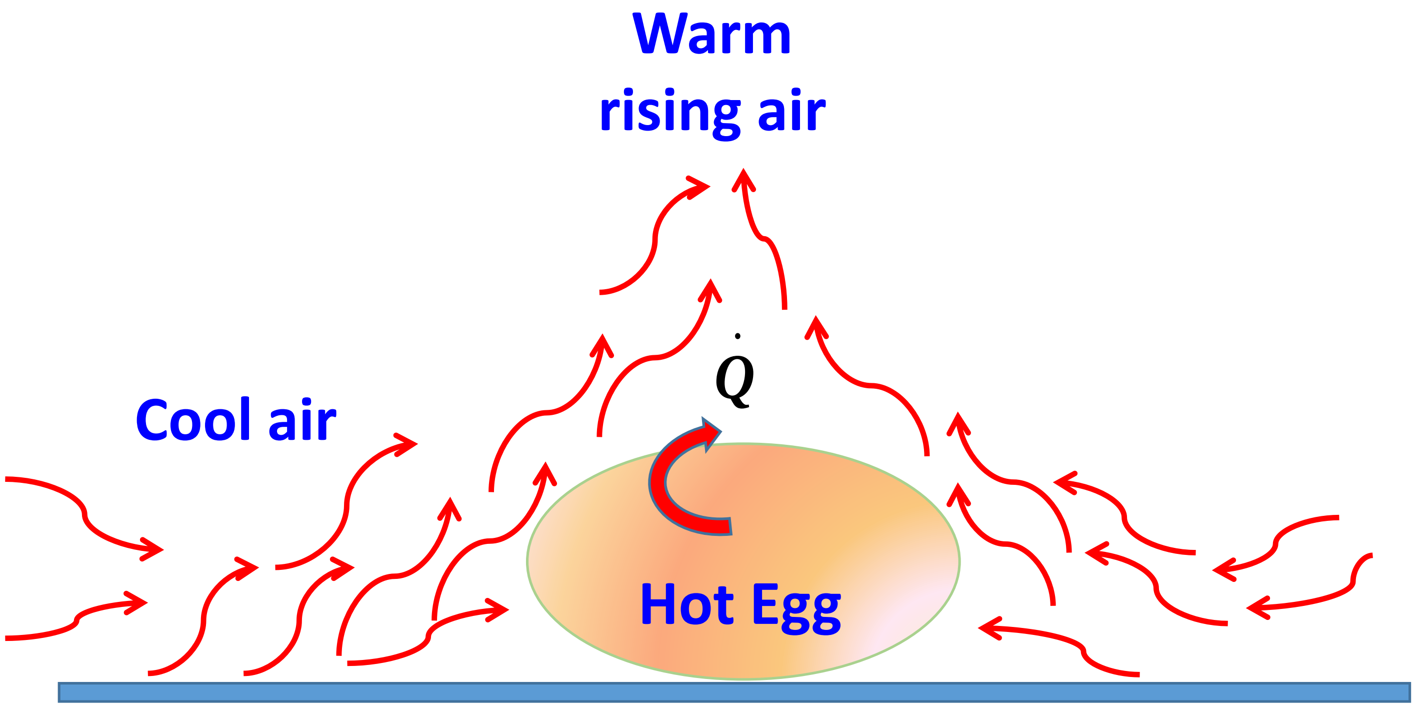
# NATURAL CONVECTION




Cooling of boiled egg in a cooler environment by natural convection

The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current

Heat transfer that is enhanced as a result of this natural convection current is called natural convection heat transfer

**Natural convection occurs because of the presence of**

**a. Density difference**

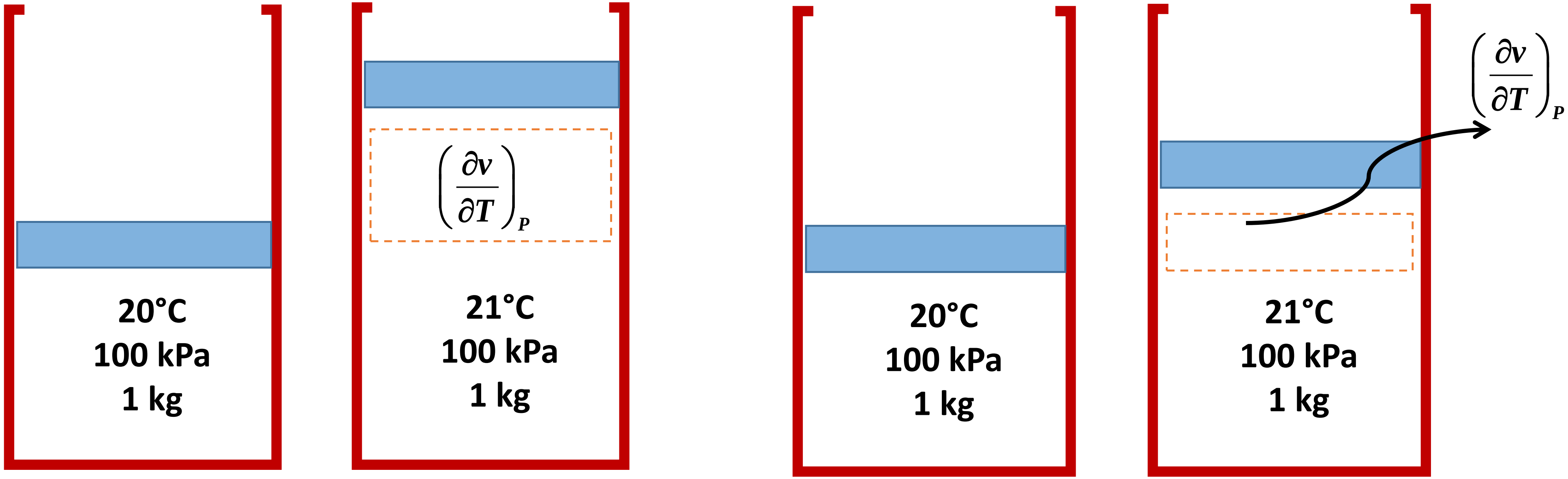
**b. Gravity**

**There is no gravity in space. Therefore, there is no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.**

**In heat transfer studies, the primary variable is temperature. Therefore, we need to express net buoyancy force in terms of temperature differences.**

**Density difference is to be expressed in terms of temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure* .**

***The property which provides this information is volume expansion coefficient, β***




a) A Substance with a large β

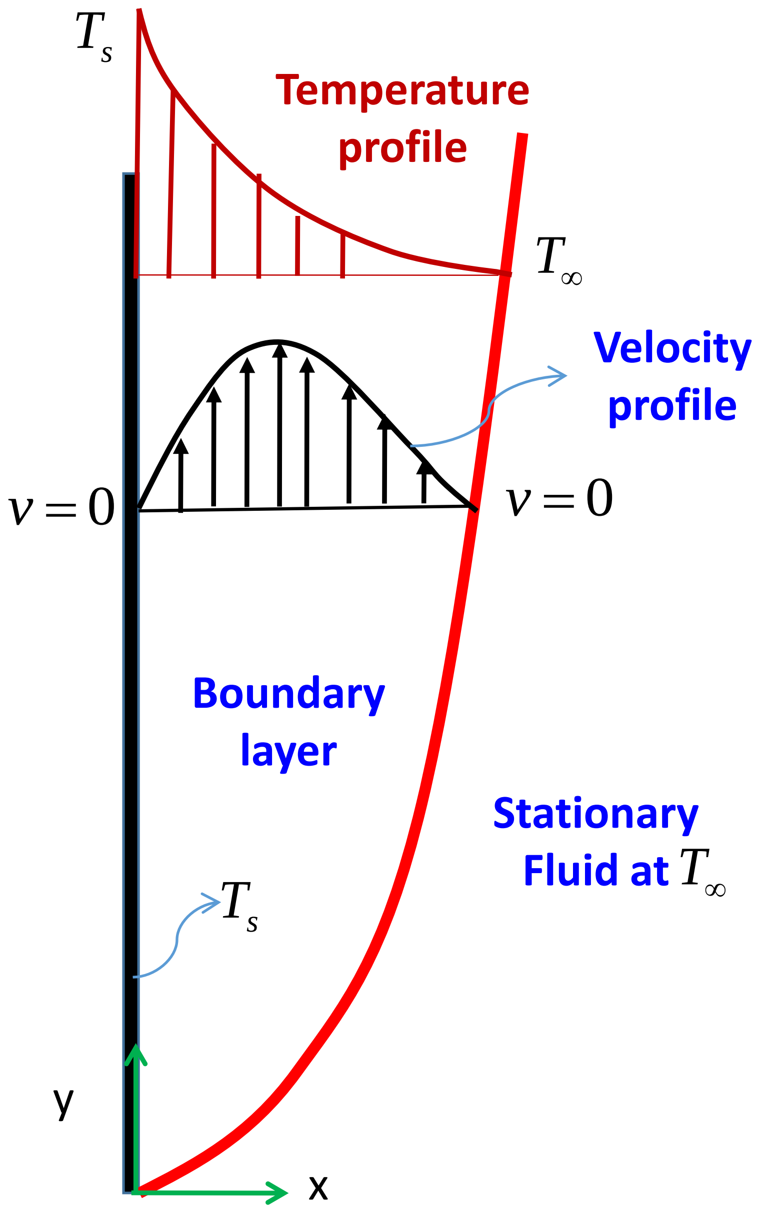
b) A Substance with a small β

**Coefficient of volume expansion is a measure of change in volume of the substance with temperature at constant pressure**

$$\beta = \frac{1}{V}\left(\frac{dV}{dT}\right)_P = -\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)_P$$

# Equation of motion and Grashoff number

## Consider a vertical hot body immersed in a quiescent cold fluid




### Assumptions

- Natural convection Flow is laminar
- Flow is two dimensional
- Flow is steady
- Fluid is Newtonian and properties are constant
- one exception: the density difference between the fluid inside and outside the boundary layer that gives rise to the buoyancy force and sustains flow (This is known as Boussinesq approximation)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho g$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$u << v$ **Complete x-momentum equation vanishes**

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\rho g$$

$$\frac{\partial^2 v}{\partial x^2}\gg\frac{\partial^2 v}{\partial y^2}$$

$\frac{dP_\infty}{dy}$ is impressed upon the boundary layer, hence, $\frac{dP}{dy}=\frac{dP_\infty}{dy}$

Hydrostatic pressure gradient dictated by the reservoir fluid of density $\rho_\infty$

$$\therefore\ \frac{dP_\infty}{dy}=-\rho_\infty g$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\frac{\partial^2 v}{\partial x^2}+(\rho_\infty-\rho)g$$

$$(\rho_\infty-\rho)g$$

Body force terms – flow is driven by the density field generated by the temperature field

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\frac{\partial^2 v}{\partial x^2}+(\rho_\infty-\rho)g$$

$$\beta=\frac{1}{V}\left(\frac{dV}{dT}\right)_P=-\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)_P=-\frac{1}{\rho}\frac{(\rho_\infty-\rho)}{(T_\infty-T)}$$

$$(\rho_\infty-\rho)=\beta\rho(T-T_\infty)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\frac{\partial^2 v}{\partial x^2}+\beta\rho(T-T_\infty)g$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\nu\frac{\partial^2 v}{\partial x^2}+g\beta(T-T_\infty)$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{\partial^2 T}{\partial x^2}\gg\frac{\partial^2 T}{\partial y^2}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\frac{\partial^2 T}{\partial x^2}$$

# Governing equations and boundary conditions

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial x^2}$$

$$u = v = 0; T = T_s \; at \; x = 0$$

$$v = 0; T = T_\infty \; at \; x \to \infty$$

# Non-dimensional numbers

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2}$$

$$Re_H = \frac{u_\infty H}{\nu}$$

$$Ra_H = \frac{g\beta(T - T_\infty)H^3}{\alpha\nu}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Ra_H = Gr_H Pr$$

# Similarity approach

$$x^* = \frac{x}{L}; y^* = \frac{y}{L}; u^* = \frac{u}{u_\infty}; v^* = \frac{v}{u_\infty}; T^* = \frac{T - T_\infty}{T_s - T_\infty};$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

# Similarity approach

$$x^* = \frac{x}{H};\; y^* = \frac{y}{H};\; u^* = \frac{u}{u_\infty};\; v^* = \frac{v}{u_\infty};\; T^* = \frac{T - T_\infty}{T_s - T_\infty};$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\frac{u_\infty^2}{H}\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \nu\frac{u_\infty}{H^2}\frac{\partial^2 v^*}{\partial x^{*2}} + g\beta T^*(T_s - T_\infty)$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \nu\frac{u_\infty}{H^2}\frac{H}{u_\infty^2}\frac{\partial^2 v^*}{\partial x^{*2}} + g\beta T^*(T_s - T_\infty)\frac{H}{u_\infty^2}$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{\nu}{u_\infty H}\frac{\partial^2 v^*}{\partial x^{*2}} + T^*\frac{g\beta(T_s - T_\infty)H}{u_\infty^2}$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{\nu}{u_\infty H}\frac{\partial^2 v^*}{\partial x^{*2}} + T^*\frac{g\beta(T_s - T_\infty)H^3}{\nu^2}\frac{\nu^2}{u_\infty^2 H^2}$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{1}{Re_H}\frac{\partial^2 v^*}{\partial x^{*2}} + T^* Gr_H\frac{1}{Re_H^2}$$

$$\frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = Gr_H$$

$$\frac{u_\infty H}{\nu} = Re_H$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{1}{Re_H}\frac{\partial^2 v^*}{\partial x^{*2}} + T^*\frac{Gr_H}{Re_H^2}$$

## Similarity approach

$$x^* = \frac{x}{H};\, y^* = \frac{y}{H};\, u^* = \frac{u}{u_\infty};\, v^* = \frac{v}{u_\infty};\, T^* = \frac{T - T_\infty}{T_s - T_\infty};$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{u_\infty(T_s - T_\infty)}{H}\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{\alpha(T_s - T_\infty)}{H^2}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

$$\frac{u_\infty H}{\nu} = Re_H$$

$$\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{\alpha(T_s - T_\infty)}{H^2}\frac{H}{u_\infty(T_s - T_\infty)}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

$$\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{\alpha}{H}\frac{1}{u_\infty}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

$$\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{1}{Pr}\frac{1}{Re_H}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

$$\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{\alpha}{\nu}\frac{\nu}{u_\infty H}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

# Outcome of the mass, momentum and energy equations

Mass and momentum and energy equations

Solve these COUPLED equations to get

u, v, T

$\delta, \delta_T, Nu$

$\delta, \delta_T$ – Hydrodynamic and thermal boundary layer thickness

$Nu$ – Nusselt number

$$u^*, v^*, T^* = f(Re_H, Pr, Ra_H)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$\left(u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*}\right) = \frac{1}{Re_H}\frac{\partial^2 v^*}{\partial x^{*2}} + T^*\frac{Gr_H}{Re_H^2}$$

$$\left(u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{1}{Pr}\frac{1}{Re_H}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

- The role played by the Reynolds number in forced convection is played by the Grashoff number in natural convection
- Grashoff number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection
- For vertical plates, critical Grashoff number is around $10^9$

$$\frac{Gr_H}{Re_H^2} \ll 1$$

Forced convection dominates

$$\frac{Gr_H}{Re_H^2} \gg 1$$

Natural convection dominates

$$\frac{Gr_H}{Re_H^2} \approx 1$$

Both natural convection and forced are important

# NATURAL CONVECTION OVER SURFACES

$$Nu = \frac{hL_c}{k} = C(Ra_H)^n$$

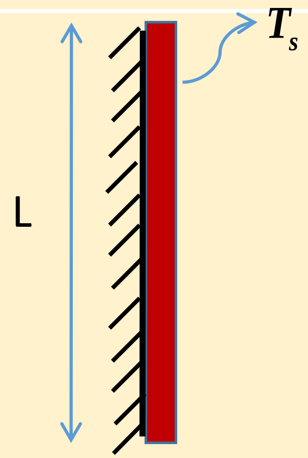
$$Nu \sim (Ra_H)^{\frac{1}{4}}$$

$$Gr_H = \frac{g\beta(T_s - T_\infty){L_c}^3}{\nu^2}$$

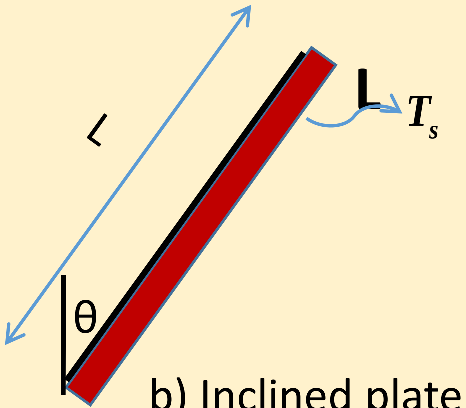
$$Ra_H = \frac{g\beta(T_s - T_\infty){L_c}^3}{\alpha\nu}$$

$$Ra_H = Gr_H Pr$$

**Properties of fluid are calculated at mean film temperature** $\frac{(T_s+T_\infty)}{2}$

| Geometry | Characteristic length | *Ra* range | *Nu* |
|---|---|---|---|
|  a) Vertical plate | L | $10^4$-$10^9$ | $Nu - 0.59 Ra_L^{1/4}$ (6.1) |
| | | $10^9$-$10^{13}$ | $Nu - 0.1 Ra_L^{1/3}$ (6.2) |
| | | Entire Range | $Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}} \right\}$ (6.3) Complex but more accurate |



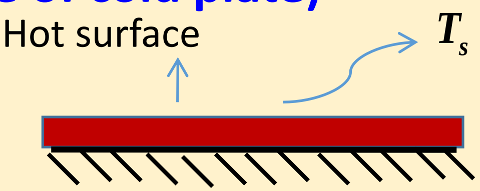
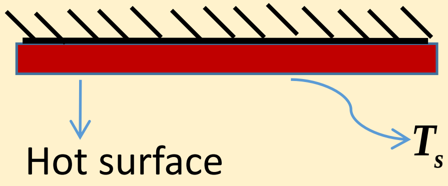


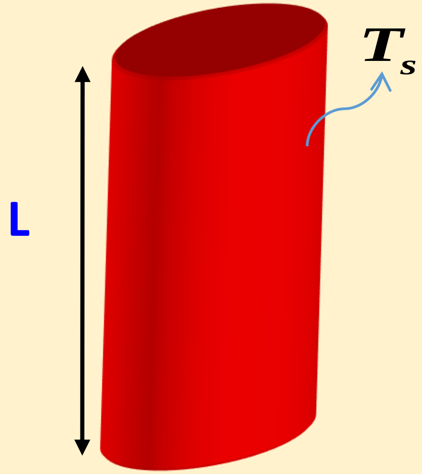


b) Inclined plate

**Use vertical plate equation for the upper surface of a cold plate and the lower surface of a hot plate**

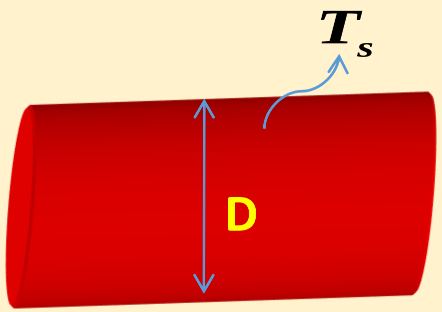
**Replace g by gcosθ for Ra<$10^9$**

| Geometry | Characteristic length | *Ra* range | *Nu* | |
|---|---|---|---|---|
| **Horizontal plate**<br>**Surface are A and Perimeter P**<br>**a)upper surface of hot plate (lower surface of cold plate)**<br><br> | $A_s/P$ | $10^4$-$10^7$<br>$10^7$-$10^{11}$ | $Nu - 0.54 Ra_L^{1/4}$<br>$Nu - 0.15 Ra_L^{1/3}$ | (6.4)<br>(6.5) |
| **a)Lower surface of hot plate (Upper surface of cold plate)**<br><br> | $A_s/P$ | $10^5$-$10^{11}$ | $Nu - 0.27 Ra_L^{1/4}$ | (6.6) |



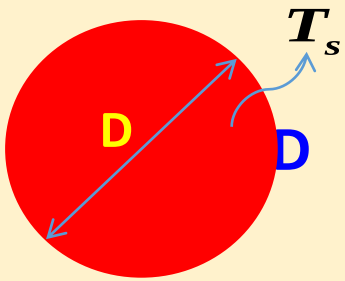

**A vertical cylinder can be treated as a vertical plate when**

$$D \geq \frac{35L}{Gr_L^{1/4}}$$




$$Ra_D \leq 10^{12}$$

$$Nu = \left\{ 0.6 + \frac{0.387\,Ra^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}} \right\} \qquad (6.7)$$




$$Ra_D \leq 10^{11}$$
$$\left(Pr \geq 0.7\right)$$

$$Nu = \left\{ 2 + \frac{0.589\,Ra_D^{1/4}}{\left[1 + \left(0.469/Pr\right)^{9/16}\right]^{4/9}} \right\} \qquad (6.8)$$

**These correlations are for constant wall temperature boundary condition**

**Problem: A 6 m long section of an 8 cm diameter horizontal hot water pipe shown in fig below passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.**

**Solution: A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.**

**Assumptions: 1) Steady state conditions 2) Air is ideal gas**

**3) The local atmospheric pressure is 1 atm.**

**Properties: The Properties of air at the film temperature of**

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70 + 20}{2} = 45°\text{C}\ (318K)$$

$$k = 0.02699 \quad W/m.°\text{C} \quad Pr = 0.7241$$

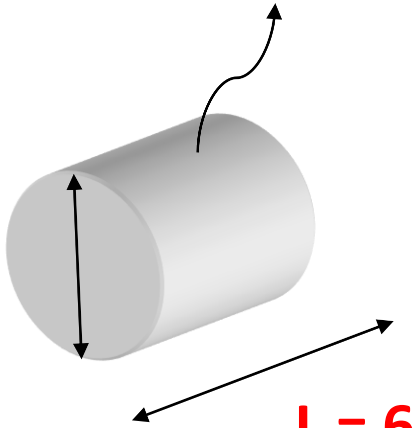
$$\nu = 1.749 \times 10^{-5}\ m^2/s$$

$$\beta = \frac{1}{T_f} = \frac{1}{318}$$

$$T_\infty = 20°\text{C}$$

$$T_s = 70°\text{C}$$

D = 8 mm



L = 6 m

Analysis: The characteristic length in this case is the outer diameter of the pipe, $L_c$ = D = 0.08 m. Then the Rayleigh number becomes

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\alpha v}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}Pr$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70+20}{2} = 45℃\ (318K)$$

$$k = 0.02699 \quad W/m.℃ \quad Pr = 0.7241$$

$$\nu = 1.749 \times 10^{-5}\ m^2/s$$

$$\beta = \frac{1}{T_f} = \frac{1}{318}$$

$$Ra_D = \frac{9.81\left(\frac{1}{318}\right)(70-20)\left(8\times 10^{-3}\right)^3}{(1.749 \times 10^{-5})^2}(0.7241)$$

$$Ra_D = 1.869 \times 10^6$$

The natural convection Nusselt number in this case can be determined from

$$Nu = \left(0.6 + \frac{0.387Ra_D^{\frac{1}{6}}}{\left[1+\left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right)^2$$

$$Nu = \left(0.6 + \frac{0.387\left(1.869\times 10^6\right)^{\frac{1}{6}}}{\left[1+\left(\frac{0.559}{0.7241}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right)^2$$

$$Nu = 17.4$$

$$Nu = \frac{hD}{k}$$

$$Nu = 17.4$$

$$17.44 = \frac{h\left(8 \times 10^{-3}\right)}{0.02699}$$

$$h = 5.869\ W/m^2.℃$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$A_s = \pi DL = \pi\left(8 \times 10^{-3}\right)(6) = 1.508m^2$$

$$\dot{Q} = 5.869(1.508)(70 - 20)$$

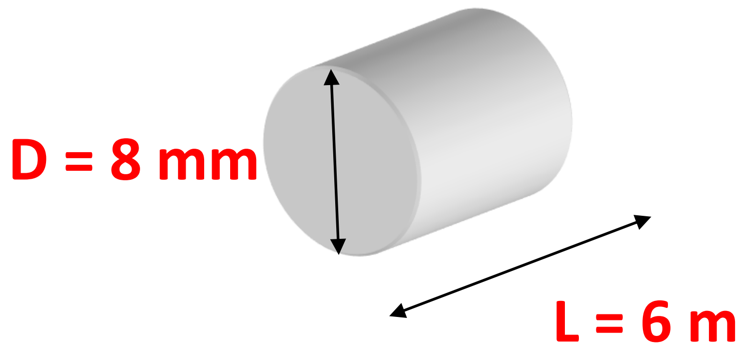
$$\dot{Q} = 443\ W$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70 + 20}{2} = 45℃\ (318K)$$

$$k = 0.02699\quad W/m.℃\quad Pr = 0.7241$$

$$\nu = 1.749\ \times 10^{-5}\ m^2/s$$

$$\beta = \frac{1}{T_f} = \frac{1}{318}$$




**Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.**

Discussion: The pipe will lose heat to the surrounding by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity $\varepsilon$ = 1) and the inner surface of the walls of the room to be at room temperature, the radiation heat transfer is determined to be

$$\dot{Q} = \sigma\varepsilon A_s\left(T_s^4 - T_\infty^4\right) = 5.67 \times 10^{-8}(1)(1.508)\left[(70+273)^4 - (20+273)^4\right]$$

$$\dot{Q} = 553\ W$$

$$A_s = \pi DL = \pi\left(8 \times 10^{-3}\right)(6) = 1.508 m^2$$

which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most system cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low

**Problem:** Consider a 0.6 m X 0.6 m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Fig. determine the rate of heat transfer from the plate by natural convection if the plate is a) vertical, b) horizontal with hot surface facing up, and c) horizontal surface facing down.

**Solution:** A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

**Assumption:** 1) Steady operating conditions exist. 2) Air is ideal gas. 3) The local atmospheric pressure is 1 atm.

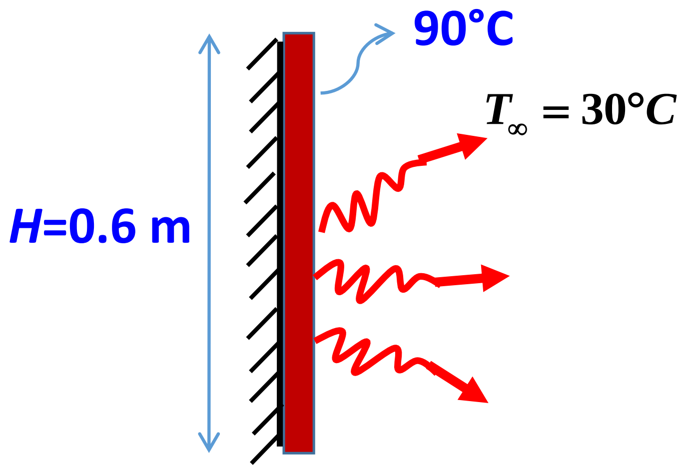
Properties: The properties of air the at the film temperature of

$$T_f = \frac{T_s + T_\infty}{2} = \frac{90 + 30}{2} = 60^\circ\mathrm{C}\ (318K)$$

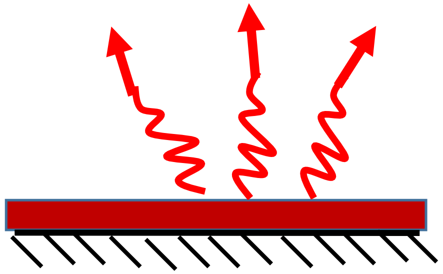
$$k = 0.02808 \quad W/m.^\circ\mathrm{C} \quad Pr = 0.7202$$

$$\nu = 1.896 \times 10^{-5}\ m^2/s$$

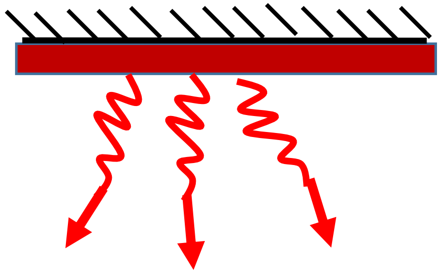
$$\beta = \frac{1}{T_f} = \frac{1}{333}$$




a) Vertical



b) Hot surface facing up



b) Hot surface facing down

**Analysis: a) Vertical- The characteristic length in this case is the height of the plate, $H$ = D = 0.6 m. Then the Rayleigh number becomes**

$$Ra_H = \frac{g\beta(T_s - T_\infty)H^3}{\alpha\nu}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Ra_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} Pr$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{90+30}{2} = 60℃\ (318K)$$

$$k = 0.02808 \quad W/m.℃ \quad Pr = 0.7202$$

$$\nu = 1.896 \times 10^{-5}\ m^2/s$$

$$\beta = \frac{1}{T_f} = \frac{1}{333}$$

$$Ra_H = \frac{9.81\left(\frac{1}{333}\right)(90-30)(0.6)^3}{(1.896 \times 10^{-5})^2}(0.722)$$

$$Ra_H = 7.656 \times 10^8$$

**The natural convection Nusselt number in this case can be determined from**

$$Nu = \left(0.825 + \frac{0.387 Ra_H^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right)^2$$

$$Nu = \left(0.825 + \frac{0.387\left(7.656 \times 10^8\right)^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{0.7202}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right)^2$$

$$Nu = 113.4$$

Note that the simpler relation Eq would be

$$Nu = 0.59Ra_H^{\frac{1}{4}}$$

$$Ra_H = 7.656 \times 10^8$$

$$Nu = 0.59\left(7.656 \times 10^8\right)^{\frac{1}{4}}$$

$$Nu = 98.14$$

which is 13% lower. Then,

$$Nu = \frac{hD}{k}$$

$$113.4 = \frac{h(0.6)}{0.02808}$$

$$h = 5.306\ W/m^2.℃$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$A_s = L^2 = (0.6)^2 = 0.36\,m^2$$

$$\dot{Q} = 5.306(0.36)(90 - 30)$$

$$\dot{Q} = 115\,W$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{90 + 30}{2} = 60℃\ (318K)$$

$$k = 0.02808 \quad W/m.℃ \quad Pr = 0.7202$$

$$\nu = 1.896\ \times 10^{-5}\ m^2/s$$

$$\beta = \frac{1}{T_f} = \frac{1}{333}$$

**b) Horizontal with hot surface facing up. The characteristic length and the Rayleigh number in this case are**

$$L_c = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4}$$

$$L_c = \frac{L}{4}$$

$$Ra_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} Pr$$

$$Ra_H = \frac{9.81\left(\frac{1}{333}\right)(90-30)(0.15)^3}{(1.896\times 10^{-5})^2}(0.722)$$

$$Ra_H = 1.196 \times 10^7$$

**The natural convection Nusselt number in this case can be determined from**

$$Nu = 0.59 Ra_H^{\frac{1}{4}}$$

$$Nu = 0.59\left(1.196 \times 10^7\right)^{\frac{1}{4}}$$

$$Nu = 31.76$$

$$Nu = \frac{hD}{k}$$

$$31.76 = \frac{h(0.15)}{0.02808}$$

$$h = 5.946\ W/m^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$A_s = L^2 = (0.6)^2 = 0.36\,m^2$$

$$\dot{Q} = 5.946(0.36)(90-30)$$

$$\dot{Q} = 128.4\ W$$

c) Horizontal with hot surface facing down. The characteristic length, the heat transfer surface and the Rayleigh number in this case are the same as those determined in b). But the natural convection Nusselt number is to be determined from Eq

$$Nu = 0.27 Ra_H^{\frac{1}{4}}$$

$$Nu = 0.27(1.196 \times 10^7)^{\frac{1}{4}}$$

$$Nu = 15.88$$

$$Ra_H = 1.196 \times 10^7$$

$$Nu = \frac{hD}{k}$$

$$15.88 = \frac{h(0.15)}{0.02808}$$

$$h = 2.973\ W/m^2.°\mathrm{C}$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$\dot{Q} = 2.973(0.36)(90 - 30)$$

$$A_s = L^2 = (0.6)^2 = 0.36\,m^2$$

$$\dot{Q} = 64.2\ W$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty in reaching the plate, which results in a reduced rate of heat transfer.