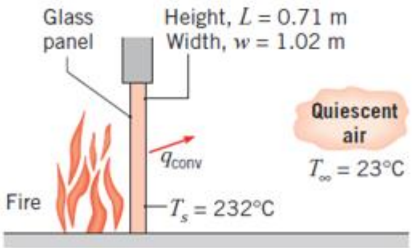
Tutorial #4 (ME 346 S1)

Q1. A glass-door firescreen, used to reduce exfiltration of room air through a chimney, has a height of 0.71 m and a width of 1.02 m and reaches a temperature of 232⁰C. If the room temperature is 23⁰C, estimate the convection heat rate from the fireplace to the room.

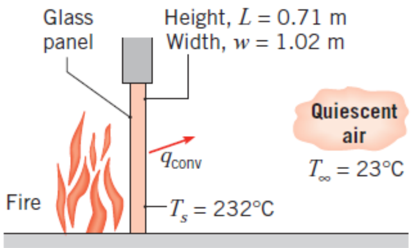


Solution:

Known: Glass screen situated in fireplace opening.

Find: Heat transfer by convection between screen and room air.

Schematic:



Assumptions:

1. Screen is at a uniform temperature T_s.
2. Room air is quiescent.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 400$ K): $k = 33.8 \times 10^{-3}$ W/m·K, $\nu = 26.4 \times 10^{-6}$ m²/s, $\alpha = 38.3 \times 10^{-6}$ m²/s, $Pr = 0.690$, $\beta = (1/T_f) = 0.0025\ \text{K}^{-1}$.

Analysis: The rate of heat transfer by free convection from the panel to the room is given by Newton's law of cooling

$$q = \bar{h} A_s (T_s - T_\infty)$$

where $\bar{h}$ may be obtained from knowledge of the Rayleigh number. Using Equation 9.25,

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$$

$$= \frac{9.8\ \text{m/s}^2 \times 0.0025\ \text{K}^{-1} \times (232 - 23)°\text{C} \times (0.71\ \text{m})^3}{38.3 \times 10^{-6}\ \text{m}^2/\text{s} \times 26.4 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.813 \times 10^9$$

and from Equation 9.23 it follows that transition to turbulence occurs on the panel. The appropriate correlation is then given by Equation 9.26

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387(1.813 \times 10^9)^{1/6}}{[1 + (0.492/0.690)^{9/16}]^{8/27}}\right\}^2 = 147$$

Hence

$$\bar{h} = \frac{\overline{Nu}_L \cdot k}{L} = \frac{147 \times 33.8 \times 10^{-3}\ \text{W/m} \cdot \text{K}}{0.71\ \text{m}} = 7.0\ \text{W/m}^2 \cdot \text{K}$$

and

$$q = 7.0\ \text{W/m}^2 \cdot \text{K}\ (1.02 \times 0.71)\ \text{m}^2\ (232 - 23)°\text{C} = 1060\ \text{W} \qquad \triangleleft$$

Comments:

1. Radiation heat transfer effects are often significant relative to free convection. Using Equation 1.7 and assuming $\varepsilon = 1.0$ for the glass surface and $T_{sur} = 23°\text{C}$, the net rate of radiation heat transfer between the glass and the surroundings is

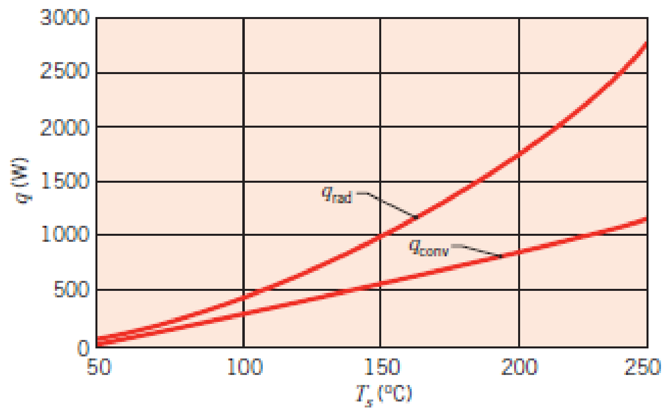
$$q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_{sur}^4)$$

$$q_{rad} = 1(1.02 \times 0.71)\text{m}^2 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (505^4 - 296^4)\ \text{K}^4$$

$$q_{rad} = 2355\ \text{W}$$

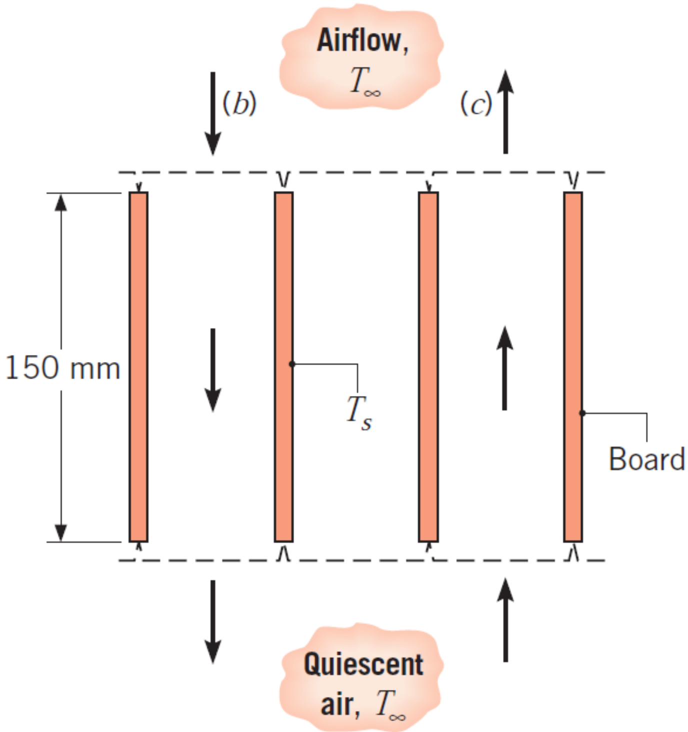
   Hence in this case radiation heat transfer exceeds free convection heat transfer by more than a factor of 2.

2. The effects of radiation and free convection on heat transfer from the glass depend strongly on its temperature. With $q \propto T_s^4$ for radiation and $q \propto T_s^n$ for free convection, where $1.25 < n < 1.33$, we expect the relative influence of radiation to increase with increasing temperature. This behavior is revealed by computing and plotting the heat rates as a function of temperature for $50 \leq T_s \leq 250°\text{C}$.



For each value of T_s used to generate the foregoing free convection results, air properties were determined at the corresponding value of T_f.

Q2. A vertical array of circuit boards of 150-mm height is to be air cooled such that the board temperature does not exceed 60^{0}C when the ambient temperature is 25^{0}C.



Assuming isothermal surface conditions, determine the allowable electrical power dissipation per board for the cooling arrangements:

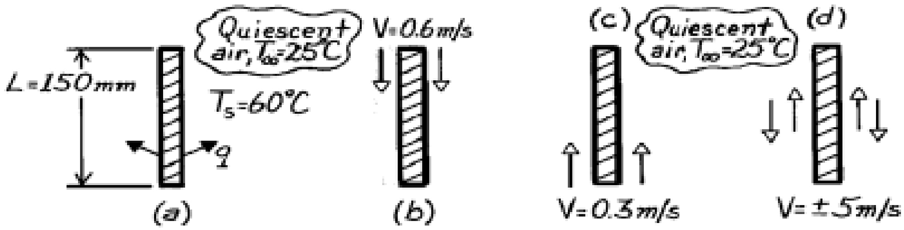
(a) Free convection only (no forced airflow).
(b) Airflow with a downward velocity of 0.6 m/s.
(c) Airflow with an upward velocity of 0.3 m/s.
(d) Airflow with a velocity (upward or downward) of 5 m/s.

Solution:

KNOWN: Vertical array of circuit boards 0.15m high with maximum allowable uniform surface temperature for prescribed ambient air temperature.

FIND: Allowable electrical power dissipation per board, $q'[W/m]$, for these cooling arrangements: (a) Free convection only, (b) Air flow downward at 0.6 m/s, (c) Air flow upward at 0.3 m/s, and (d) Air flow upward or downward at 5 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Board horizontal spacing sufficient that boundary layers don't interfere, (3) Ambient air behaves as quiescent medium, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 \approx 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}\ m^2/s$, $k = 0.0274$ W/m·K, $\alpha = 24.7 \times 10^{-6}\ m^2/s$, $Pr = 0.705$, $\beta = 1/T_f$.

ANALYSIS: (a) For *free convection* only, the allowable electrical power dissipation rate is

$$q' = \bar{h}_L (2L)(T_s - T_\infty) \tag{1}$$

where $\bar{h}_L$ is estimated using the appropriate correlation for free convection from a vertical plate. Find the Rayleigh number,

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\,m/s^2 (1/315K)(60-25)K(0.150m)^3}{17.4\times10^{-6} m^2/s \times 24.7\times10^{-6} m^2/s} = 8.551\times10^6. \tag{2}$$

Since $Ra_L < 10^9$, the flow is laminar. With Eq. 9.27 find

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670\,Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{\left(0.670\left[8.551\times10^6\right]^{1/4}\right)}{\left[1+(0.492/0.705)^{9/16}\right]^{4/9}} = 28.47 \tag{3}$$

$$\bar{h}_L = (0.0274\,W/m\cdot K/0.150m)\times 28.47 = 5.20\,W/m^2\cdot K.$$

Hence, the allowable electrical power dissipation rate is,

$$q' = 5.20\,W/m^2\cdot K(2\times0.150m)(60-25)°C = 54.6\,W/m$$ <

(b) With *downward velocity* V = 0.6 m/s, the possibility of mixed forced-free convection must be considered. With $Re_L = VL/\nu$, find

$$\left(Gr_L/Re_L^2\right) = \left(\frac{Ra_L}{Pr}/Re_L^2\right) \tag{4}$$

$$\left(Gr_L/Re_L^2\right) = \left(8.551\times10^6/0.705\right)/\left(0.6\,m/s\times0.150m/17.40\times10^{-6} m^2/s\right)^2 = 0.453.$$

Since $\left(Gr_L / Re_L^2\right) \sim 1$, flow is mixed and the average heat transfer coefficient may be found from a correlating equation of the form

$$\overline{Nu}^n = Nu_F^n \pm Nu_N^n \qquad (5)$$

where n = 3 for the vertical plate geometry and the minus sign is appropriate since the natural convection (N) flow opposes the forced convection (F) flow. For the forced convection flow, Re_L = 5172 and the flow is laminar; using Eq. 7.30,

$$\overline{Nu}_F = 0.664\, Re_L^{1/2}\, Pr^{1/3} = 0.664(5172)^{1/2}(0.705)^{1/3} = 42.50. \qquad (6)$$

Using $\overline{Nu}_N = 28.47$ from Eq. (3), Eq. (5) now becomes

$$\overline{Nu}^3 = \left(\frac{\overline{h}L}{k}\right)^3 = (42.50)^3 - (28.47)^3 \qquad \overline{Nu} = 37.72$$

$$\overline{h} = \left(\frac{0.0274\ W/m\cdot K}{0.150m}\right) \times 37.72 = 6.89\ W/m^2\cdot K.$$

Substituting for $\overline{h}$ into the rate equation, Eq. (1), the allowable power dissipation with a downward velocity of 0.6 m/s is

$$q' = 6.89\ W/m^2\cdot K(2\times 0.150m)\ (60-25)°C = 72.3\ W/m. \qquad <$$

(c) With an *upward velocity* V = 0.3 m/s, the positive sign of Eq. (5) applies since the N-flow is assisting the F-flow. For forced convection, find

$$Re_L = VL/\nu = 0.3\, m/s \times 0.150m / \left(17.40\times 10^{-6} m^2/s\right) = 2586.$$

The flow is again laminar, hence Eq. (6) is appropriate.

$$\overline{Nu}_F = 0.664(2586)^{1/2}(0.705)^{1/3} = 30.05.$$

From Eq. (5), with the positive sign, and $\overline{Nu}_N$ from Eq. (4),

$$\overline{Nu}^3 = (30.05)^3 + (28.47)^3 \quad \text{or} \quad \overline{Nu} = 36.88 \quad \text{and} \quad \overline{h} = 6.74\ W/m^2\cdot K.$$

From Eq. (1), the allowable power dissipation with an upward velocity of 0.3 m/s is

$$q' = 6.74\ W/m^2\cdot K(2\times 0.150m)\ (60-25)°C = 70.7\ W/m. \qquad <$$

(d) With a *forced convection* velocity V = 5 m/s, very likely forced convection will dominate. Check by evaluating whether $\left(Gr_L / Re_L^2\right) \ll 1$ where $Re_L = VL/\nu$ = 5 m/s × 0.150m/(17.40 × 10^{-6} m^2/s) = 43,103. Hence,

$$\left(Gr_L / Re_L^2\right) = \left(\frac{Ra_L}{Pr} / Re_L^2\right) = \left(8.551\times 10^6 / 0.705\right) / 43{,}103^2 = 0.007.$$

The flow is not mixed, but pure forced convection. Using Eq. (6), find

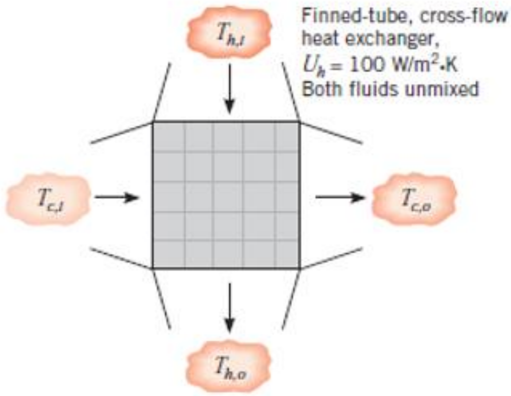
$$\overline{h} = (0.0274\ W/m\cdot K / 0.150m) 0.664(43{,}103)^{1/2}(0.705)^{1/3} = 22.4\ W/m^2\cdot K$$

and the allowable dissipation rate is

$$q' = 22.4\ W/m^2\cdot K(2\times 0.150m)\ (60-25)°C = 235\ W/m. \qquad <$$

COMMENTS: Be sure to compare dissipation rates to see relative importance of mixed flow conditions.

Q3. Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at 300 °C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 °C to 125 °C. The overall heat transfer coefficient based on the gas-side surface area is *Uh* =100 W/m^2 K. Determine the required gas-side surface area *Ah* using the NTU method.

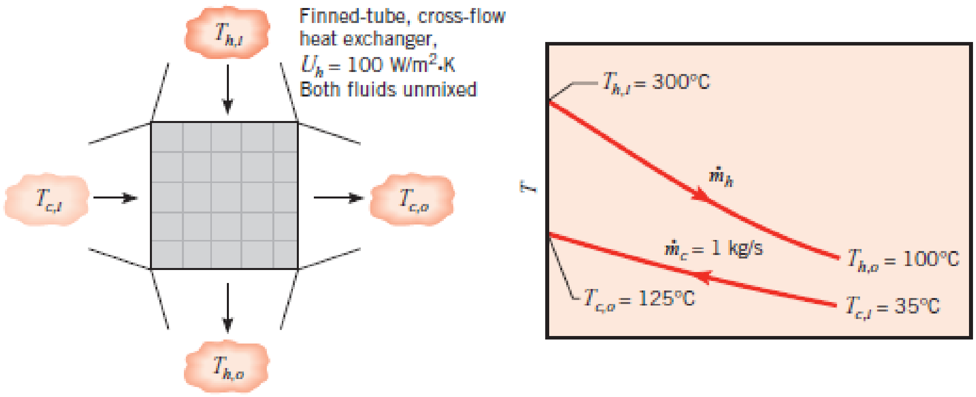

$T_{h,i}$
Finned-tube, cross-flow
heat exchanger,
$U_h = 100$ W/m^2·K
Both fluids unmixed
$T_{c,i}$
$T_{c,o}$
$T_{h,o}$

Solution:

Known: Inlet and outlet temperatures of hot gases and water used in a finned-tube, cross-flow heat exchanger. Water flow rate and gas-side overall heat transfer coefficient.

Find: Required gas-side surface area.

Schematic:



Assumptions:

1. Negligible heat loss to the surroundings and kinetic and potential energy changes.
2. Constant properties.

Properties: Table A.6, water ($\overline{T}_c = 80°\text{C}$): $c_{p,c} = 4197$ J/kg·K.

Analysis: The required surface area may be obtained from knowledge of the number of transfer units, which, in turn, may be obtained from knowledge of the ratio of heat capacity rates and the effectiveness. To determine the minimum heat capacity rate, we begin by computing

$$C_c = \dot{m}_c c_{p,c} = 1 \text{ kg/s} \times 4197 \text{ J/kg}\cdot\text{K} = 4197 \text{ W/K}$$

Since $\dot{m}_h$ is not specified, C_h is obtained by combining the overall energy balances, Equations 11.6b and 11.7b:

$$C_h = \dot{m}_h c_{p,h} = C_c \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}} = 4197 \frac{125 - 35}{300 - 100} = 1889 \text{ W/K} = C_{\min}$$

From Equation 11.18

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) = 1889 \text{ W/K } (300 - 35)°\text{C} = 5.00 \times 10^5 \text{ W}$$

From Equation 11.7b the actual heat transfer rate is

$$q = C_c(T_{c,o} - T_{c,i}) = 4197 \text{ W/K } (125 - 35)°\text{C}$$

$$q = 3.78 \times 10^5 \text{ W}$$

Hence from Equation 11.19 the effectiveness is

$$\varepsilon = \frac{q}{q_{max}} = \frac{3.78 \times 10^5 \text{ W}}{5.00 \times 10^5 \text{ W}} = 0.755$$

With

$$\frac{C_{min}}{C_{max}} = \frac{1889}{4197} = 0.45$$

it follows from Figure 11.14 that

$$\text{NTU} = \frac{U_h A_h}{C_{min}} \approx 2.0$$

or

$$A_h = \frac{2.0\,(1889 \text{ W/K})}{100 \text{ W/m}^2 \cdot \text{K}} = 37.8 \text{ m}^2 \quad \triangleleft$$

Comments:

1. Equation 11.32 may be solved iteratively or by trial and error to yield NTU = 2.0, which is in excellent agreement with the estimate obtained from the charts.

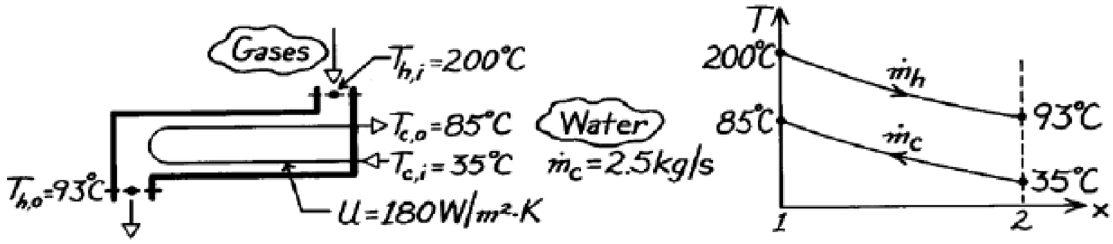
Q4: Hot exhaust gases are used in a shell-and-tube exchanger to heat 2.5 kg/s of water from 35 to 85°C. The gases, assumed to have the properties of air, enter at 200°C and leave at 93°C. The overall heat transfer coefficient is 180 W/m^2 K. Using the effectiveness–NTU method, calculate the area of the heat exchanger.

Solution:

KNOWN: Shell and tube heat exchanger for cooling exhaust gases with water.

FIND: Required surface area using ε-NTU method.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Gases have properties of air.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_c$ = (85 + 35)°C/2 = 333 K): c_p = 4185 J/kg·K.

ANALYSIS: Using the ε-NTU method, the area can be expressed as

$$A = NTU \cdot C_{min} / U \qquad (1)$$

where NTU must be found from knowledge of ε and $C_{min}/C_{max} = C_r$. The capacity rates are:

$$C_c = \dot{m}_c c_{p,c} = 2.5\,kg/s \times 4185\ J/kg\cdot K = 10,463\,W/K$$

Equating the energy balance relation for each fluid,

$$C_h = C_c\left(T_{c,o}-T_{c,i}\right)/\left(T_{h,i}-T_{h,o}\right) = 10,463\,W/K\,(85-35)/(200-93) = 4889\,W/K.$$

Hence,

$$C_r = C_{min}/C_{max} = C_h/C_c = 4889/10,463 = 0.467.$$

The effectiveness of the exchanger, with $q_{max} = C_{min}(T_{h,i} - T_{c,i})$ and $C_{min} = C_h$, is

$$\varepsilon = q/q_{max} = C_h\left(T_{h,i}-T_{h,o}\right)/C_h\left(T_{h,i}-T_{c,i}\right) = (200-93)/(200-35) = 0.648.$$

Considering the HXer to be a single shell with 2,4….tube passes, Eqs. 11.30b,c are appropriate to evaluate NTU.

$$NTU = -\left(1+C_r^2\right)^{-1/2} \ell n\frac{E-1}{E+1} \qquad\qquad E = \frac{2/\varepsilon_1-\left(1+C_r\right)}{\left(1+C_r^2\right)^{1/2}}.$$

Substituting numerical values,

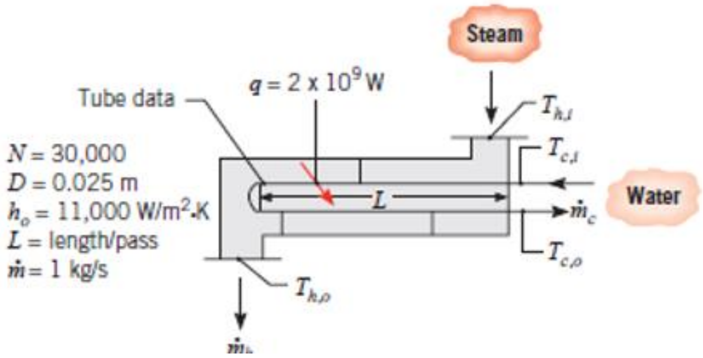
$$E = \frac{2/0.648-(1+0.467)}{\left(1+0.467^2\right)^{1/2}} = 1.467 \quad NTU = -\left(1+(0.467)^2\right)^{-1/2}\ell n\frac{1.467-1}{1.467+1} = 1.51.$$

Using the appropriate numerical values in Eq. (1), the required area is

$$A = 1.51\times 4889\,W/K/180\,W/m^2\cdot K = 40.9\,m^2. \qquad <$$

COMMENTS: Figure 11.12 could also have been used with C_r and ε to find NTU.

Q5. The condenser of a large steam power plant is a heat exchanger in which steam is condensed to liquid water. Assume the condenser to be a *shell-and-tube* heat exchanger consisting of a single shell and 30,000 tubes, each executing two passes. The tubes are of thin wall construction with D = 25 mm, and steam condenses on their outer surface with an associated convection coefficient of h_o = 11,000 W/m^2 K. The heat transfer rate that must be effected by the exchanger is $q = 2 \times 10^9$ W, and this is accomplished by passing cooling water through the tubes at a rate of 3×10^4 kg/s (the flow rate per tube is therefore 1 kg/s). The water enters at 20^{0}C, while the steam condenses at 50^{0}C. What is the temperature of the cooling water emerging from the condenser? What is the required tube length L per pass?



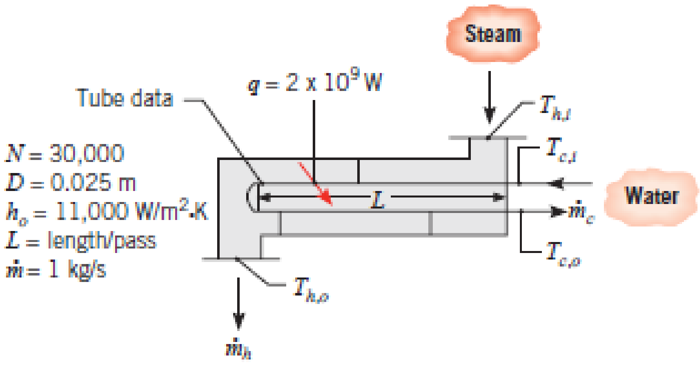
Solution:

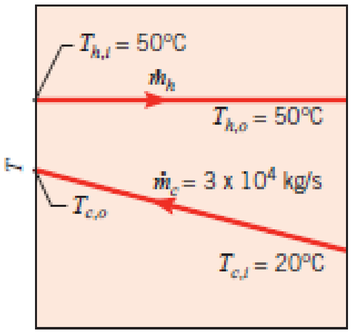
Known: Heat exchanger consisting of single shell and 30,000 tubes with two passes each.

Find:

1. Outlet temperature of the cooling water.
2. Tube length per pass to achieve required heat transfer.

Schematic:





Assumptions:

1. Negligible heat transfer between exchanger and surroundings and negligible kinetic and potential energy changes.
2. Tube internal flow and thermal conditions fully developed.
3. Negligible thermal resistance of tube material and fouling effects.
4. Constant properties.

Properties: Table A.6, water (assume $\overline{T}_c \approx 27°C = 300$ K): $\rho = 997$ kg/m³, $c_p = 4179$ J/kg·K, $\mu = 855 \times 10^{-6}$ N·s/m², $k = 0.613$ W/m·K, $Pr = 5.83$.

Analysis:

1. The cooling water outlet temperature may be obtained from the overall energy balance, Equation 11.7b. Accordingly,

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 20°C + \frac{2 \times 10^9 \text{ W}}{3 \times 10^4 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K}}$$

$$T_{c,o} = 36.0°C \qquad \triangleleft$$

2. The problem may be classified as one requiring a *heat exchanger design calculation*. First, we determine the overall heat transfer coefficient for use in the NTU method.
   From Equation 11.5

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

where h_i may be estimated from an internal flow correlation. With

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 1 \text{ kg/s}}{\pi (0.025 \text{ m}) 855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 59{,}567$$

the flow is turbulent and from Equation 8.60

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{0.4} = 0.023(59{,}567)^{0.8}(5.83)^{0.4} = 308$$

Hence

$$h_i = Nu_D \frac{k}{D} = 308 \frac{0.613 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} = 7543 \text{ W/m}^2 \cdot \text{K}$$

$$U = \frac{1}{[(1/7543) + (1/11{,}000)] \text{ m}^2 \cdot \text{K/W}} = 4474 \text{ W/m}^2 \cdot \text{K}$$

Using the design calculation methodology, we note that

$$C_h = C_{max} = \infty$$

and

$$C_{min} = \dot{m}_c c_{p,c} = 3 \times 10^4 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 1.25 \times 10^8 \text{ W/K}$$

from which

$$\frac{C_{min}}{C_{max}} = C_r = 0$$

The maximum possible heat transfer rate is

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1.25 \times 10^8 \text{ W/K} \times (50 - 20) \text{ K} = 3.76 \times 10^9 \text{ W}$$

from which

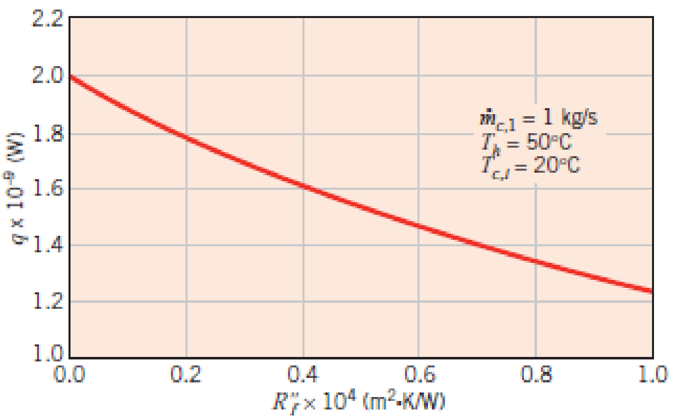
$$\varepsilon = \frac{q}{q_{max}} = \frac{2 \times 10^9 \text{ W}}{3.76 \times 10^9 \text{ W}} = 0.532$$

From Equation 11.35b or Figure 11.12, we find NTU = 0.759. From Equation 11.24, it follows that the tube length per pass is

$$L = \frac{\text{NTU} \cdot C_{min}}{U(N2\pi D)} = \frac{0.759 \times 1.25 \times 10^8 \text{ W/K}}{4474 \text{ W/m}^2 \cdot \text{K} (30{,}000 \times 2 \times \pi \times 0.025 \text{ m})} = 4.51 \text{ m} \qquad \triangleleft$$

Comments:

1. Recognize that L is the tube length per pass, in which case the total length per tube is 9.0 m. The entire length of tubing in the condenser is $N \times L \times 2 = 30{,}000 \times 4.51 \text{ m} \times 2 = 271{,}000$ m or 271 km.
2. Over time, the performance of the heat exchanger would be degraded by fouling on both the inner and outer tube surfaces. A representative maintenance schedule would call for taking the heat exchanger off-line and cleaning the tubes when fouling factors reached values of $R''_{f,i} = R''_{f,o} = 10^{-4}$ m$^2 \cdot$K/W. To determine the effect of fouling on performance, the ε−NTU method may be used to calculate the total heat rate as a function of the fouling factor, with $R''_{f,o}$ assumed to equal $R''_{f,i}$. The following results are obtained:



To maintain the requirement of $q = 2 \times 10^9$ W with the maximum allowable fouling and the restriction of $\dot{m}_{c,1} = 1$ kg/s, the tube length or the number of tubes would have to be increased. Keeping the length per pass at $L = 4.51$ m, $N = 48{,}300$ tubes would be needed to transfer 2×10^9 W for $R''_{f,i} = R''_{f,o} = 10^{-4}$ m$^2 \cdot$K/W. The corresponding increase in the total flow rate to $\dot{m}_c = N\dot{m}_{c,1} = 48{,}300$ kg/s would have the beneficial effect of reducing the water outlet temperature to $T_{c,o} = 29.9°$C, thereby ameliorating potentially harmful effects associated with discharge into the environment. The additional tube length associated with increasing the number of tubes to $N = 48{,}300$ is 165 km, which would result in a significant increase in the capital cost of the condenser.

3. The steam plant generates 1250 MW of electricity with a wholesale value of \$0.05 per kW·h. If the plant is shut down for 48 hours to clean the condenser tubes, the loss in revenue for the plant's owner is $48\text{ h} \times 1250 \times 10^6\text{ W} \times \$0.05/(1 \times 10^3\text{ W}\cdot\text{h}) =$ \$3 million.

4. Assuming a smooth surface condition within each tube, the friction factor may be determined from Equation 8.21, $f = (0.790 \ln(59{,}567) - 1.64)^{-2} = 0.020$. The pressure drop within one tube of length $L = 9$ m may be determined from Equation 8.22a, where $u_m = 4\dot{m}/(\rho\pi D^2) = (4 \times 1\text{ kg/s})/(997\text{ kg/m}^3 \times \pi \times 0.025^2\text{ m}^2) = 2.04$ m/s.

$$\Delta p = f\frac{\rho u_m^2}{2D}L = 0.020\frac{997\text{ kg/m}^3(2.04\text{ m/s})^2}{2(0.025\text{ m})}9.0\text{ m} = 15{,}300\text{ N/m}^2$$

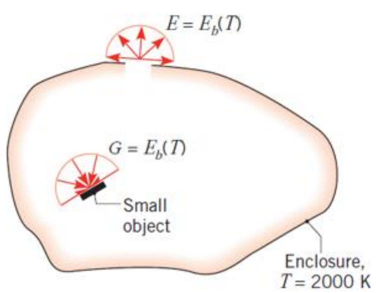
Therefore, the power required to pump the cooling water through the 48,300 tubes may be found by using Equation 8.22b and is

$$P = \frac{\Delta p\dot{m}}{\rho} = \frac{15{,}300\text{ N/m}^2 \times 48{,}300\text{ kg/s}}{997\text{ kg/m}^3} = 742{,}000\text{ W} = 0.742\text{ MW}$$

The cooling water pump is driven by an electric motor. If the combined efficiency of the pump and motor is 87%, the annual cost to overcome friction losses in the condenser tubes is $24\text{ h/day} \times 365\text{ days/yr} \times 0.742 \times 10^6\text{ W} \times \$0.05/1 \times 10^3\text{ W}\cdot\text{h}/0.87 =$ \$374,000.

5. Optimal condenser designs are based on the desired thermal performance and environmental considerations as well as on the capital cost, operating cost, and maintenance cost associated with the device.

Q6. Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000 K.



Calculate the emissive power of the radiation that emerges from a small aperture on the enclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated?

What is the wavelength λ_2 above which 10% of the emission is concentrated? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure?

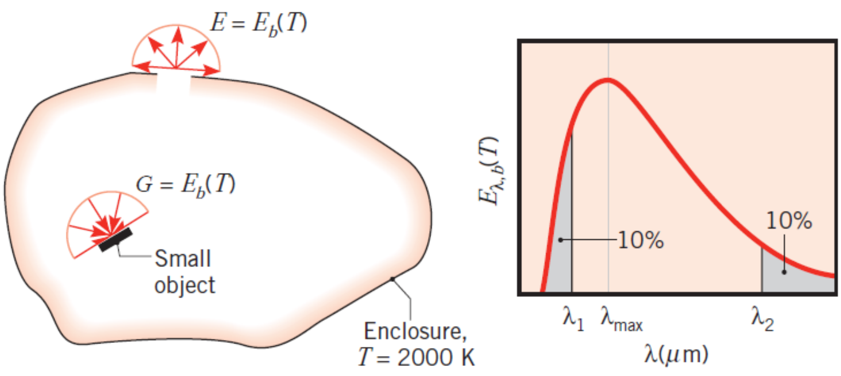
Solution:

Known: Large isothermal enclosure at uniform temperature.

Find:

1. Emissive power of a small aperture on the enclosure.
2. Wavelengths below which and above which 10% of the radiation is concentrated.
3. Spectral emissive power and wavelength associated with maximum emission.
4. Irradiation on a small object inside the enclosure.

Schematic:



Assumptions: Areas of aperture and object are very small relative to enclosure surface.

Analysis:

1. Emission from the aperture of any isothermal enclosure will have the characteristics of blackbody radiation. Hence, from Equation 12.32,

$$E = E_b(T) = \sigma T^4 = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(2000\ \mathrm{K})^4$$
$$E = 9.07 \times 10^5\ \mathrm{W/m^2} \qquad \triangleleft$$

2. The wavelength λ_1 corresponds to the upper limit of the spectral band $(0 \rightarrow \lambda_1)$ containing 10% of the emitted radiation. With $F_{(0\rightarrow\lambda 1)} = 0.10$ it follows from Table 12.2 that $\lambda_1 T = 2195\ \mu\mathrm{m \cdot K}$. Hence

$$\lambda_1 = 1.1\ \mu\mathrm{m} \qquad \triangleleft$$

The wavelength λ_2 corresponds to the lower limit of the spectral band $(\lambda_2 \rightarrow \infty)$ containing 10% of the emitted radiation. With

$$F_{(\lambda_2\rightarrow\infty)} = 1 - F_{(0\rightarrow\lambda_2)} = 0.1$$
$$F_{(0\rightarrow\lambda_2)} = 0.9$$

it follows from Table 12.2 that $\lambda_2 T = 9382\ \mu\mathrm{m \cdot K}$. Hence

$$\lambda_2 = 4.69\ \mu\mathrm{m} \qquad \triangleleft$$

3. From Wien's displacement law, Equation 12.31, $\lambda_{max} T = 2898\ \mu\mathrm{m \cdot K}$. Hence

$$\lambda_{max} = 1.45\ \mu\mathrm{m} \qquad \triangleleft$$

The spectral emissive power associated with this wavelength may be computed from Equation 12.30 or from the third column of Table 12.2. For $\lambda_{max} T = 2898\ \mu\mathrm{m \cdot K}$ it follows from Table 12.2 that

$$I_{\lambda,b}(1.45\ \mu\mathrm{m},\ T) = 0.722 \times 10^{-4} \sigma T^5$$

Hence

$$I_{\lambda,b}(1.45\ \mu\mathrm{m},\ 2000\ \mathrm{K}) = 0.722 \times 10^{-4}\ (\mu\mathrm{m \cdot K \cdot sr})^{-1} \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\ (2000\ \mathrm{K})^5$$
$$I_{\lambda,b}(1.45\ \mu\mathrm{m},\ 2000\ \mathrm{K}) = 1.31 \times 10^5\ \mathrm{W/m^2 \cdot sr \cdot \mu m}$$

Since the emission is diffuse, it follows from Equation 12.16 that

$$E_{\lambda,b} = \pi I_{\lambda,b} = 4.12 \times 10^5\ \mathrm{W/m^2 \cdot \mu m} \qquad \triangleleft$$

4. Irradiation of any small object inside the enclosure may be approximated as being equal to emission from a blackbody at the enclosure surface temperature. Hence $G = E_b(T)$, in which case

$$G = 9.07 \times 10^5\ \mathrm{W/m^2} \qquad \triangleleft$$

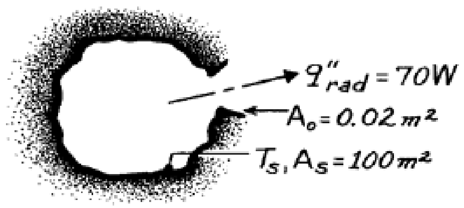
Q7. An enclosure has an inside area of 100 m^2, and its inside surface is black and is maintained at a constant temperature. A small opening in the enclosure has an area of 0.02 m^2. The radiant power emitted from this opening is 70 W. What is the temperature of the interior enclosure wall? If the interior surface is maintained at this temperature, but is now polished, what will be the value of the radiant power emitted from the opening?

Solution:

KNOWN: Isothermal enclosure of surface area, A_s, and small opening, A_o, through which 70W emerges.

FIND: (a) Temperature of the interior enclosure wall if the surface is black, (b) Temperature of the wall surface having $\varepsilon = 0.15$.

SCHEMATIC:



ASSUMPTIONS: (1) Enclosure is isothermal, (2) $A_o \ll A_s$.

ANALYSIS: A characteristic of an isothermal enclosure, according to Section 12.4, is that the radiant power emerging through a small aperture will correspond to blackbody conditions. Hence

$$q_{rad} = A_o E_b(T_s) = A_o \sigma T_s^4$$

where q_{rad} is the radiant power leaving the enclosure opening. That is,

$$T_s = \left(\frac{q_{rad}}{A_o \sigma}\right)^{1/4} = \left(\frac{70\text{W}}{0.02\text{m}^2 \times 5.670 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 498\text{K}. \qquad <$$

Recognize that the radiated power will be independent of the emissivity of the wall surface. As long as $A_o \ll A_s$ *and* the enclosure is isothermal, then the radiant power will depend only upon the temperature.

COMMENTS: It is important to recognize the unique characteristics of isothermal enclosures. See Fig. 12.11 to identify them.