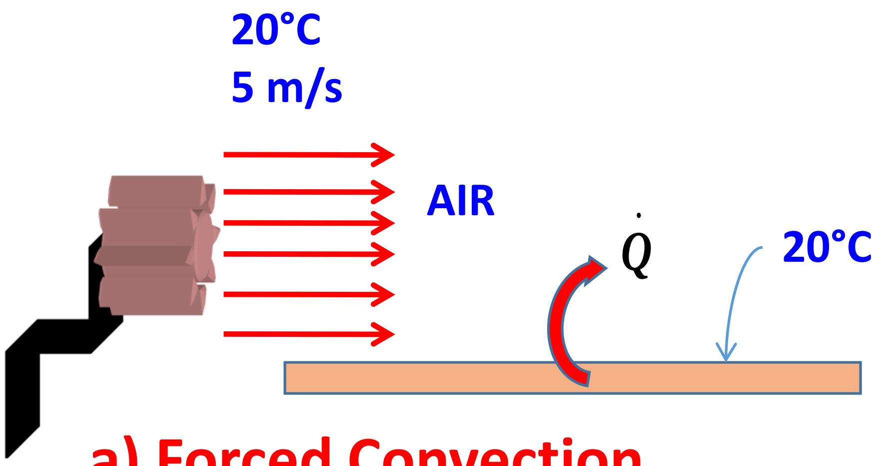
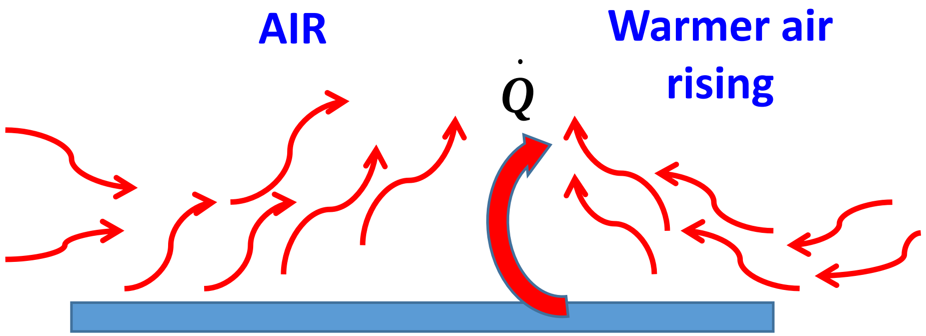
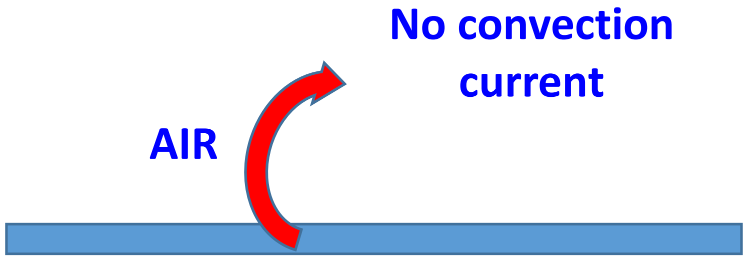
CONVECTIVE HEAT TRANSFER - CONVECTION 1



a) Forced Convection



b) Free Convection



c) Conduction

Convection heat transfer involves

- fluid motion
- heat conduction

The fluid motion enhances the heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in fluid.

Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction.

In fact, the higher the fluid velocity, the higher the rate of heat transfer.

Convection heat transfer strongly depends on

- fluid properties dynamic viscosity μ, thermal conductivity k, density and specific heat C_p
- fluid velocity V
- Geometry and the roughness of the solid surface
- Type of fluid flow (such as being laminar or turbulent).

NEWTON'S LAW OF COOLING

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

h = Convection heat transfer coefficient

A_s = Heat transfer surface area

T_s = Temperature of the surface

T_∞ = Temperature of the fluid sufficiently far from the surface

LOCAL HEAT FLUX q''_{conv}

$$q''_{conv} = h_l(T_s - T_\infty)$$

h_l is the local convection coefficient

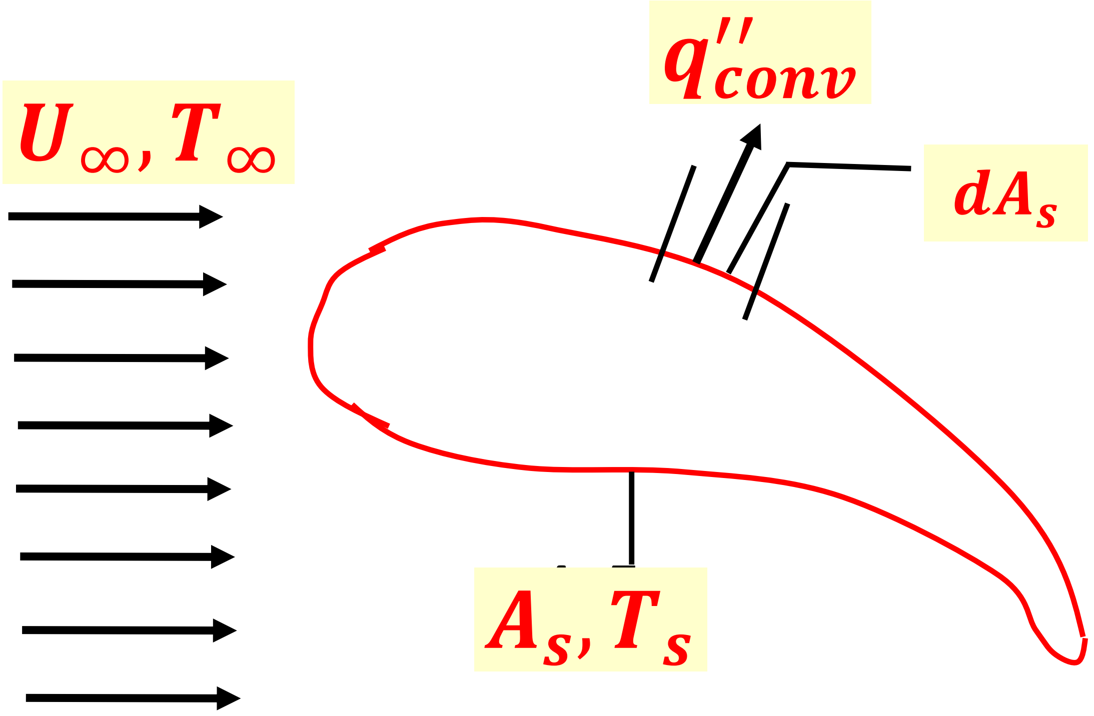
TOTAL HEAT TRANSFER RATE $\dot{Q}_{conv}$

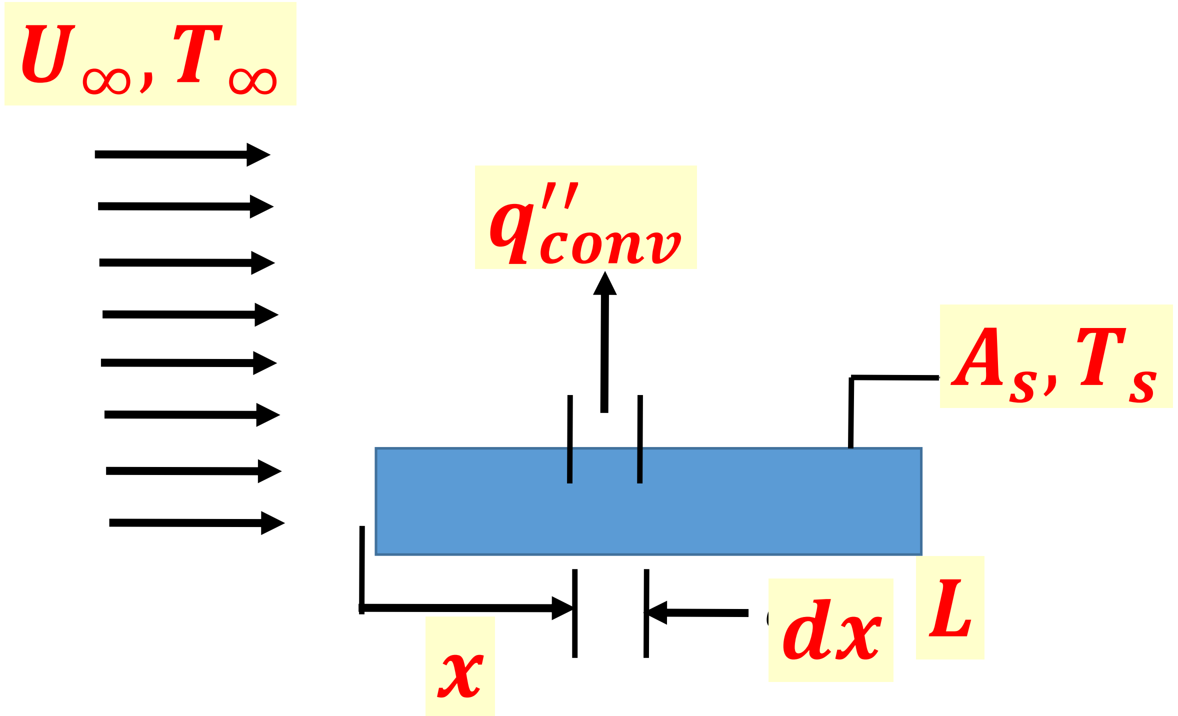
$$\dot{Q}_{conv} = \int_{A_s} q''_{conv}\, dA_s$$

$$q''_{conv} = h_l(T_s - T_\infty)$$

$$\dot{Q}_{conv} = (T_s - T_\infty) \int_{A_s} h_l\, dA_s$$

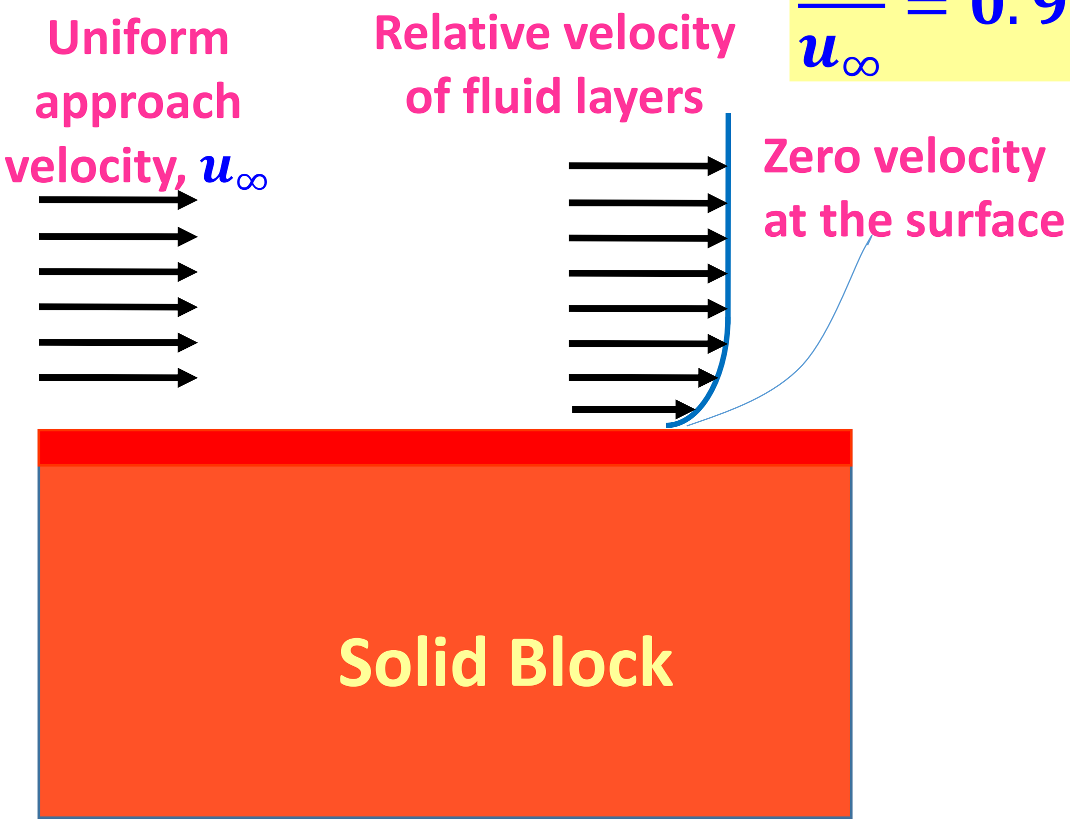
Local and total convection transfer (a) Surface of arbitrary shape. (b) Flat plate.





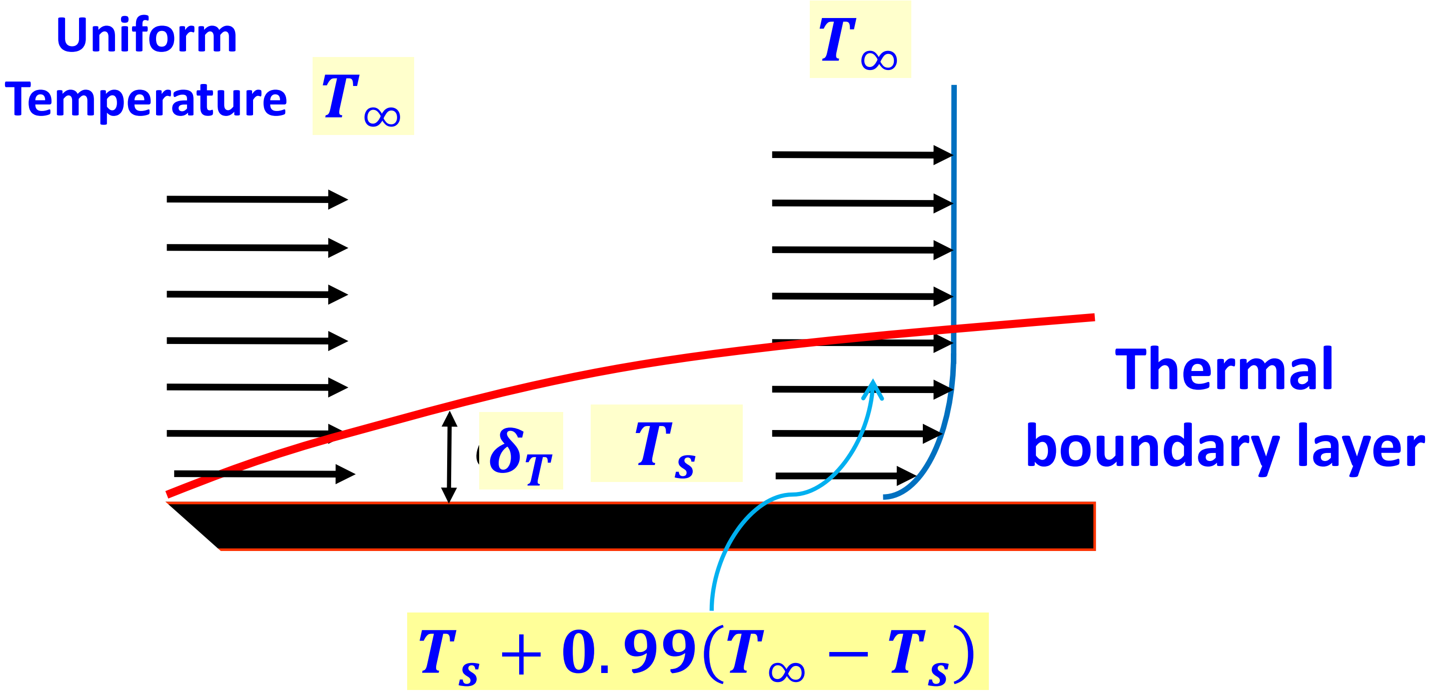
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

$$\frac{u}{u_\infty} = 0.99$$



Free Stream

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$



A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the point of contact.

Therefore, a fluid and a solid surface will have the same temperature at the point of contact. This is known as NO-TEMPERATURE-JUMP CONDITION.

An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction*, since the fluid layer is motionless,

$$\dot{q}''_{conv} = \dot{q}''_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

T **represents the temperature distribution in the fluid**

$\left.\frac{\partial T}{\partial y}\right|_{y=0}$ **is the temperature gradient at the surface.**

$$q''_{conv} = h_l (T_s - T_\infty)$$

$$h_l = \frac{-k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

Problem: Experimental results for the local heat transfer heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = x^{-0.1}$$

where x (m) is the distance form the leading edge of the plate.

Develop an expression for the ratio of the average heat transfer coefficient for a plate of length x to the local heat transfer coefficient h_x at x. Show, in a quantitative manner, the variation of h_x and as a function of x.

SOLUTION:

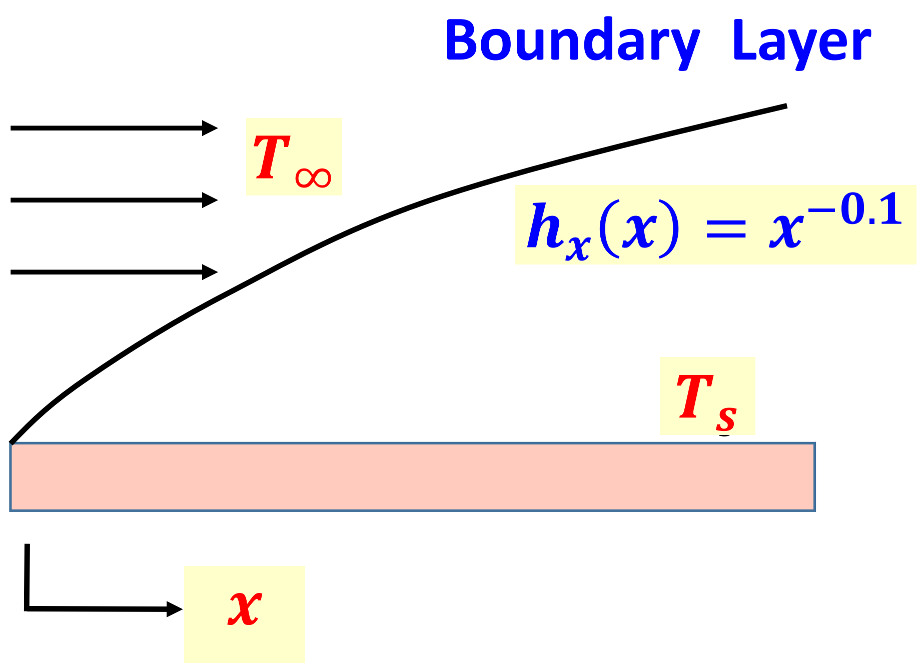
Known: Variation of the local heat transfer coefficient, $h_x(x)$.

Find:

The ratio of the average heat transfer coefficient ($\bar{h}_x$) to the local value $h_x(x)$.

sketch of the variation of h_x and with x.

Schematic:

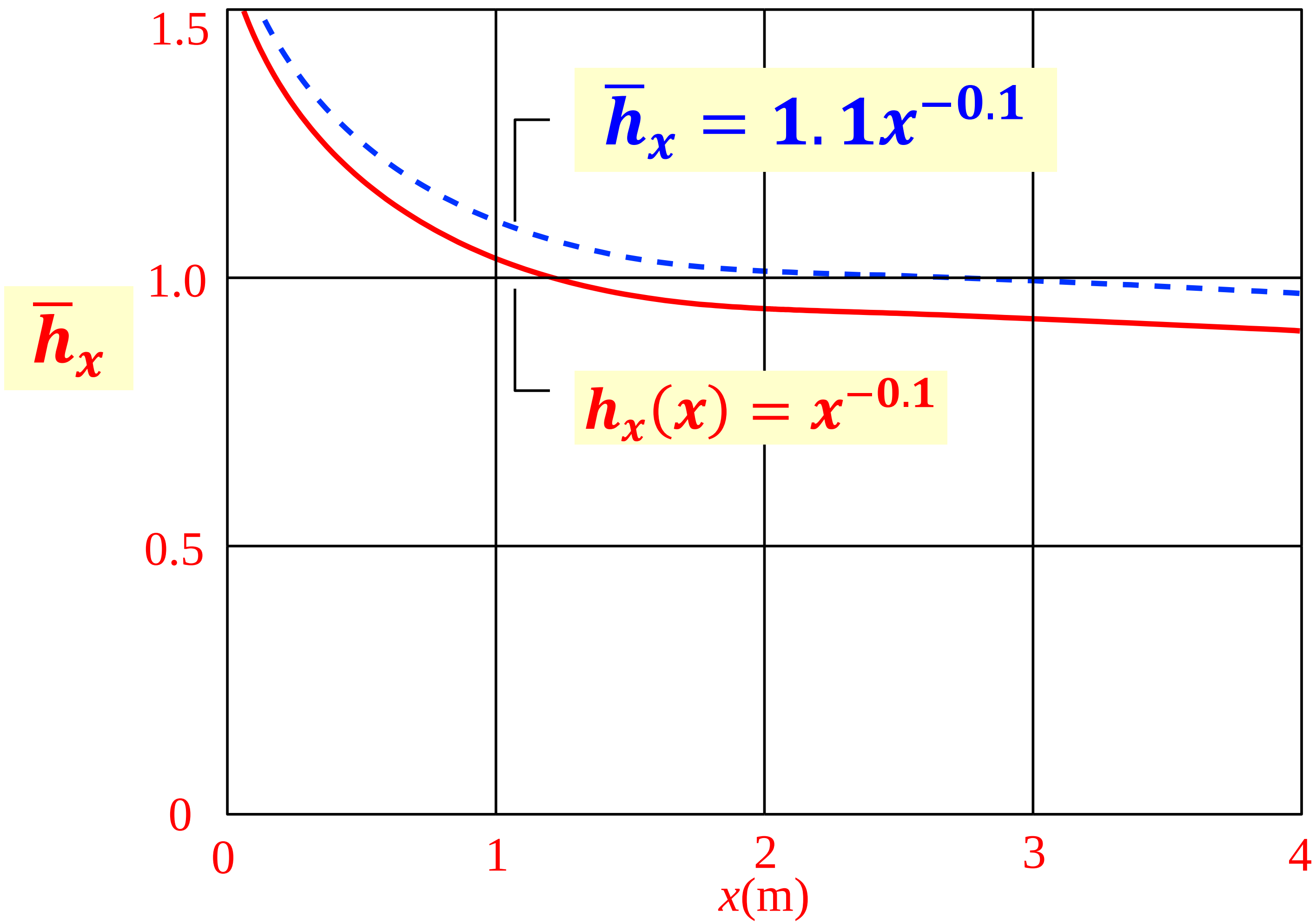


Analysis: Average value of the convection heat transfer coefficient over the region from 0 to x

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x(x)dx$$

$$h_x(x) = x^{-0.1}$$

$$\bar{h}_x = \frac{1}{x}\int_0^x x^{-0.1}dx = \frac{1}{x}\left(\frac{x^{0.9}}{0.9}\right) = 1.1x^{-0.1}$$



Comments:

Boundary layer development causes both the local and average coefficients to decrease with increasing distance from the leading edge. The average coefficient up to x must therefore exceed the local value at x.

NUSSELT NUMBER

$$Nu = \frac{hL_c}{k_f}$$

k_f is the thermal conductivity of the fluid

L_c is the characteristic length

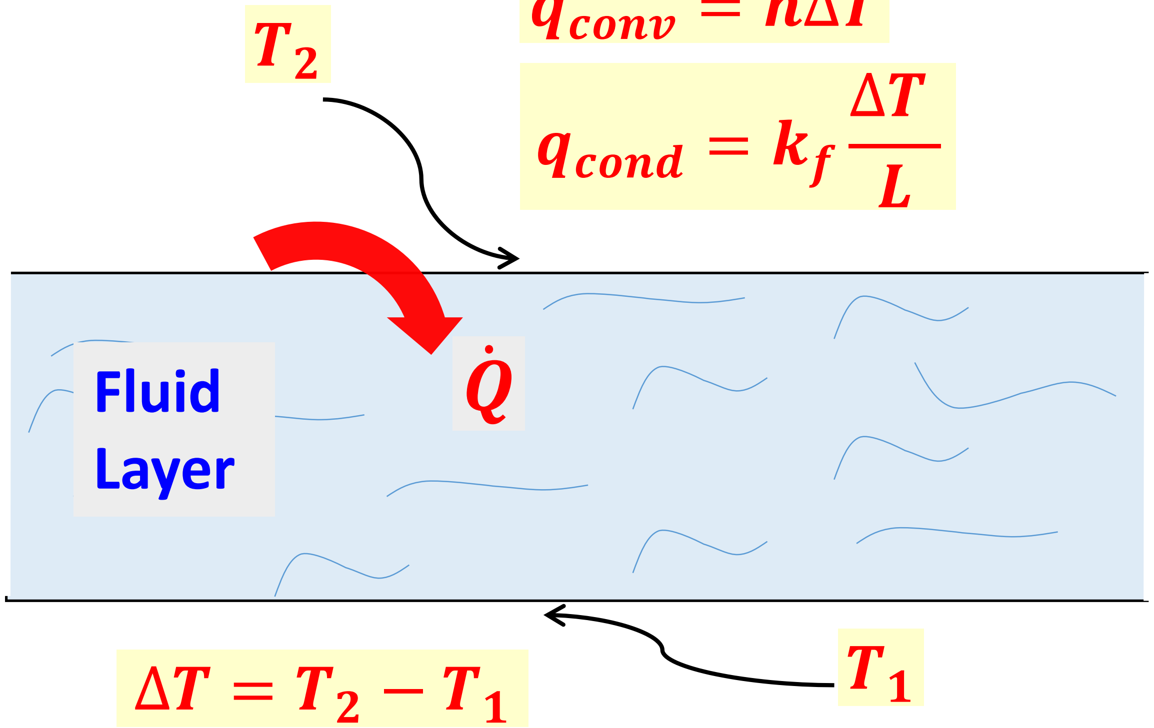
Heat transfer through the fluid layer will be by convection when the fluid involves some motion and by conduction when the fluid layer is motionless.

$$\frac{q_{conv}}{q_{cond}} = \frac{h\Delta T}{k_f \frac{\Delta T}{L}} = \frac{hL}{k_f} = Nu$$

Heat transfer through a fluid layer of thickness *L* and temperature difference

$$q_{conv} = h\Delta T$$

$$q_{cond} = k_f \frac{\Delta T}{L}$$



Nusselt number - enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

Larger the Nusselt number, the more effective the convection.

$Nu = 1$ for a fluid layer - heat transfer across the layer by pure conduction

WILHEM NUSSELT (1882-1957)

- German engineer
- Doctoral thesis – CONDUCTIVITY OF INSULATING MATERIALS
- Professor – HEAT AND MOMENTUM TRANSFER IN TUBES
- 1915 – pioneering work in BASIC LAWS OF TRANSFER
- DIMENSIONLESS GROUPS – SIMILARITY THEORY OF HEAT TRANSFER
- FILM CONDENSATION OF STEAM ON VERTICAL SURFACES
- COMBUSTION OF PULVERISED COAL
- ANALOGY OF HEAT TRANSFER & MASS TRANSFER IN EVAPORATION

Professor at Technical university of Karlsruhe - 1920 -1925

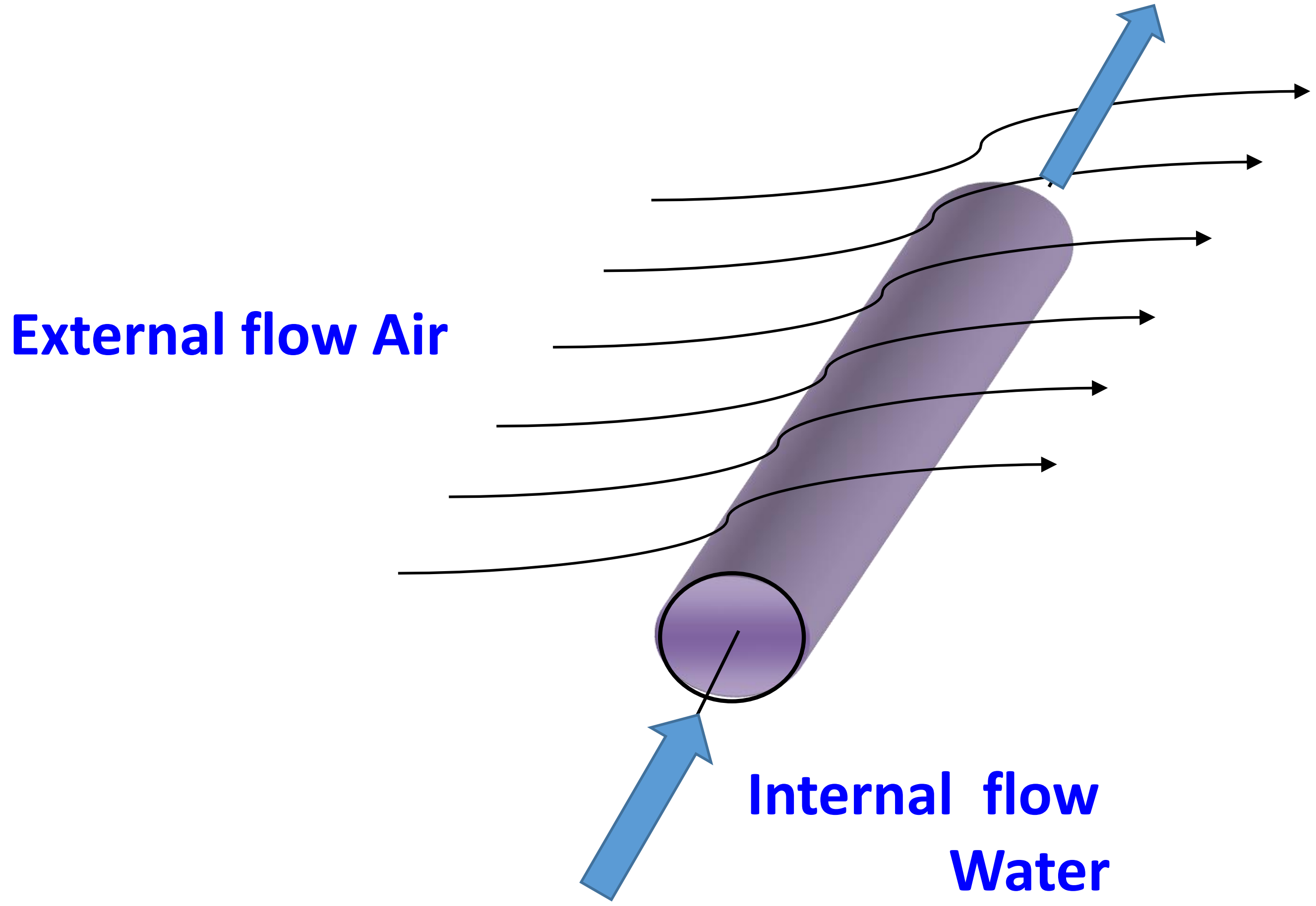
Professor at Technical university of Munchen - 1926 -1952

Worked till the age of 70 years. Lived for 75 years and died in Munchen on September 1, 1957.

INTERNAL AND EXTERNAL FLOWS

EXTERNAL FLOW - **The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe**

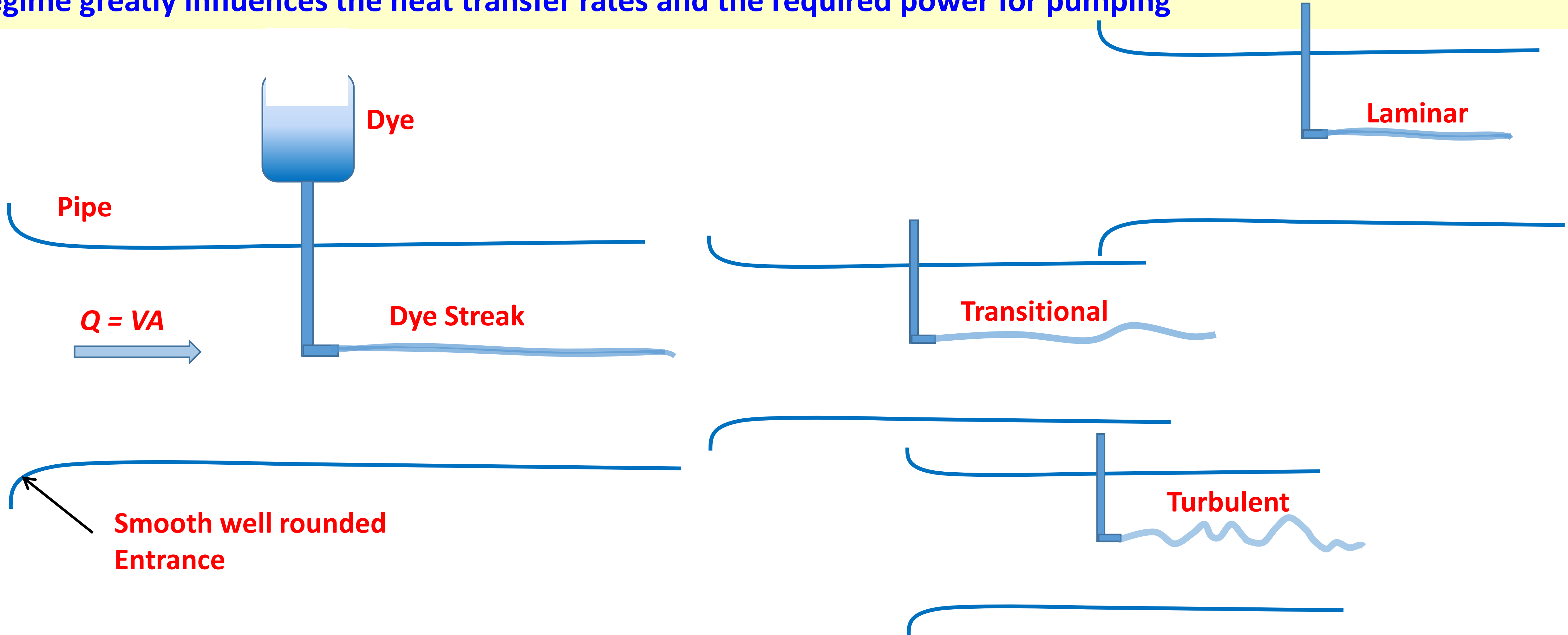
INTERNAL FLOW - **flow in a pipe or duct, if the fluid is completely bounded by solid surfaces**



LAMINAR VERSUS TURBULENT FLOW

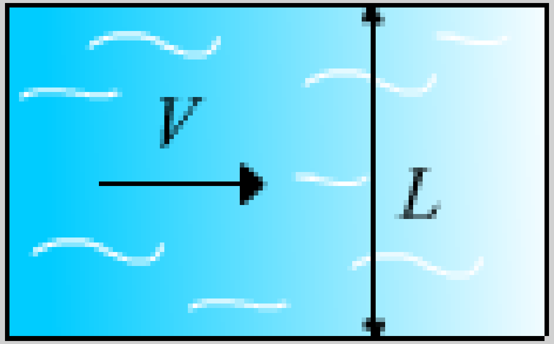
Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called laminar. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

The highly disordered fluid motion that typically occurs at high velocities characterized by velocities fluctuations is called turbulent. The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping



Osborne Reynolds in 1880's, discovered that the flow regime depends mainly on the ratio of the **inertia forces to viscous forces** in the fluid.

The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.



$$Re = \frac{\textit{Inertia forces}}{\textit{Viscous forces}} = \frac{\rho V^2/L}{\mu V/L^2} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

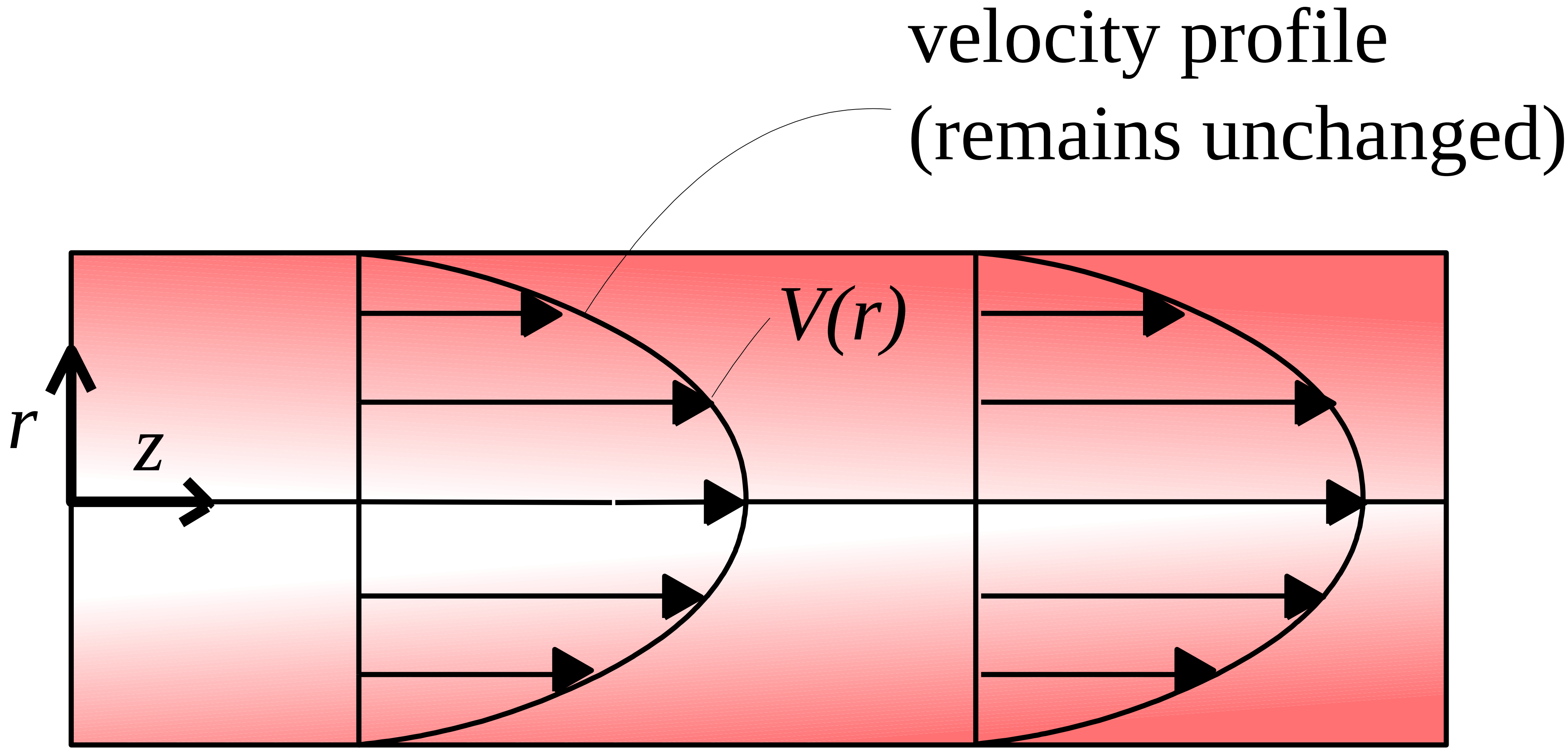
$$Re = \frac{\textit{Inertia Force}}{\textit{Viscous Force}} = \frac{\rho V L}{\mu}$$

L is the diameter of the pipe in internal flows

L is the length of the flat plate in external flows

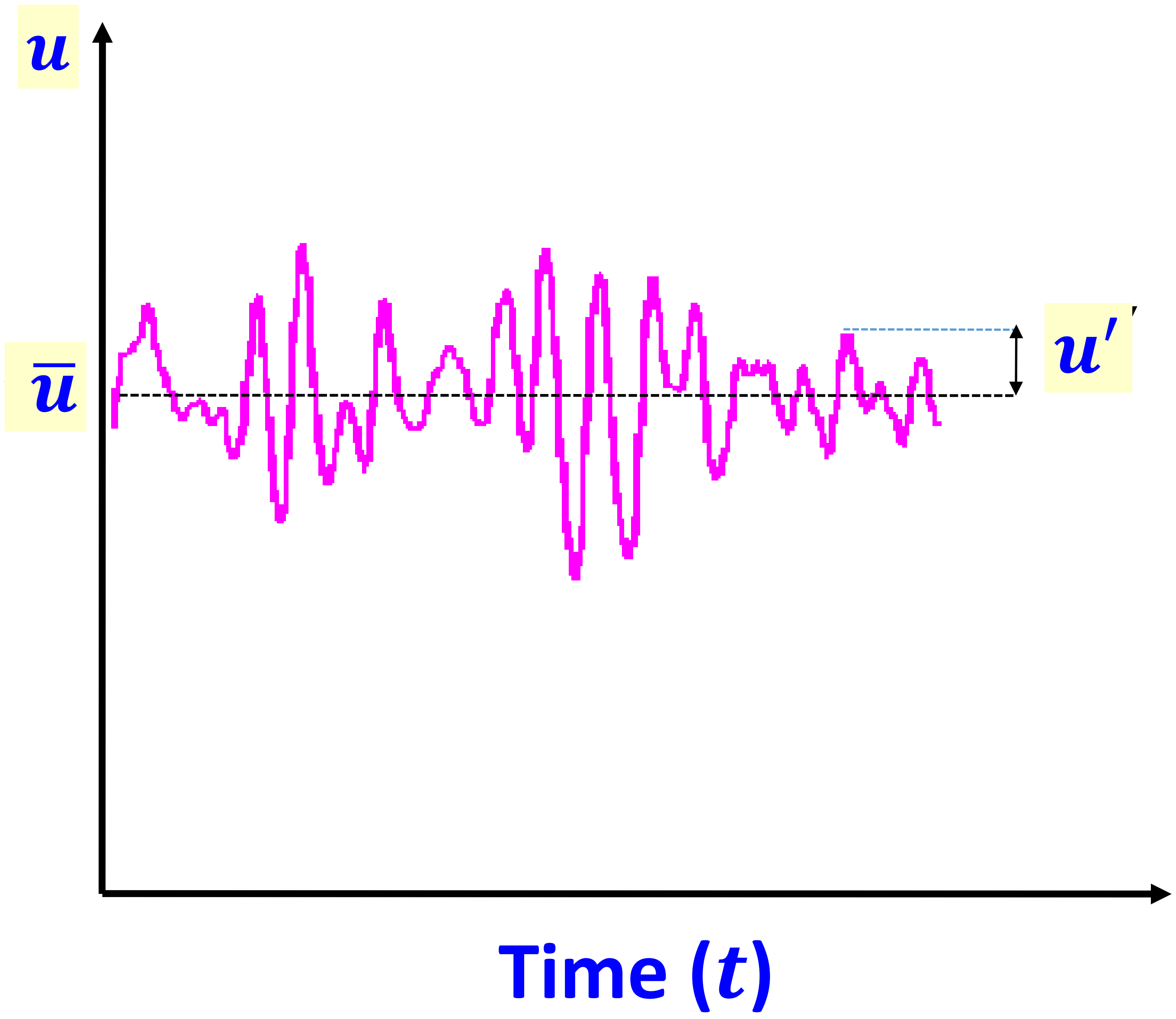
One, Two and Three Dimensional Flows

***V*(*x*, *y*, *z*) in cartesian or *V*(*r*, *θ*, *z*) in cylindrical coordinates**



One-dimensional flow in a circular pipe

TURBULENT FLOW



$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$w = \bar{w} + w'$$
$$P = \bar{P} + P'$$
$$T = \bar{T} + T'$$

The eddying motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow).

$$\tau_t = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$$

TURBULENT FLOW

Random eddy motion of groups of particles resembles the random motion of molecules in a gas-colliding with each other after traveling a certain distance and exchanging momentum and heat in process.

Therefore, momentum and heat transport by eddies in turbulent boundary layers is analogous to the molecular momentum and heat diffusion.

$$\tau_t = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$$

$$q_t = -\rho C_p \overline{v'T'} = -k_t \frac{\partial \bar{T}}{\partial y}$$

μ_t - Turbulent viscosity - momentum transport by turbulent eddies

k_t - Turbulent thermal conductivity - thermal energy transport by turbulent eddies.

Total shear stress τ_{total} and total heat flux q_{total} can be expressed as

$$\tau_t = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \varepsilon_M)\frac{\partial \bar{u}}{\partial y}$$

$$q_{total} = (-k - k_t)\frac{\partial \bar{T}}{\partial y} = -\rho C_p(\alpha + \varepsilon_H)\frac{\partial \bar{T}}{\partial y}$$

$$\varepsilon_M = \frac{\mu_t}{\rho}$$

$$\varepsilon_H = \frac{k_t}{\rho C_p}$$

Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer.

The eddy motion loses its intensity close to the wall, and diminishes at the wall because of the no-slip condition.

Therefore, the velocity and temperature profiles are nearly uniform in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface.

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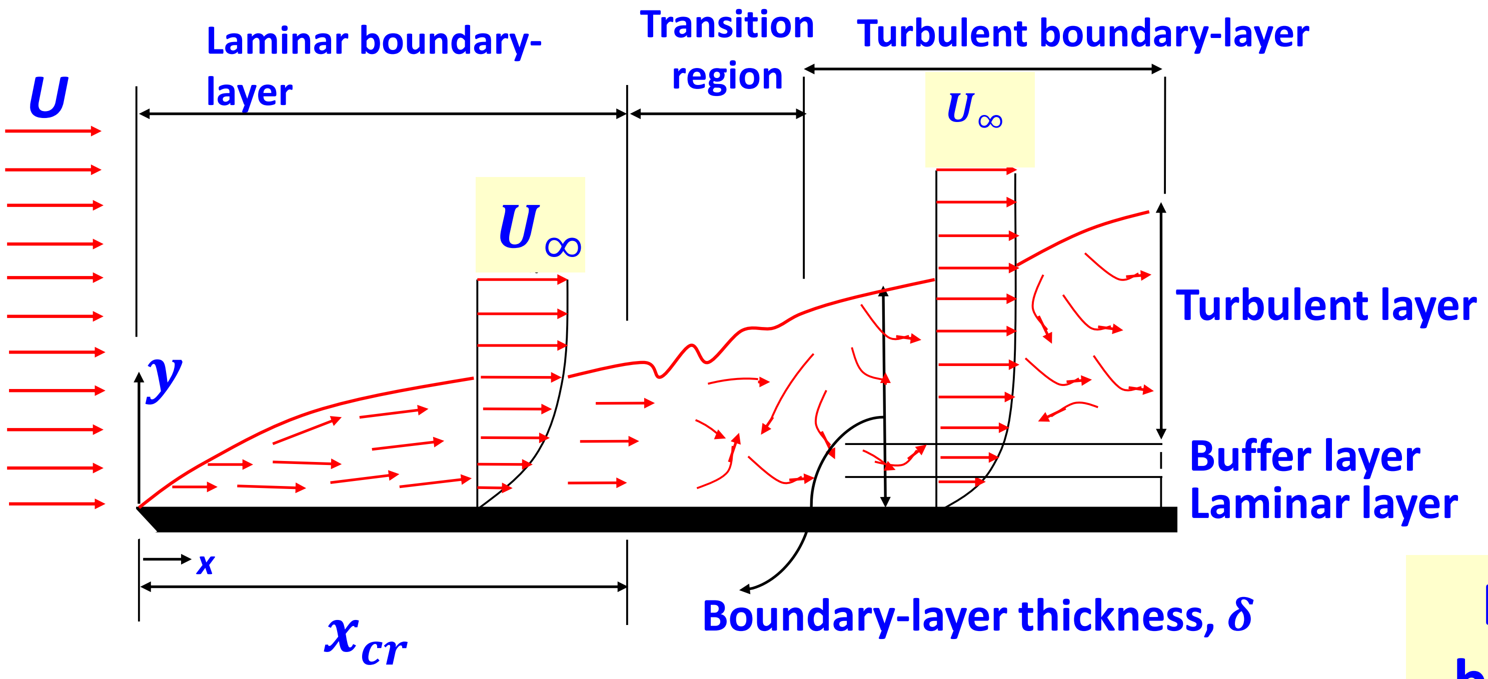
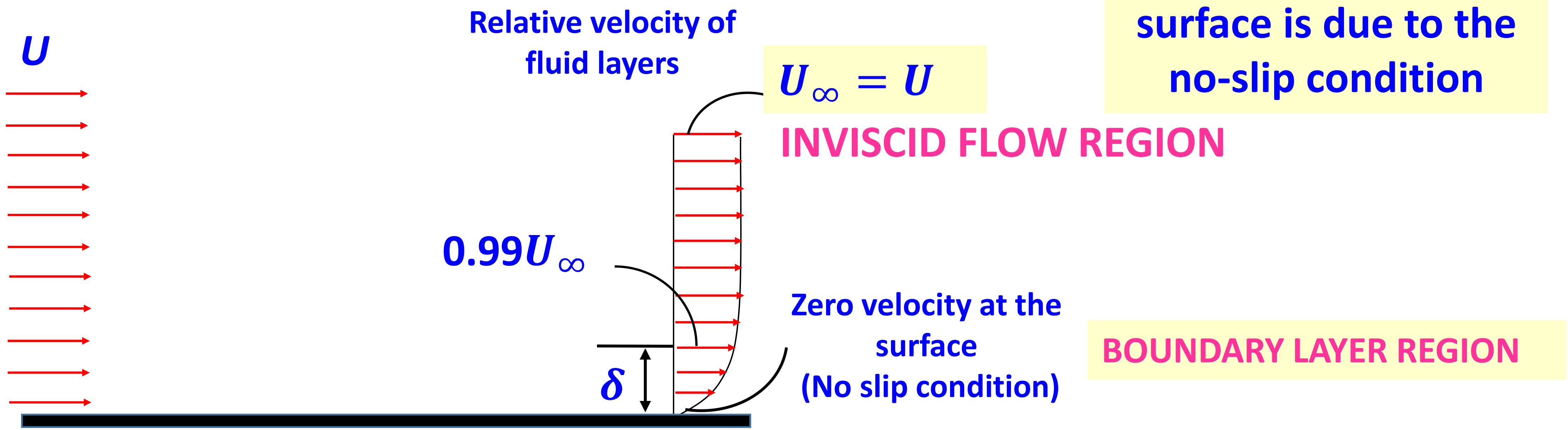
VELOCITY BOUNDARY LAYER



Development of a boundary layer on a surface is due to the no-slip condition



Surface Shear Stress

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

Skin friction coefficient

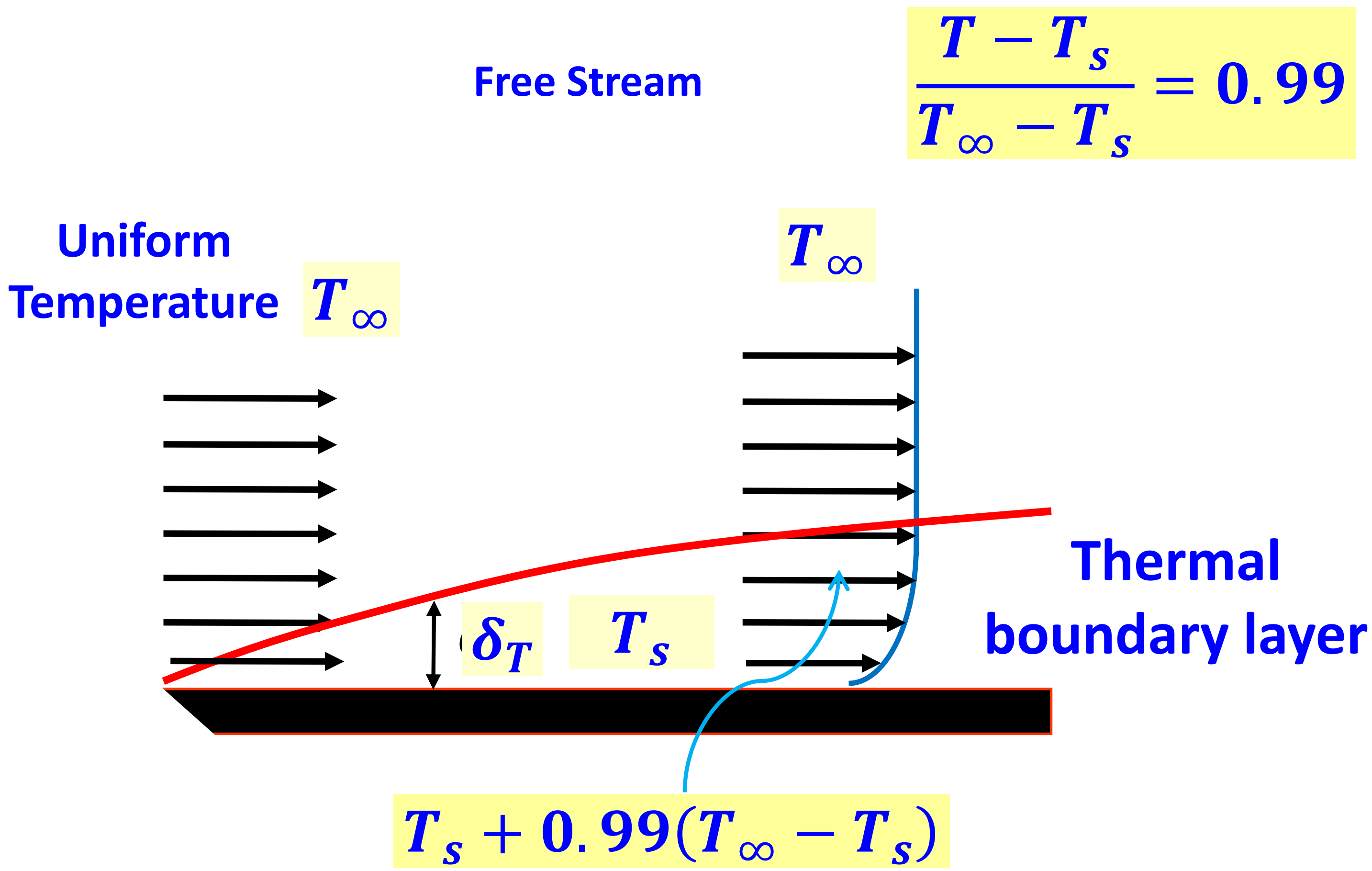
$$\tau_s = C_f \frac{\rho U^2}{2}$$

Friction force over the entire surface

$$F_f = \tau_s A_s = C_f A_s \frac{\rho U^2}{2}$$

THERMAL BOUNDARY LAYER

Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface)



The thickness of the thermal boundary layer, at any location along the surface is define as *the* distance from the surface at which the temperature difference $T - T_s$ equals $0.99(T_\infty - T_s)$ For the special case of $T_s = 0$, we have $T = 0.99$ at the outer edge of the thermal boundary layer, which is analogous to $u = 0.99U_\infty$ for the velocity boundary layer.

- Shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.
- In flow over a heated (or cooled) surface, both velocity and thermal boundary layers will develop simultaneously.
- Noting that the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

PRANDTL NUMBER

The relative thickness of the velocity and the thermal boundary layers is described by the *dimensionless* parameter Prandtl number, defined as

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho C_p}} = \frac{\mu C_p}{k}$$

TYPICAL RANGES OF PRANDTL NUMBERS FOR COMMON FLUIDS

Fluid	Pr
Liquid metals	0.004-0.030
Gases	0.7-1.0
Water	1.7-13.7
Light organic fluids	5-50
Oils	50-100,000
Glycerin	2000-100,000

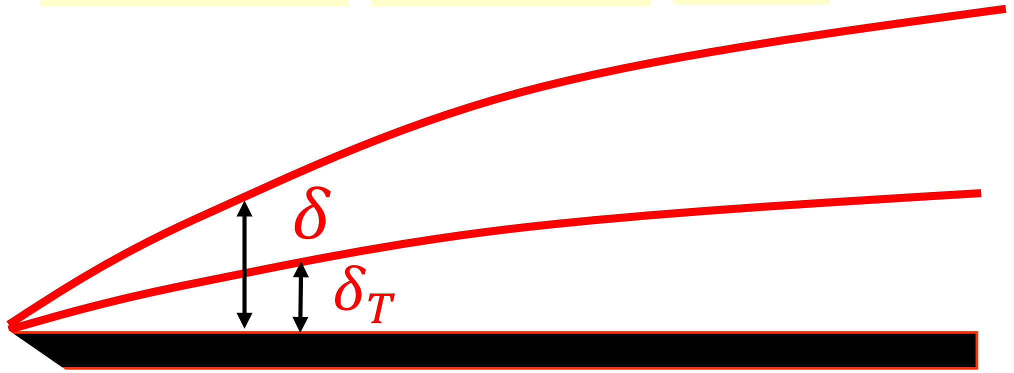
$$\frac{\delta}{\delta_T} \approx Pr^n$$

n is a positive exponent

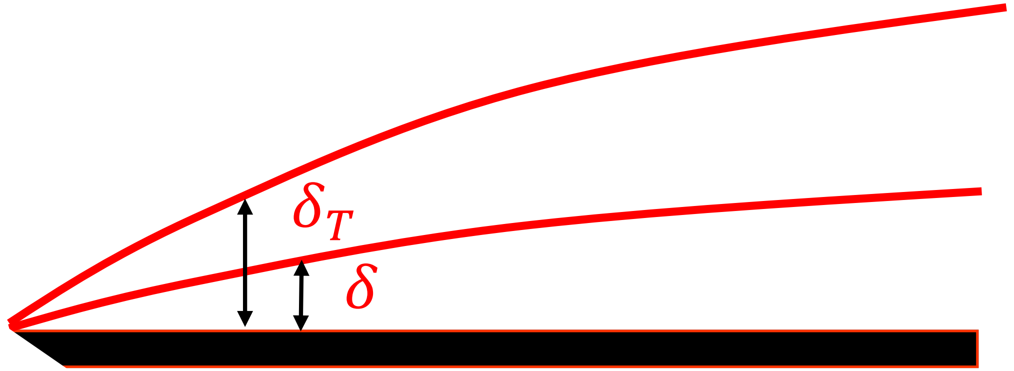
$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$Pr \simeq 1$	$\delta = \delta_T$	Gases	Momentum & heat dissipate through the fluid at same rate
$Pr > 1$	$\delta > \delta_T$	Oils	Heat diffuses very slowly in oils relative to momentum
$Pr < 1$	$\delta < \delta_T$	Liquid Metal	Heat diffuses very quickly in liquid metals relative to momentum

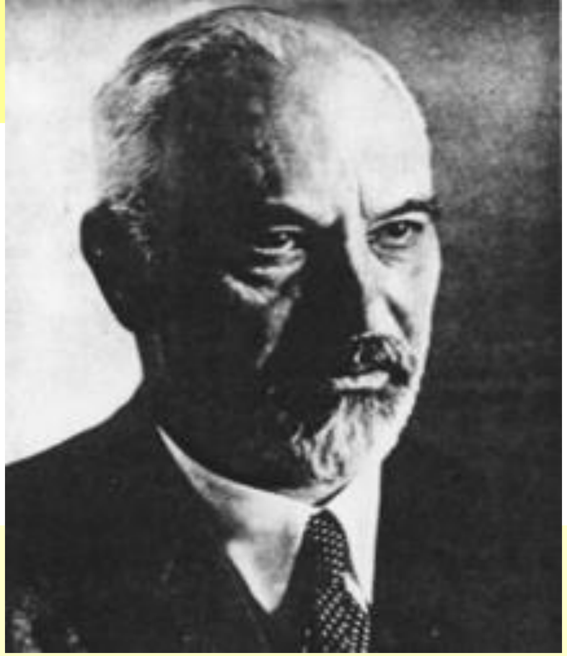
$Pr > 1$ $\delta > \delta_T$ Oils



$Pr < 1$ $\delta < \delta_T$ Liquid Metal



Ludwig Prandtl 1875-1953



Ludwig Prandtl - born at Freising, Bavaria - 1875
German Physicist - famous for his work in aeronautics Professor of Applied Mechanics at Gottingen for forty-nine years (from 1904 until his death there on August 15, 1953)

His discovery in 1904 of the Boundary Layer which adjoins the surface of a body moving in a fluid led to an understanding of skin friction drag and of the way in which streamlining reduces the drag of airplane wings and other moving bodies.