1. ELECTRIC CHARGE AND CURRENT

Electric charges in motion constitute electric current. Metals such as gold, silver, copper, aluminum etc., called conductors, have large number of free electrons. These free electrons move around in all directions from atom to atom under normal conditions but when a potential difference is applied between two points (two ends preferably), the electrons move only in one direction. The electrons are negatively charged particles and the conventional current is considered as the flow of positive charges. Hence the direction of flow of electrons is opposite to the direction of conventional current, which takes place in a direction from a point of higher potential to a point of lower potential.

The strength of the current is the rate at which the electric charges are flowing. If a charge *Q* coulomb passes through a given cross-section of the conductor in *t* second, the current *I* through the conductor is given by

$$I = \frac{Q}{t}\frac{\text{coulomb}}{\text{second}} = \frac{Q}{t} \qquad \dots (1)$$

The SI unit of current is ampere.

Illustration 1.

An electrical device sends out 78 coulombs of charge through a conductor in 6 seconds. Find the current flow.

Solution:

Given that charge flowing Q = 78 C,

time of flow t = 6 s

The current $I = \frac{Q}{t} = \frac{78\,\text{C}}{6\,\text{s}} = \mathbf{13\ A}$

Illustration 2.

What is the quantity of electricity required to provide a current of 10 A for one hour?

Solution:

Given that the current I = 10 A,

time of flow t = 1 hour = 3600 s

The quantity of electricity = the amount of charge flowing

$Q = It$

= (10 A)(3600 s) = **36000 C**

2. POTENTIAL DIFFERENCE

In current electricity, dry cells or secondary cells or generators are employed to create a potential difference in order to cause an electric current flow in closed circuits just as a water pump is used to create pressure difference in order to drive water in water pipes.

The unit of potential difference is **volt**. The volt is defined as that potential difference between two points of a conductor carrying a current of one ampere when the power dissipated between these points is equal to one watt.

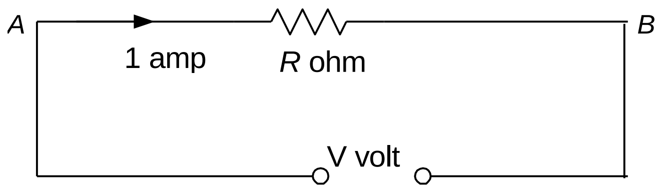
3. RESISTANCE

Electrical resistance may be defined as the property of a substance, which opposes the flow of an electric current through it.

The unit of resistance is **ohm**. Symbol is Ω. Ohm is that resistance between two points of a conductor when a potential difference of one volt is applied between these points produces in this conductor a current of one ampere.

4. OHM'S LAW

Ohm's law is the most fundamental of all the laws in electricity.



Statement: The current which flows in a conductor is proportional to the potential difference which causes its flow provided the temperature of the conductor is constant.

If a potential difference of V volt exists between the ends A and B of a conductor AB current of I ampere flows through the conductor and

$$V \propto I \quad \text{or} \quad V = IR \qquad \text{... (2)}$$

where the constant R is the resistance of the conductor. In this Ohm's law relation, V is in volts, I is in amperes and R is in ohms.

Illustration 3.

The current in a conductor is 5 A when the voltage between the ends of the conductor is 12 V.

(i) What is the resistance of the conductor?

(ii) What will be the current in the same conductor if the voltage is increased to 42 V?

Solution:

(i) Given that I = 5 A; V = 12 V; R =?

$$R = \frac{V}{I} = \frac{12V}{5A} = \mathbf{2.4\ \Omega}$$

(ii) If the voltage applied becomes 42 V

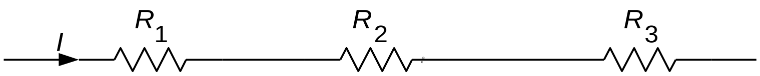
$$I = \frac{V}{R} = \frac{42V}{2.4ohm} = \mathbf{17.5\ A}$$

5. GROUPING OF RESISTANCES

5.1 RESISTORS IN SERIES

The series circuit is one in which the same current flows in all the components of the circuit. If resistors R_1, R_2, R_3, ... are connected in series, the equivalent (or effective) resistance of the combination is the sum of the resistances so connected.

$$R = R_1 + R_2 + R_3 + ... \qquad \dots (\mathbf{3})$$



In a series combination of resistors

(i) the equivalent resistance is equal to the sum of the individual resistances,

(ii) the same current flows through all the components and

(iii) the sum of the separate voltage drops (*IR* drop) is equal to the applied voltage across the combination. If V be the applied voltage, V_1, V_2, V_3, ... be the *IR* drops across resistances R_1, R_2, R_3, ... respectively.

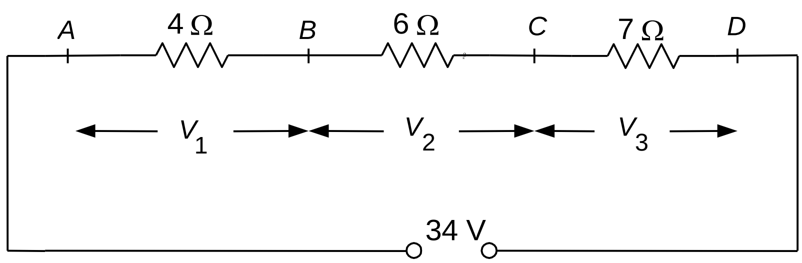
$$V = IR_1 + IR_2 + IR_3 + ...$$

$$= V_1 + V_2 + V_3 + ...$$

Illustration 4.

Three resistors of values 4 ohm, 6 ohm and 7 ohm are in series and a potential difference of 34 V is applied across the grouping. Find the potential drop across each resistor.

Solution:



The current through the circuit $= \dfrac{34V}{(4+6+7)ohm} = 2$ A

potential difference across 4 ohm resistor = IR = 2 A × 4 ohm = 8 V

potential difference across 6 ohm resistor = 2A × 6 ohm = 12 V

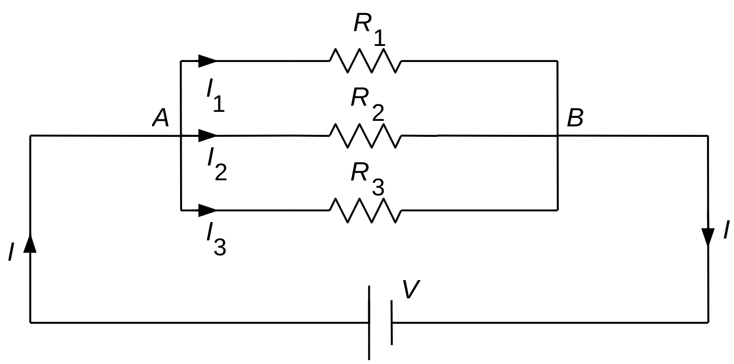
potential difference across 7 ohm resistor = 2 A × 7 ohm = 14 V

5.2 RESISTORS IN PARALLEL

A parallel circuit of resistors is one in which the same voltage is applied across all the components.

If resistors R_1, R_2, R_3, ... are connected in parallel then reciprocal of the equivalent resistance is the sum of the reciprocals of the resistance of separate components.

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+.... \qquad \ldots \textbf{(4)}$$



(i) the total current taken from the supply is equal to the sum of the currents in separate branches.

(ii) the potential difference across each resistor is the same V volt which is the applied voltage.

(iii) the branch currents I_1, I_2, I_3, ... are in the ratio,

$$\frac{1}{R_1}:\frac{1}{R_2}:\frac{1}{R_3}:\ldots$$

(iv) the equivalent resistance is smaller than the smallest of the resistances in parallel.

Illustration 5.

Two resistances 3 ohm and 2 ohm are in parallel connection and a potential difference of 12 V is applied across them. Find

(a) the equivalent resistance of the parallel combination,

(b) the circuit current and

(c) the branch currents.

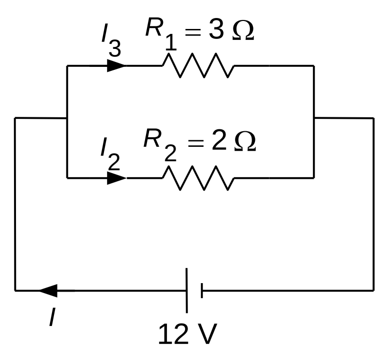
Solution:

(a) Two resistors R_1 and R_2 are in parallel. Their equivalent resistance R is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

or $$R=\frac{R_1R_2}{R_1+R_2}$$

$$=\frac{2\times3}{2+3}=\frac{6}{5}=\mathbf{1.2\ \Omega}$$



(b) The circuit current $=\dfrac{\text{Circuit voltage}}{\text{Circuit resistance}}$

$$=\frac{12V}{1.2\Omega}=\mathbf{10\ A}$$

(c) The current through 2 ohm resistor

$$I_2 = I \times \frac{3}{2+3} = 10 \times \frac{3}{5} = \mathbf{6\ A}$$

The current through 3 ohm resistor

$$I_3 = I \times \frac{2}{2+3} = 10 \times \frac{2}{5} = \mathbf{4\ A}$$

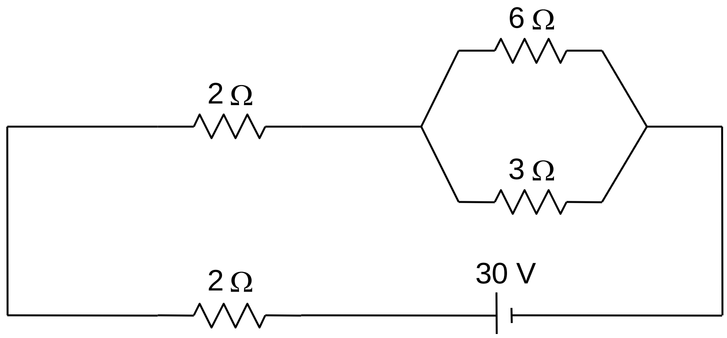
(Also $I_3 = I - I_2 = 10$ A – 6 A = 4 A)

5.3 SERIES-PARALLEL GROUPINGS

A series-parallel circuit is a combination of resistors in series as well as parallel connections. The following examples will illustrate the solutions of such problems.

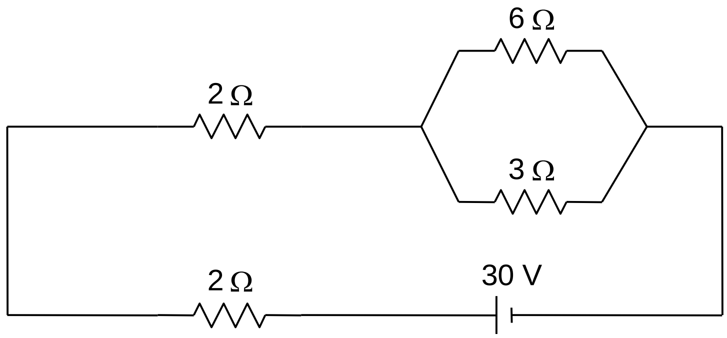
Illustration 6.

Determine the current taken from the 30 *V* supply and the current through the 6 ohm resistor.



Solution:

As a first step to solution let us reduce the parallel combination of 6 ohm and 3 ohm into a single resistance.



The parallel combination = $\frac{6 \times 3}{6+3} \Omega = 2\Omega$

Now the circuit reduces to three resistors, each 2 ohm, in series to a 30 V supply.

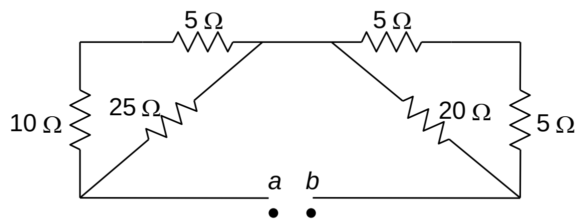
Hence the circuit current = $\frac{30V}{6\Omega}$ = **5 A**

The current through 6 ohm resistor = $5 \times \frac{3}{6+3}$ A

$= \frac{15}{9}$ = **1.7 A**

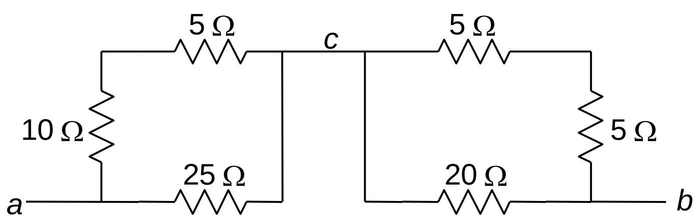
Illustration 7.

Find the equivalent resistance of the circuit given across *ab*.



Solution:

As a first step the circuit may be redrawn as follows.

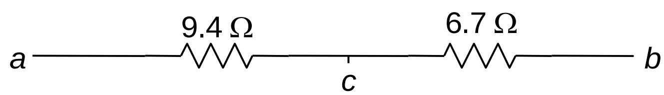


The left block is equivalent to 15 ohm and 25 ohm in parallel

i.e., $\frac{25\times15}{25+15} = 9.4\Omega$

The right block is equivalent to 10 ohm and 20 ohm in parallel

i.e., $\frac{10\times20}{10+20} = \frac{200}{30} = 6.7\Omega$



The circuit now reduces as two resistors in series i.e., 9.4 + 6. 7 = **16.1 Ω**

6. RESITIVITY

The resistance of a conductor is found to be directly proportional to its length and inversely proportional to its cross-sectional area at constant temperature.

$$R \propto L$$

$$\propto \frac{1}{A}$$

$$R = \frac{\rho L}{A} \text{ ohm} \qquad \ldots (5)$$

where ρ is a constant called the resistivity or specific resistance of the material of the wire. L the length is in metres and A the area of cross-section is in m^2.

The unit of resistivity is ohm-metre.

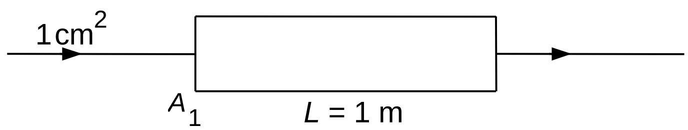
Resistivity is also the resistance of a cube of 1 m side measured between opposite faces of the cube.

Illustration 8.

A certain rectangular block has dimensions 100cm × 1cm × 1cm. Find the resistance of the block (i) across square faces and (ii) across two opposite rectangular faces. Specific resistance of the material is 40×10^{-8} ohm-metre

Solution:

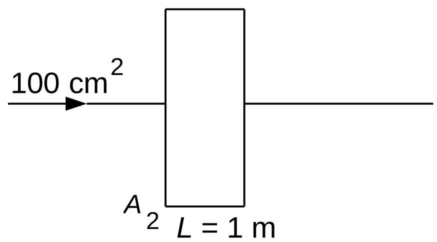
(i) Resistance of the block across the square faces



$$R = \frac{\rho \times L}{A} = \frac{(40 \times 10^{-8}\, ohm\text{-}m)\,(1m)}{1 \times 10^{-4}\, m^2}$$

$= \mathbf{40 \times 10^{-4}\ \Omega}$

(ii) Resistance across the two opposite rectangular faces



$$R = \frac{\rho L}{A} \frac{(40 \times 10^{-8}\, ohm\text{-}m)\,(10^{-2}\, m)}{100 \times 10^{-4}\, m^2}$$

$= \mathbf{40 \times 10^{-8}\ \Omega}$

7. TEMPERATURE DEPENDENCE OF RESISTANCES

The resistance of most conductors and of all pure metals increases with temperature. But in carbon the resistance decreases with temperature. There are some alloys where there is no change of resistance with temperature. If R_0 and R be the resistances of a conductor at 0°C and θ°C, then it is found that

$$R = R_0 (1 + \alpha\theta) \qquad \ldots (6)$$

where α is a constant called the temperature coefficient of resistance.

$$\alpha = \frac{R - R_0}{R_0 \cdot \theta}$$

and the unit of α is K^{-1} or °C^{-1}.

If R_1 and R_2 be the resistances of a conductor at temperatures θ_1°C and θ_2°C, then

$$R_1 = R_0 (1 + \alpha\theta_1)$$

$$R_2 = R_0 (1 + \alpha\theta_2)$$

and $$\alpha = \frac{R_2 - R_1}{R_1\theta_2 - R_2\theta_1}$$

Illustration 9.

A metal wire of diameter 2 mm and of length 100 m has a resistance of 0.5475 ohm at 20°C and 0.805 ohm at 150°C. Find the values of (i) temperature coefficient of resistance (ii) its resistance at 0°C (iii) its resistivities at 0°C and 20°C.

Solution:

(i) If R_{20} and R_{150} be the resistances at temperatures 20°C and 150°C respectively and α be the temperature coefficient of resistance

$$R_{20} = 0.5475 = R_0 (1 + \alpha \times 20) \quad \text{... (i)}$$

$$R_{150} = 0.805 = R_0 (1 + \alpha \times 150) \quad \text{... (ii)}$$

Now, $$\alpha = \frac{R_{150} - R_{20}}{R_{20} \times 150 - R_{150} \times 20} = \frac{0.805 - 0.5475}{0.5475 \times 150 - 0.805 \times 20} \text{ or } \boldsymbol{\alpha = 3.9 \times 10^{-3}\ {}^{\circ}C^{-1}}$$

(ii) Substituting this value of α in equation (i), $\boldsymbol{R_0 = 0.5079\ \Omega}$

(iii) Now $R_0 = \dfrac{\rho_0 L}{A}$

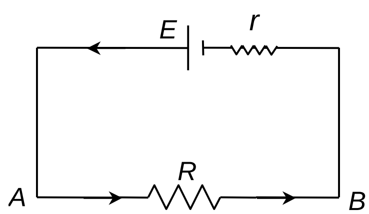
$$0.5079 = \frac{\rho_0 (100)}{\pi(1 \times 10^{-3})^2} \text{ or } \boldsymbol{\rho_0 = 1.596 \times 10^{-8}\ \Omega.m}$$

$$\rho_{20} = \rho_0 (1 + \alpha \times 20)$$

$$= 1.596 \times 10^{-8}\ [1 + (3.9 \times 10^{-3} \times 20)]$$

$$= \boldsymbol{1.720 \times 10^{-8}\ \Omega.m}$$

8. EMF OF A CELL AND ITS INTERNAL RESISTANCE



If a cell of emf E and internal resistance r be connected with a resistance R the total resistance in the circuit is (R+ r).

The current through the circuit $I = \dfrac{E}{R+r}$

Potential difference across the ends A and B of

$$R = IR = \frac{ER}{R+r}$$

Thus, although the emf of the cell is E, the effective potential difference it can deliver is less than E and it is given by

$$V_{AB} = E - Ir$$

The quantity V_{AB} is called the terminal potential difference of the cell and this is also the potential difference across the external resistance R.

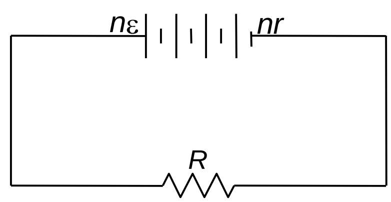
If $R \to \infty$, $V_{AB} \to E$, the emf of the cell.

9. GROUPING OF CELLS

9.1 CELLS IN SERIES

Let there be n cells each of emf ε, arranged in series.

Let r be the internal resistance of each cell.

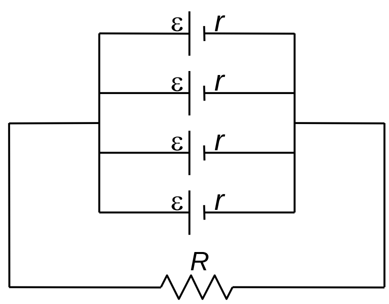


The total emf is $n\varepsilon$ and the total internal resistance is nr. If R be the external load, the current I through the circuit $I = \dfrac{n\varepsilon}{R+nr}$ … (7)

9.2 CELLS IN PARALLEL

If m cells each of emf ε and internal resistance r be connected in parallel and if this combination be connected to an external resistance R, then the emf of the circuit = ε.

The internal resistance of the circuit = the resistance due to m resistances each of r in parallel $= \dfrac{r}{m}$.



Now the current through the external resistor $R = \dfrac{\varepsilon}{R+\dfrac{r}{m}} = \dfrac{m\varepsilon}{mR+r}$. … (**8**)

9.3 MIXED GROUPING OF CELLS

Let n identical cells be arranged in series and let m such rows be connected in parallel. Obviously the total number of cells is nm.

The emf of the system = $n\varepsilon$

The internal resistance of the system $=\frac{nr}{m}$

The current through the external resistance R

$$I=\frac{n\varepsilon}{R+\frac{nr}{m}}=\frac{mn\varepsilon}{mR+nr} \qquad \ldots (\mathbf{9})$$

Illustration 10.

Six cells are connected (a) in series, (b) in parallel and (c) in 2 rows each containing 3 cells. The emf of each cell is 1.08 V and its internal resistance is 1 ohm. Calculate the currents that would flow through an external resistance of 5 ohm in the three cases.

Solution:

(a) The cells in series.

Given that ε = 1.08 V, n = 6, r = 1 ohm, R = 5 ohm

The total emf = $n\varepsilon = 6 \times 1.08$ V

The total internal resistance $nr = 6 \times 1 = 6$ ohm

The current in the circuit $I_S=\frac{n\varepsilon}{R+nr}=\frac{6\times1.08}{5+6}=\mathbf{0.589\ A}$

(b) The cells in parallel.

Here $\varepsilon = 1.08\ V$, m = 6, r = 1 ohm, R = 5 ohm

$$I_p=\frac{m\varepsilon}{mR+r}=\frac{6\times1.08}{6\times5+1}=\frac{6.48}{31}=\mathbf{0.209\ A}$$

(c) The cells in multiple arc with n = 3, m = 2

$$I=\frac{mn\varepsilon}{mR+nr}=\frac{6\times1.08}{(2\times5)+(3\times1)}$$

$$=\frac{6.48}{13}=\mathbf{0.498\ A.}$$

10. ARRANGEMENT OF CELLS FOR MAXIMUM CURRENT

Considering the above case it is required to find the condition for maximum current if the product mn is given.

In this case the product mn, ε, r and R are constants and m and n alone can be varied to get I maximum.

For I_{max} denominator ($mR + nr$) should be minimum in equation (9). This happens when $mR = nr$ or $R = \frac{nr}{m}$.

Hence the current through the external resistance R is a maximum when it is equal to internal resistance of the battery $\frac{nr}{m}$.

Note: If cells of different emf and internal resistance are in parallel there is no simple formula to give the total emf and the internal resistance and any calculations involving circuits in such cases can be done with the help of Kirchhoff's laws which will be discussed later.

Illustration 11.

How would you arrange 20 cells each of emf 1.5 V and internal resistance 1 ohm to give the maximum current through an external resistance of 5 ohm? Also find this current.

Solution:

Let n cells be in series and let there be m such groups in parallel.

Total number of cells $mn = 20$

The external resistance $R = 5$ ohm

The internal resistance of each cell $r = 1$ ohm

The condition for maximum current is $R = \frac{nr}{m}$

or, $5 = \frac{n \times 1}{m} = \frac{n}{m}$

or $n = 5\,m$

Now $mn = m\,(5m) = 20$

or $m^2 = 4$

$m = 2$

$n = 100$

To get the maximum current the cells have to be arranged in 2 rows, each row consisting of 10 cells in series.

The maximum current $= \frac{mn\varepsilon}{mR + nr} = \frac{20 \times 1.5}{2 \times 5 + 10 \times 1} = \frac{30}{20}$ = **1.5 A**

11. KIRCHHOFF'S LAW

To solve the problem of complex circuit, Gaustav Kirchhoff gave following two laws

(i) First law

Accroding to it "the algebric sum of currents meeting at a junction is zero i.e.,

$$\sum I = 0 \qquad \ldots (\mathbf{10})$$

This laws is also known as current law or junction rule.

(ii) Second law

According to it "The algebric sum of the changes in potential in a closed loop or mesh is zero."

$$\sum V = 0 \qquad \ldots \textbf{(11)}$$

This law is also known as potential law or loop rule.

Application of Kirchhoff's laws to circuit solutions

We may choose to traverse a closed circuit (loop) in clockwise or anticlockwise direction. If the current flowing in one direction and the consequent *IR* drop is taken as positive, the current flowing in the opposite direction and the consequent *IR* drop is taken as negative.

We may follow the rule as given below for guidance.

(i) The resistive drops (*IR* drops) in a loop due to current flowing in **clockwise direction may be taken as positive drops**.

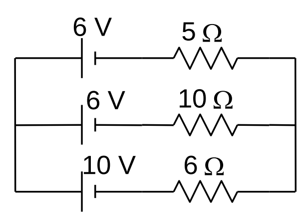
(ii) The resistive drops in a loop due to the current flowing in **anticlockwise direction is taken as negative drops**.

(iii) Similarly the battery emf causing the current to flow in clockwise direction is taken as positive emf and the battery emf causing the current to flow in counter clockwise direction in the loop is taken as negative emf.

As an illustration of the above rules let us follow the worked example.

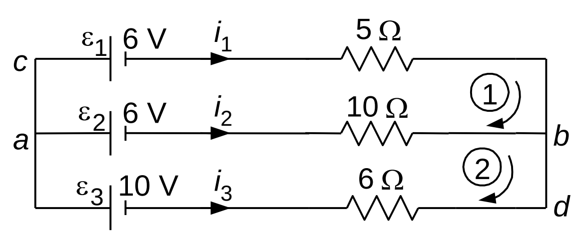
Illustration 12.

Find the current in the resistors of the circuit given. The internal resistances of the batteries are included in the external resistances.



Solution:

The circuit given can not be simplified further because it contains resistors not in simple series or parallel connection. Hence Kirchhoff's rules have to be applied. Since the currents have not been marked we have to do that first. No special care need be taken to indicate the exact current directions since those chosen incorrectly will simplify to give negative numerical values.



Applying the junction rule to junction a

$$i_1 + i_2 + i_3 = 0 \qquad \ldots \textbf{(i)}$$

Taking the loop acba

IR drop across 5 Ω = $+5i_1$

IR drop across 10 Ω = $-10i_2$

emf of $\varepsilon_1 = +6$ V

emf of $\varepsilon_2 = -6$ V

Applying the loop rule

$$5i_2 - 10i_2 = +6 - 6 = 0 \quad \text{or} \quad i_1 - 2i_2 = 0 \qquad \ldots \textbf{(ii)}$$

Considering the loop *abda*

$10i_2 - 6i_3 = 10 - 6$

$10i_2 - 6i_3 = 4$

$5i_2 - 3i_3 = 2$ … **(iii)**

To find the unknowns i_1, i_2 and i_3, solve the three equations (i), (ii) and (iii). We get

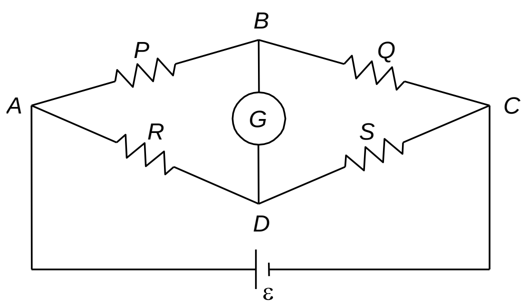
$i_1 = \frac{2}{7}\text{A}$, $i_2 = \frac{1}{7}\text{A}$

$i_3 = -\frac{3}{7}\text{A}$. The direction of flow of i_3 is opposite to that marked in the circuit.

12. ELECTRICAL DEVICES

12.1 WHEATSTONE'S BRIDGE

For measurement of a resistance, a network made up of four resistance arms *P*, *Q*, *R* and *S* is arranged as shown. Arms *AB* and *BC* having resistances *P* and *Q* respectively are known as ratio arms.



A galvanometer *G* is connected across *B* and *D*. A battery is connected across *A* and *C*. When the values of resistances *P*, *Q*, *R* and *S* are such that no current flows through the galvanometer *G* the bridge is said to be balanced. In that case *B* and *D* are at the same potential and we have the condition

$$\frac{P}{Q} = \frac{R}{S}$$

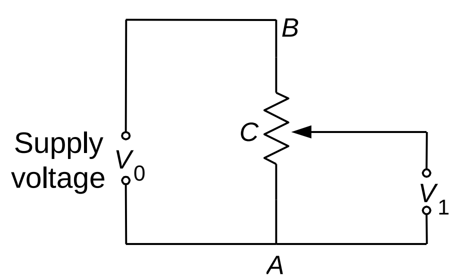
Usually *S* is an unknown resistance and *P*, *Q* and *R* are known.

12.2 POTENTIAL DIVIDER AND POTENTIOMETER

The potential divider is a resistance connected across the supply and is used to obtain a variable voltage from a constant voltage supply.

Resistance *AB* = *R* ohm (fixed)

C is a sliding contact.



Let the resistance of $AC = R_1$ and of $CB = R_2$

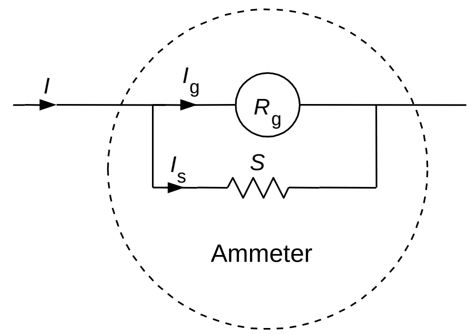
Potential drop across AC: $V_1 = \left(\frac{R_1}{R_1+R_2}\right) \times V_0 = \left(\frac{R_1}{R}\right) V_0$.

Potential drop across CB: $V_2 = \left(\frac{R_2}{R_1+R_2}\right) \times V_0 = \left(\frac{R_2}{R}\right) V_0$.

12.3 AMMETER

An ammeter is a modified form of suspended coil galvanometer. While galvanometers can permit only very small currents to pass through them, ammeters can allow, depending upon their construction, much heavy currents to flow through them.

A suitable shunt resistance S (of very small value compared to R_g) in parallel with that of galvanometer of resistance R_g achieves this objective.



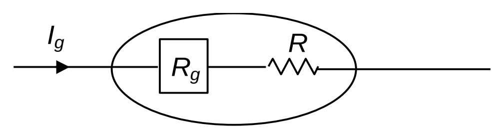
If the ammeter is designed to measure a maximum current I (full scale deflection current), then the shunt S required for the purpose is given by $I_g \cdot R_g = (I - I_g)S$

where I_g is the maximum permissible current through the galvanometer.

The resistance of ammeter is small (smaller than that of the shunt S) and for current measuring purposes it is included in series in a circuit. An ideal ammeter has zero resistance.

12.4 VOLTMETER

Voltmeter is also a modified form of a galvanometer. It is used to measure potential differences.



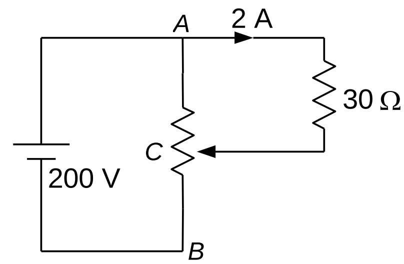
A suitable high resistance R is included in series with the galvanometer of resistance R_g to enable the instrument to measure voltages. If the maximum range of the voltmeter is V_0 and the maximum permissible current through the galvanometer is I_g, then the value of R is given by

$$I_g = \frac{V_0}{R + R_g}$$

To measure the potential difference across two points in a circuit the voltmeter is connected in parallel with it. An ideal voltmeter has infinite resistance.

Illustration 13.

A potential divider of resistance 500 ohm is used to obtain variable voltages from a supply main of 200 V. Determine the position of the tapping point *C* to get a current of 2 A through a resistance of 30 ohm connected across *A* and *C* as shown.



Solution:

Let the resistance of the potential divider between *A* and *C* be *R* ohm.

The potential difference across the 30 ohm resistor = 2*A* × 30 ohm = 60 V

∴ the voltage drop across *AC* of the potential divider = 60 V

Current flowing through $R = \frac{60}{R} A$

Now, the voltage drop across *BC* = 200 V – 60 V = 140 V

The current through $BC = \frac{140}{(500-R)} A$

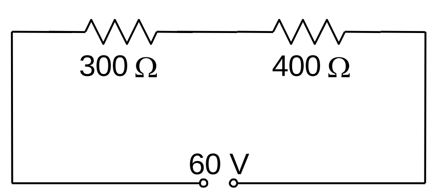
Now, $\frac{140}{(500-R)} = \frac{60}{R} + 2$

Solving *R* = 434.7 Ω

Hence the tapping point *C* lies in such a position that the length *AC* is $\frac{434.7}{500}$ = **0.8694** of the length *AB*.

Illustration 14.

In the circuit shown a voltmeter reads 30 volt when it is connected across the 400 ohm resistance. Calculate what the same voltmeter would read when it is connected across the 300 ohm resistance.



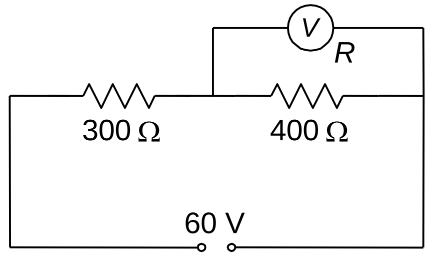
Solution:

The voltmeter is in parallel with 400 ohm resistance.

Let its resistance be *R* ohm.

It is clear that the resistances *R* and 400 ohm combining in parallel produce equivalent resistance of value of 300 ohm so that the potential drop across this equivalent resistance is half of 60 V.

$\therefore \qquad \dfrac{R \times 400}{R+400} = 300$



$400R = 300R + 120000$

$100R = 120000$

$R = 1200\ \Omega$

Next the same voltmeter is connected across the 300 ohm resistance. Now the equivalent resistance of 300 ohm and 1200 ohm of voltmeter in parallel connection will be

$$\frac{300 \times 1200}{300+1200} = 240\Omega$$

The total circuit resistance = 240 + 400 = 640

Potential drop across 240 ohm $= \dfrac{240}{640} \times 60V$ = **22.5 V**

Illustration 15.

A battery of emf 1.4 V and internal resistance 2 ohm is connected to a resistor of 100 ohm resistance through an ammeter. The resistance of the ammeter is $\dfrac{4}{3}$ ohm. A voltmeter has also been connected to find the potential difference across the resistor.

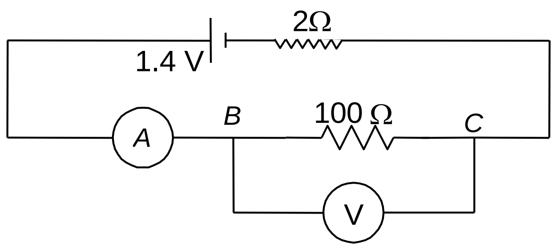
(i) Draw the circuit diagram.

(ii) The ammeter reads 0.02 A. What is the resistance of the voltmeter?

(iii) The voltmeter reads 1.1 V. What is the error in the reading?

Solution:

(i) The circuit diagram is shown.



(ii) Let the resistance of the voltmeter be R ohm. The equivalent resistance of voltmeter (R ohm) and 100 ohm in parallel is $\dfrac{100 \times R}{100+R} = \dfrac{100R}{100+R}$

The resistance of the ammeter $= \dfrac{4}{3}\Omega$

The total resistance of the circuit $= \dfrac{100R}{100+R} + \dfrac{4}{3} + 2\Omega$

The current in the circuit as read by the ammeter = 0.02 A

Now, $0.02 = \dfrac{1.4}{\dfrac{100R}{100+R} + \dfrac{4}{3} + 2}$ or, $\dfrac{100R}{100+R} + \dfrac{4}{3} + 2 = \dfrac{1.4}{0.02} = 70$

$$\frac{100R}{100+R} = 70 - \frac{10}{3} = \frac{200}{3}$$

$300R = 200R + 20000$

$100R = 20000$

$R =$ **200 Ω**

Resistance of the voltmeter = **200 Ω**

(iii) The effective resistance between B and C $= \frac{100 \times 200}{100+200} = \frac{200}{3} \Omega$

The potential drop across this resistance = circuit current $\times \frac{200}{3}$

$$= 0.02 \times \frac{200}{3} = \frac{4}{3} \text{V} = 1.33 \text{V}$$

The reading of the voltmeter = 1.1 V

The error in the reading of the voltmeter = 1.1 – 1.33 = **– 0.23 V**

13. HEATING EFFECT OF CURRENT

13.1 JOULE'S LAW OF ELECTRICAL HEATING

When an electric current flows through a conductor electrical energy is used in overcoming the resistance of the wire. If the potential difference across a conductor of resistance R is V volt and if a current of I ampere flows the energy expended in time t seconds is given by

$$W = VIt \text{ joule}$$

$$= I^2 Rt \text{ joule}$$

$$= \frac{V^2}{R} t \text{ joule}$$

The electrical energy so expended is converted into heat energy and this conversion is called the heating effect of electric current.

The heat generated in joules when a current of I ampere flows through a resistance of R ohm for t seconds is given by

$$H = I^2 Rt \text{ joule} \qquad \ldots \textbf{(12)}$$

This relation is known as Joule's law of electrical heating.

13.2 ELECTRICAL POWER

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by

$$\left.\begin{aligned} P &= VI \text{ watt} \\ &= I^2R \text{ watt} \\ &= \frac{V^2}{R} \text{ watt} \end{aligned}\right\}$$

The power P is in watts when I is in amperes, R is in ohms and V is in volts.

The practical unit of power is 1 kW = 1000 W.

The formula for power $P = I^2R = VI = \frac{V^2}{R}$ is true only when all the electrical power is dissipated as heat and not converted into mechanical work, etc., simultaneously.

13.3 UNIT OF ELECTRICAL ENERGY CONSUMPTION

1 unit of electrical energy = 1 kilowatt-hour

$$= 1 \text{ kWh} = 36 \times 10^5 J$$

Number of units consumed $= \frac{watt \times hour}{1000} = kWh$

Illustration 16.

What is the resistance of the filament of a bulb rated at (100 W – 250 V)? What is the current through it when connected to 250 V line? What will be power if it is connected to a 200 V line?

Solution:

Power $P = VI = \frac{V^2}{R}$

Resistance $R = \frac{V^2}{P} = \frac{250 \times 250}{100} = \mathbf{625\ \Omega}$

The current through the lamp $= \frac{P}{V} = \frac{100\,W}{250\,V}$

$= \mathbf{0.4\ A}$

The power of the lamp when it is connected to a 200 V line is

$$P = \frac{V^2}{R} = \frac{200 \times 200}{625} = \mathbf{64\ W}$$

Illustration 17.

Forty electric bulbs are connected in series across a 220 V supply. After one bulb is fused the remaining 39 are connected again in series across the same supply. In which case will there be more illumination and why?

Solution:

Let r be the resistance of each bulb and 40 bulbs in series will have a resistance of 40 r ohm. When connected across a supply voltage V, the power of the system with 40 bulbs will be

$$P_{40} = \frac{V^2}{40r}$$

When one of the bulbs is fused, the resistance of the remaining 39 bulbs in series = 39 r and the power of the system when connected to the same supply

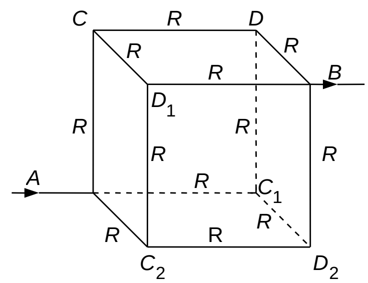
$$P_{39} = \frac{V^2}{39r}$$

It is clear that $\frac{V^2}{39r} > \frac{V^2}{40r}$

$\therefore$ power of **39 bulbs** in series is greater.

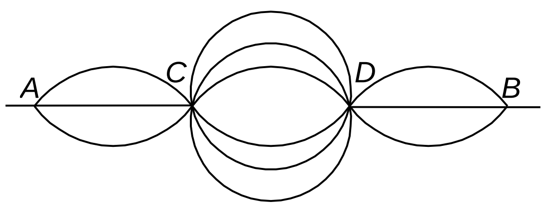
Illustration 18.

The Figure shows a cube made of wires each having a resistance R. The cube is connected into a circuit across a body diagonal AB as shown. Find the equivalent resistance of the network in this case.



Solution:

Let us search the points of same potential. Since the three edges of the cube from A viz., AC, AC_1 and AC_2 are identical in all respects the circuit points C, C_1 and C_2 are at the same potential. Similarly for the point B the sides BD, BD_1 and BD_2 are symmetrical and the points D, D_1 and D_2 are at the same potential.



Next let us bring together the points C, C_1 and C_2 and also D, D_1 and D_2.

Then the cube will look as follows.

The resistance between A and C = $\frac{R}{3}$

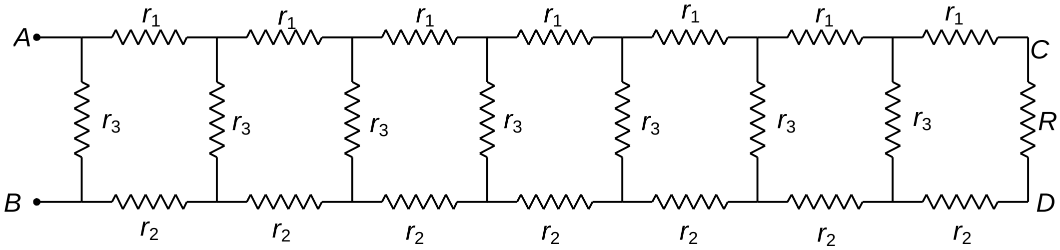
The resistance between C and D = $\frac{R}{6}$

The resistance between D and B = $\frac{R}{3}$

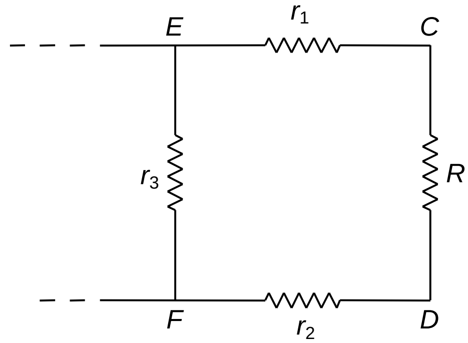
The circuit is equivalent to $\frac{R}{3}$, $\frac{R}{6}$ and $\frac{R}{3}$ in series which is equal to $\boldsymbol{\frac{5}{6}}$ R.

Illustration 18.

Study the following circuit. Values of r_1, r_2 and r_3 are 1 ohm, 2 ohm and 3 ohm respectively. A resistance R is connected across the points C and D. What should be the value of R for which the resistance of the network across AB is R?



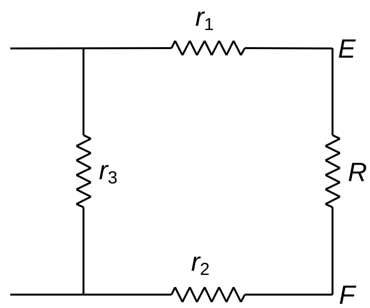
Solution:



Let us consider the extreme right square of the loop.

Resistance across EF = $(r_1 + R + r_2)$ and r_3 in parallel

$$= \frac{r_3(r_1+r_2+R)}{(r_1+r_2+r_3+R)}$$



This value should be equal to R, so that we by the repeated operation of this type we will be left with only one square which will be the left extreme one and it will have a value R

i.e., $$\frac{r_3(r_1+r_2+R)}{(r_1+r_2+r_3+R)} = R$$

Substituting the numerical values

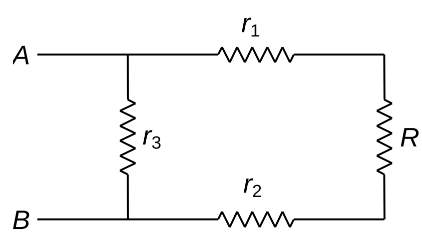
$$\frac{3(1+2+R)}{(1+2+3+R)} = R$$

(or) $$\frac{3(3+R)}{(6+R)} = R$$

$$9 + 3R = 6R + R^2$$

(or) $R^2 + 3R - 9 = 0$

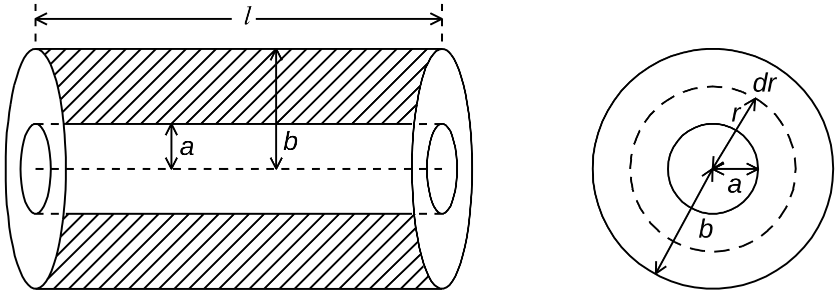
$$R = \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm 3\sqrt{5}}{2}$$



$$\therefore R = \frac{\mathbf{3(\sqrt{5}-1)}}{\mathbf{2}}\ \Omega$$

Illustration 19.

A homogeneous poorly conducting medium of resistivity ρ fills up the space between two thin coaxial ideally conducting cylinders. The radii of the cylinders are equal to a and b with a < b, the length of each cylinder is *l*. Neglecting the edge effects, find the resistance of the medium between the cylinders.



Solution:

The current will be conducted radially outwards from the inner conductor (say) to the outer. The area of cross-section for the conduction of the current is therefore the area of an elementary cylindrical shell and which varies with radius. The length of the conducting shell is measured radially from radius a to radius b.

Consider an elementary cylindrical shell of radius r and thickness dr. Its area of cross-section (normal to flow of current) = (2πrl) and its length = dr.

Hence the resistance of the elementary cylindrical shell of the medium is $dR = \frac{\rho dr}{2\pi rl} = \frac{\rho}{2\pi l}\left[\frac{dr}{r}\right]$

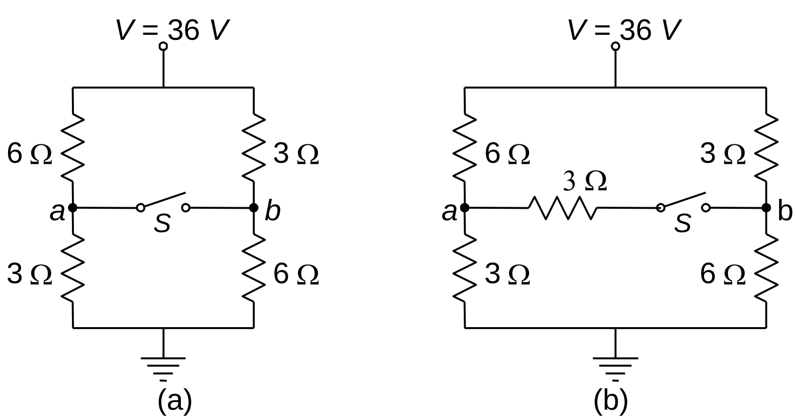
The resistance of the medium is obtained by integrating for r from a to b.

Hence required resistance

$$R = \frac{\rho}{2\pi l}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi l}\left[\log_e r\right]_a^b \;= \left(\frac{\boldsymbol{\rho}}{\boldsymbol{2\pi l}}\right)\boldsymbol{\log_e \frac{b}{a}}$$

Illustration 20.

A convention is often employed in circuit diagrams where the battery (or other power source) is not shown explicitly but the points connected to the source are indicated by voltage and ground respectively. The following two circuit diagrams are drawn on this convention. Assume the battery resistance is negligible.



(a) In Figure (a), what is the potential difference V_{ab} when the switch S is open?

(b) What is the current through switch S when it is closed?

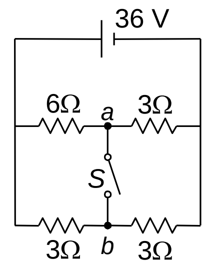
(c) In Figure (b), what is the potential difference V_{ab} when switch S is open?

(d) What is the current through switch S when it is closed?

(e) What is the equivalent resistance in the circuit (b), when (i) switch S is open and (ii) switch S is closed?

Solution:

The given circuit is equivalent to



(a) Potential at the point a = $V_a = 36 - \left(\frac{6}{9} \times 36\right) = 12V$

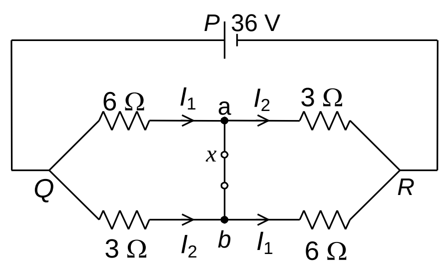
Potential at the point b = $V_b = 36 - \left(\frac{3}{9} \times 36\right) = 24V$

Hence V_{ab} = Potential difference between a and b

$= V_a - V_b = 12 - 24 = -12$ V

(b) When the switch S is closed, the currents and potentials will readjust to new values.

The equivalent circuit is now



Let the current distributions be I_1 and I_2 as shown. Let the current in the switch be i_{ab} from a to b and x the resistance of switch. Then for the loop QabQ,

$$6I_1 + I_{ab}x - 3I_2 = 0 \quad \ldots \text{(i)}$$

For the loop PQaRP, $36 = 6I_1 + 3I_2$ … (ii)

Also $i_{ab} = I_1 - I_2$ … (iii)

From (1) and (3), we get, $\frac{I_1}{I_2} = \frac{3+x}{6+x}$ … (iv)

Proceeding to the limit $x \to 0$ without $i_{ab} \to \infty$, from (4), we get

$$\frac{I_1}{I_2} = \frac{1}{2} \text{ or } I_2 = 2I_1$$

Substituting in (2), we get

$I_1 = 3$ A and $I_2 = 6$ A.

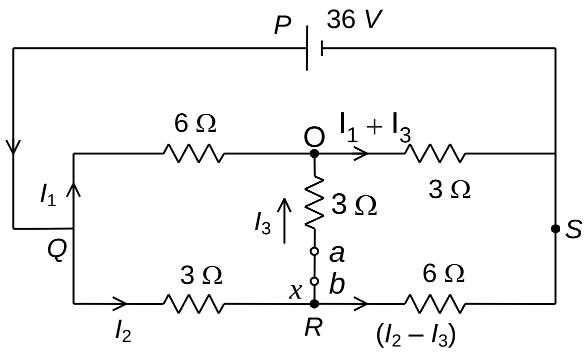
Hence the current through the switch $i_{ab} = I_1 - I_2 = -3$ A

The current flows in the switch from b to a.

(c) In figure (b) we have a resistance of 3 Ω added to the switch circuit. However this will NOT affect the current and potential distributions when the switch S is open.

Hence the potential difference $V_{ab} = -12$ V (as in the case (a) above).

(d) When the switch S is closed, the currents and potentials will redistribute to new values. Let the currents be I_1, I_2 and I_3 as shown.



For the loop QROQ,

$6I_1 - (3 + x)I_3 - 3I_2 = 0$ … (i)

For the loop PQOSP,

$36 = 6I_1 + 3(I_1 + I_3) = 9I_1 + 3I_3$

or $3I_1 + I_3 = 12$ … (ii)

For the loop QRSOQ,

$3I_2 + 6(I_2 - I_3) - 3(I_1 + I_3) - 6I_1 = 0$

or $9I_1 - 9I_2 + 9I_3 = 0$

or $I_1 - I_2 + I_3 = 0$ … (iii)

Solving (1), (2) and (3) for I_1, I_2 and I_3, we get

$$I_1 = \frac{24}{7} = 3.43 \text{ A}, \qquad I_2 = \frac{36}{7} = 5.14 \text{ A}$$

and $$I_3 = \frac{12}{7} = 1.71 \text{ A}$$

Hence the current that flows through the switch when it is closed = 1.71 A (flowing from b to a).

(e) When the switch S is open in circuit diagram (b) the total current in the circuit = the same total current as in circuit diagram (a) with switch S open. Hence equivalent resistance is the same as in case (a) above, and

$$R_{eq} = \frac{9 \times 9}{9 + 9} = \frac{81}{18} = 4.5\ \Omega$$

When the switch S is closed in diagram (b), the total current drawn from the battery is

$$I = I_1 + I_2 = \frac{24}{7} + \frac{36}{7} = \frac{60}{7} \text{ amp.}$$

Hence the equivalent resistance in the circuit is

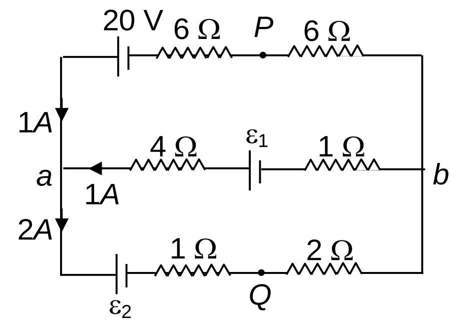
$$R_{eq} = \frac{E}{I} = \frac{36}{60/7} = \frac{21}{5} = 4.2\ \Omega$$

Illustration 21.

(a) Find the emfs ε_1 and ε_2 in the circuit of the following diagram and the potential difference between the points a and b.

(b) If in the above circuit, the polarity of the battery ε_1, be reversed, what will be the potential difference between a and b?

Solution:



(a) It is clear that 1 A current flows in the circuit from b to a.

Applying Kirchhoff's law to the loop PabP,

$$20 - E_1 = 6 + 1 - 4 - 1 = 2$$

Hence $E_1 = 18$ V

Also applying Kirchhoff's law to the loop PaQbP,

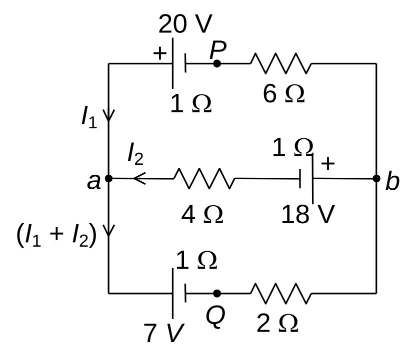
$$20 - E_2 = 6 + 1 + (1 \times 2) + (2 \times 2) = 13$$

Hence $E_2 = 7$ V

Thus the potential difference between the points a and b is

$$V_{ab} = 18 - 1 - 4 = 13 \text{ V}$$

(b) On reversing the polarity of the battery E_1, the current distributions will be changed.



Let the currents be I_1 and I_2 as shown.

Applying Kirchhoff's law for the loop PabP,

$$20 + E_1 = (6 + 1)\, I_1 - (4 + 1)\, I_2$$

Or $38 = 7I_1 - 5I_2$ …. (i)

Similarly for the loop abQa,

$$4I_2 + I_2 + 18 + 2(I_1 + I_2) + (I_1 + I_2) + 7 = 0$$

or $3I_1 + 8I_2 = -25$ …. (ii)

Solving (1) and (2) for I_1 and I_2, we get

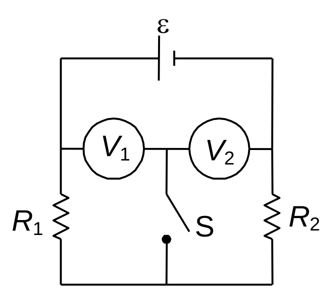
$$I_1 = 2.52 \text{ and } I_2 = -4.07 \text{ A}$$

Hence, $V_{ab} = -5 \times (4.07) + 18$

$$= -20.35 + 18 = -2.35 \text{ V}$$

Illustration 22.

In the circuit V_1 and V_2 are two voltmeters of resistances 3000 ohm and 2000 ohm respectively. The resistances R_1 = 2000 ohm and R_2 = 3000 ohm and the emf of the battery ε = 200 V. The battery has negligible internal resistance. Find the readings of the voltmeters V_1 and V_2 when



(i) the switch S is open and

(ii) the switch S is closed.

Solution:

(i) When S is open

V_1 and V_2 in series have a resistance

$= 3000 + 2000 = 5000\ \Omega$

R_1 and R_2 in series have a resistance

$= 2000 + 3000 = 5000\ \Omega$

5000 ohm and 5000 ohm in parallel are equivalent to

$$\frac{5000 \times 5000}{10000} = 2500\ \Omega$$

Circuit current = $\frac{200}{2500} = \frac{2}{25} A$

Current in the branch of V_1 and V_2

$$= \frac{1}{2}\left(\frac{2}{25}\right) = \frac{1}{25} A$$

p.d. across $V_1 = \left(\frac{1}{25} A\right)$ (3000 ohm) = 120 V

p.d. across $V_1 = \left(\frac{1}{25} A\right)$ (2000 ohm) = 80 V

$\therefore$ the voltmeters V_1 and V_2 read 120 V and 80 V respectively.

Similarly V_2 and R_2 in parallel have an equivalent resistance of 1200 ohm.

As these two equivalent resistances are same

p.d. across AS = p.d. across SB

$\therefore$ p.d. across AS = 100 V

This is registered by V_1 SB

Similarly p.d. across = 100 V

This is registered by V_1

Similarly p.d. across SB = 100 V

This is registered by V_2.

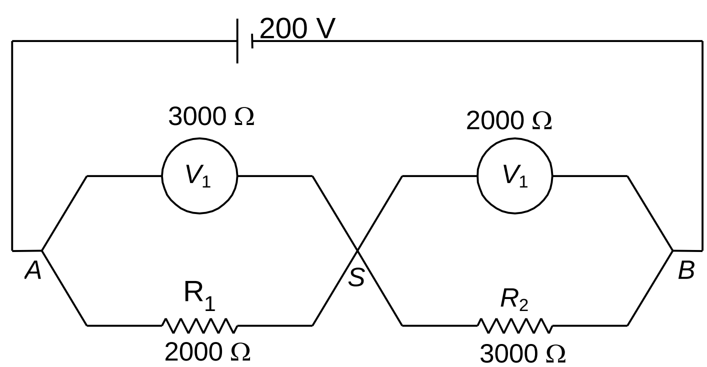


Illustration 22.

A fuse made of lead wire has an area of cross-section 0.2 mm^2. On short circuiting, the current in the fuse wire reaches 30 amp. How long after the short circuiting will the fuse begin to melt?

Specific heat capacity of lead = 134.4 J kg^{-1} K^{-1}.

Melting point of lead = 327°C

Density of lead = 11340 kg/m^3

Resistivity of lead = 22×10^{-8} ohm-m

Initial temperature of the wire= 20°C

Neglect heat loss.

Solution:

If L be the length of the wire, its resistance

$$R = \frac{\rho L}{A} = \frac{\left(22\times10^{-8}\right)L}{\left(0.2\times10^{-6}\right)m^2}$$

Heat produced in the wire in one second = $I^2R = (30)^2$ R J

Heat required to raise the temperature of the wire to 327°C

$$Q = ms\Delta T$$

$$= (LAd)\ (134.4)\ (307)\ J$$

Time required to melt the wire

$$= \frac{Q}{I^2R} = \frac{LAd\times134.4\times307}{I^2\times\rho L}\times A$$

$$= \frac{A^2}{I^2}\cdot\frac{d}{\rho}\times134.4\times307$$

$$= \frac{\left(0.2\times10^{-6}\right)^2}{900}\times\frac{11340}{22\times10^{-8}}\times134.4\times307$$

$$= 0.0945\ s$$

Illustration 23.

An electric kettle has two heating coils. When one of the coils is switched on, the kettle begins to boil in 6 minutes and when the other is switched on, the boiling begins in 8 minutes. In what time will the boiling begin if both coils are switched on simultaneously (i) in series and (ii) in parallel?

Solution:

Let the resistance of the two coils be R_1 and R_2 respectively. Let the supply voltage be V. Let Q be the heat required to boil the kettle.

Using the first coil, let t_1 be the time taken. Now

$$Q = \frac{V^2}{R_1}\times t_1;\quad t_1 = 6 \text{ minutes} \qquad \dots (i)$$

Using the second coil, let t_2 be the time taken

$$Q = \frac{V^2}{R_2}\times t_2;\ t_2 = 8 \text{ minutes} \qquad \dots (ii)$$

Now $\quad \dfrac{V^2}{R_1}\times t_1 = \dfrac{V^2}{R_2}\times t_2$

or $$\frac{R_1}{R_2}=\frac{t_1}{t_2}=\frac{6}{8}=\frac{3}{4}$$

$$R_2=\frac{4}{3}R_1 \qquad \ldots \text{(iii)}$$

(i) When the two coils are in series to the supply

$$Q = \frac{V^2}{R_1+R_2}\times T_1=\frac{V^2}{R_1+\frac{4}{3}R_1}T_1=\frac{V^2}{\frac{7}{3}R_1}T_1 \qquad \ldots \text{(iv)}$$

where T_1 is the time taken.

From (1) and (4), we get $\frac{V^2}{R_1}\times 6\times 60=\frac{V^2}{R_1}\times\frac{3}{7}T_1$

$$6\times 60 = \frac{3}{7}T_1$$

$$T_1 = \frac{6\times60\times7}{3}s$$

$$= 14 \text{ minutes}$$

(ii) When the two coils are in parallel connection the equivalent resistance

$$= \frac{R_1R_2}{R_1+R_2}=\frac{\frac{4}{3}R_1^2}{\frac{7}{3}R_1}=\frac{4}{7}R_1$$

$\therefore$ heat developed in T_2 is

$$Q = \frac{V^2}{\frac{4}{7}R_1}\cdot T_2 \qquad \ldots \text{(v)}$$

From (i) and (v)

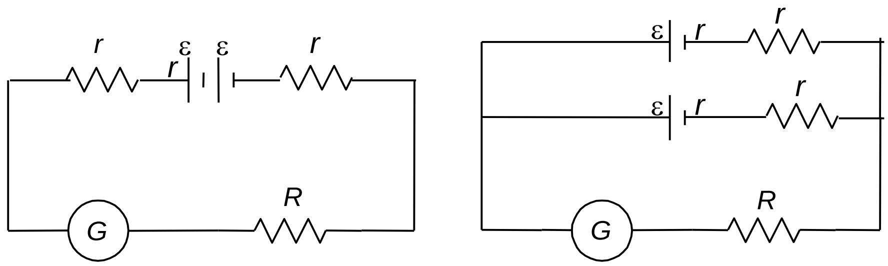
$$\frac{7}{4}\frac{V^2}{R_1}\cdot T_2=\frac{V^2}{R_1}\times t_1$$

$$T_2=t_1\times\frac{4}{7}=\frac{24}{7} \text{ minutes} = \mathbf{3\frac{3}{7}min}$$

Illustration 24.

A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1.5 V. When the batteries are connected in series the galvanometer records a current of 1 ampere and when the batteries are in parallel the current is 0.6 ampere. What is the internal resistance of the battery?

Solution:



Emf of each cell is ε = 1.5 V

The internal resistance of each cell be r.

Let the resistance of the galvanometer be G.

Let the unknown resistance in series with the galvanometer be R.

(i) Let the cells be in series.

The emf of the circuit = 2ε = 3 V

The resistance of the circuit = (R + G + 2r) Ω

The current in the circuit $\frac{3}{R+G+2r} = 1A$ (given) … (i)

(ii) When the cells are in parallel the emf of the circuit = ε = 1.5 V

The resistance of the circuit = $R + G + \frac{r \cdot r}{r+r}$

$$= \left(R + G + \frac{r}{2}\right)\Omega$$

The current in the circuit $\frac{1.5}{R+G+\frac{r}{2}} = 0.6A$ … (ii)

From equations (1) and (2), we get

R + G + 2r = 3

and $R+G+\frac{r}{2} = \frac{1.5}{0.6} = 2.5$

Subtracting these two equations, we get

$\frac{3}{2}r = 0.5 \qquad \Rightarrow r = \frac{1}{3}\Omega$

Internal resistance of each cell = $\frac{1}{3}\Omega$

Illustration 25.

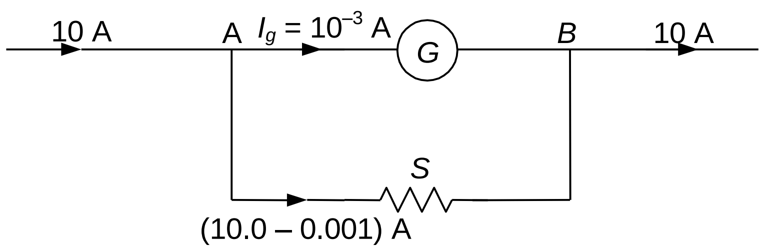
(i) A galvanometer having a coil of resistance of 100 ohms gives a full scale deflection when a current of one milliampere is passed through it. What is the value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 amperes?

(ii) A resistance of the required value is available but it will get burnt if the energy dissipated in it is greater than 1 watt. Can it be used for the above described conversion of the galvanometer?

(iii) When this modified galvanometer is connected across the terminals of a battery it shows a current of 4 ampere. The current drops to 1 ampere when a resistance of 1.5 ohm is connected in series with the modified galvanometer. Find the emf and the internal resistance of the battery.

Solution:

(i) The value of shunt resistance.



Let the shunt resistance required be S Ω. The galvanometer permits the full-scale deflection current of $I_g = 1 \times 10^{-3}$ A through it when the circuit is 10 A.

Then, $(10 - 0.001)\,S = 0.001 \times 100$

$$S = \frac{0.1}{\frac{9999}{1000}} = \frac{100}{9999} \approx \frac{\mathbf{1}}{\mathbf{100}}\,\Omega$$

(ii) Power dissipated by the shunt = i^2R

$$= (9999 \times 10^{-3})^2 \times \frac{100}{9999}$$

$$= 9999 \times 10^{-6} \times 100$$

$$= 0.9999 \text{ W}$$

This is less than the maximum power of 1 W, which the resistor can dissipate. Hence the resistance can be safely used.

(iii) Emf of the battery and its internal resistance: The combined resistance of the galvanometer and the shunt is given by $\dfrac{\frac{1}{100} \times 100}{\frac{1}{100} + 100}\,\Omega \approx \dfrac{1}{100}\,\Omega$

This combined resistance and the internal resistance of the battery in series give a total resistance of $\left(\frac{1}{100} + r\right)$ ohm to the circuit.

If ε be the emf of the battery, then

$$\frac{\varepsilon}{r+0.01} = 4A \qquad \ldots \text{(i)}$$

Now with an additional resistance of 1.5 Ω in series

$$\frac{\varepsilon}{r+0.01+1.5} = 1A \qquad \ldots \text{(ii)}$$

From equations (1) and (2), we get $\frac{r+1.51}{r+0.01} = 4$

3r = 1.47

Internal resistance r = 0.49 Ω

Substituting this value of r in equation (1),

ε = 4 × 0.5 = 2 V