Elite quality

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2. Basic model:

The model has three agents: households, colleges and firms. Households invest in education in order to keep or improve their elite status. They do not necessarily want productive skills, they focus in acquiring skills that allow their children to remain or enter the elite. The solution of the households' problem yields the demand for education. The demand for education is a demand for elite skills. Colleges are not necessarily in the business of education, they aim at supplying skills to produce members of the elite. The solution of Colleges' problem yields the supply of education. Therefore, in our model, an equilibrium in the education market is elite formation. Firms are non-competitive and managed in accordance with stockholders preferences. Stockholders are elite members, who have preferences to hire managers from its own pool. Firms maximize elite interests rather than profits.

Households:

Consider an individual born at time t. Individuals live for two periods. He consumes C_{1t} in period t and C_{2t+1} in period t+1, and derives utility:

$$U(C_{1t}) + \beta U(C_{2t+1}),\ U'(.) > 0, U''(.) < 0 \ (1)$$

Where $\beta = \frac{1}{1+\rho}, \rho > 0$, is the discount rate.

In the first period of their lives, individuals allocate family wealth αY_t, where $\alpha \in (0,1]$, to consumption C_{1t} and to invest in their education E_t . Wealth is a simple way to define elite membership, the higher its wealth [high α], the more likely the family is part of the elite. The individual inherits natural abilities that combined with their social status is summarized by the parameter $\theta \in [\underline{\theta}, \overline{\theta}]$, which can be thought as elite status when θ is greater than an exogenous threshold. The first period budget constraint is:

$$C_{1t} + \frac{E_t}{\theta} = \alpha Y_t \ (2)$$

In the final period of their lives, individuals consume C_{2t+1} with the income earned based on their education $(1 + r_{t+1})E_t$, where r_{t+1} is the stock market return . This yields the following budget constraint:

$$C_{2t+1} = (1 + r_{t+1})E_t \ (3)$$

The individual maximizes Eq.(1) subject to Eqs. (2) and (3). The first order condition is the following Euler equation:

$$U_{C_{1t}}\left(\alpha Y_t - \frac{E_t}{\theta}\right) = \theta\beta(1 + r_{t+1})U'((1 + r_{t+1})E_t) \ \ (4)$$

Equation (4) yields the demand for education. We rewrite it as:

$$E_t = F(\alpha Y_t, \theta, r_{t+1}) \ \ (5)$$

Note that the demand for education depends on family wealth αY_t, elite status θ and stock market return r_{t+1}. It is important to stress that the demand for education has nothing to do with the ordinary definition of human capital (e.g., Ben-Porath, 1967).

The impact of each term on the demand for education is as follows:

$$\frac{dE_t}{d(\alpha Y_t)} = F_1 = \frac{U"(C_{1t})}{\Delta} > 0 \ \ (5A)$$

$$\frac{dE_t}{d\theta} = F_2 = \frac{\theta^{-2}E_tU"(C_{1t}) - \beta(1+r_{t+1})U\prime(C_{2t+1})}{\Delta} > 0 \ \ (5B)$$

$$\frac{dE_t}{dr_{t+1}} = F_3 = -\theta\beta\,\frac{[U"(C_{2t+1})E_t + U\prime(C_{2\,t+1})]}{\Delta} \gtrless 0 \ \ (5C)$$

Where

$$\Delta = \frac{U"(C_{1t})}{\theta} + \ \theta\beta U"(C_{2t+1})(1 + r_{t+1}) < 0 \ (5D)$$

Given the assumption of the concavity of the utility function, the demand for education is an increasing function of family wealth αY_t and elite status θ. The effect of an increase of the stock market return r_{t+1} is ambiguous, however.

From Eqs. (4) or (5) we can notice that the demand for education is dynamic, i.e., it involves periods t and t+1. The way to treat this dynamic equation is as follows; we present and solve the problems of Colleges and firms. The solution of the Colleges' problem yield the supply of education and the solution of firms' problem yield the stock market return. We substitute these solutions into Eq. (5), which summarizes the whole model, and then we can solve the model by studying the dynamics and steady-state equilibria.

Colleges

The business of Colleges is to provide skills to elite formation. Elite colleges set their tuition fees in accordance with their reputation R. The higher their reputation, the higher the tuition fee so as that: $R_t = \tau\alpha Y_t, \tau \in (0,1)$. A high tuition fee [high τ], selects members of the elite based on their wealth.

Colleges spend resources providing education. These costs are increasing and convex, $C(E_t), C' > 0$. Therefore, Colleges net income ['profits'] is,

$$\varphi_t = R_t - C(E_t) = \tau\alpha Y_t - \frac{\delta}{2}{E_t}^2 \quad (6)$$

Using budget (2) into Eq. (6) and maximizing with respect to E_t yields the supply of education:

$$E_t = \frac{\tau\theta}{\delta} \quad (7)$$

Note that the supply of education increases with the tuition fee and elite status and decreases with the marginal cost of education. Here in this simple set up, education has nothing to do with human capital formation. The next section will expand the College problem to consider the case in which they supply productive skills.

We can rewrite Eq.(7) to express the parameter θ as a function of E_t

$$\theta = \frac{\delta E_t}{\tau} \quad (8)$$

As mentioned before, in order to deal with the dynamic structure of the model, we will substitute Eq. (8) into Eq. (5).

Firms

Stock market firms are few and have some degree of market control, i.e., they are not competitive. This leaves some room for incompetence and mismanagement. On top of that we assume that Stockholders are members of the elite, which allocate greater part of their wealth in stocks. Firms aim at satisfying stockholders preferences given by the parameterσ. These preferences may not necessarily be conducive to profit maximization.

Firms hire college-educated personnel for managerial positions. A representative firm's profits is

$$\pi_t = \aleph(K_t, k_t) - rk_t - \sigma E_t k_t \quad (9)$$

Where $\aleph(K_t, k_t)$ is the part of the real profits that is decreasing in the industry-wide capital stock, K (which reflects competition), and is increasing in firm's own capital stock, k. Notice that the term that reflects stockholder preferences for a specific style of management based on elite status, $\sigma E_t k_t$. This term enters negatively because managers have no productive skills, therefore the term reduces total profits. The next section will expand the firm's problem to consider the case in which managers have productive skills.

Profit maximization with respect to k yields

$$r_t = \aleph_{k_t}(K_t, k_t) - \sigma E_t \quad (10)$$

The rate of return of capital r_t (a proxy for the stock market return) depends on how much stockholders preferences for elite education. Rewriting Eq.(10) as

$$\sigma E_t = \aleph_{k_t}(K_t, k_t) - r_t \quad (10')$$

One can see that stockholders preferences for elite education depends on the difference between the marginal productivity of capital and its marginal cost.

As mentioned before, in order to deal with the dynamic structure of the model, we will substitute Eq. (10) into Eq. (5).

Dynamics and Steady State Equilibrium

In order to find the model's equilibrium, substitute Eqs. (8) and (10) into Eq. (5):

$$E_t = F\left(\alpha Y_t, \frac{\delta}{\tau} E_t, \aleph_{k_t}(K_{t+1}, k_{t+1}) - \sigma E_{t+1}\right) \quad (11)$$

Equation (11) implies a relationship between E_t, E_{t+1} . It is a first difference equation that describes the dynamic behavior of education. It yields an education locus.

The properties of the education locus depend on the derivative:

$$\frac{dE_{t+1}}{dE_t} = \frac{\left(1-F_2\frac{\delta}{\tau}\right)}{F_3(-\sigma)} = \frac{\left(1-\left(\frac{\delta}{\tau}\right)\frac{\theta^{-2}E_t U''(C_{1t})-\beta(1+r_{t+1})U'(C_{2t+1})}{\Delta}\right)}{\left[\sigma\theta\beta\frac{[U''(C_{2t+1})E_t+U'(C_{2\,t+1})]}{\Delta}\right]} \gtrless 0 \quad (12)$$

Figure 1. Equilibria

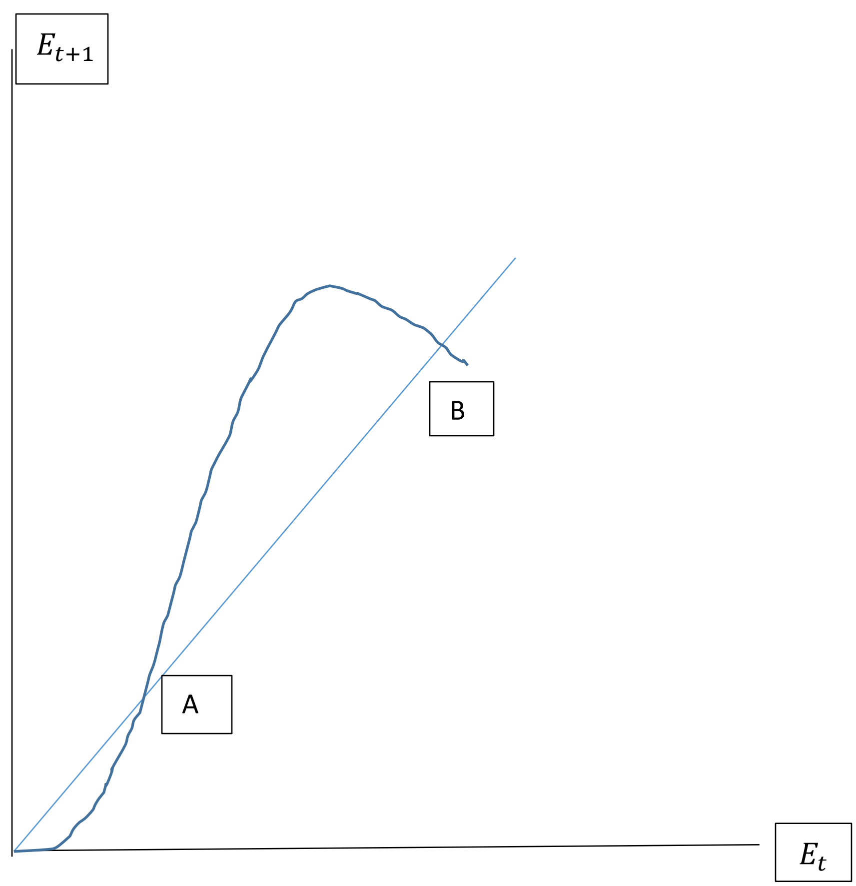

Figure 1 depicts Eq.(12) in a graph in the space E_t, E_{t+1} with a 45 degree line that summarizes the dynamic and the steady state (where $E_t = E_{t+1} = \mathrm{E}*$) behavior of the model. There are two steady-states A and B. Equilibrium A with low level of education is unstable. Equilibrium B with high level of education is stable. Any changes in the parameters in Eq. (12) move the locus and therefore the equilibria.

Given the steady state equilibrium of E^*, we substitute it into Eq.(10) to obtain the equilibrium stock market return, and into Eq. (8) to obtain the equilibrium status level, i.e., elite quality.

Solving the model algebraically for the steady-state yields:

$$E_{1,2}^* = \frac{1+\aleph_{k_t}\binom{+}{-}\sqrt{\left(1+\aleph_{k_t}\right)^2+\frac{4\sigma}{\beta^2\left(\frac{\delta}{\tau}\right)\left[1-\left(\frac{\delta}{\tau}\right)\alpha Y\right]}}}{2\sigma} \quad \text{(N.1)}$$

A numerical example helps to clarify the workings of the model allowing us to do the comparative statics analysis of the steady-state equilibrium.

Assume the following parameter values: $\alpha = 0.4; \beta = 0.94;\ \delta = 0.5; \sigma = 0.5;\ \tau = 0.3;\ \aleph_k = 0.1; Y = 10$. With these parameters, we have the equilibria

$$A = E_1^* = 0.13 \text{ , and}$$

$$B = E_2^* = 2.7 \text{ (N.2)}$$

As equilibrium B is the only stable equilibrium it yields equilibrium status and equilibrium stock market return as, respectively:

$$\theta^* = \frac{\delta}{\tau} E_2^* = 4.3 \quad \text{(N.3)}$$

$$r^* = \aleph_{k_t} - \sigma E_2^* = -1.25 \quad \text{(N.4)}$$

The comparative statics analysis of the steady-state solution (N.1) yield the following multipliers for the stable and higher equilibrium level of education E_2^*:

$$\frac{dE_2^*}{d(\alpha Y)} > 0; \frac{dE_2^*}{d\aleph_{k_t}} > 0; \frac{dE_2^*}{d\delta} > 0;\ \frac{dE_2^*}{d\beta} < 0; \frac{dE_2^*}{d\sigma} < 0; \frac{dE_2^*}{d\tau} < 0 \text{ (N.5)}$$

According to (N.5) equilibrium education E_2^* grows with wealth αY, the marginal productivity of capital $\aleph_{k_t}$ and college costs to provide education δ. The parameters that reduce E_2^* are: the discount factor β, stockholder preferences for a specific style of management based on elite status σ, and tuition fee τ.

As per (N.3) equilibrium status grows with equilibrium education E_2^* , consequently we have the following multipliers:

$$\frac{d\theta^*}{d(\alpha Y)} > 0; \frac{d\theta^*}{d\aleph_{k_t}} > 0; \frac{d\theta^*}{d\delta} > 0;\ \frac{d\theta^*}{d\beta} < 0; \frac{d\theta^*}{d\sigma} < 0; \frac{d\theta^*}{d\tau} < 0 \text{ (N.6)}$$

According to (N.4) equilibrium stock market return decreases with equilibrium education E_2^*. Therefore, we have the following multipliers [the only one that is ambiguous is the marginal productivity of capital $\aleph_{k_t}$]:

$$\frac{dr^*}{d(\alpha Y)} < 0; \frac{dr^*}{d\aleph_{k_t}} \gtrless 0; \frac{dr^*}{d\delta} < 0;\ \frac{dr^*}{d\beta} > 0; \frac{dr^*}{d\sigma} > 0; \frac{dr^*}{d\tau} > 0 \text{ (N.7)}$$

3. Model Extensions

1) Including education in the utility function

In this set up we rewrite the utility function as:

$$U(E_t, C_{1t}) + \beta U(C_{2t+1}),\ \ U_{E_t} > 0, U_{E_tE_t} < 0, U_{C_{1t}} > 0, U_{C_{1t}C_{1t}} < 0, U'(C_{2t+1}) > 0, U''(C_{2t+1}) < 0 \quad (13)$$

The first order condition is

$$U_{C_{1t}}\left(E_t, \alpha Y_t - \frac{E_t}{\theta}\right) = \theta\left[U_{E_t}\left(E_t, \alpha Y_t - \frac{E_t}{\theta}\right) + \beta(1 + r_{t+1})U_{C_{2t+1}}((1 + r_{t+1})E_t)\right] \quad (14)$$

The education locus changes to:

$$\frac{dE_{t+1}}{dE_t} = \frac{\left(1 - F_2\frac{\delta}{\tau}\right)}{F_3(-\sigma)} = \frac{\left(1 - \left(\frac{\delta}{\tau}\right)\frac{\left[U_{E_t}\left(E_t, \alpha Y_t - \frac{E_t}{\theta}\right) + \beta(1+r_{t+1})U_{C_{2t+1}}((1+r_{t+1})E_t)\right] - \theta^{-2}E_t U_{C_{1t}C_{1t}}(E_t, C_{1t}) + \theta^{-1}E_{t+1}U_{E_tC_{1t}}(E_t, C_{1t})}{\Delta'}\right)}{\left[\sigma\left(\frac{\theta}{1-\theta}\right)\beta\frac{[U''(C_{2t+1})E_t + U'(C_{2\,t+1})]}{\Delta'}\right]} \gtrless 0$$

(15)

Where $\Delta' = 2U_{E_tC_{1t}}(E_t, C_{1t}) - \theta^{-1}U_{C_{1t}C_{1t}}(E_t, C_{1t}) - \beta(1 + r_{t+1})U''(C_{2t})$

2) Including valuable skills in college's supply of education

In this set up, Colleges offer some useful education in the form of productive skills, ϑE_t

$$\varphi_t = \tau\alpha Y_t + \vartheta E_t - \frac{\delta}{2}{E_t}^2 \quad (16)$$

Therefore, the supply of education becomes

$$E_t = \frac{\tau\theta + \vartheta}{\delta} \quad (17)$$

And we can express the parameter θ as

$$\theta = \frac{\delta E_t - \vartheta}{\tau} \quad (18)$$

Substituting Eq. (18) into Eq. (11) yields

$$E_t = F\left(\alpha Y_t, \frac{\delta E_t - \vartheta}{\tau}, \aleph_{k_t}(K_{t+1}, k_{t+1}) - \sigma E_{t+1}\right) \quad (19)$$

Curiously, it is important to stress that in spite of this change, the education locus remains the same as in Eq. (12):

$$\frac{dE_{t+1}}{dE_t} = \frac{\left(1 - F_2 \frac{\delta}{\tau}\right)}{F_3(-\sigma)} = \frac{\left(1 - \left(\frac{\delta}{\tau}\right)\frac{\theta^{-2} E_t U''(C_{1t}) - \beta(1 + r_{t+1})U'(C_{2t+1})}{\Delta}\right)}{\left[\sigma\theta\beta \frac{[U''(C_{2t+1})E_t + U'(C_{2\,t+1})]}{\Delta}\right]} \overset{\geq}{<} 0 \quad (12)$$

3) Including productive education in firms' production function

In this set up Firms' benefit from productive education through the term εE_t as

$$\pi_t = (1 + \varepsilon E_t)\aleph(K_t, k_t) - r k_t - \sigma E_t k_t \quad (20)$$

Profit maximization with respect to k yields

$$r_t = (1 + \varepsilon E_t)\aleph_{k_t}(K_t, k_t) - \sigma E_t \quad (21)$$

Substituting Eq. (21) into Eq. (11) yields

$$E_t = F\left(\alpha Y_t, \frac{\delta}{\tau} E_t, (1 + \varepsilon E_t)\aleph_{k_t}(K_t, k_t) - \sigma E_{t+1}\right) \quad (22)$$

Consequently, the education locus changes to

$$\frac{dE_{t+1}}{dE_t} = \frac{\left(1 - F_2 \frac{\delta}{\tau}\right)}{F_3\left(\varepsilon \aleph_{k_t} - \sigma\right)} = \frac{\left(1 - \left(\frac{\delta}{\tau}\right)\frac{\theta^{-2} E_t U''(C_{1t}) - \beta(1 + r_{t+1}) U'(C_{2t+1})}{\Delta}\right)}{\left[-\theta\beta\left(\varepsilon \aleph_{k_t} - \sigma\right)\frac{[U''(C_{2t+1}) E_t + U'(C_{2\,t+1})]}{\Delta}\right]} \underset{<}{\geq} 0 \quad (23)$$