Matthew Dow

Course: AEM 669 Advanced Astrodynamics

Homework #4

Due Date: March 10, 2024

Problem D1:

a) Using the given initial conditions, the spacecraft was propagated for ~0.48 nondimensional time units before stopping at the first x-axis crossing. The trajectory is shown in Figure 1 on page 8
   i) The STM was integrated to this final time value using coupled integration with the spacecraft state. The STM at time tf is shown in figure 2 on page 8

b) The table below shows the original final state as well as the perturbed final states using the given initial perturbations and the STM calculated using Coupled Integration:

Case	x	y	vx	vy
Reference	-0.25187	-0.00631	-0.76609	-2.13504
Case #1 X offset	-0.25182	-0.00704	-0.76474	-2.13579
Case #2 Vy offset	-0.23686	0.008643	-0.77009	-2.12732

   i) The plot of the three trajectories (reference, case 1, and case 2) are shown in Figure 3 on page 9
   ii) The plot and table both show that the offset of the initial x value seemed to affect the final position of the spacecraft much more than the offset in the initial vy value.

c) The table below shows the final states of each case in part c. Since the initial state is not changing, but the flight duration is increasing, it is expected that the spacecraft would follow the same path over time, but the length of the path would increase. This can be seen in the table by the x and y values continuing to trend in the same direction for this small duration. The velocity values do not correlate well with the reference. These results can also be seen in Figure 4 on page 10, where all 3 curves are on top of each other, with Case 2's curve being the longest.

Case	x	y	vx	vy
Reference	-0.25187	-0.00631	-0.76609	-2.13504
Case #1 1% tf offset	-0.623642	-1.04240	5.37697	-1.17526
Case #2 10% tf offset	-0.65677	-1.13473	5.92437	-1.08973

Problem D2:

a) The dimensional time of flight from the initial conditions to the first x-axis crossing is ~2.086 days. The plots of these trajectories is shown in Figures 5, 6, and 7 on pages 11,12, and 13 respectively.

Case	# Iterations	DV vec [km/s]	DV mag [km/s]	TOF [days]
1	5	(-0.159,0.045)	0.166	2.086
2	7	(0.623,-0.116)	0.634	2.086
3	6	(-0.545,-0.100	0.554	2.086

i) A comparison was run for the targeter using the baseline STM and the STM from the previous iteration, and it showed that the targeter was able to find a solution in both cases, but reached the solution in fewer iterations when using the STM from the previous iteration, not the baseline.

b) The same cases were then run with variable time while keeping the x velocity constant, producing the following results:

Case	# Iterations	DV vec [km/s]	DV mag [km/s]	TOF [days]
1	5	(0,0.100)	0.100	2.287
2	8	(0,-0.362)	0.362	1.458
3	6	(0,0.131)	0.131	2.868

i) The plotted trajectories for each of these cases is in Figures 8, 9, and 10 on pages 14,15, and 16 respectively.

ii) As for how far away the target can be and still converge, I assume that the targeter would most likely start to have trouble converging to a solution once the target starts to depart from the system's local area (within the Moon's orbit)

Problem D3:

a) Fixed-Time, Velocity Free Targeter:

i) E vector before corrections:

1) $\bar{e} = [-\ 0.0788, 0.0233, 0.1398] = [-\ 30282, 8972, 53758]km$

ii)

Iteration	DV vec [km/s]	DV mag [km/s]	e [km]
0	(0,0,0)	0	[-30281,8972,53575]
1	(0.043,-0.093,0.161)	0.191	[-7867,11135,-10882]
2	(0.041,-0.098,0.149)	0.183	[138,-27.6,-10.8]
3	(0.041,-0.097,0.149)	0.183	[-0.029,0.021,-0.015]
4	(0.0411,-0.0975,0.1494)	0.1831	[-1.0e-9,1.5e-10,-1.7e-10]

iii) For these iterations, the STM from the previous guess was used to update the initial conditions. If the baseline was used, it would not align with the states that are in the current guess, and the Jacobian used in the correction process would not be as accurate.

iv) For these initial conditions, 4 iterations (not counting the initial guess) were used to converge to the solution

v) Because there are the same amount of free variables and constraints (3 each), this solution is unique.

vi) The trajectories for each iteration in the targeter are shown in Figure 11 on page 17

b) Fixed-X-Velocity, Y&Z + TOF Free:

i) Before corrections:

1) $\bar{e} = [-\ 0.0788, 0.0233, 0.1398] = [-\ 30282, 8972, 53758]km$

2) TOF = 2.303 days

ii) Before corrections, the e vector is the same as in part a

iii) This version of the targeter required for the free variable vector to consist of the YZ velocity components plus TOF instead of all 3 velocity components. This also required for the TOF to be updated during each iteration, which required the Jacobian of the system to take into account the state derivative vector in addition to the STM.

iv) There are still the same amount of free variables (3) as part a (Vy,Vz,TOF instead of Vx,Vy,Vz)

v) The trajectories for each iteration in the targeter are shown in Figure 12 on page 18. The requested results from each iteration are shown in the table below:

Iteration	TOF [days]	Delta V [km/s]	Delta V Mag [km/s]	Error Mag [km]

0	10	(0,0,0)	0	62349
1	10.865	(0,-0.122,0.208)	0.2417	23057
2	11.189	(0,-0.171,0.251)	0.304	2215.9
3	11.287	(0,-0.184,0.268)	0.3247	258.83
4	11.291	(0,-0.1847,0.2683)	0.3258	0.60699
5	11.291	(0,-0.1847,0.2683)	0.3258	8.943e-6
6	11.291	(0,-0.1847,0.2683)	0.3258	1.359e-9

vi) It can be seen in the table that the targeter converges in 6 iterations, converging to a solution that requires a TOF of 11.291 days.

vii) Because there are the same amount of free variables and constraints (3 each), this solution is unique.

c) Now altering the targeter to work with a case where time is free as well as all 3 components of velocity:

i) From the version used in part b, the changes that were required were

1) Changing the free variable vector to align with the new problem
2) Update the Jacobian to account for the newly added free variable
3) Changing the update equation (newton's method) to use the minimum norm of the Jacobian instead of the inverse since the Jacobian is now rectangular

ii) There are now 4 free variables (Vx,Vy,Vz,TOF) and 3 constraints, which means that an infinite number of solutions exists, and this targeter will converge upon the minimum norm solution

iii) The trajectories for each iteration in the targeter are shown in Figure 13 on page 19. A screenshot of the values related to each iteration is also shown in Figure 14 on page 20

Problem D4:

a) Using the Earth-Moon L1 as the reference point, an initial offset position of (0.01,0) is chosen. From problem C2 on the previous homework, an initial velocity of (0,-0.0837226945866448) was found in order to ensure that only oscillatory behavior is present in the linearized system. **Propagating these initial conditions until the X-axis is hit again, it can be seen in Figure 15 on page 21 that the axis crossing is not perpendicular, meaning this will not create a periodic orbit.** A targeting problem was set up as follows:
   i) Initial position and x velocity are fixed
   ii) Initial Y velocity and TOF are **Free Variables**
   iii) The target is the opposite side of some periodic orbit, in which both the y position and x velocity are known to be zero (This is our constraints)
   iv) The targeter found the solution to this in 5 iterations, and that process is shown in Figure 16 on page 22.
   v) The solution was an orbit with the above fixed initial conditions with a y velocity of -0.07824048 and a TOF of ~5.88 days
   vi) This initial state was then propagated for 2x the solution's TOF, completing 1 full revolution of the solved period orbit (with a period of ~11.763 days). This trajectory is shown below in Figure 17 on page 23.
   vii) The error in the final state after 1 revolution is as follows: $\bar{e}$ **=[1.11e-7,9.1e-8,-3.2e-7,1.7e-7]**

b) The STM and eigenvalues were calculated at the 1 revolution point of this orbit, and these values are included in Figure 18 on page 23 below
   i) As for the eigenvalues, the first 2 are real, and the others are complex conjugates including nonzero real and imaginary components.

c) By using the solved velocity from the previous case and reducing the x positional offset by 0.001 each time, 3 more Lyapunov orbits were produced, and their trajectories are shown in Figure 19 on page 24
   i) The eigenvalues for each of these 3 orbits is shown in Figures 20, 21, and 22 respectively, all of which are on page 25

Solutions:

Problem D1: All work done with code in file Homework4-1.py
Problem D2:

a) All work done with code in file Homework4-2a.py
b) All work done with code in file Homework4-2b.py

Problem D3:

a) All work done with code in file Homework4-3a.py
b) All work done with code in file Homework4-3b.py
c) All work done with code in file Homework4-3c.py

Problem D4:

a) All work done with code in file Homework4-4.py

Figures

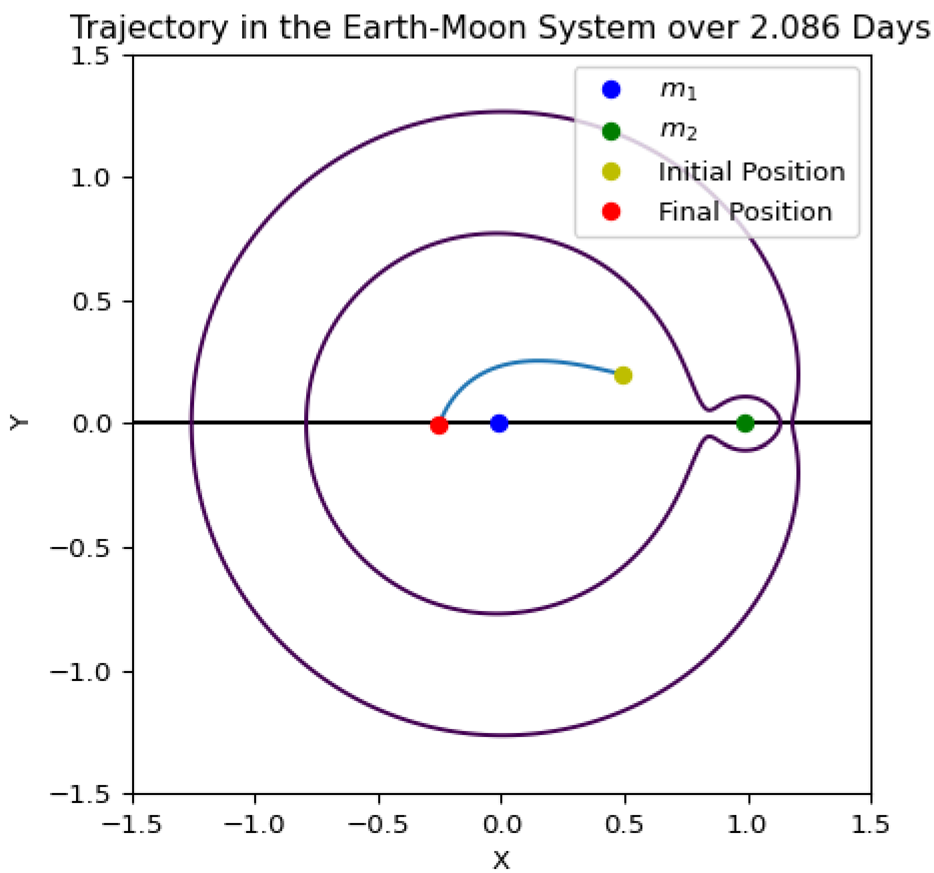


Figure 1

```
final_STM from CI:
 [[ 1.06804089e-02 -1.49694601e-01  0.00000000e+00  2.78003236e-01
   -1.55236083e-01  0.00000000e+00]
  [ 4.22719208e+00  3.72915641e+00  0.00000000e+00  5.74725087e-02
    1.42209807e+00  0.00000000e+00]
  [ 0.00000000e+00  0.00000000e+00 -4.81438113e-01  0.00000000e+00
    0.00000000e+00 -2.87404458e-02]
  [-2.70435864e+01 -2.11030312e+01  0.00000000e+00  5.37642099e-01
   -8.40466735e+00  0.00000000e+00]
  [ 7.50300473e+00  7.47511240e+00  0.00000000e+00 -1.99702837e+00
    3.85615101e+00  0.00000000e+00]
  [ 0.00000000e+00  0.00000000e+00 -4.14181354e+00  0.00000000e+00
    0.00000000e+00 -2.32436360e+00]]
```

Figure 2

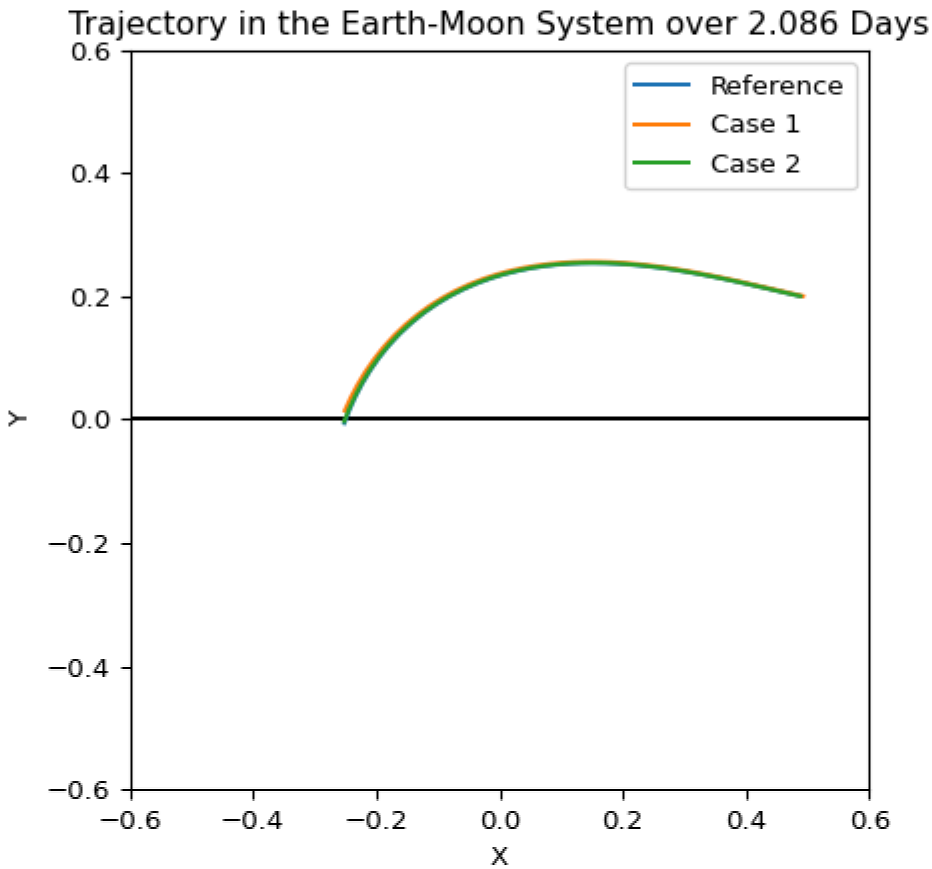

Trajectory in the Earth-Moon System over 2.086 Days
Reference
Case 1
Case 2
Y
X
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6

Figure 3

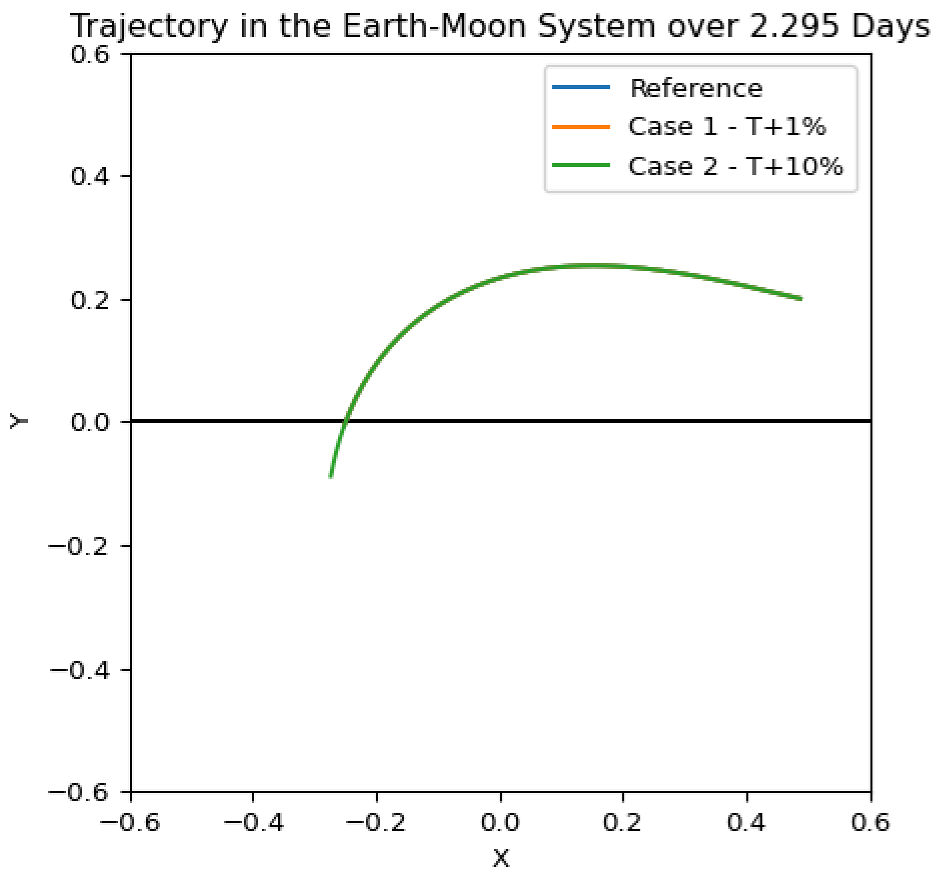
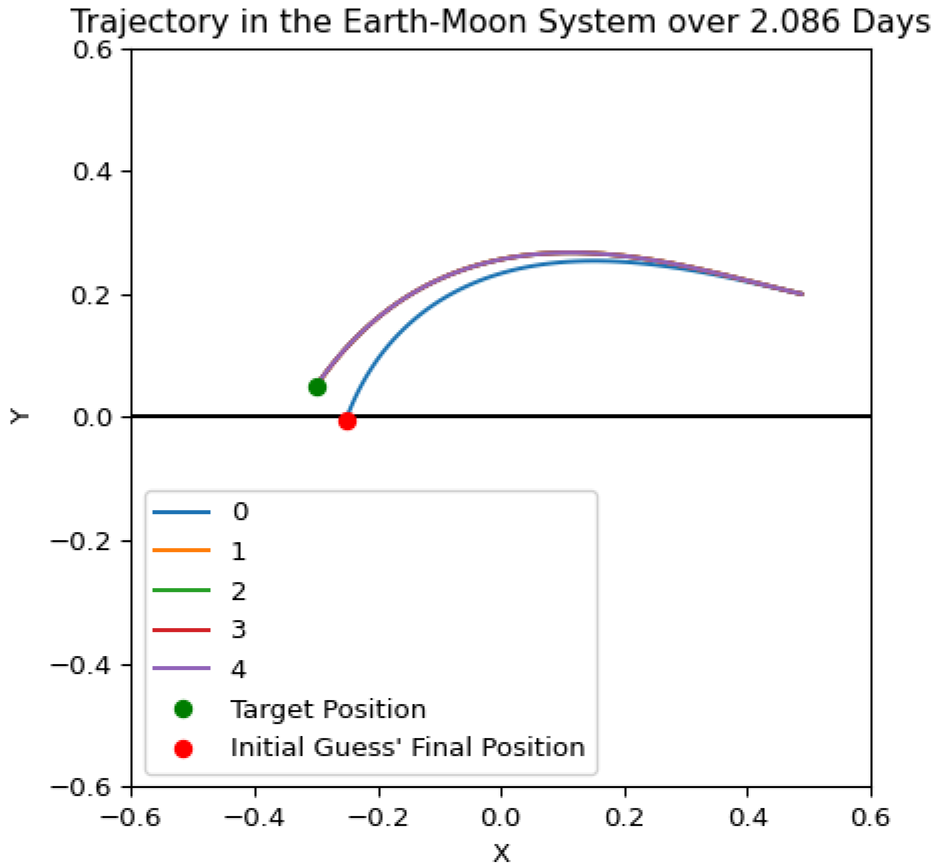

Trajectory in the Earth-Moon System over 2.295 Days
Reference
Case 1 - T+1%
Case 2 - T+10%
Y
X
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6

Figure 4



Trajectory in the Earth-Moon System over 2.086 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
Target Position
Initial Guess' Final Position

Figure 5

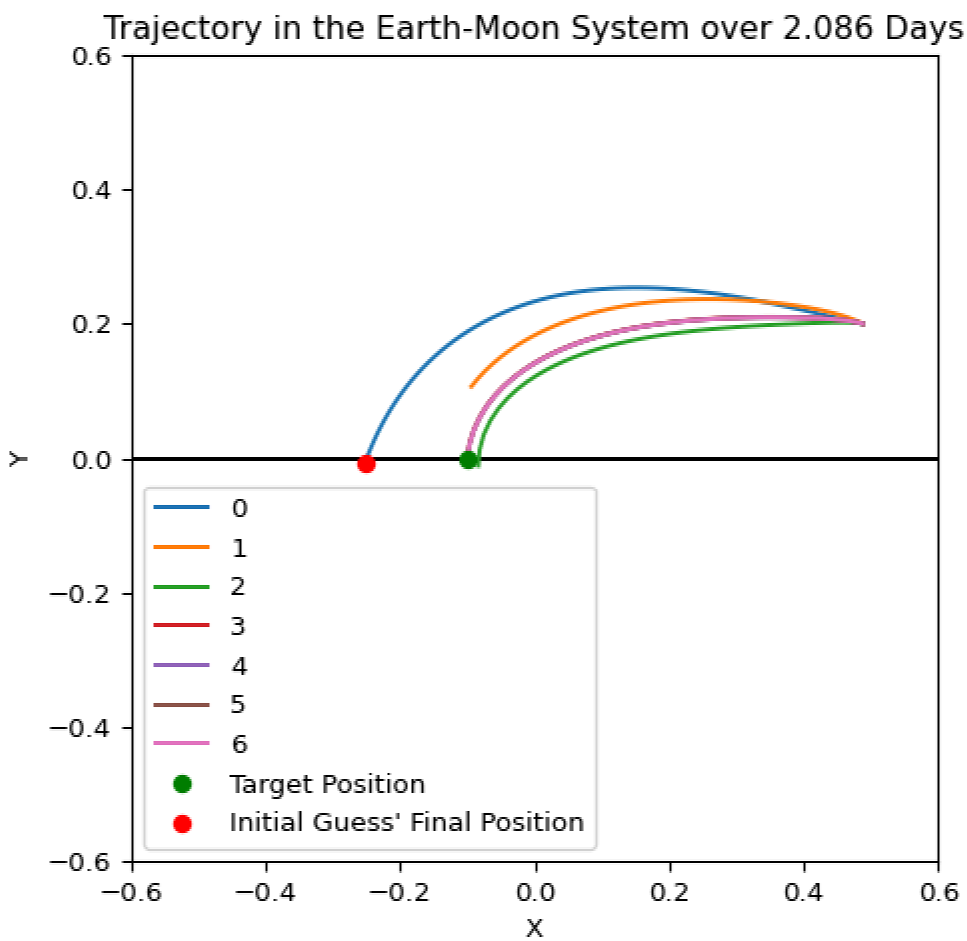
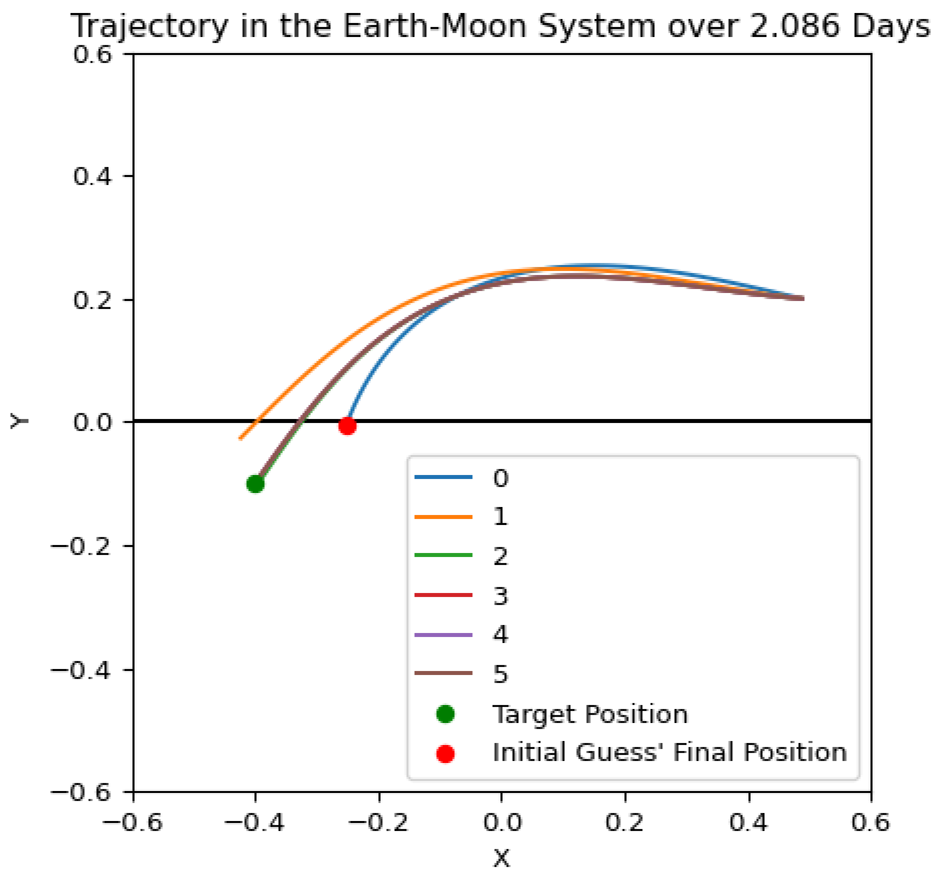

Trajectory in the Earth-Moon System over 2.086 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
5
6
Target Position
Initial Guess' Final Position

Figure 6



Trajectory in the Earth-Moon System over 2.086 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
5
Target Position
Initial Guess' Final Position

Figure 7

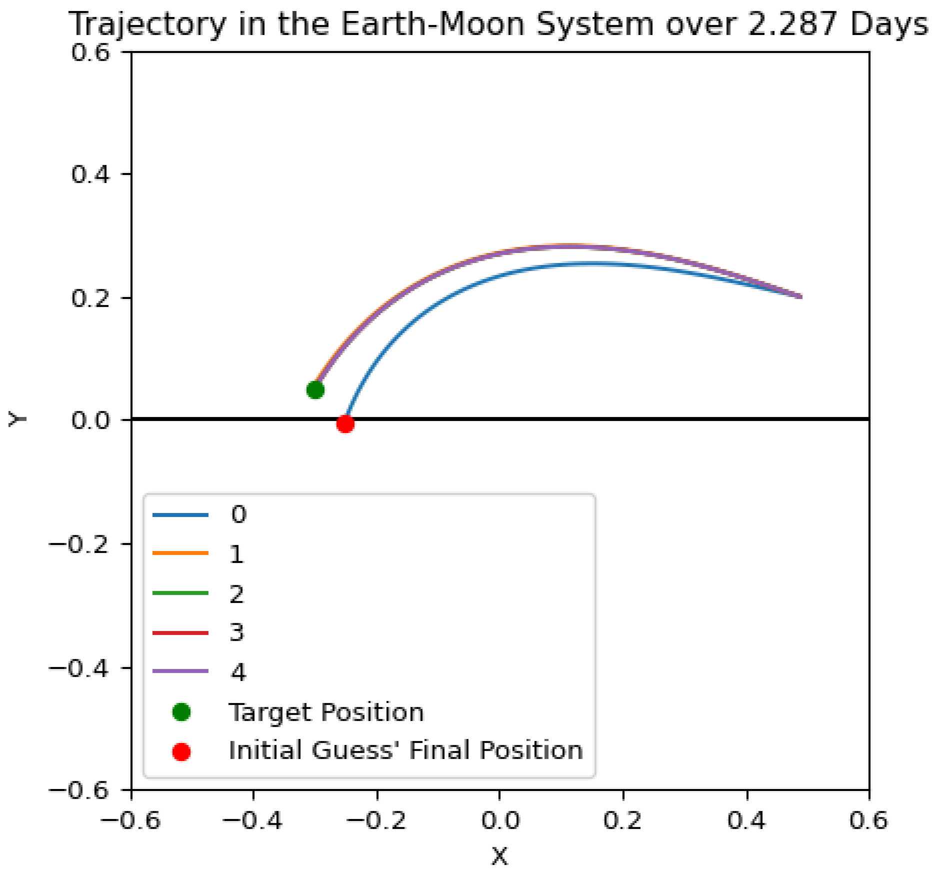

Trajectory in the Earth-Moon System over 2.287 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
Target Position
Initial Guess' Final Position

Figure 8

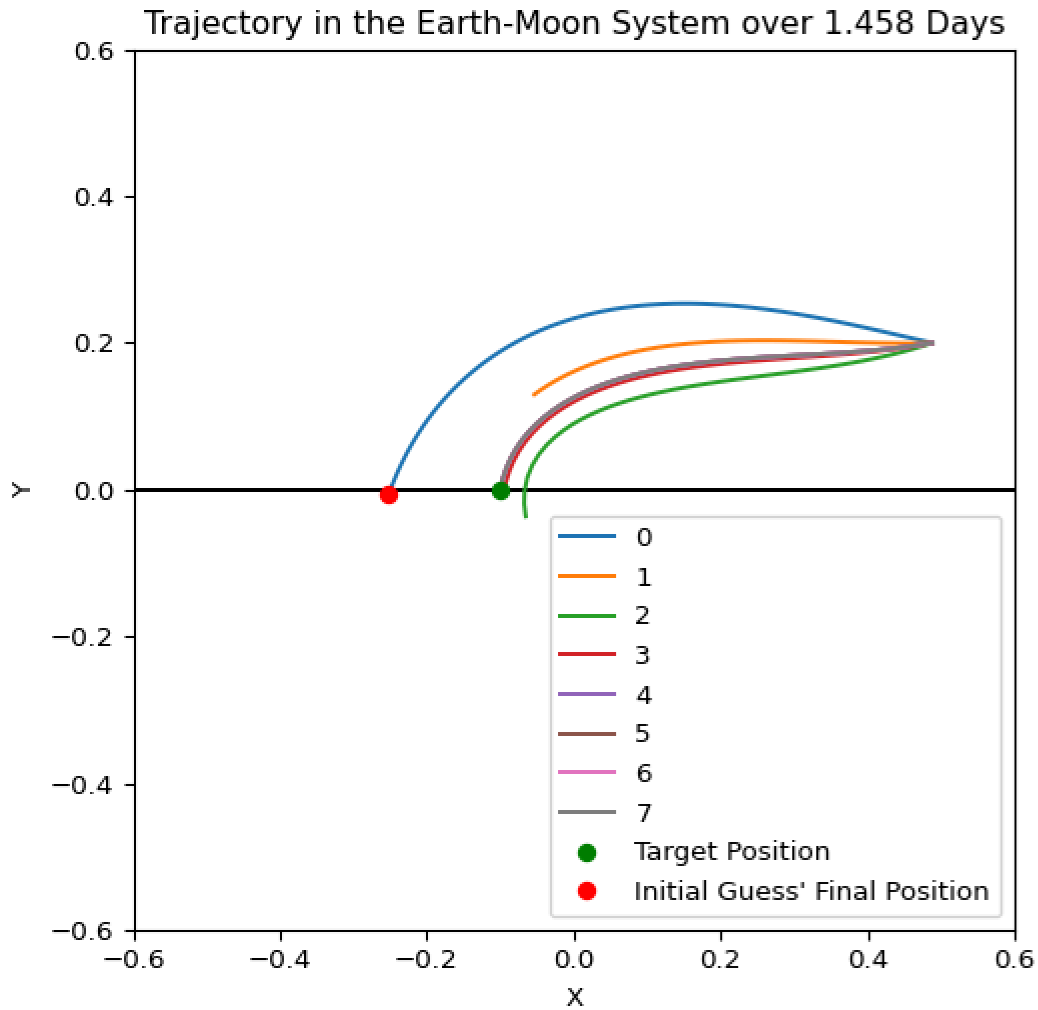

Trajectory in the Earth-Moon System over 1.458 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
5
6
7
Target Position
Initial Guess' Final Position

Figure 9

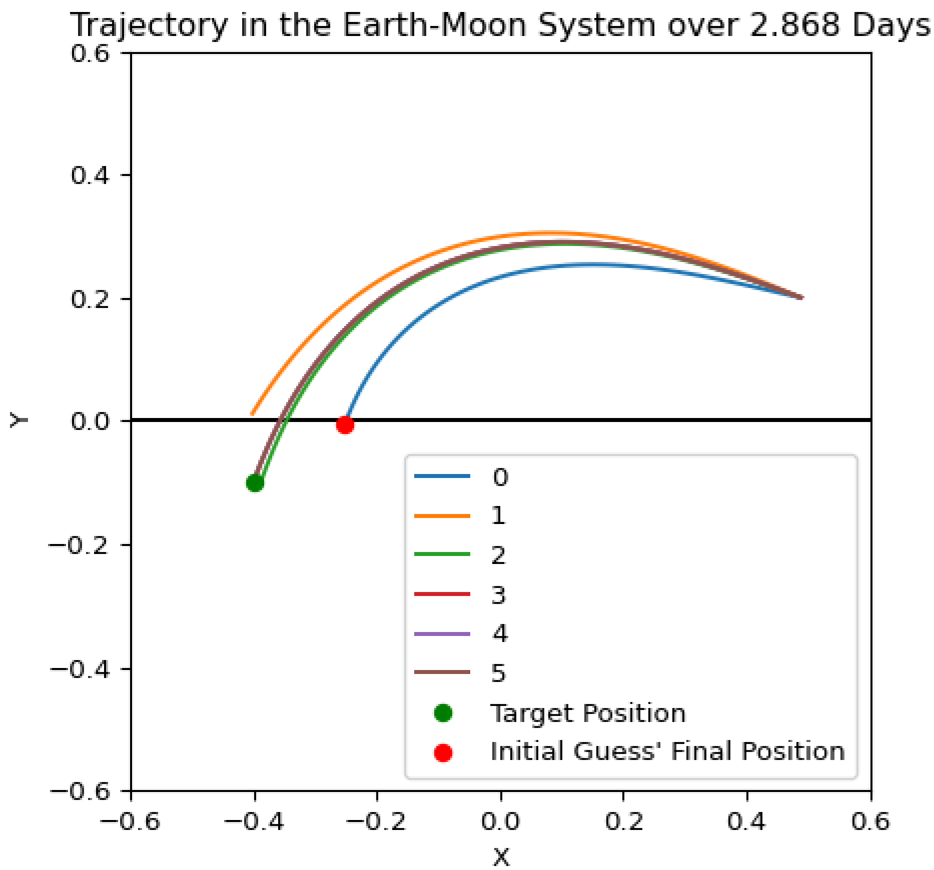

Trajectory in the Earth-Moon System over 2.868 Days
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Y
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X
0
1
2
3
4
5
Target Position
Initial Guess' Final Position

Figure 10

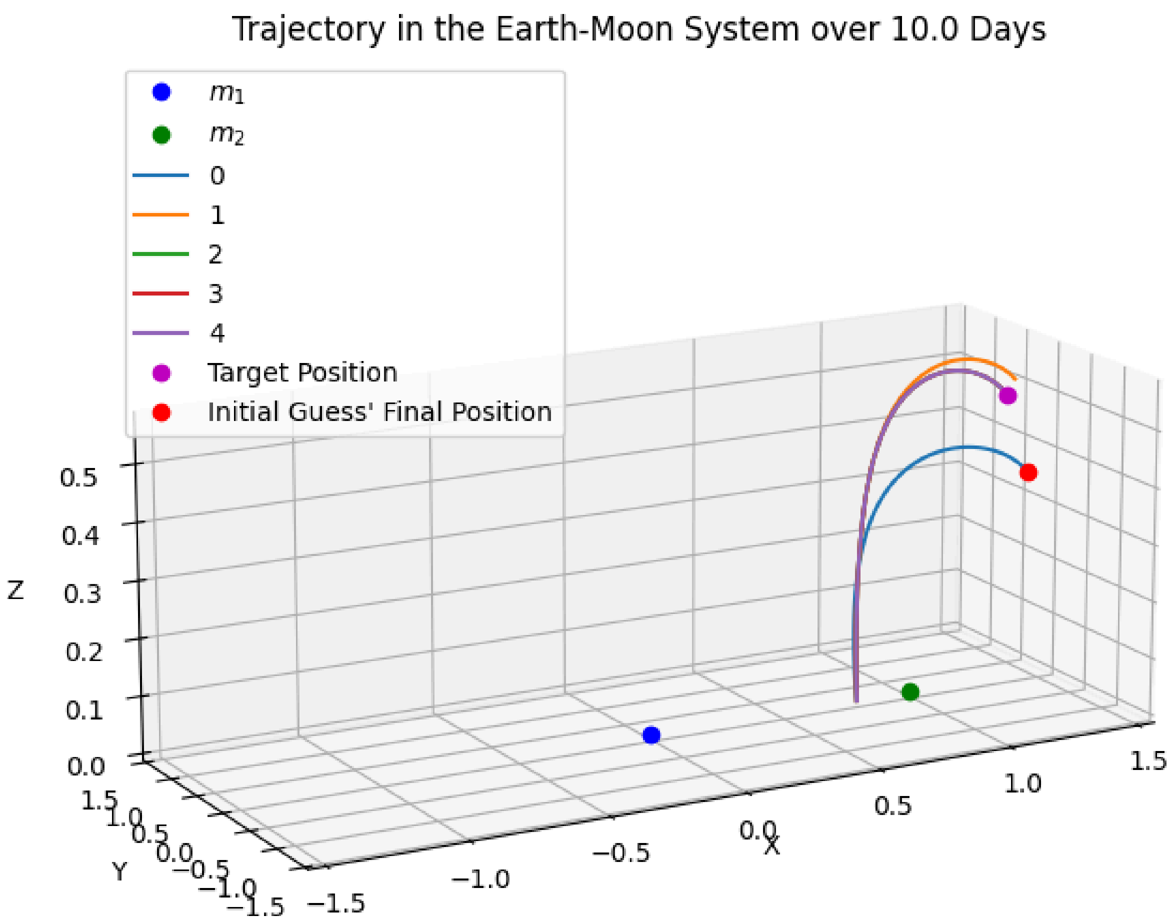

Trajectory in the Earth-Moon System over 10.0 Days
m_1
m_2
0
1
2
3
4
Target Position
Initial Guess' Final Position
Z
Y
X
0.0
0.1
0.2
0.3
0.4
0.5
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

Figure 11

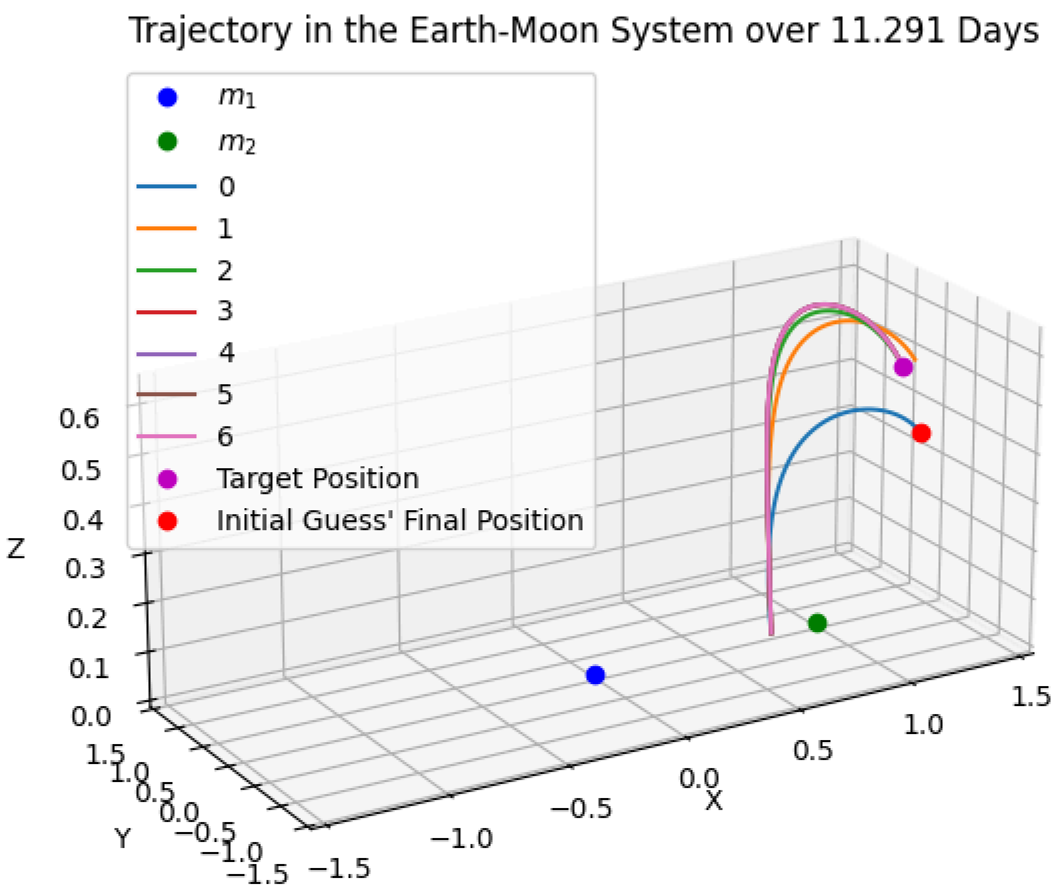

Trajectory in the Earth-Moon System over 11.291 Days
m_1
m_2
0
1
2
3
4
5
6
Target Position
Initial Guess' Final Position
Z
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Y
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
X
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

Figure 12

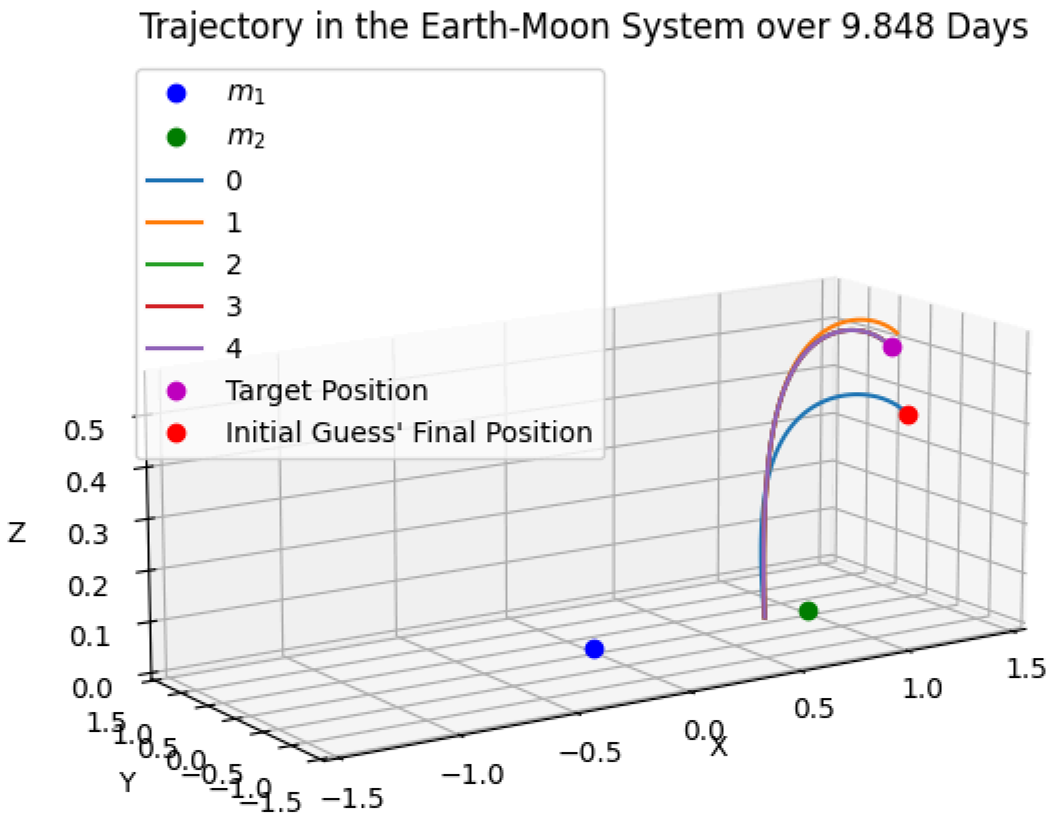
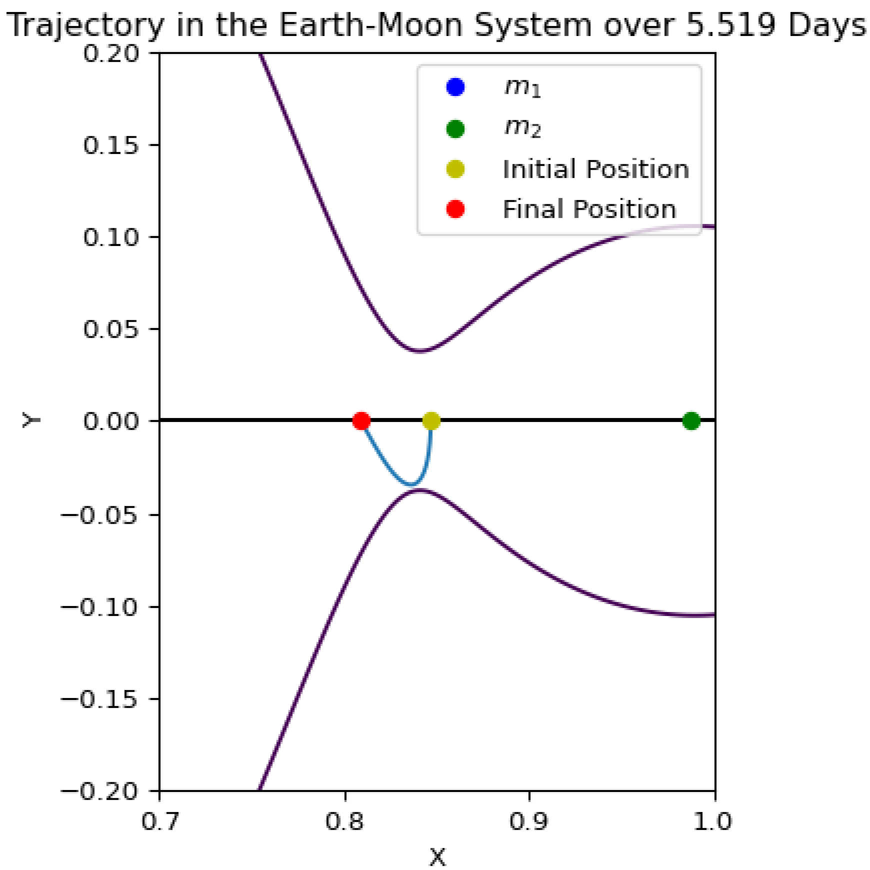

Trajectory in the Earth-Moon System over 9.848 Days
m_1
m_2
0
1
2
3
4
Target Position
Initial Guess' Final Position
Z
0.0
0.1
0.2
0.3
0.4
0.5
X
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
Y

Figure 13

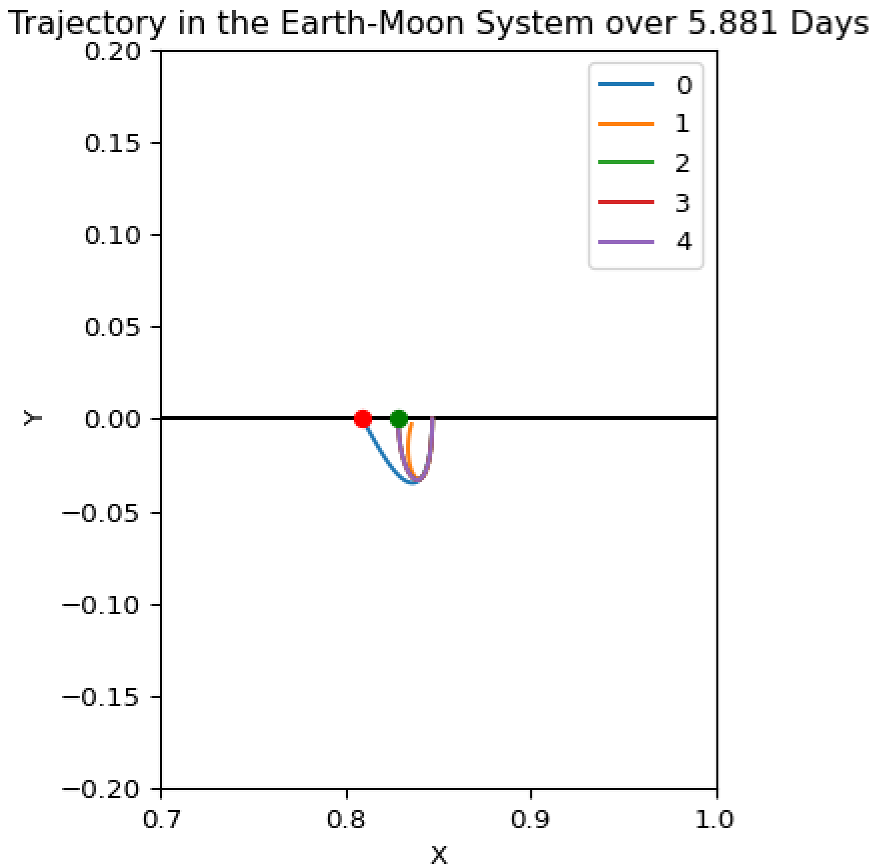
```
current iteration 0
evec_dim [-30281.97706211   8972.04362373  53757.86315496]
emag_dim 62349.04612392908
Delta V vec: [0. 0. 0.]
Delta V mag: 0.0
TOF: 10.000000000000002
current iteration 1
evec_dim [ -7008.35871876  10856.76826967 -10885.41345804]
emag_dim 16896.115983993885
Delta V vec: [ 0.05075996 -0.08774602  0.15189121]
Delta V mag: 0.18261127344664807
TOF: 9.84134046892331
current iteration 2
evec_dim [138.9746159  -39.58036117 -16.25716756]
emag_dim 145.4126691628604
Delta V vec: [ 0.04761801 -0.09132714  0.14012716]
Delta V mag: 0.17390728507669853
TOF: 9.848248779097652
current iteration 3
evec_dim [-0.0303757   0.02846911 -0.01726791]
emag_dim 0.04507054554726788
Delta V vec: [ 0.04747384 -0.09110303  0.13999005]
Delta V mag: 0.1736396911533379
TOF: 9.848447265408284
current iteration 4
evec_dim [ 3.41415785e-10 -2.13384865e-10  5.54800650e-10]
emag_dim 6.854936905418212e-10
Delta V vec: [ 0.04747385 -0.09110307  0.13999005]
Delta V mag: 0.1736397106618747
TOF: 9.84844724864144
Target Achieved:  True
iteration amount 5
V0d_dim [0.04747385 0.40889693 0.63999005]
Delta V vec: [ 0.04747385 -0.09110307  0.13999005]
Delta V mag: 0.17363971066187528
Achieved Time (d): 9.84844724864144
evec [-0.07591321 -0.00347249  0.13621427]
evec_dim [-29181.03680871  -1334.82623713  52360.76349276]
```

Figure 14



Trajectory in the Earth-Moon System over 5.519 Days
m_1
m_2
Initial Position
Final Position
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Y
0.7
0.8
0.9
1.0
X

Figure 15



Trajectory in the Earth-Moon System over 5.881 Days
0
1
2
3
4
Y
X
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.7
0.8
0.9
1.0

Figure 16

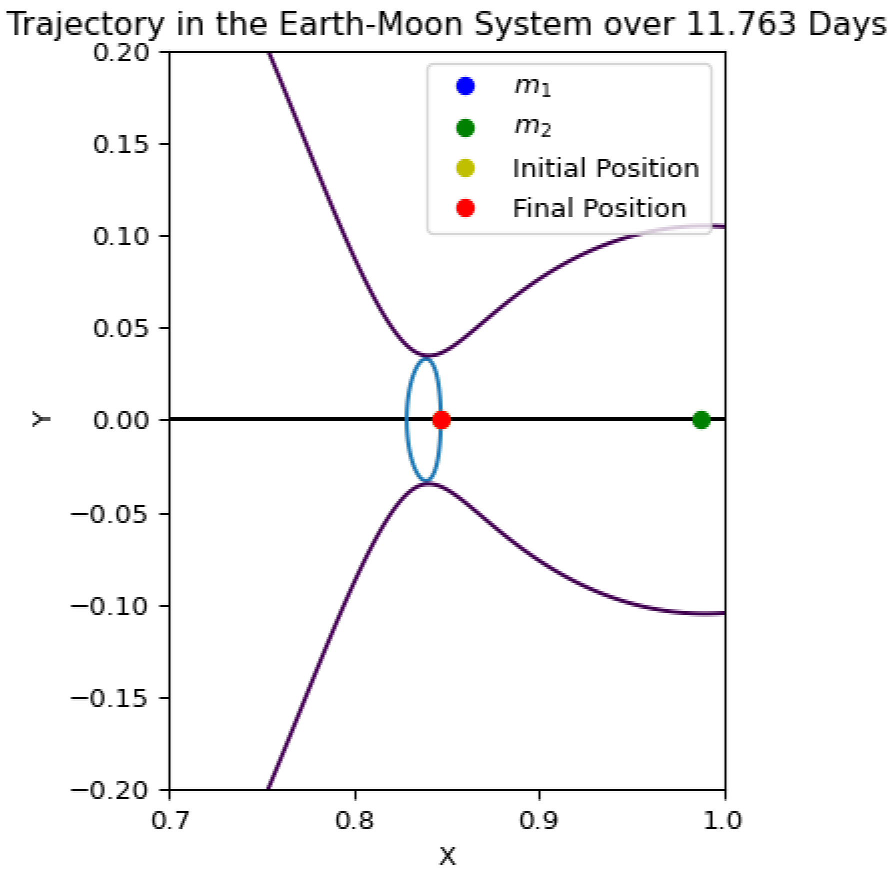


Figure 17

```
final_STM[0,1,3,4][0,1,3,4] [[ 1618.83781715  -177.41474516   381.44604359   220.38821366]
 [ -936.26838781   103.50503536  -220.38833394  -127.5772607 ]
 [ 5000.07171388  -547.46382658  1178.06062567   681.11305352]
 [-2483.20359289   272.3121768   -585.47783579  -337.27173462]]
eigs [2.56113135e+03+0.j         3.90452584e-04+0.j
 9.99999869e-01+0.00040794j 9.99999869e-01-0.00040794j]
```

Figure 18

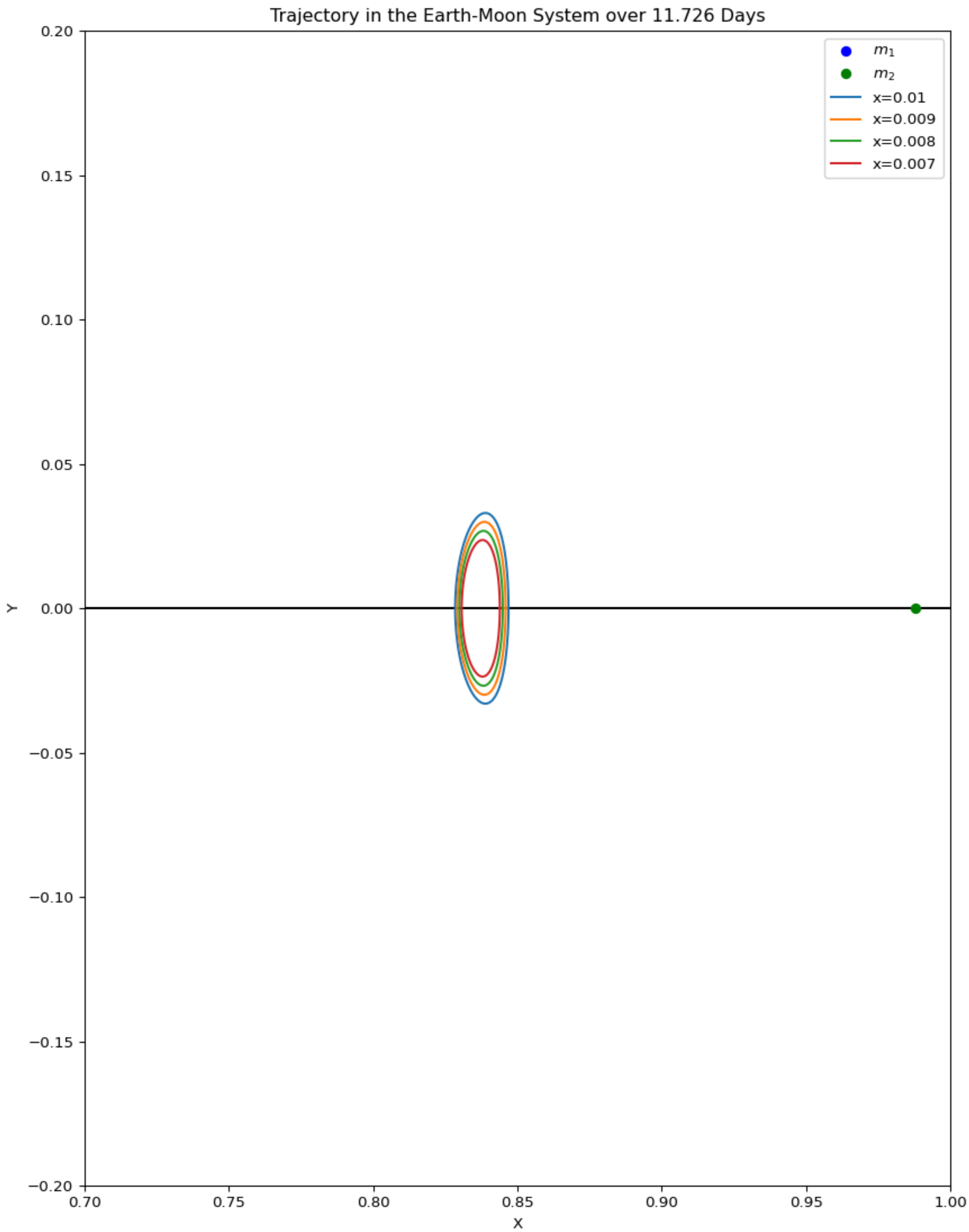

Trajectory in the Earth-Moon System over 11.726 Days
m_1
m_2
x=0.01
x=0.009
x=0.008
x=0.007
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Y
0.70
0.75
0.80
0.85
0.90
0.95
1.00
X

Figure 19

```
eigs [2.58105938e+03+0.j         3.87437949e-04+0.j
 9.99999893e-01+0.00034635j 9.99999893e-01-0.00034635j]
```

Figure 20

```
eigs [2.59942652e+03+0.j         3.84700374e-04+0.j
 9.99999911e-01+0.00029023j 9.99999911e-01-0.00029023j]
```

Figure 21

```
eigs [2.61611807e+03+0.j         3.82245880e-04+0.j
 9.99999925e-01+0.00023963j 9.99999925e-01-0.00023963j]
```

Figure 22