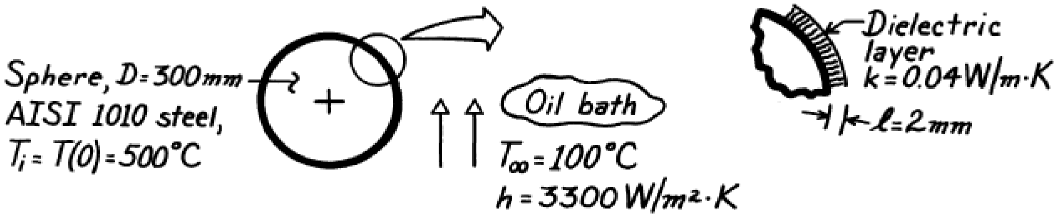
Q1. A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/mK. The coated sphere is initially at a uniform temperature of 500 °C and is suddenly quenched in a large oil bath for which $\boldsymbol{T}_\infty$ = 100°C and h = 3300 W/m^2 K. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance (ρcV) is small compared to that of the steel sphere.

Sol:

KNOWN: Solid steel sphere (AISI 1010), coated with dielectric layer of prescribed thickness and thermal conductivity. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath.

FIND: Time required for sphere to reach 140°C.

SCHEMATIC:



PROPERTIES: *Table A-1*, AISI 1010 Steel $\left(\bar{T} = [500 + 140]^\circ C/2 = 320^\circ C \approx 600K\right)$:

$\rho = 7832\ kg/m^3$, $c = 559\ J/kg \cdot K$, $k = 48.8\ W/m \cdot K$.

ASSUMPTIONS: (1) Steel sphere is space-wise isothermal, (2) Dielectric layer has negligible thermal capacitance compared to steel sphere, (3) Layer is thin compared to radius of sphere, (4) Constant properties, (5) Neglect contact resistance between steel and coating.

ANALYSIS: The thermal resistance to heat transfer from the sphere is due to the dielectric layer and the convection coefficient. That is,

$$R'' = \frac{\ell}{k} + \frac{1}{h} = \frac{0.002m}{0.04\ W/m \cdot K} + \frac{1}{3300\ W/m^2 \cdot K} = (0.050 + 0.0003) = 0.0503 \frac{m^2 \cdot K}{W},$$

or in terms of an overall coefficient, $U = 1/R'' = 19.88\ W/m^2 \cdot K$. The effective Biot number is

$$Bi_e = \frac{UL_c}{k} = \frac{U(r_o/3)}{k} = \frac{19.88\ W/m^2 \cdot K \times (0.300/6)m}{48.8\ W/m \cdot K} = 0.0204$$

where the characteristic length is $L_c = r_o/3$ for the sphere. Since $Bi_e < 0.1$, the lumped capacitance approach is applicable. Hence, Eq. 5.5 is appropriate with h replaced by U,

$$t = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{\theta_i}{\theta_o} = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{T(0) - T_\infty}{T(t) - T_\infty}.$$

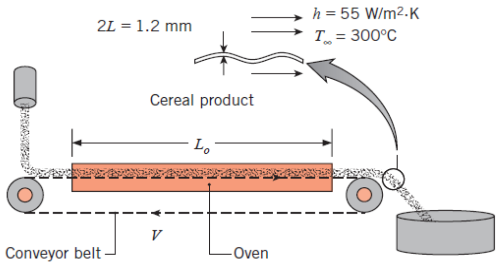
Substituting numerical values with $(V/A_s) = r_o/3 = D/6$,

$$t = \frac{7832\ kg/m^3 \times 559\ J/kg \cdot K}{19.88\ W/m^2 \cdot K}\left[\frac{0.300m}{6}\right] \ln\frac{(500 - 100)^\circ C}{(140 - 100)^\circ C}$$

$$t = 25{,}358s = 7.04h. \qquad <$$

COMMENTS: (1) Note from calculation of R'' that the resistance of the dielectric layer dominates and therefore nearly all the temperature drop occurs across the layer.

Q2: A flaked cereal is of thickness $2L$ = 1.2 mm. The density, specific heat, and thermal conductivity of the flake are ρ= 700 kg/m^3, cp =2400 J/kg K, and k= 0.34 W/m K, respectively. The product is to be baked by increasing its temperature from Ti = 20°C to Tf =220°C in a convection oven, through which the product is carried on a conveyor. If the oven is Lo = 3 m long and the convection heat transfer coefficient at the product surface and oven air temperature are h = 55 W/m^2 K and T= 300°C, respectively, determine the required conveyor velocity, V. An engineer suggests that if the flake thickness is reduced to $2L$ =1.0 mm the conveyor velocity can be increased, resulting in higher productivity. Determine the required conveyor velocity for the thinner flake.

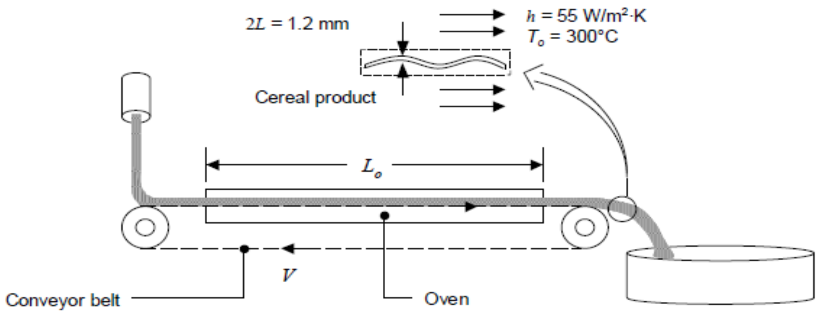


Sol:

KNOWN: Thickness and properties of flaked food product. Conveyor length. Initial flake temperature. Ambient temperature and convection heat transfer coefficient. Final product temperature

FIND: Required conveyor velocities for thick and thin flakes.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties. (2) Lumped capacitance behavior. (3) Negligible radiation heat transfer. (4) Negligible moisture evaporation from product. (5) Negligible conduction between flake and conveyor belt.

PROPERTIES: Flake: ρ = 700 kg/m^3, c_p = 2400 J/kg·K, and k = 0.34 W/m·K.

ANALYSIS: The Biot number is

$$Bi = \frac{hL}{k} = \frac{55\ \text{W/m}^2\cdot\text{K}\times 0.6\times10^{-3}\ \text{m}}{0.34\ \text{W/m}\cdot\text{K}} = 0.098$$

Hence the lumped capacitance assumption is valid. The required heating time is

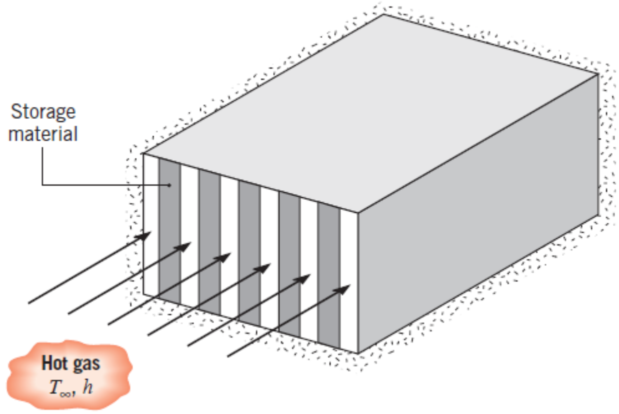
$$t = \frac{\rho Vc}{hA_s}\ln\frac{\theta_i}{\theta} = \frac{\rho Lc}{h}\ln\frac{\theta_i}{\theta} = \frac{700\ \text{kg/m}^3\times0.6\times10^{-3}\ \text{m}\times2400\ \text{J/kg}\cdot\text{K}}{55\ \text{W/m}^2\cdot\text{K}}\ln\frac{(20-300)}{(220-300)} = 23\ \text{s}$$

Therefore the required conveyor velocity is $V = L_o/t$ = 3m/23s = 0.13 m/s . <

If the flake thickness is reduced to $2L$ = 1 mm, the lumped capacitance approximation remains valid and the heating time is 19 s. The associated conveyor velocity is 0.16 m/s. <

COMMENTS: (1) Assuming large surroundings, a representative value of the radiation heat transfer coefficient is $h_r = \sigma\left(T_i + T_o\right)\left(T_i^2 + T_o^2\right) = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(293+573\right)\left(293^2+573^2\right)\text{K}^4 = 20.3$ W/m^2·K. Radiation heat transfer would be significant and would serve to increase the product heating rate, increasing the allowable conveyor belt speed. (2) The food product is likely to enter the oven in a moist state. Additional thermal energy would be required to remove the moisture during heating, reducing the rate at which the product temperature increases. (3) The effects noted in Comments 1 and 2 would tend to offset each other. A detailed analysis would be required to assess the impact of radiation and evaporation on the required conveyor velocity.

Q3. A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.



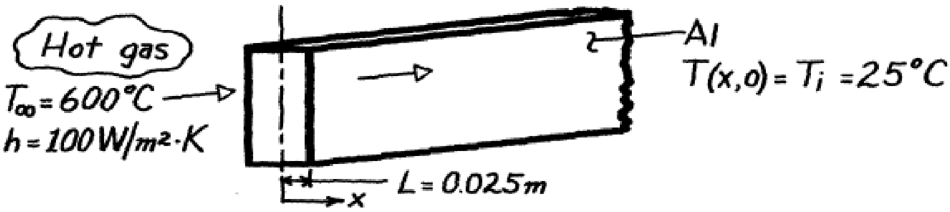
Each layer of the storage material is an aluminum slab of width W =0.05 m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of T=600°C and h =100 W/m^2 K throughout the channel. How long will it take to achieve 75% of the maximum possible energy storage? What is the temperature of the aluminum at this time?

Sol:

KNOWN: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

FIND: Time required to achieve 75% of maximum possible energy storage. Temperature of storage medium at this time.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Table A-1*, Aluminum, pure $\left(\overline{T} \approx 600\text{K} = 327^\circ\text{C}\right)$: k = 231 W/m·K, c = 1033 J/kg·K, ρ = 2702 kg/m^3.

ANALYSIS: Recognizing the characteristic length is the half thickness, find

$$\text{Bi} = \frac{\text{hL}}{\text{k}} = \frac{100\ \text{W/m}^2\cdot\text{K} \times 0.025\text{m}}{231\ \text{W/m}\cdot\text{K}} = 0.011.$$

Hence, the lumped capacitance method may be used. From Eq. 5.8,

$$\text{Q} = (\rho\text{Vc})\theta_\text{i}\left[1 - \exp(-\text{t}/\tau_\text{t})\right] = -\Delta\text{E}_\text{st} \qquad (1)$$

$$-\Delta\text{E}_\text{st,max} = (\rho\text{Vc})\theta_\text{i}. \qquad (2)$$

Dividing Eq. (1) by (2),

$$\Delta\text{E}_\text{st} / \Delta\text{E}_\text{st,max} = 1 - \exp(-\text{t}/\tau_\text{th}) = 0.75.$$

Solving for $\tau_\text{th} = \dfrac{\rho\text{Vc}}{\text{hA}_\text{s}} = \dfrac{\rho\text{Lc}}{\text{h}} = \dfrac{2702\ \text{kg/m}^3 \times 0.025\text{m} \times 1033\ \text{J/kg}\cdot\text{K}}{100\ \text{W/m}^2\cdot\text{K}} = 698\text{s}.$

Hence, the required time is

$$-\exp(-\text{t}/698\text{s}) = -0.25 \qquad \text{or} \qquad \text{t} = 968\text{s}. \qquad <$$

From Eq. 5.6,

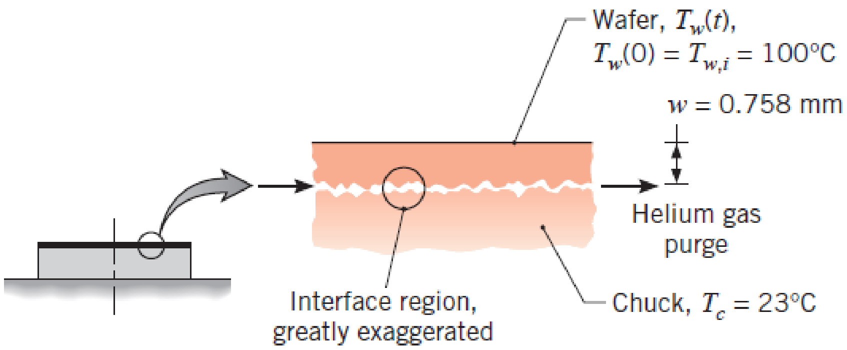
$$\frac{\text{T} - \text{T}_\infty}{\text{T}_\text{i} - \text{T}_\infty} = \exp(-\text{t}/\tau_\text{th})$$

$$\text{T} = \text{T}_\infty + (\text{T}_\text{i} - \text{T}_\infty)\exp(-\text{t}/\tau_\text{th}) = 600^\circ\text{C} - \left(575^\circ\text{C}\right)\exp(-968/698)$$

$$\text{T} = 456^\circ\text{C}. \qquad <$$

COMMENTS: For the prescribed temperatures, the property temperature dependence is significant and some error is incurred by assuming constant properties. However, selecting properties at 600K was reasonable for this estimate.

Q4. A tool used for fabricating semiconductor devices consists of a chuck (thick metallic, cylindrical disk) onto which a very thin silicon wafer (*ρ* = 2700 kg/m3, *c* =875 J/kg K, *k* =177 W/m_K) is placed by a robotic arm. Once in position, an electric field in the chuck is energized, creating an electrostatic force that holds the wafer firmly to the chuck. To ensure a reproducible thermal contact resistance between the chuck and the wafer from cycle to cycle, pressurized helium gas is introduced at the center of the chuck and flows (very slowly) radially outward between the asperities of the interface region.



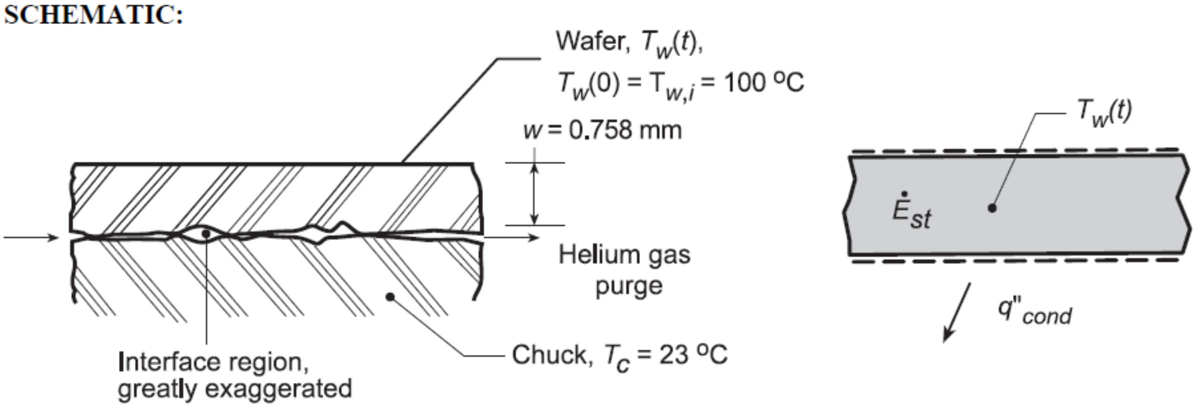
An experiment has been performed under conditions for which the wafer, initially at a uniform temperature Tw,i = 100°C, is suddenly placed on the chuck, which is at a uniform and constant temperature Tc = 23°C. With the wafer in place, the electrostatic force and the helium gas flow are applied. After 15 s, the temperature of the wafer is determined to be 33°C. What is the thermal contact resistance (m^2 K/W) between the wafer and chuck? Will the value of increase, decrease, or remain the same if air, instead of helium, is used as the purge gas?

Sol:

KNOWN: Wafer, initially at 100°C, is suddenly placed on a chuck with uniform and constant temperature, 23°C. Wafer temperature after 15 seconds is observed as 33°C.

FIND: (a) Contact resistance, R''_{tc}, between interface of wafer and chuck through which helium slowly flows, and (b) Whether R''_{tc} will change if air, rather than helium, is the purge gas.

SCHEMATIC:



PROPERTIES: Wafer (silicon, typical values): $\rho = 2700\ \text{kg/m}^3$, $c = 875\ \text{J/kg·K}$, $k = 177\ \text{W/m·K}$.

ASSUMPTIONS: (1) Wafer behaves as a space-wise isothermal object, (2) Negligible heat transfer from wafer top surface, (3) Chuck remains at uniform temperature, (4) Thermal resistance across the interface is due to conduction effects, not convective, (5) Constant properties.

ANALYSIS: (a) Perform an energy balance on the wafer as shown in the Schematic.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}_g = \dot{E}_{st} \tag{1}$$

$$-q''_{cond} = \dot{E}''_{st} \tag{2}$$

$$-\frac{T_w(t) - T_c}{R''_{tc}} = \rho w c \frac{dT_w}{dt} \tag{3}$$

Separate and integrate Eq. (3)

$$-\int_0^t \frac{dt}{\rho w c R''_{tc}} = \int_{T_{wi}}^{T_w} \frac{dT_w}{T_w - T_c} \tag{4}$$

$$\frac{T_w(t) - T_c}{T_{wi} - T_c} = \exp\left[-\frac{t}{\rho w c R''_{tc}}\right] \tag{5}$$

Substituting numerical values for $T_w(15\text{s}) = 33°\text{C}$,

$$\frac{(33-23)^\circ \text{C}}{(100-23)^\circ \text{C}} = \exp\left[-\frac{15\,\text{s}}{2700\,\text{kg/m}^3 \times 0.758\times10^{-3}\,\text{m} \times 875\,\text{J/kg}\cdot\text{K} \times R''_{tc}}\right] \tag{6}$$

$$R''_{tc} = 0.0041\,\text{m}^2\cdot\text{K/W}$$ <

(b) R''_{tc} will increase since $k_{air} < k_{helium}$. See Table A.4.

COMMENTS: Note that $Bi = R_{int}/R_{ext} = (w/k)/\ R''_{tc} = 0.001$. Hence the spacewise isothermal assumption is reasonable.