1. INTRODUCTION

Until now, we have studied only circuits with direct current (dc)- which flows only in one direction, the primary source of emf in such circuit is a battery and when resistance is connected across the terminals of the battery, a current is established in the circuits, which flows in a unique direction from the positive terminal to the negative terminal via the external resistance.

But most of the electric power generated and used in the world is in the form of **alternating current (ac),** the magnitude of which changes continuously with time and direction is reversed periodically as shown in figure and it is given by

$$i = i_0 \sin(\omega t + \phi) \qquad \dots (\mathbf{1})$$

Here i is instantaneous value of current i.e. magnitude of current at any instant of time and i_0 is the maximum value of current which is called peak current or the current amplitude and the current repeats its value after each time interval $T = \frac{2\pi}{\omega}$ as shown in figure. This time interval is called the time period and ω is angular frequency which is equal to 2π times of frequency f.

$$\omega = 2\pi f$$

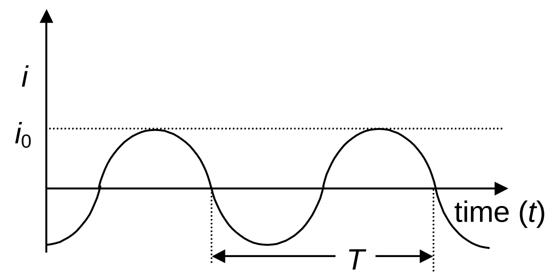


Fig. (a)

The current is positive for half the time period and negative for remaining half period, it means direction of current is reversed after each half time period. The frequency of ac in India is 50 Hz.

An alternating voltage is given by

$$V = V_0 \sin(\omega t + \phi) \qquad \dots (\mathbf{2})$$

It also varies alternatively as shown in the figure (b), where V is instantaneous voltage and V_0 is peak voltage. It is produced by ac generator also called as ac dynamo.

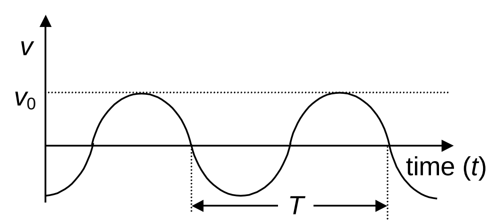
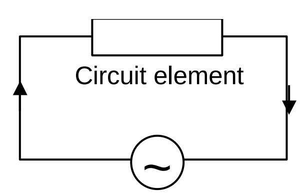


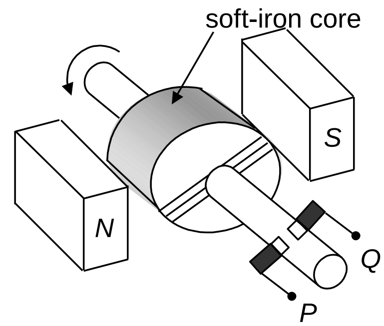
Fig. (b)

AC Circuit: An ac circuit consists of circuit element i.e., resistor, capacitor, inductor or any combination of these and a generator that provides the alternating current as shown in figure. The ac source is represented by symbol —(~)— in the circuit.



2. AC GENERATOR OR AC DYNAMO

The basic principle of the ac generator is a direct consequence of Faraday's laws of electro-magnetic induction. When a coil of N turns and area of cross section A is rotated in a uniform magnetic field B with constant angular velocity ω as shown in figure, a sinusoidal voltage (emf) is induced in the coil.



Suppose the plane of the coil at $t = 0$ is perpendicular to the magnetic field and in time t, it rotates through an angle θ.

Therefore, flux through the coil at time t is

$$\phi = NBA\cos\theta$$

$$= NBA\cos\omega t$$

$$\frac{d\phi}{dt} = -NBA\omega\sin\omega t \quad \ldots (\mathbf{3})$$

The emf induced in the coil is,

$$\varepsilon = \frac{-d\phi}{dt} = NBA\omega\sin\omega t$$

$\varepsilon = \varepsilon_0 \sin\omega t$, where $\varepsilon_0 = NBA\omega$

ε_0 is maximum value of emf, which is called peak emf or voltage amplitude.

and current, $i = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\sin\omega t = i_0\sin\omega t \quad \ldots (\mathbf{4})$

where i_0 is peak current.

3. AVERAGE AND RMS VALUE OF ALTERNATING CURRENT

3.1 AVERAGE CURRENT (MEAN CURRENT)

As we know an alternating current is given by

$$i = i_0 \sin(\omega t + \phi) \quad \ldots (\text{i})$$

The mean or average value of ac over any time t is given by

$$i_{avg} = \frac{\int_0^t i\,dt}{\int_0^t dt}$$

Using equation (i)

$$i_{avg} = \frac{\int_0^t i_0 \sin(\omega t + \phi)}{\int_0^t dt}$$

In one complete cycle average current

$$i_{avg} = -\frac{i_0}{T}\left[\frac{\cos(\omega t + \phi)}{\omega}\right]_0^T$$

$$= -\frac{i_0}{T}\left[\frac{\cos(\omega T+\phi)-\cos\phi}{\omega}\right]$$

$$= -\frac{i_0}{T}\left[\frac{\cos(2\pi+\phi)-\cos\phi}{\omega}\right] = 0 \quad (\text{as } \omega T = 2\pi)$$

Since ac is positive during the first half cycle and negative during the other half cycle so i_{avg} will be zero for long time also. Hence the dc instrument will indicate zero deflection when connected to a branch carrying ac current. So it is defined for either positive half cycle or negative half cycle.

$$i_{avg} = \frac{\int_0^{T/2} i_0 \sin(\omega t+\phi)}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \approx 0.637\, i_0 \qquad \ldots (\mathbf{5})$$

Similarly $\quad v_{avg} = \dfrac{2v_0}{\pi} \approx 0.637\, v_0 \qquad \ldots (\mathbf{6})$

3.2 R.M.S. VALUE OF ALTERNATING CURRENT

The notation rms refers to root mean square, which is given by square root of mean square current.

i.e., $\quad i_{rms} = \sqrt{i^2_{avg}}$

$$i^2_{avg} = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t+\phi)\, dt = \frac{i_0^2}{2T}\int_0^T [1-\cos 2(\omega t+\phi)]\, dt$$

$$= \frac{i_0^2}{2T}\left[t - \frac{\sin 2(\omega t+\phi)}{2\omega}\right]_0^T$$

$$= \frac{i_0^2}{2T}\left[T - \frac{\sin(4\pi+2\phi)-\sin 2\phi}{2\omega}\right] = \frac{i_0^2}{2}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} \approx 0.706\, i_0 \qquad \ldots (\mathbf{7})$$

Similarly the rms voltage is given by $V_{rms} = \dfrac{V_0}{\sqrt{2}} \approx 0.706\, v_0 \qquad \ldots (\mathbf{8})$

The significance of rms current and rms voltage may be shown by considering a resistance R carrying a current $i = i_0 \sin(\omega t + \phi)$

The voltage across the resistor will be

$$V = Ri = (i_0 R) \sin(\omega t + \phi)$$

The thermal energy developed in the resistor during the time t to $t + dt$ is

$$i^2 R\, dt = i_0^2 R \sin^2(\omega t + \phi)\, dt$$

The thermal energy developed in one time period is

$$U = \int_0^T i^2 R dt \quad = \quad R\int_0^T i_0^2 \sin^2(\omega t + \phi)\, dt$$

$$= RT\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t+\phi)\, dt\right] = i_{rms}^2\, RT \qquad \ldots (\mathbf{9})$$

It means the root mean square value of ac is that value of steady current, which would generate the same amount of heat in a given resistance in a given time.

So in ac circuits, current and ac voltage are measured in terms of their rms values. Like when we say that the house hold supply is 220 V ac it means the rms value is 220 V and peak value is $220\sqrt{2} = 311$ V.

Illustration 1.

If the voltage in an ac circuit is represented by the equation, $V = 220\sqrt{2}\ \sin(314\, t - \phi)$

Calculate (a) peak and rms value of the voltage, (b) average voltage, (c) frequency of ac.

Solution:

(a) As in case of ac,

$$V = V_0 \sin(\omega t - \phi)$$

The peak value

$$V_0 = 220\ \sqrt{2} = 311 \text{ V}$$

and as in case of ac,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}\ ;\ V_{\text{rms}} = \mathbf{220\ V}$$

(b) In case of ac

$$V_{av} = \frac{2}{\pi}V_0 = \frac{2}{\pi}\times 311 = \mathbf{198.17\ V}$$

(c) As $\omega = 2\pi f$, $2\pi f = 314$

$$\text{i.e., } f = \frac{314}{2\times\pi} = \mathbf{50\ Hz}$$

Illustration 2.

The electric current in a circuit is given by $i = i_0\ (t/T)$ for some time. Calculate the rms current for the period $t = 0$ to $t = T$.

Solution:

The mean square current is

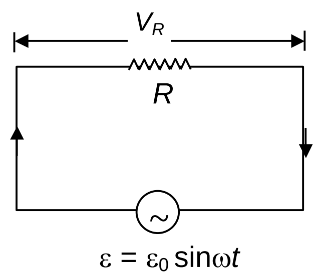
$$i_{avg}^2 = \frac{1}{T}\int_0^T i_0^2\,(t/T)^2\, dt = \frac{i_0^2}{T^3}\int_0^T t^2 dt = \frac{i_0^2}{3}$$

Thus, the rms current is

$$i_{\text{rms}} = \sqrt{i^2_{avg.}} = \frac{i_0}{\sqrt{3}}$$

4. SERIES AC CIRCUIT

4.1 WHEN ONLY RESISTANCE IS IN AC CIRCUIT



Consider a simple ac circuit consisting of a resistor of resistance R and an ac generator, as shown in the figure.

According to Kirchhoff's loop law at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero

$$\varepsilon - V_R = 0$$

$$\varepsilon - i_R R = 0$$

$$\varepsilon_0 \sin\omega t - i_R R = 0$$

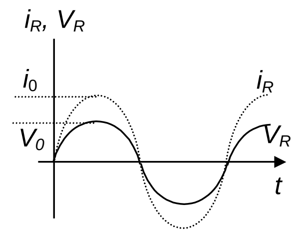
$$i_R = \frac{\varepsilon_0}{R} \sin \omega t = i_0 \sin \omega t \qquad \text{.... (i)}$$

where i_0 is the maximum current. $i_0 = \frac{\varepsilon_0}{R}$

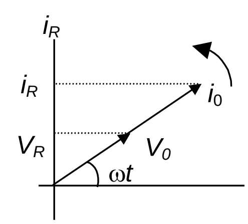
From above equations, we see that the instantaneous voltage drop across the resistor is

$$V_R = i_0 R \sin \omega t \qquad \text{... (ii)}$$

We see in equation (i) & (ii), i_R *and* V_R both vary as sin ωt and reach their maximum values at the same time as shown in figure (a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The lengths of the arrows correspond to V_0 and i_0. The projections of the arrows onto the vertical axis give V_R and i_R. In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in figure (b), because **i_R and V_R are in phase**.



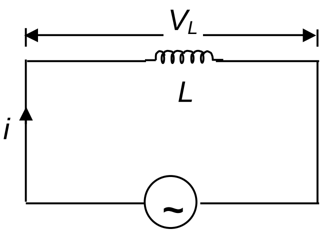
Wave diagram
Fig. (a)



Phasor diagram
Fig. (b)

4.2 WHEN ONLY INDUCTOR IS IN AN AC CIRCUIT

Now consider an ac circuit consisting only of an inductor of inductance L connected to the terminals of an ac generator, as shown in the figure. The induced emf across the inductor is given by Ldi/dt. On applying Kirchhoff's loop rule to the circuit



$\varepsilon = \varepsilon_0 \sin \omega t$

$$\varepsilon - V_L = 0 \;\Rightarrow\; \varepsilon - L\frac{di}{dt} = 0$$

When we rearrange this equation and substitute

$\varepsilon = \varepsilon_0 \sin \omega t$, we get

$$L\frac{di}{dt} = \varepsilon_0 \sin \omega t \qquad \ldots \text{(iii)}$$

Integration of this expression gives the current as a function of time

$$i_L = \frac{\varepsilon_0}{L}\int \sin \omega t \, dt = -\frac{\varepsilon_0}{\omega L}\cos \omega t + C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \qquad i_L = -\frac{\varepsilon_0}{\omega L}\cos \omega t$$

When we use the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express equation as

$$i_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \ldots \text{(iv)}$$

From equation (iv) , we see that the current reaches its maximum values when $\cos \omega t = 1$.

$$i_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_L} \qquad \ldots \text{(v)}$$

where the quantity X_L, called the inductive reactance, is

$$X_L = \omega L \qquad \ldots (\mathbf{10})$$

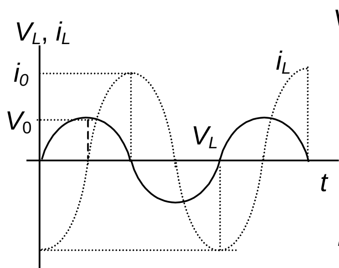
The expression for the rms current is similar to equation (v), with ε_0 replaced by ε_{rms}.

Inductive reactance, like resistance, has unit of ohms.

$$V_L = L\frac{di}{dt} = \varepsilon_0 \sin \omega t = I_0 X_L \sin \omega t \qquad \ldots (\mathbf{11})$$

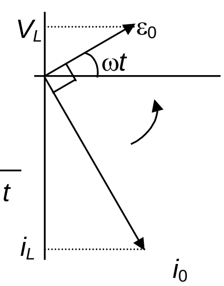
We can think of equation (v) as Ohm's law for an inductive circuit.

On comparing result of equation (iv) with equation (iii), we can see that **the current and voltage are out of phase with each other by π/2 rad, or 90°**. A plot of voltage and current versus time is given in figure (a). The voltage reaches its maximum value one quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in figure (b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.



Wave diagram

Fig. (a)



Phasor diagram

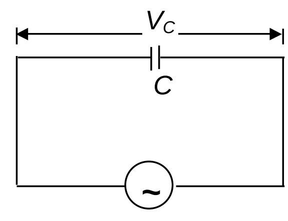
Fig. (b)

4.3 WHEN ONLY CAPACITOR IS IN AN AC CIRCUIT

Figure shows an ac circuit consisting of a capacitor of capacitance C connected across the terminals of an ac generator. On applying Kirchhoff's loop rule to this circuit gives

$$\varepsilon - V_C = 0$$

$$V_C = \varepsilon = \varepsilon_0 \sin \omega t \qquad \ldots\text{(vi)}$$



where V_C is the instantaneous voltage drop across the capacitor. From the definition of capacitance, $V_C = Q/C$, and this value for V_C substituted into equation gives

$$Q = C\varepsilon_0 \sin \omega t$$

Since $i = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit.

$$i_C = \frac{dQ}{dt} = C\varepsilon_0\omega \cos\omega t$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by equation (vi). Using the trigonometric identity $\cos \omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative from

$$i_C = \omega C\varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots\text{(vii)}$$

From equation (vii), we see that the current in the circuit reaches its maximum value when $\cos \omega t = 1$.

$$i_0 = \omega C\varepsilon_0 = \frac{\varepsilon_0}{X_C}$$

where X_C is called the capacitive reactance.

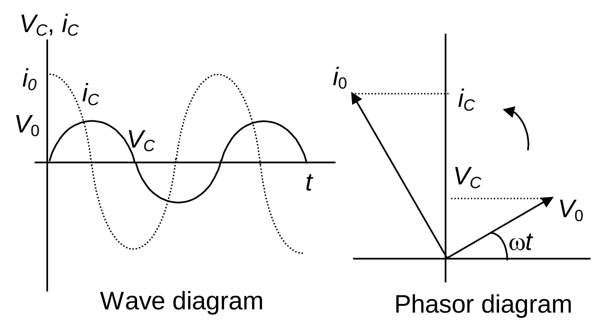
$$X_C = \frac{1}{\omega C} \qquad \ldots\textbf{(12)}$$

The SI unit of X_C is also ohm. The rms current is given by an expression similar to equation with V_0 replaced by V_{rms}.

Combining equation (vi) & (vii), we can express the instantaneous voltage drop across the capacitor as

$$V_C = V_0 \sin \omega t = I_0\, X_C \sin \omega t \qquad \ldots\textbf{(13)}$$

Comparing the result of equation (v) with equation (vi), we see that the **current is π/2 rad = 90° out of phase with the voltage** across the capacitor. A plot of current and voltage versus time, shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in the figure (b).Thus we see that for a sinusoidally applied emf, the current always leads the voltage across a capacitor by 90°.



Wave diagram
Fig. (a)

Phasor diagram
Fig. (b)

4.4 VECTOR ANALYSIS (PHASOR ALGEBRA)

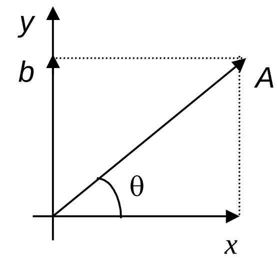
The complex quantities normally employed in ac circuit analysis, can be added and subtracted like coplanar vectors. Such coplanar vectors, which represent sinusoidally time varying quantities, are known as phasors.

In cartesian form, a phasor A can be written as,

$$A = a + jb$$

where a is the x-component and b is the y component of phasor A.

The magnitude of A is, $|A| = \sqrt{a^2 + b^2}$



and the angle between the direction of phasor A and the positive x-axis is,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

when a given phasor A, the direction of which is along the x-axis is multiplied by the operator j, a new phasor $j A$ is obtained which will be 90° anticlockwise from A, i.e., along y-axis. If the operator j is multiplied now to the phasor jA, a new phasor j^2A is obtained which is along x-axis and having same magnitude as of A. Thus,

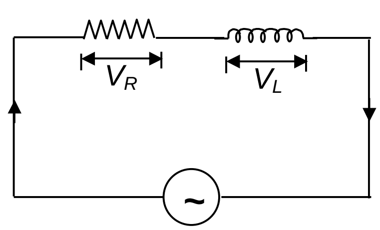
$$j^2 A = -A$$

$$j^2 = -1 \qquad \text{or } j = \sqrt{-1}$$

Now using the j operator, let us discuss different circuits of an ac.

4.5 SERIES L-R CIRCUIT

Now consider an ac circuit consisting of a resistor of resistance R and an inductor of inductance L in series with an ac source generator.

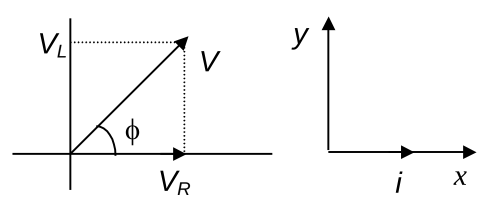


Suppose in phasor diagram current is taken along positive x-direction. The V_R is also along positive x-direction and V_L along positive y-direction as we know potential difference across a resistance in ac is in phase with current and it leads in phase by 90° with current across the inductor, so we can write

$$V = V_R + jV_L = iR + j(iX_L)$$

$$= iR + j(i\omega L)$$

$$= iZ$$



Here, $Z = R + jX_L = R + j\,(\omega L)$ is called as impedance of the circuit. Impedance plays the same role in ac circuits as the ohmic resistance does in dc circuits. The modulus of impedance is,

$$|Z| = \sqrt{R^2 + (\omega L)^2} \qquad \dots \textbf{(14)}$$

The potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L}{V_R}\right| = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \ldots \textbf{(15)}$$

Illustration 3.

An alternating voltage of 220 volt r.m.s. at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 ohms in series. Calculate (i) the current, (ii) potential difference across the resistance, (iii) potential difference across the inductance, (iv) the time lag.

Solution:

The impedance of *L*-*R* series circuit is given by

$$Z = [R^2 + (\omega L)^2]^{1/2} = [(R)^2 + (2\pi f L)^2]^{1/2}$$

$$= [(6)^2 + (2 \times 3.14 \times 40 \times 0.01)^2]^{1/2} = 6.504 \text{ ohm.}$$

(i) R.M.S. value of current

$$I_{\text{rms}} = \frac{\varepsilon_{rms}}{Z} = \frac{220 \text{ volt}}{6.504 \text{ ohm}} = \mathbf{33.83\ amp}$$

(ii) The potential difference across the resistance is given by

$$V_R = I_{\text{rms}} \times R = 33.83 \times 6 = \mathbf{202.98\ volts}$$

(iii) Potential difference across inductance is given by

$$V_L = I_{\text{rms}} \times (\omega L) = 33.83 \times (2 \times 3.14 \times 40 \times 0.01) = \mathbf{96.83\ volts}$$

(iv) Phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ $\therefore$ $\phi = \tan^{-1}(0.4189) = 22°46'$

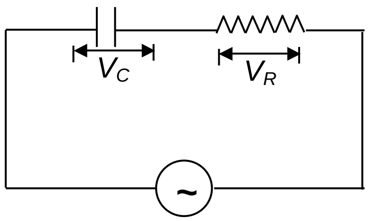
$$\text{Now time lag} = \frac{\phi}{360} \times T = \frac{\phi}{360} \times \frac{1}{f} = \frac{22°46'}{360 \times 40} = \mathbf{0.01579\ second}$$

4.6 SERIES C-R CIRCUIT

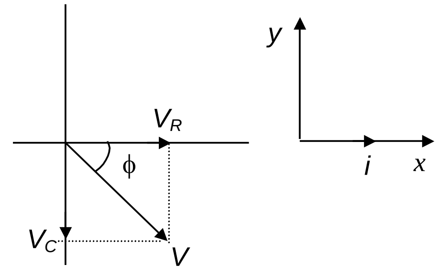
Now consider an ac circuit consisting of a resistor of resistance *R* and an capacitor of capacitance *C* in series with an ac source generator.

Suppose in phasor diagram current is taken along positive *x*-direction. Then V_R is also along positive *x*-direction but V_C is along negative *y*-direction as potential difference across a capacitor in ac lags in phase by 90° with the current in the circuit. So we can write.

$$V = V_R - jV_C = iR - j\,(iX_C)$$



$$= iR - j\left(\frac{i}{\omega C}\right) = iZ$$



Here, impedence is, $Z = R - j\left(\frac{1}{\omega C}\right)$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad \ldots \textbf{(16)}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right| = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{1/\omega C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right) \qquad \ldots \textbf{(17)}$$

Illustration 4.

An A.C. source of angular frequency ω is fed across a resistor *R* and a capacitor *C* in series. The current registered is *i*. If now the frequency of the source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance of resistance at the original frequency.

Solution:

At angular frequency ω, the current in *R*-*C* circuit is given by

$$i_{\text{rms}} = \frac{\varepsilon_{rms}}{\sqrt{\{R^2 + (1/\omega^2 C^2)\}}} \qquad \ldots \text{(i)}$$

When frequency is changed to ω/3, the current is halved. Thus

$$\frac{i_{rms}}{2} = \frac{\varepsilon_{rms}}{\sqrt{\{R^2 + 1/(\omega/3)^2 C^2\}}} = \frac{\varepsilon_{rms}}{\sqrt{\{R^2 + (9/\omega^2 C^2)]}} \qquad \ldots \text{(ii)}$$

From equation (i) and (ii), we have $\dfrac{1}{\sqrt{\{R^2 + (1/\omega^2 C^2)\}}} = \dfrac{2}{\sqrt{\{R^2 + (9/\omega^2 C^2.)\}}}$

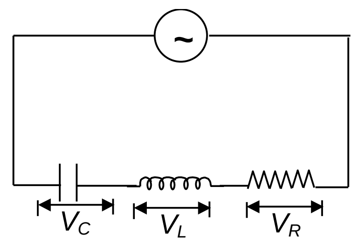
Solving this equation, we get $3R^2 = \dfrac{5}{\omega^2 C^2}$

Hence, the ratio of reactance to resistance is $\dfrac{(1/\omega C)}{R} = \sqrt{\left(\dfrac{\mathbf{3}}{\mathbf{5}}\right)}$

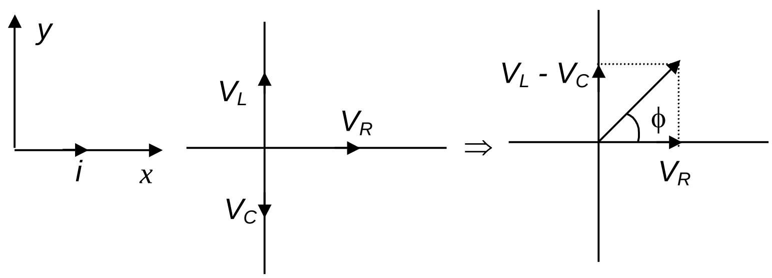
4.7 SERIES L-C CIRCUIT

Now consider an ac circuit consisting of a resistor of resistance R , a capacitor of capacitance C and an inductor of inductance L are in series with an ac source generator.

Suppose in a phasor diagram current is taken along positive x-direction. Then V_R is along positive x-direction, V_L along positive y-direction and V_C along negative y-direction, as potential difference across an inductor leads the current by 90° in phase while that across a capacitor, lags it in phase by 90°.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$



So, we can write, $V = V_R + jV_L - jV_C = iR + j(iX_L) - j(iX_C)$

$$iR + j\,[i\,(X_L - X_C)] = iZ$$

Here impedance is,

$$Z = R + j\,(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The modulus of impedance is, $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ … (**18**)

and the potential difference leads the current by an angle,.

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad \text{… (19)}$$

The steady current in the circuit is given by $i = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\sin(\omega t + \phi)$

where ϕ is given from equation (19)

The peak current is $i_0 = \dfrac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

It depends on angular frequency ω of ac source and it will be maximum when

$$\omega L - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{1}{LC}}$$

and corresponding frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad \dots \textbf{(20)}$$

This frequency is known as **resonant frequency** of the given circuit. At this frequency peak current will be $i_0 = \dfrac{V_0}{R}$

If the resistance R in the LCR circuit is zero, the peak current at resonance is $i_0 = \dfrac{V_0}{0}$

It means, there can be a finite current in pure LC circuit even without any applied emf, when a charged capacitor is connected to pure inductor.

This current in the circuit is at frequency, $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$

Illustration 5.

A resistance R, and inductance L and a capacitor C all are connected in series with an a.c. supply. The resistance of R is 16 ohm and for a given frequency, the inductive reactance of L is 24 ohm and capacitive reactance of C is 12 ohm. If the current in the circuit is 5 ampere, find

(a) the potential difference across R, L and C

(b) the impedance of the circuit

(c) the voltage of a.c. supply

(d) phase angle

Solution:

(a) Potential difference across resistance

$V_R = iR = 5 \times 16 =$ **80 volt**

Potential difference across inductance

$V_L = i \times (\omega L) = 5 \times 24 =$ **120 volt**

Potential difference across condenser

$V_C = i \times (1/\omega C) = 5 \times 12 =$ **60 volt**

(b) $Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$

$$= \sqrt{[(16)^2 + (24-12)^2]} = \mathbf{20\ ohm}$$

(c) The voltage of a.c. supply is given by

$r = IZ = 5 \times 20 =$ **100 volt**

(d) Phase angle

$$\phi = \tan^{-1}\left[\frac{\omega L - (1/\omega C)}{R}\right]$$

$$= \tan^{-1}\left[\frac{24-12}{16}\right]$$

$$= \tan^{-1}(0.75) = \mathbf{36°46'}$$

Illustration 6.

A series circuit consists of a resistance of 15 ohms, an inductance of 0.08 henry and a condenser of capacity 30 micro farad. The applied voltage has a frequency of 500 radian/s. Does the current lead or lag the applied voltage and by what angle.

Solution:

Here $\omega L = 500 \times 0.08 = 40$ ohm

and $\dfrac{1}{\omega C} = \dfrac{1}{500 \times (30 \times 10^{-6})}$

$= 66.7$ ohm

$$\tan\phi = \frac{[\omega L - (1/\omega C)]}{R} = \frac{40-66.7}{15} = 1.78$$

$$\phi = \mathbf{60.65°}$$

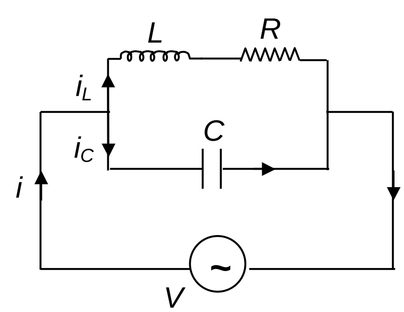
Thus the current leads the applied voltage by 60.65°.

5. PARALLEL AC CIRCUIT

Let us consider an alternating source connected across an inductance L in parallel with a capacitor C.

The resistance in series with the inductance is R and with the capacitor as zero. Let the instantaneous value of emf applied be V and the corresponding current is i, i_L and i_C. Then,

$$i = i_L + i_C$$



or, $$\frac{V}{Z} = \frac{V}{R + j\omega L} - \frac{V}{j/\omega C} = \frac{V}{R + j\omega L} - \frac{(\omega C)V}{j} = \frac{V}{R + j\omega L} - \frac{j(\omega C)V}{j^2}$$

$$= \frac{V}{R + j\omega L} + j(\omega C)V \quad (\text{as } j^2 = -1)$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as **admittance (Y)**. Therefore,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C \quad = \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$\therefore$ The magnitude of the admittance,

$$Y = |Y| = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2} \qquad \ldots \textbf{(21)}$$

The admittance will be minimum, when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad \ldots \textbf{(22)}$$

is known as **resonance frequency**. At resonance frequency admittance is minimum or the impedence is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as rejector circuit.

Note: *If $R = 0$, resonance frequency is* $\frac{1}{2\pi\sqrt{LC}}$ *same as resonance frequency in series circuit.*

At resonance, the reactive component of Y is real. The reciprocal of the admittance is called the **parallel resistor** or the **dynamic resistance**. The dynamic resistance is thus, reciprocal of the real part of the admittance.

Dynamic resistance = $\frac{R^2 + \omega^2 L^2}{R}$

Substituting $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$

we have, dynamic resistance = $\frac{L}{CR}$

$\therefore$ peak current through the supply = $\frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor = $\dfrac{V_0}{1/\omega C} = \omega C V_0$. The ratio of the peak current through capacitor and through the supply is known as ***Q*-factor**.

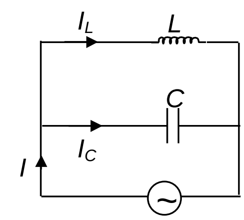
Thus, Q-factor = $\dfrac{V_0 \omega C}{V_0 CR/L} = \dfrac{\omega L}{R}$ … (**23**)

This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\dfrac{\omega L}{R}$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

Note: At resonance the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90°.

Illustration 7.

For the circuit shown in figure. Current in inductance is 0.8 *A* while in capacitance is 0.6 A. What is the current drawn from the source

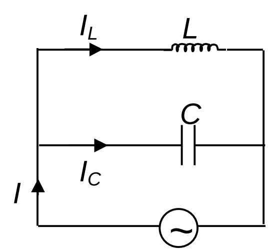


Solution:

In this ac circuit $\varepsilon = \varepsilon_0 \sin \omega t$ is applied across an inductance and capacitance in parallel, current in inductance will lag the applied voltage while across the capacitor will lead,

and so, $I_L = \dfrac{V}{X_L} \sin\left(\omega t - \dfrac{\pi}{2}\right) = -0.8 \cos \omega t$

$I_C = \dfrac{V}{X_C} \sin\left(\omega t + \dfrac{\pi}{2}\right) = +0.6 \cos \omega t$



So the current drawn from the source,

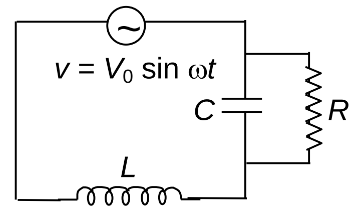
$I = I_L + I_C = -0.2 \cos \omega t$ i.e., $|I_0| =$ **0.2 A**

Illustration 8

An emf $V_0 \sin \omega t$ is applied to a circuit which consists of a self inductance L of negligible resistance in series with a variable capacitor C. The capactior is shunted by a variable resistance R. Find the value of C for which the amplitude of the current is independent of R.

Solution:

To make the problem easy, let us make use of phasor algebra. The complex impedance, of the circuit as shown in the figure.



$$Z = j\omega L + Z'.$$

where Z' is complex impedence due to C and R in parallel and is given by

$$\frac{1}{Z'} = \frac{1}{R} + j\omega C = \frac{1 + j\omega CR}{R}$$

or $$Z' = \frac{R}{1 + j\omega CR} = \frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2}$$

$\therefore$ $$Z = j\omega L + \frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2}$$

$$= \frac{R}{1 + \omega^2 C^2 R^2} + j\left(\omega L - \frac{\omega CR^2}{1 + \omega^2 C^2 R^2}\right)$$

The magnitude of Z is thus given by

$$Z = \sqrt{\left[\frac{R^2}{(1 + \omega^2 C^2 R^2)^2} + \left(\omega L - \frac{\omega CR^2}{1 + \omega^2 C^2 R^2}\right)^2\right]}$$

or $$Z^2 = \frac{R^2}{(1 + \omega^2 C^2 R^2)^2} + \omega^2 L^2 + \frac{\omega^2 C^2 R^4}{(1 + \omega^2 C^2 R^2)^2} - \frac{2\omega^2 LCR^2}{1 + \omega^2 C^2 R^2}$$

$$= \frac{R^2 - 2\omega^2 LCR^2}{1 + \omega^2 C^2 R^2} + \omega^2 L^2$$

The peak value of current will be independent of R, if Z or Z^2 is also independent of R. It is possible when $R^2 - 2\omega^2 LCR^2 = 0$, or $C = \mathbf{1/2\omega^2 L}$

6. POWER IN AN CIRCUIT

In case of a steady current the rate of doing work is given by,

$$P = Vi$$

In an alternating circuit current and voltage both vary with time, so the work done by the soruce in time interval dt is given by

$$dW = vidt$$

Suppose in an ac the current is leading the voltage by an angle ϕ. Then we can write,

$$V = V_0 \sin \omega t$$

and $$i = i_0 \sin (\omega t + \phi)$$

$$dW = V_0 i_0 \sin \omega t \sin (\omega t + \phi)\, dt$$

$$= V_0 i_0 (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)\, dt.$$

The total work done in a complete cycle is

$$W = V_0\, i_0 \cos\phi \int_0^T \sin^2 \omega t\, dt + V_0 i_0 \sin\phi \int_0^T \sin \omega t \cos \omega t\, dt$$

$$= \frac{1}{2} V_0\, i_0 \cos\phi \int_0^T (1 - \cos 2\omega t)\, dt + \frac{1}{2} V_0\, i_0 \sin\phi \int_0^T \sin 2\omega t\ dt = \frac{1}{2} V_0 i_0\, T \cos\phi$$

The average power delivered by the soruce is, therefore,

$$P = \frac{W}{T} = \frac{1}{2} V_0 i_0 \cos\phi = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)(\cos\phi) = \varepsilon_{rms}\, i_{rms} \cos\phi$$

or $\langle P \rangle_{\text{one cycle}} = V_{rms}\, i_{rms} \cos\phi$ … **(24)**

Here, the term cos ϕ is known as **power factor**.

It is said to be leading if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1} 0.5 = 60°$). The product of V_{rms} and i_{rms} gives the apparent power. While the true power is obtained by multiplying the apparent power by the power factor cosϕ. Thus,

and apparent power = $V_{rms} \times i_{rms}$

True power = apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($V_{rms} \times i_{rms}$). For $\phi = 90°$, the power is zero. The current is then stated wattles. Such a case will arise when resistance in the circuit is zero. The circuit is purely inductive or capacitive.

Illustration 9.

A series *LCR* with $R = 20\ \Omega$, $L = 1.5$ H and $C = 35\ \mu$F is connected to a variable frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit. What is the average power transferred to the circuit in one complete cycle?

Solution:

When the frequency of the supply equals the natural frequency of the circuit, resonance occurs.

$\therefore \quad Z = R = 20$ ohm.

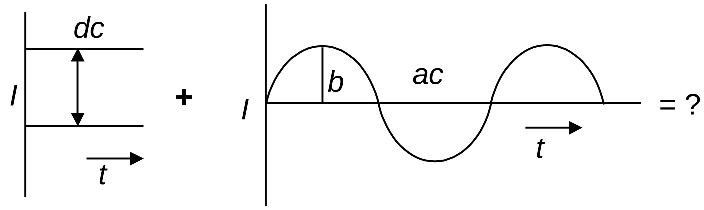
$$i_{rms} = \frac{E_{rms}}{Z} = \frac{200}{20} = 10\, A$$

Average power transferred/cycle

$P = E_{rms}\, i_{rms} \cos 0° = 200 \times 10 \times 1 =$ **2000 watt**

Illustration 10.

If a direct current of value a ampere is superimposed on an alternating current I = b sin ωt flowing through a wire, what is the effective value of the resulting current in the circuit?



Solution:

As current at any instant in the circuit will be, $I = I_{dc} + I_{ac} = a + b\sin\omega t$

So, $$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b\sin\omega t)^2\ dt\right]^{1/2}$$

i.e., $$I_{eff} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)\,dt\right]^{1/2}$$

but as $\frac{1}{T}\int_0^T \sin\omega t\ dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2}$

So, $$I_{eff} = \left[\boldsymbol{a^2 + \frac{1}{2}b^2}\right]^{\boldsymbol{1/2}}$$

Illustration 11.

When 100 volt dc is applied across a coil, a current of 1 amp flows through it. But when 100 V ac of 50 Hz is applied to the same coil, only 0.5 amp flows. Calculate the resistance of inductance of the coil.

Solution:

In case of a coil, i.e., L-R circuit.

$I = \frac{V}{Z}$ with $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$

So when dc is applied, ω = 0, so Z = R

and hence $I = \frac{V}{R}$ i.e., $R = \frac{V}{I} = \frac{100}{0.5} = 200\ \Omega$

but $Z = \sqrt{R^2 + \omega^2 L^2}$ i.e., $\omega^2 L^2 = Z^2 - R^2$

i.e., $(2\pi fL)^2 = 200^2 - 100^2 = 3 \times 10^4$ (as $\omega = 2\pi f$)

So, $$L = \frac{\sqrt{3}\times 10^2}{2\pi\times 50} = \frac{\sqrt{3}}{\pi}\ \text{H} = 0.55\ \text{H}$$

Illustration 12.

A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp?

Solution:

As resistance of the lamp R = $\dfrac{V_s^2}{W} = \dfrac{100^2}{50}$ = 200 Ω and the maximum current I = $\dfrac{V}{R} = \dfrac{100}{200} = \dfrac{1}{2}A$; so when the lamp is put in series with a capacitance and run at 200 V ac, from V = IZ we have,

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\ \Omega$$

Now as in case of C-R circuit, $Z = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$, i.e., $R^2 + \left(\dfrac{1}{\omega C}\right)^2 = 160000$

or, $\left(\dfrac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$

So, $\dfrac{1}{\omega C} = \sqrt{12} \times 10^2$ or $C = \dfrac{1}{100\pi \times \sqrt{12} \times 10^2}$ F

i.e., $C = \dfrac{100}{\pi\sqrt{12}}\ \mu F = 9.2\ \mu F$

Illustration 13.

A 12 ohm resistance and an inductance of 0.05/π henry with negligible resistance are connected in series. Across the end of this circuit is connected a 130 volt alternating voltage of frequency 50 cycles/second. Calculate the alternating current in the circuit and potential difference across the resistance and that across the inductance.

Solution:

The impedance of the circuit is given by

$$Z = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{[R^2 + (2\pi f L)^2]}$$

$$= \sqrt{[(12)^2 + \{2 \times 3.14 \times 50 \times (0.05/3.14)\}^2]} = \sqrt{(144 + 25)} = 13 \text{ ohm}$$

Current in the circuit i = E/Z = $\dfrac{130}{13}$ = 10 amp.

Potential difference across resistance

$V_R = iR = 10 \times 12 = 120$ volt

Inductive reactance of coil $X_L = \omega L = 2\pi fL$

$\therefore X_L = 2\pi \times 50 \times \left(\dfrac{0.05}{\pi}\right) = 5$ ohm.

Potential difference across inductance $V_L = i \times X_L = 10 \times 5 = 50$ volt.

Illustration 14.

A resistance of 10 ohm is joined in series with an inductance of 0.5 henry. What capacitance should be put in series with the combination to obtain the maximum current? What will be the potential difference across the resistance, inductance and capacitor? The current is being supplied by 200 volts and 50 cycles per second mains.

Solution:

The current in the circuit would be maximum when

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad C = \frac{1}{\omega^2 L}$$

$$\therefore \; C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\times 3.14\times 50)^2 \times 0.5} = 20.24 \times 10^{-6} \text{ farad}$$

Here $\omega L = 1/\omega C$. So the impedance Z of the circuit

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} = R = 10 \text{ ohm}$$

$$I = \frac{E}{R} = \frac{200}{10} = 20 \text{ amp.}$$

Potential difference across resistance

$V_R = I \times R = 20 \times 10 = 200$ volt

Potential difference across inductance

$V_L = \omega L \times I = (2\pi \times 50 \times 0.5) \times 20 = 3142$ volt

Potential difference across condenser

$$V_C = \frac{1}{\omega C} = I \times \omega L = 3142 \text{ volt}$$

Illustration 15.

A 100 volt a.c. source of frequency 500 hertz is connected to LCR circuit with L = 8.1 milli-henry, C = 12.5 microfarad and R = 10 ohm, all connected in series. Find the potential difference across the resistance.

Solution:

The impedance of LCR circuit is given by

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

where $X_L = \omega L = 2\pi f L$

$= 2 \times 3.14 \times 500 \times (8.1 \times 10^{-3}) = 25.4$ ohm and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2.3.14 \times 400 \times (12.5 \times 10^{-6})} = 25.4 \text{ ohm}$$

$\therefore \quad Z = \sqrt{[(10)^2 + (25.4 - 25.4)^2]} = 10$ ohm

$\therefore \quad I_{rms} = \frac{E_{rms}}{Z} = \frac{100\text{ volt}}{10\text{ ohm}} = 10$ amp.

Potential difference across resistance

$V_R = I_{rms} \times R = 10 \text{ amp} \times 10 \text{ ohm}$

$= 10$ volt.

Illustration 16.

An LCR series circuit with 100 Ω resistance is connected to an AC source of 200 V and angular frequency 300 radians per second. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and power dissipated in LCR circuit.

Solution:

$\tan 60° = \frac{\omega L}{R}$ or $\tan 60° = \frac{1/\omega C}{R}$

$\because \quad \omega L = \frac{1}{\omega C}$

Impendence of circuit $Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} = R$

Current in the circuit

$I_0 = \frac{V_0}{Z} = \frac{V_0}{R} = \frac{200}{100} = 2$ Amp.

Average power $\overline{P} = \frac{1}{2} V_0 I_0 \cos\phi$

But, $\tan\phi = \frac{\omega L - (1/\omega C)}{R} = 0 \quad (\cos\phi = 1)$

Now, $\overline{P} = \frac{1}{2} \times 200 \times 2 \times 1 = 200$ watt.

Illustration 17.

A current of 4 A flows in a coil when connected to a 12 V d.c. source. If the same coil is connected to a 12 V, 50 rad/s, a.c. source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF condenser is connected in series with the coil.

Solution:

When the coil is connected to a d.c. source, its resistance R is given by

$R = \frac{V}{I} = \frac{12}{4} = 3\ \Omega$

When it is connected to a.c. source, the impedance Z of the coil is given by

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{2.4} = 5\Omega$$

For a coil, Z = $\sqrt{[R^2 + (\omega L)^2]}$

∴ $5 = \sqrt{[(3)^2 + (50L)^2]}$

or $25 = [(3)^2 + (50\ L)^2]$

Solving we get L = 0.08 henry

When the coil is connected with a condenser in series, the impedance Z′ is given by

$$Z' = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$= \left[(3)^2 + \left(50 \times 0.08 - \frac{1}{50 - 2500 \times 10^{-6}}\right)^2\right]^{1/2} = 5 \text{ ohm}$$

Power developed P = $V_{rms} \times I_{rms} \times \cos\theta$

where $\cos\phi = R/Z' = 3/5 = 0.6$

∴ $P = 12 \times 2.4 \times 0.6 = 17.28$ watt

Illustration 18.

A LCR circuit has L = 10 mH, R = 3 ohm and C = 1 μF connected in series to an ac source of the voltage 15 V. Calculate current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonant frequency.

Solution:

Resonant frequency $\omega_R = 1/\sqrt{(L.C.)}$

Here, $L = 10\text{ mH} = 10 \times 10^{-3}$ H

and $C = 1\ \mu F = 1 \times 10^{-6}$ F

∴ $\omega_R = \sqrt{\left(\frac{1}{(10\times10^{-3})\,(1\times10^{-6}}\right)} = 10^4$/second.

Now, 10% less frequency will be

$$\omega = 10^4 - 10^4 \times \frac{10}{100} = 9 \times 10^3\text{/second.}$$

At this frequency,

$X_L = \omega L = 9 \times 10^3 \times (10 \times 10^{-3}) = 90$ ohm

$X_C = \frac{1}{\omega C} = \frac{1}{(9\times10^3)(1\times10^{-6})}$ = 111.11 ohm

$\therefore \quad Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

$= \sqrt{[(3)^2 + (90 - 111.11)^2]}$ = 21.32 ohm

Current amplitude,

$I_0 = \frac{E_0}{Z} = \frac{15}{21.32}$ = 0.704 amp.

Average power, P = $\frac{1}{2}$ E_0 I_0 cos ϕ

where cos ϕ = $\frac{R}{Z}$ = $\frac{3}{21.32}$ = 0.141

$P = \frac{1}{2} \times 15 \times 0.704 \times 0.141 = 0.744$ watt.

Illustration 19.

A 20 volts 5 watt lamp is used in ac main of 220 volts 50 c.p.s. Calculate the (i) capacitance of capacitor, (ii) inductance of inductor, to be put in series to run the lamp. (iii) What pure resistance should be included in place of the above device so that the lamp can run on its voltage. (iv) Which of the above arrangements will be more economical and why?

Solution:

The current required by the lamp

$I = \frac{\text{wattage}}{\text{voltage}} = \frac{5}{20}$ = 0.25 amp.

The resistance of the lamp

$R = \frac{\text{voltage}}{\text{current}} = \frac{20}{0.25}$ = 80 ohm

So for proper running of the lamp, the current through the lamp should be 0.25 amp.

(i) When the condenser C is placed in series with lamp, then

$$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}$$

The current through the circuit

$$I = \frac{200}{\sqrt{[R^2 + (1/\omega C)^2}} = 0.25 \quad \text{or} \quad \frac{200}{\sqrt{(80)^2 + \left(\frac{1}{4\pi^2 \times 50^2 \times C^2}\right)}} = 0.25$$

Solving it for C, we get

$C = 4.0 \times 10^{-6}$ F = 4.0 μF

(ii) When inductor L henry is placed in series with the lamp, then

$$Z = \sqrt{[R^2 + (\omega L)^2]}$$

or $\dfrac{200}{\sqrt{[R^2 + (\omega L)^2]}} = 0.25$ or $\dfrac{200}{\sqrt{[(80)^2 + (4\pi^2 \times 50^2 \times L^2)]}} = 0.25$

Solving it for L, we get L = 2.53 henry

(iii) When resistance r ohm is placed in series with lamp of resistance R, then

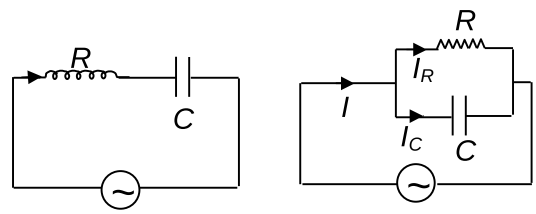
$$\frac{200}{R+r} = 0.25 \quad \text{or} \quad \frac{200}{80+r} = 0.25 \quad \Rightarrow \quad r = 720 \text{ ohms}$$

(iv) It will be more economical to use inductance or capacitance in series with the lamp to run it as it consumes no power while there would be dissipation of power when resistance is inserted in series with the lamp.

Illustration 20.

For a resistance R and capacitance C in series, the impedence is twice that of a parallel combination of the same elements. What is the frequency of applied emf?

Solution:



(A) (B)

As shown in figure (a), in case of series combination,

$$Z_s = \sqrt{R^2 + X_C^2} = [R^2 + (1/\omega C)^2]^{1/2}$$

In case of parallel combination,

$$I_R = \frac{V}{R}\sin\omega t \text{ and } I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

So, $I = I_R + I_C = \dfrac{V}{R}\sin\omega t + \dfrac{V}{X_C}\cos\omega t$

i.e., $I = I_0 \sin(\omega t + \phi)$

with $I_0 \cos\phi = \dfrac{V}{R}$ and $I_0 \sin\phi = \dfrac{V}{X_C}$

So, $I_0 = \left[\left(\dfrac{V}{R}\right)^2 + \left(\dfrac{V}{X_C}\right)^2\right]^{1/2} = \dfrac{V}{Z_P}$

i.e., $\dfrac{1}{Z_P} = \left[\dfrac{1}{R^2} + \left(\dfrac{1}{X_C}\right)^2\right]^{1/2}$ i.e., $Z_P = \dfrac{R}{\sqrt{1+\omega^2 C^2 R^2}}$

and as according to given problem,

$Z_S = 2Z_P$, i.e., $Z_s^2 = 4Z_P^2$

i.e., $$\frac{(R^2\omega^2C^2+1)}{\omega^2C^2} = 4\frac{R^2}{(1+R^2\omega^2C^2)}$$

i.e., $(1 + R^2\omega^2C^2)^2 = 4R^2\omega^2C^2$ or, $1 + R^2\omega^2 C^2 = 2R\omega C$

or, $(R\omega C - 1)^2 = 0$ or, $\omega = \frac{1}{RC}$ i.e., $f = \frac{1}{2\pi RC}$