Numerical Example and Steady-State (S-S) (1)

$$u(c_{1t}) = c_{1t}^{1/2}, \quad u(c_{2t+1}) = c_{2t+2}^{1/2}$$

assume S-S; considering Eq. (4):

$$\left(\alpha Y - \frac{E}{\theta}\right)^{-1/2} = \theta\beta(1+r)\left[(1+r)E\right]^{-1/2}$$

$$\left((1+r)E\right)^{1/2} = \theta\beta(1+r)\left(\alpha Y - \frac{E}{\theta}\right)^{1/2}$$

$$(1+r)E = \left[\theta\beta(1+r)\left(\alpha Y - \frac{E}{\theta}\right)^{1/2}\right]^2$$

$$(1+r)E = \left[\theta\beta(1+r)\right]^2\left(\alpha Y - \frac{E}{\theta}\right)$$

$$E\left[(1+r) + \theta\left[\beta(1+r)\right]^2\right] = \left[\theta\beta(1+r)\right]^2 \alpha Y$$

$$E = \frac{(\theta\beta)^2(1+r)^2\alpha Y}{(1+r)\left[1+\theta\beta^2(1+r)\right]}$$

$$E = \frac{(1+r)(\theta\beta)^2\alpha Y}{\left[(1+r)\theta\beta^2+1\right]}$$ Subst. (8) and (10)

$$E = \frac{(1+\chi_F - \sigma E)\beta^2\left(\frac{\delta}{2}E\right)^2\alpha Y}{\left[(1+\chi_K - \sigma E)\beta^2\left(\frac{\delta}{2}E\right)+1\right]}$$

$$(1+\chi_k-\sigma E)\,\beta^2\frac{\delta}{\tau}E^2+E-(1+\chi_k-\sigma E)\,\beta^2\left(\frac{\delta}{\tau}\right)^2E^2\alpha Y=0$$

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$$(1+\chi_k-\sigma E)\,\beta^2\frac{\delta}{\tau}E^2\left[1-\frac{\delta}{\tau}\alpha Y\right]=-E$$

$$E(1+\chi_k-\sigma E)\,\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)+1=0$$

$$-E^2\sigma\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)+E(1+\chi_k)\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)+1=0$$

$\times\dfrac{(-1)}{\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)}$ Yields $E^2\sigma-E(1+\chi_k)-\dfrac{1}{\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)}=0$

$$E_{1,2}=\frac{1+\chi_k\pm\sqrt{(1+\chi_k)^2+\dfrac{4\sigma}{\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)}}}{2\sigma}$$

Assume the following parameter values (calculations next page)

$\alpha=0.4$; $\beta=0.94$; $\sigma=0.5$; $\tau=0.3$; $\delta=0.5$; $\chi_k=0.1$; $Y=10$

Note that the sufficient condition for positive solutions E_1 and E_2 is:

$$\frac{4\sigma}{\beta^2\frac{\delta}{\tau}\left(1-\frac{\delta}{\tau}\alpha Y\right)}<0$$

$$\alpha Y = (0.4)\,10 = 4$$

$$\frac{\delta}{\tau} = \frac{0.5}{0.3} = 1.6$$

$$\frac{\delta}{\tau}\alpha Y = (1.6)\,4 = 6.4$$

$$1 - \frac{\delta}{\tau}\alpha Y = 1 - 6.4 = -5.4$$

$$\beta^2 (1.6)(-5.4) = \underbrace{0.88}_{1.4}(1.6)(-5.4) = -7.6$$

$$4\delta = 4(0.5) = 2$$

$$\frac{2}{-7.6} = -0.26$$

$$1.21 - 0.26 = 0.95$$

$$\sqrt{0.95} = 0.97$$

$$E_{1,2} = \frac{1 + \mathcal{X}_K \pm 0.97}{1} = 1.1 \pm 0.97 \Rightarrow \begin{array}{l} E_2 = 2.07 \\ E_1 = 0.13 \end{array}$$