HEAT TRANSFER

Conduction 1

1. WHAT IS HEAT TRANSFER?
2. HOW IS HEAT TRANSFERRED?
3. WHY IS IT IMPORTANT TO STUDY IT?
4. HOW IS HEAT TRANSFER DIFFERENT FROM THERMODYNAMICS

WHAT IS HEAT TRANSFER

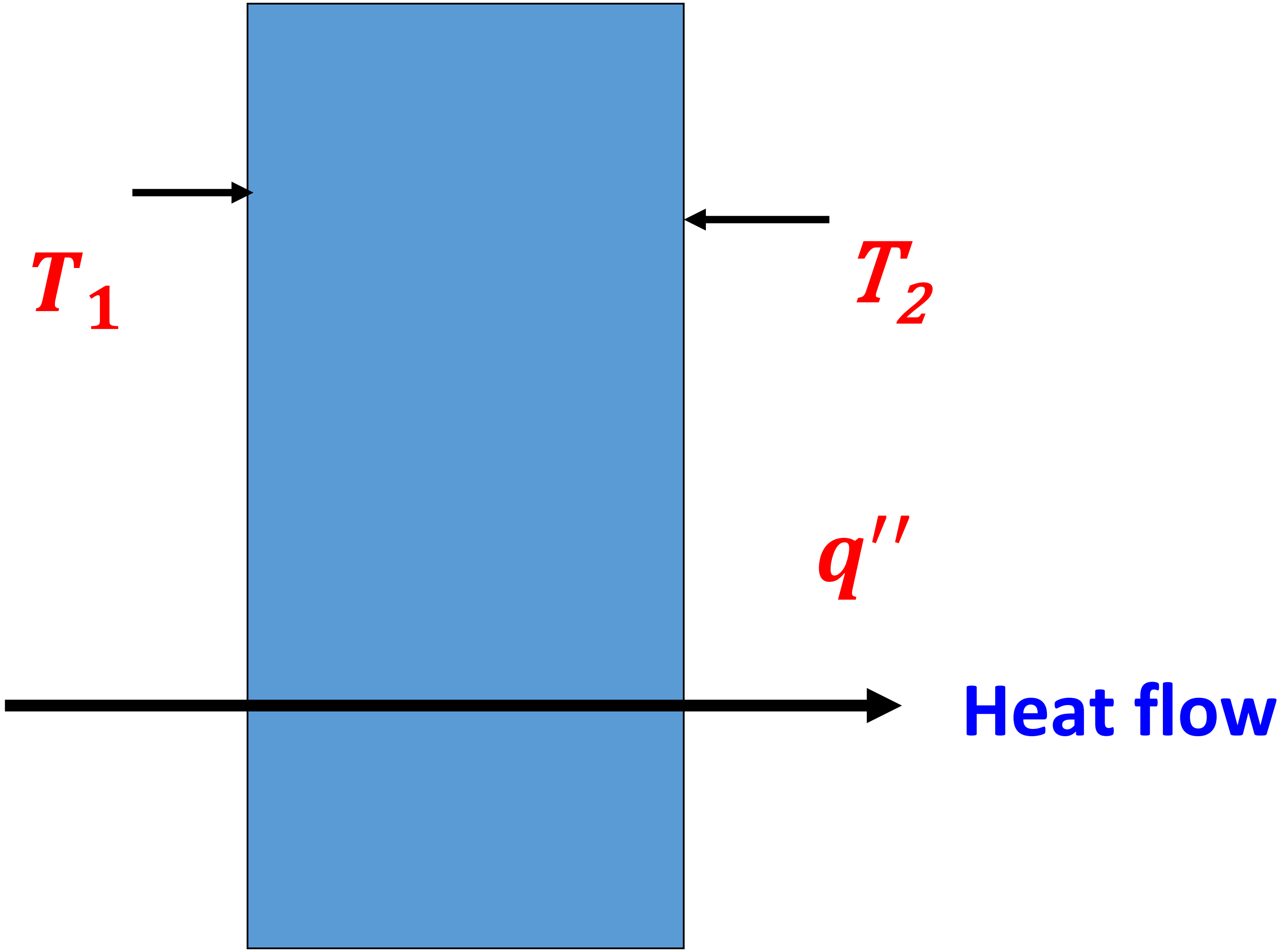
- HEAT TRANSFER IS ENERGY IN TRANSIT DUE TO A TEMPERATURE DIFFERENCE

MODES OF HEAT TRANSFER

- CONDUCTION
- CONVECTION
- RADIATION

CONDUCTION

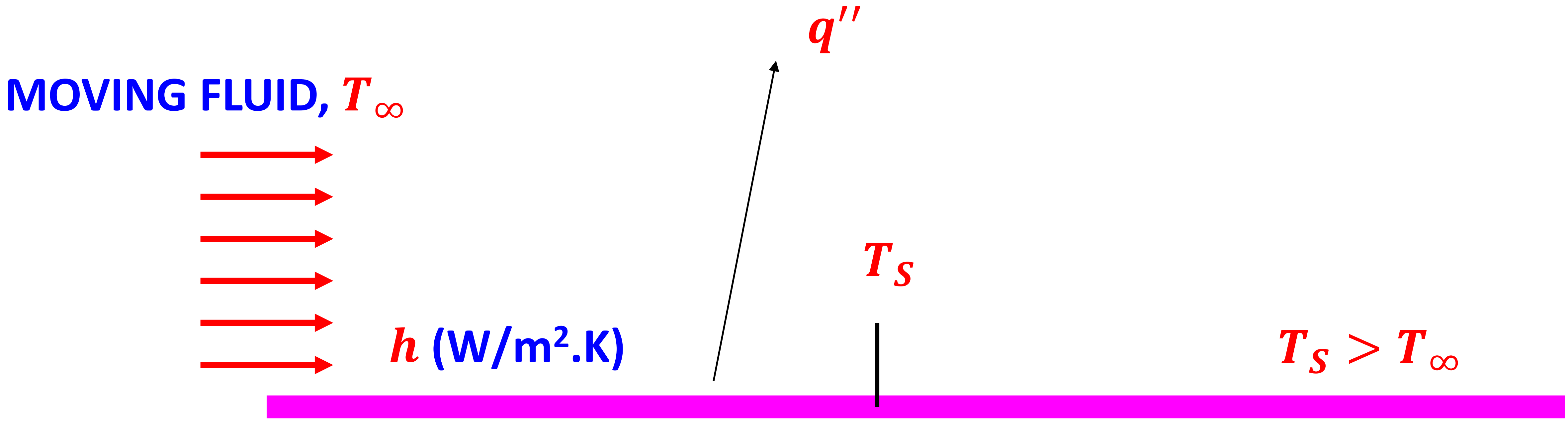
- Conduction through a solid or a stationary fluid



CONVECTION

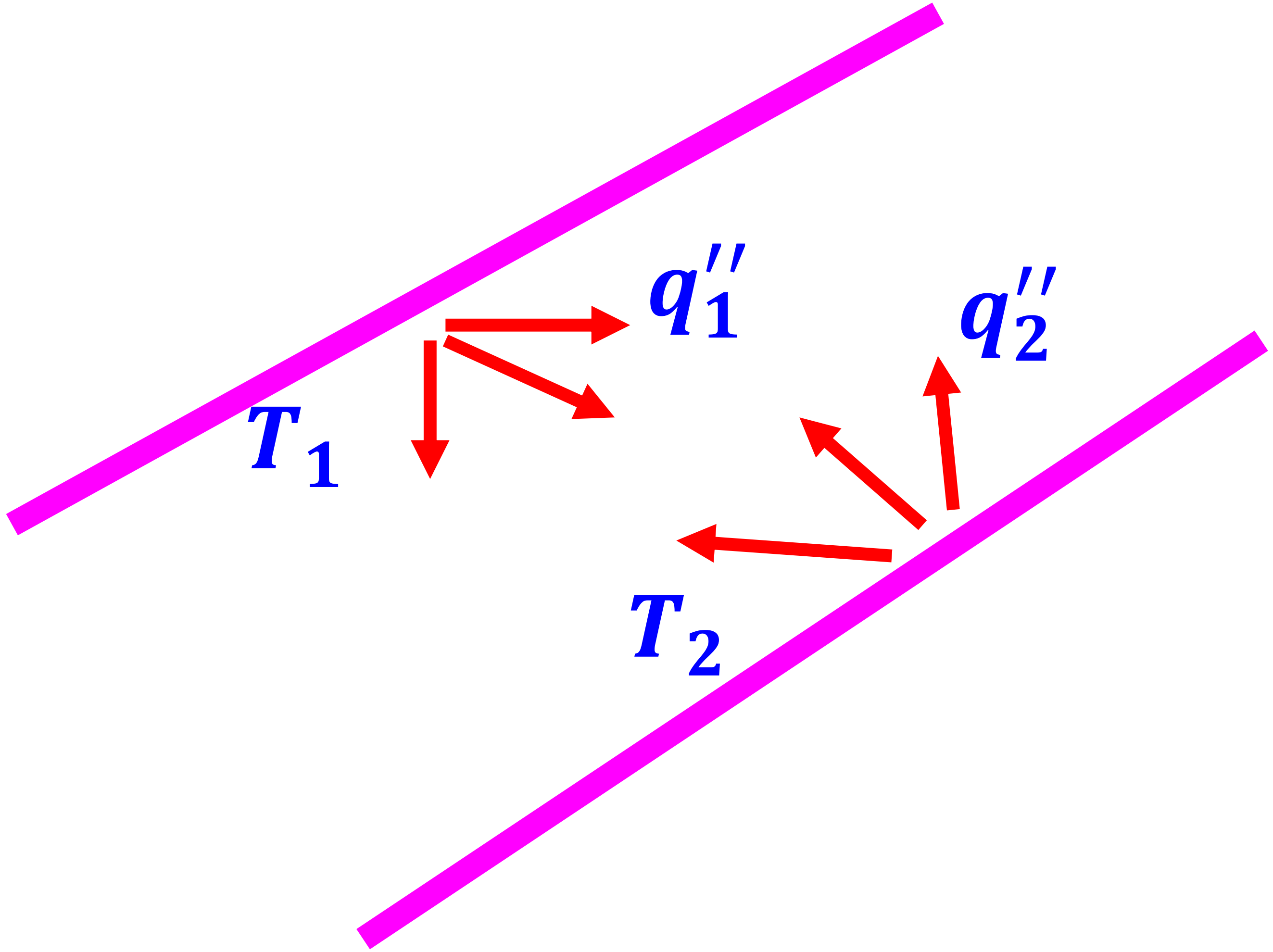
- **CONVECTION FROM A SURFACE TO A MOVING FLUID**

$$q'' = h(T_S - T_\infty)$$



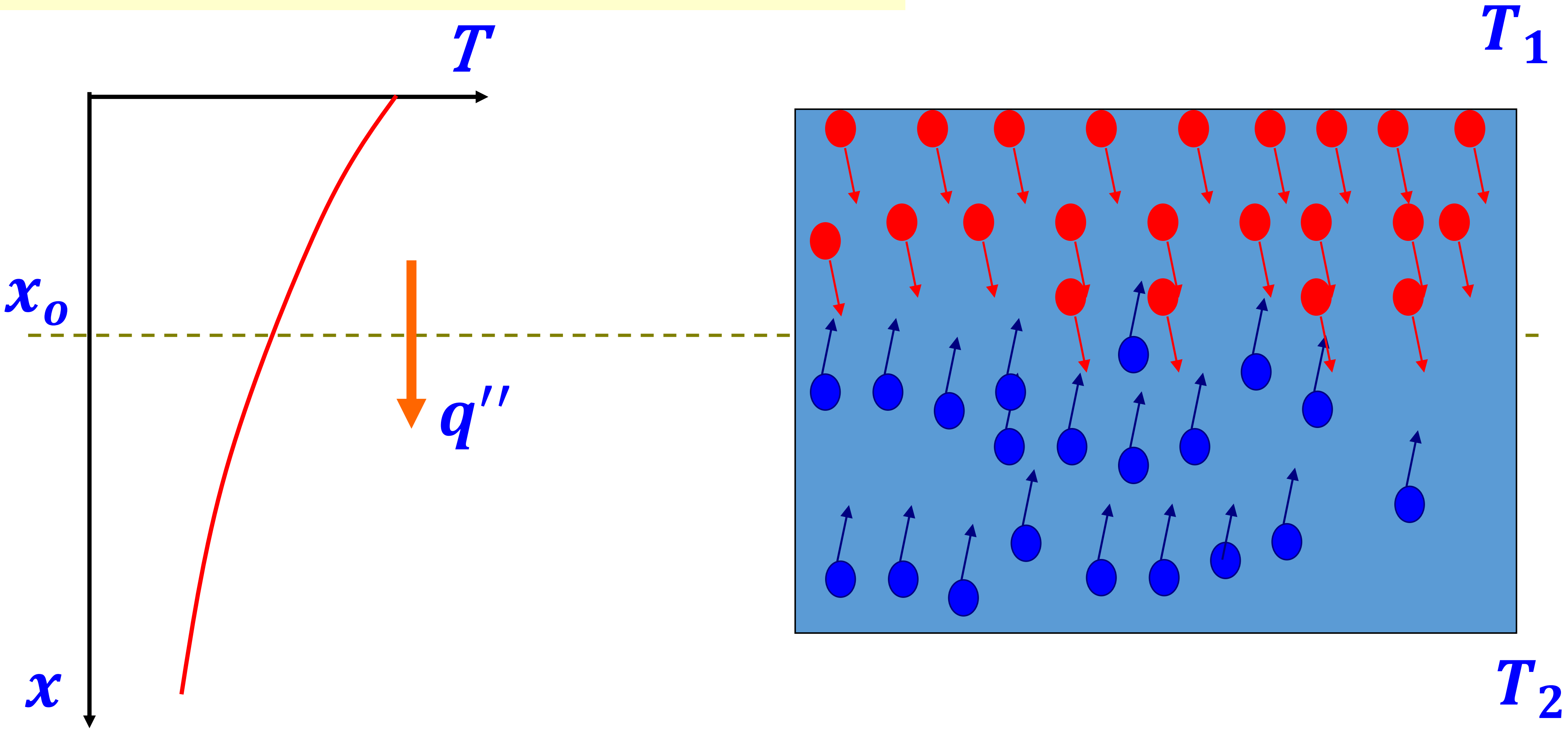
RADIATION

- **NET RADIATION HEAT EXCHANGE BETWEEN TWO SURFACES**



CONDUCTION – ATOMIC OR MOLECULAR ACTIVITY

- **A GAS WITH TEMPERATURE GRADIENT**
- **NO BULK MOTION** $T_1 > T_2$



FOURIER’S LAW OF CONDUCTION

- **1-D Conduction**

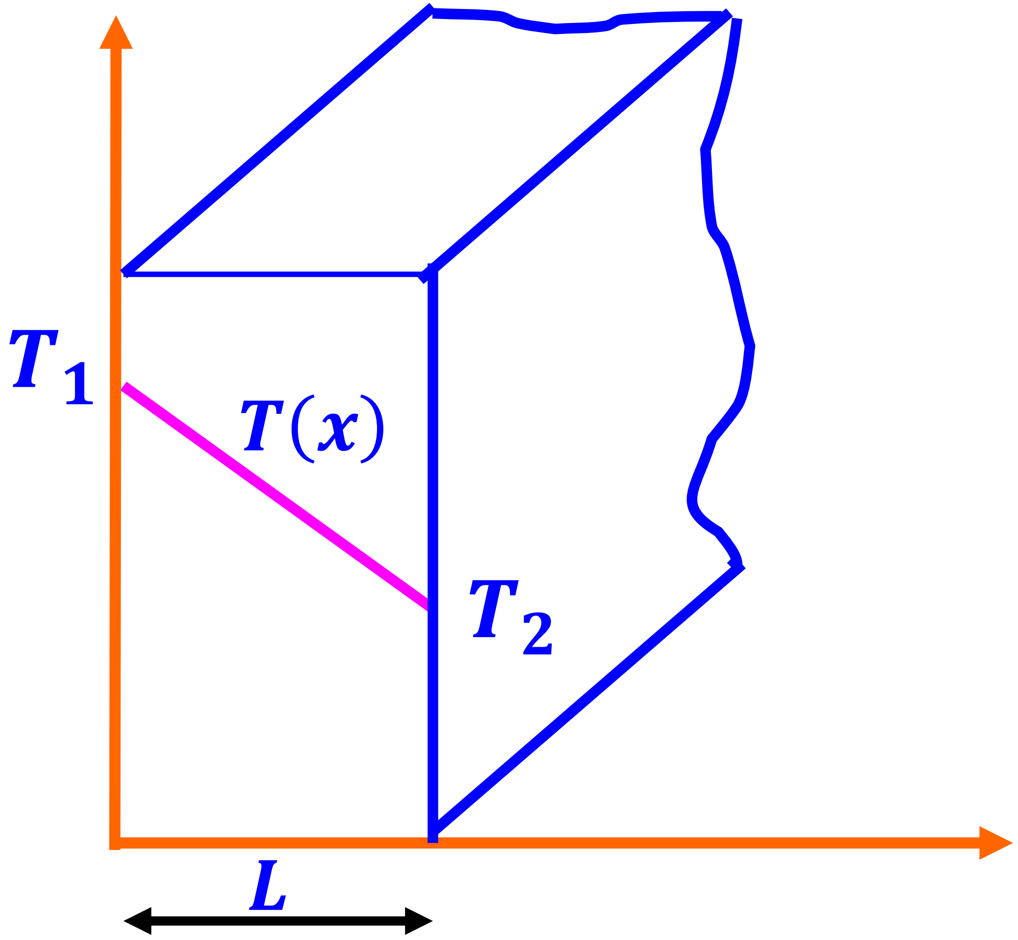
$$q''_x = -k\frac{dT}{dx}$$

$$\frac{dT}{dx} = \frac{T_1 - T_2}{0 - L}$$

$$\frac{dT}{dx} = \frac{T_1 - T_2}{0 - L}$$

$$q''_x = -k\frac{T_1 - T_2}{0 - L}$$

$$q''_x = k\frac{T_1 - T_2}{L}$$

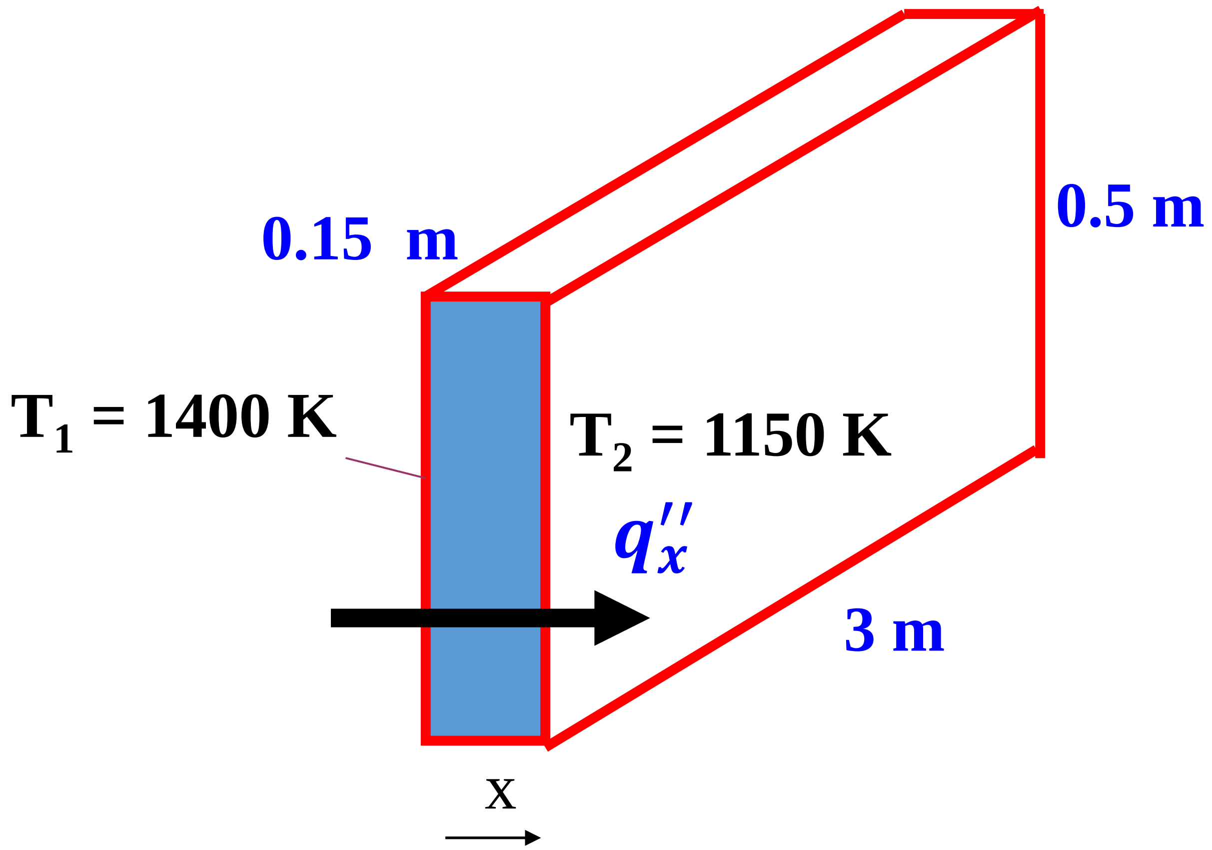


Problem: The wall of an industrial furnace is constructed from 0.15 m thick fire clay brick having a thermal conductivity of 1.7 W/m.k. Measurements made during steady state operation reveal temperatures of 1400 K and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 3 m on a side

KNOWN: Steady state conditions with prescribed wall thickness, area, thermal conductivity and surface temperatures

FIND: Wall Heat Loss

SCHEMATIC



ASSUMPTIONS

Steady state conditions

One dimensional heat conduction through the wall

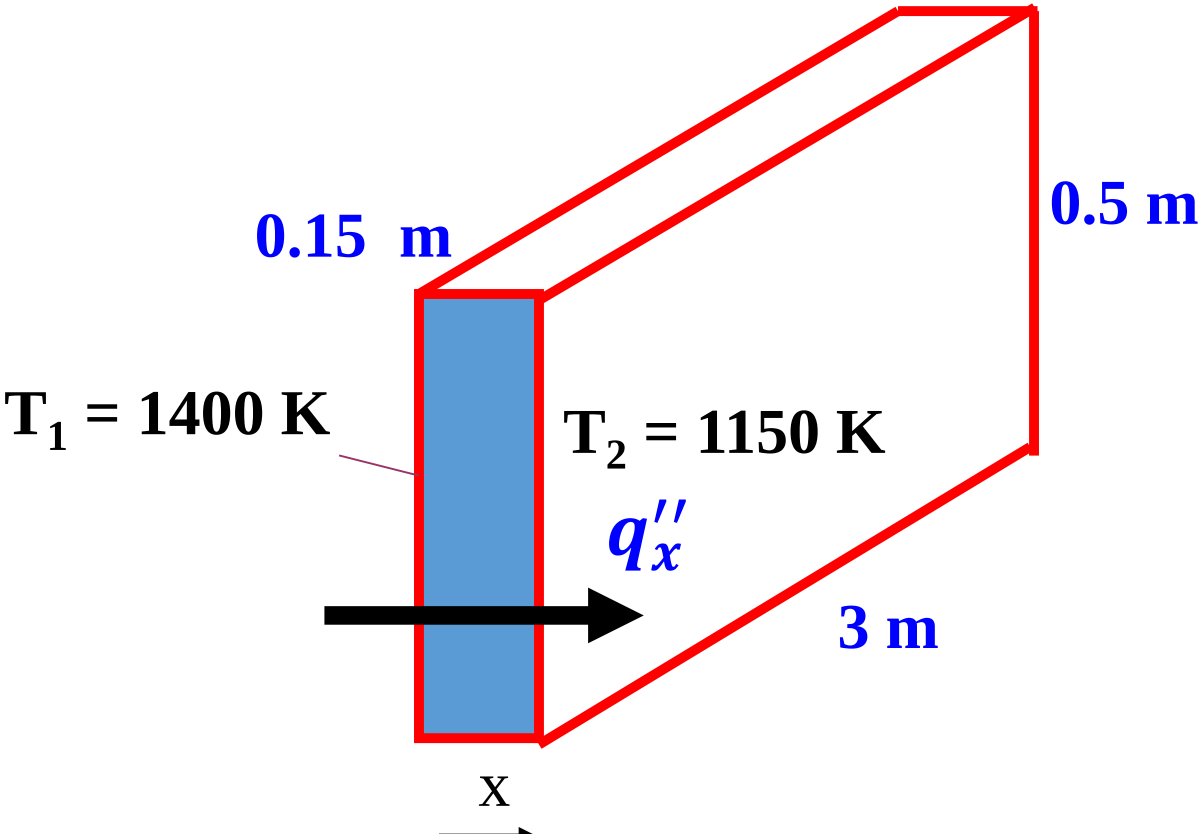
Constant thermal conductivity

$$q_x'' = k\frac{T_1 - T_2}{L} = 1.7\frac{1400 - 1150}{0.15} = 1.7\frac{250}{0.15} = 2833$$

$$q_x'' = 2833\frac{W}{m^2}$$

$$q_x = q_x'' W H = 2833 \times 3.0 \times 0.5$$

$$q_x = 4250\ W$$



CONVECTION

Energy transfer due to

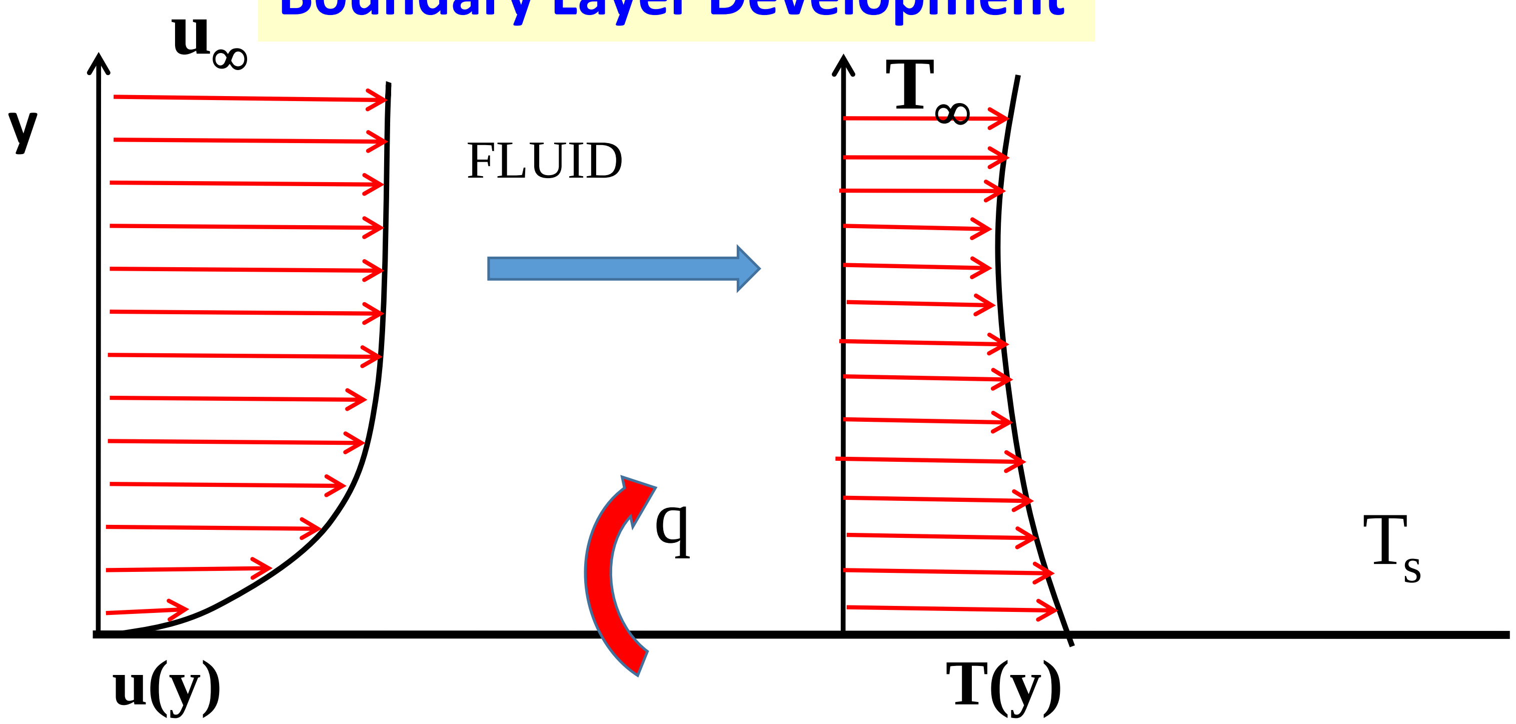
Random Molecular Motion (Diffusion) + Motion Of Fluid (Advection)

Diffusion + Advection - Convection

Diffusion - Within The Velocity Boundary Layer; Advection - Outside The Boundary Layer

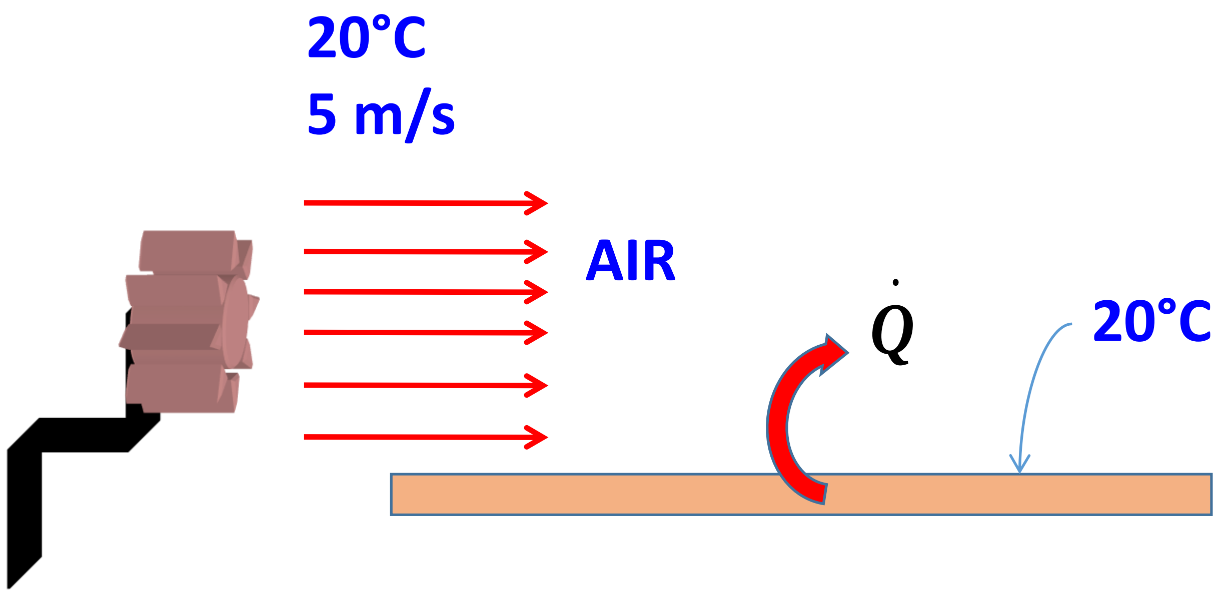
FLUID MECHANICS IS INDISPENSABLE FOR CONVECTION

Boundary Layer Development

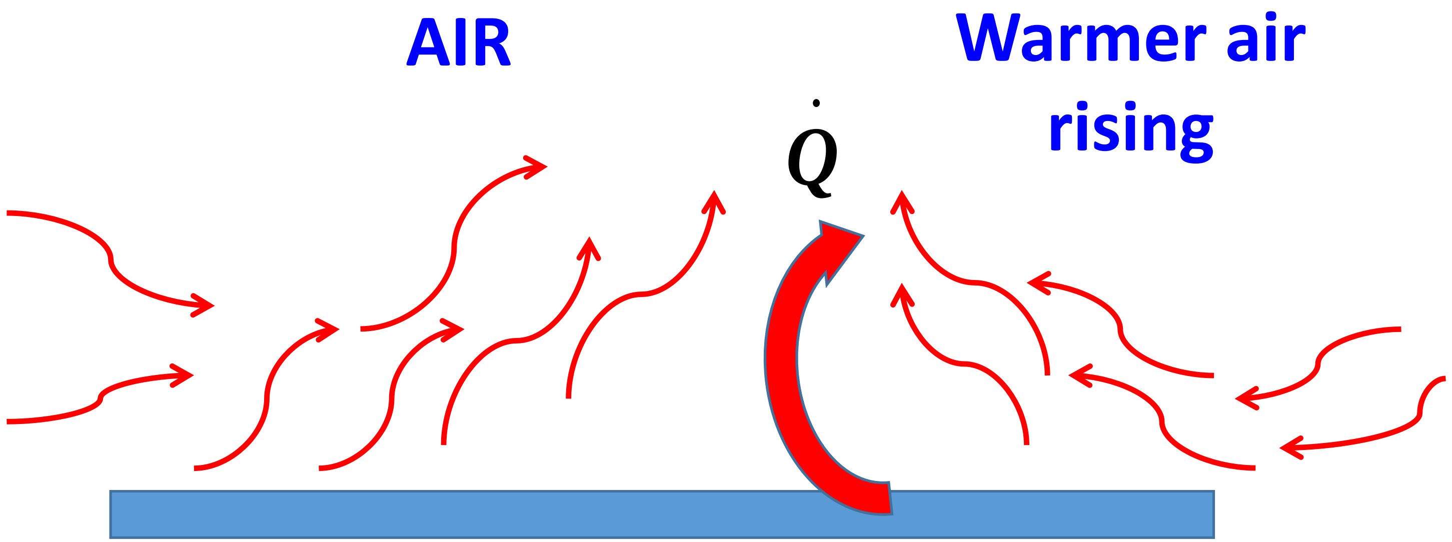


TYPES OF CONVECTION

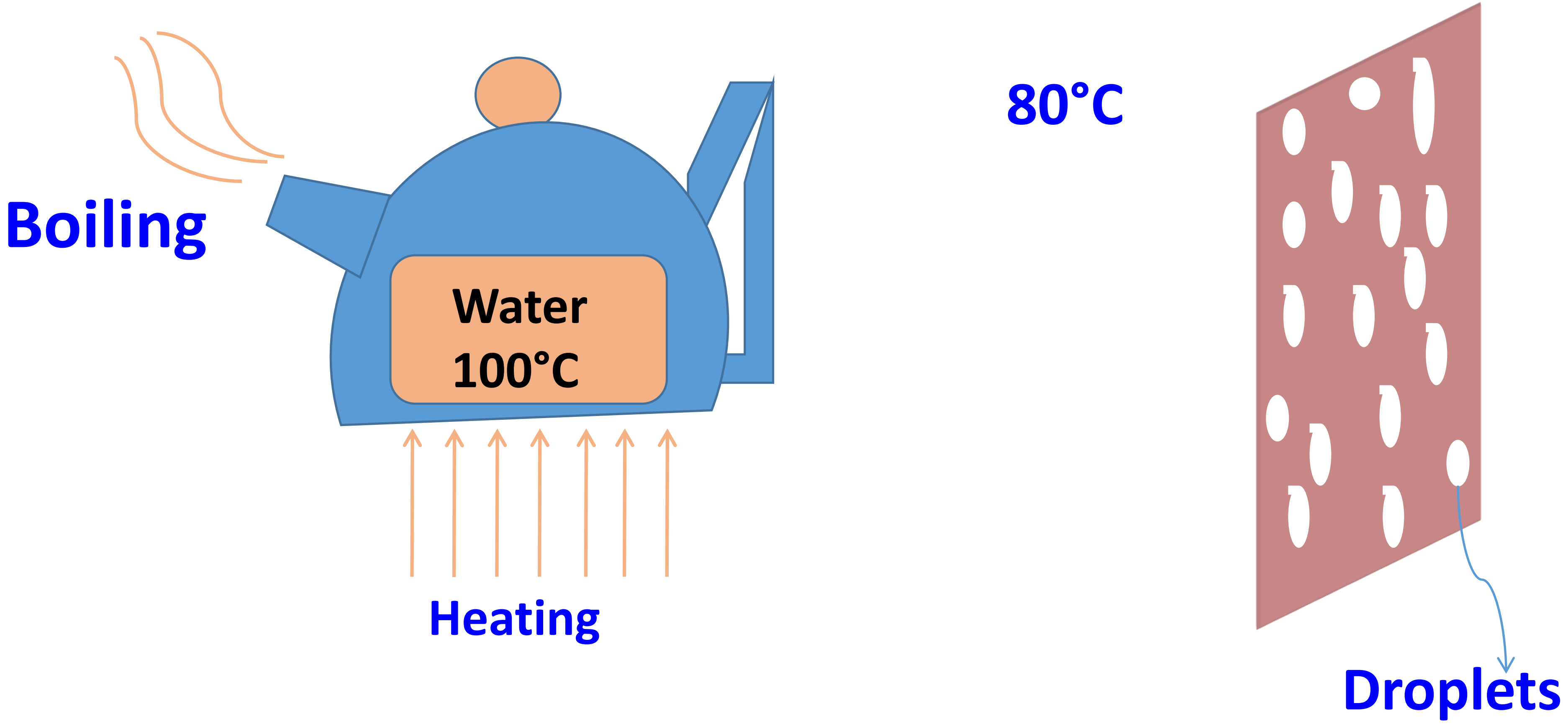
FORCED CONVECTION



NATURAL CONVECTION



BOILING AND CONDENSATION – involve phase change



NEWTON'S LAW OF COOLING

$$q'' = h(T_S - T_\infty)$$

- q'' – positive – heat is transferred from the surface $T_S > T_\infty$

- q'' – negative – heat is transferred to the surface $T_S < T_\infty$

- h - f (surface geometry, fluid, nature of flow)

TYPICAL VALUES OF HEAT TRANSFER COEFFICIENT

Process	h ($W/m^2.K$)
Free convection	
Gases	2 - 25
Liquids	50 - 1000
Forced Convection	
Gases	25 - 250
Liquids	50 - 20000
Boiling and condensation	2500 - 1,00,000

RADIATION

- Energy emitted by matter that is at a finite temperature
- Radiation – solid, liquid , gas
- Energy of radiation – transported – E.M.Waves
- No medium is required (vacuum is perfect medium)

EMISSIVE POWER - Rate at which energy is released per unit area

STEFAN-BOLTZMAN LAW

$$E_b = \sigma T_s^4$$

T_s- Absolute temperature of the surface (K)

σ - Stefan-Boltzman constant (5.67 x 10^{-8} $W/m^2.K^4$)

E_b - Emissive Power (W/m^2)

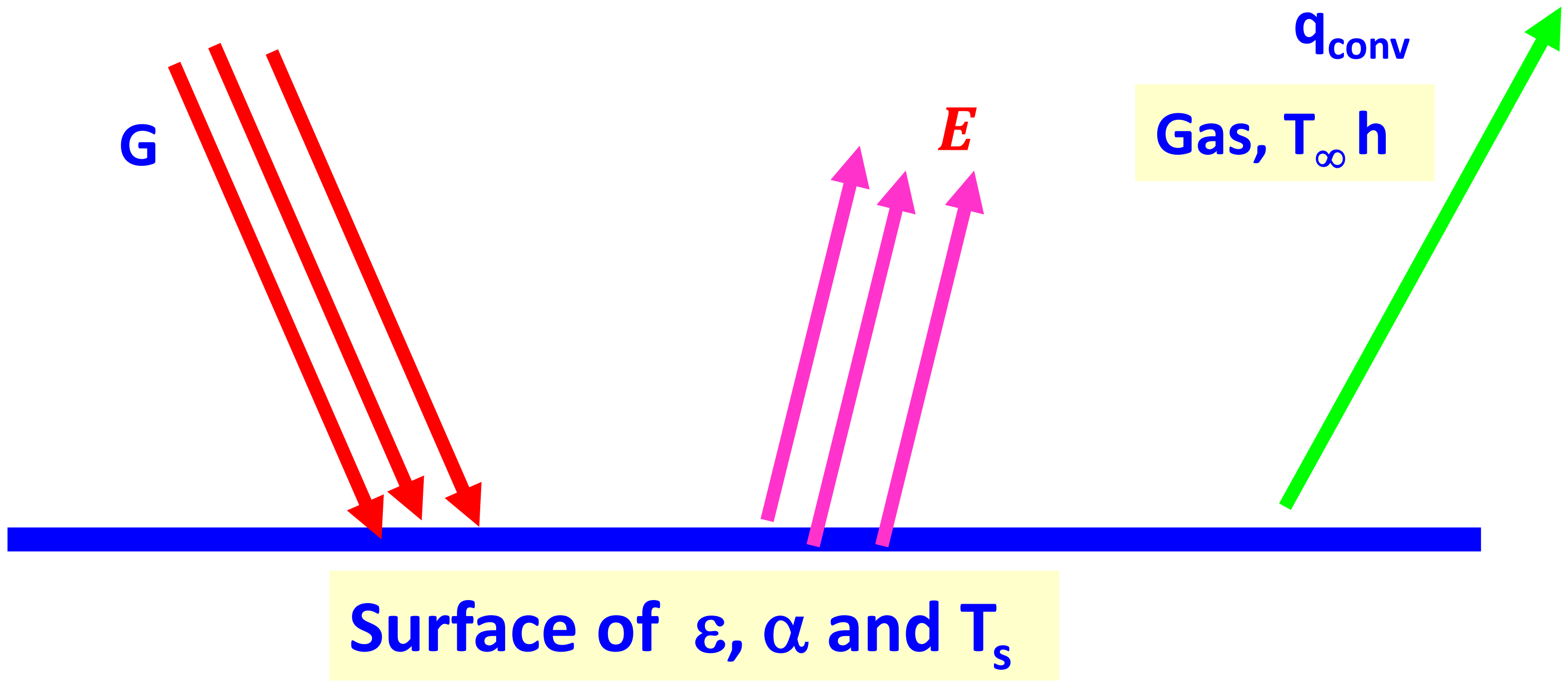
BLACK SURFACE – Stefan Boltzman Law

$$E_b = \sigma T_s^4$$

REAL SURFACE – Less Than Black Surface

$$E = \varepsilon \sigma T_s^4$$

ε- **Emissivity** $0 \leq \varepsilon \leq 1$



G - IRRADIATION – Rate at which radiation is incident on a unit area of the surface (receiving/sending)

G_{abs} – ABSORPTION – A portion of the irradiation may be absorbed by the surface

$$G_{abs} = \alpha\, G$$

$$0 \leq \alpha \leq 1$$

RELATIONSHIP TO THERMODYNAMICS

THERMODYNAMICS

- equilibrium states of matter (no temp gradient)
- amount of energy required in the form of heat for a system to pass from one equilibrium state to another

HEAT TRANSFER

- thermodynamic non-equilibrium process
- rate at which heat transfer occurs

SUMMARY

Mode	Mechanism	Rate equation
Conduction	diffusion of energy due to random molecular motion	$q''_x = -k\frac{dT}{dx}$
Convection	diffusion of energy due to random molecular motion + energy transfer due to bulk motion	$q'' = h(T_S - T_\infty)$
Radiation	energy transfer by electromagnetic waves	$E = \varepsilon\sigma T_s^4$

CONSERVATION OF ENERGY FOR A CONTROL VOLUME AT ANY TIME '*t*'

AMOUNT OF THERMAL & MECHANICAL ENERGY ENTERING THE CONTROL VOLUME $\dot{E}_{in}$

+

AMOUNT OF THERMAL ENERGY GENERATED WITHIN THE CONTROL VOLUME $\dot{E}_g$

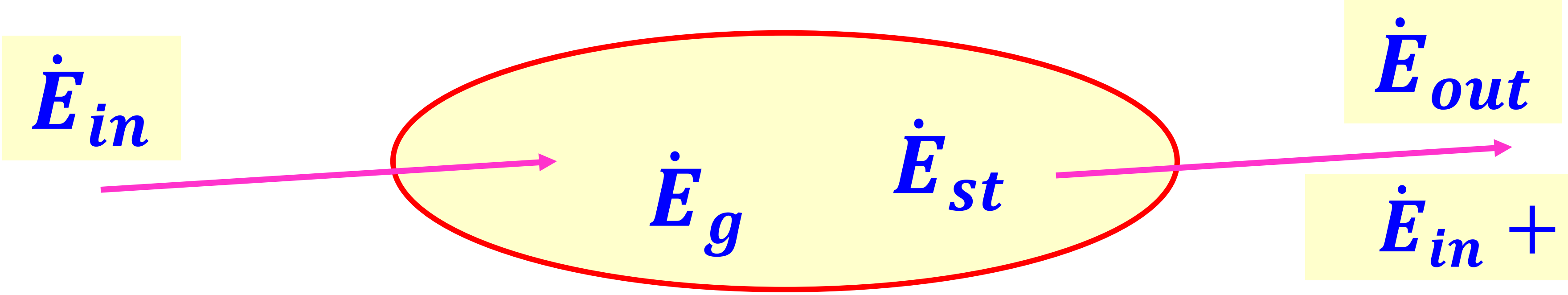
-

AMOUNT OF THERMAL & MECHANICAL ENERGY LEAVING THE CONTROL VOLUME $\dot{E}_{out}$

=

INCREASE IN THE AMOUNT OF ENERGY STORED IN THE CONTROL VOLUME $\dot{E}_{st}$

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$



$$\dot{E}_{in} + \dot{E}_{g} - \dot{E}_{out} = \dot{E}_{st}$$

INFLOW AND OUTFLOW TERMS – SURFACE PHENOMENA

Ex: heat transfer by conduction, convection, radiation; may also involve work interactions at the boundaries

ENERGY GENERATION TERM – VOLUMETRIC PHENOMENA

Ex: chemical, electrical, and nuclear

ENERGY STORAGE TERM – VOLUMETRIC PHENOMENA

Thermal component (sensible component) + latent component

Ex: internal energy, potential, and nuclear

APPLICATIONS OF CONSERVATION LAWS

1. Appropriate control volume must be defined
2. Appropriate time basis must be chosen
3. Relevant energy processes must be identified
4. Conservation equation

APPLICATIONS OF HEAT TRANSFER

Heat transfer is definitely a relevant subject.

Heat transfer is commonly encountered in engineering systems and other aspects of life and we can see that many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer

Examples:

Air-conditioning system, the refrigerator, the water heater, the iron and also the computer, the TV, and the DVD.

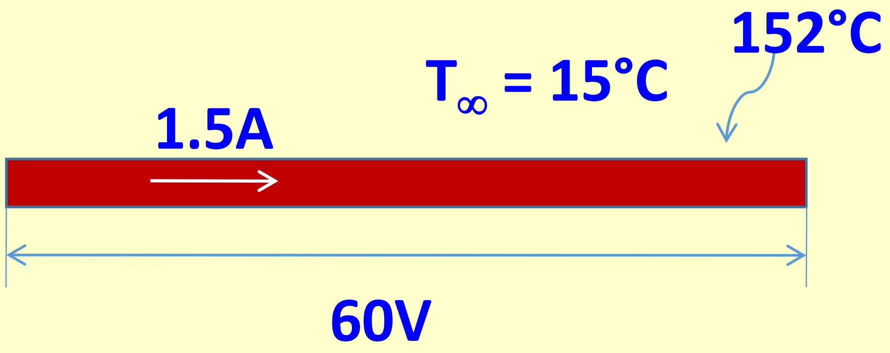
Heat transfer plays an important role in the design of devices such as car radiators, various components of power plants, solar collectors, and even spacecraft.

The thickness of insulation in the walls and roof of the houses, on hot water or steam pipes, or on water heaters is also determined on the basis of heat transfer analysis with economic consideration.

Problem: A 2 m long, 0.3 cm diameter electrical wire extends across a room at 15^0 C as given in schematic. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152^0 C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Known: wire dimensions, room temperature, surface temperature of the wire, voltage drop and electric current through the wire.

Find: convection heat transfer coefficient between the outer surface of the wire and the air in the room.



Assumptions:

- Steady operating conditions exist since the temperature readings do not change with time
- Radiation heat transfer is negligible.

Analysis

When steady operating conditions are reached, the rate of heat loss from the wire will equal the rate of heat generation in the wire as a result of resistance heating.

$$\dot{Q} = \dot{E}_{generated} = VI = 60 \times 1.5 = 90\ W$$

That is , the surface area of the wire is

$$A = \pi DL = \pi \times \left(0.3 \times 10^{-2}\right) \times 2.0 = 0.01885\ m^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q} = hA(T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficient is to be determined to be

$$90 = h(0.01885)(152 - 15)$$

$$h = 34.9\ \frac{W}{m^2 C}$$

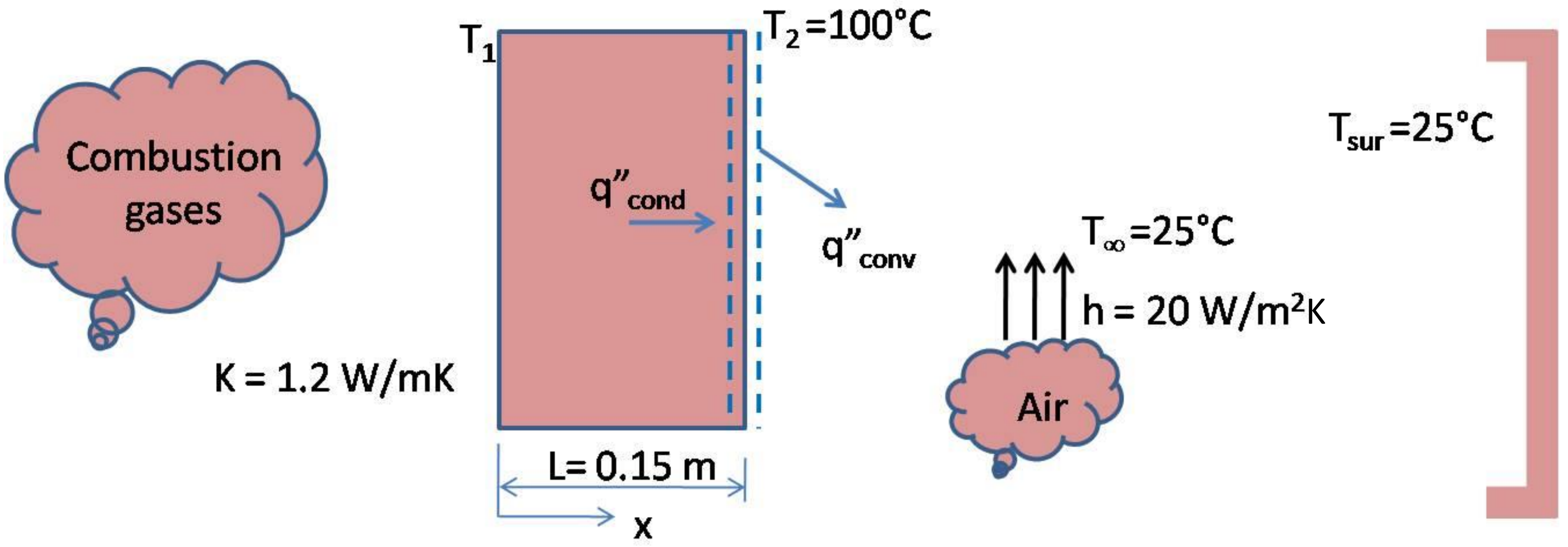
Comments:

Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

Problem: The hot combustion gases of a furnace are separated from the ambient air and its surroundings, which are at 25^0 C, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m.K. Under steady state conditions an outer surface temperature of 100^0 C is measured. Free convection heat transfer to the air adjoining the surface is characterized by a convection coefficient of h = 20 $W/m^2.K$. What is the brick inner surface temperature. Neglect any heat transfer by radiation.

Known: outer surface temperature of a furnace wall of prescribed thickness, thermal conductivity, ambient conditions

Find: Wall inner surface temperature



Assumptions:

1. Steady state conditions
2. One dimensional heat transfer by conduction across the wall
3. Radiation heat transfer is neglected

Analysis:

The inside surface temperature may be obtained by performing an energy balance at the outer surface.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{E}_{in} + \dot{E}_{g} - \dot{E}_{out} = \dot{E}_{st}$$

it follows that, on a unit area basis,

$$q''_{cond} - q''_{conv} = h(T_2 - T_\infty) = 0$$

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty)$$

$$1.2\left(\frac{T_1 - 100}{0.15}\right) = 20(100 - 25)$$

$$T_1 = 287.5°\mathrm{C}$$

Comments

Brick surface temperature is high

STEADY STATE CONDUCTION

Objectives

The aim of this lecture is to understand the Fourier's law of conduction (both physically and mathematically) and introduce various thermal properties like thermal conductivity and thermal diffusivity

INTRODUCTION

- Heat transfer has direction as well as magnitude.
- The rate of heat conduction in a specified direction is proportional to the temperature gradient.
- Heat conduction in a medium is three dimensional and time dependent

$$T = f(x, y, z, t)$$

Heat conduction in a medium is said to be

– **STEADY** - temperature does not vary with time.

– **UNSTEADY** (transient) - temperature varies with time

- **Heat conduction in a medium is three dimensional and time dependent**

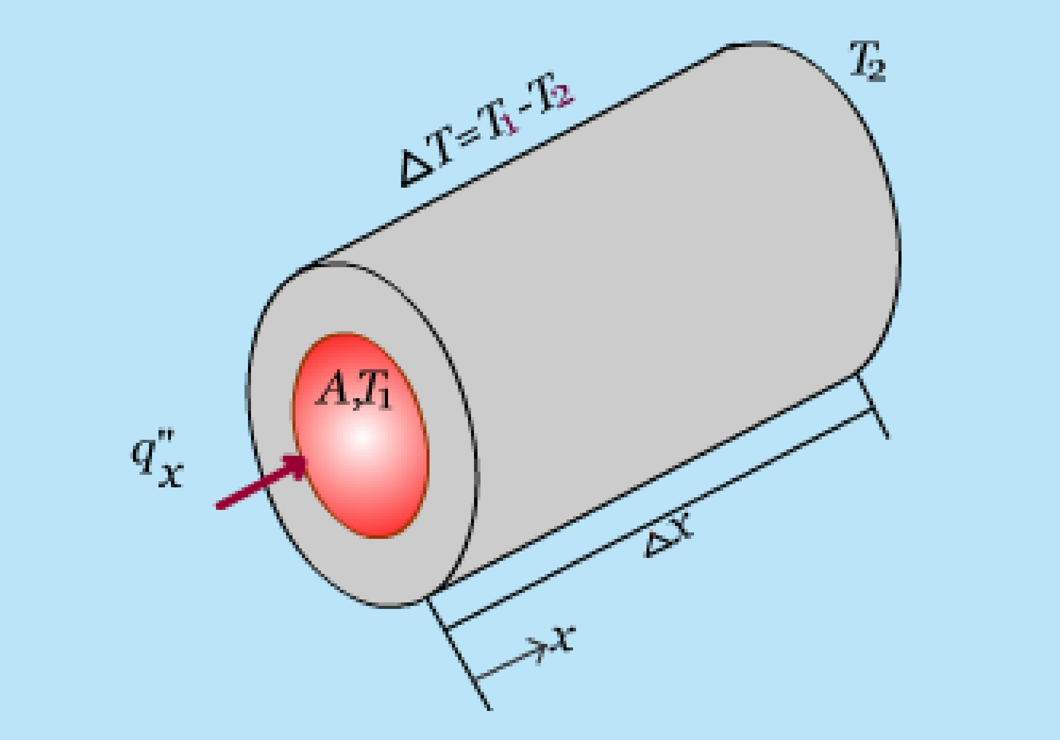
$$T = f(x, y, z, t)$$

- **One dimensional** $$T = f(x) = f(y) = f(z)$$

- **Two dimensional** $$T = f(x, y) = f(y, z) = f(x, z)$$

- **Three dimensional** $$T = f(x, y, z)$$

CONDUCTION RATE EQUATION



A cylindrical rod of known material is insulated on its lateral surface.

Its end faces are maintained at different temperatures with $T_1 > T_2$

The heat transfer rate q_x *depends upon,*

- The temperature difference, ΔT
- The rod length, Δ*x*
- The cross sectional area, *A*

The heat transfer rate varies as,

$$Q_x \, \alpha \, A \frac{\Delta T}{\Delta x}$$

the proportionality may be converted to an equality by introducing a coefficient that is a measure of the material behavior.

where *k* is the thermal conductivity (*W/m.K*)

$$Q_x = k \, A \frac{\Delta T}{\Delta x}$$

Applying limits

$$\lim_{\Delta x \to 0} Q_x = \lim_{\Delta x \to 0} k\, A \frac{\Delta T}{\Delta x}$$

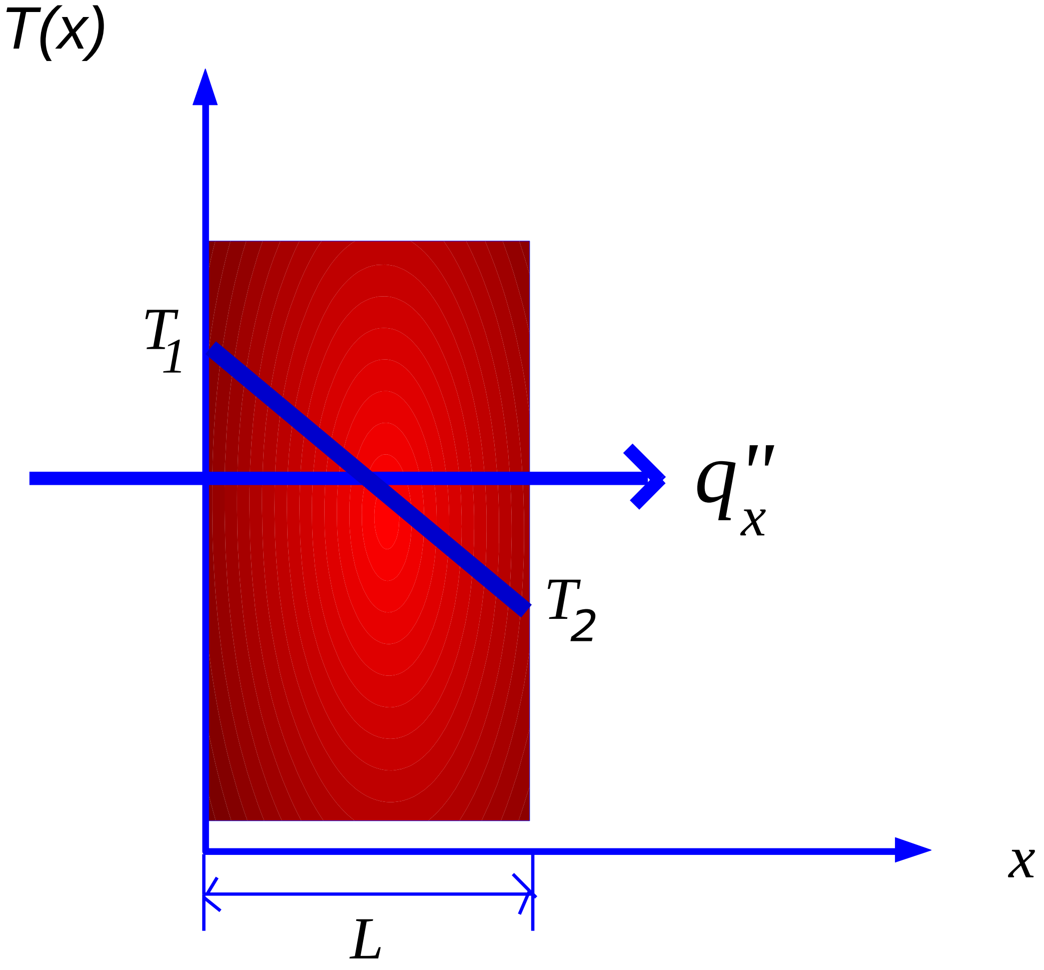
$$Q_x = -k\, A \frac{dT}{dx}$$

The minus sign is necessary because heat is always transferred in the direction of decreasing temperature.

The heat flux is given by

$$q''_x = \frac{Q_x}{A} = -k\, \frac{dT}{dx}$$

Fourier's law, as written in the above equation, follows that heat flux is normal to the cross sectional area A called an isothermal surface as illustrated in Figure



Relation between co-ordinate system, heat flow direction and temperature gradient in one dimension

Generalizing the conduction rate equation for three dimension gives,

$$q'' = -k\nabla T = -k\left(\hat{\imath}\frac{\partial T}{\partial x} + \hat{\jmath}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

In Cartesian coordinates, the general expression for q'' is

$$q'' = \hat{\imath}q''_x + \hat{\jmath}q''_y + \hat{k}q''_z$$

$$q''_x = -k\,\frac{\partial T}{\partial x} \qquad q''_y = -k\,\frac{\partial T}{\partial y} \qquad q''_z = -k\,\frac{\partial T}{\partial z}$$

NOTE: Here we have assumed that,

The medium in which conduction occurs is isotropic.

The thermal conductivity is independent of the coordinate direction in an isotropic medium.

THERMAL CONDUCTIVITY

- Thermal conductivity of a material is defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
- The thermal conductivity of a material is a measure of how fast heat will flow in that material.
- A large value for thermal conductivity indicates that the material is a good heat conductor,
- A low value indicates that the material is a poor heat conductor or insulator.

$$Q_x = -k\,A\frac{dT}{dx}$$

Range Of Thermal Conductivity For Various States Of Matter At Normal Temperature And Pressure

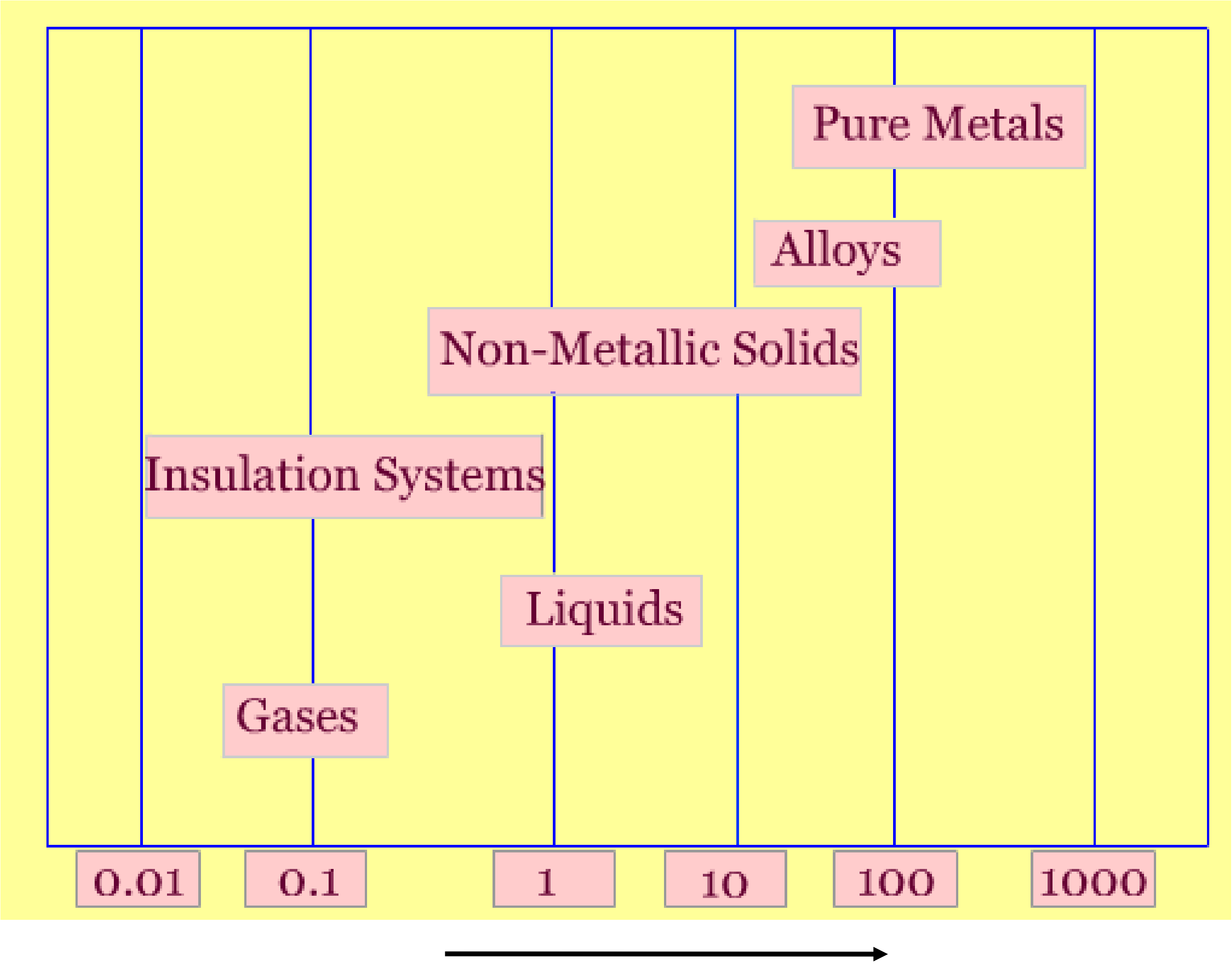
Conductivity of a solid may be more than four orders of magnitude larger than that of a gas.

This trend is largely due to differences in intermolecular spacing for the two states

$$k_{air} = 0.02635\ \frac{W}{m.K}$$

$$k_{water} = 0.6\ \frac{W}{m.K}$$

$$k_{copper} = 400\ \frac{W}{m.K}$$



Thermal Conductivity K (W/mK)

THE SOLID STATE

A solid may comprised free electrons and of atoms bound in a periodic arrangement called the lattice. Accordingly, transport of thermal energy is due to two effects:

- the migration of free electrons and
- lattice vibrational waves.
- These effects are additive, such that the thermal conductivity k is the sum of the electronic component k_e and the lattice component k_i

$$\boxed{k = k_e + k_l}$$

k_e is inversely proportional to the electrical resistivity ρ_e .

- For pure metals, which are of low ρ_e, k_e is much larger than k_l.
- For alloys, which are of substantially larger ρ_e, the contribution of k_l to k is no longer negligible.
- For non-metallic solids, k is determined primarily by k_l , which depends on the frequency of interactions between the atoms of the lattice.
- The regularity of the lattice arrangement has an important effect on k_l , with crystalline (well-ordered) materials like quartz having a higher thermal conductivity than amorphous materials like glass.

- In fact, for crystalline, non-metallic solids such as diamond and beryllium oxide, k_l can be quite large, exceeding values of k associated with good conductors, such as aluminium.

INSULATION SYSTEMS

Thermal insulation systems are comprised of low thermal conductivity materials combined to achieve an even lower system thermal conductivity.

In fiber-, powder-, flake-type insulations, the solid material is finely dispersed throughout an air space to achieve effective thermal conductivity.

Effective thermal conductivity of such systems depends on

- the thermal conductivity
- surface radiative properties of the solid material,
- the nature and volumetric fraction of the air or void space.
- A special parameter of the system is its bulk density (solid mass/total volume), which depends strongly on the manner in which the solid material is interconnected

THE FLUID STATE

- Since the intermolecular spacing is much larger and the motion of the molecules is more random for the fluid state than for the solid state, thermal energy transport is less effective.
- Therefore, the thermal conductivity of gases and liquids is smaller than that of solids.

THERMAL DIFFUSIVITY

- In the analysis of heat transfer problems, it will be necessary to use many properties of matter.
- These properties are generally referred to as thermophysical properties and consists of two distinct categories,
  - transport properties – include the diffusion rate coefficients such as *k*, (for heat transfer), and, the kinematic viscosity (for momentum transfer)
  - thermodynamic properties - pertain to the equilibrium state of a system such as density (ρ) and specific heat (C_p)
- The product ρC_p commonly termed volumetric heat capacity, measures the ability of a material to store thermal energy.

THERMAL DIFFUSIVITY

The ratio of thermal conductivity to heat capacity is an important property termed the thermal diffusivity , which has units of *m^2/s*.

$$\alpha = \frac{k}{\rho C_P}$$

- **It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy.**

NOTE:

- **Materials of large α will respond quickly to changes in their thermal environment**
- **Materials of small α will respond more sluggishly to reach new equilibrium condition.**

For copper, $\alpha = 117 \times 10^{-6}\,\frac{m^2}{s}$

For air, $\alpha = 22.5 \times 10^{-6}\,\frac{m^2}{s}$

For wood, $\alpha = 0.177 \times 10^{-6}\,\frac{m^2}{s}$

THREE DIMENSIONAL HEAT DIFFUSION EQUATION IN CARTESIAN CO-ORDINATES AND VARIOUS BOUNDARY AND INTIAL CONDITIONS

OBJECTIVES

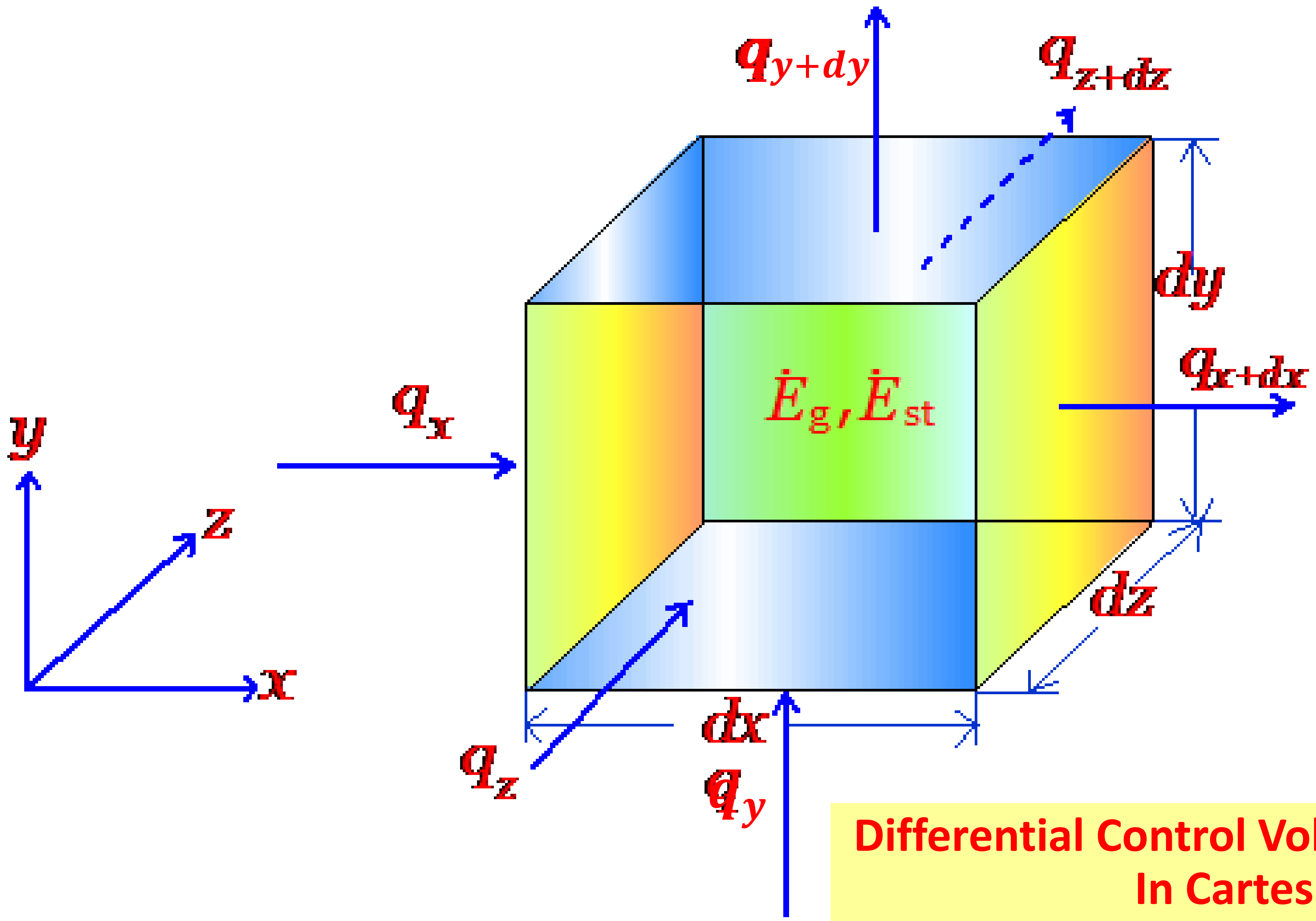
- To derive the from first principle and various boundary and initial conditions are stated.

THE HEAT DIFFUSION EQUATION

- A major objective in a conduction analysis is to determine the temperature field in a medium resulting from conditions imposed on its boundaries.
- We now consider the manner in which the temperature distribution can be determined.
- We define a differential control volume, identify the relevant energy transfer processes, and introduce the appropriate rate equations.
- The result is a differential equation whose solution, for prescribed boundary conditions, provides the temperature distribution in the medium.
- To determine the differential equation whose solution, for prescribed boundary conditions, provides the temperature distribution in the medium.

Consider a homogenous medium with no bulk motion and temperature distribution *T*(*x,y,z*)
Consider a infinitesimally small control volume *dx.dy.dz* , as shown in Figure

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$



Differential Control Volume For Conduction Analysis In Cartesian Coordinates

The conduction heat rates perpendicular to each of the control surfaces at the *x*, *y* and *z* coordinate locations are indicated by the terms q_x, q_y and q_z respectively.

The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher order

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx \qquad q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy \qquad q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

Within the medium there may also be an energy source term. This term is represented as

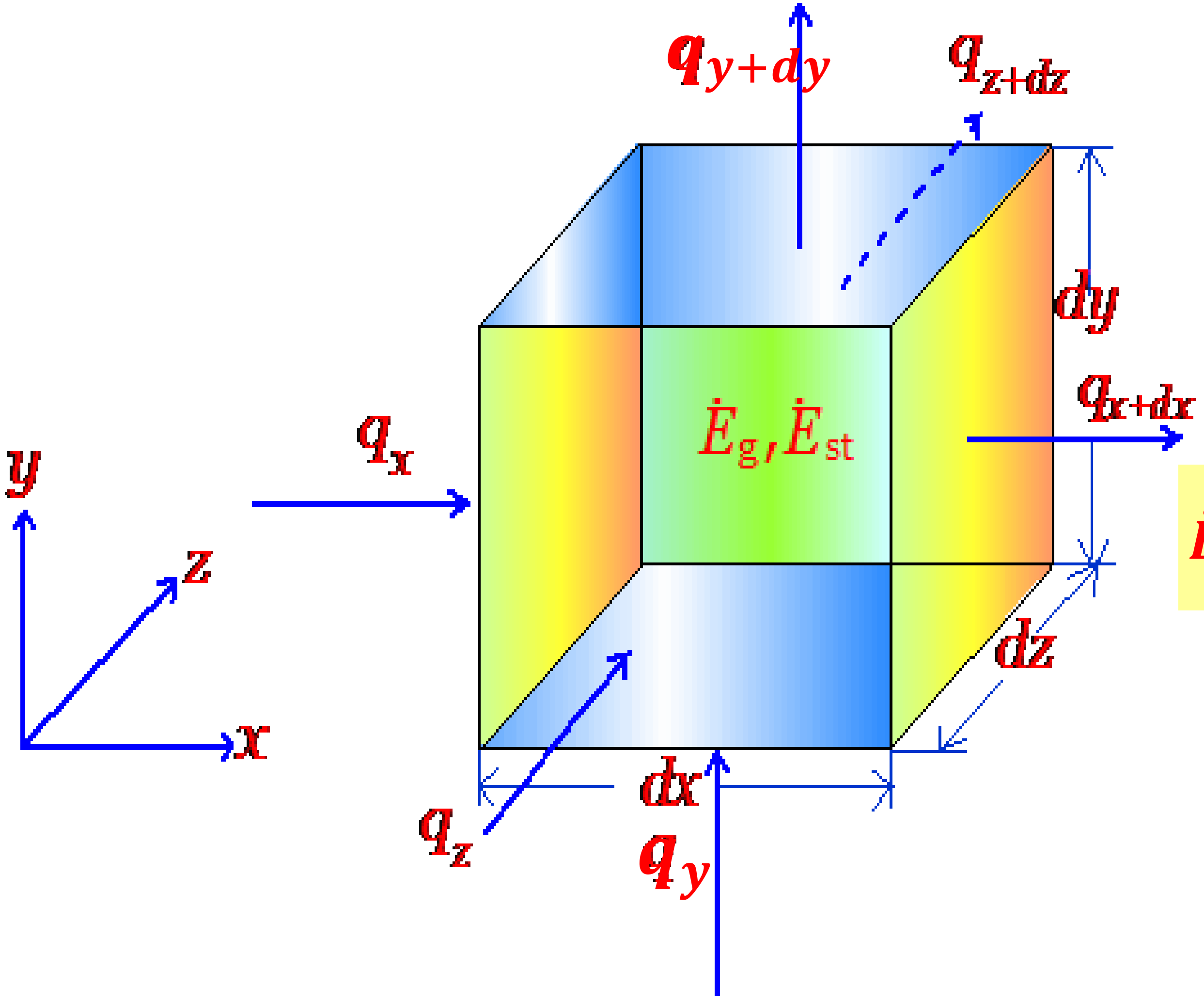
$$\dot{E}_g = \dot{q} dxdydz$$

$\dot{q}$-rate at which energy is generated per unit volume of medium (W/m^3)

There may occur changes in the amount of the internal thermal energy stored by the material in the control volume. The energy storage term may be expressed as

$$\dot{E}_{st} = \rho C_p \frac{\partial T}{\partial t} dxdydz$$

where $\rho C_p \frac{\partial T}{\partial t}$ is the time rate of change of the thermal (sensible) energy of the medium per unit volume.



$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{E}_{in} = q_x + q_y + q_z$$

$$\dot{E}_g = \dot{q}dxdydz$$

$$\dot{E}_{out} = q_x + \frac{\partial q_x}{\partial x}dx + q_y + \frac{\partial q_y}{\partial y}dy + q_z + \frac{\partial q_z}{\partial z}dz$$

$$\dot{E}_{st} = \rho C_p \frac{\partial T}{\partial t}dxdydz$$

$$q_x + q_y + q_z + \dot{q}dxdydz - q_x - \frac{\partial q_x}{\partial x}dx - q_y - \frac{\partial q_y}{\partial y}dy - q_z - \frac{\partial q_z}{\partial z}dz = \rho C_p \frac{\partial T}{\partial t}dxdydz$$

$$\dot{q}dxdydz - \frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz = \rho C_p \frac{\partial T}{\partial t}dxdydz$$

$$\dot{q}dxdydz - \frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz = \rho C_p \frac{\partial T}{\partial t}dxdydz$$

$$q_x = -k\,\frac{\partial T}{\partial x}dydz \qquad q_y = -k\,\frac{\partial T}{\partial y}dxdz \qquad q_z'' = -k\,\frac{\partial T}{\partial z}dxdy$$

$$\dot{q}dxdydz - \frac{\partial}{\partial x}\left(-k\,\frac{\partial T}{\partial x}\right)dxdydz - \frac{\partial}{\partial y}\left(-k\,\frac{\partial T}{\partial y}\right)dydxdz - \frac{\partial}{\partial z}\left(-k\,\frac{\partial T}{\partial z}\right)dzdxdy = \rho C_p \frac{\partial T}{\partial t}dxdydz$$

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Equation is the general form of the heat diffusion equation in Cartesian coordinates.
From the solution of this heat equation, we can obtain the temperature distribution $T(x, y, z)$ as a function of time.
If the thermal conductivity is a constant, the heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \alpha = \frac{k}{\rho C_P}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

Equation is the general form of the heat diffusion equation in Cartesian coordinates.
If the thermal conductivity is a constant, the heat equation is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\alpha=\frac{k}{\rho C_P}$$

Under steady-state conditions , there can be no change in the amount of energy storage; hence equation reduces to

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

If the heat transfer is one dimensional (e.g., in the *x* direction) and there is no energy generation , equation reduces to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$$

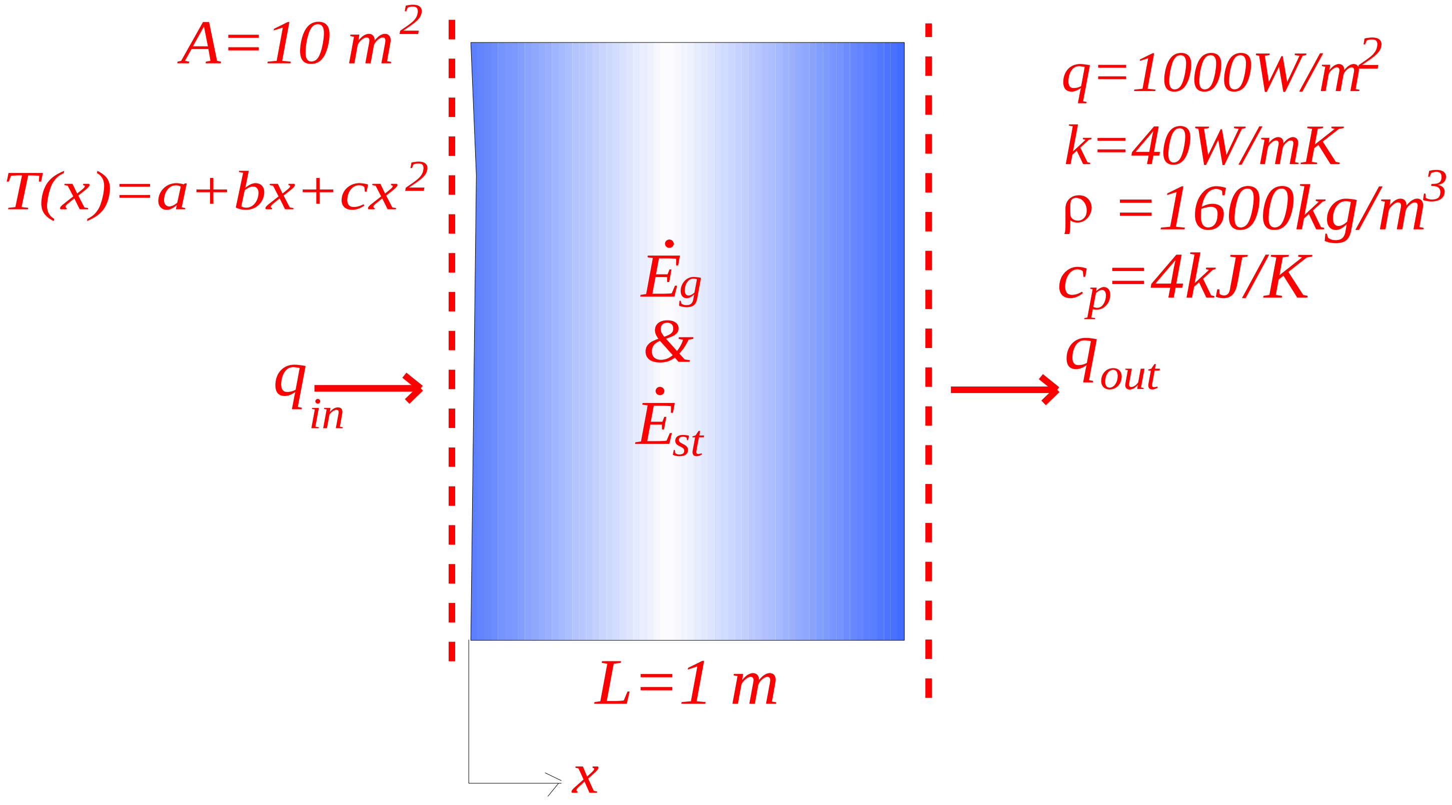
The most important implication of this result is that under steady state, one dimensional conditions with no energy generation, the heat flux is a constant in the direction of heat transfer

The temperature distribution across a wall of 1 m thick at a certain instant of time is given as *T(x)* = $a+bx+cx^2$ where *T* is in degrees Celsuis and x is in meters, while

a = 800 °C, *b* = -350 °*C/m*, and *c* = -60 °C/m^2 . A uniform heat generation, = 1000 W/m^3, is present in the wall of area 10 m^2 having the properties *ρ* = 1600 kg/m^3, *k* = 40 *W/m.K*, and C_p = 4 *kJ/kg.K*.

- Determine the rate of heat transfer entering the wall (*x* = 0) and leaving the wall (*x* = 1*m*)
- Determine the rate of change of energy storage in the wall
- Determine the time rate of temperature change at *x* = 0.25 and 0.5 *m*

$$T(x) = 800 - 350x - 60x^2$$



Known: Temperature distribution *T(x)* at an instant of time *t* in a one dimensional wall with uniform heat generation.

Find:

- Heat rates entering, q_{in}(x = 0), and leaving, q_{out}(x = 1), the wall
- Rate of change of energy storage in the wall,
- Time rate of temperature change at x = 0.25 and 0.5 m.

Assumptions:

- One dimensional conduction in the x -direction.
- Homogenous medium with constant properties.
- Uniform heat generation.

Analysis:

Recall that once the temperature distribution is known for a medium, it is a simple matter to determine the conduction heat transfer rate at any point in the medium, or at its surfaces, by Fourier's law.

Hence the desired heat rates may be determined by using the prescribed temperature distribution with Equation. Accordingly,

$$T(x) = 800 - 350x - 60x^2$$

$$q_{in} = q_x(0) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

$$q_{in} = q_x(0) = -kA(-350 - 120x)_{x=0}$$

$$q_{in} = q_x(0) = -40 \times 10(-350)$$

$$q_{in} = 140\ kW$$

$$q_{out} = q_x(L) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=L}$$

$$q_{out} = q_x(L) = -kA(-350 - 120x)_{x=1}$$

$$q_{out} = q_x(L) = -40 \times 10(-350 - 120)$$

$$q_{out} = 188\ kW$$

The rate of change of energy storage $\dot{E}_{st}$ in the wall may be determined by applying an overall energy balance to the wall. Using Equation for a control volume about the wall

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{E}_g = \dot{q}AL = 1000 \times 10 \times 1 = 10\ kW$$

$$140 + 10 - 188 = \dot{E}_{st}$$

$$\dot{E}_{st} = -38\ kW$$

The time rate of change of change of the temperature at any point in the medium may be determined from the heat equation rewritten as

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{k}{\rho C_p}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\dot{q}}{\rho C_p}=\frac{\partial T}{\partial t}$$

From the prescribed temperature distribution, it follows that

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)=\frac{\partial}{\partial x}(-350-120x)=-120\,\frac{℃}{m^2}$$

$$T(x)=800-350x-60x^2$$

Note that this derivative is independent of position in the medium. Hence the time rate of temperature change is also independent of position and is given by

$$\frac{k}{\rho C_p}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\dot{q}}{\rho C_p}=\frac{\partial T}{\partial t}$$

ρ = 1600 kg/m^3
k = 40 $W/m.K$
C_p = 4 $kJ/kg.K$

$$\dot{q}=1000\,\frac{W}{m^3}$$

$$\frac{40}{1600\times 4000}(-120)+\frac{1000}{1600\times 4000}=\frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t}=-5.94\times 10^{-4}\,\frac{℃}{s}$$

- Heat rates entering, $q_{in}(x = 0)$, and leaving, $q_{out}(x = 1)$, the wall
- Rate of change of energy storage in the wall,
- Time rate of temperature change at x = 0.25 and 0.5 m.

$$q_{in} = 140\ kW$$

$$q_{out} = 188\ kW$$

$$\dot{E}_{st} = -38\ kW$$

$$\frac{\partial T}{\partial t} = -5.94 \times 10^{-4}\ \frac{°C}{s}$$

Comments:

- From the above result it is evident that the temperature at every point within the wall is decreasing with time.
- Fourier's law can always be used to compute the conduction heat rate from knowledge of the temperature distribution, even for unsteady conditions with internal heat generation

BOUNDARY AND INITIAL CONDITIONS

- To determine the temperature distribution in a medium, it is necessary to solve the appropriate form of the heat equation.

- However, such a solution depends on the physical conditions existing at the boundaries of the medium and, if the situation is time dependent, on conditions existing in the medium at some initial time.

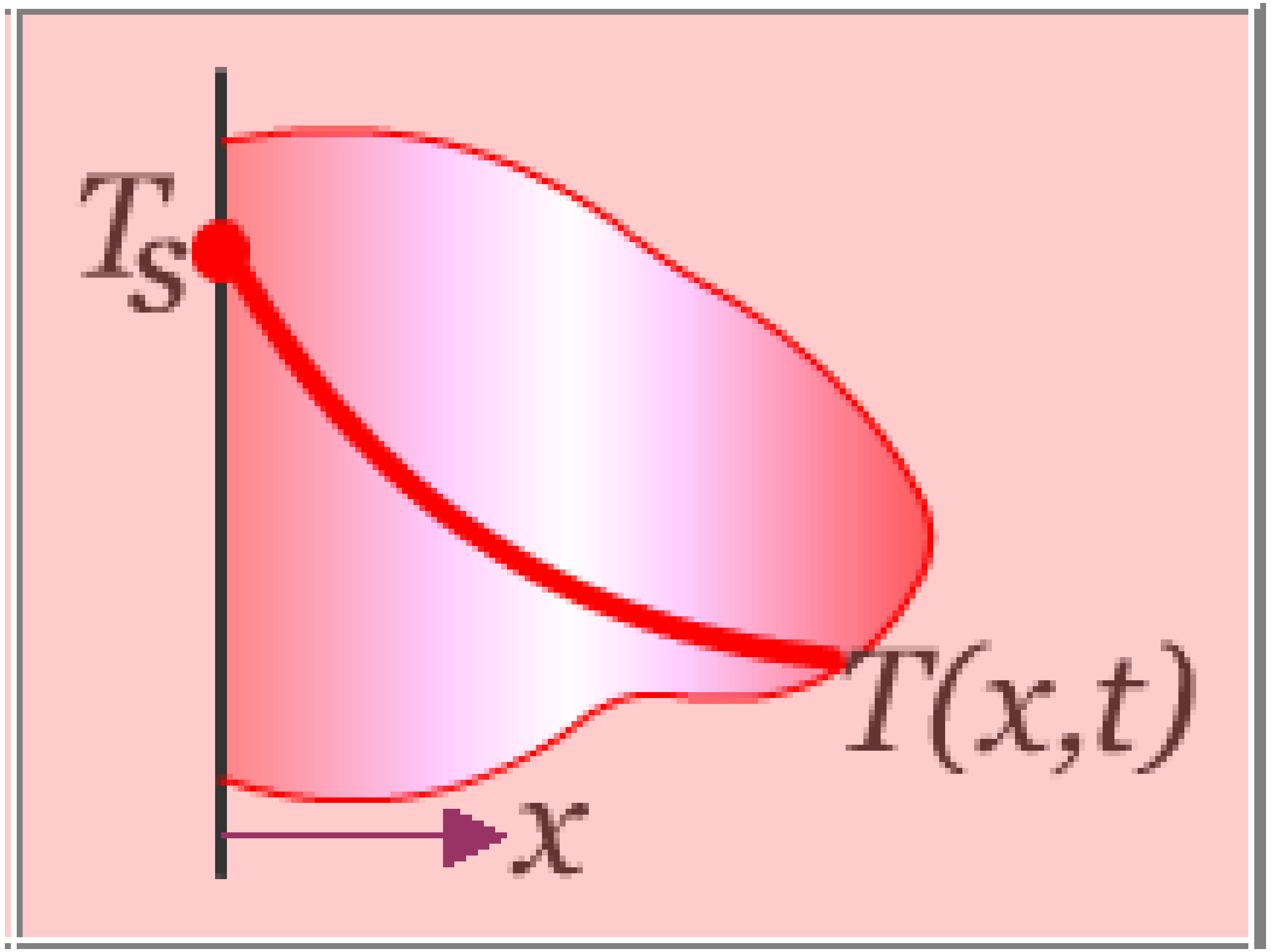
- Because the heat equation is second order in the spatial coordinates, two boundary conditions must be expressed for each coordinate to describe the system.

- Because the equation is first order in time, however, only one condition, termed the initial condition, must be specified.

BOUNDARY CONDITIONS FOR THE HEAT DIFFUSION EQUATION AT THE SURFACE (x=0)

The conditions are specified at the surface $x = 0$ for a one-dimensional system.

Heat transfer is in the positive x direction with the temperature distribution, which may be time dependent, designated as $T(x, t)$.



CONSTANT SURFACE TEMPERATURE

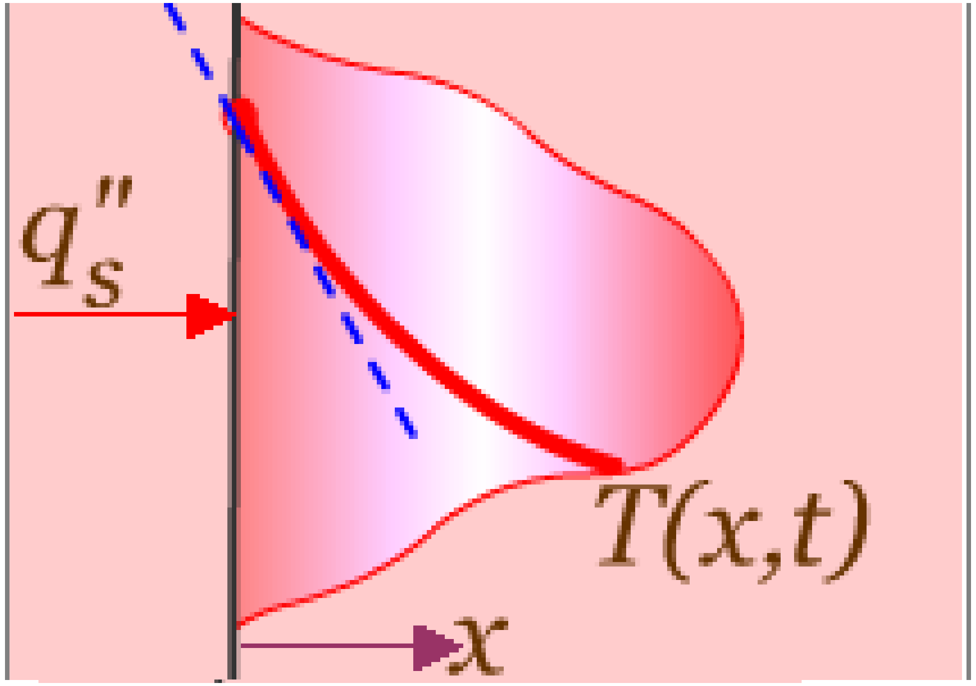
$$T(0, t) = T_s$$

The first condition corresponds to a situation for which the surface is maintained at a fixed temperature T_s.

It is commonly termed a Dirichlet condition, or a boundary condition of the first kind.

Example: when the surface is in contact with a melting solid or a boiling liquid. In both cases there is heat transfer at the surface, while the surface remains at the temperature of the phase change process.

BOUNDARY CONDITIONS FOR THE HEAT DIFFUSION EQUATION AT THE SURFACE (x=0)

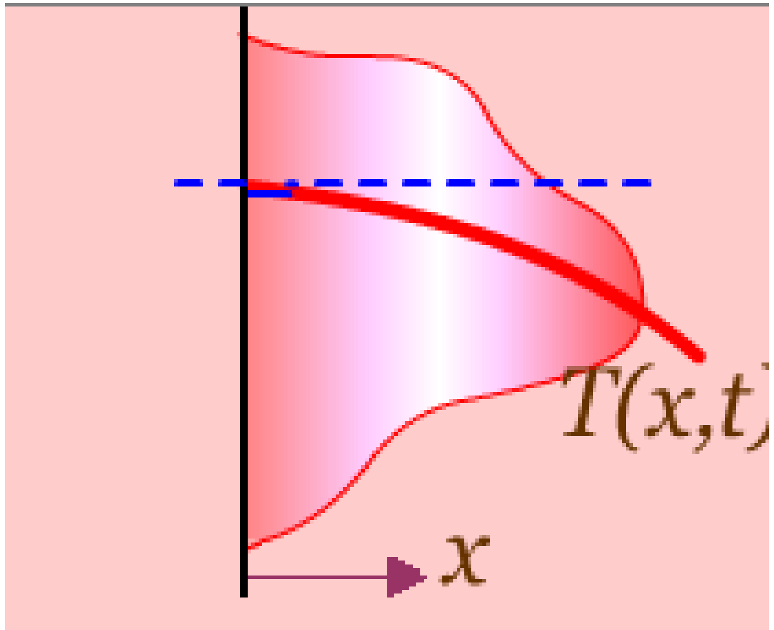


CONSTANT HEAT FLUX

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s''$$

The second condition corresponds to the existence of a fixed or constant heat flux q_s'' at the surface.

The second condition is termed as Neumann condition, or a boundary condition of the second kind, and may be realized by bonding a thin film or patch electric heater to the surface.

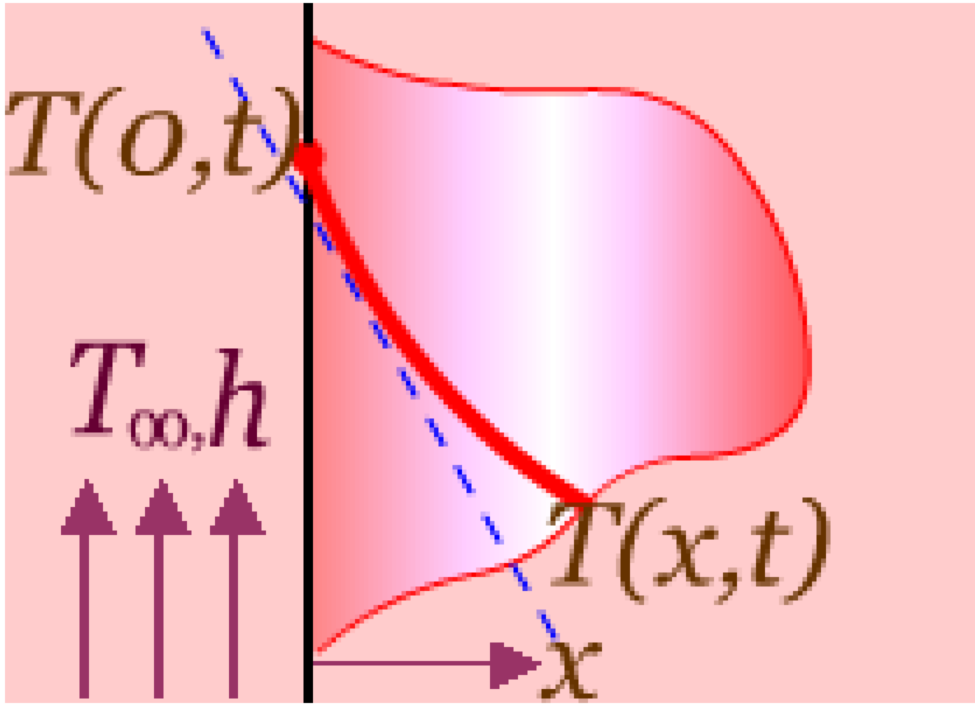


ADIABATIC OR INSULATED SURFACE

A special case of this condition corresponds to the perfectly insulated, or adiabatic, surface for which

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$

BOUNDARY CONDITIONS FOR THE HEAT DIFFUSION EQUATION AT THE SURFACE (x=0)



CONVECTION SURFACE CONDITION

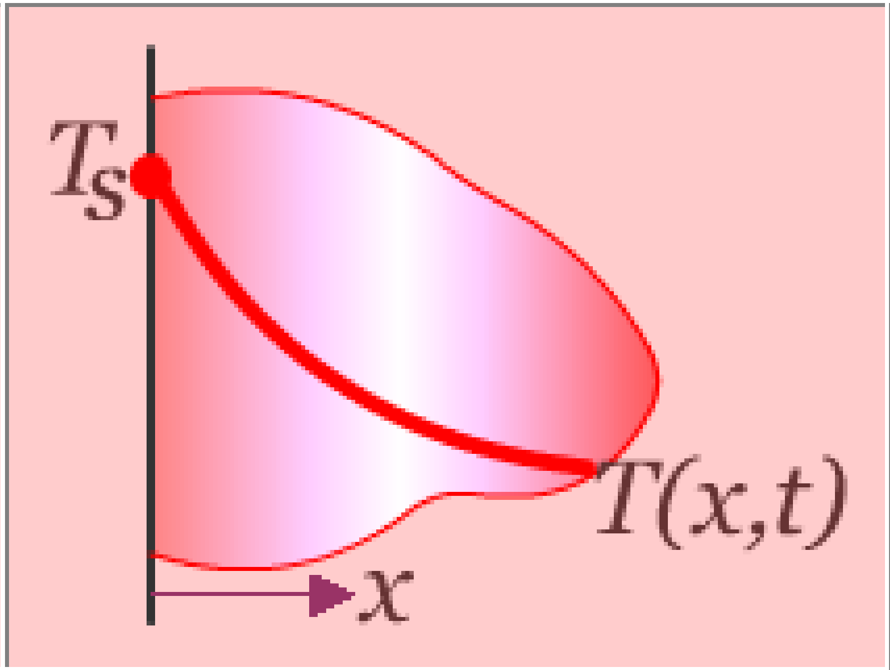
The boundary condition of the third kind corresponds to the existence of convection heating (or cooling) at the surface and is obtained from the surface energy balance.

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)]$$

BOUNDARY CONDITIONS FOR THE HEAT DIFFUSION EQUATION AT THE SURFACE (x=0)

1. Constant surface temperature

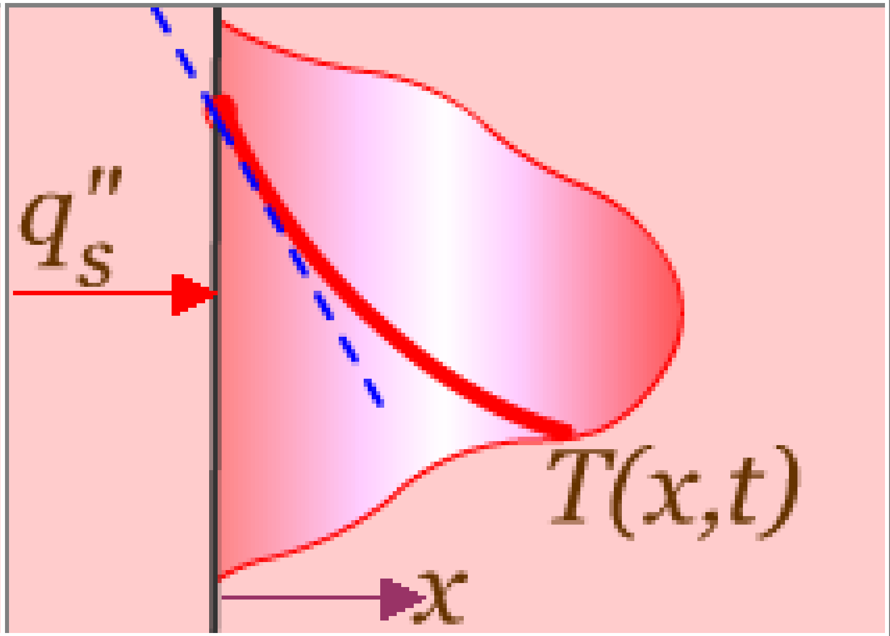
$$T(0,t) = T_s$$



2. Constant Surface Heat Flux

a. Finite Heat Flux

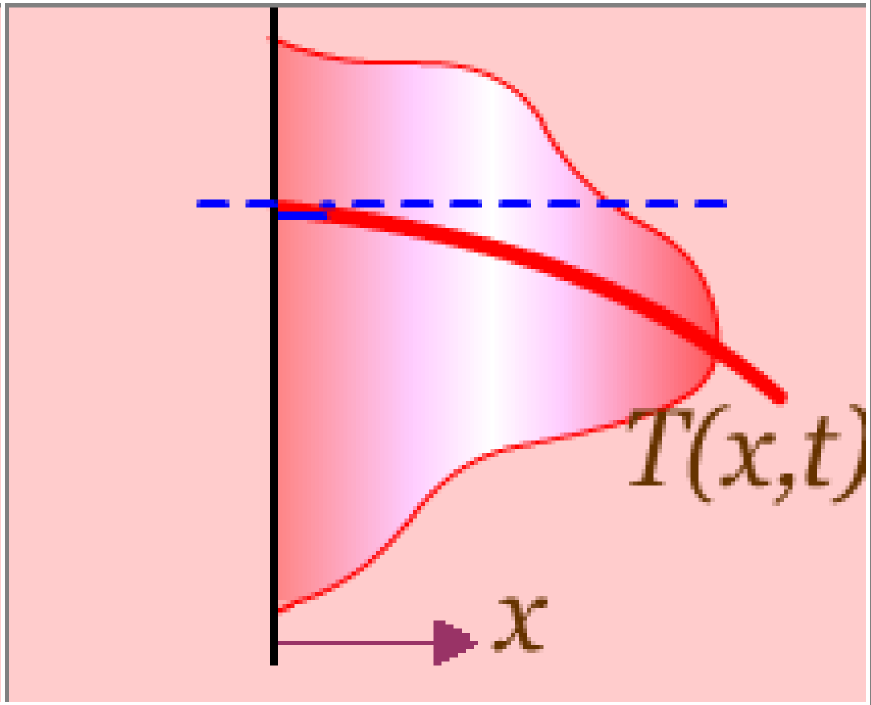
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s''$$



BOUNDARY CONDITIONS FOR THE HEAT DIFFUSION EQUATION AT THE SURFACE (x=0)

b. Adiabatic or insulated surface

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$



3. Convection Surface condition

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right]$$