Economics 1011B
Sections 9: New Keynesian Model and Labor Markets

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January 25, 2025

Today's Outline

- New Keynesian Model
  - Setup
  - Production
  - Household
  - Equilibrium, Sticky vs. Flexible
  - Monetary Neutrality
- Search and Matching Model
  - Matching function
  - Value functions
  - Equilibrium
  - Law of motion for unemployment
- Bonus Slides: Wage rigidity and the Shimer Puzzle

New Keynesian Model

From IS-MP to New Keynesian models

- Last class lecture: discussed IS-MP, a nice, simple framework for thinking about macro policy.
- But IS-MP felt somewhat detached from the rest of this course! No optimizing agents, no dynamics (short vs. long-run). How can we reconcile these things?
- Somewhat historical: IS-MP-type models dominant until the 1970s (see last section's bonus slides).
- This week: 'New Keynesian' framework, still the dominant paradigm as a workhorse macro framework in modern academic economics.
- New Keynesian models typically involve a lot of math, so we will walk through a very 'stylized' one that minimizes the amount of math while retaining the lessons.
- Essentially: New Keynesian model = neoclassical growth with market power/monopolistic competition, monetary policy, and sticky prices.

Krugman NK Model: Setup

- Infinite horizon: time indexed by $t = 1, 2, 3, \dots$.

- Production: simplify production by assuming firm receives Y_t^* exogenously for free each period, sells output $Y_t \leq Y_t^*$ at a price P_t, rebates revenue to household.

- Consumption: Household chooses consumption C_t in each period. Can invest in bonds that pay a nominal return of $i_{1,t}$ between 1 and t, or hold cash that can be carried across periods but generates no return. Must hold enough cash to finance spending.

- Central bank sets money supply M_t each period.

- Assume $Y_2^* = Y_3^* = \cdots = Y^*$ and $M_2 = M_3 = \cdots = M^*$ (exogenous constants Y^*, M^*)
  $\implies$ Periods $t \geq 2$ are identical: makes the model a little more tractable!

Krugman NK Model: Production

- Purely for simplicity, we'll assume an 'endowment economy' that does not explicitly model firm production from inputs.

- This is an analytical shortcut that just saves us some math. Sometimes called a "fruit tree" economy – firm just receives the goods they are selling from thin air.

- Firms receive output Y_t^* every period and sell some amount $Y_t \leq Y_t^*$ at a price P_t.

- Zero input cost, so firms rebate all revenue (which is profit) $P_t Y_t$ directly back to household.

- That's it!

Krugman NK Model: Household Problem

- Household chooses consumption C_t each period to maximize lifetime discounted utility:

$$\max_{\{C_t\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} \beta^{t-1} U(C_t) \quad \text{subject to} \quad \sum_{t=1}^{\infty} \frac{P_t C_t}{1+i_{1,t}} = \sum_{t=1}^{\infty} \frac{P_t Y_t}{1+i_{1,t}} \qquad \text{(lifetime budget)}$$

$$P_t C_t \leq M_t \qquad \text{(cash-in-advance)}$$

where $1 + i_{1,t} = \prod_{s=1}^{t-1}(1 + i_{s,s+1})$ is the nominal (compounded) interest rate between 1 and t.

- Assets (do not appear in lifetime BC): household can hold bonds B_t that pay nominal return of $i_{t,t+1}$ between t and $t+1$ or money M_t, which has 0 nominal return.

- Cash-in-advance constraint: the household must hold enough cash M_t to finance expenditure $P_t C_t$. If $i_{t,t+1} >= 0$, CIA constraint will bind in t (review q: why?)

- Let's assume utility takes the 'power utility' form $U(C_t) = \frac{C_t^{1-1/\sigma} - 1}{1-1/\sigma}$

Krugman NK Model: Household FOC

- Solving household problem yields **nominal** Euler equation between t and $t+h$:

$$\beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1/\sigma} = \frac{i_{1,t}}{i_{1,t+h}} \frac{P_{t+h}}{P_t}$$

- Substituting in the Fisher equation $(1 + r_{t,t+h}) = \frac{(1+i_{t,t+h})}{P_{t+h}/P_t}$, this is the same as:

$$\beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1/\sigma} = \frac{1}{1 + r_{t,t+h}}$$

- These are totally equivalent ways to represent the household FOC! Consumption depends on nominal interest rates and prices (nominal Euler equation), or equivalently the real interest rate (real Euler equation).

- Worded differently: the real interest rate is sufficient to capture the influence of both nominal interest rates and prices on consumption.

Krugman NK Model: IS and LM Curves

- The New Keynesian IS curve relates output and the nominal interest rate: obtain by imposing goods market clearing $Y_t = C_t$ in Euler equation for periods t and $t+h$:

- For periods $t = 1$ and $t = 2$, the IS curve is:

$$Y_1 = \left(\beta \frac{1 + i_{1,2}}{P_2 / P_1} \right)^{-\sigma} Y_2$$

- The New Keynesian LM curve relates the nominal interest rate to the money supply: obtain by plugging in CIA constraint and ZLB constraint into IS curve:

$$i_{1,t} = \max \left\{ \beta^{1-t} \frac{M_t}{M_1} \left(\frac{Y_1}{Y_t} \right)^{1-1/\sigma} - 1, 0 \right\}$$

Central Bank

- Central bank chooses target money supply M_t.

- To reach this money supply, chooses a quantity of bonds $B_t = P_t Y_t - M_t$.

- Nominal interest rate $i_{1,t}$ is determined in equilibrium by supply and demand for bonds / money.

Krugman NK Model: Solution for $t > 1$

- Easy to work backwards to solve the model. Recall that we assumed output and monetary policy was the same from periods 2 on.

- This implies that period 2 on is at the steady state: $C_t = C_{t+1}$ and $P_t = P_{t+1}$ for any $t > 1$. By the Fisher equation, this implies $i_{t,t+1} = r_{t,t+1}$ (zero inflation at steady state).

- For $t > 1$, we have:

$$\begin{aligned} C_t &= Y_t = Y^* && \text{(goods market clearing)} \\ 1 + i_{t,t+1} &= 1 + i^* = \beta^{-1} && \text{(Euler equation at steady state)} \\ P_t &= M_t / C_t = M^* / Y^* && \text{(cash-in-advance constraint binds)} \end{aligned}$$

- For $t = 1$, we'll distinguish between a sticky price and flexible price equilibrium.

Krugman NK Model: Solution for $t = 1$ (Flexible Prices)

- Start out by assuming prices are flexible; that is, the price P_t adjusts so that supply Y_t^* equals demand C_t for goods in each period.

- Endogenous variables: P_1 and $i_{1,2}$.

- Two cases: $i > 0$ and $i = 0$.
  - If $i > 0$, CIA binds (why? bonds dominate money as investment). CIA and LM curves characterize P_1 and $i_{1,2}$.

  - If $i = 0$, ZLB condition $i_{1,2} = 0$ and IS curves characterize $i1, 2$ and P_1.

- What does monetary policy do in this (flexible price) model?
  - If $i > 0$: $M_1 \uparrow \implies i_{1,2} \downarrow \implies P_1 \uparrow \implies Y_1$ unchanged

  - If $i = 0$: $M_1 \uparrow \implies P_1, Y_1$ unchanged

- Monetary neutrality: money can't impact real variables, only nominal variables. Krugman model with flexible prices exhibits monetary neutrality.

Krugman NK Model: Solution for $t = 1$ (Sticky Prices)

- What changes with sticky prices? As before, assume P_1 is perfectly sticky (exogenous) so that C_1 (equivalently Y_1) is now endogenous.

- Two unknowns, C_1 and $i_{1,2}$.

- Like before, Euler equation and LM curves characterize solution:

$$C_1 = \beta^{-\sigma}\left(\frac{1+i_{1,2}}{1+\pi_{1,2}}\right)^{-\sigma} Y^*$$

$$i_{1,2} = \max\left\{\beta^{-1}\frac{M^*}{M_1}\left(\frac{C_1}{Y^*}\right)^{1-1/\sigma} - 1, 0\right\}$$

- Key difference: Assume $Y_2 = Y^* \implies 1 + r_{1,2}$ endogenous. Lowest possible $r_{1,2}$ is $-\pi_{1,2}$ by the Fisher equation and $i_{1,2} \geq 0$. If $r_{1,2}^{\text{full employment}} < -\pi_{1,2}$, economy is in "liquidity trap", cannot achieve full employment with monetary policy.

Krugman NK Model: Wrapping Up

- What does this model teach us?

- Lesson 1: If prices are flexible, model exhibits monetary neutrality: monetary policy has no impact on real variables (and therefore welfare)

- Lesson 2: If prices are sticky, monetary policy can have real effects (can help fight recessions), as long as $i > 0$.

- Lesson 3: Zero lower bound on nominal interest rates constraints efficacy of monetary policy, even when prices are sticky: if 0 return on cash dominates return on bonds, CIA does not bind, LM curve is flat.

Labor Markets

Motivation

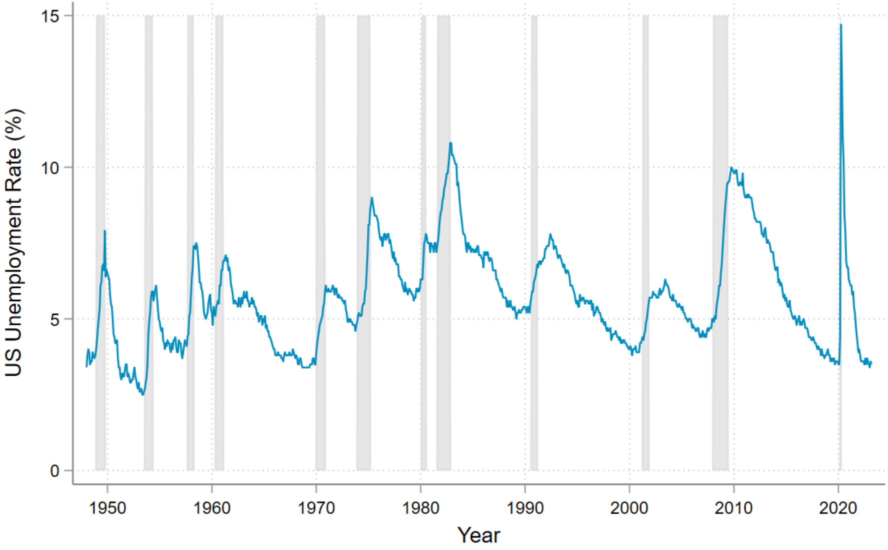
- In my (subjective) view, labor markets are the most important interaction most people have with the macroeconomy.

- To a first-order approximation, most people's status, life satisfaction, and income are determined by their participation in labor markets.

- Labor markets also exhibit strong cyclical fluctuations. Recessions characterized by sharp, sudden increases in (involuntary) unemployment: cannot be captured by neoclassical models that assume labor supply = labor demand (labor market clearing).

- Over longer horizons, wage growth is an important determinant of inequality and changes in standards of living.

US Unemployment Rate over Time



Shaded region denotes NBER-dated recessions.

Search and Matching Model of Labor Markets

- The search and matching model we analyze is the starting-off point for thinking about "frictional" labor markets - a situation where labor markets do not clear, i.e. labor supply does not equal labor demand.

- Indeed, in our search model, households can be involuntarily unemployed - they can want a job and not get one - and firms can have unfilled vacancies - two very important characteristics of the real world.

- The search model will give us a lens through which to view unemployment as a stock whose level is determined by flows - the "ins and outs" of unemployment.

- Useful lens for us as academics, for policymakers, forecasters, etc.

Matching Function

- Individuals can be either employed (e) or unemployed (u). Normalize population to 1, so that $e + u = 1$.

- There is a mass v of vacancies, which firms can post at a cost c (more later).

- Matching function: plays a role like a production function. Given unemployed u and vacancies v, how many matches are created (i.e. how many people are given jobs this period)? Assume CRS Cobb-Douglas:

$$m(u, v) = Mu^{1-\eta} v^{\eta}$$

- Here, M is an exogenous parameter (matching efficiency) that plays a role like productivity in a production function. η controls the curvature of the matching function: holding fixed u, it tells us how much

- Define $\theta = v/u$, call it market tightness: ratio of vacancies (demand for new hires) over unemployment (supply for new hires).

Job Finding, Vacancy-Filling, Separation Rates

- Define the job finding rate, f, as:

$$f(\theta) \equiv m(u, v)/u = m(1, \theta) = \theta^{\eta} \tag{1}$$

  - Probability that an unemployed person finds a job.

- Define the vacancy-filling rate, q, as:

$$q(\theta) \equiv m(u, v)/v = m(1/\theta, 1) = \theta^{\eta - 1} = \frac{f(\theta)}{\theta} \tag{2}$$

  - Probability that a vacancy is filled.

- Employed individuals have a probability s of losing their job this period. Just for simplicity, we will assume s is exogenous, not a decision made by firms.

- Vacant jobs have a probability q of being filled in the next period.

Dynamic Problems and Value Functions

- Value functions are a compact, recursive way to solve dynamic optimization problems: problems where choices today affect choices tomorrow.

- In some sense, you can think of value functions are making a dynamic optimization problem simpler by breaking payoffs into two components:
    1. Payoff today: sometimes called the flow value
    2. Payoffs in the future: sometimes called the continuation value

- Search and matching provides our first concrete example of value functions (though we could have used them to represent many other models we have worked through so far, i.e. neoclassical growth, q-theory, asset pricing).

- Very generally - value functions only solvable through numerical approximation. We make strong simplifying assumptions (just for this class) to get nice closed-form solutions, but not necessary.

Value Functions for Unemployment and Employment

- Unemployed individuals earn a wage-equivalent z from being unemployed (utility from sitting at home) this period. Employed individuals earn wage w.
- Let U denote the (expected PDV) utility value of being unemployed and let W denote the value of being employed.
- We will express U and W as value functions, which are recursively defined: the value of employment is this period's value plus next period's value, which is determined by the probability of remaining employed/unemployed.
- Value function for unemployed:

$$U = z + \left[fW + (1-f)U\right] \tag{3}$$

- Value function for employed:

$$W = w + \left[(1-s)W + sU\right] \tag{4}$$

Value Functions for Vacancies and Workers

- We can likewise define value functions corresponding to the value of a vacancy and of a worker to the firm.

- Workers are paid a wage w and produce output p, $p > w$. The surplus (flow value) of a worker to the firm is $p - w > 0$. Workers separate with probability s. Hence, the value function representing the value of a worker to the firm can be written as:
$$J = (p - w) + \left[(1 - s)J + sV\right] \tag{5}$$

- The firm pays an exogenous cost c to post a vacancy this period. A vacancy is filled with probability q and becomes a worker. So the value of a vacancy to the firm is:
$$V = -c + \left[qJ + (1 - q)V\right] \tag{6}$$

- Close the model with free-entry condition – firms will post vacancies until the value of a vacancy is zero:
$$V = 0 \tag{7}$$

Solving the Model

- Seven equations (1-7) and unknowns $(U, W, J, V, f, q, \theta)$.

- Can use these equations to express any endogenous variable in terms of parameters (s, p, w, z, M, η).

- Once we know these variables, and in particular f, we can figure out how unemployment behaves according to the model.

- Law of motion for unemployment: $u_{t+1} = (1-f)u_t + s_t(1-u_t)$

- At the steady state, job-finding and unemployment rates constant: $f_t = f_{t+1} = f^*$, $u_t = u_{t+1} = u^*$. Imposing the steady state into law of motion for u:

$$u^* = s/(s+f^*)$$

- Big idea of the model: unemployment is a stock variable that is determined by the flows into unemployment (separations) and the flows out of unemployment (job finding rate).

Solving the Model

- What does this model give us? A search and matching model allows us to think about why labor markets are characterized by involuntary unemployment: takes time for workers to find jobs, posted vacancies to find suitable applicants.

- Useful insight: unemployment as a stock, determined by "ins" and "outs": job finding and separation rates. If we can understand those, we can understand unemployment fluctuations.

- In the real world, unemployment fluctuations driven largely by variation in job-finding rates, not separations (partially why we made s exogenous).

- Not so informative for wage determination: in our model, wages totally exogenous. More generally, macroeconomists typically assume a "bargaining rule" that makes wages endogeneous, but most frequently a trivial function of productivity and the value of leisure, so still not very satisfying for wages.

Bonus Slides

Bonus Slides: Shimer Puzzle

- Does the search and matching model do a good job fitting business cycle data?

- Shimer (2005, link): under the usual approach to make wages endogenous, the answer is no.

- Given plausible calibration of parameters in a very similar (though slightly richer) search and matching model, wages are "too flexible" - wages adjust in response to productivity too much, yielding too little adjustment in terms of unemployment.

- Should remind you of what we found sticky prices did to the New Keynesian model – sticky prices yield more volatile output. Likewise, in a search and matching model, stickier wages

- Although it is easy to add in a bit of wage stickiness, it turns out to be a lot harder to convince macroeconomists about the underlying mechanisms that generate wage stickiness.

Bonus Slides: Modern Search

- Many variants of search models exist - though the one we went through, sometimes called the Diamond-Mortensen-Pissarides (DMP) model - is the most popular.

- Much recent research concerns extensions of the baseline model to account for things like on-the-job search, worker heterogeneity/human capital.

- Very different setups that nonetheless count as search models exist - one important one is called the wage posting model, where firms make a decision to choose a wage themselves – high wages attract more qualified workers and/or are filled more quickly, but cost more. Canonical reference is Burdett and Mortensen (1998).

- If you're curious, this paper is a wonderful (though a bit dated) survey of search models in and outside of macroeconomics. It's totally readable.