1. Calcula els límits següents quan $n \to +\infty$

(a) $lim \quad \left[\frac{2n-1}{n+4} - \frac{3n^2+n}{6n^2+4n-1}\right]$ (sol: $\frac{3}{2}$)

(b) $lim \quad \left[\frac{3n+1}{4n-2} \cdot \frac{4n^2+1}{3n^2-4}\right]$ (sol: 1)

(c) $lim \quad \left[\frac{2n^2+4}{n+2} + \frac{n^2+3n}{2n+5}\right]$ (sol: $+\infty$)

(d) $lim \quad \left[\frac{2n^3-1}{3n^3+4n} \cdot \frac{n^2+n}{3n-1}\right]$ (sol: $+\infty$)

(e) $lim \quad 3^{\frac{6n-1}{2n}}$ (sol: 27)

(f) $lim \quad \sqrt{\frac{2n+1}{n+5}}$ (sol: $\sqrt{2}$)

(g) $lim \quad 2^{\frac{n-5}{n^2+1}}$ (sol: 1)

(h) $lim \quad \left(\frac{2n-1}{2n+n}\right)^{\frac{5n-3}{2n+1}}$ (sol: $\frac{4}{9}\sqrt{\frac{3}{2}}$)

(i) $lim \quad \left(\sqrt{\frac{4n^2+3n}{n^2-5n}}\right)^{\frac{2n+1}{3n-4}}$ (sol: $\sqrt[3]{4}$)

(j) $lim \quad \left(\frac{n-2}{n+2}\right)^5$ (sol: 1)

(k) $lim \quad \left(\frac{n-2}{n+2}\right)^{\frac{5}{n}}$ (sol: 1)

(l) $lim \quad \left(n + \frac{1}{n^2}\right)^6$ (sol: $+\infty$)

(m) $lim \quad \left[\frac{3n^2+2}{6n^2+3n+1} - \frac{n+1}{n}\right]^2$ (sol: $\frac{1}{4}$)

(n) $lim \quad \left[\frac{-3n^3+2n^2-n+1}{4n^3-n^2+7n-5}\right]^2$ (sol: $\frac{9}{16}$)

(o) $lim \quad \left[\frac{n^5+3n^2+1}{2n^3+4n^2-n-2}\right]^{-3}$ (sol: 0)

(p) $lim \quad \left[\frac{2n^2+3n-1}{4n^4+2n^3-n^2+n+5}\right]^{-1}$ (sol: $+\infty$)

(q) $lim \quad \frac{1+2+3+...+n}{n^2-(3n-1)^2}$ (sol: $\frac{-1}{16}$)

(r) $lim \quad \frac{a+2a+3a+...+na}{(a+n)^2}$, $a \in R$ (sol: $\frac{a}{2}$)

(s) $lim \quad \frac{1+2+3+...+2n}{3n^2}$ (sol: $\frac{1}{3}$)

(t) $lim \quad \sqrt[n]{2}$ (sol: 1)

(u) $lim \quad \sqrt[2n^2]{\frac{2n+1}{n+1}}$ (sol: 1)

2. Donades les successions: $a_n = \frac{n^2-2n}{n^3+n}$ i $b_n = \frac{n^3-4n}{n^2+2n}$ calcula:

(a) $lim \quad a_n$ (sol: 0)

(b) $lim \quad b_n$ (sol: $+\infty$)

    (c) $lim \quad a_n \cdot b_n$ (sol: 1)

    (d) $lim \quad \frac{a_n}{b_n}$ (sol: 0)

    (e) $lim \quad a_n - b_n$ (sol: $-\infty$)

3. Donades les successions: $a_n = \frac{n^2+1}{n}$ i $b_n = \frac{2n-1}{n+2}$ calcula:

    (a) Calcula els seus límits (sol: $+\infty$ i 2)

    (b) $lim \quad a_n + b_n$ (sol: $+\infty$)

    (c) $lim \quad a_n - b_n$ (sol: $+\infty$)

    (d) $lim \quad a_n \cdot b_n$ (sol: $+\infty$)

    (e) $lim \quad \frac{a_n}{b_n}$ (sol: $+\infty$)

4. Calcula els límits següents:

    (a) $lim \quad \left(\frac{7n^12 - 8n^8 + n^5 - 3n}{2n^10 + 4n^9 - n^7 + 2n^5 + 4} \right) \cdot \left(\frac{-n^5 + 2n^4 - n^3 + 7n^2 - 1}{3n^7 + n^6 - 4n^3 - 5n + 2} \right)$ (sol: ind)

    (b) $lim \quad \frac{1 + \frac{n}{1-n}}{\frac{n-1}{n+1} - \frac{n+1}{n-1}}$ (sol: $\frac{1}{4}$)

    (c) $lim \quad \frac{(3n+1)^3 - (3n-1)^3}{2n^2 + 2}$ (sol: 27)

    (d) $lim \quad \left[\frac{2n^2 + 3n - 1}{3n^2 + n + 2} \right]^{\frac{n+1}{2n-3}}$ (sol: $\sqrt{\frac{2}{3}}$)

    (e) $lim \quad \left[\frac{2n^2 + 1}{4n^2 + 1} \right]^{\frac{3n^2+1}{4n}}$ (sol: 0)

    (f) $lim \quad \left[\frac{n^3 - 3n^2 + 7}{3n^3 + 4n^2 + 5n + 1} \right]^{\frac{1+n-2n^2}{4n+3}}$ (sol: $+\infty$)

    (g) $lim \quad \left[\frac{2n+1}{-n+3} \right]^{\frac{n+1}{2n-3}}$ (sol: $\nexists$)

    (h) $lim \quad \left[3 + \frac{4}{n^2+1} \right]^{\frac{n^2+1}{2n}}$ (sol: $+\infty$)

    (i) $lim \quad \frac{\sqrt{n^2+1} - n}{\sqrt{n^2+n} - 1}$ (sol: 0)

    (j) $lim \quad \frac{(n+2)^2 + \sqrt{n^3+n+1}}{\sqrt{n^4 + 2n^2}}$ (sol: 1)

    (k) $lim \quad \frac{\sqrt{n^3+1} - \sqrt{n}}{n + \sqrt{3n^3}}$ (sol: $\frac{\sqrt{3}}{3}$)

    (l) $lim \quad \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}}$ (sol: 0)

    (m) $lim \quad \left(\frac{n+1}{n^2} \cdot \frac{n^2+1}{n} \right)$ (sol: 1)

    (n) $lim \quad \frac{(2n+n^2)^3 - n^4}{n^5 + n - 2}$ (sol: $+\infty$)

    (o) $lim \quad \frac{\sqrt{n+1} - \sqrt{n+4}}{\sqrt{n-5}}$ (sol: 0)

    (p) $lim \quad \frac{(n+1)^3 - 3n^2 + 2}{n^4 + (1-n)^3}$ (sol: 0)

(q) $lim \quad \frac{\sqrt{n^3+2n+1}-n^2}{\sqrt{2+n^4}}$ (sol: -1)

(r) $lim \quad \frac{\sqrt{n^5+4n}+(n+5)^3}{\sqrt{n^7+2n^2+4}}$ (sol: 0)

(s) $lim \quad \left[3 + \frac{4}{n^2+1}\right]^{\frac{1-n^2}{3n+2}}$ (sol: 0)

(t) $lim \quad \left[n^3 + 4n^2 - 3n + 7\right]^{1+2n-n^2-3n^3}$ (sol: 0)

(u) $lim \quad \left[\frac{2n^2+1}{4n^3+2n}\right]^{-7n^2+1}$ (sol: $+\infty$)

(v) $lim \quad \left[\frac{1}{\sqrt{n+1}}\right]^{n+1}$ (sol: 0)

(w) $lim \quad \left[\frac{\sqrt{1+2n}}{1+8n} \cdot \sqrt[5]{\frac{3n^3+2n^2+n-2}{96n^3+10}}\right]$ (sol: $\frac{1}{4}$)

(x) $lim \quad \frac{(-1)^n \cdot n}{2n^2+3}$ (sol: 0)

(y) $lim \quad \left[\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + ... + \frac{n}{n^2}\right]$ (sol: $\frac{1}{2}$)

5. Calcula els límits següents:

(a) $lim \quad \frac{(-1)^n \cdot (2n^2+3)}{n^4+n^3+1}$ (sol: 0)

(b) $lim \quad \frac{\sqrt{n+3}}{3n+7}$ (sol: 0)

(c) $lim \quad \frac{\sqrt{n+1}+\sqrt{n-1}}{2n+1}$ (sol: 0)

(d) $lim \quad \left(\sqrt{(n+1)(n+2)} - \sqrt{n^2+1}\right)$ (sol: $\frac{3}{2}$)

(e) $lim \quad \left(\sqrt{3n^2+4n+7} - \sqrt{3n^2-5n+2}\right)$ (sol: $\frac{3\sqrt{3}}{2}$)

(f) $lim \quad \left(\sqrt{2n^2+1} - \sqrt{n^2+1}\right)$ (sol: $+\infty$)

(g) $lim \quad \left(\sqrt{n(n+1)} - n\right)$ (sol: $\frac{1}{2}$)

(h) $lim \quad \left(\sqrt{(n+1)(n+2)} - n\right)$ (sol: $\frac{3}{2}$)

(i) $lim \quad \left(\sqrt{n^2-10n+8} - (n+3)\right)$ (sol: -2)

(j) $lim \quad \left(5n - \sqrt{1+n+2n^2}\right)$ (sol: $+\infty$)

(k) $lim \quad \frac{\sqrt{n^2+a^2}-a}{\sqrt{n^2+b^2}-b}, a,b \in R$ (sol: 1)

(l) $lim \quad \frac{\sqrt{2+n}-3}{n}$ (sol: 0)

6. Calcula els límits següents:

(a) $lim \quad \left[1 + \frac{1}{n}\right]^{3n}$ (sol: e^3)

(b) $lim \quad \left[1+\frac{1}{n}\right]^{2n^2+3}$ (sol: $+\infty$)

(c) $lim \quad \left[1+\frac{1}{n}\right]^{\frac{n^2+1}{2n-1}}$ (sol: $\sqrt{e}$)

(d) $lim \quad \left[1+\frac{1}{n+3}\right]^{n+4}$ (sol: e)

(e) $lim \quad \left[1+\frac{1}{n^2+3n+1}\right]^{2n^2-n+1}$ (sol: e^2)

(f) $lim \quad \left[1-\frac{2}{n}\right]^{2n}$ (sol: $\frac{1}{e^4}$)

(g) $lim \quad \left[1-\frac{3}{n+1}\right]^{n+3}$ (sol: $\frac{1}{e^3}$)

(h) $lim \quad \left[\frac{n+1}{n-1}\right]^{n}$ (sol: e^2)

(i) $lim \quad \left[\frac{2n-3}{2n+7}\right]^{5n-4}$ (sol: $\frac{1}{e^25}$)

(j) $lim \quad \left[\frac{5n+4}{5n}\right]^{5n}$ (sol: e^4)

(k) $lim \quad \left[\frac{n^2+3}{n^2}\right]^{4n^2}$ (sol: e^{12})

(l) $lim \quad \left[\frac{1-5n}{3-5n}\right]^{n+1}$ (sol: $\sqrt[5]{e^2}$)

(m) $lim \quad \left[\frac{n^2+2n+1}{n^2+3n+2}\right]^{7n+1}$ (sol: $\frac{1}{e^7}$)

7. Indica quins valors ha de tenir a per obtenir els límits següents:

(a) $lim \quad ((a^2-1)n^3-3n^2+1)=+\infty$ (sol: $a\in\mathbb{R}-[-1,1]$)

(b) $lim \quad \frac{3an^2-7n+1}{(a+1)n^2+3}=2$ (sol: $a=2$)

(c) $lim \quad (\sqrt{n^2+an+1}-\sqrt{n^2-1})=2$ (sol: $a=4$)

(d) $lim \quad \left[\frac{2n+1}{2n-1}\right]^{an-3}=e^{-4}$ (sol: $a=-4$)

8. Calcula a i b perquè es compleixin:

(a) $lim \quad \frac{(1-a)n^3+2bn^2+3n-1}{(n^2-1}=3$ (sol: $a=1$, $b=\frac{3}{2}$)

(b) $lim \quad \left[\frac{an+1}{3n+2}\right]^{bn-3}=e^5$ (sol: $a=3$, $b=-15$)

9. Calcula els límits següents:

(a) $lim \quad \left[\frac{1-n-3n^2}{4+5n-3n^2}\right]^{2n^2-3n+1}$ (sol: 0)

(b) $lim \quad \left[\frac{3n^2+1}{3n^2-1}\right]^{\frac{n}{n^2+1}}$ (sol: 1)

(c) $lim \quad \left[\dfrac{\sqrt{n}}{n+2}\right]^{\frac{-3n}{n+2}}$ (sol: $+\infty$)

(d) $lim \quad \left[\dfrac{n^3+n^2+2}{n^2+2}\right]^{\frac{8n^2-1}{n^2+1}}$ (sol: $+\infty$)

(e) $lim \quad \left[\sqrt{n+1}-\sqrt{n-1}\right]^{1-\sqrt{n+1}}$ (sol: $+\infty$)

(f) $lim \quad \left[\dfrac{4n+3}{1-4n}+2\right]^{\sqrt{n^2+1}}$ (sol: $\frac{1}{e}$)