BLACK BODY RADIATION

- A body at an absolute temperature above zero emits radiation in all directions over a wide range of wavelengths.

- The amount of radiation energy emitted from a surface at a given wavelength depends on the
  - Material of the body
  - Condition of its surface
  - Surface temperature
  - The amount of radiation energy emitted from a surface at a given

- Ideal case: Maximum amount of radiation that can be emitted by a surface at a given temperature

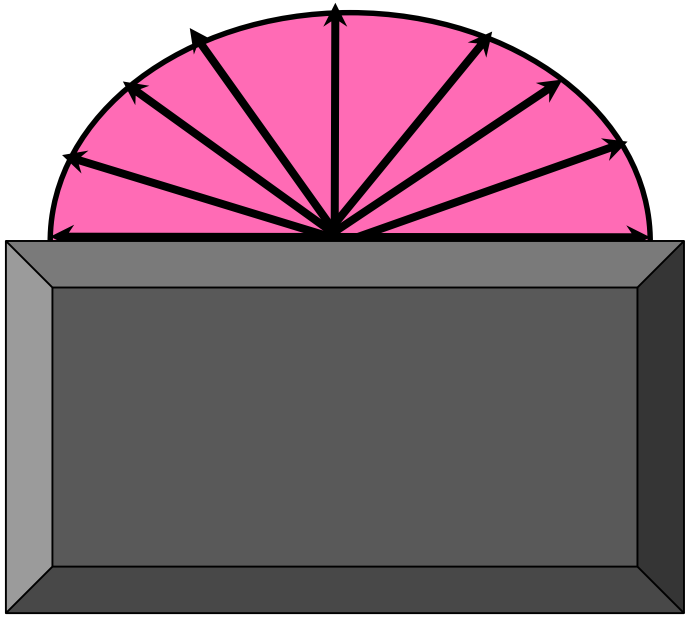
- BLACK BODY serves as a standard against which the radiative properties of real surfaces may be compared.

BLACK BODY RADIATION

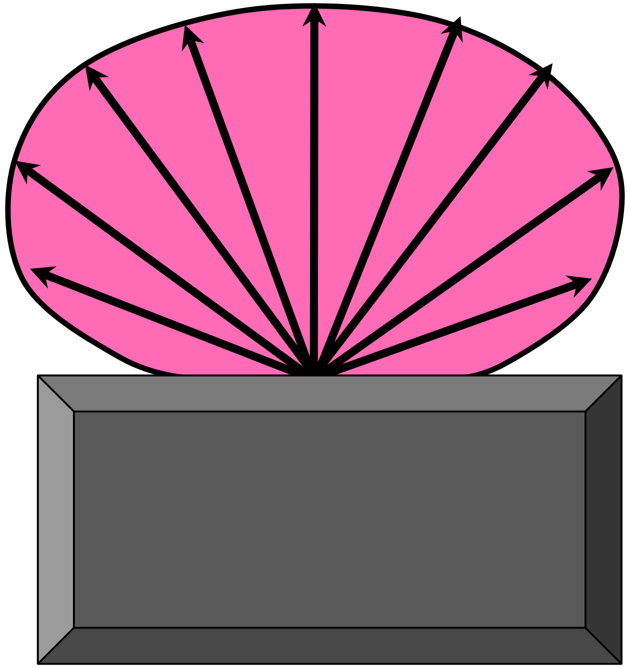
BLACK BODY IS A

- A blackbody absorbs all incident radiation, regardless of wavelength and direction
- For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody
- Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the black body is a diffuse emitter.

UNIFORM



NONUNIFORM



STEFAN-BOLTZMAN LAW

Radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b = \sigma T^4 \ \frac{W}{m^2}$$

Stefan-Boltzman constant $\sigma = 5.67 \times 10^{-8} \ W/m^2K^4$

T – absolute temperature of the surface in K

This relation was theoretically verified in 1884 by Ludwig Boltzman.

This gives the total blackbody emissive power which is the sum of the radiation emitted over all wavelengths.

Historical perspective of Stefan-Boltzmann's Law

Josef Stefan (1835-1893) Austrian physicist

- Serving as professor at the University of Vienna, Stefan determined in 1879 that, based on his experiments, blackbody emission was proportional to temperature to the fourth power.

Ludwig Erhard Boltzmann (1844-1906)

- Austrian physicist. After receiving his doctorate from the University of Vienna he held professorships in Vienna, Graz (both in Austria), Munich and Leipzig (in Germany).
- His greatest contributions were in the field of statistical mechanics (Boltzmann statistics).
- He derived the fourth-power law from thermodynamic considerations in 1889.

DISTINCTION BETWEEN IDEALISED BLACK BODY AND AN ORDINARY BLACK SURFACE

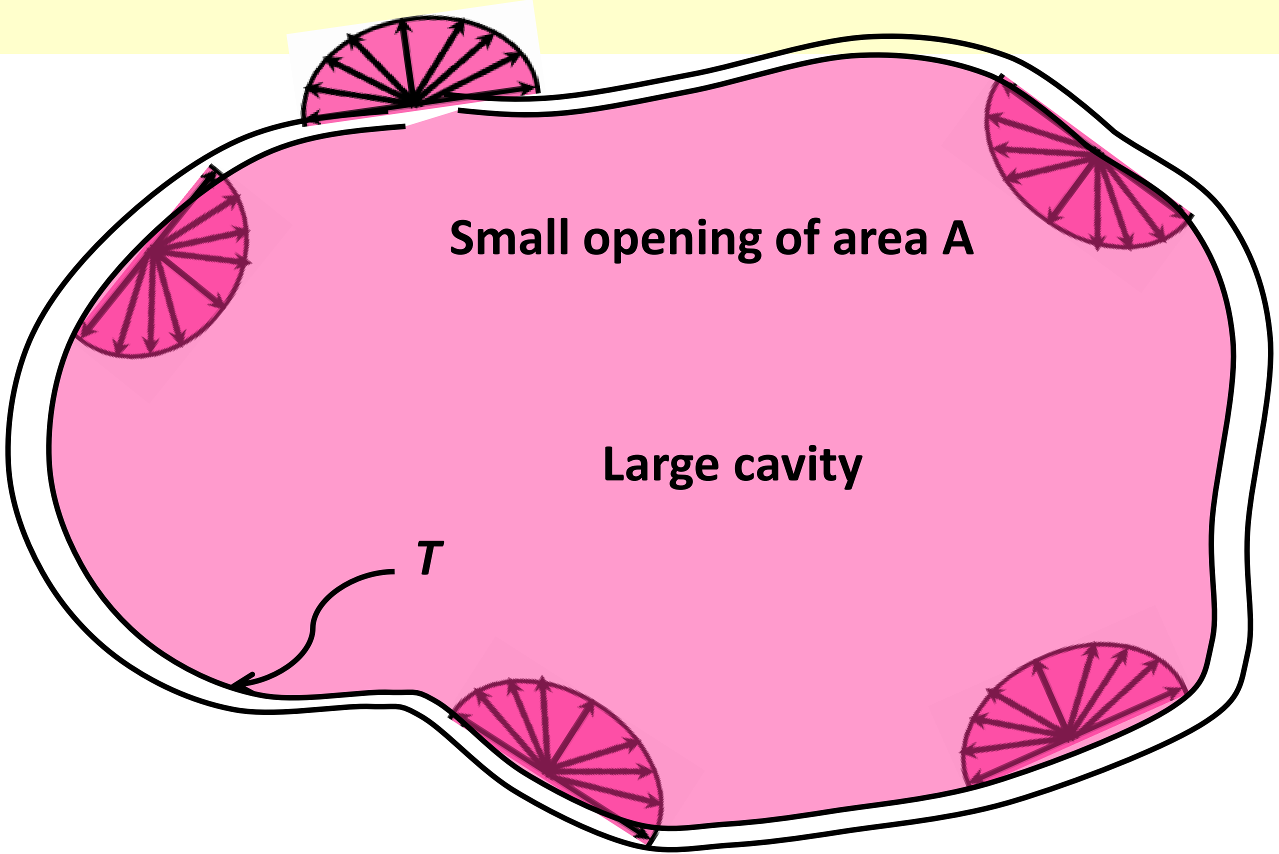
- **Any surface that absorbs light (the visible portion of radiation) would appear in black to the eye and the surface that reflects it completely would appear white**
- **Considering that visible radiation occupies a very narrow band of spectrum from 0.4 to 0.76 μm, we cannot make judgments about the blackness of a surface on the basis of visual observations.**
- **Snow and white paint reflect light and thus appear white. But they are essentially black for infrared radiation since they strongly absorb long wavelength radiation.**
- **Surfaces coated with lampblack paint approach idealised blackbody behaviour**

A large isothermal cavity at temperature T with a small opening of area A closely resembles a blackbody of surface area A at the same temperature

Radiation coming in through the opening of area A undergoes multiple reflections, and thus it has several chances to be absorbed by the interior surfaces of the cavity before any part of it can possibly escape.

If the surface of the cavity is isothermal at temperature T, the radiation emitted by the interior surfaces streams through the opening after undergoing multiple reflections, and thus it has diffuse nature.

CAVITY ACTS AS A PERFECT ABSORBER AND EMITTER



SPECTRAL BLACKBODY EMISSIVE POWER – is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength λ

The Planck Distribution – Spectral Distribution of Black Body emission

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_o^2}{\lambda^5\left[exp\left(\frac{hc_o}{\lambda kT}\right)-1\right]}$$

h = 6.6256 x 10^{-34} J.s – Universal Planck Constant

k = 1.3805 x 10^{-23} J/K – Universal Boltzmann Constant

C_o = 2.998 x 10^{8} m/s

T = Absolute temperature of the blackbody (K)

Max Planck (1858-1947)

- German physicist. Planck studied in Berlin with H. L. F. von Helmholtz and G. R. Kirchhoff, but obtained his doctorate at the University of Munich before returning to Berlin as professor in theoretical physics

- He later became head of the Kaiser Wilhelm Society (today the Max Planck Institute)

- For his development of the quantum theory he was awarded the Nobel Prize in Physics in 1918

Since Black body is a diffuse emitter, Spectral Emissive Power is given by

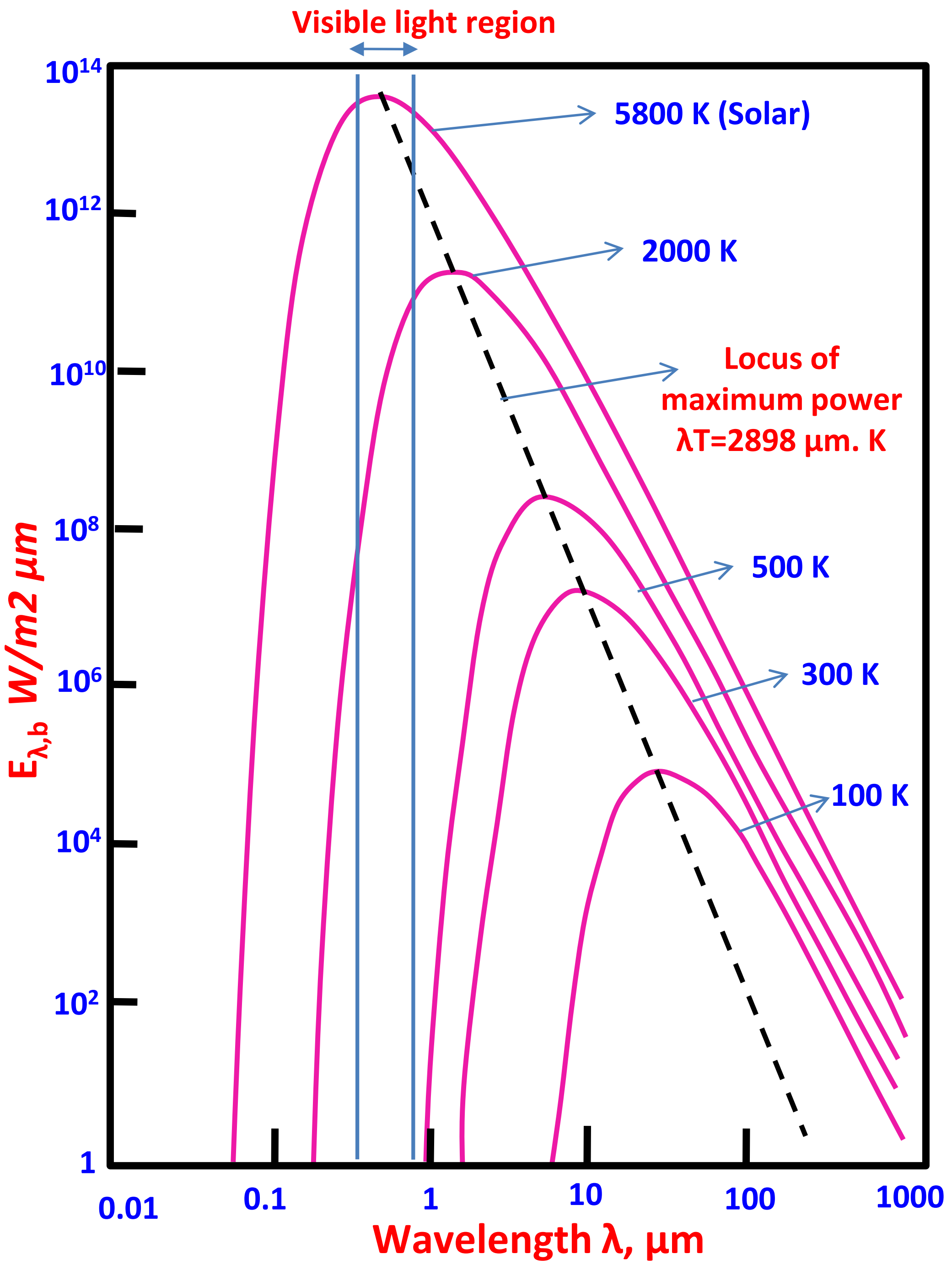
$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{2\pi h c_o^2}{\lambda^5\left[exp\left(\frac{hc_o}{\lambda kT}\right) - 1\right]}$$

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_o^2}{\lambda^5\left[exp\left(\frac{hc_o}{\lambda kT}\right) - 1\right]}$$

$$E_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$

$$C_1 = 2\pi h c_o^2 = 3.742 \times 10^8 \frac{W.\mu m^4}{m^2}$$

$$C_2 = \frac{hc_o}{k} = 1.439 \times 10^4 \mu m.K$$



IMPORTANT FEATURES

At any temperature, there is no contribution by waves other than UV, VL, IF to the thermal radiation

The emitted radiation varies continuously with wavelength

At any wavelength the magnitude of the radiation increases with increasing temperature

Spectral region in which the radiation is concentrated depends on the temperature, comparatively more radiation appearing at shorter wavelengths as the temperature increases

Significant fraction of the radiation emitted by sun which may be approximated by black body at 5800 K is in the visible region of the spectrum (0.39 – 0.77 μm)

$T > 800\ K$, emission is predominantly in the infrared region (0.77 – 100 μm) of the spectrum and is not visible to the eye

WEIN'S DISPLACEMENT LAW

$$\lambda_{max}T = C_3 = 2897.8\ \mu m.K$$

- Maximum spectral power is displaced to shorter wavelengths with increasing temperature
- Solar radiation – middle of the spectrum (λ = 0.5 μm), since sun emits as a blackbody at approximately 5800 K
- Blackbody at 1000 K, peak emission – 2.9 μm
- With increasing temperature, shorter wavelengths become more prominent, until eventually significant emission occurs over the entire visible spectrum
- Tungsten filament lamp – 2900 K (λ_{max} = 1.0 μm) emits white light, although most of the emission remains in Infra-red region

Wilhelm Wien (1864-1928)

- German physicist, who served as professor of physics at the University of Giessen and later at the University of Munich.
- Besides his research in the area of electromagnetic waves, his interests included other rays, such as electron beams, X-rays and α-particles.
- For the discovery of his displacement law he was awarded the Nobel Prize in Physics in 1911.

STEFAN-BOLTZMANN LAW

$$E_b = \int_0^{\theta} \frac{C_1}{\lambda^5\left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} d\lambda = \sigma T^4$$

$$E_b = \sigma T^4$$

$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2K^4}$ - Stefan Boltzman constant

Total Intensity associated with blackbody emission is

$$I_b = \frac{E_b}{\pi}$$

This enables calculation of the amount of radiation emitted in all directions and over all wavelengths simply from the knowledge of the temperature of the blackbody.

BAND EMISSION

Fraction of the total emission from a blackbody that is in a certain wavelength interval

$$F_{(0-\lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\int_0^{\infty} E_{\lambda,b} d\lambda} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T^4}$$

$$\lambda T = m; \quad (d\lambda)T = dm \quad d\lambda = \frac{dm}{T}$$

$$\lambda = 0 \quad \lambda T = m = 0$$
$$\lambda = \lambda \quad \lambda T = m$$

$$F_{(0-\lambda)} = \int_0^{\lambda T} \frac{E_{\lambda,b} d(\lambda T)}{\sigma T^4 (T)} = \int_0^{\lambda T} \frac{E_{\lambda,b} d(\lambda T)}{\sigma T^5}$$

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5 \left[exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]}$$

$$F_{(0-\lambda)} = \int_0^{\lambda T} \frac{E_{\lambda,b} d(\lambda T)}{\sigma T^4 (T)} = \int_0^{\lambda T} \frac{1}{\sigma T^5} \frac{C_1}{\lambda^5 \left[exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} d(\lambda T)$$

$$F_{(0-\lambda)} = \int_0^{\lambda T} \frac{C_1 d(\lambda T)}{\sigma (\lambda T)^5 \left[exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} = f(\lambda T)$$

Since the integrand $\left(\frac{E_{\lambda,b}}{\sigma T^4}\right)$ is exclusively a function of wavelength temperature product λT. The results are presented in tabular form to obtain $F_{(0-\lambda)}$ as a function of λT

$$F_{(0-\lambda)} = \int_0^{\lambda T} \frac{E_{\lambda,b}\, d(\lambda T)}{\sigma T^4 (T)} = \int_0^{\lambda T} \frac{1}{\sigma T^5} \frac{C_1}{\lambda^5 \left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} d(\lambda T)$$

$$F_{(0-\lambda)} = \int_0^{\lambda T} \frac{C_1 d(\lambda T)}{\sigma (\lambda T)^5 \left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} = f(\lambda T)$$

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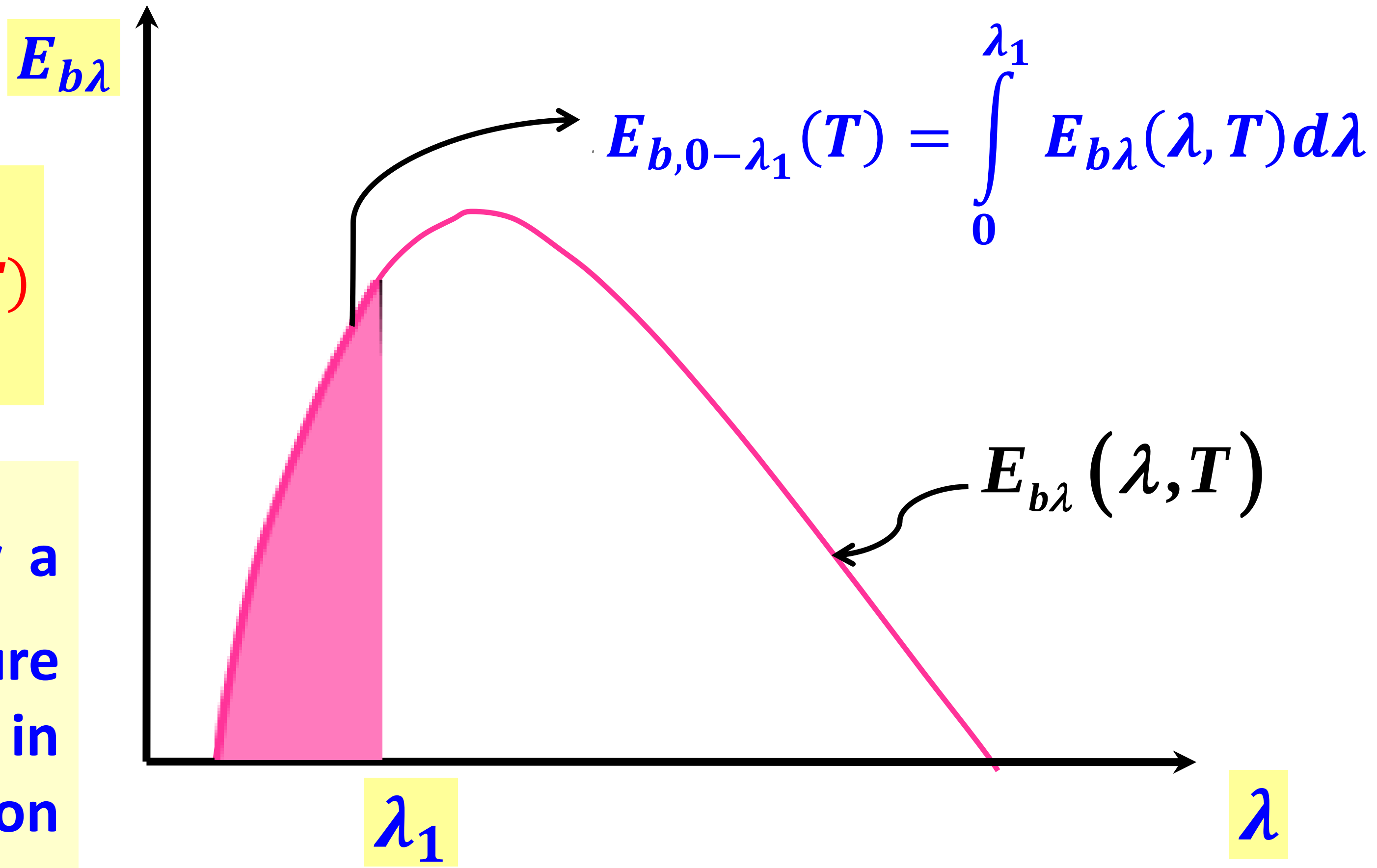


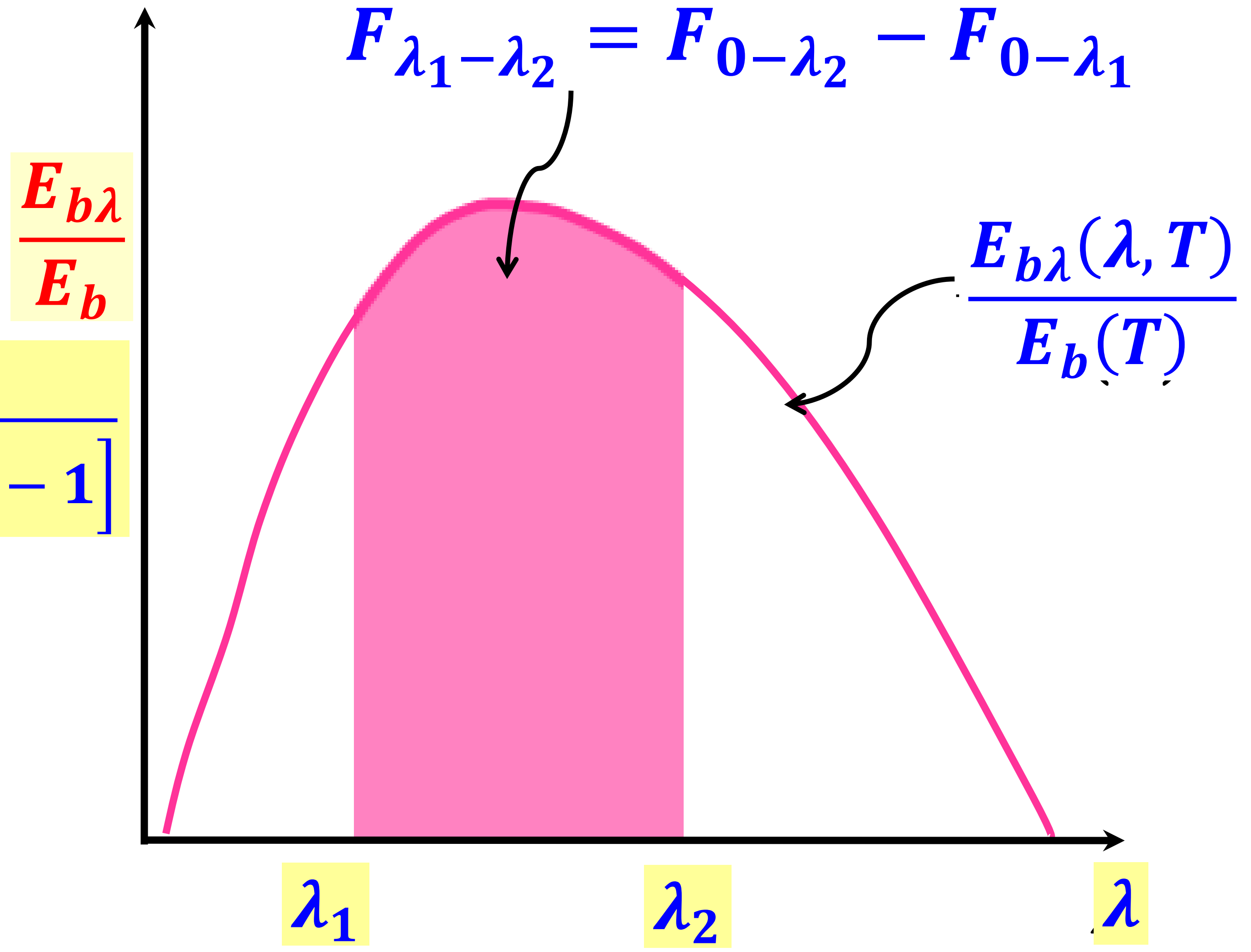
Fraction of radiation between any two wavelengths λ_1 and λ_2

$$F_{(\lambda_1-\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_0^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4}$$

$$F_{(\lambda_1-\lambda_2)} = F_{0-\lambda_2} - F_{0-\lambda_1}$$

$$I_{\lambda,b}(\lambda,T) = \frac{E_{\lambda,b}(\lambda,T)}{\pi} = \frac{C_1}{\pi\lambda^5\left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$

$$\frac{I_{\lambda,b}(\lambda,T)}{\sigma T^5} = \frac{C_1}{\sigma\pi(\lambda T)^5\left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$



λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

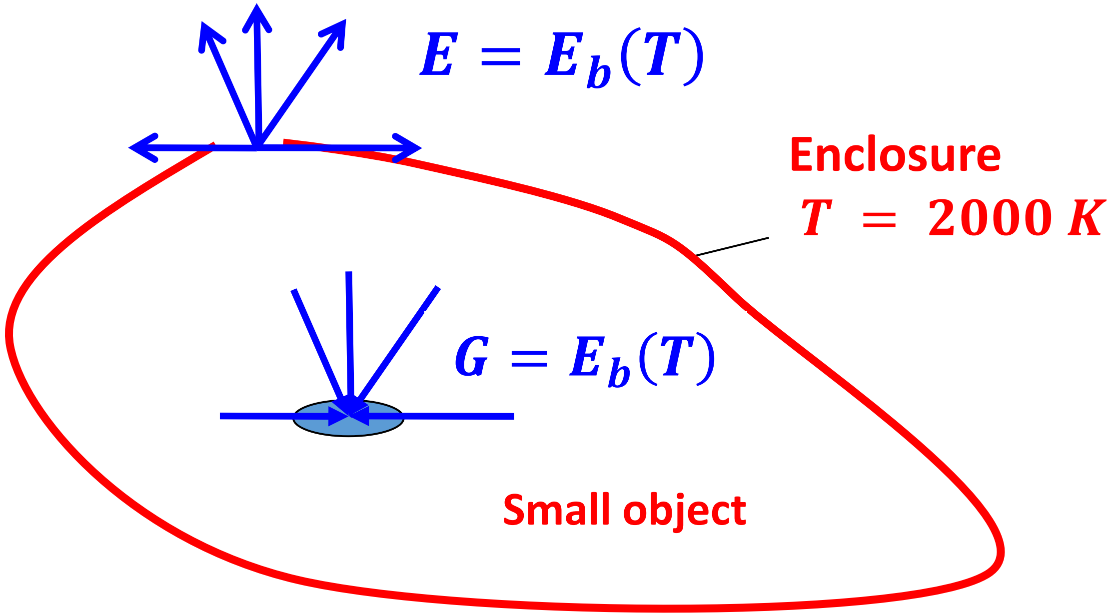
Problem: Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000K. Calculate the emissive power of the radiation that emerges from a small aperture on the enclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated ? What is the wavelength λ_2 above which 10% of the emission is concentrated ? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure ?

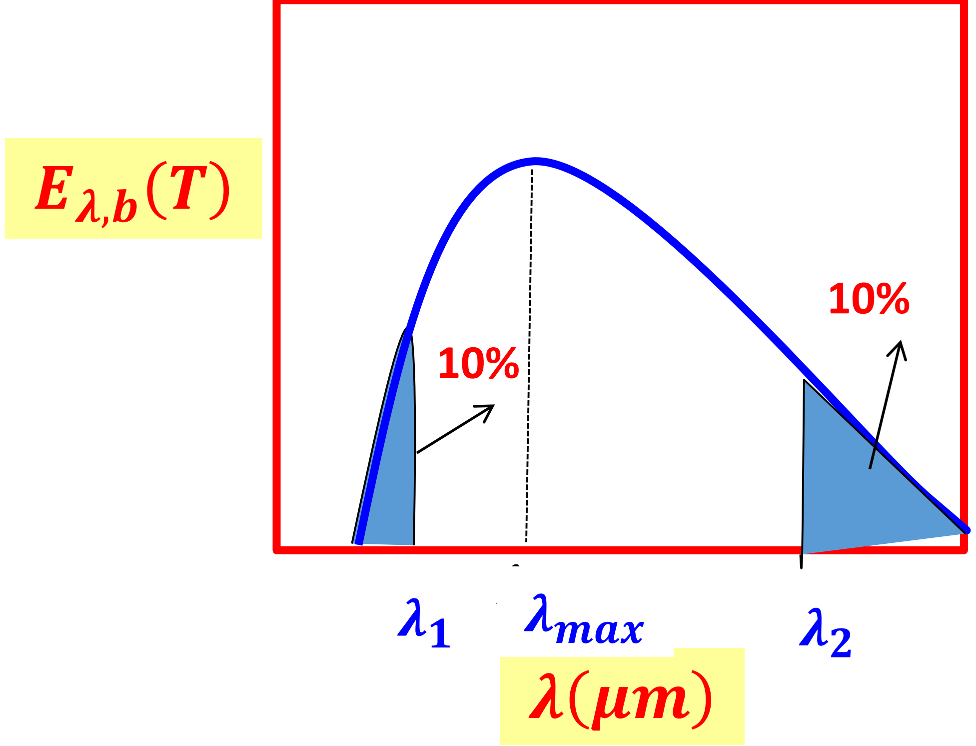
Known: Large isothermal enclosure at uniform temperature

Find :

1. Emissive power of a small aperture on the enclosure
2. Wavelengths below which and above which 10% of the radiation is concentrated
3. Spectral emissive power and wavelength associated with maximum emission
4. Irradiation on a small object inside the enclosure

Schematic





Assumptions: Areas of aperture and object are very small relative to enclosure surface

Analysis

1. Emission from the aperture of any isothermal enclosure will have the characteristics of black body radiation. Hence,

$$E = E_b(T) = \sigma T^4 = 5.67 \times 10^{-8}(2000)^4$$

$$E = 9.07 \times 10^5\ W/m^2$$

2. The wavelength λ_1 corresponds to the upper limit of the spectral band $(0\rightarrow\lambda_1)$ containing 10% of the emitted radiation. With $F_{(0-\lambda_1)} = 0.1$, it follows from the Table that

Enclosure
$T = 2000\ K$

$$\lambda_1 T = 2195\ \mu m.K$$
$$\lambda_1 = 1.1\ \mu m$$

The wavelength λ_2 corresponds to the lower limit of the spectral band $(\lambda_2\rightarrow\infty)$ containing 1-% of the emitted radiation. With

$$F_{(\lambda_2-\infty)} = 1 - F_{(0-\lambda_1)}$$
$$F_{(\lambda_2-\infty)} = 1 - 0.1$$
$$F_{(\lambda_2-\infty)} = 0.9$$

It follows from the Table that $\lambda_2 T = 9382\ \mu m.K$ $\quad \lambda_2 = 4.69\ \mu m$

Enclosure
$T = 2000\ K$

3. From Wien's law, $\lambda_{max}T = 2898\ \mu m.K$ $\lambda_{max} = 1.45\ \mu m$

The spectral emissive power associated with this wavelength may be obtained from the third column of the Table. For $\lambda_{max}T = 2898\ \mu m.K$, it follows from Table that

$$I_{\lambda,b}(1.45\mu m, T) = 0.722 \times 10^{-4}\sigma T^5$$

$$I_{\lambda,b}(1.45\mu m, T) = 0.722 \times 10^{-4}(1/\mu m.K.sr) \times 5.67 \times 10^{-8}\left(W/m^2.K^4\right)(2000)^5\left(K^5\right)$$

$$I_{\lambda,b}(1.45\mu m, T) = 1.31 \times 10^5 \frac{W}{m^2.sr.\mu m}$$

Since the emission is diffuse, it follows that

$$E_{\lambda,b} = \pi I_{\lambda,b} = \pi \times 1.31 \times 10^5 = 4.12 \times 10^5 \frac{W}{m^2.\mu m}$$

4. Irradiation of any small object inside the enclosure may be approximated as being equal to emission from a blackbody at the enclosure surface temperature. Hence,

$$G = E_b(T) = \sigma T^4 = 5.67 \times 10^{-8}(2000)^4$$

$$G = 9.07\ \times 10^5 \frac{W}{m^2}$$

A surface emits as a blackbody at 1500 K. What is the rate per unit area (W/m^2) at which it emits radiation over all directions corresponding to $0° \leq \theta \leq 60°$ and over the wavelength interval $2\ \mu m \leq \lambda \leq 4 \mu m$?

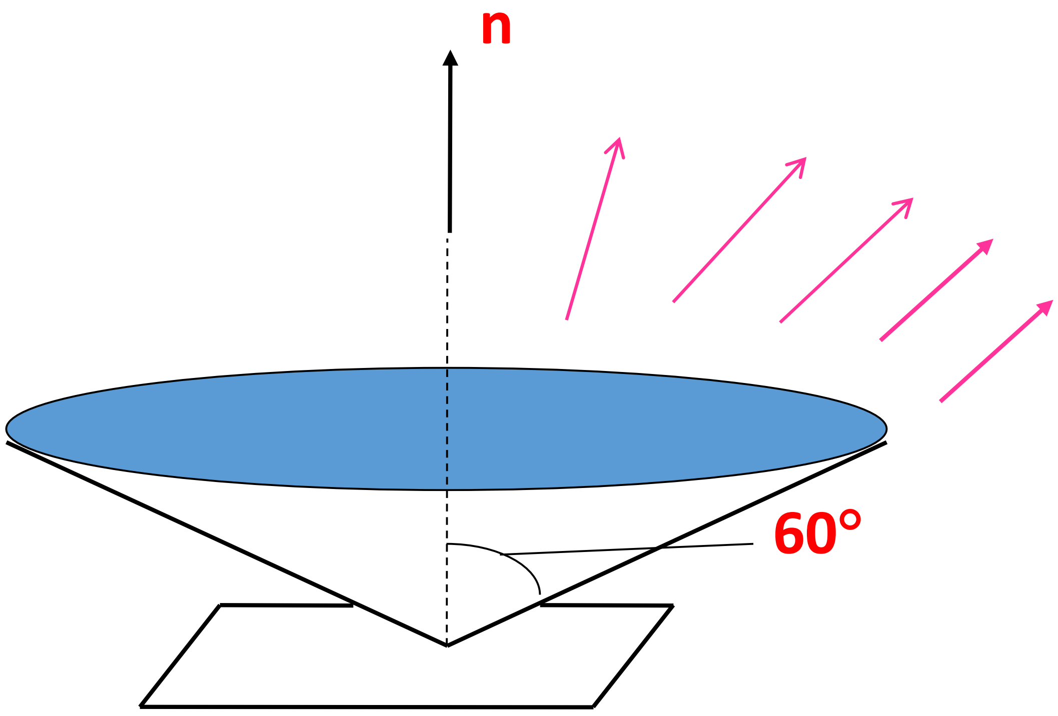
Solution

Known: temperature of a surface that emits as a blackbody

Find: Rate of emisison per unit area over all directions between $\theta = 0°$ and $60°$ and over all wavelengths between $\lambda = 2$ and $4\ \mu m$

Schematic:

Assumptions: surface emits as a blackbody



Blackbody at 1500 K

Analysis

$$\Delta E = \int_{2}^{4}\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{3}} I_{\lambda,b} sin\theta cos\theta d\theta d\phi d\lambda$$

Since a blackbody emits diffusely,

$$\Delta E = \int_{2}^{4} I_{\lambda,b}\left(\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{3}} sin\theta cos\theta d\theta d\phi\right) d\lambda = \int_{2}^{4} I_{\lambda,b}\left(2\pi \int_{0}^{\frac{\pi}{3}} \frac{sin2\theta}{2} d\theta\right) d\lambda$$

$$\Delta E = \int_{2}^{4} I_{\lambda,b}\left(2\pi\left[-\frac{cos2\theta}{4}\Big|_{0}^{\frac{\pi}{3}}\right]\right) d\lambda = \int_{2}^{4} I_{\lambda,b}\left(\frac{2\pi}{4}\left[-\left(-\frac{1}{2}-1\right)\right]\right) d\lambda = \frac{3}{4}\int_{2}^{4} I_{\lambda,b} d\lambda$$

$$\Delta E = \frac{3}{4} E_b \int_{2}^{4} \frac{E_{\lambda,b}}{E_b} d\lambda = \frac{3}{4} E_b (F_{0-4} - F_{0-2})$$

$$E_{\lambda,b} = \pi I_{\lambda,b}$$

$$\Delta E = \frac{3}{4} E_b (F_{0-4} - F_{0-2})$$

$$\lambda_1 T = 2\mu m \times 1500K = 3000\mu m.K: \quad F_{0-2} = 0.273$$
$$\lambda_2 T = 4\mu m \times 1500K = 6000\mu m.K: \quad F_{0-4} = 0.738$$

$$\Delta E = \frac{3}{4} E_b (0.738 - 0.273)$$

$$\Delta E = \frac{3}{4} E_b (0.465)$$

$$\Delta E = \frac{3}{4} (5.67 \times 10^{-8})(1500)^4 (0.465)$$

$$\Delta E = 10^5 \frac{W}{m^2}$$

Comments: The total, hemispherical emissive power is reduced by 25% and 53.5 % due to the directional and spectral restrictions, respectively

SURFACE EMISSION OF REAL SURFACES

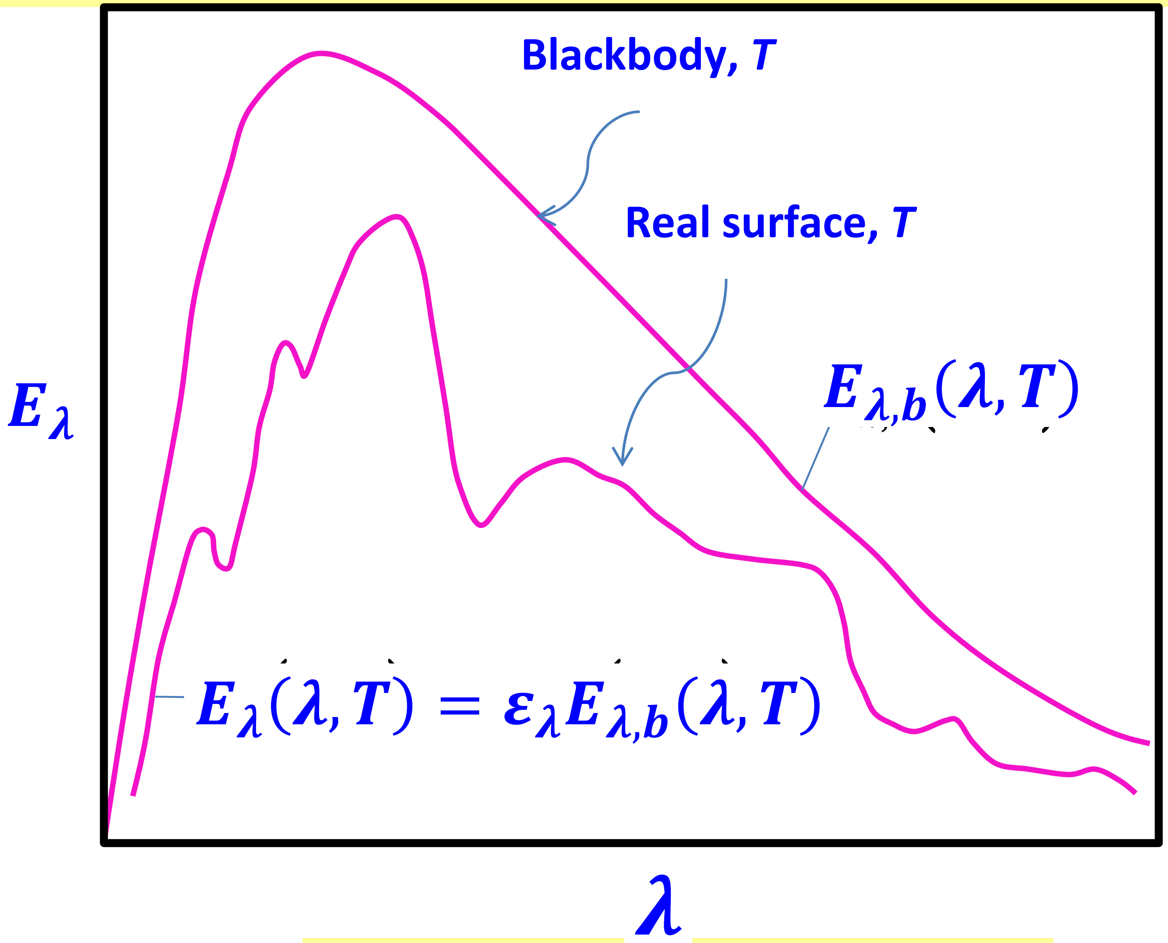
BLACK BODY IS AN IDEAL EMITTER – no surface can emit more radiation than a black body at the same temperature

BLACK BODY IS CHOSEN AS A REFERENCE

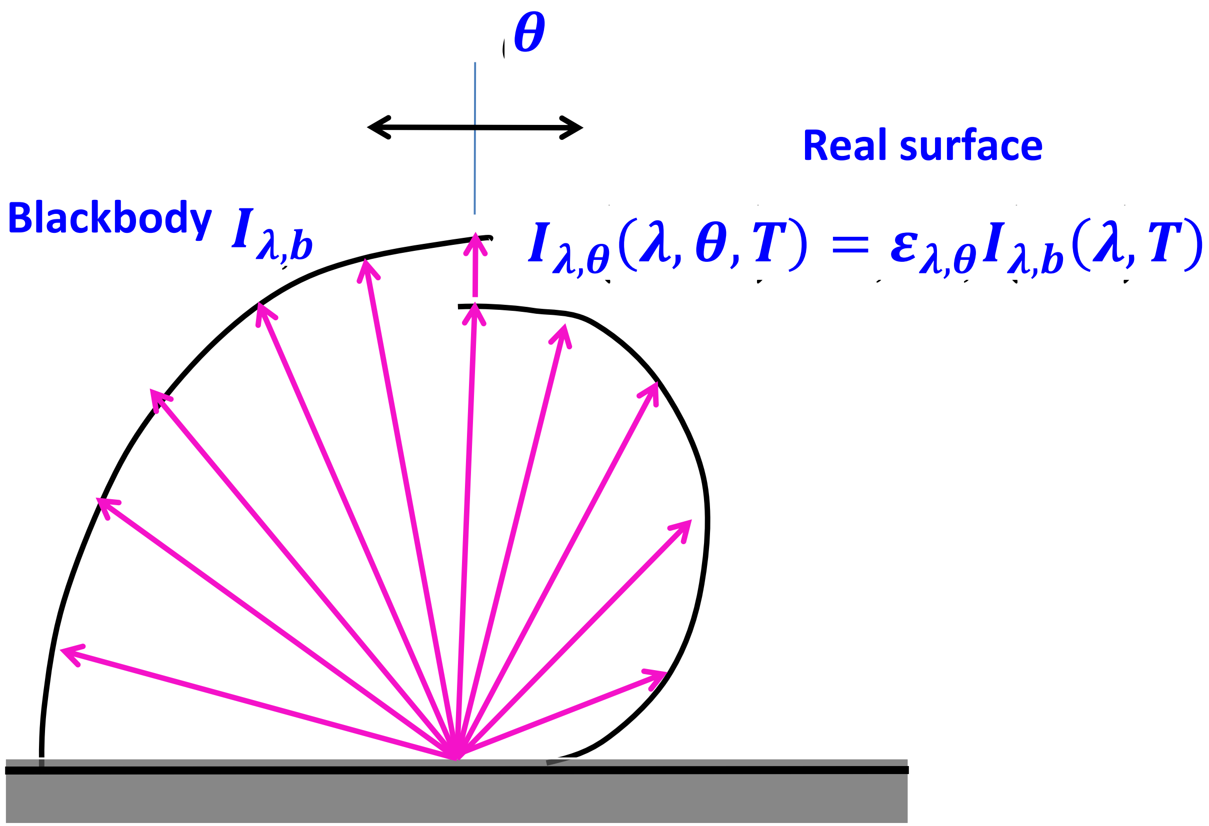
Emissivity ε – Surface radiative property

$$\varepsilon = \frac{\text{Radiation emitted by a surface}}{\text{Radiation emitted by a blackbody at the same temperature}}$$

Comparison of black body and real surface emission



Spectral distribution



Directional distribution

REAL SURFACE

Spectral radiation emitted by a real surface – different – Black body (Planck Distribution)

Directional distribution need not be diffuse

Spectral Directional Emissivity (Emission in a given wave length and direction) $\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)$

$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)$ **of a surface at temperature T**

$$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) = \frac{\textit{Intensity of radiation emitted at wavelength } \lambda \textit{ and in the direction of } \theta \textit{ and } \phi}{\textit{Intensity of radiation emitted by a blackbody at the same values of } \lambda \textit{ and } T}$$

$$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{\lambda,b}(\lambda, T)}$$

Spectral directional emissivity $\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)$

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)}$$

Total directional emissivity – Spectral average of $\varepsilon_{\lambda,\theta}\ (\lambda,\theta,\phi,T)$ *i.e.,* $\varepsilon_{\theta}\ (\theta,\phi,T)$

$$\varepsilon_{\lambda,\theta}(\theta,\phi,T) = \frac{I_{e}(\theta,\phi,T)}{I_{b}(T)}$$

Spectral Hemispherical Emissivity $\varepsilon_{\lambda}\ (\lambda,T)$

$$\varepsilon_{\lambda}(\lambda,T) = \frac{E_{\lambda}(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$$

Total Hemispherical Emissivity

$$\varepsilon(T) = \frac{E(T)}{E_{b}(T)}$$

Relation between Spectral Hemispherical Emissivity $\varepsilon_\lambda(\lambda, T)$ and Directional Emissivity $\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)$

$$\varepsilon_\lambda(\lambda, T) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,e}(\lambda, \theta, \phi, T)\sin\theta\cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,b}(\lambda, T)\sin\theta\cos\theta d\theta d\phi}$$

$I_{\lambda,b}(\lambda, T)$ is independent of θ and ϕ

$$\varepsilon_\lambda(\lambda, T) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{\lambda,b}(\lambda, T)}\sin\theta\cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta d\phi}$$

Spectral directional emissivity $\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)$

$$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{\lambda,b}(\lambda, T)}$$

$$\varepsilon_\lambda(\lambda, T) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)\sin\theta\cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta d\phi}$$

Assuming $\varepsilon_{\lambda,\theta}$ is independent of ϕ - reasonable assumption

$$\varepsilon_\lambda(\lambda, T) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T)\sin\theta\cos\theta d\theta d\phi}{2\pi\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta}$$

$$\varepsilon_\lambda(\lambda,T)=\frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)sin\theta cos\theta d\theta d\phi}{2\pi\int_0^{\frac{\pi}{2}}sin\theta cos\theta d\theta}$$

$$\int_0^{\frac{\pi}{2}} sin\theta cos\theta d\theta=\int_0^{\frac{\pi}{2}}\frac{sin2\theta}{2}d\theta=-\frac{cos2\theta}{4}\Big|_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} sin\theta cos\theta d\theta=-\frac{1}{4}[cos\pi-cos0]=-\frac{1}{4}[-1-1]$$

$$\varepsilon_\lambda(\lambda,T)=\frac{2\pi\int_0^{\frac{\pi}{2}}\varepsilon_{\lambda,\theta}(\lambda,\theta,T)sin\theta cos\theta d\theta}{2\pi\frac{1}{2}}$$

$$\varepsilon_\lambda(\lambda,T)=2\int_0^{\frac{\pi}{2}}\varepsilon_{\lambda,\theta}(\lambda,\theta,T)sin\theta cos\theta d\theta$$

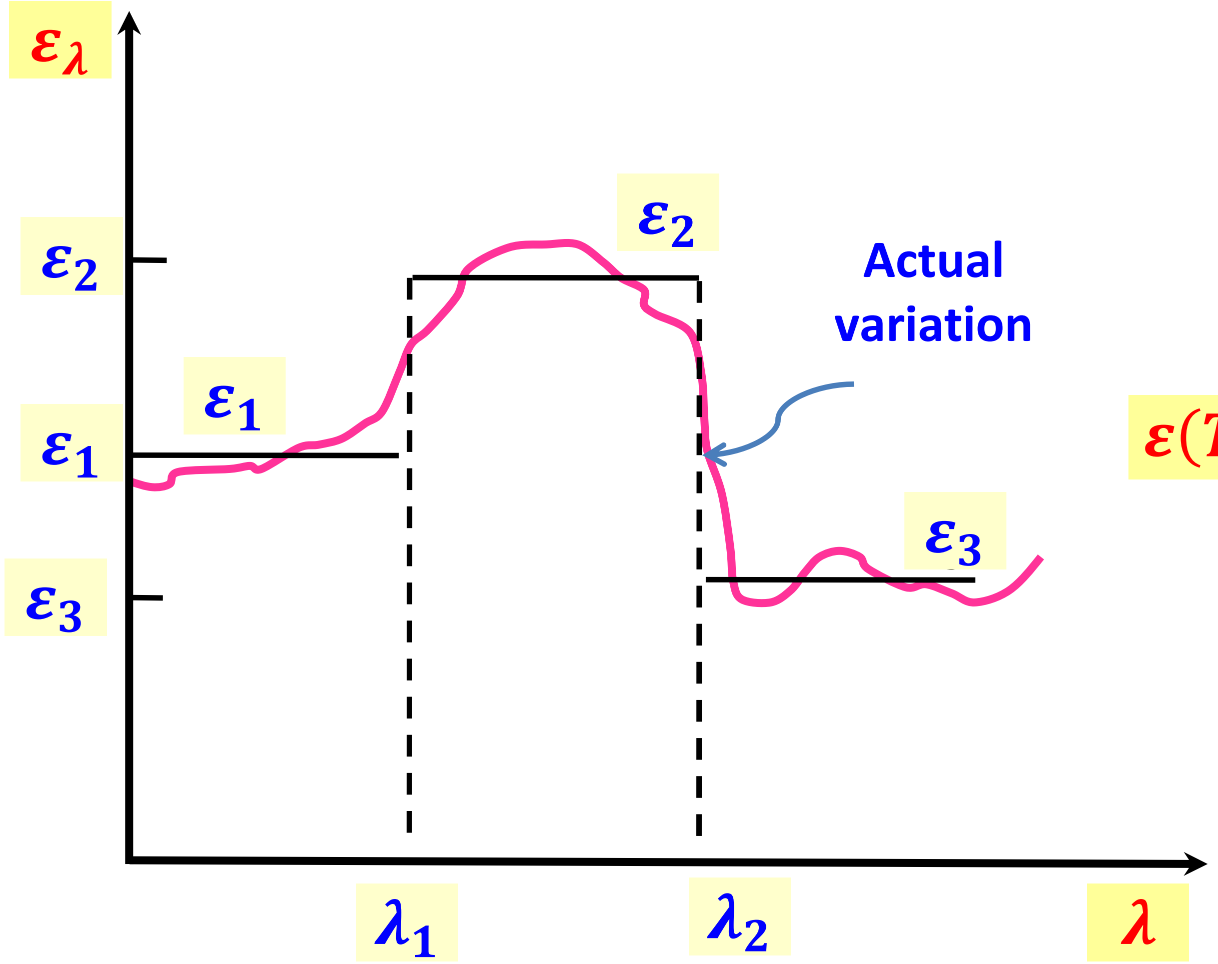
Total Hemispherical Emissivity

$$\varepsilon(T)=\frac{E(T)}{E_b(T)}$$

$$\varepsilon_\lambda(\lambda,T)=\frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$$

$$\varepsilon_\lambda(\lambda,T)=\frac{E(T)}{E_b(T)}=\frac{\int_0^{\infty}\varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)}$$

Emissivity function is known - average emissivity may be computed



$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = \text{constant} & 0 \le \lambda < \lambda_1 \\ \varepsilon_2 = \text{constant} & \lambda_1 \le \lambda < \lambda_2 \\ \varepsilon_3 = \text{constant} & \lambda_2 \le \lambda < \infty \end{cases}$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2 F_{\lambda_1-\lambda_2}(T) + \varepsilon_3 F_{\lambda_2-\infty}(T)$$

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{E_b}$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2 F_{\lambda_1-\lambda_2}(T) + \varepsilon_3 F_{\lambda_2-\infty}(T)$$

Problem: Determine the average emissivity of the surface and its emissive power, if the spectral emissivity function of an opaque surface at 800 K is approximated as

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3 & 0 \le \lambda < 3\mu m \\ \varepsilon_2 = 0.8 & 3\mu m \le \lambda < 7\mu m \\ \varepsilon_3 = 0.1 & 7\mu m \le \lambda < \infty \end{cases}$$

Solution: The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis: The variation of emissivity of the surface with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined by breaking the integral into three parts

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2 F_{\lambda_1-\lambda_2}(T) + \varepsilon_3 F_{\lambda_2-\infty}(T)$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2 \left(F_{0-\lambda_2}(T) - F_{0-\lambda_1}(T)\right) + \varepsilon_3 \left(1 - F_{0-\lambda_2}(T)\right)$$

$$\lambda_1 T = 3\mu m \times 800K = 2400\mu m.K: \quad F_{0-3} = 0.140256$$
$$\lambda_2 T = 7\mu m \times 800K = 5600\mu m.K: \quad F_{0-7} = 0.701046$$

$$\varepsilon(T) = 0.3(0.140256) + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)$$

$$\varepsilon(T) = 0.521$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2\left(F_{0-\lambda_2}(T) - F_{0-\lambda_1}(T)\right) + \varepsilon_3\left(1 - F_{0-\lambda_2}(T)\right)$$

$$\lambda_1 T = 3\mu m \times 800K = 2400\mu m.K: \quad F_{0-3} = 0.140256$$
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$$\varepsilon(T) = 0.3(0.140256) + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)$$

$$\varepsilon(T) = 0.521$$

The surface will emit as much radiation at 800 K as a gray surface having a constant emissvity of $\varepsilon(T) = 0.521$

The emissive power of the surface is

$$E = \varepsilon\sigma T^4 = 0.521$$

$$E = \varepsilon\sigma T^4 = 0.521 \times 5.67 \times 10^{-8}(800)^4 = 12100 \qquad E = 12100\, W/m^2$$

Comments: The surface will emit 12.1 kJ of radiation energy per second per m^2 area of the surface.

Problem: Measurements of the spectral, directional emissivity of a metallic surface at temperature T = 2000 K and λ = 1.0 μm yield a directional distribution that may be approximated as

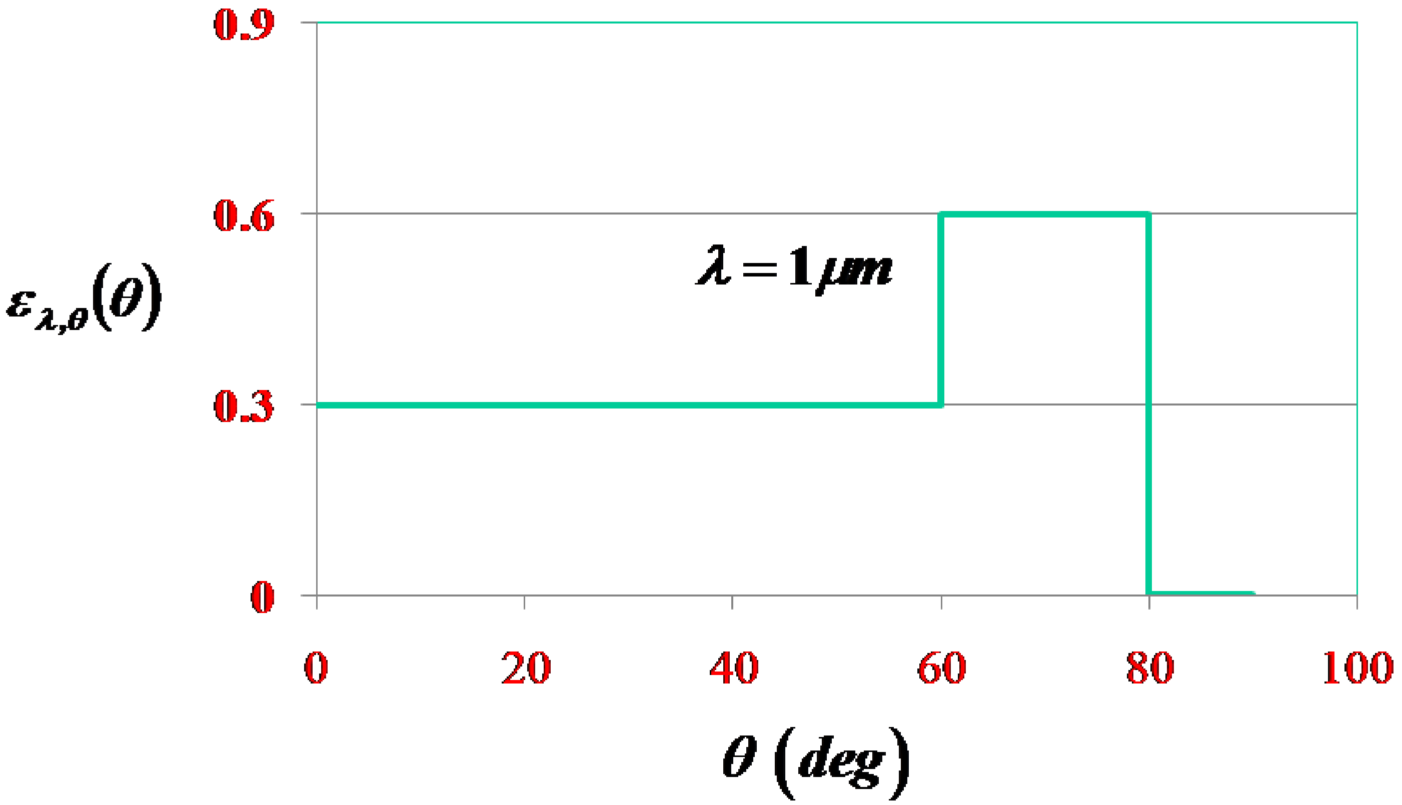
Determine

1. corresponding values of the spectral, normal emissivity; the spectral hemispherical emissivity;
2. the spectral intensity of radiation emitted in the normal direction; and the spectral emissive power.

Known: directional distribution of $\varepsilon_{\lambda,\theta}$ at λ = 1 μm for a metallic surface at 2000 K

Find: 1. Spectral, normal emissivity $\varepsilon_{\lambda,n}$ and spectral, hemispherical emissivity ε_{λ}

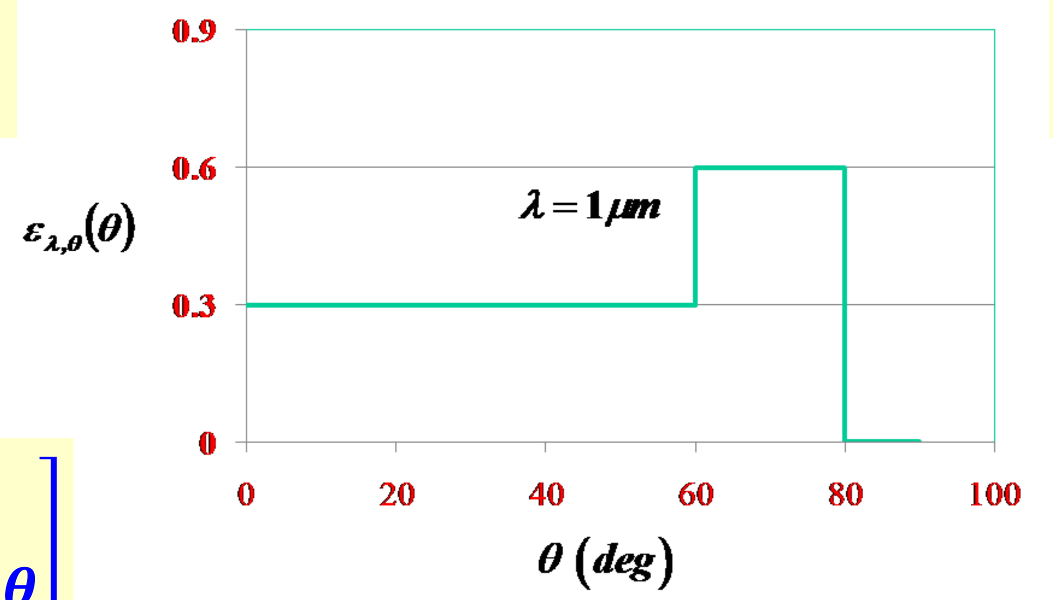
2. Spectral, normal intensity $I_{\lambda,n}$ and spectral, emissive power E_{λ}



From the measurement of $\varepsilon_{\lambda,\theta}$ at λ =1 μm, we see that

$$\varepsilon_{\lambda,n} = \varepsilon_{\lambda,\theta} = \varepsilon_{\lambda,\theta}(1\ \mu m, 0^o) = 0.3$$

The spectral, hemispherical emissivity is



$$\varepsilon_\lambda(1\mu m) = 2\int_0^{\frac{\pi}{2}} \varepsilon_{\lambda,\theta} sin\theta cos\theta d\theta$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3\int_0^{\frac{\pi}{3}} sin\theta cos\theta d\theta + 0.63\int_{\frac{\pi}{3}}^{\frac{4\pi}{9}} sin\theta cos\theta d\theta\right]$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3\int_0^{\frac{\pi}{3}} \frac{sin2\theta}{2} d\theta + 0.63\int_{\frac{\pi}{3}}^{\frac{4\pi}{9}} \frac{sin2\theta}{2} d\theta\right]$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3 \times -\frac{cos2\theta}{4}\bigg|_0^{\frac{\pi}{3}} + 0.63 \times -\frac{cos2\theta}{4}\bigg|_{\frac{\pi}{3}}^{\frac{4\pi}{9}}\right]$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3 \times -\frac{\cos 2\theta}{4}\Big|_0^{\frac{\pi}{3}} + 0.63 \times -\frac{\cos 2\theta}{4}\Big|_{\frac{\pi}{3}}^{\frac{4\pi}{9}}\right]$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3 \times -0.25\left(\cos\frac{2\pi}{3} - 1\right) + 0.63 \times -0.25\left(\cos\frac{8\pi}{9} - \cos\frac{2\pi}{3}\right)\right]$$

$$\varepsilon_\lambda(1\mu m) = 2\left[0.3 \times -0.25(-0.5 - 1) + 0.63 \times -0.25\left(\cos\frac{8\pi}{9} + 0.5\right)\right]$$

$$\varepsilon_\lambda(1\mu m) = 2[0.1125 + 0.63 \times -0.25(-0.9397 + 0.5)]$$

$$\varepsilon_\lambda(1\mu m) = 2[0.1125 + 0.06925]$$

$$\varepsilon_\lambda(1\mu m) = 0.3635$$

The spectral intensity of radiation emitted at λ =1 μm in the normal direction is

$$I_{\lambda,n}(1\mu m, 0^o, 2000K) = \varepsilon_{\lambda,\theta}(1\mu m, 0^o) I_{\lambda,b}(1\mu m, 2000K)$$

$\varepsilon_{\lambda,\theta}(1\mu m, 0^o) = 0.3$ from figure

$I_{\lambda,b}(1\mu m, 2000K)$ is obtained from Table

$$\lambda T = 2000\mu m.K, \qquad \frac{I_{\lambda,b}}{\sigma T^5} = 0.493 \times 10^{-4}(\mu m.K.sr)^{-1}$$

$$I_{\lambda,b} = 0.493 \times 10^{-4}(\mu m.K.sr)^{-1} \times 5.67 \times 10^{-8}\left(\frac{W}{m^2K^4}\right)(2000K)^4$$

$$I_{\lambda,b} = 8.95 \times 10^4\left(\frac{W}{m^2\mu m.sr}\right)$$

$$I_{\lambda,n}(1\mu m, 0^o, 2000K) = \varepsilon_{\lambda,\theta}(1\mu m, 0^o) I_{\lambda,b}(1\mu m, 2000K)$$

$$I_{\lambda,n}(1\mu m, 0^o, 2000K) = 0.3 \times 8.95 \times 10^4$$

$$I_{\lambda,n}(1\mu m, 0^o, 2000K) = 2.69 \times 10^4 \frac{W}{m^2\mu m.sr}$$

The spectral emissive power for λ = 1 μm and T = 2000 K

$$I_{\lambda,b} = 8.95 \times 10^4 \left(\frac{W}{m^2 \mu m.sr}\right)$$

$$\varepsilon_\lambda(1\mu m) = 0.3635$$

$$E_\lambda(1\mu m, 2000K) = \varepsilon_\lambda(1\mu m)E_{\lambda,b}(1\mu m, 2000K)$$

$$E_{\lambda,b}(1\mu m, 2000K) = \pi I_{\lambda,b}(1\mu m, 2000K)$$

$$E_{\lambda,b}(1\mu m, 2000K) = \pi sr \times 8.95 \times 10^4 \frac{W}{m^2 \mu m.sr}$$

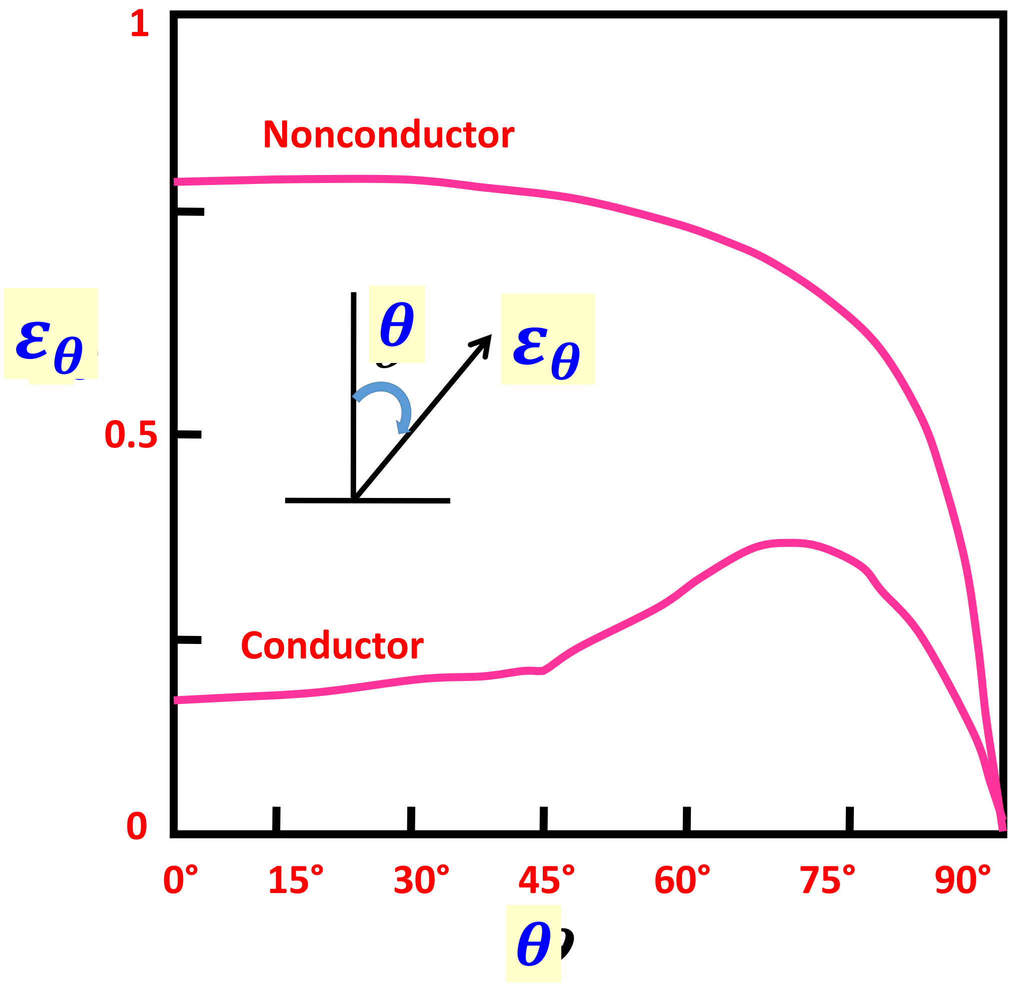
$$E_{\lambda,b}(1\mu m, 2000K) = 2.81 \times 10^5 \frac{W}{m^2 \mu m}$$

$$E_\lambda(1\mu m, 2000K) = \varepsilon_\lambda(1\mu m)E_{\lambda,b}(1\mu m, 2000K)$$

$$E_{\lambda,b}(1\mu m, 2000K) = 0.3635 \times 2.81 \times 10^5 \frac{W}{m^2 \mu m}$$

$$E_{\lambda,b}(1\mu m, 2000K) = 1.01 \times 10^5 \frac{W}{m^2 \mu m}$$

Real surfaces do not emit radiation in a perfectly diffuse manner as a blackbody does, they often come close



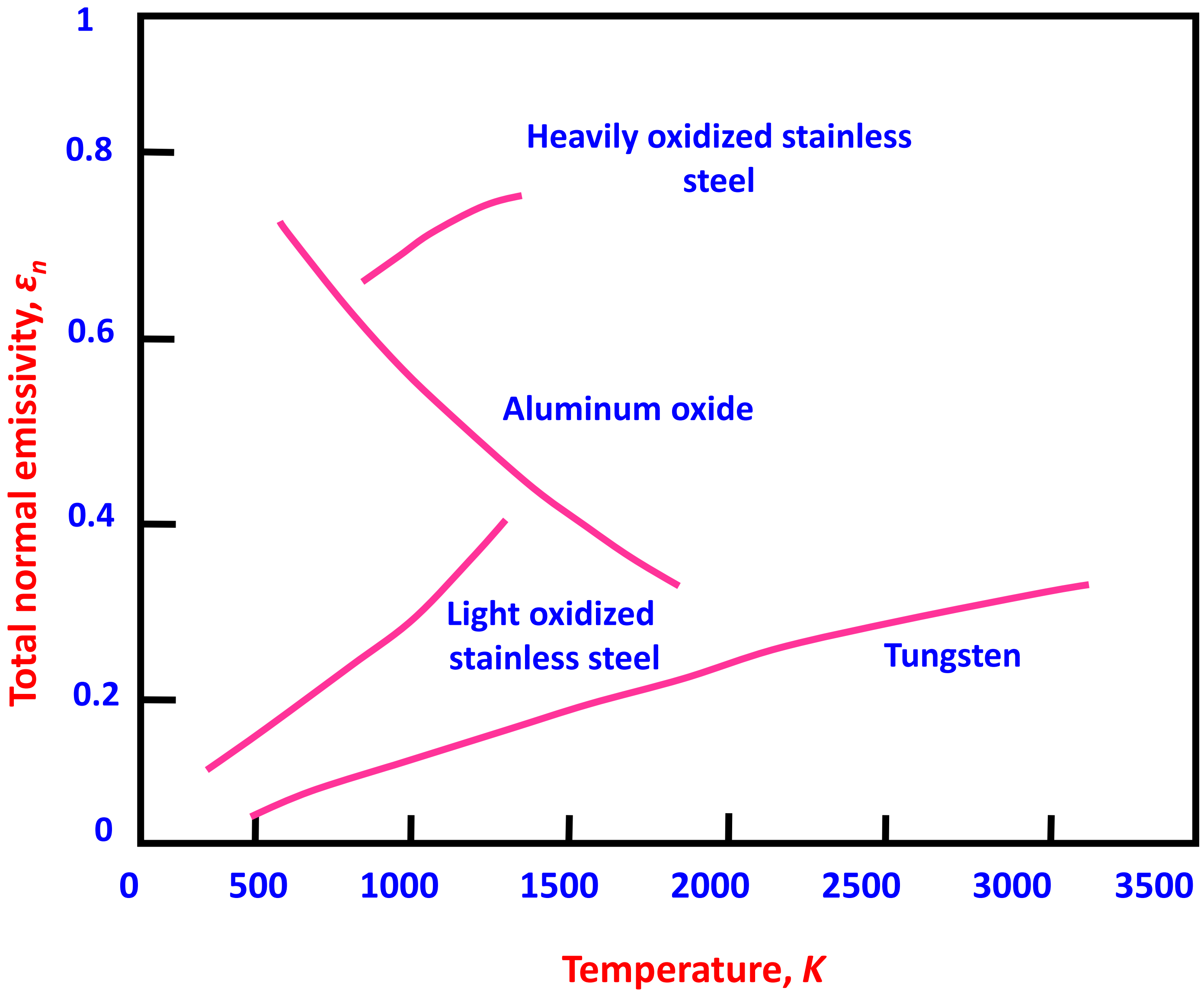
Representative directional distribution of the total, directional emissivity

Directional emissivity of a surface in the normal direction is representative of the hemispherical emissivity of the surface

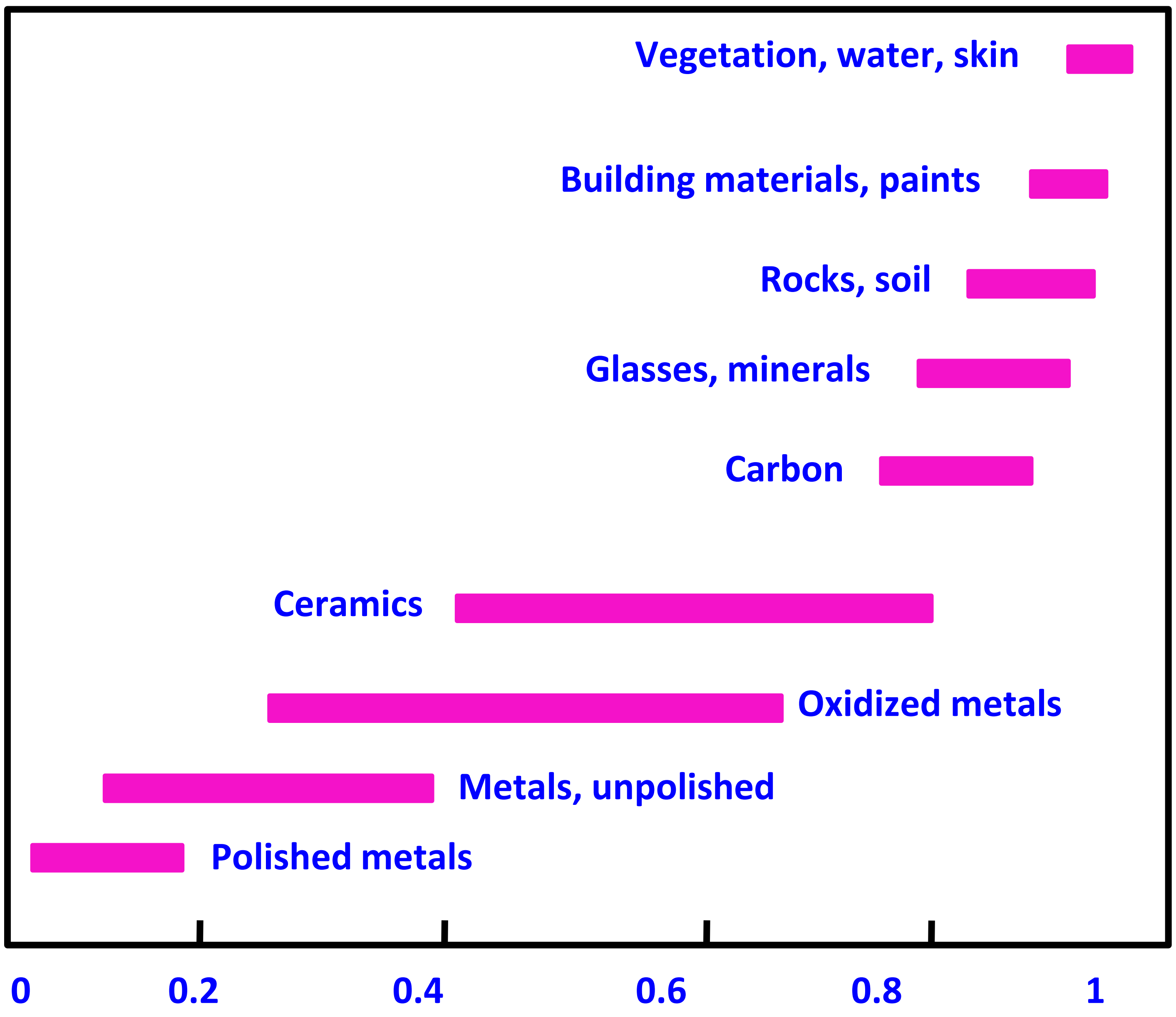
It is common practice to assume the surfaces to be diffuse emitters with an emissivity equal to the value in the normal direction $(\theta = 0^o)$

$$\varepsilon = \varepsilon_n$$

Temperature dependence of the total, normal emissivity of ε_n



TYPICAL RANGES OF EMISSIVITIES FOR VARIOUS MATERIALS



Emissivity of metallic surfaces is generally small, achieving values as low as 0.02 for highly polished gold and silver

The presence of oxide layers may significantly increase the emissivity of metallic surfaces. Stainless steel has emissivity of 0.3 and 0.7 at 900 K depending on whether it is polished or heavily oxidized

Emissivity of non-conductors is comparatively large, generally exceeding 0.6.

Emissivity of conductors increases with increasing temperature. However, depending on the specific material, the emissivity of non-conductors may either increase or decrease with increasing temperature

Variation of ε_n with T is consistent with the spectral distribution of $\varepsilon_{\lambda,n}$
Although the spectral distribution of $\varepsilon_{\lambda,n}$ is approximately independent of temperature, there is proportionately more emission at lower wavelengths with increasing temperature.
Hence, if $\varepsilon_{\lambda,n}$ increases with decreasing wavelength for a particular material, ε_n will increase with increasing temperature of that material

SURFACE ABSORPTION, REFLECTION AND TRANSMISSION

Spectral Irradiation G_λ (W/m^2.μm) – Rate at which radiation of wavelength λ is incident on a surface per unit area of the surface and per unit wavelength $d\lambda$ about λ

It may be incident from all possible directions and it may originate from several different sources

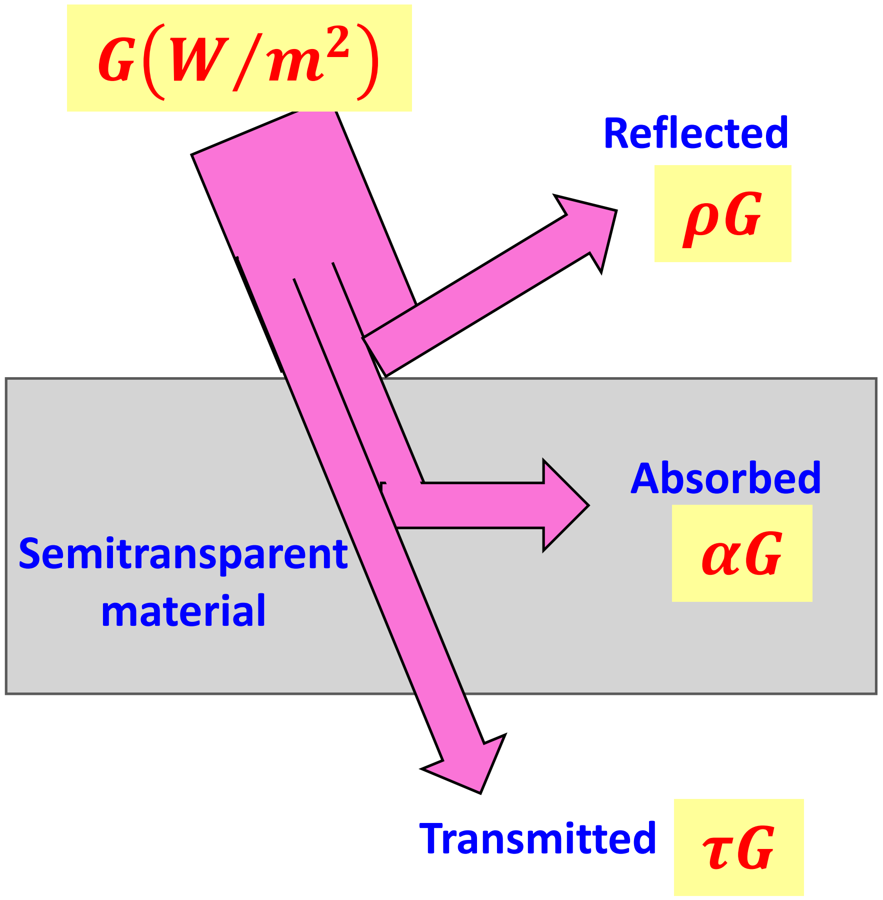
Total Irradiation G(W/m^2) consists of all spectral contributions

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda$$

Generally, Irradiation interacts with a semi-transparent medium such as layer of water or glass plate

Spectral component of irradiation – Reflected, absorbed, transmitted

ABSORPTIVITY (α), REFLECTIVITY (ρ) AND TRANSMISSIVITY (τ)



$$\alpha = \frac{\text{Absorbed Radiation}}{\text{Incident Radiation}} = \frac{G_{abs}}{G} \qquad 0 \le \alpha \le 1$$

$$\rho = \frac{\text{Reflected Radiation}}{\text{Incident Radiation}} = \frac{G_{ref}}{G} \qquad 0 \le \rho \le 1$$

$$\tau = \frac{\text{Transmitted Radiation}}{\text{Incident Radiation}} = \frac{G_{trans}}{G} \qquad 0 \le \tau \le 1$$

G - Radiation incident on the surface

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted.

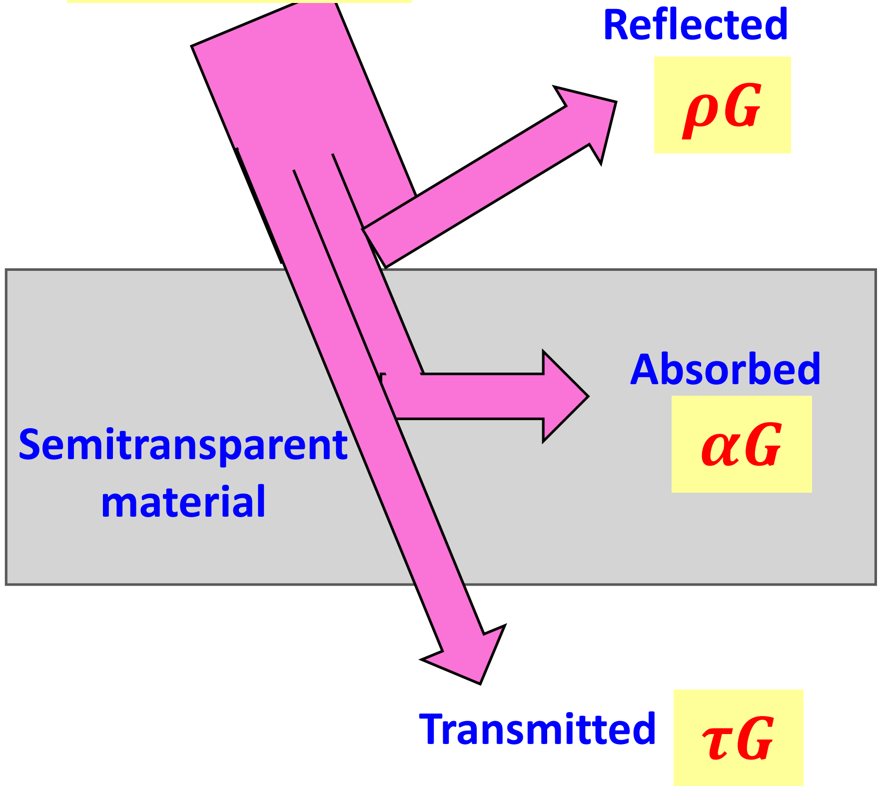
Absorptivity α - fraction of irradiation absorbed by the surface

Reflectivity ρ - fraction of irradiation reflected by the surface

Transmissivity τ - fraction of irradiation transmitted by the surface

ABSORPTIVITY (α), REFLECTIVITY (ρ) AND TRANSMISSIVITY (τ)

$G(W/m^2)$



$$\alpha = \frac{\text{Absorbed Radiation}}{\text{Incident Radiation}} = \frac{G_{abs}}{G} \qquad 0 \le \alpha \le 1$$

$$\rho = \frac{\text{Reflected Radiation}}{\text{Incident Radiation}} = \frac{G_{ref}}{G} \qquad 0 \le \rho \le 1$$

$$\tau = \frac{\text{Transmitted Radiation}}{\text{Incident Radiation}} = \frac{G_{trans}}{G} \qquad 0 \le \tau \le 1$$

G - Radiation incident on the surface

$$G_{abs} + G_{ref} + G_{trans} = G$$

$$\frac{G_{abs}}{G} + \frac{G_{ref}}{G} + \frac{G_{trans}}{G} = 1$$

$$\alpha + \rho + \tau = 1$$

For opaque surfaces

$$\tau = 0 \qquad \alpha + \rho = 1$$

These definitions are for total hemispherical properties, since G represents the radiation flux incident on the surface from all the directions over the hemispherical space and over all wavelengths

α, ρ and τ are the average properties of a medium for all directions

Absorptivity – a property that determines the fraction of the irradiation absorbed by a surface

SPECTRAL DIRECTIONAL ABSORPTIVITY

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi)=\frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

SPECTRAL HEMISPHERICAL ABSORPTIVITY

$$\alpha_{\lambda}(\lambda)=\frac{G_{\lambda,abs}(\lambda)}{G_{\lambda}(\lambda)}$$

Spectral directional absorptivity, $\alpha_{\lambda,\theta}(\lambda,\theta,\phi)$ – fraction of the spectral intensity incident in the direction of θ and ϕ that is absorbed by the surface

Dependence of absorptivity on the surface temperature - neglected

SPECTRAL HEMISPHERICAL ABSORPTIVITY

$$\alpha_{\lambda}(\lambda)=\frac{G_{\lambda,abs}(\lambda)}{G_{\lambda}(\lambda)}=\frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi)sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}}I_{\lambda,i}(\lambda,\theta,\phi)sin\theta cos\theta d\theta d\phi}$$

SPECTRAL HEMISPHERICAL ABSORPTIVITY

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}$$

$\alpha_\lambda(\lambda)$ depends on

- Directional distribution of the incident radiation
- Wavelength of the incident radiation
- Nature of the absorbing surface

If incident radiation is diffusely distributed and $\alpha_{\lambda,\theta}$ is independent of ϕ

$$G_\lambda(\lambda) = \pi I_{\lambda,i}$$

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)} = \frac{2\pi I_{\lambda,i}(\lambda)\int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta}{\pi I_{\lambda,i}(\lambda)}$$

$$\alpha_\lambda(\lambda) = 2\int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta$$

Total Hemispherical absorptivity, α - integrated average over both direction and wavelength

$$\alpha_\lambda(\lambda) = \frac{G_{abs}}{G} = \frac{\int_0^\infty \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi d\lambda}{\int_0^\infty \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi\, d\lambda}$$

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)} = \frac{\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

α depends on

Spectral Distribution of the incident radiation

Directional distribution of the incident radiation

Nature of the absorbing surface

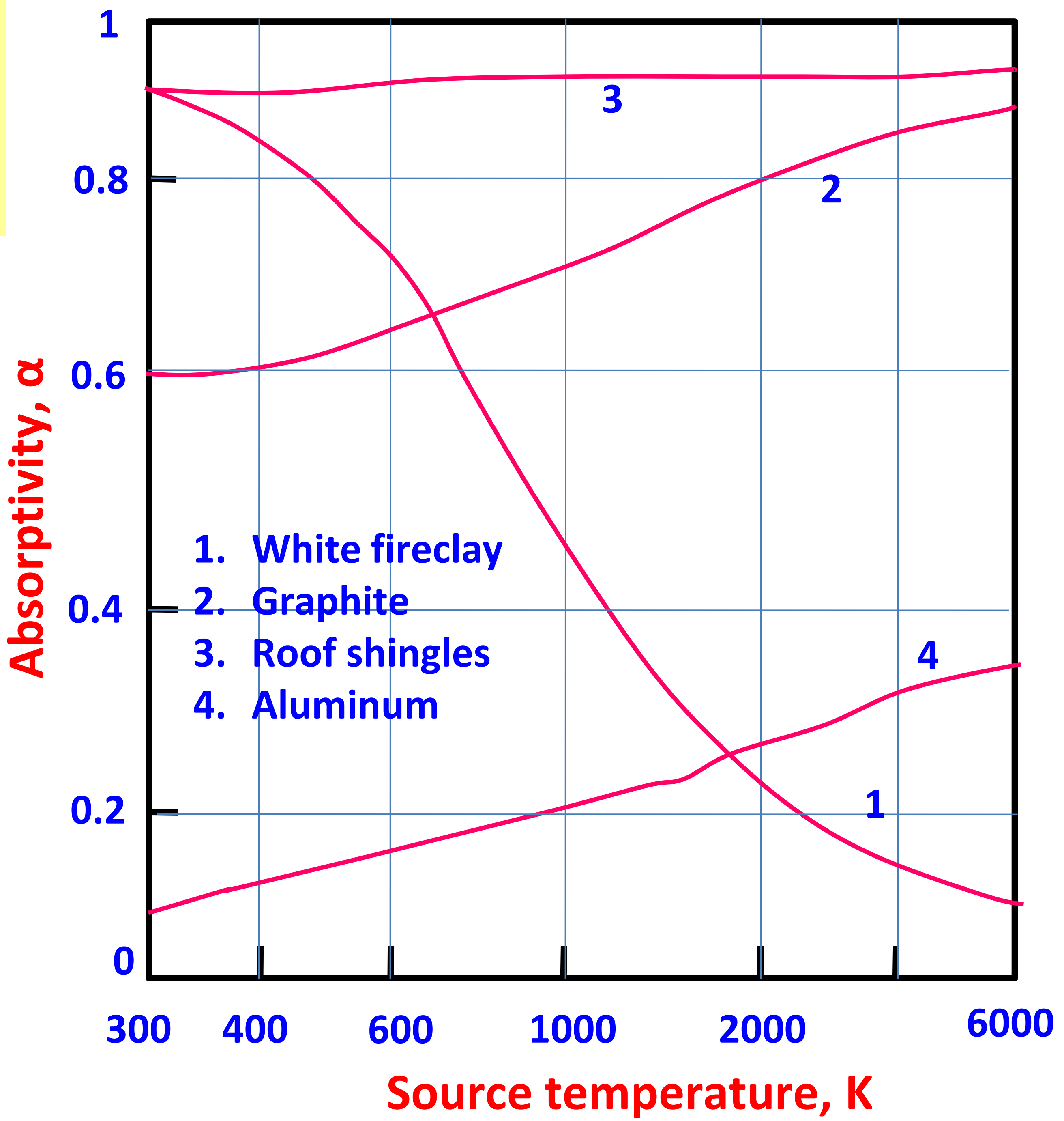
α is approximately independent of temperature of receiving surface

ε is dependent of temperature of receiving surface

Variation of absorptivity with the temperature of the source of irradiation for various common materials at room temperature

Unlike emissivity, absorptivity of a material is practically independent of surface temperature

Absorptivity depends strongly on the temperature of the source at which the incident radiation is originating

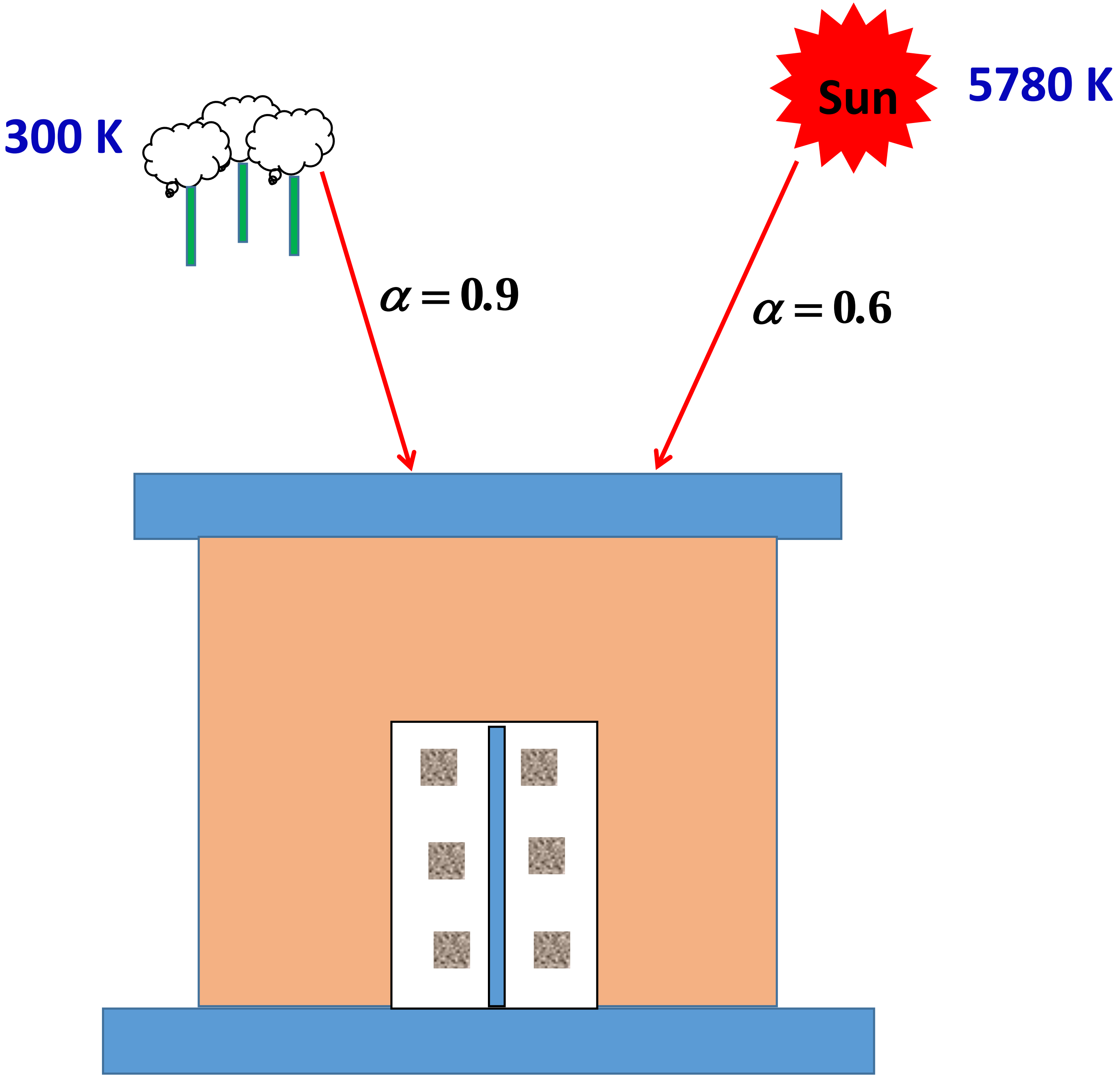


Absorptivity depends strongly on the temperature of the source at which the incident radiation is originating

Absorptivity of the concrete roof of a house is about

0.6 for solar radiation (source temperature: 5780 K) and

0.9 for radiation originating from the surrounding trees and buildings (source temperature: 300 K)



Reflectivity – a property that determines the fraction of the irradiation reflected by a surface

Reflectivity is inherently bidirectional

- Depends on the direction of the incident radiation
- Depends on the direction of the reflected radiation

SPECTRAL DIRECTIONAL REFLECTIVITY, $\rho_{\lambda,\theta}(\lambda,\theta,\phi)$ – fraction of the spectral intensity incident in the direction of θ and ϕ that is reflected by the surface

$$\rho_{\lambda,\theta}(\lambda,\theta,\phi)=\frac{I_{\lambda,i,ref}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

Dependence of reflectivity on the surface temperature - neglected

Spectral hemispherical reflectivity, $\rho_{\lambda}(\lambda)$ –

$$\rho_{\lambda}(\lambda)=\frac{G_{\lambda,ref}(\lambda)}{G_{\lambda}(\lambda)}$$

SPECTRAL HEMISPHERICAL REFLECTIVITY $\rho_\lambda(\lambda)$

$$\rho_\lambda(\lambda) = \frac{G_{\lambda,ref}(\lambda)}{G_\lambda(\lambda)}$$

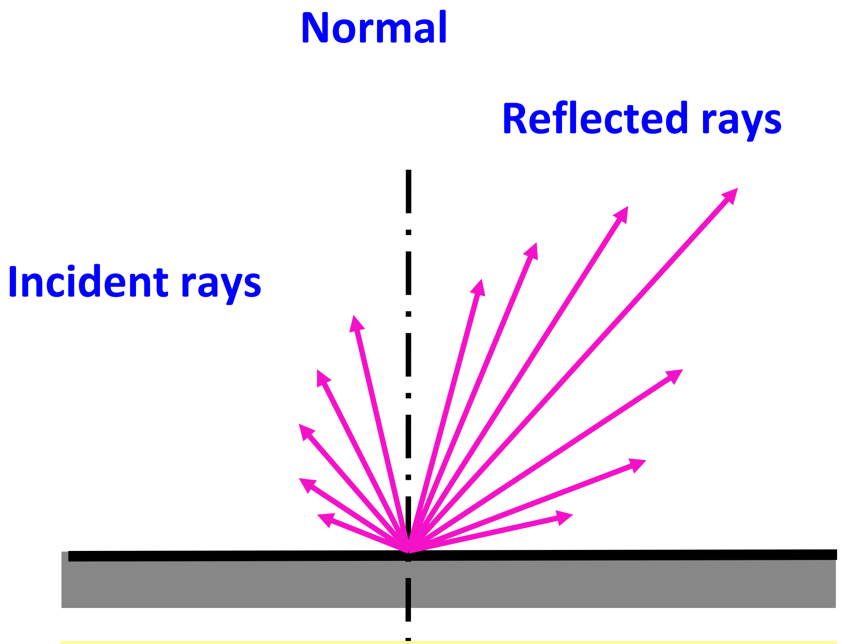
$$\rho_\lambda(\lambda) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \rho_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}$$

Total Hemispherical reflectivity , ρ - integrated average over both direction and wavelength

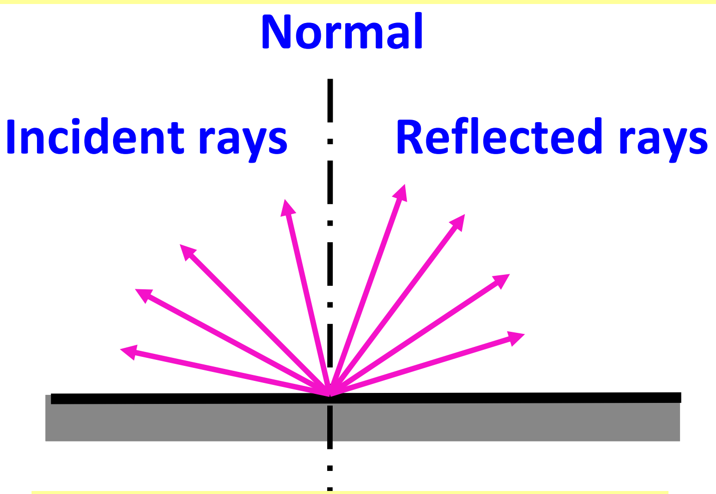
$$\rho = \frac{G_{ref}}{G} = \frac{\int_0^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \rho_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi d\lambda}{\int_0^{\infty}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi\, d\lambda}$$

$$\rho = \frac{\int_0^{\infty} \rho_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^{\infty} G_\lambda(\lambda) d\lambda}$$

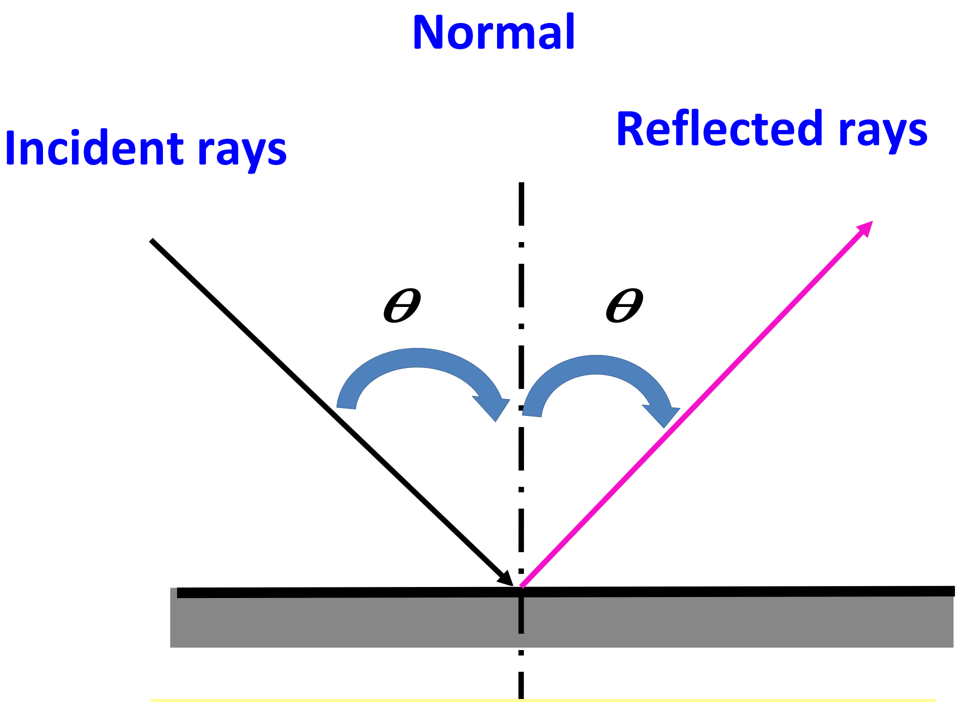
Reflectivity is inherently bidirectional



Real surface



Diffuse surface



Specular surface

Specular surface - angle of reflection is equal to angle of incidence of the radiation beam

Ex: smooth and polished surfaces

Smooth surface: surface roughness is smaller than the wavelength of the incident radiation

Diffuse surface – radiation is reflected equally in all directions

Ex: Rough surfaces

Transmissivity – a property that determines the fraction of the irradiation transmitted through the surface

SPECTRAL HEMISPHERICAL TRANSMISSIVITY $\tau_\lambda(\lambda)$

$$\tau_\lambda(\lambda) = \frac{G_{\lambda,tr}(\lambda)}{G_\lambda(\lambda)}$$

Total Hemispherical transmissivity, τ - integrated average over both direction and wavelength

$$\tau = \frac{G_{tr}}{G}$$

Relation between τ and $\tau_\lambda(\lambda)$

$$\tau = \frac{\int_0^\infty G_{\lambda,tr}(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda} = \frac{\int_0^\infty \tau_\lambda(\lambda)\,G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

MEDIUM IS OPAQUE TO THE INCIDENT RADIATION

$G_{\lambda,tr}(\lambda) = 0$ – absorption and reflection – surface phenomena

NO net effect of the reflection process on the medium

ABSORPTION has the effect of increasing the internal thermal energy of the medium

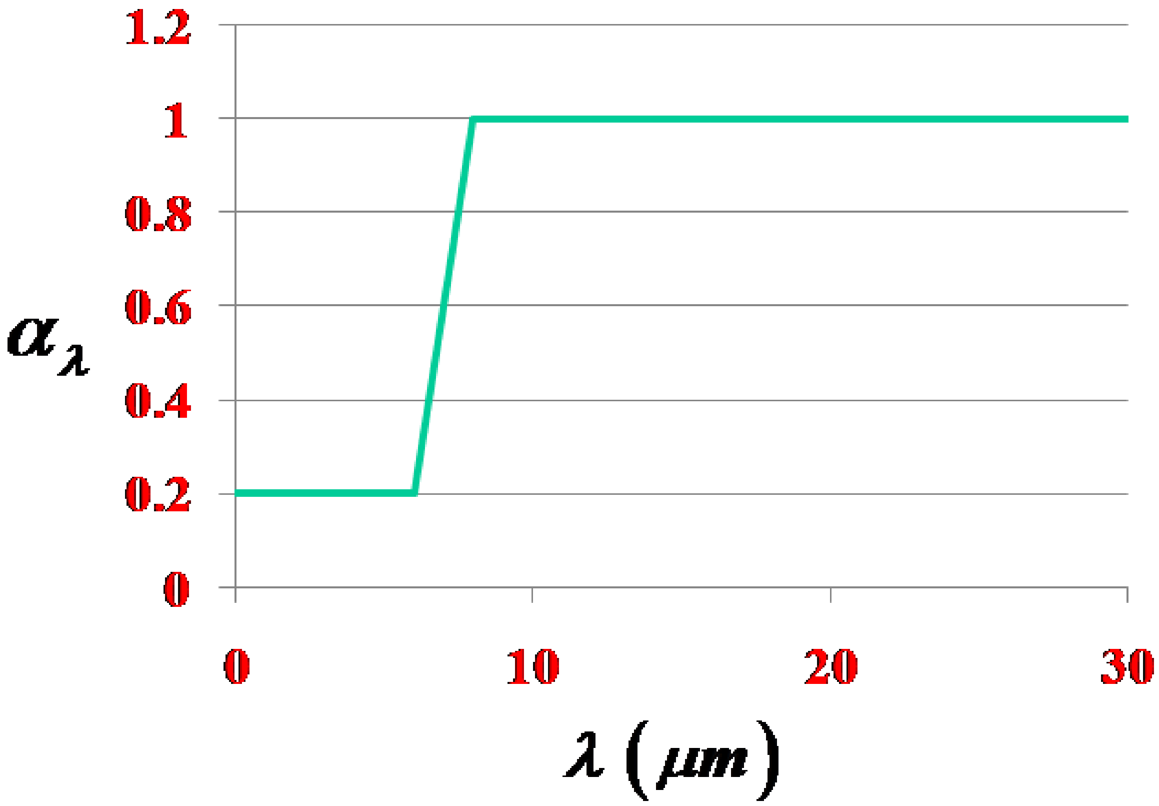
PERCEPTION OF COLOUR

Surface temperature < 1000 K – Emission is in infra-red region – not seen by eye

Colour is due to selctive reflection and absorption of the visible portion of the irradiation that is incident from the sun or an artificial source of light

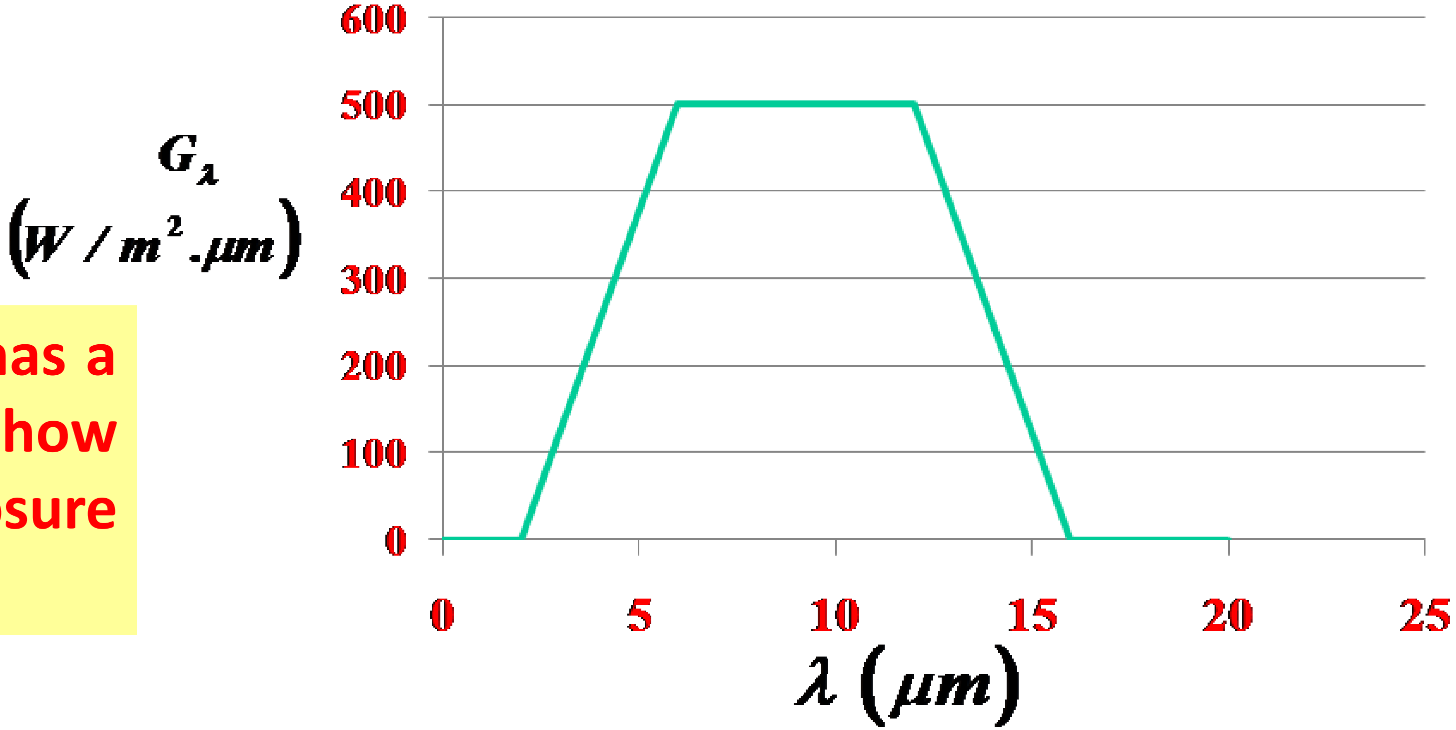
Leaf is green – Chlorophyll, - absorbs the blue and red and preferential reflection in the green

COLOUR OF THE SURFACE – Not an indicator of the overall capacity of an absorber or reflector, since much of the irradiation may be in the IR region



The spectral, hemispherical absorptivity of an opaque surface and the spectral irradiation at the surface are as shown.

How does the spectral, hemispherical reflectivity vary with wavelength ? What is the total, hemispherical absorptivity of the surface ?



If the surface is initially at 500K and has a total,hemispherical emissivity of 0.8, how will its temperature change upon exposure to the irradiation ?

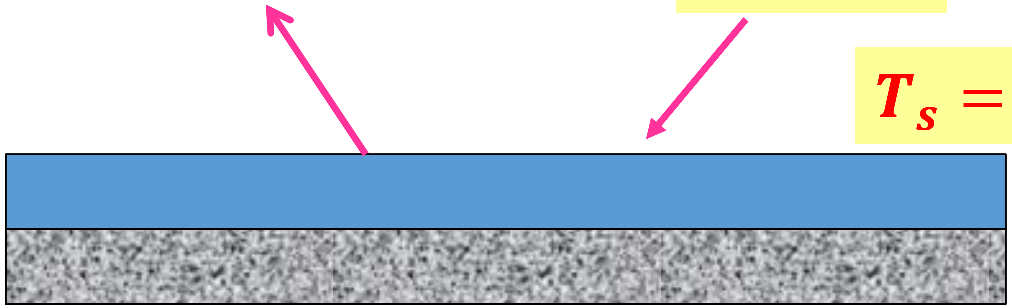
KNOWN: Spectral, hemispherical absorptivity and irradiation of a surface. Surface temperature (500K) and total, hemispherical emissivity (0.8)

FIND:
1. Spectral distribution of reflectivity
2. Total, hemispherical absorptivity
3. Nature of surface temperature change.

$E = \varepsilon E_b$

αG

$T_s = 500\ K \quad \varepsilon = 0.8$



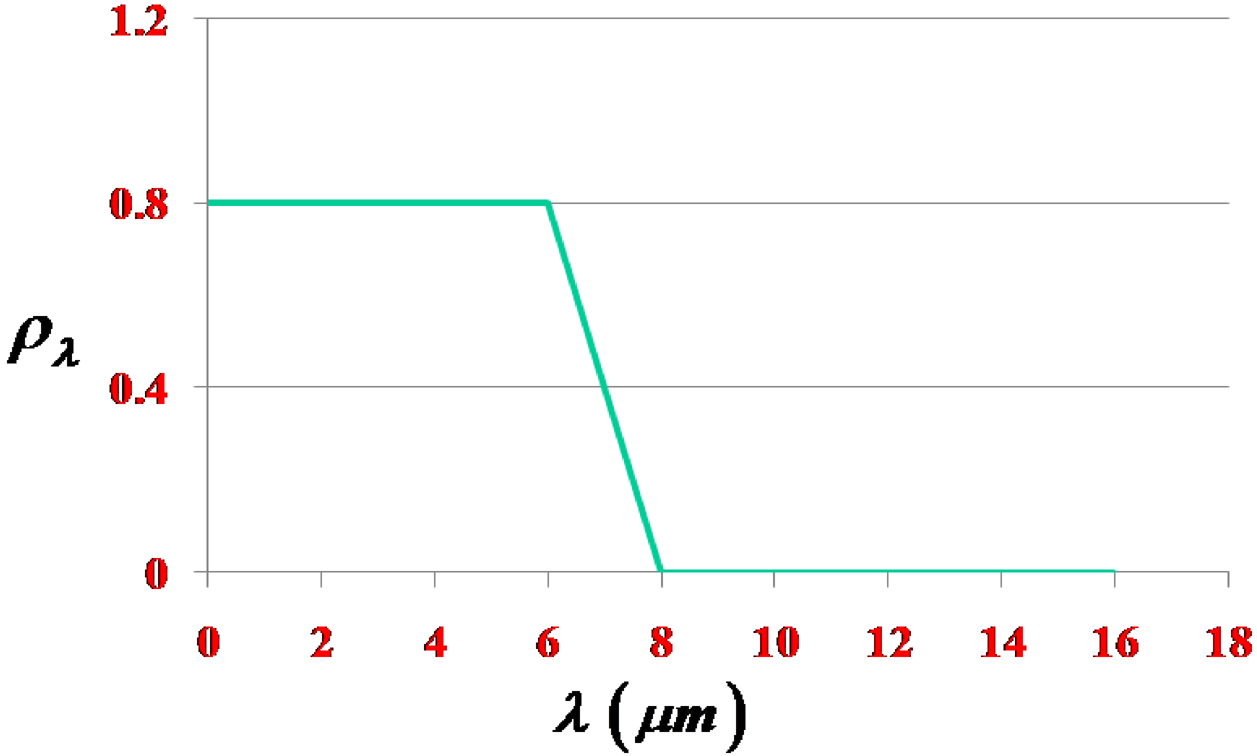
Assumptions

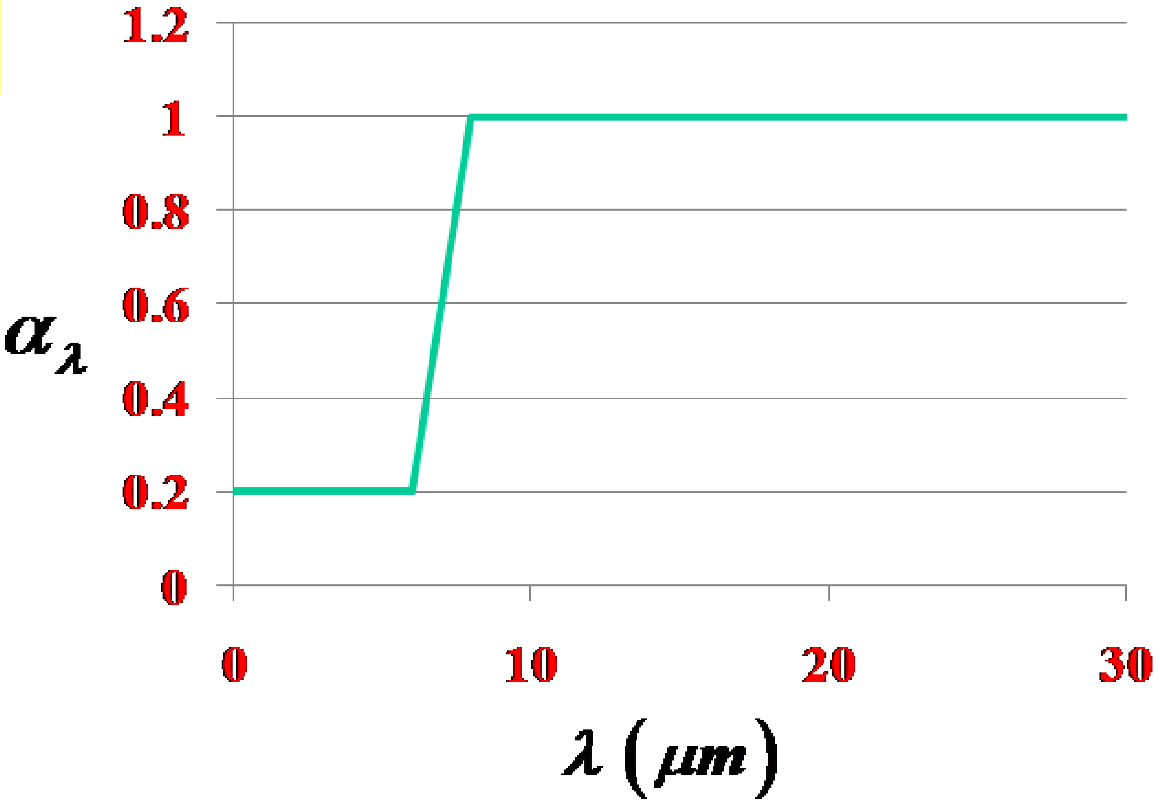
1. Surface is opaque $(\tau = 0)$
2. Surface convection effects are negligible
3. Back surface is insulated

$$\alpha + \rho + \tau = 1$$

$$\rho = 1 - \alpha \qquad \tau = 0$$

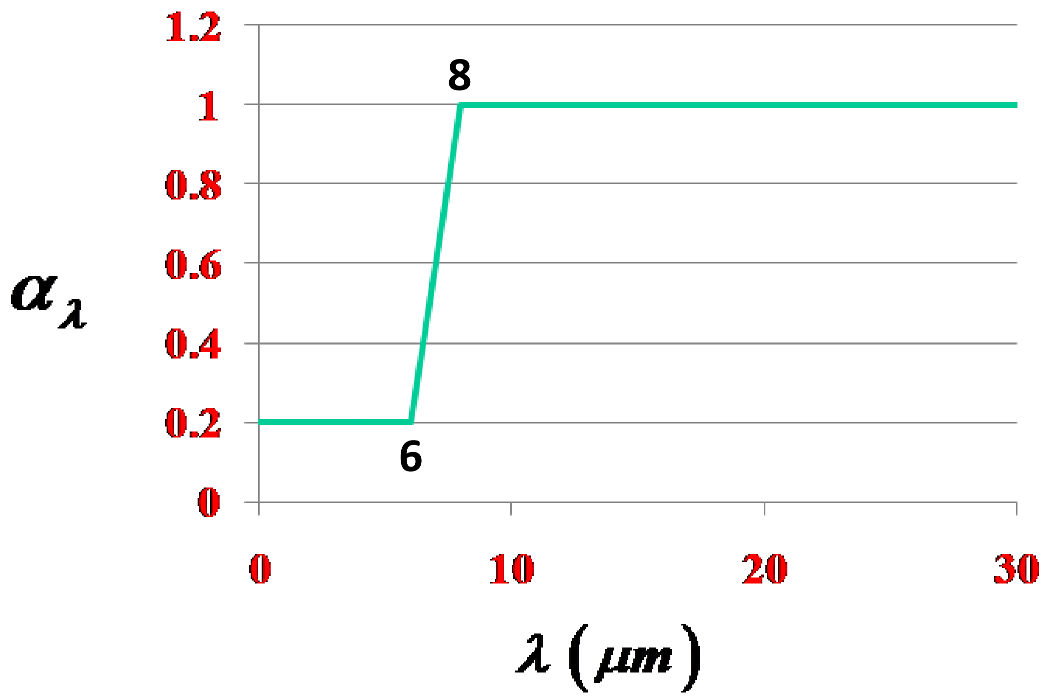
On a spectral basis: Thus a plot of ρ_λ can be made

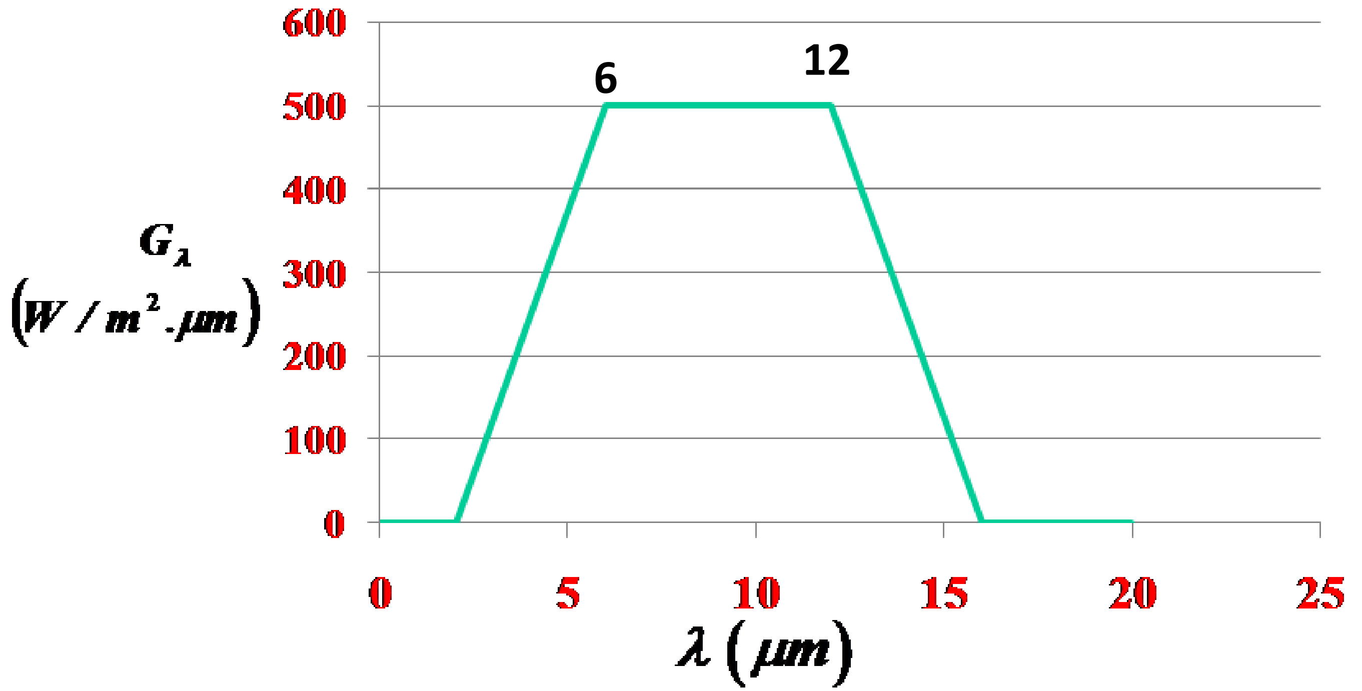




$$\rho_\lambda = 1 - \alpha_\lambda$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$





$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\,G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

$$\alpha = \frac{0.2\int_2^6 G_\lambda(\lambda)d\lambda + 500\int_6^8 \alpha_\lambda(\lambda)d\lambda + 1.0\int_8^{16} G_\lambda(\lambda)d\lambda}{\int_2^6 G_\lambda(\lambda)d\lambda + \int_6^{12} \alpha_\lambda(\lambda)d\lambda + 1.0\int_{12}^{16} G_\lambda(\lambda)d\lambda}$$

$$\alpha = \frac{0.2\left[\frac{1}{2}(500)(6-2)\right] + 500\left[0.2(8-6) + \frac{1}{2}(1-0.2)(8-6)\right] + \left[1\times 500(12-8) + 1\left(\frac{1}{2}\right)500(16-12)\right]}{\frac{1}{2}(500)(6-2) + 500(12-6) + \left(\frac{1}{2}\right)500(16-12)}$$

$$\alpha = \frac{200+600+3000}{1000+3000+1000} = \frac{3800}{5000} = 0.76$$

$$\alpha = 0.76$$

Neglecting convection effects, the net heat flux to the surface is

$$q''_{net} = \alpha G - E = \alpha G - \varepsilon\sigma T^4$$

$$q''_{net} = 0.76(5000) - 0.8\left(5.67 \times 10^{-8}\right)(500)^4$$

$$q''_{net} = 3800 - 2835 = 965\ W/m^2$$

$$q''_{net} = 965\ W/m^2$$

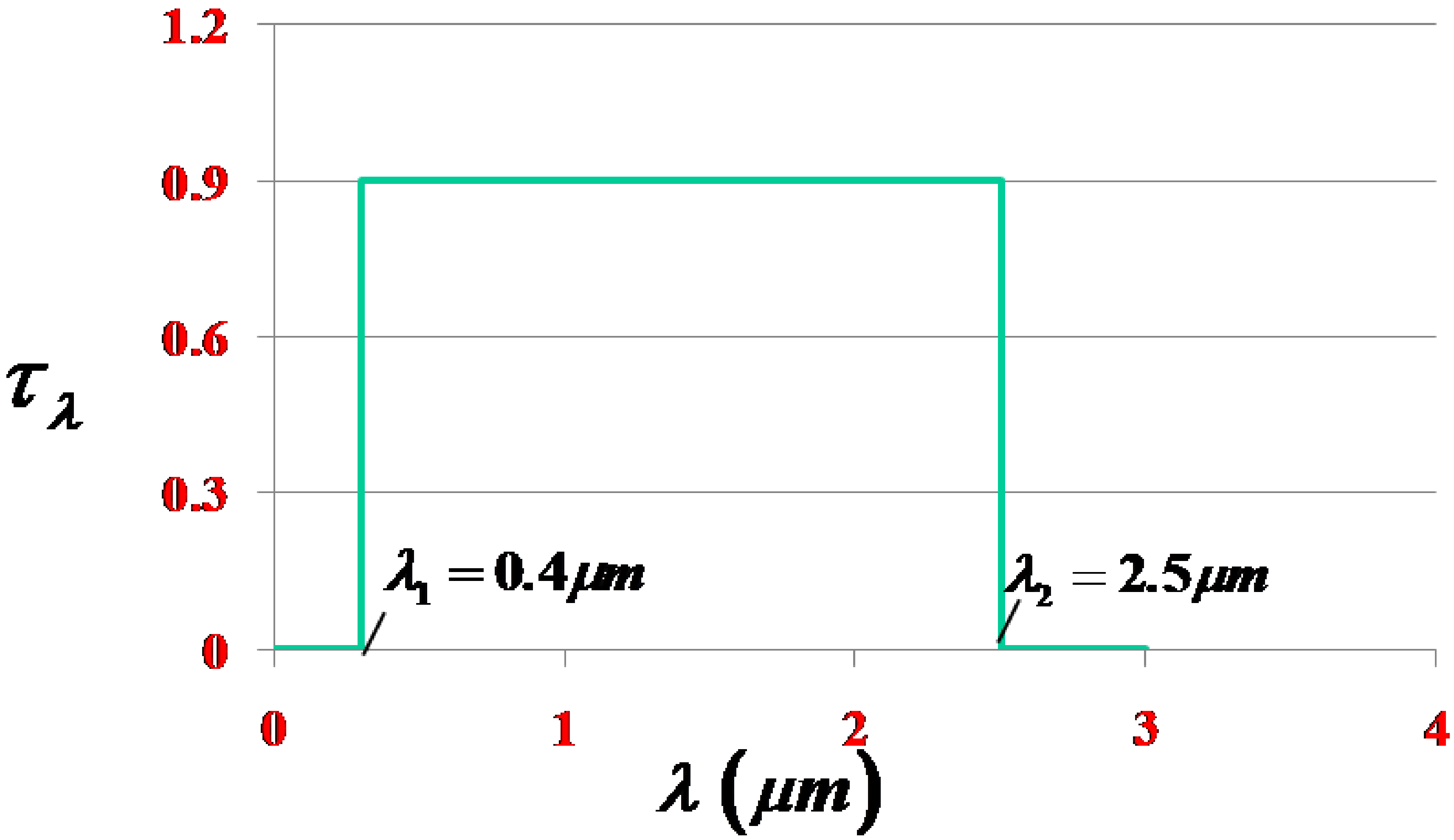
Since, the net heat flux to the surface is greater than zero, ie., positive, the surface temperature will increase with time.

Problem: The cover glass on a flat plate solar collector has a low iron content, and its spectral transmissivity may be approximated by the following distribution. What is the total transmissivity of the cover glass to the solar radiation.

Known: spectral transmissivity of solar collector cover glass

Find: Total transmissivity of cover glass to solar radiation

Assumptions: Spectral distribution of solar irradiation is proportional to that of blackbody emission at 5800 K



Total transmissivity of the cover is

$$\tau = \frac{\int_0^\infty G_{\lambda,tr}(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda} = \frac{\int_0^\infty \tau_\lambda(\lambda)\, G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

where the irradiation G_λ is due to solar emisison. Having assumed that the sun emits as a blackbody at 5800 K, it follows that

$$G_\lambda(\lambda) \quad \alpha \quad E_{\lambda,b}(5800\ K)$$

With the proportionality constant cancelling from the numerator and denominator of the expression for τ, we obtain

$$\tau = \frac{\int_0^\infty \tau_\lambda(\lambda) \quad E_{\lambda,b}(5800\ K)d\lambda}{\int_0^\infty \quad E_{\lambda,b}(5800\ K)d\lambda}$$

For prescribed transmissivity variation with wavelength

$$\tau = \frac{0.9\int_{0.4}^{2.5} \quad E_{\lambda,b}(5800\ K)\, d\lambda}{\int_{0.4}^{2.5} \quad E_{\lambda,b}(5800\ K)d\lambda}$$

$$\tau = \frac{0.9\int_{0.4}^{2.5} E_{\lambda,b}(5800\ K)\, d\lambda}{\int_{0.4}^{2.5} E_{\lambda,b}(5800\ K) d\lambda}$$

$$\lambda_1 T = 0.4\mu m \times 5800K = 2320\mu m.K: \ \ F_{0-0.4} = 0.1245$$
$$\lambda_2 T = 2.5\mu m \times 5800K = 14500\mu m.K: \ F_{0-2.5} = 0.9664$$

$$\tau = 0.9[\,F_{0-2.5} - F_{0-0.4}] = 0.9(0.9660 - 0.1245)$$

$$\tau = 0.84$$

Comments :

It is important to recognize that the irradiation at the cover plate is not equal to the emissive power of a blackbody at 5800 K, $G_\lambda(\lambda) \neq E_{\lambda,b}(5800\ K)$.

It is simply assumed to be proportional to this emissive power, in which case it is assumed to have a spectral distribution of the same form.

With $G_\lambda(\lambda)$ appearing in both the numerator and denominator of the expression for τ, it is then possible to replace G_λ by $E_{\lambda,b}$

KIRCHOFF'S LAW

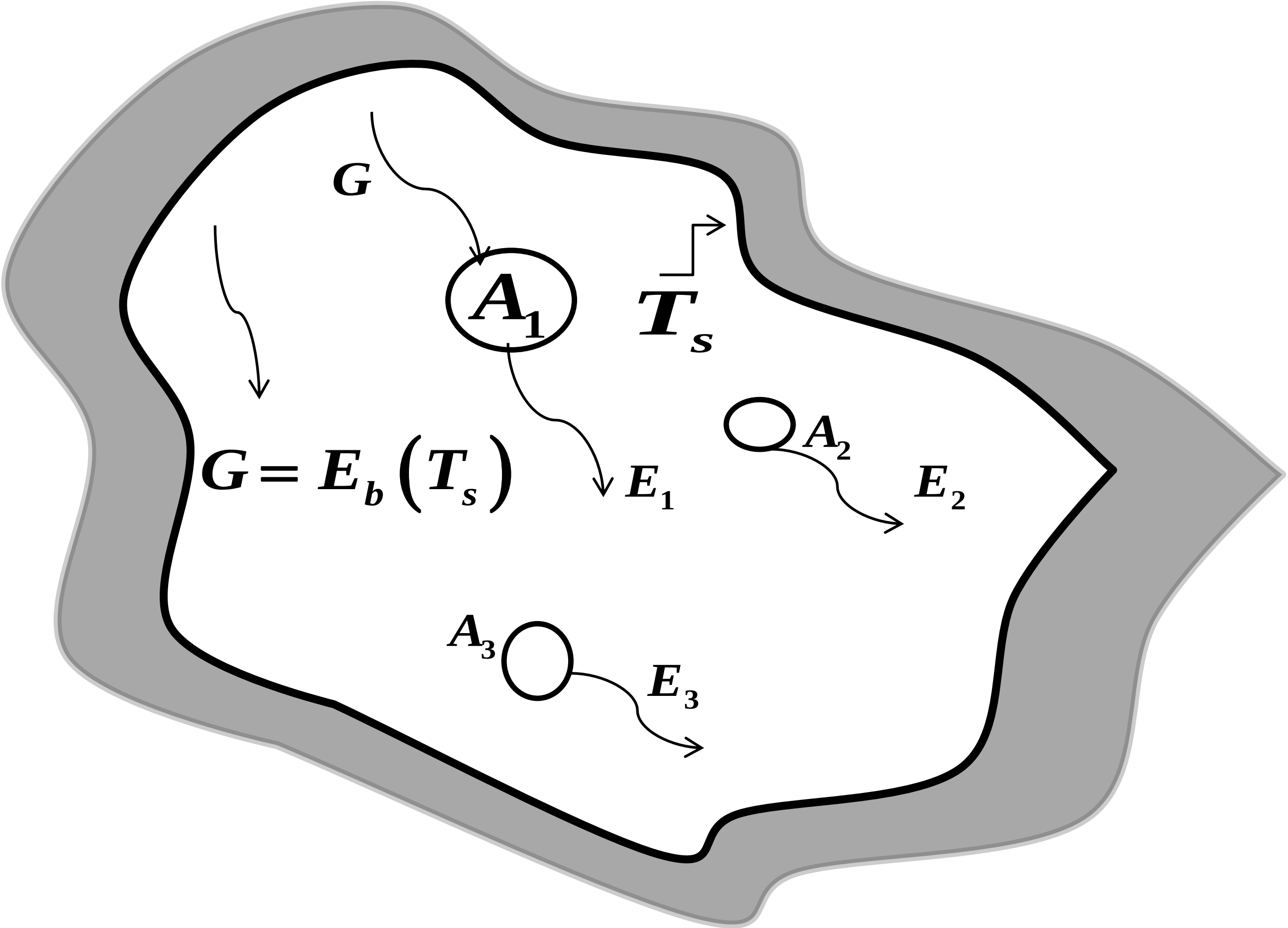
- Consider a large, isothermal enclosure of surface temperature T_s within which several small bodies are confined.

- Since these bodies are small relative to the enclosure, they have negligible influence on the radiation field, which is due to the cumulative effect of emission and reflection by the enclosure surface.

- Regardless of the radiative properties, such a surface forms a blackbody cavity.

- Accordingly, regardless of its orientation, the irradiation experienced by any body in the cavity is diffuse and equal to emission from a blackbody at T_s.

$$G = E_b(Ts)$$



Under steady state conditions, thermal equilibrium must exist between the bodies and the enclosure.

Hence, $T_1 = T_2 = T_3 = \ldots = Ts$ and the net rate of energy transfer to each surface must be zero.

Applying energy balance to a control surface about body 1, it follows that

$$\alpha_1 G A_1 = E_1(T_s) A_1$$

$$G = E_b(T_s)$$

$$\alpha_1 E_b(T_s) = E_1(T_s)$$

Since this result must apply to each of the confined bodies, we then obtain

$$E_b(T_s) = \frac{E_1(T_s)}{\alpha_1} = \frac{E_2(T_s)}{\alpha_2} = \frac{E_3(T_s)}{\alpha_3} \ldots \ldots ..$$

This relation is known as KIRCHOFF'S LAW. No real surface can have an emissive power exceeding that of a black surface at the same temperature, and the notion of the black body as an ideal emitter is confirmed.

But, total hemispherical emissivity is given by

$$\varepsilon(T) = \frac{E(T)}{E_b(T)}$$

$$\frac{\varepsilon_1}{\alpha_1} = \frac{\varepsilon_2}{\alpha_2} = \frac{\varepsilon_3}{\alpha_3} = 1$$

Hence, for any surface in the enclosure $\varepsilon = \alpha$

Total hemispherical emissivity is equal to total hemispherical absorptivity

The restrictive conditions inherent in this derivation are

the surface irradiation has been assumed to correspond to emission from a blackbody at the same temperature as the surface

The derivation can also be repeated for radiation at a specified wavelength to obtain spectral form of Kirchoff's law

$$\varepsilon_{\lambda}(T) = \alpha_{\lambda}(T)$$

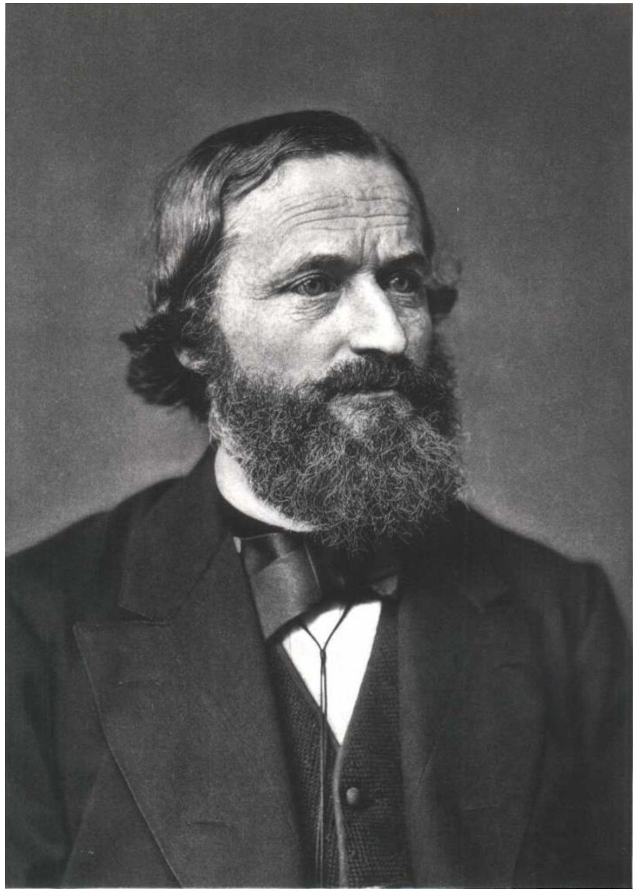
This relation is valid when the irradiation or the emitted radiation is independent of direction. The form of the Kirchoff's law that involves no restrictions is the *spectral directional* form expressed as

$$\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T)$$

The emissivity of the surface at a specified wavelength, direction and temperature is always equal to its absorptivity at the same wavelength, direction and temperature

It is very tempting to use Kirchoff's law in radiation analysis since the relation $\varepsilon = \alpha$ together with $\rho = 1 - \alpha$ enables us to determine all three properties of opaque surface from a knowledge of only one property. Although, this equation $\varepsilon = \alpha$ gives acceptable results in most cases, in practice, care should be exercised when there is considerable difference between the surface temperature and the temperature of the source of incident radiation.

HISTORICAL PERSPECTIVE OF KIRCHOFF'S LAW



Gustav Kirchhoff was born in East Prussia, Germany

Kirchhoff formulated his circuit laws, which are now ubiquitous in electrical engineering, in 1845, while still a student.

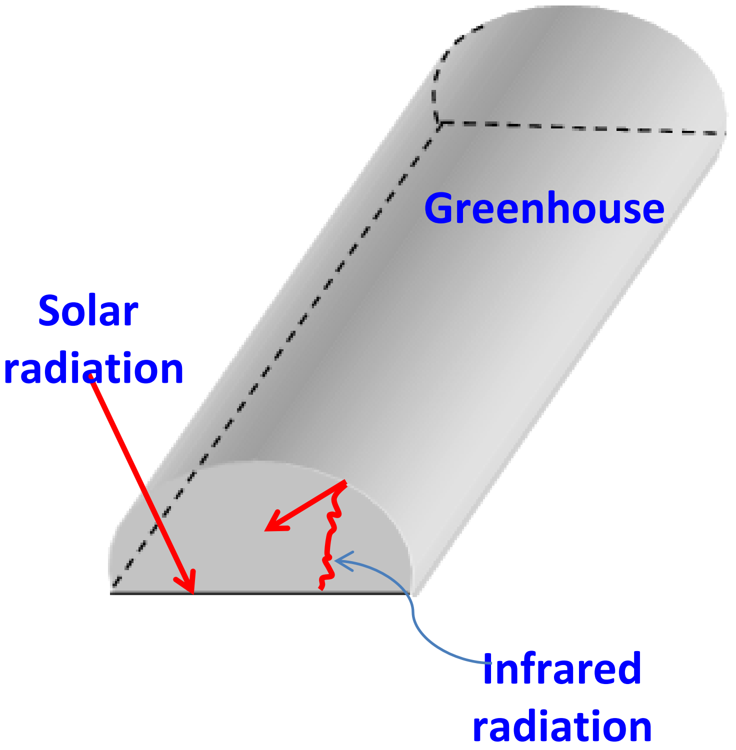
He completed this study as a seminar exercise; it later became his doctoral dissertation.

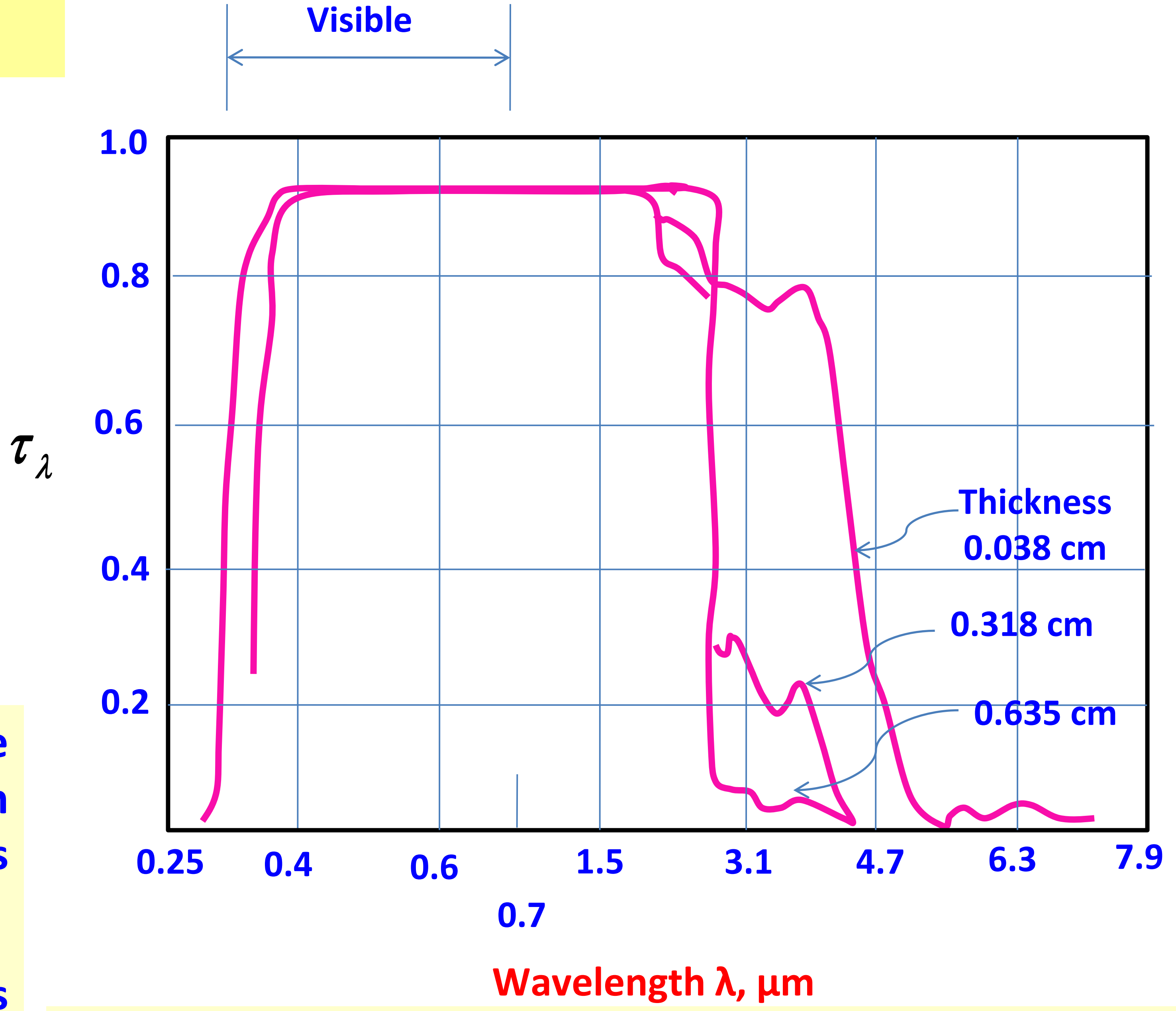
He proposed his law of thermal radiation 1859, and gave a proof in 1861.

He was called to the University of Heidelberg in 1854, where he collaborated in spectroscopic work with Robert Bunsen.

Together Kirchhoff and Bunsen discovered caesium and rubidium in 1861.

GREEN HOUSE EFFECT





Glass has a transparent window in the wavelength range 0.3 μm < λ < 3 μm in which over 90% of the solar radiation is emitted

Entire radiation emitted by the surfaces at room temperature falls in the infrared region (1-1000 m)

Glass allows the solar radiation to enter but does not allow the infrared radiation from the interior surfaces to escape

GRAY SURFACE

Gray surface is a surface in which its properties are independent of wavelength

Diffuse surface is a surface in which its properties are independent of direction

Emissivity of a gray, diffuse surface is total hemispherical emissivity of that surface because of the independence of wavelength and direction

REAL SURFACE	DIFFUSE SURFACE	GRAY SURFACE	DIFFUSE GRAY SURFACE
$\varepsilon_\theta \neq \text{constant}$ $\varepsilon_\lambda \neq \text{constant}$	$\varepsilon_\theta = \text{constant}$	$\varepsilon_\lambda = \text{constant}$	$\varepsilon_\theta = \varepsilon_\lambda = \text{constant}$

COMMENTARY ON KIRCHOFF'S LAW

Problem of predicting radiant energy exchange between surfaces is greatly simplified if $\varepsilon = \alpha$ may be assumed to apply for each surface

Spectral hemispherical emissivity was defined as

$$\varepsilon_\lambda(\lambda, T) = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} sin\theta cos\theta d\theta d\phi}$$

Spectral hemispherical absorptivity was defined as

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}$$

The pertinent question is

Spectral hemispherical emissivity $\overset{????????}{\triangleq}$ Spectral hemispherical absorptivity

The pertinent question is

$$\varepsilon_\lambda(\lambda, T) \text{ Spectral hemispherical emissivity} \overset{????????}{\triangleq} \alpha_\lambda(\lambda) \text{ Spectral hemispherical absorptivity}$$

$$\frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} sin\theta cos\theta d\theta d\phi} \overset{???}{\triangleq} \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \alpha_{\lambda,\theta}(\lambda,\theta,\phi)\, I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) sin\theta cos\theta d\theta d\phi}$$

Since, $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$, it follows by inspection that $\varepsilon_\lambda = \alpha_\lambda$ is applicable if either of the following conditions are met

1. The irradiation is diffuse ($I_{\lambda,i}$ is independent of θ and ϕ)
2. The surface is diffuse ($\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$ are independent of θ and ϕ)

The first condition is reasonably assumption for many engineering calculations

The second condition is reasonable for many surfaces, particularly for electrically non-conducting surfaces

Assuming the existence of either diffuse irradiation or a diffuse surface, consider what additional conditions must be satisfied for $\varepsilon = \alpha$ to be valid.

$\varepsilon(T)$ Total hemispherical emissivity $\overset{????????}{\triangleq}$ α Spectral hemispherical absorptivity

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)} \overset{????????}{\triangleq} \alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

Since, $\varepsilon_\lambda = \alpha_\lambda$, it follows by inspection that $\varepsilon = \alpha$ applies if either of the following conditions are met

1. The irradiation corresponds to emission from a blackbody at the surface temperature T, in which case $G_\lambda(\lambda) = E_b(T)$ and $G = E_b(T)$
2. The surface is *gray* ($\varepsilon_\lambda = \alpha_\lambda$ are independent of λ)

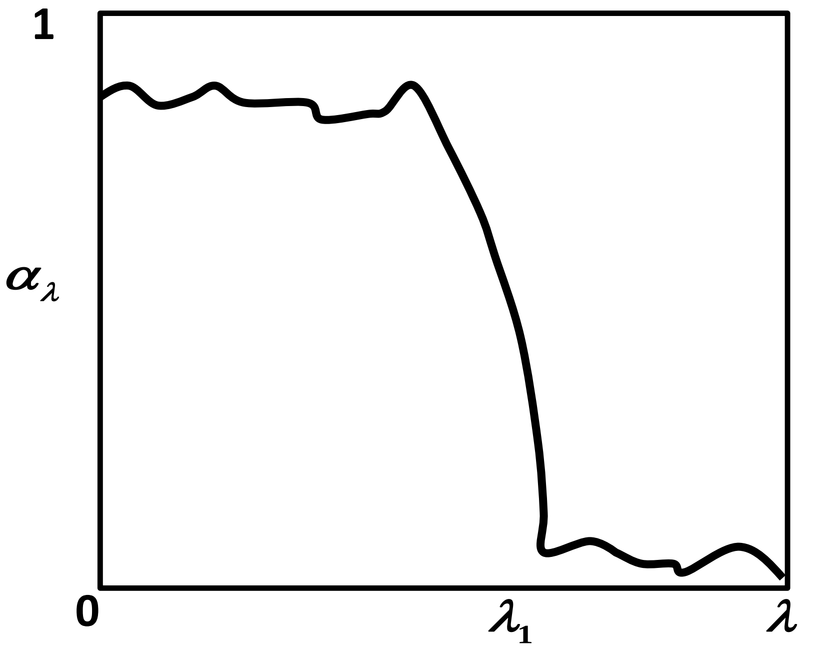
Because the total absorptivity of a surface depends on the spectral distribution of the irradiation, it cannot be stated unequivocally that $\alpha = \varepsilon$

For example, a particular surface may be highly absorbing to radiation in one spectral region and virtually non-absorbing in another region

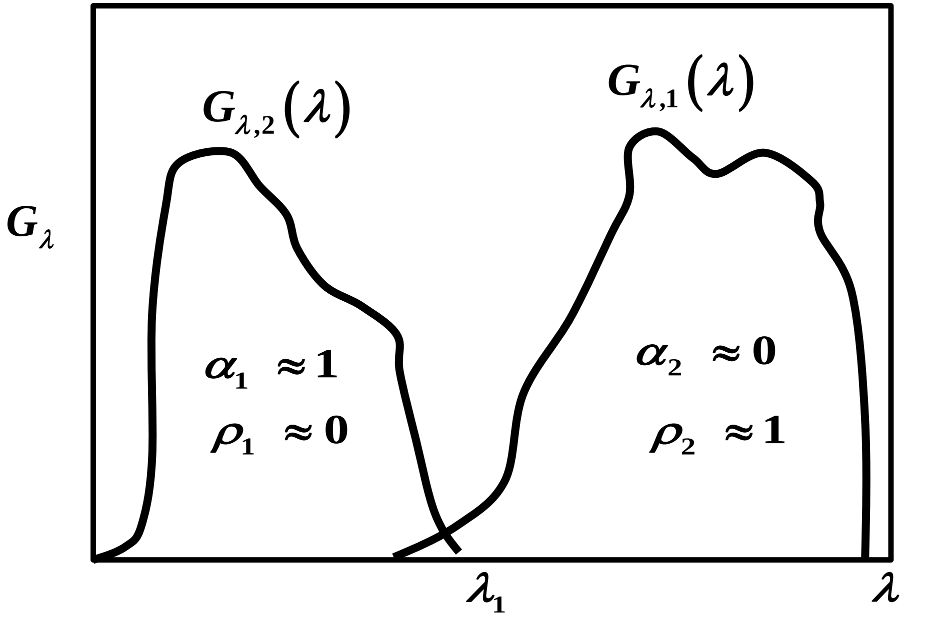
Accordingly for two possible irradiation fields $G_{\lambda,1}(\lambda)$ and $G_{\lambda,2}(\lambda)$of Fig below, the values of α would differ drastically

In contrast, the value of ε is independent of the irradiation.

Hence there is no basis for stating that $\alpha = \varepsilon$



Spectral absorptivity of a surface



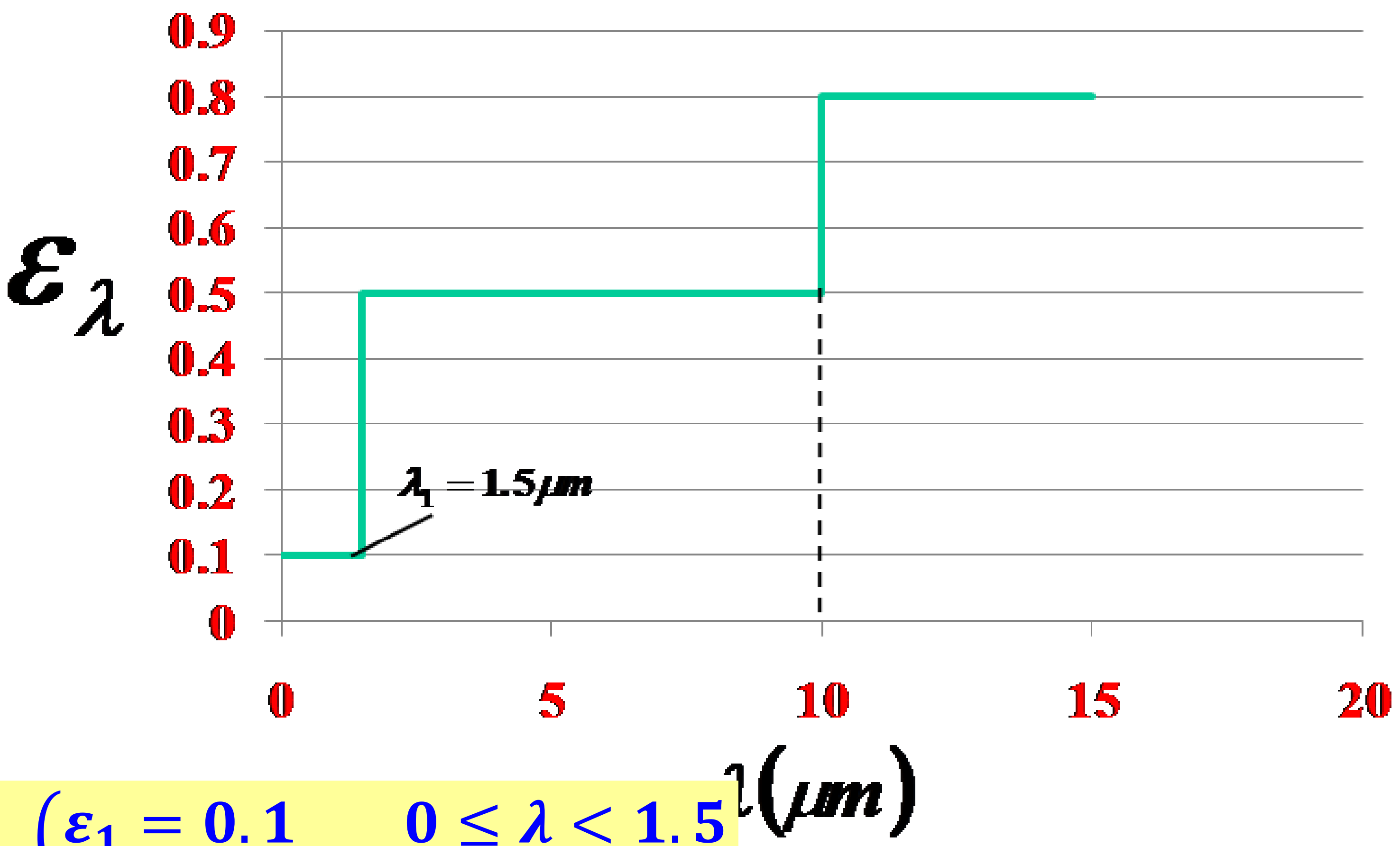
Spectral irradiation at the surface

Problem: A diffuse, fire brick wall of temperature 500 K has the spectral emissivity shown and is exposed to a bed of coals at 2000 K. Determine the total, hemispherical emissivity and emissive power of the fire brick wall. What is the total absorptivity of the wall to irradiation resulting from emission by the coals

Known: Brick wall of surface temperature $T_s = 500\,K$ and prescribed $\varepsilon_\lambda(\lambda)$ is exposed to coals at $T_c = 2000K$

Find:

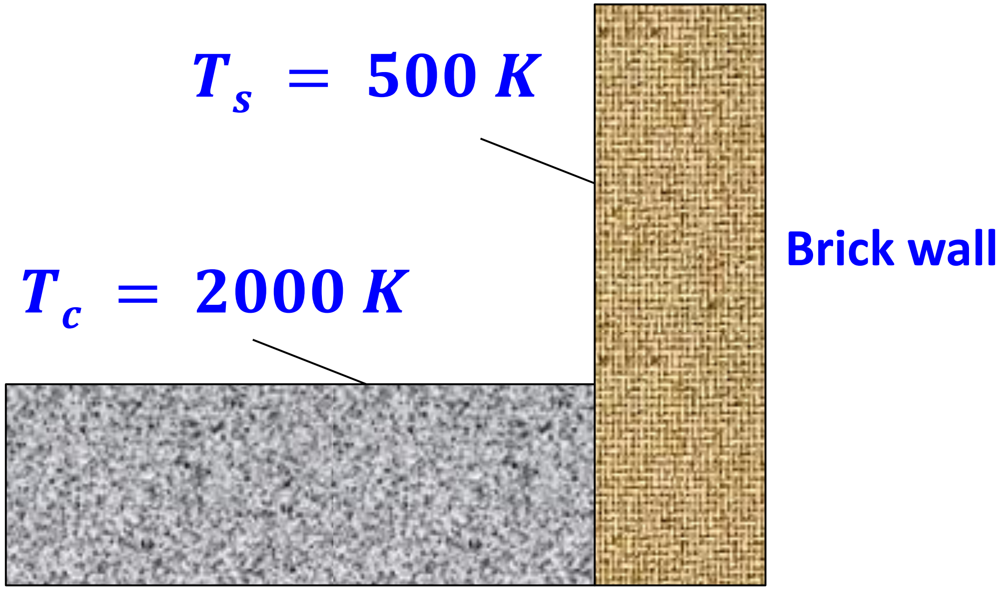
1. Total hemispherical emissivity of the fire brick wall
2. Total emissive power of the brick wall
3. Absorptivity of the wall to irradiation from the coals



$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.1 & 0 \le \lambda < 1.5 \\ \varepsilon_2 = 0.5 & 1.5 \le \lambda < 10 \\ \varepsilon_3 = 0.8 & 10 \le \lambda < \infty \end{cases}$$

Assumptions

1. Brick wall is opaque and diffuse
2. Spectral distribution of irradiation at the brick wall approximates that due to emission from a blackbody at 2000 K



$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.1 & 0 \le \lambda < 1.5 \\ \varepsilon_2 = 0.5 & 1.5 \le \lambda < 10 \\ \varepsilon_3 = 0.8 & 10 \le \lambda < \infty \end{cases}$$

1. **Total hemispherical emissivity of the fire brick wall**

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{E_b}$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1}(T) + \varepsilon_2 F_{\lambda_1-\lambda_2}(T) + \varepsilon_3 F_{\lambda_2-\infty}(T)$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1} + \varepsilon_2 \left(F_{0-\lambda_2} - F_{0-\lambda_1}\right) + \varepsilon_3 \left(1 - F_{0-\lambda_2}\right)$$

$$\lambda_1 T = 0.4 \mu m \times 500K = 750\ \mu m.K: \ \ F_{0-1.5} = 0.0$$
$$\lambda_2 T = 10\ \mu m \times 500K = 5000 \mu m.K: \ F_{0-10} = 0.634$$

$$\varepsilon(T) = \varepsilon_1 F_{0-\lambda_1} + \varepsilon_2 \left(F_{0-\lambda_2} - F_{0-\lambda_1}\right) + \varepsilon_3 \left(1 - F_{0-\lambda_2}\right)$$

$$\varepsilon(T) = 0.1(0) + 0.5(0.634 - 0.0) + 0.8(1 - 0.631)$$

$$\varepsilon(T) = 0.61$$

2. Total emissive power of the brick wall

$$E(T_s) = \varepsilon\sigma T_s^4$$

$$E(T_s) = 0.61(5.67 \times 10^{-8})(500)^4 = 2162\ W/m^2$$

$$E(T_s) = 2162\ W/m^2$$

3. The total absorptivity of the wall to radiation from the coals is

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

Since the surface is diffuse, $\alpha_\lambda(\lambda) = \varepsilon_\lambda(\lambda)$. Moreover, since the spectral distribution of the irradiation approximates that due to emission from a blackbody at 2000K, $G_\lambda(\lambda)\alpha\ E_{\lambda,b}(\lambda, T_c)$. It follows that

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.1 & 0 \le \lambda < 1.5 \\ \varepsilon_2 = 0.5 & 1.5 \le \lambda < 10 \\ \varepsilon_3 = 0.8 & 10 \le \lambda < \infty \end{cases}$$

$$\alpha = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)\, E_{\lambda,b}(\lambda, T_c) d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, T_c) d\lambda}$$

$\lambda_1 T = 0.4 \mu m \times 2000K = 3000\ \mu m.K$: $F_{0-1.5} = 0.273$

$\lambda_2 T = 10\ \mu m \times 2000K = 20000 \mu m.K$: $F_{0-10} = 0.986$

$$\alpha(T_c) = \varepsilon_1 F_{0-\lambda_1} + \varepsilon_2\left(F_{0-\lambda_2} - F_{0-\lambda_1}\right) + \varepsilon_3\left(1 - F_{0-\lambda_2}\right)$$

$$\alpha(T_c) = 0.1(0.273) + 0.5(0.986 - 0.273) + 0.8(1 - 0.986)$$

$$\alpha(T_c) = 0.395$$

1. Total hemispherical emissivity of the fire brick wall $\varepsilon(T) = 0.61$

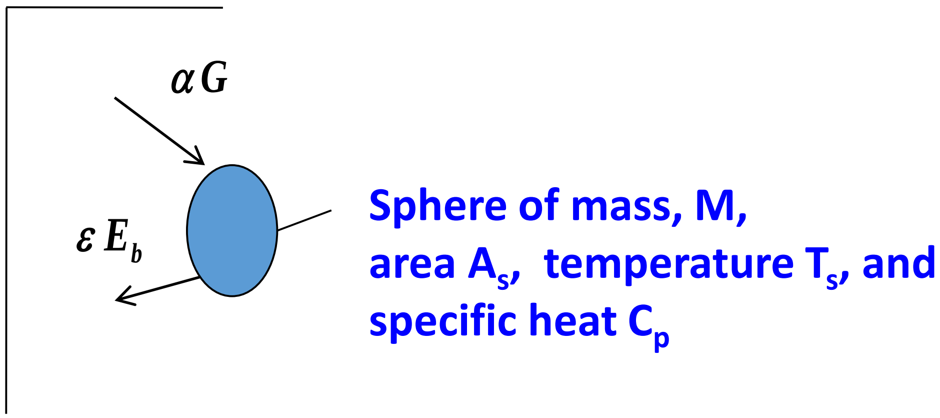
2. Total emissive power of the brick wall $E(T_s) = 2162\ W/m^2$

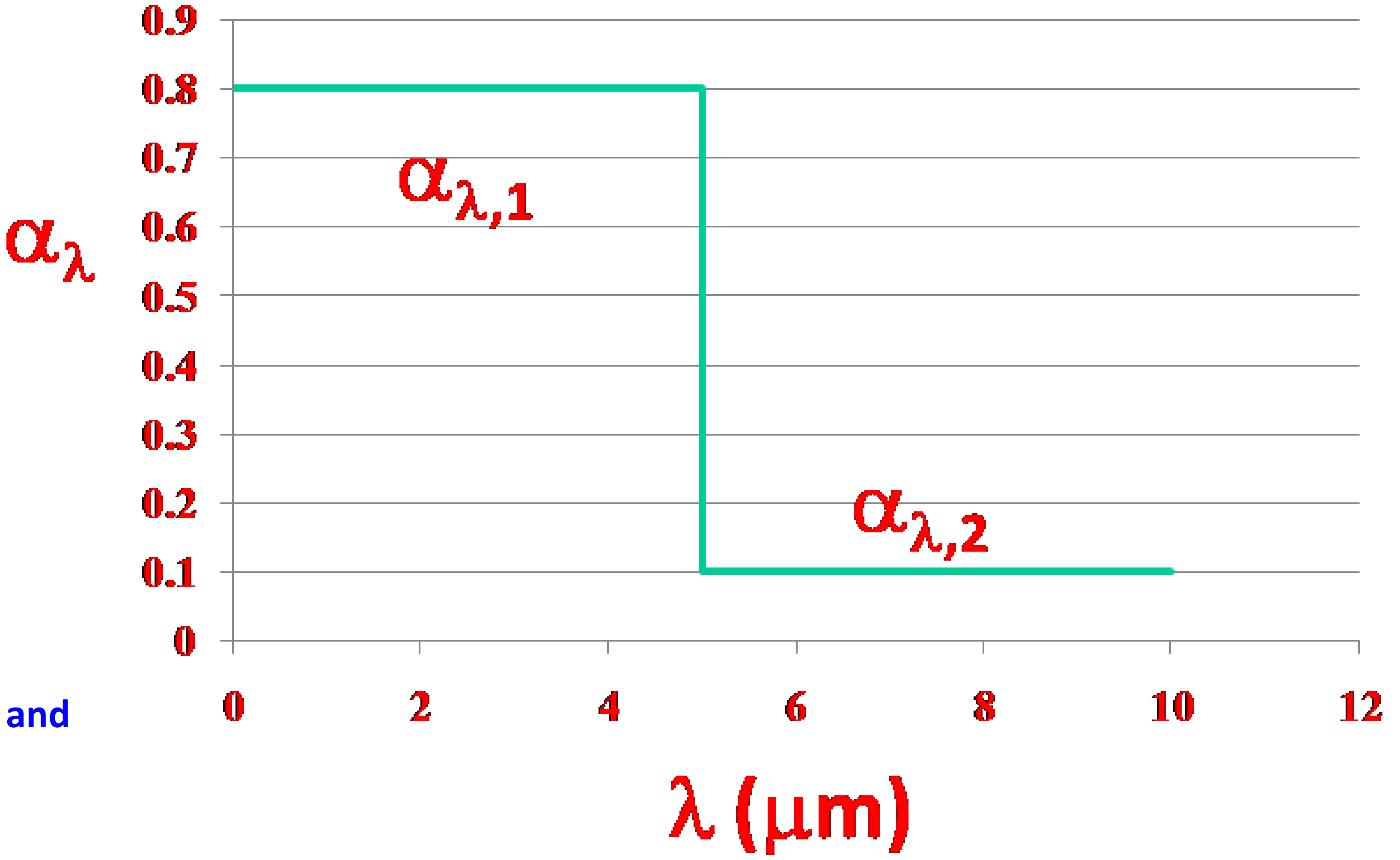
3. The total absorptivity of the wall to radiation from the coals is $\alpha(T_c) = 0.395$

Comments: the emissivity depends on the surface temperature T_s, while the absorptivity depends on the spectral distribution of the irradiation, which depends on the temperature of the source T_c.

The surface is not gray, $\alpha \neq \varepsilon$. This results is to be expected. Since emission is associated with $T_s = 500\ K$, its spectral maximum occurs at $\lambda_{max} \approx 6$ µm.
In contrast, since irradiation is associated with emission from a source at $T_c = 2000K$, its spectral maximum occurs at $\lambda_{max} \approx 1.5\ \mu m$.
Even though $\varepsilon_\lambda = \alpha_\lambda$, ε and α decrease with increasing T_s and T_c, respectively, and it is only for $T_s = T_c$ that $\varepsilon = \alpha$.

Problem: A small, solid metallic sphere has an opaque, diffuse coating for which $\alpha_\lambda = 0.8$ for $\lambda \leq 5\ \mu m$ and $\alpha_\lambda = 0.1$ for $\lambda > 5\ \mu m$. The sphere, which is initially at a uniform temperature of 300 K, is inserted into a large furnace whose walls are at 1200 K. Determine the total, hemispherical absorptivity and emissivity of the coating for the initial condition and for the final, steady state condition.





Known: small metallic sphere with spectrally selective absorptivity, initially at $T_s = 300\ K$, is inserted into a large furnace at $T_f = 1200$ K

Find:

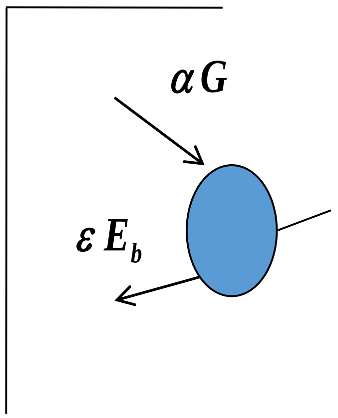
1. Total, hemispherical absorptivity and emissivity of sphere coating for the initial condition
2. values of α and ε after sphere has been in furnace a long time

Assumptions:

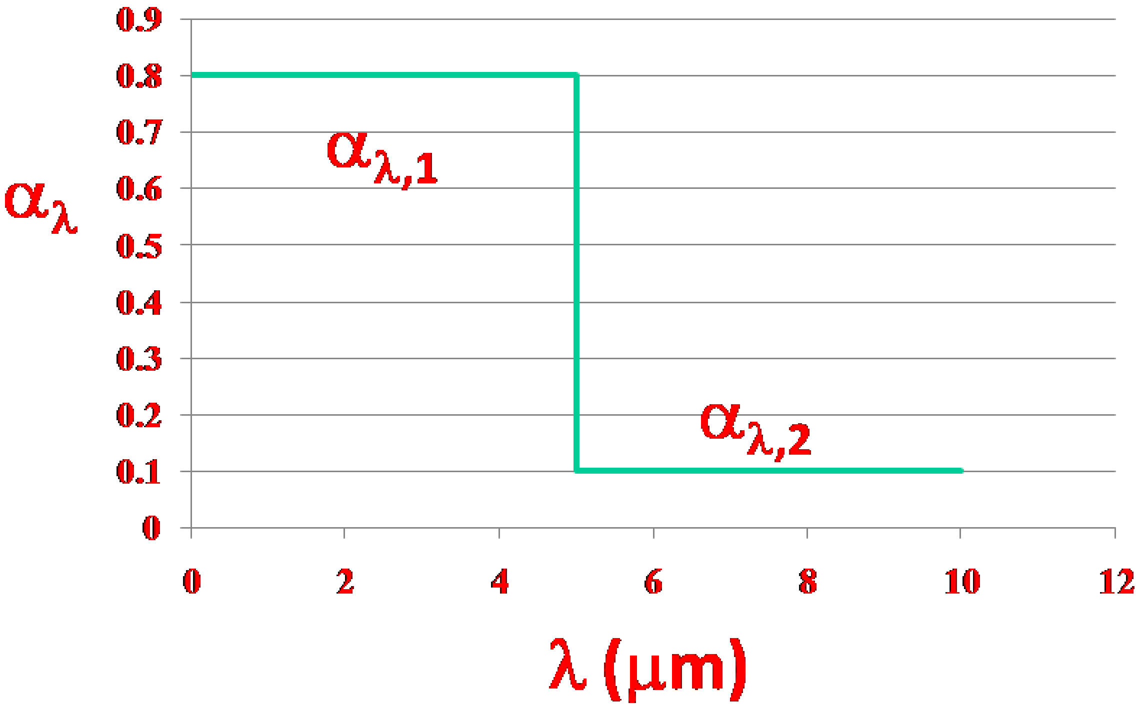
1. Coating is opaque and diffuse
2. since furnace surface is much larger than that of sphere, irradiation approximates emission from a blackbody at T_f

Total hemispherical absorptivity

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$



Sphere of mass, M, area A_s, temperature T_s, and specific heat C_p



Total hemispherical absorptivity

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

Since furnace surface is much larger than that of sphere, irradiation approximates emission from a blackbody at T_f

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, E_{\lambda,b}(\lambda, T_f) d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, T_f) d\lambda} \qquad \alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, E_{\lambda,b}(\lambda, T_f) d\lambda}{E_b(T_f)} \qquad \alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, E_{\lambda,b}(\lambda, 1200K) d\lambda}{E_b(1200K)}$$

$$\alpha = \alpha_{\lambda_1} \frac{\int_0^{\lambda_1} E_{\lambda,b}(\lambda, 1200K)\, d\lambda}{E_b(1200K)} + \alpha_{\lambda_2} \frac{\int_0^\infty E_{\lambda,b}(\lambda, 1200K)\, d\lambda}{E_b(1200K)}$$

$$\alpha = \alpha_{\lambda_1} F_{(0-\lambda_1)} + \alpha_{\lambda_2} F_{(\lambda_1-\infty)}$$

$$\alpha = \alpha_{\lambda_1} F_{(0-\lambda_1)} + \alpha_{\lambda_2} \left(1 - F_{(0-\lambda_1)}\right)$$

$$\lambda_1 T_f = 5.0 \mu m \times 1200K = 6000\ \mu m.K: \quad F_{0-5.0} = 0.738$$

$$\alpha = 0.8 \times 0.738 + 0.1(1 - 0.738)$$

$$\alpha = 0.62$$

Total hemispherical emissivity

$$\alpha = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)\, E_{\lambda,b}(\lambda, T_s) d\lambda}{E_b(T_s)}$$

Since the surface is diffuse, $\varepsilon_\lambda = \alpha_\lambda$

$$\varepsilon = \alpha_{\lambda_1} \frac{\int_0^{\lambda_1} E_{\lambda,b}(\lambda, 300K)\, d\lambda}{E_b(300K)} + \alpha_{\lambda_2} \frac{\int_0^\infty E_{\lambda,b}(\lambda, 300K)\, d\lambda}{E_b(300K)}$$

$$\varepsilon = \alpha_{\lambda_1} F_{(0-\lambda_1)} + \alpha_{\lambda_2} F_{(\lambda_1-\infty)}$$

$$\varepsilon = \alpha_{\lambda_1} F_{(0-\lambda_1)} + \alpha_{\lambda_2} \left(1 - F_{(0-\lambda_1)}\right)$$

$\lambda_1 T_s = 5.0\mu m \times 300K = 1500\ \mu m.K$: $F_{0-5.0} = 0.014$

$$\varepsilon = 0.8 \times 0.014 + 0.1(1 - 0.014)$$

$$\alpha = 0.11$$

Comments

1. The equilibrium condition that eventually exists ($T_s = T_f$) corresponds precisely to the condition for which Kirchoff's law was derived. Hence $\alpha = \varepsilon$
2. Because the spectral characteristics of the coating and the furance temperature remain fixed, there is no change in the value of α with increasing time. However, as Ts increases with time, the value of ε will change. After a sufficiently long time, $T_s = T_f$ and $\varepsilon = \alpha$ (ε = 0.62)