1. THE MAGNETIC FIELD

In earlier lessons we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounds an electric charge. The region of space surrounding a moving charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic substance. In order to describe any type of field, we must define its magnitude, or strength, and its direction.

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is represented by $\vec{B}$.

Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of lines per unit area crossing a small area perpendicular to the direction of the induction being numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred as the magnetic flux linked with that area. For this reason $\vec{B}$ is also called magnetic flux density.

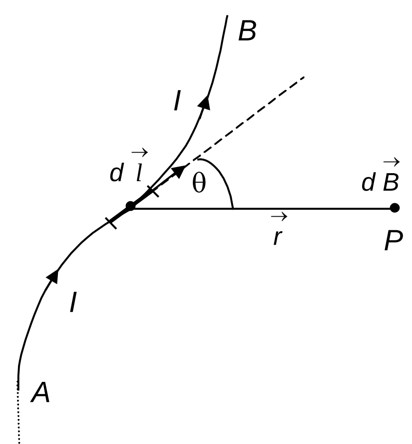
There are two methods of calculating magnetic field at some point. One is **Biot Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and another is **Ampere's law,** which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

The unit of magnetic field is weber / m^2 and is known as tesla (T) in the SI system.

2. BIOT-SAVART LAW

Biot- Savart law gives the magnetic induction due to an infinitesimal current element.

Let *AB* be a conductor of an arbitrary shape carrying a current *I*, and *P* be a point in vacuum at which the field is to be determined. Let us divide the conductor into infinitesimal current–elements. Let $\vec{r}$ be a displacement vector from the element to the point *P*.



According to 'Biot-Savart Law', the magnetic field induction $d\vec{B}$ at *P* due to the current element $d\vec{l}$ is given by

$$d\vec{B} \propto \frac{I(d\vec{l}\times\vec{r})}{r^3}$$

or,
$$d\vec{B} = k\frac{I(d\vec{l}\times\vec{r})}{r^3},$$

where *k* is a constant of proportionality. Here $d\vec{l}$ vector points in the direction of current *I*.

In S.I units, $k = \frac{\mu_0}{4\pi} = 10^{-7} \frac{\text{Wb}}{\text{amp} \times \text{metre}}$

$$\therefore \qquad d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3} \qquad \ldots (\mathbf{1})$$

Equation (1) is the vector form of the Biot Savart Law. The magnitude of the field induction at *P* is given by

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2},$$

where θ is the angle between $d\vec{l}$ and $\vec{r}$.

If the medium is other than air or vacuum, the magnetic induction is

$$d\vec{B} = \frac{\mu_0 \mu_r}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3} \qquad \ldots (\mathbf{2})$$

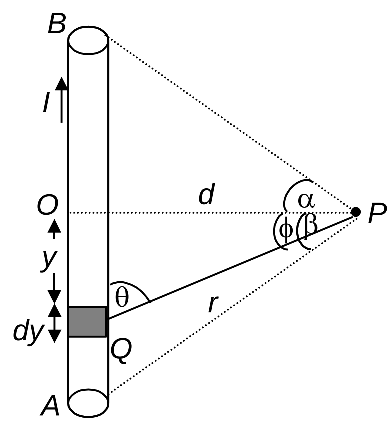
where μ_r is relative permeability of the medium and is a dimensionless quantity.

3. FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

3.1 WHEN THE WIRE IS OF FINITE LENGTH

Consider a straight wire segment carrying a current *I* and there is a point *P* at which magnetic field is to be calculated is as shown in the figure. This wire segment makes angle α and β at that point. Consider an element of length *dy* at a distance *y* from *O* and distance of this element from point *P* is *r* and line joining *P* to *Q* makes an angle θ with the direction of current as shown in figure. Using Biot-Savart Law magnetic field at point *P* due to small current element is given by

$$dB = \frac{\mu_0 I}{4\pi} \left(\frac{dy \sin\theta}{r^2} \right)$$



As every element of the wire contributes to $\vec{B}$ in the same direction, we have

$$B = \frac{\mu_0 I}{4\pi} \int_A^B \frac{dy \sin\theta}{r^2} \qquad \ldots (\text{i})$$

From the triangle *OPQ* as shown in diagram, we have

$$y = d \tan\phi$$

or, $\quad dy = d \sec^2\phi d\phi$

Moreover, in same triangle,

$r = d \sec\phi$ and $\theta = (90° - \phi)$, where ϕ is angle between line *OP* and *PQ*

Now equation (i) can be written in this form

$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{I}{d}\int_{-\beta}^{\propto}\cos\phi\, d\phi$$

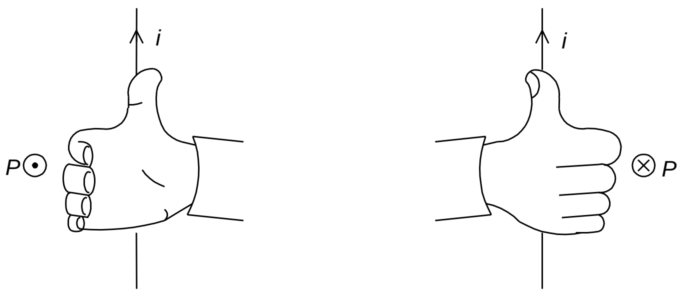
$$\text{or,} \qquad B = \frac{\mu_0}{4\pi}\frac{I}{d}\int_{-\beta}^{\alpha}\cos\phi\, d\phi$$

$$\text{or,} \qquad B = \frac{\mu_0}{4\pi}\frac{I}{d}\,[\sin\alpha + \sin\beta] \qquad \ldots (\mathbf{3})$$

Direction of $\vec{B}$: The direction of magnetic field is determined by the cross product of the vector $i d\vec{l}$ with $\vec{r}$. Therefore, at point *P*, the direction of the magnetic field due to the whole conductor will be perpendicular to the plane of paper and going into the plane.

Right-hand Thumb Rule: The direction of *B* at a point *P* due to a long, straight wire can be found by the right-hand thumb rule. The direction of magnetic field is perpendicular to the plane containing wire and perpendicular from the point. The orientation of magnetic field is given by the direction of curl fingers if we stretch thumb along the wire in the direction of current. Refer figure.

Conventionally, the direction of the field perpendicular to the plane of the paper is represented by ⊗ if into the page and by ⊙ if out of the page.



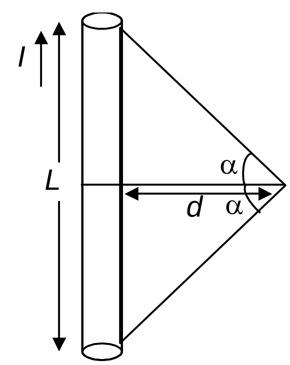
Now consider some special cases involving the application of equation (3)

Case I: When the point *P* is on perpendicular bisector

In this case angle $\alpha = \beta$, using result of equation (3), the magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}\sin\alpha$$

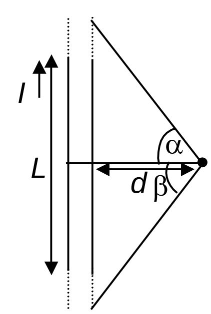
where $\sin\alpha = \dfrac{L}{\sqrt{L^2 + 4d^2}}$



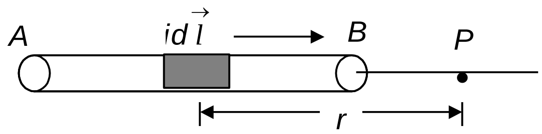
Case II: When wire is of infinite length

In this case $\alpha = \beta = 90°$, using result of equation (3), the magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$



Case III: When the point P lies along the length of wire (but not on it):



If the point is along the length of the wire (but not on it), then as $d\vec{l}$ and $\vec{r}$ will either be parallel or antiparallel, i.e., $\theta = 0$ or π, so $id\vec{l} \times \vec{r} = 0$ and hence using equation (1)

$$\vec{B} = \int_A^B d\vec{B} = 0$$

Illustration 1.

Calculate the magnetic field induction at a point distance, $\frac{a\sqrt{3}}{2}$ metre from a straight wire of length 'a' metre carrying a current of I amp. The point is on the perpendicular bisector of the wire.

Solution:

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}[\sin\alpha + \sin\beta]$$

$$= 10^{-7}\left[\frac{I}{(a\sqrt{3}/2}\left(\frac{1}{2}+\frac{1}{2}\right)\right] = \frac{2I}{\sqrt{3a}} \times 10^{-7}\ \text{T},$$

Perpendicular to the plane of figure (inward).

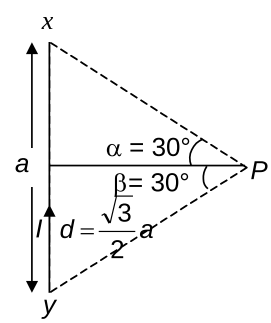
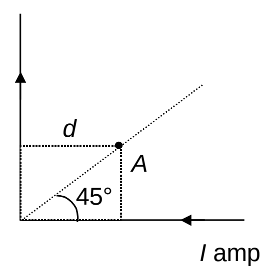


Illustration 2.

A long straight conductor is bent at an angle of 90° as shown in the figure. Calculate the magnetic field induction at A.



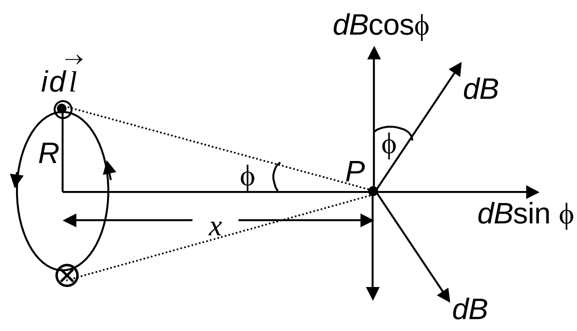
Solution:

For each portion, $\beta = 45°$ and $\alpha = 90°$

$\therefore$ total field at $A = \frac{\mu_0}{4\pi}\left[\frac{2I}{d}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\right]$

4. MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

Consider a circular loop of radius R and carrying a steady current I. We have to find out magnetic field at the axial point P, which is at distance x form the centre of the loop.



Consider an element $Id\vec{l}$ of the loop as shown in figure, and the distance of point P from current element is r. The magnetic field at P due to this current element from the equation (1) can be given by,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

In case of point on the axis of a circular coil, as for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the axis cancel each-other while along the axis add up.

$\therefore$ $$B = \int dB\sin\phi = \frac{\mu_0}{4\pi}\int\frac{Idl\sin\theta}{r^2}\sin\phi$$

Here, θ is angle between the current element $Id\vec{l}$ and $\vec{r}$, which is $\frac{\pi}{2}$ everywhere and $\sin\phi = \frac{R}{r} = \frac{R}{\sqrt{R^2+x^2}}$

$\therefore$ $$B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+x^2)^{3/2}}\int_0^{2\pi R} dL$$

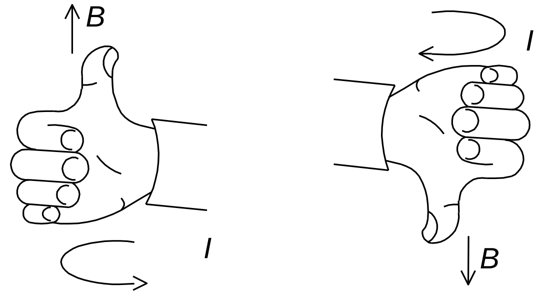
or, $$B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+x^2)^{3/2}}(2\pi R)$$

or, $$B = \frac{\mu_0}{4\pi}\frac{2\pi IR^2}{(R^2+x^2)^{3/2}} \qquad \ldots (4)$$

If the coil has N turns, then

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{(R^2+x^2)^{3/2}}$$

Direction of $\vec{B}$: Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained by using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.



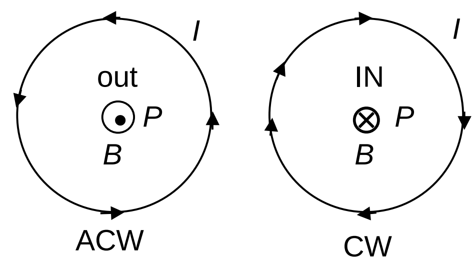
Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure given.

Now consider some special cases involving the application of equation (4)

Case I: Field at the centre of the coil

In this case distance of the point P from the centre $(x) = 0$, the magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0}{2}\frac{I}{R}$$



Case II: Field at a point far away from the centre

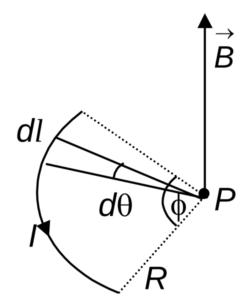
It means $x >> R$, $B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{x^3}$

5. FIELD AT THE CENTRE OF A CURRENT ARC

Consider an arc of radius R carrying current I and subtends an angle ϕ at the centre.

According to Biot-Savart Law, the magnetic field induction at the point P is given by

$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{Idl}{R^2} \qquad \text{Here,} \quad dl = Rd\theta$$



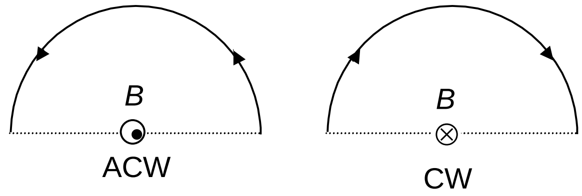
$$\therefore \qquad B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{IRd\theta}{R^2}$$

or, $$B = \frac{\mu_0}{4\pi}\frac{I\phi}{R} \qquad \ldots (\mathbf{5})$$

If 'l' is the length of the circular arc, we have

$$B = \frac{\mu_0}{4\pi}\frac{Il}{R^2} \qquad \ldots (\mathbf{6})$$

Consider some special cases involving the application of equation (5)

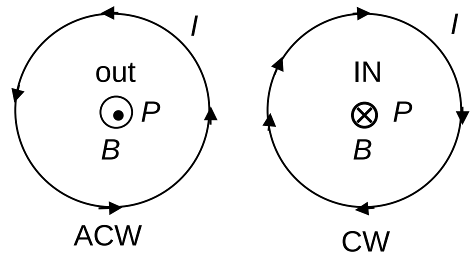


Case I: If the loop is the semi-circular,

In this case $\phi = \pi$, so

$$B = \frac{\mu_0}{4\pi}\frac{\pi I}{R}$$

and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.



Case II: If the loop is a full circle with *N* turns,

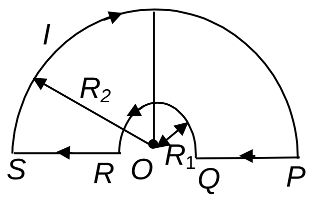
In this case $\phi = 2\pi$ so,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{R}$$

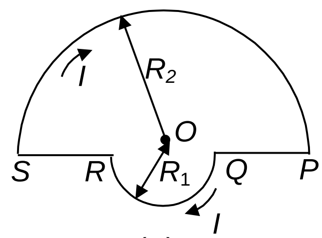
and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.

Illustration 3.

The wire loop *PQRSP* formed by joining two semi-circular wires of radii R_1 and R_2 carries a current I as shown in the figure given below. What is the magnetic induction at the centre O in cases (A) and (B)



(A)



(B)

Solution:

(a) As the point O is along the length of the straight wires, so the field at O due to them will be zero and hence magnetic field is only due to semicircular portions.

$$\therefore \ |\vec{B}| = \frac{\mu_0}{4\pi}\pi I \left[\frac{\pi I}{R_2}\otimes + \frac{\pi I}{R_1}\odot\right]$$

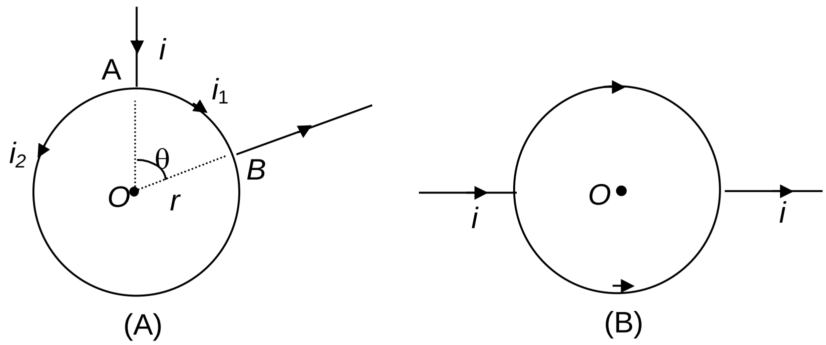
$$\text{or, } |\vec{B}| = \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_2}-\frac{1}{R_1}\right] \text{ out of the page}$$

(b) $$|\vec{B}| = \frac{\mu_0}{4\pi} I\,\pi \left[\frac{1}{R_1}+\frac{1}{R_2}\right] \text{in to the page}$$

$$\vec{M} = \frac{1}{2}\boldsymbol{\pi I\,[R_2^2 + R_1^2]}\ \textbf{into the page}.$$

Illustration 4

A battery is connected between two points *A* and *B* on the circumference of a uniform conducting ring of radius *r* and resistance *R* as shown in the figure given below. One of the arcs *AB* of the ring subtends an angle θ at the centre. What is the value of the magnetic field at the centre due to the current in the ring?



Solution:

As the field due to arc at the centre is given by

$$B = \frac{\mu_0}{4\pi}\frac{I\phi}{r}$$

$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{I_1\theta}{r}\otimes + \frac{\mu_0}{4\pi}\frac{I_2(2\pi-\theta)}{r}\odot$$

But $(V_A - V_B) = I_1R_1 = I_2R_2$

or, $I_2 = I_1\dfrac{R_1}{R_2} = I_1\dfrac{L_1}{L_2}$ $[\because R \propto L]$

$$I_2 = I_1\frac{\theta}{(2\pi-\theta)} \quad [\because L = r\theta]$$

$$\therefore \qquad B_R = \frac{\mu_0}{4\pi}\frac{I_1\theta}{r}\otimes + \frac{\mu_0}{4\pi}\frac{I_1\theta}{r}\odot = \mathbf{0}$$

i.e., **the field at the centre of the coil is zero and is independent of θ.**

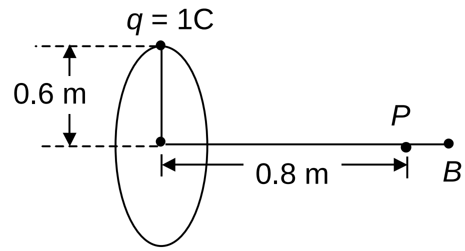
Illustration 5.

A charge of one Coulomb is placed at one end of a non-conducting rod of length 0.6 m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency $10^4\pi$ rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path.

Now half of the charge is removed from one end and placed on the other end. The rod is rotated in a vertical plane about horizontal axis passing through the mid-point of the rod with the same angular frequency. Calculate the magnetic field at a point on the axis at a distance of 0.4 m from the centre of the rod.

Solution:

As the revolving charge q is equivalent to a current

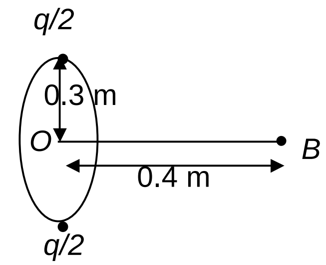


$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4\pi}{2\pi} = 5 \times 10^3 \text{ A}$$

Now $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi IR^2}{(R^2+x^2)^{3/2}}$

$$\therefore \quad B = 10^{-7} \times \frac{2\pi \times 1 \times 5 \times 10^3\,(.6)^2}{[(.6)^2+(.8)^2]^{3/2}} = \mathbf{1.13 \times 10^{-3}\ T}$$

If half of the charge is placed at the other end and the rod is rotated at the same frequency



$$I' = \left(\frac{q}{2}\right)f + \left(\frac{q}{2}\right)f = qf = I = 5 \times 10^3 \text{ A}$$

In this case, $R' = 0.3$ m and $x' = 4$m

$$\therefore \qquad B' = 10^{-7} \times \frac{2\pi \times 1 \times 5 \times 10^3 \times (.3)^2}{[(.3)^2+(.4)^2]^{3/2}} = \mathbf{2.\,3 \times 10^{-3}\ T}$$

6. AMPERE'S LAW

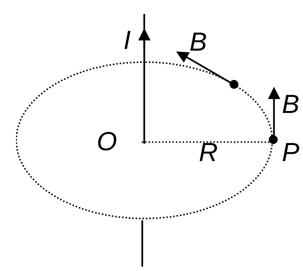
This law is useful in finding the magnetic field due to currents under certain conditions of symmetry. Consider a closed plane curve enclosing some current-carrying conductors.

The line integral $\oint \vec{B} \cdot \vec{dl}$ taken along this closed curve is equal to μ_0 times the total current crossing the area bounded by the curve.

i.e., $$\oint \vec{B}.\,d\vec{l} = \mu_0 I \qquad \dots (7)$$

where I = total current (algebraic sum) crossing the area.

As a simple application of this law, we can derive the magnetic induction due to a long straight wire carrying current I.



Suppose the magnetic induction at point P, distant R from the wire is required.

Draw the circle through P with centre O and radius R as shown in figure.

The magnetic induction $|\vec{B}|$ at all points along this circle will be the same and will be tangential to the circle, which is also the direction of the length element $\vec{dl}$.

Thus, $\oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\oint dl = B \times 2\pi R$

The current crossing the circular area is I.

Thus, by Ampere's law, $B \times 2\pi R = \mu_0 I$

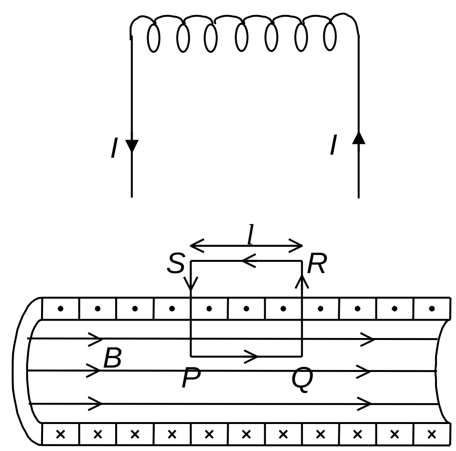
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R} \qquad \ldots \textbf{(8)}$$

7. THE MAGNETIC FIELD OF A SOLENOID

7.1 MAGNETIC FIELD INSIDE A LONG SOLENOID

A solenoid is a wire wound closely in the form of a helix, such that the adjacent turns are electrically insulated.

The magnetic field inside a very tightly wound long solenoid is uniform everywhere along the axis of the solenoid and is zero outside it.



To calculate the magnetic field at a point P inside the solenoid, let us draw a rectangle $PQRS$ as shown in figure. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid.

$$\Rightarrow \quad \int_P^Q \vec{B}\cdot\vec{dl} = Bl$$

On the remaining three sides, $\vec{B}\cdot\vec{dl}$ is zero everywhere as $\vec{B}$ is either zero (outside the solenoid) or perpendicular to $\vec{dl}$ (inside the solenoid).

$\Rightarrow$ the circulation of $\vec{B}$ along $PQRS$ is

$$\oint \vec{B}\cdot\vec{dl} = Bl$$

If n is the number of turns per unit length along the length of solenoid, a total of nl turns cross the rectangle $PQRS$. Each turn carries a current I.

$\Rightarrow$ Net current crossing $PQRS = nlI$

Using Ampere's law,

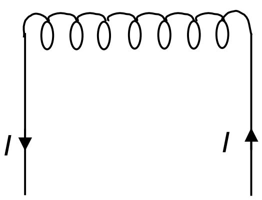
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 nIl$$

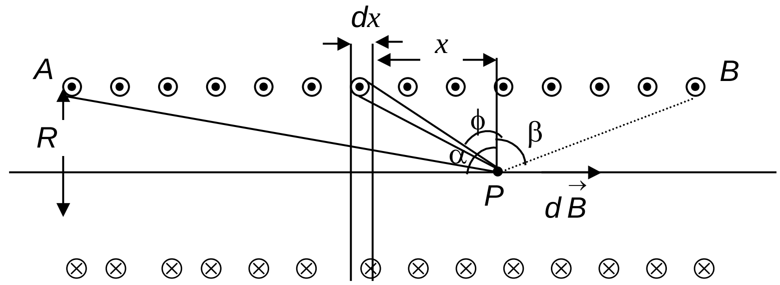
$\Rightarrow \quad Bl = \mu_0\, nIl$

$\Rightarrow \quad B = \mu_0 nI \qquad \ldots \textbf{(9)}$

7.2 MAGNETIC FIELD AT A POINT ON THE AXIS OF A SHORT SOLENOID

Consider a solenoid of length l and radius r containing N closely spaced turns and carrying a steady current I. We have to find out an expression for the magnetic field at an axial point P lying in the space enclosed by the solenoid as shown in the figure below.





The field at a point on the axis of a solenoid can be obtained by the superposition of fields due to a large number of identical coils all having their centre on the axis of the solenoid.

Let us consider a coil of width dx at a distance x from the point P on the axis of the solenoid as shown in the above diagram.

The field at P due to this coil is given by

$$dB = \frac{\mu_0}{4\pi}\,\frac{2\pi dNIR^2}{(R^2+x^2)^{3/2}}$$

If n be the number of turns per unit length, $dN = ndx$.

From the above figure,

$x = R\tan\phi$

or, $\quad dx = R\sec^2\phi\, d\phi$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\,\frac{2\pi(ndx)IR^2}{(R^2+R^2\tan^2\phi)^{3/2}} = \frac{\mu_0}{4\pi}(2\pi nI)\cos\phi\, d\phi$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(2\pi\, nI)\int_{-\alpha}^{\beta}\cos\phi\, d\phi$$

$$\text{or,} \quad B = \frac{\mu_0}{4\pi}(2\pi nI)[\sin\alpha + \sin\beta] \qquad \ldots \textbf{(10)}$$

Now consider some cases involving the application of equation (10)

Case I: If the solenoid is of infinite length and the point is well inside the solenoid,

In this case, $\alpha = \beta = \dfrac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(2\pi nI)[1+1] = \mu_0 nI$$

Case II: If the solenoid is of infinite length and the point is near one end

In this case, $\alpha = 0$ and $\beta = \frac{\pi}{2}$

$$\therefore \qquad B = \frac{\mu_0}{4\pi}(2\pi nI)[1+0] = \frac{1}{2}(\mu_0 nI)$$

Case III: If the solenoid is of finite length and the point is on the perpendicular bisector of its axis

In this case, $\alpha = \beta$

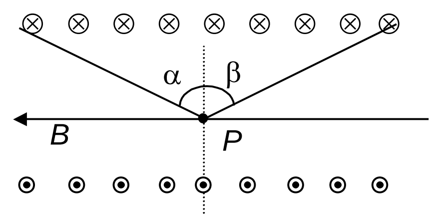
$$\therefore \qquad B = \frac{\mu_0}{4\pi}(4\pi nI)\sin\alpha, \qquad \text{where, } \sin\alpha = \frac{L}{\sqrt{L^2+4R^2}}$$

Illustration 6.

A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5×10^{-3} ampere. Calculate the magnetic field on the axis at the middle and at the end of the solenoid.

Solution:

In case of a solenoid, the field at point on the axis as shown in the above diagram is given by



$$B = \frac{\mu_0}{4\pi}(2\pi nI)[\sin\alpha + \sin\beta]$$

Here, $n = \frac{N}{L} = \frac{1000}{0.4} = 2.5 \times 10^3$ turns/m

$\therefore\ B = 10^{-7} \times 2\pi \times 2.5 \times 10^3 \times 5 \times 10^{-3} (\sin\alpha + \sin\beta)$

or, $B = 2.5\pi \times 10^{-6}[\sin\alpha + \sin\beta]$

(a) When the point is at the middle on the axis, $\alpha = \beta$ with

$$\sin\alpha = \frac{L}{\sqrt{L^2+4r^2}} = \frac{.4}{\sqrt{(.4)^2+(.3)^2}} = \frac{4}{7.2}$$

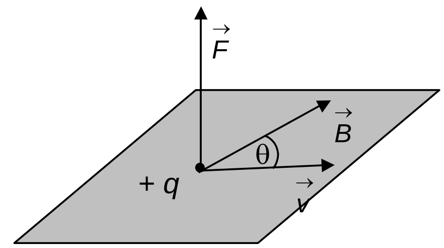
$\therefore \qquad B = 2.5\pi \times 10^{-6}[\sin\alpha + \sin\beta]$ = $\mathbf{8.72 \times 10^{-6}}$ **T**

(b) When the point is at the end on the axis, $\alpha = 0$

with $\sin\beta = \frac{L}{\sqrt{L^2+r^2}} = \frac{0.4}{\sqrt{(.4)^2+(.3)^2}} = \frac{4}{5}$

$\therefore \qquad B = 2.5\pi \times 10^{-6} \times \frac{4}{5}$ = $\mathbf{6.28 \times 10^{-6}}$ **T**

8. FORCE ON A CHARGED PARTICLE IN A MAGNETIC FIELD

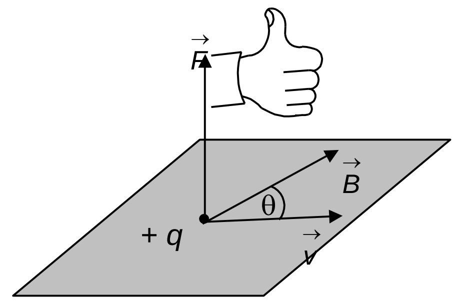


When a charge q moves with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ as shown in the figure, it experiences a magnetic force $\vec{F}$ given by

$$\vec{F} = q\,(\vec{v} \times \vec{B}) \qquad \ldots (11)$$

$$|\vec{F}| = qvB\sin\theta$$

where θ is the angle between velocity $\vec{v}$ and magnetic field $\vec{B}$. (smaller angle)



The force is directed at right angle to the plane containing the vectors $\vec{v}$ and $\vec{B}$.

The right hand palm rule: For determining the direction of the cross product $\vec{v} \times \vec{B}$, you point the fore fingers of your right hand along the direction of $\vec{v}$, and palm in the direction of magnetic field $\vec{B}$ then curl the fingers. The thumb then points in the direction of $\vec{v} \times \vec{B}$.

Since $F = q\vec{v} \times \vec{B}$, $\vec{F}$ is in the direction of $\vec{v} \times \vec{B}$ if q is positive and opposite the to direction of $\vec{v} \times \vec{B}$ if q is negative.

Some important points

(1) The magnetic force will be maximum when sinθ = 1

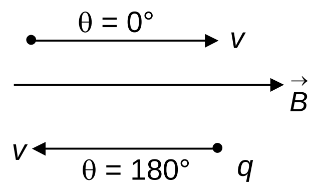
$\Rightarrow$ θ = 90°, i.e., the charge is moving perpendicular to the field.

$F_{max} = qvB$

In this situations $\vec{F}$, $\vec{v}$ and $\vec{B}$ are mutually perpendicular to each other.

(2) The magnetic force will be minimum when sinθ = 0, i.e., θ = 0° or 180°. It means the charge is moving parallel to the field.

Thus, $F_{min} = 0$



(3) In case of motion of charged particle in a magnetic field, as the force is always perpendicular to motion,

$$W = \int \vec{F}.d\vec{s} = \int Fds\cos 90° = 0$$

So work done by the force due to magnetic field on a moving charged particle is always zero. According to work-energy theorem,

$W = \Delta$ KE, so the kinetic energy will not change and hence the speed v will remain constant. However, in this situation, the force changes the direction of the motion. Therefore, the velocity $\vec{v}$ of the charged particle changes .

8.1 DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE

(1) Magnetic force is always perpendicular to the field while electric force is collinear with the field.

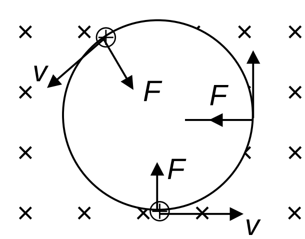
(2) Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion while electric force (qE) is independent of the state of rest or motion of the charged particle.

(3) Magnetic force does no work when the charged particle is displaced while the electric force does work in displacing the charged particle.

(4) Magnetic force is always non-central while the electric force may or may not be.

9. MOTION OF CHARGED PARTICLE IN UNIFORM MAGNETIC FIELD

9.1 WHEN THE CHARGED PARTICLE IS GIVEN VELOCITY PERPENDICULAR TO THE FIELD



Let a particle of charge q and mass m is moving with a velocity v and enters at right angles to a uniform magnetic field $\vec{B}$ as shown in figure.

The force on the particle is qvB and this force will always act in a direction perpendicular to v. Hence, the particle will move on a circular path. If the radius of the path is r then

$$\frac{mv^2}{r} = Bqv \quad \text{or,} \quad r = \frac{mv}{qB} \qquad \ldots \textbf{(12)}$$

Thus, radius of the path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of magnetic field.

Time period: is the time taken by the charge particle to complete one rotation of the circular path which is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \qquad \ldots \textbf{(13)}$$

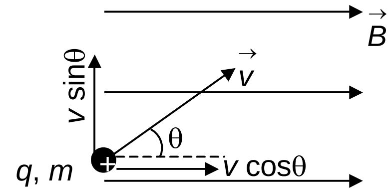
The time period is independent of the speed v.

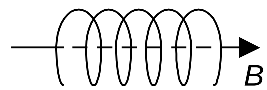
Frequency: The frequency is number of revolution of charged particle in one second, which is given by

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \ldots \textbf{(14)}$$

9.2 WHEN THE CHARGED PARTICLE IS MOVING AT AN ANGLE TO THE FIELD

In this case the charged particle having charge q and mass m is moving with velocity v and it enters the magnetic field B at angle θ as shown in figure. Velocity can be resolved in two components, one along magnetic field and the other perpendicular to it. Let these components are $v_{\parallel}$ and $v_{\perp}$





$v_{\parallel} = v\cos\theta$ and $v_{\perp} = v\sin\theta$

The parallel component $v_{\parallel}$ of velocity remains unchanged as it is parallel to $\vec{B}$. Due to the $v_{\perp}$ the particle will move on a circular path. So the resultant path will be combination of straight-line motion and circular motion, which will be helical as shown in figure.

The radius of path is $(r) = \dfrac{mv_{\perp}}{qB} = \dfrac{mv\sin\theta}{qB}$ … **(15)**

Time period $(T) = \dfrac{2\pi r}{v_{\perp}} = \dfrac{2\pi mv\sin\theta}{v\sin\theta\, qB} = \dfrac{2\pi m}{qB}$ … **(16)**

Frequency $(f) = \dfrac{Bq}{2\pi m}$ … **(17)**

Pitch: Pitch of helix described by charged particle is defined as the distance moved by the centre of circular path in the time in which particle completes one revolution.

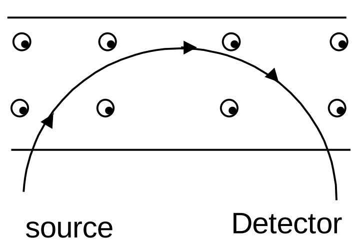
pitch $= (v_{\parallel})$ (time period)

$= v\cos\theta\,\dfrac{2\pi m}{Bq} = \dfrac{2\pi mv\cos\theta}{qB}$ … **(18)**

Illustration 7.

A uniform magnetic field with a slit system as shown is to be used as a momentum filter for high-energy charged particles. With a field of B tesla it is found that the filter transmits α-particle each of energy 5.3 MeV. The magnetic field is increased to 2.3 B tesla and deutrons are passed into the filter. What is energy of each deutron transmitted by the filter?

Solution:



$p = mv$

$K = \dfrac{1}{2}mv^2$

$K = \dfrac{m^2v^2}{2m}$

$p = \sqrt{2mK}$

$r = \dfrac{mV}{qB} = \dfrac{p}{qB} = \dfrac{\sqrt{2mK}}{qB} \Rightarrow K = \dfrac{a^2B^2r^2}{2B}$

$K_{\alpha} = \dfrac{r^2(2e)^2B^2}{2(4m)}$

$K_D = \dfrac{r^2(e^2)\times(2.3B)^2}{2(2m)}$

$\Rightarrow$ $K_D =$ **14.02 MeV**

Illustration 8.

A beam of protons with a velocity 4×10^{-5} m/s enters a uniform magnetic field of 0.3 T at an angle 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix $m_p = 1.67 \times 10^{-27}$ kg

Solution:

$$r = \frac{m(v\sin\theta)}{qB} = 1.2 \text{ cm}$$

$$T = \frac{2\pi r}{v\sin\theta} = 2.175 \times 10^{-7} \text{ s}$$

$$\therefore \quad p = v\cos\theta . T$$

$$\Rightarrow \quad \rho = 4 \times 10^5 \times \frac{1}{2} \times 2.175 \times 10^{-7} = \mathbf{4.35\ cm}$$

10. MOTION OF A CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$, so the net force on it will be

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] \qquad \ldots (\mathbf{19})$$

Which is '**Lorentz force equation**'.

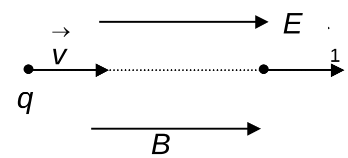
Now let us consider two special cases involving the application of above equation

Case I: When $\vec{V}$, $\vec{E}$ and $\vec{B}$ all the three are collinear:

In this situation as the particle is moving parallel or anti-parallel to the field, the magnetic force on it will be zero and only electric force will act, so

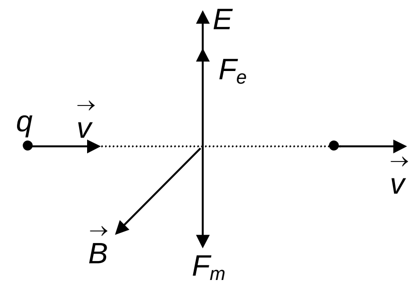
$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

Hence the particle will pass through the field following a straight-line path (parallel to the field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in the figure given below.



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are collinear.

Case II: $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular. In case situation of $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

or, $\vec{a} = \left(\frac{\vec{F}}{m}\right) = 0$, then the particle will pass through the field with the same velocity.

In this situation,

$$F_e = F_m \qquad \text{or, } qE = qvB$$

or, $$v = \frac{E}{B}$$

This principle is used in velocity-selector to get a charged beam having a specific velocity.

Illustration 9.

A particle of mass 1×10^{-26} kg and charge $+ 1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^{6} m/s in $+ x$ direction enters a region in which a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ are present such that $E_x = E_y = 0$; $E_z = 102.4$ kV/m and $B_x = B_z = 0$; $B_y = 8 \times 16^2$ Wb/m^2. The particle enters in a region at the origin at time $t = 0$. Find the location (x, y and z) of the particle at $t = 5 \times 10^{-6}$ s.

Solution:

$$\vec{F}_e = q\vec{E} = 1.6 \times 10^{-9} \times 102.4 \times 10^{3} (-\hat{k})$$

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = 1.6 \times 10^{-19} \times 1.2 \times 10^{6} \times 8 \times 10^{-2} (\hat{K})$$

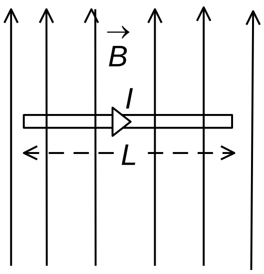
$$\therefore \qquad \vec{F} = \vec{F}_e + \vec{F}_B = 0$$

Hence, the particle will move along $+ x$-axis with constant velocity $\vec{v} = 1.20 \times 10^{6}\ \hat{i}$ m/s

$x_0 = vt = 6.40$ m

$\therefore$ **(6.4 , 0, 0)**

11. FORCE ON A CURRENT CARRYING WIRE IN A MAGNETIC FIELD



Let L be the length of a straight conductor carrying a current I and placed perpendicular to a uniform magnetic field of induction B.

A current in a conductor is due to the movement of electrons and the direction of the conventional current is opposite to that of the direction of motion of electrons. If n be the number of moving charges per unit volume, each charge is of q, and travelling with drift velocity v_d, the charge passing through any cross-section per second is nqv_dA

$$I = nqv_dA$$

where A is the cross-sectional area.

The number of charges in length L of a conductor

$$N = nLA$$

The force on each charge

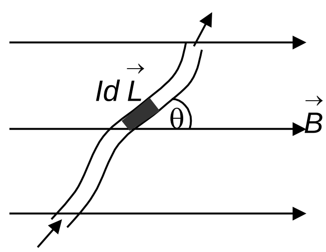
$$F = Bqv_d$$

The force on all charges i.e., the force on the conductor

$$F = Bqv_d \times nLA \quad \text{or} \quad F = BIL$$

In vector form $\vec{F} = I(\vec{L} \times \vec{B})$ … (**20**)

where L is a vector in the direction of the current I; the magnitude of $\vec{L}$ equals the length L of the segment. Note that this expression applies only to a straight segment of wire in a uniform magnetic field.

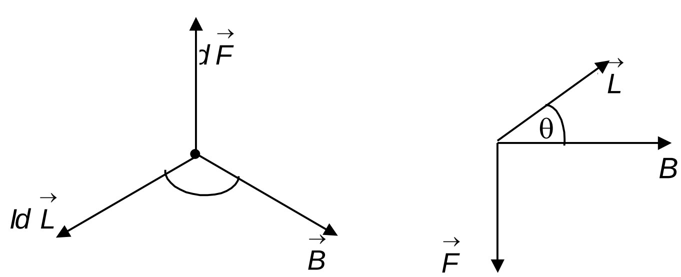


Now consider an arbitrarily shaped wire segment of uniform cross-section in a magnetic field $\vec{B}$, as shown in figure. Then the magnetic force on a very small segment dL in the presence of magnetic field $\vec{B}$ is given by

$$d\vec{F} = d\vec{L} \times \vec{B}$$

The magnitude of force is $dF = BIdL \sin\theta$, where θ is the angle between the vectors $Id\vec{L}$ and $\vec{B}$.

Direction of force: The direction of force is always perpendicular to the plane containing $Id\vec{L}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{a} \times \vec{b})$ with $\vec{a} = Id\vec{L}$ and $\vec{b} = \vec{B}$.



The direction of force when current element $I d\vec{L}$ and $\vec{B}$ are perpendicular to each-other can also be determined by applying either of the following rules.

(a) Fleming's Left-hand Rule: Strech the fore-finger, central finger and thumb of the left hand mutually perpendicular. Then if the fore-finger points in the direction of the field ($\vec{B}$) and the central in the direction of current *I*, the thumb will point in the direction of force (or motion).

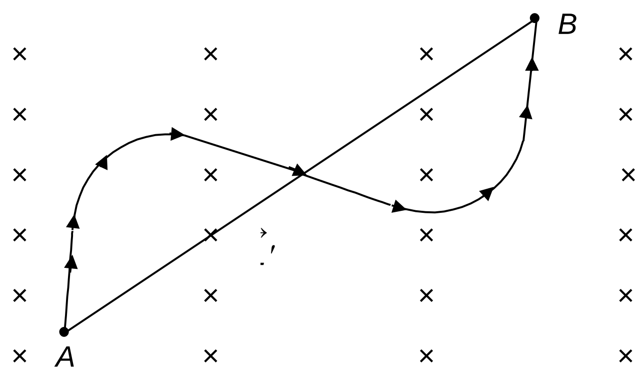
(b) Right-hand Palm rule: Stretch the fingers and thumb of the right hand at right angles to each-other. If the fingers point in the direction of current *I*, and the palm in the direction of the field $\vec{B}$ then thumb will point in the direction of force.

11.1 FORCE ON A CURVED CURRENT CARRYING WIRE

In this case a current-carrying conductor is placed in a uniform magnetic field $\vec{B}$. the force is given by $\vec{F} = \int I d\vec{L} \times \vec{B} = I \int d\vec{L} \times \vec{B}$

and for a conductor $\int d\vec{L}$ represents the vector sum of all the lengths elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to the final point. So a current-carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\vec{F} = I\left[\int d\vec{L}\right] \times \vec{B} = I\,\vec{L} \times \vec{B}$$



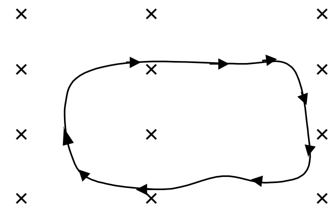
11.2 FORCE ON A CLOSED LOOP OF AN ARBITRARILY SHAPED CONDUCTOR

Consider a current-carrying conductor in the form of a loop of any orbitrary shape is placed in a uniform field $\vec{B}$. In this case the vector sum of the current element must be taken over the closed loop.

$\vec{F} = \oint I d\vec{L} \times \vec{B} = I \oint d\vec{L} \times \vec{B}$ for a closed loop, the vector sum of $d\vec{L}$ is always zero.

$\therefore \quad \vec{F} = 0$

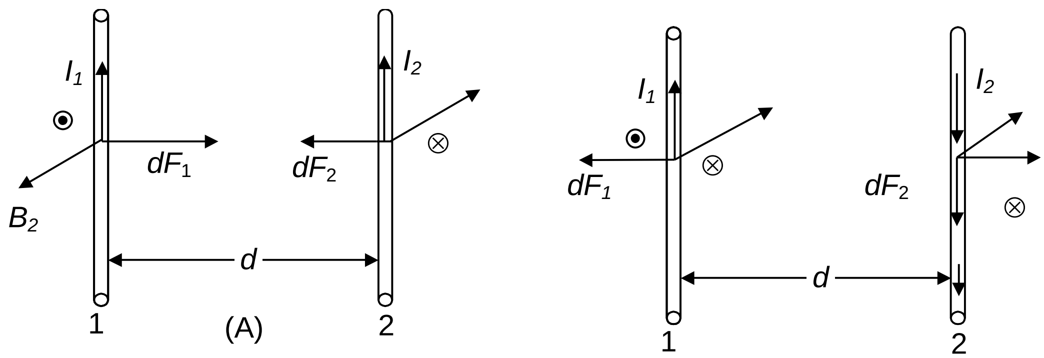
i.e., the magnetic force on a current loop in a uniform magnetic field is always zero.



11.3 FORCE BETWEEN TWO LONG STRAIGHT PARALLEL CURRENT CARRYING CONDUCTORS

Let us consider two very long parallel straight wires carrying currents I_1 and I_2.

Each wire is placed in the region of magnetic induction of other and hence will experience a force. The net force on a current-carrying conductor due to its own field is zero. So if there are two long parallel current-carrying wires 1 and 2 (as shown below), the wire-1 will be in the field of wire-2 and vice-versa.



The force on dl_2 length of wire-2 due to field of wire-1, $dF_2 = I_2\, dL_2\, B_1$

$$= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}\, dL_2 \;\left[\because B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{d}\right]$$

or, $$\frac{dF_2}{dL_2} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d} \qquad \ldots \textbf{(21)}$$

It will be true for wire-1 in the field of wire-2. The direction of force in accordance with the right-hand screw rule will be as shown above.

So the force per unit length in case of two parallel current-carrying wires separated by a distance 'd' is

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$$

If I_1 and I_2 are along the same direction, the forces between the wires is attractive in nature and if I_1 and I_2 are oppositely directed the force is repulsive. The direction of forces is given by Fleming's left hand rule.

Definition of 'Ampere'

We have $$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$$

If $I_1 = I_2 = 1A$; d = 1m; dL = 1m; then

$$dF = 2 \times 10^{-7}\ \text{N}$$

Hence, 'Ampere' is defined as the current which when passing though each of two parallel infinitely long straight conductors placed in free space at a distance of 1 m from each-other produces between them force of 2×10^{-7} N for one metre of their length.

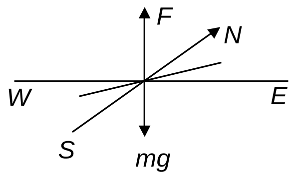
Illustration 10

A straight wire of length 30 cm and mass 60 mg lies in a direction 30° east of north. The earth's magnetic field at this is in horizontal and has a magnitude of 0.8×10^{-4} T . What current must be passed through the wire so that it may float in air? [g = 10 m/s^2]

Solution:

$P = BIL \sin\theta$

$$\therefore\ I = \frac{mg}{BL\sin\theta} = \frac{60\times10^{-6}\times10}{.8\times10^{-4}\times30\times10^{2}\times\frac{1}{2}} = \mathbf{50A}$$



12. CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

12.1 MAGHNETIC MOMENT

According to magnetic effects of current, in case of current-carrying coil for axial point,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{(R^2+x^2)^{3/2}} \qquad \text{when } x >> R, \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{x^3}$$

If we compare this result with the field due to a small bar magnet for a distant axial point, i.e., $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$,

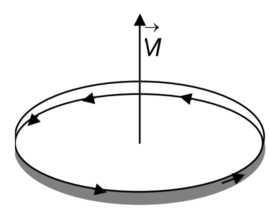
where M is magnetic moment of the bar magnet

we find that a current-carrying coil for a distant point behaves as a magnetic dipole of moment

$$\vec{M} = NI\,\pi R^2 = NIA \qquad \dots \textbf{(22)}$$

where A is area of the loop. So the magnetic moment of a current carrying coil is defined as the product of current in the coil with the area of coil in the vector form.

Magnetic moment of a current loop is a vector quantity and direction is perpendicular to the plane of the loop. Its dimensions are [L^2A] and units are A-m^2. Magnetic moment in case of a charged particle having charge q and moving in a circle of radius R with speed v is given by $\frac{1}{2}\,qvR$

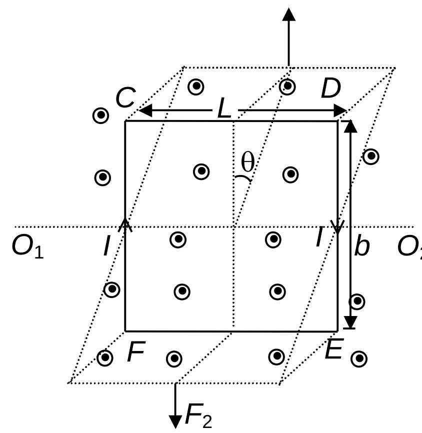


As we know, $I = qf = q\frac{v}{2\pi R}$ and $|A| = \pi R^2$

$$\therefore \qquad |A| = I\,|\vec{S}| = \frac{1}{2}qvR$$

$$|\vec{M}| = I\,|\vec{S}| = \frac{1}{2}qvR$$

12.2 TORQUE ON A CURRENT LOOP



Consider a rectangular coil *CDEF* of length *L* and width *b* is placed vertically, while a uniform magnetic induction *B* passes normally through it as shown. The coil is capable of rotation about an axis O_1O_2.

If the loop is oriented in the magnetic field such that the normal to the plane of the coil makes an angle θ with the direction of $\vec{B}$, then the torque experienced by the loop

$$\tau = \frac{b}{2}(ILB)\sin\theta + \frac{b}{2}(ILB)\sin\theta$$

i.e., $\tau = ILb\sin\theta = IAB\sin\theta$

where $A = Lb$ is the area of the loop.

The maximum torque experienced is $\tau = IAB$, when $\theta = 90°$

and for a coil of *N* turns

$$\tau = NIAB$$

Here $NIA = M$ = Magnetic moment of the loop.

In vector notation $\vec{\tau} = \vec{M} \times \vec{B}$. … (**23**)

This result holds good for plane loops of all shapes rectangular, circular or otherwise.

12.3 WORK DONE IN ROTATING A CURRENT LOOP

When a current loop is rotated in a uniform magnetic field through an angle θ about an axis then work done will be

$$\int_0^w dW = \int \tau d\theta = \int_0^\theta MB\sin\theta\, d\theta$$

$$W = -\left[MB\cos\theta\right]_0^\theta = MB(1-\cos\theta) \qquad \ldots (\mathbf{24})$$

Illustration 11.

For a given length *L* of a wire carrying a current *I*, how many circular turns would produce the maximum magnetic moment and of what value?

Solution:

for a circular coil having *N* turns,

$$A = \pi R^2 I N$$

Now, $L = (2\pi R)N$

$$\therefore \quad R = \frac{L}{2\pi N}$$

Substituting the above value of R in equation (1), we get

$$M = \pi NI \times \frac{L^2}{4\pi^2 N^2} \quad \text{or,} \quad M = \frac{IL^2}{4\pi N} \qquad \ldots \text{(i)}$$

From equation (ii), it is clear that M will be maximum when N = minimum = 1, i.e., the coil has only one turn and

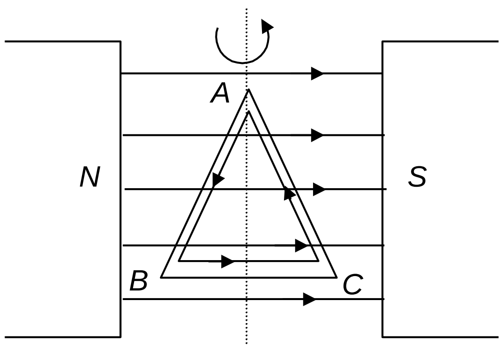
$$(M)_{max} = \frac{1}{4\pi} IL^2$$

Illustration 12.

A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical in plane magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 ampere is passed through it and the magnetic field is parallel to its plane.

Solution:

As the coil is in the form of an equilateral triangle, its area



$$S = \frac{1}{2} \times L x L \sin\ 60°$$

$$= \frac{1}{2} \times (0.02)^2 \times \frac{V^2}{2} = \sqrt{3} \times 10^{-4}\ \text{m}^2$$

So its magnetic moment

$$M = IA\ = 0.1\ \sqrt{3}\ \times 10^{-4}$$

$$= \sqrt{3}\ \times 10^{-5}\ \text{A-m}^2$$

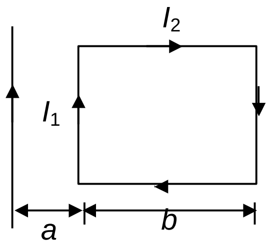
Now, the couple on a current – carrying coil in a magnetic field is given by $\tau = MB \sin\theta$

since the plane of the coil is parallel to the magnetic field, the angle between $\vec{M}$ and $\vec{B}$ will be 90° and hence $\tau = MB \sin 90° = MB$

$\therefore \quad \tau = (\sqrt{3}\ \times 10^{-5}) \times 5 \times 10^{-2} = \mathbf{5\sqrt{3}\ \times 10^{-7}}$ **N-m.**

Illustration 13.

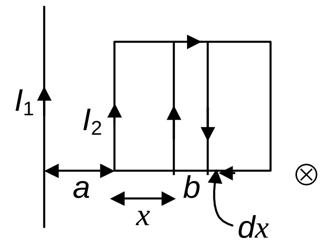
The arrangement is as shown below



(a) Find the potential energy of the loop

(b) Find the work done to increase the spacing between the wire and the loop from a to $2a$.

Solution:



(a) Magnetic moment of a small element of the loop. $dM = I_2 L dx$

The direction of the magnetic moment is perpendicular to the plane of paper pointing inwards.

$dU = -d\vec{M}\,\vec{B} = -dM\,B$, where B is the magnetic field at the position of this element

i.e., $$B = \frac{\mu_0}{4\pi}\frac{2I_1}{a+x}$$

$$\therefore \quad dU = -\frac{\mu_0}{4\pi}\,2\,I_1\,I_2\,l\left(\frac{dx}{a+x}\right)$$

$$\therefore \quad U = \frac{\mu_0}{4\pi}\,2I_1I_2\,l\int_o^b \frac{dx}{a+x} = \frac{\mu_0}{4\pi}2I_1I_2l\;\log_e\left(\frac{a+b}{a}\right)$$

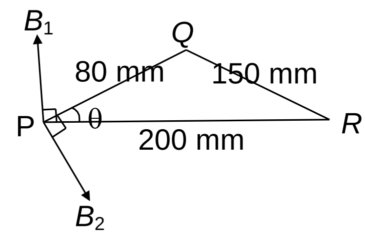
(b) $$u_i = -\frac{\mu_0}{4\pi}2\,I_1\,I_2\,l\quad\log_e\left(\frac{a+b}{a}\right)$$

$$u_f = \frac{\mu_0}{4\pi}\,2I_1\,I_2\,l\;\left(\log_e\frac{2a+b}{2a}\right)$$

$$\therefore \quad W = \Delta U = U_f - U_i = \frac{\mu_0}{4\pi}\,\mathbf{2I_1\,I_2\,l\,\log_e}\left(\frac{2(a+b)}{2a+b}\right)$$

Illustration 14.

Two parallel wires carry currents 5 A and 10 A respectively in opposite directions. A plane lamina ABCD intersects the wires at right angles at points Q and R. Find the magnitude of the total magnetic induction at point P located in the lamina as shown in the Figure.



Solution:

The magnetic induction at P due to the 10 A current,

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi\times10^{-7}\times10}{2\pi\times200\times10^{-3}}\ \text{T} = 10\ \mu\text{T}$$

The magnetic induction at P due to the 5 A current,

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \text{T} = 12.5\ \mu\text{T}$$

Resultant magnetic induction,

$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos(180° - \theta)}$$

where $QR^2 = PQ^2 + PR^2 - 2PQ \cdot PR \cos\theta$

$$\cos\theta = \frac{-150^2 + 80^2 + 200^2}{2 \times 80 \times 200} = +0.7469$$

$$B = 10^{-6}\sqrt{10^2 + 12.5^2 - 2 \times 10 \times 12.5 \times 0.7469} = 8.34 \times 10^{-6}\ \text{T}$$

$$= 8.34\ \mu\text{T}$$

Illustration 15.

An insulating circular disc of radius 'a' has a uniformly distributed static charge of surface density σ. The disc rotates about its centre with an angular velocity ω. Find the magnetic field at the centre of the disc.

Solution:

The disc may be considered to be made up of concentric circular rings with radius increasing from 0 to a. Consider one such elementary ring of radii r and r + dr.

Face area $= 2\pi r dr$

Charge $= 2\pi r dr\sigma$

Angular velocity $= \omega$

Period $T = \frac{2\pi}{\omega}$

The elementary charge when the disc rotates to equivalent to a circular current loop. The amount of charge passing through the current loop during T = $2\pi r dr\sigma$

$$\therefore \qquad I = \frac{2\pi r dr\sigma}{T} = \frac{2\pi r dr\sigma\omega}{2\pi}$$

$$= r dr\omega\sigma$$

Magnitude of magnetic field at centre $dB = \frac{\mu_0 I}{2r}$

$$= \frac{\mu_0 r dr\omega\sigma}{2r} = \mu_0 \frac{\omega\sigma dr}{2}$$

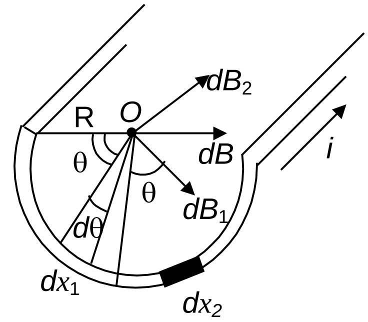
$\therefore$ total magnetic field due to all rings = $B = \int dB$

$$= \frac{\mu_0\omega\sigma}{2}\int_0^a dr = \mu_0 \frac{\omega\sigma a}{2}$$

Illustration 16.

Find the magnetic induction B at a point on the axis due to an infinite thin conductor with semicircular cross-section of radius 'r' carrying a uniform current i.

Solution:



Let R be the radius of semicircular cross-section and O, a point on the axis. Considering an element dx_1 carrying current di_1

$$di_1 = \frac{idx_1}{\pi r}$$

Induction dB_1 at O due to di_1 is $dB_1 = \frac{\mu_o di}{2\pi R}$

This is directed normal to line joining the element to O.

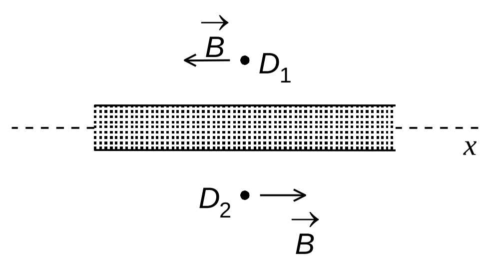
$$dB_1 = \frac{\mu_o i}{2\pi R}\frac{dx_1}{\pi R} = \frac{\mu_o i}{2\pi^2 R^2} R d\theta$$

$$= \frac{\mu_o i}{2\pi^2 R} d\theta$$

Field dB_2 due to corresponding element dx_2 points in a direction shown. Resolving $\overline{d\mathbf{B}_1}$ and $\overline{d\mathbf{B}_2}$, $dB_1 \sin\theta = d\mathbf{B}_2 \sin\theta$ acting on same line and $d\mathbf{B}_2\cos\theta = d\mathbf{B}_1\cos\theta$ acting on opposite direction and hence cancelling each other.

$$\therefore \qquad B = \int_0^{\pi} d\mathbf{B}_1 \sin\theta = \frac{\mu_0 i}{2\pi^2 R}\int_0^{\pi} \sin\theta d\theta = \frac{\boldsymbol{\mu_0 i}}{\boldsymbol{\pi^2 R}}$$

Illustration 17.



Given Figure shows a cross-section of an infinite conducting sheet carrying a current per unit x-length of λ, the current emerges perpendicularly out of the page.

(a) Use the Biot-Savart law and symmetry to prove that for all points D_1 above the sheet, and all points D_2 below it, the magnetic field $\vec{B}$ is parallel to the sheet and directed as shown.

(b) Use Ampere's law to prove that $B = \frac{1}{2}\mu_0\lambda$ at all points D_1 and D_2.

Solution:

(a) Let the field be not parallel to the sheet as shown in figure (I)

Reverse the direction of the current:

According to the Biot-Savart law, the field reverses and will be as shown in figure (ii)

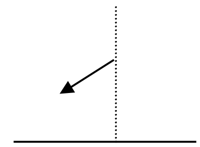
Rotate the sheet by 180° about a line that is perpendicular to the sheet. Then the field will rotate with it to be finally as in figure (iii).

Only if the field is parallel to the sheet, then the final direction

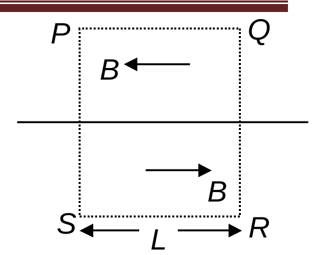
Fig. (i) Fig. (ii)

of the field be the same as the original direction.

If the current is out of the page, any inifinitesimal portion of the sheet in the form of a long straight wire produces a field that is to the left above the sheet and to the right below the sheet. The field then is framed as in figure given in question.



(iii)



(iv)

(b) Integrate the tangential component of the magnetic field around the rectangular loop shown with dotted lines (figure iv).

The upper and lower edges are the same distance from the current sheet and each has length L. This means the field has the same magnitude along these edges. It points to the left along the upper edge PQ and to the right along the lower edge SR.

The contributions to the integral $\oint \vec{B}.d\vec{l}$ are

BL from PQ

BL from SR

Zero from PS

Zero from QR

(as PS and QR are perpendicular to the sides, the cosine term in $\vec{B}.d\vec{S} = |\vec{B}|.|d\vec{S}|\cos(\angle\vec{B}.d\vec{S})$ is zero).

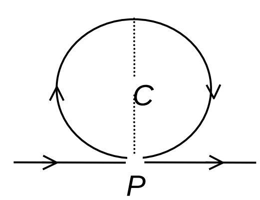
$$\therefore \qquad \oint \vec{B}.d\vec{l} = BL + BL = 2BL$$

Total current through the loop = λL

Ampere's law $\Rightarrow 2BL = \mu_0 xL$

$$\Rightarrow \boldsymbol{B = (\mu_0 \lambda)/2}$$

Illustration 18.

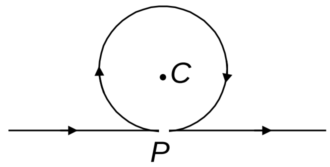


(a) A long wire is bent into the shape shown in the figure without cross contact at P. Determine the magnitude and direction of $\vec{B}$ at the centre C of the circular portion when the current i flow as indicated.

(b) The circular part of the wire is rotated without distortion about its dashed diameter perpendicular to the straight portion of the wire. The magnetic moment associated with circular loop is now in the direction of the current in the straight part of the wire. Determine $\vec{B}$ at C in this case

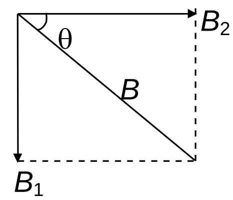
Solution:

(a) Let C be the centre of the circle. The magnetic induction at C due to the long straight part of the conductor is



$$B_1 = \frac{\mu_o}{4\pi} \cdot \frac{2i}{R}$$

This field is perpendicular to the plane of the paper and points outwards (towards the reader). The field at C due to the circular current loop



$$B_2 = \frac{\mu_o}{2} \cdot \frac{i}{R}$$

This field is also directed as B_1.

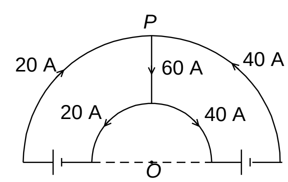
Hence the resultant field B = B_1 + B_2 = $\frac{\mu_o}{4\pi} \cdot \frac{2i}{R} + \frac{\mu_o}{2} \cdot \frac{i}{R} = \frac{\mu_o}{2R} i\left(1 + \frac{1}{\pi}\right)$

(b) Due to the rotation of the circular loop the plane of the loop is perpendicular to the straight part. Now the fields B_1 and B_2 are mutually perpendicular, B_2 acts on the plane of the paper from left to right.

The resultant field B = $\sqrt{B_1^2 + B_2^2} = \frac{\mu_o}{2} \frac{i}{R} \sqrt{1^2 + \frac{1}{\pi^2}} = \boldsymbol{\frac{\mu_o}{2\pi} \cdot \frac{i}{R} \sqrt{1 + \pi^2}}$

Illustration 19.

Two concentric coplanar semicircular conductors form part of two current loops as shown in the figure. If their radii are 11 cm and 4 cm calculate the magnetic induction at the centre.



Solution:

Magnetic induction at C = $\frac{1}{4} \frac{\mu_o}{2} \left(\frac{40}{r_1} - \frac{40}{r_2}\right) - \frac{1}{4} \frac{\mu_o}{2} \left(\frac{20}{r_1} - \frac{20}{r_2}\right)$

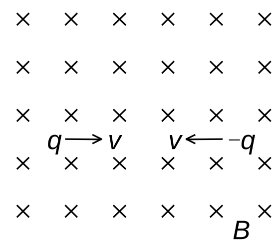
$$= \frac{4\pi \times 10^{-7}}{8} \left[20\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]$$

$$= \frac{4\pi \times 10^{-7}}{8} \left[20\left(\frac{1}{4 \times 10^{-2}} - \frac{1}{11 \times 10^{-2}}\right)\right]$$

= 5×10^{-5} weber/m^2 (inward)

Illustration 20.

Two particles P and Q, each having a mass M are placed at a separation D in a uniform magnetic field B as shown in figure. They have opposite charges of equal magnitude. At time t = 0 the particles are projected towards each other, each with a speed v. Suppose the coulomb force between the charges is switched off. (a) Find the maximum value v_m of the projection speed so that the two particles do not collide (b) What would be the minimum and maximum separation between the particles if $v = \frac{v_m}{4}$? (c) At what instant will a collision occur between the particles if $v = 2v_m$?

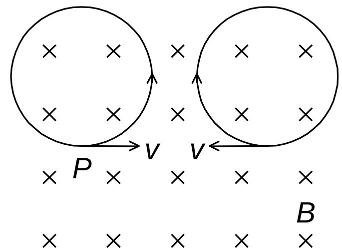


Solution:

Solution For particle P

(a) Force acting on P due to the magnetic field = Bqv.

This particle P will describe a circle in the clockwise direction whose radius is obtained from the equation



$$Bqv = \frac{mv^2}{r_1} \qquad \Rightarrow \qquad r_1 = \frac{mv}{qB}$$

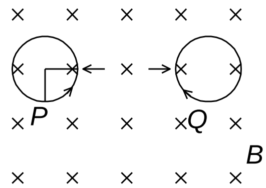
The particle Q will describe a circle in the anticlockwise direction $r_2 = \frac{mv}{qB}$

The two particles do not collide if $r_1 + r_2 \le d$

$$\frac{mv}{qB} + \frac{mv}{qB} \le d$$

Maximum value of the projection speed, $v_{max} = \frac{qBd}{2m}$

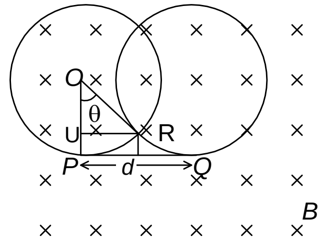
(b) If $v = \frac{v_{max}}{4}$, then $r_1 = \frac{m}{qB}\left(\frac{qBd}{8m}\right) = \frac{d}{8}$



$$r_2 = \frac{d}{8}$$

Minimum separation between the two particles

$$= d - r_1 - r_2 = \mathbf{\frac{3d}{4}}$$



Maximum separation between the two particles

$$= d + r_1 + r_2 = \frac{5d}{4}$$

(c) If v = $2v_{max}$, then $r_1 = \frac{m}{qB}\left(\frac{2qBd}{2m}\right)$ = d ; r_2 = d

Angular velocity, $\omega = \frac{qBd}{md} = \frac{\boldsymbol{qB}}{\boldsymbol{m}}$

The particle will collide at point R, UR = $\frac{d}{2}$

$$\sin\theta = \frac{\frac{d}{2}}{d}$$

$$\theta = 30^\circ = \frac{\pi}{6}$$

A collision will occur between the particles at time t

$$t = \frac{\frac{\pi}{6}}{\omega} = \frac{\boldsymbol{m}\pi}{\boldsymbol{6qB}}$$

Illustration 21.

A fixed horizontal wire carries 200 A current below which another wire of linear density 10 g/m carrying a current is kept at a depth 2 cm and parallel to the first wire. If the second wire hangs in air, find the current in it. If the current in the first wire increases to 220 A, what will be the instantaneous acceleration on the second wire?

Solution:

For the second wire to hang in air without any support its weight acting downwards should be balanced by an upward force of equal magnitude. In this case this upward force is provided by the mutual attractive force between the two parallel conductors carrying current. For this to happen, the current should flow in the same direction in both the wires.

Now, $\frac{\mu_0}{4\pi}\cdot\frac{2I_1I_2}{r}$ = mg

$$10^{-7}\times\frac{2\times200\times I_2}{2\times10^{-2}} = (10 \times 10^{-3}\text{kg})\ (9.8\ \text{m/s}^2)$$

or I_2 = 49 A

The current carried by the second wire = 49 A

If the current carried by the first wire I_1 = 220 A, then the force experienced by 1 m length of the second wire

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} = \frac{2\times220\times49}{2\times10^{-2}}\times10^{-7}\,N$$

Considering a metre length of second wire

F – mg = ma, where a is the acceleration.

Now,

$$a = \frac{F}{m} - g = \frac{2\times220\times49}{2\times10^{-2}}\times\frac{10^{-7}}{10\times10^{-3}} - 9.8 = 10.78 - 9.8 = 0.98 \text{ m/s}^2$$

The instantaneous acceleration experienced by the second wire = 0.98 m/s^2 in the upward direction.

Illustration 22.

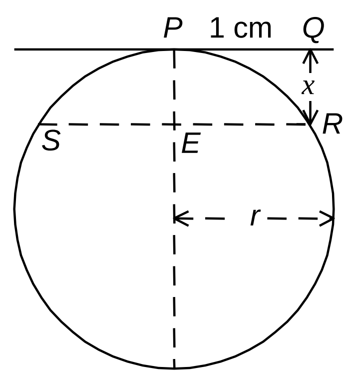
A proton of velocity 1.0×10^7 m/s is projected at right angles to a uniform magnetic field of induction 100 mT.

(a) How much is the particle path deflected from a straight line after it has traversed a distance of 1 cm?

(b) How long does it take for the proton to traverse a 90° arc?

(Mass of proton = 1.65×10^{-27} kg; charge of a proton = 1.6×10^{-19} C.)

Solution:



(a) It is clear that the proton would describe a circular path under the given conditions. The radius of the path is given by

$$r = \frac{mv}{qB} = \frac{(1.65\times10^{-27}kg)(1.0\times10^{7}m/s)}{(1.6\times10^{-19}C)(100\times10^{-3}T)} = \frac{1.65}{1.6} \text{ m}$$

If QR = x be the deflection of the particle after traversing a path of 1 cm, then by the properties of the circle

SE × ER = PE × (2r – PE)

or (0.01 m) (0.01 m) = x (2r – x) = 2rx – x^2

Neglecting x^2 which is a small quantity compared to 2rx,

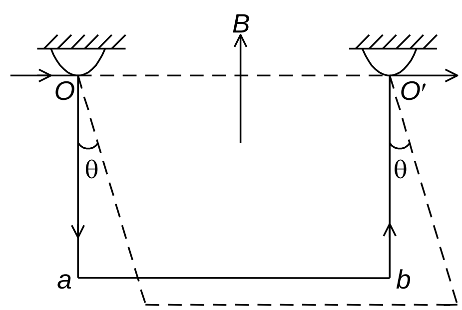
$$x = \frac{1\times10^{-4}}{2r} = \frac{1\times10^{-4}}{2\times\frac{1.65}{1.6}} = \frac{1.6}{2\times1.65}\times10^{-4}m = 4.848 \times 10^{-5} \text{ m}$$

(b) Let t be the time taken by the particle to traverse a 90° arc. Then

$$t = \frac{2\pi r}{4}\times\frac{1}{v} = \frac{\pi}{2}\times\frac{1.65}{1.6}\times\frac{1}{10^7} = 1.62 \times 10^{-7} \text{ s}$$

Illustration 23.

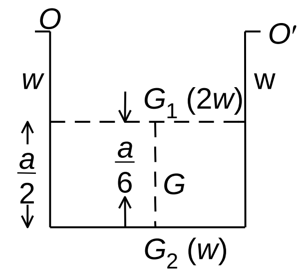
A copper wire with cross-sectional area 2.5 mm^2 and bent to make three sides of a square can turn about a horizontal axis OO′. The wire is located in a uniform vertical magnetic field. Find the magnetic induction, if on passing a current I = 16 A deflects by an angle $\theta = 20^\circ$ (Specific gravity of copper = 8.9)



Solution:

The deflection of the system is due to the force on the wire, due to the magnetic field B and the force is given by = BIa,

where a is the side of the square and this force acts in the horizontal direction.



The moment of the force about the axis of rotation OO′ = BIa × a cos θ = Bia^2 cos θ

where θ = 20°.

The weight of the wire is mg and this acts through the centre of gravity of the wire, G lying at a distance of $\frac{a}{2}+\frac{a}{6}=\frac{2a}{3}$ from OO′.

∴ moment of the weight about OO′

$$= mg \cdot \frac{2}{3}\ a \sin\theta$$

For the equilibrium of the system

$$\frac{2}{3}\ mga \sin\theta = Bia^2 \cos\theta$$

or $$B = \frac{2}{3}\frac{mga\sin\theta}{Ia^2\cos\theta} = \frac{2}{3}\frac{mg}{Ia}\tan\theta$$

The mass m of the wire is given by

m = Length of the wire × cross-section × density

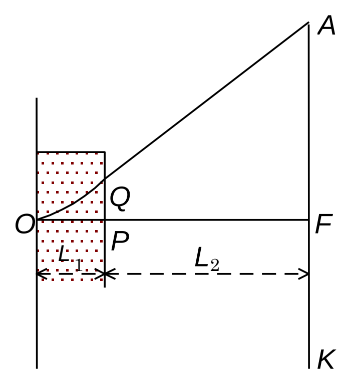
$= 3a \times (2.5 \times 10^{-6}\ m^2)\ (8900\ kg/m^3)$

∴ $$B = \frac{2}{3}\cdot\frac{3a(2.5\times10^{-6})(8900)(9.8)}{16\times a}\tan 20^\circ$$

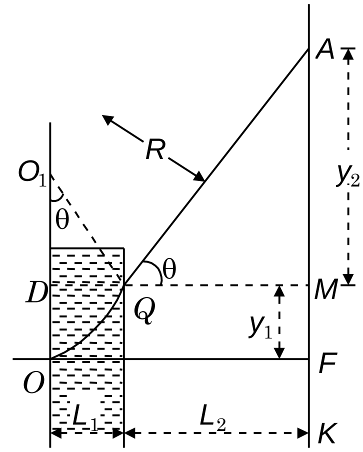
$= 9.9 \times 10^{-3}$ T

Illustration 24.

A beam of electrons accelerated by a potential difference of 100 V flies into a homogeneous magnetic field applied perpendicular to the plane of the paper and towards the observer. The width of the magnetic field OP = 2 cm. In the absence of the magnetic field the electron beam produces a spot at a point F on fluorescent screen AK, which is at a distance L_2 = 6 cm from the edge of the magnetic field. When the magnetic field is switched on the spot moves on to A along the path OQA. Find the displacement FA of the spot if the induction of the magnetic field $B = 1.5 \times 10^{-3}\ \frac{Wb}{m^2}$



Solution:



The path OQ of the electron travelled in the magnetic field B is an arc of the circle of radius R is given by

$$R=\frac{mv}{eB} \quad \ldots \text{(i)}$$

where m and e are mass and charge of an electron and v its velocity.

The velocity gained by the accelerated electron

$$v=\sqrt{\frac{2eU}{m}} \quad \ldots \text{(ii)}$$

where U is the accelerating potential difference.

From equations (1) and (2),

$$R=\frac{1}{B}\sqrt{\frac{2Um}{e}} \quad \ldots \text{(iii)}$$

The centre of curvature of the arc OQ lies at O_1 and

$OO_1 = QO_1 = R$

QA is a straight line and tangent to the curve at Q.

Displacement FA = FM + MA = $y_1 + y_2$.

To find y_1

$y_1 = FM = OD = OO_1 - O_1D$

$$= R-\sqrt{R^2-DQ^2} = R-\sqrt{R^2-L_1^2} \quad \ldots \text{(iv)}$$

To find y_2

Triangles AQM and O_1 DQ are similar.

$$\frac{AM}{QM}=\frac{DQ}{O_1D}$$

$$\frac{y_2}{L_2}=\frac{L_1}{\sqrt{R^2-L_1^2}} \text{ or } \quad y_2=\frac{L_1L_2}{\sqrt{R^2-L_1^2}}$$

Total displacement

FA = $y_1 + y_2$

$$= \left(R-\sqrt{R^2-L_1^2}\right)+\frac{L_1L_2}{\sqrt{R^2-L_1^2}} \quad \ldots \text{(v)}$$

$$R = \frac{1}{B}\sqrt{\frac{2Um}{e}}$$

$$= \frac{1}{1.5\times10^{-3}}\sqrt{\frac{2\times100\times9.1\times10^{-31}}{1.6\times10^{-19}}}$$

$$= 22.48 \times 10^{-3} \text{ m} = 2.25 \text{ cm}$$

Now,

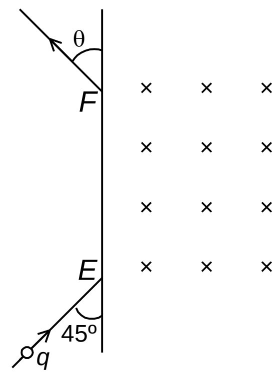
$$FA = 2.25 - \sqrt{(2.25)^2 - 2^2} + \frac{2\times6}{\sqrt{(2.25)^2-2^2}}$$

$$= 2.25 - 1.03 + \frac{12}{1.03}$$

$$= 2.25 - 1.03 + 11.65 = 12.87 \text{ cm}$$

Illustration 25.

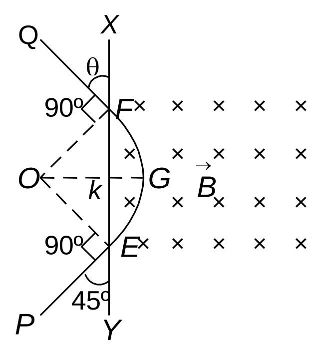
A particle of mass m = 1.6×10^{-27} kg and charge q = 1.6×10^{-19} C enters a region of uniform magnetic field of strength 1 T along the direction shown in Figure. The speed of the particle is 10^7 m/s.



(i) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(ii) If the direction of the magnetic field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

Solution



(i) Given that

the mass of the particle = 1.6×10^{-27} kg

the charge on the particle = 1.6×10^{-19} C

the magnetic field B = 1 T

velocity of projection v = 10^7 m/s

Let XY be the boundary of the magnetic field. The particle, coming inside the magnetic field region experiences magnetic force in a mutually perpendicular direction to $\vec{v}$ and $\vec{B}$. This causes the particle to describe a circular arc EGF inside the magnetic region.

The radius of the arc R = OE = OF

PE is a tangent to the circle at E.

$\therefore$ OEP = 90°

Let G be the midpoint of the arc. GO is perpendicular to EF.

GOE = 45° (angle between perpendiculars of EF and PE is equal to the angle between EF and PE).

Similarly QF is a tangent to the circle at F and by symmetry it is clear that $QFX = 45^\circ$, $FOG = 45^\circ$

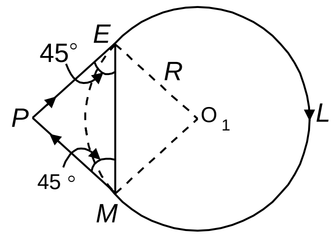
Now EF = EK + KF = 2R cos 45°

But $R = \frac{mv}{Bq} = \frac{(1.6\times10^{-27}kg)(10^7 m/s)}{(1T)(1.6\times10^{-19}C)} = 0.1$ m

∴ $EF = 2R\cos 45^\circ = 2\times 0.1\times\frac{1}{\sqrt{2}} = 0.141$ m

(ii) Now the direction of magnetic field is reversed i.e., directed along the outward normal to the plane of the paper.

The charged particle at E experiences a magnetic force directed along EO_1 perpendicular to PE and would again describe a circular path with PE as tangent at E. The motion is in clockwise direction and would emerge out at M along MP.



$EO_1M = 90^\circ$

since $EO_1 \perp PE$ and $MO_1 \perp PM$

The chord EM subtends an angle of 90° at O_1 and hence arc ELM

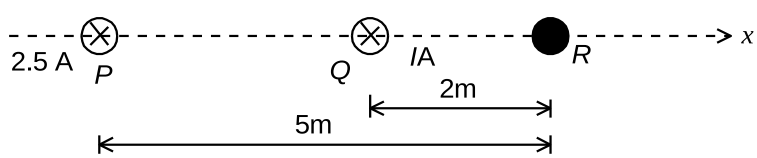
$$= \frac{3}{4}\times 2\pi R = \frac{3}{2}\pi R$$

Now the time spent by the particle inside the magnetic region = time to describe the arc ELM

$$= \frac{arcELM}{v} = \frac{3\pi R}{2v} = \frac{3\pi\times0.1}{2\times10^7} = 4.71\times10^{-8}\text{ s}$$

Illustration 26.

Two long parallel wires carrying currents 2.5 ampere and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 metre and 2 metre respectively from a collinear point R (see Figure).



(1) An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(2) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 amperes may be placed so that the magnetic induction at R is zero.

Solution:

(1) Magnetic field B_1 due to P at R

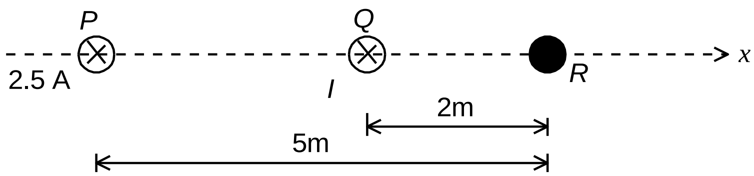
$$B_1 = \frac{\mu_0}{2\pi}\frac{I}{d} = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5}\right)$$

Magnetic field B_2 due to Q at B

$$B_2 = \frac{\mu_0}{2\pi}\cdot\frac{I}{d} = \frac{\mu_0}{2\pi}\left(\frac{I}{2}\right)$$

Resultant magnetic field at R

$$B = B_1 + B_2 = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right) \qquad \text{.... (i)}$$



This field B at R acts into the plane of the paper and perpendicular to PX. Force experienced by electron moving along PX is

$$\vec{F} = e(\vec{v} \times \vec{B})$$

$\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Because of the negative charge of electron (i.e.q = negative) the force F acts perpendicular to the plane of paper directed upwards.

Now, F = evB

$$\therefore \qquad B = \frac{F}{ev} = \frac{3.2\times10^{-20}}{1.6\times10^{-19}\times4\times10^{5}} = 0.5\times10^{-6} \qquad \text{.... (ii)}$$

Equating this to (1), we get

$$\frac{\mu_0}{2\pi}\left[\frac{2.5}{5} + \frac{I}{2}\right] = 0.5\times10^{-6}$$

Which gives

$$I = 2\left[\frac{2\pi}{\mu_0}(0.5\times10^{-6}) - \frac{2.5}{5}\right] = 2\left[\frac{2\pi\times(0.5\times10^{-6})}{4\pi\times10^{-7}} - \frac{2.5}{5}\right] = 2\left[\frac{2.5}{1} - \frac{2.5}{5}\right] = \mathbf{4A}$$

(2) In this case, we consider the following alternatives:

(a) When the current 2.5 A is directed into the plane of paper. If r is the distance of this current wire from R. we have $B_3 = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{2.5}{r}\right)$

Now since $B_1 + B_2 + B_3 = 0$ we get,

$$\frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{4}{2} + \frac{2.5}{r}\right) = 0$$

which gives r = –1 m

Thus the third wire is located at 1 m from R on RX.

(b) When the current 2.5 A is directed out from the plane of paper upwards.

Here $I_3 = -2.5$ A

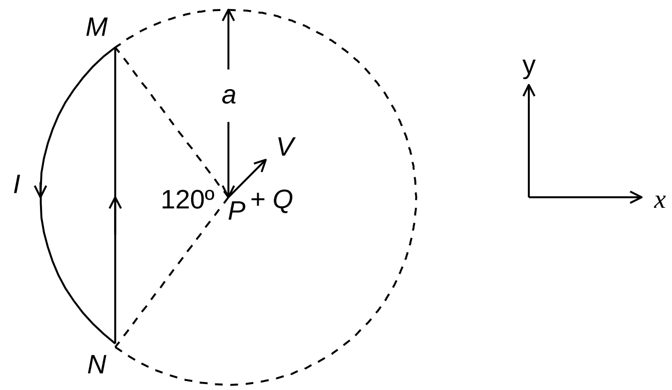
Hence we will similarly have

$$\frac{\mu_0}{2\pi}\left(\frac{2.5}{5}+\frac{4}{2}+\frac{2.5}{r}\right)=0$$

which gives r = 1 m i.e. the third wire is located 1 m from R on RQ.

Illustration 27.

A wire loop carrying a current I is placed in the x-y plane as shown in the figure.



(a) If a particle with charge + Q and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

Solution:

(a) Let us calculate the total magnetic field B at P due to the whole loop. The loop consists of a linear part MN and a circular part MIN, let the fields due to these be B_1 and B_2 respectively.

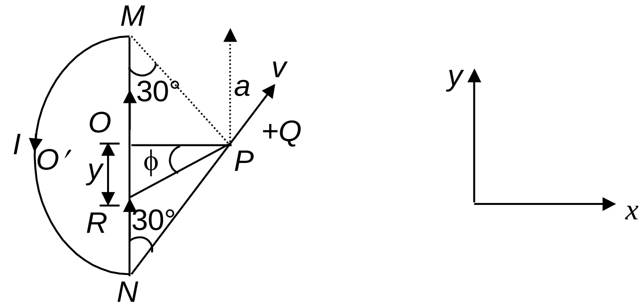
Then clearly $B = B_1 + B_2$

Since the circular part subtends $\underline{|90^\circ}$ with every radius joining it to P, the field

$$B_2 = \frac{\mu_0 i}{6a} \qquad \ldots\text{(i)}$$

For calculating the field B_1, due to the linear part, consider a small element of length dy at R at a distance y form O.

Then the elementary magnetic field due to this element at



$$P = dB = \frac{\mu_0 i dy \cos\phi}{4\pi\left(\frac{a^2}{4}+y^2\right)} \qquad \ldots(ii)$$

where $\phi = \angle OPR$

From the figure,

$$\left.\begin{aligned} y &= \frac{a}{2}\tan\phi \\ \text{Hence } dy &= \frac{a}{2}\sec^2\phi\, d\phi \end{aligned}\right\} \text{[because PO = PN sin 30° = } \frac{a}{2} \qquad \ldots\text{(iii)}$$

Substituting for y and dy in (2),

$$dB = \frac{\mu_0 I \cos\phi \frac{a}{2}\sec^2\phi\, d\phi}{4\pi\left(\frac{a^2}{4}+\frac{a^2}{4}\tan^2\phi\right)}$$

or $$dB = \frac{\mu_0 I}{4\pi}\left[\frac{\frac{a}{2}\sec^2\phi\cos\phi\, d\phi}{\frac{a^2}{2}\sec^2\phi}\right]$$

$$= \frac{\mu_0 I}{2\pi a}\cos\phi\, d\phi$$

Hence the total field due to the whole current line MN at P is

$$B_1 = \int_{-\pi/3}^{+\pi/3}\frac{\mu_0 I}{2\pi a}\cos\phi\, d\phi = \frac{\mu_0 I}{2\pi a}\left[\sin\phi\Big|_{-\pi/3}^{+\pi/3}\right] = \frac{\mu_0 I}{2\pi a}\left[2x\frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}\mu_0 I}{2\pi a}$$

The field B_1 is into the plane of paper perpendicular to it, and field B_2 is out of the plane of paper also perpendicular to it. The two have opposite directions. Hence, taking B_1 as +ve, the net field

$$B = B_1 - B_2 = \frac{\sqrt{3}\mu_0 I}{2\pi a} - \frac{\mu_0 I}{6a} = \frac{\mu_0 I}{6\pi a}\left[3\sqrt{3}-\pi\right]$$

Hence force on the charge Q, F = B.Q.v = $\frac{\mu_0 IQv}{6\pi a}\left(3\sqrt{3}-\pi\right)$

Hence the acceleration of the charge = F/m = $\frac{\mu_0 IQv}{6\pi ma}(3\sqrt{3}-\pi)$

(b) Area of the given loop = (area of the sector PNIMP - area of the triangle MPN)

Area of sector PNIMP = $\frac{1}{3}\pi a^2$

Area of the triangle MPN = ON x OP = $a^2 \sin 60^\circ \cos 60^\circ$

Hence area of the given loop $= \frac{1}{3}\pi a^2 - a^2 \sin 60^\circ \cos 60^\circ$

$$= \frac{a^2}{12}\left[4\pi - 3\sqrt{3}\right]$$

The required torque = $A\vec{I} \, x \vec{B}$

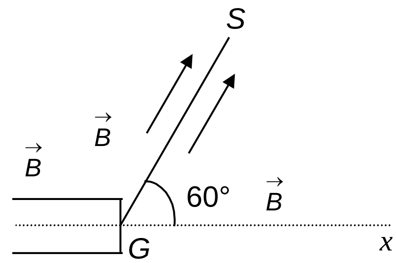
$$= AI\,(\vec{i}\, x \vec{j})\; x\; B\vec{i}$$

$$= ABI\,(\vec{j}) = Bi\left[\frac{a^2}{12}(4\pi - 3\sqrt{3})\right]\vec{j}$$

The direction of this torque is along y-axis.

Illustration 28.

An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S, where GS = 0.1 m and the line GS makes an angle of 60° with x-axis as shown in Figure. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.



Solution:

The velocity of electrons can be resolved into the following two components:

Component parallel to the magnetic field, v = v cos 60^0

Component perpendicular to the magnetic field, $v_\perp$ = v sin 60^o

Now the component v will make the electro to revolve in a circular path perpendicular to B such that

$$\frac{mv_\perp^{\ 2}}{r} = qBv_\perp$$

$$\therefore \qquad r = \frac{mv_\perp^{\ 2}}{qB} \qquad \ldots(i)$$

The component $v_\perp$ will make the electron to move in the direction of B. Time taken by the electron to cover the distance GS is

$$t = \frac{\text{distance}\, GS}{v} = \frac{0.1m}{v\cos 60^\circ} \qquad \ldots(ii)$$

In order that the electrons hit the spot S, time taken by electron to cover the distance GS must be equal to integral multiples of time period of circular motion

$$t = n(T) \qquad \ldots \text{(iii)}$$

where $T = \dfrac{2\pi r}{v\sin 60^\circ}$ … (iv)

Substituting the value of r from (1) in (4)

$$T = \frac{2\pi m}{qB}$$

$\therefore$ equation (3) becomes $t = n\left(\dfrac{2\pi m}{qB}\right)$

Substituting from (2) and rearranging

$$\frac{B}{n} = \frac{2\pi m}{q}\left(\frac{v\cos 60^\circ}{0.1}\right)$$

For $B_{minimum}$ we set n = 1,

$$\therefore B_{minimum} = \frac{2\pi m}{q}\left(\frac{v\cos 60^\circ}{0.1}\right) = \frac{\pi m}{0.1q}v \qquad \ldots \text{(v)}$$

But $\dfrac{1}{2}mv^2 = E$ or $v = \sqrt{2E/m}$

$$B_{min} = \frac{\pi m}{0.1q}\sqrt{\frac{2E}{m}} = \frac{\pi\sqrt{(2E/m)}}{q \times 0.1}$$

$= 4.736 \times 10^{-3}$ tesla