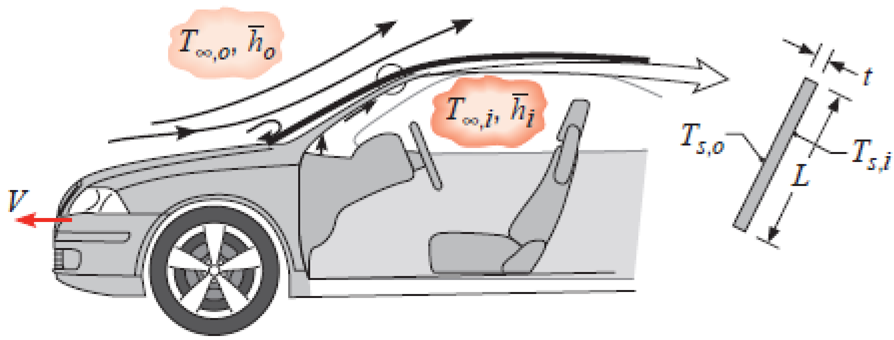
ME 346 – Heat Transfer: Tutorial 3

Q1: The defroster of an automobile functions by discharging warm air on the inner surface of the windshield. To prevent condensation of water vapor on the surface, the temperature of the air and the surface convection coefficient ($T_{\infty,i}$, $\bar{h}_i$) must be large enough to maintain a surface temperature $T_{s,i}$ that is at least as high as the dewpoint ($T_{s,i} \geq T_{dp}$). Consider a windshield of length L =800 mm and thickness t= 6 mm and driving conditions for which the vehicle moves at a velocity of V = 70 mph in ambient air at $T_{\infty,o}$= -15 °C. From laboratory experiments performed on a model of the vehicle, the average convection coefficient on the outer surface of the windshield is known to be correlated by an expression of the form $\overline{Nu}_L = 0.030\, Re_L^{0.8} Pr^{1/3}$, where $Re_L = VL/\nu$. Air properties may be approximated as k = 0.023 W/m K, ν=12.5 x 10^{-6} m^2/s, and Pr = 0.71. If T_{dp} = 10°C and $T_{\infty,i}$ =50°C, what is the smallest value of $\bar{h}_i$ required to prevent condensation on the inner surface?

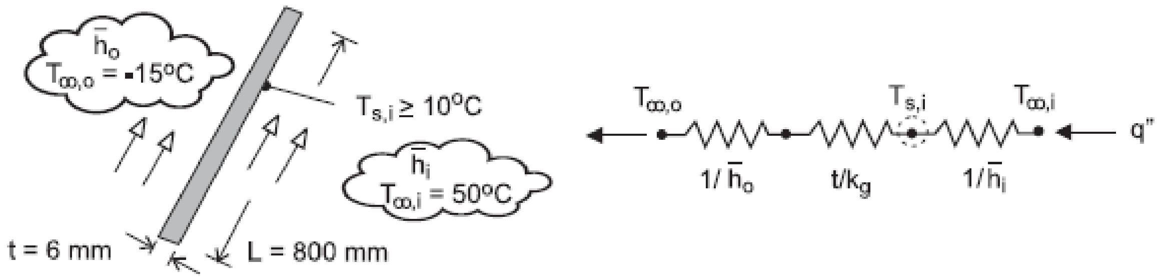


Sol:

KNOWN: Ambient, interior and dewpoint temperatures. Vehicle speed and dimensions of windshield. Heat transfer correlation for external flow.

FIND: Minimum value of convection coefficient needed to prevent condensation on interior surface of windshield.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

PROPERTIES: *Table A-3*, glass: $k_g = 1.4$ W/m·K. Prescribed, air: $k = 0.023$ W/m·K, $\nu = 12.5 \times 10^{-6}$ m^2/s, Pr = 0.70.

ANALYSIS: From the prescribed thermal circuit, conservation of energy yields

$$\frac{T_{\infty,i} - T_{s,i}}{1/\bar{h}_i} = \frac{T_{s,i} - T_{\infty,o}}{t/k_g + 1/\bar{h}_o}$$

where $\bar{h}_o$ may be obtained from the correlation

$$\overline{Nu}_L = \frac{\bar{h}_o L}{k} = 0.030 Re_L^{0.8} Pr^{1/3}$$

With V = (70 mph × 1585 m/mile)/3600 s/h = 30.8 m/s, Re_D = (30.8 m/s × 0.800 m)/12.5 × 10^{-6} m^2/s = 1.97 × 10^6 and

$$\bar{h}_o = \frac{0.023\,\mathrm{W/m \cdot K}}{0.800\ \mathrm{m}} 0.030\left(1.97 \times 10^6\right)^{0.8} (0.70)^{1/3} = 83.1\,\mathrm{W/m^2 \cdot K}$$

From the energy balance, with $T_{s,i} = T_{dp} = 10°C$

$$\bar{h}_i = \frac{\left(T_{s,i} - T_{\infty,o}\right)}{\left(T_{\infty,i} - T_{s,i}\right)}\left(\frac{t}{k_g} + \frac{1}{\bar{h}_o}\right)^{-1}$$

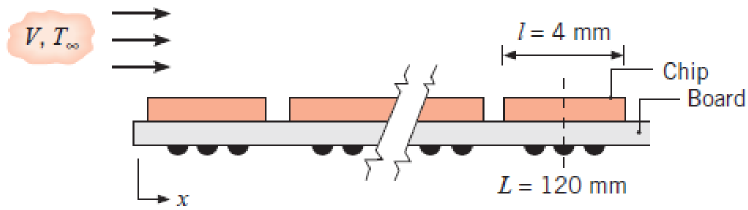
$$\bar{h}_i = \frac{(10+15)°C}{(50-10)°C}\left(\frac{0.006\ \mathrm{m}}{1.4\,\mathrm{W/m \cdot K}} + \frac{1}{83.1\,\mathrm{W/m^2 \cdot K}}\right)^{-1}$$

$$\bar{h}_i = 38.3\,\mathrm{W/m^2 \cdot K}$$ <

COMMENTS: The output of the fan in the automobile's heater/defroster system must maintain a velocity for flow over the inner surface that is large enough to provide the foregoing value of $\bar{h}_i$. In addition, the output of the heater must be sufficient to maintain the prescribed value of $T_{\infty,i}$ at this velocity.

Q2: Forced air at T_∞ = 25°C and V = 10 m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm x 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.85} Pr^{1/3}$$



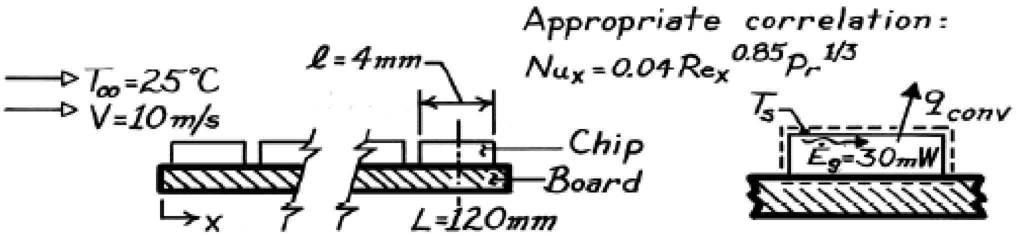
Estimate the surface temperature of the chip if it is dissipating 30 mW.

Sol:

KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L, (5) Negligible radiation.

PROPERTIES: *Table A-4*, Air (assume $T_s = 45°C$, $T_f = (45 + 25)/2 = 35°C = 308K$, 1atm): $\nu = 16.69 \times 10^{-6}\ m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip (see above),

$$q_{conv} = \dot{E}_g = 30W. \qquad (1)$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \bar{h}\, A_{chip} \qquad (2)$$

where $A_{chip} = \ell^2$. Assume that the *average* heat transfer coefficient $(\bar{h})$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L. That is, $\bar{h}_{chip} \approx h_x(L)$ where the local coefficient can be evaluated from the special correlation for this situation,

$$Nu_x = \frac{h_x x}{k} = 0.04\left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

and substituting numerical values with x = L, find

$$h_x = 0.04\frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

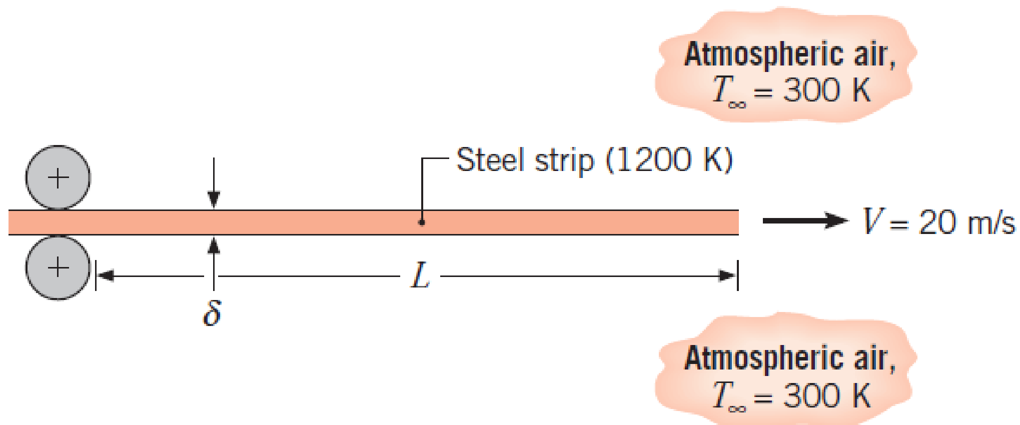
$$h_x = 0.04\left[\frac{0.0269\ W/m\cdot K}{0.120\ m}\right]\left[\frac{10\ m/s \times 0.120\ m}{16.69\times10^{-6}\ m^2/s}\right]^{0.85}(0.703)^{1/3} = 107\ W/m^2\cdot K.$$

The surface temperature of the chip is from Eq. (2),

$$T_s = 25°C + 30\times10^{-3}\ W/\left[107\ W/m^2\cdot K\times(0.004m)^2\right] = 42.5°C. \qquad <$$

COMMENTS: (1) Note that the estimated value for T_f used to evaluate the air properties was reasonable. (2) Alternatively, we could have evaluated $\bar{h}_{chip}$ by performing the integration of the local value, h(x).

Q3: A steel strip emerges from the hot roll section of a steel mill at a speed of 20 m/s and a temperature of 1200 K. Its length and thickness are L = 100 m and δ= 0.003 m, respectively, and its density and specific heat are 7900 kg/m^3 and 640 J/kg K, respectively.



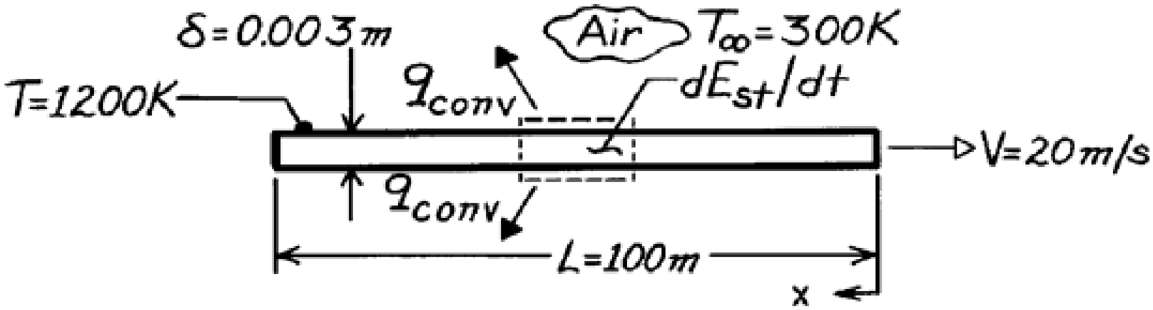
Accounting for heat transfer from the top and bottom surfaces and neglecting radiation and strip conduction effects, determine the time rate of change of the strip temperature at a distance of 1 m from the leading edge and at the trailing edge. Determine the distance from the leading edge at which the minimum cooling rate is achieved.

Sol:

KNOWN: Length, thickness, speed and temperature of steel strip.

FIND: Rate of change of strip temperature 1 m from leading edge and at trailing edge. Location of minimum cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is 5×10^5.

PROPERTIES: Steel (given): $\rho = 7900\ \text{kg/m}^3$, $c_p = 640\ \text{J/kg·K}$. *Table A-4*, Air $\left(\bar{T} = 750\text{K, 1 atm}\right)$: $\nu = 76.4 \times 10^{-6}\ \text{m}^2/\text{s}$, $k = 0.0549\ \text{W/m·K}$, $Pr = 0.702$.

ANALYSIS: Performing an energy balance for a control mass of unit surface area A_s riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s\left(T - T_\infty\right) = \rho\delta A_s c_p\left(dT/dt\right)$$

$$dT/dt = \frac{-2h_x\left(T - T_\infty\right)}{\rho\delta c_p} = -\frac{2(900\text{K})h_x}{7900\ \text{kg/m}^3\,(0.003\ \text{m})640\ \text{J/kg}\cdot\text{K}} = -0.119h_x\,(\text{K/s}).$$

At $x = 1$ m, $\quad Re_x = \dfrac{Vx}{\nu} = \dfrac{20\ \text{m/s}(1\text{m})}{76.4\times10^{-6}\ \text{m}^2/\text{s}} = 2.62\times10^5 < Re_{x,c}$. Hence,

$$h_x = (k/x)0.332Re_x^{1/2}Pr^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{1\ \text{m}}(0.332)\left(2.62\times10^5\right)^{1/2}(0.702)^{1/3} = 8.29\text{W/m}^2\cdot\text{K}$$

and at x = 1m, $dT/dt = -0.987$ K/s. <

At the trailing edge, $Re_x = 2.62\times10^7 > Re_{x,c}$. Hence

$$h_x = (k/x)0.0296Re_x^{4/5}Pr^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{100\ \text{m}}(0.0296)\left(2.62\times10^7\right)^{4/5}(0.702)^{1/3} = 12.4\text{W/m}^2\cdot\text{K}$$

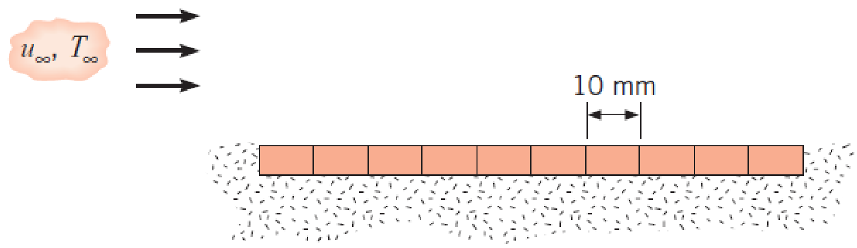
and at x = 100 m, $dT/dt = -1.47$ K/s. <

The minimum cooling rate occurs just before transition; hence, for $Re_{x,c} = 5\times10^5$

$$x_c = 5\times10^5(\nu/V) = \frac{5\times10^5\times76.4\times10^{-6}\text{m}^2/\text{s}}{20\ \text{m/s}} = 1.91\ \text{m}$$ <

COMMENTS: The cooling rates are very low and would remain low even if radiation were considered. For this reason, hot strip metals are quenched by water and not by air.

Q4: An array of 10 silicon chips, each of length L = 10 mm on a side, is insulated on one surface and cooled on the opposite surface by atmospheric air in parallel flow with T_∞ = 24°C and u_∞ = 40 m/s. When in use, the same electrical power is dissipated in each chip, maintaining a uniform heat flux over the entire cooled surface.



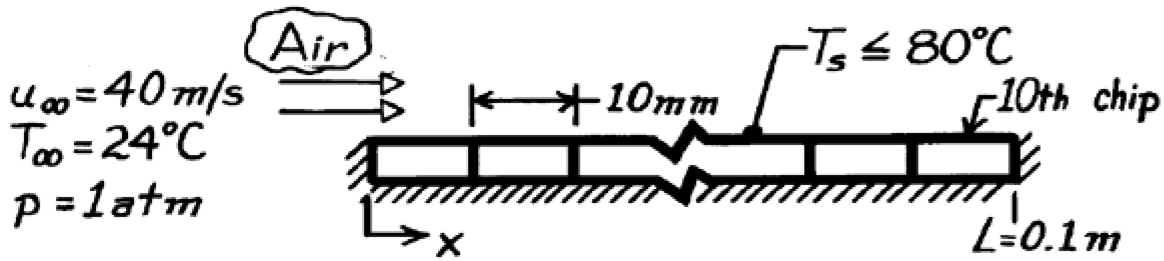
If the temperature of each chip may not exceed 80°C, what is the maximum allowable power per chip? What is the maximum allowable power if a turbulence promoter is used to trip the boundary layer at the leading edge? Would it be preferable to orient the array normal, instead of parallel, to the airflow?

Sol:

KNOWN: Surface dimensions for an array of 10 silicon chips. Maximum allowable chip temperature. Air flow conditions.

FIND: Maximum allowable chip electrical power (a) without and (b) with a turbulence promoter at the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Film temperature of 52°C, (3) Negligible radiation, (4) Negligible heat loss through insulation, (5) Uniform heat flux at chip interface with air, (6) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: $Re_L = u_\infty L/\nu = 40\text{ m/s}\times 0.1\text{ m}/18.4\times10^{-6}\text{ m}^2/\text{s} = 2.174\times10^5$. Hence, flow is laminar over all chips without the promoter.

(a) For *laminar flow*, the minimum h_x exists on the last chip. Approximating the average coefficient for Chip 10 as the local coefficient at x = 95 mm, $\overline{h}_{10} = h_{x=0.095m}$.

$$\overline{h}_{10} = 0.453\frac{k}{x}Re_x^{1/2}Pr^{1/3}$$

$$Re_x = \frac{u_\infty x}{\nu} = \frac{40\text{ m/s}\times 0.095\text{ m}}{18.4\times10^{-6}\text{ m}^2/\text{s}} = 2.065\times10^5$$

$$\overline{h}_{10} = 0.453\frac{0.0282\text{ W/m}\cdot\text{K}}{0.095}\left(2.065\times10^5\right)^{1/2}(0.703)^{1/3} = 54.3\text{W/m}^2\cdot\text{K}$$

$$q_{10} = \overline{h}_{10}A\left(T_s - T_\infty\right) = 54.3\frac{\text{W}}{\text{m}^2\cdot\text{K}}(0.01\text{ m})^2(80-24)^\circ\text{C} = 0.30\text{ W.}$$

Hence, if all chips are to dissipate the same power and T_s is not to exceed 80°C.

$$q_{max} = 0.30\text{ W.}$$ <

(b) For *turbulent flow*,

$$\overline{h}_{10} = 0.0308\frac{k}{x}Re_x^{4/5}Pr^{1/3} = 0.0308\frac{0.0282\text{W/m}\cdot\text{K}}{0.095\text{ m}}\left(2.065\times10^5\right)^{4/5}(0.703)^{1/3} = 145\text{W/m}^2\cdot\text{K}$$

$$q_{10} = \overline{h}_{10}A\left(T_s - T_\infty\right) = 145\frac{\text{W}}{\text{m}^2\cdot\text{K}}(0.01\text{ m})^2(80-24)^\circ\text{C} = 0.81\text{ W.}$$

Hence, $q_{max} = 0.81$ W. <

COMMENTS: It is far better to orient array normal to the air flow. Since $\overline{h}_1 > \overline{h}_{10}$, more heat could be dissipated per chip, and the same heat could be dissipated from each chip.

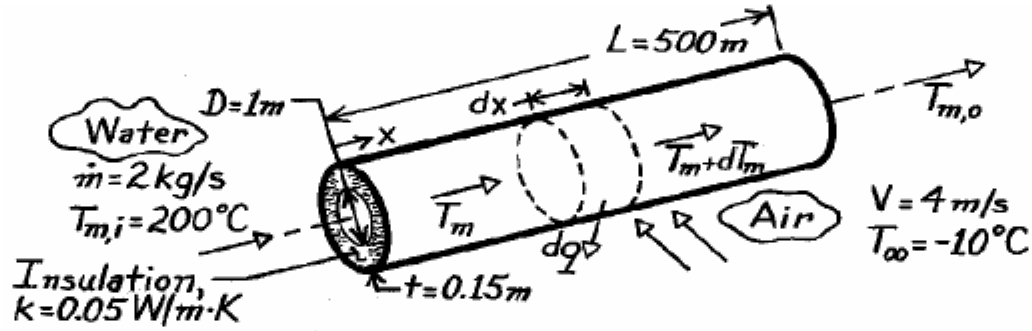
Q5 : Pressurized water at $T_{m,i}$ = 200°C is pumped at m = 2 kg/s from a power plant to a nearby industrial user through a thin-walled, round pipe of inside dia-meter D = 1 m. The pipe is covered with a layer of insulation of thickness t = 0.15 m and thermal con-ductivity k = 0.05 W/m K. The pipe, which is of length L = 500 m, is exposed to a cross flow of air at T = - 10°C and V = 4 m/s. Obtain a differential equation that could be used to solve for the variation of the mixed mean temperature of the water $T_m(x)$ with the axial coordinate. As a first approximation, the internal flow may be assumed to be fully developed throughout the pipe. Express your results in terms of m , V, T_m, D, t, k, and appropriate water (w) and air (a) properties. Evaluate the heat loss per unit length of the pipe at the inlet. What is the mean temperature of the water at the outlet?

Sol:

KNOWN: Flow conditions associated with water passing through a pipe and air flowing over the pipe.

FIND: (a) Differential equation which determines the variation of the mixed-mean temperature of the water, (b) Heat transfer per unit length of pipe at the inlet and outlet temperature of the water.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature drop across the pipe wall, (2) Negligible radiation exchange between outer surface of insulation and surroundings, (3) Fully developed flow throughout pipe, (4) Water is incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A-6*, Water ($T_{m,i}$ = 200°C): $c_{p,w}$ = 4500 J/kg·K, $\mu_w = 134 \times 10^{-6}$ N·s/m^2, k_w = 0.665 W/m·K, Pr_w = 0.91; *Table A-4*, Air (T_∞ = -10°C): $\nu_a = 12.6 \times 10^{-6}$ m^2/s, k_a = 0.023 W/m·K, Pr_a = 0.71, $Pr_s \approx 0.7$.

ANALYSIS: (a) Following the development of Section 8.3.1 and applying Eq. 1.12e to a differential element in the water, we obtain

$$dq = -\dot{m}\, c_{p,w}\, dT_m$$

where $$dq = U_i dA_i \left(T_m - T_\infty\right) = U_i \pi\, D\, dx \left(T_m - T_\infty\right).$$

Substituting into the energy balance, it follows that

$$\frac{d\,T_m}{dx} = -\frac{U_i \pi\, D}{\dot{m}\, c_p}\left(T_m - T_\infty\right). \qquad (1)$$ <

The overall heat transfer coefficient based on the inside surface area may be evaluated from Eq. 3.36 which, for the present conditions, reduces to

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{D}{2k}\ell n\left(\dfrac{D+2t}{D}\right) + \dfrac{D}{D+2t}\,\dfrac{1}{h_o}}. \qquad (2)$$

For the *inner water flow*, Eq. 8.6 gives

$$Re_D = \frac{4\,\dot{m}}{\pi\, D \mu_w} = \frac{4 \times 2\ \text{kg/s}}{\pi(1\ \text{m}) \times 134 \times 10^{-6}\ \text{kg/s}\cdot\text{m}} = 19,004.$$

Continued …

Hence, the flow is turbulent. With the assumption of fully developed conditions, it follows from Eq. 8.60 that

$$h_i = \frac{k_w}{D} \times 0.023\, Re_D^{4/5}\, Pr_w^{0.3}. \tag{3}$$

For the *external air flow*

$$Re_D = \frac{V(D+2t)}{\nu} = \frac{4\ m/s(1.3m)}{12.6\times10^{-6}\ m^2/s} = 4.13\times10^5.$$

Using Eq. 7.53 to obtain the outside convection coefficient,

$$h_o = \frac{k_a}{(D+2t)} \times 0.076\, Re_D^{0.7}\, Pr_a^{0.37}\, \left(Pr_a/Pr_s\right)^{1/4}. \tag{4}$$

(b) The heat transfer per unit length of pipe at the inlet is

$$q' = \pi\, D\, U_i \left(T_{m,i} - T_\infty\right). \tag{5}$$

From Eqs. (3 and 4),

$$h_i = \frac{0.665\ W/m\cdot K}{1\ m} \times 0.023(19{,}004)^{4/5}\ (0.91)^{0.3} = 39.4\ W/m^2\cdot K$$

$$h_o = \frac{0.023\ W/m\cdot K}{(1.3\ m)} \times 0.076\left(4.13\times10^5\right)^{0.7} (0.71)^{0.37}\, (1)^{1/4} = 10.1\ W/m^2\cdot K.$$

Hence, from Eq. (2)

$$U_i = \left[\frac{1}{39.4\ W/m^2\cdot K} + \frac{1\ m}{0.1\ W/m\cdot K}\ell n\left(\frac{1.3}{1}\right) + \frac{1}{1.3}\times\frac{1}{10.1\ W/m^2\cdot K}\right]^{-1} = 0.37\ W/m^2\cdot K$$

and from Eq. (5)

$$q' = \pi(1\ m)\ \left(0.37\ W/m^2\cdot K\right)\ (200+10)^\circ C = 244\ W/m. \qquad <$$

Since U_i is a constant, independent of x, Eq. (1) may be integrated from x = 0 to x = L. The result is Eq. 8.45a.

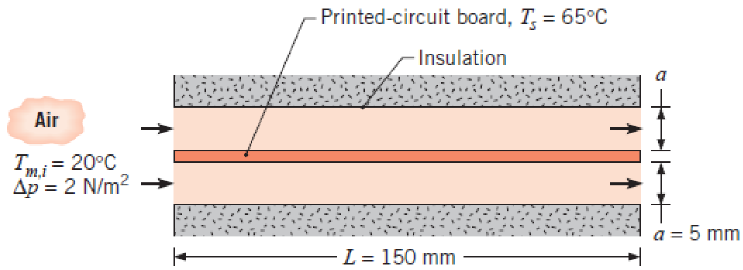
$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi\, DL}{\dot{m}\, c_{p,w}} U_i\right) = \exp\left(-\frac{\pi\times 1m\times 500m}{2\ kg/s\times 4500\ J/kg\cdot K}\times 0.37\ W/m^2\cdot K\right)$$

Hence $$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = 0.937.$$

$$T_{m,o} = T_\infty + 0.937\left(T_{m,i} - T_\infty\right) = 187^\circ C. \qquad <$$

COMMENTS: The largest contribution to the denominator on the right-hand side of Eq. (2) is made by the conduction term (the insulation provides 96% of the total resistance to heat transfer). For this reason the assumption of fully developed conditions throughout the pipe has a negligible effect on the calculations. Since the reduction in T_m is small (13°C), little error is incurred by evaluating all properties of water at $T_{m,i}$.

Q6: A printed circuit board (PCB) is cooled by laminar, fully developed airflow in adjoining, parallel-plate channels of length L and separation distance a. The channels may be assumed to be of infinite extent in the transverse direction, and the upper and lower surfaces are insulated. The temperature T_s of the PCB board is uniform, and airflow with an inlet temperature of $T_{m,i}$ is driven by a pressure difference Δp. Calculate the average heat removal rate per unit area (W/m2) from the PCB.

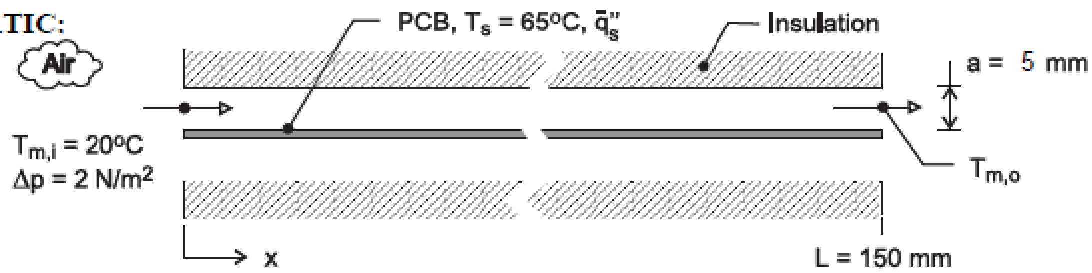


Sol:

KNOWN: Printed-circuit board (PCB) with uniform temperature T_s cooled by laminar, fully developed flow in a parallel-plate channel. The air flow with an inlet temperature of $T_{m,i}$ is driven by a pressure difference, Δp.

FIND: The average heat removal rate per unit area, $\overline{q}_s''\left(W/m^2\right)$, from the PCB.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar, fully developed flow, (2) Upper and lower walls of the channel are insulated and of infinite extent in the transverse direction, (3) PCB has uniform surface temperature, (4) Constant properties, (5) Ideal gas with negligible viscous dissipation.

PROPERTIES: *Table A-4*, Air (T_m = 293 K, 1 atm): $\rho = 1.192\ kg/m^3$, c_p = 1007 J/kg·K, $\nu = 1.531 \times 10^{-5}\ m^2/s$, k = 0.0258 W/m·K, Pr = 0.709.

ANALYSIS: The energy equations for determining the heat rate from one surface of the board are Eqs. 8.34 and 8.41b

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) = \overline{q}_s'' A_s \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{P_h L \overline{h}}{\dot{m} c_p}\right) \tag{2}$$

where A_s = Lw and P = w, since heat transfer is only from one surface, where w is the width in the transverse direction. For the fully developed flow condition, the velocity is estimated from the friction pressure drop relation, Eq. 8.22a,

$$\Delta p = f\left(\rho u_m^2 / 2\right)\left(L / D_h\right) \tag{3}$$

where the hydraulic diameter for the channel cross section is

$$D_h = \frac{4 A_c}{P} = \frac{4(w a)}{2(w+a)} = 2a \qquad\qquad a << w$$

The friction factor f from Table 8.1 for the cross section b/a = ∞ is

$$f \cdot Re_{Dh} = 96 \tag{4}$$

where the Reynolds number is

$$Re_{Dh} = u_m D_h / \nu \tag{5}$$

and the flow rate through one channel is

$$\dot{m} = \rho A_c u_m = \rho(wa)u_m \tag{6}$$

For fully developed laminar flow from Table 8.1.

$$\overline{Nu}_D = \bar{h} D_h / k = 4.86 \tag{7}$$

Substituting Eqs. (4) and (5) into Eq. (3) and solving for u_m yields

$$u_m = \Delta p D_h^2 / 48 \nu \rho L = 2 N/m^2 \times (0.01 m)^2 / 48 \times 1.531 \times 10^{-5} m^2/s \times 1.192 kg/m^3 \times 0.15 m = 1.52 m/s$$
$$Re = u_m D_h / \nu = 1.52 m/s \times 0.01 m / 1.531 \times 10^{-5} m^2/s = 994$$

Thus the flow is laminar, as assumed. From Eqs. (6), (7), and (2), $\dot{m}/w = \rho u_m a = 1.192 kg/m^3 \times 1.52$ m/s $\times$ 0.005 m = 0.00907 kg/s·m. $\bar{h} = \overline{Nu}_D k / D_h = 4.86 \times 0.0258$ W/m·K/0.01m = 12.5 W/m^2·K. $T_{m,o} = T_s - (T_s - T_{m,i})\exp(-L\bar{h}/(\dot{m}/w)c_p)$ = 65°C - 45°Cexp(-0.15m × 12.5W/m^2·K/0.00907kg/s·m× 1007 J/kg·K) = 28.4°C.

From Eq. (1)

$$q' = \frac{\dot{m}}{w} c_p (T_{m,o} - T_{m,i}) = 0.00907 kg/m \cdot s \times 1007 J/kg \cdot K \times (28.4 - 20)°C = 76.5 W/m \quad <$$
$$q'' = q'/L = 510 W/m^2$$

COMMENTS: (1) The thermophysical properties of the air are evaluated at the average mean temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.

(2) The fully developed flow length, $x_{fd,t}$, for the channel follows from Eq. 8.23,

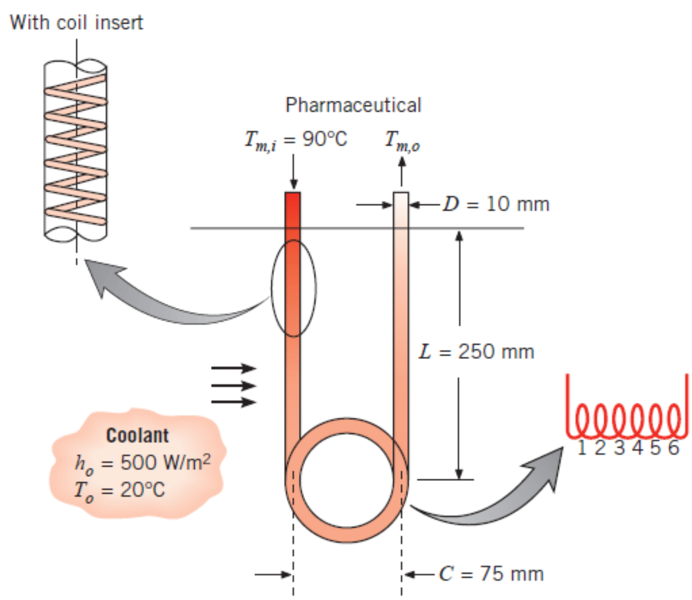
$$x_{fd,t} = D_h \times 0.05 Re_{Dh} Pr$$

$$x_{fd,t} = 2 \times 0.010 \text{ m} \times 0.05 \times 7954 \times 0.709 = 5.6 \text{ m}$$

Since L << $x_{fd,t}$, we conclude that the flow is not likely to be fully developed.

Q7: A bayonet cooler is used to reduce the temperature of a pharmaceutical fluid. The pharmaceutical fluid flows through the cooler, which is fabricated of 10-mm-diameter, thin-walled tubing with two 250-mm-long straight sections and a coil with six and a half turns and a coil diameter of 75 mm. A coolant flows outside the cooler, with a convection coefficient at the outside
surface of h_o= 500 W/m^2 K and a coolant temperature of 20°C. Consider the situation where the pharma-ceutical fluid enters at 90°C with a mass flow rate of 0.005 kg/s. The pharmaceutical has the following properties: ρ = 1200 kg/m^3 , μ = 4 x 10^{-3} N s/m^2 , c_p = 2000 J/kg K, and k = 0.5 W/m K.
(a) Determine the outlet temperature of the pharmaceutical fluid.

(b) It is desired to further reduce the outlet temperature of the pharmaceutical. However, because the cooling process is just one part of an intricate processing operation, flow rates cannot be changed. A young engineer suggests that the outlet temperature might be reduced by inserting stainless steel coiled springs into the straight sections of the cooler with the notion that the springs will disturb the flow adjacent to the inner tube wall and, in turn, increase the heat transfer coefficient at the inner tube wall. A senior engineer asserts that insertion of the springs should double the heat transfer coefficient at the straight inner tube walls. Determine the outlet tem-
perature of the pharmaceutical fluid with the springs inserted into the tubes, assuming the senior engineer is correct in his assertion.
(c) Would you expect the outlet temperature of the pharmaceutical to depend on whether the springs have a left-hand or right-hand spiral? Why?

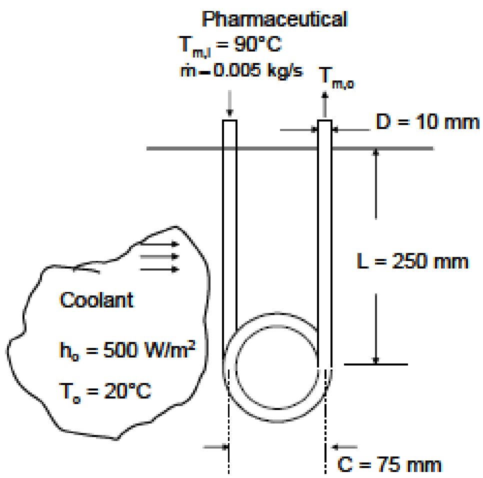


Sol:

KNOWN: Geometry and dimensions of a tube with straight and coiled sections. Temperature and convection coefficient of coolant flowing outside the tube. Inlet temperature, mass flow rate, and properties of pharmaceutical fluid in tube.

FIND: (a) Outlet temperature of pharmaceutical, (b) Outlet temperature with inner heat transfer coefficient doubled in straight sections, (c) Effect of left- or right-handed spiral.

SCHEMATIC:



ASSUMPTIONS: (1) Tube wall thermal resistance is negligible. (2) Flow is fully-developed in coiled section. (3) Flow in last straight section is unaffected by swirl introduced in coiled section. (4) Constant properties.

PROPERTIES: Pharmaceutical fluid (given): $\rho = 1200\ \text{kg/m}^3$, $\mu = 4 \times 10^{-3}\ \text{N·s/m}^2$, $c_p = 2000\ \text{J/kg·K}$, $k = 0.5\ \text{W/m·K}$, $Pr = \mu c_p/k = 16$.

ANALYSIS:

(a) The Reynolds number is

$$Re_D = \frac{4\,\dot{m}}{\pi\,D\,\mu} = \frac{4 \times 0.005\ \text{kg/s}}{\pi \times 0.01\ \text{m} \times 4 \times 10^{-3}\text{N}\cdot\text{s/m}^2} = 159$$

Thus the flow is laminar.

<u>1st Straight Section.</u> The development length in the straight section is

$$x_{fd,h} = 0.05\,Re_D\,D = 0.05 \times 159 \times 0.01\ \text{m} = 0.08\ \text{m}$$

$$x_{fd,t} = x_{fd,h} \cdot Pr = 0.08\ \text{m} \times 16 = 1.3\ \text{m}$$

The flow is thermally developing. With Pr > 5, we can use Equation 8.57 with Equation 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668\,(D/L)\,Re_D\,Pr}{1 + 0.04\left[(D/L)\,Re_D\,Pr\right]^{2/3}} = 7.29$$

Thus $h_i = \overline{Nu}_D k/D = 7.29 \times 0.5\ \text{W/m}\cdot\text{K}/0.01\ \text{m} = 365\ \text{W/m}^2\cdot\text{K}$.

Continued…

The mean temperature at the end of the first straight section can be found from Equation 8.45a,

$$T_{m,o1} = T_\infty + (T_{m,i} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

where $\overline{U} = [1/h_i + 1/h_o]^{-1} = \left[1/365\ \text{W/m}^2\cdot\text{K} + 1/500\ \text{W/m}^2\cdot\text{K}\right]^{-1} = 211\ \text{W/m}^2\cdot\text{K}$.

Thus $\quad T_{m,o1} = 20°\text{C} + (90°\text{C} - 20°\text{C})\exp\left(-\frac{211\ \text{W/m}^2\cdot\text{K}\times\pi\times 0.01\ \text{m}\times 0.25\ \text{m}}{0.005\ \text{kg/s}\times 2000\ \text{J/kg}\cdot\text{K}}\right) = 79.3°\text{C}$

<u>Coiled Section.</u> The critical Reynolds number in the coiled section is given by Equation 8.74,

$$Re_{D,C,h} = Re_{D,C}\left[1 + 12(D/C)^{0.5}\right]$$

where $Re_{D,C} = 2300$. Since this must be greater than 2300, the flow in the coiled section, with $Re_D = 159$, is still laminar. The length of the coiled section is $6.5\,\pi\,C = 6.5\,\pi\,(0.075\ \text{m}) = 1.53$ m. Since development lengths are 20 to 50% shorter in coiled tubes than in straight tubes the flow can be approximated as fully developed. The Nusselt number is given by Equation 8.76, with

$$a = \left[1 + \frac{957\,(C/D)}{Re_D^2\ Pr}\right] = \left[1 + \frac{957\,(75\ \text{mm}/10\ \text{mm})}{(159)^2\times 16}\right] = 1.018$$

and $b = 1 + 0.477/Pr = 1 + 0.477/16 = 1.030$. Note that $Re_D\,(D/C)^{1/2} = 58$, therefore the criteria for using Equations 8.76 and 8.77 are satisfied. Thus assuming $\mu_s = \mu$,

$$Nu_D = \left[\left(3.66 + \frac{4.343}{a}\right)^3 + 1.158\left(\frac{Re_D\ (D/C)^{1/2}}{b}\right)^{3/2}\right]^{1/3}$$

$$= \left[\left(3.66 + \frac{4.343}{1.018}\right)^3 + 1.158\left(\frac{159\ (10\ \text{mm}/75\ \text{mm})^{1/2}}{1.030}\right)^{3/2}\right]^{1/3} = 9.96$$

and $h_i = Nu_D k/D = 498\ \text{W/m}^2\cdot\text{K}$.

Then $\quad \overline{U} = [1/h_i + 1/h_o]^{-1} = \left[1/498\ \text{W/m}^2\cdot\text{K} + 1/500\ \text{W/m}^2\cdot\text{K}\right]^{-1} = 250\ \text{W/m}^2\cdot\text{K}$.

The outlet temperature of the coiled section can be found from Equation 8.45a, with $A_s = (\pi\,D)(6.5\,\pi\,C) = 0.048\ \text{m}^2$, and the inlet temperature is the outlet temperature of the straight section:

$$T_{m,o2} = T_\infty + (T_{m,o1} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

$$T_{m,o2} = 20°\text{C} + (79.3°\text{C} - 20°\text{C})\exp\left(-\frac{250\ \text{W/m}^2\cdot\text{K}\times 0.048\ \text{m}^2}{0.005\ \text{kg/s}\times 2000\ \text{J/kg}\cdot\text{K}}\right) = 37.9°\text{C}$$

Continued…

2nd Straight Section. The overall heat transfer coefficient would be the same as in the 1st straight section. The outlet temperature can be calculated from Equation 8.45a with the inlet temperature equal to the outlet temperature of the coiled section.

$$T_{m,o3} = T_\infty + (T_{m,o2} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

$$T_{m,o3} = 20°C + (37.9°C - 20°C)\exp\left(-\frac{211\ W/m^2\cdot K \times \pi \times 0.01\ m \times 0.25\ m^2}{0.005\ kg/s \times 2000\ J/kg\cdot K}\right)$$

$$T_{m,o3} = 35.1°C$$ <

(b) Repeating the calculations with h_i in the straight sections doubled, in the 1st straight section:

$$\overline{U} = \left[1/730\ W/m^2\cdot K + 1/500\ W/m^2\cdot K\right]^{-1} = 297\ W/m^2\cdot K$$

$$T_{m,o1} = 75.4°C$$

In the coiled section, $\overline{U}$ is unchanged, and

$$T_{m,o2} = 36.7°C$$

In the 2nd straight section, $\overline{U} = 297\ W/m^2\cdot K$ and

$$T_{m,o3} = 33.2°C$$

(c) Yes, the orientation of the springs could have an effect, because they introduce swirl that interacts with the swirl introduced in the coiled section. However, the effect is probably small.

COMMENTS: The analysis is only approximate. In particular, the flow in the last section would be affected by the swirl introduced in the coiled section, which would in turn affect the heat transfer.

Q8: An experiment is designed to study microscale forced convection. Water at $T_{m,i}$ = 300 K is to be heated in a straight, circular glass tube with a 50-μm inner diameter and a wall thickness of 1 mm. Warm water at T_m = 350 K, V = 2 m/s is in cross flow over the exte-rior tube surface. The experiment is to be designed to cover the operating range 1 ≤ ReD ≤ 2000, where ReD is he Reynolds number associated with the internal flow.

(a) Determine the tube length L that meets a design requirement that the tube be twice as long as the thermal entrance length associated with the high-est Reynolds number of interest. Evaluate water properties at 305 K.

(b) Determine the water outlet temperature, $T_{m,o}$, that is expected to be associated with ReD = 2000. Evaluate the heating water (water in cross flow over the tube) properties at 330 K.

(c) Calculate the pressure drop from the entrance to the exit of the tube for ReD = 2000.

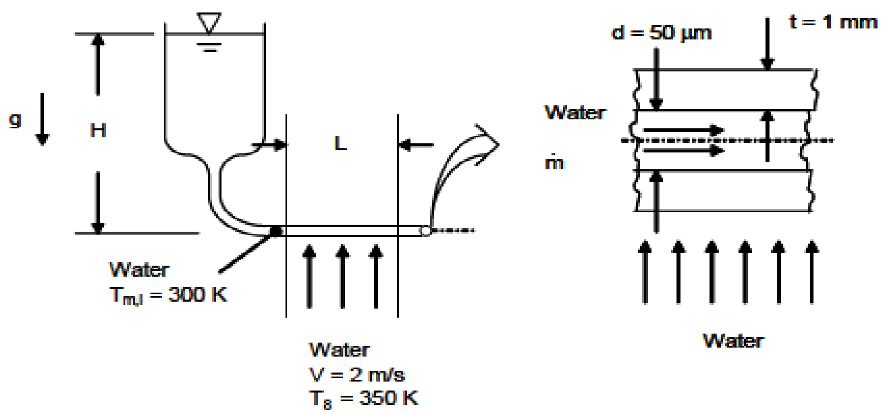
(d) Based on the calculated flow rate and pressure drop in the tube, estimate the height of a column of water (at 300 K) needed to supply the neces-sary pressure at the tube entrance and the time needed to collect 0.1 liter of water. Discuss how the outlet temperature of the water flowing from the tube, $T_{m,o}$, might be measured.

Sol:

KNOWN: Inner diameter of microscale tube, wall thickness of tube, temperature of water inside the tube, and temperature of water in cross flow over the tube.

FIND: (a) Required tube length at $Re_D = 2000$, (b) Water outlet temperature, (c) Pressure drop associated with the flow of water inside the tube, (d) Height of water column needed to supply the required inlet pressure and time needed to collect 0.1 liter of water. Discuss measurement of outlet water temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties and steady-state conditions, (2) Incompressible liquid and negligible viscous dissipation, (3) Negligible microscale or nanoscale effects.

PROPERTIES: Table A.6, water: ($\overline{T}_m$ = 305 K): k = 0.620 W/m·K, c_p = 4178 J/kg·K, μ = 769 × 10^{-6} N·s/m^2, Pr = 5.2, ρ = 995 kg/m^3: ($\overline{T}$ = 330 K): k = 0.650 W/m·K, c_p = 4194 J/kg·K, μ = 489 × 10^{-6} N·s/m^2, Pr = 3.15, ρ = 984 kg/m^3. Table A.3 glass: k = 1.4 W/m·K.

ANALYSIS: (a) At $Re_D = 2000$, Equation 8.3 yields $x_{fd,h} = 0.05Re_DPrD = 0.05 \times 2000 \times 5.2 \times 50 \times 10^{-6}$ m $= 26 \times 10^{-3}$ m. Therefore, $L = 2x_{fd,h} = 2 \times 26 \times 10^{-3}$ m $= 52 \times 10^{-6}$ m = 52 mm. <

(b) Equation 8.45a is

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right) \tag{1}$$

where we will use $\overline{U} = \overline{U}_i$, $A_s = A_{s,i}$. Note that $Re_D = 4\dot{m}/(\pi D\mu)$ so that $\dot{m} = Re_D\,\pi D\mu/4$ $= 2000 \times \pi \times 50 \times 10^{-6}$ m $\times 769 \times 10^{-6}$ N·s/m^2/4 $= 60.4 \times 10^{-6}$ kg/s. Therefore, $u_m = \dot{m}/(\rho A_c) = 60.4 \times 10^{-6}$ kg/s × 4/ (995 kg/m^3 × π × (50 × 10^{-6} m)^2) = 31 m/s. From Equation 3.36,

Continued…

$$\overline{U}_i = \frac{1}{\frac{1}{h_i} + \frac{d/2}{k_g}\ln\left[\frac{(d/2+t)}{d/2}\right] + \frac{d/2}{(d/2+t)}\frac{1}{h_o}} \tag{2}$$

$A_{s,i} = \pi dL = \pi \times 50 \times 10^{-6}$ m $\times$ 52 $\times 10^{-3}$ m = 8.17 $\times 10^{-6}$ m^2. From Equation 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668(50\times10^{-6}/53\times10^{-3})\times2000\times5.3}{1+0.04\left[(50\times10^{-6}/53\times10^{-3})\times2000\times5.3\right]^{2/3}} = 4.371$$

and $\overline{h}_D = h_i = \overline{Nu}_D \dfrac{k}{D} = 4.371\times0.620\,W/m\cdot K/50\times10^{-6}m = 54.2\times10^3\,W/m^2\cdot K$.

For the cross flow of water over the tube, $Re_D = VD\rho/\mu$ = 2 m/s $\times$ (50 $\times 10^{-6}$ m + 2 $\times$ 1 $\times 10^{-3}$ m)(984 kg/m^3)/489 $\times 10^{-6}$ N·s/m^2 = 8253. From Equation 7.54,

$$\overline{Nu}_D = 0.3 + \frac{0.62(8253)^{1/2}(3.15)^{1/3}}{\left[1+(0.4/3.15)^{2/3}\right]^{1/4}}\left[1+\left(\frac{8253}{282,000}\right)^{5/8}\right]^{4/5} = 85.14$$

and

$$\overline{h}_D = h_o = \overline{Nu}_D k/(d+2t) = 85.14\times0.65\,W/m\cdot K/(50\times10^{-6}m + 2\times1\times10^{-3}m) = 27.0\times10^3\,W/m^2\cdot K$$

Therefore,

$$\overline{U}_i = \frac{1}{\left[\begin{array}{l}\dfrac{1}{54.2\times10^3\,W/m\cdot K} + \dfrac{50\times10^{-6}m/2}{1.4\,W/m\cdot K}\ln\left[\dfrac{(50\times10^{-6}m/2+1\times10^{-3}m)}{50\times10^{-6}m/2}\right] \\ + \dfrac{50\times10^{-6}m/2}{(50\times10^{-6}m/2+1\times10^{-3}m)}\times\dfrac{1}{27.0\times10^3\,W/m^2\cdot K}\end{array}\right]} = 11.7\times10^3\,W/m^2\cdot K$$

Equation (1) becomes

$$\frac{350K - T_{m,o}}{350K - 300K} = \exp\left(-\frac{11.7\times10^3\,W/m^2\cdot K\times8.17\times10^{-6}m^2}{60.4\times10^{-6}kg/s\cdot4194J/kg\cdot K}\right)$$

or, $T_{m,o}$ = 316 K <

Continued…

(c) For laminar flow, Equation 8.19 yields $f = 64/Re_D = 64/2000 = 32 \times 10^{-3}$. Equation 8.22a yields

$$\Delta p = f\frac{\rho u_m^2}{2D}L = \frac{32\times10^{-3}\times995\,\mathrm{kg/m^3}\times(31\,\mathrm{m/s})^2\times52\times10^{-3}\,\mathrm{m}}{2(50\times10^{-6}\,\mathrm{m})} = 15.9\times10^6\,\mathrm{Pa} \qquad <$$

(d) The pressure generated by the water column must offset the pressure drop in the tube. Therefore,

$$\rho g H = \Delta p \quad \text{or} \quad H = \Delta p/\rho g = 15.9\times10^6\,\mathrm{N/m^2}/(995\,\mathrm{kg/m^3}\times9.8\,\mathrm{m/s^2}) = 1630\,\mathrm{m} = 1.63\,\mathrm{km} \qquad <$$

The time required for a particular volume of water to flow through the system is

$$t = \frac{V\rho}{m} = \frac{0.1\times\dfrac{1\,\mathrm{m^3}}{1000\,\mathrm{ml}}\times995\,\mathrm{kg/m^3}}{60.4\times10^{-6}\,\mathrm{kg/s}} = 1650\,\mathrm{s} \qquad <$$

COMMENTS: (1) Microscale experimentation is often very difficult to perform. In addition to the difficulty in measuring the water outlet temperature, establishing a constant flow rate with such a large inlet pressure would be very difficult. (2) Turbulent conditions in microscale systems are rare in nature, and are difficult to achieve experimentally. (3) The glass tube wall is relatively thick. Therefore, conduction in the axial direction is likely to be significant. (4) The average mean water temperature inside the tube is $\overline{T}_m = (T_{m,i} + T_{m,o})/2 = (300\text{ K} + 316\text{ K})/2 = 308$ K. The assumed mean temperature of 305 K is good.