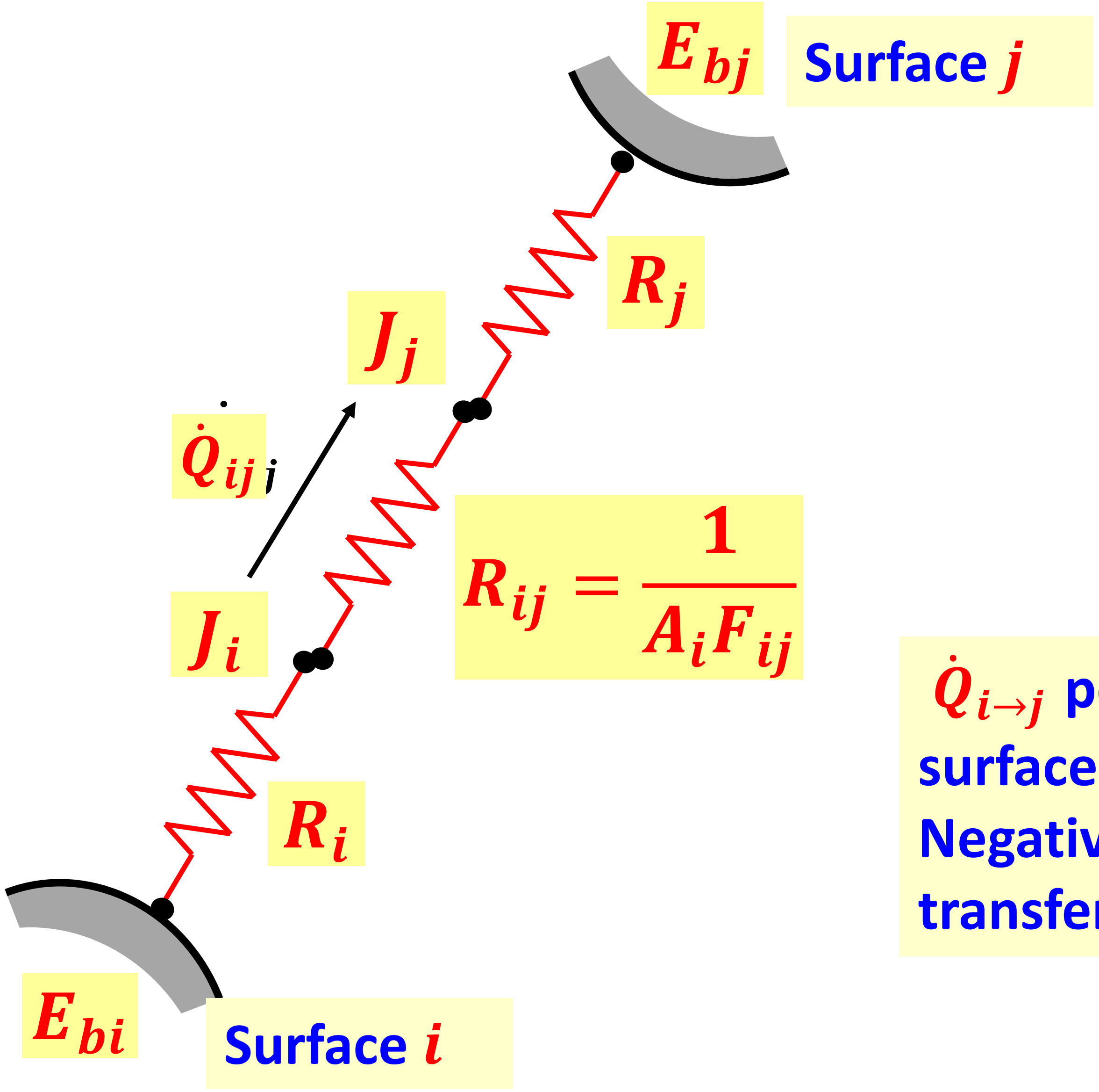
$$\dot{Q}_{i\to j} = A_i J_i F_{ij} - A_j J_j F_{ji}$$

Applying the reciprocity relation $A_i F_{i\to j} = A_j F_{j\to i}$

$$\dot{Q}_{i\to j} = A_i F_{i\to j}(J_i - J_j)$$

$$\dot{Q}_{i\to j} = \frac{(J_i - J_j)}{\dfrac{1}{A_i F_{ij}}} = \frac{(J_i - J_j)}{R_{ij}}$$

Net radiation heat transfer between any two diffuse, gray and opaque surfaces



$$\dot{Q}_{i\rightarrow j} = \frac{(J_i - J_j)}{\dfrac{1}{A_i F_{ij}}} = \frac{(J_i - J_j)}{R_{ij}}$$

$$R_{ij} = \frac{1}{A_i F_{ij}}$$ Space resistance

$$(J_i - J_j)$$ Potential difference

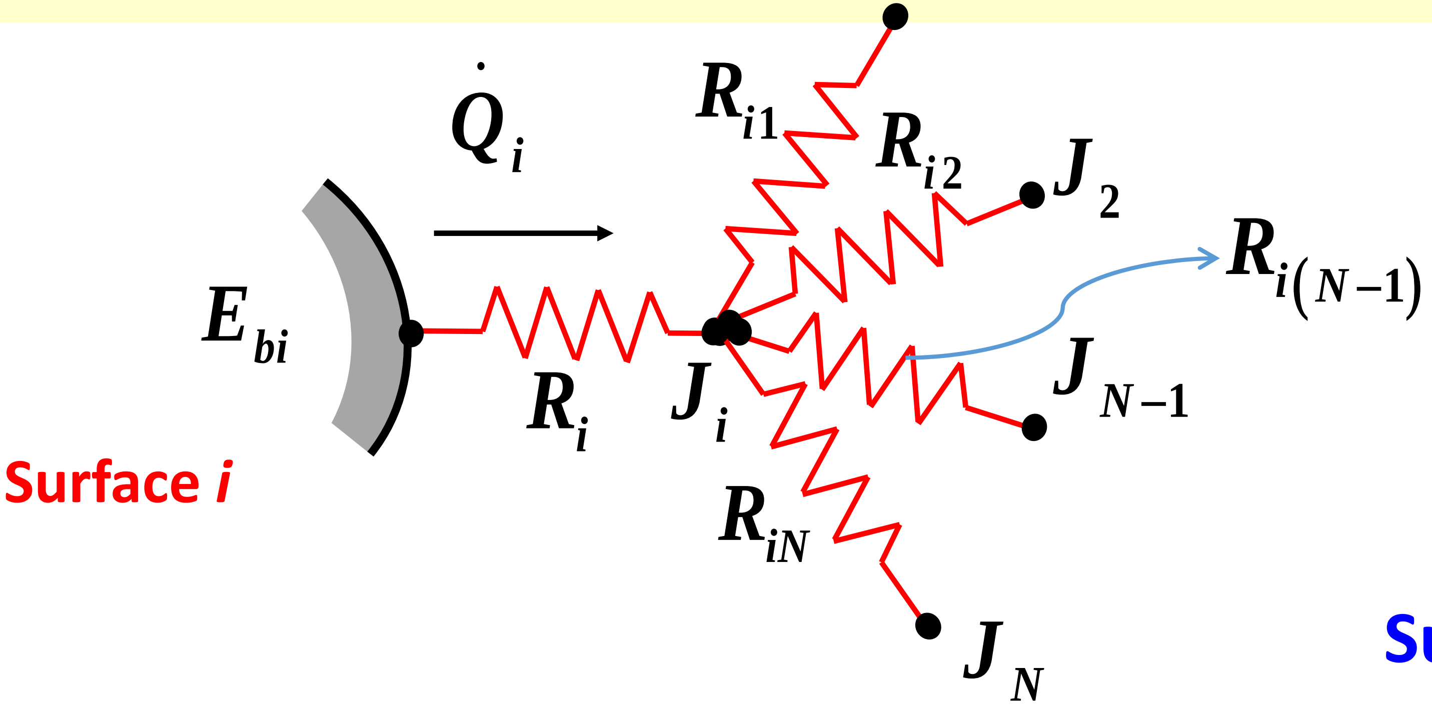
$\dot{Q}_{i\rightarrow j}$ positive represents that the heat transfer is from surface i to j

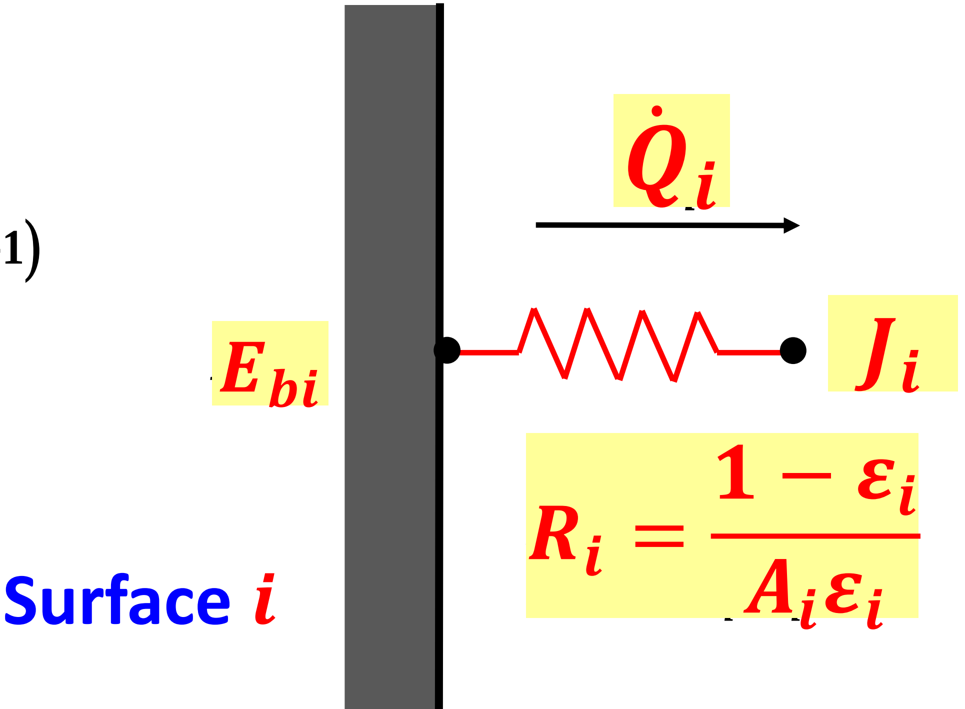
Negative value indicates the opposite – the heat transfer is from surface j to i

In an N-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface i be equal to the sum of the net heat transfers from surface i to each of the N surfaces of the enclosure

$$\dot{Q}_i = \sum_{j=1}^{j=N} \dot{Q}_{i\rightarrow j} = \sum_{j=1}^{j=N} A_i F_{ij}(J_i - J_j) = \sum_{j=1}^{j=N} \frac{(J_i - J_j)}{\frac{1}{A_i F_{ij}}} = \sum_{j=1}^{j=N} \frac{(J_i - J_j)}{R_{ij}}$$

Network representation of net radiation heat transfer from surface i to the remaining surfaces of an N-surface enclosure

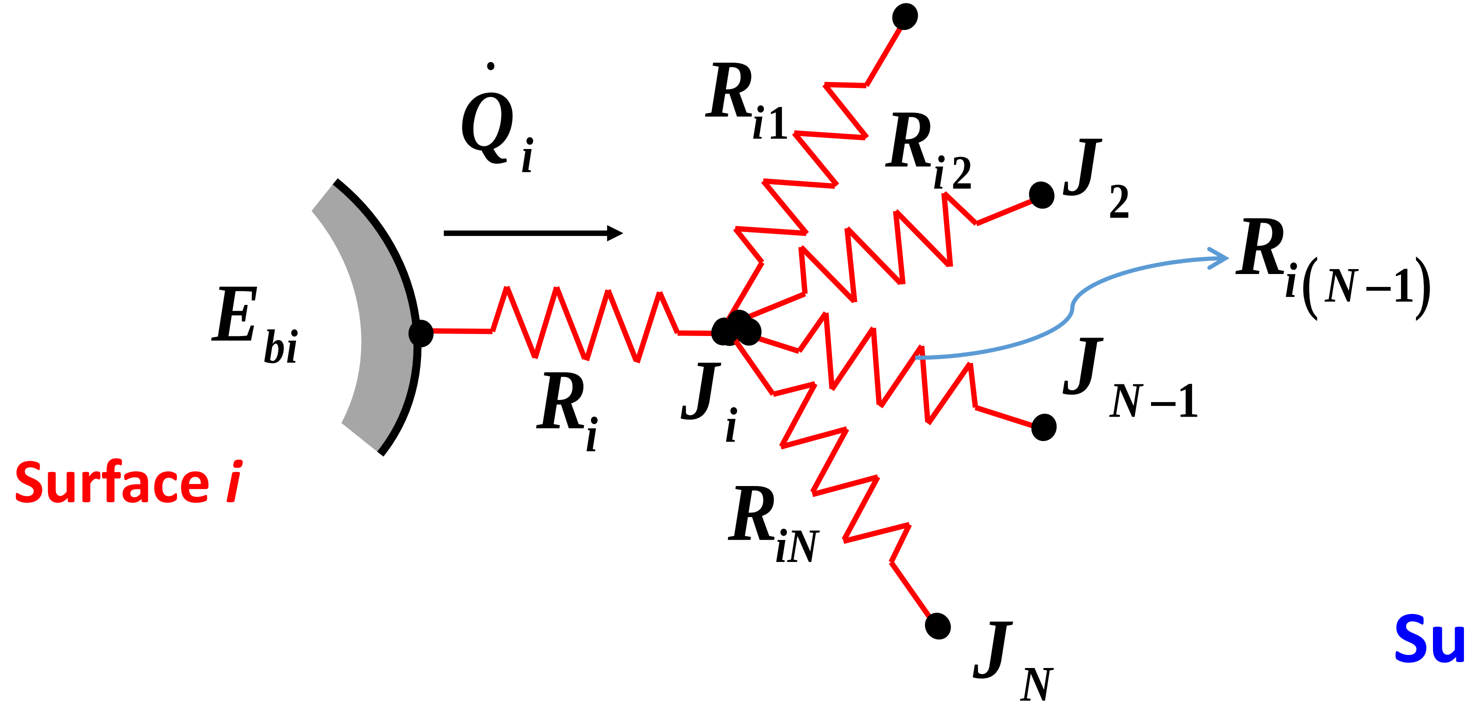


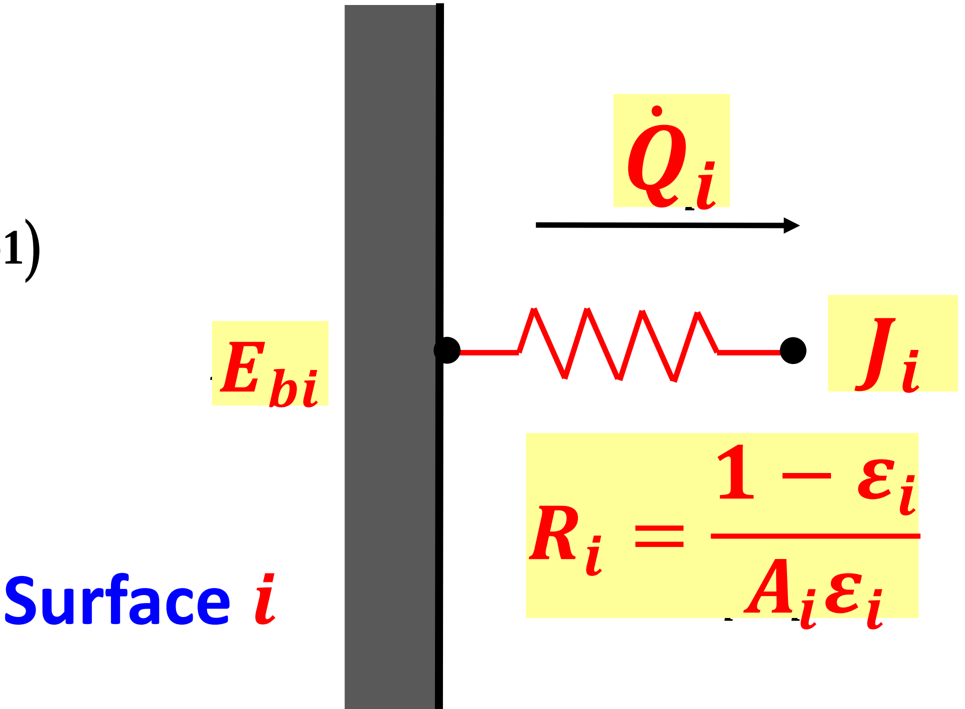


Combining surface and space resistance

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} = \frac{E_{bi} - J_i}{\frac{1-\varepsilon_i}{A_i \varepsilon_i}} = \sum_{j=1}^{j=N} \dot{Q}_{i\to j} = \sum_{j=1}^{j=N} A_i F_{ij}(J_i - J_j) = \sum_{j=1}^{j=N} \frac{(J_i - J_j)}{\frac{1}{A_i F_{ij}}} = \sum_{j=1}^{j=N} \frac{(J_i - J_j)}{R_{ij}}$$

The net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces including itself, through the corresponding space resistances





METHODS OF SOLVING RADIATION PROBLEMS

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates.

Method 1 – For surfaces with specified heat transfer rates

$$\dot{Q}_i = \sum_{j=1}^{j=N} \dot{Q}_{i\to j} = \sum_{j=1}^{j=N} A_i F_{ij}\left(J_i - J_j\right)$$

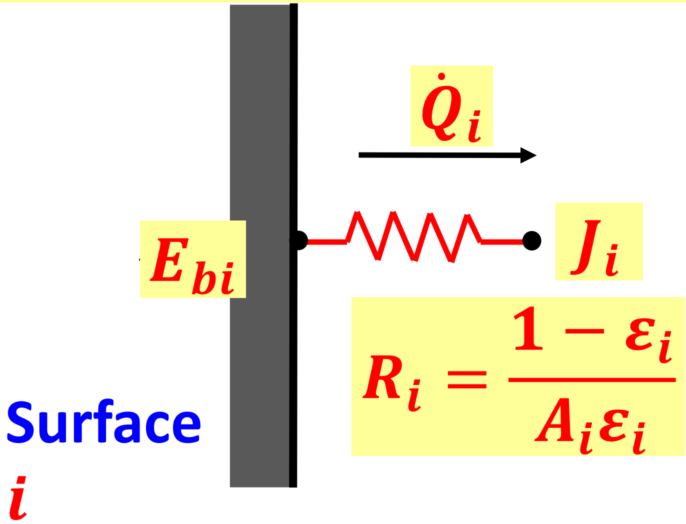
Method 2 – Surfaces with specified temperature T_i

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} = \frac{E_{bi} - J_i}{\dfrac{1-\varepsilon_i}{A_i\varepsilon_i}} = \sum_{j=1}^{j=N} \dot{Q}_{i\to j} = \sum_{j=1}^{j=N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{j=N} \frac{\left(J_i - J_j\right)}{\dfrac{1}{A_i F_{ij}}} = \sum_{j=1}^{j=N} \frac{\left(J_i - J_j\right)}{R_{ij}}$$

$$E_{bi} - J_i = \frac{1-\varepsilon_i}{A_i\varepsilon_i} = \sum_{j=1}^{j=N} A_i F_{ij}\left(J_i - J_j\right)$$

$$\sigma T_i^4 = J_i + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1}^{j=N} F_{ij}\left(J_i - J_j\right)$$

For insulated (re-radiating surfaces) $\dot{Q}_i = 0$



The temperatures of insulated or reradiating surfaces can be determined from is $\sigma T_i^4 = J_i$

For black surfaces, $\varepsilon_i = 1 \quad \therefore R_i = 0 \quad E_{bi} = \sigma T_i^4 = J_i$

The term corresponding to $j = i$ will drop out from either relation since $J_i - J_j = J_i - J_i = 0$

The equations give N linear algebraic equations for the determination of the N unknown radiosities for an N surface enclosure

Once the radiosities $J_1, J_2, J_3 \ldots. JN$ are available, the unknown heat transfer rates can be determined

$$\dot{Q}_i = \sum_{j=1}^{j=N} \dot{Q}_{i \rightarrow j} = \sum_{j=1}^{j=N} A_i F_{ij} (J_i - J_j)$$

Once the radiosities $J_1, J_2, J_3 \ldots. JN$ are available, the unknown surface temperatures can be determined

$$\sigma T_i^4 = J_i + \frac{1-\varepsilon_i}{\varepsilon_i} \sum_{j=1}^{j=N} F_{ij} (J_i - J_j)$$

The temperatures of insulated or reradiating surfaces can be determined

$$\sigma T_i^4 = J_i$$

Matrix method - matlab, mathcad may be used

Network method – electrical network analogy

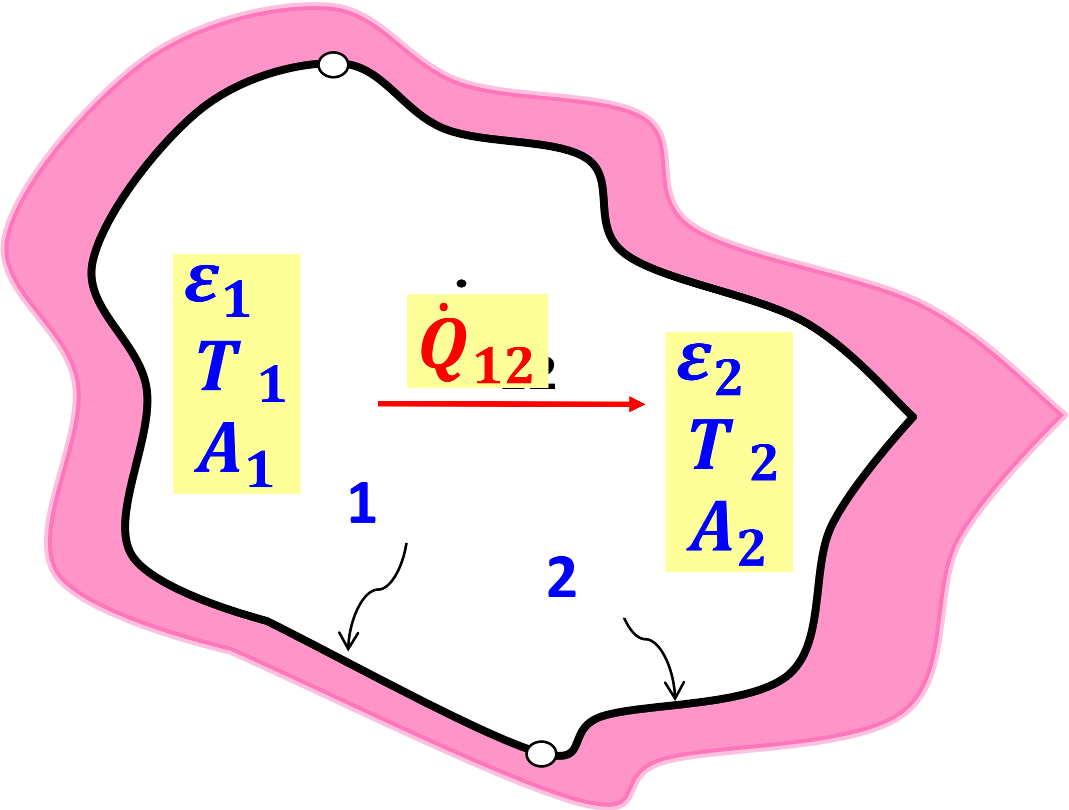
A.K. Oppenheim in the 1950s – because of its simplicity and emphasis on the physics of the problem

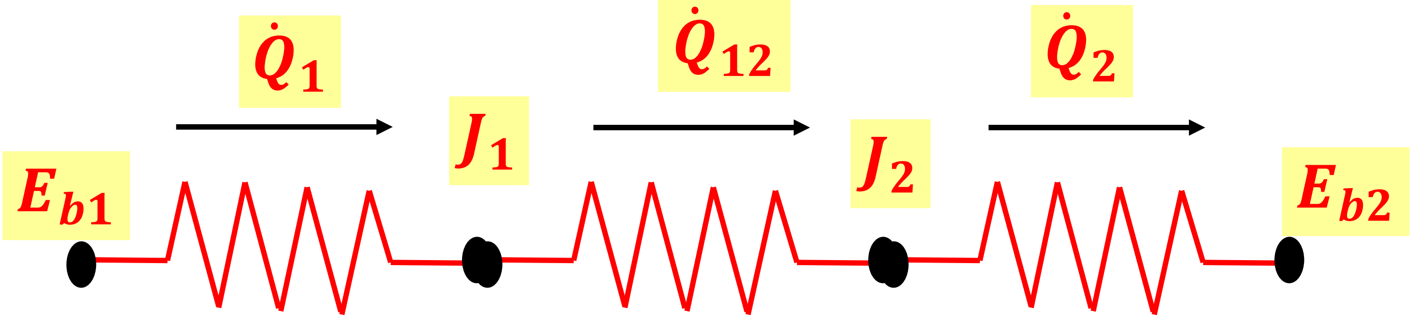
Draw a surface resistance associated with each surface of an enclosure and connect them with space resistances

Then solve the radiation problem by treating it as an electrical network problem where the radiation heat transfer replaces the current and radiosity replaces the potential

The network method is not practical for enclosures with more than three or four surfaces, however, because of the increased complexity of the network

Radiation heat transfer in two surface enclosures





$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} \qquad R_{12} = \frac{1}{A_1F_{12}} \qquad R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

$$\dot{Q}_i = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

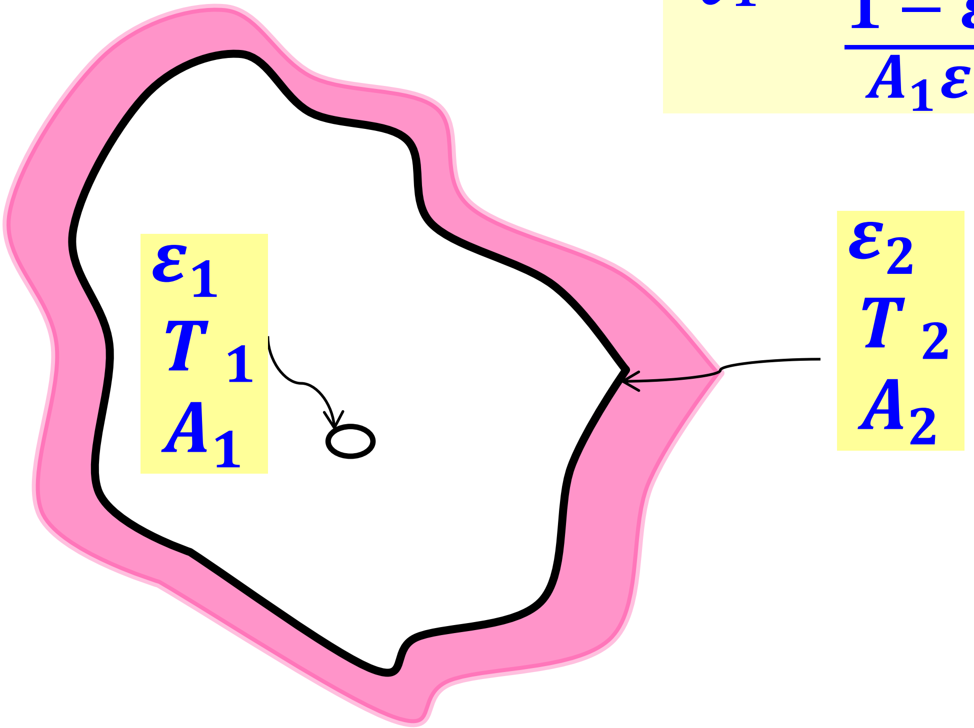
$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_2 + R_3}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

Small object in a cavity

$$\dot{Q}_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1$$



$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{A_1\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + A_1\dfrac{1}{A_1 F_{12}} + A_1\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{1}{F_{12}} + \cancel{\dfrac{A_1}{A_2}\dfrac{1-\varepsilon_2}{\varepsilon_2}}} \qquad \frac{A_1}{A_2} \approx 0$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} - 1 + \dfrac{1}{F_{12}}} \qquad F_{12} = 1$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} - 1 + 1} \qquad \dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1}}$$

$$\dot{Q}_1 = \sigma\varepsilon_1 A_1\left(T_1^4 - T_2^4\right)$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

It is interesting to note that the heat transfer rate is independent of the area of the small object and the emissivity of the small object as long as the object is small compared to the size

Infinitely large parallel plates

$A_1 \quad \varepsilon_1 \quad T_1$

$A_2 \quad \varepsilon_2 \quad T_2$

$$\dot{Q}_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$A_1 = A_2 \qquad F_{12} = 1$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{A_1\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + A_1\dfrac{1}{A_1 F_{12}} + A_1\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

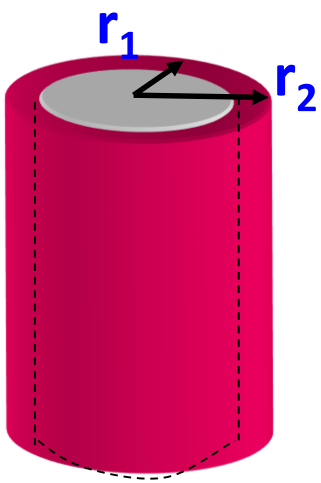
$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{1}{F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2}}$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} - 1 + \dfrac{1}{1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$\dot{Q}_1 = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

Infinitely long concentric cylinders



$$\dot{Q}_1 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\frac{A_1}{A_2} = \frac{2\pi r_1 L}{2\pi r_2 L} = \frac{r_1}{r_2}$$

$$F_{12} = 1$$

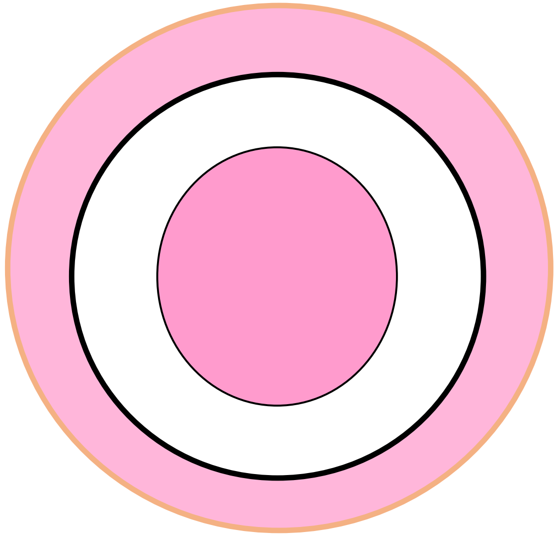
$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{A_1\frac{1-\varepsilon_1}{A_1\varepsilon_1} + A_1\frac{1}{A_1F_{12}} + A_1\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} - 1 + \frac{1}{1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}$$

$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

Concentric spheres



$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{A_1\frac{1-\varepsilon_1}{A_1\varepsilon_1} + A_1\frac{1}{A_1F_{12}} + A_1\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

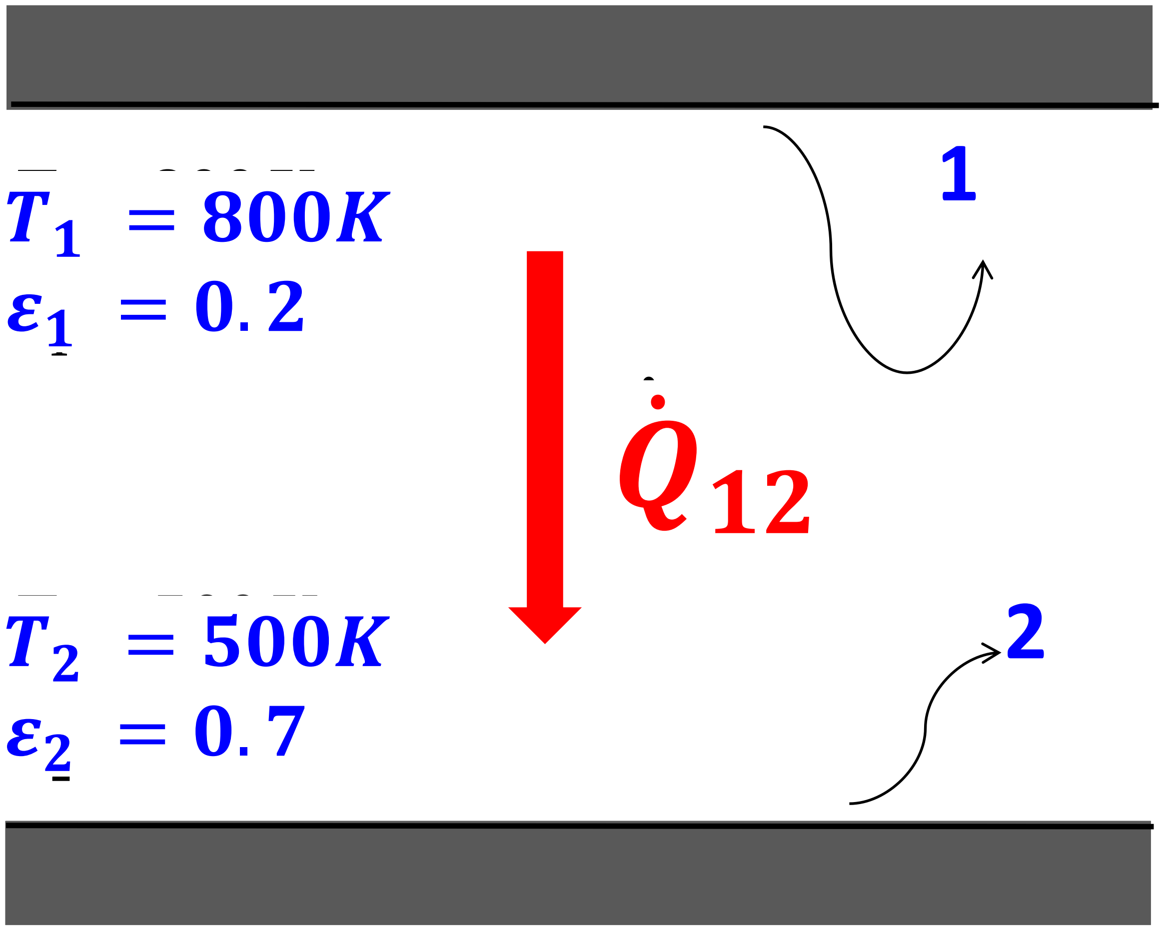
$$F_{12} = 1$$

$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} - 1 + \frac{1}{1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}$$

$$\dot{Q}_1 = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

Problem: Determine the net radiation heat transfer between two large parallel plates maintained at uniform temperature as shown in figure below



Solution: Two large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates is to be determined.

Assumptions: Both the surfaces are opaque, diffuse and gray

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A_1} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

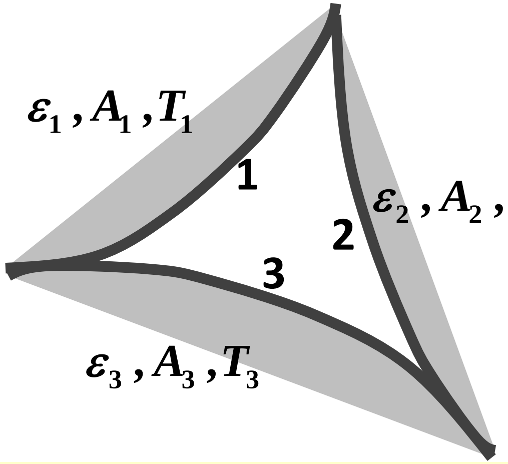
$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

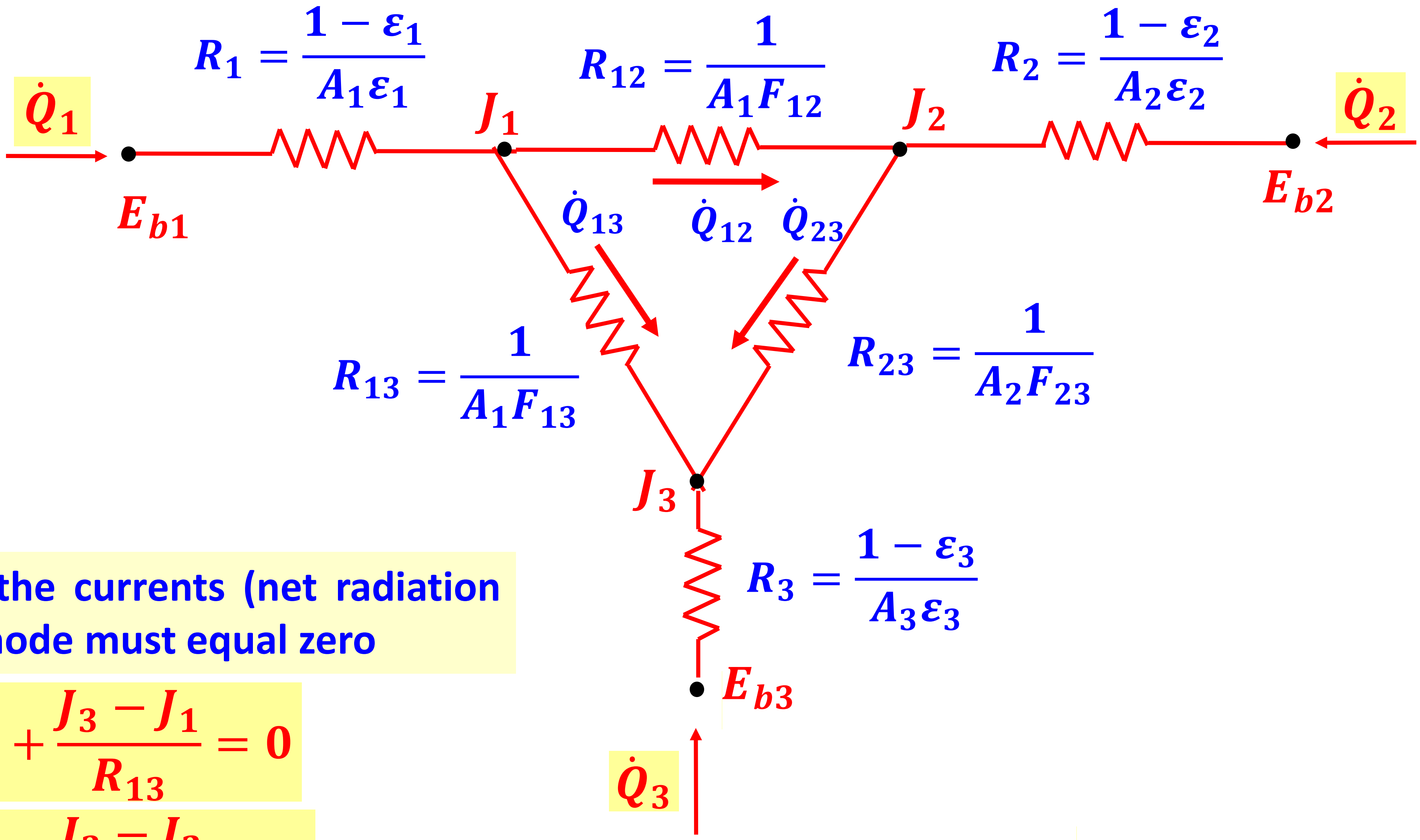
$$\dot{q}_{12} = \frac{5.67 \times 10^{-8}\left(800^4 - 500^4\right)}{\frac{1}{0.2} + \frac{1}{0.7} - 1}$$

$$\dot{q}_{12} = 3625\,\frac{W}{m^2}$$

Discussion: Note that heat at a net rate of 3625 W is transferred from plate 1 to plate 2 by radiation per unit surface area of either plate.

Radiation heat transfer in three surface enclosures





The algebraic sum of the currents (net radiation heat transfer) at each node must equal zero

$$\frac{E_{b1}-J_1}{R_1}+\frac{J_2-J_1}{R_{12}}+\frac{J_3-J_1}{R_{13}}=0$$

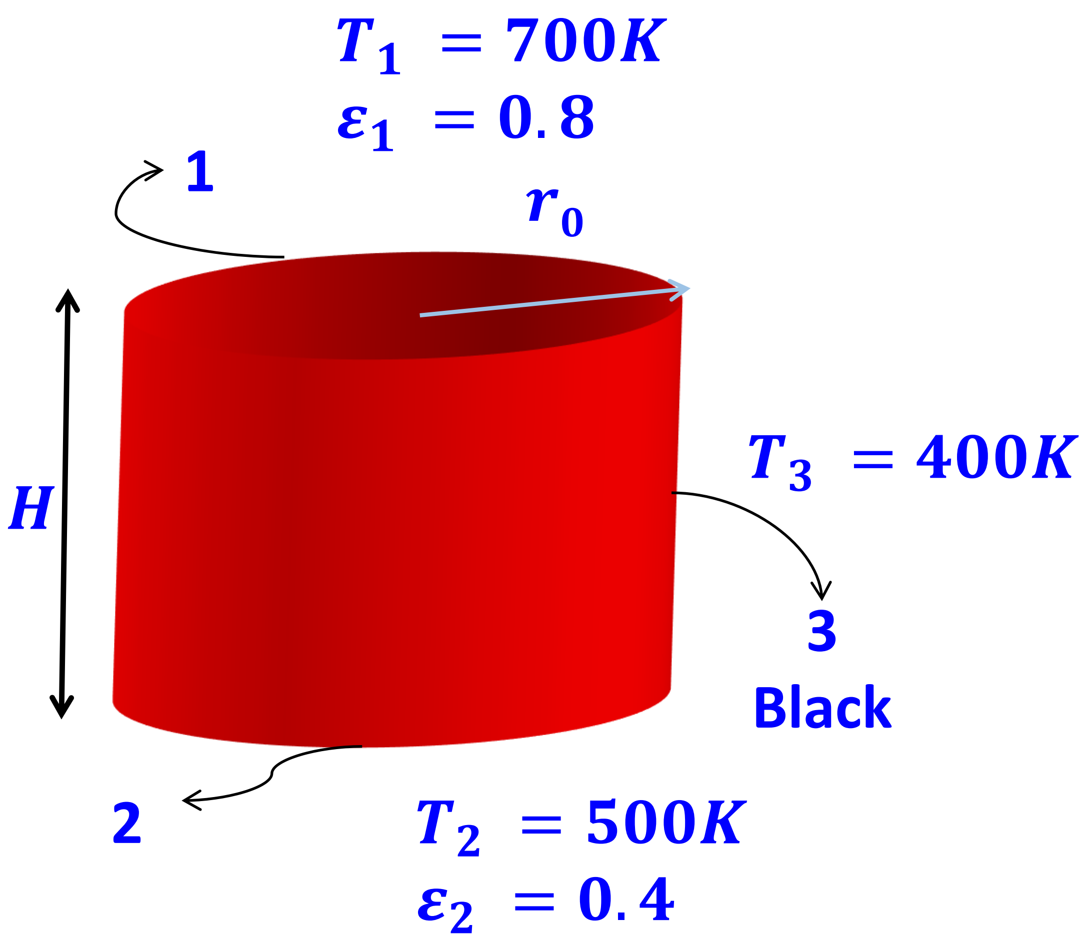
$$\frac{J_1-J_2}{R_{21}}+\frac{E_{b2}-J_2}{R_2}+\frac{J_3-J_2}{R_{23}}=0$$

$$\frac{J_1-J_3}{R_{13}}+\frac{J_2-J_3}{R_{23}}+\frac{E_{b3}-J_3}{R_3}=0$$

RADIATION HEAT TRANSFER IN A CYLINDRICAL FURNACE

Consider a cylindrical furnace with $r_0 = H = 1\,m$, as shown in figure. The top surface (surface 1) and the base surface (surface 2) of the furnace has emissivities $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$, respectively, and are maintained at uniform temperatures $T_1 = 700\,K$ and $T_2 = 500\,K$. The side surface closely approximates a blackbody and is maintained at a temperature of $T_3 = 400K$.

Determine the net rate of radiation heat transfer at each surface during steady operation and explain how these surfaces can be maintained at specified temperatures.

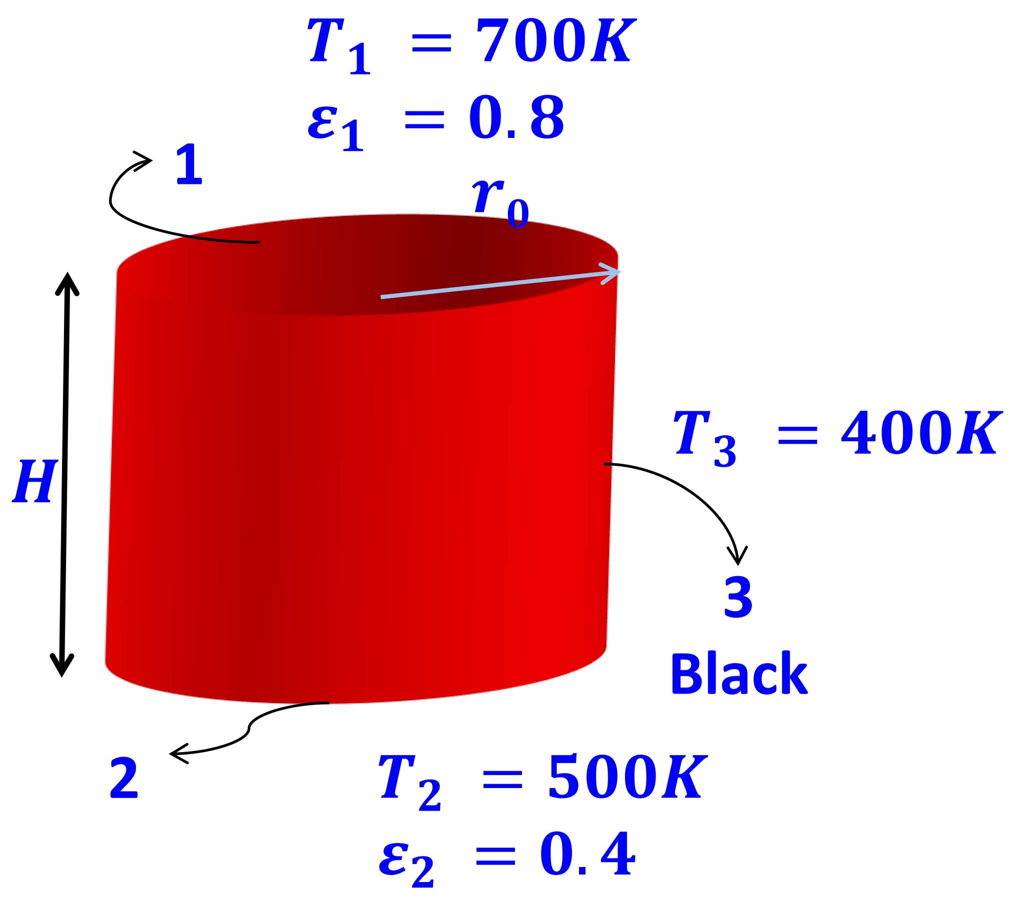


Solution: The surfaces of a cylindrical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer at each surface during steady operation is to be determined.

Assumptions: 1. steady operating conditions exist. 2. The surfaces are opaque, diffuse and gray. 3. Convection heat transfer is not considered.

Analysis: We will solve this problem systematically using the direct method to demonstrate its use. The cylindrical furnace can be consider to be a three surface enclosure with surface areas of

$$A_1 = A_2 = \pi r_o^2 = \pi(1)^2 = 3.14\, m^2$$
$$A_3 = 2\pi r_o H = 2\pi(1)(1) = 6.28\, m^2$$



The view factor from the base to the top surface is $F_{12} = 0.38$.

$$R_i = \frac{r_i}{L};\ R_j = \frac{r_j}{L};\ S = 1 + \frac{1+R_j^2}{R_i^2};$$

$$F_{i\to j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{\frac{1}{2}}\right\}$$

$$R_i = \frac{r_i}{L} = \frac{1}{1} = 1;\ R_j = \frac{r_j}{L} = \frac{1}{1} = 1;\ S = 1 + \frac{1+R_j^2}{R_i^2} = 1 + \frac{1+1}{1} = 3;$$

$$F_{i\to j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{\frac{1}{2}}\right\} = \frac{1}{2}\left\{3 - \left[3^2 - 4\left(\frac{1}{1}\right)^2\right]^{\frac{1}{2}}\right\} = 0.38$$

Then the view factor from the base to the side surface is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \qquad 0 + 0.38 + F_{13} = 1 \qquad F_{13} = 0.62$$

Since the base surface is flat and thus $F_{11} = 0$. Noting that the top and bottom surfaces are symmetric about the side surface, $F_{21} = F_{12} = 0.38$ and $F_{23} = F_{13} = 0.62$.
The view factor F_{31} is determined from the reciprocity relation,

$$A_1F_{13} = A_3F_{31} \qquad F_{31} = \frac{A_1F_{13}}{A_3} = \frac{3.14 \times 0.62}{6.28} \qquad F_{31} = 0.31$$

$$A_1 = A_2 = 3.14\, m^2$$
$$A_3 = 6.28\, m^2$$

$F_{32} = F_{31} = 0.31$, By symmetry

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

$$\frac{J_1 - J_2}{R_{21}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0$$

$$\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0$$

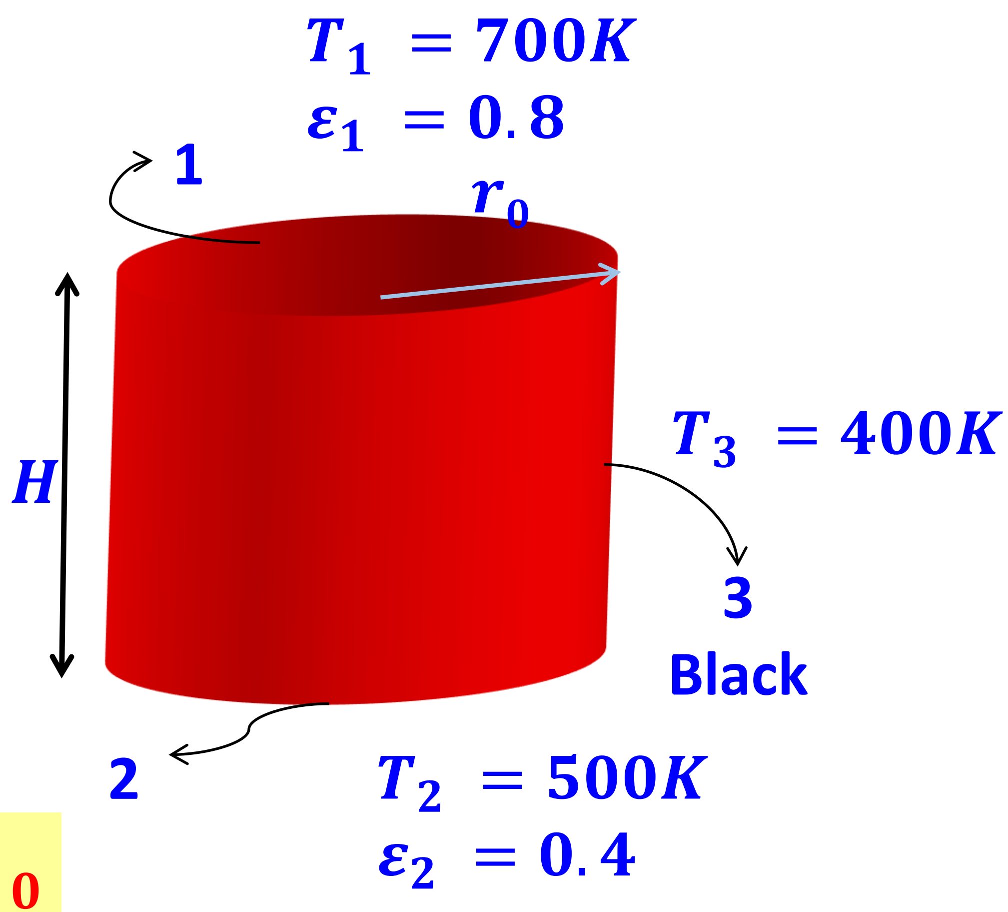
$$\frac{E_{b1} - J_1}{\frac{1 - \varepsilon_1}{A_1\varepsilon_1}} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} + \frac{J_3 - J_1}{\frac{1}{A_1F_{13}}} = 0$$

$$\frac{J_1 - J_2}{\frac{1}{A_1F_{12}}} + \frac{E_{b2} - J_2}{\frac{1 - \varepsilon_2}{A_2\varepsilon_2}} + \frac{J_3 - J_2}{\frac{1}{A_2F_{23}}} = 0$$

$$\frac{J_1 - J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_2F_{23}}} + \frac{E_{b3} - J_3}{\frac{1 - \varepsilon_3}{A_3\varepsilon_3}} = 0$$

$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\frac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\frac{1}{A_1F_{13}}}=0$$

$$A_1 = A_2 = 3.14\ m^2$$
$$A_3 = 6.28\ m^2$$



$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}}+\frac{E_{b2}-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\frac{1}{A_2F_{23}}}=0$$

$$F_{21} = F_{12} = 0.38$$
$$F_{23} = F_{13} = 0.62$$

$$\frac{J_1-J_3}{\frac{1}{A_1F_{13}}}+\frac{J_2-J_3}{\frac{1}{A_2F_{23}}}+\frac{E_{b3}-J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}}=0$$

$E_{b3} = J_3$ $\quad$ $\sigma T_3^4 = J_3$ $\quad$ $5.67\times10^{-8}(400)^4 = J_3$

$$J_3 = 1451.52\ W/m^2$$

$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\frac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\frac{1}{A_1F_{13}}}=0 \qquad \frac{\sigma T_1^4-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\frac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\frac{1}{A_1F_{13}}}=0$$

$$\frac{5.67\times10^{-8}(700)^4-J_1}{\frac{1-0.8}{3.14(0.8)}}+\frac{J_2-J_1}{\frac{1}{3.14(0.38)}}+\frac{1451.52-J_1}{\frac{1}{3.14(0.62)}}=0$$

$$170987.69-12.56J_1+1.1932J_2-1.1932J_1+2825.82-1.9468J_1=0$$

$$15.7J_1-1.1932J_2=173813.51$$

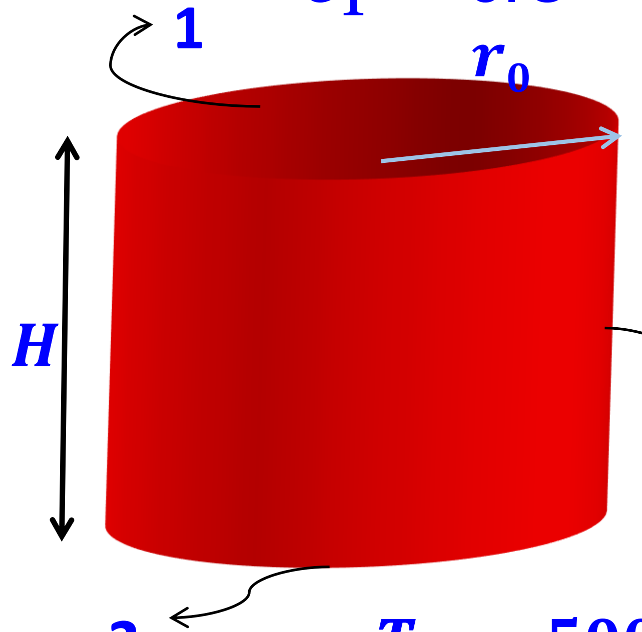
$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\frac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\frac{1}{A_1F_{13}}}=0$$

$$A_1 = A_2 = 3.14\ m^2$$
$$A_3 = 6.28\ m^2$$

$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}}+\frac{E_{b2}-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\frac{1}{A_2F_{23}}}=0$$

$$F_{21} = F_{12} = 0.38$$
$$F_{23} = F_{13} = 0.62$$

$$\frac{J_1-J_3}{\frac{1}{A_1F_{13}}}+\frac{J_2-J_3}{\frac{1}{A_2F_{23}}}+\frac{E_{b3}-J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}}=0$$



$$E_{b3} = J_3 \qquad \sigma T_3^4 = J_3 \qquad 5.67\times10^{-8}(400)^4 = J_3$$

$$J_3 = 1451.52\ W/m^2$$

$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}}+\frac{E_{b2}-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\frac{1}{A_2F_{23}}}=0 \qquad \frac{J_1-J_2}{\frac{1}{A_1F_{12}}}+\frac{\sigma T_2^4-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\frac{1}{A_2F_{23}}}=0$$

$$\frac{J_1-J_2}{\frac{1}{3.14(0.38)}}+\frac{5.67\times10^{-8}(500)^4-J_2}{\frac{1-0.4}{3.14(0.4)}}+\frac{1451.52-J_2}{\frac{1}{3.14(0.62)}}=0$$

$$1.1932J_1-1.1932J_2+7418.25-2.0933J_2+2825.82-1.9468J_2=0$$

$$1.1932J_1-5.2333J_2=-10244.07$$

$$15.7J_1 - 1.1932J_2 = 173813.51$$

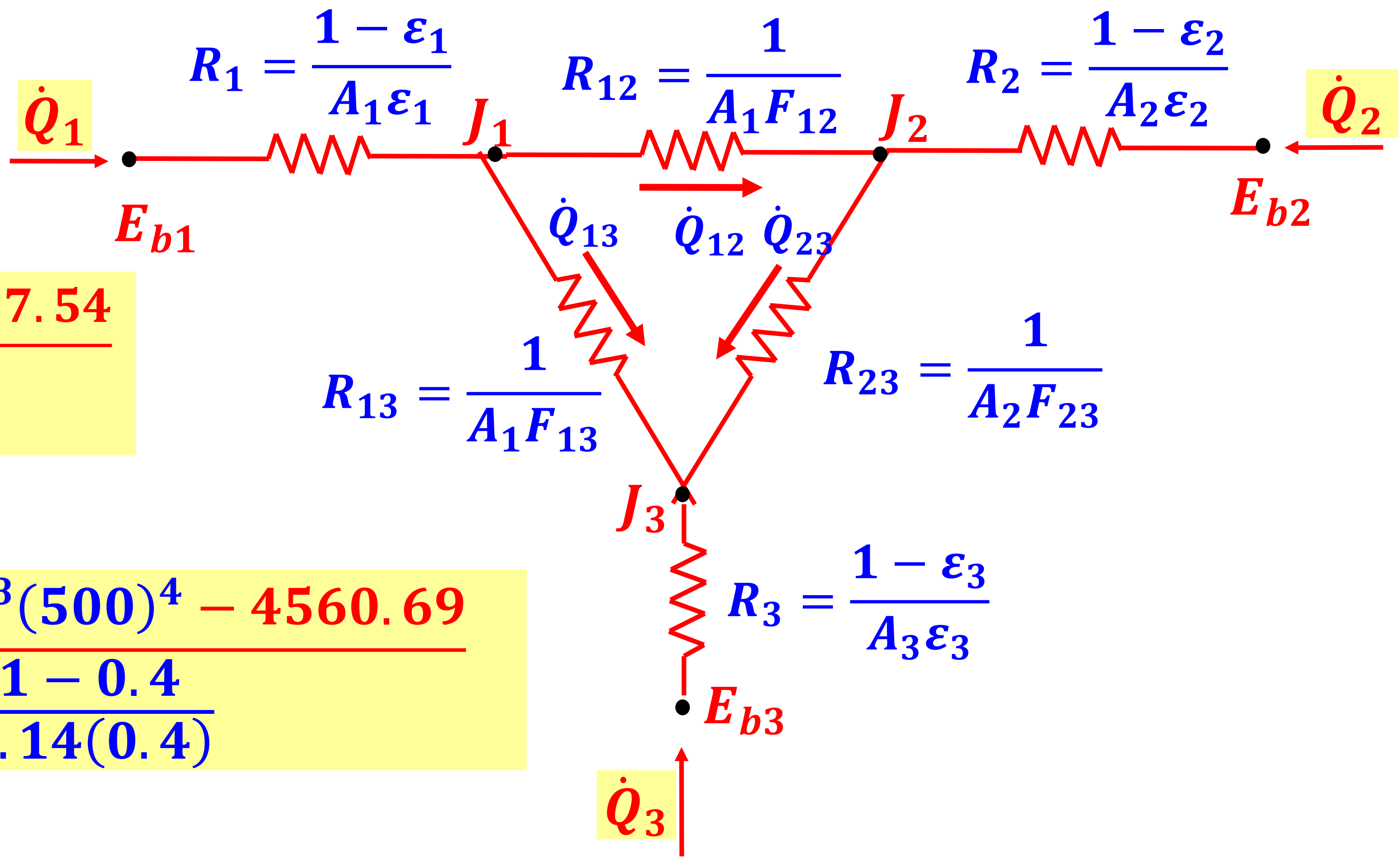
$$1.1932J_1 - 5.2333J_2 = -10244.07$$

$$J_1 = 11417.54\ W/m^2$$

$$J_2 = 4560.69\ W/m^2$$

$$J_3 = 1451.52\ W/m^2$$

$$\dot{Q}_1 = \frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}$$



$$\dot{Q}_1 = \frac{5.67\times10^{-8}(700)^4 - 11417.54}{\frac{1-0.8}{3.14(0.8)}}$$

$$\dot{Q}_1 = 27583.39\ W$$

$$\dot{Q}_2 = \frac{E_{b2} - J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{5.67\times10^{-8}(500)^4 - 4560.69}{\frac{1-0.4}{3.14(0.4)}}$$

$$\dot{Q}_2 = -2128.79\ W$$

$$J_1 = 11417.54\ W/m^2$$

$$J_2 = 4560.69\ W/m^2$$

$$J_3 = 1451.52\ W/m^2$$

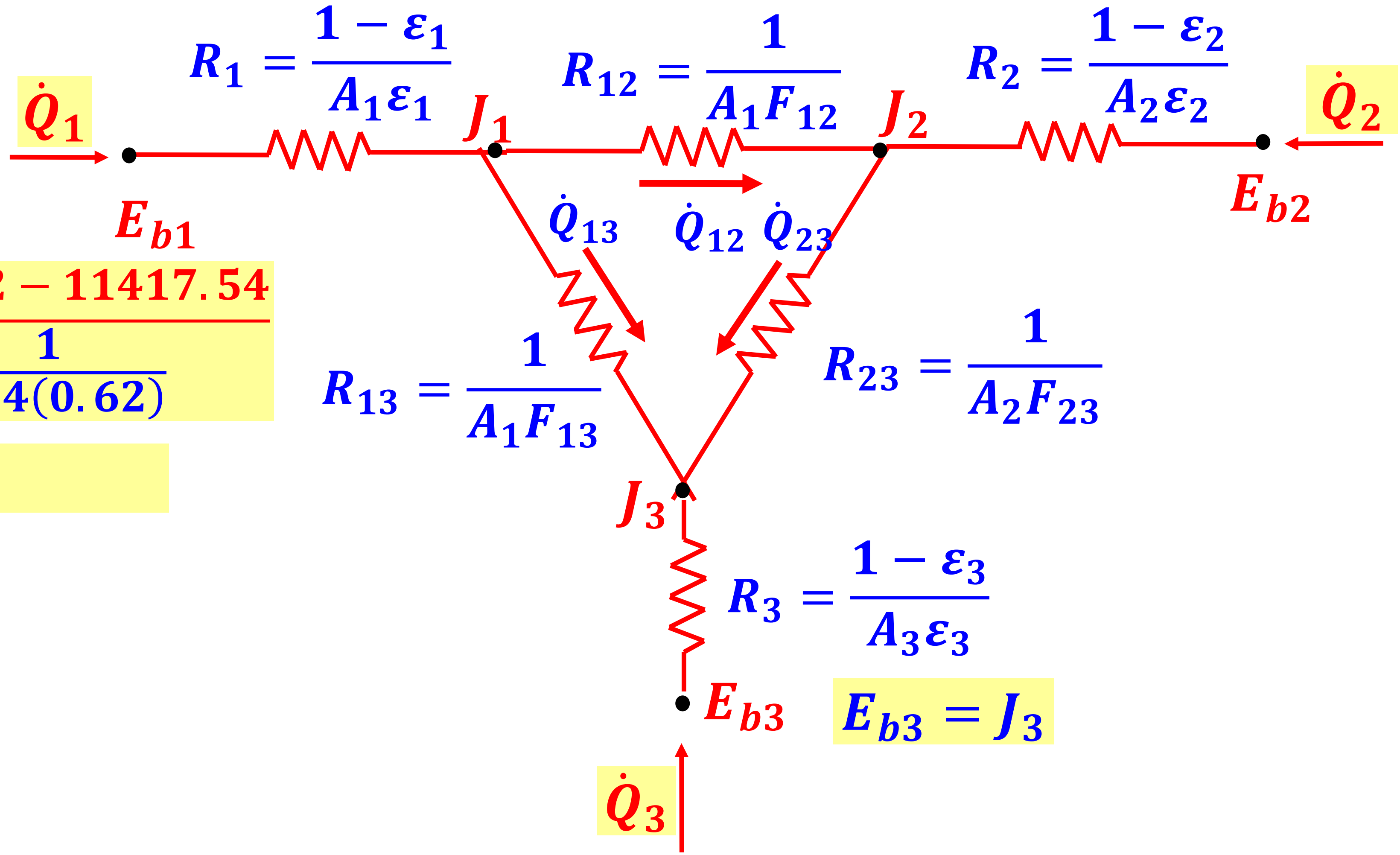
$$A_1 = A_2 = 3.14\ m^2$$

$$A_3 = 6.28\ m^2$$

$$F_{21} = F_{12} = 0.38$$

$$F_{23} = F_{13} = 0.62$$

$$\dot{Q}_3 = \frac{J_3 - J_2}{\frac{1}{A_2 F_{23}}} + \frac{J_3 - J_1}{\frac{1}{A_1 F_{13}}}$$



$$\dot{Q}_3 = \frac{1451.52 - 4560.69}{\frac{1}{3.14(0.62)}} + \frac{1451.52 - 11417.54}{\frac{1}{3.14(0.62)}}$$

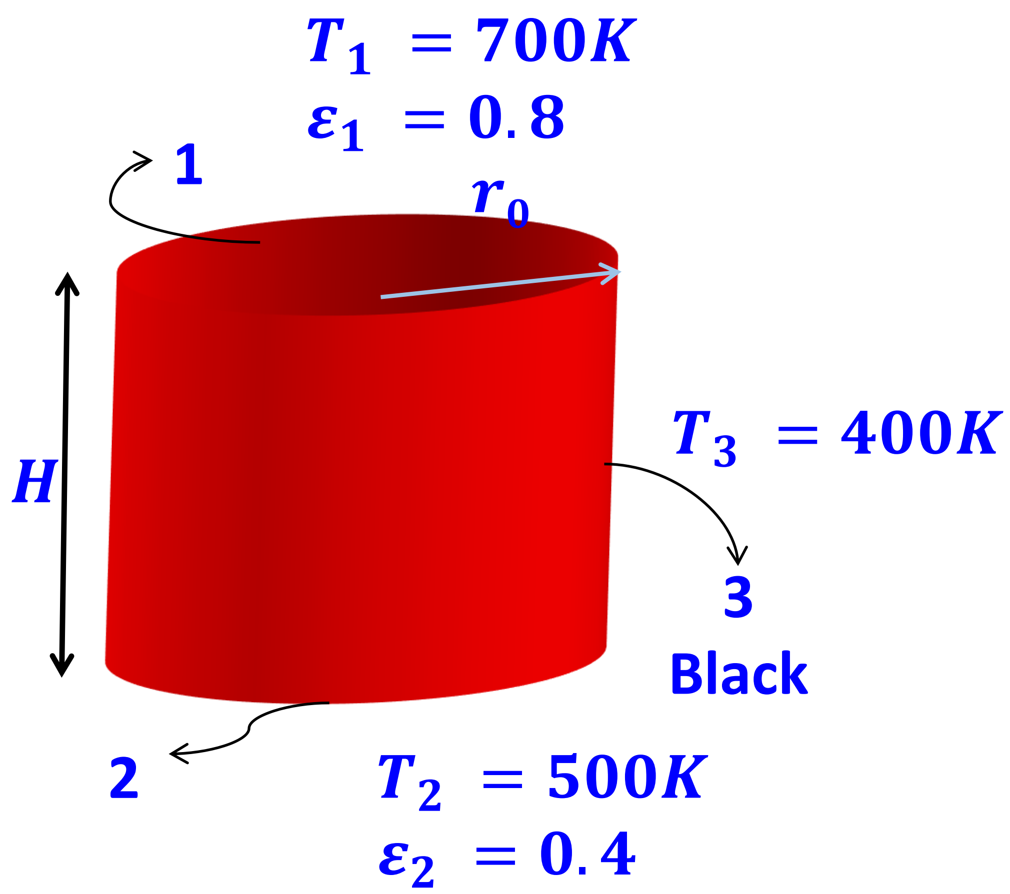
$$\dot{Q}_3 = -6052.93 - 19401.85$$

$$\dot{Q}_3 = -25454.78\ W$$

$$\dot{Q}_1 = 27583.39\ W$$

$$\dot{Q}_2 = -2128.79\ W$$

$$\dot{Q}_3 = -25454.78\ W$$



Note that the direction of the net radiation heat transfer is from the top surface to the base and side surfaces, and the algebraic sum of these three quantities must be equal to zero. That is,

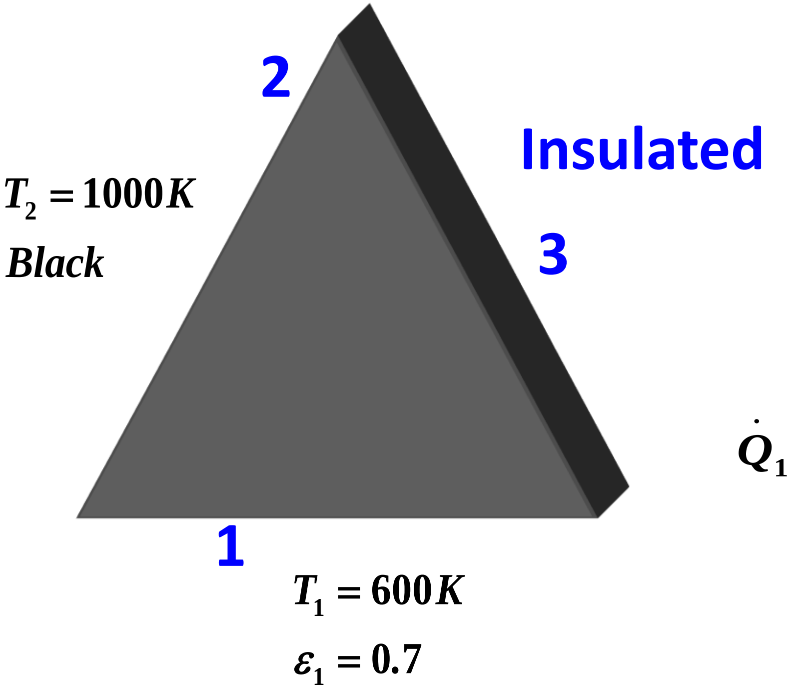
$$\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 27583.39\ W - 2128.79\ W - 25454.78\ W$$

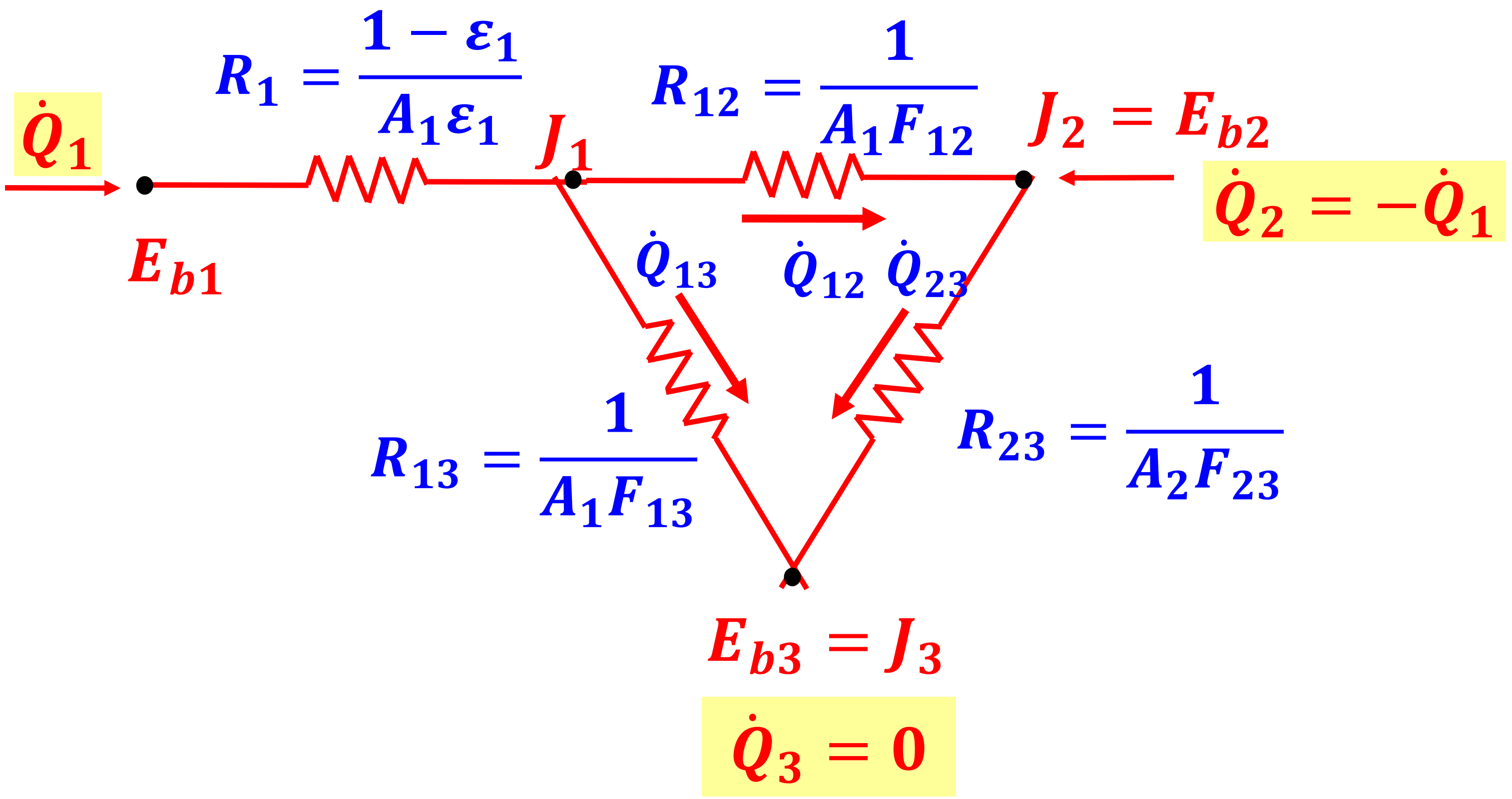
$$\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 0$$

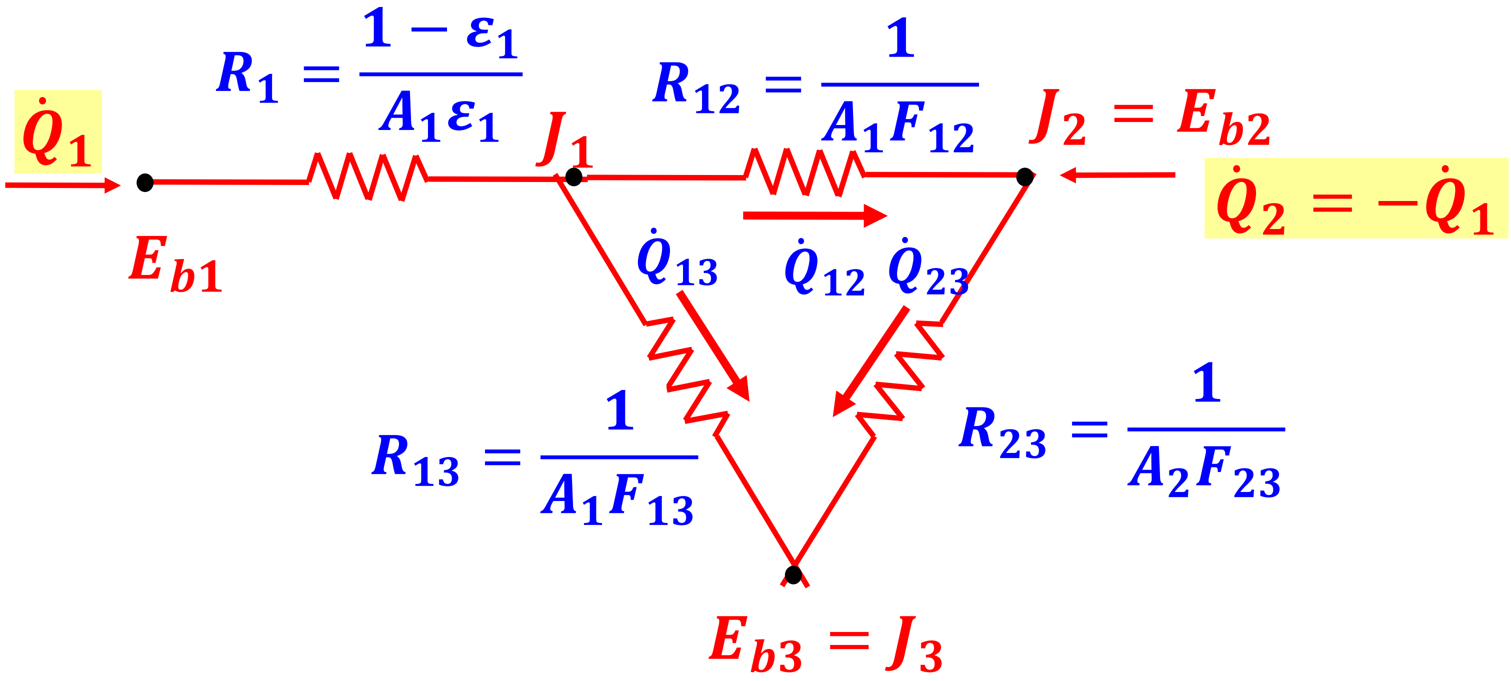
The direct method presented here is straightforward, and it does not require the evaluation of radiation resistances. Also, it can be applied to enclosures with any number of surfaces in the same manner.

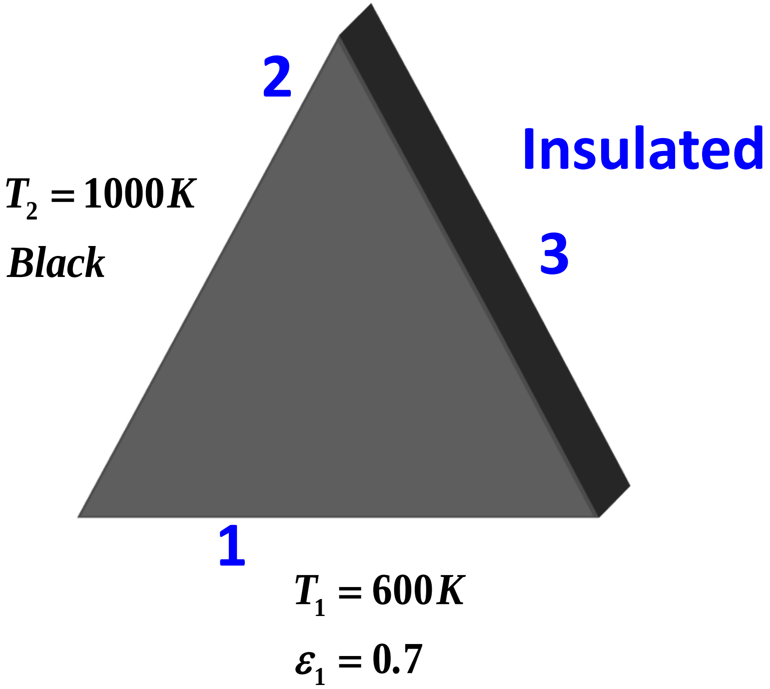
Problem: A furnace is shaped like a long equilateral triangular duct as in Fig. The width of each side is 1 m. The base surface has an emissivity of 0.7 and is maintained at uniform temperature of 600 K. The heated left side surface closely approximates a blackbody at 1000K. The right side surface is well insulated. Determine the rate at which heat must be supplied to the heated side externally per unit length of the duct in order to maintain these operating conditions.

$$A_1 = A_2 = A_3 = 1 \times 1 = 1m^2$$









$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} + \frac{J_2-J_1}{\frac{1}{A_1F_{12}}} + \frac{J_3-J_1}{\frac{1}{A_1F_{13}}} = 0$$

$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} + \frac{E_{b2}-J_1}{\frac{1}{A_1F_{12}}} + \frac{E_{b3}-J_1}{\frac{1}{A_1F_{13}}} = 0$$

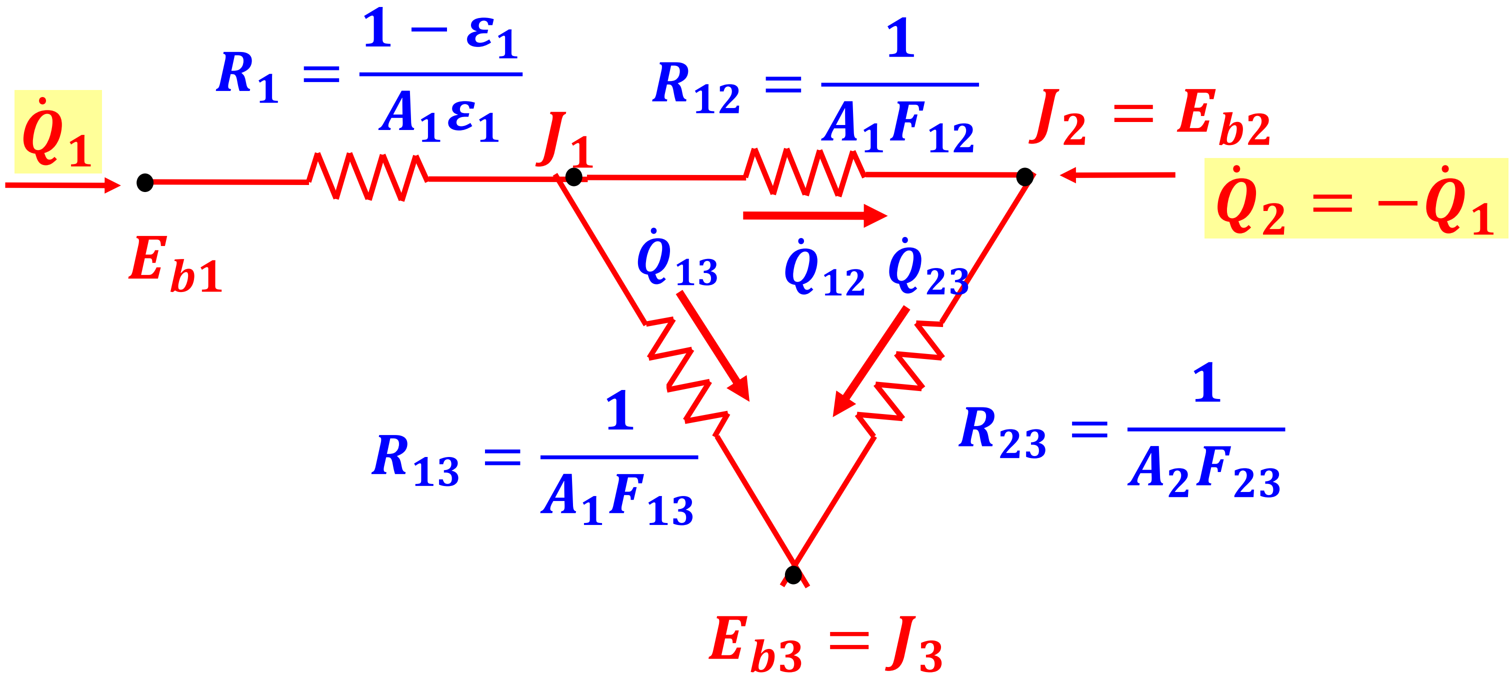
$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}} + \frac{E_{b2}-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}} + \frac{J_3-J_2}{\frac{1}{A_2F_{23}}} = 0$$

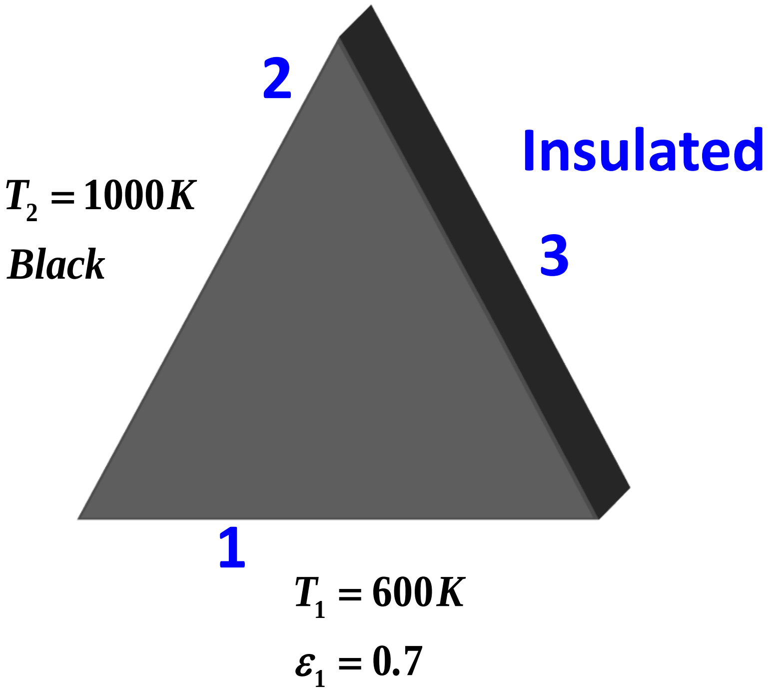
$$\frac{5.67\times10^{-8}(600)^4-J_1}{\frac{1-0.7}{(1)(0.7)}} + \frac{5.67\times10^{-8}(1000)^4-J_1}{\frac{1}{(1)(0.5)}} + \frac{E_{b3}-J_1}{\frac{1}{(1)(0.5)}} = 0$$

$$\frac{J_1-J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2-J_3}{\frac{1}{A_2F_{23}}} + \frac{E_{b3}-J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}} = 0$$

$$17146.08 - 2.3333J_1 + 28350 - 0.5J_1 + 0.5(E_{b3}-J_1) = 0$$

$$45496.08 - 3.3333J_1 + 0.5E_{b3} = 0$$





$$\frac{E_{b1}-J_1}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\dfrac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\dfrac{1}{A_1F_{13}}}=0$$

$$\frac{J_1-J_2}{\dfrac{1}{A_1F_{12}}}+\frac{E_{b2}-J_2}{\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\dfrac{1}{A_2F_{23}}}=0$$

$$\frac{J_1-J_3}{\dfrac{1}{A_1F_{13}}}+\frac{J_2-J_3}{\dfrac{1}{A_2F_{23}}}+\frac{E_{b3}-J_3}{\dfrac{1-\varepsilon_3}{A_3\varepsilon_3}}=0$$

$$\frac{J_1-J_2}{\dfrac{1}{A_1F_{12}}}+\frac{E_{b2}-J_2}{\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{J_3-J_2}{\dfrac{1}{A_2F_{23}}}=0$$

$$\frac{J_1-J_2}{\dfrac{1}{(1)(0.5)}}+\frac{0}{\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}+\frac{E_{b3}-J_1}{\dfrac{1}{(1)(0.5)}}=0$$

$$J_2 = E_{b2}$$

$$(J_1-J_2)(0.5)+(E_{b3}-J_1)(0.5)=0$$

$$J_2 = E_{b3}$$

$$E_{b3} = 5.67\times10^{-8}(1000)^4 \qquad E_{b3} = 56700\ W/m^2$$

$$45496.08 - 3.3333J_1 + 0.5E_{b3} = 0$$

$$E_{b3} = 56700\ W/m^2$$

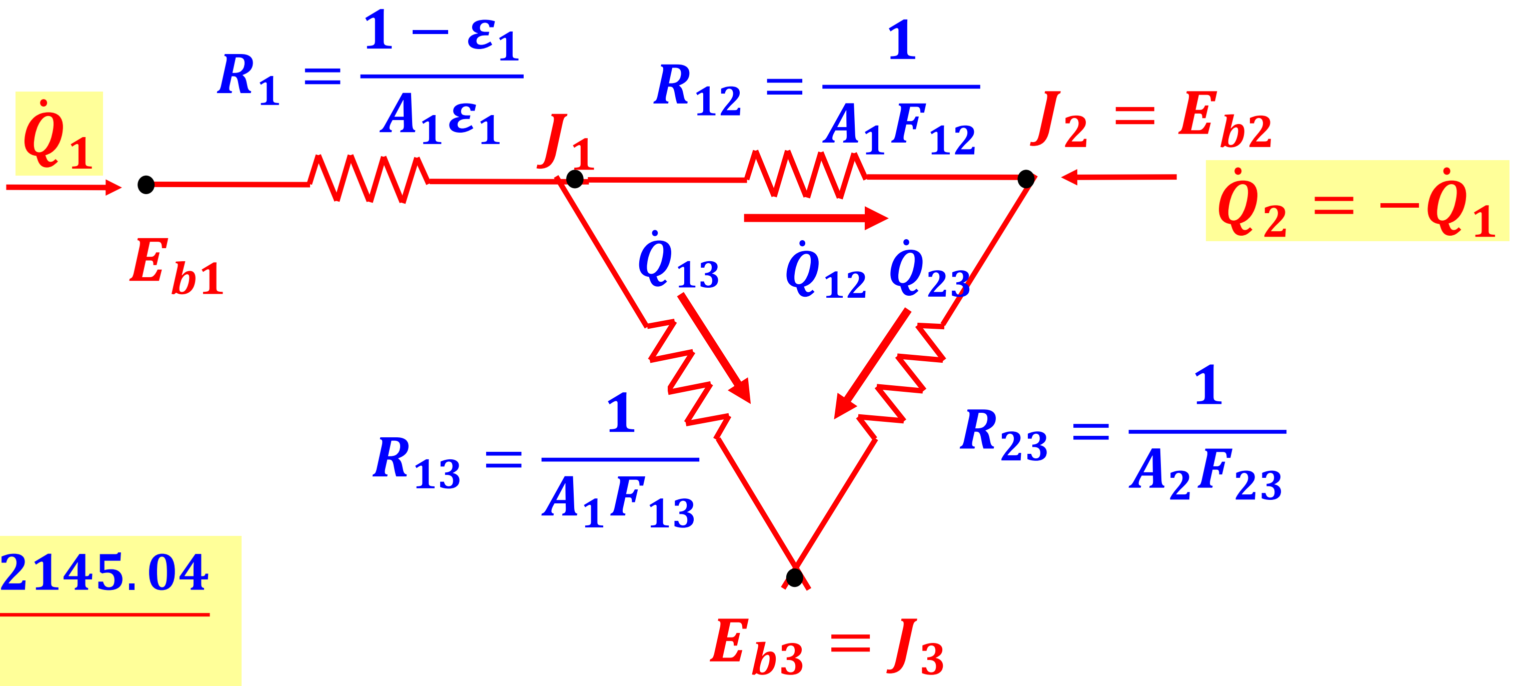
$$45496.08 - 3.3333J_1 + 0.5(56700) = 0$$

$$45496.08 - 3.3333J_1 + 0.5(56700) = 0$$

$$45496.08 - 3.3333J_1 + 28350 = 0$$

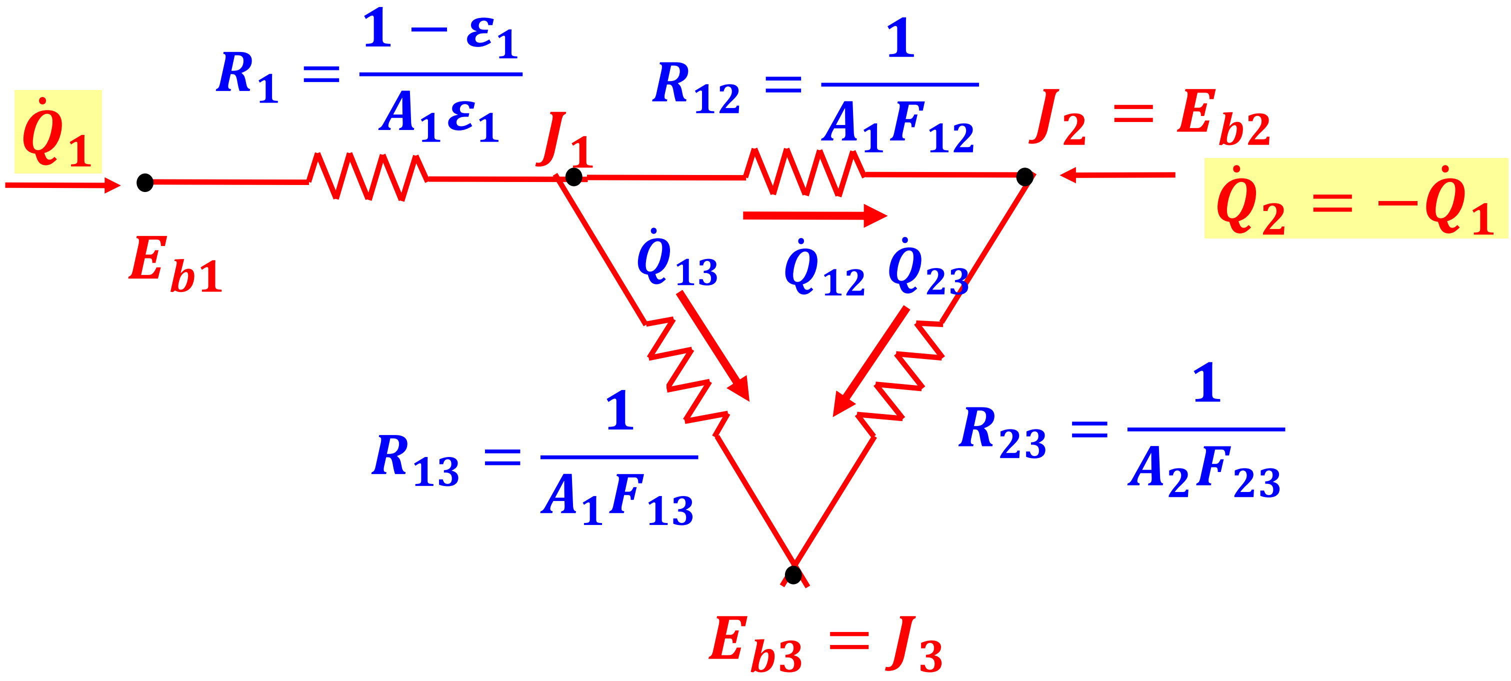
$$3.3333J_1 = 73846.08$$

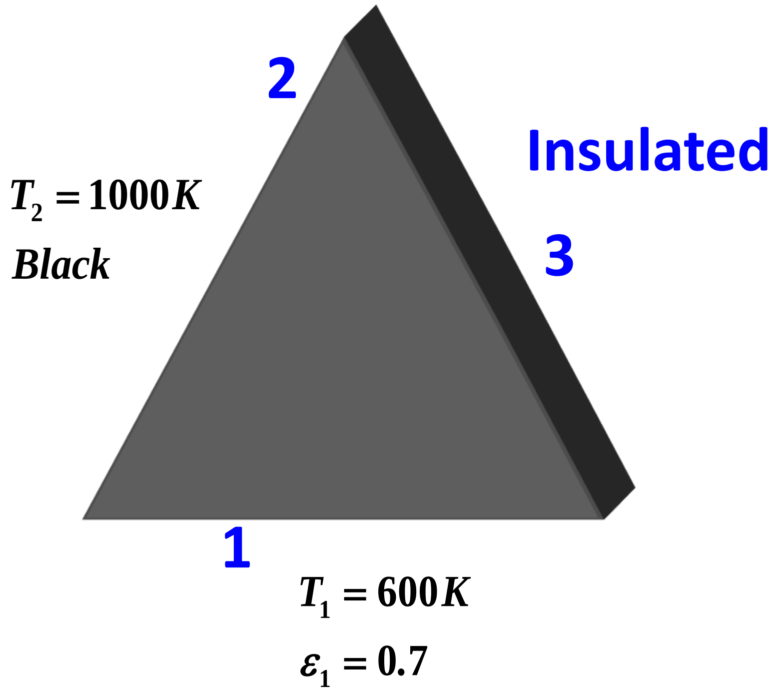
$$J_1 = 22154.05\ W/m^2$$



$$\dot{Q}_1 = \frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} = \frac{5.67\times10^{-8}(600)^4 - 22145.04}{\frac{1-0.7}{(1)(0.7)}}$$

$$\dot{Q}_1 = 34525.7\ W/m^2$$





$$\frac{E_{b1}-J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} + \frac{J_2-J_1}{\frac{1}{A_1F_{12}}} + \frac{J_3-J_1}{\frac{1}{A_1F_{13}}} = 0$$

$$\frac{J_1-J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2-J_3}{\frac{1}{A_2F_{23}}} + \frac{E_{b3}-J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}} = 0$$

$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}} + \frac{E_{b2}-J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}} + \frac{J_3-J_2}{\frac{1}{A_2F_{23}}} = 0$$

$$\frac{J_1-E_{b3}}{\frac{1}{(1)(0.5)}} + \frac{5.67\times10^{-8}(1000)^4 - J_3}{\frac{1}{(1)(0.5)}} + \frac{0}{\frac{1}{(1)(0.5)}} = 0$$

$$\frac{J_1-J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2-J_3}{\frac{1}{A_2F_{23}}} + \frac{E_{b3}-J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}} = 0$$

$$(J_1-E_{b3})(0.5) + (E_{b3}-J_1)(0.5) = 0 \qquad J_2 = E_{b3}$$

$$\frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}} + \frac{E_{b3} - J_3}{\frac{1 - \varepsilon_3}{A_3 \varepsilon_3}} = 0$$

Solution : Two of the surfaces of a long equilateral triangular furnace are maintained at uniform temperatures while the third surface is insulated. The external rate of heat transfer to the heated side per unit length of the duct during steady operation is to be determined.

Assumptions: 1. Steady operating conditions exist. 2. The surfaces are opaque, diffuse and gray 3. Convection heat transfer is not considered

Analysis: The furnace can be considered to be a three surface enclosure with a radiation network shown in the fig, since the duct is very long and thus the end effects are negligible. We observe that the view factor from any surface to any other surface in the enclosure is 0.5 because of symmetry.

Surface 3 is a reradiating surface since the heat transfer at that surface is zero

Then, we must have, the entire heat lost by surface 1 must be gained by surface 2.

$$\dot{Q}_2 = -\dot{Q}_1$$

The radiation network in this case is a simple series-parallel connection, and we can determine $\dot{Q}_1$ directly from .

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(1 \quad 1 \quad 1 \right)^{-1}} = \frac{E_{b1} - E_{b2}}{\left(\quad \right)^{-1}}$$

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13}} + \dfrac{1}{R_{23}}\right)^{-1}} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \left(A_1F_{12} + \dfrac{1}{\dfrac{1}{A_1F_{13}}} + \dfrac{1}{\dfrac{1}{A_2F_{23}}}\right)^{-1}}$$

$$\dot{Q}_1 = \frac{5.67\times 10^{-8}\left(600^4 - 1000^4\right)E_{b1} - E_{b2}}{\dfrac{1-0.7}{1\times 0.7} + \left(1\times 0.5 + \dfrac{1}{\dfrac{1}{1\times 0.5}} + \dfrac{1}{\dfrac{1}{1\times 0.5}}\right)^{-1}}$$

$$\dot{Q}_1 = -28\ kW$$

Therefore, heat at a rate of 28kW must be supplied to the heated surface per unit length of the duct to maintain steady operation in the furnace.

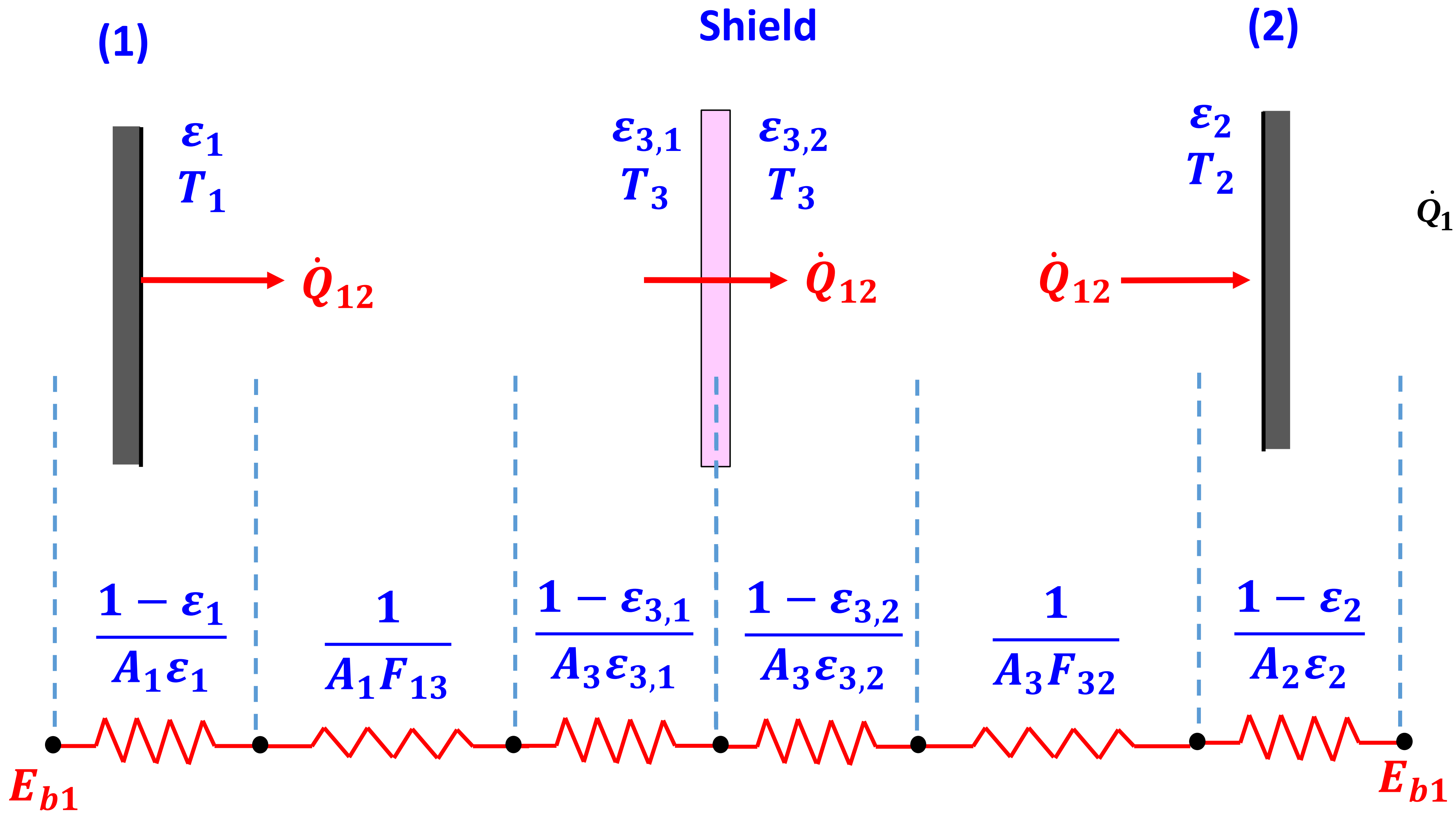
RADIATION SHIELDS AND RADIATION EFFECTS

- **Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high-reflectivity (low emissivity) sheet of material between the two surfaces.**
- **Such highly reflective thin plates or shells are called radiation shields**

- **Multilayer radiation shields constructed of about 20 sheets per cm thickness separatead by evacuated space are commonly used in cyrogenic and space applications**

- **Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself.**

- **THE ROLE OF THE RADIATION SHIELD IS TO REDUCE THE RATE OF RADIATION HEAT TRASNFER BY PLACING ADDITIONAL RESISTANCES IN THE PATH OF RADIATION HEAT FLOW**



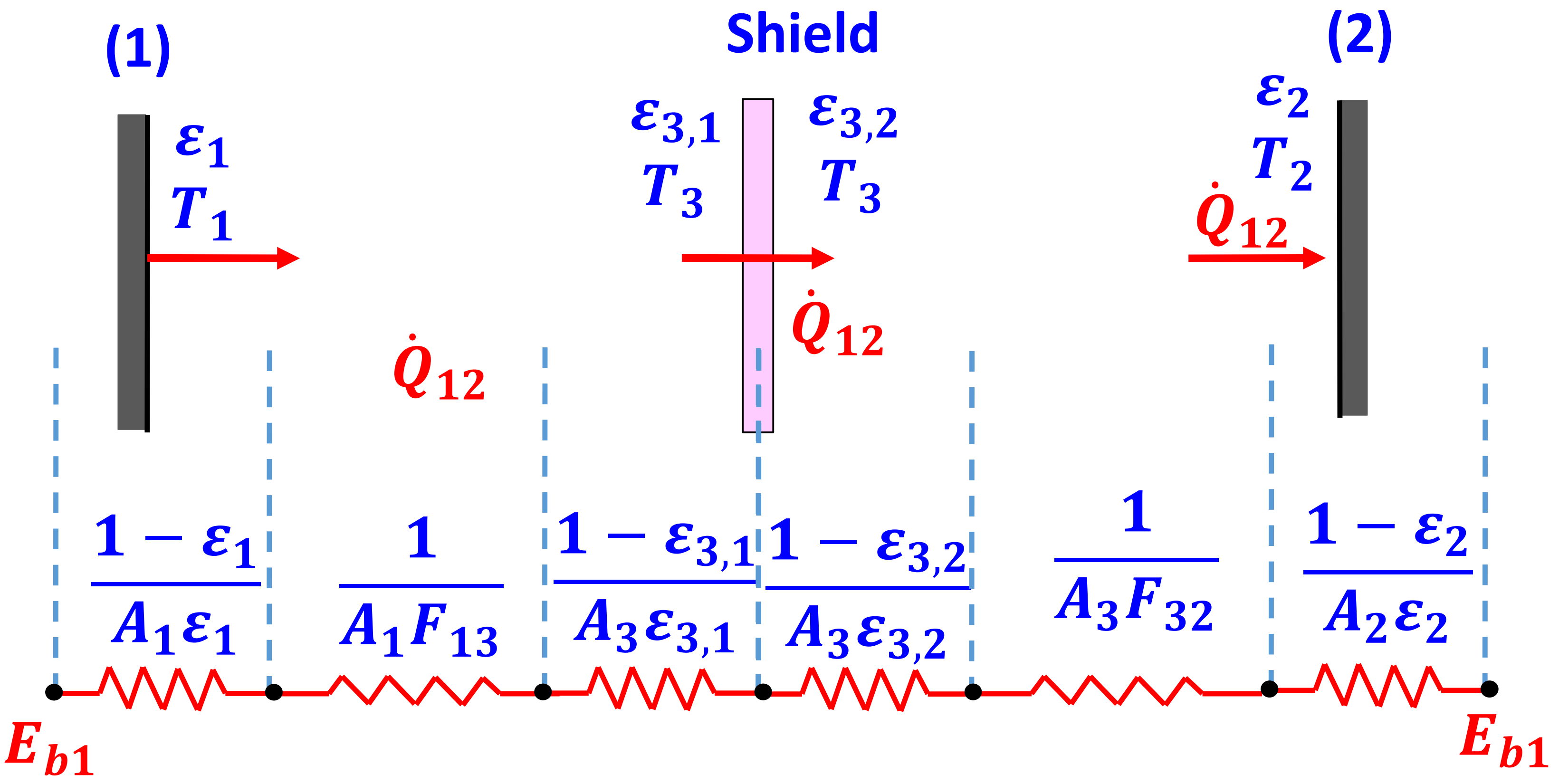
$$\dot{Q}_{12,\ no\ shield} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$\dot{Q}_{12,no\ shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$A_1 = A_2 = A$

$F_{12} = 1$

$$\dot{Q}_{12,no\ shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

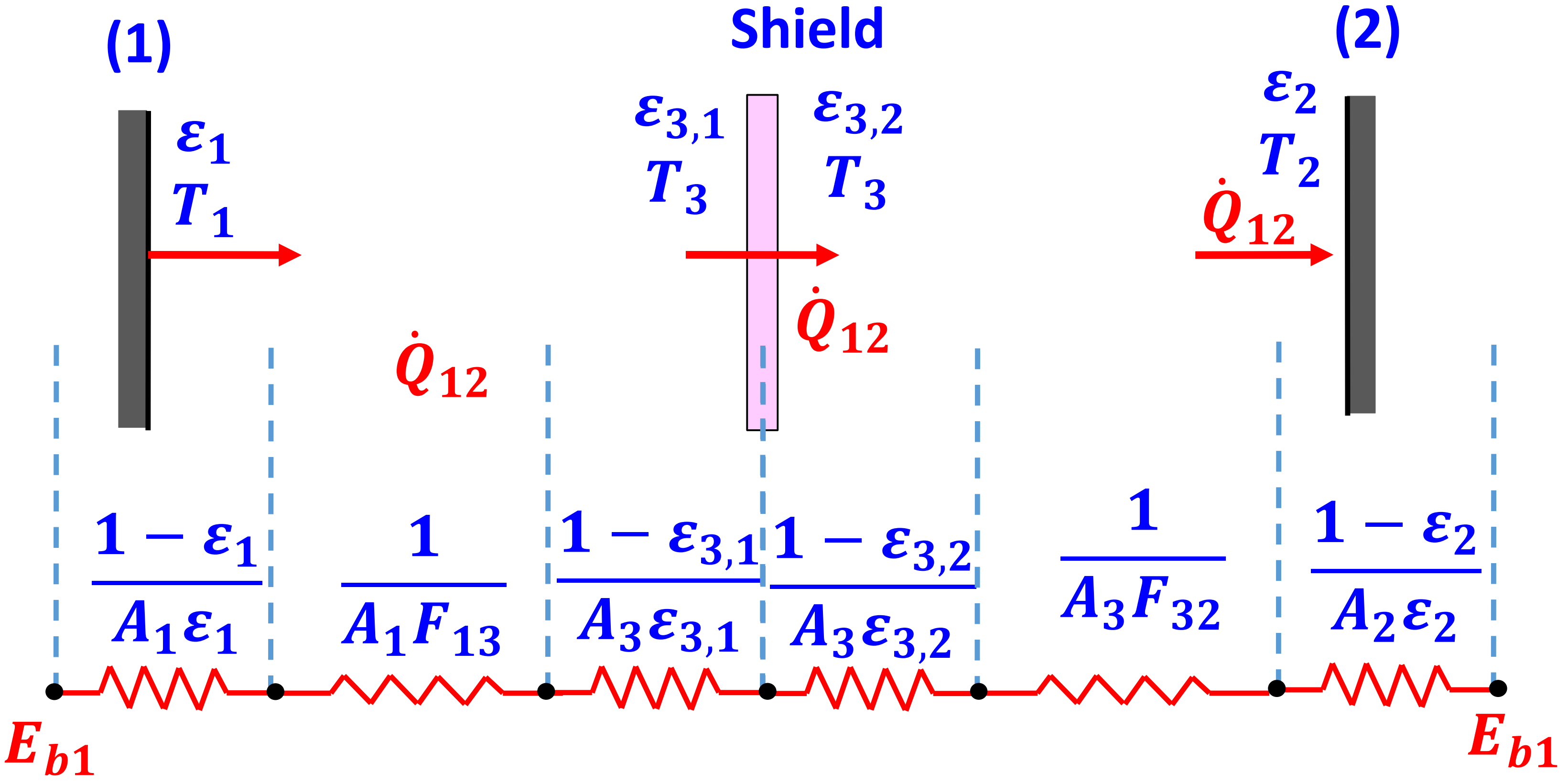


$$A_1 = A_2 = A_3 = A$$
$$F_{12} = F_{32} = 1$$

$$\dot{Q}_{12,shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{13}} + \frac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \frac{1}{A_3F_{32}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\dot{Q}_{12,shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\dot{Q}_{12,no\ shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$



$$\dot{Q}_{12,shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\dot{Q}_{12,no\ shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$\left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)$$

Additional resistance introduced by shield

$$\dot{Q}_{12,shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

N radiation shields

$$\dot{Q}_{12,shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right) + \cdots \dots \dots .. + \left(\frac{1}{\varepsilon_{N,1}} + \frac{1}{\varepsilon_{N,2}} - 1\right)}$$

If all the emissivities are equal

$$\dot{Q}_{12,shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{(N+1)\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{(N+1)} \dot{Q}_{12,\ no\ \ shield}$$

1 shield reduces the rate of radiation heat transfer to one-half, 9 shields reduce it to one-tenth, and 19 shields reduce it to one-twentieth (or 5%) of what it was when there were no radiation shields

The equilibrium temperature of the radiation shield T_3 is determined by expressing the following equation for Q_{13} or Q_{23} (which involves T_3) after evaluating Q_{12} and noting $Q_{12} = Q_{13} = Q_{23}$ when steady state conditions are reached

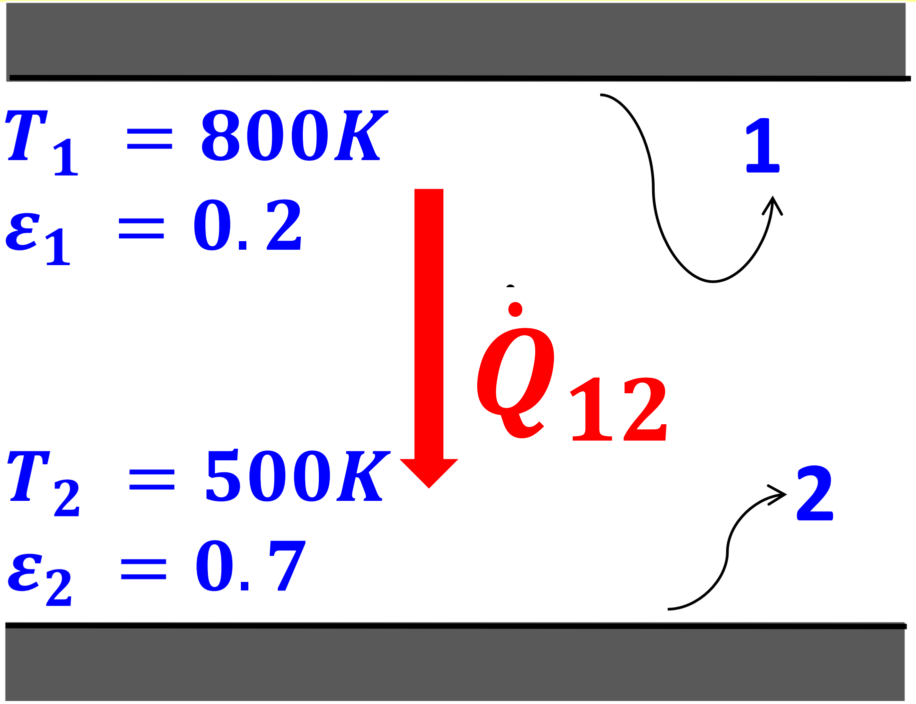
$$\dot{Q}_{12,one\ shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

Radiation shields used to reduce the rate of radiation heat transfer between concentric cylinders and spheres can be handled in a similar manner.

Problem: A thin aluminium sheet with an emissivity of 0.1 on both sides is placed between two very large parallel plates that are maintained at uniform temperatures $T_1 = 800\ K$ and $T_2 = 500\ K$ and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively as in Fig. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and compare the result to that without the shield.

Solution: A thin aluminium sheet is placed between two large parallel plates maintained at uniform temperatures. The net rates of radiation heat transfer between the two plates with and without the radiation shield are to be determined.

Assumptions: The surfaces are opaque, diffuse and gray



$$\dot{Q}_{12,no\ shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$\dot{Q}_{12,no\ shield} = \frac{5.67 \times 10^{-8}\left[(800)^4 - (500)^4\right]}{\frac{1}{0.2} + \frac{1}{0.7} - 1}$$

$$\dot{Q}_{12,no\ shield} = 3625\ \frac{W}{m^2}$$

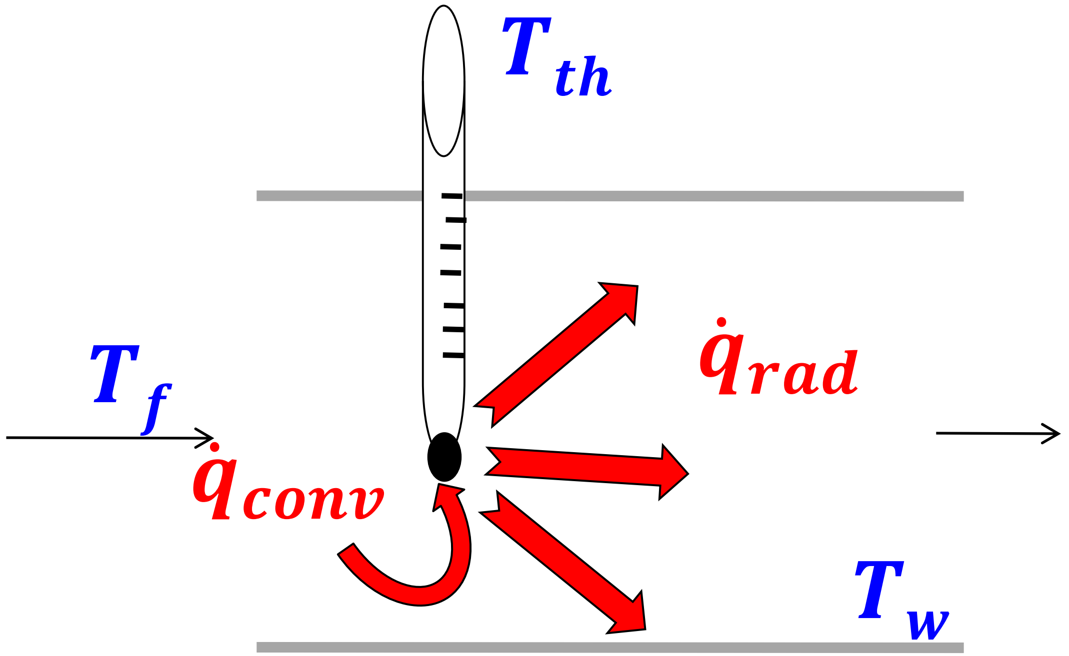
$$\dot{Q}_{12,one\ \ shield} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\dot{q}_{12,one\ \ shield} = \frac{\dot{Q}_{12,one\ \ shield}}{A} = \frac{5.67 \times 10^{-8}\left[(800)^4 - (500)^4\right]}{\left(\frac{1}{0.2} + \frac{1}{0.7} - 1\right) + \left(\frac{1}{0.1} + \frac{1}{0.1} - 1\right)}$$

$$\dot{q}_{12,one\ \ shield} = 806\,\frac{W}{m^2}$$

Discussion: Radiation heat transfer reduces to about one fourth of what it was as a result of placing a radiation shield between the two parallel plates

RADIATION EFFECT IN TEMPERATURE MEASUREMENTS



$$\dot{q}_{conv,to\ sensor} = \dot{q}_{rad\ from\ sensor}$$

$$h(T_f - T_{th}) = \varepsilon_{th}\,\sigma\left(T_{th}^4 - T_w^4\right)$$

$$T_f = T_{th} + \frac{\varepsilon_{th}\,\sigma\left(T_{th}^4 - T_w^4\right)}{h}$$

Radiation effect Correction term $\dfrac{\varepsilon_{th}\,\sigma\left(T_{th}^4 - T_w^4\right)}{h}$

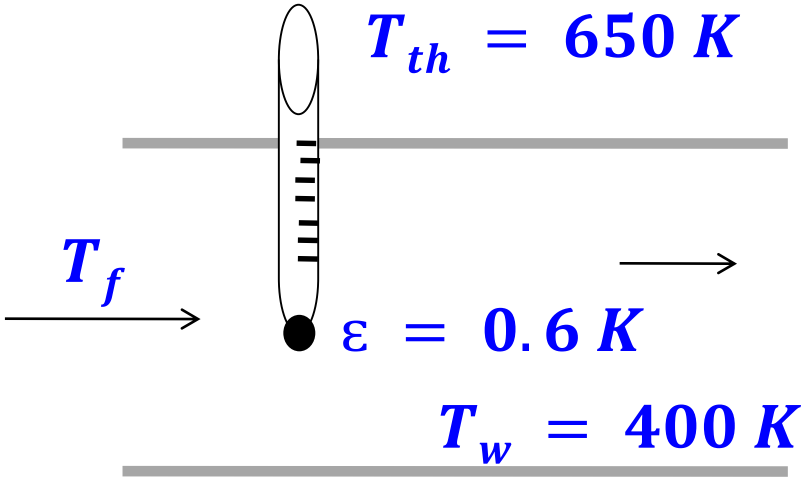
The radiation correction term is most significant when the convection heat transfer coefficient is small and the emissivity of the surface of the sensor is large

Sensor should be coated with a material of high reflectivity (low emissivity) to reduce the radiation effect

Placing the sensor in a radiation shield without interfering with the fluid flow also reduces the radiation effect

Sensors of temperature measurement devices used outdoors must be protected from direct sunlight since the radiation effect in that case is sure to reach unacceptable levels

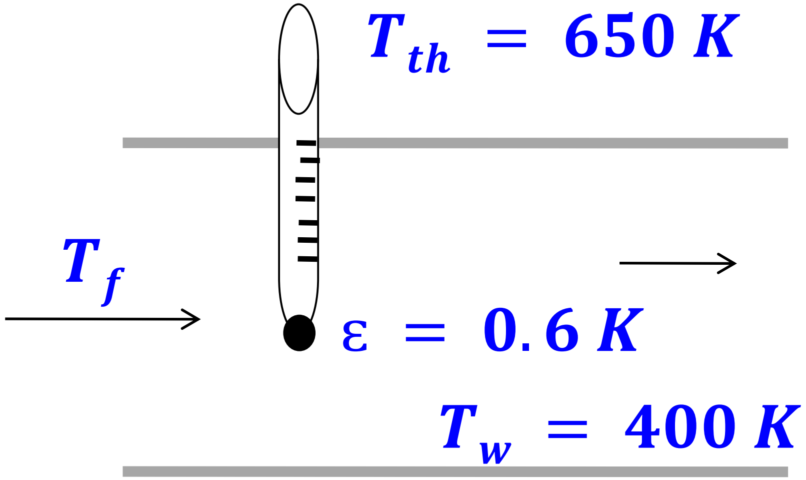
Problem: A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at 400 K shows a temperature reading of 650 K. Assuming the emissivity of the thermocouple junction to be 0.6 and the convection heat transfer coefficient as 80 W/m².°C, determine the actual temperature of the air.



Solution: The temperature of air in a duct is measured. The radiation effect on the temperature measurment is to be quantified, and the actual air temperature is to be determined.

Assumptions: The surfaces are opaque, diffuse and gray

Analysis: The walls of the duct are at a considerably lower temperature than the air in it, and thus we expect the thermocouple to show a reading lower than the actual air temperature as a result of the radiation effect. The actual air temperature is determined from



$$T_f = T_{th} + \frac{\varepsilon_{th}\,\sigma\left(T_{th}^4 - T_w^4\right)}{h}$$

$$T_f = 650 + \frac{0.6 \times 5.67 \times 10^{-8}\left[(650)^4 - (400)^4\right]}{80}$$

$$T_f = 715\ K$$

Note that the radiation effect causes a difference of 65°C (65 K) in temperature reading in this case