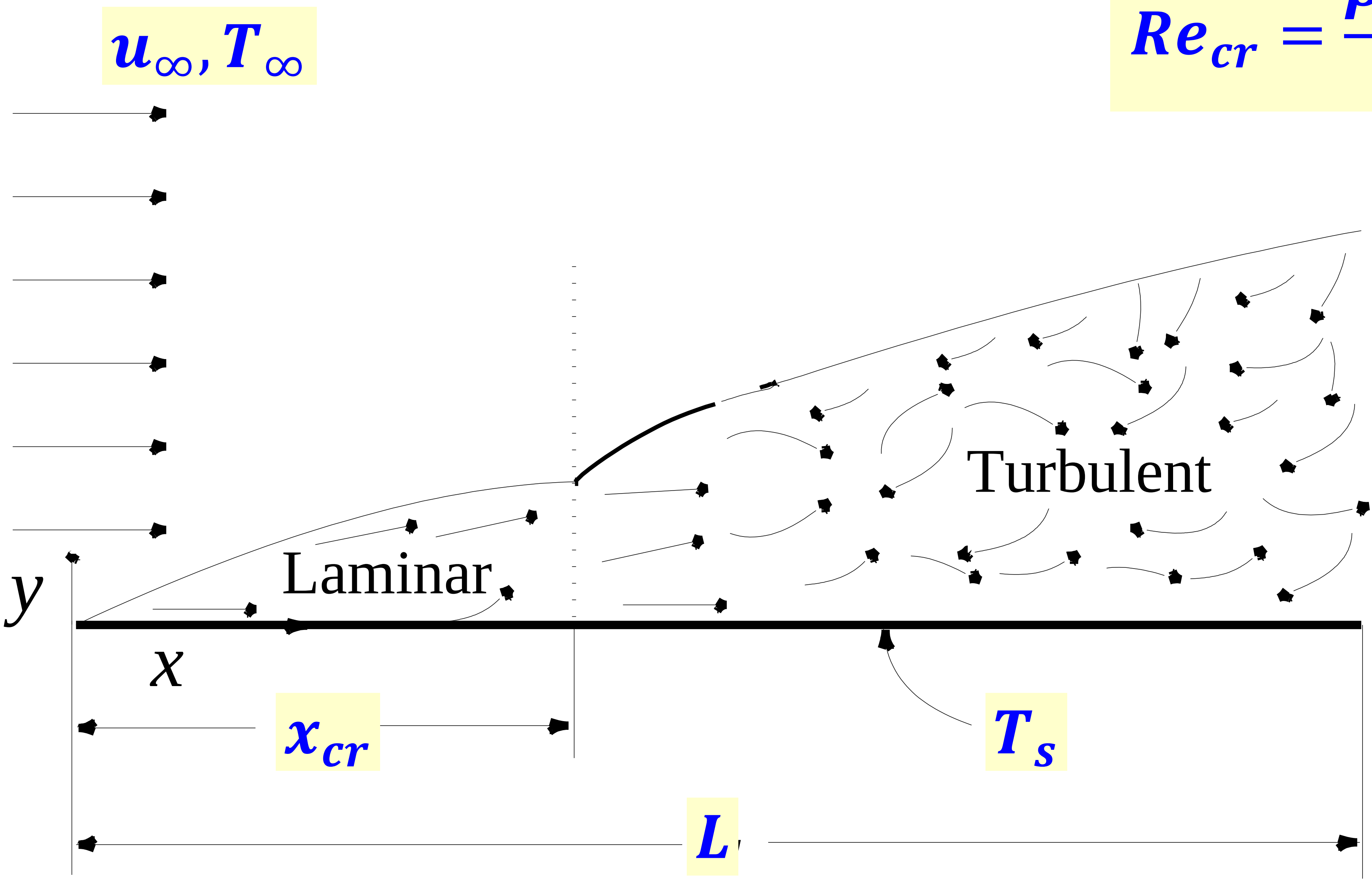
PARALLEL FLOW OVER FLAT PLATES

$$Re_{cr} = \frac{\rho u_\infty x_{cr}}{\mu} = 5 \times 10^5$$

$$Re_x = \frac{\rho u_\infty x}{\mu}$$



LAMINAR

$Re_x < 5 \times 10^5$

$$\frac{\delta_x}{x} = 5 Re_x^{-\frac{1}{2}}$$

$$C_{f,x} = 0.664 Re_x^{-\frac{1}{2}}$$

TURBULENT

$5 \times 10^5 < Re_x < 10^7$

$$\frac{\delta_x}{x} = 0.382 Re_x^{-\frac{1}{5}}$$

$$C_{f,x} = 0.0592 Re_x^{-\frac{1}{5}}$$

AVERAGE SKIN FRICTION COEFFICIENT

$$\bar{C}_{f,L} = \frac{1}{L}\int_0^L C_{f,x}\,dx = \frac{1}{L}\int_0^L 0.664 Re_x^{-\frac{1}{2}}\,dx = \frac{1}{L}\int_0^L 0.664\left(\frac{\rho u_\infty x}{\mu}\right)^{-\frac{1}{2}} dx$$

$$\bar{C}_{f,L} = \frac{0.664}{L}\left(\frac{\rho u_\infty}{\mu}\right)^{-\frac{1}{2}} \left.\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right|_0^L = 1.328\left(\frac{\rho u_\infty L}{\mu}\right)^{-\frac{1}{2}}$$

$$C_f = 1.328 Re_L^{-\frac{1}{2}}$$

LAMINAR

$Re_x < 5 \times 10^5$

$$\bar{C}_{f,L} = 1.328 Re_L^{-\frac{1}{2}}$$

TURBULENT

$5 \times 10^5 < Re_x < 10^7$

$$\bar{C}_{f,L} = 0.074 Re_x^{-\frac{1}{5}}$$

$$\bar{C}_{f,L} = \frac{1}{L}\int_{0}^{x_{cr}} C_{f,x\,laminar}\,dx + \frac{1}{L}\int_{x_{cr}}^{L} C_{f,x\,turbulent}\,dx$$

LAMINAR

$Re_x < 5 \times 10^5$

$$C_{f,x} = 0.664 Re_x^{-\frac{1}{2}}$$

$$\bar{C}_{f,L} = \frac{1}{L}\left[0.664\left(\frac{\rho u_\infty}{\mu}\right)^{\frac{1}{2}}\int_{0}^{x_{cr}} \frac{dx}{x^{\frac{1}{2}}} + 0.0592\left(\frac{\rho u_\infty}{\mu}\right)^{0.8}\int_{x_{cr}}^{L} \frac{dx}{x^{0.2}}\right]$$

TURBULENT

$5 \times 10^5 < Re_x < 10^7$

$$C_{f,x} = 0.0592 Re_x^{-\frac{1}{5}}$$

$$\bar{C}_{f,L} = 0.074 Re_L^{-\frac{1}{5}} - \frac{1740}{Re_L}$$

$5 \times 10^5 < Re_x < 10^7$

Rough Surface, Turbulent

$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

HEAT TRANSFER COEFFICIENT

LAMINAR

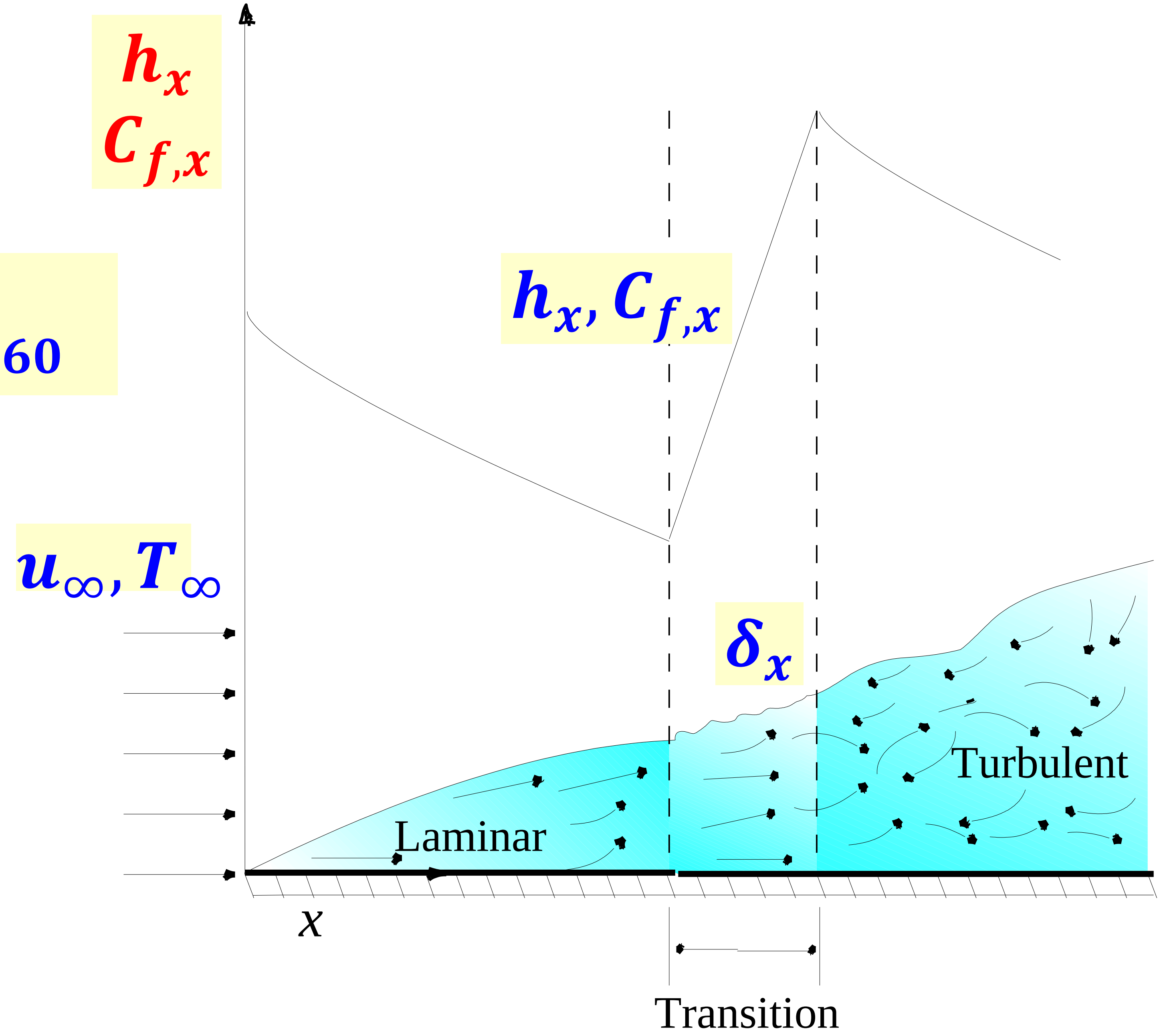
$Re_x < 5 \times 10^5 \quad Pr > 0.6$

$$Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

TURBULENT

$5 \times 10^5 < Re_x < 10^7 \quad 0.6 < Pr > 60$

$$Nu_x = 0.0296 Re_x^{0.8} Pr^{\frac{1}{3}}$$



AVERAGE HEAT TRANSFER COEFFICIENT

LAMINAR $Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$

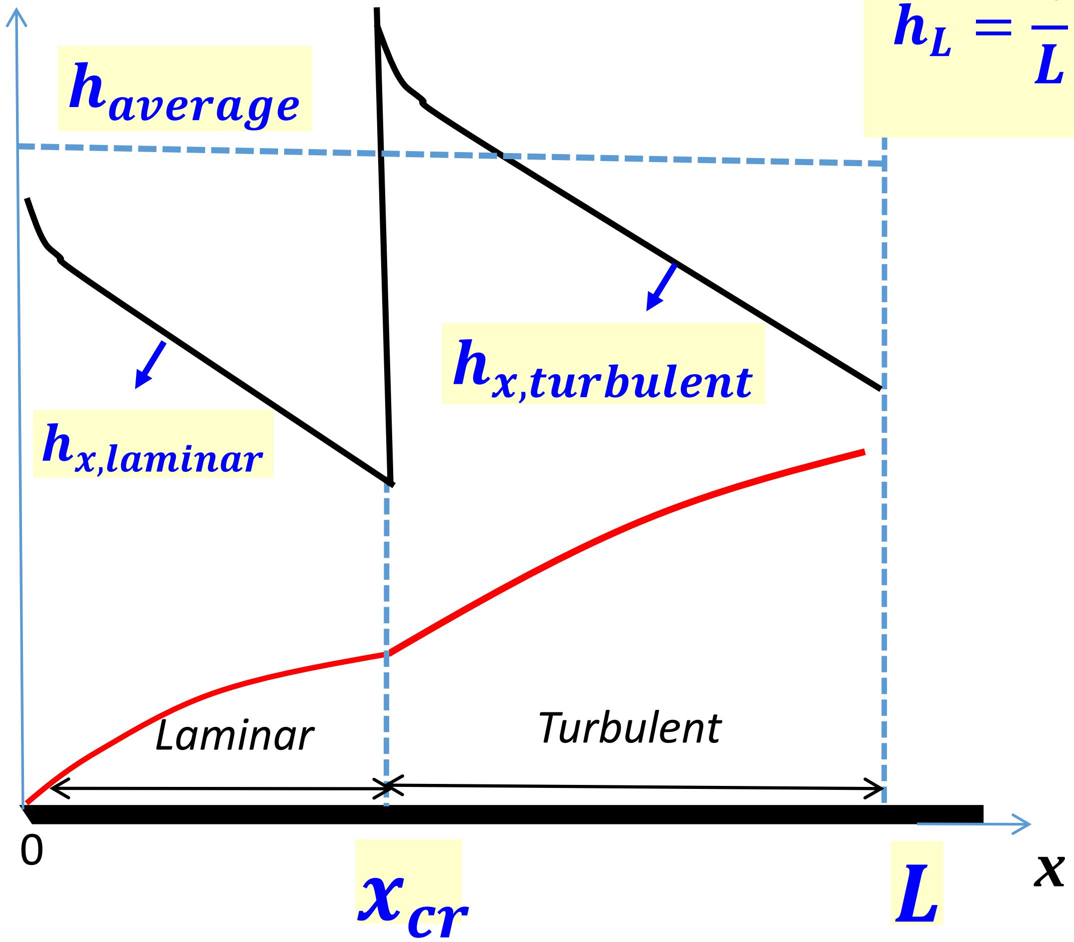
TURBULENT $Nu_x = 0.0296 Re_x^{0.8} Pr^{\frac{1}{3}}$

$$\bar{h}_L = \frac{1}{L}\int_0^{x_{cr}} h_{x\ laminar}\, dx + \frac{1}{L}\int_{x_{cr}}^{L} h_{x\ turbulent}\, dx$$

$$\bar{h}_L = \frac{k}{L}\left[0.332\left(\frac{\rho u_\infty}{\mu}\right)^{\frac{1}{2}}\int_0^{x_{cr}}\frac{dx}{x^{\frac{1}{2}}} + 0.0296\left(\frac{\rho u_\infty}{\mu}\right)^{0.8}\int_{x_{cr}}^{L}\frac{dx}{x^{0.2}}\right]Pr^{\frac{1}{3}}$$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k}\left(0.037 Re_L^{0.8} - 871\right)Pr^{\frac{1}{3}}$$

$5 \times 10^5 < Re_x < 10^7 \quad 0.6 < Pr > 60$



AVERAGE HEAT TRANSFER COEFFICIENT

Liquid metals

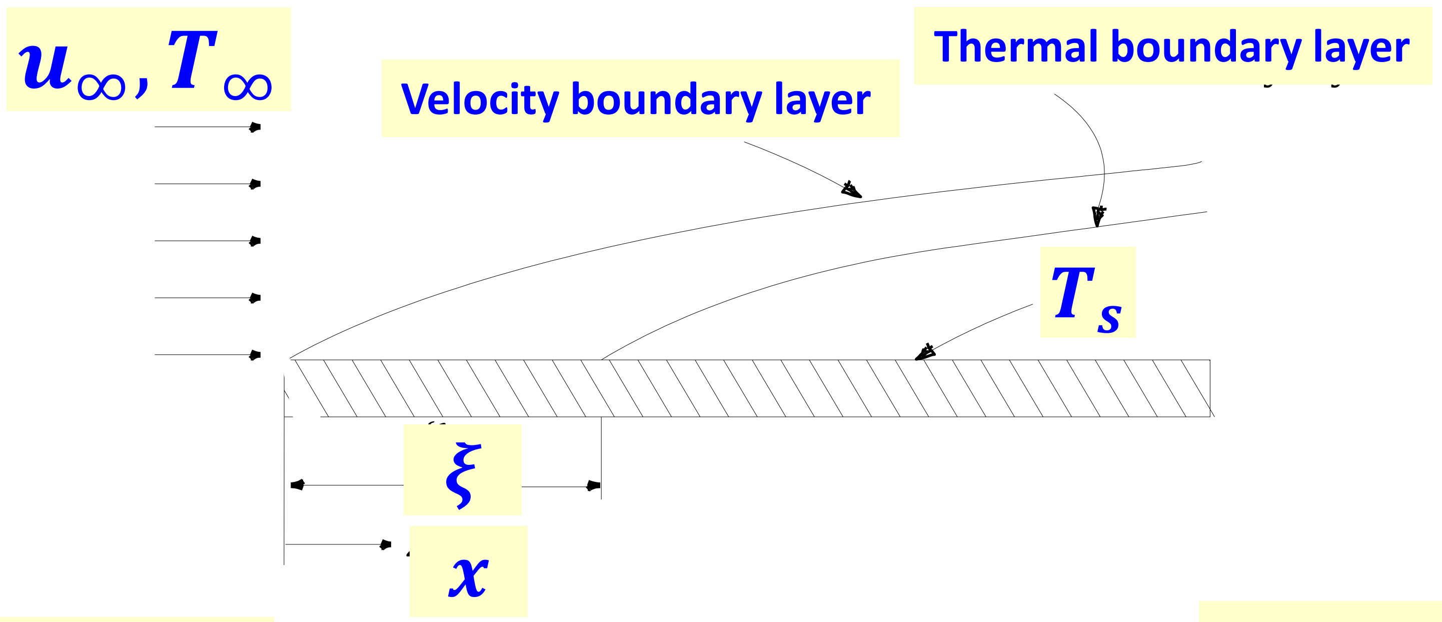
$$Nu_x = 0.565 Re_x^{\frac{1}{2}} Pr^{\frac{1}{2}}$$

$$Pr < 0.05$$

Churchill's correlation for all Prandtl numbers

$$Nu_x = \frac{0.3387 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.0468}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}$$

FLAT PLATE WITH UNHEATED STARTING LENGTH



$$\dot{Q} = q_s'' A_s$$

$$q_s'' = h_x[T_s(x) - T_\infty]$$

$$T_s(x) = T_\infty + \frac{q_s''}{h_x}$$

$$Nu_x = \frac{h_x x}{k}$$

LAMINAR

T_s = constant

$$Nu_x = \frac{Nu_{x(\xi=0)}}{\left[1-\left(\frac{\xi}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}} = \frac{0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}}{\left[1-\left(\frac{\xi}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}}$$

q_s'' = constant

$$Nu_x = 0.453 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

TURBULENT

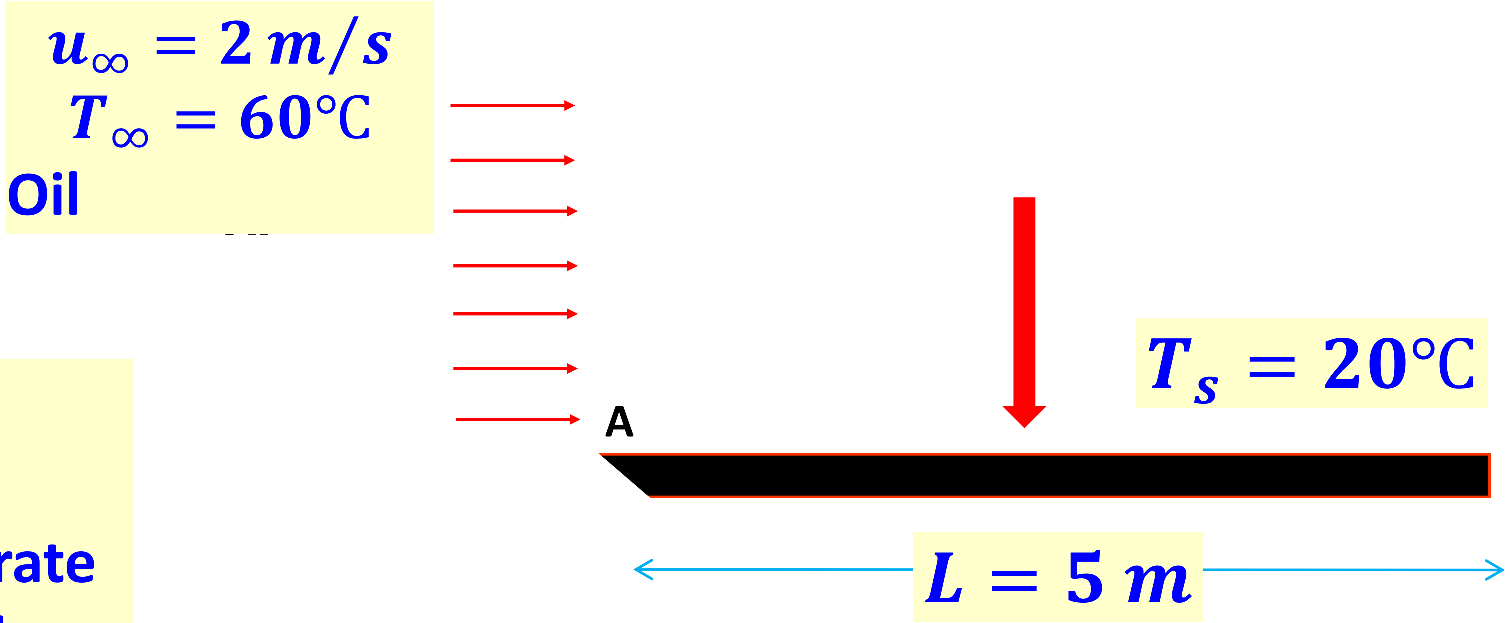
T_s = constant

$$Nu_x = \frac{Nu_{x(\xi=0)}}{\left[1-\left(\frac{\xi}{x}\right)^{\frac{9}{10}}\right]^{\frac{1}{9}}} = \frac{0.0296 Re_x^{0.8} Pr^{\frac{1}{3}}}{\left[1-\left(\frac{\xi}{x}\right)^{\frac{9}{10}}\right]^{\frac{1}{9}}}$$

q_s'' = constant

$$Nu_x = 0.0308 Re_x^{0.8} Pr^{\frac{1}{3}}$$

Problem: Engine oil at 60° C flows over the upper surface of a 5-m long flat plate whose temperature is 20° C with a velocity of 2 m/s (Fig 2.12). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



Known: Engine oil flows over a flat plate.

Find: The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions:

The flow is steady and incompressible.

The critical Reynolds number is $Re_{cr} = 5 \times 10^5$

$$T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°\text{C}$$

$$\rho = 876\, kg/m^3 \quad Pr = 2870$$

$$k = 0.144\ W/m.°\text{C} \quad \nu = 242\ \times 10^{-6}\ m^2/s$$

$$Re_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu} = \frac{2\times 5}{242\ \times 10^{-6}} = 41322$$

This is the less than the critical Reynolds number $Re_{cr} = 5\times 10^5$

LAMINAR

$Re_x < 5\times 10^5$

$$\overline{C}_{f,L} = 1.328 Re_L^{-\frac{1}{2}}$$

$T_f = (T_s + T_\infty)/2 = (20+60)/2 = 40℃$

$\rho = 876\, kg/m^3\ Pr = 2870$

$k = 0.144\ W/m.℃\ \nu = 242\ \times 10^{-6}\ m^2/s$

$$\overline{C}_{f,L} = 1.328(41322)^{-\frac{1}{2}}$$

$$\overline{C}_{f,L} = 6.533\times 10^{-3}$$

DRAG FORCE

$$F_D = \overline{C}_{f,L} A_s \frac{\rho u_\infty^2}{2} = 6.533\times 10^{-3}(5\times 1)\frac{876\times 2^2}{2}$$

$$F_D = 57.23\ N$$

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = 0.664 Re_L^{0.5} Pr^{\frac{1}{3}} = 0.664(41322)^{\frac{1}{2}}(2870)^{\frac{1}{3}} = 1918$$

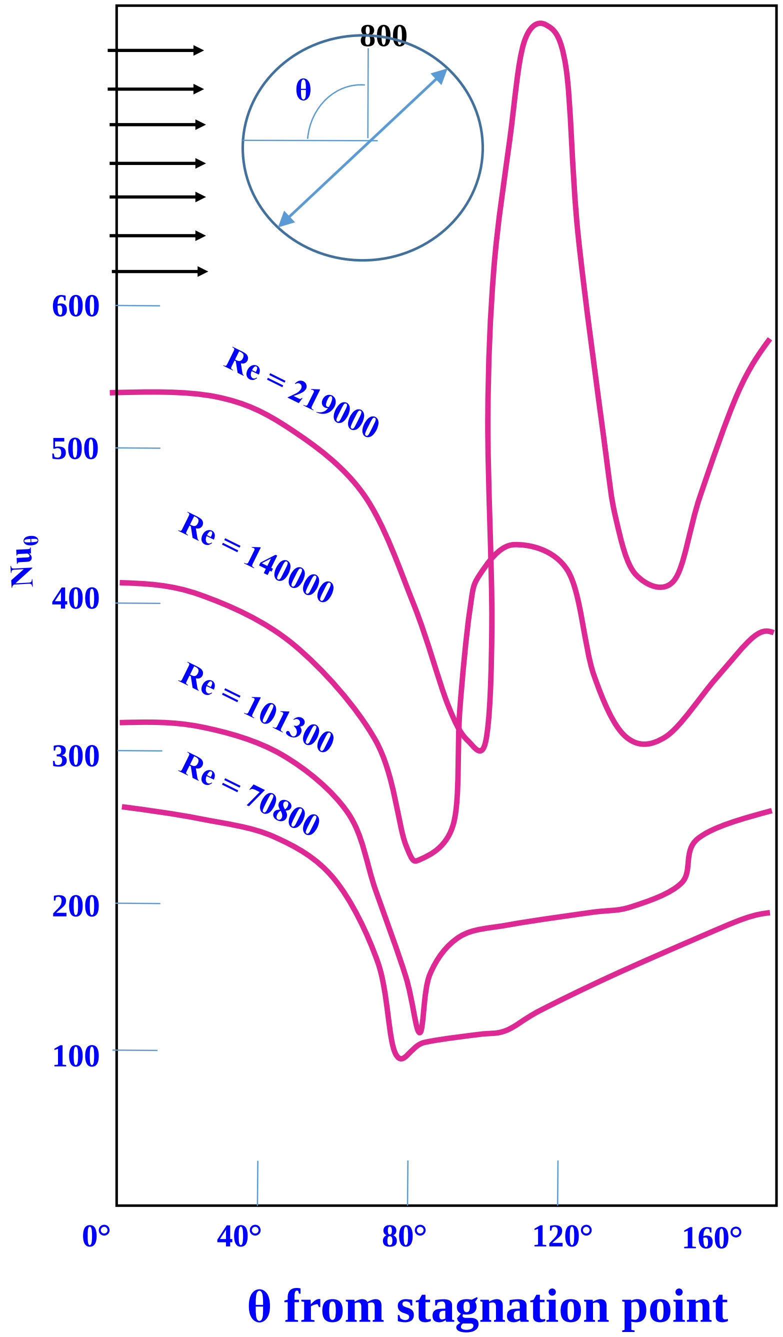
$$\overline{h}_L = \frac{k\overline{Nu}_L}{L} = \frac{0.144\times 1918}{5} = 55.2$$

$$\overline{h}_L = 55.2\ W/m^2℃$$

$$\dot{Q} = \overline{h}_L A_s(T_\infty - T_s) = 55.2\times(5\times 1)(60-20) = 11049$$

$$\dot{Q} = 11049\ W$$

Note that, heat transfer is always from the higher-temperature medium to the lower-temperature one. In this case, it is from the oil to the plate. The heat transfer rate is per m width of the plate. The heat transfer for the entire plate can be obtained by multiplying the value obtained by the actual width of the plate.



Churchill and Bernstein correlation

$$Nu_{cyl} = 0.3 + \frac{0.62 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \left[1 + \left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

Nu is relatively high at the stagnation point

Decreases with increasing as a result of the thickening of the laminar boundary layer.

Nu reaches a minimum at 80°, which is the separation point in laminar flow.

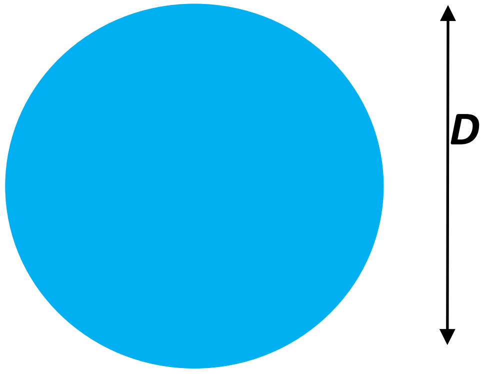
Nu increases with increasing as a result of the intense mixing in the separated flow region (the wake).

The curves at the top corresponding to $Re = 140000\ to\ 219000$ differ from the first two curves in that they have *two* minima.

The sharp increase at about 90° is due to the transition from laminar to turbulent flow.

The later decrease is again due to the thickening of the boundary layer.

Nu reaches its second minimum at about 140°, which is the flow separation point in turbulent flow, and increases with as a result of the intense mixing in the turbulent wake region.

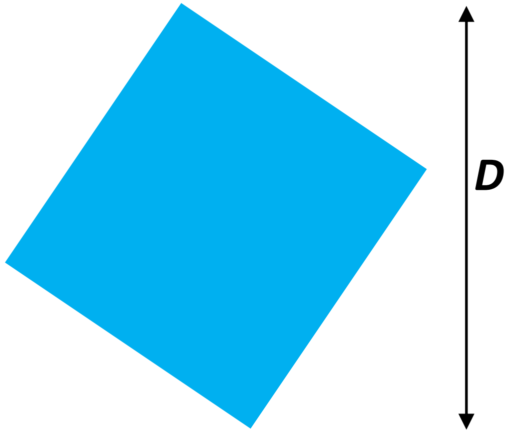


Gas or liquid

$$Re = 0.4 - 4 \qquad Nu = 0.989\, Re^{0.33} Pr^{1/3}$$
$$Re = 4 - 40 \qquad Nu = 0.911\, Re^{0.385} Pr^{1/3}$$
$$Re = 40 - 4000 \qquad Nu = 0.683\, Re^{0.466} Pr^{1/3}$$
$$Re = 4000 - 40000 \qquad Nu = 0.193\, Re^{0.618} Pr^{1/3}$$
$$Re = 40000 - 400000 \qquad Nu = 0.102\, Re^{0.675} Pr^{1/3}$$

Gas

$$Re = 5000 - 100000 \qquad Nu = 0.102\, Re^{0.675} Pr^{\frac{1}{3}}$$

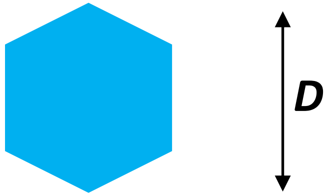


Gas

$$Re = 5000 - 100000 \qquad Nu = 0.264\, Re^{0.588} Pr^{\frac{1}{3}}$$

Gas

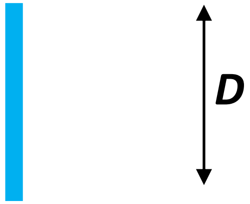
$$Re = 5000 - 100000 \quad Nu = 0.153\, Re^{0.638} Pr^{\frac{1}{3}}$$



Gas

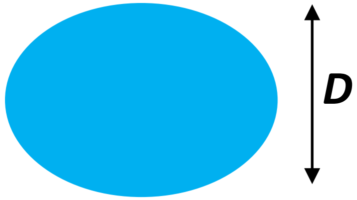
$$Re = 5000 - 19500 \quad Nu = 0.16\, Re^{0.638} Pr^{\frac{1}{3}}$$

$$Re = 19500 - 100000 \quad Nu = 0.0385\, Re^{0.782} Pr^{\frac{1}{3}}$$



Gas

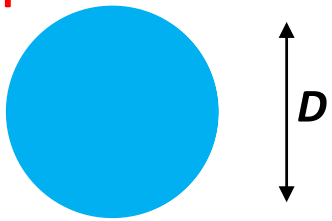
$$Re = 4000 - 15000 \quad Nu = 0.228\, Re^{0.731} Pr^{\frac{1}{3}}$$



Gas

$$Re = 2500 - 15000 \quad Nu = 0.248\, Re^{0.612} Pr^{\frac{1}{3}}$$

sphere



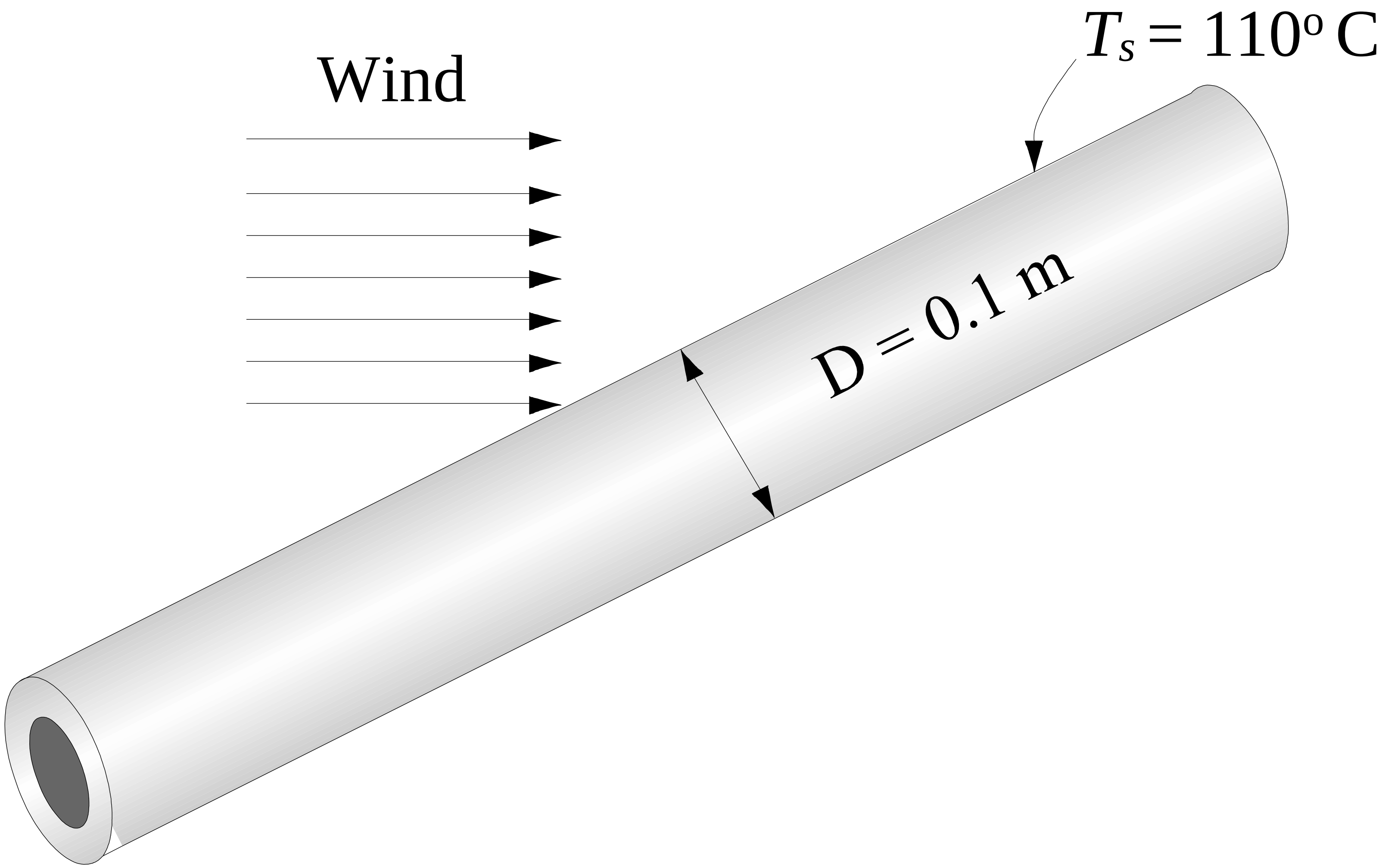
Gas

$$Nu_{sphere} = 2 + \left[0.4Re^{0.5} + 0.06Re^{\frac{2}{3}}\right]Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{\frac{1}{4}}$$

$$3.5 \le Re \le 80000 \qquad 0.7 \le Pr \le 380$$

Problem: A long 10-cm diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig. 2.23). Determine the rate of heat loss from the pipe per unit length of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

Schematic:



Known: A steam pipe is exposed to windy air.

Find: The rate of heat loss from the steam is to be determined.

Assumptions:

Steady operating conditions exist.

Radiation effects are negligible.

Air is an ideal gas.

$$T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60°\text{C}$$

$$Pr = 0.7202$$

$$k = 0.02808\ W/m.°\text{C} \quad \nu = 1.896\ \times 10^{-5}\ m^2/s$$

$$Re_L = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu} = \frac{8 \times 0.1}{1.896\ \times 10^{-5}} = 4.219 \times 10^4$$

Churchill and Bernstein correlation

$$Nu_{cyl} = 0.3 + \frac{0.62 Re^{\frac{1}{2}}\ Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \left[1 + \left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$Nu_{cyl} = 0.3 + \frac{0.62(4.219 \times 10^4)^{\frac{1}{2}}(0.7202)^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{0.7202}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \left[1 + \left(\frac{4.219 \times 10^4}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$Nu_{cyl} = 124.44$$

$$Nu_{cyl} = 124.44$$

$$T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60℃$$

$$Pr = 0.7202$$

$$k = 0.02808\ W/m.℃ \quad \nu = 1.896\ \times 10^{-5}\ m^2/s$$

$$Nu_{cyl} = \frac{h_{cyl} D}{k}$$

$$124.44 = \frac{h_{cyl} \times 0.1}{0.02808}$$

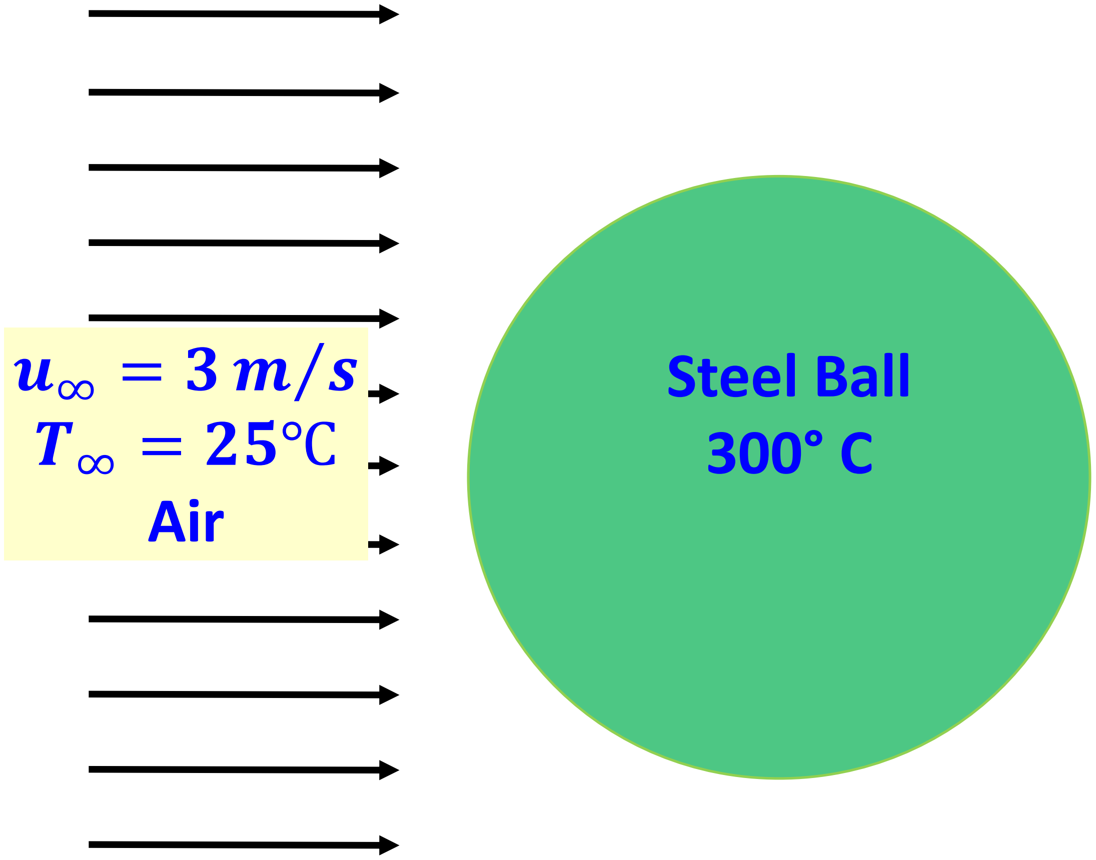
$$h_{cyl} = 34.94\, W/m^2℃$$

$$A_s = \pi DL = \pi(0.1)(1) = 0.314\, m^2$$

$$\dot{Q} = h_{cyl} A_s (T_\infty - T_s) = 34.94 \times (0.314)(110 - 10) = 1097.3$$

$$\dot{Q} = 1097.3\, W$$

A 25-cm-diameter stainless steel ball is removed from the oven at a uniform temperature of 300°C . The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take. $\rho = 8055\ kg/m^3\ \ C_p = 480\ J/kg.{}^{\circ}C$



Known: A hot stainless steel ball is cooled by forced air.

Find: The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions:

Steady operating conditions exist.

Radiation effects are negligible.

Air is an ideal gas.

The outer surface temperature of the ball is uniform at all times.

The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

The properties of air at the free–stream, temperature of 25°C and 1 atm

$Pr = 0.7296 \quad \mu = 1.849 \times 10^{-5}\ Pa.s$

$k = 0.00251\ W/m.℃ \quad \nu = 1.562 \times 10^{-5}\ m^2/s$

$$Re_L = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu} = \frac{3 \times 0.25}{1.562 \times 10^{-5}} = 4.802 \times 10^4$$

$$Re_L = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu} = \frac{3 \times 0.25}{1.562 \times 10^{-5}}$$

$$Re_L = 4.802 \times 10^4$$

The properties of air at the free–stream, temperature of 25°C and 1 atm

$$Pr = 0.7296 \quad \mu = 1.849 \times 10^{-5}\ Pa.s$$

$$k = 0.00251\ W/m.°C \quad \nu = 1.562 \times 10^{-5}\ m^2/s$$

$$\mu_s = \mu_{@250°C} = 2.76 \times 10^{-5}\ Pa.s$$

$$Nu_{sphere} = 2 + \left[0.4Re^{0.5} + 0.06Re^{\frac{2}{3}}\right]Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{\frac{1}{4}}$$

$$3.5 \le Re \le 80000 \qquad 0.7 \le Pr \le 380$$

$$Nu_{sphere} = 2 + \left[0.4\left(4.802 \times 10^4\right)^{0.5} + 0.06\left(4.802 \times 10^4\right)^{\frac{2}{3}}\right](0.7296)^{0.4}\left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{\frac{1}{4}}$$

$$Nu_{sphere} = 135.12$$

$$Nu_{sphere} = \frac{h_{sphere} D}{k} \qquad 135.12 = \frac{h_{sphere} \times 0.25}{0.02808} \qquad h_{sphere} = 13.8\ W/m^2°C$$

$$A_s = \pi D^2 = \pi(0.25)^2 = 0.1963\ m^2$$

$$\dot{Q} = h_{sphere} A_s (T_\infty - T_s) = 13.8 \times (0.1963)(250 - 25) = 610 \qquad \dot{Q} = 610\ W$$

$$m = \rho V = \rho \frac{1}{6} \pi D^3 = 8055 \times \frac{1}{6} \pi (0.25)^3 = 65.9\ kg$$

$$m = 65.9\ kg$$

$$Q_{total} = mC_p(T_2 - T_1) = 65.9(480)(300 - 200) = 3163000\ J$$

$$Q_{total} = 3163000\ J$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t = \frac{Q_{total}}{\dot{Q}} = \frac{3163000}{610} = 5185\ \mathrm{s} = 1\ \mathrm{hr}\ \ 26\ \mathrm{min}$$

Comments:

The time of cooling should also be determined more accurately using the transient temperature relations introduced in i.e. transient heat conduction. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naïve to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

Hot air at 470 °C flows overs a flat plate 40 cm × 20 cm and 3 mm thick at a velocity of 2 m/s along the 40 cm side. The intial temperature is 30 °C. The specific heat and density of the plate is 450 J/kg. K and 8000 kg/m³. Calculate the initial temperature rise of the plate deg C.min, if the plate receives the heat due to convection and radiation from both sides. Assume the emissivity of the plate as 0.85.

Given

$T_i = 30\ °C = 30 + 273 = 303K$

$T_\infty = 470\ °C = 470 + 273 = 743K$

Plate

$$C_p = 450 \frac{J}{kgK}$$

$$\rho = 8000 \frac{kg}{m^3}$$

$\varepsilon = 0.85$

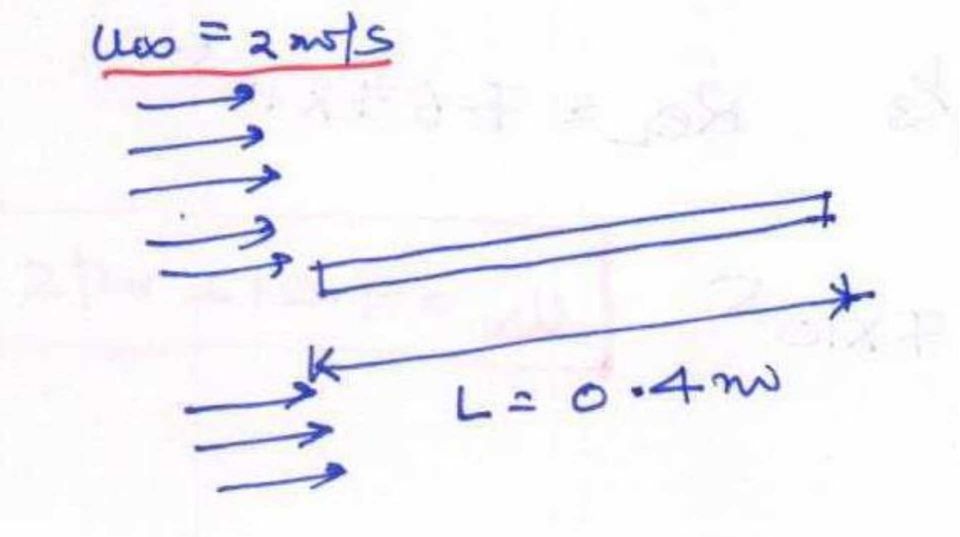
Properties of air

$T_f = (T_s + T_\infty)/2 = (303 + 743)/2 = 60°C$

$$\rho = 0.66719 \frac{kg}{m^3}$$

$Pr = 0.68354$

$k = 0.045372\ W/m.°C$ $\mu = 278.518 \times 10^{-7}$ Pa.s



Assumptions:

Steady operating conditions exist.

Air is an ideal gas.

Properties of air

$T_f = (T_s + T_\infty)/2 = (303 + 743)/2 = 60°C$

$\rho = 0.66719 \ \frac{kg}{m^3}$

$Pr = 0.68354$

$k = 0.045372 \ W/m.°C \quad \mu = 278.518 \ \times 10^{-7}$ Pa.s

Given

$T_i = 30 \ °C = 30 + 273 = 303K$

$T_\infty = 470 \ °C = 470 + 273 = 743K$

Plate

$C_p = 450 \frac{J}{kgK}$

$\rho = 8000 \ \frac{kg}{m^3}$

$\varepsilon = 0.85$

$$Re_L = \frac{\rho u_\infty D}{\mu} = \frac{0.66719 \times 2 \times 0.4}{278.518 \ \times 10^{-7}} = 19095$$

$Re_L = 19095$ **Laminar**

$Re_L < 5 \times 10^5$ **(Laminar)**

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = 0.664 Re_L^{0.5} Pr^{\frac{1}{3}} = 0.664(19095)^{\frac{1}{2}}(0.68354)^{\frac{1}{3}} = 80.83$$

$$\overline{h}_L = \frac{k\overline{Nu}_L}{L} = \frac{0.045372 \times 80.83}{0.4} = 9.169$$

$\overline{h}_L = 9.169 \ W/m^2°C$

Energy balance

$$\dot{E}_{in} - \cancel{\dot{E}_{out}}^{\,0} + \cancel{\dot{E}_g}^{\,0} = \dot{E}_{st}$$

$$hA_s(T_\infty - T) + \sigma \in A_s(T_\infty^4 - T^4) = \rho V c_p \frac{dT}{dt}$$

at $t = 0$ $T = T_i$ find $\frac{dT}{dt}$

$$(9.169)(0.16)(470 - 30) + 5.67\times10^{-8} \times 0.85 \times 0.16\left[743^4 - 303^4\right]$$

$$= 8000 \times 2.4\times10^{-4} \times 450 \times \frac{dT}{dt}$$

$$645.5 + 2285.05 = 864\frac{dT}{dt}$$

$$\boxed{\frac{dT}{dt} = 3.3918\ \frac{°C}{sec}}$$

both sides

$A_s = LW \times 2$

$= 0.4 \times 0.2$

$\boxed{A_s = \cancel{28}\ 0.08\ m^2}$

$A_s = 0.16\ m^2$

$V = LWt$

$= 0.4 \times 0.2 \times 3\times10^{-3}$

$\boxed{V = 2.4\times10^{-4}\ m^3}$