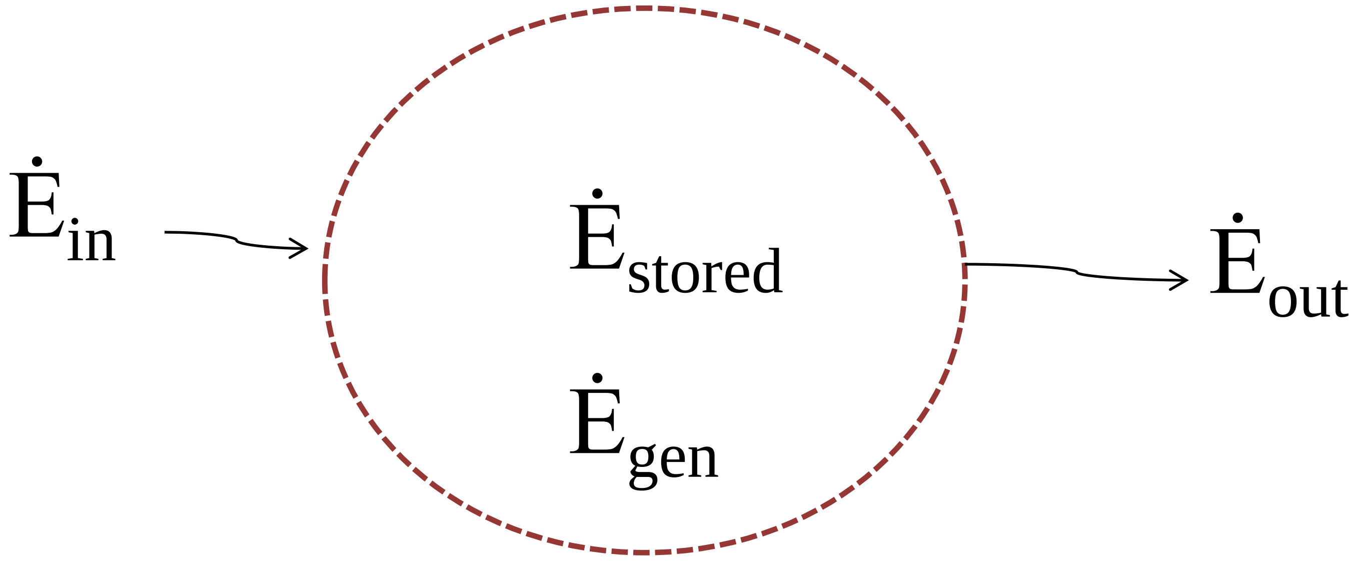
Control volume energy conservation

- Consider a control volume and write the energy balance



- $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = \dot{E}_{stored}$

- $\dot{E}_{stored} = 0 \quad \Rightarrow$ steady state

Thermal Energy Equation

- At an instant of time, t: the rate at which thermal energy enters a control volume, plus the rate at which thermal energy is generated within the control volume, minus the rate at which thermal energy leaves the control volume must equal the rate of increase of thermal energy stored within the control volume
- $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = dE_{stored}/dt = \dot{E}_{stored}$

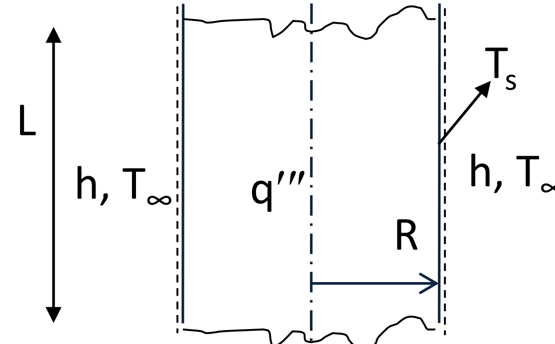
Thermal Energy Equation

- Over a time interval, Δt: The amount of thermal energy which enters a control volume, plus the amount of thermal energy is generated within the control volume, minus the amount of thermal energy which leaves the control volume must equal the increase in amount of thermal energy stored in the control volume
- $E_{in} + E_{gen} - E_{out} = \Delta E_{stored}$

Notes on Thermal Energy Equation

- In and out flow terms would include all thermal energy transfer across the control surface
  - Conduction, convection and radiation exchange are reckoned relative to a pre-defined control surface
  - Control volume on an interface should have E_{gen} and E_{stored} terms set to zero
- Thermal energy generation and storage terms are volumetric phenomena
  - Generation associate with a specific conversion process of one form of energy, such as electrical, chemical, nuclear, etc to thermal energy
  - Thermal energy storage is associated with an increase or decrease in the thermal energy of matter occupying the control volume
- Generation is the consequence of a conversion process within the control volume
- Storage term is associated with the change in internal energy of the matter within the control volume
  - For steady-state situations, rate of energy storage must be zero

Example -1



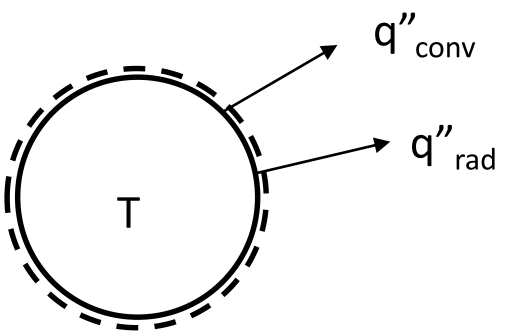
- Long cylindrical nuclear fuel element subjected to convective cooling on its surface
- Thermal energy is generated uniformly at every point within the fuel rod at the rate of q''' W/m^3, as a consequence of radioactivity of the nuclear fuel. The control volume enveloping the rod allows to predict its steady-state surface temperature.
- $\cancel{\dot{E}_{in}} + \dot{E}_{gen} - \dot{E}_{out} = \cancel{\dot{E}_{stored}}$

Example-1

- $-\dot{E}_{out} + \dot{E}_{gen} = 0$
- $-2\pi RLh(T_s - T_\infty) = q'''(\pi R^2 L)$
- $T_s = T_\infty + \dfrac{q'''R}{2h}$

Example-2

- Unsteady situation
- Copper sphere, initially at an elevated temperature T_i, is cooling by convection to the ambient air and radiation to the surroundings
- $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = \dot{E}_{stored} = mCdT/dt$

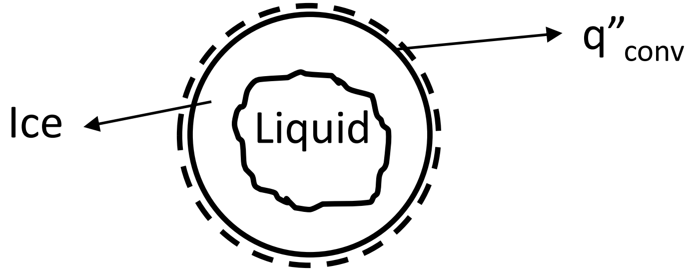


Example-2

- $\cancel{\dot{E}_{in}} + \cancel{\dot{E}_{gen}} - \dot{E}_{out} = \dot{E}_{stored} = mCdT/dt$
- $-4\pi R^2 \{h(T - T_\infty) + \varepsilon\sigma (T^4 - T_{sur}^4)\} = (4/3 * \pi R^3 \rho c_p)\, dT/dt$
- $dT/dt = -(3/R\, \rho c_p) \{h(T - T_\infty) + \varepsilon\sigma (T^4 - T_{sur}^4)\}$
- Upon integration we will be find the solution
- What is our assumption behind this equation?

Example-3

- Not all unsteady problems involve transient variation in temperature… let us look at phase change in a capsule
- Spherical capsule containing a liquid phase change material that is initially at a temperature (very) close to its freezing point is immersed in a refrigerant bath and experiences convective heat transfer – So, the liquid begins to freeze, and layer of ice begins to develop
- Energy storage term involves changes in the proportions of liquid and solid in the capsule.

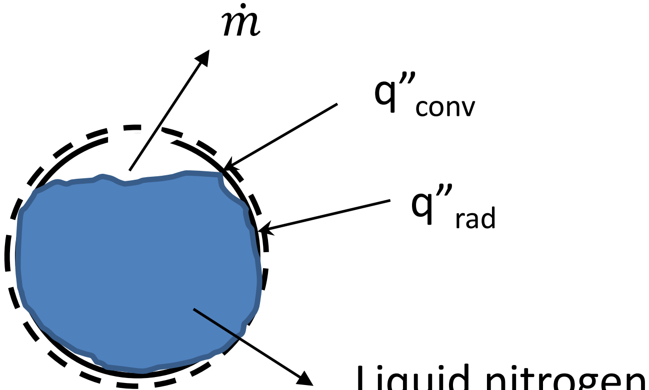


Example -3

- Let us focus on the storage term
- At any instant, we may write $E_{stored} = m_s e_s + m_l e_l$ where m and e are mass and internal energy
- Conservation of mass requires, $m = m_s + m_l$ or writing as a rate equation, $0 = dm_s/dt + dm_l/dt$
- Taking the time derivative of E_{stored}, $dE_{stored}/dt = e_s\, dm_s/dt + e_l\, dm_l/dt$
- $\dot{E}_{stored} = -(e_l - e_s)\, dm_s/dt = -(e_l - e_s) m_{cap}\, d(m_s/m_{cap})/dt$
- Therefore, $\dot{E}_{stored} = -m_{cap} L\, df_s/dt$ where $L = (e_l - e_s)$ is the latent heat of fusion and $f_s = (m_s/m_{cap})$ is the fraction of mass that is solid
- Equate it to q''_{conv}

Example - 4

- Heat transfer to the container causes the nitrogen to boil and escape at high velocity through the small vent at the top
- $q_{conv} + q_{rad} - \dot{m}e_{gas} + \dot{E}_{gen} = \dot{m}e_{liq}$
- Storage term turns out to be negative as the mass of liquid nitrogen is diminishing at the rate of $\dot{m}$



Example-4

- $\dot{E}_{gen}$ represents the rate at which thermal energy is converted into kinetic energy of the escaping gas as well as the rate at which thermal energy is expended in doing work at the outflow boundary.
- $\dot{E}_{gen} = -\dot{m}\,(V^2/2) - \dot{m}\,(p/\rho_{gas})$

Methodology for Analysis

- To perform a heat transfer analysis of a physical system, we can apply the conservation of energy requirement following these basic steps:
  - Define appropriate control volume
  - Determine whether to invoke the energy balance on a rate basis or for a specific time interval
  - Identify the significant processes representing energy flows across control surfaces and volumetric phenomena within the control volume; show these processes with appropriately labeled arrows, and
  - Write the conservation of energy requirement in the appropriate form and associate proper rate equations