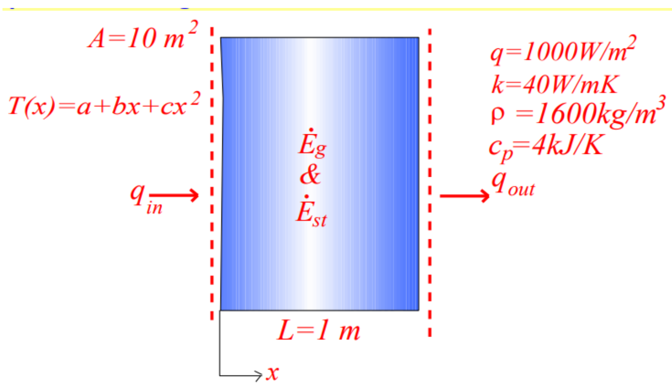
ME 346 – Heat Transfer: Tutorial 1

Q1: The temperature distribution across a wall of 1 m thick at a certain instant of time is given as $T(x) = a+bx+cx^2$ where T is in degrees Celsuis and x is in meters, while a = 800 °C, b = -350 °C/m, and c = -60 °C/m^2 . A uniform heat generation, = 1000 W/m^3 , is present in the wall of area 10 m^2 having the properties ϱ = 1600 kg/m^3 , k = 40 W/m.K, and Cp = 4 kJ/kg.K.
• Determine the rate of heat transfer entering the wall (x = 0) and leaving the wall (x = 1m)
• Determine the rate of change of energy storage in the wall
• Determine the time rate of temperature change at x = 0.25 and 0.5 m
(**Conservation of Energy**)



Known: Temperature distribution T(x) at an instant of time t in a one dimensional wall with uniform heat generation.
Find:
• Heat rates entering, qin(x = 0), and leaving, qout(x = 1), the wall
• Rate of change of energy storage in the wall,
• Time rate of temperature change at x = 0.25 and 0.5 m.
Assumptions:
• One dimensional conduction in the x -direction.
• Homogenous medium with constant properties.
• Uniform heat generation.
Analysis: Recall that once the temperature distribution is known for a medium, it is a simple matter to determine the conduction heat transfer rate at any point in the medium, or at its surfaces, by Fourier's law.
Hence the desired heat rates may be determined by using the prescribed temperature distribution with Equation. Accordingly,

$T(x) = 800 - 350x - 60x^2$

$$q_{in} = q_x(0) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

$$q_{in} = q_x(0) = -kA(-350 - 120x)_{x=0}$$

$$q_{in} = q_x(0) = -40 \times 10(-350)$$

$$q_{in} = 140\ kW$$

$$q_{out} = q_x(L) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=L}$$

$$q_{out} = q_x(L) = -kA(-350 - 120x)_{x=1}$$

$$q_{out} = q_x(L) = -40 \times 10(-350 - 120)$$

$$q_{out} = 188\ kW$$

The rate of change of energy storage *Eᵔ st* in the wall may be determined by applying an overall energy balance to the wall. Using Equation for a control volume about the wall

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{E}_g = \dot{q}AL = 1000 \times 10 \times 1 = 10\ kW$$

$$140 + 10 - 188 = \dot{E}_{st}$$

$$\dot{E}_{st} = -38\ kW$$

The time rate of change of change of the temperature at any point in the medium may be determined from the heat equation rewritten as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{k}{\rho C_p}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\dot{q}}{\rho C_p} = \frac{\partial T}{\partial t}$$

From the prescribed temperature distribution, it follows that

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}(-350 - 120x) = -120\frac{℃}{m^2}$$

$$T(x) = 800 - 350x - 60x^2$$

Note that this derivative is independent of position in the medium. Hence the time rate of temperature change is also independent of position and is given by

$$\frac{k}{\rho C_p}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\dot{q}}{\rho C_p} = \frac{\partial T}{\partial t}$$

$\rho = 1600\ kg/m^3$
$k = 40\ W/m.K$
$C_p = 4\ kJ/kg.K$

$$\dot{q} = 1000\frac{W}{m^3}$$

$$\frac{40}{1600 \times 4000}(-120) + \frac{1000}{1600 \times 4000} = \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = -5.94 \times 10^{-4}\ \frac{℃}{s}$$

• Heat rates entering, qin(x = 0), and leaving, qout(x = 1), the wall
• Rate of change of energy storage in the wall,
• Time rate of temperature change at x = 0.25 and 0.5 m

$$q_{in} = 140\ kW$$

$$q_{out} = 188\ kW$$

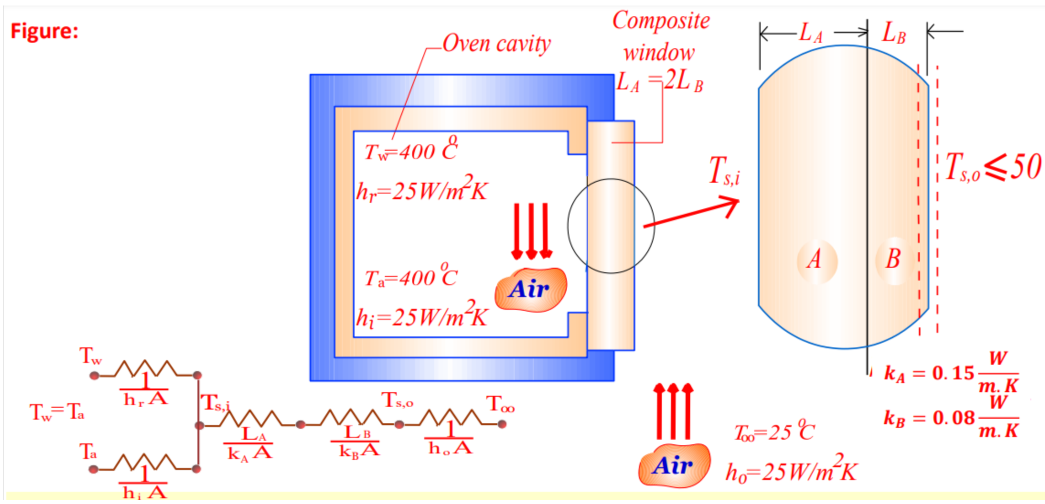
$$\dot{E}_{st} = -38\ kW$$

$$\frac{\partial T}{\partial t} = -5.94 \times 10^{-4}\ \frac{℃}{s}$$

Comments:

• From the above result it is evident that the temperature at every point within the wall is decreasing with time.
• Fourier's law can always be used to compute the conduction heat rate from knowledge of the temperature distribution, even for unsteady conditions with internal heat generation

Q2: A leading manufacturer of household appliances is proposing a self-cleaning oven design that involves use of a composite window separating the oven cavity from the room air. The composite is to consist of two high temperature plastics (A and B) of thicknesses $L_A = 2L_B$ and thermal conductivities $k_A = 0.15\ W\ m.K$ and $k_B = 0.08\ W\ m.K$. During the self-cleaning process, the oven wall and air temperatures,$Tw = Ta = 400$ ℃, while the room air temperature is 25 ℃. The inside convection and radiation heat transfer coefficients h_i and h_r , as well as the outside convection coefficient ho , are each approximately 25 W/m^2 .K. What is the minimum window thickness, L = $L_A + L_B$, needed to ensure a temperature that is 50 ℃ or less at the outer surface of the window? This temperature must not be exceeded for safety reasons



Known: The properties and relative dimensions of plastic materials used for a composite oven window, and conditions associated with self-cleaning operation

Find : Composite thickness $L_A + L_B$ needed to ensure safe operation

Assumptions:

• Steady state conditions exist

• Conduction through the window is one dimensional

• Contact resistance is negligible

• Radiation absorption within the window is negligible; hence no internal heat generation

• Radiation exchange between window outer surface and surroundings is negligible

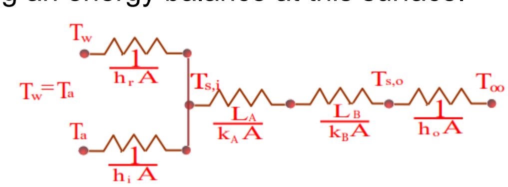
• Each plastic is homogeneous with constant properties

Analysis: The thermal circuit can be constructed by recognizing that resistance to heat flow is associated with convection at the outer surface, conduction in the plastics, and convection and radiation at the inner surface. Accordingly, the circuit and the resistances are of the following form. Since the outer surface temperature of the window, Ts,o is prescribed, the required window thickness may be obtained by applying an energy balance at this surface.

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\frac{T_a - T_{s,o}}{\sum R_t} = h_o A(T_{s,o} - T_\infty)$$



The total thermal resistance between the oven cavity and the outer surface of the window includes an effective resistance associated with convection and radiation, which act in parallel at the inner surface of the window, and the conduction resistances of the window materials. Hence,

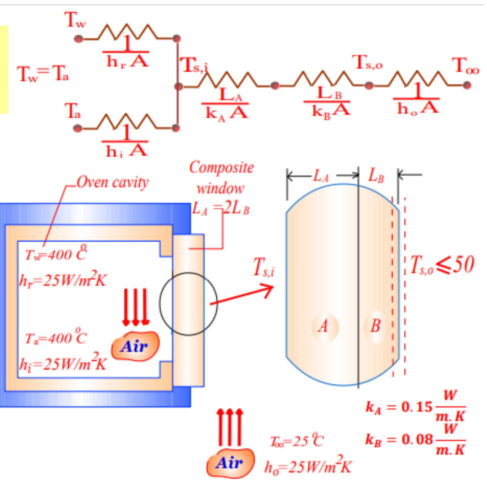
$$\sum R_t = \frac{1}{h_iA + h_rA} + \frac{L_A}{k_AA} + + \frac{L_B}{k_BA}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{1}{h_iA}} + \frac{1}{\frac{1}{h_rA}}$$

$$\frac{T_a - T_{s,o}}{\frac{1}{h_iA + h_rA} + \frac{L_A}{k_AA} + + \frac{L_B}{k_BA}} = h_oA(T_{s,o} - T_\infty)$$

$$\frac{1}{R} = h_iA + h_rA \qquad R = \frac{1}{h_iA + h_rA}$$

$$\frac{T_a - T_{s,o}}{\frac{1}{h_iA + h_rA} + \frac{L_A}{k_AA} + \frac{L_B}{k_BA}} = h_oA(T_{s,o} - T_\infty)$$



$$\frac{T_a - T_{s,o}}{\frac{1}{h_i + h_r} + \frac{L_A}{k_A} + \frac{L_B}{k_B}} = h_o(T_{s,o} - T_\infty)$$

$$L_A = 2L_B$$

$$\frac{T_a - T_{s,o}}{\frac{1}{h_i + h_r} + \frac{L_A}{k_A} + \frac{L_A}{2k_B}} = h_o(T_{s,o} - T_\infty)$$

$$\frac{400 - 50}{\frac{1}{25 + 25} + \frac{L_A}{0.15} + \frac{L_A}{2(0.08)}} = 25(50 - 25)$$

$$\frac{400 - 50}{25(50 - 25)} = \frac{1}{50} + \frac{L_A}{0.15} + \frac{L_A}{0.16} \qquad 0.54 = L_A\left(\frac{1}{0.15} + \frac{1}{0.16}\right) \qquad L_A = 0.0418m$$

$$L_A = 0.0418\ m \qquad L_A = 2L_B$$

$$L_B = 0.0209\ m$$

$$L = L_A + L_B = 0.0418 + 0.0209 = 0.0627\ m$$

$$L = 62.7\ mm$$

Comments:

1. The self cleaning operation is a transient process, as far as the thermal response of the window is concerned, and steady state conditions may not be reached in the time required for cleaning. However, the steady state condition provides the maximum possible value of Ts,o and hence is well suited for the design calculation.
2. Radiation exchange between the oven walls and the composite window actually depends on the inner surface temperature Ts,i , and although it has been neglected, there is radiation exchange between the window and the surroundings, which depends on Ts,o.

A more complete analysis may be made to concurrently determine Ts,i and Ts,o . Approximating the oven cavity as a large enclosure relative to the window and applying an energy balance, at the inner surface it follows that

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \qquad q''_{rad,i} + q''_{conv,i} = q''_{cond}$$

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B}}$$

Approximating the kitchen walls as a large isothermal enclosure relative to the window, with $T_{w,o} = T_\infty$ and this time applying energy balance at the outer surface, it follows that

$$q''_{rad,o} + q''_{conv,o} = q''_{cond}$$

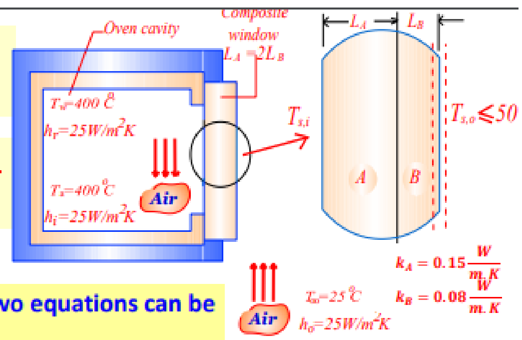
$$\varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right) = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B}}$$

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B}}$$

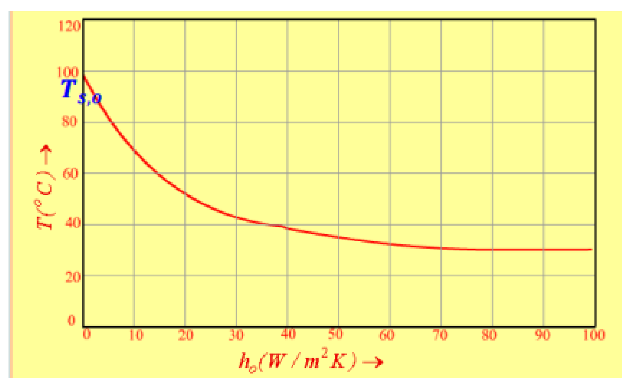
$$\varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right) = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B}}$$

$T_{w,o} = T_\infty$ $\quad$ $\varepsilon = 0.9$

As all other quantities are known, the above two equations can be $T_{s,i}$ and $T_{s,o}$.



We wish to explore the effect on $T_{s,o}$ of varying velocity, and hence the convection coefficient, associated with airflow over the outer surface. With ε = 0.9 and all other conditions remaining the same, the above equations have been solved for values of h_o in the range 0 - 100 W/m2 K and the results are represented graphically.



Increasing h_o reduces the corresponding convection resistance, and a value of h_o = **30** $W\ m2K$ would yield a safe to touch temperature of $T_{s,o}$ = **43**℃. Further increase in the outer surface heat transfer coefficient would not decrease the temperature. However, beyond h_o = **70** $W\ m2K$, the temperature of $T_{s,o}$ = **30**℃ would remain almost constant

Q3: Derive Heat Diffusion equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k.r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k.r\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q_V=\rho c_p\frac{\partial T}{\partial t}$$

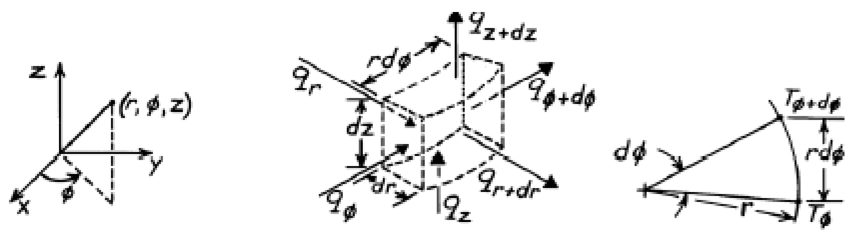
where

k is the materials conductivity [W.m^{-1}.K^{-1}]

q_V is the rate at which energy is generated per unit volume of the medium [W.m^{-3}]

ρ is the density [kg.m^3]

c_p is the specific heat capacity [J.kg^{-1}.K^{-1}]



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Consider the differential control volume identified above having a volume given as V = dr·rdφ·dz. From the conservation of energy requirement,

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_z - q_{z+dz} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}(dr\cdot rd\phi\cdot dz) \qquad \dot{E}_g = \rho Vc\,\partial T/\partial t = \rho(dr\cdot rd\phi\cdot dz)c\,\partial T/\partial t. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi, \quad q_{z+dz} = q_z + \frac{\partial}{\partial z}(q_z)dz. \qquad (4,5,6)$$

Using Fourier's law, the expressions for the conduction heat rates are

$$q_r = -kA_r\partial T/\partial r = -k(rd\phi\cdot dz)\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\partial\phi = -k(dr\cdot dz)\partial T/r\partial\phi \qquad (8)$$

$$q_z = -kA_z\partial T/\partial z = -k(dr\cdot rd\phi)\partial T/\partial z. \qquad (9)$$

Note from the above, right schematic that the gradient in the φ-direction is ∂T/r∂φ and not ∂T/∂φ. Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1),

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial z}(q_z)dz + \dot{q}\,dr\cdot rd\phi\cdot dz = \rho(dr\cdot rd\phi\cdot dz)c\frac{\partial T}{\partial t}. \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k(rd\phi\cdot dz)\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k(drdz)\frac{\partial T}{r\partial\phi}\right]d\phi - \frac{\partial}{\partial z}\left[-k(dr\cdot rd\phi)\frac{\partial T}{\partial z}\right]dz$$

$$+\dot{q}\,dr\cdot rd\phi\cdot dz = \rho(dr\cdot rd\phi\cdot dz)c\frac{\partial T}{\partial t}. \qquad (11)$$

Dividing Eq. (11) by the volume of the CV, Eq. 2.24 is obtained.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t} \qquad <$$

Q4: Derive Heat Diffusion equation in spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(k.r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\Phi}\left(k\frac{\partial T}{\partial\Phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k.\sin\theta\frac{\partial T}{\partial\theta}\right)+q_v=\rho c_p\frac{\partial T}{\partial t}$$

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is $V = dr \cdot r\sin\theta d\phi \cdot r d\theta$, and the conduction terms are identified in Figure 2.13. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \tag{1}$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right] \qquad \dot{E}_{st} = \rho V c\frac{\partial T}{\partial t} = \rho\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right]c\frac{\partial T}{\partial t}. \tag{2,3}$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \tag{4,5,6}$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k\left[r\sin\theta d\phi\cdot r d\theta\right]\partial T/\partial r \tag{7}$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k\left[dr\cdot r d\theta\right]\partial T/r\sin\theta\partial\phi \tag{8}$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k\left[dr\cdot r\sin\theta d\phi\right]\partial T/r\partial\theta. \tag{9}$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right] = \rho\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right]c\frac{\partial T}{\partial t} \tag{10}$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k\left[r\sin\theta d\phi\cdot r d\theta\right]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k\left[dr\cdot r d\theta\right]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k\left[dr\cdot r\sin\theta d\phi\right]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right] = \rho\left[dr\cdot r\sin\theta d\phi\cdot r d\theta\right]c\frac{\partial T}{\partial t} \tag{11}$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.27 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

Q5: A steel tube (k = 46 W/m-K) has an inside diameter of 3 cm and a wall thickness of 2mm. A fluid flows inside the tube producing a convection heat transfer coefficient of 1500 W/m2-K on the inside surface while on the outside surface a second fluid flows producing a heat transfer coefficient of 200 W/m2-K on the outer surface of the tube. The inside fluid is at 223 ℃ while the outside fluid is at 57 ℃. Calculate the overall heat transfer coefficient based on the tube inside area and also based on the tube outside area. Also, calculate the heat loss per unit length of the tube. **(1D, Steady state conduction with no internal generation in cylinder)**

Steel Tube : $k = 46\ W/m\text{-}K$

$D_i = 3\cancel{0}\ cm = 0.03\ m$ Wall thickness 't' $= 0.2\ cm = 2 \times 10^{-3}\ m$

$D_o = D_i + 2t = 3.4 \times 10^{-2}\ m = 0.034\ m$

$h_i = 1500\ W/m^2 K$ $T_{\infty_i} = 223\ °C$

$h_o = 200\ W/m^2 K$ $T_{\infty_o} = 57\ °C$

Find :- U_i, U_o and q' (q/L)

Assumptions :-

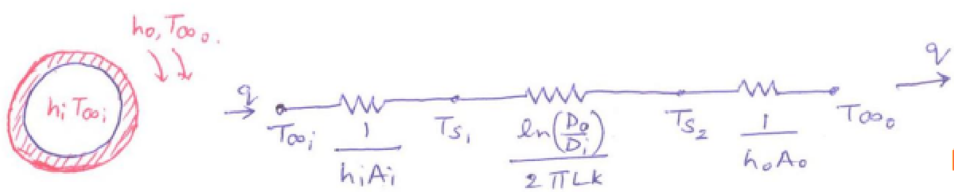
1. Steady state
2. 1-D radial conduction
3. No. volumetric heat generation

(4) Uniform h_i and h_o values

(5) Negligible radiation H.T.

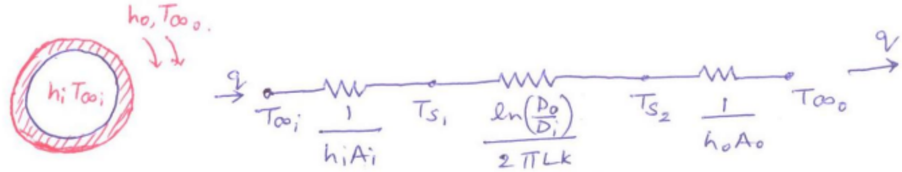
(6) Constant Thermal conductivity & other properties

Schematic :-



Probl

Schematic :-



where $A_i = 2\pi r_i L = \pi D_i L$ & $A_o = \pi D_o L$

Calculation of individual resistances.

$A_i = \pi D_i L = 0.09425L$
$A_o = \pi D_o L = 0.10681\,L$

$$\frac{1}{h_iA_i} = \frac{1}{1500 \times 0.09425L} = \frac{7.073}{(10^3)L}\ \frac{K}{W}$$

$$\frac{1}{h_oA_o} = \frac{1}{200 \times 0.10681\,L} = \frac{0.04681}{L}\ \frac{K}{W}$$

$$\frac{\ln\left(D_o/D_i\right)}{2\pi Lk} = \frac{\ln\left(\frac{34}{30}\right)}{2\pi(46)L} = \frac{4.3305}{(10^4)L}\ \frac{K}{W}.$$

Problem 8

$$\sum R = \frac{1}{h_iA_i} + \frac{\ln(D_o/D_i)}{2\pi Lk} + \frac{1}{h_oA_o} \qquad (6/8)$$

$$= \frac{7.073 \times 10^{-3}}{L} + \frac{4.3305 \times 10^{-4}}{L} + \frac{0.04681}{L}$$

$$= \frac{0.0543}{L}\ K/W$$

$$\sum \text{Resistance} = \frac{0.0543}{L}\ K/W$$

Now $U_iA_i = U_oA_o = \frac{1}{\sum R_{th}} = \frac{1}{\left(\frac{0.0543}{L}\right)} = (18.41)L\ \frac{W}{K}$

Now $U_i = \frac{18.41\cancel{L}}{0.09425\cancel{L}} = \boxed{195.33\ W/m^2K}$

$U_o = \frac{18.41\,L}{0.10681\,L} = \boxed{172.36\ W/m^2K}$

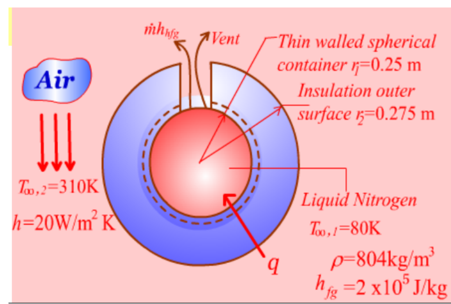
$$q = \frac{\Delta T}{\Sigma R_{th}} = \frac{T_{\infty i} - T_{\infty o}}{\Sigma R_{th}} = \frac{223-57}{\left(\frac{0.0543}{L}\right)}$$

$$\therefore q' = \frac{166}{0.0543} = \boxed{3057.1 \text{ W/m}}$$

The overall heat transfer coefficient is not a constant, and $U_i > U_o$ as expected.

The outside convection resistance is the dominant one.

Q6: A spherical thin walled metallic container is used to store liquid nitrogen at 80 K. The container has a diameter of 0.5 m and is covered with an evacuated, reflective insulation composed of silica powder. The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 310 K. The convection coefficient is known to be 20 W/m2 K. The latent heat of vaporization and the density of the liquid nitrogen are 2×10^5 J/kg and 804 kg/m3 , respectively. Thermal conductivity of evacuated silica powder (300 K) is 0.0017 W/mK. What is the rate of heat transfer to the liquid nitrogen? What is the rate of liquid boil-off? **(1D, Steady state conduction with no internal generation in sphere)**



Known: Liquid nitrogen is stored in spherical container that is insulated and exposed to ambient air.

Find:

• The rate of heat transfer to the nitrogen.

• The mass rate of nitrogen boil-off.

Assumptions:

1. Steady state conditions and one dimensional transfer in the radial direction
2. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen
3. Constant properties
4. Negligible radiation exchange between outer surface of insulation and surroundings

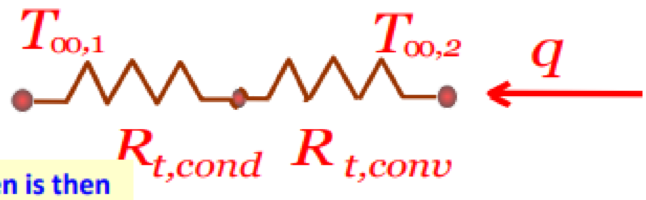
Analysis:

1. The thermal circuit involves a conduction and convection resistance in series and is of the form

$$R_{t,cond}=\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}$$

$$R_{t,conv}=\frac{1}{hA}=\frac{1}{h(4\pi r_2^2)}$$

The rate of heat transfer to the nitrogen



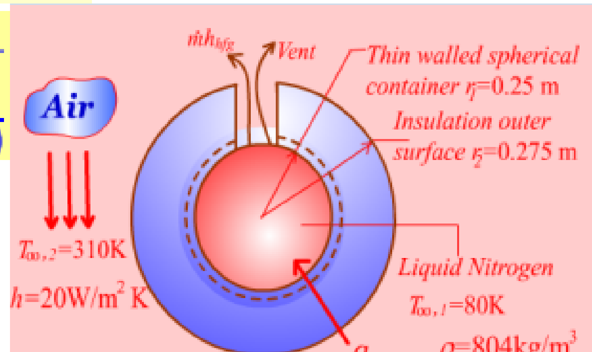
The rate of heat transfer to the liquid nitrogen is then

$$q_r=\frac{T_{\infty,2}-T_{\infty,1}}{R_{t,cond}+R_{t,conv}}=\frac{T_{\infty,2}-T_{\infty,1}}{\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}+\frac{1}{h(4\pi r_2^2)}}$$

$$q_r=\frac{310-80}{\frac{\left(\frac{1}{0.25}-\frac{1}{0.275}\right)}{4\pi(0.0017)}+\frac{1}{20(4\pi(0.275)^2)}}$$

$$q_r=\frac{230}{17.02+0.05}\qquad q_r=13.47\ W$$

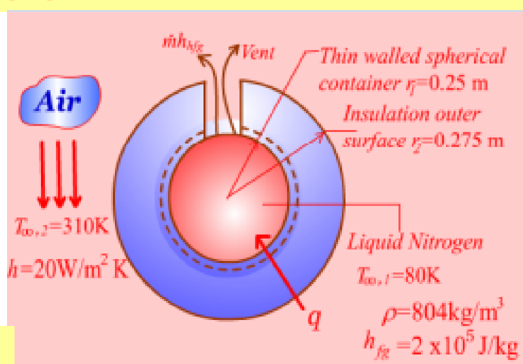
$$R_{t,cond}\gg R_{t,conv}$$



$$k=0.0017\frac{W}{m.k}$$

The rate of liquid boil off

Performing an energy balance for a control surface about the nitrogen



$$\dot{E}_{in}+\dot{E}_g-\dot{E}_{out}=\dot{E}_{st}$$

$$\dot{E}_{in}=\dot{E}_{out}\qquad q_r=13.47\ W$$

$$q_r=\dot{m}h_{fg}\qquad 13.47=\dot{m}(2\times10^5)$$

$$\dot{m}=6.74\times10^{-5}\frac{kg}{s}$$

$$\dot{m}=6.74\times10^{-5}\frac{kg}{s}\times3600\frac{s}{hour}\times24\frac{hours}{day}$$

$$\dot{m}=5.82\frac{kg}{day}$$

$$V=\frac{\dot{m}}{\rho}=\frac{5.82\frac{kg}{day}}{804\frac{kg}{m^3}}=0.0724\frac{m^3}{day}=7.24\frac{litres}{day}$$

With a container volume of $\frac{4}{3}(0.25)^3=0.065\ m^3=65$ litres, the daily loss is $(7.24/$

Q7: A 3 mm diameter and 6 m long electric wire is tightly wrapped with a 2 mm thick plastic cover whose thermal conductivity is k = 0.15 W/m.°C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at 27 °C with a heat transfer coefficient of h=12 W/m2°C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature. (**Critical Radius of Insulation**)

Known : Size of the electric wire, thermal conductivity of the wire, current and voltage supplied to the wire, ambient conditions and heat transfer coefficient.
Find : Convection heat transfer coefficient between the outer surface of the wire and the air in the room.
1. Heat transfer is steady since there is no indication of any change with time.
2. Heat transfer is one dimensional since there is thermal symmetry about the center line and no variation in the axial direction.
3. Thermal conductivities are constant.
4. The thermal contact resistance at the interface is negligible.
5. Heat transfer coefficient incorporates the radiation effects, if any.

Analysis: Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heating is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined from

$$\dot{Q} = \mathrm{VI} = 8 \times 10 = 80W$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. The values of these two resistances are determined to be

$$A_2 = 2\pi r_2 L = 2\pi \times 3.5 \times 10^{-3}(6) = 0.132\ m^2$$

3 mm diameter and 2 mm thick plastic cover

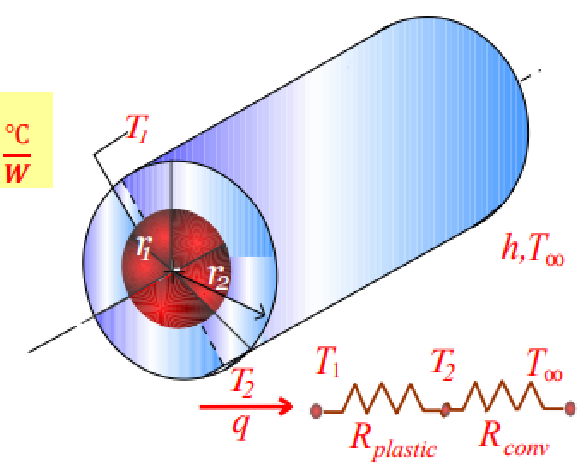
$$R_{conv} = \frac{1}{hA_2} = \frac{1}{12 \times 0.132} = 0.63\,\frac{℃}{W}$$

$$R_{platic} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk} = \frac{ln\left(\frac{3.5}{1.5}\right)}{2\pi(0.15)6} = 0.15\,\frac{℃}{W}$$

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$80 = \frac{T_1 - 27}{0.63 + 0.15}$$

$$T_1 = 89.4℃$$

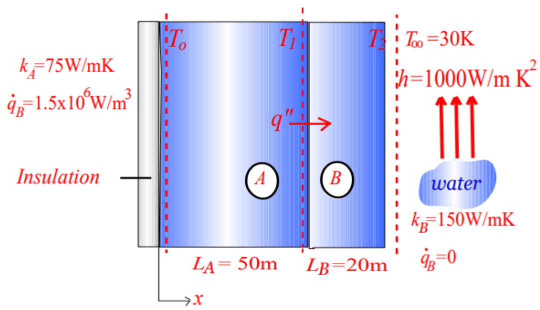


Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation. To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover.

$$r_{cr} = \frac{k}{h} = \frac{0.15}{12} = 0.0125m = 12.5\ mm$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will enhance heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer will increase when the interface temperature T1 is held constant, or T1 will decrease when is held constant, which is the case here.

Q8: A plane wall is a composite of two materials, A and B. The wall of material A has uniform heat generation $\dot{q} = 1.5\times10^6\ W/m^3$ $k_A = 75\ W\ m.K$ and thickness $L_A = 50\ mm$. The wall material B has no generation with $k_B = 150\ W\ m.K$ and thickness $L_B = 20\ mm$. The inner surface of material A is well insulated, while the outer surface of material B is cooled by water stream $T_\infty = 30$ ℃ and $h = 1000\ W/m2\ .\ K$. Sketch the temperature distribution that exists in the composite under steady state conditions. Determine the temperature T_o of the insulated surface and the temperature T_2 of the cooled surface. (**Rectangle Composite Stack with 1D Heat Generation**)

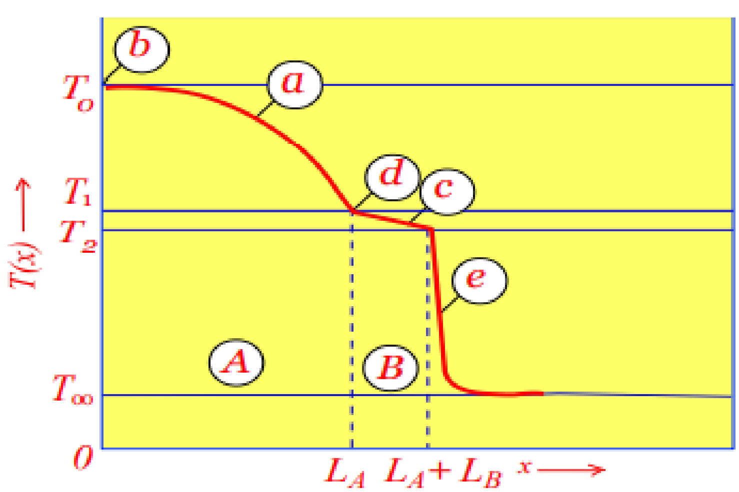


Known : Plane wall of material A with internal heat generation is insulated on one side and bounded by a second wall of material B, which is without heat generation and is subjected to convection cooling

Find : Convection heat transfer coefficient between the outer surface of the wire and the air in the room.

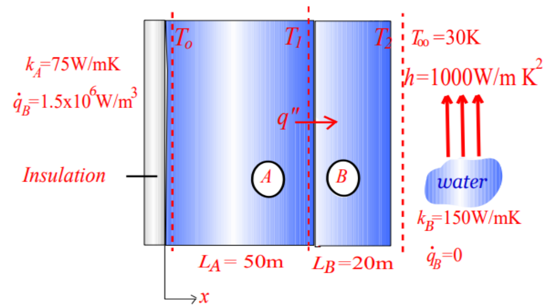
Assumptions:

1. Heat transfer is steady since there is no indication of any change with time.
2. Heat transfer is one dimensional since there is thermal symmetry about the center line and no variation in the axial direction.
3. Thermal conductivities are constant.
4. The thermal contact resistance at the interface is negligible.
5. Heat transfer coefficient incorporates the radiation effects, if any



From the prescribed physical conditions, the temperature distribution in the composite is known to have the following features, as shown:

- Parabolic in material A
- Zero slope at insulated boundary
- Linear in material A
- The temperature distribution in the water is characterized by large gradients near the surface
- Slope change = kA / kB = 2 at interface



The outer surface temperature ***T*2** may be obtained by performing an energy balance on the control volume about material B. Since there is no generation in this material, it follows that, for steady state conditions and a unit surface area, the heat flux into the material at $x = LA$ must equal the heat flux from the material due to convection at $x = LA + LB$

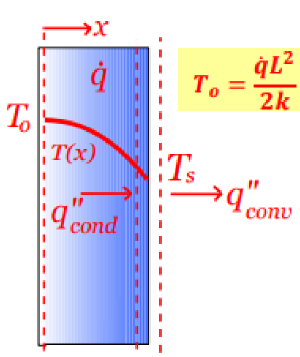
$$q'' = h(T_2 - T_\infty)$$

The heat flux may be determined by performing a second energy balance on a control volume about material A. In particular, since the surface at $x = 0$ is adiabatic, there is no inflow and the rate at which the energy is generated must equal the outflow. Accordingly, for a unit surface area,

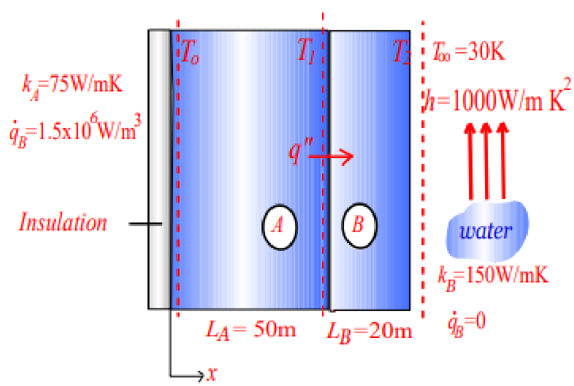
$$\dot{q}L_A = q'' \qquad q'' = \dot{q}L_A = 1.5\times10^6(0.05) = 75000\,W/m^2 \qquad q'' = 75000\,W/m^2$$

$$\dot{q}L_A = h(T_2 - T_\infty) \qquad T_2 = T_\infty + \frac{\dot{q}L_A}{h} \qquad T_2 = 30 + \frac{1.5\times10^6(0.05)}{1000} = 105 \qquad T_2 = 105℃$$

Adiabatic Surface At Midplane



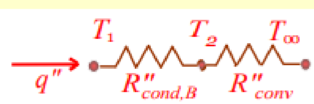
$$T_o = \frac{\dot{q}L^2}{2k} + T_s$$

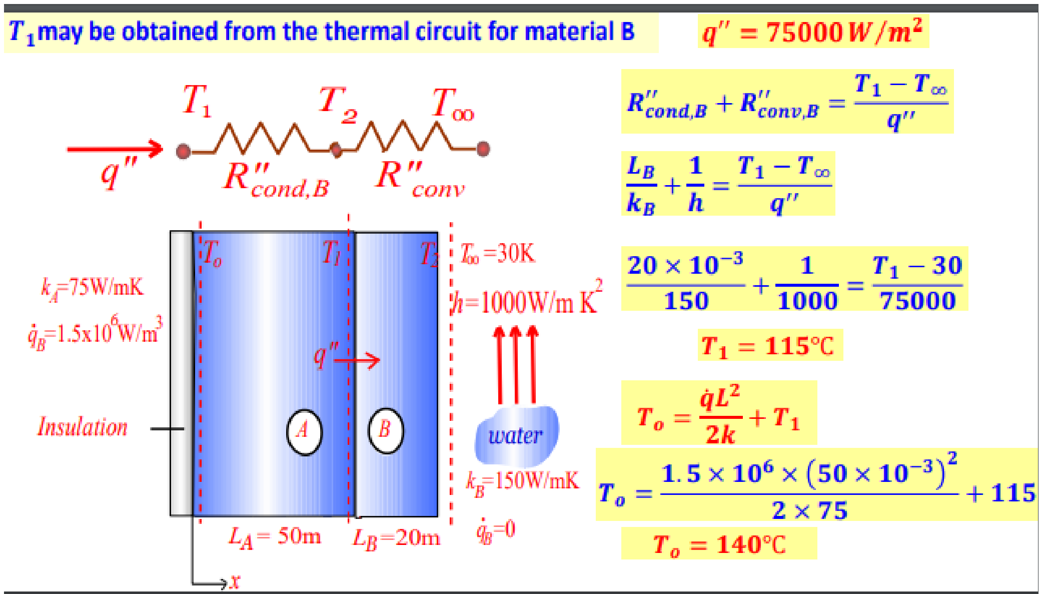


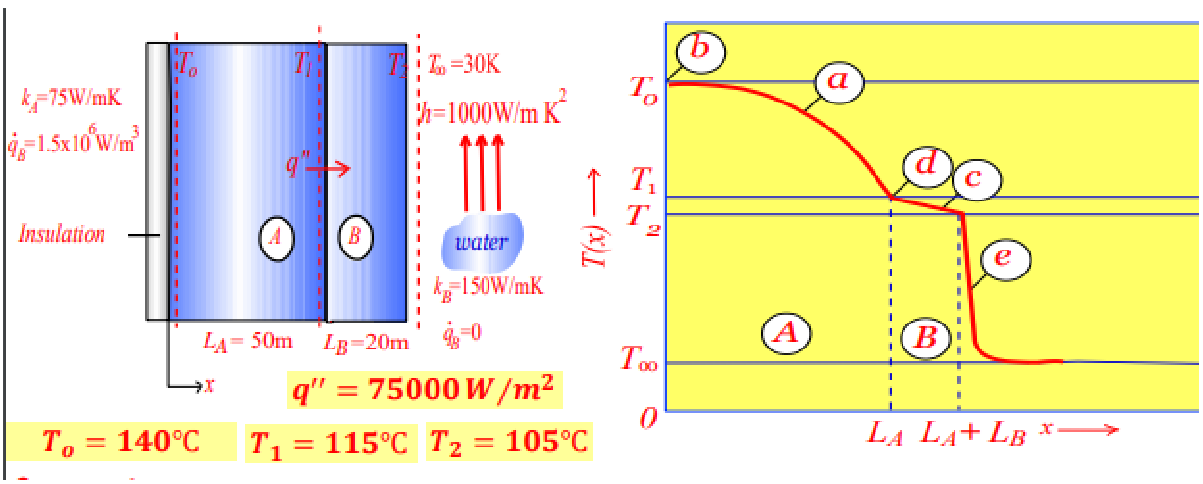
Temperature at the insulated surface $(x = 0)$

$$T_o = \frac{\dot{q}L^2}{2k} + T_1$$

T_1 may be obtained from the thermal circuit for material B







Comments:

1. Material A, having heat generation, cannot be represented by a thermal circuit element
2. Since the resistance to heat transfer by convection is significantly larger than due to conduction in material B, R T $cond,B$ " $Rconv,B$ " = 7. 5 , the surface-to-fluid temperature difference is much larger than the temperature drop across material B, This result is consistent with the temperature distribution plotted in part (1).

Q9:

3. An experimental boiling device consists of a semiconductor slab of thickness L_1 and thermal conductivity k_1, perfectly insulated on its backside, which is covered with a second slab of a thermally conducting, electrically insulating ceramic of thickness L_2 and conductivity k_2, pressed upon which is a third material slab of thickness L_3 and conductivity k_3 (See Figure). Heat is generated at a uniform rate q"' (W/m3) in slab by passing electrical current through the slab. The h eat is conducted through the other two slabs and dissipated by boiling saturated liquid (at T_{sat}) on the exposed surface of the metallic wall. The boiling heat flux is known to follow the relation

$q'' = C\,(T_{wall} - T_{sat})^3$ where C is a known empirical constant and T_{wall} is the temperature of the boiling surface.

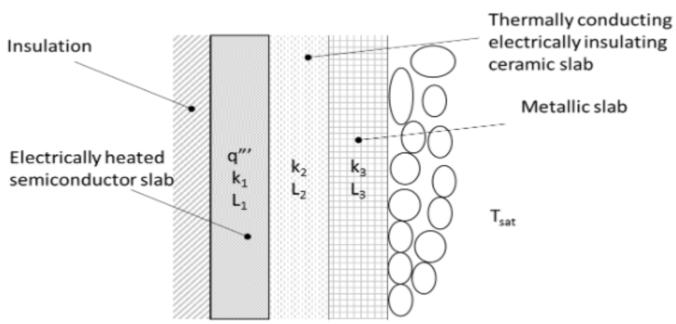
(a) Derive an expression for T_{wall} in terms of q"', C, T_{sat} and the thicknesses and thermal conductivities of the slabs

(b) Derive an expression for the highest temperature in the boiling device

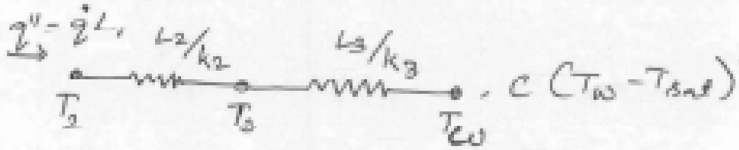
Hints:

i) Thermal resistance per unit area for a plane wall of thickness L and conductivity is L/k

ii) The steady, one-dimensional heat diffusion equation for a medium of conductivity k is



③ ⓐ Heat transfer ~~between~~ across the ceramic slab and metallic slab is 1-d and can be represented by ~~thermal~~ Thermal resistance network



$$\therefore \quad q'' = \dot{q}L_1 = c\,(T_w - T_{sat})^3$$

$$\Rightarrow T_w = \left[\frac{\dot{q}L_1}{c}\right]^{1/3} + T_{sat}$$

(b) $\dot{q} L_1 = \dfrac{T_2 - T_\omega}{\dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}} \Rightarrow T_2 = \dot{q} L_1 \left[\dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}\right] + T_\omega$

$$\Rightarrow T_2 = \dot{q} L_1 \left[\frac{L_2}{k_2} + \frac{L_3}{k_3}\right] + \left[\frac{\dot{q} L_1}{C}\right]^{1/3} + T_{ext}.$$

Highest temp. is incurred at the insulating surface, i.e, $T_{max} = T_1$. Across the semi-conductor slab

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k_1} = 0. \Rightarrow \frac{dT}{dx} = -\frac{\dot{q}}{k_1} x + C_1.$$

B.C. $\left.\dfrac{dT}{dx}\right|_{x=0} = 0 \Rightarrow C_1 = 0. \Rightarrow T = -\dfrac{\dot{q} x^2}{2k_1} + C_2$

B.C. $T|_{x=L_1} = T_2 \Rightarrow T_2 = -\dfrac{\dot{q} L_1^2}{2k_1} + C_2$

$$\Rightarrow C_2 = T_2 + \frac{\dot{q} L_1^2}{2k_1}$$

$\therefore T = \dfrac{\dot{q}}{2k_1}(L_1^2 - x^2) + T_2$

T_{max} at T_1, $\boxed{T_1 = \dfrac{\dot{q} L_1^2}{2k_1} + T_2}$

$$\therefore \boxed{T_{max} = \frac{\dot{q} L_1^2}{2k_1} + \dot{q} L_1 \left[\frac{L_2}{k_2} + \frac{L_3}{k_3}\right] + \left[\frac{\dot{q} L_1}{C}\right]^{1/3} + T_{ext}}$$