Analysis of Image Denoising Performance with Different Wavelet Systems

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***Abstract*—Wavelet systems has been very popular for image denoising techniques. There are many denoising algorithms exists right now with different wavelets and wavelet-type systems. One of the major area that has been extensively studied by different author is to choose data dependent threshold selection to have best denosing performance with introducing minimal distortion or blur to the image. In this study, we present some popular denoising schemes and implement one of them. Finally we study the implemented denoising algorithm performance with different wavelet systems and different noise levels for a given aerial image.**

***Index Terms*—Trappings effects, thermal effects, low frequency S-parameters, CAD non-linear model, RF pulsed operation.**

I. Introduction

There are many ways a image can be corrupted with different noises because of transmission, acquisition, compression [1], [2], [3], quantization, illumination conditions [4], malfunctioning instruments, ill positions etc. Types of noise can be from additive Gaussian noise to almost multiplicative speckle noise. For any kind of image based applications including computer vision alogorithms, image denoising is on of the first steps to be considered. One of the earlies methods introduced for image denoising were based on statistical filters [5]. Different statistical approaches for image denosing includes median based image denoising methods [6], [7], statistical modeling based image denoising [7]. Other approaches include derivative based methods, axiomatic based methods, variation approaches, fuzzy logic based image denoising, mathematical morphology based image denoising [7]. Recently neural network based denoising methods are also being very popular for some applications[8], [9].

It has been many years that the wavelet or wavelet system based denoising schemes has become one of the popular tools to suppress the noise owing to its effectiveness and producing better results. Because of its properties, wavelet transform of the real signals or images are usually very sparse. It is also very popular for compression schemes as they have a few large coefficients that contain the most of the energy of the signal and other small coefficients which can be discarded[10]. Unlike the natural signals, spectral energy of the noise are usually distributed among all the coefficients in the wavelet domain. The noise power can be suppressed significantly with an appropriate threshold while the original signal can be preserved because the wavelet transform of a noisy signal is a sum of the wavelet transform of the noise and the original signal.

In the very beginning of the different thresholding schemes, hard and soft thresholding functions were proposed by Donoho and Johnstone [11] at the same time with garrote [11], and semisoft thresholding functions [12] that are more powerful, are used in noise suppression applications. In these simple thresholding methods, the coefficients with low energy are set to zero. In hard thresholding, the coefficients with high energy are kept unchanged. The dominant coefficients are reduced by the absolute threshold value in soft thresholding. Along with a good thresholding function selection of a proper threshold value also is a key factor for an efficient thresholding scheme. Researchers has proposed different ways to choose proper thresholds. *Universal-threshold* methods in which the threshold value is chosen uniquely for all wavelet coefficients (VisuShrink) of for a noisy image is introduced with the above-mentioned hard and soft thresholding function as the first practical technique in signal denoising applications [11]. Researcher have also proposed, subband-adaptive methods where the threshold value is chosen uniquely for each of the detail subband [13], [14] and spatially adaptive threshold selection where for each of the detail coefficients has different thresholds [15].

In the current study, we implement the simple threshold choosing method by Donoho and Johnstone [11] for image denoising and compare the performances for different wavelet systems, with different scales and different thresholding functions. We perform analysis on a given aerial image.

II. Dataset

For this project, an aerial image has been used to analyze the performance of two threshold choosing algorithms for different wavelets with different settings. Fig. 1 the whole aerial image. The given image is a 12020 by 11920 image with 3 color channels. Noise has been simulated considering noise as additive zero mean Gaussian random variable for each pixel.

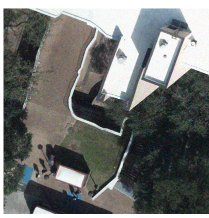
$$\boldsymbol{J} = \boldsymbol{I} + \boldsymbol{N}; \tag{1}$$

Where, I, J and N are the original, noisy and noise image respectively. The variances for the noise have been calculated as $\sigma_N^2 = \mathrm{var}(I)/\omega$ for $\omega = 20, 15, 10, 5, 1.5, 1$. Figure 2 shows the noisy 512×512 segments with different noise levels.

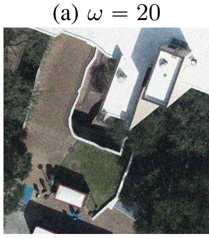
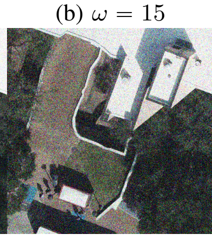
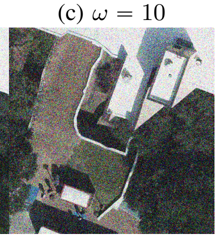
III. Methods

Motivated by the assumption that only very few wavelet coefficients contribute to the signal, threshold rules has beeb proposed by [11], that retain only observed data that exceeds a multiple of the noise level. 'hard' and 'soft' threshold non-linearities can be defined as follows,

Fig. 1: **The aerial image represents a region around the UH campus.**

(a) $\omega = 20$ (b) $\omega = 15$ (c) $\omega = 10$

(d) $\omega = 5$ (e) $\omega = 1.5$ (f) $\omega = 1$

Fig. 2: Nosiy images with different standard deviations.

$$\eta_H(w, \lambda) = w\mathrm{u}(w > \lambda)$$
$$\eta_S(w, \lambda) = \mathrm{sgn}(w) \times (|w| - \lambda)_+$$

Where λ is the threshold parameter for denoising, w represents the detail wavelet coefficients. Here u refers to unit step function. A Simple implementation for the universal sequence $\lambda_n^u = (2 \log n)^{1/2}$ as an alternative to the use of minimax thresholds where n is the number of elements in the image matrix. The sequence is easy to implement as it does not require no costly development of look-up tables. In our analysis we use this threshold parameter for thresholding.

TABLE I: Table shows the Mean Squared Errors for different wavelets with different noise level for image with Universal Threshold [11] and hard thresholding technique.

wavelets	w = 20	w = 15	w = 10	w = 5	w = 1.5	w = 1
Haar	277.39	338.87	446.35	738.73	1790.43	2381.28
Daubechies 3	258.14	316.13	423.69	708.91	1744.52	2332.88
Daubechies 5	259.63	317.79	425.51	710.9	1745.24	2332.21
Biorthogonal 2.6	216.28	270.08	371.88	648.2	1675	2265.48
Coiflets 3	254.55	312.3	419.4	704.16	1739.54	2328.2

TABLE II: Table shows the Mean Squared Errors for different wavelets with different noise level for image with Universal Threshold with rescaling [11] and hard thresholding technique.

wavelets	w = 20	w = 15	w = 10	w = 5	w = 1.5	w = 1
Haar	215.28	270.57	370.71	647.07	1670.61	2257.66
Daubechies 3	204.8	257.96	358.27	630.26	1645.1	2232.82
Daubechies 5	205.87	259.24	359.81	632.23	1647.31	2234.19
Biorthogonal 2.6	181.95	232.3	328.64	594.4	1604.35	2192.5
Coiflets 3	202.78	255.76	355.67	627.37	1642.61	2230.73

Again we perform the experiments by rescaling the thresholds with the noise level. Noise level is estimated taking the median absolute deviation of the wavelet coefficients at the finest level and dividing it by 0.6745. Denoising is performed for each of the channels of the image separately.

TABLE III: Table shows the Mean Squared Errors for different wavelets with different noise level for image with Universal Threshold [11] and soft thresholding technique.

wavelets	w = 20	w = 15	w = 10	w = 5	w = 1.5	w = 1
Haar	172.90	230.71	345.91	683.10	2076.56	2928.20
Daubechies 3	172.01	229.77	344.93	682.05	2075.60	2927.33
Daubechies 5	172.15	229.92	345.09	682.26	2075.85	2927.60
Biorthogonal 2.6	176.01	235.23	352.90	695.80	2104.85	2963.61
Coiflets 3	171.94	229.71	344.88	682.09	2075.82	2927.63

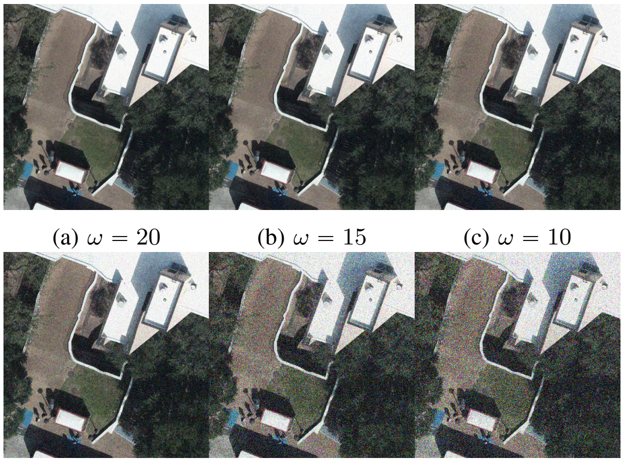
TABLE IV: Table shows the Mean Squared Errors for different wavelets with different noise level for image with Universal Threshold with Rescaling and soft thresholding technique[11].

wavelets	w = 20	w = 15	w = 10	w = 5	w = 1.5	w = 1
Haar	196.66	260.8	386.67	747.74	2204.35	3084.54
Daubechies 3	196.66	260.79	386.67	747.74	2204.3	3084.48
Daubechies 5	196.66	260.79	386.66	747.69	2204.15	3084.26
Biorthogonal 2.6	197.04	261.18	387.09	748.23	2205.2	3085.61
Coiflets 3	196.64	260.78	386.66	747.71	2204.24	3084.39

IV. Results and Discussion

Table I and II shows the mean square errors (MSE) for different wavelet systems with hard thresholding without rescaling is lower than with rescaling for threshold with the noise level respectively. But the Figure 3 and 4 shows that the smoothing operation is better in with the rescaling scheme. The later one is performing well but at the same time it is removing some of the features of the image while denoising. We can see the smoothing operation is increased scale by scale for both of the case in Figure 3 and 4.

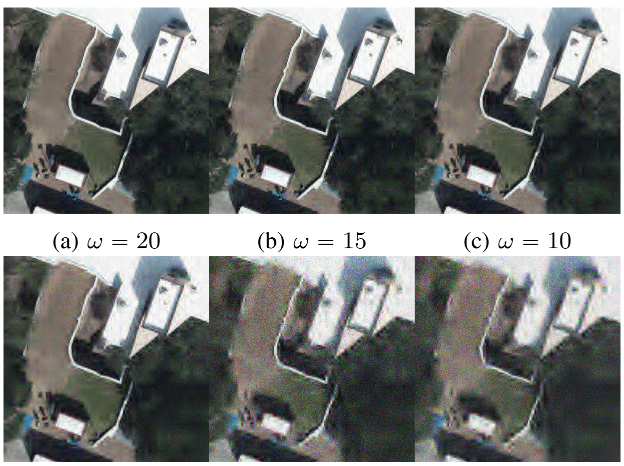
Table III and IV also shows thresholding without rescaling and has lower MSE but less noise removing performance for soft thresholding as can be seen in Figure 5 and 6. All the



(a) $\omega = 20$ (b) $\omega = 15$ (c) $\omega = 10$

(d) $\omega = 5$ (e) $\omega = 1.5$ (f) $\omega = 1$

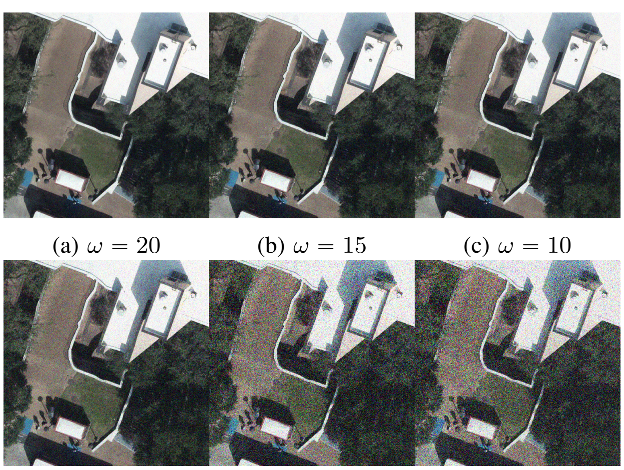
Fig. 3: Result after hard thresholding with universal threshold without rescaling.



(a) $\omega = 20$ (b) $\omega = 15$ (c) $\omega = 10$

(d) $\omega = 5$ (e) $\omega = 1.5$ (f) $\omega = 1$

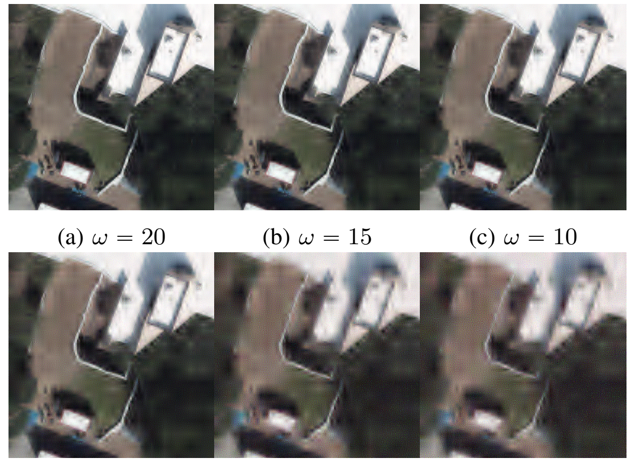
Fig. 4: Result after hard thresholding with universal threshold with rescaling.



(a) $\omega = 20$ (b) $\omega = 15$ (c) $\omega = 10$

(d) $\omega = 5$ (e) $\omega = 1.5$ (f) $\omega = 1$

Fig. 5: Result after soft thresholding with universal threshold without rescaling.



(a) $\omega = 20$ (b) $\omega = 15$ (c) $\omega = 10$

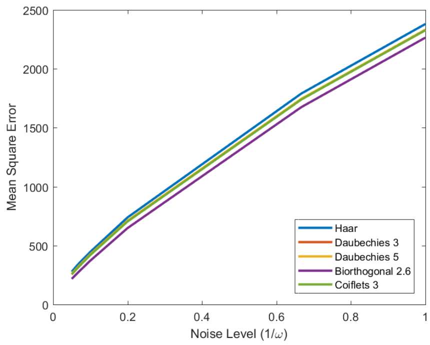
(d) $\omega = 5$ (e) $\omega = 1.5$ (f) $\omega = 1$

Fig. 6: Result after soft thresholding with universal threshold with rescaling.

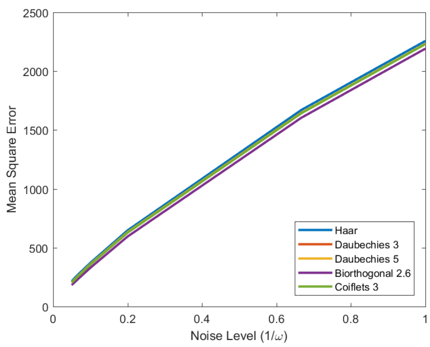
example image segments in Fig 3, 4, 5 and 6 are smoothed using Daubechies 3 for different noise levels with with wavelet scale level 5. It can be seen that soft thresholding has better performance over the hard thresholding. The denised images visually looks smooth and yet they have lower MSE. It can also be inferred that universal threshold with rescaling used in this study has an upward bias which cause some wavelet coefficients forced to be zero that contains image informations. When there is higher noise, the algorithm tends to also smooth out the details of the image. Without rescaling the image informations are kept almost unchanged. The noise removal performance is not very good as well without rescaling. The estimator in

Figure 7 shows that the MSE increases with the increase of the noise level for all wavelets and all thresholding schemes. Figure 7a shows that for hard thresholding without rescaling scheme Biorthoganl 2.6 wavelet has best performance in this case.and Haar wavelet has the worst performance. From Figure 7a to 7d the difference between different wavelets decreases in terms of performance. In figure 7d with soft thresholding with rescaling, all the wavelets have almost same MSEs.

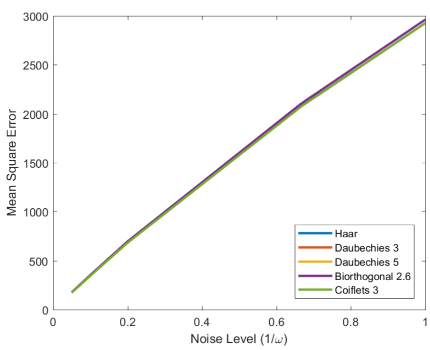
On the other hand, Figure 8 shows that the MSE increases with the increase of the noise level for all wavelets and all thresholding schemes. Figure 8a shows that for hard thresholding without rescaling scheme does perform same for all scale in this case. From Figure 8a to 8d the difference between different scales increases in terms of MSE. In figure 7d with soft thresholding with rescaling, all the wavelets have most different MSEs. So, it can be inferred from here that, for soft thresholding scheme with rescaling, the selection of the scale might play very important role. One has to carefully choose the wavelet scale level for this case. Again it can be seen that with very higher scale levels for example in 8 the differences between MSEs with finer levels (5 and 6 in this case) are very small and further increase in the scale might not increase the performance further.



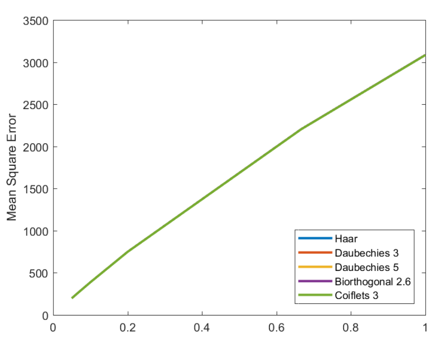
(a) Hard Thresholding without Rescaling.



(b) Hard Thresholding with Rescaling

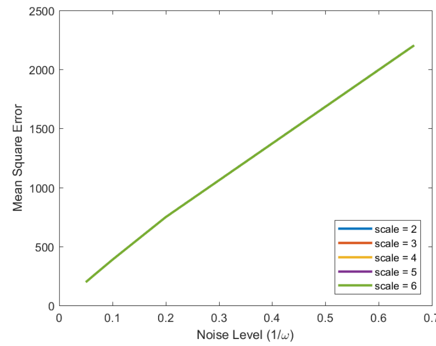


(c) Soft Thresholding without Rescaling

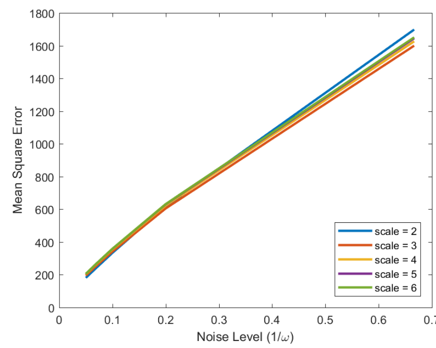


(d) Soft Thresholding with Rescaling

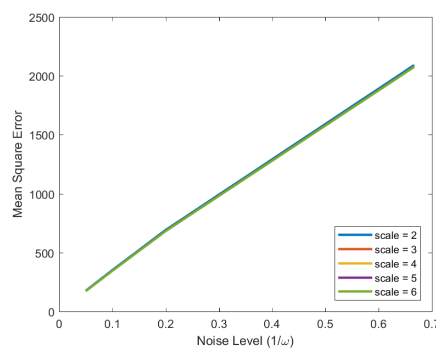
Fig. 7: MSE Plots for Different Wavelets at Scale = 5



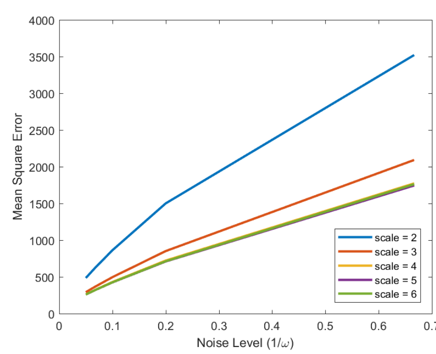
(a) Hard Thresholding without Rescaling.



(b) Hard Thresholding with Rescaling



(c) Soft Thresholding without Rescaling



(d) Soft Thresholding with Rescaling

Fig. 8: MSE Plots for Different Scales with Daubechies 3 Wavelet

V. Conclusion

Because of the sparsity nature of wavelet transform of natural signals and images, Wavelet or wavelet system based denoising techniques has been dominating the images as well as other signal denoising techniques for decades. There are many variant and intelligent techniques with wavelet systems. In this study we analyzed one of the earliest techniques in wavelet denoising. We analyzed with different wavelets and different scales for one of the wavelets. From our analysis it can be inferred that the performance of soft thresholding is technique is much better than the hard thresholding. And also with finer scales the performance in soft thresholding are similar. Choice of scales for wavelets might play a very important role in the denoising performance. From all the results, it can also be inferred that universal threshold with rescaling used in this study has an upward bias which cause some wavelet coefficients being ignored that contains image informations. Finally, the algorithm to choose the level dependent universal threshold with/without rescaling is one of the earliest proposed method and has some drawbacks. Many authors have provided other approaches that has better performance which can also be investigated for specific applications.

References

1 Rehman, M., Sharif, M., and Raza, M., "Image compression: A survey," *Research Journal of Applied Sciences, Engineering and Technology*, vol. 7, no. 4, pp. 656–672, 2014.

2 Raza, M., Adnan, A., Sharif, M., and Haider, S. W., "Lossless compression method for medical image sequences using super-spatial structure prediction and inter-frame coding," *Journal of applied research and technology*, vol. 10, no. 4, pp. 618–628, 2012.

3 Naeem, Q., Sharif, M., and Raza, M., "Improving audio data quality and compression," in *Emerging Technologies, 2008. ICET 2008. 4th International Conference on*. IEEE, 2008, pp. 332–337.

4 Sharif, M., Mohsin, S., Jamal, M. J., and Raza, M., "Illumination normalization preprocessing for face recognition," in *Environmental Science and Information Application Technology (ESIAT), 2010 International Conference on*, vol. 2. IEEE, 2010, pp. 44–47.

5 Gonzalez, R. C. and Woods, R. E., "Digital image processing," 2012.

6 Tukey, J. W., "Explorative data analysis," 1977.

7 Irum, I., Shahid, M., Sharif, M., and Raza, M., "A review of image denoising methods." *Journal of Engineering Science & Technology Review*, vol. 8, no. 5, 2015.

8 Zhang, K., Zuo, W., Chen, Y., Meng, D., and Zhang, L., "Beyond a gaussian denoiser: Residual learning of deep cnn for image denoising," *IEEE Transactions on Image Processing*, vol. 26, no. 7, pp. 3142–3155, 2017.

9 Zhang, K., Zuo, W., and Zhang, L., "Ffdnet: Toward a fast and flexible solution for cnn based image denoising," *arXiv preprint arXiv:1710.04026*, 2017.

10 Crouse, M. S., Nowak, R. D., and Baraniuk, R. G., "Wavelet-based statistical signal processing using hidden markov models," *IEEE Transactions on signal processing*, vol. 46, no. 4, pp. 886–902, 1998.

11 Donoho, D. L. and Johnstone, J. M., "Ideal spatial adaptation by wavelet shrinkage," *biometrika*, vol. 81, no. 3, pp. 425–455, 1994.

12 Gao, H.-Y., "Wavelet shrinkage denoising using the non-negative garrote," *Journal of Computational and Graphical Statistics*, vol. 7, no. 4, pp. 469–488, 1998.

13 Donoho, D. L. and Johnstone, I. M., "Adapting to unknown smoothness via wavelet shrinkage," *Journal of the american statistical association*, vol. 90, no. 432, pp. 1200–1224, 1995.

14 Chang, S. G., Yu, B., and Vetterli, M., "Adaptive wavelet thresholding for image denoising and compression," *IEEE transactions on image processing*, vol. 9, no. 9, pp. 1532–1546, 2000.

15 Mihcak, M. K., Kozintsev, I., Ramchandran, K., and Moulin, P., "Low-complexity image denoising based on statistical modeling of wavelet coefficients," *IEEE Signal Processing Letters*, vol. 6, no. 12, pp. 300–303, 1999.