Harvard Econometrics Math Camp
Worksheet #1

These review problems are primarily concerned with deriving some useful results from the setup we discussed during the first hour or so of lecture. It may be helpful to refer to the econometrics math camp slides (or the much beefier econometrics math camp notes) for details on the properties of σ-algebras, measures, etcetera.

I had two objectives in mind when writing these problems for you. First, I want to demonstrate how the usual "laws of probability" you know and love are almost immediate given mathematical structure for characterizing probability spaces that we developed this morning. Second, I want you to meet some more folks in your cohort and work through some problems with them. In my (unsolicited) view, this second objective is way more important than the first.

1. Let $(\Omega, \mathcal{A}, P)$ be a probability space. Please prove the following properties:
   (a) For any $A \in \mathcal{A}$, $P(A^c) = 1 - P(A)$
   (b) $P(\Omega) = 1$
   (c) For all $A \in \mathcal{A}$, $0 \leq P(A) \leq P(\Omega)$
   (d) If $A_1, A_2 \in \mathcal{A}$ with $A_1 \subseteq A_2$, then $P(A_1) \leq P(A_2)$

2. Let $(\Omega, \mathcal{A}, P)$ be a probability space. Consider K disjoint events C_k that partition the sample space Ω, and let $A \in \mathcal{A}$ be an event. Please prove the law of total probability, which states that $P(A) = \sum_{i=1}^{K} P(A|C_i)P(C_i)$.

3. Let $(\Omega, \mathcal{A}, P)$ be a probability space, and consider two events $A, B \in \mathcal{A}$. Recall that Bayes' rule can be stated as $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$. Please derive this rule.