1. THE HYDROGEN ATOM

By the end of nineteenth century most of the efforts of Physicists were directed towards the analysis of the discrete spectrum of radiation emitted when electrical discharges were passed in gases. The hydrogen atom being composed of a nucleus (having 1 proton) and one electron has the simplest spectrum of all the elements. It was found that various lines in optical and non optical regions were systematically spaced in various series. Interestingly it turned out that all the wavelengths of atomic hydrogen were given by a single empirical relation, the Rydberg formula.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \text{ where } R = 1.0967758 \times 10^7 \text{ m}^{-1}$$

where $n_1 = 1$ and $n_2 = 2, 3, 4, \ldots$ gives the Lyman series (ultraviolet region)

$n_1 = 2$ and $n_2 = 3, 4, 5 \ldots$ gives the Balmer series (optical/visible region)

$n_1 = 3$ and $n_2 = 4, 5\ 6, \ldots$ gives the Paschen series (infrared region)

$n_1 = 4$ and $n_2 = 5, 6, 7, \ldots$ gives the Bracket series (for infrared region)

and so on for other series lying in farthest infrared.

The Bohr's theory of hydrogen atom: In 1913 Niels Bohr developed a physical theory of atomic hydrogen from which the Rydberg formula could be derived. Bohr's model for atomic hydrogen is based on certain assumptions, which are as follows:

(i) In the hydrogen atom electron revolves around the nucleus in circular orbits.

(ii) Electron revolves only in those orbits around nucleus where the angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$. These orbits are called stationary orbits. This assumption is called Bohr's quantization rule.

(iii) The energy of electron can take only definite values in a given stationary orbit. The electron can jump from one stationary orbit to other. If it jumps from an orbit of higher energy to an orbit of lower energy it emits a photon of radiation. Similarly an electron can take energy from a source and jumps from a lower energy orbit to a higher energy orbit.

In both the cases energy of radiation involved is given by the Einstein-Planck equation.

$$\Delta E = \frac{hc}{\lambda}$$

Energy of a hydrogen atom: Let the mass of electron be m and it is revolving in an orbit of radius r, then using the quantization rule we get

$$mvr = n\frac{h}{2\pi}, \quad \text{where } n \text{ is a positive integer} \qquad \ldots \text{(i)}$$

Also, from the equation of motion of electron we have

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \qquad \ldots \text{(ii)}$$

Though $Z = 1$ for hydrogen, however generally we leave Z in the equation, as the theory is equally applicable to other atoms with all but one of their electrons are removed.

Solving (i) & (ii) we get

$$v = \frac{Ze^2}{2\varepsilon_0 hn} \quad \ldots \text{(iii)}$$

and $$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} \quad \ldots \text{(iv)}$$

So allowed radii are proportional to n^2. Putting $Z = 1$ and $n = 1$ the quantity $\frac{\varepsilon_0 n^2}{\pi m e^2} = 0.53$ Å is called Bohr's radius and is radius of smallest circle allowed to the electron. Using equation (iii) we write kinetic energy of the electron in n^{th} orbit is

$$K = \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2} \quad \ldots \text{(v)}$$

The potential energy of the atom is

$$U = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{4\varepsilon_0^2 h^2 n^2}$$

So total energy of the atom is

$$E = K + U$$

$$E = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2} \quad \ldots \text{(vi)}$$

In deriving the energy of an atom we have considered kinetic energy of electron and potential energy of the electron nucleus pair.

From (vi), the total energy of the atom in the state $n = 1$ is $E_1 = -\frac{mZe^4}{8\varepsilon_0^2 h^2}$

For hydrogen atom $Z = 1$ and we get

$E_1 = -13.6$ eV, this is the energy of electron when it moves in the smallest allowed orbit. It is also evident from equation (vi) that energy of the atom in the n^{th} energy state is proportional to $\frac{1}{n^2}$. So we can write.

$$E_n = -\frac{13.6}{n^2}\ eV$$

So we get energy in the state $n = 2$ is – 3.4 eV. In the state $n = 3$ it is – 1.5 eV etc. The state of an atom with the lowest energy is called its ground state and states with higher energies are called excited states.

Important results for hydrogen like atoms are

$$V = \frac{Ze^2}{2\varepsilon_0 hn} \quad \ldots \textbf{(1)}$$

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} = \frac{0.53 n^2}{Z} \text{ Å} \qquad \ldots (\mathbf{2})$$

$$E = -\frac{m Z^2 e^4}{8\pi\varepsilon_0 h^2 n^2} = -\frac{13.6 Z^2}{n^2} \text{ eV} \qquad \ldots (\mathbf{3})$$

Hydrogen spectra: Now on the basis of Bohr's model of hydrogen atom it is possible to explain the spectra of hydrogen. If an electron jumps from m^{th} orbit to the n^{th} orbit, the energy of the atom changes from E_m to E_n. The extra energy $E_m - E_n$ is emitted as a photon of electromagnetic radiation. The corresponding wavelength λ is given by

$$\frac{1}{\lambda} = \frac{E_m - E_n}{hc} = \frac{m Z^2 e^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \ldots (\mathbf{4})$$

where $R = \dfrac{me^4}{8\varepsilon_0^2 h^3 c}$ is called the Rydberg constant. Putting the values of different constant, the Rydberg constant R comes out to be $1.0973 \times 10^7 \text{ m}^{-1}$.

So on the basis of energy levels involved in the transition we can divide the entire hydrogen spectrum in various series.

Lyman series: When an electron jumps from any of the higher states to the ground state ($n = 1$), the series of spectral lines emitted fall in *uv* region and A is called as Lyman series. The wavelength λ of any line of the series can be given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \qquad n = 2, 3, 4, \ldots$$

Balmer series: When an electron makes a transition from any of the higher states to the state with $n = 2$ (first excited state), the series of spectral lines emitted fall in visible region and is called Balmer series. The wavelength of any of Balmer lines is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad n = 3, 4, 5, \ldots$$

Paschen series: When an electron jumps from any of the higher states to the state with $n = 3$ (2nd excited state), the series of spectral lines emitted fall in near infra-red region and is called Paschen series. The wavelength λ for any of the line of Paschen series is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \qquad n = 4, 5, 6, \ldots$$

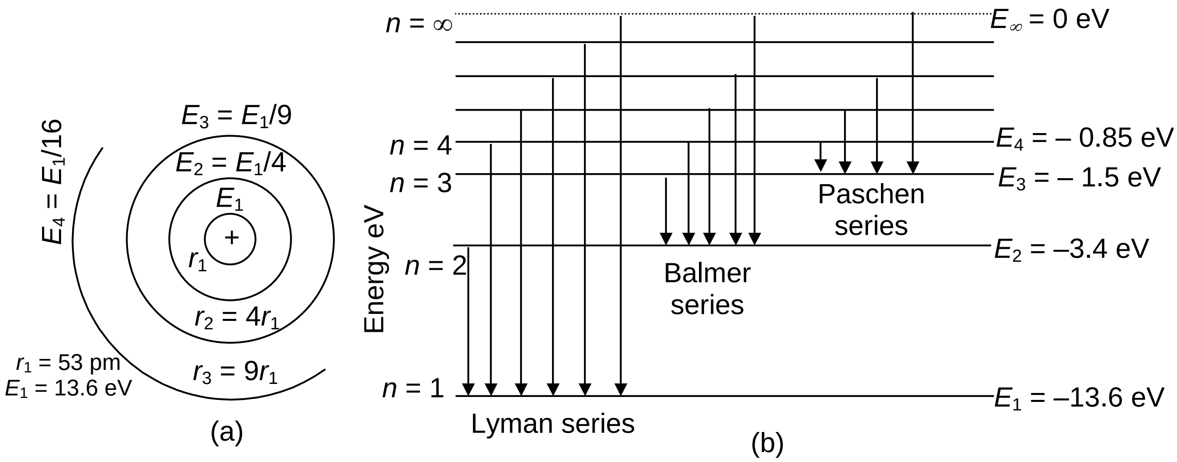
Bracket Series: When an electron jumps from any of the higher states to the state with $n = 4$, (3rd excited state) the series of spectral lines emitted fall in far infrared region and constitute Bracket series. The wavelength λ of any spectral lines of Bracket series is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \qquad n = 4, 5, 6, 7, \ldots$$

Pfund series: Pfund series is constituted by spectral lines emitted when electron jumps from any of the higher energy states to the state with $n = 5$ (4th excited state).

$$\frac{1}{\lambda}=RZ^2\left(\frac{1}{5^2}-\frac{1}{n^2}\right), \qquad n=6,7,8,\ldots$$

Figure below shows schematically the allowed orbits together with the energies of the hydrogen atom. It also shows the allowed energies separately.



Ionisation potential: Negative total energy of hydrogen atom means that hydrogen nucleus and electron constitute a bounded system. Therefore a positive or zero energy of hydrogen atom would mean that electron is not bound to the nucleus i.e., atom is ionised. The minimum energy needed to ionise an atom is called ionisation energy, and the potential difference through which an electron should be accelerated to acquire this energy is called ionisation potential. The ionisation energy of hydrogen atom in ground state is 13.6 eV and ionization potential is 13.6 V.

Binding Energy: Binding energy of a system is defined as energy liberated when its constituents are brought from infinity to form the system. For hydrogen atom binding energy is same as its ionization energy.

Excitation potential: The energy required to take an atom from its ground state to an excited state is called excitation energy of that excited state, and the potential through which the electron should be accelerated to acquire this, is called excitation potential.

Illustration 1.

The longest wavelength in the Lyman series for hydrogen is 1215Å. Calculate the Rydberg constant.

Solution:

$$\frac{1}{\lambda}=R\left(\frac{1}{n_l^2}-\frac{1}{n_u^2}\right)$$

For the Lyman series $n_l = 1$; the longest wavelength will correspond to the value $n_u = 2$.

$$\frac{1}{1215\text{Å}}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right) \quad \text{or} \quad R = \mathbf{1.097\times 10^{-3}\ Å^{-1}}$$

Illustration 2.

How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the n = 5 state?

Solution:

Consider the problem for arbitrary n. If $n_u > n_l$ is any pair of unequal integers in the range 1 to n, it is clear that there is at least one route from state n down to the ground state that includes the transition $n_u \to n_l$. Thus, the number of photons is equal to the number of such pairs, which is

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

For $n = 5$, there are 5(4)/2 = 10 photons.

The above reasoning fails if there is "degeneracy," i.e., if two different pairs of quantum numbers correspond to the same energy difference. In that case the number of distinct photons is smaller than $n(n - 1)/2$.

Illustration 3.

In a transition to a state of excitation energy 10.19 eV, a hydrogen atom emits a 4890Å photon. Determine the binding energy of the initial state.

Solution:

The energy of the emitted photon is

$$h\nu = \frac{hc}{\lambda} = \frac{12.40\times10^3\,\text{eV}.\AA}{4.89\times10^3\,\AA} = 2.54\text{ eV}$$

The excitation energy (E_x) is the energy to excite the atom to a level above the ground state. Therefore, the energy of the level is

$$E_n = E_1 + E_x = -13.6\text{ eV} + 10.19\text{ eV} = -3.41\text{ eV}$$

The photon arises from the transition between energy states such that $E_u - E_l = h\nu$; hence

$$E_u - (-3.41\text{ eV}) = 2.54\text{ eV} \quad \text{or} \quad E_u = -0.87\text{ eV}$$

Therefore, the binding energy of an electron in the state is 0.87 eV.

Note that the transition corresponds to

$$n_u = \sqrt{\frac{E_1}{E_u}} = \sqrt{\frac{13.6\,\text{eV}}{0.87\text{ eV}}} = \mathbf{4} \quad \text{and}$$

$$n_l = \sqrt{\frac{E_1}{E_l}} = \sqrt{\frac{13.6\,\text{eV}}{3.14\text{ eV}}} = \mathbf{2}$$

2. WAVE PARTICLE DUALITY

Nearly two decades after the 1905 discovery of the particle properties of wave, Louis de-Broglie proposed that moving object have wave as well as particle characteristics. de-Broglie ideas soon received respectful attention despite a complete lack of experimental mandate. The existence of de-Broglie waves was experimentally demonstrated by 1927 and the duality principle they represent provided the starting point for successful development of quantum mechanics.

de-Broglie suggested that a moving body behaves in certain ways as though it has a wave nature. A photon of light of frequency ν has a momentum

$$p = \frac{h\nu}{C} = \frac{h}{\lambda} \qquad \ldots (5)$$

The wavelength of a photon is therefore specified by its momentum as

$$\lambda = \frac{h}{p} \qquad \ldots (6)$$

de-Broglie suggested that equation (6) is a completely general one that applied to material particles as well as photons. The momentum of a particle of mass m and velocity v is $p = mv$, and its de-Broglie wavelength is accordingly.

$$\lambda = \frac{h}{mv} \qquad \ldots (7)$$

The greater the particle's momentum; the shorter is its wavelength. In equation (7) m is the relativistic mass, which is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$ where m_0 is the rest mass of the particle.

The wave and particle aspects of moving bodies can never be observed at the same time. So one can not ask which is the correct description. All that can be said is that in certain situations a moving body resembles a wave and in others it resembles a particle. Which set of properties is most conspicuous depends on how its de-Broglie wavelength compares with its dimensions and the dimensions of whatever it interacts with.

Illustration 4.

Find the de Broglie wavelengths of (*a*) a 46-g golf ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s.

Solution:

(a) since $v << c$, we can let $m = m_0$. Hence

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\,\text{Js}}{(0.046\text{ kg})\,(30\text{ m/s})} = 4.8 \times 10^{-34}\text{ m}$$

The wavelength of the golf ball is so small compared with its dimensions that we would not except to find any wave aspects in its behavior.

(b) Again $v << c$, so with $m = m_0 = 9.1 \times 10^{-31}$ kg, we have

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\,\text{J.s}}{(9.1 \times 10^{-31}\text{kg})(10^7\text{m/s})} = 7.3 \times 10^{-11}\text{ m}$$

The dimensions of atoms are comparable with this figure – the radius of the hydrogen atom, for instance is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behavior.

3. PHOTOELECTRIC EFFECT

When light of sufficiently small wavelength is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called photoelectric effect and the electrons ejected are called photoelectrons. An experimental setup was arranged to study photoelectric effect, and the results obtained from the experiment are

(i) When light of sufficiently small wavelength falls a metal surface, the metal emits photoelectrons. This emission of photoelectrons is instantaneous.

(ii) The photoelectric current i.e., the number of photoelectrons emitted per second depends on the intensity of the incident light.

(iii) The maximum kinetic energy with which electrons come out of the metal depends only on frequency of incident light and is independent of intensity of light.

(iv) There is a threshold wavelength for a given metal such that if the wavelength of incident light is greater than the threshold, there will be no emission of photoelectrons.

Einstein's theory of photoelectric effect: Soon after the publication of results of photoelectric effect, efforts were made to explain the result. Wave theory, which considered light as wave, failed on all counts to explain photoelectric effect. The main cause for this failure was inability of wave theory to consider the energy of light quantised and not distributed continuously. In 1900 Planck proposed that radiation from a hot body consists of small packets of energy called 'quanta'. The energy of a quantum is given by $h\nu$ (ν being frequency of radiation). Einstein getting a hint from this proposed that light waves also are consisted of packets of energy or quanta whose energy is also given by '$h\nu$'. He called these quanta as photons.

Einstein postulated that a photon of incident light interacts with a metal electron and transfers its energy to electron in two ways. A part of the energy of the incident photon is used up in liberating the metal electron against the attractive forces of surrounding ions inside the metal; the remaining energy is spent in giving kinetic energy to ejected photoelectrons. If ν be the frequency of incident light W_0 be the minimum energy required to liberate an electron from the surface and E_K be the maximum kinetic energy of the emitted free electrons, then

$$h\nu = W_0 + E_K \qquad \ldots \textbf{(8)}$$

W_0 is called the work function and it obviously depends on the nature of metal. Equation (1) is called Einstein photoelectric equation.

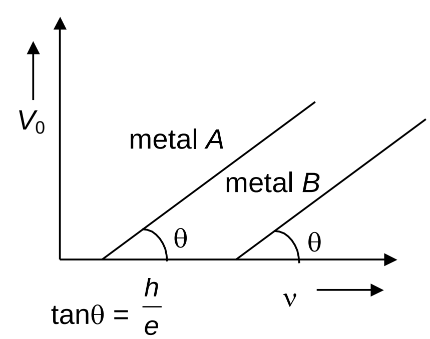
Stopping potential: This is the smallest magnitude of anode potential which just stops the electron with maximum kinetic energy from reaching the anode.

As $K.E_{max.} = h\nu - W_0$

So, if stopping potential for a given photoelectric emission is V_0 then

$$eV_0 = KE_{max} = h\nu - W_0$$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{W_0}{e} \qquad \ldots \textbf{(9)}$$



If we plot a curve between V_0 and ν we can get the value of Planck's constant by measuring slope.

Illustration 5.

The emitter in a photoelectric tube has a threshold wavelength of 6000Å. Determine the wavelength of the light incident on the tube if the stopping potential for this light is 2.5V.

Solution:

The work function is

$$W_0 = h\nu_{th} = \frac{hc}{\lambda_{th}} = \frac{12.4\times10^3 \text{ eV.}\mathring{A}}{6000\mathring{A}} = 2.07 \text{ eV}$$

The photoelectric equation then gives

$$eV_s = h\nu - W_0 = \frac{hc}{\lambda} - W_0 \quad \text{or} \quad 2.5 \text{ eV} = \frac{12.4\times10^3 \text{ eV.Å}}{\lambda} - 2.07 \text{ eV}$$

Solving , λ = **2713 Å.**

Illustration 6.

Prove that the photoelectric effect cannot occur for free electrons.

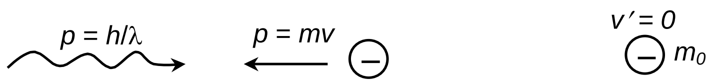
Solution:

Look at the hypothetical process in the centre-of-mass shown in figure, which is defined as that system in which the initial momentum is zero. From conservation of energy,

$$E_{\text{initial}} = E_{\text{final}} \quad \text{or} \quad h\nu + mc^2 = m_0c^2$$

which implies $m_0 > m$. Since this cannot be true, the process cannot occur.

The electrons participating in the photoelectric effect are not free. The heavy matter present takes off momentum but absorbs a negligible amount of energy.

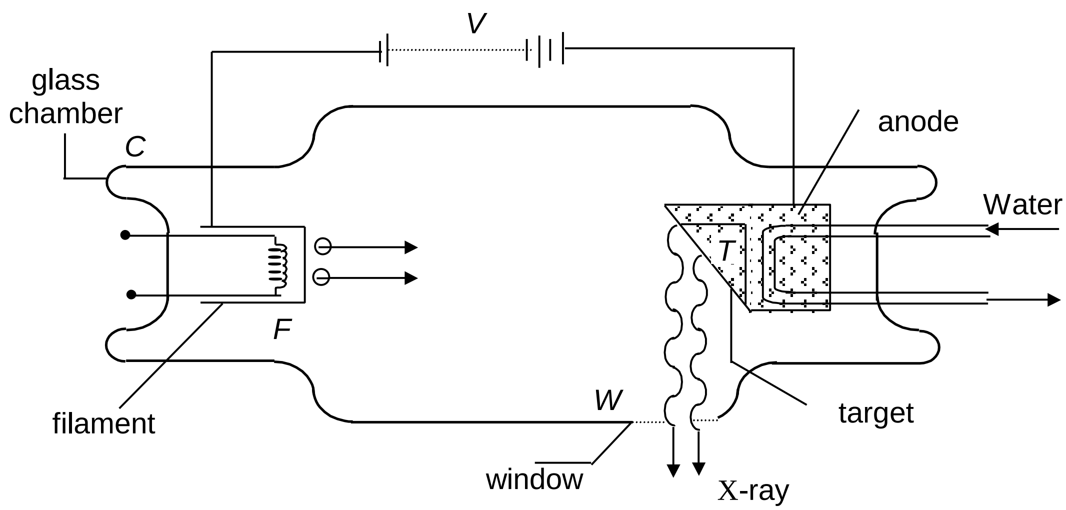


(a) Before absorption **(b) After absorption**

4. X-RAYS

PRODUCTION OF X-RAYS

When highly energetic electrons are made to strike a metal target, electromagnetic radiation comes out. A large part of this radiation has wavelength of the order of 1Å and is called X-rays. The device which is used to produce X-rays is called Coolidge tube as shown below.



A filament F and a metallic target T are fixed in an evacuated glass chamber C. The filament is heated electrically and emits electrons by thermionic emission. A constant potential difference of several kilovolts is maintained between the filament and the target using a DC power supply so that the target is at a higher potential than the filament. The electrons emitted by the filament are, therefore, accelerated by the electric field set up between the filament and the target and hit the target with a very high speed. These electrons are stopped by the target and in the process X-rays are emitted. These X-rays are brought out of the tube through a window W made of thin mica or mylar or some such material which does not absorb X-rays appreciably. In process, large amount of heat is developed, and thus an arrangement is provided to cool down the tube continuously by running water.

4.1 CONTINUOUS AND CHARACTERISTIC X-RAYS

When X-rays coming from a Coolidge tube are investigated for the wavelength present. We find that X-rays can be divided in two categories based on the mechanism of their generation. These X-rays are called continuous and characteristic X-rays. These X-rays have their origin in the manner in which the highly energetic electron loses its kinetic energy. As the fast moving electrons enter metal target, they starts losing their energy by collisions with the atoms of metal target. At each such collision either of the following two processes take place.

(i) Electron loses its kinetic energy and a part of this lost kinetic energy is converted into a photon of electromagnetic radiation and the increases the kinetic energy of the target atoms, which ultimately heats up the target. This electromagnetic radiation is nothing but continuous X-rays. The fraction of kinetic energy converting into energy of photon varies from one collision to the other and the energy of such photon will be maximum when electron converts all its energy into a photon in the first collision itself.

If electron are accelerated through a potential difference V, then maximum energy of emitted photon can be

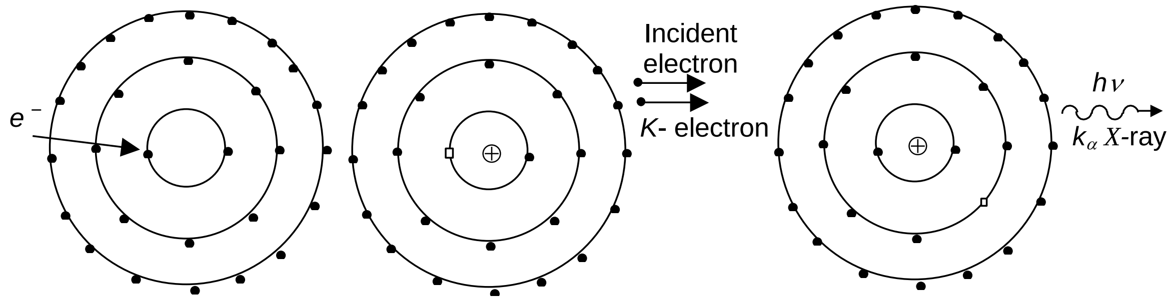
$$E_{max} = eV$$

$$\frac{hc}{\lambda_{min}} = eV$$

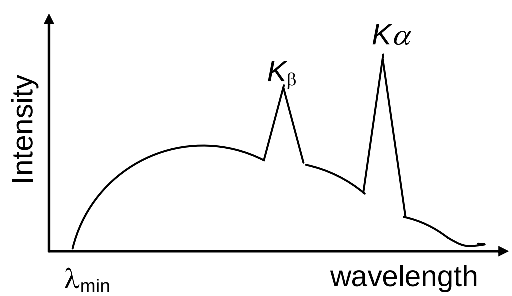
$$\lambda_{min} = \frac{hc}{eV} \qquad \ldots \textbf{(10)}$$

λ_{min} is also called cut off wavelength. Since electron may loose very small energy in a given collision, the upper value of λ will approach to infinity. However both the cases e.g. electron converting all its energy in one go and loosing very small energy will have very small probability. That explains the origin of continuous X-ray. We can also see from the discussion, which we have had on continuous X-rays that λ_{min} depends only on accelerating voltage applied on the electron and not on the material of the target.

(ii) The electron knocks out an inner shell electron of the atom with which it collides. Let us take a hypothetical case of a target atom whose *K*- shell electron has been knocked out as shown.



This will create a vacancy in *K*-shell. Sensing this vacancy an electron from a higher energy state may make a transition to this vacant state. When such a transition takes place the difference of energy is converted into photon of electromagnetic radiation, which is called characteristic X-rays. Now depending upon which shell electron makes a transition to K-shell we may have different *K* X-rays e.g. if electron from *L* shell jumps to *K* shell we have $K\alpha$, if electron from *M* shell jumps to K shell we have K_β X-ray and so on. Similarly if vacancy has been caused in *L* shell we may have L_α, L_β X-ray etc depending upon, whether we have transition of electron from *M* shell or N shell. Since emission of characteristic X-ray involves the inner material energy levels of target atom, hence the wavelength of characteristic X-ray will depend on the target. If we plot curve between intensity of different wavelength component of X-ray coming out of a Coolidge tube, and, wavelength, it is like figure given below



As we can see from the curve, at certain clearly defined wavelengths the intensity of X-rays is very large. These X-rays are known as characteristic X-rays. It is also clear from the curve that $\lambda_{K\beta}$ is greater than $\lambda_{K\alpha}$, however intensity of K_α transition is more as compared to the intensity of K_β transition, it is primarily because transition probability of K_α is more as compared to transition probability of K_β. At other wavelengths intensity varies gradually and these are called continuous X-rays.

Moseley's law: Moseley conducted many experiments on characteristic X-rays, the findings of which played an important role in developing the concept of atomic number. Moseley's observations can be expressed as

$$\sqrt{\nu} = a\,(Z - b) \qquad \ldots \textbf{(11)}$$

where a and b are constants. Z is the atomic number of target atom and ν is the frequency of characteristic X-rays. Moseley's law can be easily understood on the basis of Bohr's atomic model. Let us consider an atom from which an electron from K–shell has been knocked out, an L shell electron which is about to make transition to the vacant site will find the charge of nucleus is screeened by the spherical cloud of remaining one electron in the K shell. If the effect of outer electrons and other L-electrons are neglected then electron making the transition will find a charge $(Z\text{-}1)e$ at the centre. Hence we may expect Bohr's model to give expected result if we replace Z by $(Z\text{-}1)$.

According to Bohr's model, the energy released during a transition from $n = 2$ to $n = 1$ is given by

$$\Delta E = Rhc\,(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$h\nu = Rhc \left(\frac{3}{4}\right) (Z-1)^2$$

$$\nu = \sqrt{\frac{3Rc}{4}}\,(Z-1)$$

Which is same as Moseley's equation with $b = 1$ & $a = \sqrt{\frac{3Rc}{4}}$.

4.2 PROPERTIES OF X-RAYS

(i) X-rays being an electromagnetic wave travel with a speed equal to the speed of light.

(ii) X-rays are not responsive to electric or magnetic field.

(iii) X-rays when pass through gases, produce ionisation.

(iv) X-rays affect photographic plates and exhibit the phenomenon of fluorescence.

Illustration 7.

An experiment measuring the K_α lines for various elements yields the following data:

Fe : 1.94Å *Co* : 1.79Å *Ni* : 1.66Å *Cu* : 1.54Å

Determine the atomic number of each of the elements from these data.

Solution:

The Moseley relation gives

$$v^{1/2} = (4.97 \times 10^7\ \text{Hz}^{1/2})\,(Z-1) \quad \text{or} \quad Z = 1 + \frac{v^{1/2}}{4.97 \times 10^7\ \text{Hz}^{1/2}}$$

and using $v = c/\lambda$ we obtain

$$Z = 1 + \frac{c^{1/2}}{\lambda^{1/2}}\left(\frac{1}{4.97 \times 10^7\ \text{Hz}^{1/2}}\right) = 1 + \frac{34.85}{\lambda^{1/2}} \quad (\lambda \text{ in Å})$$

The results are given in Table.

Element	λ , Å	Z
Fe	1.94	$26.02 \approx 26$
Co	1.79	$27.04 \approx 27$
Ni	1.66	$28.04 \approx 28$
Cu	1.54	$29.08 \approx 29$

Before Moseley's work, Ni whose atomic weight is 58.69, was listed in the periodic table before Co, whose atomic weight is 58.94, and it was believed that the atomic numbers for Ni and Co were 27 and 28, respectively. By using the above experimental data, Moseley showed that this ordering and the corresponding atomic numbers should be reversed.

Illustration 8.

When 0.50Å X-rays strike a material, the photoelectrons from the K shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2} T. What is the binding energy of K-shell electrons?

Solution:

The velocity of the photoelectrons is found from $F = ma$:

$$evB = m\frac{v^2}{R} \qquad \text{or} \quad v = \frac{e}{m}BR$$

The kinetic energy of the photoelectrons is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6\times10^{-19}C)^2\ (2\times10^{-2}T)^2\ (23\times10^{-3}m)^2}{(9.11\times10^{-31}\text{kg})} = 2.97\times10^{-15}\ J$$

or $$K = (2.97\times10^{-15}\,J)\,\frac{1\,\text{keV}}{1.6\times10^{-16}\,J} = 18.6\,\text{eV}$$

The energy of the incident photon is $E_v = \dfrac{hc}{\lambda} = \dfrac{12.4\ \text{keV.Å}}{0.50\text{Å}} = 24.8\,\text{eV}$

The binding energy is the difference between these two values:

$BE = Ev - K = 24.\ \text{keV} - 18.6\ \text{keV} =$ **6.2 keV**

Illustration 9.

Stopping potentials of 24, 100, 110 and 115 kV are measured for photoelectrons emitted from a certain element when it is irradiated with monochromatic X–ray. If this element is used as a target in an X-ray tube, what will be the wavelength of the $K\alpha$ line?

Solution:

The stopping potential energy, eV_s, is equal to the difference between the energy of the incident photon and the binding energy of the electron in a particular shell:

$$eV_s = E_p - E_B$$

The different stopping potentials arise from electrons being emitted from different shells, with the smallest value (24 kV) corresponding to ejection of a K-shell electron. Subtracting the expression for the two smallest stopping potentials, we obtain

$$eV_{sL} - eV_{sK} = (E_p - E_{BL}) - (E_p - E_{BK}) = E_{BK} - E_{BL}$$

or $$100 \text{ keV} - 24 \text{ keV} = E_{BK} - E_{BL}$$

The difference, 76 keV, is the energy of the K_α line. The corresponding wavelength is

$$\lambda = \frac{hc}{E_{BK} - E_{BL}} = \frac{12.4 \text{ keV.Å}}{76 \text{ keV}} = \mathbf{0.163\ Å}$$

5. NUCLEAR PHYSICS

So far the only knowledge of nucleus we have as a tiny positively charged object whose primary contributions are to provide the atom with most of its mass and to hold its electrons in captivity. The chief properties of atoms, molecules, solids and liquids can all be traced to the behaviour of atomic electrons not to the behaviour of nuclei. However, the nucleus turns out to be of paramount importance in the grand scheme of things. To start with the very existence of the various elements is due to the ability of nuclei to possess multiple electric charge. Furthermore, the energy involved in almost all natural processes can be traced to nuclear reactions and transformations. In the following sections we will study about the nucleus and phenomenon associated with nucleus.

5.1 NUCLEAR CHARACTERISTICS

Nuclear mass

It was observed in Rutherford's α-particle scattering experiment that mass of an atom is concentrated within a very small positively charged region at the centre called nucleus. The total mass of nucleons in the nucleus is called as nuclear mass.

Nuclear mass = mass of protons + mass of neutrons

Size and shape of the nucleus

The nucleus is nearly spherical. Hence its size is usually given in terms of radius. The radius of nucleus was measured by Rutherford and it was found to have following relation

$$R = R_0 A^{1/3}$$

where $R_0 = 1.1\ fm = 1.1 \times 10^{-15}$ m and A is mass number of particular element.

Nuclear charge

Nucleus is made of protons and neutrons. Protons have positive charge of magnitude equal to that of electron and neutrons are uncharged. So, nuclear charge = Ze

Nuclear density

The ratio of the mass of the nucleus to its volume is called nuclear density. As the masses of proton and neutron are roughly equal, the mass of a nucleus is roughly proportional to *A*.

As volume of a nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

$$V \propto A$$

density within a nucleus is independent of *A*.

5.2 DIFFERENT TYPES OF NUCLEI

There are different types of nuclei depending upon the number of protons or the total number of nucleons in them.

Isotopes

The atoms of an element having same atomic number but different mass number are called isotopes of that element i.e. different isotopes of the same element have same number of protons inside the nucleus but different number of neutrons inside the nucleus. Though isotopes have same chemical properties but their nuclear properties are highly different. Examples of isotopes are ${}_1H^1$, ${}_1H^2$, ${}_1H^3$ and ${}_8O^{16}$, ${}_8O^{17}$, ${}_8O^{18}$ etc.

Isotones

Atoms whose nuclei have same number of neutrons are called isotones. For them, both the atomic number *Z* and atomic mass *A* are different but the value of difference ($A - Z$) is same. Examples of isotones are ${}_1H^3$, and ${}_2He^4$, ${}_1H^2$, and ${}_2He^3$ etc.

Isobars

Atoms of same mass number but different atomic number are called isobars examples of isobars are ${}_1H^3$ and ${}_2He^3$, ${}_6C^{14}$ and ${}_7N^{14}$ etc.

5.3 NUCLEAR FORCES

The strong forces of attraction, which firmly hold the nucleons in the nucleus, are known as nuclear forces. Though the exact theory of nuclear forces is still to be understood completely, yet it is undoubtedly established that these forces exist between the nucleons i.e. between a neutron and a proton, between two protons and between two neutrons. The stability of nucleus is due to the presence of these forces. Nuclear forces have following important characteristics.

(i) They are attractive i.e. nucleons exert attractive force on each other hence they are also called cohesive forces.

(ii) They are extremely strong. These forces are strongest possible force in nature.

(iii) They are charge independent.

(iv) They are short-range forces i.e. they act only over a short range of distances.

(v) They are spin dependent i.e. nuclear forces acting between two nucleons depend on the mutual orientation of the spins of the nucleons.

They are saturated i.e. their magnitude does not increase with the increase in the number of nucleons, beyond a certain number.

5.4 EINSTEIN'S MASS ENERGY EQUIVALENCE PRINCIPLE

Before the discovery of Einstein's mass energy equivalence principle, mass and energy were considered independent physical quantities. Einstein on the basis of theory of relativity showed that mass of a body is not independent of energy but they are inter convertible. According to Einstein if a substance loses an amount Δm of its mass, an equivalent amount ΔE of energy is produced, where $\Delta E = (\Delta m)\, c^2$

where c is the speed of light. This is called Einstein's mass-energy equivalence principle. In nuclear physics mass is usually represented in terms of energy according to the conversion formula $E = mc^2$. For example the mass of an electron is 9.1×10^{-31} kg and the equivalent energy is 511 KeV/c^2. Similarly, the mass of a proton is 938 MeV/c^2, and the mass of a neutron is 939 MeV/c^2. The energy corresponding to the mass of a particle when it is at rest is called its rest mass energy. Another useful unit of mass in nuclear physics is unified atomic mass unit, denoted by the symbol u. It is $(1/12)^{th}$ of the mass of a neutral carbon atom in its lowest energy state which has six protons, six neutrons and six electrons. We have

$$1u = 1.67 \times 10^{-27} \text{ kg} = 931.478 \text{ MeV/c}^2$$

5.5 BINDING ENERGY OF NUCLEUS

(i) The total energy required to liberate all the nucleons from the nucleus (i.e., the disintegrate the nucleus completely into its constituent particles) is called binding energy of the nucleus. Clearly, this is the same energy with which the nucleons are held together within the nucleus. The origin of binding energy results from strong nuclear exchange forces. In other words, we may think of existence of binding energy in other useful way also. A nucleus is made by the coming together of various nucleons. It has been observed experimentally that the mass of the nucleus is always less than the sum of the masses of its constituents when measured in free state. For example, deutron $({}_1H^2)$ is composed of one proton and 1 neutron. The question arises where the difference in mass has gone? The answer is that this decrease in mass has been converted into energy binding the nucleons together according to the following relation:

$$\Delta E = \Delta mc^2$$

where, ΔE = binding energy of nucleus, Δm = decrease in mass, called mass defect and c = velocity of light.

Hence in the formation of stable nucleus, the following equation holds good.

Mass of protons + Mass of neutrons = Mass of nucleus + Binding energy

Example: Consider a deutron $({}_1H^2)$ nucleus. It is the nucleus of heavy hydrogen or deuterium $({}_1H^2)$. It contains 1 proton and 1 neutron. We shall compare the mass of one free proton and one free neutron with their mass when combined to form deutron and thus find out mass defect and binding energy of deutron.

	Mass one free neutron	=	1.675×10^{-27} kg = 1.008665 a.m.u
	Mass of one free proton	=	1.673×10^{-27} kg = 1.007825 a.m.u.
	Their total	=	3.348×10^{-27} = 2.01649 a.m.u.
	Mass of deutron	=	3.348×10^{-27} = 2.014103 a.m.u.
$\therefore$	Mass defect, Δm	=	0.002387 a.m.u.

It shows that when a proton and a neutron come together to form a deutron, a small mass of 0.002387 disappears. In fact, this mass is converted into biding energy according to the following relation:

$$\Delta E = \Delta mc^2 = \frac{0.002387 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} = 2.22 \times 10^6 \text{ eV} = 2.22 \text{ MeV}$$

Binding energy curve

Expression for biding energy per nucleon:

In order to compare the stability of various nuclei, we calculate binding energy per nucleon. Higher is the binding energy per nucleon more stable is the nucleus.

We have seen that the mass defect during the formation of a nucleus:

$\Delta m = Zm_p + (A - Z)\, m_n - m$, where m_p, m_n and m are masses of proton, neutron and nucleus respectively.

∴ Total binding energy of nucleus

$$\Delta E = \Delta mc^2 = [Zm_p + (A - Z)\, m_n - m] \times c^2$$

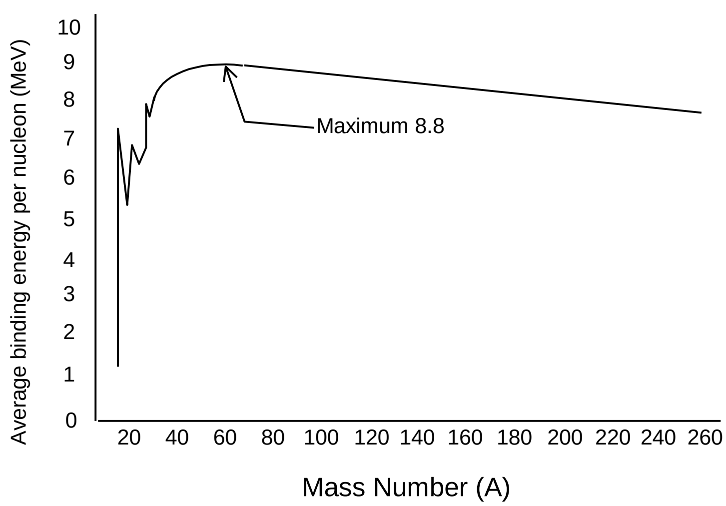
∴ Mean binding energy per nucleon

$$= \frac{\Delta E}{A} = \frac{\Delta mc^2}{A} = \left[\frac{Z}{A}(m_p - m_n) + m_n - \frac{m}{A}\right] \times c^2$$

If the mass m of the nucleas is found experimentally, we can find mean binding energy per nucleon since all other factors are known to us.

Binding energy curve:

A graph between the binding energy per nucleon and the mass number of nuclei is called as the binding energy curve.



The following points may be noted from the biding energy curve:

(a) The binding energy per nucleon is maximum (≈ 8.8 MeV) for the nucleus having mass number 56. So, this nucleus is most stable i.e. iron is the most stable element of periodic table.

(b) The light nuclei with $A < 20$ are least stable.

(c) The curve has certain peaks indicating that certain nuclei like ${}^4_2He$, ${}^{12}_6C$ and ${}^{16}_8O$ are much more stable than the nuclei in their vicinity.

(d) For atomic number $Z > 56$, the curve takes a downside turn indicating lesser stability of these nuclei.

(e) Nuclei of intermediate mass are most stable. This means maximum energy is needed to break them into their nucleons.

(f) The binding energy per nucleon has a low value for both very light and very heavy nuclei. Hence, if we break a very heavy nucleus (like uranium) into comparatively lighter nuclei then the binding energy per nucleon will increase. Hence a large quantity of energy will be liberated in this process. This phenomenon is called nuclear fission.

(g) Similarly, if we combine two or more very light nuclei (e.g. nucleus of heavy hydrogen ${}_1H^2$) into a relatively heavier nucleus (e.g. ${}_2He^4$), then also the binding energy per nucleon will increase i.e., again energy will be liberated. This phenomenon is called nuclear fusion.

5.6 NUCLEAR FISSION

The phenomenon of breaking a heavy nucleus into two light nuclei of almost equal masses along with the release of huge amount of energy is called nuclear fission. The process of nuclear fission was first discovered by German Scientists Otto Hahn and Strassman is 1939. They bombarded uranium nucleus (${}_{92}U^{235}$) with slow neutrons and found that ${}_{92}U^{236}$ was split into two medium weight parts with the release of enormous energy. These fragments have atomic numbers far less than the target nucleus (${}_{92}U^{235}$). The nuclear fission of ${}_{92}U^{235}$ is given by the following nuclear reaction:

$${}_{92}U^{235} + {}_0n^1 \longrightarrow [{}_{92}U^{236}] \longrightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3{}_0n^1 + \text{energy.}$$

The fission of ${}_{92}U^{235}$ nucleus when bombarded with a neutron takes place in following manner. When a neutron strikes ${}_{92}U^{235}$ nucleus, it is absorbed by it, producing a highly unstable ${}_{92}U^{236}$ nucleus. Instead of emitting α or β particles or γ rays, this unstable nucleus is split into two middle weight parts viz ${}_{56}Ba^{144}$ and krypton (${}_{36}Kr^{89}$). During this fission, three neutrons are given out and a small mass defect occurs which is converted into enormous amount of energy. The following points are worth noting about nuclear fission process:

(a) The energy released in the fission of uranium is about 200 MeV per nucleus. This can be easily verified. If we obtain atomic mass unit values of reactants and products in the fission of ${}_{92}U^{235}$ nucleus, we find that there occurs a mass defect of 0.214 a.m.u. Which is converted into energy. Energy released per fission of ${}_{92}U^{235}$ nucleus $= 0.214 \times 931$.

$$\approx 200 \text{ MeV}$$

(b) The products of uranium fission are not always barium and krypton. Sometimes, they are Strontium and Xenon. There are other pairs as well. However, in each case, neutrons are emitted and tremendous amount of energy is released.

(c) Energy is released in the form of kinetic energy of fission fragments. Some of the energy is also released in the form of γ-rays, heat energy sound energy and light energy.

(d) The pressure and temperature is very high in fission process.

5.7 NUCLEAR FUSION

The process of combining two light nuclei to form a heavy nucleus is known as nuclear fusion. An important feature of nuclear fusion is that there is a release of huge amount of energy in the process. This can be easily understood. When two light nuclei are combined to form a heavy nucleus there occurs a small mass defect. This small mass defect results in the release of huge amount of energy according to the relation ΔE = mc^2. For example by the fusion of two nuclei of heavy hydrogen, the following reaction is possible.

$$_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + \Delta E_1$$

The nucleus of tritium $^3_1$H so formed can again fuse with a deuterium nucleus.

$$^3_1H + ^2_1H \longrightarrow ^4_2He + ^1_0n + \Delta E_2$$

Nuclear fusion is a very difficult process to achieve. This is because when positively charged nuclei come close to each other for fusion they required very high energy to counter repulsive force between them. So a high temperature is required for fusion. Though the energy output in the process of nuclear fission is much more than in a nuclear fusion the energy liberated by the fusion of a certain mass of heavy hydrogen is much more than the energy liberated by the fission of equal mass of uranium.

6. RADIOACTIVITY

The phenomenon of spontaneous emission of radiations from radioactive substances is known as Radioactivity. This is exhibited naturally by certain heavy elements like uranium, radium, thorium, etc is called natural radioactivity. However it was later established that it can be induced in lighter elements as well using modern techniques and this is called induced radioactivity.

Rutherford analysed the radiations coming from radioactive sources and showed that this consists of three types of rays namely α, β and γ rays. The prime reason for the emission of these rays is that nucleus can have excited states; these excited states can decay by the emission of high-energy photons (γ rays) to the ground states, directly or via lower energy states. In addition nuclei in both excited and ground states can spontaneously emit other particles (α and β) to reach lower energy configuration.

α-decay

In alpha decay an α particle is ejected from a nucleus and the parent nucleus loses two protons and two neutrons. Therefore its atomic number *z* decreases by two units and its mass number *A* decreases by four units, so that the daughter *D* and parent *P*, are different chemical elements. Applying conservation of charge and nucleons we can write alpha decay symbolically as

$$^A_ZP \rightarrow {}^{A-4}_{Z-2}D + {}^4_2He$$

β-decay

It is possible for a nuclear process to occur where the charge *Ze* of a nucleus changes but the number of nucleons remains unchanged. This can happen with a nucleus emitting an electron (β-decay), emitting a positron (β-decay) or capturing an inner atomic electron (electron capture). In each of these processes either a proton is converted into a neutron or vice-versa. It is also found that in each of these processes an extra particle called a neutrino appears as one of the decay products. The properties of a neutrino are electric charge = *O*, rest mass ≈ *O*, intrinsic spin, $\frac{1}{2}$, and, as with all massless particles it has speed *C* (speed of light)

γ- decay

A gamma ray emission does not affect either the charge or the mass number

The statistical radioactive law

In a typical radioactive decay an initially unstable nucleus called the parent, emits a particle and decays into a nucleus called the daughter, effectively, the birth of the daughter arises from the death of the parent. The daughter may be either the same nucleus in a lower energy state, as in the case of a γ-decay or an entirely new nucleus as arises from α and β decays. No matter what types of particles are emitted all nuclear decays follow the same radioactive decay law. If there are initially no unstable parents nuclei present, the number N of parents that will be left after a time t is

$$N = N_0 e^{-\lambda t} \qquad \dots (\mathbf{12})$$

The constant λ is called the decay constant or disintegration constant and depends on the particular decay process.

Equation (12) is statistical, not a deterministic, law, it gives the expected number N of parent that survive after a time t. However for a large number of unstable nuclei, the actual number and expected number of survivors will almost certainly differ by no more than an insignificant fraction The rapidity of decay of a particular radioactive sample is usually measured by the half life $T_{1/2}$, defined as the time interval in which the number of parent nuclei at the beginning of the interval is reduced by a factor of one half. The half-life is readily obtained in terms of λ as

$$T_{1/2} = \frac{\ln 2}{\lambda} \qquad \dots (\mathbf{13})$$

Another quantity that measures the rapidity of decay is the average or mean lifetime of a nucleus, T_{av}, is given by

$$T_{av} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} \qquad \dots (\mathbf{14})$$

$\therefore$ Average life = 1.44 times the half-life.

Activity of a radioactive substance

The activity of a radioactive substance is the rate of decay or the number of disintegrations per second. It is denoted by A.

$$\therefore \qquad A = \frac{dN}{dt} \qquad \dots (\mathbf{15})$$

$$= \frac{d}{dt}(N_0 e^{-\lambda t})$$

$$= -\lambda N_0 e^{-\lambda t}$$

$$\therefore \qquad A = -\lambda N$$

Also, $$A = A_0 e^{-\lambda t} \qquad \dots (\mathbf{16})$$

The activity of a sample is measured in unit called Curie. A Curie is defined as that quantity of radioactive substance in which the number of disintegrations per second is 3.7×10^{10}. This is also equal to the activity of one gram of radium.

The activity at time t is given by $A = \dfrac{A_0}{2^{t/T_{1/2}}}$.

Illustration 9.

Determine the approximate density of a nucleus.

Solution

If the nucleus is treated as a uniform sphere,

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \approx \frac{A\times(\text{mass of a nucleon})}{\frac{4}{3}\pi R^3}$$

$$= \frac{A(1.7\times10^{-27}\,\text{kg})}{\frac{4}{3}\pi\,(1.4\times10^{-15}A^{1/3}m)^3} = \mathbf{1.5\times10^{-17}\,\frac{kg}{m^3}}$$

A cube inch of nuclear material would weigh about 1 billion tons!

Illustration 10.

What is the activity of one gram of $^{226}_{88}\mathbf{Ra}$, whose half-life is 1622 years?

Solution

The number of atoms in 1g of radium is

$$N = (1\text{g})\left(\frac{1\text{g}-\text{mole}}{226\text{ g}}\right)\left(6.025\times10^{23}\,\frac{\text{atoms}}{\text{g}-\text{mole}}\right) = 2.666\times10^{21}$$

The decay constant is related to the half-life by

$$\lambda = \frac{0.693}{T_{1/2}} = \left(\frac{0.693}{1622\,y}\right)\left(\frac{1\,y}{365\,d}\right)\left(\frac{1\,d}{8.64\times10^4\,s}\right) = 1.355\times10^{-11}s^{-1}$$

The activity is then found from

Activity = λN = $(1.355\times10^{-11}\text{ s}^{-1})(2.666\times10^{21}) = 3.612\times10^{10}$ disingegrations/s

The definition of the curie is 1 Ci = 3.7×10^{10} disintegrations/s. This is approximately equal to the value found above.

Illustration 10.

An unstable element is produced in nuclear reactor at a constant rate R. If its half-life β^- decay is $T_{1/2}$, how much time, in terms of $T_{1/2}$, is required to produce 50% of the equilibrium quantity?

Solution

We have

$$\text{Rate of increase of element} = \frac{\text{number of nuclei by reactor}}{\text{s}} - \frac{\text{number of nuclei decaying}}{\text{s}}$$

$$\frac{dN}{dt} = R - \lambda N \quad \text{or} \quad \frac{dN}{dt} + \lambda N = R$$

The solution to this is the sum of the homogeneous solution, $N_h = ce^{-\lambda t}$, where c is a constant, and a particular solution, $N_l = \frac{R}{\lambda}$.

$$N = N_h + N_p = ce^{-\lambda t} + \frac{R}{\lambda}$$

The constant c is obtained from the requirement that the initial number of nuclei be zero:

$$N(0) = 0 = c + \frac{R}{\lambda} \quad \text{or} \quad c = -\frac{R}{\lambda}$$

so that $$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

The equilibrium value is $(t \to \infty) = R/\lambda$. Setting N equal to 1/2 of this value gives

$$\frac{1}{2}\left(\frac{R}{\lambda}\right) = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

$$e^{-\lambda t} = \frac{1}{2} \Rightarrow t = \frac{\ln 2}{\lambda} = \boldsymbol{T_{1/2}}$$

The result is independent of R.

Illustration 11.

If the average life time of an excited state of hydrogen is of the order of 10^{-8} s estimate how many orbits an electron makes when it is in the state n = 2 and before it suffers a transition to state n = 1.

Take Bohr radius = 5.3×10^{-11} m.

Solution:

Velocity of electron in the n^th orbit of hydrogen atom

$$v_n = \frac{v_1}{n} = \frac{2.19 \times 10^6}{n} \text{ m/s}$$

If n = 2, $v_n = \frac{2.19 \times 10^6}{n}$ m/s

Radius of n = 2 orbit, $r_n = n^2 r_1$ = 4 × Bohr radius

$= 4 \times 5.3 \times 10^{-11}$ m

Number of revolutions made in 1 sec

$$= \frac{v_n}{2\pi r} = \frac{2.19 \times 10^6}{2 \times 2\pi \times 4 \times 5.3 \times 10^{-11}}$$

Number of revolutions made in 10^{-8} s $= \frac{2.19 \times 10^6 \times 10^{-8}}{2 \times 2\pi \times 4 \times 5.3 \times 10^{-11}} = 8.22 \times 10^6$ revolutions

Illustration 12.

The wavelength of the first member of the Balmer series in hydrogen spectrum is 6563 Å. What is the wavelength of the first member of Lyman series?

Solution:

Balmer series

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

Lyman series

$$\frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{4}{3R} \times \frac{5R}{36} = \frac{20}{108} = \frac{5}{27}$$

$$\lambda_2 = \frac{5}{27} \times \lambda_1 = \frac{5}{27} \times 6563 = 1215 \text{ Å}$$

Illustration 13.

A single electron, orbits around a stationary nucleus of charge Ze, where Z is a constant and e is the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to 3rd Bohr orbit. Find,

(i) the value of Z.

(ii) the energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity.

(iv) the kinetic energy, potential energy and angular momentum of the electron in the first Bohr orbit.

(v) the radius of the first Bohr orbit.

(Ionisation energy of hydrogen atom = 13.6 eV.

Bohr radius = 5.3×10^{-11} m, Velocity of light = 3×10^{8} m/s and

Planck’s constant = 6.6×10^{-34} Js)

Solution:

(i) For a general hydrogen-like atom

$$E_{n_2} - E_{n_1} = Z^2 E_0\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ eV}$$

where E_0 is the ionisation energy of hydrogen atom

$$\Delta E = Z^2 \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 47.2 \qquad \text{or} \qquad Z^2 \times \frac{13.6}{36} \times 5 = 47.2$$

$$Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25$$

Z = 5

(ii) $E_4 - E_3 = 5^2 \times 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$eV

$$= 25 \times 13.6 \times \frac{7}{144} = 16.53 \text{ eV}$$

Energy required to excite the electron from the third to the fourth Bohr orbit

= 16.53 eV

(iii) $E_\infty - E_1 = Z^2 \times 13.6\left[\frac{1}{1^2} - \frac{1}{\infty}\right]$ = 13.6 × 25 eV

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.6\times10^{-34})(3\times10^{8})}{13.6\times25\times1.6\times10^{-19}} = 0.03640 \times 10^{-7} = 36.4 \times 10^{-10}$$

= 36.4

The wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity = 36.4 Å

(iv) K.E. of I Bohr orbit is numerically equal to the energy of the orbit

$$E_1 = -Z^2 E_o = -25 \times 13.6 eV$$

$\therefore$ K.E. $= 25 \times 13.6 \times 1.6 \times 10^{-19}$ J = 544 × 10^{-19} J

Potential energy of electron $= -2 \times K.E.$ = – 2 × 544 × 10^{-19} J = – 1088 × 10^{-19} J

Angular momentum of the electron

$$mvr = \frac{nh}{2\pi} = \frac{h}{2\pi} \quad \because n = 1$$

$$= \frac{6.6\times10^{-34}}{2\pi} = 1.05 \times 10\text{-}34 \text{ Js}$$

(v) Radius r_1 of the first Bohr orbit

$$r_n = \frac{n^2 r_o}{Z}$$

For n = 1,

$$r_1 = \frac{1^2 \times 5.3\times10^{-11}}{5} = 1.06 \times 10^{-11} \text{ m}$$

Illustration 14.

One milliwatt of light of wavelength 4560 Å is incident on a caesium surface. Calculate the electron current liberated. Assume a quantum efficiency of 0.5%.

Solution:

The energy of one photon of incident light

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{4560\times10^{-10}} = 4.34\times10^{-19} J$$

1 mW of light energy is equivalent to

$$\frac{10^{-3}}{4.34\times10^{-34}} = 2.30\times10^{15} \text{ photon/sec}$$

The quantum efficiency = 0.5%

This means that only 0.5% of these photons release photoelectrons.

$\therefore$ the number of electrons released from the surface per second

$$= 2.30\times10^{15}\times\frac{0.5}{100}$$

$$= 1.15\times10^{13} \text{ electron/sec}$$

The electron current = $1.15\times10^{13}\times1.6\times10^{-19}$ amp

= 1.84×10^{-6} amp

Illustration 15.

Find the de Broglie wavelength for an electron beam of kinetic energy 100 eV.

Solution:

Kinetic energy of electrons: $(E)_K = \frac{1}{2}mv^2$

$\Rightarrow$ velocity of electrons: $v = \sqrt{\frac{2(E)_K}{m}}$

$\Rightarrow$ $v = \sqrt{\frac{2\times100\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 5.9\times10^{6}$ m/s

Momentum : p = mv = $9.1\times10^{-31}\times 5.9\times10^{6} = 5.37\times10^{-24}$ kg m/s

$\Rightarrow$ de Broglie wavelength : $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{5.37\times10^{-24}} = 1.23 \times 10^{-10}$ m = 1.23 Å

Illustration 16.

The wavelength of K_α X–ray of tungsten is 0.21 Å. If the energy of a tungsten atom with an L electron knocked out is 11.3 keV, what will be the energy of this atom when a K electron is knocked out? (h = 4.14×10^{-15} eVs)

Solution:

Energy of K_α photon: $E = \frac{hc}{\lambda}$

$\Rightarrow \quad E = \dfrac{4.14\times10^{-15}\times3\times10^{18}\,\text{eV}-\overset{\circ}{A}}{0.21\overset{\circ}{A}} = 59.1\,\text{keV}$

Let $\quad E_K$ = energy of the atom with a vacancy in the K-shell

E_L = energy of the atom with a vacancy in the L-shell

Then, $\quad E_K - E_L = E \Rightarrow E_K = E + E_L = 59.1 + 11.3 = 70.4$ keV

Illustration 17.

Radio phosphorus-32 has a half-life of 14 days. A source containing this isotope has initial activity 10 μ curie.

(a) What is the activity of the source after 42 days?

(b) What time elapses before the activity of the source falls to 2.5 micros curie?

Solution:

(a) The number of half-lives in 42 days = $\dfrac{42}{14} = 3$

$$A = A_0 e^{-\lambda t} = A_0 \frac{1}{2^3} = \frac{10\mu\,\text{curie}}{8} = 1.25\ \mu\ \text{curie}$$

(b) $\dfrac{\text{Final activity}}{\text{Initial activity}} = \dfrac{2.5\mu\,\text{curie}}{10\mu\,\text{curie}} = \dfrac{1}{4}$

Time to decay to $\dfrac{1}{4}$ of initial activity = 2 half-lives = 28 days

Illustration 18.

It is found from an experiment that the radioactive substance emits one beta particle for each decay process. Also an average of 8.4 beta particles are emitted each second by 2.5 milligram of substance. The atomic weight of substance is 230. What is its half-life?

Solution:

The activity = 8.4 sec^{-1}

Number of atoms in kilomole (i.e., 230 kg) = 6.02×10^{26}

$$\therefore \quad N = \frac{6.02\times10^{26}}{230} \times 2.5 \times 10^{-6} = 6.54 \times 10^{18}$$

$8.4 = \lambda N$

$= \lambda \times 6.54 \times 10^{18}$

$\lambda \times 6.54 \times 10^{18}$

$$\lambda = \frac{8.4\times10^{-18}}{6.54} = 1.28 \times 10^{-18}/\text{sec}$$

$$\text{Half-life } T = \frac{0.693}{\lambda} = \frac{0.693}{1.28\times10^{-18}} = 5.41 \times 10^{17}\ \text{sec} = \frac{5.41\times10^{17}}{3.16\times10^{7}} = 1.7 \times 10^{10}\ \text{years}$$

Illustration 19.

In the deuterium-tritium fusion reaction find the rate at which deuterium and tritium are consumed to produce 1 MW. The Q-value of deuterium-tritium reaction is 17.6 MeV. You can assume that the efficiency is 100%.

Solution:

Energy released per fusion = 17.6 MeV

Number of fusion reactions to produce 1 MW

$$= \frac{10^6}{17.6 \times 1.6 \times 10^{-19} \times 10^6} = 3.55 \times 10^{17}$$

In each reaction one atom of deuterium and one atom of tritium are consumed.

Mass of deuterium consumed per second

$$= \frac{2\text{ kg}}{1\text{ kmol}} \times \frac{1\text{ kmol}}{6.023 \times 10^{26}} \times 3.55 \times 10^{17} \text{ atom/sec} = 1.179 \times 10^{-9} \text{ kg/sec}$$

Mass of tritium consumed per second $= \frac{3}{2} \times 1.179 \times 10^{-9} = 1.769 \times 10^{-9}$ kg/sec

Illustration 20.

Two deuterium nuclei fuse to form a tritium nucleus and a proton as by product. Compute the energy released.

Given: Mass of deuterium = 2.0141 U

Mass of tritium nucleus = 3.01605 U

and mass of proton = 1.00782 U

Solution:

$${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + {}_1^1H + \Delta E$$

Mass of reactants = 2.0141 × 2 = 4.02820 U

Mass of products = 3.01605 + 1.00782 = 4.02387 U

Mass defect = 4.0282 – 4.02387 = 0.00433 U

Energy released = 0.00433 × 931 = 4.03 MeV