

# CSC2529 HW4

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Code available at <https://github.com/mdpham/CSC2529/tree/main/homework4>

## 1 HDR image fusion

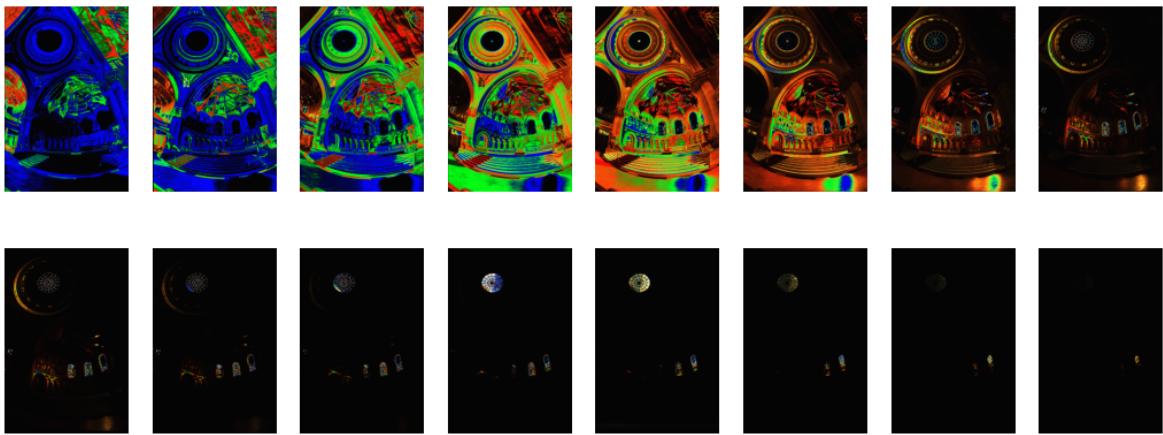


Figure 1: Debevec weight maps for LDR images

## 2 Burst Denoising and SNR

Consider a burst denoised image as the average of  $K$  images  $I_i = \tilde{I}_i + n_i$  where  $n_i$  is some independent additive noise for  $i = 1\dots K$ .

In the case of heat noise modelled by a Gaussian, define  $\mu_g = \sum_{i=1}^K \mu_i$  and  $\sigma_g = \sqrt{\sum_{i=1}^K \sigma_i^2}$  for  $n_i \sim G(\mu_i, \sigma_i^2)$ . By properties of Gaussian distributions, the sum has distribution:

$$\sum_{i=1}^K n_i \sim G(\mu_g, \sigma_g^2)$$

To obtain the average distribution of noise, divide random variables by  $K$  to get distribution of additive noise in burst image

$$N_{heat} := K^{-1} \sum_{i=1}^K n_i \sim G(K^{-1} \mu_g, K^{-2} \sigma_g^2)$$

For  $\mu_i = \mu$  and  $\sigma_i = \sigma$ , the SNR is thus

$$SNR = \frac{K^{-1} \mu_g}{\sqrt{K^{-2} \sigma_g^2}} = \frac{\sum_{i=1}^K \mu_i}{\sqrt{\sum_{i=1}^K \sigma_i^2}} = \frac{K\mu}{\sqrt{K}\sigma} = \frac{\sqrt{K}\mu}{\sigma}$$

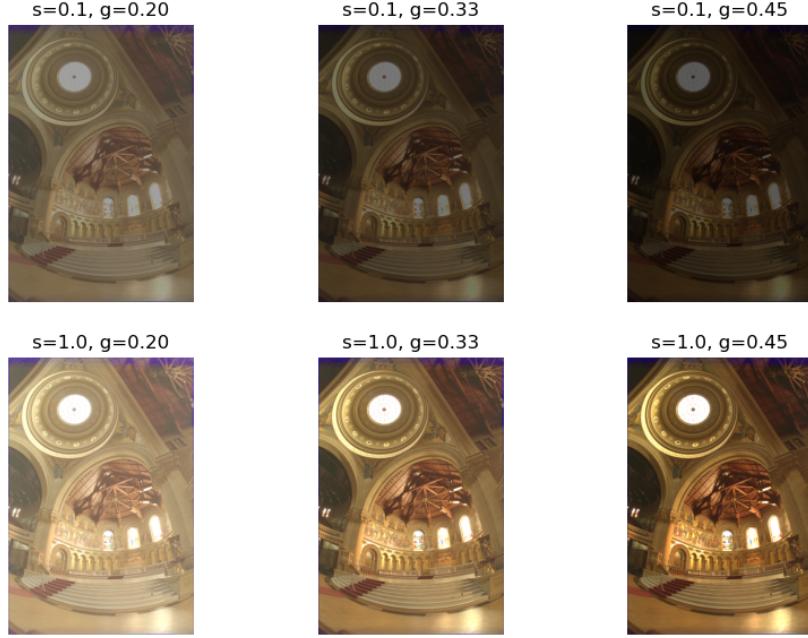


Figure 2: Scale ( $s = \{0.1, 1.0\}$ ) and gamma ( $g = \{1/5, 1/3, 1/2.2\}$ ) based tonemapping



Figure 3: Drago based tonemapping from OpenCV

In the case of shot noise modelled by a Poisson, consider  $n_i \sim P(\lambda_i)$ . By properties of Poisson distributions, the sum has distribution:

$$\sum_{i=1}^K n_i \sim P\left(\sum_{i=1}^K \lambda_i\right)$$

Define  $\lambda_p = \sum_{i=1}^K \lambda_i$ . Dividing the sum by  $K$  produces a non-Poisson distribution but we apply expectation/variance properties

$$N_{shot} := K^{-1} \sum_{i=1}^K n_i$$

where

$$E\left[\sum_{i=1}^K n_i\right] = \lambda_p = Var\left[\sum_{i=1}^K n_i\right]$$

so

$$E\left[K^{-1} \sum_{i=1}^K n_i\right] = K^{-1} E\left[\sum_{i=1}^K n_i\right] = K^{-1} \lambda_p$$

and

$$Var\left[K^{-1} \sum_{i=1}^K n_i\right] = K^{-2} Var\left[\sum_{i=1}^K n_i\right] = K^{-2} \lambda_p$$

For  $\lambda_i = \lambda$ , the SNR is thus

$$SNR = \frac{K^{-1} \lambda_p}{\sqrt{K^{-2} \lambda_p}} = \frac{K^{-1} \sum_{i=1}^K \lambda_i}{\sqrt{K^{-2} \sum_{i=1}^K \lambda_i}} = \frac{K^{-1} K \lambda}{K^{-1} \sqrt{K \lambda}} = \sqrt{K \lambda}$$

### 3 Flutter Shutter and SNR

Consider consumer camera ( $n = 10000$ ) with flutter shutter, so  $\mu = \frac{n}{2} = 5000$  and  $\sigma = 50$ .

$$SNR = \frac{\mu}{\sigma} = \frac{5000}{50} = 100$$

Consider consumer camera ( $n = 10000$ ) with burst, so  $\mu = \frac{n}{100} = 100$  and  $\sigma = 50$ . We can use the SNR formula from previous task to compute for case of Gaussian noise.

$$SNR = \frac{\sqrt{K} \mu}{\sigma} = \frac{\sqrt{100} 100}{50} = 20$$

Consider scientific camera ( $n = 1000$ ) with flutter shutter, so  $\lambda = \sigma^2 = \mu = \frac{n}{2} = 500$ .

$$SNR = \frac{\mu}{\sigma} = \frac{500}{\sqrt{500}} = \sqrt{500} \approx 22.36$$

Consider scientific camera ( $n = 1000$ ) with burst, so  $\lambda = \sigma^2 = \mu = \frac{n}{100} = 10$ . We can use the SNR formula from previous task to compute for case of Poisson noise.

$$SNR = \sqrt{K \lambda} = \sqrt{100} \sqrt{10} \approx 31.62$$

Camera	SNR	
	Flutter shutter	Burst
Consumer sCMOS ( $n = 1000$ )	100	20
	22	31

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as img
import skimage.io as io
import imageio
import cv2

from pdb import set_trace
from pathlib import Path
```

```
In [2]: hdr_dir = 'hdr_data'

def read_exposures():
    filepaths = []
    exposures = []
    with open(Path(hdr_dir, 'memorial.hdr_image_list.txt')) as f:
        lines = f.readlines()
    for l in lines[3:]:
        data = l.split(' ')
        img_png = data[0].replace('.ppm', '.png')
        expo = 1/float(data[1]) #read value is 1/shutter speed, so inverse
        filepaths.append(Path(hdr_dir, img_png))
        exposures.append(expo)
    return filepaths, exposures

filepaths, exposures = read_exposures()
print(exposures)

def read_img(filepath):
    img = io.imread(filepath).astype(float)/255
    return img

def plot_images(imgs):
    fig, axes = plt.subplots(2, 8, figsize=(13, 5))
    ax = axes.ravel()
    for i in range(len(imgs)):
        ax[i].imshow(imgs[i])
        ax[i].axis('off')
    return fig, axes

imgs = [read_img(fp) for fp in filepaths]
# imgs[0].shape
plot_images(imgs)
```

```
[32.0, 16.0, 8.0, 4.0, 2.0, 1.0, 0.5, 0.25, 0.125, 0.0325, 0.015625, 0.0078125, 0.00390625, 0.001953125, 0.0009765625]
```

```
Out[2]: <Figure size 1300x500 with 16 Axes>
array([<AxesSubplot: 0, 1>, <AxesSubplot: 0, 2>, <AxesSubplot: 0, 3>, <AxesSubplot: 0, 4>, <AxesSubplot: 0, 5>, <AxesSubplot: 0, 6>, <AxesSubplot: 0, 7>, <AxesSubplot: 1, 0>, <AxesSubplot: 1, 1>, <AxesSubplot: 1, 2>, <AxesSubplot: 1, 3>, <AxesSubplot: 1, 4>, <AxesSubplot: 1, 5>, <AxesSubplot: 1, 6>, <AxesSubplot: 1, 7>, <AxesSubplot: 2, 0>, <AxesSubplot: 2, 1>, <AxesSubplot: 2, 2>, <AxesSubplot: 2, 3>, <AxesSubplot: 2, 4>, <AxesSubplot: 2, 5>, <AxesSubplot: 2, 6>, <AxesSubplot: 2, 7>], dtype=object)
```

```
In [3]: def linearize_img(img, gamma=2.2):
    return np.power(img, 2.2)

lin_imgs = list(map(lambda x: linearize_img(x, 2.2), imgs))
plot_images(lin_imgs)
```

```
Out[3]: <Figure size 1300x500 with 16 Axes>
array([<AxesSubplot: 0, 1>, <AxesSubplot: 0, 2>, <AxesSubplot: 0, 3>, <AxesSubplot: 0, 4>, <AxesSubplot: 0, 5>, <AxesSubplot: 0, 6>, <AxesSubplot: 0, 7>, <AxesSubplot: 1, 0>, <AxesSubplot: 1, 1>, <AxesSubplot: 1, 2>, <AxesSubplot: 1, 3>, <AxesSubplot: 1, 4>, <AxesSubplot: 1, 5>, <AxesSubplot: 1, 6>, <AxesSubplot: 1, 7>, <AxesSubplot: 2, 0>, <AxesSubplot: 2, 1>, <AxesSubplot: 2, 2>, <AxesSubplot: 2, 3>, <AxesSubplot: 2, 4>, <AxesSubplot: 2, 5>, <AxesSubplot: 2, 6>, <AxesSubplot: 2, 7>], dtype=object)
```

```
In [4]: def compute_hdr_weights(img):
    center_dr = 0.5 # center of dynamic range between float[0, 1]
    w = np.zeros_like(img)
    for i in range(3):
        img_i = img[:, :, i]
        w[:, :, i] = np.exp(-4*np.square((img_i - center_dr)/center_dr))
    w = np.exp(-4*np.square((img - center_dr)/center_dr))
    return w
```

```
# hdr_debevec(lin_imgs, exposures)
```

```
weight_maps = list(map(compute_hdr_weights, lin_imgs))
plot_images(weight_maps)
```

```
Out[4]: <Figure size 1300x500 with 16 Axes>
array([<AxesSubplot: 0, 1>, <AxesSubplot: 0, 2>, <AxesSubplot: 0, 3>, <AxesSubplot: 0, 4>, <AxesSubplot: 0, 5>, <AxesSubplot: 0, 6>, <AxesSubplot: 0, 7>, <AxesSubplot: 1, 0>, <AxesSubplot: 1, 1>, <AxesSubplot: 1, 2>, <AxesSubplot: 1, 3>, <AxesSubplot: 1, 4>, <AxesSubplot: 1, 5>, <AxesSubplot: 1, 6>, <AxesSubplot: 1, 7>, <AxesSubplot: 2, 0>, <AxesSubplot: 2, 1>, <AxesSubplot: 2, 2>, <AxesSubplot: 2, 3>, <AxesSubplot: 2, 4>, <AxesSubplot: 2, 5>, <AxesSubplot: 2, 6>, <AxesSubplot: 2, 7>], dtype=object)
```

```
In [5]: def crop_boundary(img):
    # crop boundary - image data here are only captured in some of the exposures, which is why they are indicated in blue in the LDR images
    return img[29:720, 19:480, :]
```

```
# approximate true HDR image X
```

```
def hdr_debevec(imgs, exposures):
    # initialize HDR image with all zeros
```

```
    hdr = np.zeros((768, 512, 3), dtype=float)
```

```
    # fuse LDR images using weights, make sure to store your fused HDR using the name hdr
```

```
    weights = []
    for k in range(len(imgs)):
        # add machine precision to avoid log(0)
        img_k = imgs[k] + np.finfo(np.float32).eps
```

```
        t_k = exposures[k]
```

```
        # compute weights
```

```
        w_k = compute_hdr_weights(img_k)
```

```
        weights.append(w_k)
```

```
        # compute and accumulate sum
```

```
        x_k = np.zeros_like(hdr)
```

```
        for i in range(3):
            x_k[:, :, i] = np.multiply(w_k[:, :, i], (np.log(img_k[:, :, i]) - np.log(t_k)))
```

```
        hdr += x_k
```

```
    print(hdr[1, 1, :])
```

```
    scale = np.sum(weights, axis=0) # weight scale
```

```
    hdr = np.exp(hdr/scale)
```

```
    # crop and clip
```

```
    # hdr = crop_boundary(hdr)
```

```
    hdr = np.clip(hdr, 0, 1)
```

```
    return hdr
```

```
hdr_lin_img = hdr_debevec(lin_imgs, exposures)
```

```
test = hdr_lin_img
```

```
# test = np.uint8(np.clip(3*test, 0, 1) * 255)
```

```
# test[:, :, 1] = 0
```

```
# test[:, :, 0] = 0
```

```
io.imshow(test)
```

```
# np.max(hdr_lin_img)
```

```
[ -0.12349241 -3.65027509 -0.87557801]
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```
[ -3.63585318 -3.8787933 -0.8121096]
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[ -3.85156159 -4.13891611 -0.7359280]
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[ -4.0770569 -4.32344218 -0.76411056]
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