Physical Quantities, Standards and Units

Anything that can be quantified be that through a direct measurement using a physical apparatus or by calculations using other directly measured quantities is defined as a Physical Quantity. In a sense the systems used for measuring and quantifying them are very basic building blocks of Physics. All laws and theories of Physics are expressed in terms Physical quantities and their quantification. The measure of a physical quantity can be a pure number (or a ratio), or a number followed by a unit.
Measurement of a quantity would require us to follow two steps – first, we need to define a standard for that quantity, and then we need to compare the magnitude of the measured quantity with the defined standard.

The role of a system of units is therefore to provide standardization in the process of measurement. For example if 1Kg represents a standard unit of mass then the mass of a body can be measured and then expressed as numerical coefficient times 1Kg.

But how do we ensure that all instruments measuring a given quantity give the same result? (In fact, we will shortly see that this may not strictly be the case, since there may be errors associated with any measurement. But for the time being we will assume that we are talking of perfect instruments giving exact results.) To ensure standardization and error free measurements all instruments must be "calibrated" using the same standard. A calibrated instrument is supposed to give the "correct" reading when measuring a physical quantity. The current system of standards followed worldwide is the SI system (or Standard International system).

Fundamental (Base) and Derived Quantities

A fairly large number of physical quantities are used in the study of Physics. However, only seven of these are independent of other quantities in the sense that they are not defined in terms of other physical quantities. These seven quantities are called fundamental or base quantities, and their units are called fundamental units. Examples of fundamental quantities (units) are mass (kg), length (m) and time (s), and all seven are given in the table shown.

All other quantities, called derived, can be expressed as some combination (products and quotients) of the base quantities. Examples of derived quantities (units) are volume (m^3), velocity (m/s or ms^{-1}), pressure (N/m^2 or Nm^{-2}), energy (J), etc.

Following are the seven Fundamental physical quantities along with their units in the Standard International (S.I.) system.

	Basic physical quantity	**Name of SI unit**	**Symbol of SI unit**
1	Length	Metre	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electric current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd
7	Amount of substance	Mole	mol

Luminous intensity (or flux) is the amount of light falling at a point per unit area (surrounding the point) per unit time. Multiples or fractions of units are expressed by using certain prefixes before the names of the unit. The various prefixes used for this purpose are given in the following table.

Prefix	**Symbol**	**Multiple**
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tetra	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	K	10^{3}
Hecto	H	10^{2}
Deca	da	10
Deci	D	10^{-1}
Centi	C	10^{-2}
Milli	m	10^{-3}
Micro	µ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Fermi	f	10^{-15}
Atto	a	10^{-18}
Zepto	z	10^{-21}
Yocto	y	10^{-24}

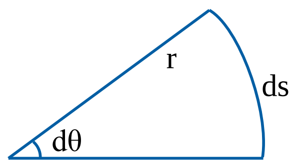
In addition to the seven basic units, there are many derived units. Some of the derived units in practice are given in the table below:

Physical quantity	**Relation with other basic quantities**	**SI units**
Area	Length square	m^2
Volume	Length cube	m^3
Density	Mass per unit volume	$kg\ m^{-3}$
Speed / Velocity	Distance travelled per unit time	ms^{-1}
Acceleration	Speed change per unit time	ms^{-2}
Force	Product of mass and acceleration	$kg\ ms^{-2}$ or (newton, N)
Pressure	Force per unit area	$Kg\ m^{-1}\ s^{-2}$or (pascal, Pa)
Energy	Product of force and distance travelled	Kgm^2s^{-2}or (joule, J)

Dimensions

It has been stated that all derived quantities can be described as products and/or quotients of base (fundamental) quantities. The powers (exponent) of the fundamental quantities occurring in the derived quantity (q) in question are the dimensions of the quantity q.

If $q = [M]^{\alpha}\,[L]^{\beta}\,[T]^{-\gamma}$, dimension of q is α in [M], β in [L] and $-\gamma$ in [T].
(When we say 3-D in space we mean dimension is 3 in length or one can observe the length, the breadth and the depth of the object).
Naturally, dimensions of fundamental quantities are one in themselves and zero in others.
Certain quantities (like angle (θ), trigonometric functions, and ratios of like dimensional quantities) can be dimensionless. This is the reason why radian and steradian are dimensionless supplementary units.



The relationship between physical quantities and various units can be checked or derived by a powerful procedure called dimensional analysis. Dimensional analysis is essentially based on the fact that 'dimensions can be treated as algebraic quantities'. To elaborate:

i. Quantities with same or different dimensions can be multiplied or divided.
ii. Quantities with different dimensions cannot be added or subtracted.
iii. In an equation (formula) the terms on both sides must be of the same dimension (Dimensional consistency).

Applications of Dimensional Analysis

There are two major application of Dimensional Analysis:

(A) Verification of a relation (equation)

Both sides of an equation must have the same dimensions. So if both sides have different dimensions the equation (and the relationship) is wrong.

For example: Is the relation $x = ut + \frac{1}{2}at^2$ correct?

(x = Displacement, u = Initial velocity, a = Acceleration, t = Time)
Dimension of x, t, u and a are $[L]^1$, $[T]^1$, $[L]^1\,[T]^{-1}$ and $[L]^1\,[T]^{-2}$

So dimensionally
$$[L]^1 = \{[L]^1\,[T]^{-1}\}\,[T] + \{[L]^1\,[T]^{-2}\}\,[T]^2$$
Dimension of both the sides of the equation are the same. So the equation is dimensionally correct.

(B) Setting up (or deriving) a relationship between quantities:

When an object moves, it is displaced by a straight line distance x. It starts from rest and has an acceleration a. What is its displacement in time t?

We start by assuming (from common sense, from observation or from physical consideration) thatx must be dependent on a and t.

Thus $x = ka^n t^m$,where k is dimensionless constant and n and m are the dimensions of a and t, respectively.This relation must by dimensionally correct:

$[x]^1 = k\,[a]^n\,[t]^m\;[a]^n = [LT^{-2}]^n = [L]^n\,[T]^{-2n}$

$[x]^1 = [L]^1$ and $t^m = [T]^m\;[x]^1 = [L]^1$ and $t^m = [T]$

So $[L]^1\,[T]^0 = [L]^n\,[T]^{-2n}\,[T]^m$

Equating dimensions of [T], m – 2n = 0 or m = 2n

Equating dimensions of [L], n = 1 and m = 2

(**Note:** The value of dimensionless quantity k has to be found experimentally or from other theoretical considerations. Actually k = ½ in this case).

Limitations of the theory of dimensions

1. If only dimensions are given, the corresponding physical quantity may not be unique. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be energy or torque.

2. Numerical constants having no dimensions such as θ, or 2π etc., cannot be deduced by the method of dimensions.

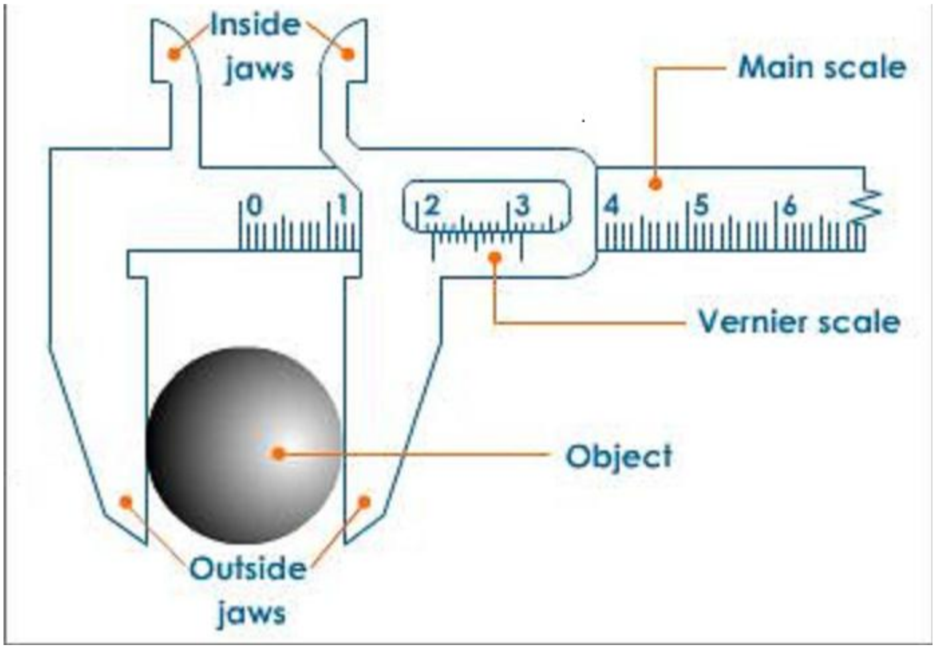
3. The method of dimensions cannot be used to derive relations other than the product of power functions. For example, the relations $s = ut + at^2$ or y = a sin(wt); cannot be derived using this theory.

4. We compare the powers of fundamental quantities to obtain a number of independent equations for finding the unknown powers. But if the physical quantity depends on more parameters than the number of independent equations, then the relation between the physical quantity and the various parameters cannot be found out using dimensional method. But we can still check the correctness of the given equation dimensionally. For example, $T = 2\pi\sqrt{\frac{I}{Mgl}}$ cannot be derived by the theory of dimensions but its dimensional correctness can be checked.

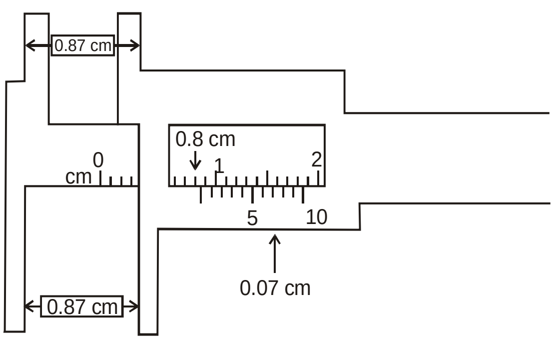
5. Even if a physical quantity depends on three physical quantities, out of which two have dimensions, the formula cannot be derived by theory of dimensions. For example, the formula $f = \frac{uv}{u-v}$, cannot be derived from the theory of dimensions, but can be checked.

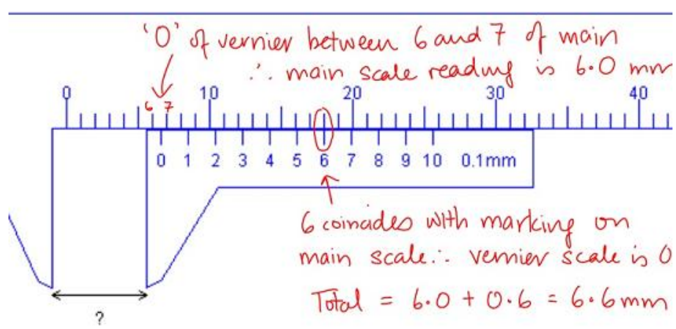
Measuring Length using Vernier Calipers

The common metre scale used for measuring length suffers from the limitation that the value of the smallest division is 1 mm or 0.1 cm. This is called the least count of the metre scale. So, if the length of an object lies between 2.3 cm and 2.4 cm, using the metre scale we cannot tell if it is 2.34 or 2.38 cm.

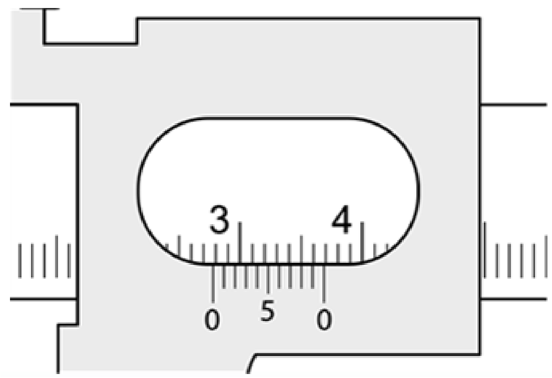


The Vernier caliper is a common instrument used in laboratories and factories for measuring length often to the precision of 0.01 cm. It consists of a main scale engraved on a fixed ruler, and a vernier scale engraved on a movable jaw (see figure). The vernier scale is free to slide along the fixed main scale. In the most common caliper, the vernier scale has 10 divisions that cover the same length as 9 divisions on the main scale.





When the jaws of the calipers are closed, the zero mark on the vernier scale must coincide with the zero mark on the main scale. The last or the 10th mark on the vernier scale then coincides with the 9th mark of the main scale (see figure below). This is read as 0.00 cm.



Least Count of the Vernier Scale

To determine how much 1 division on the vernier scale (or the least count) is equal to, we use the following procedure:

a. When the jaws of the vernier calliper are closed, i.e. when the zero of both the scale coincides, note how many divisions on the main scale coincide with all the divisions on the vernier scale. Let's say we have a vernier where m divisions on the main scale (MSD) coincide with n divisions on the vernier scale (VSD).

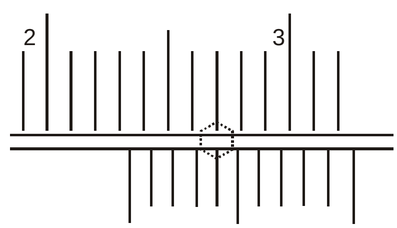
b. We then write n VSD = m MSD
or 1 VSD = (m/n) MSD

c. The least count is then
Least Count = (1 MSD – 1 VSD) least count of Main scale

In our example, as in most standard calipers
10 VSD = 9 MSD
Or 1 VSD = 0.9 MSD
Therefore least count = (1 - 0.9) mm = 0.1 mm

To use the vernier, we fix the object between jaws and then follow the steps given below:

a. Look at the vernier scale and see where its zero falls in relation to the main scale. For example, in the figure below, the zero of the vernier scale falls between 2.3 cm and 2.4 cm on the main scale.

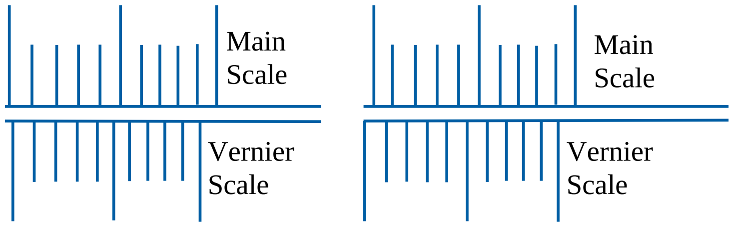


b. Next, note which mark on the vernier scale coincides with a mark (any mark) on the main scale. In our example, we find that the 4^{th} mark coincides with a mark on the main scale.

c. Thus, our reading for the length measured is 2.3 cm + 4 divisions on the vernier scale. Since in our example 1 division on the vernier scale is equal to 0.01 cm, the reading is 2.34 cm.

Zero Error

Sometimes, if the zeroes of the vernier and main scales do not coincide when the jaws of the calipers are closed, the reading showed by the instrument will not be correct. We will then need to factor into the reading the zero error of the instrument. Let's see how we can determine the zero error.



(A)
Positive Zero Error

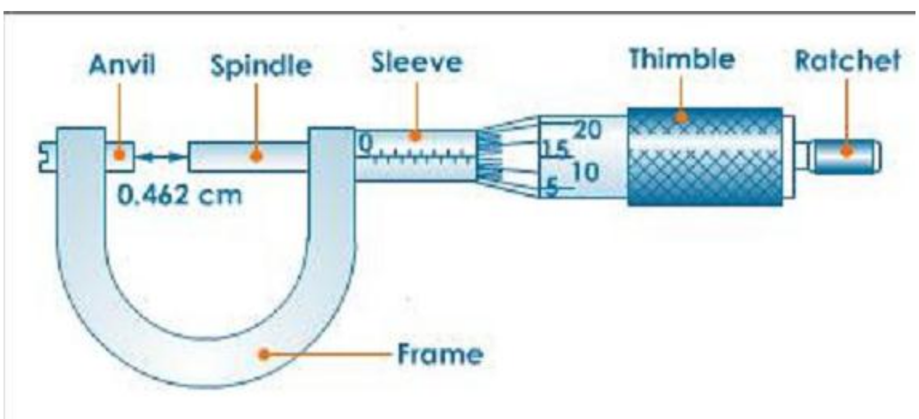
(B)
Negative Zero Error

A vernier caliper has positive zero error when the zero of the vernier scale falls to the right of the zero of the main scale. In the reverse case, i.e., when the zero of the vernier scale falls to the left of the zero of the main scale, the caliper is said to have negative zero error.

To find the zero error, we close the jaws of the calipers and then see which mark on the vernier scale coincides with any mark on the main scale. For example, in the figure (A) above, a case of positive zero error, we find that the 3^{rd} mark on the vernier scale coincides with a mark on the main scale. Since the least count is 0.01 cm, we conclude that the zero error = 0.03 cm. Similarly, in figure (B), the zero error is – 0.06 cm (note the negative sign).

To factor the zero error into the readings, we simply subtract the zero error (with the sign) from the reading. For example, if the reading in case (B) comes out to be 3.67 cm, the corrected reading will be 3.67 – (– 0.06) = 3.73 cm.

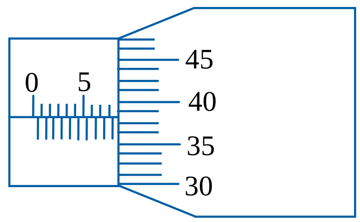
The Micrometer Screw Gauge



The micrometer screw gauge is another common precision instrument for measuring smaller lengths, and has a higher precision than the vernier calipers. Like the vernier caliper, it consists of a main scale engraved on the sleeve, and a circular scale engraved on the rotatory thimble.

The pitch of a micrometer is defined as the length by which the spindle moves forward or backward when the thimble undergoes one revolution. For example, if the pitch is 0.50 mm, then the spindle moves through 1 mm in two complete revolutions of the rotatory thimble.

The common laboratory micrometer has a pitch of 0.50 mm, so that the circular scale has 50 equal divisions, with the least count as 0.01 mm. This means that in one full revolution the thimble covers a linear length of 0.50 mm on the main scale. Normally, the smallest division on the main scale is 1 mm. So, if the edge of the thimble coincides with a mark on the main scale, the thimble takes two revolutions for its edge to coincide with the next mark on the main scale.

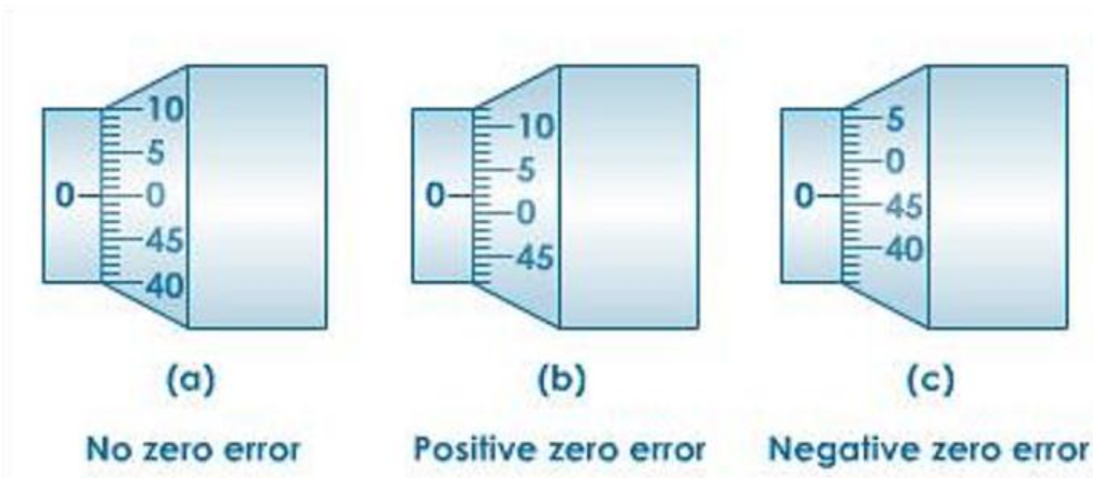


If, for example, we wish to measure the diameter of a metallic wire, we close the jaws of the gauge so that the wire is held securely between them. The first significant figure of the reading is taken from the last mark showing on the sleeve directly to the left of the edge of the thimble. In the figure shown, this is the 7 mm mark. This is therefore the Main Scale Reading (MSR = 7 mm). The remaining two significant figures are taken from the circular scale. In the figure we see that the 38th division on the circular scale coincides with the horizontal line on the main scale. Thus, the circular scale reading (CSR) is

CSR = 38 × Least count of Circular Scale

= 38 × 0.01 mm = 0.38 mm

Hence, the reading shown by the screw gauge = MSR + CSR = 7.38 mm = 0.738 cm



As with the vernier calipers, in this case also we need to check for zero error. Negative zero error would mean that when the jaws of the micrometer are closed, the edge of the thimble is to the left of the zero mark on the main scale. Likewise, if the edge is to the right of the zero mark on the main scale, the instrument has a positive zero error. The zero error value can be read from the circular scale by adjusting the edge of the thimble so that it coincides with the zero on the main scale. The zero error (along with its sign) must be subtracted from the reading to get the correct measure.

Errors in Measurements

The value of a physical quantity can be obtained either by direct measurement or from calculations involving other measured quantities. Errors are an essential part of any measurement, no matter how precise the instrument is. This means that the measured value of a quantity will mostly be different from the "true" value. Error is the difference between the measured and true values.

Of course, it can be argued that since we do not know the true value how will we find the error? The deviation of a measured value from the true value depends on many factors, some of which we can control, while others that we cannot control. While we cannot determine the true value of a quantity, we can reduce the error by various means.

Types of Error

Measurement errors are primarily of two types – systematic errors and random errors. Systematic errors are contributed to by factors that can be controlled to some extent. For example, a faulty or a biased instrument, certain environmental factors, or human errors in recording readings from an instrument, etc. all constitute systematic errors. By its very nature, systematic error will tend to be biased in one direction, mostly. For example, if a vernier caliper has negative zero error, and it is not factored into the readings, then most values measured by it will be less than the true value. In this case, the systematic error will be negative. Systematic errors can be minimized by using an instrument that is more accurate, and by being careful in avoiding human errors and fluctuations in environmental conditions while taking readings.

Even if we remove all systematic errors, repeated readings of the same quantity taken by an instrument may show different values. This constitutes random error. Random error cannot be controlled. It can be reduced by using an instrument with higher precision, meaning an instrument with better resolution or lower least count. By its nature, random error will tend to be evenly distributed on either side of zero. Since random error is probabilistic, the probability of a positive error will be more or less the same as the probability of a negative error. Thus, if we take a large number of readings of the measure of a quantity, and find the mean value, we have every possibility of minimizing the random error. The mean value will be very close to the true value.

Estimating Error

Least Count Error

Every instrument has a least count, and cannot give a measure smaller than it. Hence, every measurement has a least count error. This error depends upon the precision of the instrument. For example, the precision of a micrometer screw gauge, with least count 0.001 cm, is higher than that of a vernier calipers whose least count is 0.01 cm. So, the least count error of the micrometer is lower than that of the vernier.

Least count error is normally expressed as half the least count. A metre scale will have a least count error of 1cm, and a vernier caliper will have a least count error of 0.01cm. Suppose the reading showed by the vernier calipers is 6.23 cm. One way of representing this would be to write it as $6.23 \pm 0.01\, cms$, since the least count is 0.01 cm. This would imply that the true value of the length lies between 6.22 cm and 6.24 cm. A more precise way would be to write the value as $6.23 \pm 0.005\, cms$. This would mean that the true value lies between 6.225 cm and 6.235 cm.

Absolute Error

While performing an experiment, we normally take a number of readings for a quantity. Suppose these values are $a_1, a_2, ...a_n$. Assuming these readings to have only random error, we know that any value will have as much likelihood of exceeding the true value as it will have of being less than the true value. Thus, the mean of the readings will be a fair estimate of the true value.

$$\bar{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The difference between a reading and the mean value is called the absolute error.

$$\Delta a_i = \bar{a} - a_i$$

Since some readings will be less than, and some more than, the mean value, the absolute error may be positive or negative.

We are interested in determining the mean absolute error, which is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

Note that is the absolute value of a quantity is always positive. Thus, the mean absolute error estimated using the formula above is also positive.

The final reading for the quantity a is written as

$$\bar{a} \pm \overline{\Delta a}$$

Relative or Percentage Error

Experimental error is often represented as relative or percentage error which is defined as

Relative error = $\Delta a / a$

Percentage error $= (\Delta a / a) \times 100\,\%$

Example:

Using a vernier calipers, the following readings were taken while measuring the diameter of a cylinder: 2.74 cm, 2.72 cm, 2.78 cm, 2.75 cm, 2.74 cm, 2.73 cm. Find the mean absolute error, and the percentage error.

The mean value = $\bar{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{2.74 + 2.72 + 2.78 + 2.75 + 2.74 + 2.73}{6} = 2.743 = 2.74\,cm$

Note that we have expressed the mean value upto the second decimal, since all readings are recorded to a resolution of 0.01 cm.

The absolute errors are

$\Delta a_1 = \bar{a} - a_1 = 2.74 - 2.74 = 0.00$ cm

$\Delta a_2 = \bar{a} - a_2 = 2.74 - 2.72 = 0.02$ cm

$\Delta a_3 = \bar{a} - a_3 = 2.74 - 2.78 = -0.04$ cm

$\Delta a_4 = \bar{a} - a_4 = 2.74 - 2.75 = -0.01$ cm

$\Delta a_5 = \bar{a} - a_5 = 2.74 - 2.74 = 0.00$ cm

$\Delta a_6 = \bar{a} - a_6 = 2.74 - 2.73 = 0.01$ cm

Therefore,

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n} = \frac{0.00 + 0.02 + 0.04 + 0.01 + 0.00 + 0.01}{6}$$

$$= \frac{0.08}{6} = 0.0133 = 0.01\text{cm}$$

And Percentage error,

$$= (\overline{\Delta a} / \bar{a}) \times 100\% = \frac{0.01}{2.74} \times 100 = 0.36\%$$

Propagation of Error

The values of many physical quantities are calculated from other quantities which are measured in experiments. Since the measured values invariably have some error, the calculated value will also have an error. If we know the error in the measured value, we can determine the error in the calculated value.

a. Error of a sum or difference

Suppose the physical quantity Z is related to quantities A and B as $Z = A + B$. If the errors in A and B are $A \pm \Delta A$ and $B \pm \Delta B$, then

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B) = (A + B) \pm (\Delta A + \Delta B)$

or $\Delta Z = (\Delta A + \Delta B)$

Similarly, if $Z = A - B$, then

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B) = (A - B) \pm (\Delta A + \Delta B)$

or $\Delta Z = (\Delta A + \Delta B)$

We conclude that in either case the absolute error in Z is equal to the sum of the absolute errors in A and B.

b. Error of a product or quotient

$Z = A\,B$

$\therefore \ln(Z) = \ln(A) + \ln(B)$

Differentiating both sides, we get

$$\frac{dZ}{Z} = \frac{dA}{A} + \frac{dB}{B}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Similarly, if $Z = (A/B)$, then

$\ln(Z) = \ln(A) - \ln(B)$

$$\therefore \frac{dZ}{Z} = \frac{dA}{A} - \frac{dB}{B}$$

$$\frac{\pm\Delta Z}{Z} = \frac{\pm\Delta A}{A} - \frac{\pm\Delta B}{B}$$

Or, Maximum relative error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Conclusion: In either case, the relative error in Z is equal to the sum of the relative errors in A and B

Precision and Accuracy

Accuracy

Concentration of Cu in a sample is 25.15 ppm (parts per million). True value is 26 ppm. Absolute error (accuracy) is –0.85ppm. Sign has to be retained while expressing accuracy. Accuracy is the degree of agreement of a measurement with the true (accepted) value.

Precision

Percentages of copper in an alloy are 2.65, 2.62 and 2.64. Percentages of copper determined by another analyst are 2.72, 2.77 and 2.83. Evidently, the precision is more in the first set of values. Precision is expressed without any sign. Precision is the degree of agreement between two or more measurements made on a sample in an identical manner.

In scientific notation, a number is generally expressed in the form, $N \times 10^{n}$,where N is a number between 1.000 … and 9.999 … and n is called power (or exponent) of 10.

Let us see some examples to understand it.

1. 562.43
$= 5.6243 \times 10^{2}$
Here 2 is the power (or exponent) of 10.

2. 0.0056243
$= 5.6243 \times 10^{-3}$
In this case –3 is the power (or exponent) of 10.

3. (0.0056243 + 0.0023412)
$= 5.6243 \times 10^{-3} + 2.3412 \times 10^{-3}$
$= (5.6243 + 2.3412) \times 10^{-3}$
$= 7.9655 \times 10^{-3}$

4. $(5.031 \times 10^{2}) \times (2.687 \times 10^{5})$

$= (5.031 \times 2.687) \times 10^{2+5}$
$= 13.518297 \times 10^{7}$
$= 1.3518297 \times 10^{8}$

5. $\left(1.59195 \times 10^{10}\right) \div \left(1 \times 10^{15}\right)$
$= 1.59195 \times 10^{10-15}$
$= 1.59195 \times 10^{-5}$

Significant Figures

We have already seen that no measured reading is exact. The precision of the reading depends on the least count of the instrument. For example, if a vernier caliper gives a reading of 3.76 cm, it does not mean that the value is 3.760000… cm. So, we say that the reading is reliable or precise upto two decimal places. Any digit after the second place of decimal will be uncertain. Normally, a reading is reported to include all reliable digits plus the first digit that is uncertain. The number of digits thus reported is called significant digits or figures.

Rules to determine significant figures

1. All non-zero digits are significant. For example, 171 cm, 17.1 cm, and 1.71 cm all have three significant figures.
2. Zeroes to the left of the first non-zero digit are not significant. For example, 615, 61.5, 6.15, 0.615 and 0.0615 all have three significant figures.
3. Zeroes between non-zero digits are significant. For example, 5.01 has three significant figures.
4. Zeroes to the right of the decimal point are significant. For example, 6.00 has three significant figures.
5. When there is no decimal point in a number, the last zero may or may not be significant. For example, the number 2200 may have two, three or four significant figures.

Example: Determine the number of significant figures in each of the following numbers.

1. 807.35 **2.** 0.0035 **3.** 625 **4.** 2.358×10^{5} **5.** 0.425

Solution:
1. 807.35 has five significant figures.
2. 0.0035 has two significant figures.
3. 625 has three significant figures.
4. 2.358×10^{5} has four significant figures.
5. 0.425 has three significant figures.

Calculations involving significant figures

To arrive at the correct result of an experiment, one may have to often add, subtract, multiply or divide the values obtained in different measurements. When several values of numbers are combined, the final result cannot be more precise than the least precise value. The following rules should be applied to obtain the correct number of significant figures in any calculation.

Rule 1: Rounding off the results

If the digit coming after the desired number of significant figures is less than five, the last digit retained is left unchanged. If the digit following the last digit to be retained is more than five, the last digit retained is increased by one. If the digit following the last digit to be retained is five, the last digit retained is not changed if it is even, but increased by one if it is odd. For example, to express the results to three significant figures,

6.314 is rounded off to 6.31.

8.216 is rounded off to 8.22.

5.715 is rounded off to 5.72.

5.725 is round off to 5.72.

If the problem involves more than one step, the rounding off must be done only in the final result.

Rule 2: In addition and subtraction, final result should be reported to the same number of decimal places as that of the term with least number of decimal places. For example,

$$\begin{array}{r} 42.200 \\ +\ 2.220 \\ +\ 0.222 \\ \hline \end{array}$$

Sum = 44.642

Since 42.2 has only one digit after decimal place, the correct answer is 44.6

Similar considerations apply to subtraction also. For example,

$$\begin{array}{r} 46.3820 \\ -\ 5.4292 \\ \hline \end{array}$$

Difference = 40.9528

Since 46.382 has only three digits after decimal place, the correct answer is 40.953.

Rule 3: In multiplication and division, the final result should be reported up to the same number of significant figures as are present in the term with the least number of significant figures.

For example,
$31.037 \times 2.23 = 69.21251$
Since 2.23 has only three significant figures, the correct answer is 69.2.
Similarly, $\frac{0.28}{3.8687} = 0.072375733$
Since 0.28 has only two significant figures, the correct answer is 0.072.

Example: The length, breadth and thickness of a rectangular sheet of wood are 2.234 m, 1.560 m and 2.07 cm respectively. Find the area and volume of the sheet to correct significant figures.

Solution: Area = 2.234 m x 1.560 m = 3.48504 m^2
Since the number of significant figures in the length as well as breadth is 4, the area must also have 4 significant figures. Hence, area = 3.485 m^2.
Volume = $2.234 \text{ m} \times 1.560 \text{ m} \times 2.07 \times 10^{-2} \text{ m} = 7.214033 \times 10^{-2} \text{ m}^3$
$= 7.21 \times 10^{-2} \text{ m}^3$

In this case the volume has been reported with 3 significant figures because the thickness has the lowest number of significant figures, i.e. 3.

Example: The escape velocity from the surface of the earth is defined as $v = \sqrt{2gR}$. If the errors in measuring g and R are 6% and 4% respectively, what is the percentage error in v? If the escape velocity for a body calculated using this relation is 11.1854 km/s, to what value should the velocity be rounded off?

Solution: $v = \sqrt{2gR}$ $\quad \therefore \ln v = \frac{1}{2}\ln(2gR) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln g + \frac{1}{2}\ln R$

Differentiating both sides, we get

$$\frac{dv}{v} = \frac{1}{2}\frac{dg}{g} + \frac{1}{2}\frac{dR}{R}$$

$$\Rightarrow \frac{\Delta v}{v} = \frac{1}{2}\left(\frac{\Delta g}{g} + \frac{\Delta R}{R}\right)$$

Therefore, the percentage error in v is

$$\frac{\Delta v}{v}\% = \frac{1}{2}(6+4) = 5\%$$

$$\Delta v = 0.05 \times v = 0.05 \times 11.1854 = 0.559270 \text{ km/s}$$

Therefore, v lies between 10.6261 and 11.7447 km/s. Thus we see that the only reliable digit is the first digit. While rounding off we can consider at most 1 uncertain digit, which in this case will be the second digit. Thus, the escape velocity should be written as 11 km/s.

Example: How many significant figures are there in each of the following?
(a) 0.00043 (b) 0.14600 (c) 4.32×10^{3} (d) -154.090×10^{-27}

Solution: Since the zeroes before the first non-zero digit are not significant, 0.00043 has 2 significant figures.
Zeroes to the right of the decimal and also to the right of a non-zero digit are significant. Thus, 0.14600 has 5 significant figures.
Multiplying a number by any power of 10 does not alter the number of significant figures. Hence 4.30×10^{3} has 3 and has 6 significant figures.

Example: If x = (2.0 ± 0.2) cm and y = (3.0 ± 0.6) cm, find z = x – 2y and its uncertainty.

Solution: $z = x - 2y = 2.0 - 6.0 = -4.0$ cm
$\Delta z = \Delta x + 2\,\Delta y = 0.2 + 1.2 = 1.4$ cm

Example: The radius of a circle is r = (3.0 ± 0.2) cm. Find the circumference and its uncertainty.

Solution: Circumference, $C = 2\pi r = 2 \times 3.14 \times 3.0 = 18.840 = 18.8$ cm
$\Delta C = 2\pi\Delta r = 2 \times 3.14 \times 0.2 = 1.256 = 1.3$ cm
Since the value of r has 1 decimal place, the value of C and its uncertainty have been rounded off to 1 decimal place.

Example: If w = (4.52 ± 0.02) cm, A = (2.0 ± 0.2) cm^2, y = (3.0 ± 0.6) cm, Find $z = \frac{wy^2}{\sqrt{A}}$

Solution: Note that the number of significant figures in w is 3, and that in A and y is 2.

$$z = \frac{wy^2}{\sqrt{A}} = \frac{4.52 \times (3.0)^2}{\sqrt{2.0}} = 28.76\ cm^2 = 29\ cm^2$$

$$\text{Also,} \frac{\Delta z}{z} = \frac{\Delta w}{w} + 2\frac{\Delta y}{y} + 0.5\frac{\Delta A}{A}$$

$$= \frac{0.02}{4.52} + 2 \times \frac{0.6}{3.0} + 0.5 \times \frac{0.2}{2.0} = 0.0044 + 0.4 + 0.0 = 0.4044$$

$\therefore\ \Delta z = 0.4044 \times 28.76 = 11.63 = 12\ cm^2$
Thus, $z = (29 \pm 12)\ cm^2$