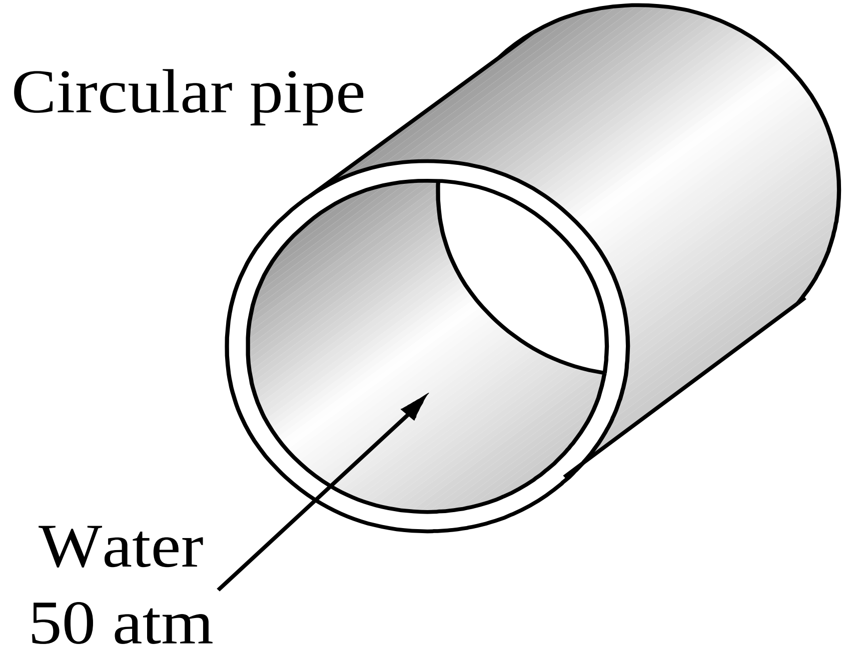
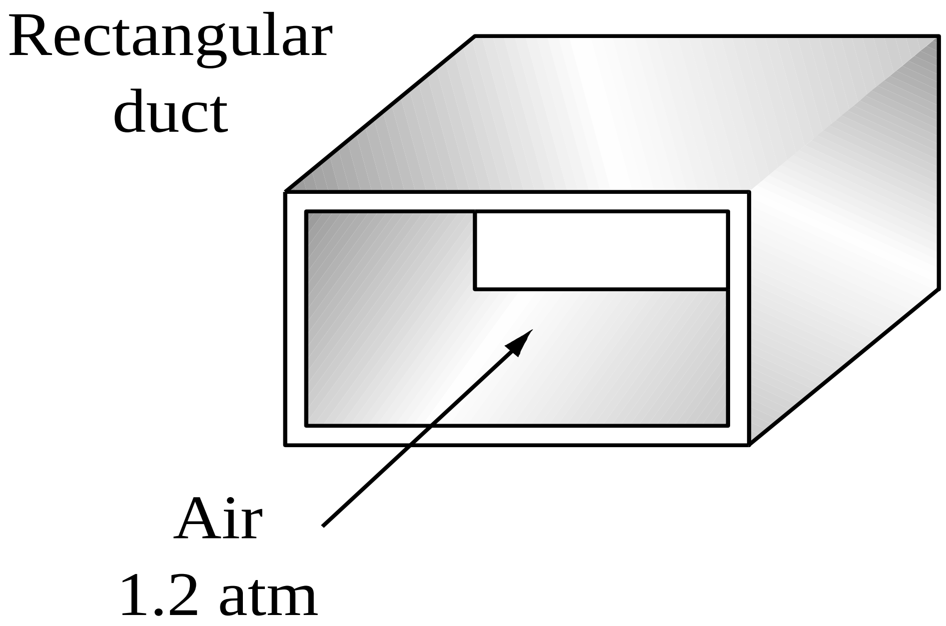
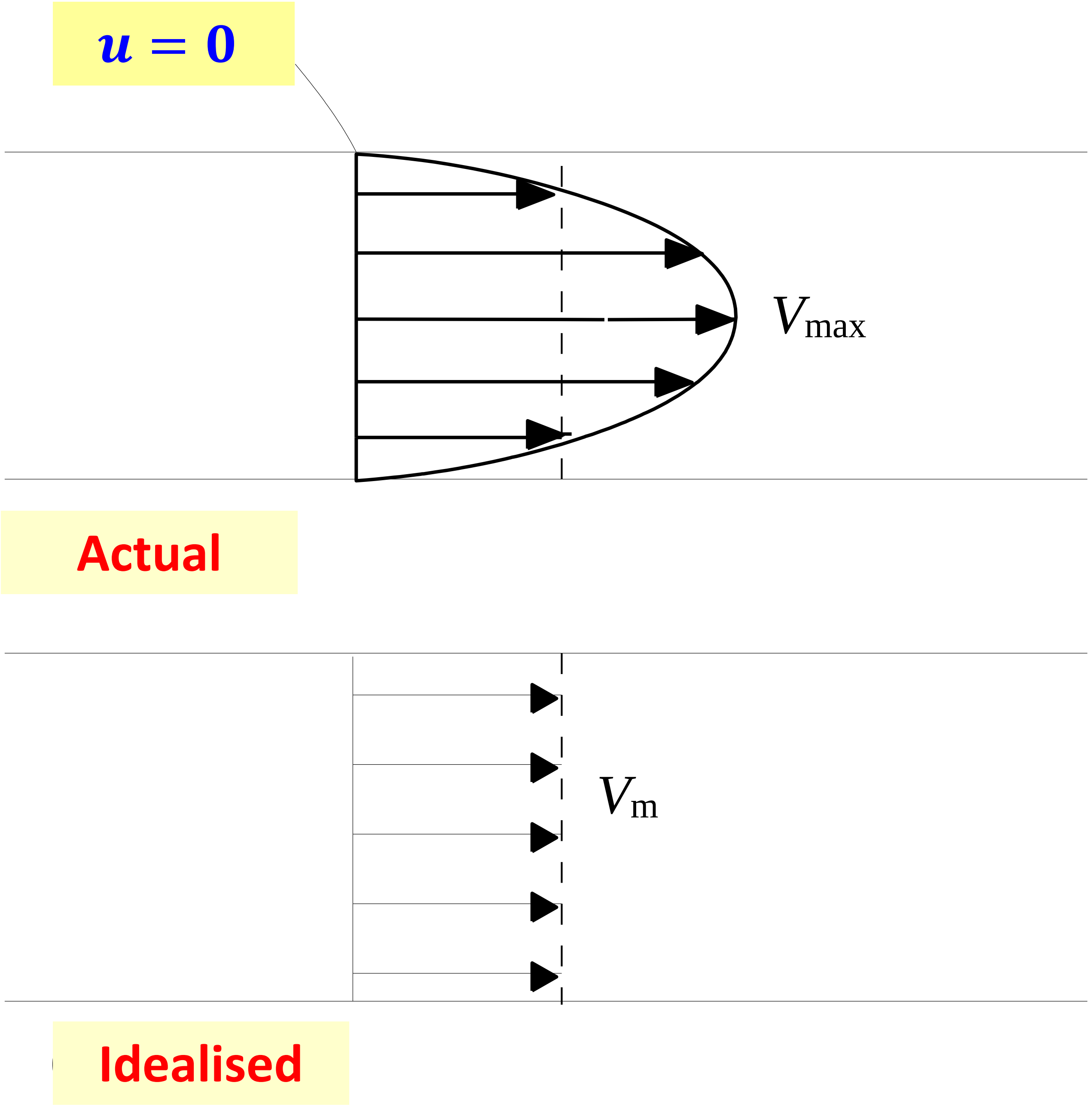
Internal flows

Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot.





MEAN VELOCITY



$$\dot{m} = \rho u_m A_c = \int_{A_c} \rho u(r,x) dA_c$$

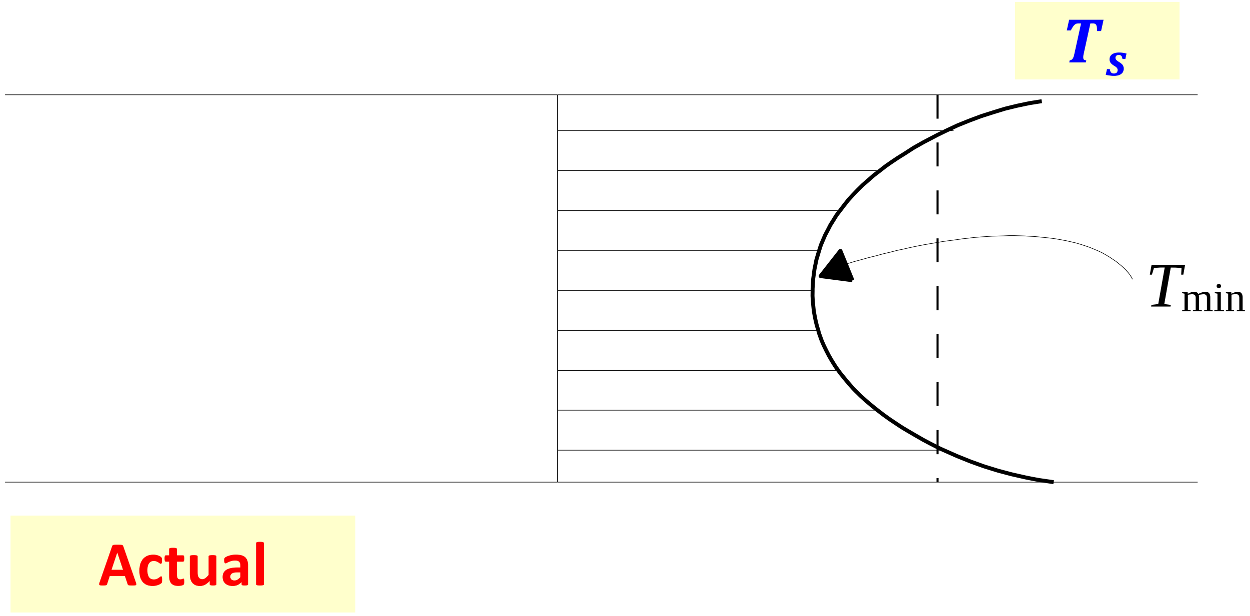
$$u_m = \frac{\int_{A_c} \rho u(r,x) dA_c}{\rho A_c}$$

$$u_m = \frac{\int_0^R \rho u(r,x) 2\pi r dr}{\rho \pi R^2}$$

$$u_m = \frac{2}{R^2} \int_0^R u(r,x) r dr$$

Actual and idealized velocity profiles for flow in a tube (the mass flow rate of the fluid is the same for both cases).

AVERAGE OR MEAN TEMPERATURE



$$\dot{E}_{fluid} = \dot{m}C_pT_m = \int_{A_c} \rho C_p T(r,x)u(r,x)dA_c$$

$$\dot{E}_{fluid} = \dot{m}C_pT_m = \frac{\int_{A_c} \rho C_p T(r,x)u(r,x)dA_c}{\rho u_m C_p(\pi R^2)}$$

$$\dot{E}_{fluid} = \dot{m}C_pT_m = \frac{\int_{A_c} \rho C_p T(r,x)u(r,x)dA_c}{\rho u_m C_p(\pi R^2)}$$

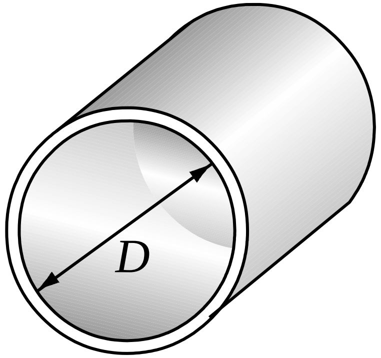
$$T_m = \frac{2}{u_m R^2}\int_0^R T(r,x)u(r,x)r\,dr$$

Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

HYDRAULIC DIAMETER

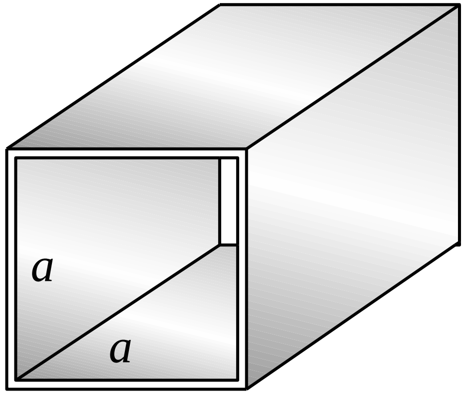
$$D_h = \frac{4A_c}{P}$$

Circular tube:



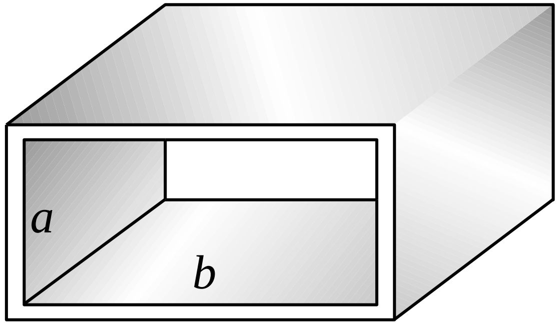
$$D_h = \frac{4\left(\frac{\pi D^2}{4}\right)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

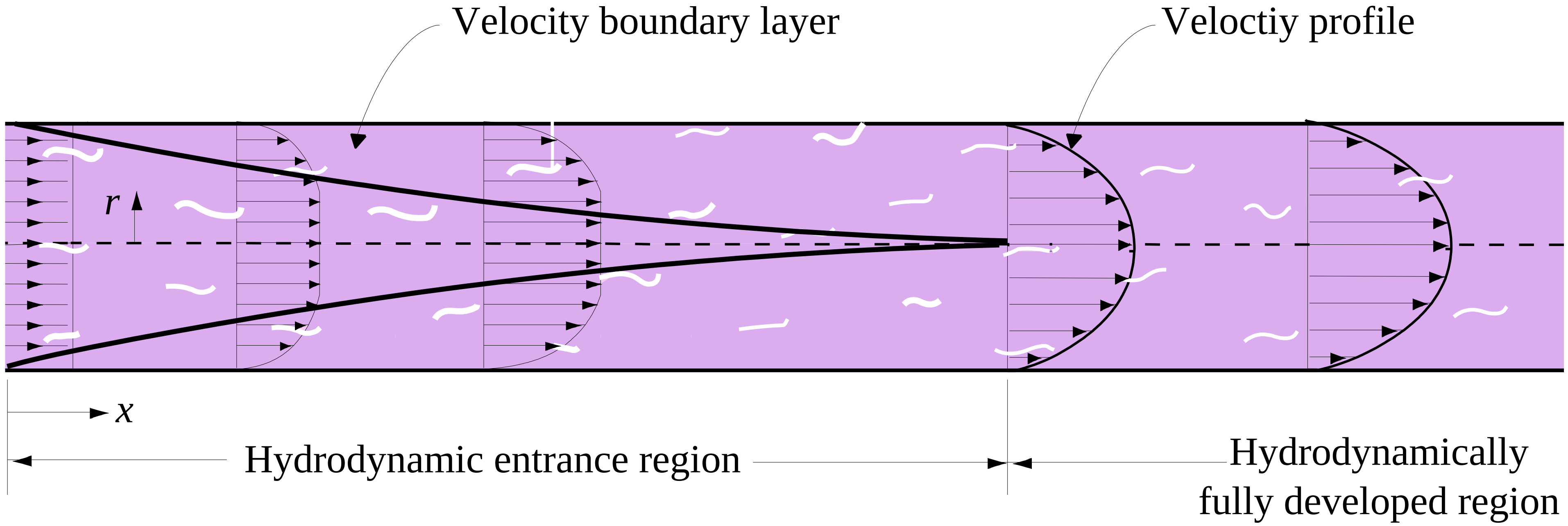
Rectangular duct:



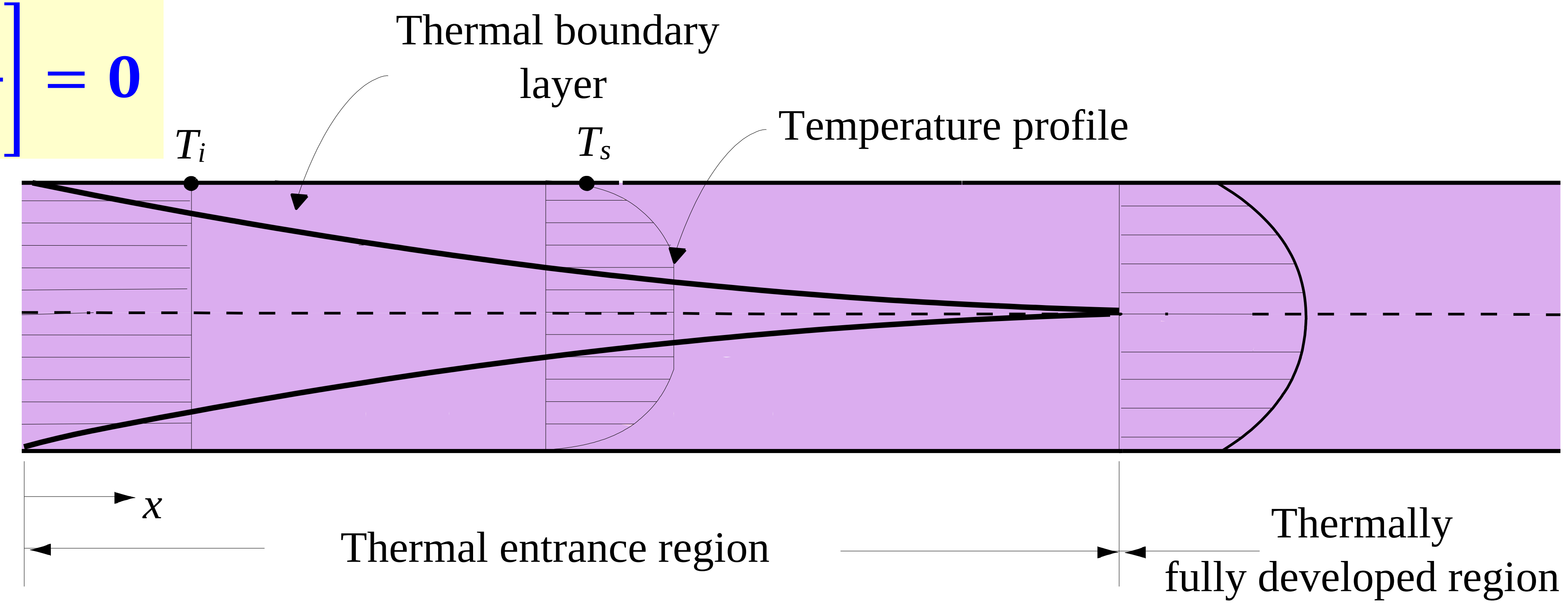
$$D_h = \frac{4ab}{2(a+b)}$$

$$\frac{\partial u(r,x)}{\partial x} = 0$$

$$u = u(r)$$



$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$



In a thermally fully developed region,

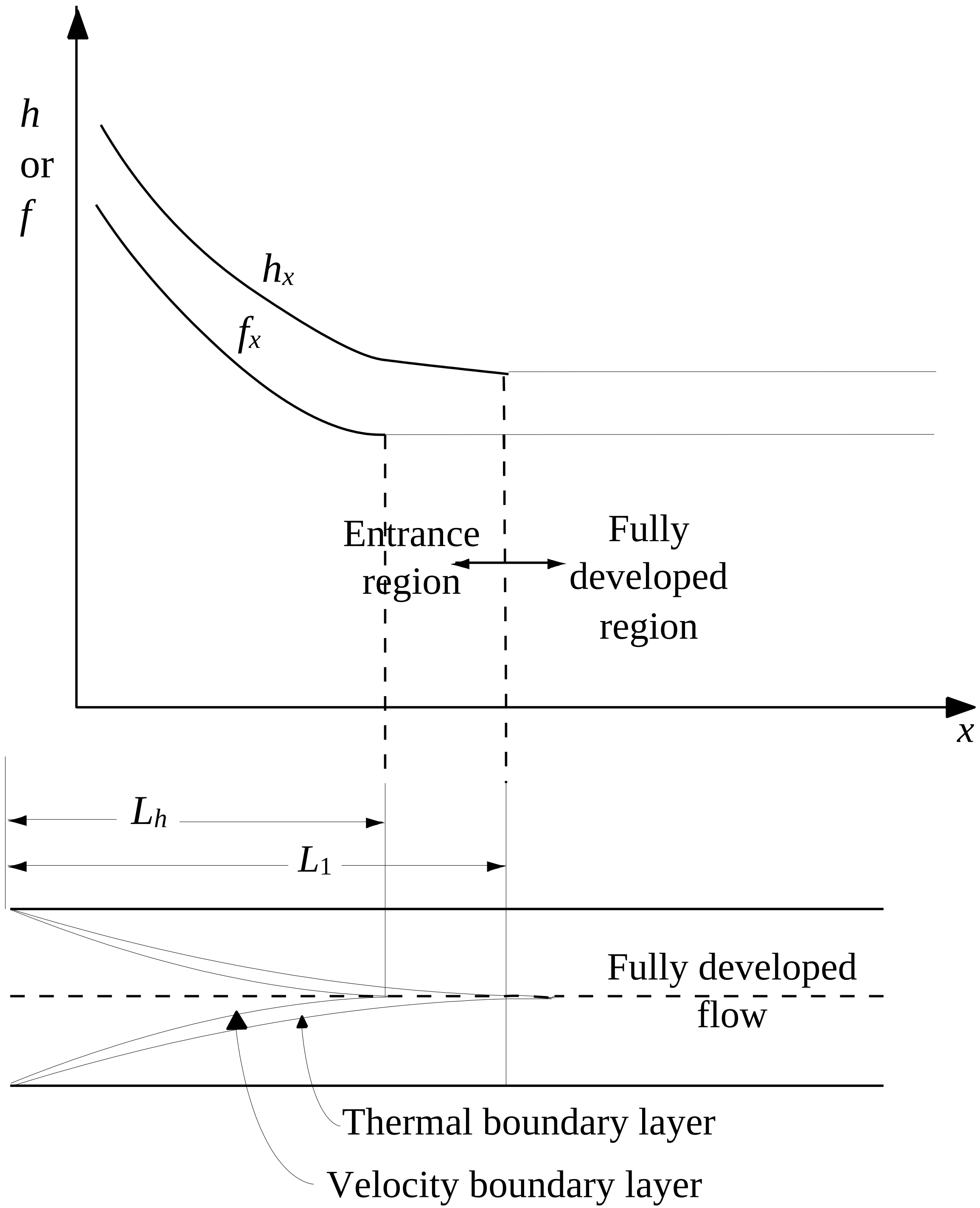
- the derivatives of $\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]$ with respect to x is zero by definition
- Thus $\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]$ is independent of x.
- Then the derivative of $\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]$ with respect to r must also be independent of x.

$$\frac{\partial}{\partial r}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]\Bigg|_{r=R} = \frac{-\left.\frac{\partial T}{\partial r}\right|_{r=R}}{T_s(x) - T_m(x)} \neq f(x)$$

$$q_s'' = h_x(T_s - T_m) = k\left.\frac{\partial T}{\partial r}\right|_{r=R}$$

$$h_x = \frac{k\left.\frac{\partial T}{\partial r}\right|_{r=R}}{(T_s - T_m)}$$

In the thermally fully developed region of a tube, the local convection coefficient is constant



Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube $(Pr > 1)$

$$L_{h,Laminar} \approx 0.05DRe$$

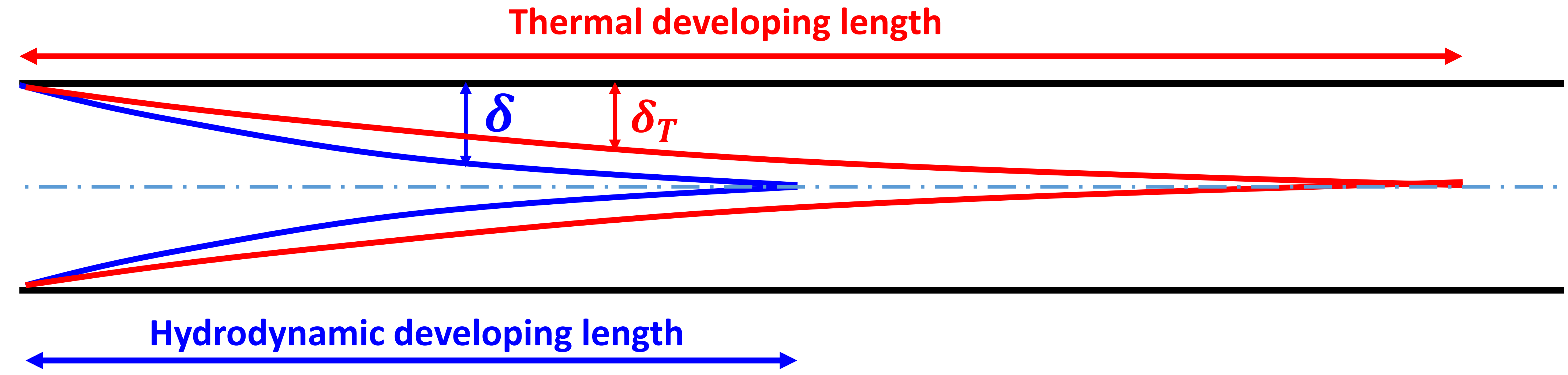
$$L_{t,Laminar} \approx 0.05DRePr = PrL_{h,Laminar}$$

$$L_{h,turbulent} = 1.359DRe^{0.25}$$

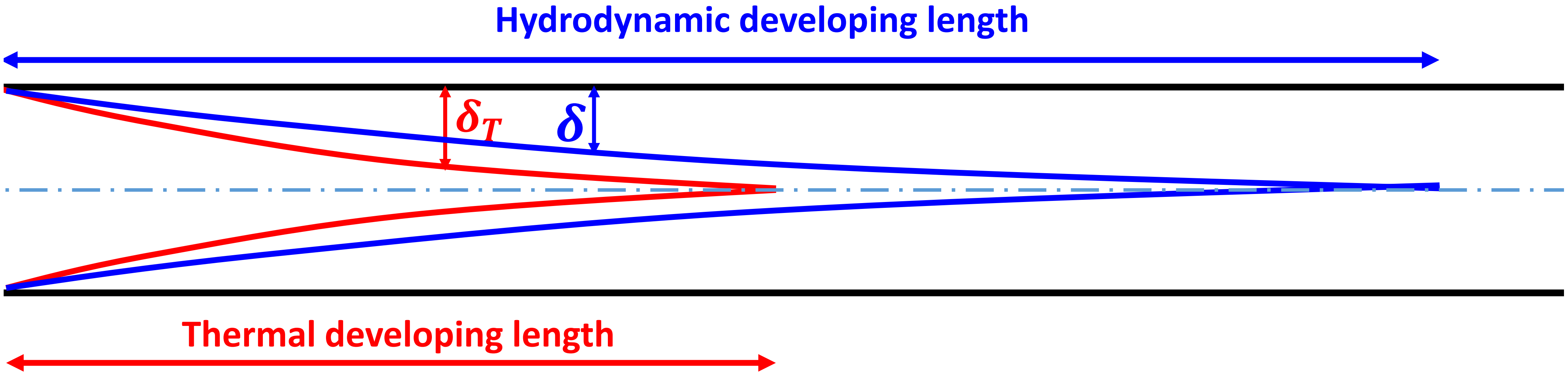
$$L_{h,turbulent} \approx L_{h,turbulent} = 10D$$

$$\frac{\delta}{\delta_T} \approx Pr^n$$

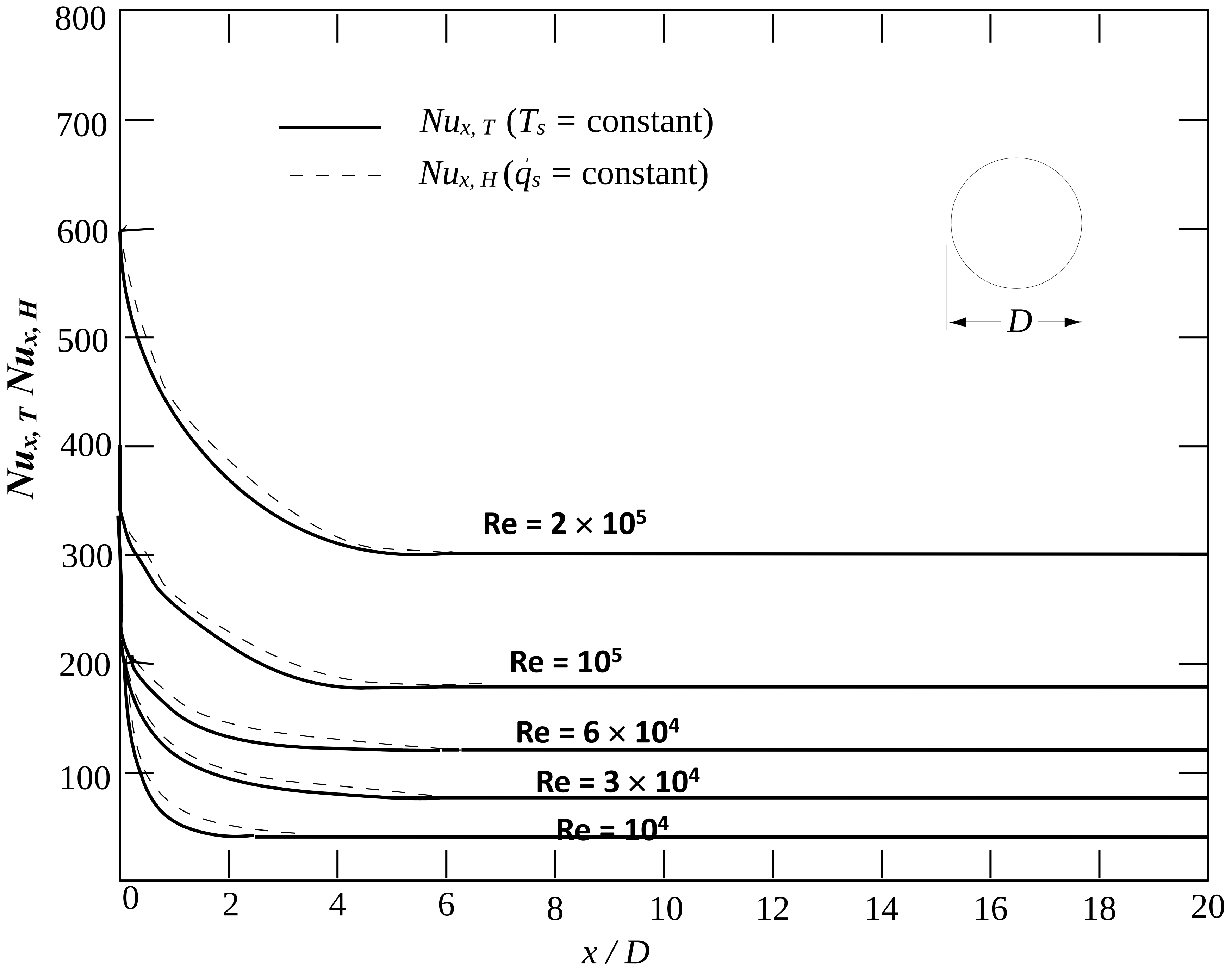
$Pr > 1$ $\quad$ $\delta > \delta_T$ $\quad$ **Oils**



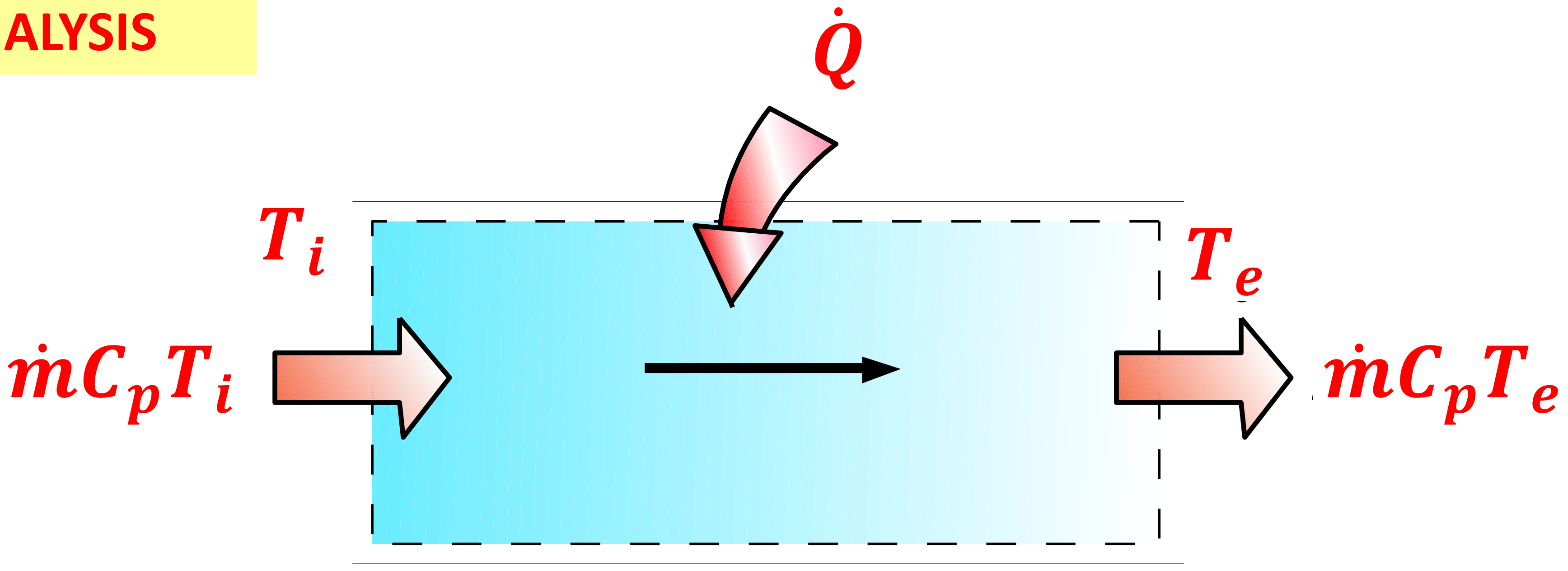
$Pr < 1$ $\quad$ $\delta < \delta_T$ $\quad$ **Liquid Metal**



Variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux



GENERAL THERMAL ANALYSIS



Energy balance

$$\dot{Q} = \dot{m}C_p(T_e - T_i)$$

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be constant surface temperature or constant surface heat flux

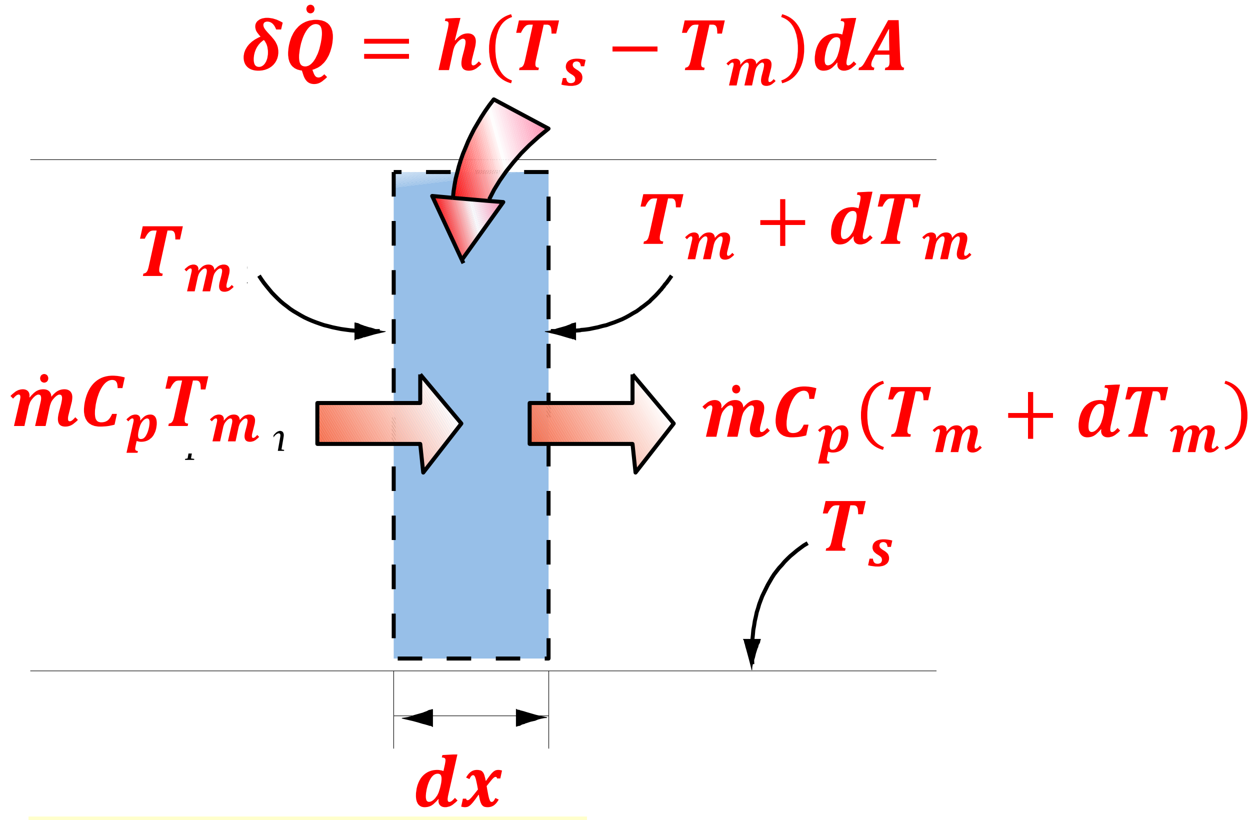
For example, the constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube.

The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

CONSTANT SURFACE HEAT FLUX

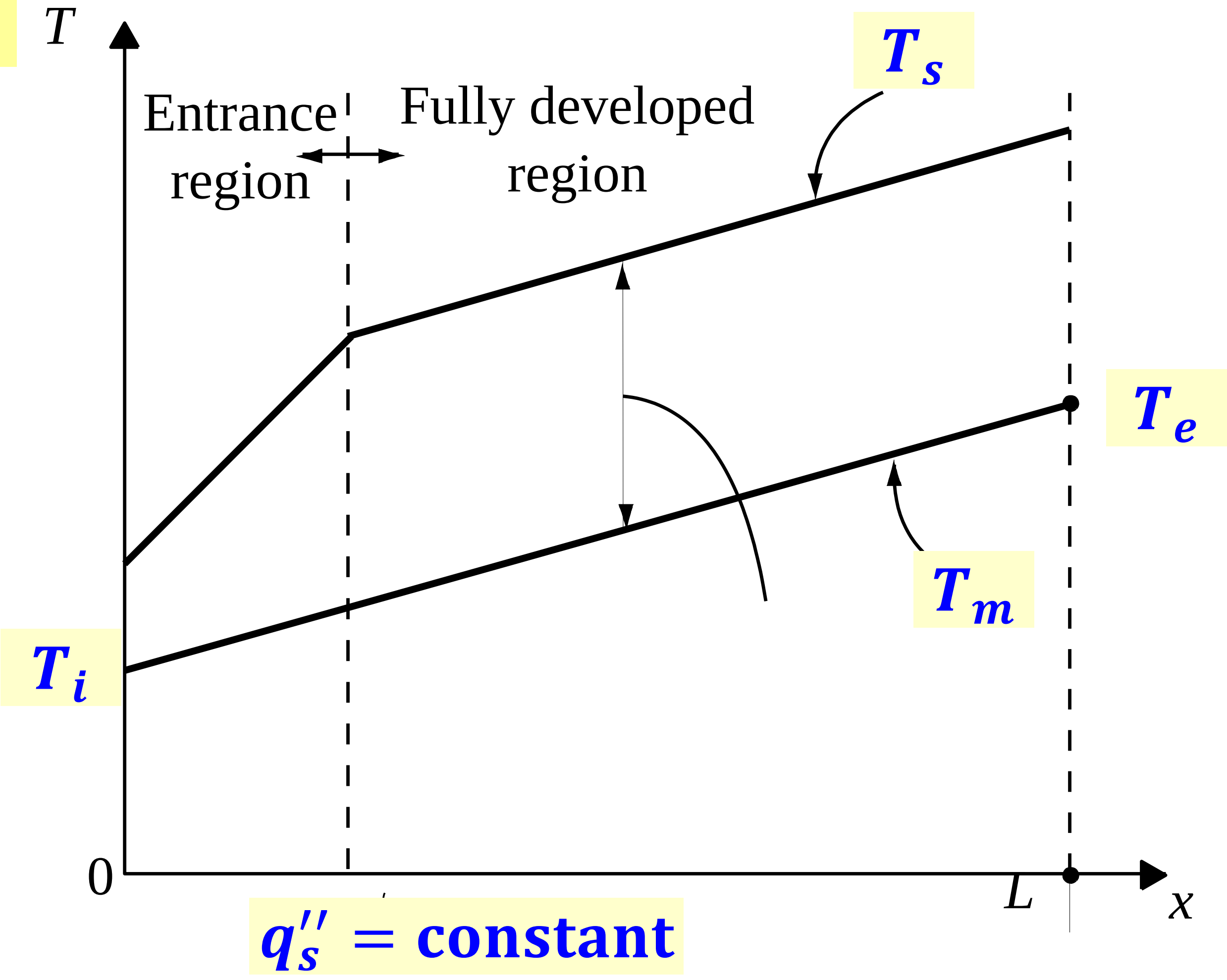
$$\dot{Q} = \dot{m}C_p(T_e - T_i) = q_s'' A_s$$

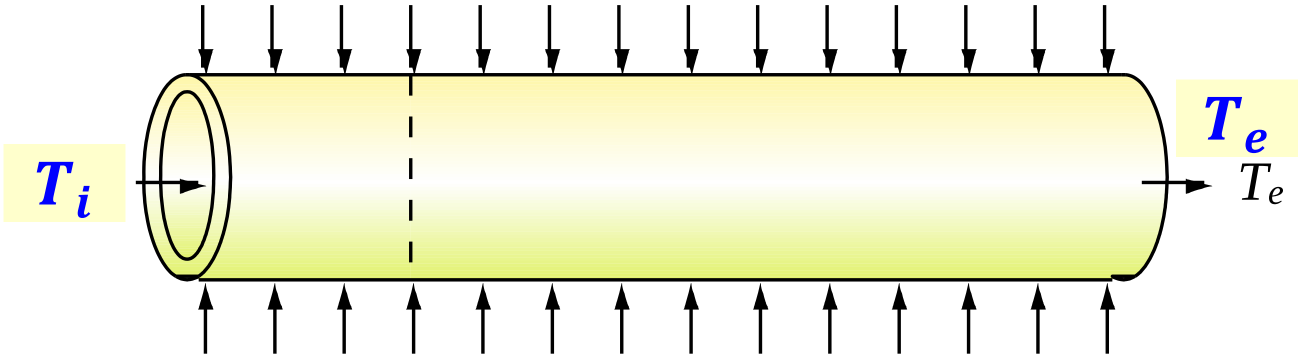
$$T_e = T_i + \frac{q_s'' A_s}{\dot{m}C_p}$$



$$\dot{m}C_p dT_m = q_s'' P dx$$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}C_p} = constant$$





$$q_s'' = h[T_s(x) - T_m(x)]$$

$$\frac{q_s''}{h} = T_s(x) - T_m(x)$$

For fully developed flow, h is constant

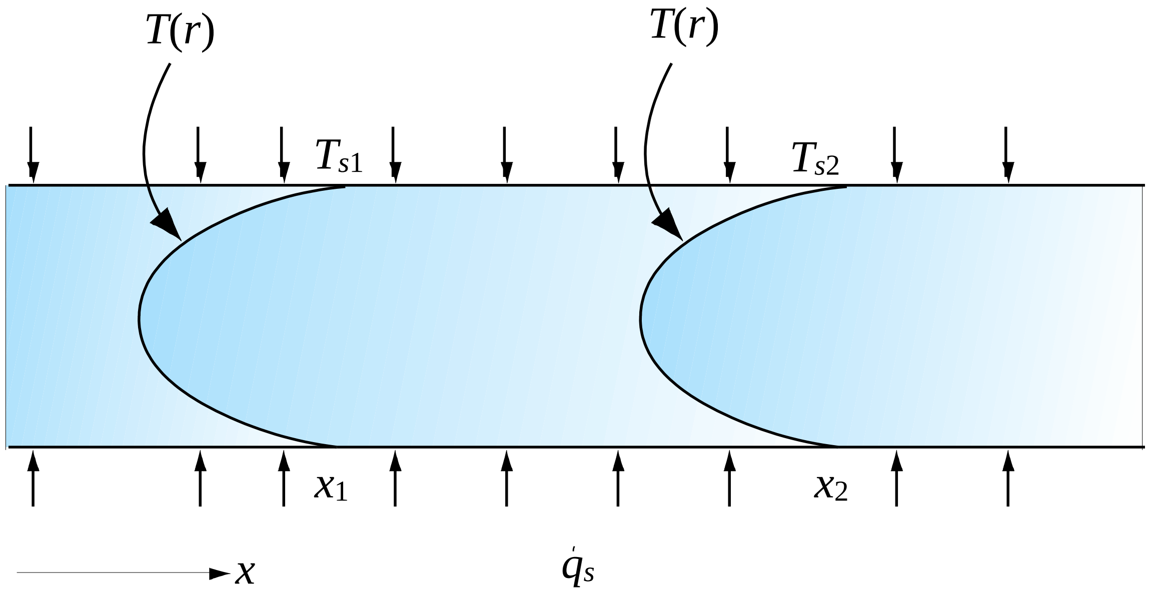
q_s'' = constant

$$\frac{dT_s}{dx} = \frac{dT_m}{dx}$$

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$

$$\frac{1}{T_s(x) - T_m(x)}\left[\frac{dT_s}{dx} - \frac{\partial T}{\partial x}\right] = 0$$

$$\frac{dT_s}{dx} = \frac{\partial T}{\partial x}$$



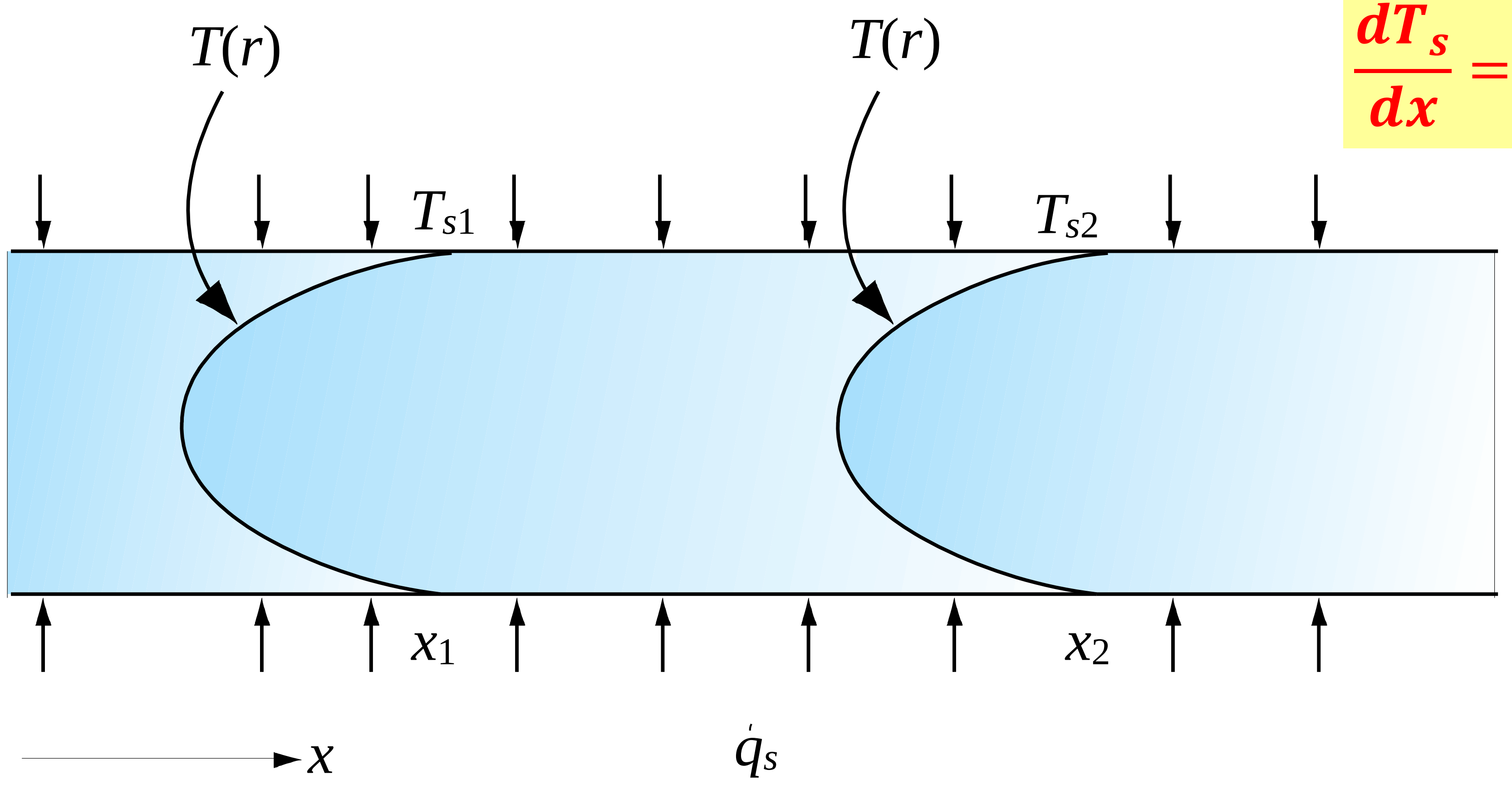
$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} C_p} = constant$$

$$\frac{dT_s}{dx} = \frac{dT_m}{dx}$$

$$\frac{dT_s}{dx} = \frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} C_p} = constant$$

Fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube

$$\frac{dT_s}{dx} = \frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} C_p} = constant$$



Fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube

For circular tube

$$\dot{m} = \rho u_m A_c = \rho u_m \left(\frac{\pi D^2}{4}\right)$$

$$\frac{dT_s}{dx} = \frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{q_s'' \pi D}{\rho u_m \left(\frac{\pi D^2}{4}\right) C_p} = \frac{4 q_s''}{\rho u_m C_p D} = constant$$

Constant Surface Temperature (T_s = constant)

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature.
Mean temperature of the fluid will increase in the flow direction as a result of heat transfer.

$$\dot{m}C_p dT_m = h(T_s - T_m)Pdx$$

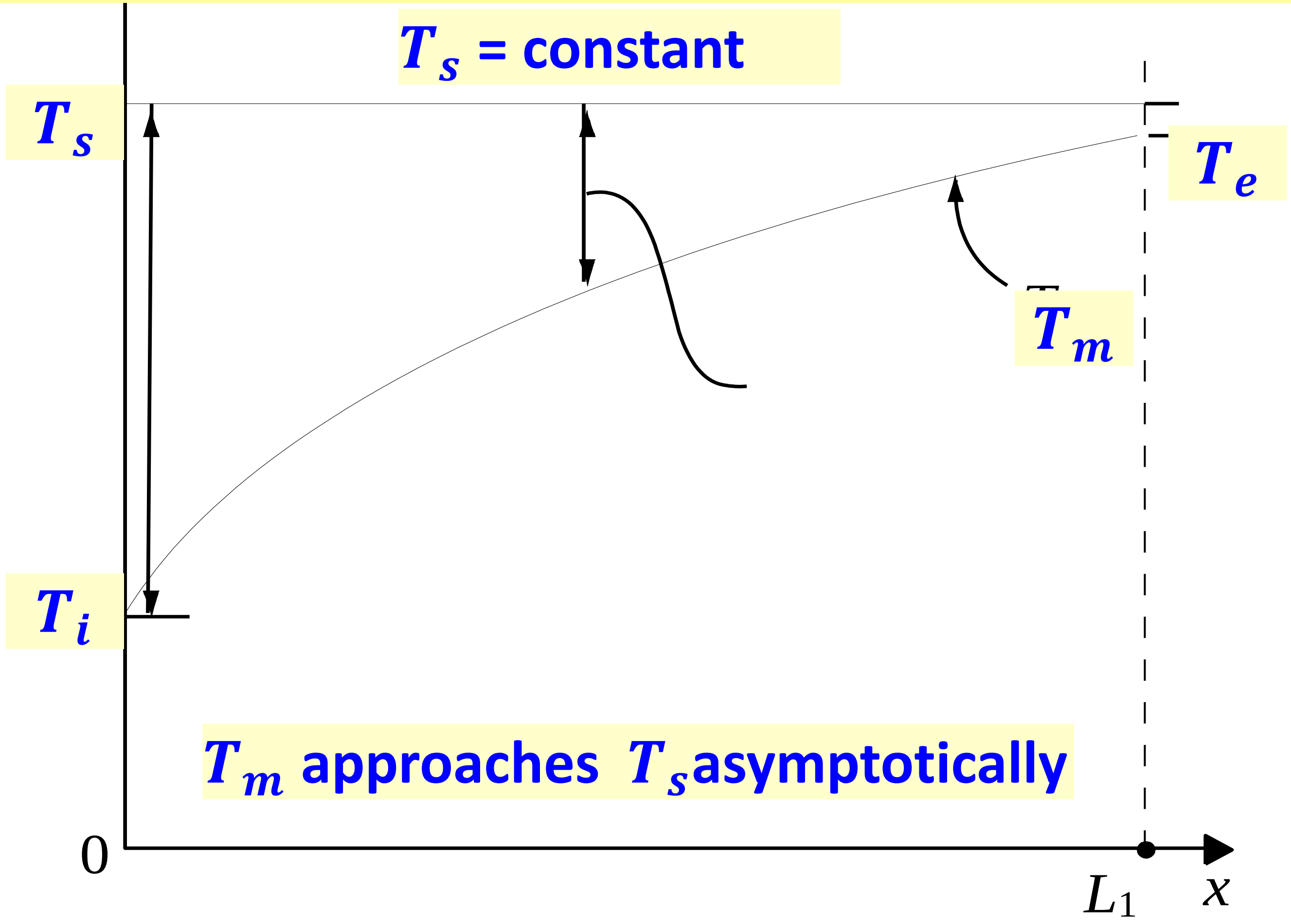
$$dT_m = -d(T_s - T_m)$$

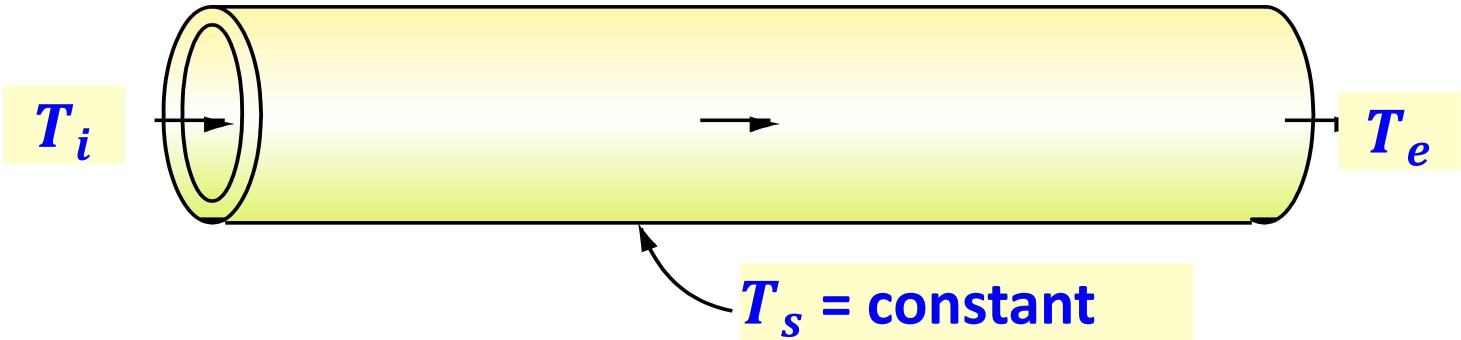
$$\dot{m}C_p[-d(T_s - T_m)] = h(T_s - T_m)Pdx$$

$$\frac{d(T_s - T_m)}{(T_s - T_m)} = -\frac{hP}{\dot{m}C_p}dx$$

$$\int_{T_i}^{T_e}\frac{d(T_s - T_m)}{(T_s - T_m)} = -\frac{hP}{\dot{m}C_p}\int_0^L dx$$

$$ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{hPL}{\dot{m}C_p}$$





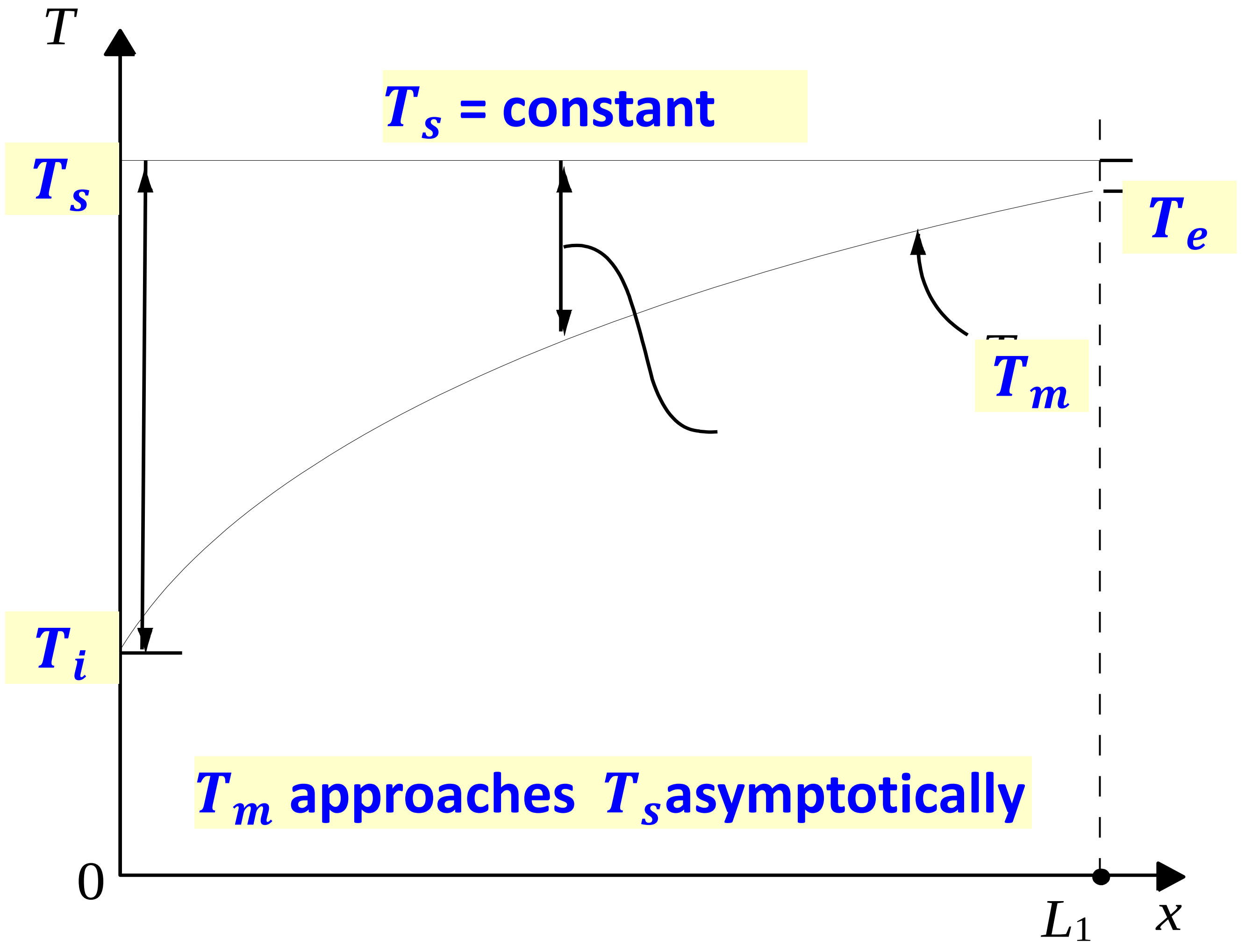
Variation of the mean fluid temperature along the tube for the case of constant temperature

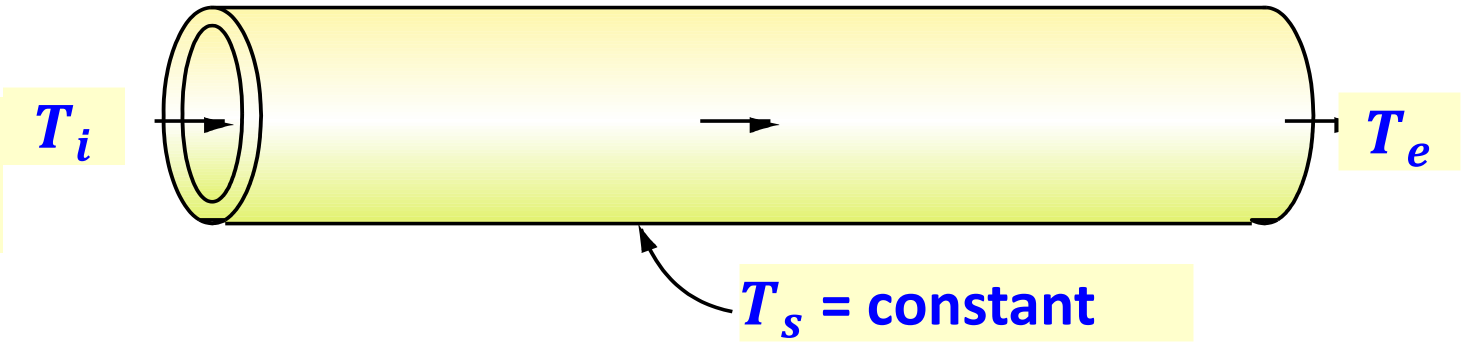
Temperature difference between the fluid and the surface decays exponentially in the flow direction, and the rate of decay depends on the magnitude of the exponent $-\dfrac{hA_s}{\dot{m}C_p}$

$$\int_{T_i}^{T}\frac{d(T_s-T_m)}{(T_s-T_m)}=-\frac{hP}{\dot{m}C_p}\int_0^x dx$$

$$ln\left(\frac{T_s-T(x)}{T_s-T_i}\right)=-\frac{hP}{\dot{m}C_p}x$$

$$T(x)=T_s-(T_s-T_i)e^{-\frac{hP}{\dot{m}C_p}x}$$

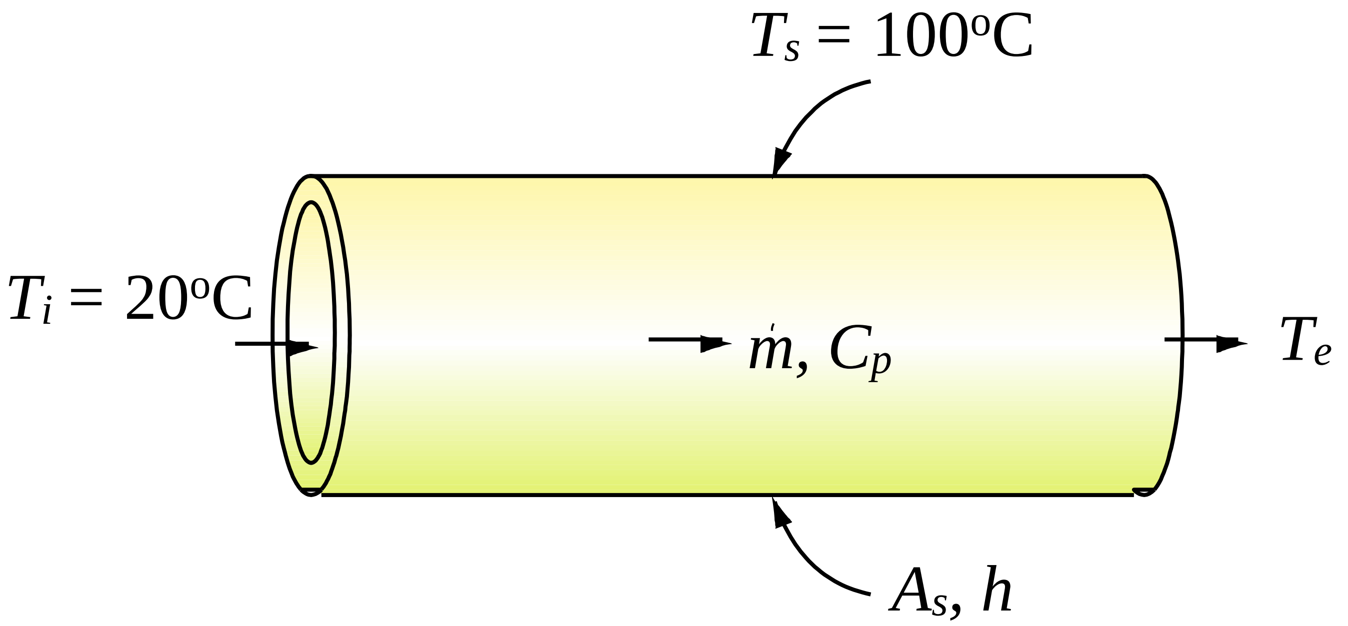




$\frac{hA_s}{\dot{m}C_p}$ is dimensionless parameter is called the number of transfer units, denoted by ***NTU***, and is a measure of the effectiveness of the heat transfer systems.

NTU of about 5 indicates that the limit is reached for heat transfer, and the heat transfer will not increase no matter how much we extend the length of the tube.

A large ***NTU*** and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, but it may be unacceptable from an economic point of view.



NTU = $hA_s/\dot{m}C_p$	T_e ,ºC
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

$$\ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{hPL}{\dot{m}C_p} = -\frac{hA_s}{\dot{m}C_p}$$

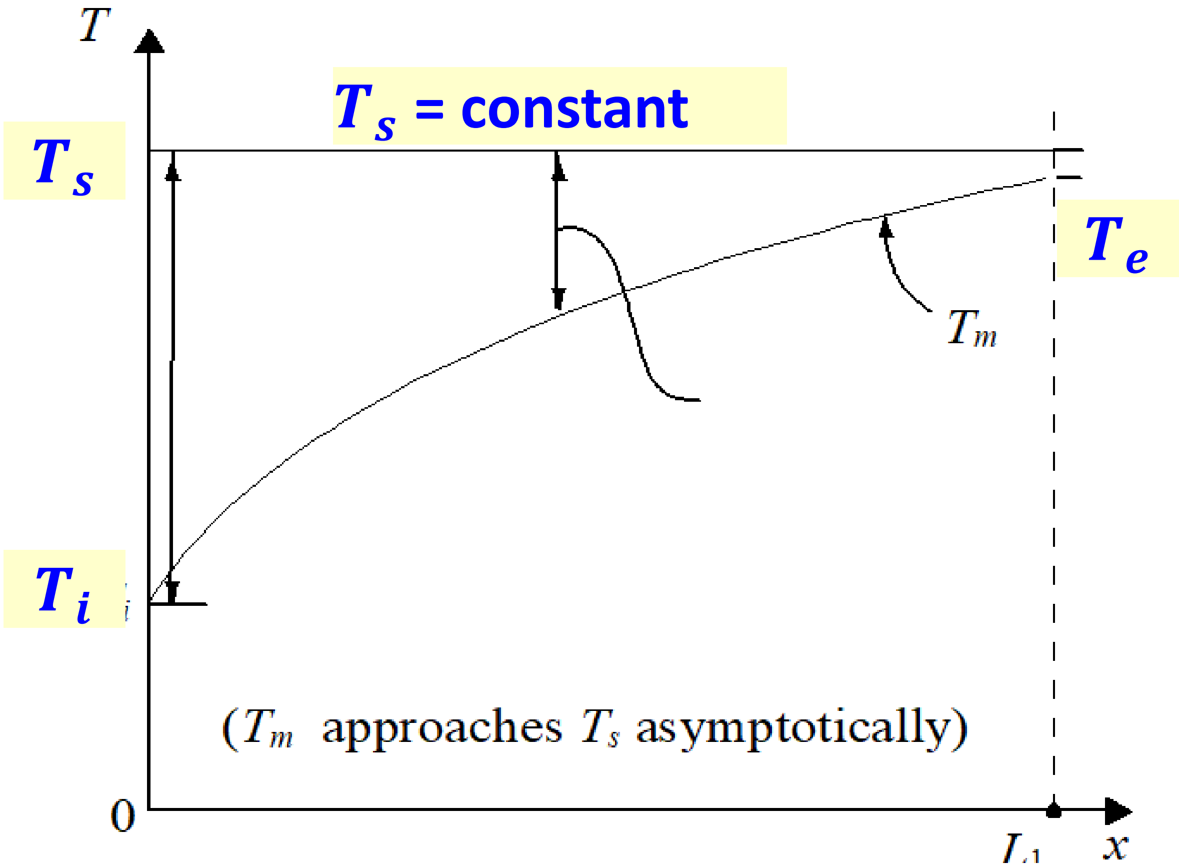
$$\dot{m}C_p = -\frac{hA_s}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = -\frac{hA_s}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}(T_e - T_i)$$

$$\dot{Q} = \frac{hA_s}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}(T_i - T_e)$$

$$\dot{Q} = hA_s\frac{(T_s - T_e) - (T_s - T_i)}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}$$

$$\dot{Q} = hA_s\Delta T_{LMTD}$$



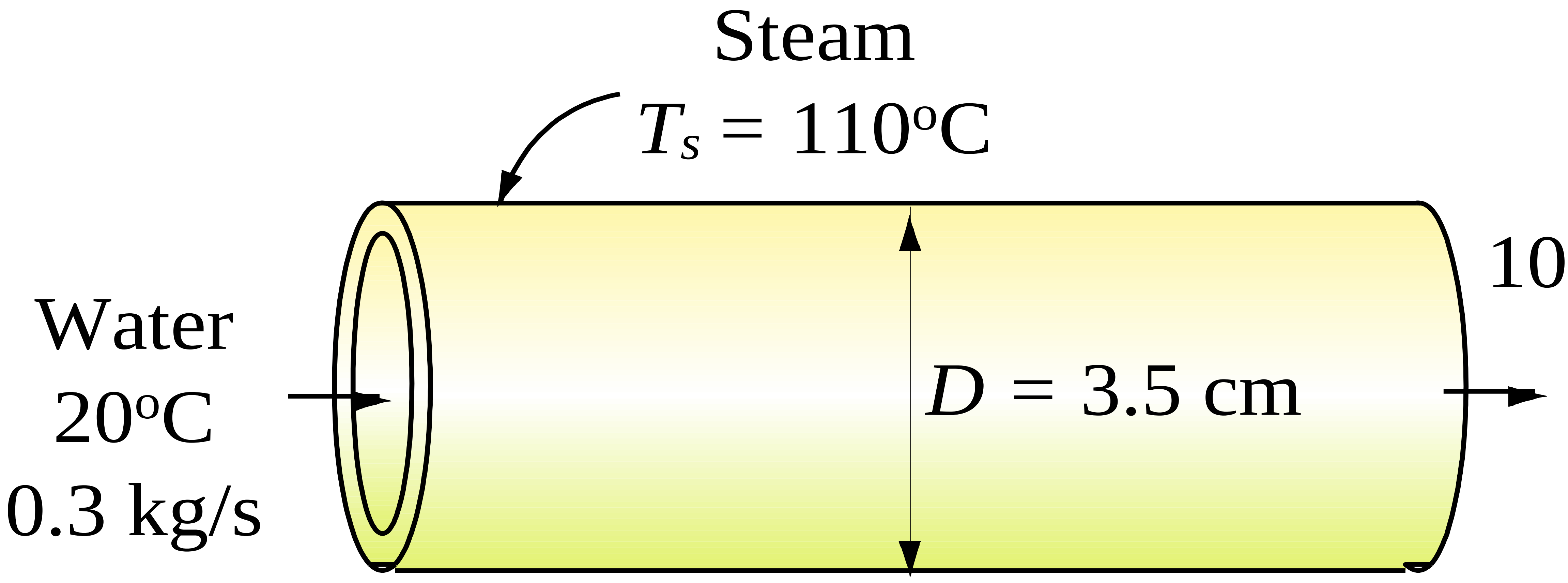
$$\Delta T_{LMTD} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}$$

Logarithmic mean temperature difference

$$\Delta T_{LMTD} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

$$\Delta T_e = T_s - T_e$$
$$\Delta T_i = T_s - T_i$$

Problem: Water enters a 3.5 cm internal diameter thin copper tube of a heat exchanger at a rate of 0.3 kg/s, and is heated by steam condensing outside at a temperature of 110° C . If the average heat transfer coefficient is 900 W/m^2.K, determine the length of the tube required in order to heat the water to 105 ° C .



Known: **Water is heated by steam in a circular tube..**

Find: **The tube length required to heat the water to a specified temperature is to be determined.**

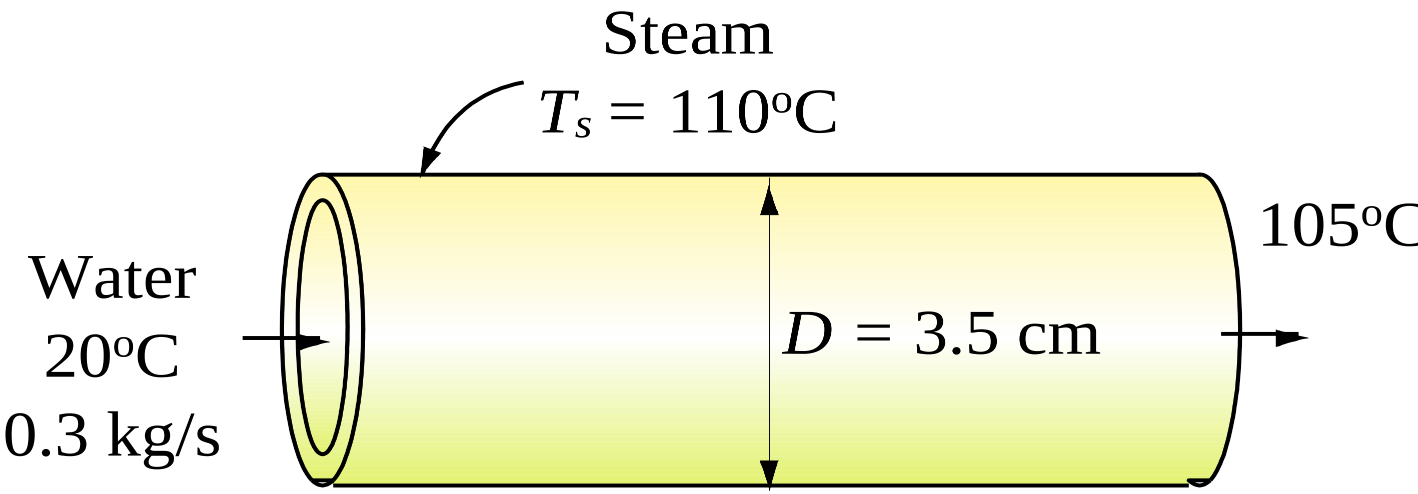
Assumptions:

Steady operations conditions exist.

Fluid properties are constant.

The convection heat transfer coefficient is constant.

The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.



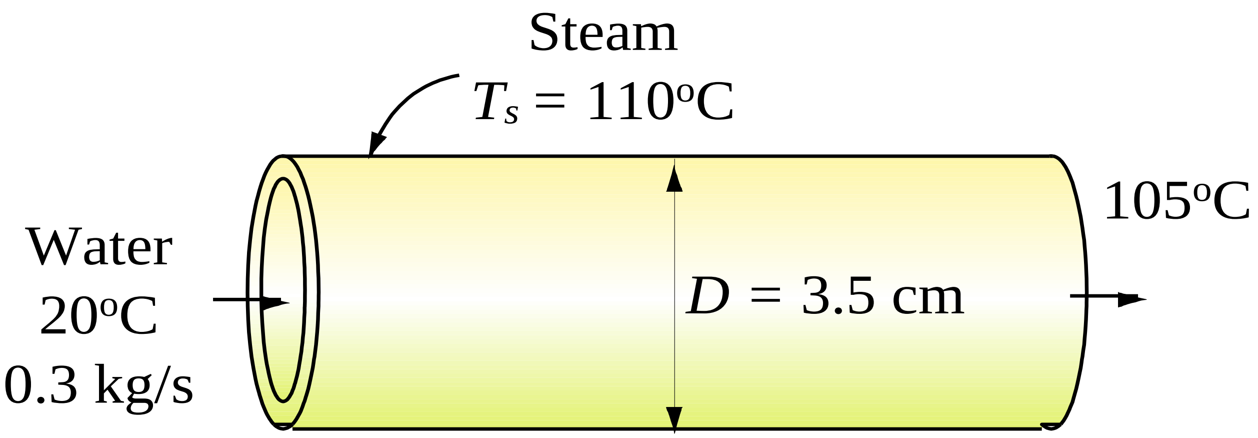
The specific heat of water of the bulk mean temperature of (20 + 110) / 2 = 65 is 4187 J/kg°C

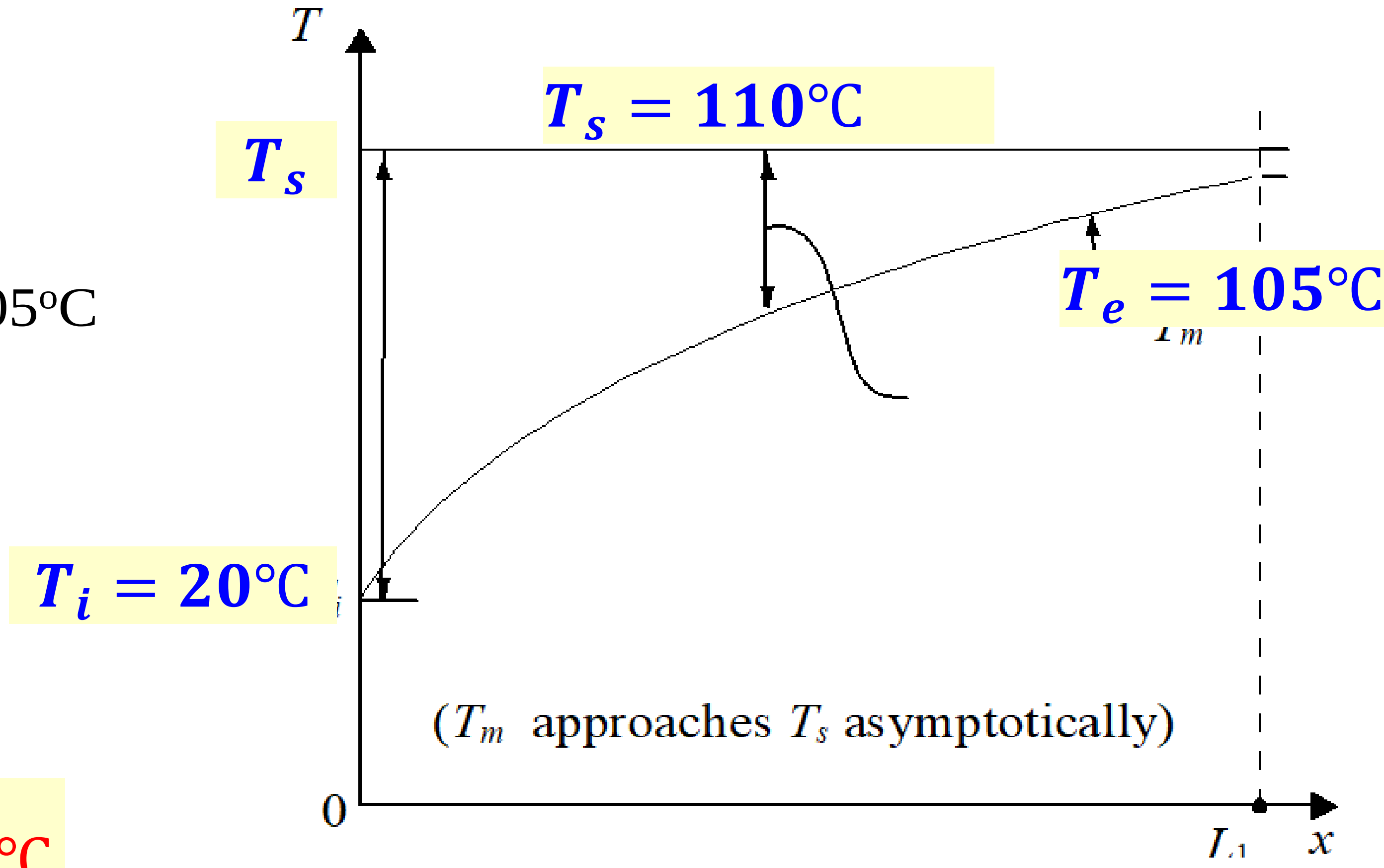
The heat of condensation of steam at 110° C is 2230 kJ/kg

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = 0.3 \times 4187(105 - 20) = 106.77\ kW$$

$$\dot{Q} = 106.77\ kW$$

$\dot{Q} = 106.77\ kW$





$$\Delta T_e = T_s - T_e = 110 - 105 = 5℃$$
$$\Delta T_i = T_s - T_i = 110 - 20 = 90℃$$

$$\Delta T_{LMTD} = \frac{\Delta T_e - \Delta T_i}{ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} = \frac{5 - 90}{ln\left(\frac{5}{90}\right)} = 29.41℃$$

$\dot{Q} = hA_s\Delta T_{LMTD}$ $\quad 106.77 \times 10^3 = (900)A_s(29.41)$ $\quad A_s = 4.03\ m^2$

$A_s = \pi DL$ $\quad 4.03 = \pi\left(3.5 \times 10^{-2}\right)L$ $\quad L = 37\ m$

OUTCOMES OF THE GOVERNING EQUATIONS

Mass and momentum equations

Solve these equations to get

v_r, v_θ, v_z and P

δ, C_f, f

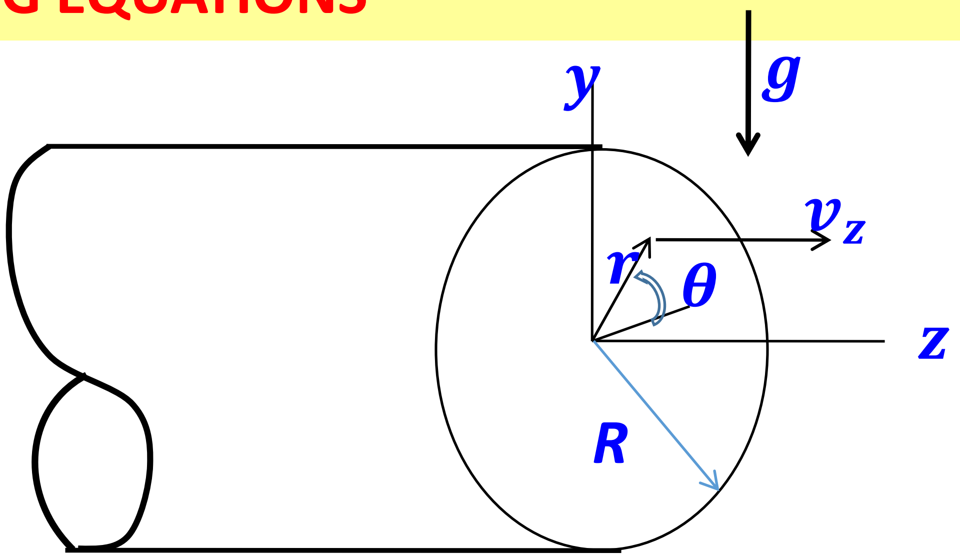
δ – Hydrodynamic boundary layer thickness

C_f - Skin friction coefficient

f - Friction factor

(Engineering necessity)

$C_f = f(Re)$ - dimensional similarity



$$f = 4C_f = 4\frac{\tau_w}{\frac{1}{2}\rho v_{zavg}^2}$$

τ_w – Wall Shear Stress

ρ – Density of the fluid

v_{zavg} – Average Velocity

$$P_{pump} = \frac{\dot{Q}\,\Delta P}{\eta_{pump}}$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

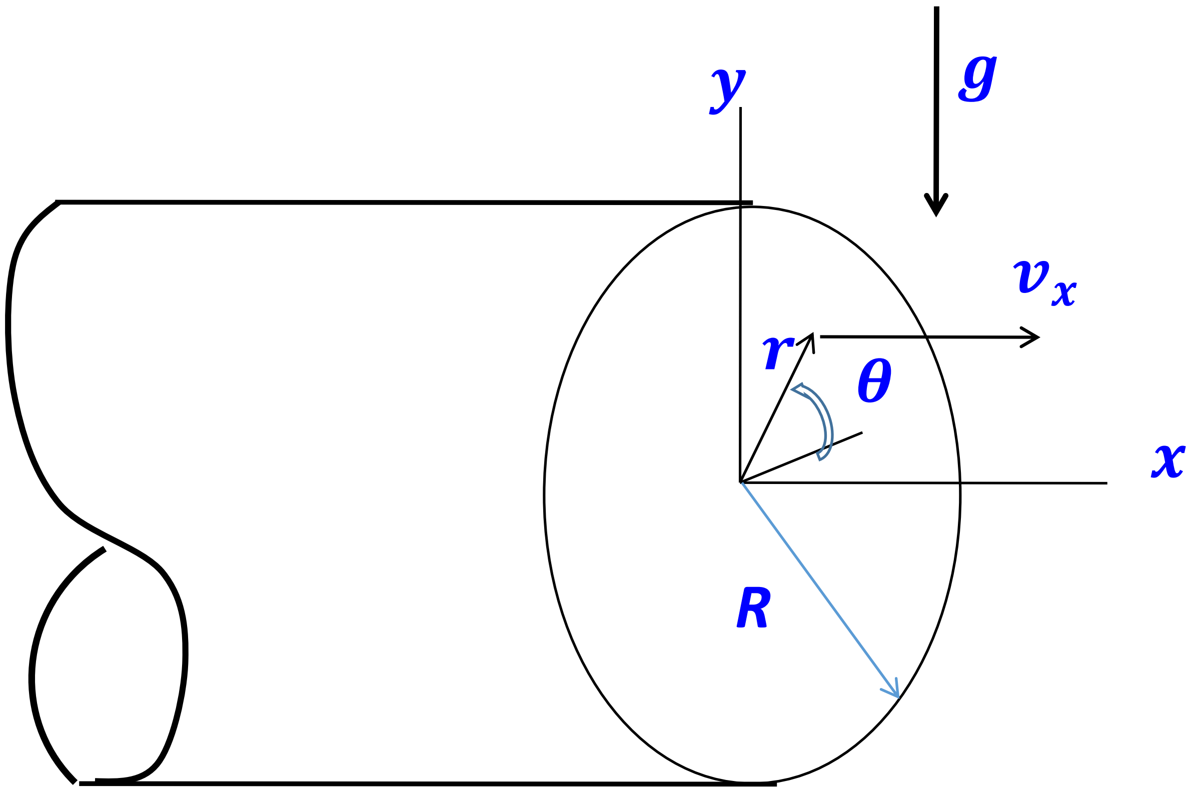
L – Length of the pipe

D – Diameter of the pipe

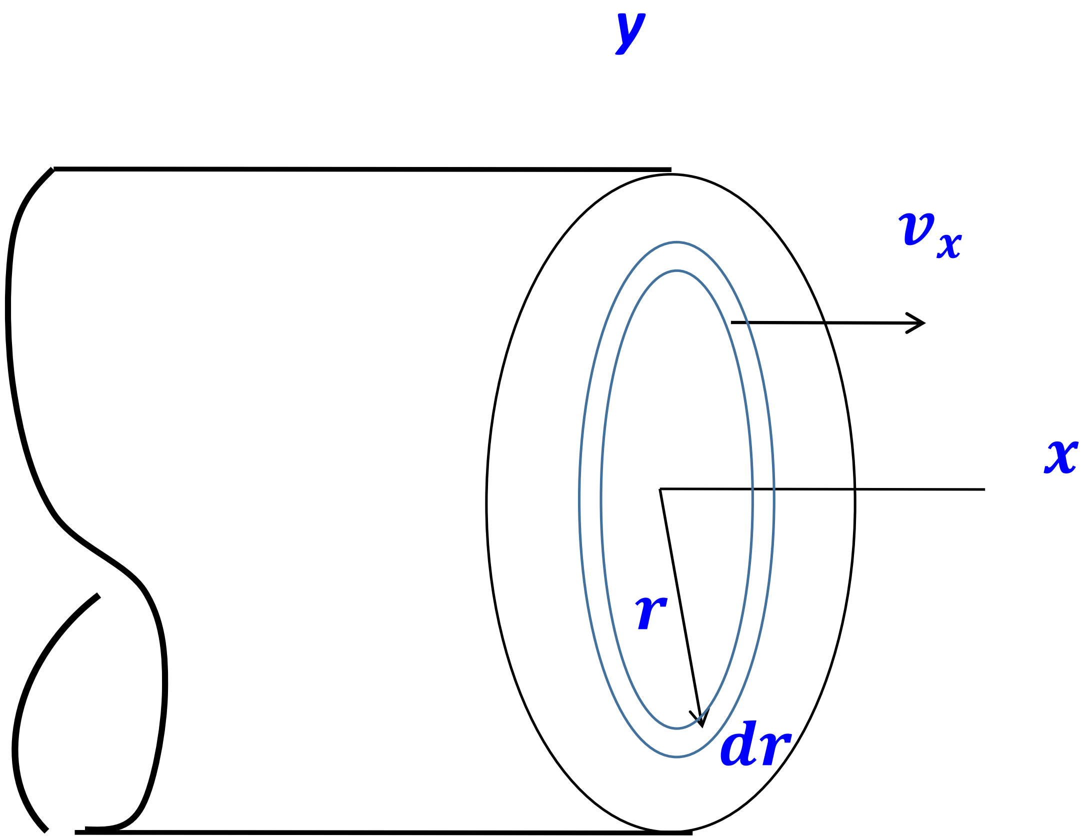
ΔP – Pressure drop

$\dot{Q}$ - Volume flow rate

STEADY FLOW IN PIPES - STEADY, LAMINAR, INCOMPRESSIBLE FLOW



a)



b)

HAGEN – POISEUILLE FLOW

Continuity equation

$$\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x} + \frac{v_r}{r} = 0$$

Navier stokes equations – viscous, incompressible, steady flow

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_x \frac{\partial v_x}{\partial x} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial x} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \theta} + \nu\left[\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right]$$

$$v_r \frac{\partial v_x}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_x}{\partial \theta} + v_x \frac{\partial v_x}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu[\nabla^2 v_x]$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial x^2}$$

Hydrodynamically fully developed flow $v_r = v_\theta = 0$ $v_x = v_x(r)$

Continuity equation

$$\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x} + \frac{v_r}{r} = 0$$

$$\frac{\partial v_z(r)}{\partial z} = 0$$

Implies hydrodynamically fully developed flow

Navier stokes equations – viscous, incompressible, steady flow

$$v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_x\frac{\partial v_x}{\partial x} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$\frac{\partial P}{\partial r} = 0$$

$$v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_x\frac{\partial v_\theta}{\partial x} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \theta} + \nu\left[\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right]$$

$$\frac{\partial P}{\partial \theta} = 0$$

Hence, pressure is a function of z only ; $P = P(z)$ only

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

$$v_r\frac{\partial v_x}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_x}{\partial \theta} + v_x\frac{\partial v_x}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r}\frac{\partial v_x}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_x}{\partial \theta^2} + \frac{\partial^2 v_x}{\partial x^2}\right]$$

$$0 = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left[\frac{d^2 v_x}{dr^2} + \frac{1}{r}\frac{dv_x}{dr}\right]$$

$$\frac{dP}{dx} = \mu\left[\frac{d^2 v_x}{dr^2} + \frac{1}{r}\frac{dv_x}{dr}\right]$$

$$\mu\left[\frac{d^2v_x}{dr^2}+\frac{1}{r}\frac{dv_x}{dr}\right]=\frac{dP}{dx}$$

Multiply by r

$$\mu\left[r\frac{d^2v_x}{dr^2}+\frac{dv_x}{dr}\right]=r\frac{dP}{dx}$$

$$\mu\frac{d}{dr}\left(r\frac{dv_x}{dr}\right)=r\frac{dP}{dx}$$

Integrating w.r.t r

$$\mu\left(r\frac{dv_x}{dr}\right)=\frac{r^2}{2}\frac{dP}{dx}+A$$

Divide with μr

$$\frac{dv_x}{dr}=\frac{r}{2\mu}\frac{dP}{dx}+\frac{A}{\mu r}$$

$$v_x=\frac{1}{4\mu}\frac{dP}{dx}r^2+\frac{A}{\mu}lnr+B$$

Boundary conditions

$$r=0 \quad \frac{dv_x}{dr}=0$$

$$r=R \quad v_x=0$$

$$\frac{dv_x}{dr}=\frac{r}{2\mu}\frac{dP}{dx}+\frac{A}{\mu r}$$

$$r=0 \quad \frac{dv_x}{dr}=0$$

$$A=0$$

$$v_x=\frac{1}{4\mu}\frac{dP}{dx}r^2+B$$

$$r=R \quad v_x=0$$

$$0=\frac{1}{4\mu}\frac{dP}{dx}R^2+B$$

$$B=-\frac{1}{4\mu}\frac{dP}{dx}R^2$$

$$v_x=\frac{1}{4\mu}\frac{dP}{dx}r^2-\frac{1}{4\mu}\frac{dP}{dx}R^2$$

$$v_x=-\frac{R^2}{4\mu}\frac{dP}{dx}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Paraboloid of revolution

Using this velocity distribution, compute

- Maximum velocity
- Average velocity
- Volumetric flow rate
- Shear stress
- Skin friction coefficient
- Friction factor

$$v_x = -\frac{R^2}{4\mu}\frac{dP}{dx}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Maximum velocity

$r = 0 \quad v_x = v_{xmax}$

$$v_{xmax} = -\frac{R^2}{4\mu}\frac{dP}{dx}$$

$$v_x = v_{xmax}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Average velocity

$$u_m = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_x r\,dr\,d\theta = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_{xmax}\left[1-\left(\frac{r}{R}\right)^2\right] r\,dr\,d\theta = \frac{2\pi}{\pi R^2}v_{xmax}\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right] r\,dr$$

$$u_m = \frac{2v_{xmax}}{R^2}\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right] r\,dr = \frac{2v_{xmax}}{R^2}\int_0^R\left[r-\frac{r^3}{R^2}\right]dr = \frac{2v_{xmax}}{R^2}\left[\frac{R^2}{2}-\frac{R^4}{4R^2}\right] = \frac{2v_{xmax}}{R^2}\left[\frac{R^2}{2}-\frac{R^2}{4}\right]$$

$$u_m = \frac{2v_{xmax}}{R^2}\frac{R^2}{4} = \frac{v_{xmax}}{2}$$

$$u_m = \frac{v_{xmax}}{2} = -\frac{R^2}{8\mu}\frac{dP}{dx}$$

Volumetric flow rate

$$\dot{Q} = \pi R^2 u_m = \pi R^2\left(-\frac{R^2}{8\mu}\frac{dP}{dx}\right)$$

$$\dot{Q} = \frac{\pi R^4}{8\mu}\left(-\frac{dP}{dx}\right)$$

$$v_x = -\frac{R^2}{4\mu}\frac{dP}{dx}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$u_m = -\frac{R^2}{8\mu}\frac{dP}{dx}$$

Shear Stress

$$\tau_w = -\mu \frac{dv_x}{dr}\bigg|_{r=R} = -\mu\left(-\frac{R^2}{4\mu}\frac{dP}{dx}\right)\left(-\frac{2R}{R^2}\right) = -\frac{R}{2}\frac{dP}{dx} = -\frac{R}{2}\left(-\frac{8\mu u_m}{R^2}\right)$$

$$\tau_w = \frac{4\mu u_m}{R}$$

Skin Friction Coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_m^2} = \frac{\frac{4\mu u_m}{R}}{\frac{1}{2}\rho u_m^2} = \frac{8\mu u_m}{\rho u_m R} = \frac{16\mu}{\rho u_m D} = \frac{16}{Re_D}$$

$$C_f = \frac{16}{Re_D}$$

$$Re_D = \frac{\rho u_m D}{\mu}$$

Friction Factor

$$f = 4C_f = \frac{64}{Re_D}$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

$$v_x = -\frac{R^2}{4\mu}\frac{dP}{dx}\left[1-\left(\frac{r}{R}\right)^2\right]$$

HAGEN – POISEUILLE FLOW

Steady, Laminar, Incompressible Flow in a circular pipe

Maximum velocity

$$v_{xmax} = -\frac{R^2}{4\mu}\frac{dP}{dx}$$

$$v_x = v_{xmax}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Average velocity

$$u_m = \frac{v_{xmax}}{2} = -\frac{R^2}{8\mu}\frac{dP}{dx}$$

Volumetric flow rate

$$\dot{Q} = \frac{\pi R^4}{8\mu}\left(-\frac{dP}{dx}\right)$$

Shear Stress

$$\tau_w = \frac{4\mu u_m}{R}$$

Skin Friction Coefficient

$$C_f = \frac{16}{Re_D}$$

$$Re_D = \frac{\rho u_m D}{\mu}$$

Friction Factor

$$f = 4C_f = \frac{64}{Re_D}$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

OUTCOMES OF THE GOVERNING EQUATIONS

Energy equation

Solve this equation using
v_r, v_θ, v_x and P
to get

T

δ_T, Nu

δ_T – Thermal boundary layer thickness

Nu – Nusselt number (Engineering necessity)

$Nu = f(Re, Pr)$ - dimensional similarity

$$Nu = \frac{hD}{k_f}$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial r}\right|_{r=R}}{(T_s - T_m)}$$

h – heat transfer coefficient
D – Diameter of the pipe
k_f – Thermal conductivity of the fluid
T_s – Surface temperature of the Plate
T_m – Bulk fluid temperature

Resistance to the transfer of heat from the pipe to the stream

$$\dot{Q} = h(\pi DL)(T_s - T_m)$$

L – Length of the pipe

Temperature Profile and the Nusselt Number in a circular pipe with constant heat flux
Steady, incompressible, laminar flow

$$\rho C_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_x\frac{\partial T}{\partial x}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial x^2}\right]+\dot{q}+\phi_v$$

Hydrodynamically fully developed flow $v_r=v_\theta=0$ $v_z=v_z(r)$ $\dot{q}=0 \quad \phi_v=0$

Thermally fully developed flow $T=T(r,x)$ $\dfrac{\partial^2 T}{\partial x^2}=0$

$$\rho C_p\left(v_x\frac{\partial T}{\partial x}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

$$v_x\frac{\partial T}{\partial x}=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

$$u_m=\frac{v_{xmax}}{2}$$

$$\frac{dT_s}{dx}=\frac{\partial T}{\partial x}=\frac{dT_m}{dx}=\frac{2q_s''}{\rho u_m C_p R}=constant$$

$$v_x=v_{xmax}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$v_x=2u_m\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$v_x\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\frac{2q_s''}{\rho u_m C_p R}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\frac{2q_s''}{\rho u_m C_p R}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\left[1-\left(\frac{r}{R}\right)^2\right]\frac{4q_s''}{\rho C_p R}=\frac{k}{\rho C_p r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\frac{4q_s''}{kR}\left[1-\left(\frac{r}{R}\right)^2\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\frac{\partial}{\partial r}(\quad)=\frac{4q_s''}{kR}\left[r-\frac{r^3}{R^2}\right]$$

$$r\frac{\partial T}{\partial r}=\frac{4q_s''}{kR}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]+C_1$$

Boundary condition

$$r=0 \quad \frac{\partial T}{\partial r}=0$$

$$C_1=0$$

$$r\frac{\partial T}{\partial r}=\frac{4q_s''}{kR}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]$$

$$\frac{\partial T}{\partial r}=\frac{4q_s''}{kR}\left[\frac{r}{2}-\frac{r^3}{4R^2}\right]$$

$$T=\frac{4q_s''}{kR}\left[\frac{r^2}{4}-\frac{r^4}{16R^2}\right]+C_2$$

Boundary condition

$$r=R \quad T=T_s$$

$$T_s=\frac{4q_s''}{kR}\left[\frac{R^2}{4}-\frac{R^4}{16R^2}\right]+C_2$$

$$T_s-\frac{4q_s''}{kR}\left[\frac{R^2}{4}-\frac{R^2}{16}\right]=C_2$$

$$T_s-\frac{4q_s''}{kR}\left[\frac{3R^2}{16}\right]=C_2$$

$$C_2=T_s-\frac{3q_s''R}{4k}$$

$$T = \frac{4q_s''}{kR}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_2$$

$$C_2 = T_s - \frac{3q_s''R}{4k}$$

$$T = \frac{4q_s''}{kR}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + T_s - \frac{3q_s''R}{4k}$$

$$T = T_s - \frac{q_s''R}{k}\left[\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right]$$

$$T_m = \frac{2}{u_m R^2}\int_0^R T(r)v_x(r)r\,dr$$

$$v_x = 2u_m\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$T_m = \frac{2}{u_m R^2}\int_0^R \left[T_s - \frac{q_s''R}{k}\left[\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right]\right]\left(2u_m\left[1 - \left(\frac{r}{R}\right)^2\right]\right)r\,dr$$

$$T_m = \frac{2}{u_m R^2}\int_0^R \left[T_s - \frac{q_s'' R}{k}\left[\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right]\right]\left(2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\right) r\,dr$$

$$T_m = \frac{4}{R^2}\int_0^R \left[T_s - \frac{q_s'' R}{k}\left[\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right]\right]\left(r - \frac{r^3}{R^2}\right) dr$$

$$T_m = \frac{4}{R^2}\int_0^R T_s\left(r - \frac{r^3}{R^2}\right) - \frac{q_s'' R}{k}\left(\frac{3}{4}r - \frac{r^3}{R^2} + \frac{r^5}{4R^4} - \frac{3}{4}\frac{r^3}{R^2} + \frac{r^5}{R^4} - \frac{r^7}{4R^6}\right) dr$$

$$T_m = \frac{4}{R^2}\left[T_s\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) - \frac{q_s'' R}{k}\left(\frac{3}{4}\frac{R^2}{2} - \frac{R^4}{4R^2} + \frac{R^6}{24R^4} - \frac{3}{4}\frac{R^4}{4R^2} + \frac{R^6}{6R^4} - \frac{R^8}{32R^6}\right)\right]$$

$$T_m = \frac{4}{R^2}T_s\left(\frac{R^2}{2} - \frac{R^2}{4}\right) - \frac{4}{R^2}\frac{q_s'' R}{k}\left(\frac{3}{4}\frac{R^2}{2} - \frac{R^4}{4R^2} + \frac{R^6}{24R^4} - \frac{3}{4}\frac{R^4}{4R^2} + \frac{R^6}{6R^4} - \frac{R^8}{32R^6}\right)$$

$$T_m = T_s - \frac{4q_s'' R}{k}\left(\frac{3}{8} - \frac{1}{4} + \frac{1}{24} - \frac{3}{16} + \frac{1}{6} - \frac{1}{32}\right)$$

Temperature Profile and the Nusselt Number in a circular pipe with constant heat flux
Steady, incompressible, laminar flow

$$T_m = T_s - \frac{4q_s''R}{k}\left(\frac{3}{8} - \frac{1}{4} + \frac{1}{24} - \frac{3}{16} + \frac{1}{6} - \frac{1}{32}\right)$$

$$T_m = T_s - \frac{4q_s''R}{k}\left(\frac{11}{96}\right)$$

$$T_m = T_s - \frac{11}{24}\frac{q_s''R}{k}$$

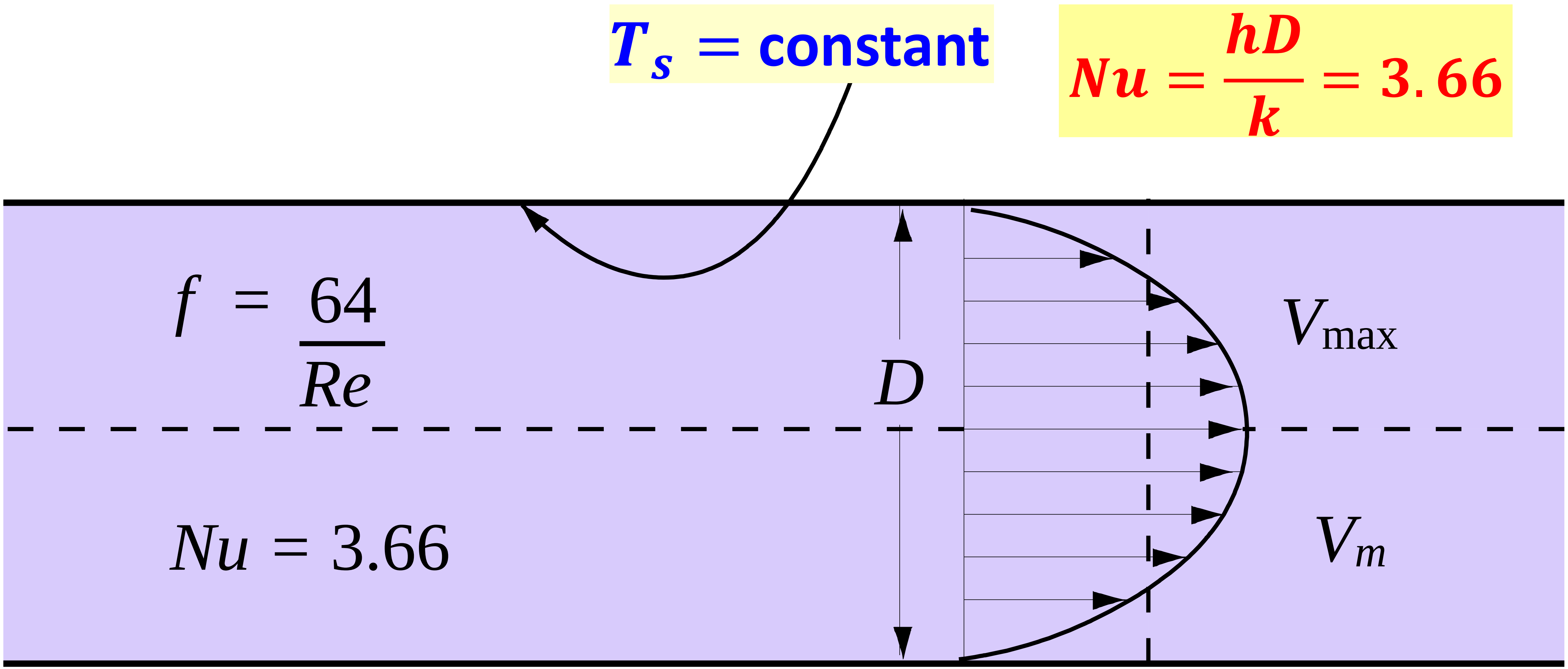
$$T_s - T_m = \frac{11}{24}\frac{q_s''R}{k}$$

$$h = \frac{q_s''}{T_s - T_m} = q_s''\frac{24}{11}\frac{k}{q_s''R} = \frac{24}{11}\frac{k}{R}$$

$$\frac{hR}{k} = \frac{24}{11} \qquad \frac{hD}{k} = \frac{48}{11} \qquad Nu = \frac{48}{11} = 4.3636$$

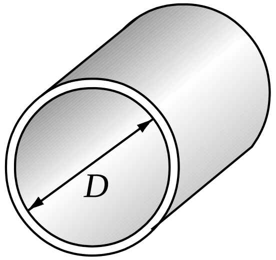
LAMINAR FLOW - CONSTANT SURFACE TEMPERATURE

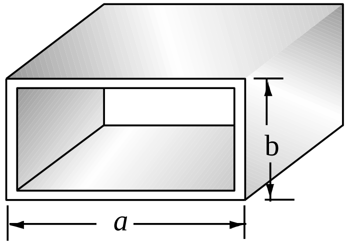
$$Nu = \frac{hD}{k} = 3.66$$

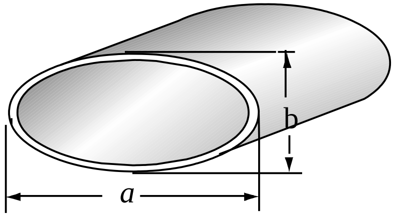


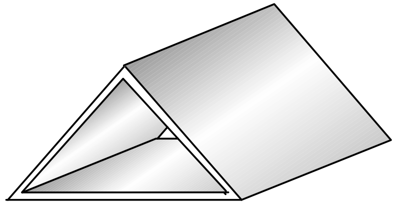
Fully developed laminar flow

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections









Tube Geometry	a/b or θ	Nusselt Number T_s = constant	Nusselt Number q''= constant	Friction Factor f
Circle	--	3.66	4.36	64.00/*Re*
Rectangle	a/b			
	1	2.98	3.61	56.92/*Re*
	2	3.39	4.12	62.20/ *Re*
	3	3.96	4.79	68.3.6/ *Re*
	4	4.44	5.33	72.92/ *Re*
	6	5.14	6.05	78.80/ *Re*
	8	5.60	6.49	82.32/ *Re*
		7.54	8.24	96.00/ *Re*
Ellipse	a/b			
	1	3.66	4.36	64.00/ *Re*
	2	3.74	4.56	67.28/ *Re*
	4	3.79	4.88	72.96/ *Re*
	8	3.72	5.09	76.60/ *Re*
	16	3.65	5.18	78.16/ *Re*
Triangle				
	10	1.61	2.45	50.80/ *Re*
	30	2.26	2.91	52.28/ *Re*
	60	2.47	3.11	53.32/ *Re*
	90	2.34	2.98	52.60/ *Re*
	120	2.00	2.68	50.96/ *Re*

$$D_h = \frac{4A_c}{P}$$

$$u_m = \frac{\dot{m}}{\rho A_c}$$

$$Re = \frac{\rho u_m D_h}{\mu}$$

$$Nu = \frac{hD_h}{k}$$

Developing Laminar Flow in the Entrance Region – constant surface temperature

$$Nu = 3.66 + \frac{0.065\left(\frac{D}{L}\right) RePr}{1 + 0.04\left[\left(\frac{D}{L}\right) RePr\right]^{\frac{2}{3}}}$$

Edwards et al., 1979

Average Nusselt number is larger at the entrance region, as expected, and it approaches asymptotically to the fully developed value of 3.66 as length tends to infinity

Takes into account the property variation

$$Nu = 1.86\left(RePr\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

Sieder and Tate (1936)

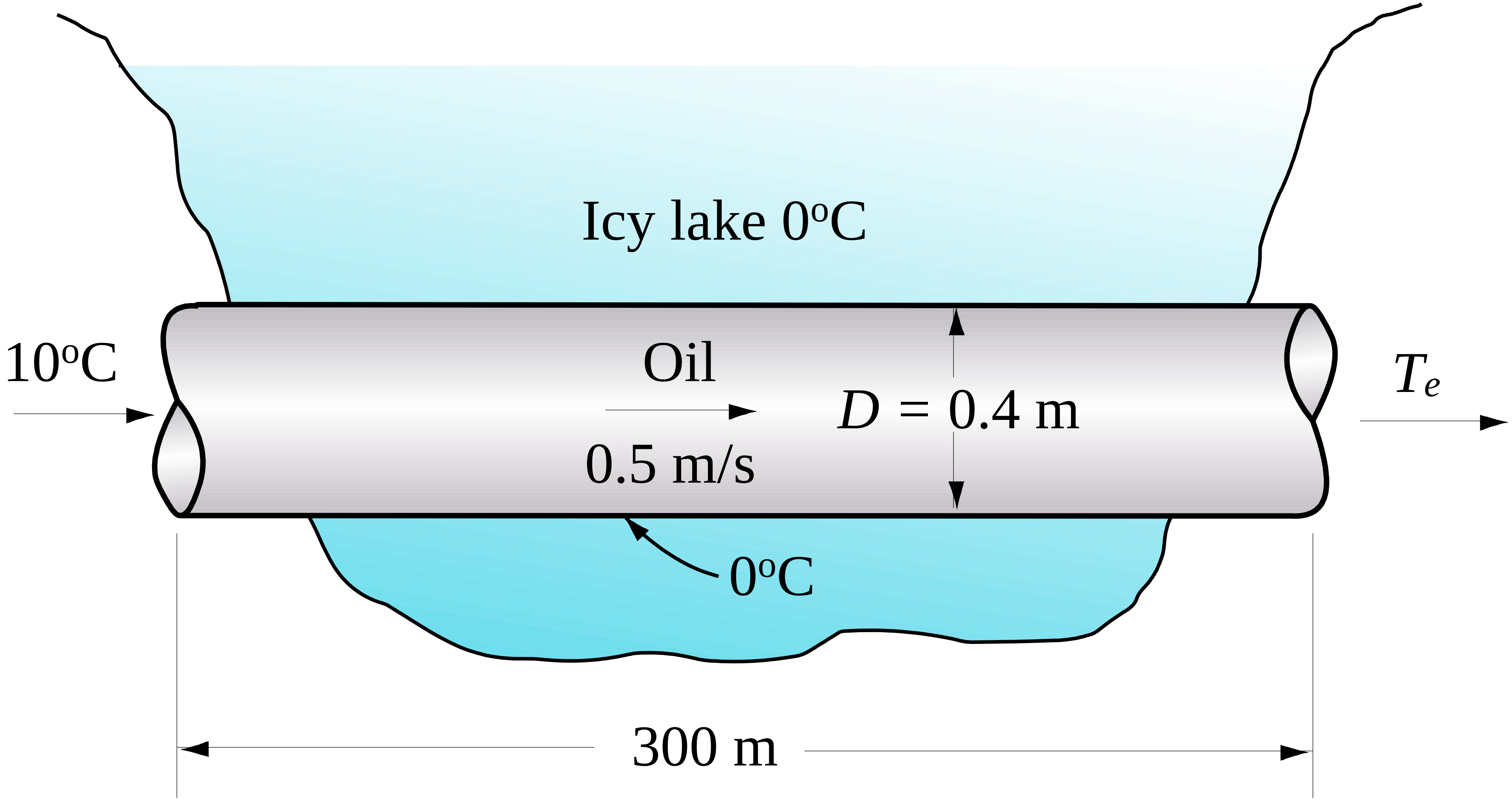
The average Nusselt number for the thermal entrance region of flow between isothermal parallel plates of length L $(Re \leq 2800)$

$$Nu = 7.54 + \frac{0.03\left(\frac{D_h}{L}\right) RePr}{1 + 0.016\left[\left(\frac{D_h}{L}\right) RePr\right]^{\frac{2}{3}}}$$

Edwards et al., 1979

Problem: Consider the flow of oil at 10 deg C in 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m long section of the pipeline passes through icy waters of a lake at 0 deg C . Measurements indicate that the surface temperature of the pipe is very nearly 0 deg C . Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

Schematic:



Known: Oil flows in a pipeline that passes through icy waters of a lake at 0 deg C .

Find: The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions:

Steady operating conditions exist.

The surface temperature of the pipe is very nearly 0 deg C.

The thermal resistance of the pipe is negligible.

The inner surfaces of the pipeline are smooth.

The flow is hydrodynamically developed when the pipeline reaches the lake.

We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 10° C , and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 10 ° C we read

$$\rho = 893.55\, kg/m^3 \quad Pr = 28750$$
$$k = 0.14595\ W/m.°\mathrm{C} \quad \nu = 2.592\ \times 10^{-3}\ m^2/s$$
$$C_p = 1839\, J/kg.°\mathrm{C}$$

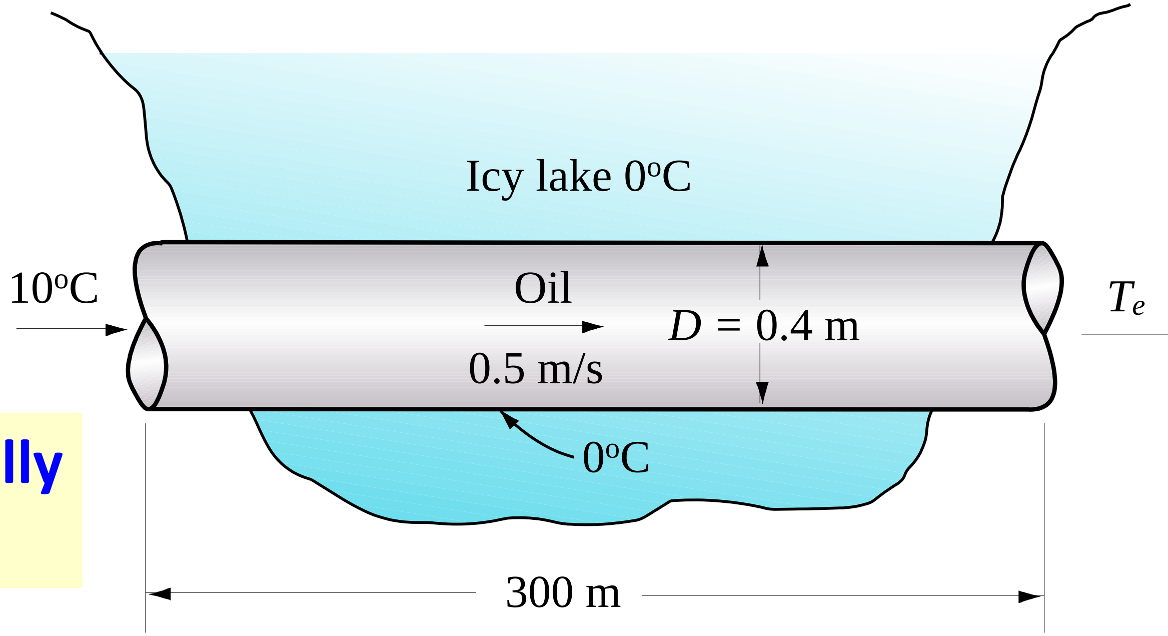
$$Re_D = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu} = \frac{0.5 \times 0.4}{2.592 \times 10^{-3}}$$

$\rho = 893.55\, kg/m^3$ $Pr = 28750$

$k = 0.14595\ W/m.°C$ $\nu = 2.592 \times 10^{-3}\ m^2/s$

$C_p = 1839\, J/kg.°C$

$Re_D = 77$ Laminar flow



Thermal entrance length

$$L_{t,Laminar} \approx 0.05DRePr$$

$$L_{t,Laminar} \approx 0.05 \times 0.4 \times 77 \times 28750$$

$$L_{t,Laminar} \approx 44275\, m$$

As 300 m is less than 44275 mm, thermally developing flow is considered.

$$Nu = 3.66 + \frac{0.065\left(\frac{D}{L}\right)RePr}{1 + 0.04\left[\left(\frac{D}{L}\right)RePr\right]^{\frac{2}{3}}}$$

$$Nu = 3.66 + \frac{0.065\left(\frac{0.4}{300}\right)77 \times 28750}{1 + 0.04\left[\left(\frac{0.4}{300}\right)77 \times 28750\right]^{\frac{2}{3}}}$$

$Nu = 24.44$ $\quad Nu = \frac{hD}{k}$ $\quad 24.44 = \frac{h \times 0.4}{0.14595}$

$$h = 8.92\, W/m^2°C$$

$$h = 8.92\, W/m^2 °C$$

$$A_s = \pi DL = \pi(0.4)(300) = 377\, m^2$$

$$\rho = 893.55\, kg/m^3 \quad Pr = 28750$$

$$k = 0.14595\ W/m.°C \quad \nu = 2.592\ \times 10^{-3}\ m^2/s$$

$$C_p = 1839\, J/kg.°C$$

$$\dot{m} = \rho A u_m = 893.55 \times \frac{\pi}{4}(0.4)^2 \times 0.5 = 56.14\, kg/s$$

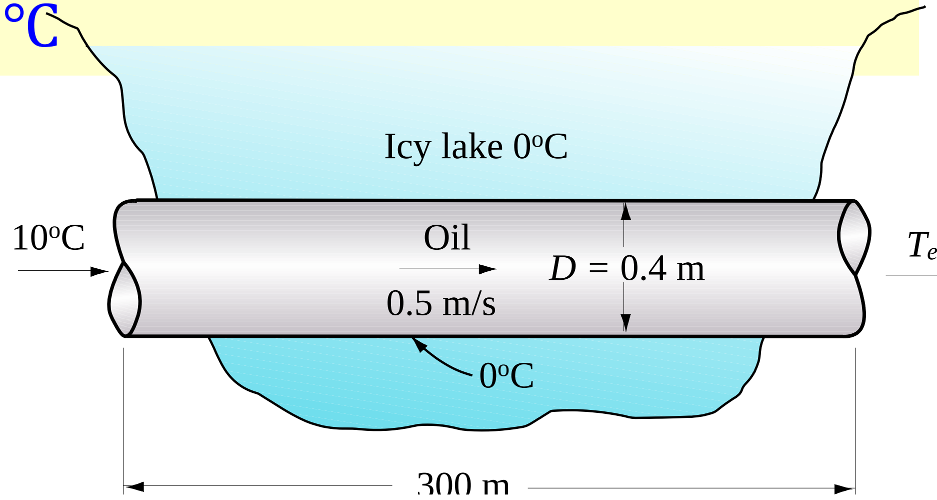
$$T(x) = T_s - (T_s - T_i)e^{-\frac{hP}{\dot{m}C_p}x}$$

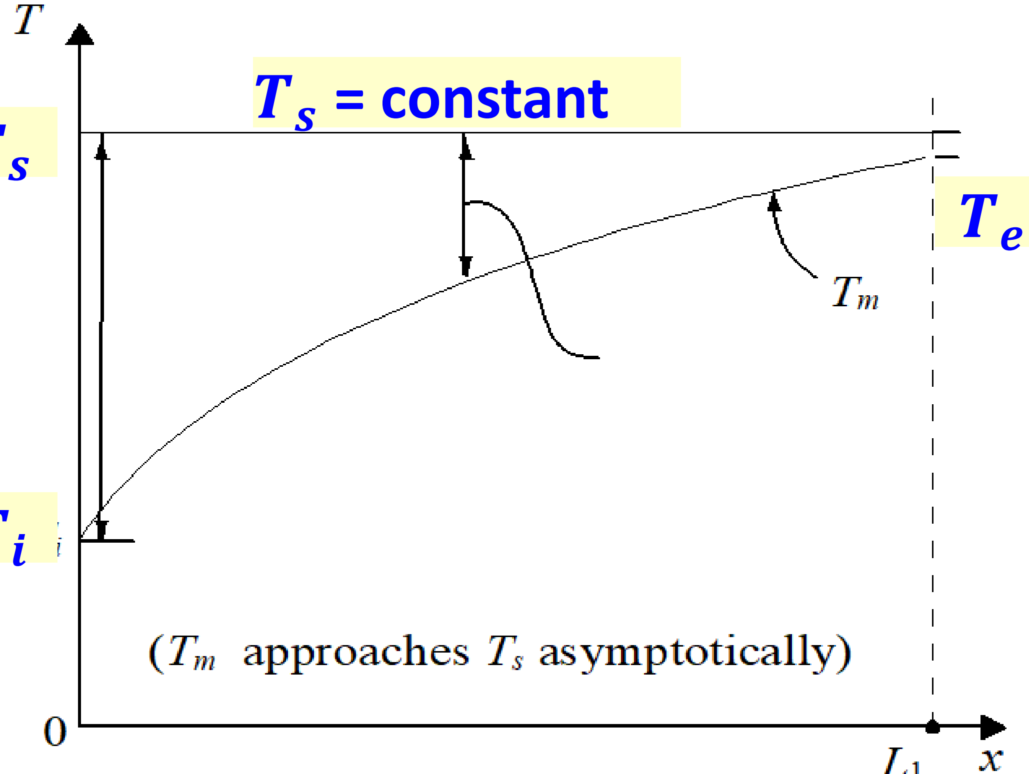
$$T_e = T_s - (T_s - T_i)e^{-\frac{hA_s}{\dot{m}C_p}}$$

$$T_e = 0 - (0 - 10)e^{-\frac{8.92 \times 377}{56.14 \times 1839}}$$

$$T_e = 9.68°C$$

Thus the mean temperature of oil drops by a mere 0.32°C as it crosses the lake. This makes the bulk mean temperature 9.84°C, which is practically identical to the inlet temperature of 10°C . Therefore, we do not need to reevaluate the properties.





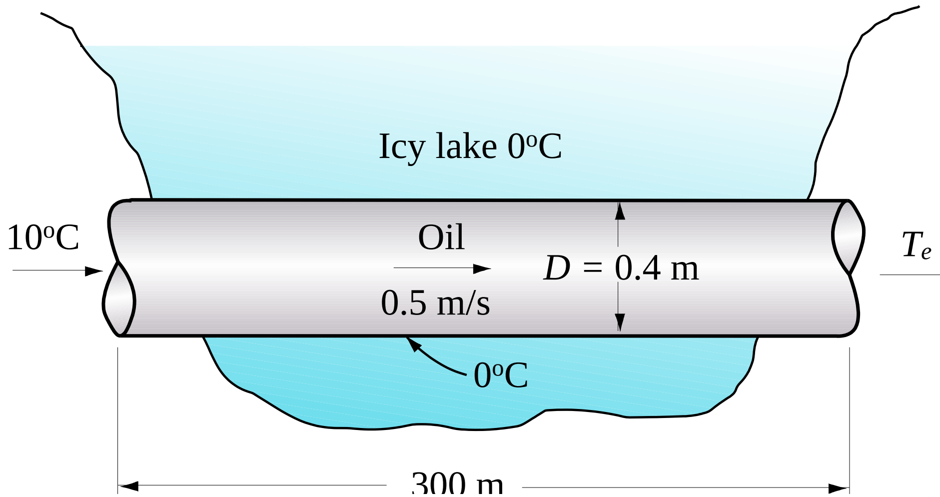
$$\Delta T_{LMTD} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}$$

$$T_e = 9.68°C$$

$$\Delta T_{LMTD} = \frac{(0 - 9.68) - (0 - 10)}{\ln\left(\frac{0 - 9.68}{0 - 10}\right)}$$

$$\Delta T_{LMTD} = -9.84°C$$

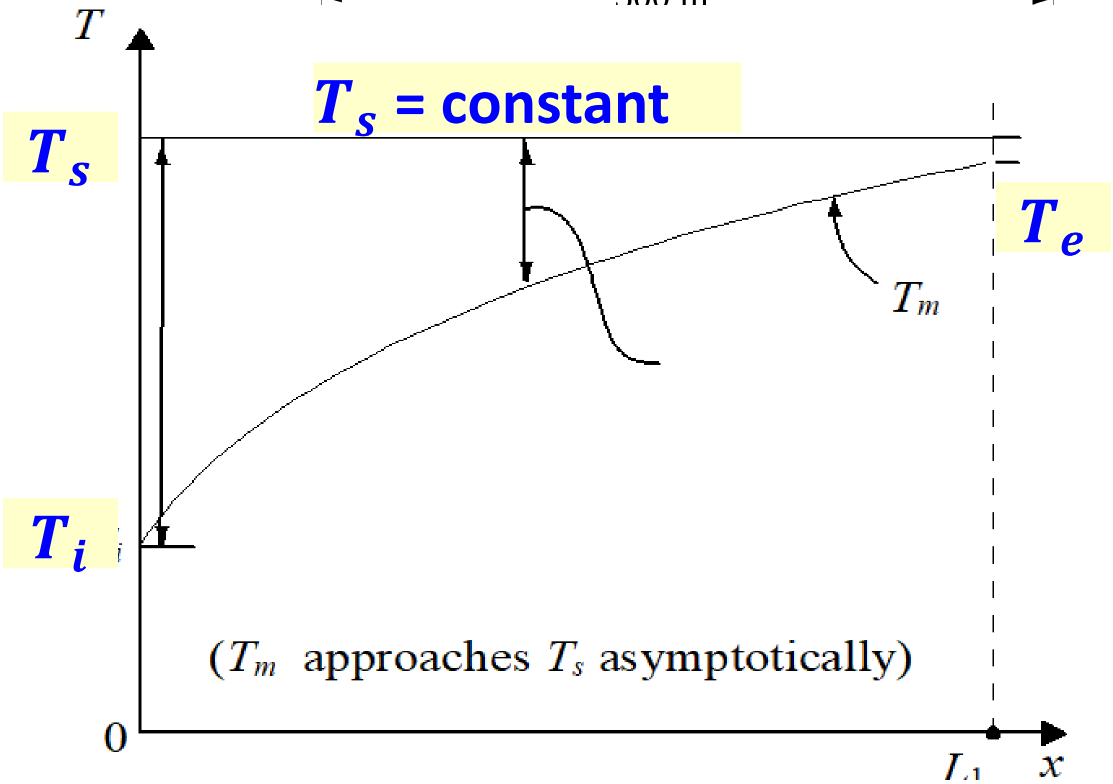
$$A_s = 377\ m^2$$



$$\dot{Q} = hA_s\Delta T_{LMTD} = 8.92(377)(-9.84)$$

$$\dot{Q} = -3.3 \times 10^4\ W \qquad \dot{Q} = -33\ kW$$

Therefore, the oil will lose heat at a rate of 33 kW as it flows through the pipe in the icy waters of the lake.



The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{Re} = \frac{64}{77}$$

$$f = 0.8312$$

$$\rho = 893.55\, kg/m^3 \quad C_p = 1839\, J/kg.℃$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

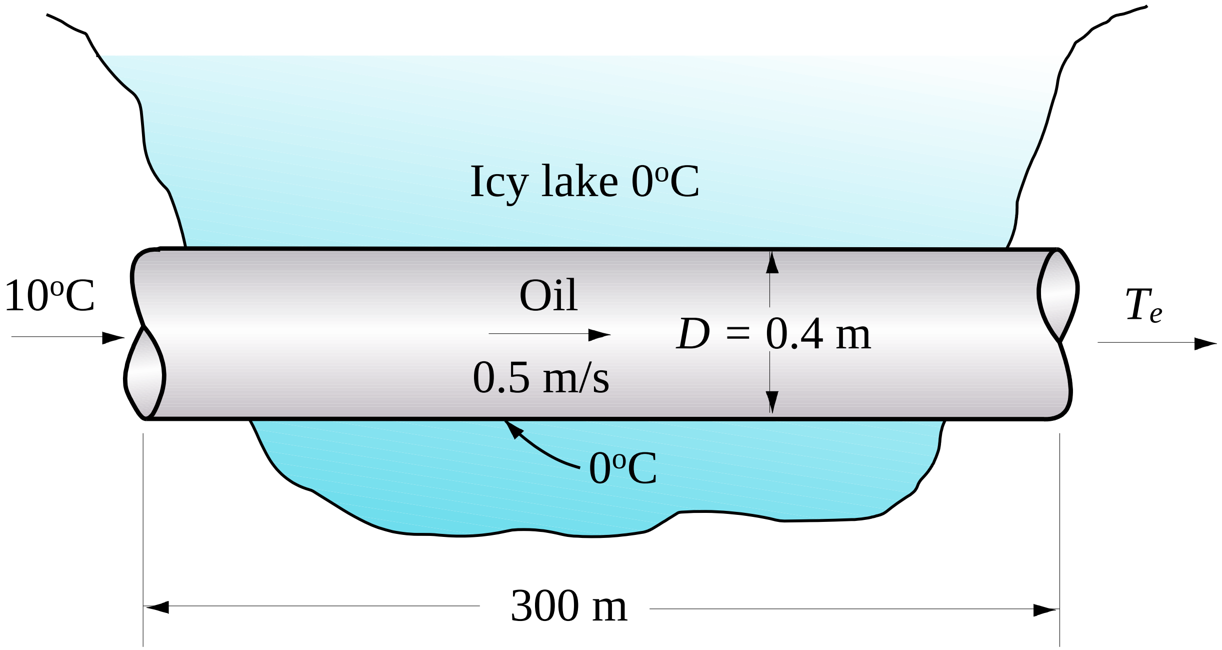
$$\Delta P = 0.8312\left(\frac{300}{0.4}\right)\frac{893.55(0.5)^2}{2}$$

$$\Delta P = 69.63\, kPa$$

$$\dot{m} = 56.14\, kg/s$$

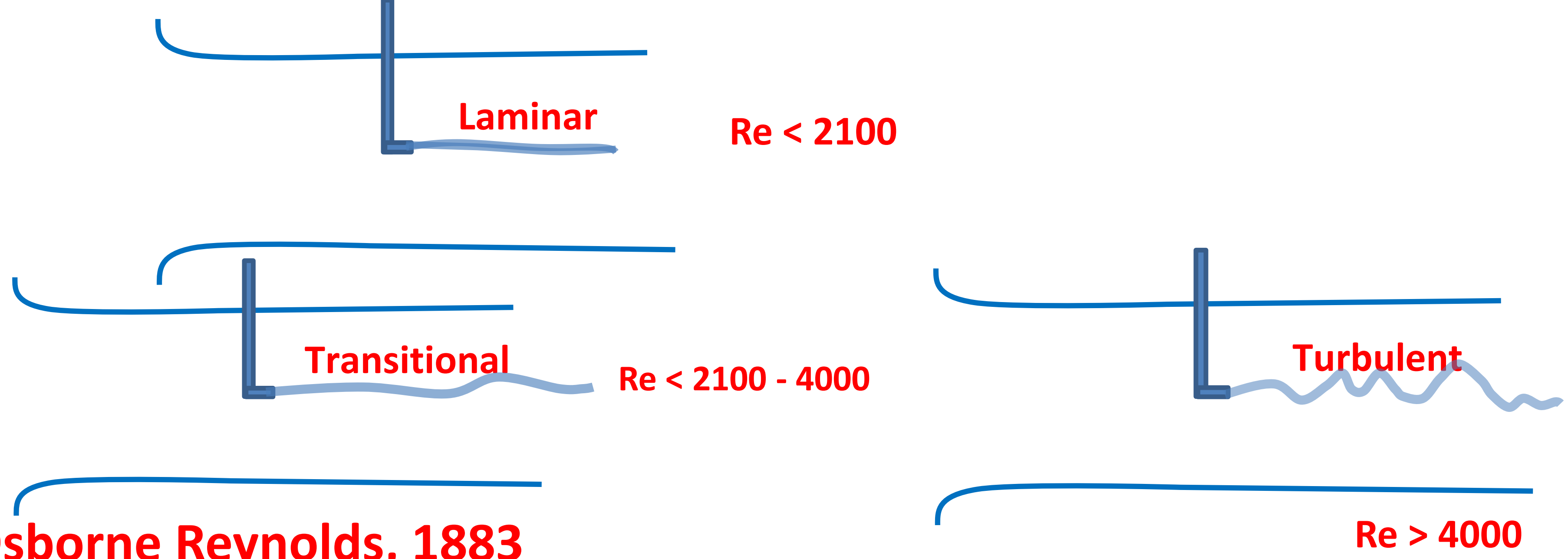
$$P_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{56.14(69.63 \times 1000)}{893.55}$$

$$P_{pump} = 4.4\, kW$$

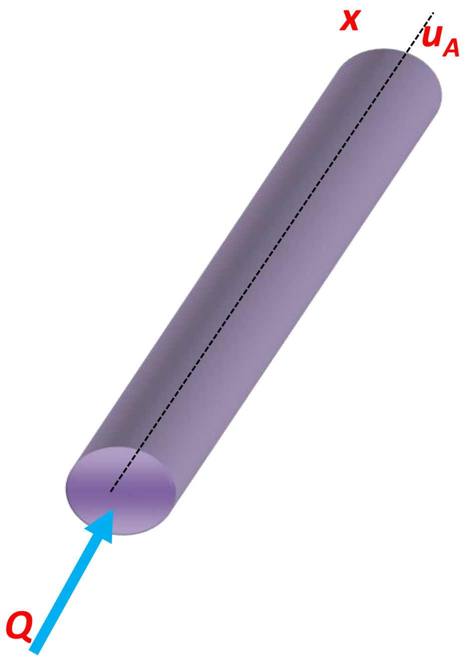


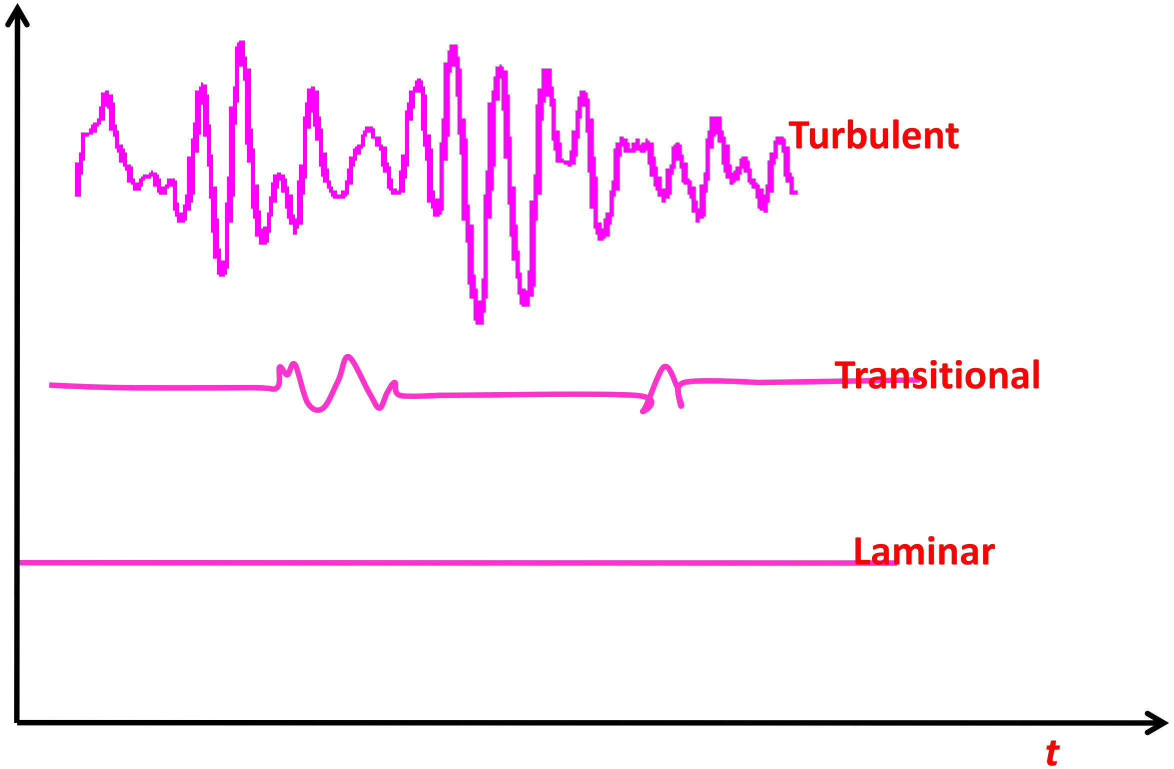
Comments:

We will need a $4.4\, kW$ pump just to overcome the friction in the pipe as the oil flows in the 300-m-long pipe through the lake.

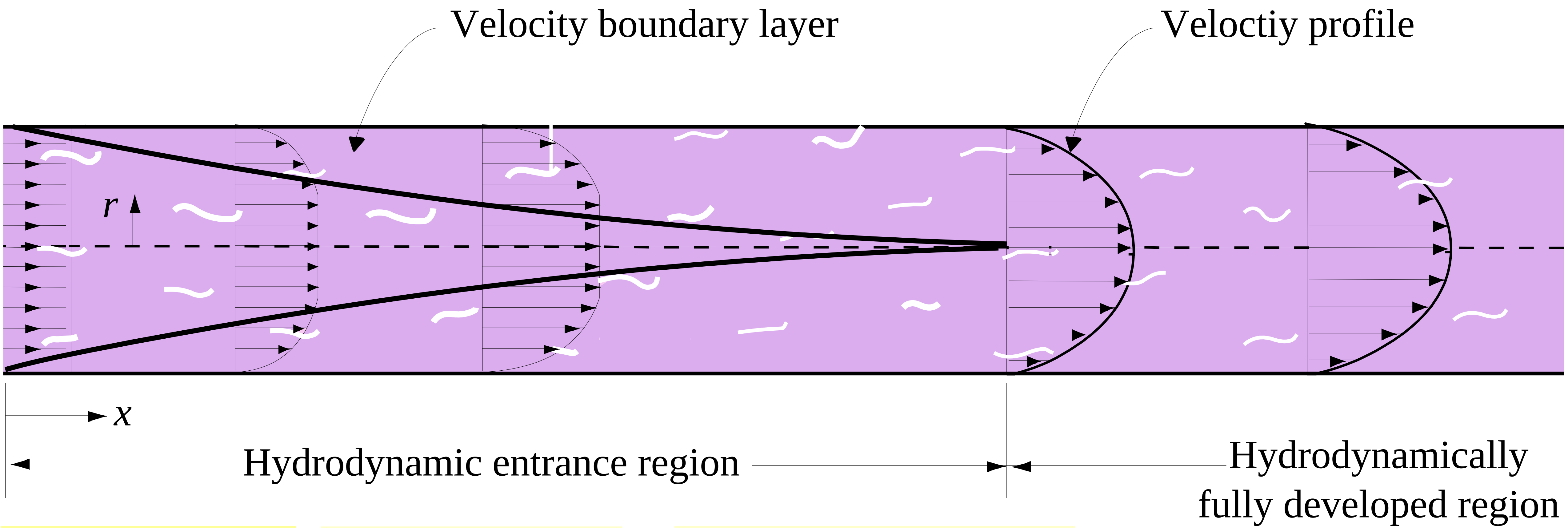


Osborne Reynolds, 1883





ENTRANCE REGION AND FULLY DEVELOPED FLOW



Laminar Flow

$$\frac{L}{D} = 0.06Re$$

$Re = 100 \quad L = 6D$

$Re = 1000 \quad L = 60D$

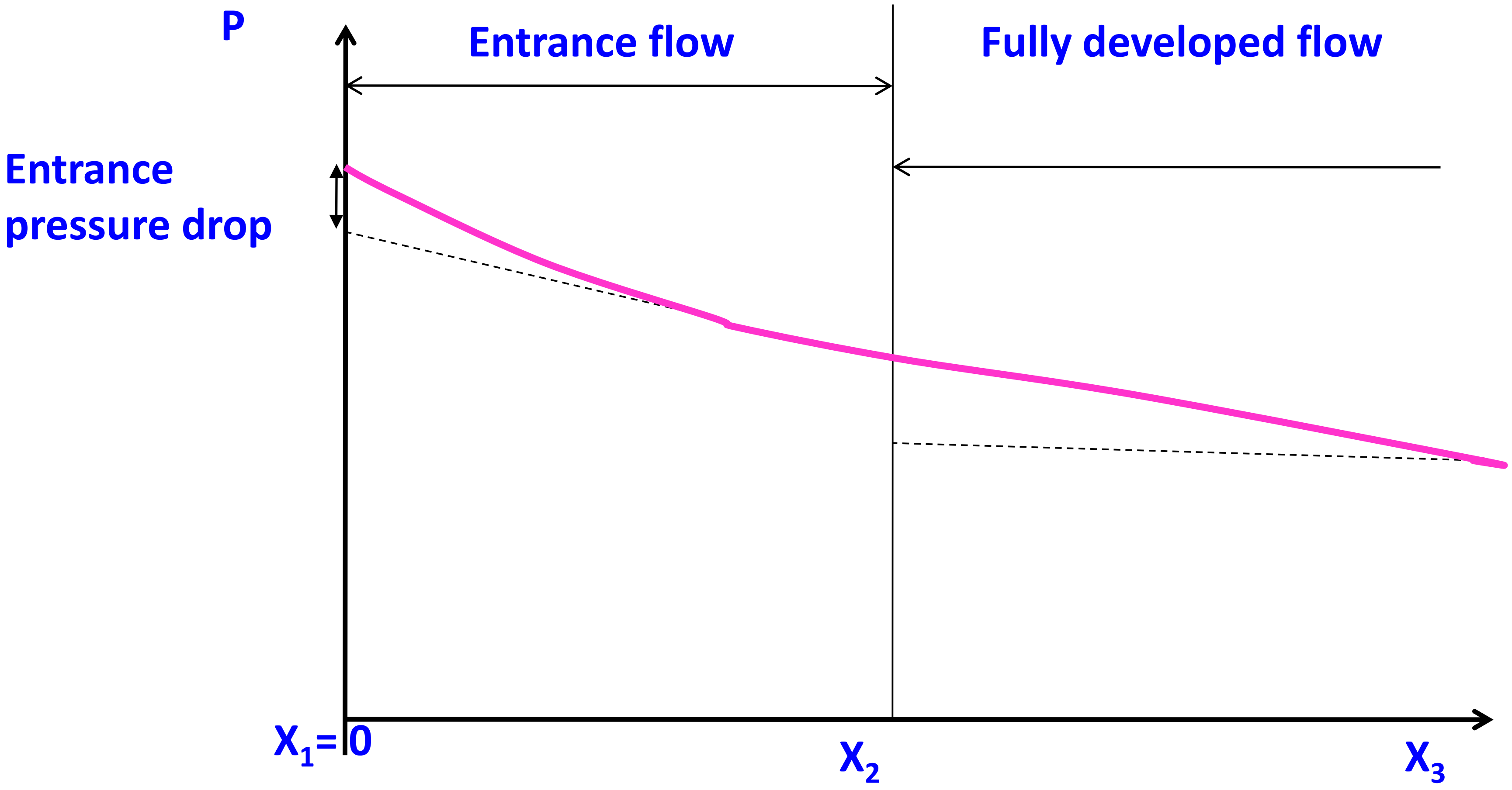
Turbulent Flow

$$\frac{L}{D} = 4.4Re^{\frac{1}{6}}$$

$Re = 10000 \quad L = 21D$

$Re = 100000 \quad L = 30D$

PRESSURE DISTRIBUTION ALONG THE HORIZONTAL PIPE



For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe, that is usually negligible

Pressure difference, between one section of the horizontal pipe and another which forces the fluid through the pipe

Viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration

If viscous effects were absent in such flows, the pressure would be constant throughout the pipe, except for the hydrostatic variation

In non-fully developed flow regions, such as the entrance region of a pipe, the fluid accelerates or decelerates as it flows (the velocity profile changes from a uniform profile at the entrance of the pipe to its fully developed profile at the end of the entrance region).

In the entrance region there is a balance between pressure, viscous, and inertial (acceleration) forces. The magnitude of the pressure gradient, is larger in the entrance region than in the fully developed region, where it is a constant,

The fact that there is a nonzero pressure gradient along the horizontal pipe is a result of viscous effects.

The need for the pressure drop can be viewed from two different standpoints.

- **In terms of a force balance, the pressure force is needed to overcome the viscous forces generated.**
- **In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid**

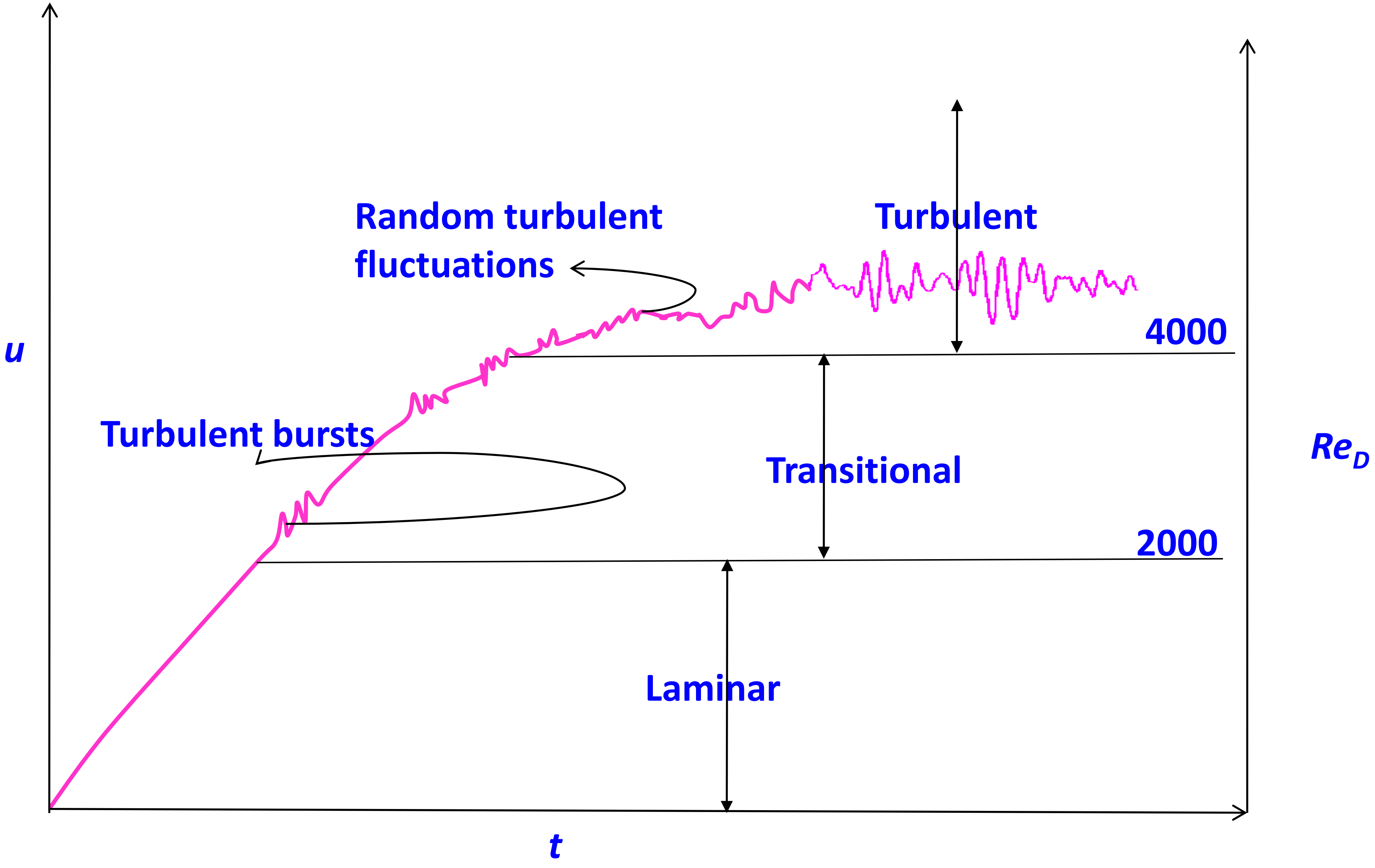
The nature of the pipe flow is strongly dependent on whether the flow is laminar or turbulent. This is a direct consequence of the differences in the nature of the shear stress in laminar and turbulent flows

The shear stress in laminar flow is a direct result of momentum transfer among the randomly moving molecules (a microscopic phenomenon)

The shear stress in turbulent flow is largely a result of momentum transfer among the randomly moving, finite-sized bundles of fluid particles (a macroscopic phenomenon).

THE NET RESULT IS THAT THE PHYSICAL PROPERTIES OF THE SHEAR STRESS ARE QUITE DIFFERENT FOR LAMINAR FLOW THAN FOR TURBULENT FLOW.

FULLY DEVELOPED TURBULENT FLOW

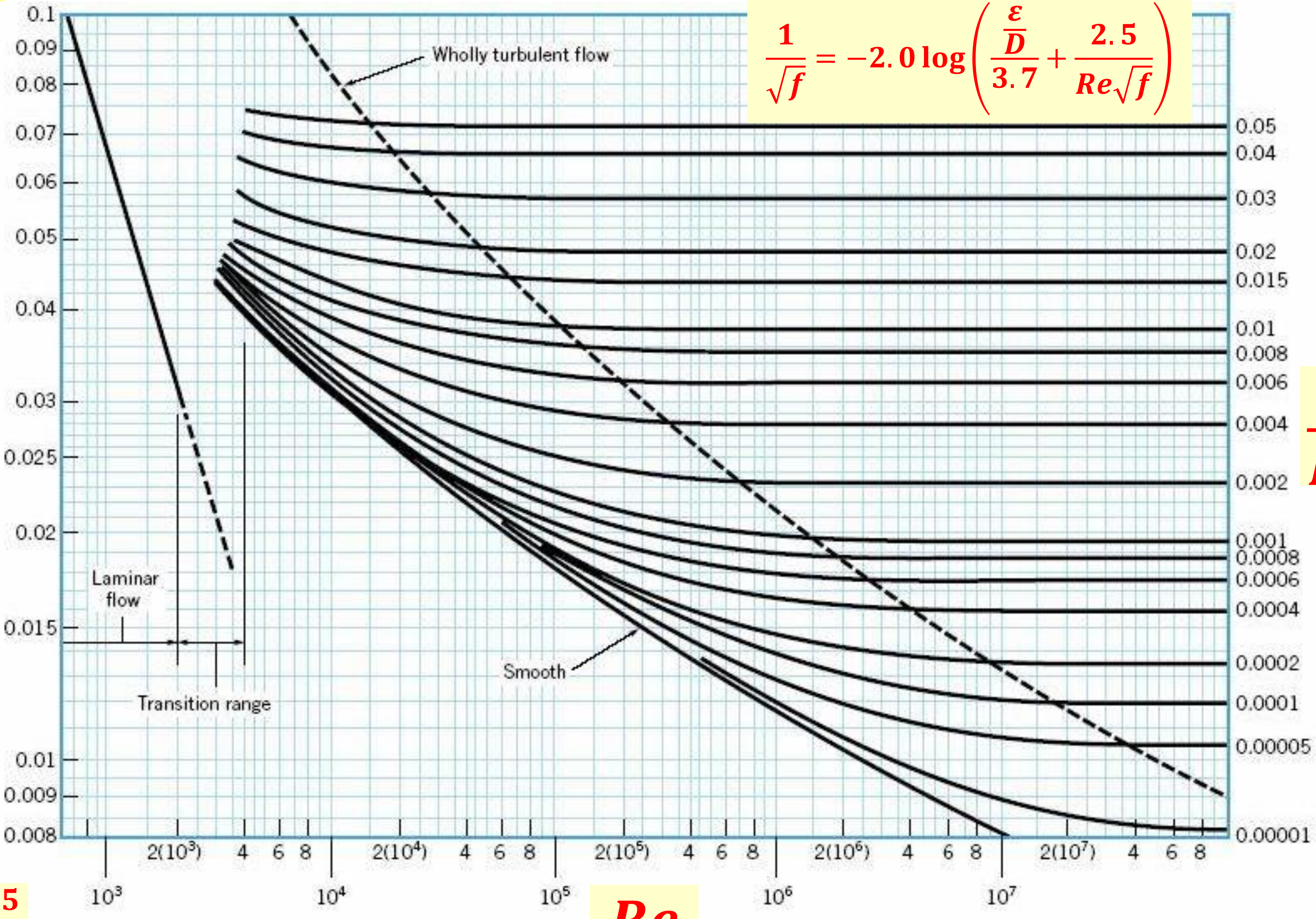


MOODY CHART

LAMINAR

$$f = \frac{64}{Re}$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$



$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.5}{Re\sqrt{f}}\right)$$

f

$\frac{\varepsilon}{D}$

Re

TURBULENT

$$f = 0.3164Re^{-0.25}$$

Turbulent Flow Isothermal Fanning Friction Factor Correlations for Smooth Circular Ducts

Number	Correlation	Remarks and Limitations	Ref
1	$C_f = \dfrac{\tau_\omega}{\frac{1}{2}\rho u_m^2} = 0.0791 Re^{\frac{-1}{4}}$	This approximate explicit equation agrees with case 3 within $\pm 2.5\%$, $4 \times 10^3 < Re < 10^5$	Blasius
2	$C_f = 0.00140 + 0.125 Re^{-0.32}$	This correlation agrees with case 3 within -0.5% *and* $+3\%$, $4 \times 10^3 < Re < 5 \times 10^6$	Drew, Koo and McAdams
3	$\dfrac{1}{\sqrt{C_f}} = 1.737 \ln\left(Re\sqrt{C_f}\right) - 0.4$ Or $\dfrac{1}{\sqrt{C_f}} = 4 \log\left(Re\sqrt{C_f}\right) - 0.4$ Approximated as $C_f = (3.64 \log Re - 3.28)^{-2}$ $C_f = 0.046 Re^{-0.25}$	Von Karman's theoretical equation with the constants adjusted to best fit Nikuradse's experimental data, also referred to as the prandtl correlation, should be valid for very high value of Re. $4 \times 10^3 < Re < 3 \times 10^6$ This approximate explicit equation agrees with the preceding within -0.4 and $+2.2\%$ for $3 \times 10^4 < Re < 10^6$	Von Karman and Nikuradse

Properties are evaluated at bulk temperatures.

Turbulent Flow Isothermal Fanning Friction Factor Correlations for Smooth Circular Ducts

Number	Correlation	Remarks and Limitations	Ref
4	$C_f = \frac{1}{(1.58 \ln Re - 3.28)^2}$	Agrees with case 3 within $\pm 0.5\%$ $for\ 3 \times 10^4 < Re < 10^7$ $and\ within\ \pm 1.8\%$ $at\ Re = 10^4$. $10^4 < Re < 5 \times 10^5$	Filonenko
5	$\frac{1}{C_f} = \left(1.7372 \ln \frac{Re}{1.964 \ln Re - 3.8215}\right)^2$	An explicit form of case 3; agrees with it within $\pm 0.1\%$, $10^4 < Re < 2.5 \times 10^8$	Techo, Ticker, and James

Properties are evaluated at bulk temperatures

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

$$\Delta P = 4C_f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

Material	Surface Roughness,	
	ft	mm
Glass, plastic	0(smooth)	
Concrete	0.003-0.03	0.9-9
Wood stave	0.0016	0.5
Rubber,		
Smoothed	0.000033	0.01
Copper or		
Brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

COLEBROOK'S FRICTION FACTOR CORRELATION

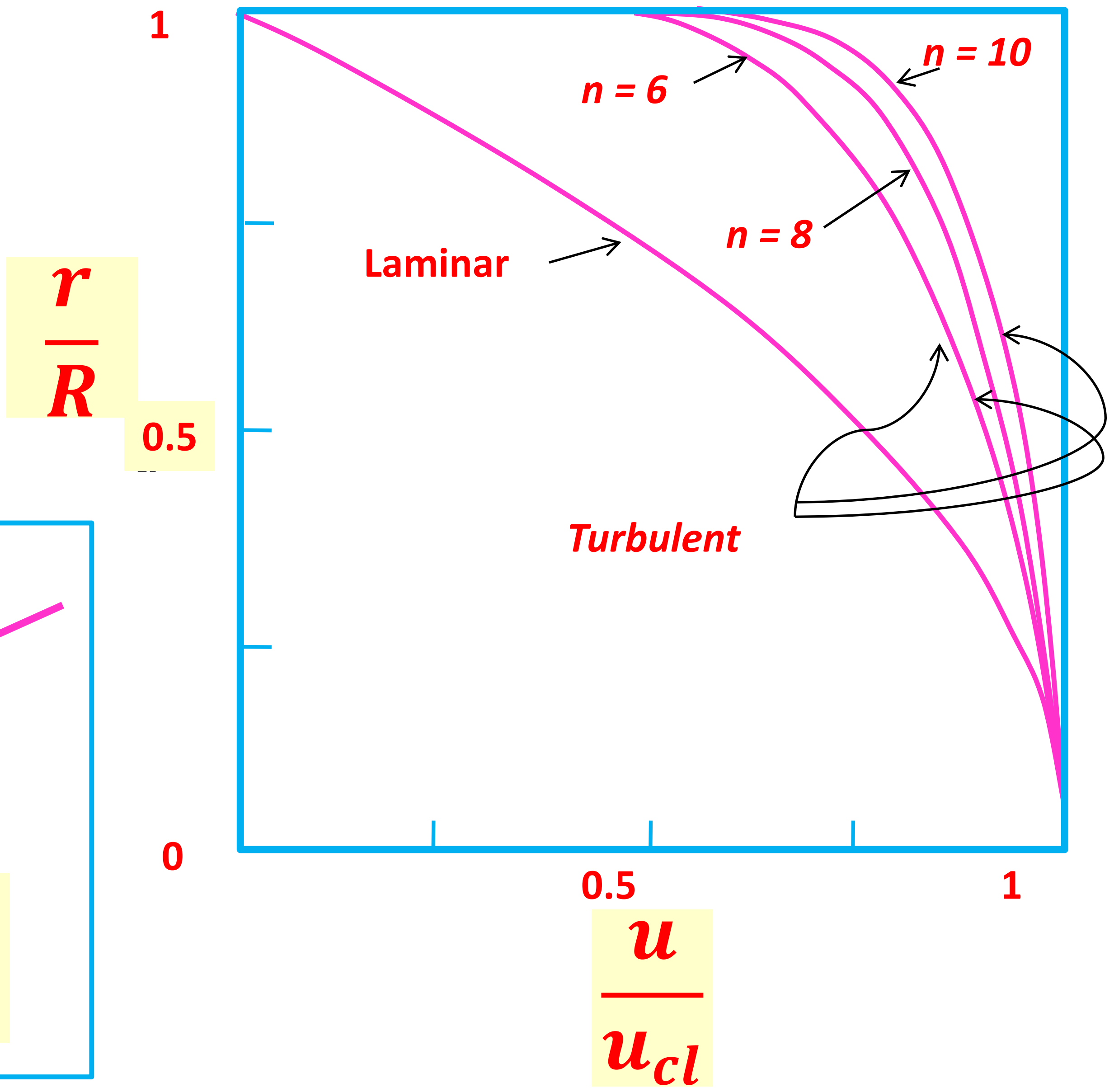
$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.5}{Re\sqrt{f}}\right)$$

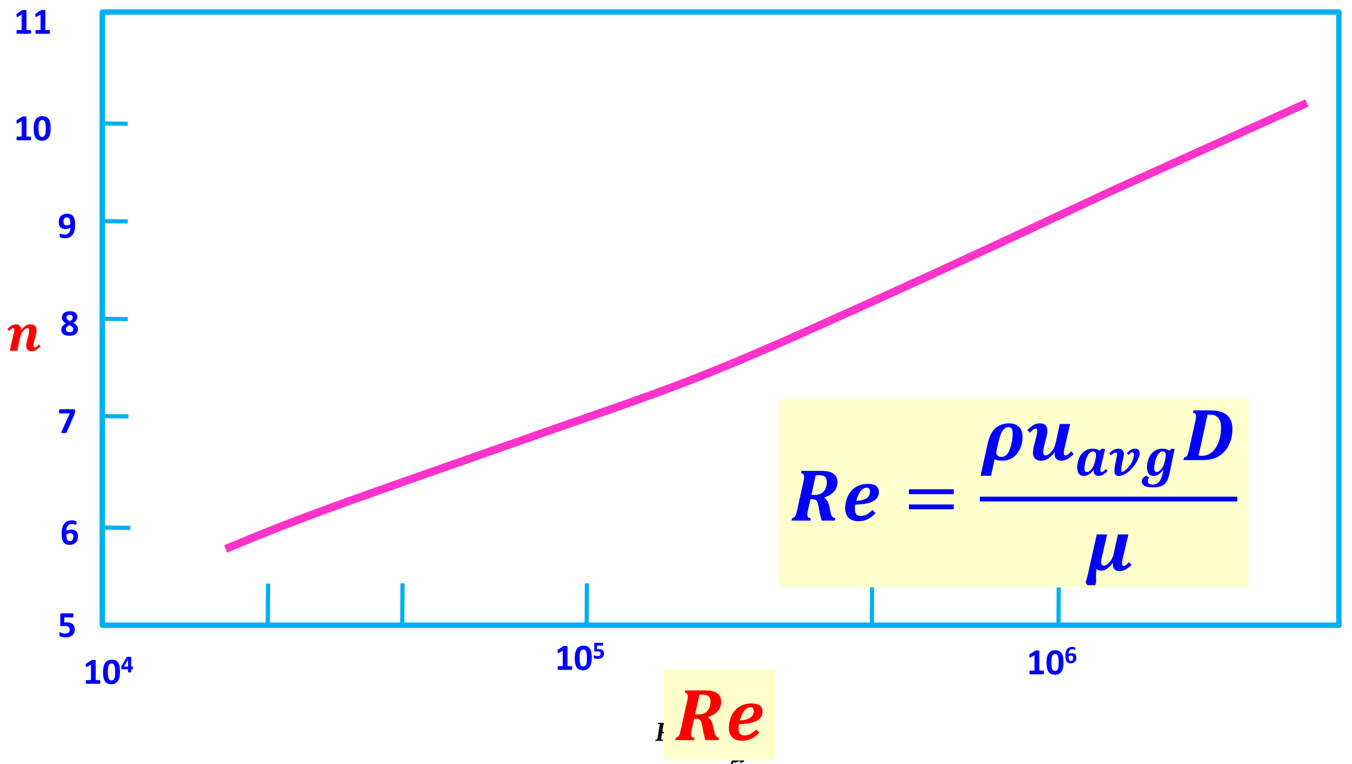
$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

VELOCITY DISTRIBUTION IN TURBULENT FLOW

$$\frac{u}{u_{cl}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

$$\frac{u_{avg}}{u_{cl}} = \frac{2n^2}{(n+1)(2n+1)}$$





$$Re = \frac{\rho u_{avg} D}{\mu}$$

Turbulent forced convection correlations in smooth straight circular Ducts

Correlations for Fully Developed Turbulent Forced Convection Through a circular duct with constant properties

No	Correlation	Remarks and Limitations
1	$Nu_b = \dfrac{(f/2)Re_b Pr_b}{1 + 8.7(f/2)^{1/2}(Pr_b - 1)}$	*Based on three layer turbulent boundary layer model,* $Pr > 0.5$
2	$Nu_b = 0.021 Re_b{}^{0.8} Pr_b{}^{0.4}$	*Based on data for common gases; recommended for Prandtl Numbers* ≈ 0.7
3	$Nu_b = \dfrac{(f/2)Re_b Pr_b}{1.07 + 12.7(f/2)^{1/2}\left(Pr_b{}^{2/3} - 1\right)}$	*Based on three layer model with constants adjusted to match Experiment data* $0.5 < Pr_b < 2000$ $10^4 < Re_b < 5 \times 10^6$
4	$Nu_b = \dfrac{(f/2)Re_b Pr_b}{1.07 + 9(f/2)^{1/2}(Pr_b - 1)Pr_b{}^{-1/4}}$	*Theoretically based; webb found case 3 better at high Pr and this one the same at other Pr*

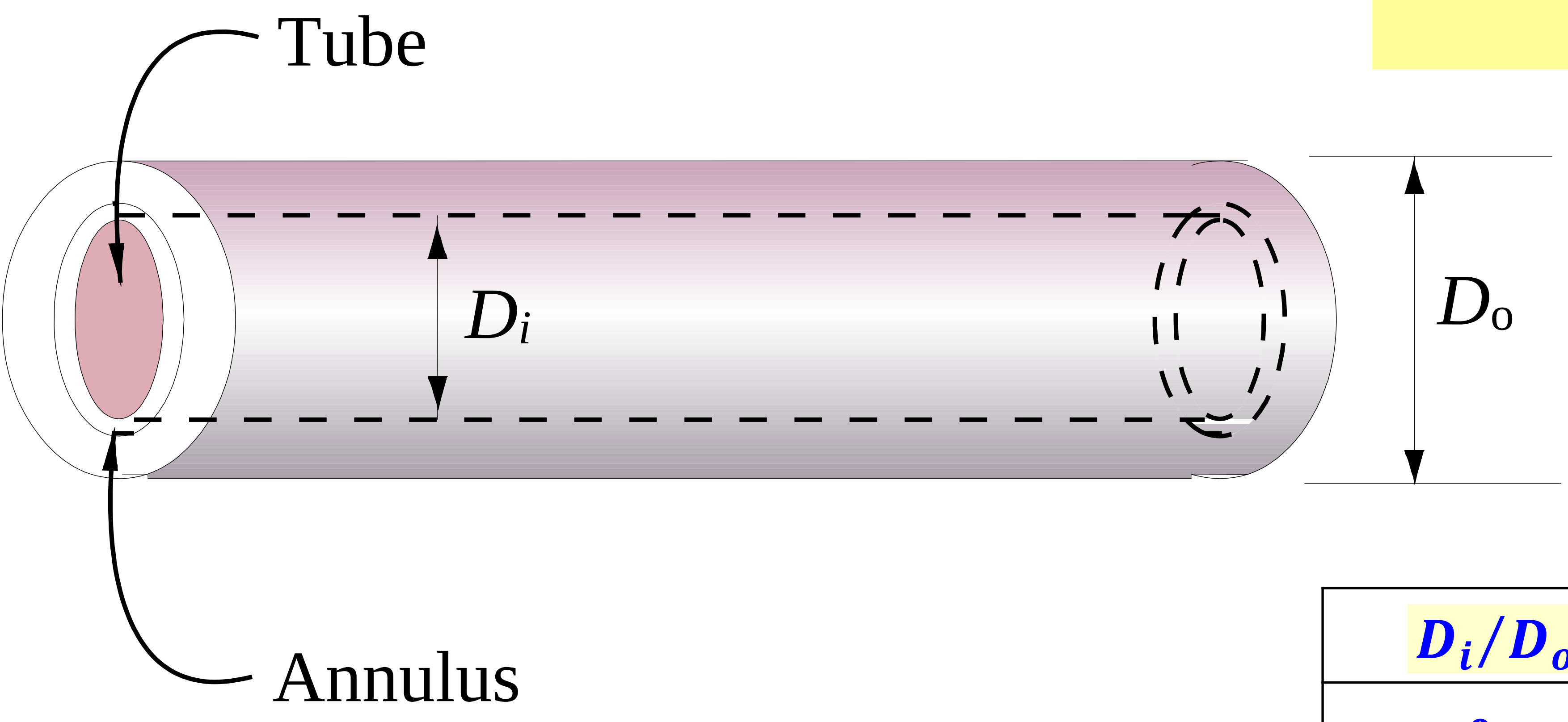
Note: Unless otherwise stated, fluid properties are evaluated at the bulk mean fluid temperature, $T_b = (T_i + T_o)/2$

Circular Ducts

No	Correlation	Remarks and Limitations
5	$Nu_b = 5 + 0.015 Re_b{}^m Pr_b{}^n$ $m = 0.88 - \frac{0.24}{(4 + Pr_b)}$ $n = {}^1/_3 + 0.5exp(-0.6Pr_b)$	Based on numerical results obtained for $0.1 < Pr_b < 10^4, 10^4 < Re_b < 10^6$ *Within 10% of case 6 for* $Re_b > 10^4$
	$Nu_b = 5 + 0.012 Re_b{}^{0.87}(Pr_b + 0.29)$	Simplified correlation for gases, $0.6 < Pr_b < 0.9$
6	$Nu_b = \frac{(f/2)(Re_b - 1000)Pr_b}{1 + 12.7(f/2)^{1/2}\left(Pr_b{}^{2/3} - 1\right)}$	Modification for case 3 to fit experimental data at low Re $(2300 < Re_b <$
	$f = (1.58 \ln Re_b - 3.28)^{-2}$	Valid for $2300 < Re_b < 5 \times 10^6$, and $0.5 < Pr_b < 2000$
	$Nu_b = 0.0214\left(Re_b{}^{0.8} - 100\right)Pr_b{}^{0.4}$	Simplified correlation for $0.5 < Pr_b < 1.5$; agrees with case 4 within -6% and +4%
	$Nu_b = 0.012\left(Re_b{}^{0.87} - 280\right)Pr_b{}^{0.4}$	Simplified correlation for $1.5 < Pr_b < 500$; agrees with case 4 within -10% and +0% for $3 \times 10^3 < Re_b < 10^6$
	$Nu_b = 0.022 Re_b{}^{0.8} Pr_b{}^{0.5}$	Modified Dittus-Boelter correlation for gases $(Pr \approx 0.5 - 1.0)$; agrees with case 6 within 0 to 4% for $Re_b \geq 5000$

Note: Unless otherwise stated, fluid properties are evaluated at the bulk mean temperature, $T_b = (T_i + T_o)/2$

Flow through Annulus



$$D_h = \frac{4A_c}{P} = \frac{4\pi\left(D_o^2 - D_i^2\right)}{\pi(D_i + D_o)}$$

$$D_h = D_o - D_i$$

$$Nu_i = \frac{h_i D_h}{k}$$

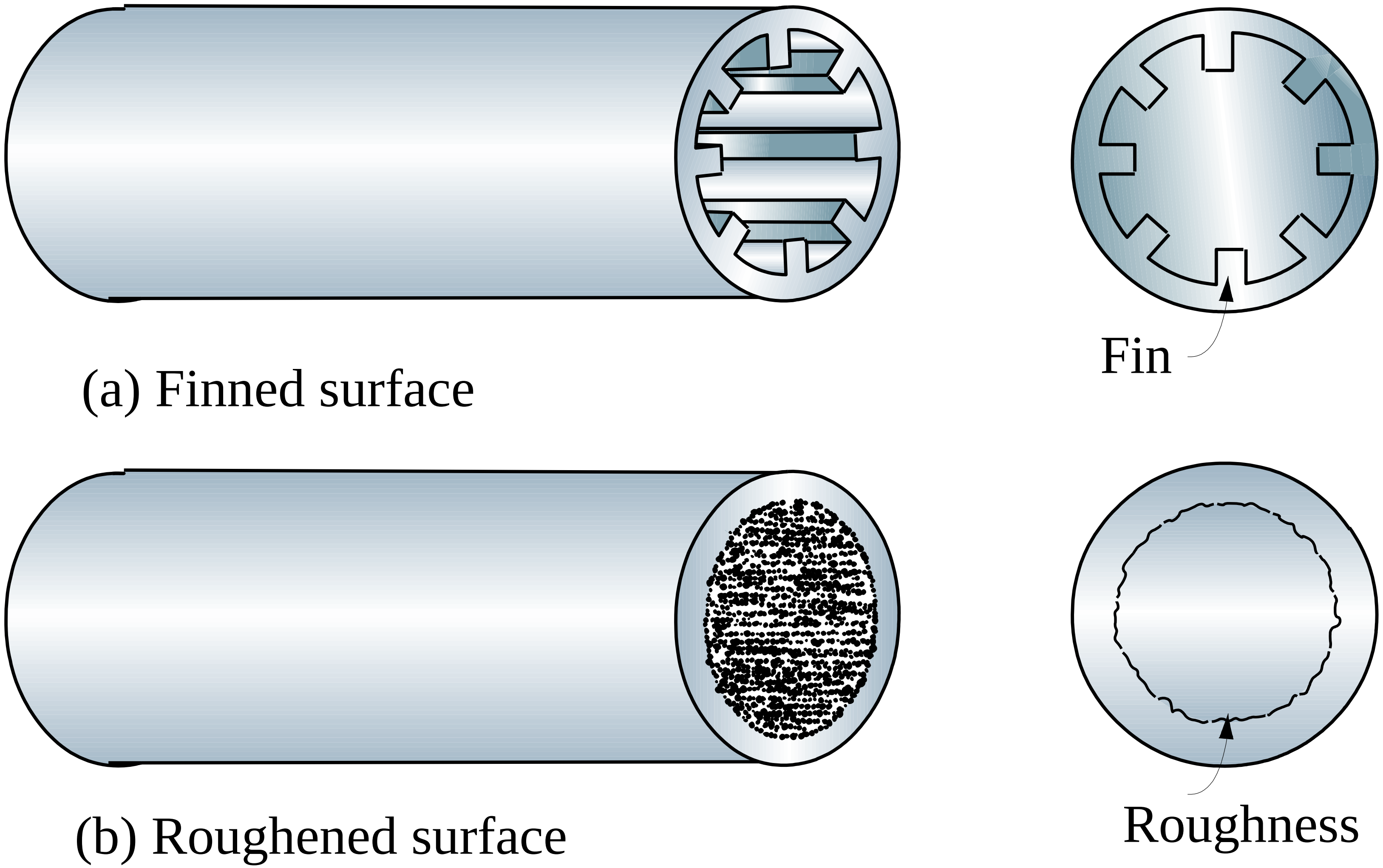
$$Nu_o = \frac{h_o D_h}{k}$$

D_i/D_o	Nu_i	Nu_o
0	--	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.33
1.00	4.86	4.86

Heat Transfer Enhancement

The convection heat transfer coefficient can also be increased by

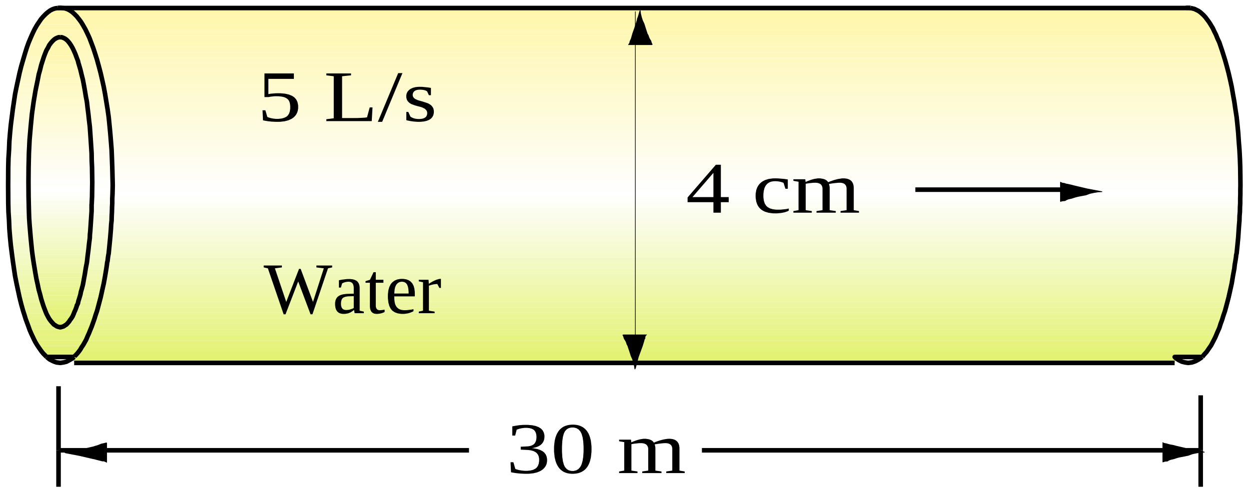
- inducing pulsating flow by pulse generators
- inducing swirl by inserting a twisted tape into the tube
- inducing secondary flows by coiling the tube



(a) Finned surface

(b) Roughened surface

Problem: Water at 15°C is flowing in a 4-cm-diameter and 30-m long horizontal pipe made of stainless steel steadily at a rate of 5 L /s. Determine the pressure drop and the pumping power requirement to overcome this pressure drop.



Known: The flow rate through a specified water pipe is given.

Find: The pressure drop and the pumping power requirements are to be determined.

Assumptions:

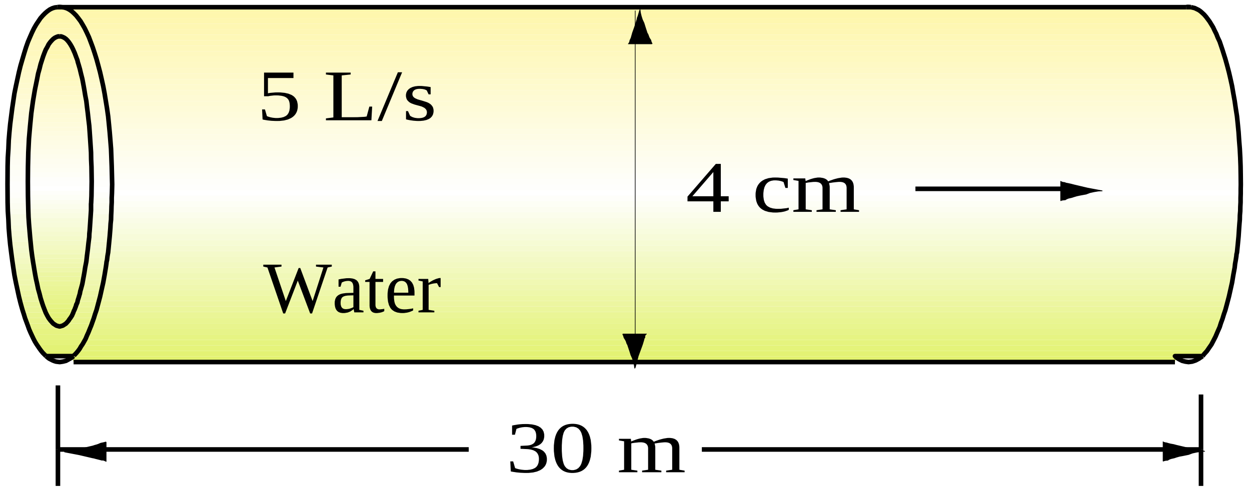
The flow is steady and incompressible.

The entrance effects are negligible, and thus the flow is fully developed.

The pipe involves no components such as bends, valves, and connectors.

The piping section involves no work devices such as a pump or a turbine.

$\rho = 999.1\ kg/m^3 \quad \mu = 1.138 \times 10^{-3}\ Pa.s$



$$u_m = \frac{\dot{Q}}{\frac{\pi}{4} D^2} = \frac{5 \times 10^{-3}}{\frac{\pi}{4} (4 \times 10^{-2})^2}$$

$$u_m = 3.98\ m/s$$

$$Re = \frac{\rho u_m D}{\mu} = \frac{999.1 \times 3.98 \times 5 \times 10^{-3}}{1.138 \times 10^{-3}}$$

$$Re = 139769$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.5}{Re\sqrt{f}}\right)$$

$$\frac{\varepsilon}{D} = \frac{0.002 \times 10^{-3}}{4 \times 10^{-2}} = 5 \times 10^{-5}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{5 \times 10^{-5}}{3.7} + \frac{2.5}{139769\sqrt{f}}\right)$$

$$f = 0.0171$$

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho u_m^2}{2}$$

$$\Delta P = 0.0171\left(\frac{30}{4 \times 10^{-2}}\right)\frac{(999.1)(3.98)^2}{2}$$

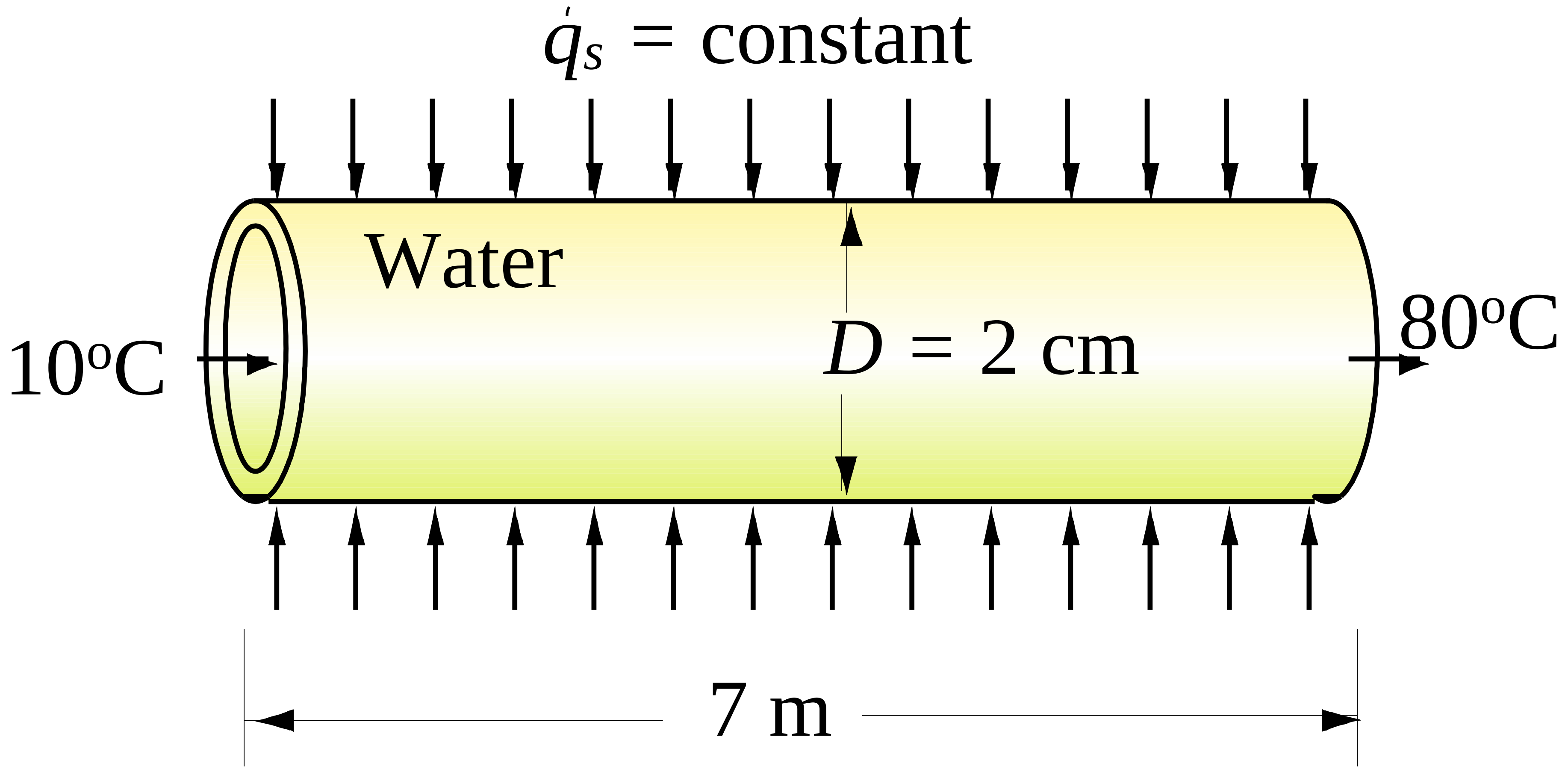
$$\Delta P = 101.48\ kPa$$

$$P_{pump} = \frac{\dot{Q}\Delta P}{\eta_{pump}}$$

$$P_{pump} = \frac{5 \times 10^{-3} \times 101.48 \times 10^3}{0.7}$$

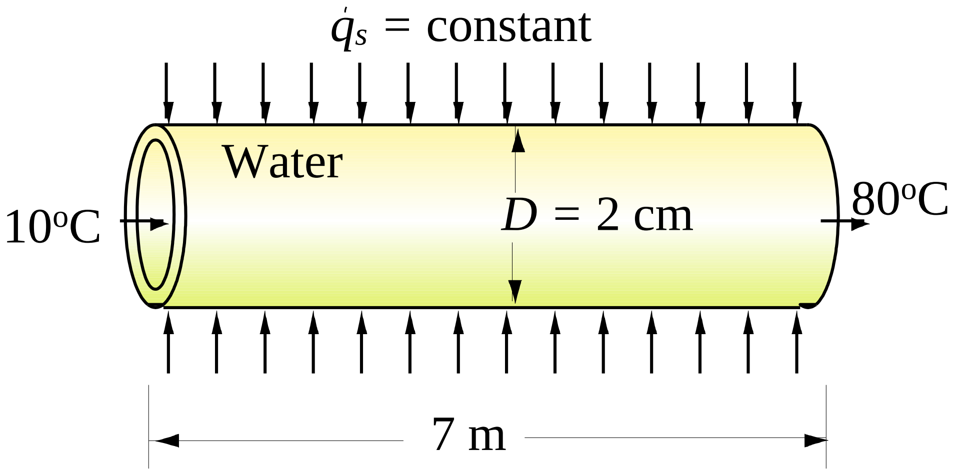
$$P_{pump} = 724.85\ W$$

Problem: Water is to be heated from 10°C to 80 °C as it flows through a 2-cm-internal-diameter, 7-m-long tube. The tube is equipped with an electric resistance heater, which provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 8 L/m, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.



Known: Water is to be heated in a tube equipped with an electric resistance heater on its surface.

Find: The power rating of the heater and the inner surface temperature are to be determined.



Assumptions:

Steady flow conditions exist.

The surface heat flux is uniform.

The inner surfaces of the tube are smooth.

Properties:

The properties of water at the bulk mean temperature of

$$T_b = ((T_i + T_e)/2) = ((10 + 80)/2) = 45°\mathrm{C}$$

$$\rho = 990.1\ kg/m^3 \quad Pr = 3.91$$

$$k = 0.637\ W/m.°\mathrm{C} \quad \nu = 0.602\ \times 10^{-6}\ m^2/s$$

$$C_p = 4180\ J/kg.°\mathrm{C}$$

$$A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} \left(2 \times 10^{-2}\right)^2$$

$$A_s = \pi DL = \pi \times 2 \times 10^{-2} \times 7$$

$$A_c = 3.1416 \times 10^{-4}\ m^2$$

$$A_s = 0.4398\ m^2$$

$$A_c = 3.1416 \times 10^{-4}\ m^2$$

$$A_s = 0.4398\ m^2$$

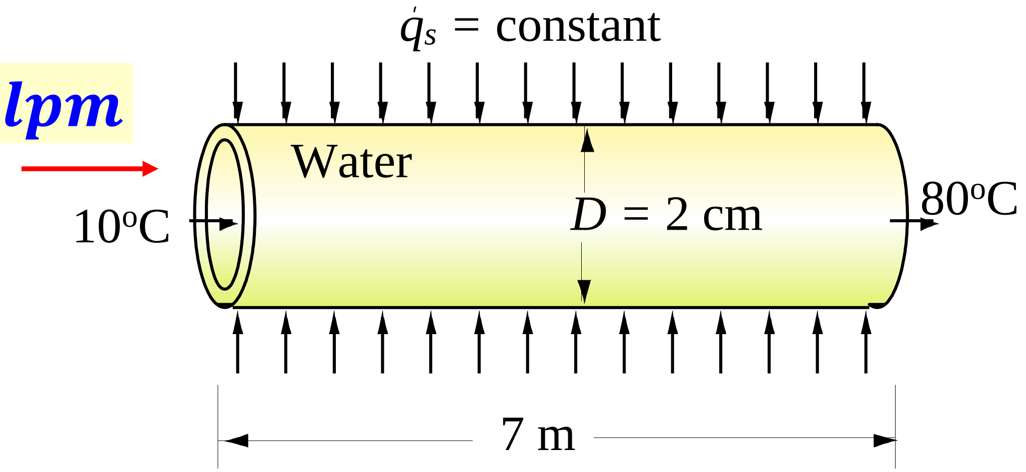
$$\rho = 990.1\ kg/m^3 \quad Pr = 3.91$$

$$k = 0.637\ W/m.°C \quad \nu = 0.602\ \times 10^{-6}\ m^2/s$$

$$C_p = 4180\ J/kg.°C$$

$$\dot{m} = \rho\dot{Q} = 990.1 \times \frac{8 \times 10^{-3}}{60}$$

$$\dot{Q} = 8\ lpm$$



$$\dot{m} = 0.132\ kg/s$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = 0.132 \times 4180(80 - 10)$$

$$\dot{Q} = 38.63\ kW$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be $38.63\ kW$.

$$u_m = \frac{\dot{Q}}{A_c} = \frac{8 \times 10^{-3}}{3.1416 \times 10^{-4}} = 0.4244\ m/s$$

$$u_m = 0.4244\ m/s$$

$$Re = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} = \frac{0.4244 \times 2 \times 10^{-2}}{0.602\ \times 10^{-6}}$$

$$Re = 14100$$

For turbulent flow, the developing length is 10 times the diameter of the pipe. Here, the developing length would be 20 cm (0.2 m). Therefore, thermally fully developed flow exists.

Modified Dittus Boelter Correlation

$\rho = 990.1\ kg/m^3 \quad Pr = 3.91$

$k = 0.637\ W/m.°C \quad \nu = 0.602 \times 10^{-6}\ m^2/s$

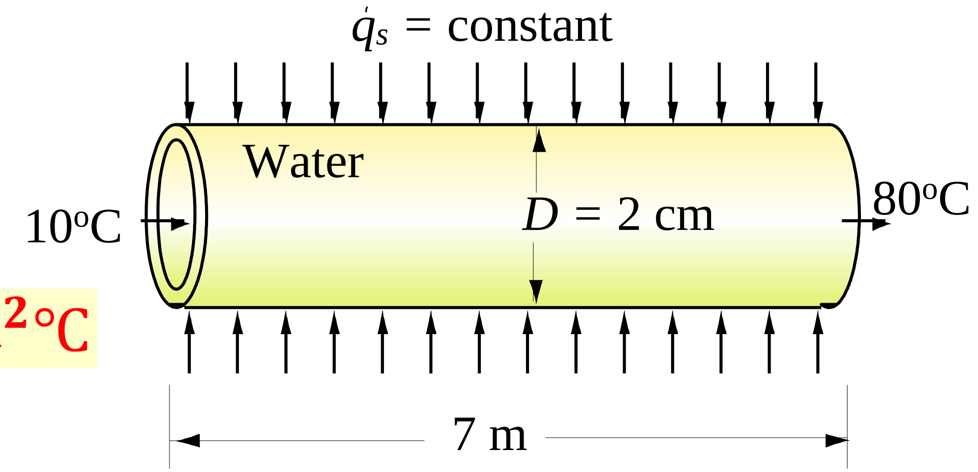
$C_p = 4180\ J/kg.°C$

$$Nu_b = 0.022 {Re_b}^{0.8} {Pr_b}^{0.5}$$

$Re = 14100$

$$Nu_b = 0.022(14100)^{0.8}(3.91)^{0.5}$$

$Nu = 90.76$



$$Nu = \frac{hD}{k} \qquad 90.76 = \frac{h \times 2 \times 10^{-2}}{0.637}$$

$h = 2890.65\ W/m^2°C$

$$q_s'' = h[T_s - T_m] \qquad \frac{q_s''}{h} = T_s - T_m$$

$\dot{Q} = q_s'' A_s \qquad A_s = 0.4398\ m^2 \qquad \dot{Q} = 38.63\ kW$

$$38.63 \times 1000 = q_s'' \times 0.4398$$

$q_s'' = 87.83\ kW/m^2$

$$\frac{q_s''}{h} = T_s - T_m$$

$$\frac{87.83 \times 1000}{2890.65} = T_s - 80$$

$T_s = 113°C$