Harvard Econometrics Math Camp
Worksheet #3

I really want to convince you that the law of iterated expectations is important. It shows up everywhere! Please work with your classmates to prove the following theorem. You will need the law of iterated expectations.

The Law of Total Variance: Let X and Y be random variables. The variance of Y can be written as:

$$Var[Y] = Var[E[Y|X]] + E[Var[Y|X]]$$

Please prove this result. There are multiple paths to the right answer, though I believe all direct, constructive proofs will require using the law of iterated expectations. You may find it helpful to begin with the definition of a conditional variance.