1. RAY OPTICS

Ray optics treats propagation of light in terms of rays and is valid only if the size of the obstacle is much greater than the wavelength of light. It concerns with the image formation and deals with the study of the simple facts such as rectilinear propagation, laws of reflection and refraction by geometrical methods.

1.1 RAY

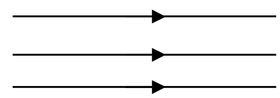
A ray can be defined as an imaginary line drawn in the direction in which light is travelling. Light behaves as a stream of energy propagated along the direction of rays. The rays are directed outward from the source of light in straight lines.

1.2 BEAM OF LIGHT

A beam of light is a collection of these rays. There are mainly three types of beams.

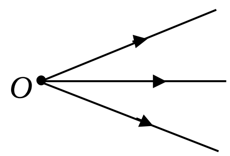
Parallel beam of light

A search light and the headlight of a vehicle emit a parallel beam of light. The source of light at a very large distance like sun effectively gives a parallel beam.



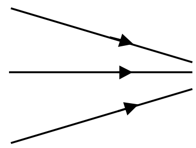
Divergent beam of light

The rays going out from a point source generally form a divergent beam.



Convergent beam of light

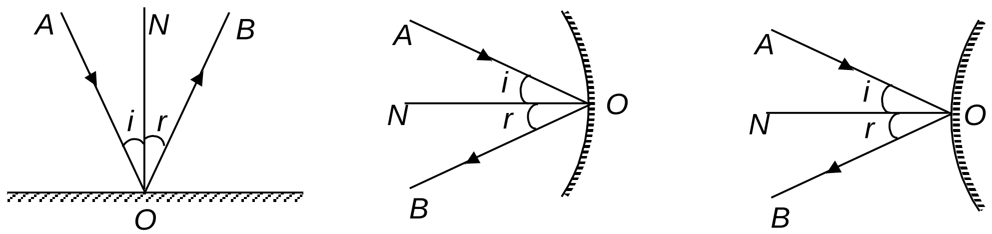
A beam of light that is going to meet (or converge) at a point is known as a convergent beam. A parallel beam of light after passing through a convex lens becomes a convergent beam.



2. REFLECTION

When a ray of light is incident at a point on the surface, the surface throws partly or wholly the incident energy back into the medium of incidence. This phenomenon is called reflection.

Surfaces that cause reflection are known as mirrors or reflectors. Mirrors can be plane or curved.



Plane mirror Concave mirror Convex mirror

In the above figures, O is the point of incidence, AO is the incident ray , OB is the reflected ray and ON is the normal at the incidence

Angle of incidence

The angle which the incident ray makes with the normal at the point of incidence is called the angle of incidence. It is generally denoted by 'i'.

Angle of reflection

The angle which the reflected ray makes with the normal at the point of incidence is called the angle of reflection. It is generally denoted by 'r'.

Glancing angle

The angle which the incident ray makes with the plane reflecting surface is called glancing angle. It is generally denoted by 'g'.

$$g = 90° - i \qquad \ldots (\mathbf{1})$$

2.1 LAWS OF REFLECTION

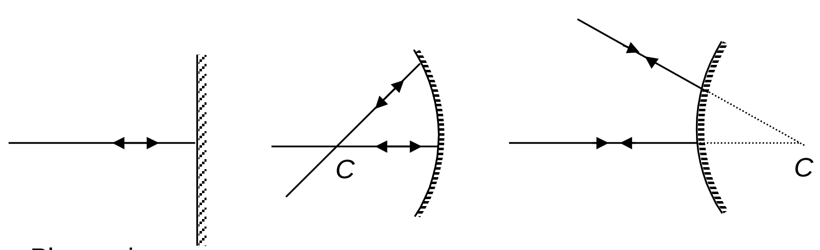
(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

(ii) The angle of incidence is equal to the angle of reflection, i.e., $\angle i = \angle r$

These laws hold good for all reflecting surfaces either plane or curved.

Some important points

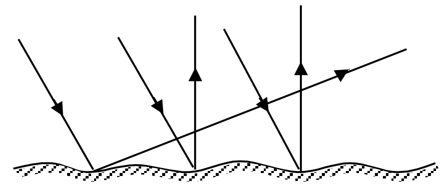
(i) If $\angle i = 0$, $\angle r = 0$, i.e., if a ray is incident normally on a boundary, after reflection it retraces its path.



Plane mirror Concave mirror Convex mirror

(ii) None of frequency, wavelength and speed changes due to reflection. However, intensity and hence amplitude ($I \propto A^2$) usually decreases.

(iii) If the surface is irregular, the reflected rays of an incident beam of parallel light rays will be in random directions. Such an irregular reflection is called diffused reflection.

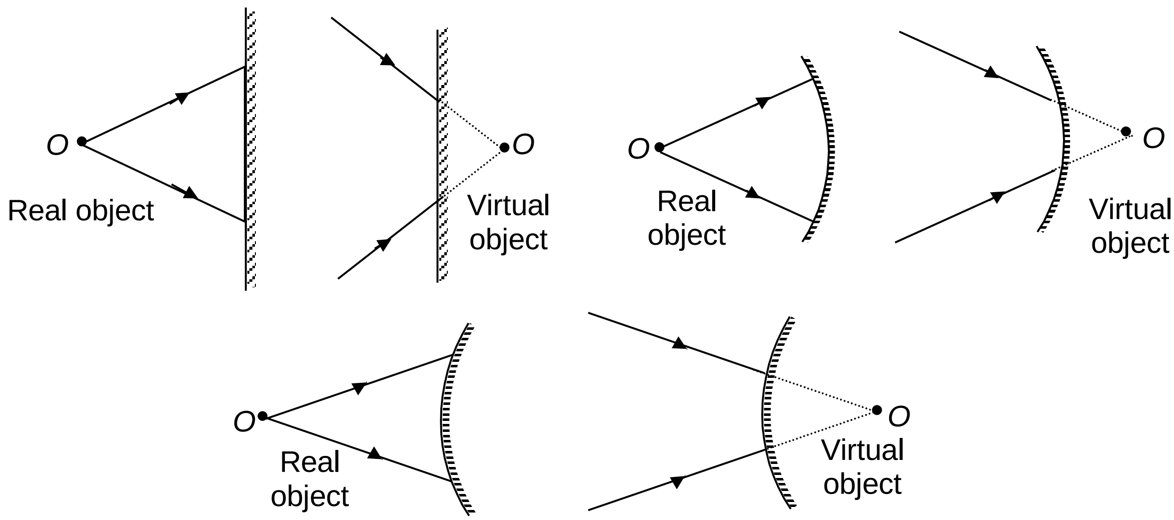


Diffused reflection

2.2 REAL AND VIRTUAL OBJECTS

If the rays from a point on an object actually diverge from it, then the object is said to be real.

If the rays incident on the mirror do not start from a point but appear to converge at a point, then that point is the virtual object for the mirror.



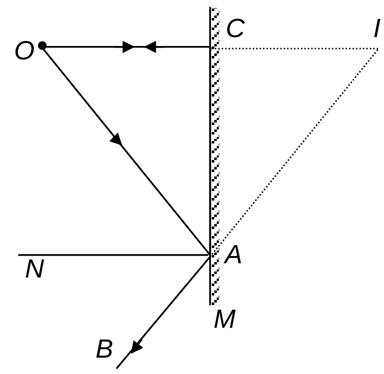
2.3 REAL AND VIRTUAL IMAGES

If the rays from a point object after reflection (or refraction) actually meet at or appear to diverge from a point I, then I is said to be the image of the object O.

Images can be real or virtual. If the reflected or refracted rays actually meet at the point I, then I is said to be a real image of the object O. But if the reflected or a refracted ray do not actually meet but only appear to diverge from the point I, then I is said to be the virtual image of the object O.

3. IMAGE FORMATION BY A PLANE MIRROR

Let us consider a point object *O* placed in front of a mirror *M*. A ray of light *OA* from *O* incident on *M* at *A*, is reflected along *AB* so that $\angle OAN = \angle NAB$. A ray *OC* incident on the mirror at *C*, is reflected back along *CO*. Thus, the ray reflected by the plane mirror *M* appears to come from a point *I* behind the mirror where *I* is the point of intersection of *BA* and *OC* produced. Thus, *I* is the image of the point *O*.



Now,

$$\angle AOC = \angle OAN$$

$$\angle NAB = \angle CIA$$

Also, $\angle \text{OAN} = \angle NAB$ ($\because$ $\angle i = \angle r$)

$\therefore$ $\angle AOC = \angle CIA$

$\therefore$ $\Delta OCA \cong \Delta\, ACI$

$$OC = CI$$

3.1 CHARACTERISTICS OF THE IMAGE FORMED BY A PLANE MIRROR

(i) The image formed is at the same distance behind the reflecting surface as the object is in front of it.

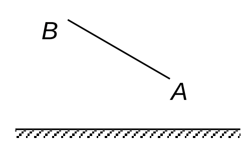
(ii) The size of the image is the same as that of the object.

(iii) The image is virtual and erect which means no light actually passes through it

(iv) The image is laterally inverted i.e., side-wise inverted. Example:- If a right handed batsman observes his stance in a plane mirror. He appears left handed. The left hand side of the image thus corresponds to the right hand side of the object and vice-versa. Thus, the image is said to be laterally inverted with respected to the object.

Illustration 1.

Find graphically the position of the observer's eye, which will allow him to see in a plane mirror of finite dimensions, the image of the straight line arranged as shown.



Solution:

A′ B ′ is the reflected image of AB by CD. The reflected rays EA_2 and FA_1 must reach the eye so that the whole image can be seen.

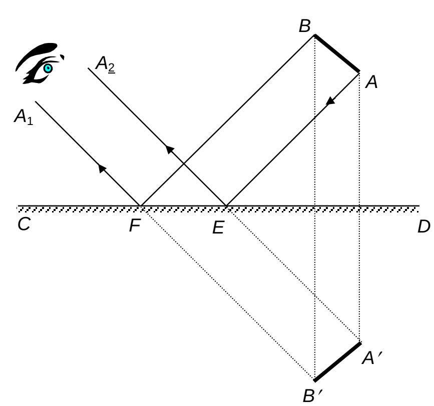
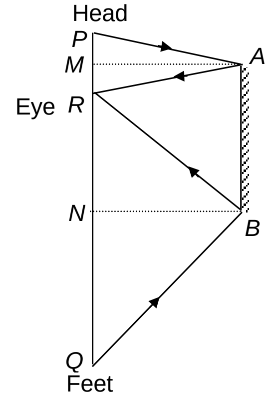


Illustration 2.

What is the minimum length of a plane mirror required for a person to see his or her full image? Is there any restriction on the position of the top edge of the mirror?

Solution:



The man can view his entire image if the light rays from the top of his head and from his feet reach his eye.

Let AB be the mirror. PQ represents the man of height h and R is the position of his eyes. Light rays from P gets reflected at A and reach his eyes. Light from Q gets reflected at B and reaches his eyes. AM and BN are normals to the mirror AB.

Now, $AB = MN = MR + RN$

$$= \frac{1}{2}(PR + RQ)\ (\because\ \Delta APM \cong \Delta ARM;$$

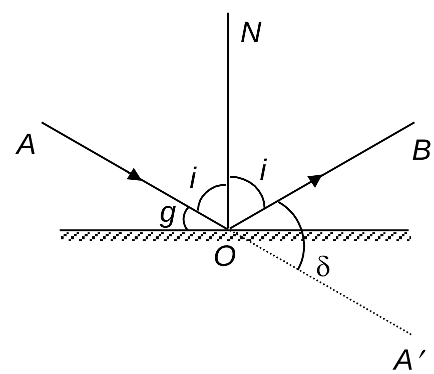
$\Delta BQN \cong \Delta BRN)$

$$= \frac{PQ}{2} = \frac{h}{2}$$

Hence the length of the mirror = $\frac{h}{2}$

It is clear from the ray diagram that the top edge of the plane mirror (A) must be at a horizontal level half-way between the eyes (R) and the top of his head (P).

3.2 DEVIATION PRODUCED BY A PLANE MIRROR



Deviation is defined as the angle between directions of the incident ray and the reflected ray (or, the emergent ray). It is generally denoted by δ.

Here, $\angle A'OB = \delta = \angle AOA' - \angle AOB = 180° - 2\,i$

Or, $\delta = 180° - 2i$ … (**2**)

We know, $g = 90° - i$

$\therefore$ $\delta = 180° - 2\,(90° - g)$

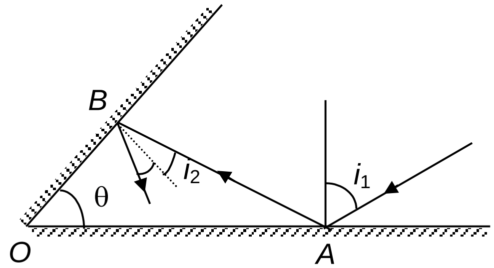
or $\delta = 2g$ … (**3**)

Illustration 3.

Two plane mirrors are inclined at angle θ with each-other. A ray of light strikes one of them. Find its deviation after it has been reflected twice-one from each mirror.

Solution:

Case I :



δ_1 = clockwise deviation at $A = 180° - 2i_1$

δ_2 = anticlockwise deviation at $B = 180° - 2i_2$

Now, from Δ *OAB*, we have

angle O + angle A + angle B = 180°

$\Rightarrow \quad i_1 - i_2 = \theta$

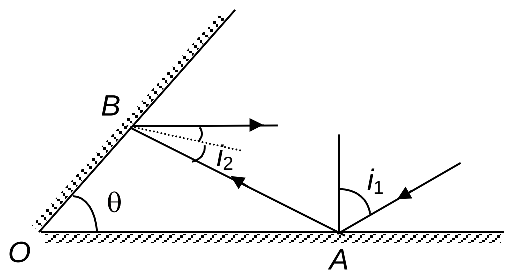
As $\quad i_1 > i_2, \delta_1 < \delta_2$

Hence, the net angle clockwise deviation = $\delta_2 - \delta_1$

$= (180° - 2i_2) - (180° - 2i_1)$

$= 2(i_1 - i_2) = \mathbf{2\theta}$

Case II :



δ_1 = clockwise deviation at $A = 180° - 2i_1$

δ_2 = clockwise deviation at $B = 180° - 2i_2$

Now, from ΔOAB, we have

Angle O + angle A + angle B = 180°

or, $\quad \theta + (90° - i_1) + (90° - i_2) = 180°$

$\Rightarrow \quad i_1 + i_2 = \theta$

Hence, net clockwise deviation = $\delta_2 + \delta_1$

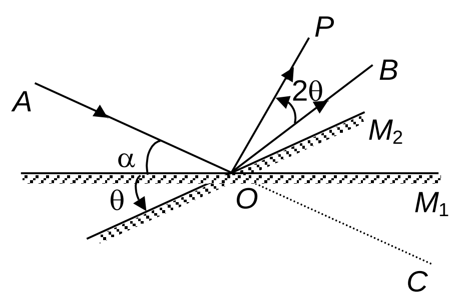
$= (180° - 2i_2) + (180° - 2i_1)$

$= 360° - 2(i_1 + i_2)$

$= 360° - 2\theta$

$\Rightarrow$ Net anti-clockwise deviation = $360° - (360° - 2\theta) = \mathbf{2\theta}$

3.3 ROTATION OF THE REFLECTED RAY BY A PLANE MIRROR



Let a ray *AO* be incident at *O* on a plane mirror M_1. Let α be the glancing angle with M_1. If *OB* is the reflected ray, then the angle of deviation ($\angle COB$) = 2α. Let the mirror be rotated through an angle θ to a position M_2, keeping the direction of the incident ray constant. The ray is now reflected from M_2 along *OP*.

The glancing angle with $M_2 = (\alpha + \theta)$.

Hence, the new angle of deviation (i.e., $\angle COP$)

$= 2(\alpha + \theta)$

The reflected ray has thus been rotated through $\angle BOP$ when the mirror is rotated through an angle θ and since

$$\angle BOP = \angle COP - \angle COB$$

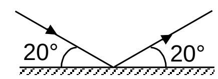
or, $$\angle BOP = 2(\alpha + \theta) - 2\alpha = 2\theta$$

Thus, keeping the incident ray fixed, if the plane mirror is rotated through an angle θ about an axis in the plane of mirror, then the reflected ray is rotated through an angle 2θ.

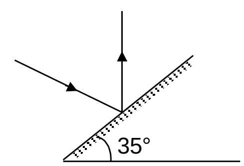
Illustration 4.

A ray of light is travelling at an angle of 20° above the horizontal. At what angle with the horizontal must a plane mirror be placed in its path so that it becomes vertically upward after reflection.

Solution:



Let us first place the mirror horizontally. The reflected ray now goes at an angle of 20° above the horizontal as shown below.



To make the reflected ray vertical, it has to be rotated anticlockwise by 70°. Hence the mirror must be rotated by $\frac{70°}{2} = 35°$

⇒ the angle of the mirror with the horizontal = **35°.**

3.4 IMAGES OF AN OBJECT FORMED BY MIRRORS INCLINED TO EACH OTHER

If two plane mirrors are kept inclined to each-other at angle θ with their reflecting surfaces facing each-other, then multiple reflections take place and more than one images are formed.

Number of images formed

(i) If $\left(\frac{360}{\theta}\right)$ is an even integer, the number of images formed

$$n = \frac{360°}{\theta} - 1 \qquad \ldots \textbf{(4)}$$

(ii) If $\left(\frac{360°}{\theta}\right)$ is an odd integer, the number of images formed

$n = \frac{360°}{\theta}$ when the object is placed unsymmetrical to the mirrors … (**5 A**)

$n = \frac{360°}{\theta}$ -1 when the object is placed symmetrical to the mirrors … (**5 B**)

Illustration 5.

Rays of light are incident on a plane mirror at 45°. At what angle with the first should a second mirror be placed such that the rays emerge from the second mirror parallel to the first mirror.

Solution:

In triangle *BOC*, we have

$2\theta + 45° = 180°$

or, $\theta = \dfrac{135°}{2} = \mathbf{67.5°}$

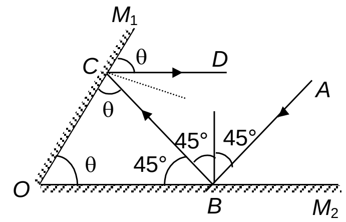


Illustration 6.

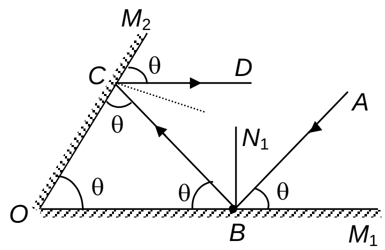
Find the angle between two plane mirrors such that a ray of light incident on the first mirror and parallel to the second mirror is reflected from the second mirror, parallel to the first mirror.

Solution:

In triangle *BOC*, we have

$3\theta = 180°$

$\theta = \mathbf{60°}$

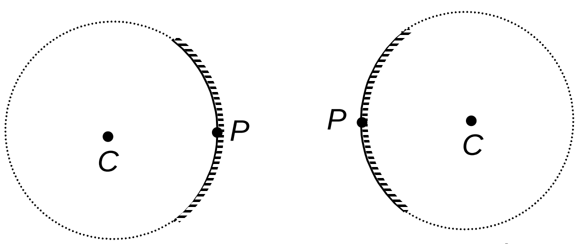


4. REFLECTION AT SPHERICAL MIRRORS

4.1 SOME IMPORTANT DEFINITIONS

Spherical Mirrors

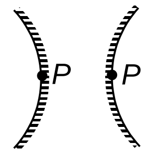
A spherical mirror is a part of a hollow sphere or a spherical surface. They are classified as concave or convex according to the reflecting surface being concave or convex respectively.



Concave mirror Convex mirror

Pole or Vertex

The geometrical centre of the spherical mirror is called the pole or vertex of the mirror.



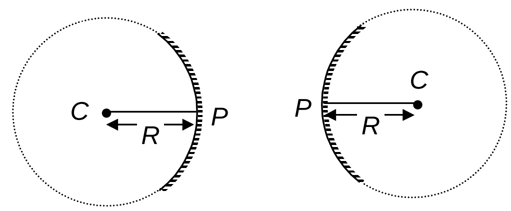
In the above figures, the point *P* is the pole.

Centre of curvature

The centre C of the sphere of which the spherical mirror is a part, is the centre of curvature of the mirror.

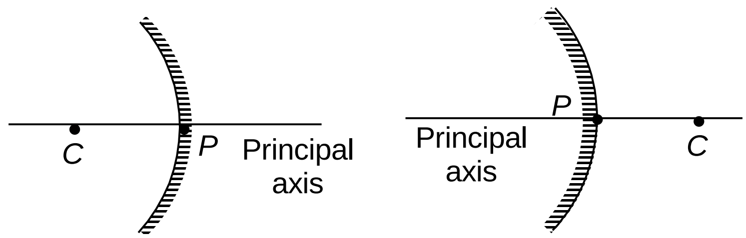
Radius of curvature (R)

Radius of curvature is the radius R of the sphere of which the mirror forms a part.



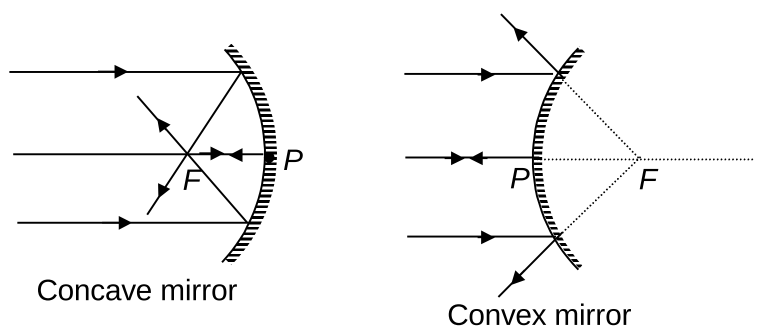
Principal axis

The line CP joining the pole and the centre of curvature of the spherical mirror is called the principal axis.



Focus (F)

If a parallel beam of rays, parallel to the principal axis and close to it, is incident on a spherical mirror; the reflected rays converge to a point F (in case of a concave mirror) or appear to diverge from a point F (in case of a convex mirror) on the principal axis. The point F is called the focus of the spherical mirror.

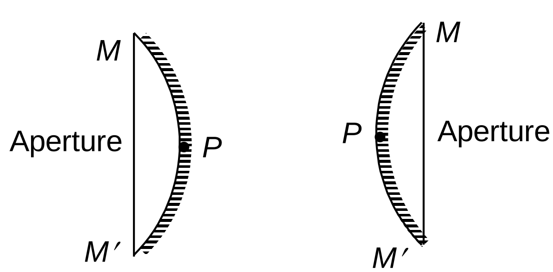


Focal Length (f)

Focal length is the distance PF between the pole P and focus F along the principle axis.

Aperture

The line joining the end points of a spherical mirror is called the aperture or linear aperture.



4.2 RELATION BETWEEN *f* AND *R*

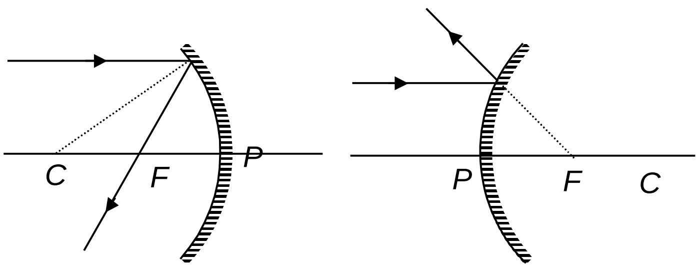
The magnitude of focal length in spherical mirrors is half the radius of curvature, i.e.,

$$f = \frac{R}{2} \qquad \dots (\mathbf{6})$$

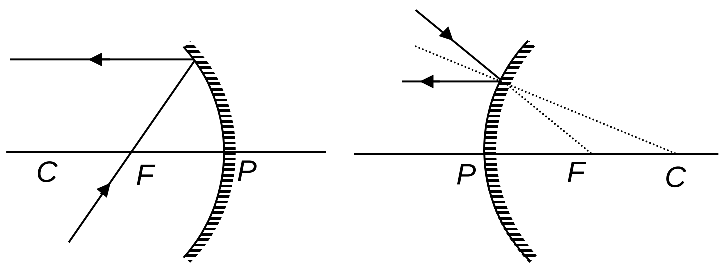
4.3 RULES FOR IMAGE FORMATION

The reflection of light rays and formation of images are shown with the help of ray diagrams. Some typical incident rays and the corresponding reflected rays are shown below.

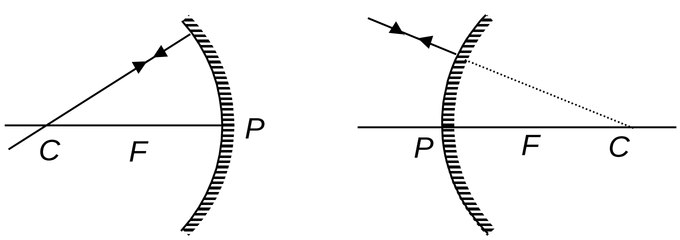
(i) A ray passing parallel to the principal axis after reflection from the spherical mirror passes or appears to pass through its focus (by the definition of focus)



(ii) A ray passing through or directed towards focus after reflection from the spherical mirror becomes parallel to the principal axis (by the principle of reversibility of light).

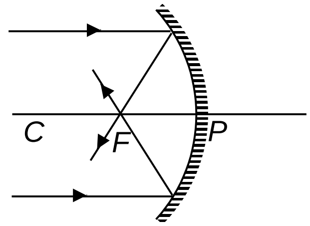


(iii) A ray passing through or directed towards the centre of curvature, after reflection from the spherical mirror, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$)



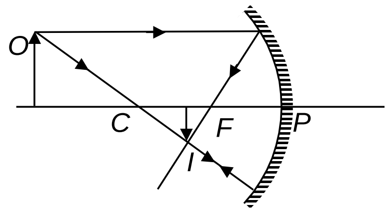
4.4 IMAGE FORMATION BY A CONCAVE MIRROR FOR A REAL LINEAR OBJECT

When the object is at infinity



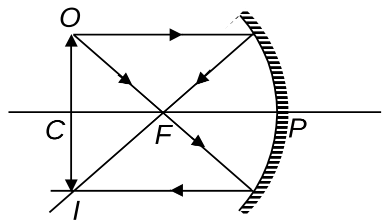
The image is formed at *F*. It is real, inverted and highly diminished.

When the object lies beyond *C* (i.e., between infinity and C)



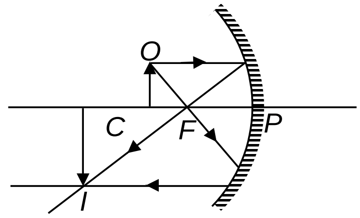
The image is formed between *F* and *C*. It is real, inverted and diminished.

When the object lies at *C*



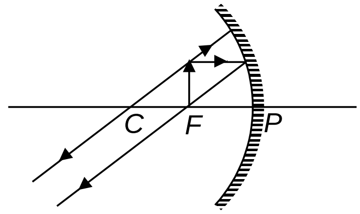
The image is formed at *C* itself. It is real, inverted and of the same size as the object is.

When the object lies between *F* and *C*



The image is formed beyond *C* (i.e., between *C* and infinity). It is real, inverted and enlarged.

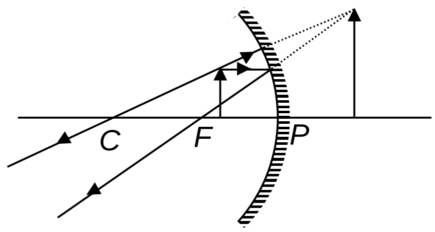
When the object is at *F*



The image is formed at infinity. It is real, inverted and highly enlarged.

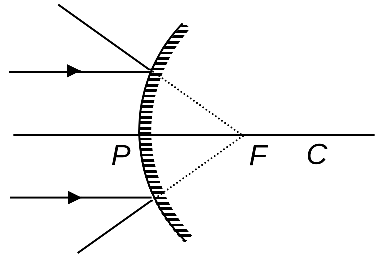
When the object lies between *P* and *F*

The image is formed behind the concave mirror. It is virtual, erect and enlarged.



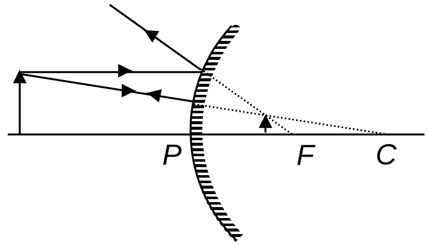
4.5 IMAGE FORMATION BY A CONVEX MIRROR FOR A REAL LINEAR OBJECT

When the object is at infinity



The image is formed at *F*. It is virtual, erect and highly diminished.

When the object lies in between infinity and *P*

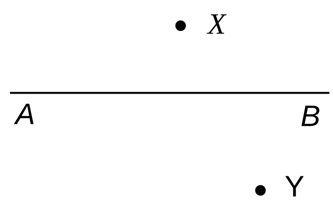


The image is formed between *P* and *F*. It is virtual, erect and diminished.

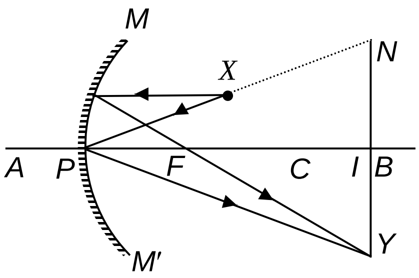
In case of image formation unless stated otherwise, object is taken to be real and we consider only rays that are close to the principal axis and that make small angles with it. Such rays are called paraxial rays. In practice this condition may be achieved by using a mirror whose size is much smaller than the radius of curvature of the surface. Otherwise the image will be distorted.

Illustration 7.

An image *Y* is formed of a point object *X* by a mirror whose principal axis is *AB* as shown below. Draw a ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagram.



Solution:



(i) From Y, we drop a perpendicular on the principal *AB* such that *YI* = *IN*

(ii) We draw a line joining points *N* and *X* so that it meets the principal axis at *P*. The point *P* will be the pole of the mirror as a ray reflected from the pole is always symmetrical about principal axis.

(iii) As the image *Y* of real object *X* is real, inverted and enlarged, the mirror must be concave and as principal section of the mirror passes through pole and is perpendicular to the principal axis. *MM′* will be the mirror.

(iv) From *X*, we draw a line parallel to the principal axis *AB* towards the mirror so that it meets the mirror at *M*. We join *M* & *Y*, so that it intersects the principal axis at *F*. *F* will be the focus of the mirror as any ray parallel to the principal axis after reflection from the mirror intersects the principal axis at the focus.

4.6 NOTATION USED

u : Distance of the object from the pole of spherical mirror.

v : Distance of the image from the pole of the spherical mirror.

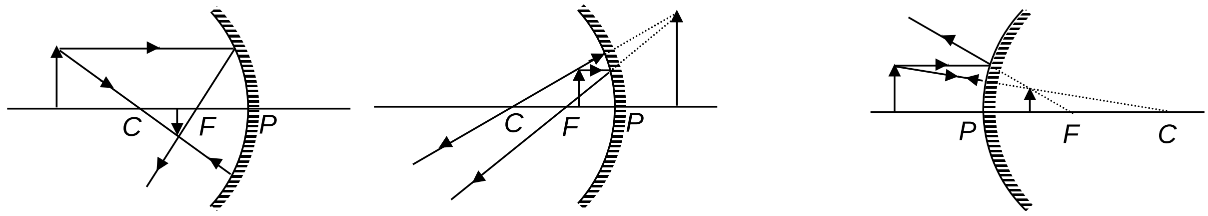
f : Focal length of the spherical mirror.

R : Radius of curvature of the spherical mirror.

4.7 SIGN CONVENTION

(i) Whenever and wherever possible, the ray of light is taken to travel from left to right.
(ii) All distances are measured from the pole of the spherical mirror along the principal axis.
(iii) Distances measured along the principal axis in the direction of the incident ray are taken to be positive while the distances measured along the principal axis against the direction of the incident rays are taken to be negative.
(iv) Distances measured above the principal axis are taken to be positive while distances measured below the principal axis are taken to be negative.

Example:



(a) (b) (c)

Figure	u	v	R	f
(a)	– ve	– ve	– ve	– ve
(b)	– ve	+ ve	– ve	– ve
(c)	– ve	+ ve	+ ve	+ ve

Important Points Regarding Sign Convection

(i) If the point (i) is valid, our convention coincides with right hand co-ordinate (or new Cartesian co-ordinate system). If the point (i) is not valid, convention is still valid but does not remain co-ordinate convention.

(ii) In this sign convention, focal length of a concave mirror is always negative while the focal length of a convex mirror is always positive.

4.8 MIRROR FORMULA

Object distance, image distance and focal length in spherical mirrors are related by the equation:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R} \qquad \ldots \textbf{(7)}$$

Illustration 8.

An object is placed in front of a concave mirror at a distance of 7.5 cm from it. If the real image is formed at a distance of 30 cm from the mirror, find the focal length of the mirror. What would be the focal length if the image is virtual.

Solution:

Case I: When the image is real.

We have

$u = -7.5$ cm ; $v = -30$ cm ; $f = ?$

We know $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

or, $f = \frac{uv}{u+v} = \frac{(-7.5)\times(-30)}{-7.5-30}$

$= \mathbf{-6\ cm}$

The negative sign shows that the spherical mirror is concave.

Case II: When the image is virtual:

In this case,

$u = -7.5$ cm

$v = +30$ cm

We know

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

or, $f = \frac{uv}{u+v} = \frac{(-75)\,(30)}{-7.5+30} = \mathbf{-10\ cm}$

4.8 LINEAR MAGNIFICATION

For linear object, the ratio of the image size (I) to the object size (O) is called linear magnification or transverse magnification or lateral magnification. It is generally denoted by m.

$$m = \frac{\text{Height of the image}}{\text{Height of the object}} = \frac{I}{O} \qquad \ldots (\mathbf{8})$$

$$m = -\frac{v}{u} = \frac{f-v}{f} = \frac{f}{f-u} \qquad \ldots (\mathbf{9})$$

Illustration 9.

An object 0.5 cm high is placed 30 cm from a convex mirror whose focal length is 20 cm. Find the position, size and nature of the image.

Solution:

We have

$u = -30$ cm ; $v = ?$ $f = +20$ cm

We know

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

or, $v = \dfrac{uf}{u-f} = \dfrac{(-30)\,(+20)}{-30-20}$ + 12 cm

The image is formed 12 cm behind the mirror. It is virtual and erect.

Now, $m = \dfrac{I}{O} = -\dfrac{v}{u} = -\dfrac{12}{-30}$

or, $I = \dfrac{2}{5} \times O = \dfrac{2}{5} \times 5 = +.2$ cm

Hence the height of the image = **+ .2 cm**

The positive sign indicates that the image is erect.

Illustration 10.

An object 0.2 cm high is placed 15 cm from a concave mirror of focal length 5 cm. Find the position and size of the image.

Solution:

We have

$u = -15$ cm; $v = ?$ $f = -5$ cm

We know that

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or,} \quad v = \frac{u\,f}{u-f} = \frac{(-15)(-5)}{-15+5} = -7.5 \text{ cm}$$

The image is formed at a distance of 7.5 cm in front of mirror.

Now, $m = \frac{I}{O} = -\frac{v}{u}$

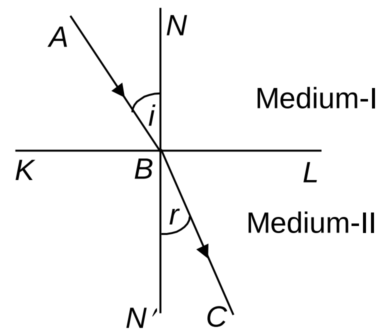
or, $\frac{I}{O} = -\frac{(-7.5)}{(-15)}$ or, $I = -0.1$ cm

The negative sign indicates that image is inverted.

5. REFRATION OF LIGHT AT PLANE SURFACE

5.1 REFRACTION

In a homogeneous medium, light rays travel in a straight line. Whenever a ray of light passes from one transparent medium to another, it gets deviated from its original path while crossing the interface of the two media (except in case of normal incidence). This phenomenon of deviation or bending of light rays from their original path while passing from one medium to another is called refraction.



$AB \rightarrow$ Incident ray.

$BC \rightarrow$ Refracted ray

$NBN' \rightarrow$ The normal to the refracting surface at the point of incident

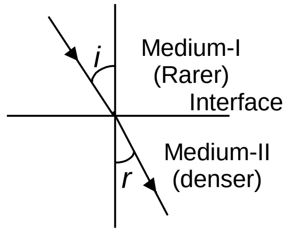
$\angle i \rightarrow$ The angle of incidence.

$\angle r \rightarrow$ The angle of refraction.

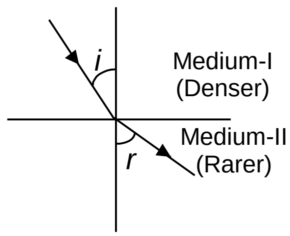
$KL \rightarrow$ Interface.

In the second medium, the ray either bends towards the normal or away from the normal with respect to its path in the first medium.

If the refracted ray bends towards the normal with respect to the incident ray, then the second medium is said to be optically denser as compared to the first medium.



If the refracted ray bends away from the normal, then the second medium is said to be (optically) rarer as compared to the first medium.



5.2 LAWS OF REFRACTION

The phenomenon of refraction takes place according to the following two laws:

(i) The incident ray, the refracted ray and the normal to the refracting surface at the point of incidence, all lie in the same plane.

(ii) The ratio of sine of the angle of incidence to the sine of the angle of refraction is a constant for any two given media and for light of given colour.

If the angle of incidence and the angle of refraction be *i* and *r* respectively, then

$$\frac{\sin i}{\sin r} = \text{constant} . \qquad \ldots (\mathbf{10})$$

This law is called Snell's law

Some Important Points

(i) According to Snell's law, $\frac{\sin i}{\sin r}$ = constant.

This constant is known as refractive index (R. I.). of the second medium with respect to the first medium. It is denoted by ${}_1\mu_2$ (or ${}_1n_2$).

(ii) Refractive index is the relative property of the two media. If the first medium carrying the incident ray is air (strictly vacuum), then the ratio $\frac{\sin i}{\sin r}$ is called the absolute refractive index of the second medium. It is dented by μ or n.

(iii) According to the wave theory of light.

$${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\text{velocity of light in medium1}}{\text{velocity of light in medium 2}} = \frac{v_1}{v_2}$$

Hence, the absolute refractive index of the medium is given by

$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{C}{V}$$

(iv) If the absolute refractive indices of the media 1 and 2 are μ_1 and μ_2 respectively, then

$${}_1\mu_2 = \frac{V_1}{V_2} = \frac{C/V_1}{C/V_2} = \frac{\mu_2}{\mu_1}$$

(v) We know ${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

or, $\mu_1 \sin i = \mu_2 \sin r \qquad \ldots (\mathbf{11})$

This is the most general formula for the refraction at a surface.

(vi) We have

$$\frac{\mu_2}{\mu_1} = \frac{C/V_2}{C/V_1} = \frac{V_1}{V_2} \qquad \text{or,} \qquad V \propto \frac{1}{\mu}$$

This shows that the higher is the refractive index of a medium, lesser is the velocity of light in that medium. Thus, a medium having higher absolute refractive index is called optically denser; while the one having smaller absolute refractive index is called to be optically rarer.

(vii) The frequency of light remains unchanged while passing from one medium to another. Hence

$$\frac{\mu_2}{\mu_1} = \frac{V_1}{V_2} = \frac{\nu\lambda_1}{\nu\lambda_2} = \frac{\lambda_1}{\lambda_2}$$

(viii) If the ray of light is incident normally on a boundary, i.e., $\angle i = 0$, then from Snell's law

$\mu_1 \sin\theta = \mu_2 \sin r$

or, $\sin r = 0$

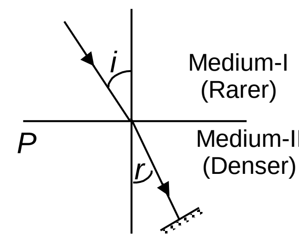
$\Rightarrow \angle r = 0$

Thus, light in the second medium will pass undeviated.

5.3 PRINCIPLE OF REVERSIBILITY OF LIGHT RAYS

A ray travelling along the path of the refracted ray is reflected along the path of the incident ray.

In the same way, a refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident ray and the refracted ray are mutually reversible. This is called the principle of reversibility of light.



When a ray of light travels from first medium to the second medium, we have

$${}_1\mu_2 = \frac{\sin i}{\sin r} \qquad \ldots \text{(i)}$$

When the path of the ray is reversed, it travels from medium-II to medium-I.

Using Snell's Law. We have

$${}_2\mu_1 = \frac{\sin r}{\sin i} \qquad \ldots \text{(ii)}$$

Multiplying (i) and (ii), we get,

$${}_1\mu_2 \times {}_2\mu_1 = 1$$

or, $${}_1\mu_2 = \frac{1}{{}_2\mu_1} \qquad \ldots \textbf{(12)}$$

Illustration 11.

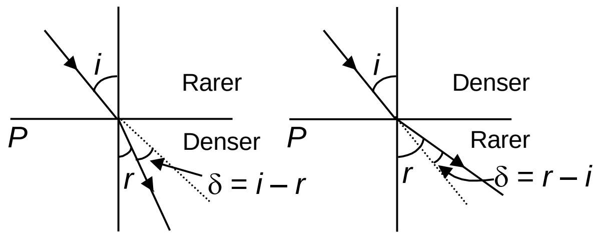
Find the refractive index of glass with respect water. Also find the refractive index of water with respect to glass. [$\mu_g = \frac{3}{2}$; $\mu_w = \frac{4}{3}$]

Solution:

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

Now, $${}_g\mu_w = \frac{1}{{}_w\mu_g} = \frac{8}{9}$$

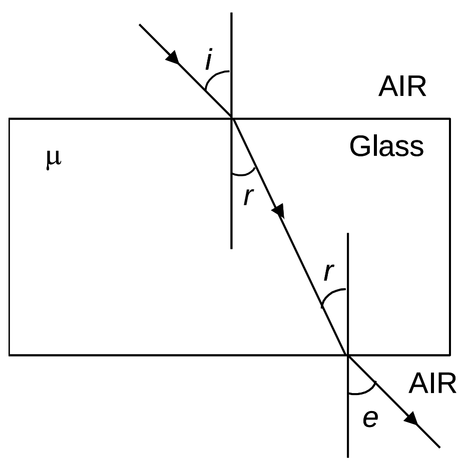
5.4 DEVIATION OF A RAY DUE TO REFRACTION



When a light ray goes from the rarer medium to the denser medium. It bends towards the normal. If a light ray goes from the denser medium to the rarer medium, it bends away from the normal.

In both cases, the magnitude of the angle of the deviation for the light ray is = $|i - r|$.

5.5 REFRACTION THROUGH A GLASS SLAB



When a light ray passes through a glass slab having parallel faces, it gets refracted twice before finally emerging out of it.

First refraction takes place from air to glass.

So, $\mu = \frac{\sin i}{\sin r}$ … (i)

The second refraction takes place from glass to air.

So, $\frac{1}{\mu} = \frac{\sin r}{\sin e}$ … (ii)

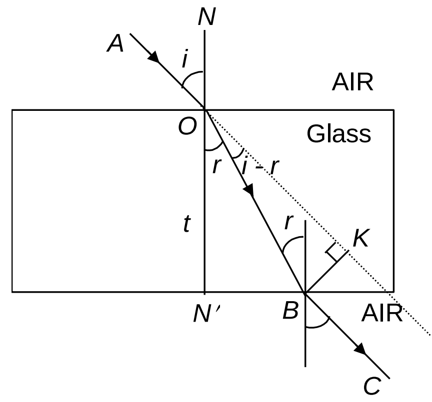
From equation (i) and equation (ii), we get

$$\frac{\sin i}{\sin r} = \frac{\sin e}{\sin r} \quad \Rightarrow i = e$$

Thus, the emergent ray is parallel to the incident ray.

5.6 LATERAL SHIFT

The perpendicular distance between the incident ray and the emergent ray, when the light is incident obliquely on a parallel sided refracting glass slab is called 'lateral shift'.



In right-angled triangle OBK, we have

$$\angle BOK = i - r$$

$$\therefore \quad \sin(i - r) = \frac{d}{OB}$$

or, $\quad d = OB \sin(i - r) \quad \ldots \text{(i)}$

In right angled triangle $ON'B$, we have

$$\cos r = \frac{ON'}{OB} \quad \text{or,} \quad OB = \frac{t}{\cos r}$$

Substituting the above value of OB in equation (i), we get

$$d = \frac{t}{\cos r} \sin(i - r) \quad \ldots \textbf{(13)}$$

Illustration 12.

A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.

Solution:

Here

$$i = 60°; \mu = 1.5 \text{ and } t = 0.1 \text{ m}$$

Now, $\quad \mu = \dfrac{\sin i}{\sin r} \quad$ or, $\quad \sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 60°}{1.5} = \dfrac{.866}{1.5} = .5773$

$\Rightarrow \quad r = 35°6'$

Now, lateral shift

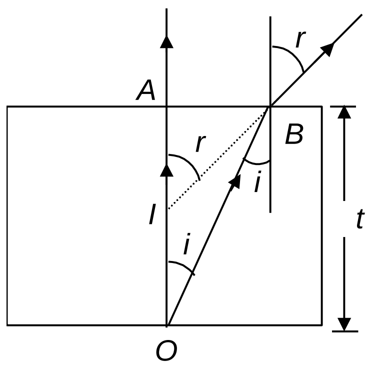
$$d = \frac{t}{\cos r} \sin(i - r) = \frac{0.1}{\cos 35°6'} \sin(60° - 35°6')$$

$$= \mathbf{0.0512 \text{ m}}$$

5.7 APPARENT DEPTH AND NORMAL SHIFT

Case I: When the object is in denser medium and the observer is in rarer medium (near normal incidence)

When an object O is in denser medium of depth 't' and absolute refractive index μ and is viewed almost normally to the surface from the outside rarer medium (say air), its image is seen at I. AO is the real depth of the object. AI is the apparent depth of the object. OI is called apparent shift.



According to Snell's law. $\frac{1}{\mu} = \frac{\sin i}{\sin r}$

or, $\frac{1}{\mu} = \frac{\tan i}{\tan r}$ ($\because$ i and r are small angles)

or, $\mu = \frac{\tan r}{\tan i}$ or, $\mu = \frac{AB}{AI} \times \frac{AO}{AB}$ or, $\mu = \frac{AO}{AI}$

$$\Rightarrow \quad \mu = \frac{\text{Real depth}}{\text{Apparent depth}} \qquad \ldots \textbf{(14)}$$

Also,

Apparent depth = $\frac{\text{Real depth}}{\mu} = \frac{t}{\mu}$

and, apparent shift (OI) = $t - t/\mu = t\,[1 - \frac{1}{\mu}]$

Illustration 13.

A tank filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Solution:

When the tank is filled with water:

Real depth = 12.5 cm; Apparent depth = 9.4 cm

$$\therefore \quad \mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{12.5}{9.4} = 1.33$$

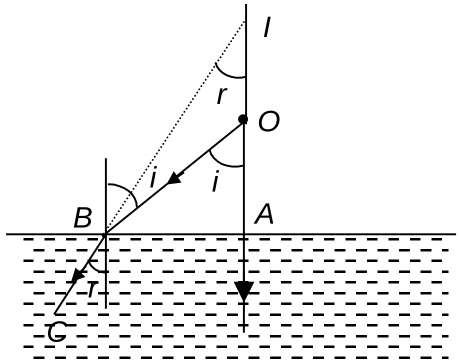
When the tank is filled with the liquid

Real depth = 12.5 cm; μ = 1.63

$$\therefore \text{ Apparent depth} = \frac{\text{Real depth}}{\mu} = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

Therefore, the distance through which the microscope to be moved = 9.4 – 7.67 = **1.73 cm**.

Case II: When the object is in rarer medium and the observer is in denser medium (Near normal incidence).



When an object O in rarer medium (say air) is seen from within a denser medium (say water), the image of O appears to be raised up to I.

The real height = AO

The apparent height = AI ; and the

Apparent shift = OI

The refraction is taking place from the rarer medium to the denser medium.

So, according to Snell's law, = $\frac{\sin\ i}{\sin\ r}$

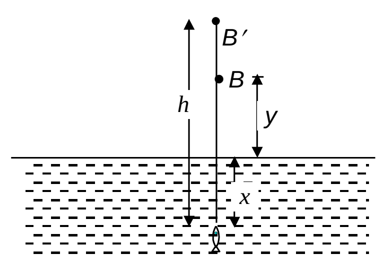
or, $\mu = \frac{\tan i}{\tan r}$ ($\because$ I and r are small angles)

or, $\mu = \frac{AB}{AO} \times \frac{AI}{AB}$ or, $\mu = \frac{AI}{AO}$

$\Rightarrow$ $\mu = \frac{\text{Apparent height}}{\text{Real height}}$ … **(15)**

Illustration 14

A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes a king fisher bird diving vertically towards water at the rate 9 m/s vertically above it. If the refractive index of water is 4/3, find the actual velocity of the dive of the bird.



Solution:

If at any instant, the fish is at a depth 'x' below water surface while the bird at a height y above the surface, then the apparent height of the bird from the surface as seen by the fish will be given by

$\mu = \frac{\text{Apparent height}}{\text{Real height}}$

or, Apparent height = μy

So, the total apparent distance of the bird as seen by the fish in water will be

$$h = x + \mu y$$

or, $$\frac{dh}{dt} = \frac{dx}{dt} + \mu\,\frac{dy}{dt}$$

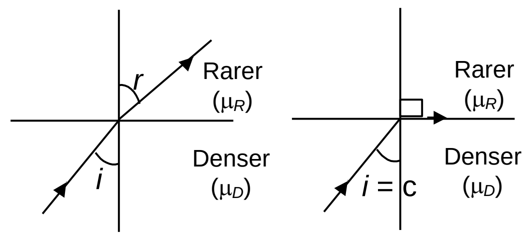
or, $$9 = 3 + \mu\left(\frac{dy}{dt}\right)$$

or, $$\frac{dy}{dt} = \frac{6}{(4/3)} = \mathbf{4.5\ m/s}$$

5.8 CRITICAL ANGLE

When a ray of light goes from a denser medium to a rarer medium, the angle of refraction is greater than the angle of incidence. If the angle of incidence is increased, the angle of refraction may eventually become 90°.

The angle of incidence for which the angle of refraction is 90° is called the critical angle for that interface. It is generally denoted by *C*.



5.9 EXPRESSION FOR CRITICAL ANGLE

Let μ_R be the refractive index of the rarer medium and μ_D be the refractive index of the denser medium. Obviously, $\mu_r < \mu_D$

From Snell's law

$$\frac{\sin C}{\sin 90°} = \frac{\mu_R}{\mu_D}$$

or, $$\frac{\sin C}{1} = \frac{\mu_R}{\mu_D}$$

or, $$\sin C = \frac{1}{\mu}. \qquad \dots \mathbf{(16)}$$

where $\mu = \dfrac{\mu_D}{\mu_R}$ is refractive index of the denser medium w.r. t the rarer medium.

5.10 TOTAL INTERNAL REFLECTION

If a ray of light travelling in a denser medium strikes a rarer medium at an angle of incidence *i* which is greater than the critical angle *C*, it gets totally reflected back into the same medium. This phenomenon is called as 'total internal reflection'.

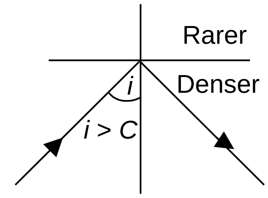
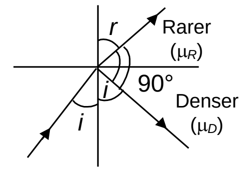


Illustration 15.

A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. If the reflected and the refracted rays are mutually perpendicular to each-other, what is the value of the critical angle?



Solution:

From Snell's law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D}$$

or, $\mu = \dfrac{\mu_D}{\mu_R} = \dfrac{\sin r}{\sin i}$ … (i)

According to the given problem,

$i + r + 90° = 180°$

or, $r = 90° - i$

Substituting the above value of r in equation (i), we get

$$\mu = \frac{\sin(90 - i)}{\sin i}$$

or, $\mu = \cot i$ … (ii)

By definition $C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

or, $C = \sin^{-1}\left(\dfrac{1}{\cot i}\right)$ (using equation (ii)

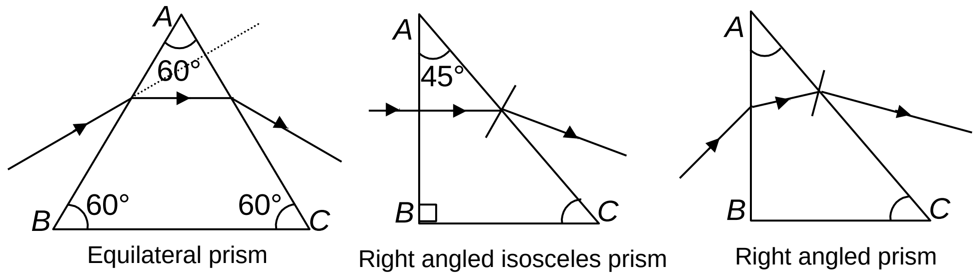
or, $C = \mathbf{\sin^{-1}(\tan \mathit{i})}$

6. REFRACTION THROUGH A PRISM

6.1 PRISM

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are plane and non-parallel.

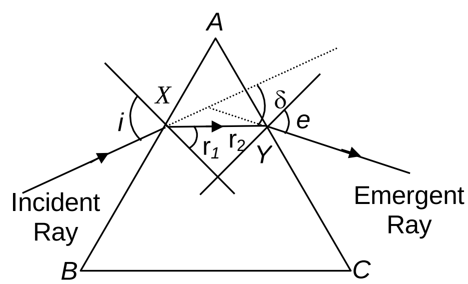
Generally equilateral, right-angled isosceles or right-angled prism are used.



6.2 ANGLE OF THE PRISM OR REFRACTING ANGLE OF THE PRISM

The angle between the two refracting faces involved is called the refracting angle or the angle (A) of the prism.

6.3 DEVIATION PRODUCED BY A PRISM:



A ray of light striking at one face of a triangular glass prism gets refracted twice and emerges out from the other face as shown above.

The angle between the emergent and the incident rays is called the angle of deviation (δ).

From ΔAXY, we have

$$A + (90° - r_1) + (90° - r_2) = 180°$$

$\Rightarrow$ $r_1 + r_2 = A$ … (**17**)

Now, deviation $\delta = (i - r_1) + (e - r_2)$

$= (i + e) - (r_1 + r_2)$

$\Rightarrow$ $\delta = (i + e) - A$ …(**18**)

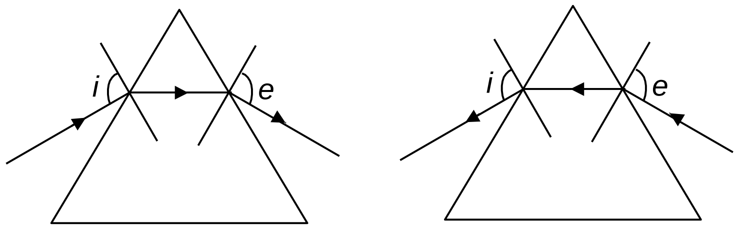
or, $\delta + A = i + e$

Important Points

(i) We have two equations from Snell's law at X and Y

$$\frac{\sin i}{\sin r_1} = \mu \text{ and } \frac{\sin r_2}{\sin e} = \frac{1}{\mu}$$

(ii) It can be easily seen that if we reverse the emergent ray, it goes back along the same path. The angles of incidence and emergence get interchanged but the angle of deviation remains the same.



Thus the same angle of deviation δ is possible for two different angles of incidence: i and e such that

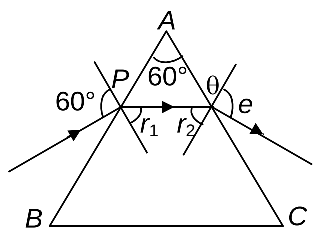
$$i + e = A + \delta$$

Illustration 16.

A ray of light is incident on one face of a prism ($\mu = 1.5$) at an angle of 60°. The refracting angle of the prism is also 60°. Find the angle of emergence and the angle of deviation. Is there any other angle of incidence, which will produce the same deviation?

Solution:

Angle of incidence = i = 60°



At point P, $\frac{\sin 60°}{\sin r_1} = \frac{1.5}{1}$

$\Rightarrow \quad \sin r_1 = \frac{1}{\sqrt{3}}$

or, $\quad r_1 \approx 35°6'$

Using $\quad r_1 + r_2 = A$, we get

$r_2 = A - r_1 = 60° - 35°6' = 24°44'$

At point Q, $\frac{\sin r_2}{\sin e} = \frac{1}{1.5}$

$\Rightarrow \sin e = 1.5 \sin 24°44 \qquad \Rightarrow \sin e = 0.63 \qquad \Rightarrow e = 39°$

$\therefore$ Deviation = $\delta = (i + e) - A = 60° + 39°$

= **39°**

If i and e are interchanged, deviation remains the same. Hence same deviation is obtained for angles of incidence 60° and 39°.

Illustration 17.

A ray of light makes an angle of 60° on one of the faces of a prism and suffers a total deviation of 30° on emergence from the other face. If the angle of the prism is 30°, show that the emergent ray is perpendicular to the other face. Also calculate the refractive index of the material of the prism.

Solution:

The angle of deviation $\delta = (i_1 + i_2) - A$

here, $\delta = 30°$, $i_1 = 60°$; $A = 30°$

Hence $30° = 60° + i_2 - 30° = 30° + i_2$

$\Rightarrow \quad i_2 = 0$

The angle of emergence is zero. This means that the emergent ray is perpendicular to the second face.

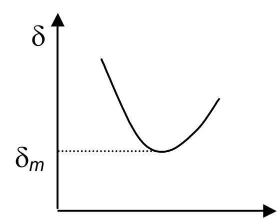
Since $i_2 = 0$, The angle of incidence at the second face is zero.

$\therefore \quad r_2 = 0$

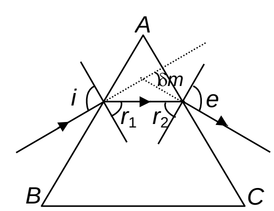
Now, $r_1 + r_2 = A$ or, $r_1 = A = 30°$

We know, $\mu = \dfrac{\sin i_1}{\sin r_1} = \dfrac{\sin 60°}{\sin 30°} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3} = \mathbf{1.732}$

6.4 MINIMUM DEVIATION AND CONDITION AND CONDITION FOR MINIMUM DEVIATION



The angle of deviation depends on the angle of incidence in a peculiar way. When the angle of incidence is small, the deviation is large. As i increase, δ decreases rapidly and attains a minimum value and then increases slowly with increase of i. The minimum value of δ so attained is called the minimum deviation (δ_m).



Theory and experiment shows that δ will be minimum when the path of the light ray through the prism is symmetrical, i.e.,

Angle of incidence = angle of emergence

or, $\angle i = \angle e$

For the refraction at the face AB, we have

$$\frac{\sin i}{\sin r_1} = \mu \text{ (Snell's Law)} \quad \text{or,} \quad \sin i = \mu \sin r_1$$

and, $\dfrac{\sin e}{\sin r_2} = \mu$ or, $\sin e = \mu \sin r_2$

$\therefore \quad \mu \sin r_1 = \mu \sin r_2$

or, $\quad r_1 = r_2$

Hence, the condition for minimum deviation is $i = e$ and $r_1 = r_2$ … (**19**)

6.5 RELATION BETWEEN REFRACTIVE INDEX AND THE ANGLE OF MINIMUM DEVIATION

When $\delta = \delta_m$, we have

$$e = i \text{ and } r_1 = r_2 = r \text{ (say)}$$

We know

$$A = r_1 + r_2 = r + r = 2r$$

or, $r = \dfrac{A}{2}$

Also, $A + \delta = i + e$

or, $A = \delta_m = i + i$

or, $i = \dfrac{A+\delta_m}{2}$

The refractive index of the material of the prism is given by

$$\mu = \frac{\sin i}{\sin r} \text{ (Snell's law)}$$

or, $$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}} \qquad \ldots (\mathbf{20})$$

Illustration 18.

A ray of light incident at 49° on the face of an equilateral prism passes symmetrically. Calculate the refractive index of the material of the prism.

Solution:

As the prism is an equilateral one, $A = 60°$. As the ray of light passes symmetrically, the prism is in the position of minimum deviation.

So, $r = \dfrac{A}{2} = \dfrac{60°}{2} = 30°$

also, $i = 49°$

$\therefore$ $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin 49°}{\sin 30°} = \dfrac{0.7547}{0.5} = \mathbf{1.5}$

Illustration 19.

The refracting angle of the prism is 60° and the refractive index of the material of the prism is 1.632. Calculate the angle of minimum deviation.

Solution:

Here, $A = 60°$; $\mu = 1.632$

Now, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

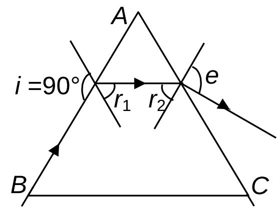
or, $1.632 = \dfrac{\sin\left(\dfrac{60°+\delta_m}{2}\right)}{\sin\dfrac{60°}{2}} = \dfrac{\sin\left(\dfrac{60+\delta_m}{2}\right)}{\sin 30°}$

or, $\sin\left(\dfrac{60°+\delta_m}{2}\right) = 1.632 \sin 30° = 1{,}632 \times .5$

or, $\sin\left(\dfrac{60°+\delta_m}{2}\right) = 0.816$ or, $\dfrac{60°+\delta_m}{2} = 54°42'$ $\delta m = \mathbf{49°24'}$

6.6 GRAZING INCIDENCE

When $I = 90°$, the incident ray grazes along the surface of the prism and the angle of refraction (r_1) inside the prism becomes equal to the critical angle for glass-air pair. This is known as grazing incidence.



6.7 GRAZING EMERGENCE

When $e = 90°$, the emergent ray grazes along the prism surface. This happens when the light ray strikes the second face of the prism at the critical angle for glass-air pair. This is known as grazing emergence.

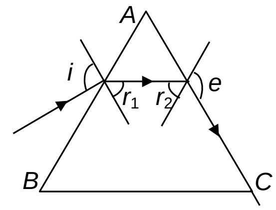


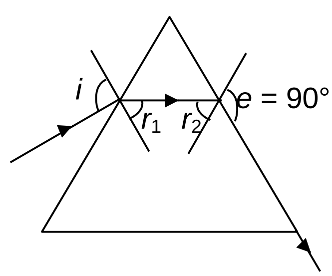
Illustration 20.

A ray of light falls on one side of a prism whose refracting angle is 60°. Find the angle of incidence in order that the emergent ray may just graze the other side.

Solution:

Given: $A = 60°$, $e = 90°$

$\therefore$ $r_2 = C$, the critical angle of the prism.



Now, $\mu = \dfrac{1}{\sin C}$

or, $\sin C = \dfrac{1}{\mu} = \dfrac{2}{3}$

$\Rightarrow$ $C = 41°49'$

Again, $A = r_1 + r_2$

$\Rightarrow \quad r_1 = A - r_2 = 60° - 41°49'$

$= 18°11'$

For the refraction at the surface *AB*, we have

$$\mu = \frac{\sin i}{\sin r_1} \text{ (Snell's Law)}$$

or, $\sin i = \mu \sin r_1$

$= 1.5 \times \sin 18°11'$

$= 1.5 \times 0.3121$

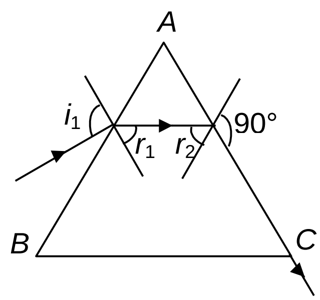
$= .46815$

$i =$ **27°55′**

Illustration 20.

The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as they come out of the prism.

Solution:



Given $A = 45°$, $\mu = 1.6$

we have $\mu = \dfrac{1}{\sin C}$

or, $\sin C = \dfrac{1}{\mu} = \dfrac{1}{1.6}$

$\Rightarrow \quad C = 38.68°$

For total internal reflection not to take place at the surface *AC*, we have

$r_2 \leq C$

or, $(r_2)_{max} = C$

Now, $r_1 + r_2 = A$

or, $r_1 = (A - r_2)$

or, $(r_1)_{min} = A - (r_2)_{max} = 45° - 38.68° = 6.32°$

For the refraction at the first face,

We have, $\mu = \dfrac{\sin i_1}{\sin r_1}$

or, $\sin i_1 = \mu \sin r_1 = 1.6 \times \sin (6.32°)$

$\Rightarrow \quad i_1 =$ **10.14°**

6.7 MAXIMUM DEVIATION

We know $\delta = (i + e) - A$

Deviation will be maximum when the angle of incidence i is maximum, i.e., $i = 90°$. Hence,

$$(\delta)_{max} = (90° + e) - A$$

This is the expression for the maximum deviation.

Illustration 22.

Find the minimum and maximum angle of deviation for a prism with angle $A = 60°$ and $\mu = 1.5$.

Solution:

Minimum deviation:

The angle of minimum deviation occurs when $i = e$ and $r_1 = r_2$ and is given by

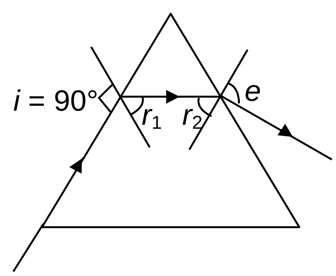
$$\mu = \frac{\sin(A+\delta_m)}{\sin\frac{A}{2}}$$

$$\Rightarrow \delta_m = 2\sin^{-1}\left(\mu\sin\frac{A}{2}\right) - A$$

Substituting $\mu = 1.5$ and $A = 60°$, we get

$$\delta_m = 2\sin^{-1}(0.75) - 60° = \mathbf{37°}$$

Maximum deviation:



The deviation is maximum when $i = 90°$ or e = 90° that is at grazing incidence or grazing emergence.

Let $i = 90°$

$$\Rightarrow \quad r_1 = C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{2}{3}\right) = 42°$$

$$\Rightarrow \quad r_2 = A - r_1 = 60° - 42° = 18°$$

Using $\frac{\sin r_2}{\sin e} = \frac{1}{\mu}$, we have

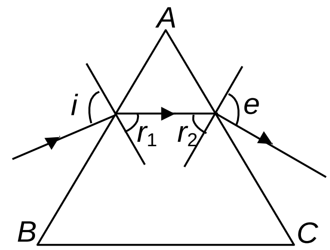
$$\sin e = \mu\sin r_2 = 1.5 \times \sin 18°$$

$$\Rightarrow \quad \sin e = 4.63$$

$$\Rightarrow \quad e = 28°$$

$$\therefore \quad \text{Deviation} = \delta_{max} = (i + e) - A = 90° + 28 - 60° = \mathbf{58°}$$

6.8 DEVIATION THROUGH A PRISM OF SMALL ANGLE



If the angle of the prism A is small, r_1 and r_2 (as $r_1 + r_2 = A$) and i and e will be small.

For the refraction at the face AB, we have $\mu = \dfrac{\sin i}{\sin r_1}$

or, $\mu = \dfrac{i}{r_1}$ (since i and r_1 are small angles, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$)

$\Rightarrow$ For refraction at the face AC, we have

$$\mu = \frac{\sin e}{\sin r_2}$$

or, $\mu = \dfrac{e}{r_2}$ ($\because$ e and r_2 are small angles, so $\sin e \approx$ and $\sin r_2 \approx r_2$)

$\Rightarrow \quad e = \mu r_2 \quad \ldots$ (ii)

Now, deviation produced by a prism

$\delta = (i + e) - A$

or, $\delta = (\mu r_1 + \mu r_2) - A$ or, $\delta = \mu\,(r_1 + r_2) - A$

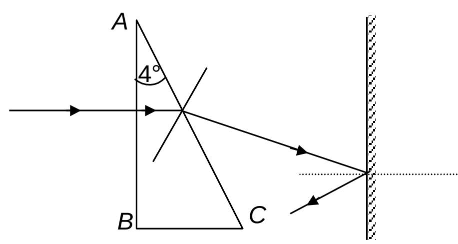
or $\delta = \mu A - A$ [$\because$ $r_1 + r_2 = A$]

or, $\delta = (\mu - 1)\,A \quad \ldots$ (**20**)

The above formula is valid for all positions of the prism provided the angle of the prism A is small (say $\le 10°$).

Illustration 23.

A prism having a refracting angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown. A horizontal ray of light is incident on the prism. What is the angle of incidence at the mirror?

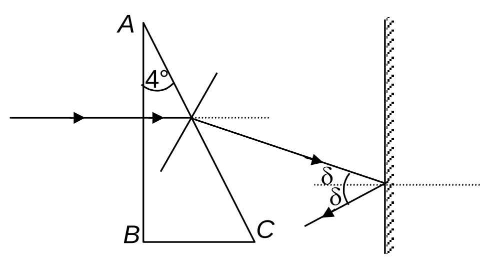


Solution:

The deviation suffered by refraction through the small angled prism is given by

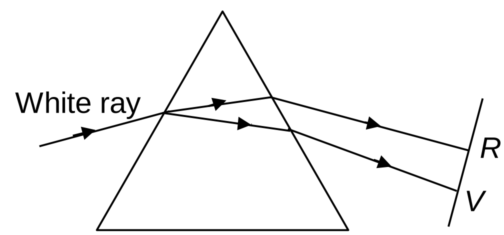
$\delta = (\mu - 1)\,A = (1.5 - 1) \times 4° = \mathbf{2°}$

This gives the angle of incidence 2° at the mirror.



7. DISPERSION OF LIGHT THROUGH A PRISM

Dispersion: When a ray of white light is passed through a prism, it splits up into its constituent colours. This phenomenon of splitting up of white light into constituent colours is called dispersion. The band of seven colours produced on the screen is called the spectrum of the source emitting the incident white light.



Rainbow the most colourful phenomenon in nature is primarily due to the dispersion of sunlight by raindrops suspended in air.

7.1 CAUSE OF DISPERSION

The different colour of light are due to different wavelengths. The wavelengths of violet colour is smaller than that of the red colour.

Cauchy obtained expression for the refractive index of a material in terms of the wavelength of the light. It is given by

$$\mu = a + \frac{b}{\lambda^2}, \qquad \dots (\mathbf{21})$$

where a & b are constants for the material

Since the wavelength of violet colour is smaller than that of red colour, from the above formula, it follows that the refractive index of the material for violet colour is more than that for red colour.

i.e., $\mu_v > \mu_R$

for a small angled prism, we have

$$\delta = (\mu - 1)A$$

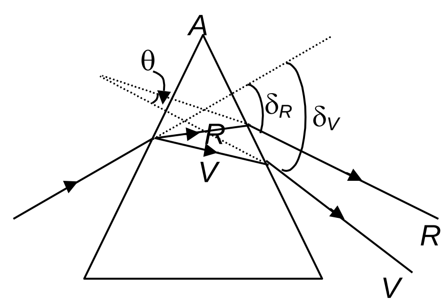
since $\mu_V > \mu_R$, we have

$$\delta_V > \delta_R$$

7.2 ANGULAR DISPERSION

In case of dispersion of light, the angle between the extreme rays (i.e., violet and red) is called angular dispersion or simply dispersion.

$$\text{Angular dispersion} = \theta = \delta_V - \delta_R \qquad \dots (\mathbf{22})$$



Let μ_V and μ_R be the refractive indices of the material of the prism for violet and red colours respectively. Let A be the angle of the prism.

We have ,

$$\delta_V = (\mu_V - 1)A;$$

$$\delta_R = (\mu_{R-1})A$$

$$\therefore \quad \delta_V - \delta_R = (\mu_V - \mu_R)A \qquad \dots (\mathbf{23})$$

7.3 DISPERSIVE POWER (ω)

The ratio of (angular) dispersion to the deviation of the mean ray (yellow) is called the dispersive power of the prism. It is denoted by ω.

$\omega = \frac{\theta}{\delta} = \frac{\delta_v - \delta_R}{\delta}$, where δ is the deviation of the mean ray; δ_V and δ_R are the deviations of the violet and the red rays respectively.

7.4 DISPERSIVE POWER IN TERMS OF REFRACTIVE INDEX

In a thin prism,

$$\delta = (\mu - 1)\, A$$

$$\therefore \quad \delta_V = (\mu_v - 1)\, A; \;\; \delta_R = (\mu_R - 1)\, A$$

Hence, $\delta_V - \delta_R = (\mu_V - \mu_R)\, A$

$$\therefore \quad \omega = \frac{\theta}{\delta} = \frac{\delta_V - \delta_R}{\delta} \quad \Rightarrow \quad \omega = \frac{\mu_V - \mu_R}{\mu - 1} \qquad \ldots \textbf{(24)}$$

Illustration 24.

Calculate the dispersive power of crown and flint glass-prism from the following data.

For crown glass, $\mu_V = 1.522$; $\mu_r = 1.514$

For flint glass, $\mu_V = 1.662$; $\mu_r = 1.644$

Solution:

For crown glass:

$$\mu_V = 1.522 \; ; \; \mu_R = 1.514$$

$$\therefore \; \mu_Y = \frac{\mu_V + \mu_R}{2} = \frac{1.522 + 1.514}{2} = 1.518$$

Hence, the dispersive power of crown glass

$$\omega = \frac{\mu_V - \mu_r}{\mu_{Y-1}} = \frac{1.522 - 1.514}{1.518} = 0.01544$$

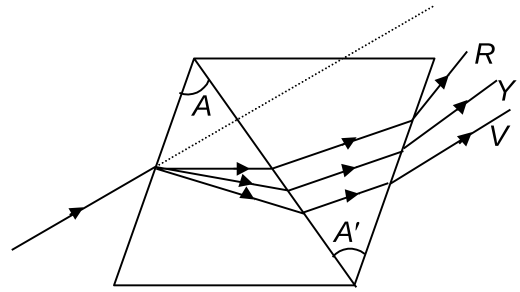
For flint glass:

$$\mu_V = 1.662 \; ; \; \mu_R = 1.644$$

$$\therefore \; \mu = \frac{\mu_V + \mu_R}{2} = \frac{1.662 + 1.644}{2} = 1.653$$

$$\therefore \; \omega = \frac{\mu_v - \mu_R}{\mu - 1} = \frac{1.662 - 1.644}{(1.653 - 1)} = \mathbf{.0276}$$

7.5 DISPERSION WITHOUT DEVIATION (DIRECT VISION SECTROSCOPE)



Let two prisms of different material of dispersive powers ω and ω' be placed in contact, with their bases turned opposite to each-other. Let A and A' be the angles of the first and the second prism respectively.

Let μ_V, μ_R and μ be the refractive indices of the material of the prism for violet, red and yellow colours respectively.

Let μ_V' μ_R', μ' be the corresponding values of the material of the second prism.

Deviation of the mean ray by the first prism is

$$\delta = (\mu - 1) A$$

Deviation of the mean ray by the second prism is

$$\delta' = (\mu' - 1) A'$$

Since the combination does not produce any deviation

$$\delta + \delta' = 0 \qquad \text{or,} \quad (\mu - 1) A + (\mu' - 1) A' = 0$$

or, $$\frac{A'}{A} = -\frac{(\mu - 1)}{(\mu' - 1)} \qquad \ldots \textbf{(25)}$$

The negative sign indicates that the two prisms are placed with their bases opposite to each-other.

NET ANGULAR DISPERSION

The angular dispersion produced by the first prism,

$$\delta_V - \delta_R = (\mu_V - \mu_R) A$$

The angular dispersion produced by the second prism,

$$\delta_V' - \delta_R' = (\mu_V' - \mu_R') A'$$

$\therefore$ Net angular dispersion

$$= (\delta_V - \delta_R) + (\delta_V' - \delta_R')$$

$$= (\mu_V - \mu_R) A + (\mu_V' - \mu_R') A'$$

$$= A\left[(\mu_V - \mu_R) + (\mu_V' - \mu_R')\frac{A'}{A}\right]$$

Substituting $\frac{A'}{A} = -\left(\frac{\mu - 1}{\mu - 1}\right)$ in the above equation,

Net angular dispersion $= A\left[\mu_V - \mu_R) - \frac{(\mu - 1)}{(\mu' - 1)} \times (\mu_v' - \mu_R')\right]$

$$= (\mu - 1)A\left[\left(\frac{\mu_V - \mu_R}{\mu - 1}\right) - \left(\frac{\mu_v' - \mu_R'}{\mu' - 1}\right)\right]$$

or, net angular dispersion = $(\mu - 1) A [\omega - \omega']$ … (**26**)

Illustration 25.

Find the angle of the flint glass prism which should be combined with a crown glass prism of 5° so as to give dispersion but no deviation.

For crown glass; $\mu_V = 1.523$; $\mu_R = 1.515$

For flint glass; $\mu_V' = 1.688$; $\mu_R' = 1.650$

Solution:

For no deviation,

$$\frac{A'}{A} = -\left(\frac{\mu - 1}{\mu' - 1}\right)$$

or, $$A' = -\left(\frac{\mu - 1}{\mu' - 1}\right) A$$

Now, $$\mu = \frac{\mu_V + \mu_R}{2} = \frac{1.523 + 1.515}{2} = 1.519$$

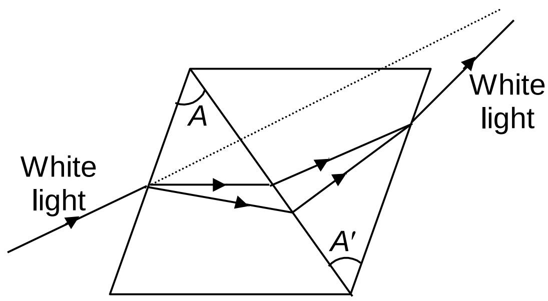
$$\mu' = \frac{\mu_V' + \mu_R'}{2} = \frac{1.668 + 1.650}{2} = 1.659$$

$$A' = -\left(\frac{1.519 - 1}{1.659 - 1}\right) 5°$$

$$= \mathbf{-3.94°}$$

7.6 DEVIATION WITHOUT DISPERSION (ARCHROMATIC COMBINATION OF PRISM)

Let two thin prisms of dispersive powers ω and ω' and angles A and A' placed in contact with their bases turned opposite to each-other.



Let μ_V, μ_R and μ be the refractive indices of the material of the first prism for violet, red and mean light respectively.

Let μ_V' μ_R' and μ' be the corresponding values for the material of the second prism.

Angular dispersion produced by the first prism

$$(\delta_V - \delta_R) = (\mu_V - \mu_R) A$$

The angular dispersion produced by the second prism.

$$(\delta_V' - \delta_R') = (\mu_V' - \mu_R') A'$$

Since the combination does not produce any dispersion

$$(\delta_V - \delta_R) + (\delta_V' - \delta_R') = 0$$

or, $A(\mu_V - \mu_R) + A'(\mu_V' - \mu_R') = 0$

or, $$\frac{A'}{A} = -\left(\frac{\mu_V - \mu_R}{\mu_V' - \mu_R'}\right) \qquad \ldots \textbf{(27)}$$

This is the condition for no dispersion or condition for achromatism.

The negative sign indicates that two prisms should be placed in opposite manner.

Another form:

We have

$$A(\mu_V - \mu_R) + A'(\mu_V' - \mu_R') = 0$$

or, $$\left(\frac{\mu_V - \mu_R}{\mu - 1}\right)(\mu - 1)A + \left(\frac{\mu_V' - \mu_R'}{\mu' - 1}\right) \times (\mu' - 1)A' = 0$$

But $(\mu - 1)A = \delta$ and $(\mu' - 1)A' = \delta'$, the deviations for mean light by the first and the second prism respectively.

Also, $\omega = \frac{\mu_V - \mu_R}{\mu - 1}$, the dispersive power of the first prism.

$\omega' = \left(\frac{\mu_V' - \mu_R'}{\mu' - 1}\right)$, the dispersive power of the second prism.

$\therefore \quad \delta\,\omega + \delta'\omega' = 0$

$$\Rightarrow \quad \frac{\omega}{\omega'} = -\frac{\delta'}{\delta} \qquad \ldots \textbf{(28)}$$

NET DEVIATION

The deviation suffered by the mean light through the first prism.

$$\delta = (\mu - 1)A$$

The deviation suffered by the mean light through the second prism

$$\delta' = (\mu' - 1)A'$$

Net deviation $= \delta + \delta' = (\mu - 1)A + (\mu' - 1\ A' = (\mu - 1)A[1 + \frac{(\mu' - 1)A'}{(\mu - 1)A}]$

$$= (\mu - 1)A\left[1 + \frac{(\mu' - 1)}{(\mu - 1)}\left(\frac{\mu_V - \mu_R}{\mu_V' - \mu_R'}\right)\right]$$

or, Net deviation $= (\mu - 1)A\ [1 - \frac{\omega}{\omega'}]$ $\qquad \ldots$ **(29)**

Illustration 26.

Find the angle of a prism of dispersive power 0.021 and refractive index 1.53 to form an achromatic combination with the prism of angle 4.2° and dispersive power 0.045 having refractive index 1.65. Also calculate the resultant deviation.

Solution:

$\omega = 0.021$; $\mu = 1.53$; $\omega' = 0.045$; $\mu' = 1.65$

$A' = 4.2°$

For no dispersion

$\omega\delta + \omega'\delta' = 0$

or, $\omega(\mu - 1)A + \omega'(\mu' - 1)A' = 0$

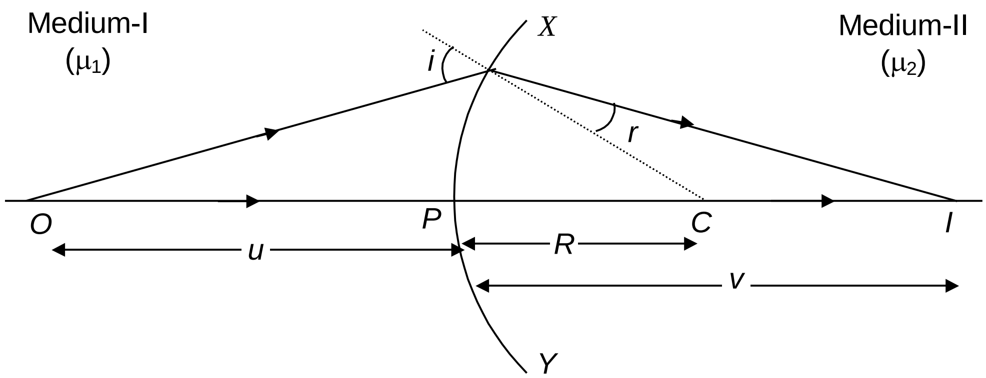
or, $A = -\dfrac{\omega' A'(\mu'-1)}{(\mu-1)} = -\dfrac{.045\times 4.2\times(1.65-1)}{.021\times(1.53-1)} = -11.4°$

Net deviation

$\delta + \delta' = (\mu - 1)A + (\mu' - 1)A'$

$= -11.04°(1.53-1) + 4.2°(1.65- u1)$ **= – 3.12°**

8. REFRACTION AT THE SINGLE SPHERICAL SURFACE



Consider a spherical surface of radius R separating two media with refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$).

If u is the object distance and v is the image distance, for light rays going from medium I to the medium II,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \qquad \dots (\mathbf{30})$$

Also, the transverse magnification in this case is

$$m = \frac{I}{O} = \frac{\mu_1}{\mu_2}\frac{v}{u} \qquad \dots (\mathbf{31})$$

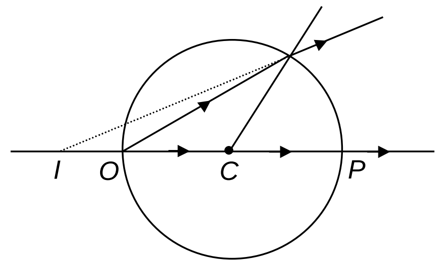
IMPORTANT POINTS

(i) These equations are valid for all refracting surfaces convex, concave or plane. In case of plane refracting surface, $R \to \infty$.

(ii) The rules for signs for a single refracting surface are same as for spherical mirrors.

Illustration 27.

If a mark of size 0.2 cm on the surface of a glass sphere of diameter 10 cm and μ = 1.5 is viewed through the diametrically opposite point, where will the image be seen and of what size?



Solution:

As the mark is on one surface, refraction will take place on the other surface (which is curved). Further refraction is taking place from glass to air. So,

$\mu_1 = 1.5$; $\mu_2 = 1$; $R = -5$ cm; $u = -10$ cm; $v = ?$

Using the formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$, we have

$$\frac{1}{v} - \frac{1.5}{(-10)} = \frac{1-1.5}{-5} \quad \text{or,} \quad v = -20 \text{ cm}$$

Hence, the image is at a distance of 20 cm from *P* towards *O*.

In case of refraction at a curved surface, we have

$$m = \frac{I}{O} = \frac{\mu_1}{\mu_2}\frac{v}{u} = \frac{(1.5)\times(-20)}{1\times(-10)} = +3$$

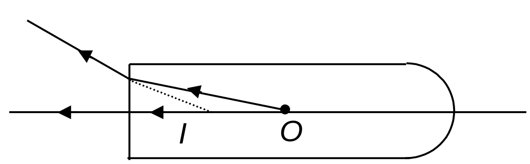
So, the image is virtual, erect and of size $I = m \times O = 3 \times 0.2 =$ **0.6 cm**

Illustration 28.

A transparent rod 40 cm long is cut flat at one end and rounded to a hemispherical surface 12 cm radius at the other end. A small object is embedded within the rod along its axis and half way between its ends. When viewed from the flat end of the rod, the object appears 12.5 cm deep. What is its apparent depth when viewed from the curved end?

Solution:

Case I: When the object is viewed from the flat surface:



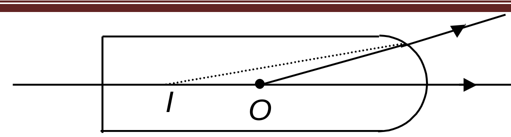
Real depth of the object = 20 cm

Apparent depth = 12.5 cm

Using, $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$,

we have, $\mu = \dfrac{20}{12.5} = 1.6$

Case II: When the object is viewed through the curved surface:



Here the refraction is taking place at the single curved surface. So we will use

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

Here, $\mu_1 = 1.6$; $\mu_2 = 1$; $u = -20$ cm ; $v = ?$ $R = -12$ cm

So $\frac{1}{v}-\frac{1.6}{(-20)}=\frac{1-1.6}{-12}$ $\Rightarrow v = -33.3$ cm

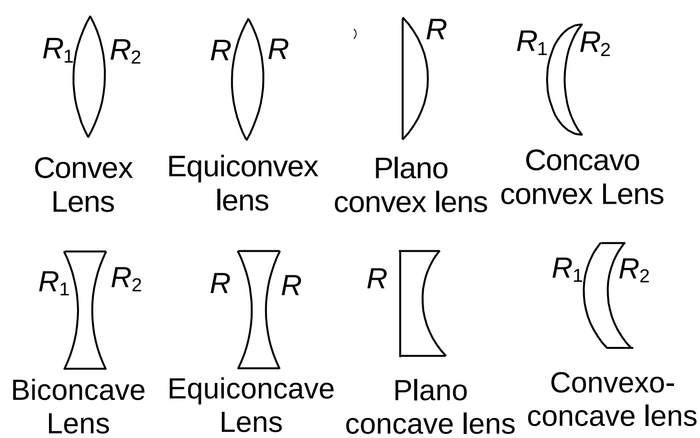
Hence the object appears 33.3 cm deep from the curved surface

9. REFRACTION THROUGH THIN LENSES

Lens: A lens is a transparent medium bounded by two refracting surfaces such that at least one of the refracting surfaces is curved.

If the thickness of the lens is negligibly small in comparison to the object distance or the image distance, the lens is called thin. Here we shall limit ourself to thin lenses.

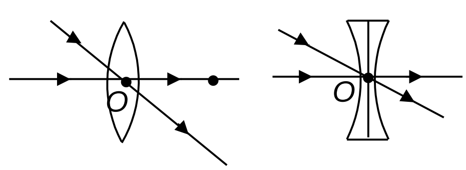
Types of lenses: Broadly, lenses are of the following types:



9.1 SOME IMPORTANT DEFINITIONS

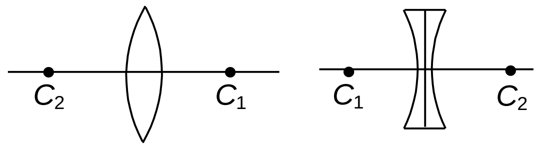
Optical centre

Optical centre is a point for a given lens through which any ray passes undeviated.



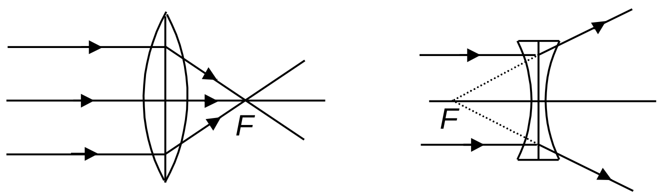
Principal axis

The line joining the centres of curvature of the two bounding surfaces is called the principal axis.



Focus (F)

A parallel beam of light, parallel and close to the principal axis of the lens after refraction passes through a fixed point on the principal axis (in case of convex lens) or appear to diverge from a point on the principal axis (in case of concave lens). This point is called the principal focus or focus of the lens. It is generally denoted by *F*.



Lenses have two foci because light can strike the lens from both the sides.

Focal length (*f*)

The distance between the optical centre and the focus of a lens is called its focal length. It is denoted by *f*.

9.2 SIGN CONVENTION

(i) Whenever and wherever possible, rays of light are taken to travel from left to right.

(ii) Distances are measured along the principal axis from the optical centre of the lens.

(iii) Distances measured along the principal axis in the direction of the incident rays are taken as positive while those measured against the direction of the incident rays are taken negative.

(iv) Distances measured above the principal axis are taken as positive and those measured below the principal axis are taken as negative.

Example

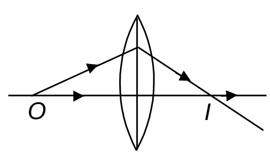


Figure I

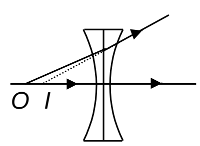


Figure II

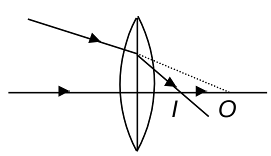
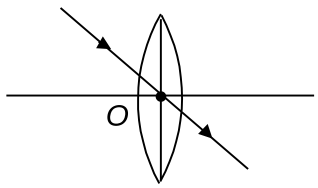


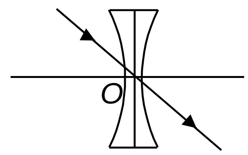
Figure III

Figure	u	v	f	R_1	R_2
(i)	– ve	+ ve	+ ve	+ ve	– ve
(ii)	– ve	– ve	– ve	– ve	+ ve
(iii)	+ ve	+ ve	+ ve	+ ve	– ve

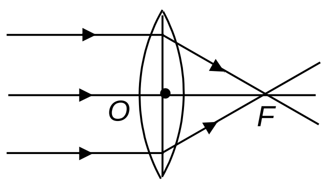
9.3 RULES FOR IMAGE FORMATION

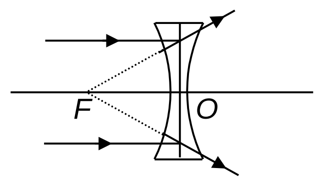
(i) A ray passing through the optical centre of the lens proceeds undeviated through the lens. (By the definition of optical centre)



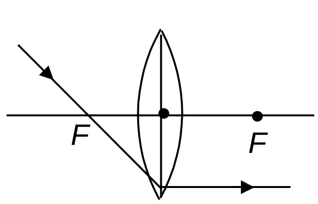


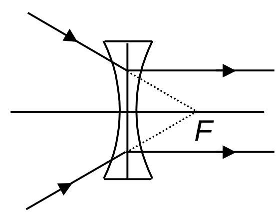
(ii) A ray passing parallel to the principal axis after refraction through the lens passes or appear to pass through the focus. (By the definition of the focus)





(iii) A ray through the focus or directed towards the focus, after refraction from the lens, becomes parallel to the principal axis. (Principle of reversibility of light)

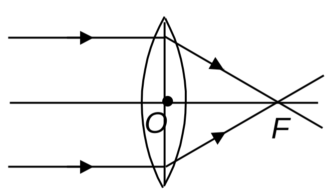




Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect, is the image of the object. If they actually intersect each-other, the image is real and if they appear to intersect the image is said to be virtual.

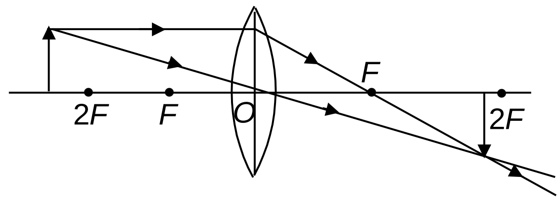
9.4 IMAGE FORMATION BY A CONVEX LENS OF THE LINEAR OBJECT

When the object is at infinity



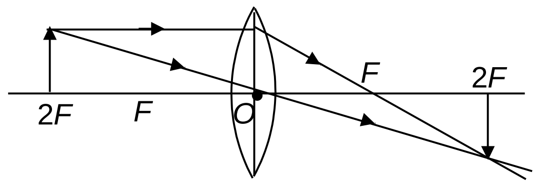
The image is formed at F. It is real, inverted and highly diminished.

When the object is beyond 2 *F*



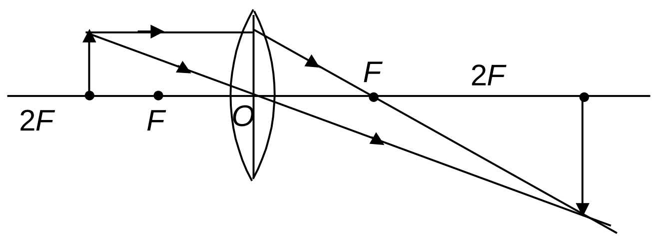
The image is formed between *F* and 2*F*. It is real, inverted and diminished.

When the object is at 2*F*



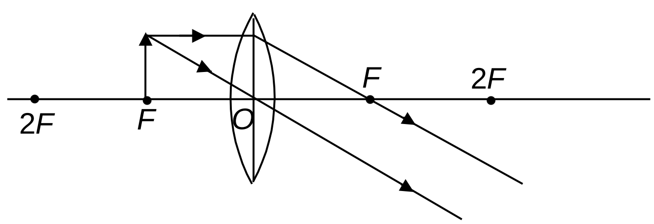
The image is formed at 2*F*. It is real, inverted and of the same size.

When the object is between *F* and 2 *F*



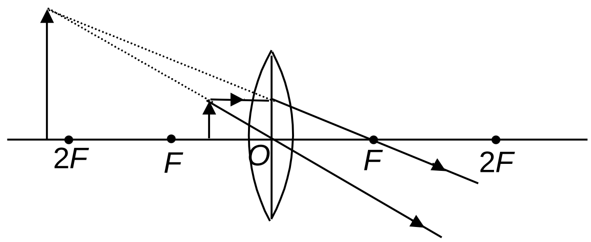
The image is formed beyond 2*F* (i.e., between 2*F* and ∞). It is real, inverted and enlarged.

When the object is at *F*



The image is formed at infinity. It is real, inverted and highly diminished.

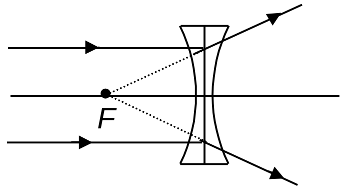
When the object is between *F* and *O*



The image is on the same side as the object is. It is virtual, erect and magnified.

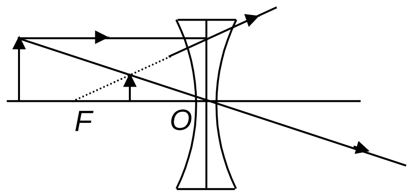
9.5 IMAGE FORMATION BY A CONCAVE LENS OF A LINEAR OBJECT

When the object is at infinity



The image is formed at the focus. It is virtual, erect and highly diminished.

When the object is infront of the lens:



The image is formed between F and the optical centre. It is virtual, erect and diminished.

9.6 LENS MAKERS FORMULA

The focal length (f) of a lens depends upon the refractive indices of the material of the lens and the medium in which the lens is present and the radii of curvature of both sides. The following relation giving focal length (f) is called as 'lens maker's formula.

$$\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right], \qquad \ldots \textbf{(32)}$$

where μ = refractive index of the material of the lens

μ_0 = Refractive index of the medium.

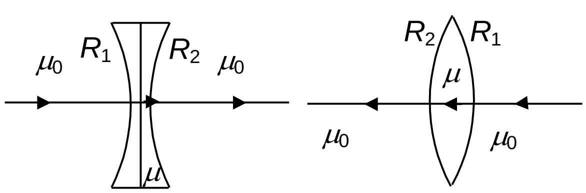
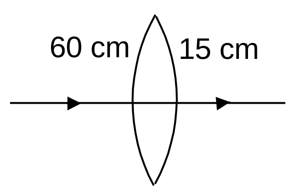


Illustration 29.

Calculate the focal length of a biconvex lens in air if the radii of its surfaces are 60 cm and 15 cm. Refractive index of glass = 1.5

Solution:

Consider a light ray going through the lens as shown. It strikes the convex side of 60 cm radius and concave side of 15 cm radius while coming out.



R_1 = + 60 cm

$R_2 = -15$ cm

$\therefore \quad \dfrac{1}{f} = \left[\dfrac{\mu}{\mu_0} - 1\right]\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

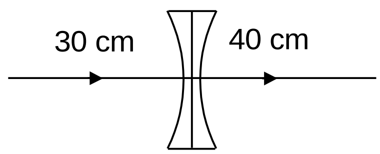
or, $\quad \dfrac{1}{f} = [\dfrac{1.5}{1} - 1]\,[\dfrac{1}{60} + \dfrac{1}{15}]$

$\Rightarrow \quad f = $ **+ 24 cm**

Illustration 30.

Calculate the focal length of a concave lens in water ($\mu_w = \dfrac{4}{3}$) if the surface have radii equal to 40 cm and 30 cm. $\mu_g = 1.5$

Solution:



$R_1 = 30$ cm; $R_2 = +40$ cm

we have

$$\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

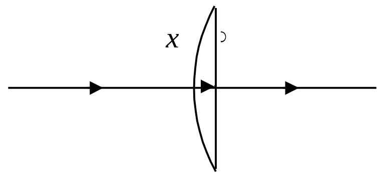
$$= \left[\frac{1.5}{4/3} - 1\right]\left[\frac{1}{-30} - \frac{1}{40}\right]$$

$\Rightarrow \quad f = -\dfrac{960}{7} = $ **– 137.1 cm**

Illustration 31.

A plane convex lens has a focal length 12 cm and is made up of glass with refractive index 1.5. Find the radius of curvature of its curved side.

Solution:



Let x = magnitude of the radius of curvature

Now, $R_1 = +x : R_2 = \infty = +12$ cm

We have $\quad \dfrac{1}{f} = \left(\dfrac{\mu}{\mu_0} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$= \left(\dfrac{1.5}{1} - 1\right)\left[\dfrac{1}{x} - \dfrac{1}{\infty}\right]$ or, $\quad \dfrac{1}{+12} = (.5) \times \dfrac{1}{x}$

$\Rightarrow x = $ **6 cm**

9.7 LENS FORMULA

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \dots (33)$$

9.8 LINEAR MAGNIFICATION (*m*)

The ratio of the size of the image formed by a lens to the size of the object is called linear magnification produced by the lens. It is denoted by *m*.

If *O* and *I* are the sizes of the object and image respectively, then

$$m = \frac{I}{O}=\frac{v}{u} \qquad \dots (34)$$

9.9 POWER OF A LENS

The power of a lens is defined as

$$P = \frac{1}{f} \qquad \dots (35)$$

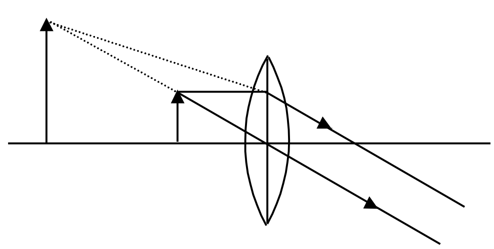
where *f* must be expressed in metres. The SI unit of power of a lens is m^{-1}. It is also known as dioptre (*D*). The focal length of a convex lens is positive. So, the power of a convex lens is positive. The focal length of a concave lens is negative. So, the power fo a concave lens is negative.

Illustration 32.

A magnifying lens has a focal length of 10 cm. (i) Where should the object be placed if the image is to be 30 cm from the lens? (ii) What will be the magnification?

Solution:

(i)



In case of magnifying lens, the lens is convergent and the image is erect, virtual and enlarged and between infinity and the object on the same side of the lens as shown in the above diagram.

So, here

$$f = +10 \text{ cm}; \; v = -30 \text{ cm}$$

Let '*x*' be the object distance

Using lens formula

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u},$$

we have, $\dfrac{1}{-30}-\dfrac{1}{((-x)}=\dfrac{1}{10}$

$\Rightarrow$ x = **7.5 cm**

(ii) $m = \dfrac{I}{O}=\dfrac{v}{u}=\dfrac{-30}{-8.5}$ = **+ 4**

Thus, the image is erect and virtual and four times of the object.

Illustration 33.

An object 25 cm high is placed infront of a convex lens of focal length 30 cm. If the height of the image formed is 50 cm, find the distance between the object and the image?

Solution:

We have

$$|m| = \frac{I}{O}=\frac{v}{u}=\frac{50}{25}=2$$

There are two possibilities

(i) If the image is inverted (i.e., real)

$$m = \frac{v}{u}=-2 \qquad \text{or,} \qquad v = -2u$$

Let x be the object distance in this case

We have

$u = -x$; $v = +2x$; $f = +30$ cm

Using lens formula, we have

$$\frac{1}{2x}-\frac{1}{-x} = \frac{1}{30} \qquad \text{or,} \qquad x = 45 \text{ cm}$$

Hence, the distance between the object and the image = 45 + 90 = **135 cm.**

(ii) If the image is erect (i.e., virtual)

$$m = \frac{v}{u}=+2$$

Let x’ be the object distance we have

$u = -x'$; $v = -2x'$; $f = +30$

Using the lens formula, we have $\dfrac{1}{-2x'}+\dfrac{1}{x}=\dfrac{1}{30}$

$\Rightarrow$ x' = **15 cm**

Hence, the distance between the object and the image = **15 cm**

9.10 COMBINATION OF LENSES AND MIRRORS

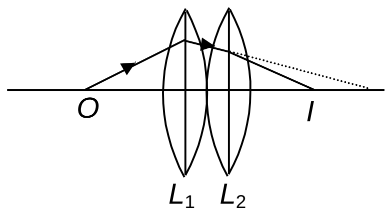
When several lenses or mirrors are used co-axially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as object for the next lens or mirror; the image formed by the second lens (or mirror) acts as object for the third and so on. The total magnification in such situations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \ldots$$

or, $\quad m = m_1 \times m_2 \times m_3 \times \ldots.. \qquad \ldots (\mathbf{36})$

9.11 TWO THIN LENSES IN CONTACT

If two or more lenses of focal lengths f_1, f_2 and so on are placed in contact, then this combination can be replaced by a lens called 'equivalent lens'. The focal length (F) of equivalent lens is given by



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \qquad \ldots (\mathbf{37})$$

Also, $\quad P = P_1 + P_2 + \ldots \qquad \ldots (\mathbf{38})$

If two thin lenses of focal lengths f_1 and f_2 are separated by a distance 'd ', then the focal length of the equivalent lens is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad \ldots (\mathbf{39})$$

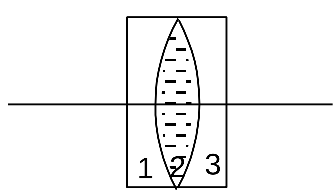
Also, $\quad P = P_1 + P_2 - dP_1P_2$

Illustration 34.

Two plano-concave lens of glass of refractive index 1.5 have radii of curvature 20 cm and 30 cm. They are placed in contact with curved surfaces towards each-other and the space between them is filled with a liquid of refractive index $\frac{4}{3}$. Find the focal length of the system.

Solution:

As shown in figure, the system is equivalent to combination of three lenses in contact,



i.e., $\quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

By lens maker's formula $\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{\infty} - \frac{1}{20}\right] = \frac{1}{40}$ cm

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left[\frac{1}{20} - \frac{1}{-30}\right] = \frac{5}{180} \text{ cm}$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{-30} - \frac{1}{\infty}\right] = -\frac{1}{60} \text{ cm}$$

$$\therefore \qquad \frac{1}{F} = -\frac{1}{40} + \frac{5}{180} - \frac{1}{60}$$

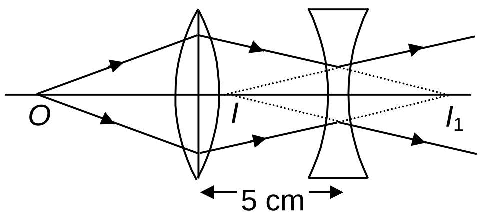
$F =$ **– 72 cm**

Thus, the system will behave as a concave lens of focal length 72 cm.

Illustration 35.

Two thin lenses, both of 10 cm focal length- one convex and other concave, are placed 5 cm apart. An object is placed 20 cm in front of the convex lens. Find the nature and position of the final image.

Solution:



For refraction at the convex lens, we have

$u = -20$ cm; $f_1 = +10$ cm; $v = v_1 = ?$

Using lens formula, we have

$$\frac{1}{v_1} - \frac{1}{(-20)} = \frac{1}{10}$$

$\Rightarrow \qquad v_1 =$o $+ 20$ cm

The convex lens produces converging rays trying to meet at I_1, 20 cm from the convex lens, i.e., 15 cm behind the concave lens.

I_1 will serve as a virtual object for the concave lens.

For refraction at the concave lens. We have

$u = +15$ cm; $v = ?$ $f = -10$ cm

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$$

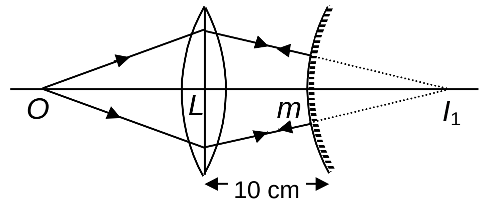
$\Rightarrow \qquad v =$ **– 30 cm**

Hence the final image is virtual and is located at 30 cm to the left of concave lens.

Illustration 36.

A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at the distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the mirror?

Solution:



For the refraction at the convex lens, we have

$u = -12$ cm; $v = ?$ $f = +10$ cm

Using lens formula, we have

$$\frac{1}{v}-\frac{1}{(-12)}=\frac{1}{10}$$

or, $v = +60$ cm

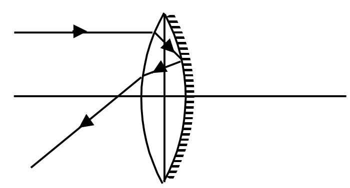
Thus, in the absence of the convex mirror, convex lens will form the image I_1, at a distance of 60 cm behind the lens. As the mirror is at a distance of 10 cm from the lens, I_1 will be at a distance of (60 – 10) = 50 cm from the mirror, i.e., $MI_1 = 50$ cm.

Now, as the final image I_2 is formed at the object itself, the rays after reflection from the mirror retraces its path, i.e., the rays on the mirror are incident normally, i.e., I_1 is the centre of the mirror so that

$$R = MI_1 = +50 \text{ cm}$$

and $f = \frac{R}{2}=\frac{50}{2}$ = **+ 25 cm**

9.12 LENS WITH ONE SURFACE SILVERED



When one surface of a thin lens is silvered, the rays are reflected back at this silvered surface. The set up acts as a spherical mirror.

The focal length of the equivalent spherical mirror is given by

$$\frac{1}{P}=\Sigma\frac{1}{f_i}, \qquad \dots (\mathbf{40})$$

where f_i = focal length of the lens or mirror, to be repeated as many times as the refraction or reflection takes place.

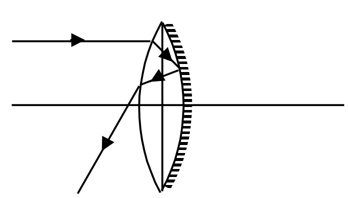
Sign convention:

In the above formula, the focal length of the converging lens or mirror is positive; and that of diverging lens or mirror is negative.

Illustration 37.

One face of an equiconvex lens of focal length 60 cm made of glass ($\mu = 1.5$) is silvered. Does it behave like a concave mirror or convex mirror?

Solution:



Let x be the radius of curvalue of each surface.

$$\frac{1}{60}=(1.5-1)\left[\frac{1}{x}-\frac{1}{-x}\right]$$

or, $\frac{1}{60} = -5 \times \frac{2}{x}$

$\Rightarrow$ $x = 60$ cm

Let 'F' be the focal length of the equivalent spherical mirror, Then

$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{F_l}$$

or, $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

Here, $F_l = +60$ cm (convex lens)

$F_m = \frac{R}{2} = +30$ cm (concave mirror)

$\therefore$ $\frac{1}{F} = \frac{2}{60} + \frac{1}{30} = \frac{2}{30}$

$\Rightarrow$ F = **+ 15 cm**

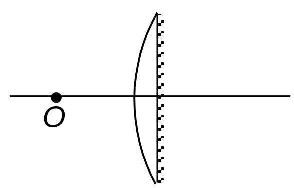
The positive sign indicates that the resulting mirror is concave.

Illustration 38.

The plane surface of a plano-convex lens of focal length 60 cm is silvered. A point object is placed at a distance 20 cm from the lens. Find the position and nature of the final image formed.

Solution:

Let f be the focal length of the equivalent spherical mirror.



we have $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$

or, $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

Here, $f_l = +60$ cm

$f_m = \infty$

$\therefore$ $\frac{1}{F} = \frac{2}{60} + \frac{1}{\infty} = \frac{1}{30}$ or, $F = +30$ cm

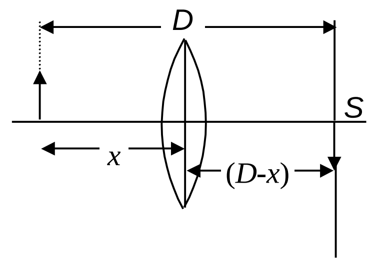
The problem is reduced to a simple case where a point object is placed in front of a concave mirror.

Now, using the mirror formula

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we have $\frac{1}{-20} + \frac{1}{v} = \frac{1}{-30}$ $\Rightarrow$ $v = 60$ cm

The image is erect and virtual.

9.13 DISPLACEMENT METHOD



Let us consider a real object and a screen fixed at a distance D apart. A convex lens is placed between them.

Let x be the distance of the object from the lens when its real image is obtained on the screen.

We have , $u = -x$

$$v = +(D-x)$$

$$f = +f$$

Using lens formula, we have

$$\frac{1}{(D-x)} - \frac{1}{-x} = \frac{1}{f}$$

or, $$\frac{1}{(D-x)} + \frac{1}{x} = \frac{1}{f}$$

or, $$\frac{D}{(D-x)x} = \frac{1}{f}$$

or, $$x^2 - Dx + Df = 0$$

or, $$x = \frac{D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D \pm \sqrt{D(D-4f)}}{2}$$

Now, there are three possibilities.

(i) If $D < 4f$; then in this case, x will be imaginary and so physically no position of lens is possible.

(ii) If $D = 4f$; then in this case, $x = \frac{D}{2} = 2f$. So only one position of the lens will be possible and in this situation.

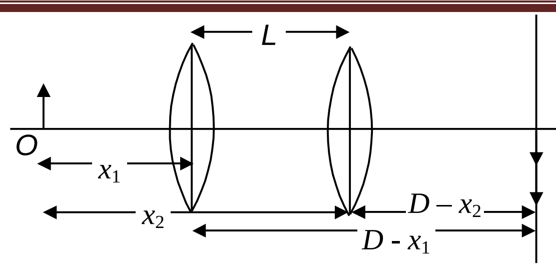
$$v = D - x = 4f - 2f = 2f \ (= x)$$

(iii) If $D > 4f$; in this situation both the roots of x will be real. Thus

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2}$$

and $$x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

So if $D > 4f$, these are two positions of lens at distances x_1 and x_2 from the object for which real image is obtained on the screen. This method is called 'Displacement method' and is used in laboratory to determine the focal length of the convex lens.



In case of displacement method, if the distance between two positions of the lens is L, then

$$L = x_2 - x_1 = \sqrt{D(D-4f)}$$

or, $L^2 = D^2 - 4\,Df$

$$\Rightarrow \quad f = \frac{D^2 - L^2}{4D} \qquad \dots \textbf{(41)}$$

We have

$$L = (x_2 - x_1)$$

and $D = x_1 + x_2$

$$\therefore \quad x_1 = \frac{D-L}{2};\ x_2 = \frac{D+L}{2}$$

Now, $x_1 + x_2 = \dfrac{D-L}{2} + \dfrac{D+L}{2} = D$

$x_1 = D - x_2$ & $x_2 = D - x_1$

Hence object and image distances are interchanged in the two positions of the lens. Let the magnification be m_1 and m_2.

$$\therefore \quad m_1 = \frac{(D-x_1)}{-x_1} = \frac{x_2}{x_1}$$

and $m_2 = \dfrac{(D-x_2)}{-x_2} = -\dfrac{x_1}{x_2}$

Now,

$$m_1 m_2 = \frac{x_2}{-x_1} \times \frac{-x_1}{x_2} = 1$$

or, $\dfrac{I_1}{O}\,\dfrac{I_2}{O} = 1$

$$\Rightarrow \quad O = \text{object size} = \sqrt{I_1 I_2} \qquad \dots \textbf{(42)}$$

i.e., the size of the object is the geometric mean of the sizes of the two images.

Illustration 39.

An object is kept at a distance of 100 cm from a screen. A convex lens placed between them produces a real & magnified image on the screen. If the lens is shifted 30 cm towards the screen, real image is again obtained. Find the focal length of the lens. Also calculate the size of the object if the image sizes are 16 mm and 9 mm respectively.

Solution:

Here, $D = 100$ cm; $L = 30$ cm

$$\therefore \quad f = \frac{D^2 - L^2}{4D} = \frac{(100)^2 - (30)^2}{4(100)} = \frac{91}{4} \text{ cm}$$

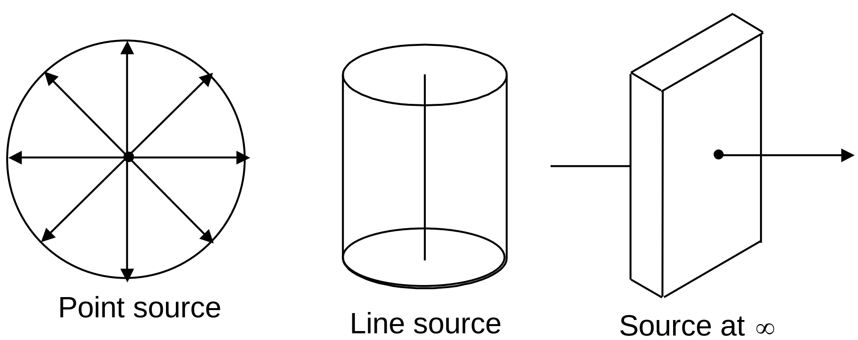
$I_1 = 16$ mm; $I_2 = 9$ mm

$$\therefore \quad \text{object size} = \sqrt{I_1 I_2} = \sqrt{16 \times 9} = \mathbf{12\ mm}$$

10. WAVE OPTICS

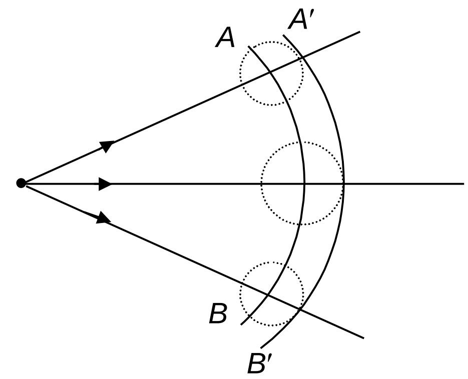
Wave optics concerns with the explanation of the observed phenomena such as interference. In wave optics, light is treated as a wave. Huygen was the first scientist who assumed that a body emits light in the form of waves. In his wave theory, Huygen used a term– the wave front. First of all, we will understand what he meant by the term, the wave front.

(i) **Wave front:** The locus of all the particles of the medium vibrating in the same phase at a given instant is called wavefront. The shape of the wavefront depends on the source producing the waves and is usually spherical, cylindrical or plane as shown below.



(ii) **Ray:** In a homogeneous medium, the wavefront is always perpendicular to the direction of wave propagation. Hence, a line drawn normal to the wavefront gives the directions of propagation of a wave and is called a ray.

10.1 HUYGEN'S WAVE THEORY



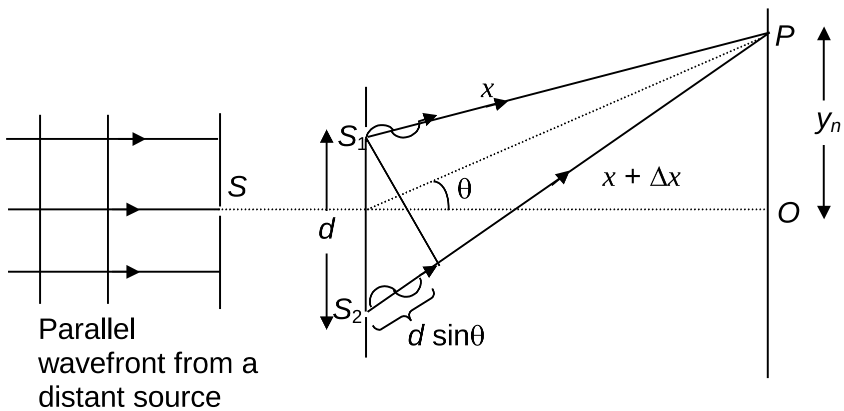
Each point of a source of light is a centre of disturbance from where waves spread in all directions. Each point on a wavefront is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium. The forward envelope of the secondary wavelets at any instant gives the new wavefront.

11. ENTERFERENCE

The terms interference in general, refers to any situation where two or more waves overlap each-other in the same region of space. But usually, interference refers to the superposition of two coherent waves of same frequency moving in the same direction.

11.1 YOUNG'S DOUBLE SLIT EXPERIMENT

It was carried out in 1802 by Thomos Young to prove the wave nature of light.



Two slits S_1 and S_2 are made in an opaque screen, parallel and very close to each-other. These two are illuminated by another narrow slit S which in turn is lit by a bright source. Light wave spread out from S and fall on both S_1 and S_2. Any phase difference between S_1 and S_2 is unaffected and remain constant. The light waves going from S_1 and S_2 to any point P on the screen interfere with each other. At some point the waves superpose in such a way that the resultant intensity is greater than the sum of the intensities due to separate waves (CONSTRUCTIVE INTERFERENCE) while at some other point, it is lesser than the sum of the separate intensities (DESTRUCTIVE INTERFERENCE). Thus the overall picture is a pattern of dark and bright bands known as fringe pattern. The dark bands are known as dark fringes and the bright bands are known as bright fringes.

11.2 THEORY OF INTERFERENCE

Let the two waves each of angular frequency ω from sources S_1 and S_2 reach the point P. If the waves have amplitudes A_1 and A_2 respectively. then

$$y_1 = A_1 \sin(\omega t - kx) \quad \ldots \text{(i)}$$

and, $y_2 = A_2 \sin(\omega t - k(x + \Delta x)]$

or, $$y_2 = A_2 \sin[\omega t - kx - \phi] \quad \ldots \text{(ii)}$$

where $$\phi = k\,\Delta x = \frac{2\pi}{\lambda}(\Delta x) \quad \ldots \text{(iii)}$$

So, by the principle of superposition, the resultant wave at P will be

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi)$$

or, $$y = [A_1 + A_2 \cos\phi] \sin[\omega t - kx] - [A_2 \sin\phi] \cos(\omega t - kx) \quad \ldots \text{(iv)}$$

Let $A_1 + A_2 \cos\phi = A\cos\alpha$

$A_2 \sin\phi = A \sin\alpha$

So, $A^2[\cos^2\alpha + \sin^2\alpha] = (A_1 + A_2\cos\phi)^2 + (A_2 \sin\phi)^2$

or, $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

and $$\alpha = \tan^{-1}\left[\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right]$$

Hence, equation (iv) becomes

$$y = A\sin(\omega t - kx)\cos\alpha - A\cos(\omega t - kx)\sin\alpha$$

or, $$y = A\sin(\omega t - kx - \alpha) \quad \ldots \text{(v)}$$

From above equation, it is clear that in case of superposition of two waves of equal frequencies propagating almost in the same direction, resultant is harmonic wave of same frequency ω and wavelength ($\lambda = \frac{2\pi}{k}$) but amplitude

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

Now as intensity of wave is given by

$$I = \frac{1}{2}\rho v\omega^2A^2 = KA^2 \quad [K = \frac{1}{2}\rho v\omega^2]$$

So the intensity of the resultant wave

$$I = K[A_1^2 + A_2^2 + 2A_1A_2\cos\phi]$$

or, $$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi \quad \ldots \textbf{(43)}$$

$[\text{As } I_1 = KA_1^2;\ I_2 = KA_2^2]$

11.3 CONDITION FOR CONSTRUCTIVE INTERFERENCE

Intensity will be maximum when

$\cos\phi$ = maximum = 1

$\Rightarrow \quad \phi = 0, \pm 2\pi, \pm 4\pi \ldots$ … (**44**)

or, $\quad \phi = \pm 2\pi n$

Where $n = 0, 1, 2, \ldots$

Now, $\dfrac{2\pi}{\lambda}(\Delta x) = \Delta\phi = \pm 2\pi n$

or, $\quad \Delta x = \pm n\lambda$, where n = 0, 1, 2, … (**45**)

and, $\quad I_{max.} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

or, $\quad I_{max.} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$\propto (A_1 + A_2)^2$

11.4 CONDITION FOR DESTRUCTIVE INTERFERENCE

Intensity will be minimum when

$\cos\phi$ = minimum = -1

i.e., $\quad \phi = \pm\pi, \pm 3\pi, \pm 5\pi$

or, $\quad \phi = \pm(2n-1)\pi$, with $n = 1, 2, 3 \ldots$ … (**46**)

or, $\quad \dfrac{2\pi}{\lambda}(\Delta x) = \pm(2n-1)\pi \; [\because \Delta\phi = \dfrac{2\pi}{\lambda}(\Delta x)]$

or, $\quad \Delta x = \pm(2n-1)\dfrac{\lambda}{2}$ with $n = 1, 2, \ldots$ … (**47**)

also, $\quad I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$

$\propto (A_1 \sim A_2)^2$

Illustration 40.

Two sources of intensity I and $4I$ are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference of (i) zero (ii) $\frac{\pi}{2}$ (iii) π

Solution:

In case of interference,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

(i) As $\phi = 0$, $\cos\phi = 1$

$\therefore I = 4I + I + 2\sqrt{4I \times I} \times 1 = \mathbf{9\,I}$

(ii) As $\phi = \frac{\pi}{2}$, $\cos\phi = 0$

$$\therefore I = 4I + I + 2\sqrt{4I \times I} \times 0 = \mathbf{5\,I}$$

(iii) As $\phi = \pi$ $\cos\phi = -1$

$$\therefore I = 4I + I + 2\sqrt{4I \times I} \times -1 = \boldsymbol{I}$$

Illustration 40.

In Young's experiment, the interference pattern is found to have an intensity ratio between the bright and the dark fringes as I. What is the ratio of

(a) intensities (b) amplitudes of the two interfering waves?

Solution:

In case of interference,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(a) $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$

According to given problem,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{9}{1}$$

or, $$\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{3}{1}$$

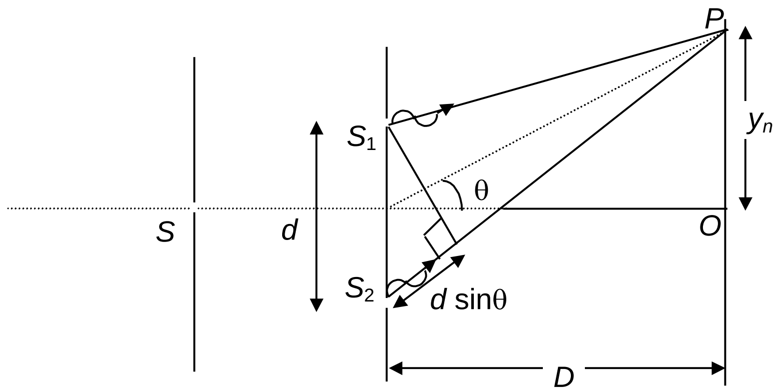
By componendo and divideno, we have

$$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{3+1}{3-1} \quad \text{or,} \quad \frac{I_1}{I_2} = \mathbf{4}$$

(b) As for a wave, $I \propto A^2$

$$\frac{I_1}{I_2} = \left[\frac{A_1}{A_2}\right]^2 = 4 \quad \text{or,} \quad \frac{A_1}{A_2} = \mathbf{2}$$

11.5 LOCATION OF BRIGHT FRINGES



Let P be the position of n^{th} maxima on the screen. The two waves arriving at P follow the path S_1P and S_2P, thus the path difference between the two waves is

$$p = S_2P - S_1P = d\sin\theta$$

From experimental conditions, we know that D > > d, therefore, the angle θ is small.

Thus

$$\sin\theta \approx \tan\theta = \frac{y_n}{D}$$

or, $p = d\sin\theta = d\left(\frac{y_n}{D}\right)$ for n^{th} maxima,

or, $p = n\lambda$

or, $d\left(\frac{y_n}{D}\right) = n\lambda$

or, $y_n = n\lambda\frac{D}{d}$ where n = 0, 1, 2, … … (48)

11.6 LOCATION OF DARK FRINGES

For n^{th} minima, we have

$p = (2n - 1)\frac{\lambda}{2}$ or, $d\left(\frac{y_n}{D}\right) = (2n - 1)\frac{\lambda}{2}$

∴ $y_n = (2n - 1)\frac{\lambda D}{2d}$ where $n = 1, 2, 3$ … … (**49**)

11.7 FRINGE WIDTH (ω)

The separation between two consecutive dark (or bright) fringes is known as fringe width (ω).

∴ $\omega = y_{n+1} - y_n = (n + 1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$

$\Rightarrow \qquad \omega = \dfrac{\lambda D}{d}$ … (**50**)

Angular fringe width or angular separation between fringes is

$$\theta = \frac{\omega}{D}$$

$\Rightarrow \qquad \theta = \dfrac{\lambda}{d}$ … (**51**)

Illustration 42.

In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0.1°. The wavelength of the light used is 6000 Å. What is the spacing between the slits?

Solution:

Angular width = $\dfrac{\omega}{D} = \dfrac{\lambda}{d}$

$$\Rightarrow \qquad \frac{\lambda}{d} = \frac{0.1\pi}{180}$$

$$\Rightarrow \qquad d = \frac{180 \times 6000 \times 10^{-10}}{0.1\,\pi}$$

$$= \mathbf{3.44 \times 10^{-4}\ m}$$

Illustration 43.

A double slits arrangement produces fringes for λ = 5890 Å that are 0.4° apart. What is the angular width if the entire arrangement is immersed in water? ($\mu_W = 4/3$)

Solution:

Let θ° be the angular width in water. We know

angular width = $\dfrac{\lambda}{d}$

$\Rightarrow$ angular width $\alpha\ \lambda$

$\dfrac{\theta\pi}{180} \times \dfrac{180}{4\pi} = \dfrac{\lambda_{water}}{\lambda_{air}}$ or, $\dfrac{\theta}{0.4} = \dfrac{\lambda_W}{\lambda_a}$ … (i)

Now, ${}_a\mu_W = \dfrac{\lambda_a}{\lambda_W} \qquad \Rightarrow \dfrac{\lambda_a}{\lambda_W} = 4/3$

Hence from equation (i), we have

$$\frac{\theta}{0.4} = \frac{3}{4} \Rightarrow \theta = \mathbf{0.3°}$$

Illustration 44.

A beam of light constituting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in a *YDSE*.

(i) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength of 6500 Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

Solution:

(i) D = 120 cm; d = 0.2 cm

Let λ_1 = 6500 Å; λ_2 = 5200 Å

Distance of the third bright fringe = $\frac{3\lambda D}{d} = \frac{3\times120\times6500\times10^{-8}}{0.2}$ = 0.117 cm

(ii) Let m^{th} bright fringe due to λ_1 and the n^{th} bright fringe due to λ_2 coincide at a distance x from the central maximum.

$\Rightarrow x = \frac{mD\lambda_1}{d}$ and $x = \frac{nD\lambda_2}{d}$

$\therefore\ m\lambda_1 = n\lambda_2 \ \Rightarrow 6500\,m = 5200\,n$

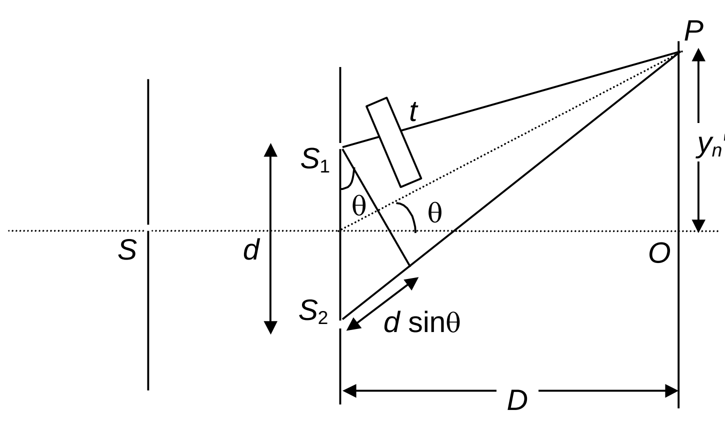
$\therefore\ \frac{m}{n} = \frac{4}{5}$

$\Rightarrow$ 4^{th} bright fringe due to λ_1 coincides with 5^{th} bright fringe due to λ_2 at distance

$x = \frac{4D\lambda_1}{d} = \frac{4\times120\times6500\times10^{-8}}{2}$ = **0.156 cm**

11.8 DISPLACEMENT OF FRINGE PATTERN

Let us analyse what happens if a thin transparent plate of thickness t and refractive index μ is placed in front of one of sources, for example, in front of S_1. This changes the path difference because light from S_1 now takes longer time to reach the screen.



Time taken = $\frac{d_{air}}{V_{air}} + \frac{d_{plate}}{V_{plate}}$

$= \frac{S_1P - t}{C} + \frac{t}{C/\mu}\left[\mu = \frac{C_{air}}{C_{medium}}\right]$

$= \frac{S_1P + t(\mu - 1)}{C}$

Hence the effective path that is equivalently covered in air O is $S_1P + (\mu\text{-}1)\, t$

$\therefore$ path difference $p = S_2P - [S_1P + t\,(\mu\text{-}1)]$

$= S_2P - S_1\,P - (\mu\text{-}1)\, t$

$\because$ $S_2P - S_1P = d \sin\theta$

$\therefore$ $p = d\sin\theta + (\mu - 1)\, t$

As $\sin\theta \approx \tan\theta = \dfrac{y'_n}{D}$

$$\therefore \quad p = \frac{dy'_n}{D} + (\mu - 1)t \qquad \text{or,} \quad y'_n = \frac{n\lambda D}{d} - (\mu - 1)\frac{tD}{d}$$

In the absence of film, the position of the n[th] maxima is given by

$$y_n = \frac{n\lambda D}{d}$$

Therefore, the fringe shift is given by

$$y_0 = y_n - y'_n = (\mu - 1)\frac{tD}{d} \qquad \ldots \textbf{(52)}$$

When a transparent sheet is introduced, the fringe pattern shifts in the direction where the film is placed.

Illustration 45.

In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of the slits and the screen is doubled. It is found that the distance between the successive maximum now is the same as the observed fringe shift upon the introduction of mica sheet. Calculate the wavelength of the light.

Solution:

Due to introduction of mica sheet, the shift on the screen

$$Y_0 = \frac{D}{d}(\mu - 1)t$$

Now, when the distance between the plane of slits and screen is changed from D to $2D$, fringe width will become,

$$\omega = \frac{2D}{d}(\lambda)$$

According to given problem,

$$\frac{D}{d}(\mu - 1)\, t = \frac{2D(\lambda)}{d}$$

$$\text{or,} \qquad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6.1)\times 1.964\times 10^{-6}}{2}$$

$= \mathbf{5892\ Å}$

11.9 COHERENT WAVES

Two waves of same frequency are said to be 'coherent' if their phase difference does not change with time, i.e., their phase difference is independent of time.

For observing interference effects, waves (or sources) must be coherent.

In case of two coherent sources, the resultant intensity at any point is given by

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$, where ϕ is the phase difference between the two coherent waves reaching the point.

In case of incoherent sources, the resultant intensity at any point is given by

$$I_R = I_1 + I_2 + \ldots$$

Illustration 46.

Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the interference is

(i) coherent (ii) incoherent

Solution:

(i) In case of two coherent sources,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

I_R will be maximum when $\cos\phi$ = maximum = 1

$\therefore\ (I_{max})_{co} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

So for n identical waves each of intensity I_0,

$$(I_{max})_{co} = (\sqrt{I_0} + \sqrt{I_0}\ \ldots)^2 = (n\sqrt{I_0})^2 = n^2 I_0$$

(ii) In case of incoherent sources,

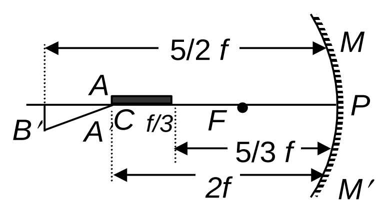
$$I_R = I_1 + I_2 + \ldots.$$

$$= I_0 + I_0 + \ldots = \boldsymbol{nI_0}$$

Illustration 47.

A thin rod of length (f/3) is placed along the principal axis of a concave mirror of focal length f such that its image, which is real and elongated, just touches the rod. What is the magnification?

Solution:



As the image is real and enlarged, the object must be between C and F. Also as the image of one end coincides with the end itself, i.e., $v_A = u_A$,

$$\frac{1}{v_A} + \frac{1}{v_A} = \frac{1}{-f}$$

i.e., $v_A = u_A = -2f$

i.e., the end A is at C.

Now as the length of the rod is (f/3), its other end B will be at a distance [2f – (f/3)] = (5/3)f from P. So if the distance of image of end B from P is v_B.

$$\frac{1}{v_B}+\frac{1}{-(5/3)\,f}=\frac{1}{-f} \quad \text{i.e., } v_B = -\frac{5}{2}\,f$$

So the size of the image

$$|v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f$$

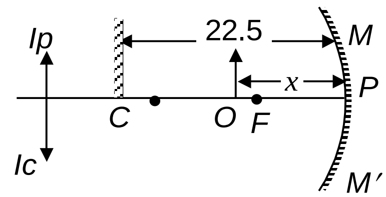
and so $m = \dfrac{(v_B - v_A)}{(u_B - u_A)} = -\dfrac{(1/2)\,f}{(1/3)\,f} = -\dfrac{\mathbf{3}}{\mathbf{2}}$

Negative sign implies that image is inverted with respect to object and so is real.

Illustration 48.

A plane mirror is placed 22.5 cm in front of a concave mirror of focal length 10 cm. Find where an object can be placed between the two mirrors, so that the first image in both the mirrors coincides.

Solution:



As shown in figure, if the object is placed at a distance x from the concave mirror, its distance from the place mirror will be (22.5 – x). So plane mirror will form equal and erect image of object at a distance (22.5 – x) behind the mirror.

Now as according to given problem the image formed by concave mirror coincides with the image formed by concave mirror, therefore for concave mirror

$$v = -[22.5 + (22.5 - x)] = -(45 - x) \quad \text{and } u = -x$$

So $$\frac{1}{-(45-x)}+\frac{1}{-x}=\frac{1}{-10} \quad \text{or} \quad \frac{45}{(45x-x^2)}=\frac{1}{10}$$

i.e., $x^2 - 45x + 450 = 0$ or $(x - 30)(x - 15) = 0$

i.e., $x = 30$ cm or $x = 15$ cm

But as the distance between two mirrors is 22.5 cm, x = 30 cm is not admissible. So the object must be at a distance of 15 cm from the concave mirror.

Illustration 49.

The material of an equilateral prism has a refractive index 1.5. Find the angle of minimum deviation, the angle of incidence when the angle of deviation is minimum and also the angle of refraction at first face.

Solution:

Given, $\mu = 1.5$, $A = 60°$

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \text{or,} \quad \sin\frac{(60+D)}{2} = 1.5 \times 0.5 = 0.75$$

$$\frac{60+D}{2} = \sin^{-1}(0.75) = 48°36'$$

$$60 + D = 97°12'$$

$$D = 37°12'$$

When the deviation is minimum $i_1 + i_2 = 2i = A + D = 60° + 37°12' = 97°12'$

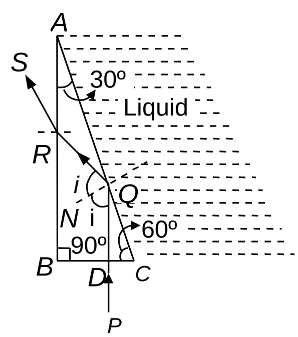
$$i_1 = \frac{97°12'}{2} = 48°36'$$

$$r_1 = \frac{A}{2} = 30°$$

Illustration 50.

The angles of a prism are 30°, 60° and 90°. What should be the refractive index of a liquid to be kept in contact with the longest face so that a ray normally incident on the smallest face gets just internally reflected at the longest face? ($\mu = 1.5$)

Solution:



The given prism ABC has the face AC the longest (opposite to 90° angle) and BC the shortest (opposite to 30° angle).

PQ is the normally incident ray at the face BC and just gets totally reflected at the longest face AC.

For the glass-liquid interface AC

$${}_g\mu_L = \frac{\mu_L}{\mu_g} = \frac{\sin i}{\sin 90^\circ}$$

In ΔCQD, $\angle CQD = 30°$

$\therefore$ $\angle DQN = 60°$

$\therefore$ $i = 60°$ $\quad \frac{\mu_L}{\mu_g} = \frac{\sin 60^\circ}{\sin 90^\circ}$

$$\mu_L = \mu_g \sin 60° = 1.5 \times \frac{\sqrt{3}}{2} = 1.299$$

Illustration 51.

A prism with refracting angle 60° has its minimum deviation as 37° when placed in air. Find its minimum deviation angle when immersed completely in water. (μ_{water} = 1.33).

Solution:

$$\mu_g = \frac{\sin\left(\frac{60+37}{2}\right)}{\sin\left(\frac{60}{2}\right)} = 1.498$$

$$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.498}{1.33} = 1.126$$

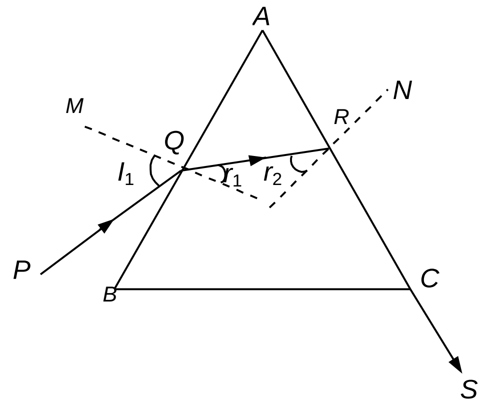
$$1.126 = \frac{\sin\left(30^\circ + \frac{D}{2}\right)}{\sin 30^\circ} \quad \sin\left(30 + \frac{D}{2}\right) = 0.563$$

$$30 + \frac{D}{2} = 34.26^\circ \quad \frac{D}{2} = 4.26^\circ \quad D = 8.52^\circ$$

Illustration 52.

The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as the ray comes out of the prism?

Solution:



Given $A = 45^\circ$ $\mu = 1.6$

$\therefore$ $\sin C = \frac{1}{\mu} = \frac{1}{1.6}$ $C = 38.68^\circ$

For total internal reflection not to take place at the face AC, angle of incidence in that face $r_2 \le C$

In the limiting case $r_2 = C$

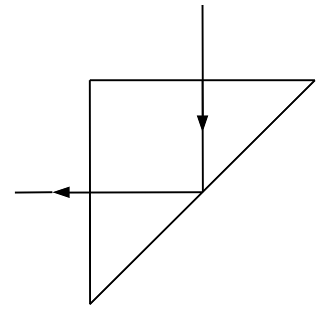
Now $r_1 = A - r_2 = 45 - 38.68^\circ = 6.32^\circ$

$\mu_a \sin i_1 = \mu_g \sin r_1$

$\sin i_1 = 1.6 \sin (6.32^\circ)$

$i_1 = \sin^{-1}(1.6 \sin 6.32^\circ) = 10.14^\circ$

Illustration 53.

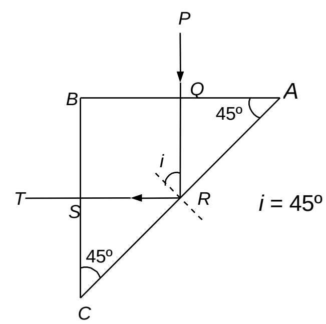


A ray of light incident normally on one of the faces of a right-angled isosceles prism is found to be totally reflected as shown.

(a) What is the minimum value of the refractive index of the material of the prism?

(b) When the prism is immersed in water trace the path of the emergent ray for the same incident ray indicating the values of all the angles. (μ of water = 4/3).

Solution:



(a) ABC is the section of the prism, B is a right angle. A and C are equal angles i.e., A = C = 45°.

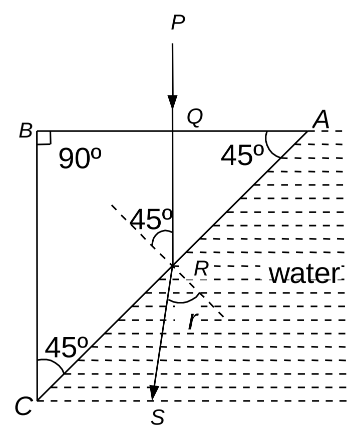
The ray PQ is normally incident on the face AB. Hence it is normally refracted and the ray QR strikes the face AC at an angle of incidence 45°. It is given that the ray does not undergo refraction but is totally reflected at the face AC. This gives a maximum value for the critical angle as 45°.

$$\sin C = \sin 45° = \frac{1}{\sqrt{2}} \text{ in the limit}$$

$$\text{since } \mu = \frac{1}{\sin C}, \qquad \frac{1}{\mu} = \sin C = \frac{1}{\sqrt{2}}$$

$$\text{or} \qquad \mu_{min} = \sqrt{2}$$

The minimum value of refractive index = $\sqrt{\mathbf{2}}$



(b) When the prism is immersed in water the critical angle for the glass-water interface is given by

$$\sin C_1 = \frac{4/3}{\sqrt{2}} = \frac{4}{3\sqrt{2}} \qquad C_1 = 70.53°$$

The angle of incidence at R continues to be 45° and since 45° < 70.53°.

There is refraction taking place now and the refracted ray is RS. The angle of refraction r is given by $\mu_g \sin i = \mu_w \sin r$

$$\sqrt{2} \sin 45° = \frac{4}{3} \sin r$$

$$\sin r = \frac{3\sqrt{2}}{4} \sin 45° = \frac{3\sqrt{2}}{4} \times \frac{1}{\sqrt{2}} = \frac{3}{4}$$

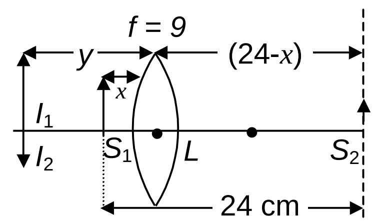
$$r = \sin^{-1}\left(\frac{3}{4}\right) = 48°36'$$

$\therefore$ The angle of refraction in water = 48°36′

Illustration 54.

The distance between two point sources of light is 24 cm. Find out where you will place a converging lens of focal length 9 cm, if the images of both sources are formed at the same point.

Solution:



As images of both sources are coincident, the lens must be placed between the sources such that S_1 forms the virtual image on same side while S_2 real image on opposite side as shown in figure. Now if x and y are the distances of sources S_1 and its image from lens L respectively.

$$\frac{1}{-y} - \frac{1}{-x} = \frac{1}{9} \qquad \ldots (1)$$

Now as the distance between S_1 and S_2 is is 24 cm, the distance of S_2 from lens L, i.e., u will be (24 – x) and as image of S_2 is coincident with that of S_1, v = y. So for S_2

$$\frac{1}{y} - \frac{1}{-(24-x)} = \frac{1}{9}$$

Adding Equation (1) and (2),

$$\frac{1}{x} + \frac{1}{(24-x)} = \frac{2}{9} \text{ i.e., } x^2 - 24x + 108 = 0$$

so that $x = \frac{1}{2}\left[24 \pm \sqrt{24^2 - 4\times 108}\right]$ or, $x = 12 \pm 6$ i.e., x = 6 cm or 18 cm

So the lens must be placed at a distance of 6 cm or 18 cm from one of the sources.

Illustration 55.

A concave lens of focal length 20 cm is placed 15 cm in front of a concave mirror of radius of curvature 26 cm and further 10 cm away from the lens is placed an object. The principal axis of the lens and the mirror are coincident and the object is on this axis. Find the position and nature of the image.

Solution:

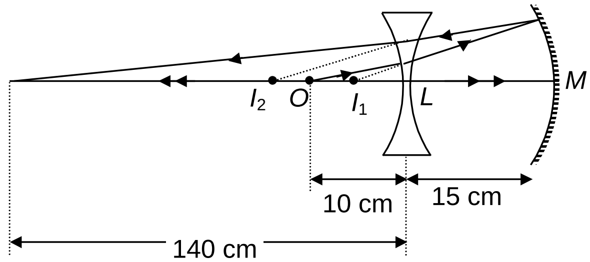
The lens will form the image I_1 of the object O at distance v from it such that

$$\frac{1}{v} - \frac{1}{-10} = \frac{1}{-20} \text{ i.e., } v = -\frac{20}{3} \text{ cm}$$

i.e., at a distance (20/3) cm in front of the lens. So the distance of this image I_1 from the mirror will be 15 + (20/3) = (65/3) cm.

The image I_1 will act as an object for the mirror, and hence the mirror will form image I_2 (of object I_1) such that

$$\frac{1}{v}+\frac{-3}{65}=\frac{1}{-13}$$ i.e., v = – 32.5 cm



i.e. The mirror will form image I_2 at a distance of 32.5 cm infront of it. However, as the lens is at a distance of 15 cm from the mirror, the image I_2 will act as an object for the lens again with u = 32.5 – 15 = 17.5 cm, so that

$$\frac{1}{v}-\frac{2}{+35}=\frac{1}{-20}$$ i.e., v = 140 cm

i.e., final image I_3 is at a distance of 140 cm in front of the lens and as here

$m = m_1 \times m_2 \times m_3$

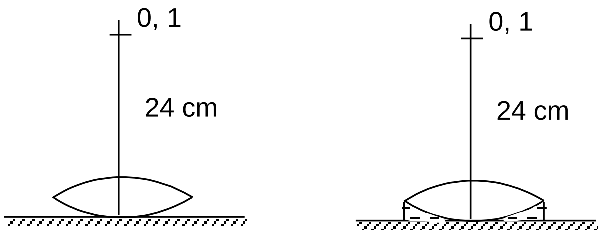
i.e., $m = \left[\frac{(-20/3)}{(-10)}\times(-1)\,\frac{(-65/2)}{(-65/3)}\times\frac{(140)}{(35/2)}\right]=\mathbf{-8}$

Final image will be inverted, real and 8 times of the object as shown in figure.

Illustration 56.

An equiconvex lens is placed on a plane mirror. An object coincides with its image when it is at a height of 24 cm from the lens. The gap between the lens and the mirror is filled with water (μ_w = 4/3). By how much distance should the object be now shifted so that is again coincides with its image? (μ_g = 1.5)

Solution:



Let F_1 be the equivalent focal length of the lens-mirror combination

$2F_1 = 24$

$F_1 = 12$ cm

$$\frac{1}{F_1}=\frac{2}{f_l}+\frac{1}{f_m}=\frac{2}{f_l}+\frac{1}{\infty} \qquad \frac{1}{12}=\frac{2}{f_l}$$

f_l = focal length of lens = 24 cm

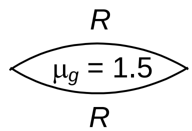
When water is filled, we have a combination of glass-lens, water-lens and a plane mirror. Let F_2 be the equivalent focal length, then $\frac{1}{F_2}=\frac{2}{f_l}+\frac{2}{f_w}+\frac{1}{f_m}=\frac{2}{f_l}+\frac{2}{f_w}+\frac{1}{\infty}$

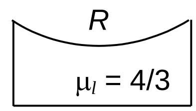
or, $$\frac{1}{F_2}=\frac{2}{24}+\frac{2}{f_w} \qquad \dots (1)$$

Applying lens maker’s formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

glass lens: $\frac{1}{f_l} = (1.5 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right)$





$$\frac{1}{24} = \frac{1}{2}\times\left(\frac{2}{R}\right)$$

R = 24 cm

Water lens $\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$ $\frac{1}{f_w} = -\frac{1}{3}\times\left(\frac{1}{R}\right)$

$f_w = -72$ cm

Substituting the value of f_w in equation (1) $\frac{1}{F_2} = \frac{2}{24} + \frac{2}{-72}$

$F_2 = 18$ cm

$2F_2 = 36$ cm

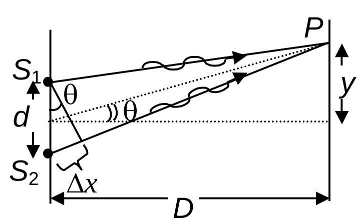
Shift, s = (36 – 24) cm = 12 cm upwards

The object will coincide with the image at a height of 36 cm from the lens and hence it should be shifted 12 cm upwards.

Illustration 57.

In an interference arrangement similar to Young's double-slit experiment, slit S_1 and S_2 are illuminated with coherent microwave sources each of frequency 1 Mhz. The sources are synchronized to have zero phase difference. The slits are separated by distance d = 150.0 m. The intensity $I_{(\theta)}$ is measured as a function of θ, where θ is defined as shown in figure. If I_0 is maximum intensity, calculate $I_{(\theta)}$ for (a) θ = 0°, (b) θ = 30° and (c) θ = 90°

Solution:



For microwaves, as c = f λ,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^6} = 300 \text{ m}$$

and as Δx = dsinθ,

$$\phi = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{300}(150\sin\theta) = \pi\sin\theta$$

So, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ and $\phi = \pi\sin\theta$, $I_1 = I_2$

reduces to $I_R = 2I_1[1 + \cos(\pi\sin\theta)] = 4I_1\cos^2(\pi\sin\theta/2)$

and as I_R will be maximum when $\cos^2[(\pi\sin\theta/2] = \text{maximum} = 1$

So that, $(I_R)_{max} = 4I_1 = I_0$ (given)

and hence, $I = I_0 \cos^2 [(\pi\sin\theta)/2]$

So (a) If $\theta = 0$ $I = I_0 \cos^2(0) = I_0$

(b) If $\theta = 30°$ $I = I_0 \cos^2(\pi/4) = (I_0/2)$

(c) If $\theta = 90°$ $\underline{I} = I_0 \cos^2 (\pi/2) = 0$

Illustration 58.

In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.

Solution:

As due to introduction of mica sheet of thickness t and refractive index μ in the path of one of the interfering beams optical path increases by $(\mu - 1)t$, the shift on the screen

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \ldots \text{(i)}$$

Now when the distance between the plane of slits and screen is charged from D to 2D, fringe width will become

$$\beta = \frac{2D}{d}(\lambda) \qquad \ldots \text{(ii)}$$

According to the given problem,

$$\frac{D}{d}(\mu - 1)t = \frac{2D}{d}(\lambda) \text{ i.e., } \lambda = \frac{1}{2}(\mu - 1)\, t$$

So, $\lambda = \frac{1}{2}(1.6 - 1) \times 1.964 \times 10^{-6} \text{ m} = 5892 \text{ Å}$

Illustration 59.

In Young's experiment, the source is red light of wavelength 7×10^{-7} m. If a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5^{th} bright fringe. Find the thickness of the plate. If the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6^{th} bright fringe due to red central light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength.

Solution:

As due to presence of glass plate path difference changes by $(\mu - 1)$ t, so according to given problem

$$(\mu_R - 1)\, t = 5\lambda_R \quad \text{i.e., } t = \frac{5 \times 7 \times 10^{-6}}{(1.5 - 1)} = 7\ \mu\text{m}$$

Now when red light is replaced by green light, $(\mu_G - 1)t = 6\lambda_R$

So $\frac{\mu_R - 1}{\mu_G - 1} = \frac{5}{6}$ i.e., $\mu_G - 1 = \frac{6}{5}(1.5 - 1)$ or $\mu_G = 1.6$

Further $5\beta_R = 10^{-3}$ i.e., $\beta_R = 2 \times 10^{-4}$

So, $\frac{\beta_G}{\beta_R} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$ i.e., $\frac{\beta_G - \beta_R}{\beta_R} = \frac{5}{7} - 1 = -\frac{2}{7}$ So, $\Delta\beta = -\frac{2}{7} \times 2 \times 10^{-4} = 0.57 \times 10^{-4}$ m

i.e., Fringe width will decrease by 0.57×10^{-4} m when red light is replaced by green light.