

## 1. KINETIC THEORY OF GASES

### 1.1 FUNDAMENTAL ASSUMPTIONS OF KINETIC THEORY OF GAS

1. All gases are made of molecules moving randomly in all directions.
2. The size of a molecule is much smaller than the average separation between the molecules.
3. No forces of attraction or repulsion are exerted on a gas molecule by other molecules or by the container except during collisions. This implies that energy of gas is purely kinetic.
4. Collisions between two molecules or between a molecule and a wall are perfectly elastic. The time spent during a collision is assumed to be negligibly small.
5. The molecules follow Newton's laws of motion.
6. In the steady state molecular density remains uniform throughout the gas and does not change with time.

A gas which satisfies all the above assumptions of kinetic theory of gases under all set of temperature and pressure is called an ideal gas or perfect gas. In our present discussion we will always consider only ideal gases.

### 1.2 INTERNAL ENERGY OF AN IDEAL GAS

In case of an ideal gas, according to kinetic theory of gases we assume no force of interaction between molecules. Hence internal potential energy is zero. Internal energy of an ideal gas is purely kinetic and it depends only on temperature. Needless to say internal energy of an ideal gas is a state function.

### 1.3 Basic Gas Laws

The state of a given mass of a gas is determined by the values of three parameters, the pressure  $P$ , the volume  $V$  and the temperature  $T$ . Since the three quantities are related to one another according to a certain law, a change in one causes change in others. If relation between any two of them is required, the third has to be constant. Three such relations can be obtained.

**Boyle's law** gives a relation between pressure and volume when temperature is kept constant. At constant temperature the pressure of a given mass of gas is inversely proportional to its volume i.e.,

$$P \propto \frac{1}{V} \quad \text{or} \quad PV = \text{constant}$$

**Charles's law** gives the relation between temperature and volume at constant pressure. The volume of a given mass of a gas is directly proportional to absolute temperature of the gas.

$$V \propto T \quad \text{when } P \text{ is constant}$$

$$\text{or} \quad \frac{V}{T} = \text{constant}$$

The pressure of a given mass of gas at constant volume varies directly as its Kelvin temperature.

$$\frac{P}{T} = \text{constant (at constant volume)}$$

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**Charles's law of pressure** says that a given volume, the pressure of a given mass of a gas is proportional to its absolute temperature. i.e.,

$$(P)_{\text{constant volume}} \propto T$$

**Avagadro's law** says that at the same temperature and pressure, equal volumes of all gases contain equal number of molecules.

One mole of a gas occupies 22.4L at STP (273K and 1 atm).

### 1.4 IDEAL GAS EQUATION

For  $n$  moles of an ideal gas at pressure  $P$ , absolute temperature  $T$ , occupying a volume  $V$ , the ideal gas equation is given by

$$PV = nRT \quad (\text{Where } R \text{ is universal constant})$$

This is called equation of the state of the gas, and this equation gives a relation between pressure ( $P$ ), volume ( $V$ ) and temperature ( $T$ ) of the gas at any given state of the system.

**Universal constant:**

One mole of a gas occupies 22.4L at STP (273K and 1 atm). This information can be used to calculate the value of universal constant.

$$PV = nRT$$

$$R = \frac{PV}{nT} = \frac{1 \text{ atm} \times 22.4 \text{ L}}{1 \text{ mole} \times 273 \text{ K}} = 0.0821 \text{ atm L K}^{-1} \text{ mol}^{-1}$$

$$R = \frac{PV}{nT} = \frac{101325 \text{ N m}^{-2} \times 22.4 \times 10^{-3} \text{ m}^3}{1 \text{ mole} \times 273 \text{ K}} = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$

### 1.5 OTHER IMPORTANT LAWS OF AN IDEAL GAS

**Graham's law of diffusion** says that when two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas i.e.,

$$\text{rate of diffusion} \propto \sqrt{\frac{1}{\text{Density of gas}}}$$

**Dalton's law of Partial Pressure** says that the pressure exerted by a mixture of several gases equals the sum of the pressures exerted by each gas occupying the same volume as that of the mixture i.e., if  $P_1, P_2, \dots, P_n$  are the pressures exerted by individual gases of the mixture, then pressure of the mixture of the gas is

$$P = P_1 + P_2 + \dots + P_n$$

#### Illustration 1

A given amount of gas is heated until its volume is doubled at constant pressure. If the initial temperature was  $27^\circ\text{C}$ , what is the final temperature?

**Solution:**

$$V_1 = V$$

$$T_1 = (27 + 273) \text{ K} = 300 \text{ K}$$

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$$V_2 = 2V$$

$$T_2 = ? \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = \frac{V_2 T_1}{V_1} = \frac{2V \times 300}{V} = 600 \text{ K} = (600 - 273) = 327^\circ\text{C}$$

### Illustration 2

An air bubble starts rising from the bottom of a lake. Its diameter is 3.6 mm at the bottom and 4 mm at the surface. The depth of the lake is 2.5 m and the temperature at the surface is  $40^\circ\text{C}$ . What is the temperature at the bottom of the lake? Atmospheric pressure = 76 cm of mercury,  $g = 9.8 \text{ m/s}^2$ , density of mercury =  $13600 \text{ mg/m}^3$ , density of water =  $1000 \text{ mg/m}^3$

### Solution:

Let us first evaluate the atmospheric pressure in terms of water barometer. If  $h_w, d_w$  be the height of water barometer and density of water respectively  $h_m$  and  $d_m$  the corresponding quantities in terms of mercury, then

$$h_w d_w g = h_m d_m g$$

$$\text{or} \quad h_w = \frac{h_m d_m}{d_w} = \frac{0.76 \times 13600}{1000} = 10.336 \text{ m of water}$$

$\therefore$  the atmospheric pressure at the surface of the lake = 10.336 m of water.

Assuming the density of water uniform over its depth, the pressure at the bottom of the lake

$$= (\text{Atmospheric pressure} + \text{depth of water}) \text{ m of water}$$

$$= (10.336 + 2.5) \text{ m of water}$$

$$= 12.836 \text{ m of water}$$

Applying the gas equation to the conditions of the bubble at the bottom and top of the lake

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{12.836 \times \frac{4\pi}{3} (1.8 \times 10^{-3})^3}{T_1} = \frac{10.336 \times \frac{4\pi}{3} (2 \times 10^{-3})^3}{T_2}$$

$$\text{or} \quad \frac{12.836 \times 1.8^3}{T_1} = \frac{10.336 \times 2^3}{273 + 40}$$

$$T_1 = \frac{12.836 \times 1.8^3}{10.336 \times 2^3} \times 313 = 283.4 \text{ K} = 10.4^\circ\text{C}$$

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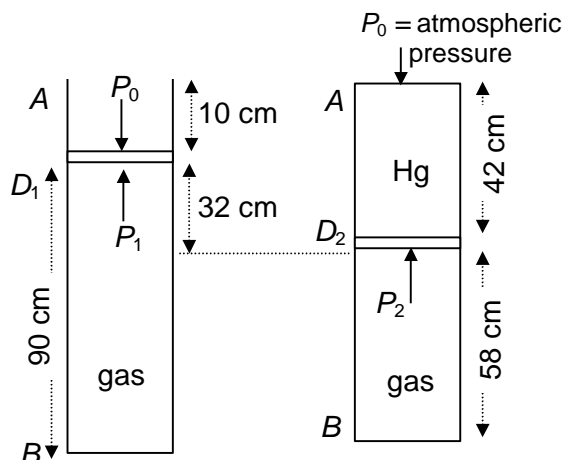
### Illustration 3

A vertical cylinder of total length 100 cm is closed at the lower end and is fitted with a movable frictionless and gas tight disc at the other end. An ideal gas is trapped under the disc. Initially, the height of the gas column is 90 cm when the disc is in equilibrium between the gas and atmosphere. Mercury is then slowly poured on the top of the disc and it just starts overflowing when the disc has descended through 32 cm. Find the atmospheric pressure. Assume the temperature of gas to remain constant and neglect the thickness and weight of the disc.

### Solution:

In the first case, the disc alone is in equilibrium with the compressed gas inside the cylinder.

In the second case, mercury standing over the disc is in equilibrium with the further compressed gas in the cylinder.



Let  $P_0$  be the atmospheric pressure and A the cross-section of the tube.

In the first case,

Volume of the compressed gas,  $V_1 = 90A$

Pressure of the gas  $P_1 = P_0$

Since the weight of the disc is to be neglected.

In the second case,

Volume of the gas =  $58A$

Pressure of the gas =  $(P_0 + 42)$  cm of mercury

By applying Boyle's law,

$$(P_0 + 42) 58A = P_0 \times 90A$$

$$90P_0 - 58P_0 = 2436$$

$$32P_0 = 2436$$

$$P_0 = 76.125 \text{ cm of mercury}$$

Atmospheric pressure = 76.125 cm of mercury.

## 2. THERMAL EXPANSION

Most of the substances expand on heating; this thermal expansion takes place in all the dimensions, length, breadth and height. For a linear object, for a small change in temperature  $dT$ , the fractional change in length ( $dl/l$ ) is directly proportional to  $dT$  i.e.,

$$\frac{dl}{l} = \alpha dT \quad \dots(1)$$

where  $\alpha$  is known as the thermal coefficient of linear expansion and it depends on the material of the rod. If the initial length of the object is  $l_0$  and change in temperature is  $\Delta T$  then equation (1) will give us the new length of the rod, which is

$$l = l_0 e^{\alpha \Delta T} \quad \dots (2)$$

If  $\alpha \Delta T$  is very small as compared to 1 then

$$l = l_0 (1 + \alpha \Delta T) \quad \dots (3)$$

Similar expressions can be written for change in area for a two-dimensional object and change in volume for a three dimensional object.

$$S = S_0 (1 + \beta \Delta T)$$

$$\& \quad V = V_0 (1 + \gamma \Delta T)$$

where  $\beta$  and  $\gamma$  are coefficients of superficial expansion and coefficient of cubical expansion respectively.

The relation between  $\alpha$ ,  $\beta$  and  $\gamma$  for an isotropic solid is  $\alpha : \beta : \gamma :: 1 : 2 : 3$ .

### 2.1 THERMAL STRESS

When a rod is held between two fixed supports and its temperature is increased, the fixed supports do not allow the rod to expand, which results in a stress which is called thermal stress.

Let a rod of length  $l$  is held between two fixed supports and its temperature is increased by  $\Delta T$ , then change in length of the rod

$$\Delta l = l \alpha \Delta T$$

where  $\alpha$  is thermal coefficient of linear expansion for the material of the rod.

If  $Y$  is the Young's modulus for the material of the rod and  $A$  is the area of cross-section of the rod then mechanical compression in the rod.

$$\Delta l' = \left( \frac{F}{A} \right) \frac{l}{Y}$$

Since support is rigid

$$\Delta l + \Delta l' = 0, \text{ which gives}$$

$$\frac{F}{A} = -\alpha Y \Delta T$$

This is thermal stress, - ve sign indicates that thermal stress is compressive in nature.

## 2.2 EXPANSION OF LIQUIDS

Liquids like solids expand on heating. The liquids have no definite shape and they experience volume expansion only. As the liquid is contained in a vessel, the observed expansion of liquid is less than the real expansion, because the capacity of vessel containing liquid increases with temperature. If  $C$  is the coefficient of real expansion of liquid and  $C'$  the coefficient of apparent expansion of liquid and  $\alpha$  coefficient of linear expansion of solid container then

$$C = C' + 3\alpha$$

The expansion of gases we will cover in section II.

### Illustration 4

The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and copper rod laid side by side at all temperatures. Find their lengths.

$$(\alpha_{Fe} = 11 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, \alpha_{Cu} = 17 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})$$

### Solution:

Since the  $\alpha_{Cu} > \alpha_{Fe}$  so length of iron rod should be greater than the length of copper rod.

Let the initial lengths of iron and copper rods be  $l_1$  and  $l_2$ , then

$$l_1 - l_2 = 10 \text{ cm} \quad \dots (i)$$

also since the difference has to be constant at all the temperatures, so

$$\Delta l = l_1 \alpha_{Fe} \Delta T = l_2 \alpha_{Cu} \Delta T$$

$$\frac{l_1}{l_2} = \frac{\alpha_{Cu}}{\alpha_{Fe}} \quad \dots (ii)$$

Solving (i) & (ii) we get

$$l_1 = \mathbf{28.3 \text{ cm}} \text{ and } l_2 = \mathbf{18.3 \text{ cm}}$$

### Illustration 5

The height of mercury column measured at  $t^\circ\text{C}$  with a brass scale which gives correct reading  $H_1$  at  $0^\circ\text{C}$ . What height  $H_0$  will the mercury column have at  $0^\circ\text{C}$ ? The coefficient of linear expansion of brass is  $\alpha$  and the coefficient of volume expansion of mercury is  $\gamma$ .

### Solution:

$H_1$  at  $t^\circ\text{C} = H_1 (1 + \alpha t)$  actually since pressure is same, so

$$\rho_0 g H_0 = \rho_t g H_1 (1 + \alpha t)$$

$$H_0 = H_1 (1 + \alpha t) \frac{\rho_t}{\rho_0} = H_1 (1 + \alpha t) (1 + \gamma t)^{-1}$$

$$H_0 = \mathbf{H_1 [1 + (\alpha - \gamma) t]}$$

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### Illustration 6

A sphere of diameter 7.0 cm and mass 266.5 g floats in a bath of liquid. As the temperature is raised, the sphere begins to sink at a temperature of 35°C. If the density of the liquid is 1.527 g/cm<sup>3</sup> at 0°C, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

### Solution:

It is given that the expansion of the sphere is negligible as compared to the expansion of the liquid. At 0°C, the density of the liquid is  $\rho_0 = 1.527 \text{ g/cm}^3$ . At 35°C, the density of the liquid equals the density of the sphere. Thus,

$$\begin{aligned}\rho_{35} &= \frac{266.5 \text{ g}}{\frac{4}{3}\pi(3.5\text{cm})^3} \\ &= 1.484 \text{ g/cm}^3\end{aligned}$$

$$\text{We have } \frac{\rho_\theta}{\rho_0} = \frac{V_0}{V_\theta} = \frac{1}{(1 + \gamma\theta)}$$

$$\text{Or, } \rho_\theta = \frac{\rho_0}{1 + \gamma\theta}$$

$$\begin{aligned}\text{Thus, } \gamma &= \frac{\rho_0 - \rho_{35}}{\rho_{35}(35^\circ\text{C})} = \frac{(1.527 - 1.484) \text{ g/cm}^3}{(1.484 \text{ g/cm}^3)(35^\circ\text{C})} \\ &= 8.28 \times 10^{-4} / ^\circ\text{C}.\end{aligned}$$

## 3. THERMOMETRY

The zeroth law of thermodynamics states that, if two bodies *A* and *B* are separately in thermal equilibrium with a third body *C*, then *A* and *B* are in thermal equilibrium with each other.

If two bodies are left for a long time so that they reach thermal equilibrium, the property that becomes common to the two bodies is temperature.

Thermometry is the branch of Physics that deals with the measurement of temperature. The device which is used to measure temperature is called thermometer.

### 3.1 TEMPERATURE SCALES

Defining a temperature scale involves

- (i) Choosing the thermometric substance
- (ii) The choice of the thermometric property of the substance.
- (iii) Choosing the upper fixed point and lower fixed point.
- (iv) Choosing the number of divisions between the two fixed points.

Let *N* be the number of divisions between the fixed upper and fixed lower point. If measure of the thermodynamic property at the upper fixed point be  $x_N$  and that at lower fixed point is  $x_0$  then if temperature at the lower fixed point is  $t_0$  and at an unknown temperature *t* the measure of the thermometric property is *x* then,

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$$t = \frac{x - x_0}{x_N - x_0} N + t_0$$

This equation defines temperature  $t$ . Some of the most common thermometric properties are the length of the liquid column (like mercury) in a glass capillary tube, the electrical resistance of the coil, the emf of a thermocouple etc.

Based on the choice of upper and lower fixed points we define various scales e.g., Celsius scale, Fahrenheit scale, Reamer scale. The relation between these scales is given by the following equation.

$$\frac{C}{100} = \frac{K - 273}{100} = \frac{F - 32}{180} = \frac{R}{4}$$

### Illustration 7

What is the temperature which has the same numerical value in (i) Centigrade scale and Fahrenheit scale and (ii) Kelvin scale and Fahrenheit scale.

#### Solution:

(i) Let  $x$  be the required temperature

$$\text{Now } \frac{x - 0}{100 - 0} = \frac{x - 32}{212 - 32} \quad \text{or} \quad \frac{x}{100} = \frac{x - 32}{180} \quad (\text{or}) \quad \frac{x}{5} = \frac{x - 32}{9}$$

$$9x = 5x - 160$$

$$4x = -160$$

$$x = -40$$

$$\therefore -40^\circ\text{C} = -40^\circ\text{F}$$

(ii) Let  $x$  be the required value in Kelvin and Fahrenheit scales.

$$\frac{x - 273}{373 - 273} = \frac{x - 32}{212 - 32}$$

$$\frac{x - 273}{100} = \frac{x - 32}{180}$$

$$9(x - 273) = 5(x - 32)$$

$$9x - 2457 = 5x - 160$$

$$4x = 2457 - 160 = 2297$$

$$x = \frac{2297}{4} = 574.25^\circ$$

$$\therefore 574.25 \text{ K} = 574.25^\circ\text{F}$$



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## Illustration 8

A thermometer has its lower and upper fixed points marked as  $10^\circ$  and  $80^\circ$ . When it reads  $40^\circ$  what is the corresponding temperature on Centigrade scale?

### Solution:

Let  $x$  be the reading on Centigrade scale

$$\frac{x-0}{100-0} = \frac{40-10}{80-10}$$

$$x = \frac{30}{70} \times 100 = \frac{300}{7} = 42.85^\circ\text{C}$$

## 4. CALORIMETRY

Neglecting any heat exchange with the surrounding the principle of calorimetry states that the total heat given by the hot objects equals the total heat received by the cold objects.

Heat is a form of energy. It is energy in transit whenever temperature difference exist. Since heat is energy in transit its unit in SI is joule. Another unit of heat is calories which is used very often. The relation between the joule and calories is given as

$$1 \text{ calories} = 4.18 \text{ joule}$$

### 4.1 SPECIFIC HEAT CAPACITY AND MOLAR HEAT CAPACITY

When heat is supplied to a body the temperature of the body increases. The increase in temperature of the body depends upon the mass of the substance, heat supplied, the material of the body as well as the surrounding conditions. We write the equation.

$$Q = ms (\Delta\theta) \quad \dots (1)$$

Where  $\Delta\theta$  is the change in temperature  $m$  is the mass of the body,  $Q$  is the heat supplied and  $s$  is a constant for given material under the given surrounding conditions. The constant  $s$  is called specific heat capacity of the substance.

The SI unit for specific heat capacity is  $\text{J/kg} - \text{K}$  or  $\text{J/kg} - ^\circ\text{C}$ . The specific heat capacity is also called specific heat in short. The amount of substance in the given body may also be measured in terms of number of moles. So equation (1) may be rewritten as

$$Q = nC\Delta\theta \quad \dots (2)$$

Where  $n$  is the number of moles in the sample. The constant  $C$  is called molar heat capacity.

### 4.2 MECHANICAL EQUIVALENT OF HEAT

Since we define heat as energy in transit hence it can be compared to work. In fact we do have a relation, which tells us that how much mechanical energy is equivalent to work. If mechanical work  $W$  joule produces the same temperature difference as is produced by a heat  $H$  calories then we write.

$$W = JH$$

Where  $J$  is called mechanical equivalent of heat. The unit of  $J$  is joule/calorie.

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### 4.3 HEAT CAPACITY & WATER EQUIVALENT

The quantity  $ms$  is called heat capacity of the body. Its unit is J/k. The mass of water having the same heat capacity as a given body is called water equivalent of the body. The unit of water equivalent is kg.

**Latent Heat:** Apart from raising the temperature, heat supplied to a body may cause a phase change such as solid to liquid or liquid to vapour. During the process of melting or vaporisation, the temperature remains constant. The amount of heat needed to melt a solid of mass  $m$  may be written as

$$Q = mL \quad \dots (3)$$

Where  $L$  is a constant for a given material for the given surrounding conditions. This constant  $L$  is called specific latent heat of fusion, commonly referred as latent heat of fusion. The equation (3) is also valid when a liquid, changes into vapour and the constant  $L$  in this case is called specific latent heat of vapourisation commonly referred as latent heat of vaporisation.

#### Illustration 9

A lead ball at  $30^\circ\text{C}$  is dropped from a height of 6.2 km. The ball is heated due to the air resistance and it completely melts just before reaching the ground. The molten substance falls slowly on the ground. Calculate the latent heat of fusion of lead. Specific heat capacity of lead =  $126 \text{ J/kg-}^\circ\text{C}$  and melting point of lead =  $330^\circ\text{C}$ . Assume that any mechanical energy lost is used to heat the ball. Use  $g = 10 \text{ m/s}^2$ .

#### Solution:

The initial gravitational potential energy of the ball

$$\begin{aligned} &= mgh \\ &= m \times (10 \text{ m/s}^2) (6.2 \times 10^3 \text{ m}) \\ &= m \times (6.2 \times 10^4 \text{ m}^2/\text{s}^2) = m \times (6.2 \times 10^4 \text{ J/kg}). \end{aligned}$$

All this energy is used to heat the ball as it reaches the ground with a small velocity. Energy required to take the ball from  $30^\circ\text{C}$  to  $330^\circ\text{C}$  is

$$\begin{aligned} &m \times (126 \text{ J/kg-}^\circ\text{C}) \times (300^\circ\text{C}) \\ &= m \times 37800 \text{ J/kg} \end{aligned}$$

and energy required to melt the ball at  $330^\circ\text{C} = mL$

where  $L$  = latent heat of fusion of lead

Thus,

$$m \times (6.2 \times 10^4 \text{ J/kg}) = m \times 37800 \text{ J/kg} + mL$$

$$\text{or, } L = 2.4 \times 10^4 \text{ J/kg.}$$

#### Illustration 10

How should one kg of water at  $5^\circ\text{C}$  be so divided that one part of it when converted into ice at  $0^\circ\text{C}$ , would by this change of state provide a quantity of heat that would be sufficient to vaporise the other part?

#### Solution:

Initially 1000 g of water is at  $5^\circ\text{C}$ .

Let  $m$  gram of it be cooled to ice at  $0^\circ\text{C}$ .

Heat released due to this =  $(m \times 1 \times 5) + (m \times 80) = 5m + 80m = 85m \text{ cal.}$

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The heat required by  $(1000 - m)$  g of water at  $5^\circ\text{C}$  to become steam at  $100^\circ\text{C}$

$$= (1000 - m) (100 - 5) + (1000 - m) 540 \text{ cal}$$

$$= (1000 - m) (95 + 540) \text{ cal}$$

$$= (1000 - m) (635) \text{ cal}$$

Now,  $85 m = (1000 - m) 635$  or,  $720 m = 635 \times 1000$

$$m = \frac{635 \times 1000}{720} = 881.9 \text{ g}$$

Hence **881.9 g** of water by turning into at  $0^\circ\text{C}$  will supply heat to evaporate **118.1 g** of water.

## 5. HEAT TRANSFER

Heat can be transferred from one place to another by three different methods, namely, conduction convection and radiation. Conduction usually takes place in solids, convection in liquids and gases, and no medium is required for radiation.

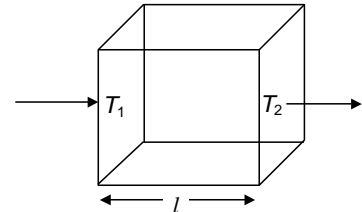
### 5.1 THERMAL CONDUCTION

This is the mode of heat transfer in which particles of the medium do not leave their position permanently i.e., they keep vibrating at their own places and transfer energy from one part of the medium to the other part.

#### 5.1.1 Thermal conductivity

The ability of a material to conduct heat is measured by thermal conductivity of the material.

Consider a solid bar of thickness  $l$  and cross-sectional area  $A$ . Let the left side of the bar is at temperature  $T_1$  and right side at  $T_2$  where  $T_1 > T_2$ .



Sooner or later the temperature of each cross-section becomes constant with respect to time. This is known as steady state. If  $\Delta Q$  amount of heat flows through a cross-section in time  $\Delta t$  under steady state conditions then the rate of heat transfer is given by

$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{l}$$

where  $K$  is a constant for the material of the rod and is called thermal conductivity of the material of the rod. It's unit is  $\text{J/s-m-K}$  or  $\text{Cal-s-m-K}$ .

If the area of cross-section is not uniform or if the steady state conditions are not reached, then the equation can only be applied to a thin layer of material perpendicular to the heat flow. If  $A$  be the area of cross-section at a place,  $dx$  be a small thickness and  $dT$  be the temperature difference across the layer of thickness  $dx$  then the rate of heat transfer is

$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

The negative sign indicates that  $\frac{dT}{dx}$  is negative along the direction of flow.

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## 5.1.2 Thermal Resistance

In the equation,  $\frac{\Delta Q}{\Delta t} = \frac{T_1 - T_2}{\left(\frac{l}{KA}\right)}$

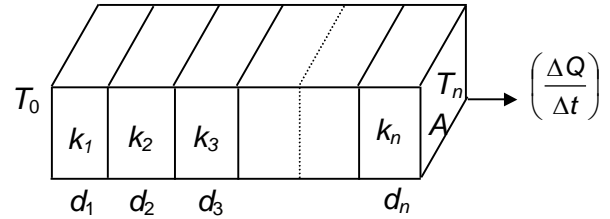
If we treat  $\frac{\Delta Q}{\Delta t}$  as thermal current and  $T_1 - T_2$  as the temperature difference due to which the thermal current is flowing, then quantity  $\frac{l}{KA}$  can be treated as somewhat similar to electrical resistance (think  $i = \frac{v}{R}$ , Ohm's law) and we call it thermal resistance. So thermal resistance of a body depends upon its length, area cross-section and thermal conductivity of the material of the body.

## 5.2 COMBINATION OF LAYERS

### 5.2.1 Layers in series

Let us consider a multiplayer medium consisted of layers having area of cross-section  $A$  each and width  $d_1, d_2, \dots, d_n$  as shown. The thermal conductivities of various layers are  $K_1, K_2, \dots, K_n$ . If a temperature difference of magnitude  $T_0 - T_n$  is maintained between the near and far faces of the multiplayer (where  $T_0$  and  $T_n$  are the temperatures of the near and far faces respectively), The rate of heat flow is given by

$$\frac{\Delta Q}{\Delta t} = \frac{A(T_0 - T_n)}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \dots + \frac{d_n}{k_n}}$$



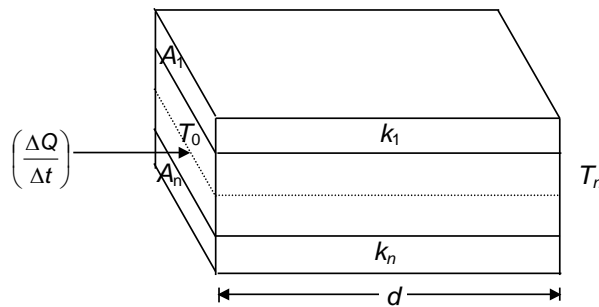
Which can be obtained directly using the concept of thermal resistance

### 5.2.2 Layers in parallel

Now let us consider a multiplayer, this time having same width  $d$  but different areas of cross-section  $A_1, A_2, \dots, A_n$  as shown. The thermal conductivities of the various layers are  $k_1, k_2, \dots, k_n$ . If a temperature difference of magnitude  $(T_0 - T_n)$  is maintained between the near and far faces of the multiplayer (where  $T_0$  and  $T_n$  are the temperatures of near and far faces respectively), the rate of heat flow is given by

$$\frac{\Delta Q}{\Delta t} = \frac{(T_0 - T_n)(k_1 A_1 + k_2 A_2 + \dots + k_n A_n)}{d}$$

Which can be obtained directly using the concept of thermal resistance.



## 5.3 CONVECTION

It is the mode of heat transfer by actual motion of matter. It is therefore possible only in fluids. Convection can be natural or forced. Gravity plays an important role in natural convection. Convection involves bulk transport of different parts of the liquid. We can see many examples of heat transfer by convection in our day to day life, right from the circulation of blood in our body to the intricacies of monsoon in India are all examples of convection. However the mathematical analysis of convection is beyond the scope of our syllabus, so we are not discussing the mathematics of convection.

## 5.4 HEAT TRANSFER THROUGH RADIATION

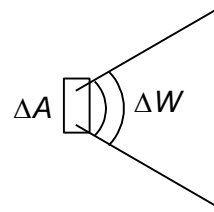
We have already defined radiation as the mode of heat transfer from one body to other even in the absence of any medium. Before discussing the laws governing radiation we have to understand following terms:

**Black body:** A black body may be defined as the one that completely absorbs the radiations of all wavelengths incident on it. The concept of black body is an idealised concept and in practice no substance behaves like black body.

**Emissive power ( $E$ ):** The emissive power of a body is defined as the energy radiated per unit area per unit time, per unit solid angle perpendicular to the area. So if energy radiated by the area  $\Delta A$  of the surface in the solid angle  $\Delta W$  in time  $\Delta t$  be  $\Delta U$  the emissive power of the body is given by

$$E = \frac{\Delta U}{(\Delta A) (\Delta W) (\Delta t)}$$

The unit of emissive power is  $\text{J/s/m}^2$



**Absorptive power ( $a$ ):** Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body.

$$\text{So } a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the radiation incident on a black body is absorbed, the absorptive power of black body is unity.

### 5.4.1. Kirchhoff's law of radiation

Kirchhoff's law states that ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. Kirchhoff's law in a way tells that a good emitter is a good absorber and a good absorber is a good emitter.

$$\text{So } \frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$$

### 5.4.2 Stefan's law

The energy emitted per second per unit area of a black body is proportional to the fourth power of absolute temperature of the emitter, and is given by

$$E = \sigma T^4 \quad \dots (i)$$

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where  $\sigma$  is a constant known as Stefan's constant and its value is  $5.67 \times 10^{-8} \text{ J/m}^2\text{-s-K}^4$  For any other body

$$E = e\sigma T^4 \quad \dots (ii)$$

where  $e$  is emissivity which is defined for a surface as the ratio of the emissive power of the surface to the emissive power of black body at the same temperature. The value of  $e$  lies between 0 and 1, it is zero for perfectly reflecting surface and unity for black body.

Using Kirchhoff's law

$$\frac{E(\text{body})}{E(\text{blackbody})} = a$$

Using (i) and (ii) we get

$$\frac{E(\text{body})}{E(\text{blackbody})} = e = a$$

So emissivity and absorptive power have the same value.

**Net Loss of Heat:** The rate at which a body radiates is determined by the temperature of the body but the rate at which it absorbs energy by radiation depends on the temperature of surroundings.

So for a body at a temperature of  $T_1$  surrounded by walls at temperature  $T_2$  the net rate of loss of energy by radiation per unit area per second by radiation is given by

$$E_{\text{net}} = e\sigma (T_1^4 - T_2^4)$$

### 5.4.3 Newton's law of cooling

For a small temperature difference between body and surrounding, the rate of cooling of the body is directly proportional to the temperature difference. This can be easily derived taking the value of  $T$  to be really close to temperature of surrounding  $T_0$ . Mathematically rate of fall of temperature with respect to time is given as

$$\frac{dT}{dt} = -bA(T - T_0)$$

where  $b$  is a constant which depends on the nature of the surface involved and surrounding conditions  $A$  is the surface area of the body and  $(T - T_0)$  is the instantaneous temperature difference between the body and the surrounding. This is known as Newton's law of cooling. The negative sign indicates that temperature decrease in time. This equation can be used in Celsius scale as well because difference in temperature is same for absolute as well as for Celsius scale.

### 5.4.4 Spectral distribution of energy in a black body radiation

A black body emits radiations of different wavelength, however the energy content of radiations of different wavelength is not equal. The relative intensities of different wavelengths depend upon the temperature of radiator. The distribution of energy among the various wavelengths in blackbody radiation was studied by Lummer and Pringsheim and following information were obtained.

(1) For a given temperature the graph between energy and wavelength is a continuous curve spreading from a minimum to a maximum and has wavelength ( $\lambda_m$ ) where the emission is maximum.

(2) as the temperature increases the wavelength at which the emission is maximum, shifts to a lower value.

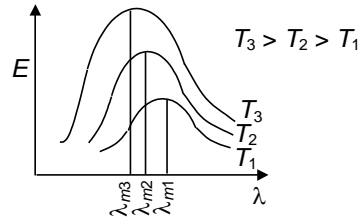
i.e.,  $\lambda_m T = b$  where  $b$  is a constant having value  $2.9 \times 10^{-3} \text{ m-K}$

This is called Wein's displacement law

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(3) as the temperature increases, the area enclosed by the curve also increases.

(4) Area under the curve is proportional to  $T^4$  and hence verifies Stefan's law.



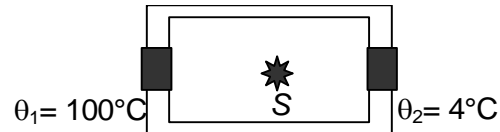
### Illustration 11

A closed cubical box is made of perfectly insulating material and the only way for heat to enter or leave the box is through two solid cylindrical metal plugs, each of cross-sectional area  $12 \text{ cm}^2$  and length  $8 \text{ cm}$  fixed in the opposite walls of the box. The outer surface of one plug is kept at a temperature of  $100^\circ\text{C}$  while the outer surface of the other plug is maintained at a temperature of  $4^\circ\text{C}$ . The thermal conductivity of the material of the plug is  $2.0 \text{ W/m-}^\circ\text{C}$ . A source of energy generating  $13 \text{ W}$  is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface.

#### Solution:

The situation is shown in figure. Let the temperature inside the box be  $\theta$ . The rate at which heat enters the box through the left plug is

$$\frac{\Delta Q_1}{\Delta t} = \frac{KA(\theta_1 - \theta)}{x}$$



The rate of heat generation in the box =  $13 \text{ W}$ . The rate at which heat flows out of the box through the right plug is

$$\frac{\Delta Q_1}{\Delta t} + 13 \text{ W} = \frac{\Delta Q_2}{\Delta t}$$

$$\text{or, } \frac{KA}{x}(\theta_1 - \theta) + 13 \text{ W} = \frac{KA}{x}(\theta - \theta_2)$$

$$\begin{aligned} \text{or, } 2 \frac{KA}{x} \theta &= \frac{KA}{x}(\theta_1 + \theta_2) + 13 \text{ W} \quad \text{or, } \theta = \frac{\theta_1 + \theta_2}{2} + \frac{(13 \text{ W})x}{2KA} \\ &= \frac{100^\circ\text{C} + 4^\circ\text{C}}{2} + \frac{(13 \text{ W}) \times 0.08 \text{ m}}{2 \times (2.0 \text{ W/m-}^\circ\text{C})(12 \times 10^{-4} \text{ m}^2)} = 52^\circ\text{C} + 216.67^\circ\text{C} \approx 269^\circ\text{C}. \end{aligned}$$

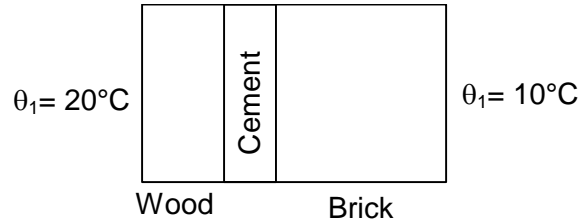
### Illustration 12

An electric heater is used in a room of total wall area  $137 \text{ m}^2$  to maintain a temperature of  $20^\circ\text{C}$  inside it, when the outside temperature is  $-10^\circ\text{C}$ . The walls have three different layers of materials. The innermost layer is of wood of thickness  $2.5 \text{ cm}$ , the middle layer is of cement of thickness  $1.0 \text{ cm}$  and the outermost layer is of brick of thickness  $25.0 \text{ cm}$ . Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are  $0.125 \text{ W/m-}^\circ\text{C}$ ,  $1.5 \text{ W/m-}^\circ\text{C}$  and  $1.0 \text{ W/m-}^\circ\text{C}$  respectively.

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### Solution:

The situation is shown in figure. The thermal resistances of the wood, the cement and the brick layers are



$$R_w = \frac{1}{K} \frac{x}{A}$$

$$= \frac{1}{0.125 \text{ W/m} - ^\circ\text{C}} \frac{2.5 \times 10^{-2} \text{ m}}{137 \text{ m}^2}$$

$$= \frac{0.20}{137} ^\circ\text{C/W},$$

$$R_C = \frac{1}{1.5 \text{ W/m} - ^\circ\text{C}} \frac{1.0 \times 10^{-2} \text{ m}}{137 \text{ m}^2} = \frac{0.0067}{137} ^\circ\text{C/W}$$

$$\text{and } R_B = \frac{1}{1.0 \text{ W/m} - ^\circ\text{C}} \frac{25.0 \times 10^{-2} \text{ m}}{137 \text{ m}^2} = \frac{0.25}{137} ^\circ\text{C/W}$$

As the layers are connected in series, the equivalent  $R = R_w + R_C + R_B$

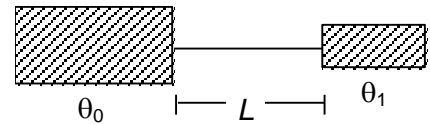
$$= \frac{0.20 + 0.0067 + 0.25}{137} ^\circ\text{C/W} = 3.33 \times 10^{-3} ^\circ\text{C/W}$$

$$\text{The heat current is } i = \frac{\theta_1 - \theta_2}{R} = \frac{20^\circ\text{C} - (-10^\circ\text{C})}{3.33 \times 10^{-3} ^\circ\text{C/W}} = \mathbf{9000 \text{ W}}$$

The heater must supply 9000 W to compensate the outflow of heat.

### Illustration 13

The figure shows a large tank of water a constant temperature  $\theta_0$  and a small vessel containing a mass  $m$  of water at an initial temperature  $\theta_1 (< \theta_0)$ . A metal rod of length  $L$ , area of cross-section  $A$  and thermal conductivity  $K$  connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel to become  $\theta_2 (\theta_1 < \theta_2 < \theta_0)$ . Specific heat capacity of water is  $s$  and all other heat capacities are negligible.



### Solution:

Suppose, the temperature of the water in the smaller vessel is  $\theta$  at time  $t$ . In the next time interval  $dt$ , a heat,  $\Delta Q$  is transferred to it where

$$\Delta Q = \frac{KA}{L} (\theta_0 - \theta) dt \quad \dots (i)$$



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This heat increases the temperature of the water of mass  $m$  to  $\theta + d\theta$  where

$$\Delta Q = ms d\theta \quad \dots (ii)$$

From (i) and (ii),

$$\frac{KA}{L}(\theta_0 - \theta) dt = ms d\theta$$

$$\text{or, } dt = \frac{Lms}{KA} \frac{d\theta}{\theta_0 - \theta} \quad \text{or, } \int_0^T dt = \frac{Lms}{KA} \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta_0 - \theta}$$

where  $T$  is the time required for the temperature of the water to become  $\theta_2$ .

$$\text{Thus, } T = \frac{Lms}{KA} \ln \frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}$$

### Illustration 14

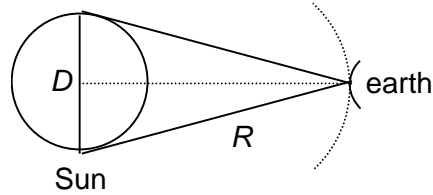
The earth receives solar radiation at a rate of  $8.2 \text{ J/cm}^2 - \text{minute}$ . Assuming that the sun radiates like a blackbody. Calculate the surface temperature of the sun. The angle subtended by the sun on the earth is  $0.53^\circ$  and the Stefan constant  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 - \text{K}^4$ .

### Solution:

Let the diameter of the sun be  $D$  and its distance from the earth be  $R$ . From the question,

$$\frac{D}{R} \approx 0.53 \times \frac{\pi}{180}$$

$$= 9.25 \times 10^{-3} \quad \dots (i)$$



The radiation emitted by the surface of the sun per unit time is  $4\pi \left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4$

At distance  $R$ , this radiation falls on an area  $4\pi R^2$  in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4} \left[ \frac{D}{R} \right]^2$$

$$\text{Thus, } \frac{\sigma T^4}{4} \left[ \frac{D}{R} \right]^2 = 8.2 \text{ J/m}^2 - \text{minute}$$

$$\text{Or, } \frac{1}{4} \times \left( 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 - \text{K}^4} \right) T^4 \times (9.25 \times 10^{-3})^2 = \frac{8.2}{10^{-4} \times 60} \frac{\text{W}}{\text{m}^2}$$

$$\text{or, } T = 5794 \text{ K} \approx 5800 \text{ K}$$

## 6. THERMODYNAMICS

### 6.1 THE FIRST LAW OF THERMODYNAMICS

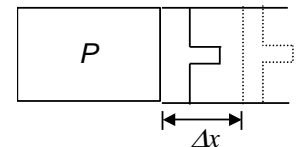
The first law of thermodynamics states that the heat supplied to a thermodynamics system gets used in increasing the internal energy and doing external work. Mathematically this law can be expressed as

$$\Delta Q = \Delta U + \Delta W$$

where  $\Delta Q$  is the heat given to the gas and  $\Delta W$  is the work done by the gas. The total energy of the gas must be conserved. The conventions which we follow for  $\Delta Q$  and  $\Delta W$  are,

- (a) if work is done by the system it is positive
- (b) if work is done on the system it is negative
- (c) if heat is given to the system it is positive
- (d) if heat is given by the system it is negative

**Work done by a gas:** Consider a gas contained in a cylinder of cross-sectional area  $A$  fitted with a movable piston. Let the pressure of the gas is  $p$ . The force exerted by the gas on the piston is  $pA$  in the outward direction. If the gas expands by a small distance  $\Delta x$  then the work done by the gas on the piston is



$$\Delta W = (PA) (\Delta x) = P\Delta V$$

where  $\Delta V$  is the change in volume of the gas. We can divide the entire process of expansion in small steps and add work done in each step. So the total work done by the gas in the process is

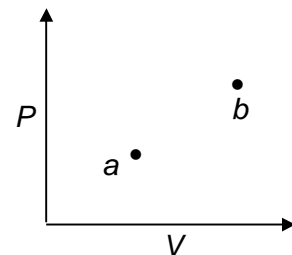
$$W = \int_{v_1}^{v_2} P dV$$

If we show the process in a diagram, the work done is equal to the area bounded by the  $p$ - $v$  curve,  $v$ -axis and ordinates at  $v_1$  &  $v_2$ .

**Specific heat capacities for gases:** The specific heat capacity of a substance is defined as the heat supplied per unit mass of the substance for the unit rise in the temperature. If an amount  $\Delta Q$  of heat is given to a mass  $m$  of the substance and its temperature rises by  $\Delta T$  then the specific heat capacity is given by

$$S = \frac{Q}{m \Delta T}$$

This definition applied for all substances. Let us consider one mole (it is convenient to describe amount of gas in mole) of an ideal gas at pressure  $p_1$ , volume  $v_1$ , and temperature  $T_1$ . We represent the state of the gas by the point  $a$  in the  $p$ - $v$  diagram as shown in the figure. Let us consider another point  $b$  in the  $p$ - $v$  diagram at which the pressure is  $p_2$ , volume is  $v_2$  and temperature is  $(T_1 + 1)$ . If we stick to our definition, then amount of heat given to gas to take it from  $a$  to  $b$  is specific heat of the gas. Now according to 1<sup>st</sup> law of thermodynamics



$$\Delta Q = \Delta U + \Delta W$$

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Since  $\Delta U$  depends only on temperature difference it will be a constant irrespective of the path the gas takes in reaching from  $a$  to  $b$ .  $\Delta W$  being path dependent will be different for different processes.

So in order to increase the temperature by  $1^\circ\text{C}$ , the heat supplied will be different for different processes. So we conclude that specific heat of a gas is path dependent. In order to define specific heat for gases we will first have to define the path. So specific heat for a gas can have infinite number of values depending upon the process by which temperature increase has been accomplished. So we can say that a gas can have infinite number of specific heats based on the process. However the most common processes in which temperature increase takes place is defined as process at constant volume. So specific heat for unit mass of gas at constant pressure is defined as  $c_p$  and at constant volume  $c_v$ . Let the volume of a gas of mass  $m$  is kept constant and heat  $\Delta Q$  is given to it. If its temperature rises by  $\Delta T$ , the specific heat capacity is given by

$C_V = \left( \frac{\Delta Q}{m \Delta T} \right)_{\text{constant volume}}$  and is called specific heat capacity of the gas at constant volume. Now suppose the

pressure of the gas is constant, if temperature of  $m$  mass of a gas is increased by  $\Delta T$  by giving heat  $\Delta Q$ , then specific heat capacity is given by

$C_p = \left( \frac{\Delta Q}{m \Delta T} \right)_{\text{constant pressure}}$  and is called specific heat capacity of the gas at constant pressure. If we take the

amount of the gas 1 mole instead of its mass  $m$  then corresponding quantities are called molar specific heat of the gas at constant volume and at constant pressure respectively. These quantities are represented by  $C_P$  and  $C_V$  and  $C_P - C_V = R$  where  $R$  is universal gas constant. The unit of specific heat capacity is  $\text{J/kg} - \text{K}$  and that of molar heat capacity is  $\text{J/mol} - \text{K}$ .

## 7. DEGREES OF FREEDOM

The degrees of freedom of a system is the number of independent quantities which must be known to specify the position and configuration of the system completely

For example, if an ant moves along straight line we say that it has one degree of freedom. If the same ant moves in a plane surface, it has two degrees of freedom since we require  $x$  and  $y$  co-ordinates to specify its position at any instant. A bird moving in free space has 3 degrees of freedom.

A monatomic gas molecule has three degrees of freedom. As a point mass it can move along any of three mutually perpendicular directions viz  $x$ ,  $y$  or  $z$  axis. Its kinetic energy is the sum of the three terms

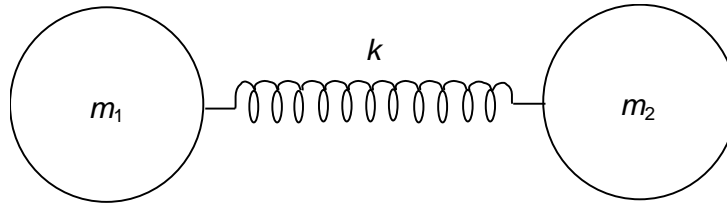
$$\frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2$$

Let us consider a rigid body (not a point mass). Its moment of inertia about each of the three mutually perpendicular axes passing through its centre of mass is not negligible. Then it has six degrees of freedom (a) its centre of mass has three translational degrees of freedom of motion. (b) In addition to the translational motion it can rotate about any of the three mutually perpendicular axes and hence it has three degrees of freedom of rotational motion. Its total kinetic energy is the sum of six terms.

$$K.E. = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 + \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$

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In addition to the above it is possible for a body to have vibrational motion about its centre of mass.



Let us consider two spheres of masses  $m_1$  and  $m_2$  connected by a spring of spring constant  $k$ . The two spheres can vibrate about their common centre of mass. Its potential energy of vibrations is  $\frac{1}{2} kx^2$  and the kinetic energy

is  $\frac{1}{2} \mu x^2$ , where  $\mu = \frac{m_1 m_2}{m_1 + m_2}$  is its reduced mass. This type of body has three degrees of freedom of translational

motion, two degrees of freedom of rotational motion and one degree of freedom of vibrational motion. The number of degrees of freedom of rotational motion is two and not three because its moment of inertia about a line passing the centres of the two spheres is small compared to the moment of inertia about the other two axes.

If the two spheres are connected by a rigid rod instead of a spring the total number of degrees of freedom is five, three translational and two rotational. Diatomic molecules can be considered as two spheres joined by a rigid rod just like a dumb-bell. This has five degrees of freedom, three translational and two rotational.

For a triatomic molecule, the number of degrees of freedom could be five or six, three translational and two rotational or three translational and three rotational, depending on whether the atoms are arranged linearly or not.

## 8. LAW OF EQUIPARTITION OF ENERGY

For a system in equilibrium at absolute temperature  $T$ , the average energy per particle corresponding to each degrees of freedom is  $\frac{1}{2} kT$ .

Where  $k$  is Boltzman constant.

Since we already know that internal energy of an ideal gas is entirely the kinetic energy of its molecules.

So internal energy of one mole of an ideal gas having  $f$  degrees of freedom.

$$U = Nf \frac{1}{2} kT = \frac{1}{2} fRT$$

Where  $R = kN$  where  $N$  is Avagadro number. If the gas is heated at constant volume until its temperature rises by  $dT$ . Then heat given

$$dQ = (1) C_V dT$$

Since  $dQ = dU$ , as volume is constant hence  $dW = 0$

So  $dU = C_V dT$

$$C_V = \frac{dU}{dT} = \frac{1}{2} fR$$

$$C_P = C_V + R = \left( \frac{f}{2} + 1 \right) R$$

## KTG, HEAT AND THERMODYNAMICS

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$$\text{Ratio of the specific heat } \gamma = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

For a monoatomic gas which has only 3 (translational) degrees of freedom  $f = 3$

$$C_V = \frac{3}{2}R$$

$$C_P = \left(1 + \frac{3}{2}\right)R = \frac{5}{2}R$$

$$\gamma = 1 + \frac{2}{f} = \frac{5}{3} = 1.67$$

For a diatomic gas which has 5 degrees of freedom (3 translational and 2 rotational), as moment of inertia about the line joining the two atoms is negligibly small and so is the rotational energy about that axis is zero So  $f = 5$

$$C_V = \frac{5}{2}R$$

$$C_P = \left(1 + \frac{5}{2}\right)R = \frac{7}{2}R$$

$$\gamma = 1 + \frac{2}{f} = 1.40$$

For a polyatomic gas which has 6 degrees of freedom (3 translational and 3 rotational)

$$f = 6$$

$$C_V = 3R$$

$$C_P = (3R + R) = 4R$$

$$\gamma = \frac{4}{3} = 1.33$$

However if the atoms of the molecule are arranged linearly like the molecule of  $CO_2$ , then degrees of freedom are again 5 and its  $C_V$ ,  $C_P$  and  $\gamma$  will be similar to those of the diatomic gases.

## 9. THERMODYNAMIC PROCESSES

### 9.1 ISOCHORIC PROCESS

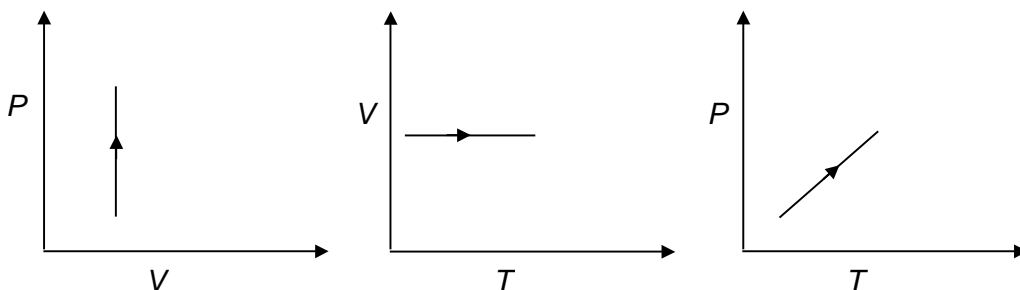
Isochoric process is the process in which volume of the gas remains constant. Since in this process volume remains constant, the work done in this process is equal to zero. Applying the 1<sup>st</sup> law of thermodynamics to this process we get

$$\Delta Q = \Delta U + \Delta W = \Delta U$$

So heat exchange in this process takes place at the expense of the internal energy of the system.

## KTG, HEAT AND THERMODYNAMICS

The  $P$ - $V$ ,  $V$ - $T$  and  $P$ - $T$  diagram for Isochoric process will be like the curves given below



### 9.2 ISOBARIC PROCESS

Isobaric process is the process in which the pressure of the gas remains constant. In this case the work done by the gas in changing its volume from  $V_1$  to  $V_2$  at constant pressure  $p$  is given by

$$W = \int_{V_1}^{V_2} P dV = P \int_{V_1}^{V_2} dV = P (V_2 - V_1)$$

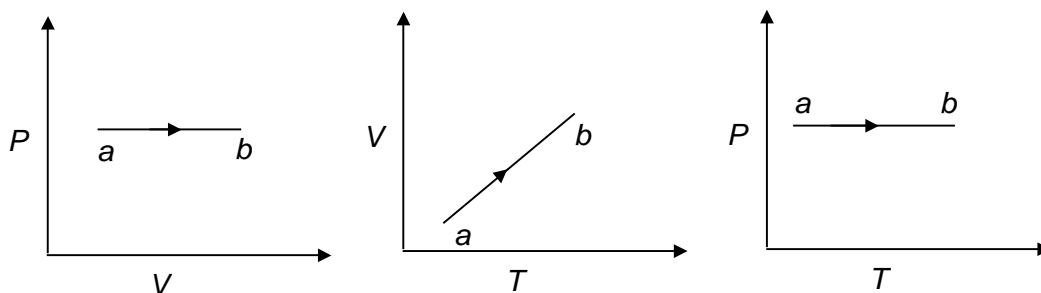
Applying 1<sup>st</sup> law of thermodynamics to isobaric processes we get

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \Delta U + P \int_{V_1}^{V_2} dV$$

$$\Delta Q = \Delta U + P (V_2 - V_1)$$

The  $P$ - $V$ ,  $V$ - $T$  and  $P$ - $T$  diagram for Isobaric process will be like the curves given below

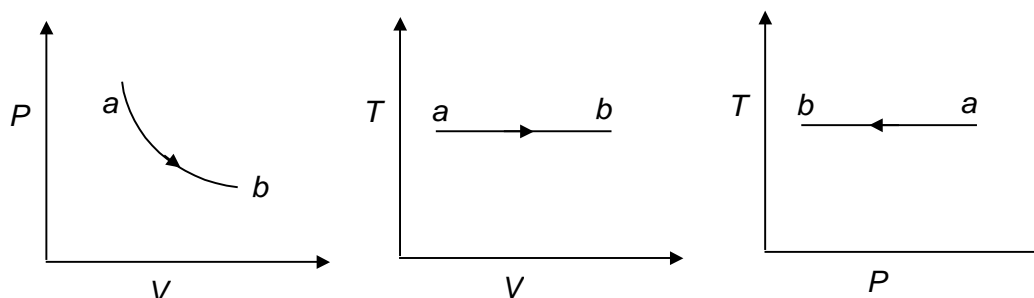


### 9.3 ISOTHERMAL PROCESS

An expansion or contraction of a gas, which occurs without change of temperature, is said to be isothermal. During isothermal process, the temperature  $T$  remains constant while the volume  $V$  and pressure  $P$  change. Since at constant temperature, there is no change in the internal energy of the gas the quantity  $\Delta U$  in the equation,  $\Delta Q = \Delta U + \Delta W$  is zero and hence  $\Delta Q = \Delta W$  for an isothermal process. So heat supplied during an isothermal expansion is equal to the work done by the gas in expanding. Similarly, during an isothermal compression there must be a continual transfer of heat away from the gas in order to keep the temperature constant and the heat transferred is equal to the work done in compressing the gas. Isothermal changes usually take place very slowly. The equation representing an isothermal process is  $PV = \text{constant}$  or  $P_1V_1 = P_2V_2$

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The  $P$ - $V$ ,  $T$ - $V$  and  $T$ - $P$  diagram for Isothermal process will be like the curves given below



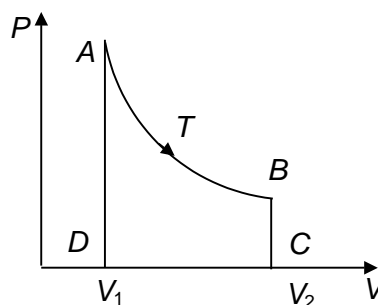
### 9.3.1 Work done during Isothermal Process

From the equation of an isothermal process we know,  $PV = nRT$  (constant). The work  $dW$  done during a small expansion  $dV$  can be written as  $dW = P dV = \frac{nRT}{V} dV$  and consequently during a finite expansion from a volume  $V_1$  to volume  $V_2$  the work done is given by

$$W = \int P dV = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= nRT \ln \left( \frac{V_2}{V_1} \right),$$

where  $n$  is the number of moles of gas involved.



When converted from natural logarithms to those of base 10, the expression for the work done by a gas during an isothermal expansion becomes

$$W = 2.303 nRT \log_{10} \left( \frac{V_2}{V_1} \right)$$

Since  $\frac{V_2}{V_1} = \frac{P_1}{P_2}$        $W = 2.303 nRT \log_{10} \left( \frac{P_1}{P_2} \right)$

Applying first law of thermodynamics to Isothermal processes we get

$$\Delta Q = \Delta U + \Delta W, \text{ but } \Delta U = 0 \text{ for Isothermal processes.}$$

Hence,  $\Delta Q = \Delta W$  for Isothermal processes.

$$\Delta Q = 2.303 nRT \log_{10} \left( \frac{V_2}{V_1} \right)$$

## 9.4 ADIABATIC PROCESS

When a gas expands or contracts without exchanging heat with the surrounding the process is called adiabatic. Such a process is possible if the gas contained in a cylinder is completely insulated by a perfect heat insulator so that no heat is absorbed from the surroundings during expansion or given off to the surroundings during compression. In the relation, for an adiabatic change

$$\begin{aligned}\Delta Q &= \Delta U + \Delta W \\ \Delta Q &= 0 \text{ and hence} \\ -\Delta U &= \Delta W\end{aligned}$$

The reduction in the internal energy of the gas (due to which the temperature falls) is equal to the work done during an adiabatic expansion. Again during an adiabatic compression the work done on the gas causes its temperature rise. Adiabatic processes are generally very fast.

### 9.4.1 Equations of Adiabatic Process

Consider one mole of an ideal gas undergoing adiabatic process. For the process we can write

$$\begin{aligned}\Delta U + \Delta W &= 0 \\ C_V dT + PdV &= 0 \quad \dots (i)\end{aligned}$$

Since equation of state for one mole of an ideal gas is given by

$$PV = RT \quad \dots (ii)$$

where  $P$ ,  $V$ ,  $T$  are pressure, volume and temperature of the gas.

Differentiating (ii) we get

$$dT = \frac{PdV + VdP}{R}$$

$$\text{So } C_V \left( \frac{PdV + VdP}{R} \right) + PdV = 0$$

$$\Rightarrow PdV \left( 1 + \frac{C_V}{R} \right) + VdP \left( \frac{C_V}{R} \right) = 0$$

$$\Rightarrow C_P \frac{dV}{V} + C_V \frac{dP}{P} = 0$$

$$\Rightarrow C_P \frac{dV}{V} + C_V \frac{dP}{P} = 0$$

$$\Rightarrow \frac{C_P}{C_V} \frac{dV}{V} + \frac{dP}{P} = 0$$

$$\Rightarrow \gamma \frac{dV}{V} + \frac{dP}{P} = 0$$



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Integrating we get

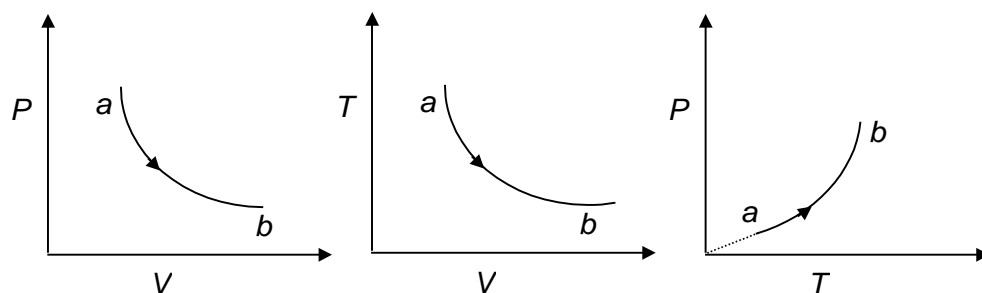
$$\gamma \ln V + \ln P = \text{constant}$$

or  $PV^\gamma = \text{constant}$

which is the equation of adiabatic process. Putting  $P = \frac{RT}{V}$

we get  $TV^{\gamma-1} = \text{constant}$  which is the equation of adiabatic process in terms of temperature and volume.

The  $P$ - $V$ ,  $T$ - $V$  and  $P$ - $T$  diagrams for adiabatic process will be like the curves given below



### 9.4.2 Work done during an Adiabatic Process

The work done by  $n$  moles of an ideal gas during adiabatic expansion is equal to the loss of its internal energy

$$nC_V(T_1 - T_2) = \frac{nR}{\gamma - 1}(T_1 - T_2) = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

where  $C_V$  is the molar heat capacity at constant volume.

Applying 1<sup>st</sup> law of thermodynamic to adiabatic processes we get

$$\Delta Q = \Delta U + \Delta W, \text{ but } \Delta Q = 0 \text{ for adiabatic processes}$$

Hence,  $\Delta U = -\Delta W$

### 9.5 ISOTHERMAL AND ADIABATIC CURVE

The relation between the pressure and volume of a gas can be represented graphically. The curve for an isothermal process is called isothermal curve or an isotherm and there are different isotherms for different temperatures for a given gas. A similar curve for an adiabatic process is called an adiabatic curve or adiabatic.

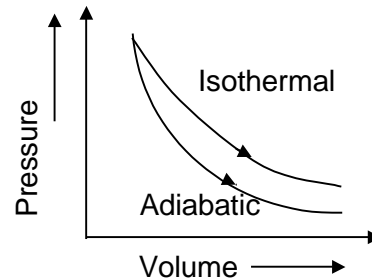
$$\text{Since } \left(\frac{dP}{dV}\right)_{\text{isothermal}} = -\frac{P}{V} \quad \text{and} \quad \left(\frac{dP}{dV}\right)_{\text{adiabatic}} = -\gamma \frac{P}{V}$$

$$\text{So } \left(\frac{dP}{dV}\right)_{\text{adiabatic}} = \gamma \left(\frac{dP}{dV}\right)_{\text{isothermal}}$$

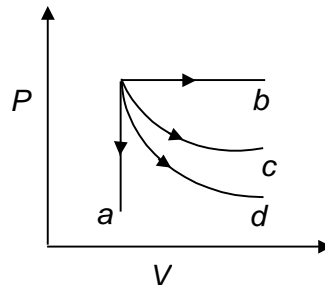
Since  $\gamma > 1$ , so adiabatic curve is steeper than the isothermal curve.

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To permit comparison between isothermal and adiabatic process an isothermal curve and an adiabatic curve of the gas are drawn on the same pressure-volume diagram starting from the same point.



**Note:** The work done in different processes can also be compared by drawing the P-V diagram for various processes as shown below.



where  $a$  is isochoric process,  $b$  is isobaric process,  $c$  is isothermal process and  $d$  is adiabatic process. Needless to say work done in an isobaric process is maximum and in adiabatic process it is minimum, for a given change in volume.

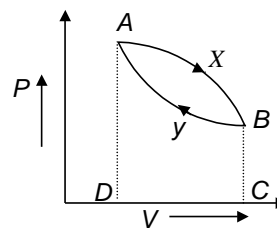
## 9.6 CYCLIC PROCESS AND EFFICIENCY CALCULATION

When a system after passing through various intermediate steps returns to its original state, then it is called a cyclic process.

Suppose a gas enclosed in a cylinder is expanded from initial state  $A$  to final state  $B$  along the path  $AXB$  as shown in the figure.

If  $W_1$  be the work done by the system during expansion, then

$$W_1 = + \text{Area } AXBCDA$$



Now let the gas is compressed from state  $B$  to state  $A$  along the path  $BYA$ , so as to return the system to the initial state. If  $W_2$  be the work done on the system during compression, then

$$W_2 = - \text{Area } BYADCB$$

According to sign convention, work done on the system during compression is negative. So net work done in the cyclic process  $AXBYA$

$$W = \text{Area } AXBCDA - \text{Area } BYADCB = \text{Area } AXBYA$$

Which is a positive quantity and hence net work will be done by the system.

So the net amount of work done during a cyclic process is equal to the area enclosed by the cyclic path. It is also evident from the figure that if the cyclic path is being traced in anticlockwise direction, the expansion curve will

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be below the compression curve and net work done during the process will be negative. This implies that the net work will now be done on the system. Applying first law of thermodynamics to cyclic process we get

$$\Delta Q = \Delta U + \Delta W$$

but  $\Delta U = 0$  for a cyclic process

So  $\Delta Q = \Delta W$

Sometimes in numerical problems we have to deal with efficiency calculation in cyclic process. Since efficiency of process is given as

$$\eta = \frac{\text{workdone}}{\text{Heat taken}}$$

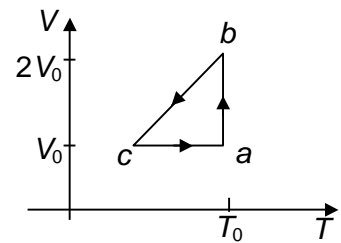
We can solve the problem quickly by replacing work done by net heat exchanged, as for a cyclic process  $\Delta Q = W$ . So if we can calculate heat exchanged in individual processes, there is no need to calculate work done separately.

### Illustration 15

A sample of an ideal gas has pressure  $p_0$ , volume  $V_0$  and temperature  $T_0$ . It is isothermally expanded to twice its original volume. It is then compressed at constant pressure to have the original volume  $V_0$ . Finally, the gas is heated at constant volume to get the original temperature (a) show the process in a  $V$ - $T$  diagram (b) Calculate the heat absorbed in the process.

### Solution:

(a) The  $V$ - $T$  diagram for the process is shown in figure. The initial state is represented by the point  $a$ . In the first step, it is isothermally expanded to a volume  $2V_0$ . This is shown in  $ab$ . Then the pressure is kept constant and the gas is compressed to the volume  $V_0$ . From the ideal gas equation,  $V/T$  is constant at constant pressure. Hence, the process is shown by a line  $bc$  which passes through the origin. At point  $c$ , the volume is  $V_0$ . In the final step, the gas is heated at constant volume to a temperature  $T_0$ . This is shown by  $ca$ . The final state is the same as the initial state.



(b) The process is cyclic so that the change in internal energy is zero. The heat supplied is, therefore, equal to the work done by the gas. The work done during  $ab$  is

$$W_1 = nRT_0 \ln \frac{2V_0}{V_0} = nRT_0 \ln 2 = p_0 V_0 \ln 2$$

Also from the ideal gas equation,

$$P_a V_a = P_b V_b \quad \text{Or, } P_b = \frac{P_a V_a}{V_b} = \frac{P_0 V_0}{2V_0} = \frac{P_0}{2}$$

In the step  $bc$ , the pressure remains constant. Hence the work done is,

$$W_2 = \frac{P_0}{2} (V_0 - 2V_0) = -\frac{P_0 V_0}{2}$$

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In the step *ca*, the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$$\begin{aligned} W &= W_1 + W_2 = p_0 V_0 [\ln 2 - 0.5] \\ &= \mathbf{0.193 p_0 V_0} \end{aligned}$$

Hence, the heat supplied to the gas is  $0.193 p_0 V_0$ .

### Illustration 16

An experiment is performed to measure the molar heat capacity of a gas at constant pressure using Regnault's method. The gas is initially contained in a cubical reservoir is size  $40 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}$  at  $600 \text{ kPa}$  at  $27^\circ\text{C}$ . A part of the gas is brought out, heated to  $100^\circ\text{C}$  and is passed through a calorimeter at constant pressure. The water equivalent of the calorimeter and its contents is  $100 \text{ g}$ . The temperature of the calorimeter and its contents increases from  $20^\circ\text{C}$  to  $30^\circ\text{C}$  during the experiment and the pressure in the reservoir decreases to  $525 \text{ kPa}$ . Specific heat capacity of water =  $4200 \text{ J/kg-K}$ . Calculate the molar heat capacity  $C_p$  from these data.

### Solution:

We have  $PV = nRT$  or,  $n = \frac{PV}{RT}$ . The amount of the gas in the reservoir is  $n_1 = \frac{P_1 V}{RT}$ . The amount of the gas in the reservoir is  $n_1 = \frac{P_1 V}{RT}$  before the gas is taken out and  $n_2 = \frac{P_2 V}{RT}$  after the gas is taken out. The amount taken out is

$$\begin{aligned} \Delta n &= n_1 - n_2 = (P_1 - P_2) \frac{V}{RT} \\ &= \frac{(600 - 525) \times 10^3 \text{ N/m}^2 \times (40 \times 10^{-2} \text{ m})^3}{(8.3 \text{ J/mol-K}) \times (300 \text{ K})} = 1.925 \text{ mol.} \end{aligned}$$

The gas is heated to  $100^\circ\text{C}$  and cools down as it passes through the calorimeter. The average final temperature of the gas is  $\frac{20^\circ\text{C} + 30^\circ\text{C}}{2} = 25^\circ\text{C}$ . Thus, the average decrease in temperature of the gas is

$$\Delta T = (100^\circ\text{C} - 25^\circ\text{C}) = 75^\circ\text{C}$$

or  $\Delta T = 75 \text{ K}$

The heat lost by the gas is

$$\Delta Q = \Delta n C_p \Delta T$$

The heat gained by the calorimeter and its contents is  $(100 \text{ g}) (4200 \text{ J/kg-K}) (30 - 20)^\circ\text{C} = 4200 \text{ J}$

Thus,  $\Delta n C_p \Delta T = 4200 \text{ J}$

$$\text{Or, } C_p = \frac{4200 \text{ J}}{(1.925 \text{ mol}) (75 \text{ K})} = \mathbf{29 \text{ J/mol-K}}$$

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### Illustration 17

Two moles of helium gas ( $\gamma = 5/3$ ) are initially at  $27^\circ\text{C}$  and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. (a) Sketch the process in a  $p - V$  diagram. (b) What is the final volume and pressure of the gas? (c) What is the work done by the gas?

### Solution:

(a) The process is shown in figure. During the part  $ab$ , the pressure is constant.

$$\text{we have, } \frac{P_a V_a}{T_a} = \frac{P_b V_b}{T_b} \quad \text{or, } T_b = \frac{V_b}{V_a} T_a = 2T_a = 600 \text{ K}$$

During the part  $bc$ , the gas is adiabatically returned to the temperature  $T_a$ . The point  $a$  and the point  $c$  are on the same isotherm. Thus, we draw an adiabatic curve from  $b$  and an isotherm from  $a$  and look for the point of intersection  $c$ . That is the final state.

(b) From the isotherm  $ac$ ,

$$P_a V_a = P_c V_c \quad \dots (i)$$

and from the adiabatic curve  $bc$ ,

$$P_b V_b^\gamma = P_c V_c^\gamma$$

$$\text{or } P_a (2V_a)^\gamma = P_c V_c^\gamma \quad \dots (ii)$$

Dividing (ii) by (i),

$$2^\gamma (V_a)^{\gamma-1} = (V_c)^{\gamma-1}$$

$$\text{or, } V_c = 2^{\gamma/(\gamma-1)} V_a = 4\sqrt{2} V_a = 113 \text{ litres.}$$

$$\begin{aligned} \text{From (i), } P_c &= \frac{P_a V_a}{V_c} = \frac{nRT_a}{V_c} \\ &= \frac{2 \text{ mol} \times (8.3 \text{ J/mol-K}) \times (300 \text{ K})}{113 \times 10^{-3} \text{ m}^3} = 4.4 \times 10^4 \text{ Pa.} \end{aligned}$$

(c) Work done by the gas in the part  $ab$

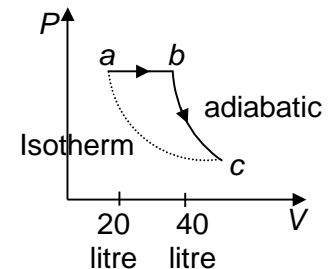
$$\begin{aligned} &= P_a (V_b - V_a) \\ &= nRT_2 - nRT_1 \\ &= 2 \text{ mol} \times (8.3 \text{ J/mol-K}) \times (600 \text{ K} - 300 \text{ K}) = 4980 \text{ J} \end{aligned}$$

$$\text{The work done in the adiabatic part } bc = \frac{P_b V_b - P_c V_c}{\gamma - 1}$$

$$= \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{4980 \text{ J}}{5/3 - 1} = 7470 \text{ J}$$

The net work done by the gas

$$= 4980 \text{ J} + 7470 \text{ J} = 12450 \text{ J}$$



# KTG, HEAT AND THERMODYNAMICS

## Fundamental Solved Examples

### Example 1.

The brass scale of a barometer gives correct reading at  $0^\circ\text{C}$ . Coefficient of thermal expansion of brass is  $0.00002/^\circ\text{C}$ . The barometer reads 75 cm at  $27^\circ\text{C}$ . What is the correct atmospheric pressure at  $27^\circ\text{C}$ ?

### Solution:

Since the metallic scale has expanded during increase in temperature a positive correction has to be applied equal to

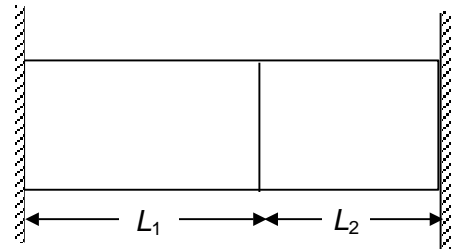
$$\begin{aligned} L_0 \alpha \Delta T &= (75 \text{ cm}) (0.00002/^\circ\text{C}) (27^\circ\text{C}) \\ &= 4050 \times 10^{-5} \text{ cm} \\ &= 0.0405 \text{ cm} \end{aligned}$$

The corrected reading =  $75 \text{ cm} + 0.0405 \text{ cm}$

$\therefore$  atmospheric pressure at  $27^\circ\text{C} = 75.0405 \text{ cm}$

### Example 2.

Two rods of different metals having the same area of cross-section  $A$  are placed end to end between two massive, as shown in the figure. The first rod has a length  $L_1$ , coefficient of linear expansion  $\alpha_1$  and Young's modulus  $Y_1$ . The corresponding quantities for the second rod are  $L_2$ ,  $\alpha_2$  and  $Y_2$ . The temperature of both the rods is now increased by  $T^\circ\text{C}$ . Find the force with which the rods act on each other (at the higher temperature) in terms of given quantities. Also find the lengths of the rods at the higher temperature. Assume that there is no change in the cross-sectional area of the rods and that the rods do not bend. There is no deformation of the walls.



### Solution:

When the rods are heated through  $T^\circ\text{C}$  they suffer expansion in length.

Increase in length of the first rod =  $L_1 \alpha_1 T$

Increase in length of the second rod =  $L_2 \alpha_2 T$

Since the rods are not allowed to expand, they develop thermal stresses which act on each other due to which they get compressed by lengths  $x_1$  and  $x_2$  respectively.

Let  $F$  be the force of compression exerted by first rod on second and  $F$  is also the force of compression exerted by the second rod on first. Now

$$x_1 = \frac{FL_1}{AY_1}$$

$$x_2 = \frac{FL_2}{AY_2}$$

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$$\text{Total compression of both rods } x_1 + x_2 = \frac{F}{A} \left( \frac{L_1}{Y_1} + \frac{L_2}{Y_2} \right)$$

$$\text{Total expansion of both rods to heating} = L_1 \alpha_1 T + L_2 \alpha_2 T$$

It is given that the total length of the rods have not been changed as the walls do not yield.

$$\therefore \quad \text{Total extension} = \text{Total compression}$$

$$L_1 \alpha_1 T + L_2 \alpha_2 T = \frac{F}{A} \left( \frac{L_1}{Y_1} + \frac{L_2}{Y_2} \right)$$

$$\text{Now, } F = \frac{AT(L_1 \alpha_1 T + L_2 \alpha_2 T)}{\frac{L_1}{Y_1} + \frac{L_2}{Y_2}} = \frac{AT(L_1 \alpha_1 + L_2 \alpha_2) Y_1 Y_2}{L_1 Y_2 + L_2 Y_1}$$

Thus the force with which the rods act on each other at the increased temperature is found.

In the compressed state at the higher temperature the length of the first rod = its original length + extension produced due to heating - compression produced by F

$$= L_1 + L_1 \alpha_1 T - x_1 = L_1 + L_1 T \left( \alpha_1 - \frac{F}{AT Y_1} \right) = L_1 + \frac{L_1 L_2 T (\alpha_1 Y_1 - \alpha_2 Y_2)}{L_2 Y_2 + L_2 Y_1}$$

$$\text{Similarly, the length of the second rod at the higher temperature is } = L_2 + \frac{L_1 L_2 T (\alpha_2 Y_2 - \alpha_1 Y_1)}{L_1 Y_2 + L_2 Y_1}$$

### Example 3.

A block of wood is floating on water at  $0^\circ \text{C}$  with a certain volume  $V$  above the water level. The temperature of water is slowly raised from  $0^\circ \text{C}$  to  $20^\circ \text{C}$ . How will the volume  $V$  change with temperature?

### Solution:

When the temperature of water at  $0^\circ \text{C}$  is increased its density increases and at  $4^\circ \text{C}$  the density becomes maximum. After  $4^\circ \text{C}$  the density decreases with temperature.

From  $0^\circ \text{C}$  to  $4^\circ \text{C}$  the volume of displaced water gradually decreases (to keep the mass of displaced water constant) and hence  $V$  gradually increases.

From  $4^\circ \text{C}$  to  $20^\circ \text{C}$  the volume of displaced water increases and hence  $V$  decreases.

### Example 4:

A one litre flask contain some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in the flask? Coefficient of linear expansion of glass =  $9 \times 10^{-6} / ^\circ \text{C}$ . Coefficient of volume expansion of mercury =  $1.8 \times 10^{-4} / ^\circ \text{C}$

### Solution:

It is given that the volume of air in the flask remains the same. This means the expansion in volume of the vessel is exactly equal to the expansion in volume of mercury.

Let  $V$  be the volume of the flask and  $v$  be the volume of mercury in it.

Now for a temperature rise  $\Delta T$

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$$V(3 \times 9 \times 10^{-6}) \Delta T = v(1.8 \times 10^{-4}) \Delta T$$

$$v = \frac{v = V \times 27 \times 10^{-6}}{1.8 \times 10^{-4}}$$

$$= \frac{27}{180} \times 1000 (V = 1000 \text{ cm}^3) = 150 \text{ cm}^3$$

### Example 5.

A calorimeter contains 0.2 kg of water at 30°C. 0.1 kg of water at 60°C is added to it, the mixture is well stirred and the resulting temperature is found to be 35°C. Find the thermal capacity of the calorimeter. Specific heat capacity of water = 4.2 kJ/kgK.

### Solution:

Let the thermal capacity of calorimeter be x.

The specific heat capacity of water = 4200 J/kgK.

The total thermal capacity of calorimeter + 0.2 kg of water originally contained in it

$$= [x + 0.2 \times 4200] \text{ J/}^\circ\text{C (pr J/K)}$$

The quantity of heat gained by calorimeter and water,

$$Q = \text{Thermal capacity} \times \text{rise in temperature}$$

$$= (x + 840) (35^\circ\text{C} - 30^\circ\text{C}) = 5(x + 840) \text{ J}$$

$$= 420 \times 25 \text{ J} = 10500 \text{ J}$$

From the principle of mixtures, heat gained by calorimeter and contents = heat lost by hot water added to it provided there is no heat loss by conduction, radiation, etc.

$$\therefore (x + 840) = 10500$$

$$x + 840 = 2100$$

$$x = 2100 - 840 = 1260 \text{ J/K}$$

### Example 6.

A copper calorimeter of water equivalent 60 g contains 600 g of water at 30°C. The calorimeter and contents are given a supply of heat at the rate of 100 cal/s. No heat is lost to the surroundings from the calorimeter. Find

(i) the time taken to bring the water to its boiling point and

(ii) the time required to boil away 50 g of water. Specific heat capacity of water = 1 cal/g°C.

Latent heat of steam = 540 cal/g.

### Solution:

The water equivalent of calorimeter = 60 g

Calorimeter + water contained in it are together equivalent to 60 + 600 = 660 g to water.

Heat required to raise the temperature of water from 30°C to 100°C

$$Q = mc\Delta T = 660 \times 1 \times 70 \text{ cal.}$$



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$$\text{Time required to get this amount of heat supply} = \frac{660 \times 70}{100} \text{ s} = 7 \text{ min } 42 \text{ s}$$

$$\begin{aligned} \text{Now to boil away 50 g of water at its boiling point the quantity of heat required} \\ = mL = 50 \times 540 \text{ cal} \end{aligned}$$

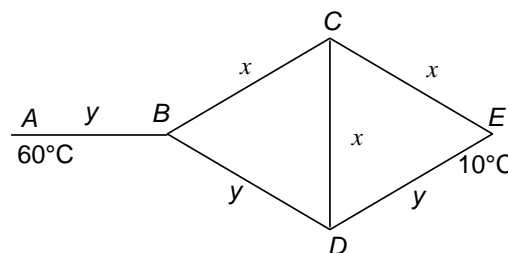
$$\text{The time required for the supply of this heat} = \frac{50 \times 540}{100} \text{ s}$$

$$= 270 \text{ s} = 4 \text{ min } 30 \text{ s}$$

$$\text{Total time required} = 7 \text{ min } 42 \text{ s} + 4 \text{ min } 30 \text{ s} = 12 \text{ min } 12 \text{ s}.$$

### Example 7.

Three rods of material x and three rods of material y are connected as shown. All the rods are of identical length and cross-sectional area. If the end A is maintained at  $60^\circ\text{C}$  and the junction E at  $10^\circ\text{C}$ , calculate the temperatures of the junctions B, C and D. The thermal conductivity of x is 0.92 c.g.s. units and that of y is 0.46 c.g.s. units.



### Solution:

Since the end A or rod AB is maintained at a temperature higher than the end B heat is conducted from A to B.

Now the total heat-entering junction B is equal to the total heat leaving it (all by conduction alone).

Let the temperatures of junctions B, C and D be  $T_1$ ,  $T_2$  and  $T_3$  respectively.

Let the cross-sectional area of each rod be A and the length of each rod be d.

$$\begin{aligned} \text{Then, heat entering joint B per second} &= \frac{k_y A (60 - T_1)}{d} \\ &= \frac{A}{d} k_y (60 - T_1) \end{aligned}$$

Heat leaving B per second = heat passing through BC + heat passing through BD

$$= K_x \cdot A \cdot \frac{(T_1 - T_2)}{d} + K_y \cdot A \cdot \frac{(T_1 - T_3)}{d}$$

$$\text{Now, } \frac{A}{d} k_y (60 - T_1) = \frac{A}{d} K_x \cdot (T_1 - T_2) + \frac{A}{d} K_y \cdot (T_1 - T_3)$$

$$\text{and } K_x = 2K_y$$

$$\therefore 60 - T_1 = 2(T_1 - T_2) + (T_1 - T_3)$$

$$\text{or } 4T_1 + 2T_2 + T_3 = 60 \quad \dots (i)$$

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Similarly for junction C

$$K_x \cdot A \cdot \frac{(T_1 - T_2)}{d} = K_x \cdot A \cdot \frac{(T_2 - 10)}{d} + K_x \cdot A \cdot \frac{(T_2 - T_3)}{d}$$

or  $T_1 - T_2 = T_2 - 10 + T_2 - T_3$

or  $T_1 + 3T_2 - T_3 = -10$  .... (ii)

For junction D

$$K_x \cdot A \cdot \frac{(T_1 - T_3)}{d} = K_x \cdot A \cdot \frac{(T_3 - T_2)}{d} + K_y \cdot A \cdot \frac{(T_3 - 10)}{d}$$

$$T_1 - T_3 = 2(T_3 - T_2) + T_3 - 10$$

$$T_1 + 2T_2 - 4T_3 = -10$$
 .... (iii)

Solving equations (1), (2) and (3), we get

$$T_1 = 30^\circ\text{C}; T_2 = 20^\circ\text{C}; T_3 = 20^\circ\text{C}.$$

Temperature of junction B =  $30^\circ\text{C}$

Temperature of junction C =  $20^\circ\text{C}$

Temperature of junction D =  $20^\circ\text{C}$

### Example 8.

Equal volumes of two liquids A and B are allowed to cool from  $80^\circ\text{C}$  to  $60^\circ\text{C}$  the times taken being 12 minutes and 16 minutes. The relative densities of two liquids are 1.1 and 1.02 respectively. The specific heat capacity of liquid A is  $2300\text{ J/kg/K}$ . Calculate the specific heat capacity of liquid B.

### Solution:

At the instant let be the temperature above that of room temperature.

Then by Newton's law of cooling  $\frac{d\theta}{dt} \propto \theta$

The rate at which a mass  $m$  of liquid loses heat  $\frac{d\theta}{dt} = -mc \cdot \frac{d\theta}{dt} = k\theta$

where  $c$  is the specific heat capacity of the liquid  $-mc \cdot \frac{d\theta}{dt} = kdt$

$$\text{Integrating, } kt = -mc \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta}$$

or  $kt = mc[\log \theta_1 - \log \theta_2]$

Let  $m_A$ ,  $m_B$  represent the respective masses and  $c_A$  and  $c_B$  the respective specific heat capacities of A and B, then

$$\frac{t_A}{t_B} = \frac{m_A c_A}{m_B c_B} \text{ since } \theta_1 \text{ and } \theta_2 \text{ are the same for both the liquids.}$$

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Since the volumes of both the liquids are the same

$$\frac{m_A}{m_B} = \frac{V \cdot \rho_A}{V \cdot \rho_B} = \frac{\text{Relative density of A}}{\text{Relative density of B}} = \frac{1.1}{1.02}$$

Substituting the known values

$$\frac{12}{16} = \frac{1.1}{1.02} \times \frac{2300}{c_B}$$

$$c_B = \frac{1.1}{1.02} \times 2300 \times \frac{16}{12} = 3307 \text{ J kg}^{-1} \text{ K}^{-1}$$

### Example 9.

A rod of length  $l$  with thermally insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as  $k = \frac{\alpha}{T}$ , where  $\alpha$  is a constant. The ends of the rod are kept at temperature  $T_1$  and  $T_2$ . Find the function  $T(x)$  where  $x$  is the distance from the end whose temperature is  $T_1$  and the heat flow density.

### Solution:

The quantity of heat flowing per unit area per second  $Q = \frac{-kdT}{dx} = \frac{\alpha}{T} \frac{dT}{dx}$

In steady state  $\int_0^l Q dx = - \int_{T_1}^{T_2} \alpha \frac{dT}{T}$

$$Q = \frac{\alpha \log_e \left( \frac{T_1}{T_2} \right)}{l} \quad \dots (i)$$

At a distance  $x$  from the end whose temperature is  $T_1$ ,

$$Q x = \left( \alpha \log_e \frac{T_1}{T_2} \right) \quad \dots (ii)$$

From equations (i) and (ii)

$$\log_e T = \frac{x}{l} \log_e \left( \frac{T_2}{T_1} \right) + \log_e T_1$$

$$\log_e T = \log_e \left[ T_1 \left( \frac{T_2}{T_1} \right)^{x/l} \right]$$

$$T(x) = T_1 \left( \frac{T_2}{T_1} \right)^{x/l}$$

$$\text{Rate flow density, } Q = \frac{\alpha \log_e (T_1/T_2)}{l}$$

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### Example 10.

A certain mass of the helium gas is at  $0^\circ\text{C}$ . It is suddenly expanded to twice its volume. Find the temperature after expansion if the ratio of the specific heats of the gas  $\gamma = \frac{5}{3}$ .

### Solution:

The temperature-volume relation of an adiabatic change can be given as  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

In this case,  $T_1 = 0^\circ\text{C} = 273\text{ K}$   $\frac{V_2}{V_1} = 2$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{\frac{5}{3}-1} = \left(\frac{1}{2}\right)^{2/3}$$

$$T_2 = T_1 \left(\frac{1}{2}\right)^{2/3} = 273 \times 0.63 = 172\text{ K} = (172 - 273)^\circ\text{C} = 101^\circ\text{C}$$

### Example 11.

A mass of 8 g of oxygen at the pressure of one atmosphere and at temperature  $27^\circ$  is enclosed in a cylinder fitted with a frictionless piston. The following operations are performed in the order given:

- (A) The gas is heated at constant pressure to  $127^\circ\text{C}$ ,
- (B) then it is compressed isothermally to its initial volume and
- (C) finally it is cooled to its initial temperature at constant volume.
- (a) What is the heat absorbed by the gas during operation (a)?
- (b) How much work is done by the gas in state (a)?
- (c) What is the work done on the gas in stage (b)?
- (d) How much heat is extracted from the gas in stage (c)?

[Specific heat capacity of oxygen  $C_v = 670\text{ J/kgK}$ ;

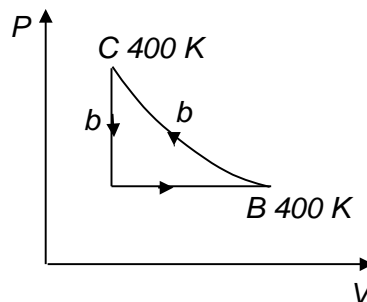
Molecular weight of oxygen = 32;  $1\text{ atm} = 1.013 \times 10^5\text{ N/m}^2$ ]

### Solution:

$$V_A = \frac{8}{32} \times 22.4 \times \frac{300}{273} \text{ litre} = \frac{560}{91} \times 10^{-3} \text{ m}^3$$

$$V_B = V_A \times \frac{400}{300} = \frac{4}{3} V_A$$

$$V_B - V_A = \frac{1}{3} V_A$$



$$(a) dQ = dU + dW = mC_v \Delta T + p(V_B - V_A)$$

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$$= \frac{8}{1000} \times 670 \times 100 + 1.013 \times 10^5 \times \left[ \frac{560 \times 10^{-3}}{3 \times 91} \right] = 536 + 207.8 = 743.8 \text{ J}$$

(b)  $dW = p(V_B - V_A) = 207.8 \text{ J}$

(c) Work done in compressing the gas isothermally  $= \frac{m}{M} RT \log_e \frac{V_B}{V_A}$

$$= \frac{8}{32} \times 8.31 \times 400 \log_e \frac{4}{3} = 831 \times 0.2877 = 239.1 \text{ J}$$

(d) Heat given out by the gas in stage (c)  $= mC_v \Delta T = \frac{8}{1000} \times 670 \times 100 = 536 \text{ J}$

### Example 12.

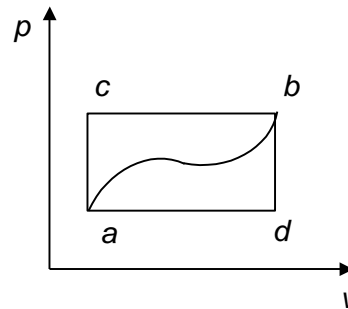
When a system is taken from state a to state b along the path acb it is found that a quantity of heat  $Q = 200 \text{ J}$  is absorbed by the system and a work  $W = 80 \text{ J}$  is done by it. Along the path adb,  $Q = 144 \text{ J}$ .

(i) What is the work done along the path adb?

(ii) If the work done on the system along the curved path ba is  $52 \text{ J}$ , does the system absorb or liberate heat and how much?

(iii) If  $U_a = 40 \text{ J}$ , what is  $U_b$ ?

(iv) If  $U_d = 88 \text{ cal}$ , what is  $Q$  for the path db and ad?



### Solution:

From the first law of thermodynamics, we have

$$Q = \Delta U + \Delta W$$

$$Q = (U_b - U_a) + W$$

Where  $U_b$  is the internal energy in the state b and  $U_a$  is the internal energy in the state a.

For the path acb, it is given that

$$Q = 200 \text{ J (absorption) and}$$

$$W = 80 \text{ J (work done by the system)}$$

$$\therefore U_b - U_a = Q - W = 200 - 80 = 120 \text{ J}$$

which is the increase in the internal energy of the system for the path acb. Whatever be the path between a and b the change in the internal energy will be  $120 \text{ J}$  only.

(i) To determine the work done along the path adb

$$Q = 144 \text{ J}$$

$$\Delta U = U_b - U_a = 120 \text{ J}$$

$$Q = (U_b - U_a) + W \quad \text{or} \quad 144 = 120 + W \quad (\text{or}) \quad W = 24 \text{ J}$$

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Since  $W$  is positive, work is done by the system.

(ii) For the curved return path  $ba$ , it is given that

$$W = -52 \text{ J (work done on the system)}$$

$$\Delta U = -120 \text{ J (negative since } \Delta U = U_b - U_a)$$

$$\therefore Q = (U_b - U_a) + W = (-120 - 52) \text{ J} = -172 \text{ J}$$

Negative sign indicates heat is extracted out of the system.

(iii) Since  $U_b - U_a = 120 \text{ J}$  and  $U_a = 40 \text{ J}$

$$U_b = U_a + 120 = 40 + 120 = 160 \text{ J}$$

(iv) For the path  $db$ , the process is isochoric since it is at constant volume.

Work done is zero.

$$Q = \Delta U + W = \Delta U = U_b - U_d = 160 - 88 = 72 \text{ J}$$

For the path  $ad$ .

$$Q = Q_{adb} - Q_{db} = 144 \text{ J} - 72 \text{ J} = 72 \text{ J}$$