## Simulation to find the Expected value for the output of the matrix game:

We use the following code to perform the simulation to obtain the graph of the expected value,

%% Given matrix

A = [3, 0; -1 1];

%% simulated outcome

NumberOfBernouliExp = 1000; X = []; Y = []; Z = [];

p1 = 0:0.02:1;

p2 = 0:0.02:1;

outcome = [];

for l = 1:length(p1)

for m = 1:length(p2)

for exp\_id = 1:NumberOfBernouliExp

% we take (1-p) and to denote

% the probability of success in binornd

% as p is the probability of selecting zero

i = binornd(1,1-p1(l)) + 1;

j = binornd(1,1-p2(m)) + 1;

outcome\_arr(exp\_id) = A(i,j);

end

outcome(l,m) = mean(outcome\_arr);

end

end

figure,mesh(p1, p2, outcome); xlabel('p\_1','fontsize',18); ylabel('p\_2','fontsize',18);

zlabel('Outcome','fontsize',18);

The corresponding output figure is as follows,

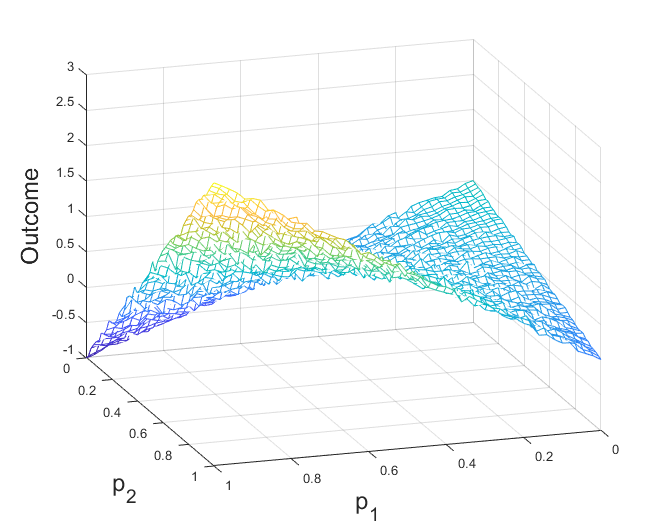


Figure : Simulated Graph for the Expected Values of Outcomes for Different mixed strategies.

## Analytical Expression for Outcome with Mixed Strategy:

For a given matrix , we know expected value of the outcome of the game, corresponding to the strategy pair {P1, P2}, will be equal to,

where and are the probability distribution vectors defined by and . For the given problem and and . Therefore, the corresponding outcome function in terms of and :

(1)

We write the following code to plot the function with regards to and :

%% Given matrix

A = [3, 0; -1 1];

%% Analytical expression for the outcome using probability

outcome\_fnc = @(x,y)(x\*y\*A(1,1)+(1-x)\*y\*A(2,1)+(1-x)\*(1-y)\*A(2,2)+(x)\*(1-y)\*A(1,2));

p1 = 0:0.001:1; p2 = 0:0.001:1;

for l = 1:length(p1)

for m = 1:length(p2)

outcome(l,m) = outcome\_fnc(p1(l),p2(m));

end

end

figure, mesh(p1, p2, outcome);

xlabel('p\_1','fontsize',18); ylabel('p\_2','fontsize',18);

zlabel('Outcome','fontsize',18)

The output is as follows,

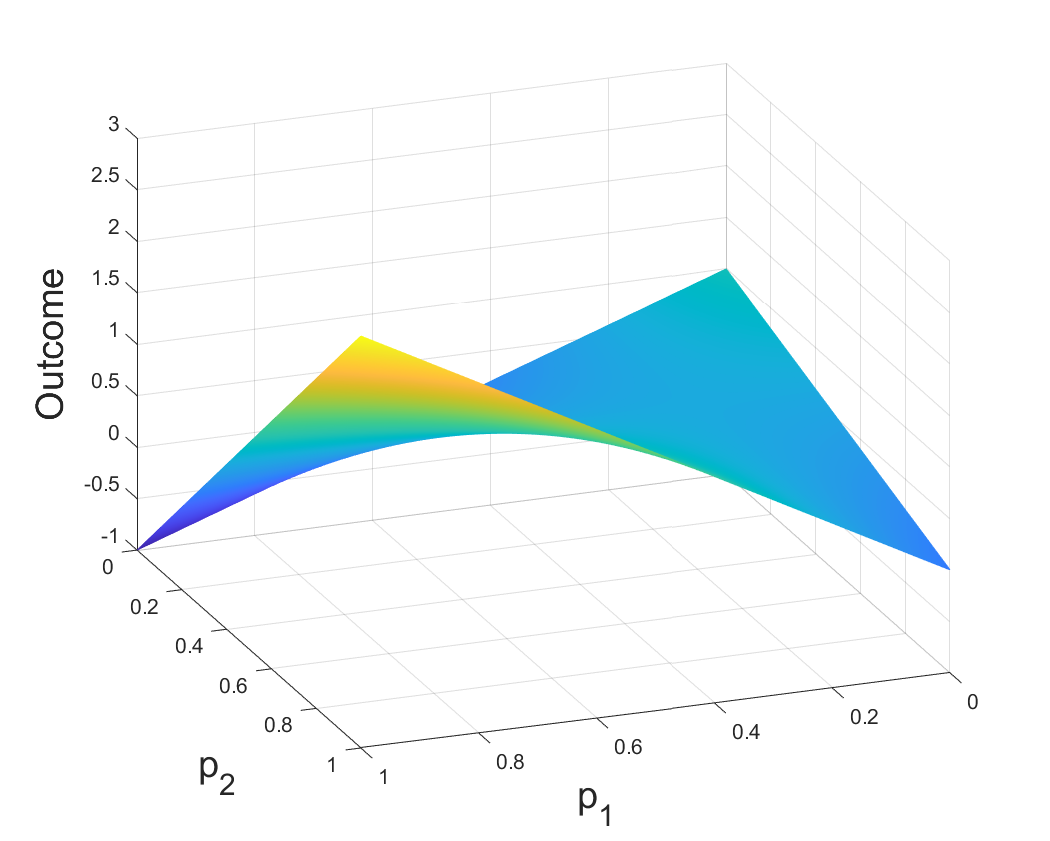


Figure : Outcome function vs mixed strategy probabilities of player 1 and player 2.

## Analytical Calculation of Saddle Point:

We rewrite the outcome function in as follows:

Where , , , and

We know, a point is a saddle point of a function of two variables if,

, and

at that point. Calculating partial derivatives,

and

For the given matrix , , , = 3+1 +1-0 = 5,

Plugging in the values, we have and .

Now plugging in the solution of and in the left hand side of the second order solution we obtain,

Which is less than zero. Therefore, and is the saddle point.

Therefore, the saddle security level is,