

Análise de Covariância ANCOVA

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2020

Panorama

- Apresentaremos uma técnica fundamentada tanto na análise de variância quanto na regressão linear denominada análise de covariância de um fator e uma covariável: ANCOVA unifatorial simples
- Uma ANCOVA simples indica se os grupos independentes diferem em uma VD enquanto se mantêm fixos os efeitos de outra variável denominada covariável (VE)
- Uma covariável, em geral, é uma variável quantitativa **sem erro de mensuração** que apresenta um relacionamento linear com a VD e não apresenta associação com o fator entre participantes (VI nominal)
- Na ANCOVA, a variável quantitativa cujo efeito é removido é denominada covariável
- Nessa aula discutiremos o delineamento entre participantes de um fator e o uso de uma covariável: ANCOVA unifatorial simples

Panorama

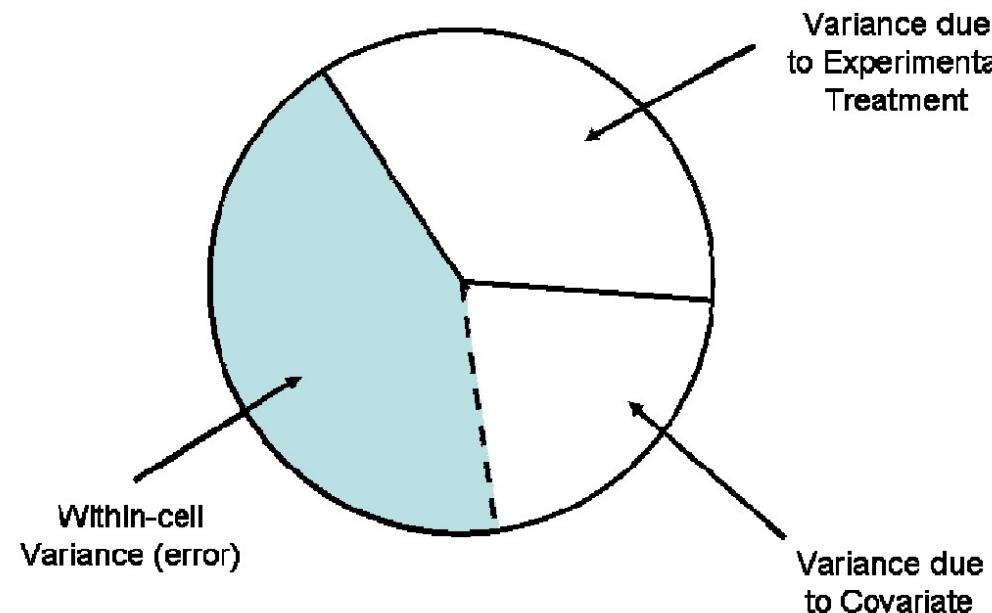
- Existem duas razões principais para usar ANCOVA
 - I. Reduzir a variância do resíduo
 - II. Ajustar as médias da covariável, de modo que o valor médio da covariável seja o mesmo em todos os grupos e depois ajustar as médias e erros-padrão da VD nos grupos (equalização estatística de condições entre participantes)

Panorama

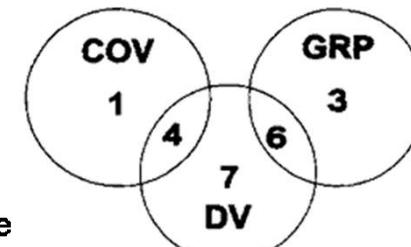
- ANCOVA unifatorial simples
 - Estatísticas descritivas
 - Tamanho de efeito
 - Teste inferencial ANCOVA que testa a ausência do efeito do fator na VD controlando pela covariável

ANCOVA

Schema of Partitioning Variation in ANCOVA



Suposição de
dissociação:
Fator (GRuPo) e
COVariável não
associados



Hipótese nula da ANCOVA

NULL AND ALTERNATIVE HYPOTHESES

The null hypothesis and the alternative hypothesis for ANCOVA are similar to those for ANOVA. Conceptually, however, these population means have been adjusted for the covariate. Thus, in reality, the null hypothesis of ANCOVA is of no difference among the *adjusted* population means.

$$H_0: \mu_1 = \mu_2 = \dots = \mu_k$$

$$H_a: \mu_i \neq \mu_k \quad \text{for some } i, k$$

Table 2.1 *Cohen's effect size benchmarks*

| Test | Relevant effect size | Effect size classes | | |
|---------------------------------|--------------------------------|---------------------|--------|-------|
| | | Small | Medium | Large |
| Comparison of independent means | $d, \Delta, \text{Hedges' } g$ | .20 | .50 | .80 |
| Comparison of two correlations | q | .10 | .30 | .50 |
| Difference between proportions | Cohen's g | .05 | .15 | .25 |
| Correlation | r | .10 | .30 | .50 |
| | r^2 | .01 | .09 | .25 |
| Crosstabulation | w, φ, V, C | .10 | .30 | .50 |
| ANOVA | f | .10 | .25 | .40 |
| Multiple regression | partial η^2 | .01 | .06 | .14 |
| | R^2 | .02 | .13 | .26 |
| | f^2 | .02 | .15 | .35 |

Notes: The rationale for most of these benchmarks can be found in Cohen (1988) at the following pages: Cohen's d (p. 40), q (p. 115), Cohen's g (pp. 147–149), r and r^2 (pp. 79–80), Cohen's w (pp. 224–227), f and η^2 (pp. 285–287), R^2 and f^2 (pp. 413–414).

Exemplo: Altura do tomateiro (*Solanum lycopersicum*)



- Um laboratório recebeu sete mudas de tomateiro.
- No momento da transplantação suas alturas foram medidas.
- Elas foram alocadas aleatoriamente a três tipos de fertilizantes.
- Após 10 semanas, as alturas dos tomateiros são medidas.
- Diferentes tipos de fertilizantes têm diferentes efeitos nas alturas dos tomateiros?
- Fonte: Norusis (2012a)

Exemplo: Altura do tomateiro

| Fertilizante | Altura inicial (mm) | Altura final (mm) |
|--------------|---------------------|-------------------|
| A | 3 | 74 |
| | 4 | 68 |
| | 5 | 77 |
| B | 2 | 76 |
| | 4 | 80 |
| C | 3 | 87 |
| | 7 | 91 |

| | A | B | C |
|---|---------------------|--------------------|----------------------|
| 1 | Fertilizante | AlturaFinal | AlturaInicial |
| 2 | A | 74 | 3 |
| 3 | A | 68 | 4 |
| 4 | A | 77 | 5 |
| 5 | B | 76 | 2 |
| 6 | B | 80 | 4 |
| 7 | C | 87 | 3 |
| 8 | C | 91 | 7 |

Tomato.xlsx

ANCOVA robusta à heterocedasticidade (White)

ANCOVA_TomateFertilizante.R

```

library(readxl)
library(gplots)
library(car)
library(lsqr)
library(emmeans)
library(multcomp)
library(ggplot2)
alfa <- 0.05
Dados <- readxl::read_excel("Tomato.xlsx")
Dados$Fertilizante <- factor(Dados$Fertilizante,
                               levels=unique(Dados$Fertilizante))
head(Dados)
with(Dados, gplots:::plotmeans(AlturaFinal ~ Fertilizante,
                               error.bars="conf.int", level=1-alfa, connect=FALSE,
                               xlab="Fertilizante", ylab="Altura Final (mm)", main="IC95%",
                               barcol="black"))
with(Dados, car:::scatterplot(AlturaInicial, AlturaFinal,
                               groups = Fertilizante,
                               regLine=TRUE, smooth=FALSE, ellipse=FALSE, col="black",
                               xlab = "Altura Inicial (mm)", ylab = "Altura Final (mm)"))
print(ggplot(Dados, aes(y = AlturaFinal, x = AlturaInicial,
                        group = Fertilizante, linetype = Fertilizante)) +
      geom_point() + geom_smooth(method = "lm"))

```

ANCOVA_TomateFertilizante.R

```
# VD: AlturaFinal VI: Fertilizante
modelo <- lm(AlturaFinal ~ Fertilizante, data=Dados)
print(Anova <- car::Anova(modelo, type=3, white.adjust=TRUE))
print(lsrr::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans::emmeans(modelo, "Fertilizante"))
print(grf <- plot(EMM, colors = "black",
                   main="Estimated Marginal Means",
                   xlab="Altura Final (mm)",
                   ylab="Fertilizante"))

mc <- multcomp::glht(modelo, linfct = mcp(Fertilizante = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc,las=3)

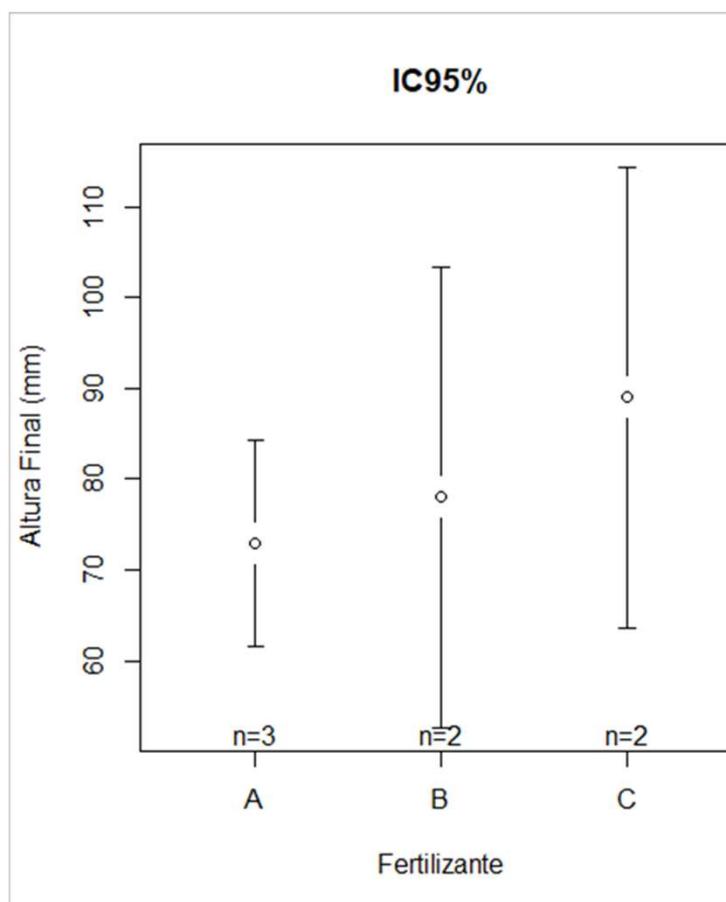
# VD: AlturaFinal - AlturaInicial VI: Fertilizante
Diferencia <- Dados$AlturaFinal - Dados$AlturaInicial
modelo <- lm(Diferencia ~ Fertilizante, data=Dados)
print(Anova <- car::Anova(modelo, type=3, white.adjust=TRUE))
print(lsrr::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans::emmeans(modelo, "Fertilizante"))
print(grf <- plot(EMM, colors = "black",
                   main="Estimated Marginal Means",
                   xlab="AlturaFinal - AlturaInicial (mm)",
                   ylab="Fertilizante"))

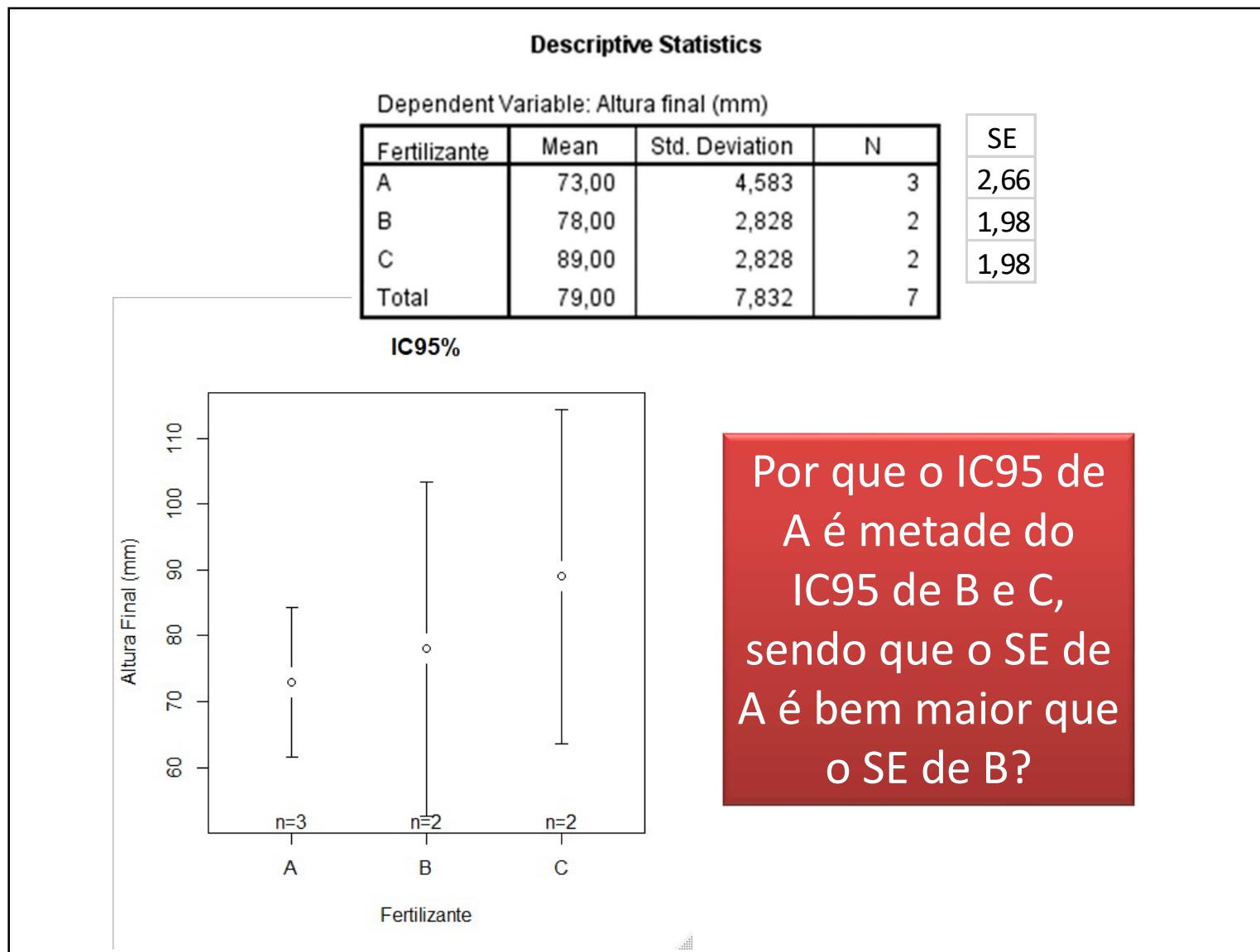
mc <- multcomp::glht(modelo, linfct = mcp(Fertilizante = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc,las=3)
```

ANCOVA_TomateFertilizante.R

```
# ANCOVA
## ANCOVA: Teste de dissociacao entre fator e covariavel
cat("\nANCOVA: Teste de dissociacao entre fator e covariavel\n")
modelo <- lm(AlturaInicial ~ Fertilizante, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
## ANCOVA: Teste de igualdade das inclinacoes das retas de regressao
cat("\nANCOVA: Teste de igualdade das inclinacoes das retas de regressao\n")
modelo <- lm(AlturaFinal ~ Fertilizante + AlturaInicial + Fertilizante*AlturaInicial,
             data=Dados)
print(Anova <- car:::Anova(modelo, type=3))
## ANCOVA: Teste do efeito do fator fixo:
##           Se as declividades sao iguais, testar se os interceptos sao iguais.
cat("\nANCOVA: Teste do efeito do fator fixo:\n")
cat("\tSe as declividades sao iguais, testar se os interceptos sao iguais.\n")
modelo <- lm(AlturaFinal ~ Fertilizante + AlturaInicial,
             data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
print(lsrr::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans::emmeans(modelo, "Fertilizante"))
print(grf <- plot(EMM, colors = "black",
                   main="Estimated Marginal Means",
                   xlab="AlturaFinal controlada por AlturaInicial (mm)",
                   ylab="Fertilizante"))
mc <- multcomp::glht(modelo, linfct = mcp(Fertilizante = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc, las=3)
```

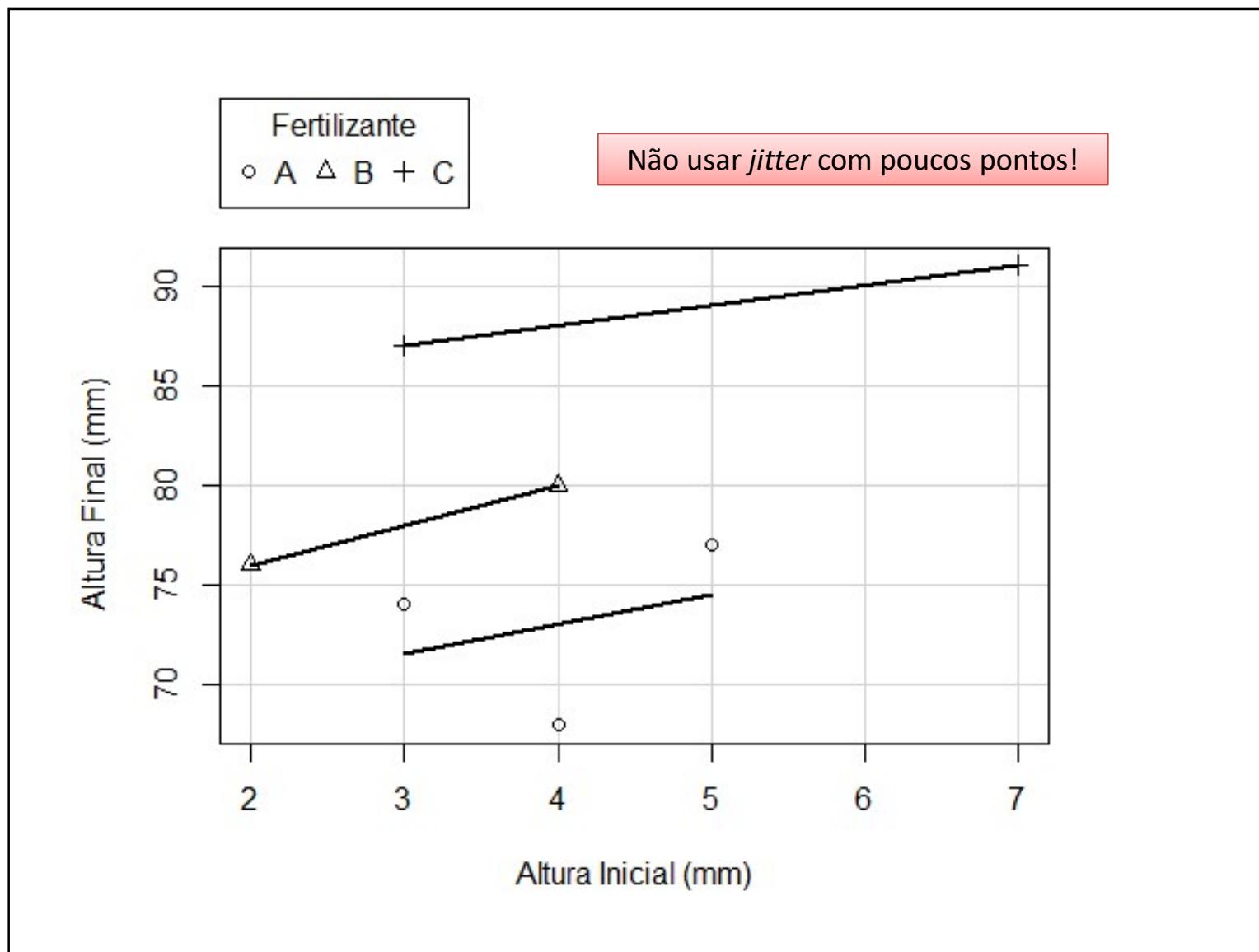
IC95 das alturas finais médias populacionais Sem modelo

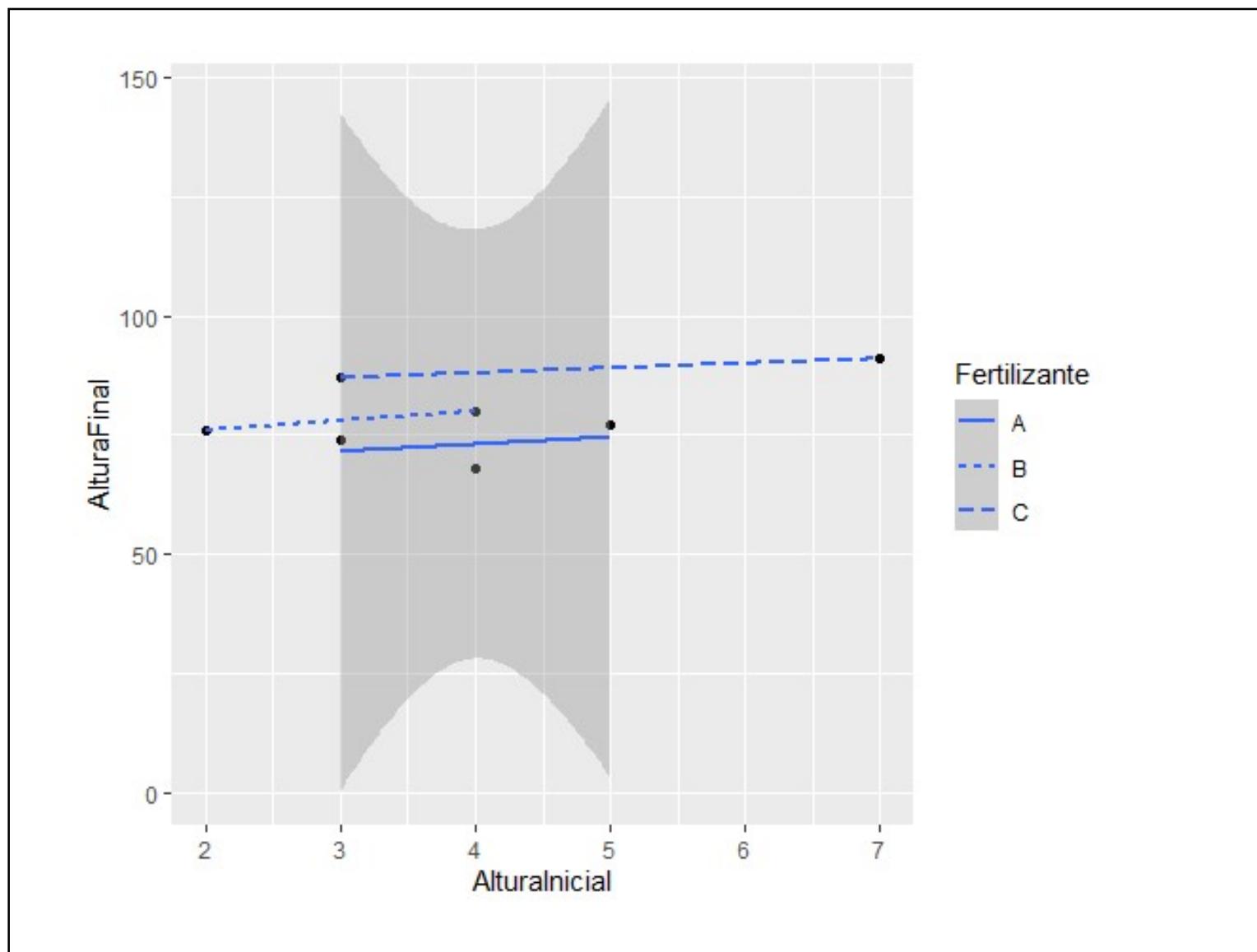




| Fertilizante | | Statistic | Std. Error |
|-------------------|---|-------------------------------------|------------|
| Altura final (mm) | A | Mean | 73,00 |
| | | 95% Confidence Interval for Mean | 2,646 |
| | | Lower Bound | 61,62 |
| | | Upper Bound | 84,38 |

- No cálculo do IC95, o quantil da distribuição t de Student com $gl = n-1$ graus de liberdade, sendo n o número de observações do grupo.
- Se o número de observações aumenta, esse quantil tende para o quantil da distribuição normal padrão.
- M e S são, respectivamente, a média e o desvio-padrão amostrais da altura final.
- O erro-padrão (*standard error of mean*) é S/\sqrt{n} .
 - $IC95 = [M - qt(0.975, n-1)*S/\sqrt{n}; M + qt(0.975, n-1)*S/\sqrt{n}]$
 - Se $n=3$, $q = 4,3$.
 - Se $n=2$, $q = 12,7$.
- Exemplo:
 - VD = Altura final do tomateiro (mm),
Grupo = Fertilizante A, $n = 3$.
 - $IC95 = [73 - 4,3*4,583/\sqrt{3}; 73 + 4,3*4,583/\sqrt{3}] = [61,62; 84,38]$.





Teste omnibus

```
Coefficient covariances computed by hccm()
Analysis of Deviance Table (Type III tests)

Response: AlturaFinal
          Df      F    Pr(>F)
(Intercept) 1 507.524 2.299e-05 ***
Fertilizante 2    7.583 0.04356 *
Residuals   4
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
            eta.sq eta.sq.part
Fertilizante 0.8423913 0.8423913
```

Testes post hoc

| Fertilizante | emmean | SE | df | lower.CL | upper.CL |
|--------------|--------|------|----|----------|----------|
| A | 73 | 2.20 | 4 | 66.9 | 79.1 |
| B | 78 | 2.69 | 4 | 70.5 | 85.5 |
| C | 89 | 2.69 | 4 | 81.5 | 96.5 |

```

Simultaneous Tests for General Linear Hypotheses

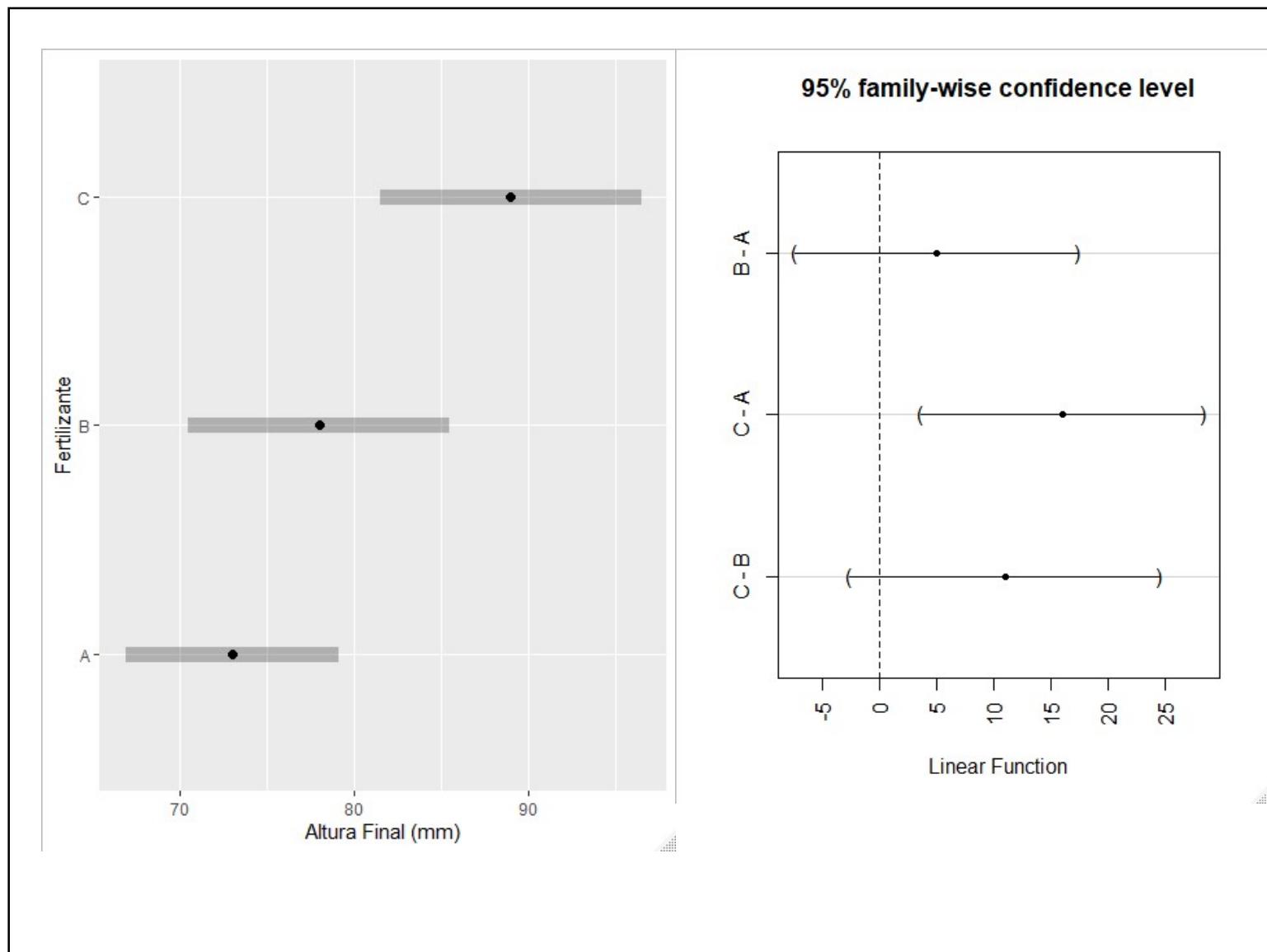
Multiple Comparisons of Means: Tukey Contrasts

Fit: lm(formula = AlturaFinal ~ Fertilizante, data = Dados)

Linear Hypotheses:
Estimate Std. Error t value Pr(>|t|)
B - A == 0     5.000    3.476   1.438   0.671
C - A == 0    16.000    3.476   4.603   0.030 *
C - B == 0    11.000    3.808   2.889   0.134
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- bonferroni method)

```

| A | B | C |
|-----|------|-----|
| "b" | "ab" | "a" |



Teste omnibus

```
Coefficient covariances computed by hccm()  
Analysis of Deviance Table (Type III tests)  
  
Response: Diferenca  
          Df      F    Pr(>F)  
(Intercept) 1 501.158 2.357e-05 ***  
Fertilizante 2  32.092 0.003442 **  
Residuals   4  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

| | eta.sq | eta.sq.part |
|--------------|-----------|-------------|
| Fertilizante | 0.8709677 | 0.8709677 |

Testes post hoc

| Fertilizante | emmmean | SE | df | lower.CL | upper.CL |
|--------------|---------|------|----|----------|----------|
| A | 69 | 1.83 | 4 | 63.9 | 74.1 |
| B | 75 | 2.24 | 4 | 68.8 | 81.2 |
| C | 84 | 2.24 | 4 | 77.8 | 90.2 |

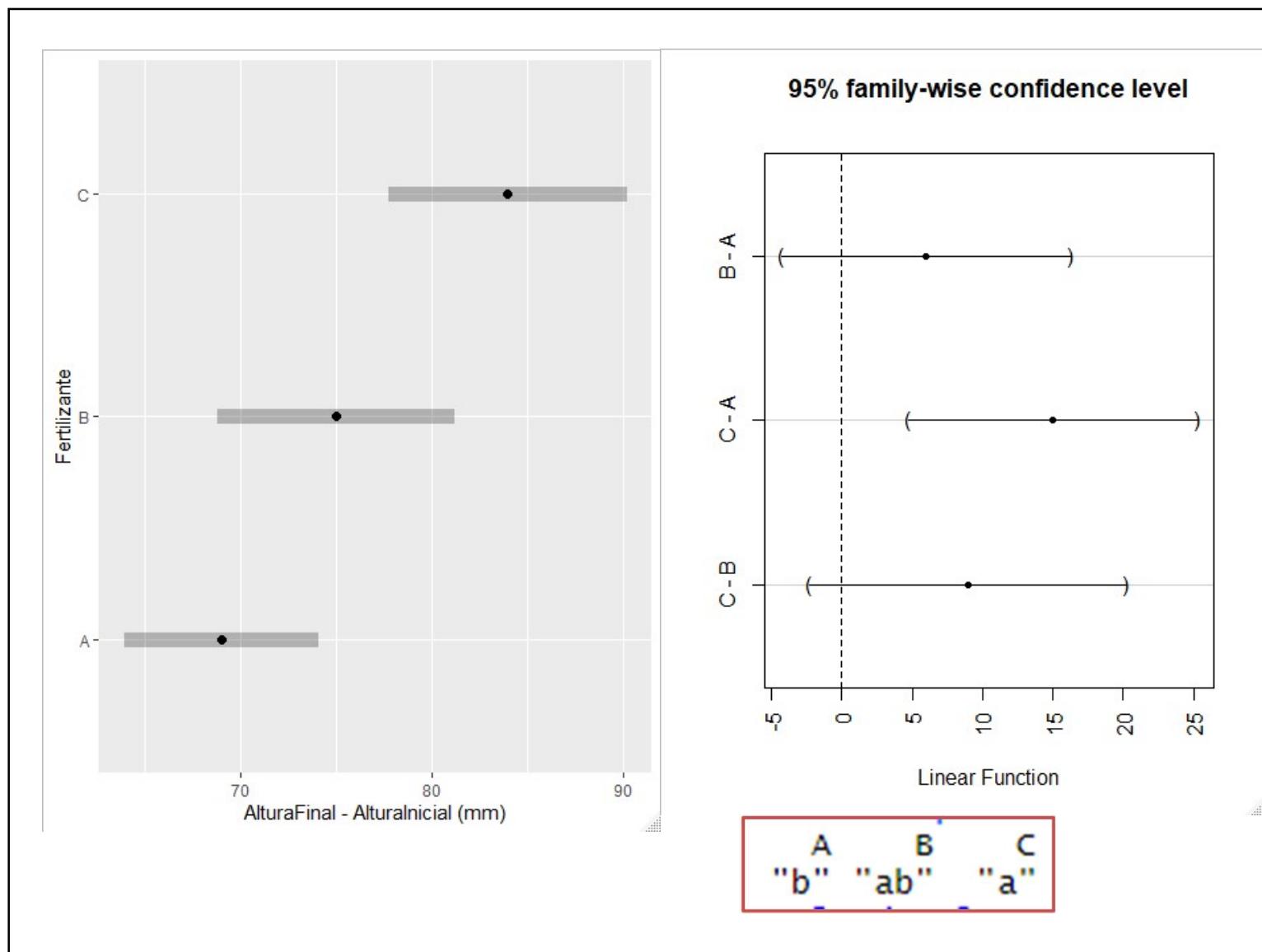
Confidence level used: 0.95

```
simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

Fit: lm(formula = Diferenca ~ Fertilizante, data = Dados)

Linear Hypotheses:
Estimate Std. Error t value Pr(>|t|)
B - A == 0    6.000    2.887   2.078   0.3186 
C - A == 0   15.000    2.887   5.196   0.0196 *
C - B == 0    9.000    3.162   2.846   0.1397 
---
signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- bonferroni method)
```



Assumptions

- 1. Independence:
observations are independent of each other.
- 2. Interval scale:
the dependent variable is measured on at least an interval scale.
- 3. Normality:
the residuals are normally distributed. Use normal quantile plots and the Shapiro-Wilk test.
- 4. Homogeneity of variance:
the groups (in our case three groups) have the same variance. Use Levene's test and Hartley's test.
- 5. Dissociation:
the covariate and the factor variable are independent of each other.
The covariate does not overlap with the effect of the factor variable(s). They each explain a different part of the variance in the dependent variable.

Assumptions

- In order to test this assumption, run an ANOVA with the factor variable(s) as independent variable(s), and the potential covariate as dependent variable.

When the result of the ANOVA test is not significant, there is no dependency between the independent variable and the covariate.

6. Homogeneity of regression slopes:

The dependent variable and any covariate(s) have the same slopes across all levels of the categorical grouping variable (factors).

- 7. Covariável sem erro de mensuração

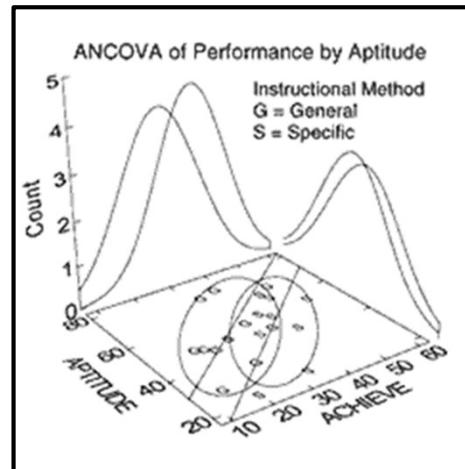
13.1.2 The covariate should be measured without error (i.e. reliably)

This means that, if you were to measure your participants' scores on the covariate on different occasions, there should be a high correlation between the scores they obtain on those different occasions. So IQ, for example, is a reliable covariate; it is made without significant measurement error: there is a high correlation between your IQ score this week and your IQ score on the next week. If you score 110 this week, you probably will *not* score exactly the same next week; remember, this is not what is implied by a high correlation. It does mean that people who have low scores this week will have low scores next week, and people who have high scores this week will have high scores next week. Age is measured without error: your age this week is perfectly correlated with your age next week. In your work at college, at some time, you may design your own questionnaires. Let's say you decide to use the data from such a questionnaire as a covariate. Do you know how reliable they are? Unless you have tested the questionnaire and know it to be reliable, you could be measuring your covariate *with error*.

Analysis of Covariance



Introduction to Statistics
Carl von Ossietzky Universität Oldenburg
Fakultät III - Sprach- und Kulturwissenschaften

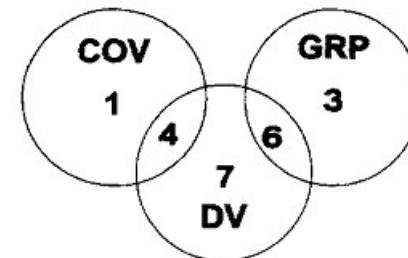


ANCOVA

Análise de covariância robusta à heteroCEDASTICIDADE (White)

ANCOVA

Teste de dissociação entre fator fixo e covariável
sem erro de mensuração



```
Coefficient covariances computed by hccm()
Analysis of Deviance Table (Type III tests)

Response: AlturaInicial
          Df      F   Pr(>F)
(Intercept) 1 32.0000 0.004813 ***
Fertilizante 2  0.2857 0.765625
Residuals    4
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

p > 0.05

não rejeitar a hipótese nula de dissociação

ANCOVA

Teste de igualdade das inclinações das retas de regressão

Anova Table (Type III tests)

Response: AlturaFinal

| | Sum Sq | Df | F value | Pr(>F) |
|----------------------------|--------|----|---------|--------|
| (Intercept) | 538.68 | 1 | 14.3648 | 0.1642 |
| Fertilizante | 30.40 | 2 | 0.4053 | 0.7432 |
| AlturaInicial | 4.50 | 1 | 0.1200 | 0.7877 |
| Fertilizante:AlturaInicial | 1.75 | 2 | 0.0233 | 0.9775 |
| Residuals | 37.50 | 1 | | |

p > 0.05

não rejeitar a hipótese nula de igualdade das inclinações das retas de regressão

ANCOVA

Teste *omnibus*

$$H_0: \mu'_A = \mu'_B = \mu'_C$$

- Igualdade médias marginais populacionais
- Hipótese alternativa: alguma igualdade é violada

ANCOVA: teste *omnibus*

Teste do efeito do fator fixo:

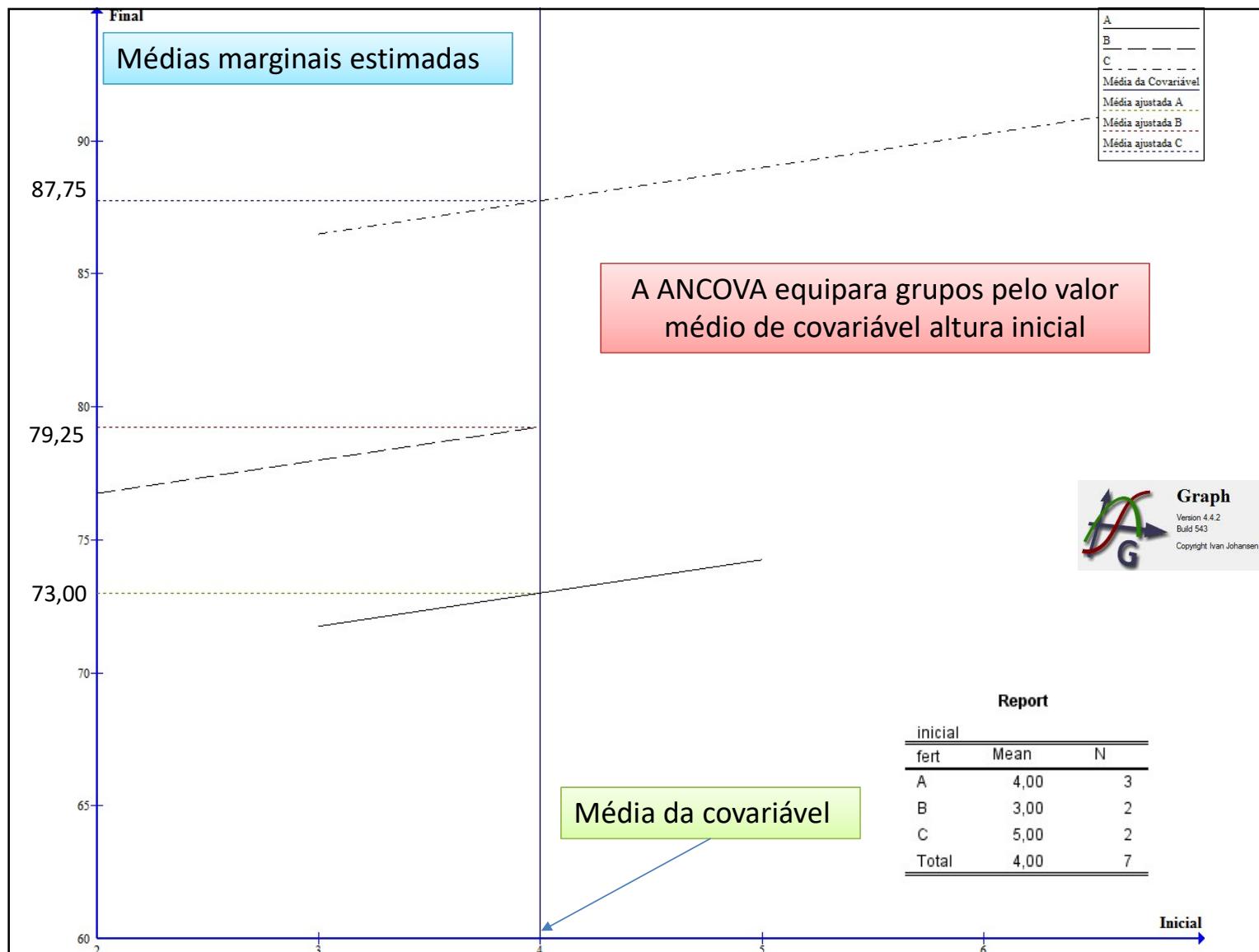
Se as inclinações são iguais, testar se os interceptos são iguais

```
Coefficient covariances computed by hccm()
Analysis of Deviance Table (Type III tests)

Response: AlturaFinal
            Df      F    Pr(>F)
(Intercept)  1 214.2502 0.0006916 ***
Fertilizante  2   7.2405 0.0710940 .
AlturaInicial 1   1.9467 0.2572933
Residuals     3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Se $p < 0.05$,
 Rejeitar a hipótese nula de ausência do
 efeito de Fertilizante na AlturaFinal
 controlando pela covariável AlturaInicial

| | eta.sq | eta.sq.part |
|---------------|------------|-------------|
| Fertilizante | 0.66083560 | 0.8610312 |
| AlturaInicial | 0.05095109 | 0.3232759 |



Média marginal ajustada

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|---------------|----------|------------|---------|--------------|
| (Intercept) | 68.000 | 4.670 | 14.562 | 0.000702 *** |
| FertilizanteB | 6.250 | 3.463 | 1.805 | 0.168876 |
| FertilizanteC | 14.750 | 3.463 | 4.259 | 0.023735 * |
| AlturaInicial | 1.250 | 1.044 | 1.197 | 0.317227 |

- Intercepto A = $68 + 0 = 68.00$
- Intercepto B = $68 + 6.25 = 74.25$
- Intercepto C = $68 + 14.75 = 82.75$
- Declividade comum = 1.25
- Altura inicial (covariável) média = 4 mm
- Média marginal ajustada da altura final A = $68 + 1.25 \cdot 4 = 73.00$
- Média marginal ajustada da altura final B = $74.25 + 1.25 \cdot 4 = 79.25$
- Média marginal ajustada da altura final C = $82.75 + 1.25 \cdot 4 = 87.75$

ANCOVA

Testes *post hoc* via médias marginais ajustadas

| Fertilizante | emmean | SE | df | lower.CL | upper.CL |
|--------------|--------|------|----|----------|----------|
| A | 73.0 | 2.09 | 3 | 66.4 | 79.6 |
| B | 79.2 | 2.76 | 3 | 70.5 | 88.0 |
| C | 87.8 | 2.76 | 3 | 79.0 | 96.5 |

Confidence level used: 0.95

```

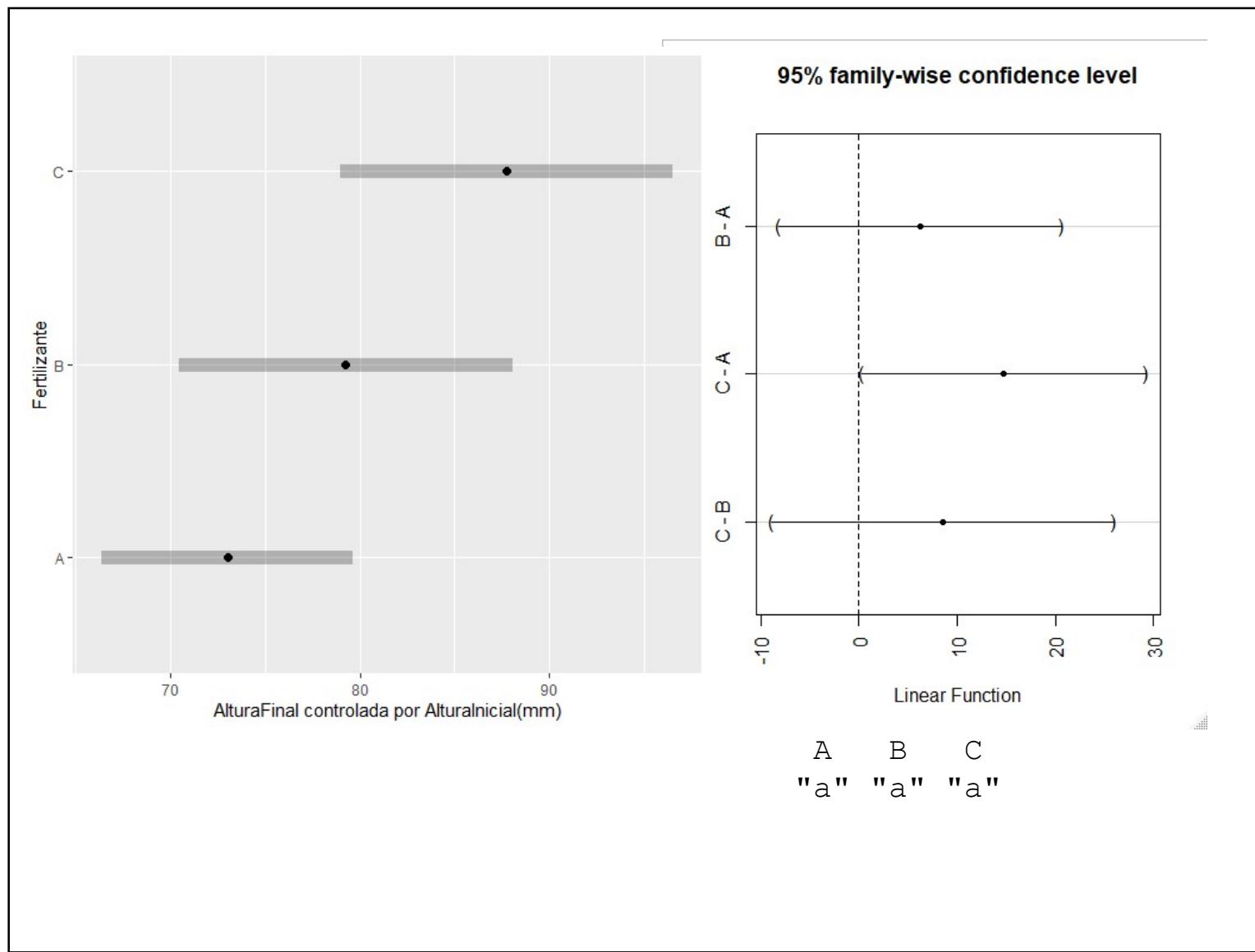
Simultaneous Tests for General Linear Hypotheses

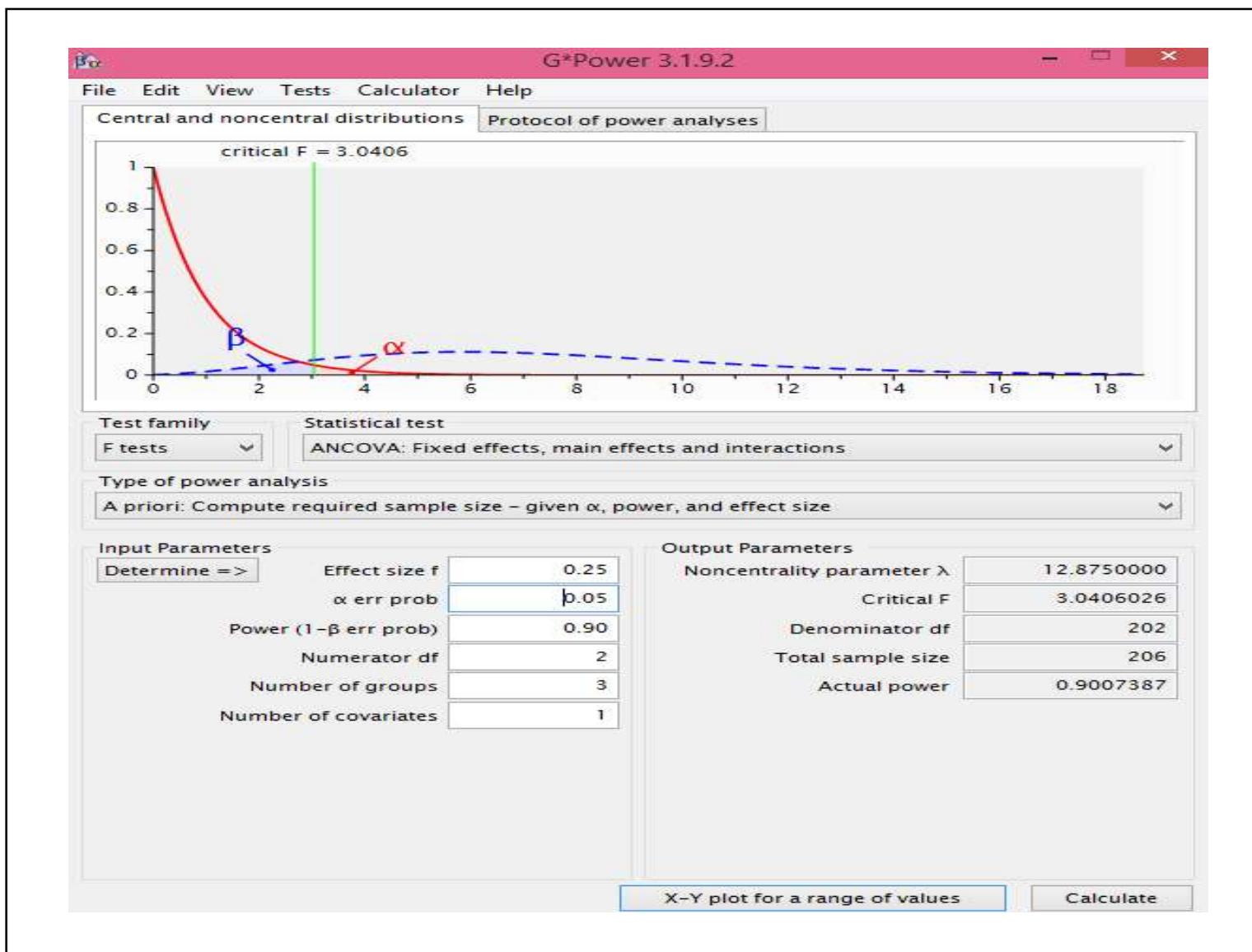
Multiple Comparisons of Means: Tukey Contrasts

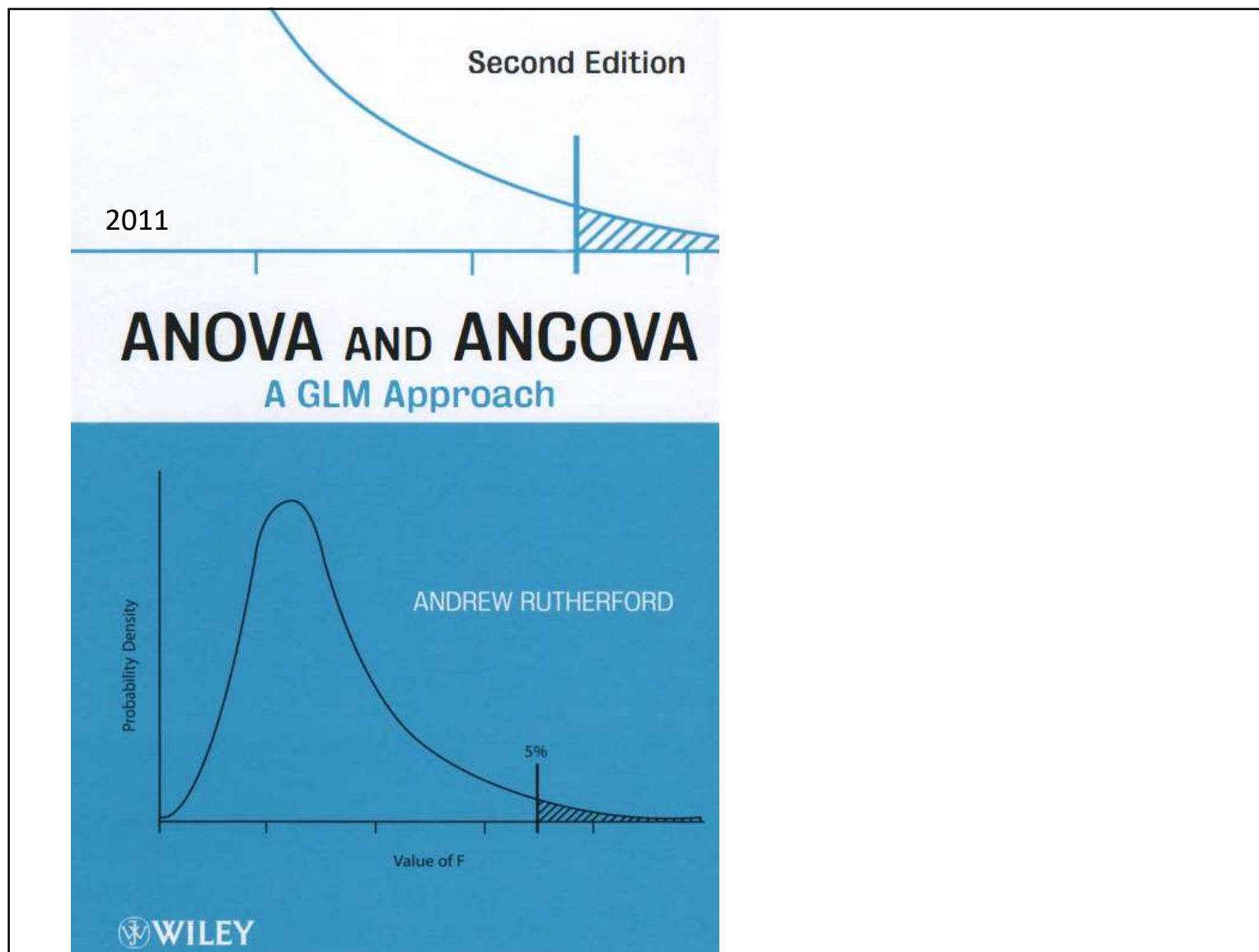
Fit: lm(formula = AlturaFinal ~ Fertilizante + AlturaInicial, data = Dados)

Linear Hypotheses:
Estimate Std. Error t value Pr(>|t|)
B - A == 0    6.250    3.463   1.805  0.5066
C - A == 0   14.750    3.463   4.259  0.0712 .
C - B == 0    8.500    4.177   2.035  0.4040
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- bonferroni method)

```







CHAPTER 10

Assumptions Underlying ANOVA, Traditional ANCOVA, and GLMs

Table 10.1 ANOVA and GLM Assumptions

| ANOVA Assumptions | GLM Assumptions |
|--|--|
| a Each condition contains a random sample of the population of such scores | Each condition contains a random sample of the population of such scores |
| b The scores in each condition are distributed normally | Errors are distributed normally |
| c The scores in each condition are independent of each other | Errors are independent |
| d The variances of the scores in each experimental condition are homogeneous | Errors are homoscedastic (errors exhibit common variance across all values of the predictor variables) |

Table 10.3 Specific ANCOVA Assumptions

-
- a** The covariate is independent of the treatments
 - b** In each treatment group the relationship between the covariate and the dependent variable is linear (the covariate and dependent variable are expressed at the first power only)
 - c** The regression coefficients of the dependent variable on the covariate in each treatment group are homogeneous
-

10.2.3 Traditional ANCOVA

In addition to all of the ANOVA assumptions, traditional ANCOVA also makes the assumptions listed in Table 10.3. These assumptions have no counterparts in GLM terms, as they are made to simplify the interpretation and/or calculation of the ANCOVA. Although orthogonal predictors are preferable and correlations can cause interpretation problems, covariate-treatment correlations do not preclude accurate and informative analysis (Cohen and Cohen, 1983; Rutherford, 1992). GLMs also can accommodate nonlinear regression of the dependent variable on the covariate (see polynomial regression, e.g. Draper and Smith, 1998; Kutner et al, 2005) and heterogeneous regressions (e.g. Rutherford, 1992; Searle, 1979, 1987). Indeed, the popularity of heterogeneous regression ANCOVA seems to be increasing (e.g., Maxwell and Delaney, 2004).

The form of ANCOVA incorporating the assumptions listed in Table 10.3 is termed traditional ANCOVA to distinguish it from less constrained forms of ANCOVA.

Traditional ANCOVA is still the most common form of ANCOVA applied in psychological research and the programs labeled ANCOVA in most commercial statistical packages implement this form of ANCOVA. Beyond the benefits accrued from simplifying ANCOVA interpretation and/or calculation, there are other reasons for choosing traditional ANCOVA. First, the good ANCOVA design practice of measuring the covariate before administering the experimental manipulation(s) usually ensures the experimental conditions cannot influence the covariate and second, most relationships between covariates and dependent variables in psychology appear to be linear, or are approximately linear. Therefore, it is very likely that two of the three traditional assumptions will be tenable for most ANCOVAs. Unfortunately, however the assumption of homogeneous regressions across experimental conditions becomes more tenuous as the number of experimental conditions increases and as the number of experimental factors increase (e.g., Winer, Brown, and Michels, 1991).

13.1. Grupos pré-existentes

- Grupos pré-existentes (intactos) são grupos nos quais os participantes não foram alocados aleatoriamente
 - E.g.: sexo, estado civil, nacionalidade, profissão
- Imagine um caso em que existem três grupos de mulheres
 - Recepcionistas de boate, secretárias que trabalham em turno parcial e cientistas de alto gabarito que trabalham em turno integral
 - Esses são grupos que ocorrem naturalmente (i.e., não podemos alocar a esses grupos, já estão neles)
 - Queremos testar a hipótese de que, quanto mais complexa a ocupação, mais alto é o nível de testosterona.

Evolutionary Neuroandrogenic: Theory and Universal Gender Differences in Cognition and Behavior, 2011

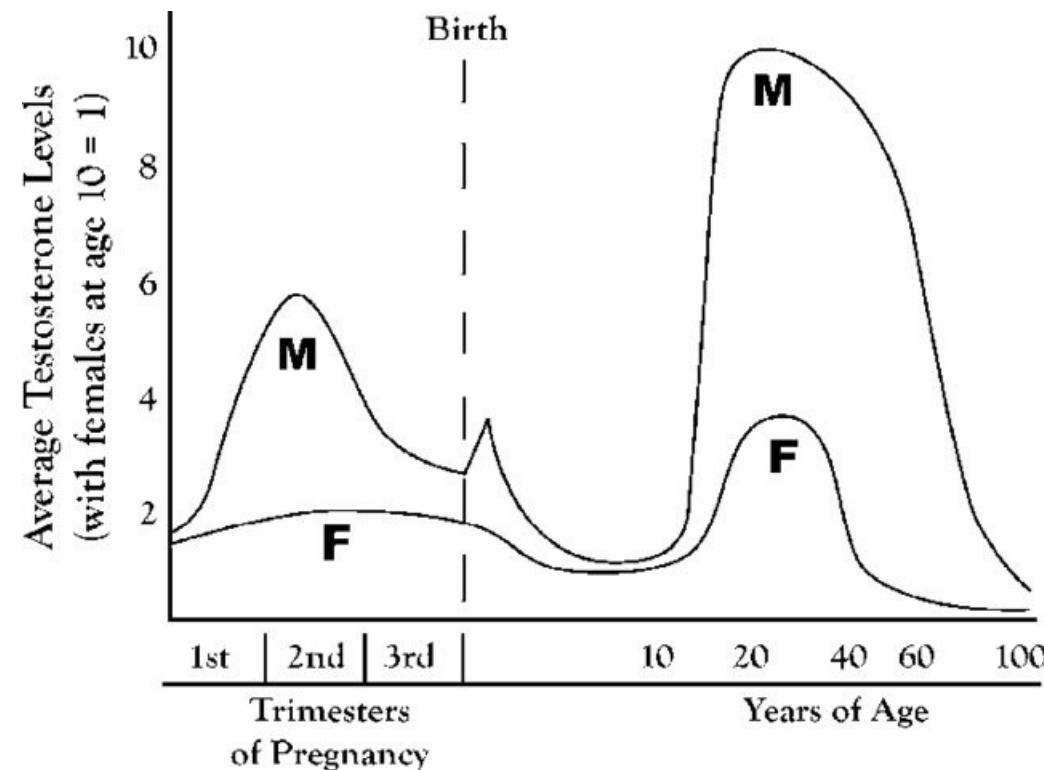
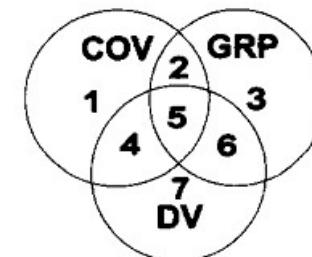
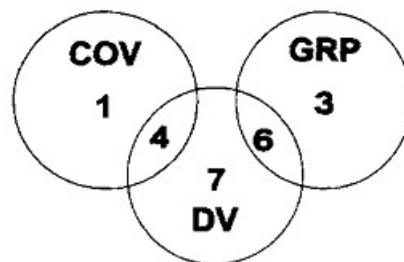


Fig. The life-cycle average testosterone levels for males (M) and females (F)

13.1. Grupos pré-existentes

- Existem, de fato, pesquisas que mostram uma leve associação entre níveis ocupacionais e testosterona
- Você pode pensar em outras variáveis que podem estar relacionadas com a variável dependente (nível de testosterona)
- Essas variáveis são denominadas covariáveis
- A idade é uma covariável que está relacionada com o nível de testosterona
- Note também que a covariável idade está associada com nível ocupacional! Isso viola uma suposição de dissociação da ANCOVA



13.1. Grupos pré-existentes

- A ANCOVA reduz a variância do erro pela remoção da variância devido ao relacionamento entre a idade (covariável) e a testosterona (VD), como também ajusta as médias da covariável para todos os níveis ocupacionais (grupos), conduzindo ao ajustamento das médias marginais da testosterona.

13.1. Grupos pré-existentes

- Em outras palavras, o que a ANCOVA faz é responder quais seriam as médias da VD nos grupos se as médias dos três grupos na covariável fossem as mesmas
- A fórmula ajusta as médias marginais da VD para o que seriam se os três grupos tivessem a mesma média de idade (covariável)
- A análise responderá “qual a probabilidade de as diferenças entre as médias marginais ajustadas dos grupos terem ocorrido devido ao erro amostral dado que a hipótese nula é verdadeira”

13.1.1. A covariável deve estar linearmente relacionada à VD

- A inclinação da reta de regressão deve ser significante para cada grupo

13.1.2. A covariável deve ser medida sem erro (ser confiável)

- Se você medisse os escores dos participantes na covariável em diferentes ocasiões, deveria existir uma alta correlação entre os escores obtidos nessas diferentes ocasiões
- E.g.: a idade é mensurada sem erro (confiável): a idade em uma semana estará perfeitamente correlacionada com a idade na próxima semana

13.1.3. As linhas de regressão para os diferentes grupos devem ser paralelas

- Se as retas de regressão entre a VD e a covariável entre os grupos não são paralelas, utilizar o procedimento que ajusta as médias dos grupos para uma média geral não faz sentido para o GLM
- O teste de igualdade (homogeneidade) das declividades das retas de regressão entre a VD e a covariável é realizada por meio do teste do efeito de interação entre a covariável e o fator de interesse do pesquisador, i.e., testar o efeito da interação Fator*Covariável, na presença dos efeitos principais do Fator e da Covariável

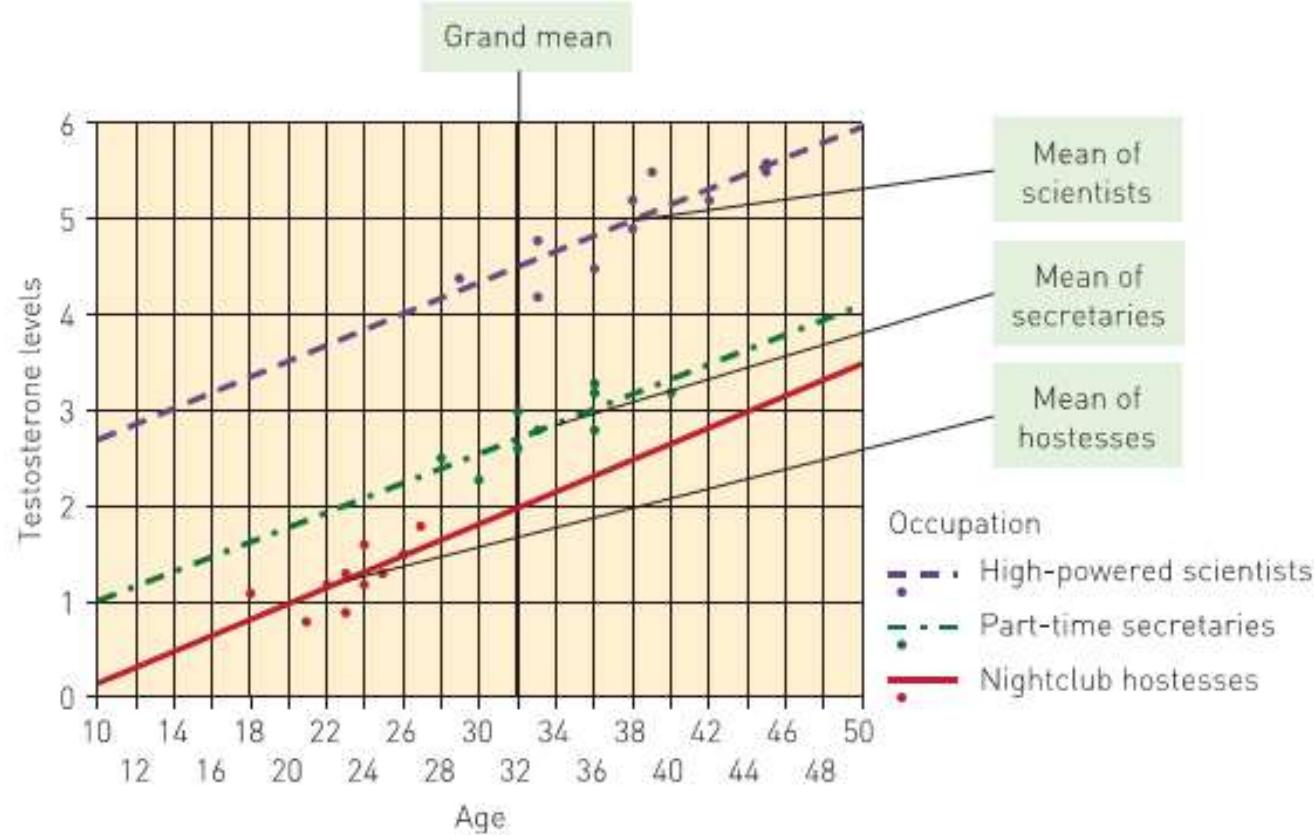


Figure 13.4 Regression lines for each separate group

Table 13.1 Unadjusted and adjusted means

| Group | Mean age | Mean testosterone | Adjusted mean testosterone |
|-------|----------|-------------------|----------------------------|
| 1 | 38 | 5.0 | 4.5 |
| 2 | 34 | 2.8 | 2.7 |
| 3 | 23 | 1.4 | 2.0 |

ANOVA would look at the difference between means

ANCOVA looks at the differences in adjusted means

Tests of Between-Subjects Effects
Dependent Variable: testosterone

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Eta Squared |
|-----------------|-------------------------|----------|--------------|---------------|-------------------|-------------|
| Corrected Model | 69.273 ^b | 3 | 23.091 | 313.806 | .000 ^a | .973 |
| Intercept | 6.521E-02 | 1 | 6.521E-02 | .886 | .355 | .033 |
| AGE | 2.981 | 1 | 2.981 | 40.509 | .000 | .609 |
| GROUP | 14.265 | 2 | 7.133 | 96.933 | .000 | .882 |
| Error | 1.913 | 26 | 7.358E-02 | | | |
| Total | 353.320 | 30 | | | | |
| Corrected Total | 71.187 | 29 | | | | |

a. Computed using alpha = .05

b. R Squared = .973 (Adjusted R Squared = .970)

Estimates

Dependent Variable: testosterone

| | Mean | Std. Error | 95% Confidence Interval | |
|-------------|-------|------------|-------------------------|-------------|
| | | | Lower Bound | Upper Bound |
| profession | | | | |
| scientists | 4.466 | .119 | 4.222 | 4.711 |
| secretaries | 2.663 | .090 | 2.477 | 2.848 |
| hostesses | 2.071 | .140 | 1.784 | 2.358 |

a. Evaluated at covariates appeared in the model: AGE = 31.6000.

O que fazer se o estudo não se satisfaz as suposições da ANCOVA?

- Primeiro, pense em controlar experimentalmente ao invés de controlar estatisticamente (o que fazemos quando executamos a ANCOVA)
- No nosso exemplo, podemos (embora isso seja difícil!) tentar encontrar alguma cientista jovem de alto gabarito e/ou velhas recepcionistas de boate ou secretárias
- Dessa forma, podemos emparelhar os grupos quanto à idade
- Vamos imaginar que os participantes foram emparelhados quanto à idade
- Isso significa que os grupos apresentam a mesma idade média
- Ainda é possível utilizar a ANCOVA com proveito – nesse caso, será pelo primeiro objetivo que mencionamos – de forma a reduzir a variância do erro – erro devido à correlação entre a VD e a covariável

Table 13.2 Matching participants on age

| Nightclub hostesses | Secretaries | Scientists |
|---------------------|-------------|------------|
| 25 | 25 | 25 |
| 30 | 30 | 31 |
| 50 | 50 | 50 |
| 41 | 41 | 42 |
| 39 | 39 | 39 |

E se as inclinações não forem iguais?

- Tutorial 7.5a - Analysis of Covariance:
 - <https://www.flutterbys.com.au/stats/tut/tut7.5a.html>
 - <https://www.flutterbys.com.au/stats/course.html>
- Marôco (2011, p. 269-74) Análise estatística com o SPSS Statistics, 5^a ed.
 - 6.5.2. E se os declives não forem homogêneos?
- Norusis (2009, p. 573-5) SPSS: Statistical Procedures Companion
 - A nested model using the interaction operator
 - Interceptos e declives distintos para cada grupo
 - GLM sem intercepto e com efeitos do fator e fator*covariável
 - VD = intercepto + declive*covariável

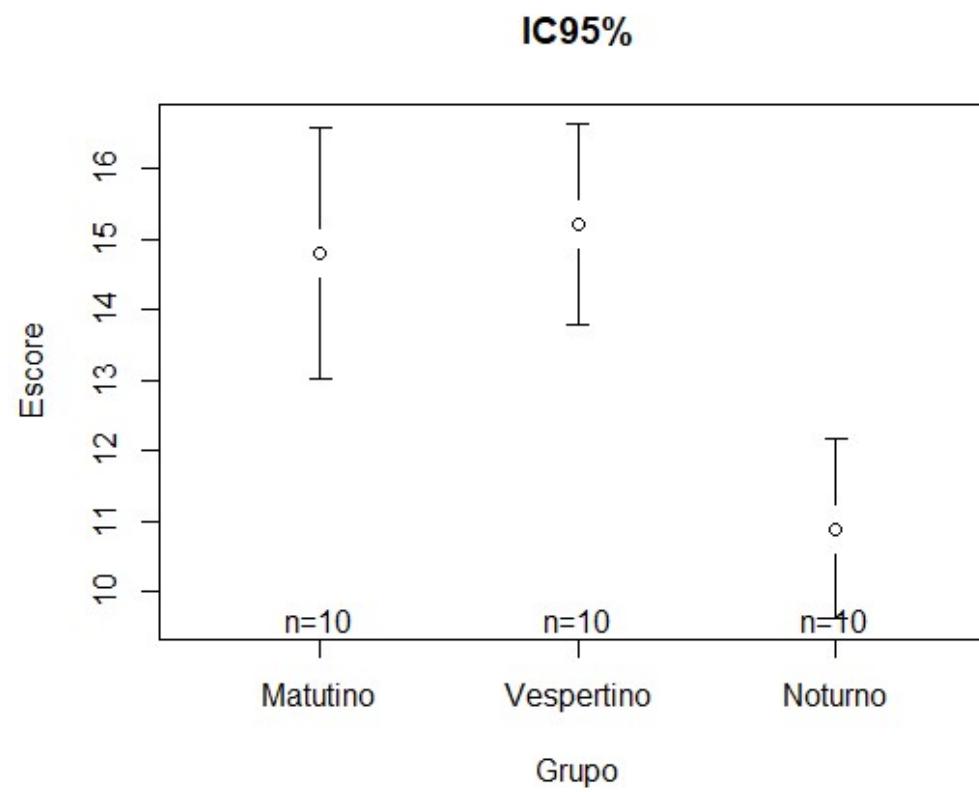
Lembrança de estudante controlada pela motivação

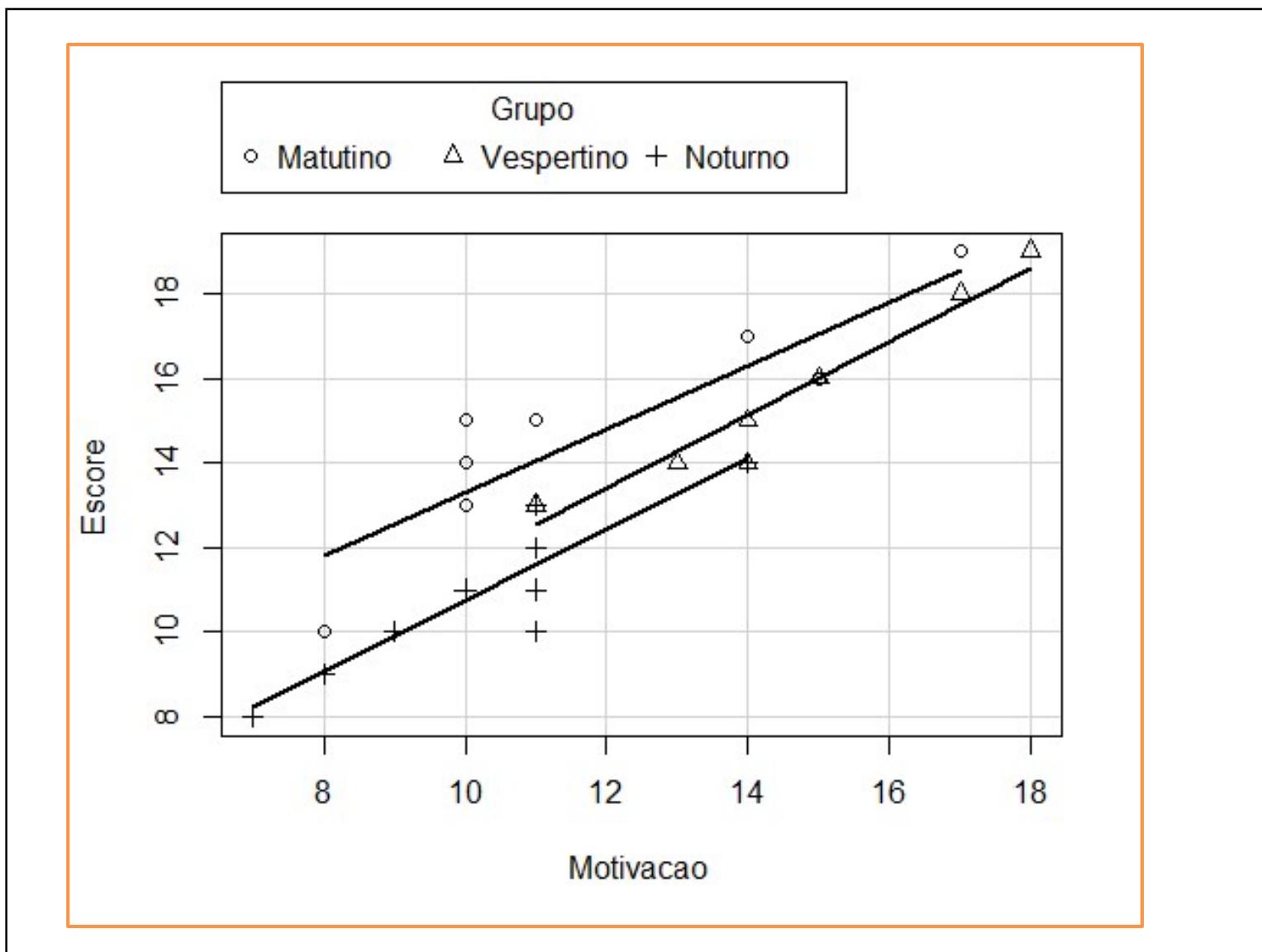


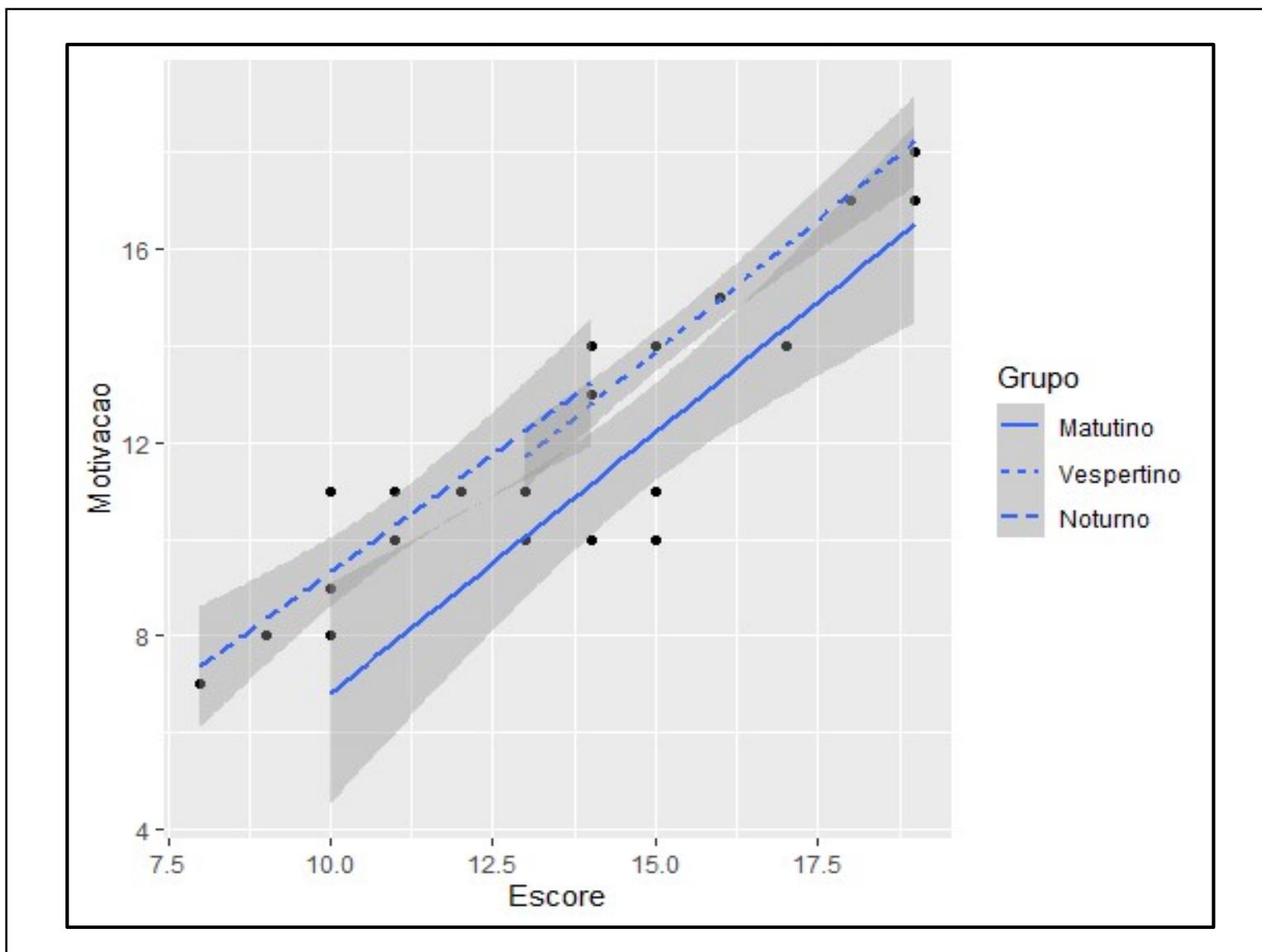
- Numa universidade, os estudantes foram alocados aleatoriamente e balanceadamente a um de 3 grupos para a realização de um trabalho de laboratório: um grupo matutino, um vespertino e outro noturno.
- Ao final da sessão, eles responderam a um questionário para verificar o quanto lembravam do que haviam feito.
- Como parte do projeto dos estudantes, decidiu-se analisar diferenças entre os grupos, controlando por motivação.

| | A | B | C |
|----|------------|--------|-----------|
| 1 | Grupo | Escore | Motivacao |
| 2 | Matutino | 15 | 10 |
| 3 | Matutino | 10 | 8 |
| 4 | Matutino | 14 | 10 |
| 5 | Matutino | 15 | 11 |
| 6 | Matutino | 17 | 14 |
| 7 | Matutino | 13 | 10 |
| 8 | Matutino | 13 | 10 |
| 9 | Matutino | 19 | 17 |
| 10 | Matutino | 16 | 15 |
| 11 | Matutino | 16 | 15 |
| 12 | Vespertino | 14 | 14 |
| 13 | Vespertino | 13 | 11 |
| 14 | Vespertino | 15 | 14 |
| 15 | Vespertino | 14 | 13 |
| 16 | Vespertino | 16 | 15 |
| 17 | Vespertino | 15 | 14 |
| 18 | Vespertino | 15 | 14 |
| 19 | Vespertino | 18 | 17 |
| 20 | Vespertino | 19 | 18 |
| 21 | Vespertino | 13 | 11 |
| 22 | Noturno | 13 | 11 |
| 23 | Noturno | 12 | 11 |
| 24 | Noturno | 11 | 11 |
| 25 | Noturno | 11 | 10 |
| 26 | Noturno | 14 | 14 |
| 27 | Noturno | 11 | 10 |
| 28 | Noturno | 10 | 9 |
| 29 | Noturno | 9 | 8 |
| 30 | Noturno | 8 | 7 |
| 31 | Noturno | 10 | 11 |

IC95 do escore médio populacional Sem modelo







ANCOVA

Teste de dissociação entre fator e covariável

Analysis of Deviance Table (Type III tests)

Response: Motivacao

| | Df | F | Pr (>F) | |
|----------------|----|----------|-----------------|----------|
| (Intercept) | 1 | 145.8000 | 2.153e-12 | *** |
| Grupo | 2 | 7.8542 | 0.002049 | ** |
| Residuals | 27 | | | |
| --- | | | | |
| Signif. codes: | 0 | '***' | 0.001 '**' | 0.01 '*' |
| | | | 0.05 '.' | 0.1 ' ' |
| | | | | 1 |

ANCOVA

Teste de igualdade das inclinações das retas de regressão

Anova Table (Type III tests)

Response: Escore

| | Sum Sq | Df | F value | Pr (>F) | | | | | | | |
|-----------------|--------|-------|---------|--------------|------|-----|------|-----|-----|-----|---|
| (Intercept) | 17.705 | 1 | 23.7344 | 5.754e-05 | *** | | | | | | |
| Grupo | 2.740 | 2 | 1.8368 | 0.181 | | | | | | | |
| Motivacao | 45.000 | 1 | 60.3237 | 5.306e-08 | *** | | | | | | |
| Grupo:Motivacao | 0.436 | 2 | 0.2925 | 0.749 | | | | | | | |
| Residuals | 17.903 | 24 | | | | | | | | | |
| --- | | | | | | | | | | | |
| Signif. codes: | 0 | '***' | 0.001 | '**' | 0.01 | '*' | 0.05 | '.' | 0.1 | ' ' | 1 |

ANCOVA: teste *omnibus*

Teste do efeito do fator

Se as declividades são iguais, testar se os interceptos são iguais

```
Analysis of Deviance Table (Type III tests)
```

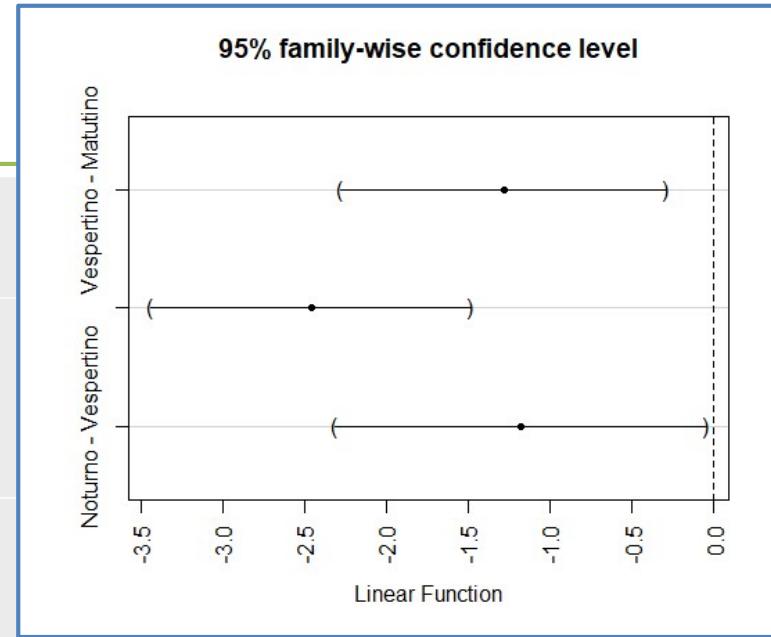
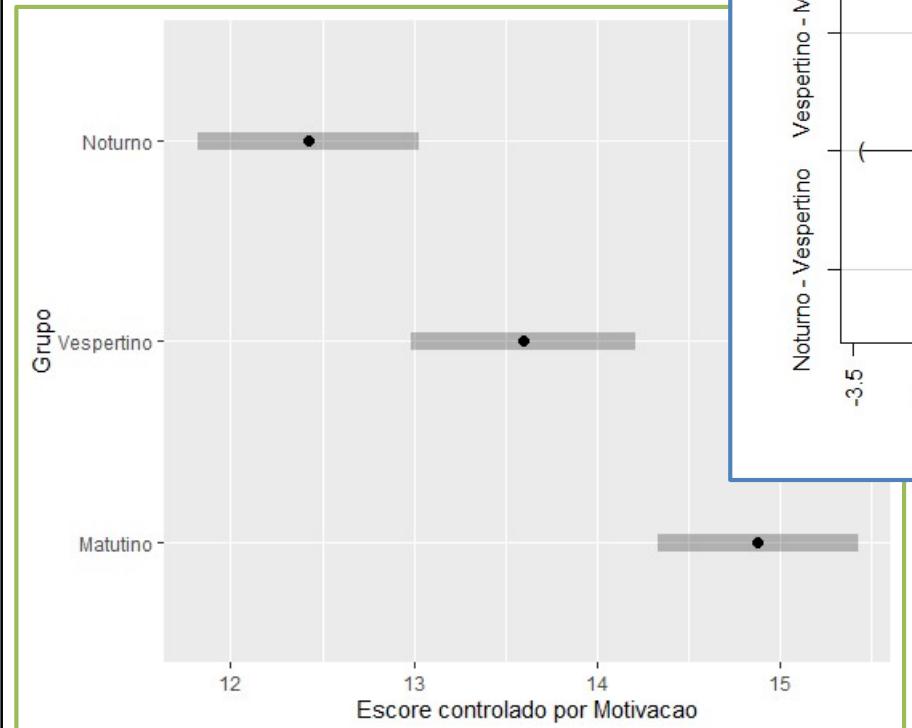
Response: Escore

| | Df | F | Pr (>F) |
|-------------|----|---------|----------------------|
| (Intercept) | 1 | 22.224 | 7.145e-05 *** |
| Grupo | 2 | 11.066 | 0.0003335 *** |
| Motivacao | 1 | 109.259 | 8.366e-11 *** |
| Residuals | 26 | | |

Signif. codes: 0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1

| | eta.sq | eta.sq.part |
|-----------|-----------|-----------------|
| Grupo | 0.1216251 | 0.607070 |
| Motivacao | 0.4368018 | 0.847296 |

Testes *post hoc*



Matutino Vespertino Noturno
"b" "a" "c"

ANCOVA

Testes *post hoc*

Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

Fit: lm(formula = Escore ~ Grupo + Motivacao, data = Dados)

Linear Hypotheses:

| | | Estimate | Std. Error | t value | Pr(> t) |
|------------|-------------------|----------|------------|---------|---------------------|
| Vespertino | - Matutino == 0 | -1.2826 | 0.4009 | -3.200 | 0.0108 * |
| Noturno | - Matutino == 0 | -2.4577 | 0.3943 | -6.233 | 4.06e-06 *** |
| Noturno | - Vespertino == 0 | -1.1751 | 0.4569 | -2.572 | 0.0485 * |

Signif. codes: 0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Adjusted p values reported -- bonferroni method)

ANCOVA_LembrancaMotivacao.R

```
library(readxl)
library(gplots)
library(car)
library(lsr)
library(emmeans)
library(multcomp)
library(ggplot2)
alfa <- 0.05
Dados <- readxl::read_excel("LembrancaMotivacao.xlsx")
Dados$Grupo <- factor(Dados$Grupo, levels=unique(Dados$Grupo))
head(Dados)
with(Dados, gplots:::plotmeans(Escore ~ Grupo,
                                error.bars="conf.int", level=1-alfa, connect=FALSE,
                                xlab="Grupo", ylab="Escore", main="IC95%",
                                barcol="black"))
with(Dados, car:::scatterplot(Motivacao, Escore,
                               groups = Grupo,
                               regLine=TRUE, smooth=FALSE, ellipse=FALSE, col="black",
                               xlab = "Motivacao ", ylab = "Escore "))
print(ggplot(Dados, aes(y = Motivacao, x = Escore,
                        group = Grupo, linetype = Grupo)) +
      geom_point() + geom_smooth(method = "lm"))
```

ANCOVA_LembrancaMotivacao.R

```
# VD: Escore VI: Grupo
modelo <- lm(Escore ~ Grupo, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
print(lsR::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans::emmeans(modelo, "Grupo"))
print(grf <- plot(EMM, colors = "black",
                  main="Estimated Marginal Means",
                  xlab="Escore",
                  ylab="Grupo"))
mc <- multcomp::glht(modelo, linfct = mcp(Grupo = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp:::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc, las=3)
# VD: Escore - Motivacao VI: Grupo
Diferenca <- Dados$Escore - Dados$Motivacao
modelo <- lm(Diferenca ~ Grupo, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
print(lsR::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans::emmeans(modelo, "Grupo"))
print(grf <- plot(EMM, colors = "black",
                  main="Estimated Marginal Means",
                  xlab="Escore - Motivacao",
                  ylab="Grupo"))
mc <- multcomp::glht(modelo, linfct = mcp(Grupo = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp:::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc, las=3)
```

ANCOVA_LembrancaMotivacao.R

```

## ANCOVA: Teste de dissociacao entre fator e covariavel
cat("\nANCOVA: Teste de dissociacao entre fator e covariavel\n")
modelo <- lm(Motivacao ~ Grupo, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
## ANCOVA: Teste de igualdade das inclinacoes das retas de regressao
cat("\nANCOVA: Teste de igualdade das inclinacoes das retas de regressao\n")
modelo <- lm(Escore ~ Grupo + Motivacao + Grupo*Motivacao,
             data=Dados)
print(Anova <- car:::Anova(modelo, type=3))
## ANCOVA: Teste do efeito do fator fixo:
##           Se as declividades sao iguais, testar se os interceptos sao iguais.
cat("\nANCOVA: Teste do efeito do fator fixo:\n")
cat("\tSe as declividades sao iguais, testar se os interceptos sao iguais.\n")
modelo <- lm(Escore ~ Grupo + Motivacao,
             data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
print(lsrr::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans:::emmeans(modelo, "Grupo"))
print(grf <- plot(EMM, colors = "black",
                   main="Estimated Marginal Means",
                   xlab="Escore controlado por Motivacao",
                   ylab="Grupo"))
mc <- multcomp::glht(modelo, linfct = mcp(Grupo = "Tukey"))
print(mcs <- summary(mc, test=adjusted("bonferroni")))
multcomp::cld(mcs, level=alfa, decreasing=TRUE)
plot(mc,las=3)

```

SPSS sintaxe

```
UNIANOVA Escore BY Grupo WITH Motivacao  
  /METHOD=SSTYPE(3)  
  /INTERCEPT=INCLUDE  
  /PLOT=PROFILE(Grupo) TYPE=LINE ERRORBAR=CI  
  MEANREFERENCE=NO YAXIS=AUTO  
  /EMMEANS=TABLES(Grupo) WITH(Motivacao=MEAN)  
  COMPARE ADJ(SIDAK)  
  /EMMEANS=TABLES(Grupo) WITH(Motivacao=MEAN)  
  COMPARE ADJ(SIDAK)  
  /PRINT ETASQ DESCRIPTIVE PARAMETER  
  HOMOGENEITY  
  /CRITERIA=ALPHA(.05)  
  /DESIGN=Grupo Motivacao.
```

Tests of Between-Subjects Effects

Dependent Variable: Escore

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|-------------------|----------------------------|----|-------------|---------|------|
| Corrected Model | 215,063 ^a | 5 | 43,013 | 57,660 | ,000 |
| Intercept | 12,929 | 1 | 12,929 | 17,332 | ,000 |
| Grupo * Motivacao | ,436 | 2 | ,218 | ,292 | ,749 |
| Grupo | 2,740 | 2 | 1,370 | 1,837 | ,181 |
| Motivacao | 93,274 | 1 | 93,274 | 125,036 | ,000 |
| Error | 17,903 | 24 | ,746 | | |
| Total | 5809,000 | 30 | | | |
| Corrected Total | 232,967 | 29 | | | |

a. R Squared = ,923 (Adjusted R Squared = ,907)

Teste do efeito do fator:
Se as inclinações são iguais, testar se os interceptos são iguais

Tests of Between-Subjects Effects

| Dependent Variable: Escore | | | | | | |
|----------------------------|-------------------------|----|-------------|---------|------|---------------------|
| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
| Corrected Model | 214,627 ^a | 3 | 71,542 | 101,425 | ,000 | ,921 |
| Intercept | 16,204 | 1 | 16,204 | 22,973 | ,000 | ,469 |
| Grupo | 28,335 | 2 | 14,167 | 20,085 | ,000 | ,607 |
| Motivacao | 101,760 | 1 | 101,760 | 144,264 | ,000 | ,847 |
| Error | 18,340 | 26 | ,705 | | | |
| Total | 5809,000 | 30 | | | | |
| Corrected Total | 232,967 | 29 | | | | |

a. R Squared = ,921 (Adjusted R Squared = ,912)

Estimates

Dependent Variable: Escore

| Grupo | Mean | Std. Error | 95% Confidence Interval | |
|------------|---------------------|------------|-------------------------|-------------|
| | | | Lower Bound | Upper Bound |
| Matutino | 14,880 ^a | ,266 | 14,334 | 15,426 |
| Vespertino | 13,597 ^a | ,297 | 12,987 | 14,208 |
| Noturno | 12,422 ^a | ,294 | 11,817 | 13,027 |

a. Covariates appearing in the model are evaluated at the following values: Motivação = 12,10.

Univariate Tests

Dependent Variable: Escore

| | Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|----------|----------------|----|-------------|--------|------|---------------------|
| Contrast | 28,335 | 2 | 14,167 | 20,085 | ,000 | ,607 |
| Error | 18,340 | 26 | ,705 | | | |

The F tests the effect of Grupo. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Pairwise Comparisons

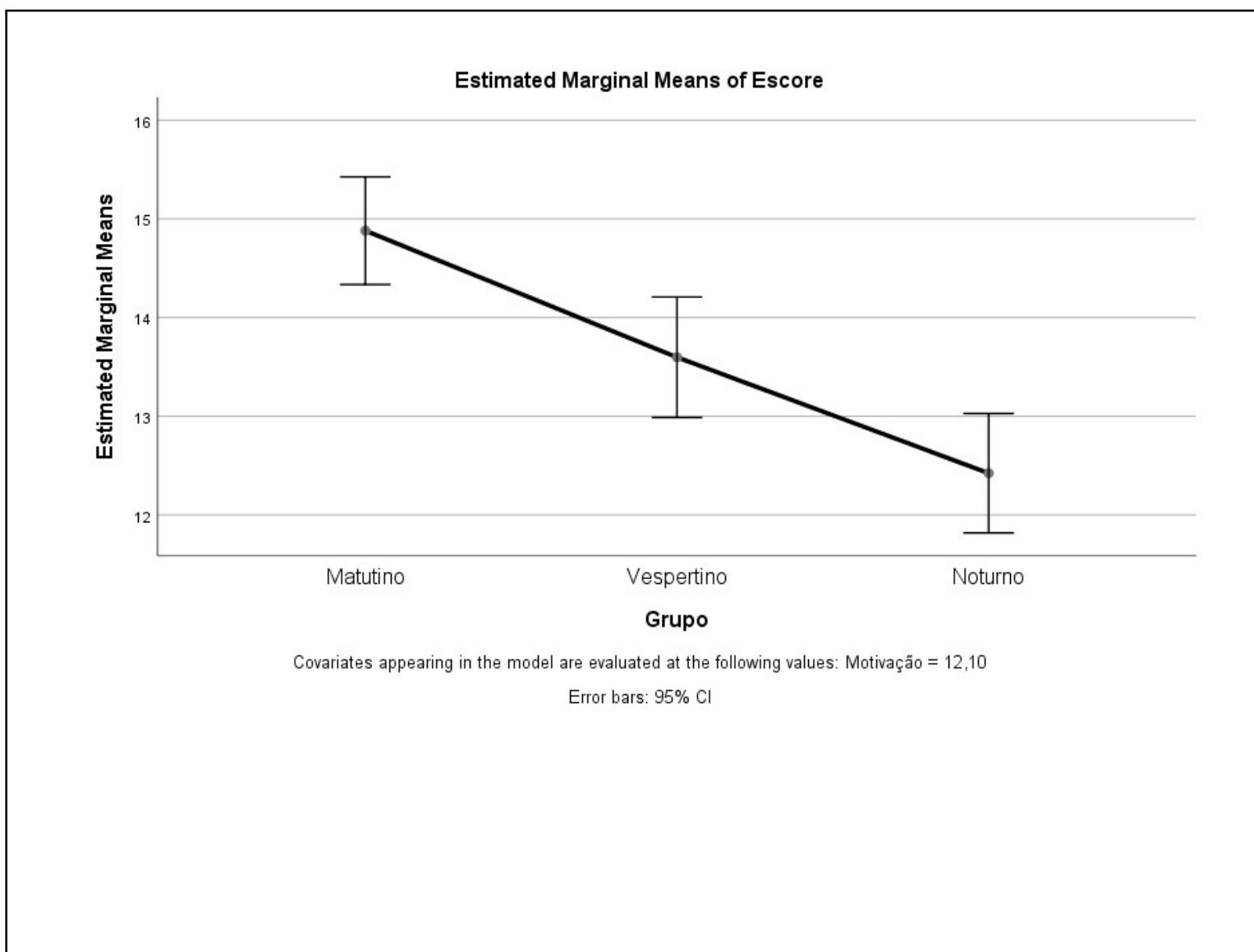
Dependent Variable: Escore

| (I) Grupo | (J) Grupo | Mean Difference (I-J) | 95% Confidence Interval for Difference ^b | | | |
|------------|------------|-----------------------|---|-------------------|-------------|-------------|
| | | | Std. Error | Sig. ^b | Lower Bound | Upper Bound |
| Matutino | Vespertino | 1,283* | ,401 | ,011 | ,260 | 2,305 |
| | Noturno | 2,458* | ,394 | ,000 | 1,452 | 3,464 |
| Vespertino | Matutino | -1,283* | ,401 | ,011 | -2,305 | -,260 |
| | Noturno | 1,175* | ,457 | ,048 | ,009 | 2,341 |
| Noturno | Matutino | -2,458* | ,394 | ,000 | -3,464 | -1,452 |
| | Vespertino | -1,175* | ,457 | ,048 | -2,341 | -,009 |

Based on estimated marginal means

*. The mean difference is significant at the ,05 level.

b. Adjustment for multiple comparisons: Sidak.



Conclusão

Lembrança de estudante controlada pela motivação

- A hipótese nula de ausência de efeito de período controlando pela covariável de motivação foi rejeitada ($F(2;26) = 11,1; p = 0,0003335$).

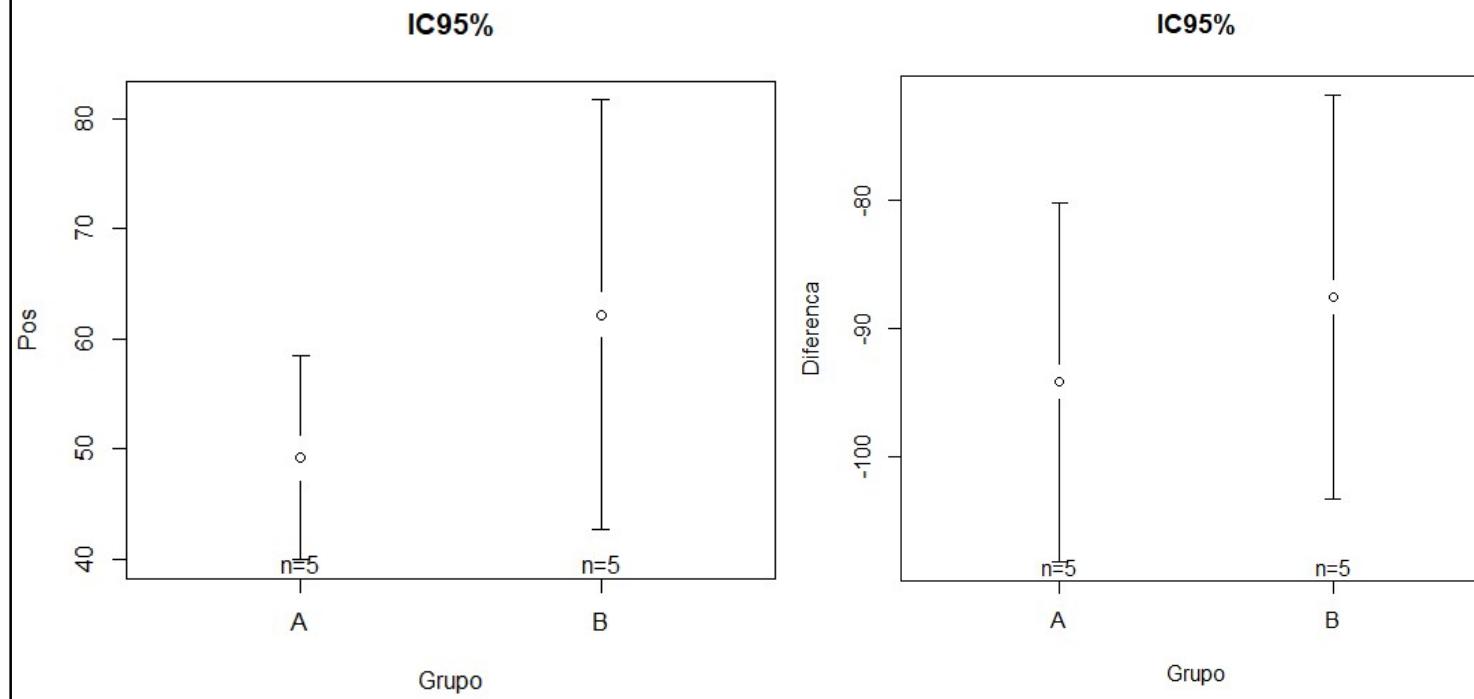
13.2. Delineamento pré e pós-teste

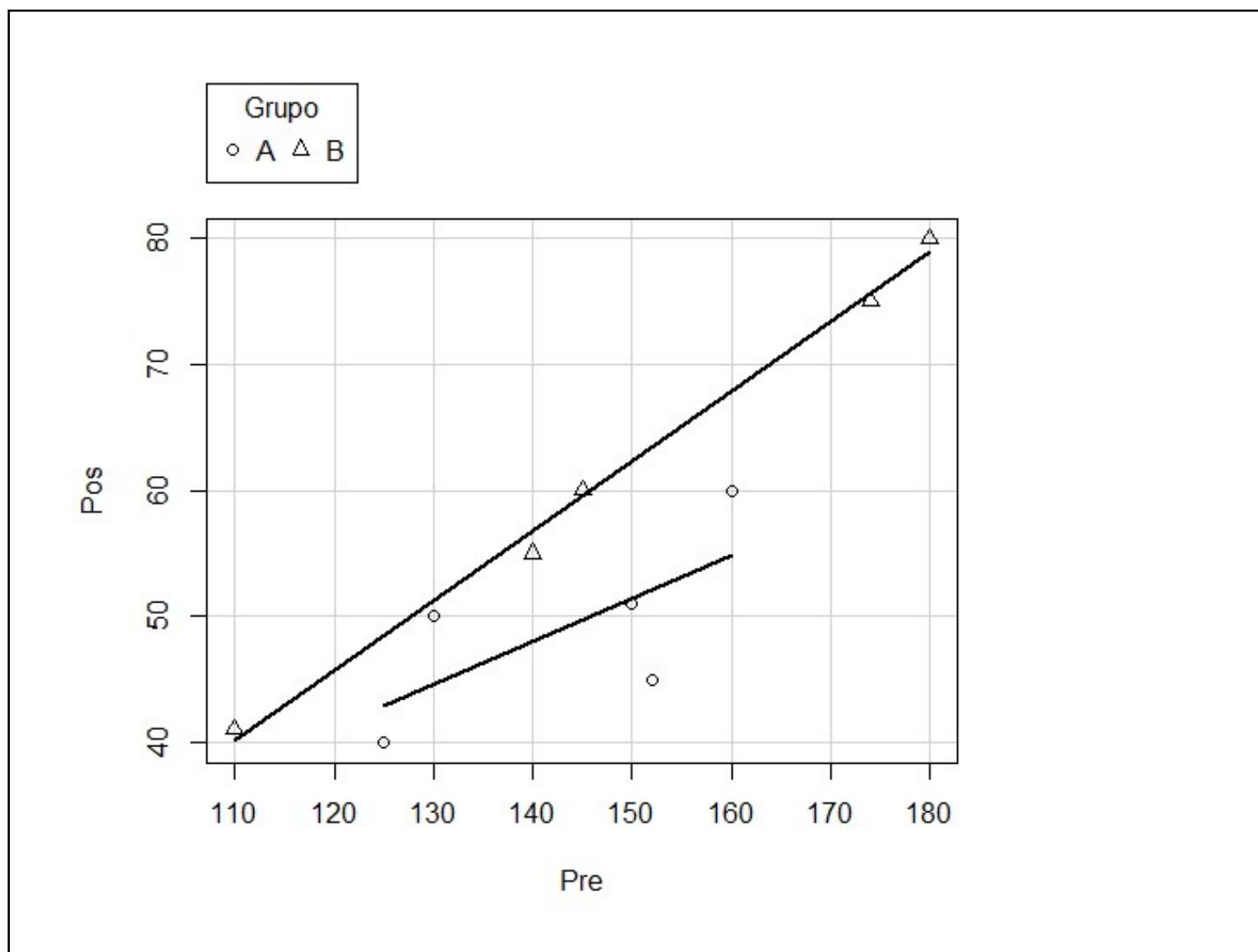
- Um dos delineamentos mais comuns em que a ANCOVA é utilizada é o pré-pós intervenção
- Consiste em observar a VD antes e depois de uma intervenção
- Nesse caso, o valor pré-intervenção da VD é utilizado como covariável
- Quando executam pré- e pós-valores da VD, os pesquisadores, às vezes, desejam controlar ou remover o efeito do pré-valor da VD para poderem perceber possíveis alterações devidas à intervenção feita
- A utilização de diferenças de escores não permite isso, uma vez que o pré-valor da VD está geralmente correlacionado ao resultado (diferença), e assim a variação nos pré-valores da VD não é removida

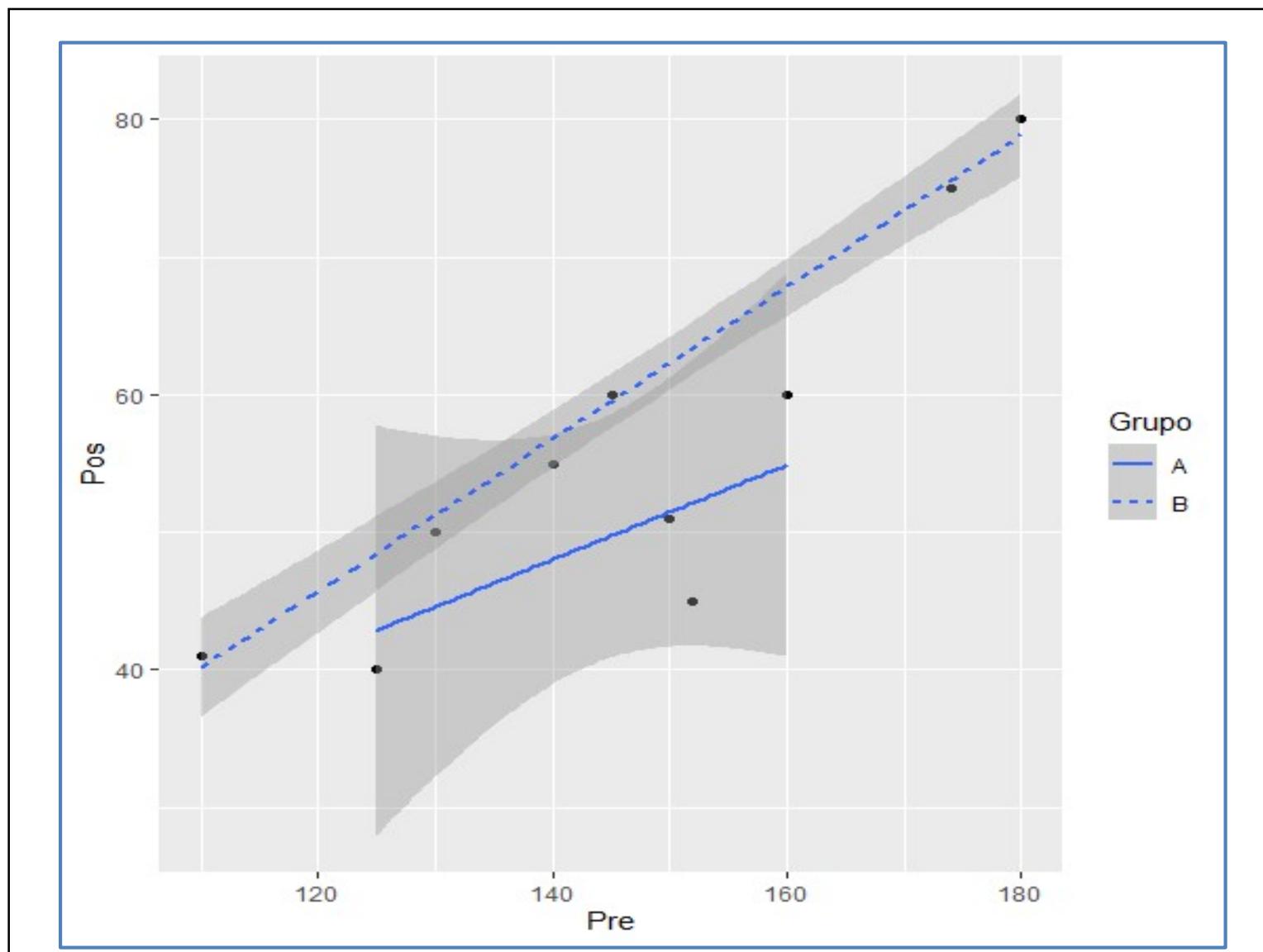
PrePos_2CondIndep.xlsx

| | A | B | C |
|----|-------|-----|-----|
| 1 | Grupo | Pre | Pos |
| 2 | A | 150 | 51 |
| 3 | A | 130 | 50 |
| 4 | A | 125 | 40 |
| 5 | A | 152 | 45 |
| 6 | A | 160 | 60 |
| 7 | B | 174 | 75 |
| 8 | B | 110 | 41 |
| 9 | B | 180 | 80 |
| 10 | B | 145 | 60 |
| 11 | B | 140 | 55 |

IC95 sem modelo







t independente

VD: pós

- Aqui pode-se ver que $t_{Welch}(5,72) = -1,67$, $p = 0,15$
- Não existe diferença estatisticamente significante entre os dois grupos nos escores pós
- O pesquisador não rejeita H_0 de que a intervenção afeta os dois grupos de forma semelhante

t independente

VD: delta = pós - pré

- Aqui pode-se ver que $t_{Welch}(7,9) = -0,87$, $p = 0,41$
- Não existe diferença estatisticamente significante entre os dois grupos nos escores diferença
- O pesquisador não rejeita H_0 de que a intervenção afeta os dois grupos de forma semelhante
- No entanto, ao executarmos uma ANCOVA, com o escore pós como VD e o escore pré como uma covariável sem erro de mensuração, podemos concluir que os dois grupos diferem na medida do pós-teste, após ajustamento para os escores pré
- Em geral, a ANCOVA tende a fornecer um teste mais poderoso de hipótese nula do que a diferença entre os escores pré e pós

ANCOVA PARA DELINEAMENTO PRÉ-PÓS INTERVENÇÃO COM DUAS CONDIÇÕES INDEPENDENTES

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Soc Sci Res. 2015 March ; 50: 277–291. doi:10.1016/j.ssresearch.2014.12.004.

The power of a paired *t*-test with a covariate

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²Arizona State University Southwest Interdisciplinary Research Center

Abstract

Many researchers employ the paired *t*-test to evaluate the mean difference between matched data points. Unfortunately, in many cases this test is inefficient. This paper reviews how to increase the precision of this test through using the mean centered independent variable x , which is familiar to researchers that use analysis of covariance (ANCOVA). We add to the literature by demonstrating how to employ these gains in efficiency as a factor for use in finding the statistical power of the test. The key parameters for this factor are the correlation between the two measures and the variance ratio of the dependent measure on the predictor. The paper then demonstrates how to compute the gains in efficiency *a priori* to amend the power computations for the traditional paired *t*-test. We include an example analysis from a recent intervention, *Families Preparing the New Generation (Familias Preparando la Nueva Generación)*. Finally, we conclude with an analysis of extant data to derive reasonable parameter values.

ANCOVA

VD:pós VI:Grupo Covariável:pré

- Aqui pode-se ver que $F(1,7) = 8,94$, $p = 0,02025$
- O nível de significância adotado é 5%
- Existe diferença estatisticamente significante entre os dois grupos no valor pós controla pelo pré
- O pesquisador rejeita H_0 de que a intervenção afeta os dois grupos de forma semelhante
- Em geral, a ANCOVA tende a fornecer um teste mais poderoso de hipótese nula do que a diferença entre os valores pré e pós

ANCOVA

Teste de dissociação entre fator e covariável

Analysis of Deviance Table (Type III tests)

Response: Pre

| | Df | F | Pr (>F) |
|----------------|--------|----------|---------------|
| (Intercept) | 1 | 361.0809 | 6.089e-08 *** |
| Grupo | 1 | 0.1594 | 0.7002 |
| Residuals | 8 | | |
| --- | | | |
| Signif. codes: | 0 | '***' | 0.001 '**' |
| | 0.01 * | 0.05 .' | 0.1 ' ' |
| | 1 | | |

ANCOVA

Teste de igualdade das inclinações das retas de regressão

Anova Table (Type III tests)

Response: Pos

| | Sum Sq | Df | F value | Pr(>F) |
|----------------|---------|-------|---------|----------------|
| (Intercept) | 0.001 | 1 | 0.0001 | 0.99453 |
| Grupo | 14.139 | 1 | 0.7043 | 0.43350 |
| Pre | 107.929 | 1 | 5.3763 | 0.05956 . |
| Grupo:Pre | 31.134 | 1 | 1.5509 | 0.25943 |
| Residuals | 120.450 | 6 | | |
| <hr/> | | | | |
| Signif. codes: | 0 | '***' | 0.001 | '**' |
| | | | 0.01 | '*' |
| | | | 0.05 | '.' |
| | | | 0.1 | ' ' |
| | | | | 1 |

ANCOVA

Teste do efeito do fator

Se as declividades são iguais, testar se os interceptos são iguais

Analysis of Deviance Table (Type III tests)

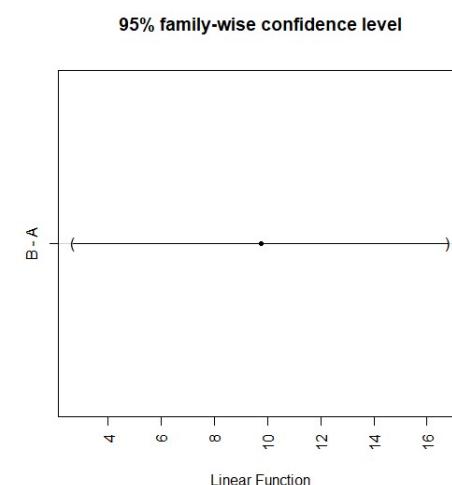
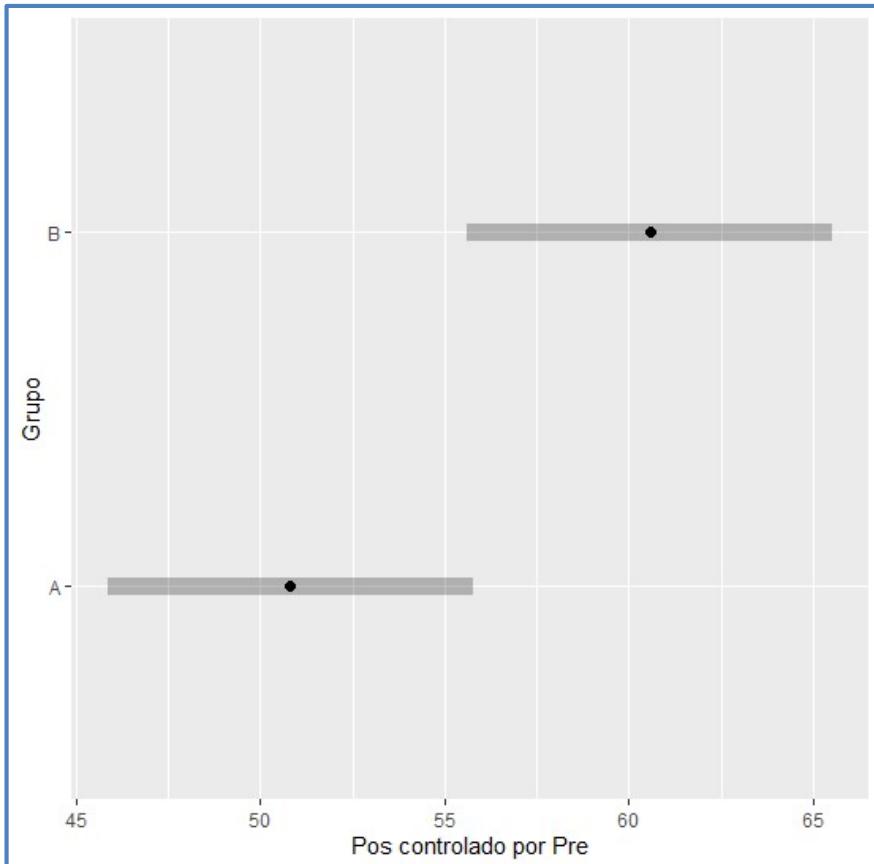
Response: Pos

| | Df | F | Pr (>F) |
|-------------|----|---------|------------------|
| (Intercept) | 1 | 6.8435 | 0.03461 * |
| Grupo | 1 | 8.9356 | 0.02025 * |
| Pre | 1 | 75.7297 | 5.306e-05 *** |
| Residuals | 7 | | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

| | eta.sq | eta.sq.part |
|-------|-----------|------------------|
| Grupo | 0.1421812 | 0.6048774 |
| Pre | 0.6482544 | 0.8746825 |

IC95 da ANCOVA



ANCOVA_PrePos.R

```
# ANCOVA robusta a heterocedasticidade
# para delineamento pre-pos intervencao
library(readxl)
library(gplots)
library(car)
library(lsr)
library(emmeans)
library(multcomp)
library(estimatr)
library(ggplot2)
alfa <- 0.05
Dados <- readxl::read_excel("PrePos_2CondIndep.xlsx")
Dados$Grupo <- factor(Dados$Grupo, levels=unique(Dados$Grupo) )
head(Dados)
with(Dados, gplots:::plotmeans(Pos ~ Grupo,
                                error.bars="conf.int", level=1-alfa, connect=FALSE,
                                xlab="Grupo", ylab="Pos", main="IC95%",
                                barcol="black"))
Diferenca <- Dados$Pos - Dados$Pre
with(Dados, gplots:::plotmeans(Diferenca ~ Grupo,
                                error.bars="conf.int", level=1-alfa, connect=FALSE,
                                xlab="Grupo", ylab="Diferenca", main="IC95%",
                                barcol="black"))
with(Dados, car:::scatterplot(Pre, Pos,
                               groups = Grupo,
                               regLine=TRUE, smooth=FALSE, ellipse=FALSE, col="black",
                               xlab = "Pre", ylab = "Pos"))
print(ggplot(Dados, aes(y = Pos, x = Pre, group = Grupo, linetype = Grupo)) +
      geom_point() + geom_smooth(method = "lm"))
```

ANCOVA_PrePos.R

```
# VD: Pos VI: Grupo  
t.test(Pos ~ Grupo, data=Dados)  
# VD: Pos - Pre VI: Grupo  
t.test(Diferenca ~ Grupo, data=Dados)
```

ANCOVA_PrePos.R

```

# ANCOVA: VD: Pos com erro de mensuracao
#           VI: Grupo
# Covariavel: Pre sem erro de mensuracao
## ANCOVA: Teste de dissociacao entre fator e covariavel
cat("\nANCOVA: Teste de dissociacao entre fator e covariavel\n")
modelo <- lm(Pre ~ Grupo, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
## ANCOVA: Teste de igualdade das inclinacoes das retas de regressao
cat("\nANCOVA: Teste de igualdade das inclinacoes das retas de regressao\n")
modelo <- lm(Pos ~ Grupo + Pre + Grupo*Pre, data=Dados)
print(Anova <- car:::Anova(modelo, type=3))
## ANCOVA: Teste do efeito do fator fixo:
##           Se as declividades sao iguais, testar se os interceptos sao iguais.
cat("\nANCOVA: Teste do efeito do fator fixo\n")
cat("\tSe as declividades sao iguais, testar se os interceptos sao iguais.\n")
modelo <- lm(Pos ~ Grupo + Pre, data=Dados)
print(Anova <- car:::Anova(modelo, type=3, white.adjust=TRUE))
print(lsr:::etaSquared(modelo))
print(sumario <- summary(modelo))
print(EMM <- emmeans:::emmeans(modelo, "Grupo"))
print(grf <- plot(EMM, colors = "black",
                   main="Estimated Marginal Means",
                   xlab="Pos controlado por Pre",
                   ylab="Grupo"))
mc <- multcomp:::glht(modelo, linfct = mcp(Grupo = "Tukey"))
plot(mc, las=3)

```

ANCOVA_PrePos.R

```
# teste t relacionado por ANCOVA:  
#   VD: Pos com erro de mensuracao - Pre com erro de mensuracao  
#   VI: Pre com erro de mensuracao - mean(Pre com erro de mensuracao)  
# Fonte: HEDGERG, EC & AYRES, S (2015) The power of a paired t-test  
#           whit a covariate. Soc Sci Res 50:277-91.  
t.test(Dados$Pos, Dados$Pre, paired=TRUE)  
with(Dados,  
  print(summary(  
    estimatr::lm_robust(Pos-Pre ~ 1 + I(Pre-mean(Pre,na.rm=TRUE))))  
))
```



BIOMETRICS 43, 895–901
December 1987

Analysis of Covariance in Parallel-Group Clinical Trials with Pretreatment Baselines

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4901 Searle Parkway, Skokie, Illinois 60077, U.S.A.

SUMMARY

Analysis of covariance (ANCOVA) techniques are often employed in the analysis of clinical trials to try to account for the effects of varying pretreatment baseline values of an outcome variable on posttreatment measurements of the same variable. Baseline measurements of outcome variables are typically random variables, which violates the usual ANCOVA assumption that covariate values are fixed. Therefore, the usual ANCOVA hypothesis tests of treatment effects may be invalid, and the ANCOVA slope parameter estimator biased, for this application. We show, however, that if the pretreatment – posttreatment measurements have a bivariate normal distribution, then (i) the ANCOVA model with residual error independent of the covariate is a valid expression of the relationship between pretreatment and posttreatment measurements; (ii) the usual (fixed-covariate analysis) ANCOVA estimates of the slope parameter and treatment effect contrasts are unbiased; and (iii) the usual ANCOVA treatment effect contrast *t*-tests are valid significance tests for treatment effects. Moreover, as long as the magnitudes of the treatment effects do not depend on the “true” pretreatment value of the outcome variable, the true slope parameter must lie in the interval (0, 1) and the ANCOVA model has a clear interpretation as an adjustment (based on between- and within-subject variability) to an analysis of variance model applied to the posttreatment – pretreatment differences.

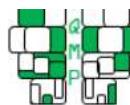
Dealing with heterogeneous regression slopes in analysis of covariance: new methodology applied to environmental effects monitoring fish survey data

Timothy J. Barrett · Maureen A. Tingley ·
Kelly R. Munkittrick · Richard B. Lowell

Abstract Analysis of covariance (ANCOVA) is a powerful statistical method which incorporates one or more covariates into the analysis to reduce error associated with measurement. ANCOVA (modeling response as a function of fish size) is frequently used to analyze environmental effects monitoring (EEM) fish survey data. In approximately 12% of fish survey data sets taken from cycles 1 to 3 of Environment Canada's EEM database for pulp and paper mills, the standard assumption of parallel regression slopes is not met. For the first three cycles of the EEM program, these data sets were classified as indicating a mill effect, but for the most part were excluded from subsequent analyses aimed at quantifying the effect. We present two different methods for initially dealing with data sets that exhibit heterogeneous

slopes so that they can be analyzed using the parallel slope model. The first method identifies data sets where heterogeneous slopes are forced by a few high-influence observations. The second approach identifies data sets where a model with heterogeneous slopes is statistically, but not practically, significant: with a high coefficient of determination for the parallel slope model. These new methodologies are applied to EEM pulp and paper data sets and about 55% of cases with heterogeneous slopes can be described by a parallel slope model. We also discuss a third method that can be used to describe mill effects when regression slopes remain heterogeneous even after applying the above two methods, enabling comparison with a critical effect size. These new methodologies could benefit the EEM program by enabling more data sets to be incorporated into meta-analyses and be used to make more equitable mill monitoring decisions in the future.

Keywords ANCOVA · Coefficient of determination · Covariate · Fish population · Influential points · Leverage



2016 ■ Vol. 12 ■ no. 3



Violation of the homogeneity of regression slopes assumption in ANCOVA for two-group pre-post designs: Tutorial on a modified Johnson-Neyman procedure

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Abstract ■ Aptitude-treatment interaction (ATI) effects are present when individuals demonstrate differential outcomes across treatments based upon aptitude—that is, any measurable individual characteristic, attribute, or ability (e.g., anxiety, learning style, motivation, prior knowledge). ATI effects may exist in data from one design commonly used in psychological and educational research—the two-group pre-post design—in which pre-intervention scores may be considered to reflect individual aptitude. Researchers may mistakenly overlook these effects, however, due to inappropriate analytical approaches. When applying analysis of covariance (ANCOVA), it is important to check for ANCOVA assumptions, including an assumption known as homogeneity of regression slopes. When heterogeneity of regression slopes is found, ATI effects are revealed. Consequently, alternative approaches to ANCOVA must be sought. Using formulae based on the Johnson-Neyman procedure to define simultaneous regions of significance is one straightforward alternative. This tutorial outlines the process for analyzing data resulting from two-group pre-post studies when data violate the ANCOVA assumption of homogeneity of regression slopes. What was initially viewed as an obstacle may result in the discovery of an ATI effect, which may be described statistically through simple mathematical calculations.

Keywords ■ Aptitude-treatment interaction effects; two-group pre-post designs; ANCOVA; Johnson-Neyman procedure. **Tools** ■ SPSS.

Acting Editor ■ Denis Cousineau (Université d'Ottawa)

Reviewers

■ One anonymous reviewer.

A cautionary note on the use of the Analysis of Covariance (ANCOVA) in classification designs with and without within-subject factors

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METHODS

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A number of statistical textbooks recommend using an analysis of covariance (ANCOVA) to control for the effects of extraneous factors that might influence the dependent measure of interest. However, it is not generally recognized that serious problems of interpretation can arise when the design contains comparisons of participants sampled from different populations (classification designs). Designs that include a comparison of younger and older adults, or a comparison of musicians and non-musicians are examples of classification designs. In such cases, estimates of differences among groups can be contaminated by differences in the covariate population means across groups. A second problem of interpretation will arise if the experimenter fails to center the covariate measures (subtracting the mean covariate score from each covariate score) whenever the design contains within-subject factors. Unless the covariate measures on the participants are centered, estimates of within-subject factors are distorted, and significant increases in Type I error rates, and/or losses in power can occur when evaluating the effects of within-subject factors. This paper: (1) alerts potential users of ANCOVA of the need to center the covariate measures when the design contains within-subject factors, and (2) indicates how they can avoid biases when one cannot assume that the expected value of the covariate measure is the same for all of the groups in a classification design.

Keywords: ANCOVA, classification design, within-subject design, between-subjects design, mixed design

TABLE 6 | Recommended procedures to follow when conducting an ANCOVA for three types of designs: (1) All factors are Within-Subject; (2) Experimental designs in which subjects are randomly selected from a uniform population and randomly assigned to different experimental conditions, and (3) Classification designs in which the different levels of Between-Subjects factor consist of samples from different populations (e.g., musicians and non-musicians) where it cannot be assumed the expected value of the covariate is the same across populations.

| All factors Within-Subject | Experimental, Between-Subjects Designs with or without a Within-Subject component (subjects randomly selected from a uniform population and randomly assigned to different experimental conditions) | Classification Designs (with or without a Within-Subject component) where it cannot be assumed that the expected value of the covariate measures is the same for each group of participants (e.g., the different levels of the Between-Subject factor represent random samples from different populations) |
|--|---|--|
| 1. Center the covariate measures 2. Conduct an ANCOVA 3. Use the ANCOVA to evaluate all effects involving covariates 4. Conduct an ANOVA 5. Use an ANOVA to evaluate all remaining effects | 1. Center the covariate measures* 2. Conduct an ANCOVA 3. Use the ANCOVA to evaluate all Between-Subjects effects and any interactions of Between-Subjects and Within-Subject effects, including Within*Covariate interactions 4. Conduct an ANOVA 5. Use an ANOVA to evaluate all remaining Within-Subject effects | 1. Center the covariate measures 2. Conduct an ANCOVA 3. Use the ANCOVA to evaluate all effects involving a covariate 4. Conduct an ANOVA 5. Use the ANOVA to evaluate all remaining effects |

Note that whenever between-subject factors are involved, it is important to first test whether the relationship between the dependent variable and the covariate is the same for all levels of the between-subjects factor (e.g., Howell, 2010, p. 600–603).

**Although it is not necessary to center the covariate measures before entering the data into a standard statistical package when all factors are Between-Subjects, it is necessary to do so when the experimental design contains Within-Subject factors because these programs do not center the covariate measures when evaluating within-subject effects. To be safe, always center the covariate measures before entering them into a statistical package.*

Uses and Abuses of Analysis of Covariance in Clinical Trials

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and H. J. Williams

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Controlled Clinical Trials 6:12-24 (1985)

ABSTRACT: Measurement of improvement in clinical trials in chronic diseases commonly compares baseline data to endpoint values by performing *t*-tests or analysis of variance (ANOVA) on raw gains or percentage changes. This procedure can be misleading and the use of an analysis of covariance (ANCOVA) should be considered.

Properly used, ANCOVA increases statistical power in a clinical trial. However, its advantage over *t*-tests can be nullified by small numbers of patients, violations of assumptions, and incorrect application of the techniques.

An evaluation of ANCOVA in chronic disease studies is given, with examples of its strengths and weaknesses as seen in several drug trials in the rheumatic diseases. Recommendations on its use and a decision tree for the nonstatistician are provided.

KEY WORDS: *Raw gain scores, analysis of covariance, precision, sample sizes, clinical trials.*

Analysis of Pre-test–Post-test Control Group Designs in Educational Research

Educational Psychology, Vol. 15, No. 2, 1995

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JOHN TODMAN, Department of Psychology, University of Dundee, UK

ABSTRACT *Pre-test–post-test control group designs are well suited to investigating effects of educational innovations and are common in educational research. They are frequently analysed by means of an ANOVA on change scores, or, what amounts to the same thing, a repeated measures ANOVA to test the treatment by occasion interaction. Although the analysis of change scores has intuitive appeal, such analyses are often inappropriate. An ANCOVA on post-test scores, with pre-test scores as co-variate usually provides a more appropriate and informative analysis. Advantages of the ANCOVA approach are explained and illustrated using SPSS-X analyses of fabricated data sets.*

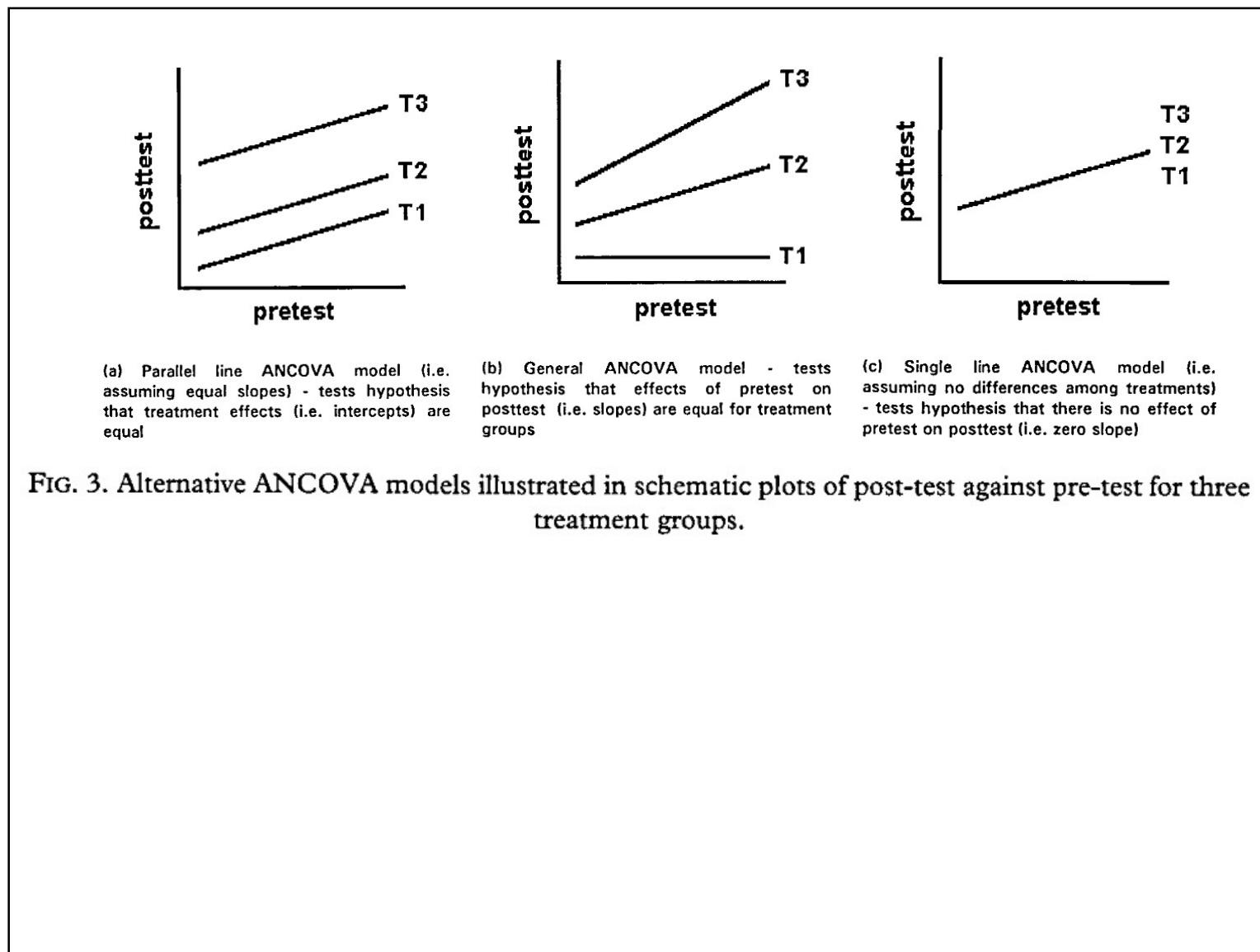


FIG. 3. Alternative ANCOVA models illustrated in schematic plots of post-test against pre-test for three treatment groups.

TABLE I. Frequencies of ANOVA and ANCOVA analyses of pre-test-post-test control group designs from 1986 to 1992

| Type of analysis | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 |
|---------------------------------------|------|------|------|------|------|------|------|
| Gain score or repeated measures ANOVA | 12 | 6 | 10 | 12 | 8 | 14 | 14 |
| ANCOVA with pre-test as co-variate | 5 | 3 | 2 | 3 | 3 | 2 | 5 |

Psychological Bulletin
1975, Vol. 82, No. 4, 511-518

Using a Repeated Measures ANOVA to Analyze the Data from a Pretest–Posttest Design: A Potentially Confusing Task

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The pretest–posttest control group design (or an extension of it) is a highly prestigious experimental design. A popular analytic strategy involves subjecting the data provided by this design to a repeated measures analysis of variance (ANOVA). Unfortunately, the statistical results yielded by this type of analysis can easily be misinterpreted, since the score model underlying the analysis is not correct. Examples from recently published articles are used to demonstrate that this statistical procedure has led to (a) incorrect statements regarding treatment effects, (b) completely redundant reanalyses of the same data, and (c) problems with respect to post hoc investigations. Two alternative strategies—gain scores and covariance—are discussed and compared.

| | | Factor B (Trials) | | |
|----------------------|-------|----------------------------|--------------------|-------------------------------------|
| | | Subject | b_1 (Pretest) | b_2 (Posttest) |
| Factor A (Groups) | a_1 | Exper- imental group | 1 2 3 | X_{111} X_{112} X_{113} |
| | a_2 | Control group | 4 5 6 | X_{214} X_{215} X_{216} |
| | | | | X_{121} X_{122} X_{123} |
| | | | | X_{224} X_{225} X_{226} |

FIGURE 1. Diagram for a pretest–posttest control group design. (Adapted from B. J. Winer, *Statistical Principles in Experimental Design* (2nd ed.), p. 519. Copyright 1971 by McGraw-Hill. Adapted by permission.)

Delineamento experimental

VD: X

Fator intra: B

Fator entre: A (randomização)

SPSS: GLM | Repeated Measures

ANOTHER OPTION: ANALYSIS OF COVARIANCE

Instead of using the pretest scores to compute gain scores, they can be used as the concomitant variable within an analysis of covariance. As compared with the analysis of gain scores, a covariance analysis will normally result in a more sensitive test of possible differences among treatments. Assuming equal pretest means (brought about by the random assignment of the subjects to groups), the adjusted between-groups sum of squares (SS) in the analysis of covariance will be identical to the between-groups SS from the gain score analysis.

are aware of the fact that several assumptions (e.g., randomization, linearity, homogeneity of regression slopes, etc.) underlie the analysis of covariance.

DISCUSSION

In the first section of this article, a repeated measures ANOVA was shown to provide potentially confusing results when applied to pretest-posttest designs involving two or more groups of subjects. In the second section, we argued that an analysis of gain scores—as an alternative to the repeated measures ANOVA—yields the same amount of useful information in a far more straightforward, parsimonious manner. In the final portion of the article, however, the analysis of covariance was shown to be a more powerful technique with greater versatility with respect to available strategies for the situation in which assumptions are violated. Since a covariance analysis can do everything that a gain score analysis can do, but not vice versa, the covariance analysis is recommended for use in place of the repeated measures ANOVA.

Misunderstanding Analysis of Covariance

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Jean P. Chapman
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Despite numerous technical treatments in many venues, analysis of covariance (ANCOVA) remains a widely misused approach to dealing with substantive group differences on potential covariates, particularly in psychopathology research. Published articles reach unfounded conclusions, and some statistics texts neglect the issue. The problem with ANCOVA in such cases is reviewed. In many cases, there is no means of achieving the superficially appealing goal of “correcting” or “controlling for” real group differences on a potential covariate. In hopes of curtailing misuse of ANCOVA and promoting appropriate use, a nontechnical discussion is provided, emphasizing a substantive confound rarely articulated in textbooks and other general presentations, to complement the mathematical critiques already available. Some alternatives are discussed for contexts in which ANCOVA is inappropriate or questionable.

Journal of Abnormal Psychology
2001, Vol. 110, No. 1, 40–48

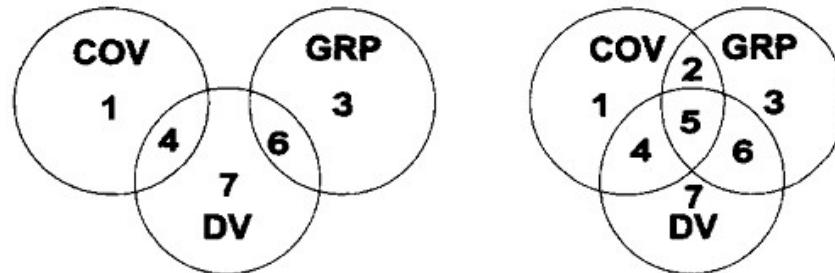
**TRUE EXPERIMENT****QUASI-EXPERIMENT**

Figure 1. Two (of several) possible relationships between the group (*Grp*), covariate (*Cov*), and dependent (*DV*) variables. In both cases, removing variance in *DV* that is associated with *Cov* will reduce the variance in *DV* that is not associated with *Grp* and thus will enhance the relationship detected between *Grp* and *DV*. In the left panel, *Grp* and *Cov* share no variance. This is the classic situation for a true experiment, with random assignment to groups. Removing the variance associated with *Cov* will not alter *Grp*. Given random assignment, individual characteristics such as height or presence of hallucinations would generally be randomly distributed across the groups, and group means should not differ except by chance. In the right panel, *Grp* and *Cov* do share variance. This is often the case when preexisting groups are studied, such as comparisons of two diagnostic groups—a quasi-experiment rather than a true experiment. In such a case, removing the variance associated with *Cov* will also alter *Grp* in potentially problematic ways.