

Q1: A workpiece (diameter 50 mm) is reduced to 49 mm in a single pass using a HSS tool having 10° rake angle. A depth of cut of 0.5 mm is used with a feed rate of 0.20 mm/rev. The cutting speed is 200 m/min. The chip thickness ratio was measured to be 0.30. The vertical and horizontal components of dynamometer-measured forces are 1080 N and 960 N, respectively. Estimate the ratio of energy spent in the primary and secondary shear zones. [6]

Q2: In an orthogonal machining process, the cutting (F_c) and thrust forces (F_t) were measured and found to be equal. The lip angle and clearance angles were 35° and 10° , respectively. The shear force F_s and shear strain (γ) are given. Under the given condition, please show that: [6]

$$\cos 2\phi = 2 - \frac{2}{\gamma} - \left(\frac{F_s}{F_c}\right)^2$$

Q3: In an orthogonal machining, it is found that the length of chip-tool contact is always equal to the chip thickness, and that the mean shear stress at the chip-tool interface is equal to the mean shear stress on the shear plane. Show that, under these circumstances, the mean coefficient of friction on the tool face (μ) must be equal to or less than $4/3$. [8]

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Solutions:

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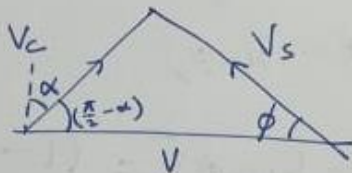
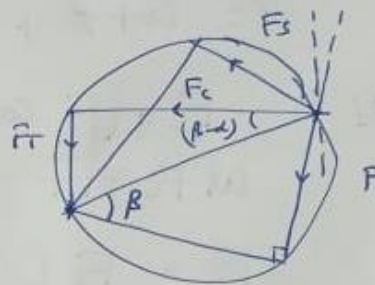
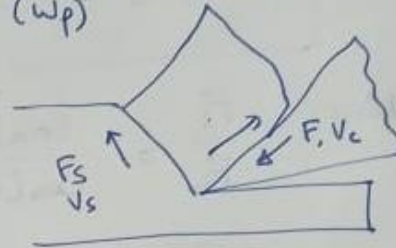
① Ratio of Energy spent in Primary & secondary zone (w_s)

in Primary zone : $F_s \cdot V_s$

In secondary zone : $F \cdot V_c$

Now: $F_s = R \cos(\phi + \beta - \alpha)$

$F = R \sin \beta$



$$\frac{V_s}{\cos \alpha} = \frac{V_c}{\sin \phi}$$

Hence the ratio:

$$\frac{w_p}{w_s} = \frac{F_s V_s}{F \cdot V_c} = \frac{\cos(\phi + \beta - \alpha)}{\sin \beta} \cdot \frac{\cos \alpha}{\sin \phi}$$

Rake angle $\alpha = 10^\circ$, chip thickness ratio = 0.3

$$\frac{\sin \phi}{\cos(\phi - \alpha)} = 0.3 \Rightarrow \tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

$$\tan \phi = \frac{0.3 \cos 10^\circ}{1 - (0.3) \sin 10^\circ} = 0.312$$

$$\phi = 17.31^\circ$$

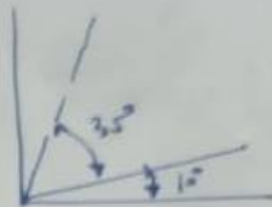
→ Using Lee-White
 $\beta = 37.69^\circ$

$$\phi + \beta - \alpha = 45^\circ$$

②

$$F_c = F_T$$

Rate angle = 45°



From Merchant's circle:

$$F_T = F_c \cdot \tan(\beta - \alpha) \Rightarrow (\beta - \alpha) = 45^\circ$$

$$\boxed{\cos 2\phi = 2 - \frac{2}{\gamma} - \left(\frac{F_s}{F_c}\right)^2}$$

Now: $\frac{F_s}{F_c} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$

$$\gamma = \cot \phi + \tan(\phi - \alpha)$$

Now: $F_s = (F_c \cos \phi - F_T \sin \phi)$

as $F_c = F_T$

$$F_s = F_c (\cos \phi - \sin \phi)$$

$$\therefore \left(\frac{F_s}{F_c}\right)^2 = (\cos \phi - \sin \phi)^2 = (1 - \sin 2\phi)$$

$$= \left(1 - \frac{2 + \tan \phi}{1 + \tan \phi}\right)$$

$$\gamma = \cot \phi + \tan(\phi - \alpha), \quad \alpha = 45^\circ$$

$$= \frac{1}{\tan \phi} + \left(\frac{\tan \phi - 1}{1 + \tan \phi}\right)$$

$$\boxed{\gamma = \frac{1 + \tan^2 \phi}{\tan \phi (1 + \tan \phi)}}$$

(3)

$$\therefore 2 - \frac{2}{\gamma} - \left(\frac{F_s}{F_c} \right)^2$$

$$= 2 - \frac{2}{\left[\frac{1 + \tan^2 \phi}{\tan \phi (1 + \tan \phi)} \right]} - \left(\frac{1 - 2 \tan \phi}{1 + \tan^2 \phi} \right)$$

$$= \left(\frac{1 - \tan^2 \phi}{1 + \tan^2 \phi} \right)$$

$$= \underline{\cos 2\phi}$$



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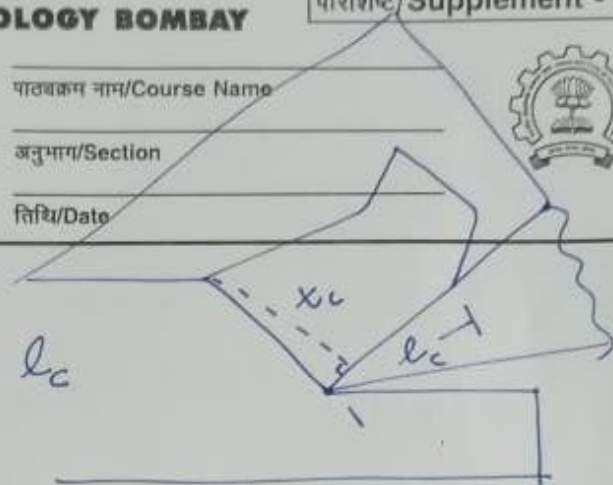
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3)

Chip-tool contact length l_c

& Chip thickness = t_c



mean shear stress @ tool-chip interface (τ_{chip})
= mean shear stress on shear plane (τ_s)

Now:

$$\text{Shear stress @ shear plane} = \frac{F_s}{\text{shear plane area}}$$

$$(\tau_s) = \frac{F_s}{(w t_o / \sin \phi)}$$

$$\text{Shear stress @ tool-chip contact} = \frac{F}{l_c \times w}$$

F: Friction force, w: width of chip

l_c : tool-chip contact length

Now: $F_s = R \cos(\phi + \beta - \alpha)$

$$F = R \sin \beta$$

(5)

$$\frac{\cos(\phi + \beta - \alpha)}{\omega \times \underline{l_s}} = \frac{\sin \beta}{\omega \times l_c} \quad l_c = \underline{\underline{t_c}}$$

$$l_c =$$

$$\cancel{\sin \beta} \quad l_s = \frac{t_o}{\sin \phi} = \frac{(t_c \sin \phi)}{\sin \phi \times \cos(\phi - \alpha)}$$

$$\Rightarrow \frac{\cos(\phi + \beta - \alpha)}{t_c} \cos(\phi - \alpha) = \frac{\sin \beta}{l_c} \quad \frac{t_o}{t_c} = r = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

$$\text{Now } \underline{l_c = t_c}$$

$$\Rightarrow \cos(\phi + \beta - \alpha) \cdot \cos(\phi - \alpha) = \sin \beta$$

$$\cos(\beta + \phi - \alpha) \cdot \cos(\phi - \alpha) = \sin \beta$$

$$\left[\cos \beta \cdot \cos(\phi - \alpha) - \sin \beta \cdot \sin(\phi - \alpha) \right] \cos(\phi - \alpha) = \sin \beta$$

→ dividing by $\cos \beta$

$$\Rightarrow \tan \beta = \left[\cos(\phi - \alpha) - \tan \beta \sin(\phi - \alpha) \right] \cos(\phi - \alpha)$$

$$\text{as } \tan \beta = \mu$$

$$\mu = \cos^2(\phi - \alpha) - \mu \sin(\phi - \alpha) \cos(\phi - \alpha)$$

$$\mu = \frac{\cos^2(\phi - \alpha)}{1 + \sin(\phi - \alpha) \cos(\phi - \alpha)}$$

For higher μ :

$$\frac{dh}{d\phi} = 0$$

$$\frac{d}{d\phi} \left[\frac{\cos^2(\phi - \alpha)}{1 + \sin(\phi - \alpha) \cos(\phi - \alpha)} \right] = 0$$

$$\Rightarrow \frac{\left[1 + \sin(\phi - \alpha) \cos(\phi - \alpha) \right] \left[-2 \cos(\phi - \alpha) \sin(\phi - \alpha) \right] - \cos^2(\phi - \alpha) \left[-\sin^2(\phi - \alpha) + \cos^2(\phi - \alpha) \right]}{\left[1 + \sin(\phi - \alpha) \cos(\phi - \alpha) \right]^2} = 0$$

$$\Rightarrow -2 \sin(\phi - \alpha) - \sin^2(\phi - \alpha) \cos(\phi - \alpha) - \cos^3(\phi - \alpha) = 0$$

$$\Rightarrow -2 + \tan(\phi - \alpha) - 1 = 0$$

$$\tan(\phi - \alpha) = -\frac{1}{2}$$

$$(\phi - \alpha) = -26.565^\circ$$

Putting $\phi - \alpha = -26.5^\circ$, $\mu = \frac{\cos^2(\phi - \alpha)}{1 + \sin(\phi - \alpha) \cos(\phi - \alpha)}$

$$\boxed{\mu_{\max} = \frac{4}{3}}$$