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Course: AEM 669 Advanced Astrodynamics

Homework #7

Due Date: April 21, 2024

Problem G1:

a) The L1 Lyapunov family of orbits was again found using continuation, and both the trajectories and the eigenvalues were plotted for a visual analysis.
   i) The found Lyapunov orbits were plotted for half orbits to demonstrate that the perpendicular crossing targeter is working properly. This plot can be seen in Figure 1 on page 6. In this plot it can be seen that some orbits are highlighted in black instead of the normal color range. These highlighted orbits represent orbits where a change in eigenvalue characteristics changed.
   ii) 10 orbits were found where the eigenvalue characteristics changed, 2 of those being near the original orbit, and the remaining 8 being near the end of the continuation range. These orbits could represent a range of bifurcation orbits where the Lyapunov family of orbits intersects or comes close to intersection with another family of periodic orbits.
   iii) At the first continuation orbit, the eigenvalues consisted of 2 positive-real eigenvalues and 2 complex eigenvalues with positive real components. At the next orbit where the eigenvalues changed, the eigenvalues now consisted of all 4 positive real values, and this structure persisted for the majority of the Lyapunov family until near the end of the continuation process where the eigenvalues changed 9 times in quick succession. In the 9 orbits close to each other where changes were detected in the eigenvalue structure, the structure swapped between various combinations of real, complex, and both positive and negative real components.

b) The eigenvalues and eigenvectors of the original Lyapunov orbit were found and are shown in Figure 2 on page 7.
   i) To ensure that an eigenvalue and eigenvector are a correctly correlated pair, one can multiply the eigenvector by the relevant A matrix and the resulting vector should be the equal to the eigenvector multiplied by the scalar eigenvalue.
   ii) 20 fixed points on this orbit were calculated, and the eigenvalues and eigenvectors for each point were calculated. The eigenvectors associated with each point was added to the plot of the orbit's trajectory, shown in Figure 3 on page 7

Problem G2:

a) Using the given initial position as the reference point of the Lyapunov orbit, the eigenvectors were calculated and the stable position eigenvector was scaled and applied as an offset to the original state to approximate a state on the stable manifold.This approximated initial state is shown in Figure 4 on page 7
b) The two half-manifolds created to represent the relevant stable manifold are shown in Figure 5 on page 7
c) By propagating one of the half-manifolds until it crosses the x-axis on the far size of the moon, the trajectory shown in Figure 6 on page 9 was created.
   i) Where this path begins at the lyapunov orbit, it seems (on the scale of the plot) that the manifold does approach the lyapunov orbit asymptotically and would be tangential to the orbit at the x-axis
   ii) By checking the distance between each position in this trajectory and the moon and finding the minimum, the closest distance to the Moon over this path was found to be ~**0.00566 non-dimensional units, which equates to ~2176 km.**

d) The same process was used to produce a manifold using the opposite side of the lyapunov orbit as the reference point.
    i) The new trajectory is shown alongside the previously produced manifold path in Figure 7 on Page 10
    ii) This new path had a minimum distance to the moon of **~0.02849 non-dimensional units, which equates to ~10951 km.**
    iii) This path also seems to be fairly tangential to the lyapunov orbit at the x-axis, which implies an asymptotical approach from this scale.
    iv) While both paths pass through the x-axis near the other side of the Moon, the first path passes much closer (5 times closer) to the Moon, but the second path crosses in a much less eccentric manner than the first path, which could potentially be useful depending on what mission is being designed.

Problem G3:

a) To convert a northern halo orbit to a southern one, the only alteration that needs to occur is to multiply the z components of the initial position and velocity by -1. The x and y components are reflected over the x-y plane.
b) After propagating this southern halo orbit, the eigenvalues of the monodromy matrix for both the northern and southern halo orbits were calculated, and are shown in Figure 8 on page 10. It can be seen that the eigenvalues for the northern and southern halo orbits are equal to each other. This makes sense because the orbits are mirrored over the x-y plane, so there is no difference in their overall dynamics besides a negative z force, so the stability of the two orbits should be the same. There would most likely be a negative relationship between some of the eigenvectors though.
c) The 2D projections of these orbits is shown in Figure 9 on page 11
d) The orbit positions and velocities are the same besides being mirrored over the x-y plane

Problem G4:

a) Starting with Halo #2 from Problem F4, the Halo family was continued towards the XY plane by decreasing the fixed initial z position value for each continuation orbit. This resulted in the trajectories shown in Figure 10 on page 11, in which the trajectory outlined in a black dash represents where the Halo family is coplanar with the XY plane.
    i) The eigenvalues associated with the monodromy matrix of each of these continued orbits was also found, and these are shown in Figure 11 on page 12.
b) The original Lyapunov orbit from previous problems was used in a continuation loop, increasing the original x value until it aligned with the planar halo orbit found in part a.
    i) The eigenvalues for each lyapunov orbit found on the way were computed, and the last few sets of eigenvalues are shown in Figure 12 on page 12, in which the last set of eigenvalues correlates with the lyapunov orbit that meets the planar halo orbit.
c) Looking at the eigenvalues for the Halo and Lyapunov orbits found, the eigenvalues for the Halo orbits seem to mostly follow the structure of 4 positive real eigenvalues and 2 complex conjugate eigenvalues with positive real components. Most of the Lyapunov family orbits seem to follow the same structure. The only changes are at the bifurcation orbit, in which from the Halo side the structure changes to all 6 positive real eigenvalues, while from the Lyapunove side the structure changes to 2 positive real

eigenvalues and 4 complex conjugates with positive real components.

i) It seems odd that the eigenvalues for that final orbit would be different, but my guess is that my targeter tolerance might be set too high for the 2 orbits to be exactly the same, and the resulting difference is non-negligible.

Problem G5:

a) An attempt was made to reproduce the left half of Figure 28 from Notes Q, and the attempted recreation is shown in Figure 13 on page 13.
   i) It can be seen that the quasi-periodic orbits close to each of the periodic orbits seem to be represented fairly well, but the quasi-periodic orbits closer to the middle of the plot are not behaving as expected. This issue persists even after altering tolerances, resolution, and time of flight, so I’m not sure what the issue is.
   ii) The first quasi-periodic orbit around the left-side periodic orbit has the first 15 crossing points labeled
   iii) The ZVCs of this jacobi constant are present in the Figure as well as the darker lines.
   iv) The quasi-periodic orbit that has the crossing points labeled used the initial conditions of x=-0.598, y=0, vx=0, vy=2.626. The trajectory created by propagating these initial conditions for a while is shown in Figure 14 on page 14

b) The coordinates of the periodic orbit on the left side of the poicare plot are [-0.635, 0, 0, 0, 2.047361637847567, 0]. These initial conditions were propagated and then periodic conditions at the axis-crossing were targeted, producing the periodic orbit shown in Figure 15 on Page 15.

Solutions:

Problem G1: All work done with code in files Homework7-1a.py and Homework7-1b.py

Problem G2: All work done with code in file Homework7-2.py

Problem G3: All work done with code in file Homework7-3.py

Problem G4: All work done with code in file Homework7-4.py

Problem G5: All work done with code in file Homework7-5.py

Figures

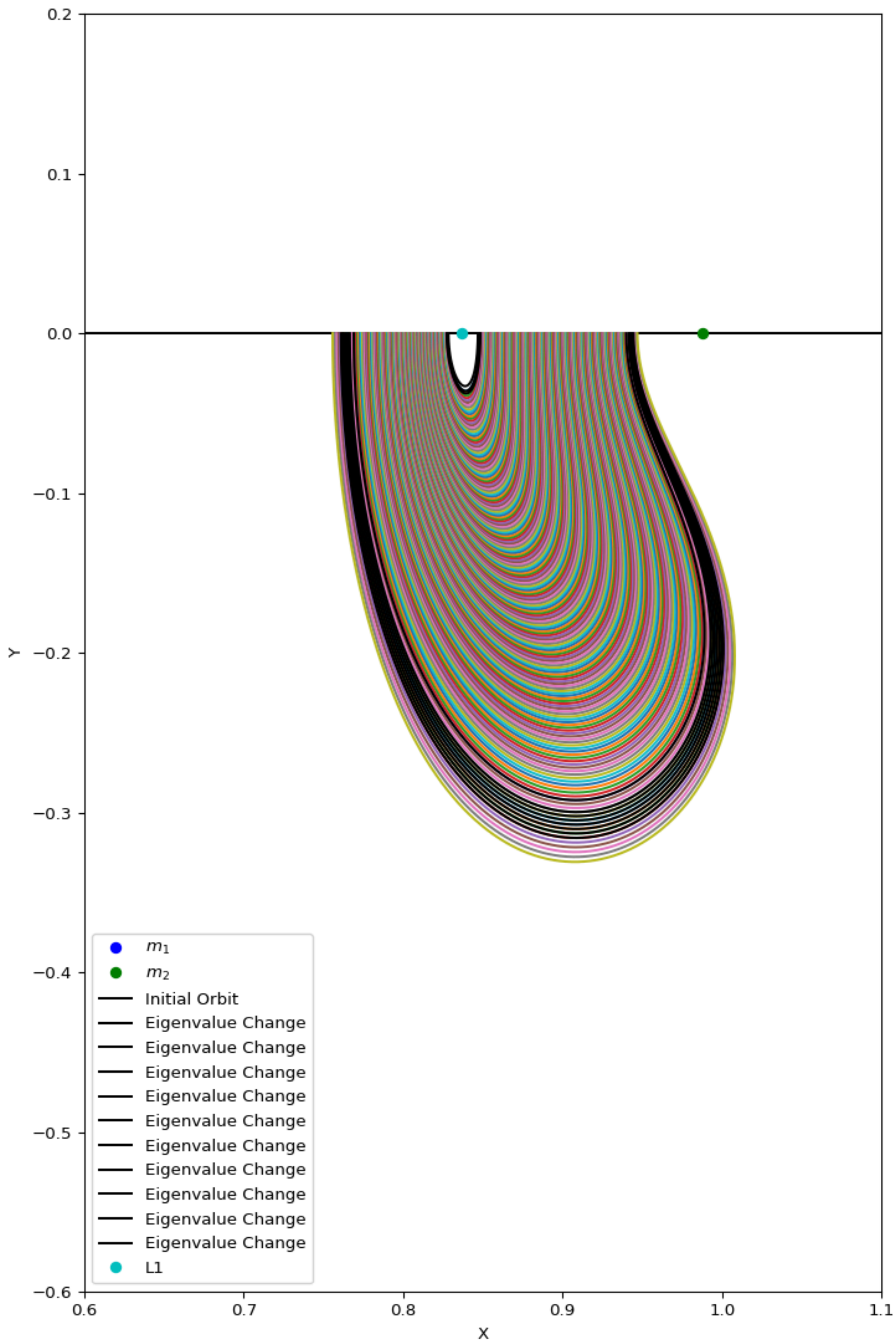

Figure 1

```
eigvals [2.56113098e+03+0.j         3.90452642e-04+0.j
 9.99999863e-01+0.00042138j 9.99999863e-01-0.00042138j]
eigvecs [[ 2.74957973e-01+0.00000000e+00j  2.74957880e-01+0.00000000e+00j
   1.49681856e-06-1.48240839e-03j  1.49681856e-06+1.48240839e-03j]
 [-1.58986763e-01+0.00000000e+00j  1.58986588e-01+0.00000000e+00j
  -9.06668544e-01+0.00000000e+00j -9.06668544e-01-0.00000000e+00j]
 [ 8.49124079e-01+0.00000000e+00j -8.49124428e-01+0.00000000e+00j
  -4.21700763e-01+3.45420300e-09j -4.21700763e-01-3.45420300e-09j]
 [-4.22030356e-01+0.00000000e+00j -4.22029779e-01+0.00000000e+00j
  -1.22605035e-05+1.08821384e-02j -1.22605035e-05-1.08821384e-02j]]
```

Figure 2

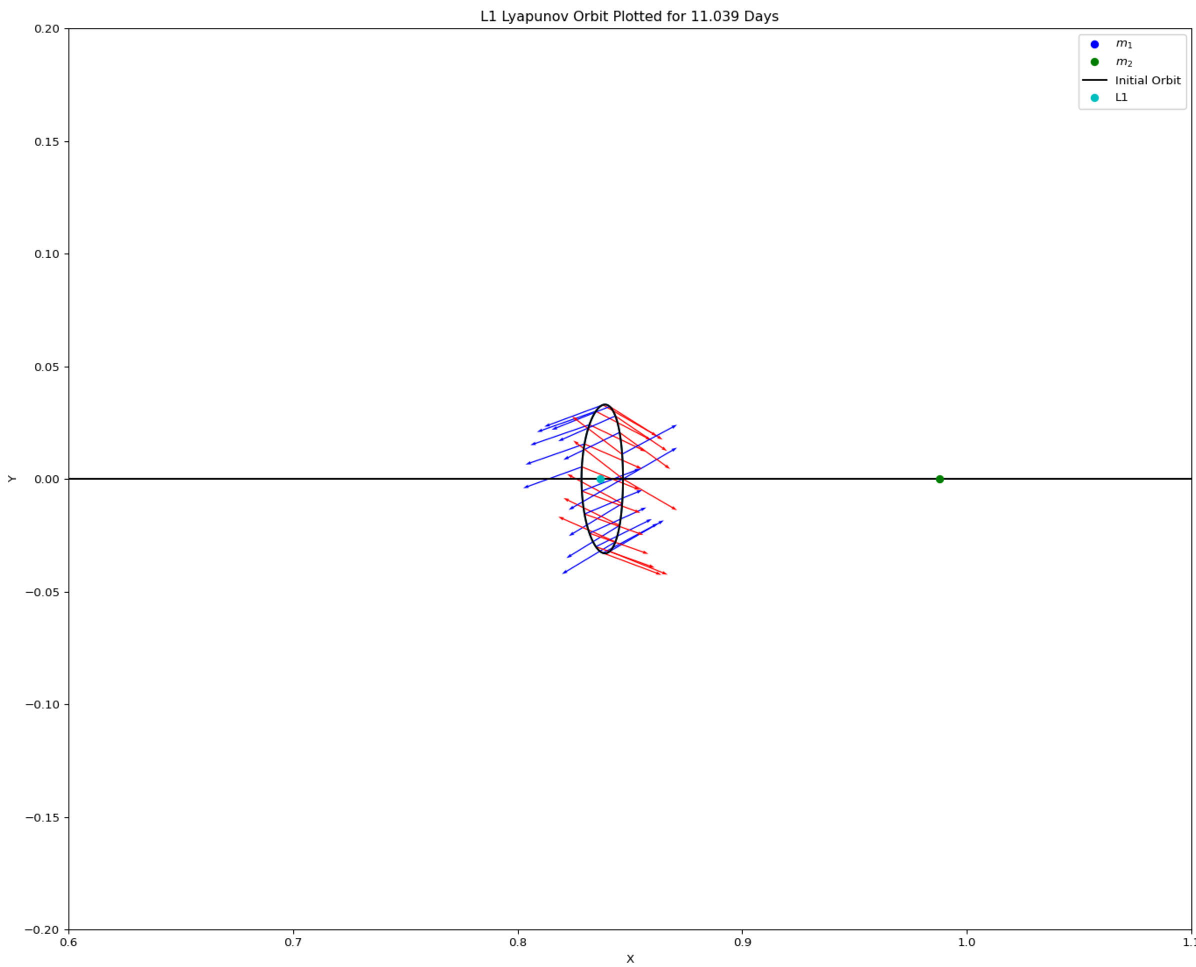

Figure 3

```
Xnew_s [ 8.47835224e-01+0.0000000e+00j  9.19753724e-04+0.0000000e+00j
  5.00696484e-09-4.9587618e-06j -7.82404750e-02+4.9587618e-06j]
```

Figure 4

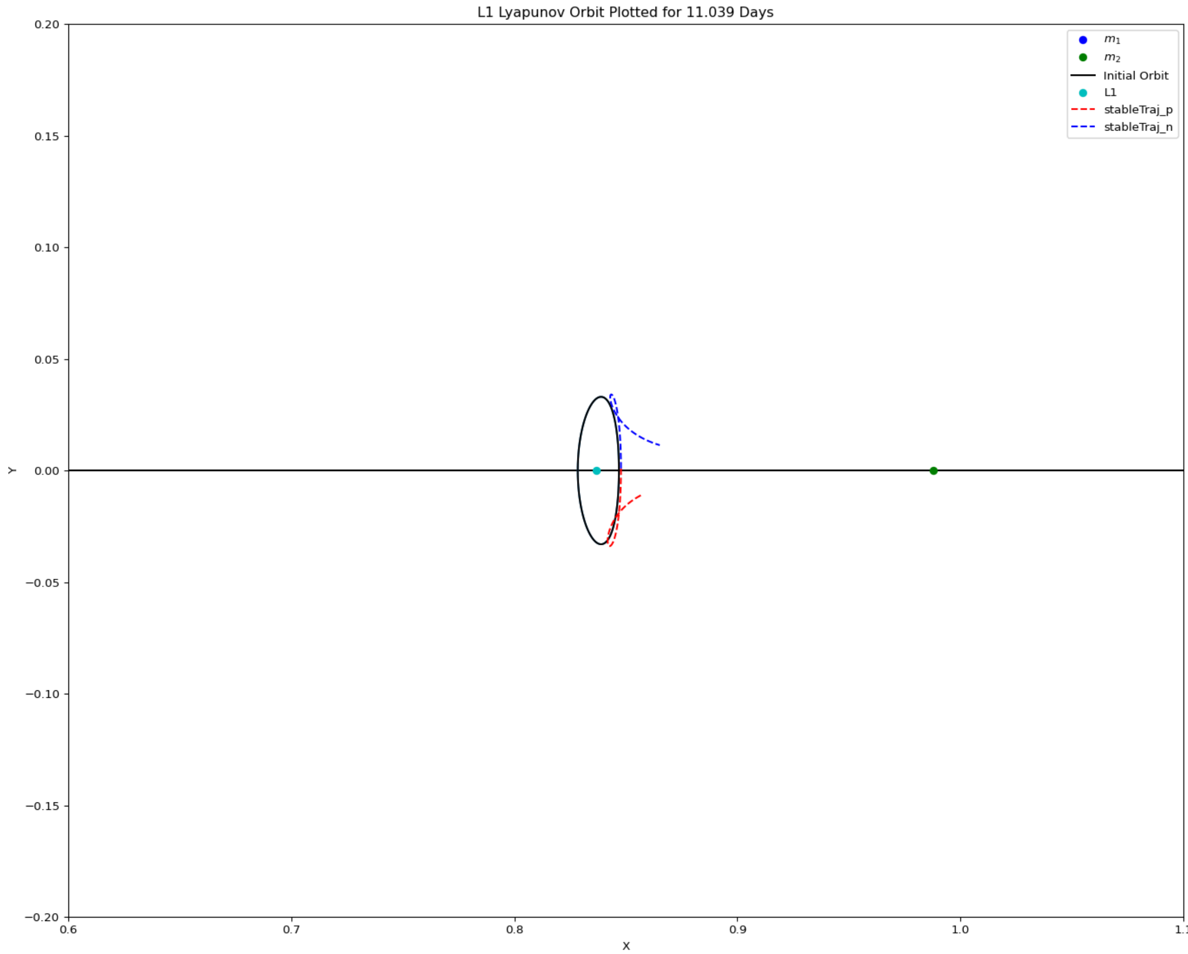
L1 Lyapunov Orbit Plotted for 11.039 Days
m_1
m_2
Initial Orbit
L1
stableTraj_p
stableTraj_n
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Y
0.6
0.7
0.8
0.9
1.0
1.1
X

Figure 5

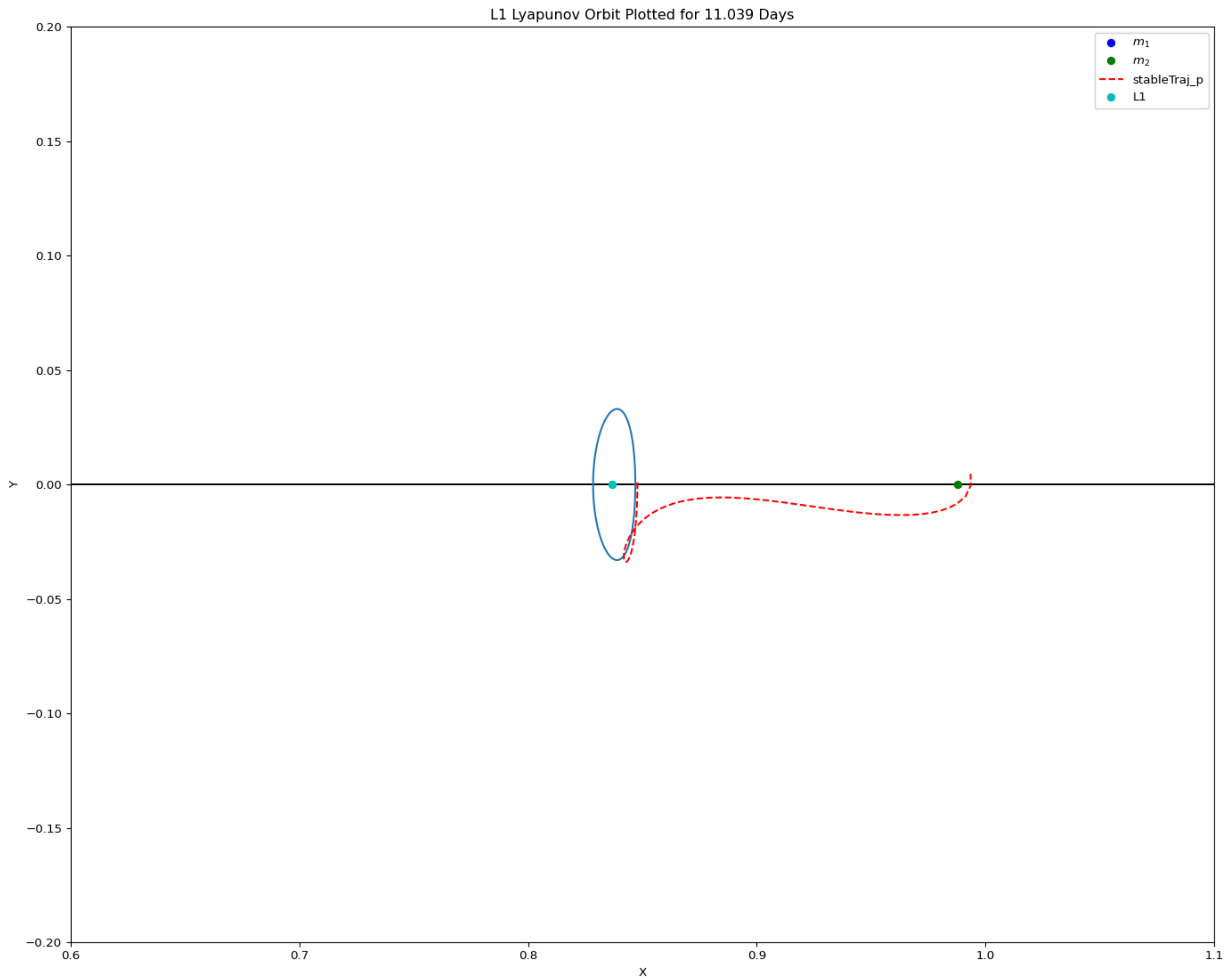
L1 Lyapunov Orbit Plotted for 11.039 Days
m_1
m_2
stableTraj_p
L1
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Y
0.6
0.7
0.8
0.9
1.0
1.1
X

Figure 6

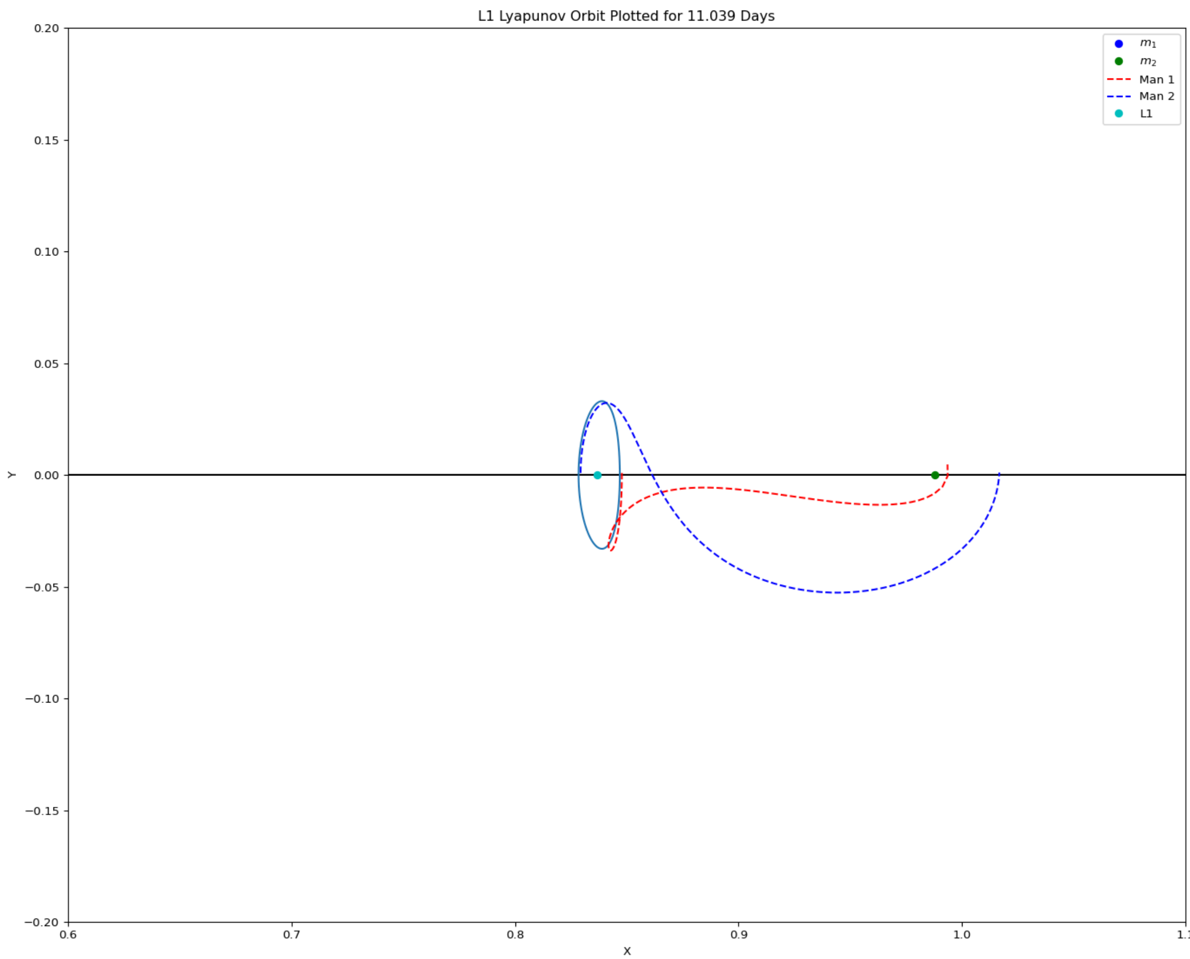

Figure 7

```
north_eigvals [1.39128747e+03+0.j          2.22179679e-02+0.j
 9.23166373e-01+0.30252847j 9.23166373e-01-0.30252847j]
south_eigvals [1.39128747e+03+0.j          2.22179679e-02+0.j
 9.23166373e-01+0.30252847j 9.23166373e-01-0.30252847j]
```

Figure 8

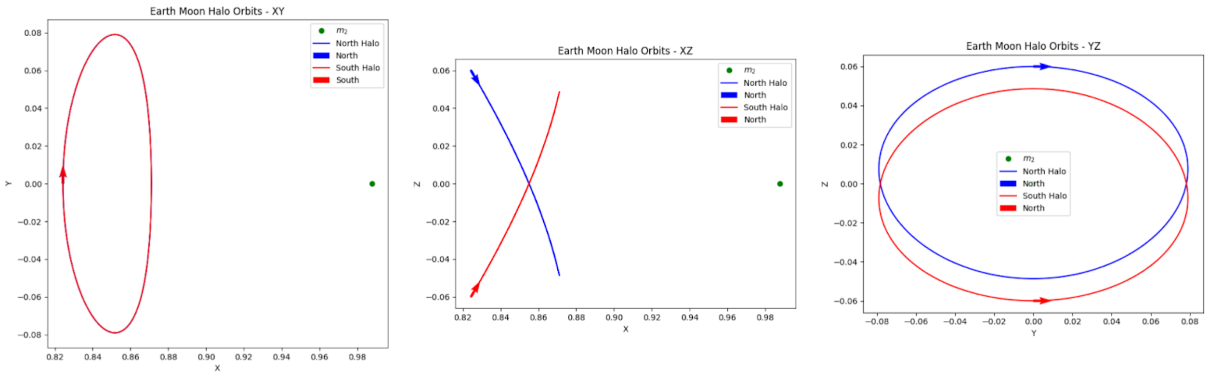

Figure 9

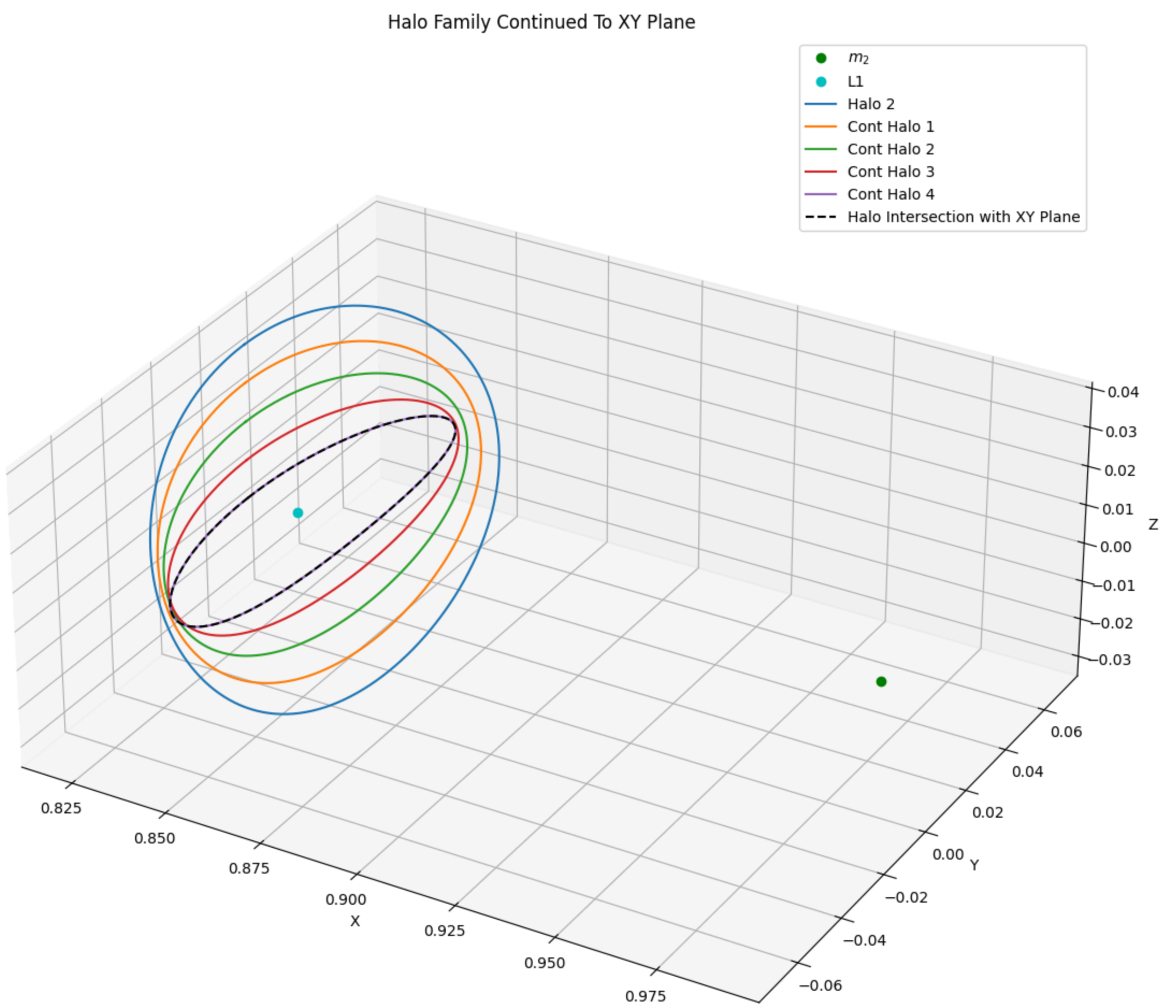

Figure 10

```
**********************************************************************
achieved_initial_state [0.82342504 0.         0.03       0.         0.14003288 0.        ]
halo_eigvals [2.07012002e+03+0.j         4.83063947e-04+0.j
 9.77711783e-01+0.20995134j 9.77711783e-01-0.20995134j
 1.00031703e+00+0.j         9.99682944e-01+0.j        ]
**********************************************************************
achieved_initial_state [0.82338161 0.         0.02       0.         0.13272117 0.        ]
halo_eigvals [2.22639898e+03+0.j         4.49155951e-04+0.j
 9.91213663e-01+0.13227008j 9.91213663e-01-0.13227008j
 1.00027829e+00+0.j         9.99721669e-01+0.j        ]
**********************************************************************
achieved_initial_state [0.82338437 0.         0.01       0.         0.12797622 0.        ]
halo_eigvals [2.32660346e+03+0.j         4.29811258e-04+0.j
 9.97974590e-01+0.06361305j 9.97974590e-01-0.06361305j
 1.00025958e+00+0.j         9.99740375e-01+0.j        ]
**********************************************************************
achieved_initial_state [ 8.23390919e-01  0.00000000e+00 -3.46944695e-18  0.00000000e+00
  1.26326391e-01  0.00000000e+00]
halo_eigvals [2.36115314e+03 4.23522025e-04 1.00055959e+00 9.99440609e-01
 1.00047079e+00 9.99529341e-01]
```

Figure 11

```
**********************************************************************
achieved_initial_state [ 0.85282838  0.          0.          0.         -0.12027394  0.        ]
lyapunov_eigvals [2.41703096e+03+0.j         4.13730885e-04+0.j
 1.00060050e+00+0.j         9.99399753e-01+0.j
 9.97069169e-01+0.07650479j 9.97069169e-01-0.07650479j]
**********************************************************************
achieved_initial_state [ 0.8532226   0.          0.          0.         -0.12298603  0.        ]
lyapunov_eigvals [2.40611213e+03+0.j         4.15608379e-04+0.j
 1.00052830e+00+0.j         9.99471866e-01+0.j
 9.97635655e-01+0.06872416j 9.97635655e-01-0.06872416j]
**********************************************************************
achieved_initial_state [ 0.85361681  0.          0.          0.         -0.12568781  0.        ]
lyapunov_eigvals [2.39505976e+03+0.j         4.17526266e-04+0.j
 1.00044180e+00+0.j         9.99558281e-01+0.j
 9.98212096e-01+0.05977057j 9.98212096e-01-0.05977057j]
**********************************************************************
achieved_initial_state [ 0.85401103  0.          0.          0.         -0.12837943  0.        ]
lyapunov_eigvals [2.38387875e+03+0.j         4.19484573e-04+0.j
 1.00033060e+00+0.j         9.99669400e-01+0.j
 9.98798368e-01+0.04900744j 9.98798368e-01-0.04900744j]
**********************************************************************
achieved_initial_state [ 0.85440525  0.          0.          0.         -0.13106105  0.        ]
lyapunov_eigvals [2.37257387e+03+0.j         4.21483340e-04+0.j
 1.00014809e+00+0.j         9.99851822e-01+0.j
 9.99394350e-01+0.03479714j 9.99394350e-01-0.03479714j]
**********************************************************************
achieved_initial_state [ 0.85479946  0.          0.          0.         -0.13373283  0.        ]
lyapunov_eigvals [2.36114983e+03+0.j         4.23522619e-04+0.j
 9.99999909e-01+0.00025751j 9.99999909e-01-0.00025751j
 9.99999924e-01+0.0002441j  9.99999924e-01-0.0002441j ]
```

Figure 12

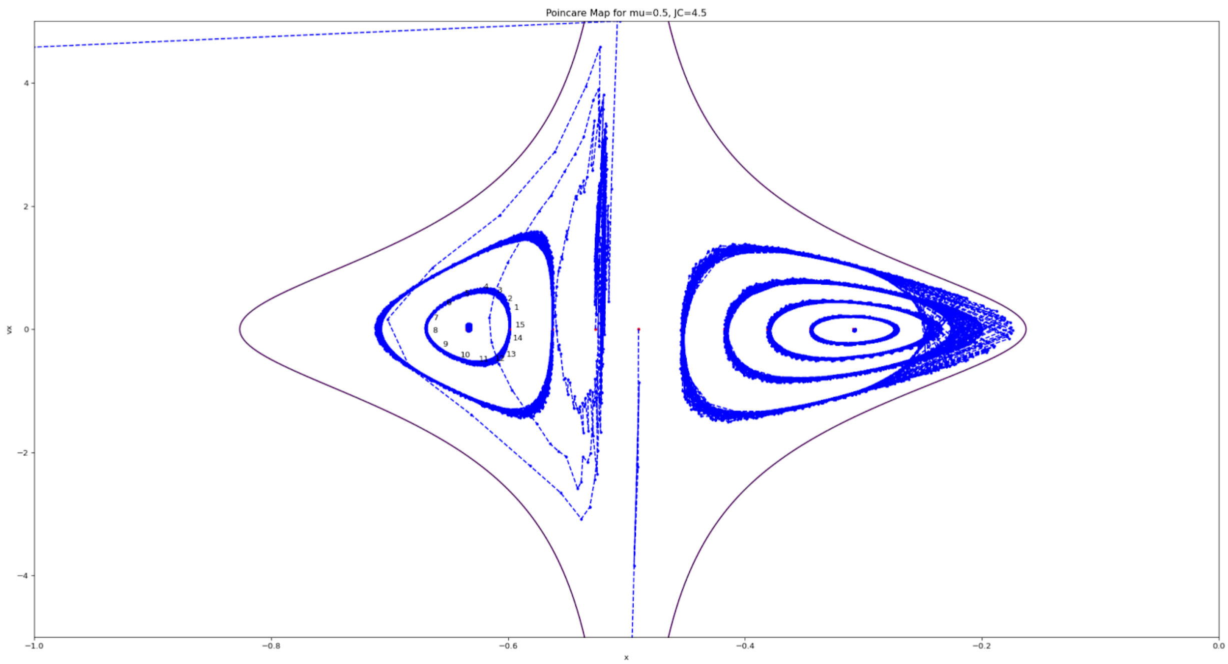

Figure 13

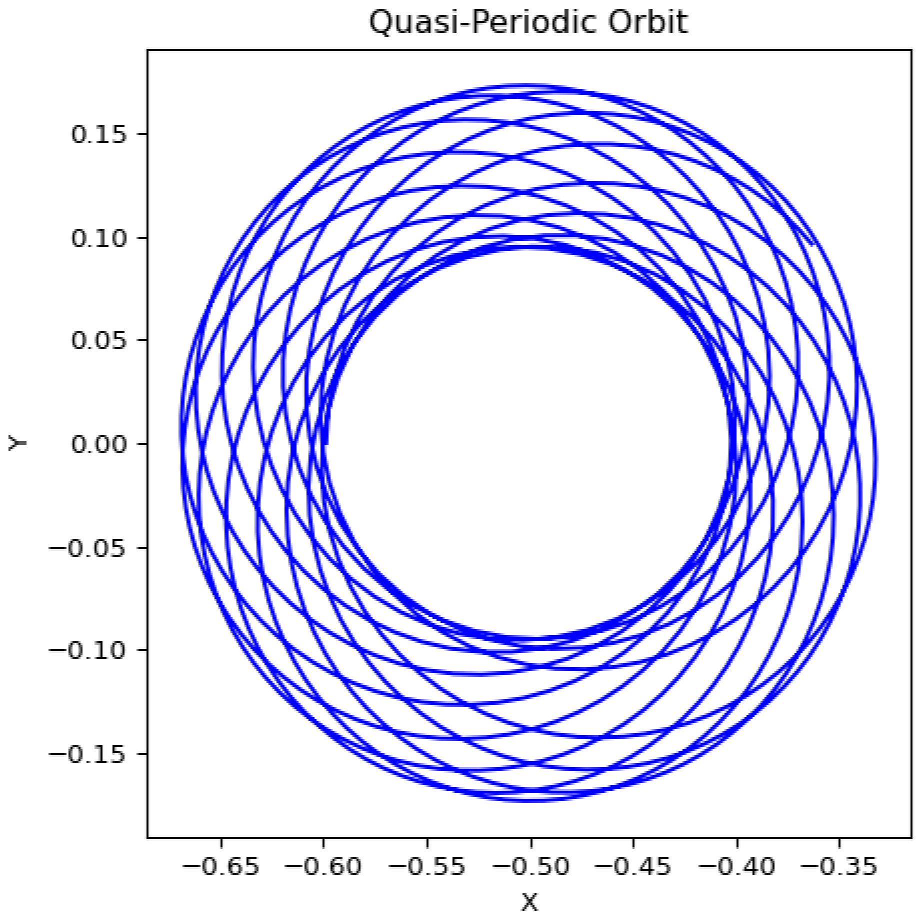
Quasi-Periodic Orbit
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
Y
−0.65
−0.60
−0.55
−0.50
−0.45
−0.40
−0.35
X

Figure 14

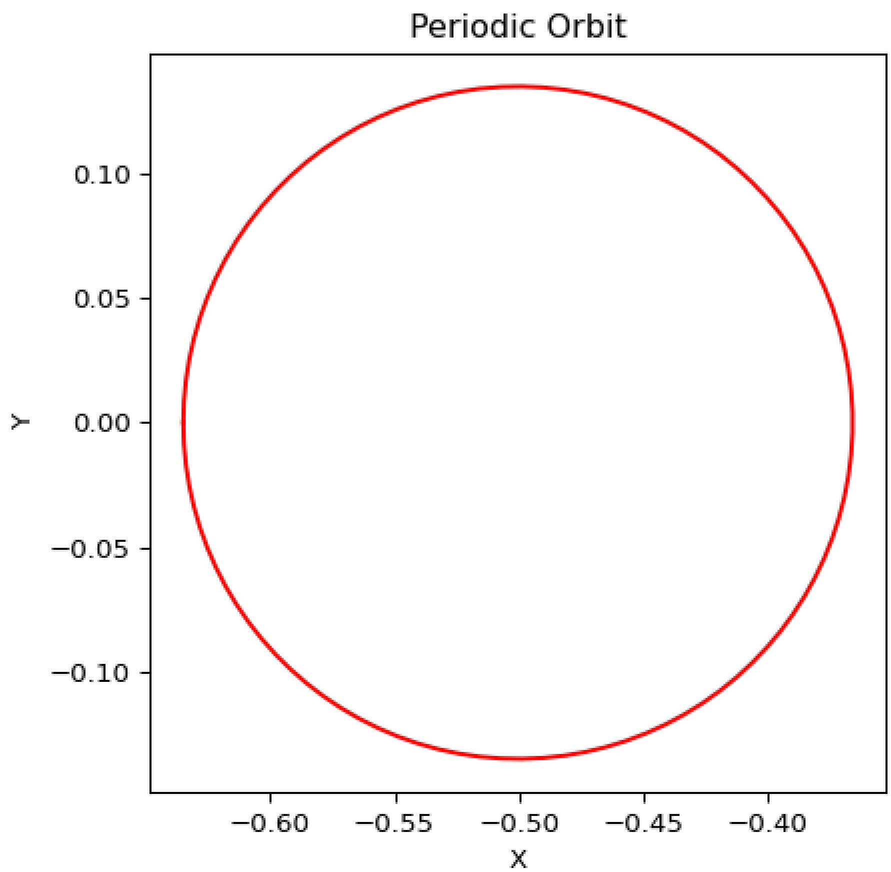

Figure 15