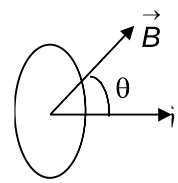
1. MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field $\vec{B}$ in that region. The magnetic flux through the area $\vec{A}$ is given by

$$\phi = \vec{B} \cdot \vec{A} \qquad \dots (\mathbf{1})$$

$$= BA\cos\theta$$



where θ is the angle which the vector B makes with the normal to the surface. If $\vec{B} \perp$ to $\vec{A}$, then the flux through the closed area $\vec{A}$ is zero. SI unit of flux is weber (Wb).

2. FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday in 1831 discovered that whenever magnetic flux linked with a closed conducting loop changes an emf is induced in it. If the circuit is closed, a current flows through it as long as magnetic flux is changing. This current is called induced current the emf induced in the loop is called induced emf and the phenomenon is known as electromagnetic induction.

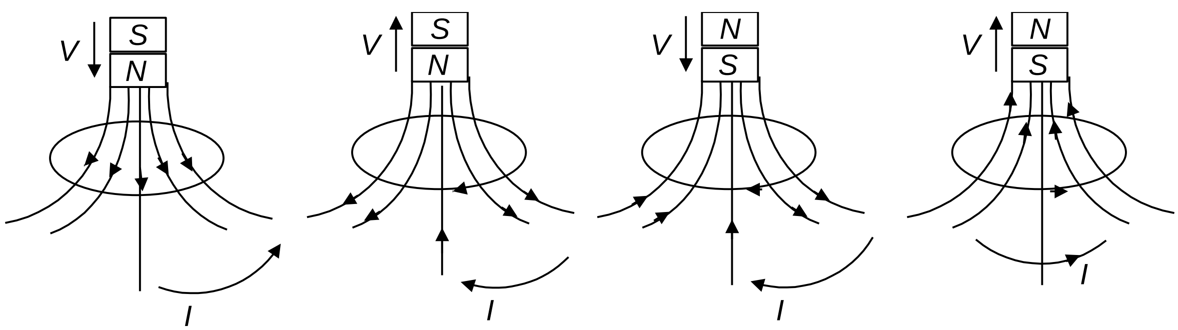


Figure shows a bar magnet placed along the axis of a conducting loop connected with a galvanometer. In this case there is no current in the galvanometer. Now if we move the magnet towards the loop there is a deflection in the galvanometer showing that there is an electric current inside the loop. If the magnet is moved away from the loop a current flows again in the loop but in the opposite direction. This current exists as long as the magnet is moving. Faraday studied this behaviour in detail and discovered the following law of nature.

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\varepsilon = -\frac{d\phi}{dt} \qquad \dots (2)$$

where $\phi = \int \vec{B}.d\vec{A}$ the flux of magnetic field through the area.

The emf so produced drives an electric current through the loop. If resistance of the loop is R, then the current

$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt}. \qquad \dots (3)$$

Illustration 1.

Show that if the flux of magnetic induction through a coil changes from ϕ_1 to ϕ_2, then the charge q that flows through the circuit of total resistance R is given by $q = \frac{\phi_2 - \phi_1}{R}$, where R is the resistance of the coil

Solution:

Let ϕ be the instantaneous flux. Then $\frac{d\phi}{dt}$ is the instantaneous rate of change of flux which is equal to the magnitude of the instantaneous emf. So current in the circuit $|i| = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$, since current is the rate of flow of charge, that is, $i = \frac{dq}{dt}$

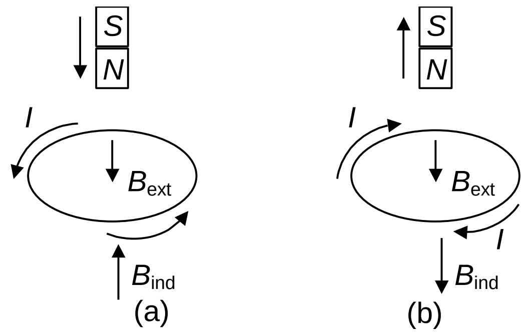
$$q = \int i dt \quad \text{or} \quad q = \int_{t=0}^{t=t} \left(\frac{d\phi}{dt}/R\right) dt$$

where τ is the time during which change takes place. But at $t = 0$, $\phi = \phi_1$ and at $t = t$, $\phi = \phi_2$

$$\therefore \qquad q = \frac{1}{R}\int_{\phi=\phi_1}^{\phi=\phi_2} d\phi = \frac{\phi_2 - \phi_1}{R}$$

3. LENZ'S LAW

The effect of the induced emf is such as to oppose the change in flux that produces it.



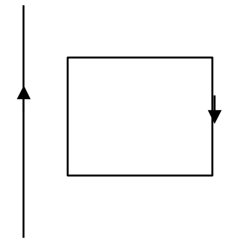
In figure (a) as the magnet approaches the loop, the positive flux through the loop increases. The induced current sets up an induced magnetic field. B_{ind} whose negative flux opposes this change. The direction of B_{ind} is opposite to that of external field B_{ext} due to the magnet.

In figure (b) the flux through the loop decreases as the magnet moves away from the loop, the flux due to the induced magnetic field tries to maintain the flux through the loop, so the direction of induced current is opposite to that first case. The direction of B_{ind} is same as that of B_{ext} due to magnet.

Lenz's law is closely related to law of conservation of energy and is actually a consequence of this general law of nature. As the north pole of the magnet moves towards the loop which opposes the motion of N-pole of the bar magnet. Thus, in order to move the magnet toward the loop with a constant velocity an external force is to be applied. The work done by this external force gets transformed into electric energy, which induces current in the loop.

There is another alternative way to find the direction of current inside the loop which is described below.

Figure shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, then there will be an emf induced inside the loop. Due to this induced emf an electric current is induced. To determine the direction of current inside the loop we put an arrow as shown. The right hand thumb rule shows the positive normal to the loop is going into the plane. Again the same rule shows that the magnetic field at the site of the loop is also going into the plane of the diagram.



Thus $\vec{B}$ and $d\vec{A}$ are in same direction. Therefore $\int \vec{B}.d\vec{A}$ is positive if i increases, the magnitude of ϕ increases. Since ϕ is positive and its magnitude increases, $\frac{d\phi}{dt}$ is positive. Thus ε is negative and hence the current is negative. Thus the current induced is opposite, to that of arrow.

4. CIRCULATION OF INDUCED EMF

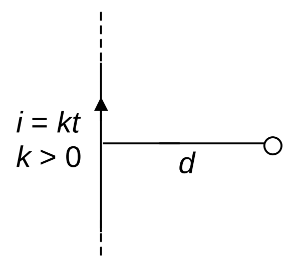
As we know that magnetic flux (ϕ) linked with a closed conducting loop = $B.A \cos\theta$

where B is the strength of the magnetic field, A is the magnitude of the area vector and θ is the angle between magnetic field vector and area vector.

Hence flux will be affected by change in any of them, which is discussed below.

4.1 BY CHANGING THE MAGNETIC FIELD

Consider a long infinite wire carrying a time varying current $i = kt$ ($k > 0$), since current in the wire is continuously increasing therefore we conclude that magnetic field due to this wire in the reason is also increasing. Let a circular loop of radius a ($a << d$) is placed at a distance d from the wire. The resistance of the loop is R.



Now magnetic field B due to wire = $\frac{\mu_0 i}{2\pi d}$ going into and perpendicular to the plane of the paper

Flux through the circular loop

$$\phi = \frac{\mu_0 i}{2\pi d} \times \pi a^2$$

$$\phi = \frac{\mu_0 a^2 kt}{2d}$$

Induced e.m.f. in the loop.

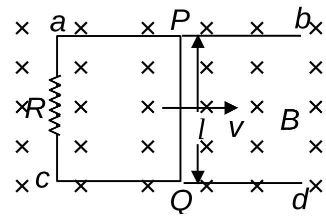
$$\therefore \quad \varepsilon = -\frac{d\phi}{dt} = \frac{-\mu_0 a^2 k}{2d}$$

Induced current in the loop $I = \dfrac{|\varepsilon|}{R} = \dfrac{\mu_0 a^2 k}{2dR}$

Direction of induced current is anticlockwise

4.2 BY CHANGING THE AREA

Figure shows a conductor kept in contact with two conducting wires *ab* and *cd* which are connected with the help of a resistance *R*. The entire setup is placed inside a region of uniform magnetic field. Let conductor start moving with uniform velocity *v* away from resistance as shown. As the conductor moves away the area of closed loop increases causing an induced e.m.f. in the closed loop.



Let at any time *t* the wire *PQ* is at a distance *x* from *ac*

So flux ϕ of magnetic field through closed loop is

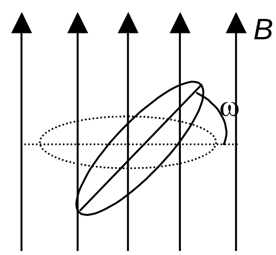
$$\varepsilon = -\frac{d\phi}{dt} = -\frac{Bldx}{dt}$$

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$$

direction of current is anticlockwise which can be found either by Faraday's law or by Lenz's law.

4.3 BY CHANGING THE ANGLE

Let us consider the case when the magnitude of the magnetic field strength and the area of the coil remains constant. When the coil is rotated relative to the direction of the field, an induced current is produced which lasts as long as coil is rotating.



We have, $\phi = BA\cos\theta$ [where *B* is the magnetic field strength, *A* is the magnitude of the area vector & θ is the angle between them]

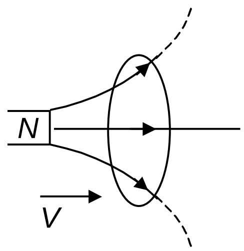
If the angular velocity with which the coil is rotating is ω then $\theta = \omega t$

Induced e.m.f in the coil

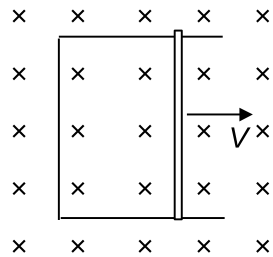
$$\varepsilon = -\frac{d\phi}{dt} = BA\omega \sin \omega t$$

Induced current in the coil $= I = \dfrac{|\varepsilon|}{R} = \dfrac{B\omega A}{R}\sin\omega t$

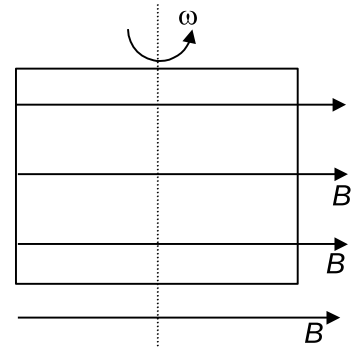
So we conclude that induced current in any circuit is caused either by the change of magnetic field vector or by the change of area vector.



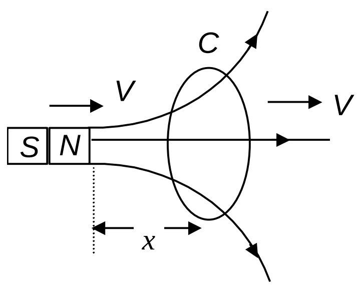
Flux changes due to change of magnetic field



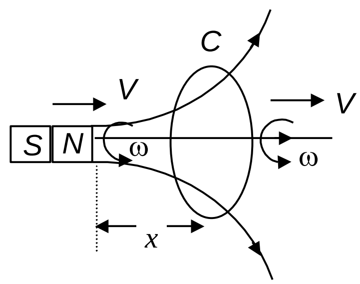
Flux changes due to change of magnitude of area



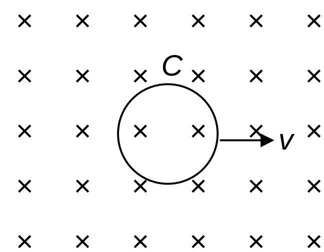
Flux changes due to change in the angle between magnetic field vector and area vector.



(a)



(b)



(c)

n figure (a), (b) & (c) the flux linked with the loop C will not change with time as here none of the quantities i.e B, A and θ changes.

Hence, $\phi \neq 0$ but $\dfrac{d\phi}{dt} = 0$

So e.m.f. induced inside the loop will be zero.

Illustration 2.

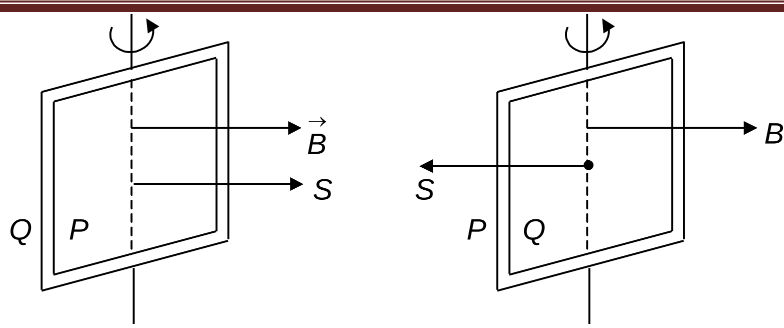
A 10 ohm coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 Gauss. The coil is turned through 180° in (1/10) s. Calculate (a) the change in flux (b) the average induced emf (c) average induced current and (d) the charge that flows in the circuit.

Solution:

(a) When the plane of coil is perpendicular to the field as shown in figure (a) the angle between area $\vec{S}$ and field $\vec{B}$ is 0°. So the flux linked with the coil,

$$\phi_1 = NSB \cos 0 = NSB$$

When the coil is turned through 180° as shown in figure (b), the flux linked with the coil will be $\phi_2 = NSB \cos 180 = -NSB$



(a) (b)

So change in flux,

$$\Delta\phi = \phi_2 - \phi_1 = -NSB - (NSB) = -2NSB$$

i.e., $|\Delta\phi| = 2 \times 10^3 \times (500 \times 10^{-4}) \times (0.4 \times 10^{-4}) =$ **4m Wb**

(b) As in turning through 180°, i.e., in change of flux $\Delta\phi$, the coil takes (1/10)s,

$$|\varepsilon_{av}| = \frac{|\Delta\phi|}{\Delta t} = \frac{2NSB}{\Delta t} = \frac{4\times10^{-3}}{10^{-1}} = \mathbf{40\ mV}$$

(c) $I_{av} = \dfrac{\varepsilon_{av}}{R} = \dfrac{0.04}{10} = \mathbf{4\ mA}$

(d) the charge that flows through the circuit $= \dfrac{\Delta\phi}{R} = \dfrac{4\times10^{-3}}{10} C = \mathbf{400\ \mu C}$

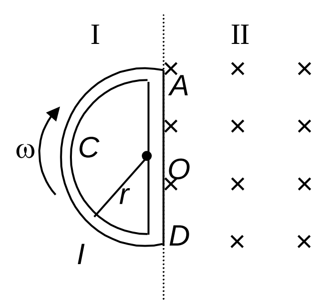
Illustration 3.

A space is divided by the line *AD* into two regions. Region I is field free and the region II has a uniform magnetic field *B* directed into the paper. *ACD* is a semi-circular conducting loop of radius *r* with centre at *O*, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through *O*, and perpendicular to the plane of the paper in the clockwise direction. The effective resistance of the loop is *R*.

(a) Obtain an expression for the magnitude of the induced current in the loop

(b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced emf and the time of rotation for two periods of rotation.



Solution:

(a) As in time *t*, the arc swept by the loop in the field, i.e., region II,

$$S = \frac{1}{2} r(r\theta) = \frac{1}{2} r^2 \omega t$$

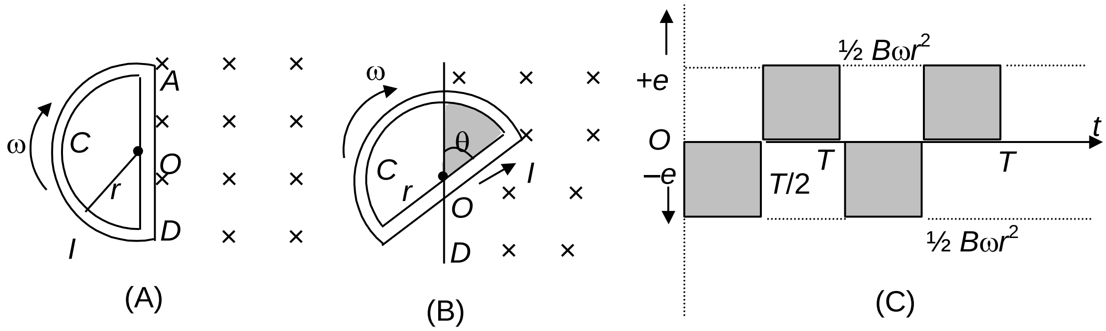
So the flux linked with the rotating loop at time *t*,

$$\phi = BS = \frac{1}{2}B\omega r^2 t \ \ [\theta = \omega t]$$

and hence the induced emf in the loop, $\varepsilon = -\frac{d\phi}{dt} = -\frac{1}{2}B\omega r^2$ = constant.

and as the resistance of the loop is R, the induced current in it,

$$I = \frac{\varepsilon}{R} = \frac{B\omega r^2}{2R}$$



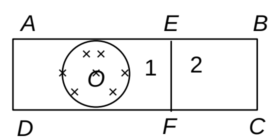
(A) (B) (C)

(b) When the loop is entering the region II, i.e., the field figure (b), the inward flux linked with it will increase, so in accordance with Lenz's law an anticlockwise current will be induced in it.

(c) Taking induced emf to the negative when flux linked with the loop is increasing and positive when decreasing, the emf versus time graph will be, as shown in figure (c).

Illustration 4.

A magnetic field perpendicular to the plane of the rectangular frame of wire is concentrated about O. if the field decreases, will there be any emf induced in loop 1 and 2?



Solution:

There is an induced emf in loop 1 because it encloses the field and so there is a change of flux. No emf is induced in loop 2 as it does not enclose any field.

Illustration 5.

Two parallel, long, straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now they start moving out with a constant velocity (v). (a) Will the induced emf be time dependent? (b) will the current be time dependent?

Solution:

Yes, φ (instantaneous flux) = $B(a + 2vt)^2$

$$\therefore \qquad \varepsilon = \frac{d\varphi}{dt} = 4Bv\,(a + 2vt)$$

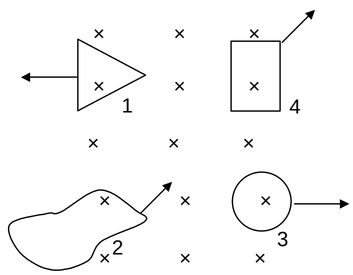
(b) No. i instantaneous current) = ε/R

Now $R = 4\,(a + 2vt)\,r$ where r = resistance per unit length

$$\therefore \quad i = \frac{4Bv\,(a+2vt)}{4r\,(a+2vt)} = \frac{Bv}{r} \quad \text{(a constant)}$$

Illustration 6.

The loops in the figure move into or out of the field which is along the inward normal to the plane of the paper. Indicate the direction of currents in loops 1, 2, 3, 4

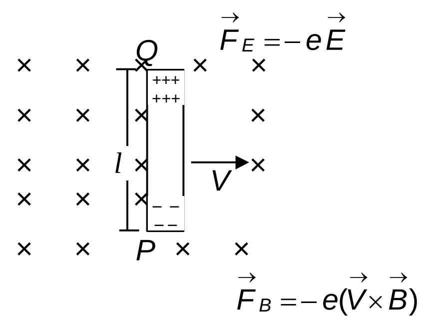


Solution:

In 1, flux decreases and so induced current must be clockwise to increase the flux. Due to the same reason currents in 3 and 4 are clockwise but in 2 current must be anticlockwise as flux is increasing.

5. MECHANICSM OF THE INDUCED EMF ACROSS THE ENDS OF A MOVING ROD

Figure shows a conducting rod of length l moving with a constant velocity v in a uniform magnetic field. The length of the rod is perpendicular to magnetic field, and velocity is perpendicular to both the magnetic field and the length of the rod. An electron inside the conductor experiences a magnetic force $\vec{F}_B = -e(\vec{v} \times \vec{B})$ directed downward along the rod. As a result electrons migrate towards the lower end and leave unbalanced positive charges at the top. This redistribution of charges sets up an electric field E directed downward. This electric field exerts a force on free electrons in the upward direction. As redistribution continues electric field grows in magnitude until a situation, when



$$|q\vec{V} \times \vec{B}| = |q\vec{E}|$$

After this, there is no resultant force on the free electrons and the potential difference across the conductor is

$$\int d\varepsilon = -\vec{E}.d\vec{l} = \int(\vec{v} \times \vec{B}).d\vec{l} \qquad \ldots\ (4)$$

Thus it is the magnetic force on the moving free electrons that maintains the potential difference. So e.m.f. developed across ends of the rod moving perpendicular to magnetic field with velocity perpendicular to the rod,

$$\varepsilon = vBl \qquad \ldots\ (5)$$

As this emf is produced due to the motion of the conductor, it is called motional emf.

In the problems related to motional e.m.f. we can replace the rod by a battery of e.m.f. vBl.

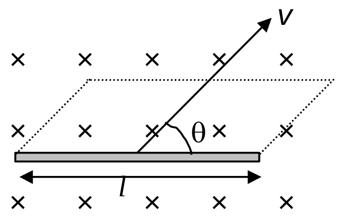
Illustration 7.

A 0.4 metre long straight conductor moves in a magnetic field of magnetic induction 0.9 Wb/m^2 with a velocity of 7 m/sec. Calculate the emf induced in the conductor under the condition when it is maximum.

Solution:

If a rod of length l is moved with velocity $\vec{v}$ an at angle θ to the length of the rod in a field $\vec{B}$ which is perpendicular to the plane of the motion, the flux linked with the area generated by the motion of rod in time t,

$$\phi = Bl(v \sin\theta)t \ \text{ so, } \ |\varepsilon| = \frac{d\phi}{dt} = Bvl \sin\theta$$

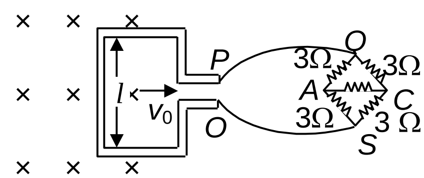


This will be maximum when sin θ = max = 1, i.e., the rod is moving perpendicular to its length and then

$$(\varepsilon)_{max} = Bvl = 0.9 \times 7 \times 0.4 = \mathbf{2.52\ V}$$

Illustration 8.

A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 Wb/m^2 as shown in figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 3 ohm. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop? Find the direction of current in the loop?



Solution:

As the network $AQCS$ is a balanced Wheatstone bridge, no current will flow through AC and hence the effective resistance of the network between QS.,

$$R_{QS} = \frac{6 \times 6}{6+6} = \mathbf{3\ ohm}$$

and as the resistance of the loop is 1 ohm, the total resistance of the circuit,

$$R = 3 + 1 = 4 \text{ ohm}$$

Now if the loop moves with speed v_0, the emf induced in the loop,

$$\varepsilon = Bv_0 l$$

So the current in the circuit, $I = \dfrac{\varepsilon}{R} = \dfrac{Bv_0 l}{R}$

Substituting the given data,

$$v_0 = \frac{IR}{Bl} = \frac{1 \times 10^{-3} \times 4}{2 \times 0.1} = 2 \times 10^{-2} \text{ ms}^{-1} = \mathbf{2\ m/s}$$

In accordance with Lenz's law the induced current in the loop will be in clockwise direction.

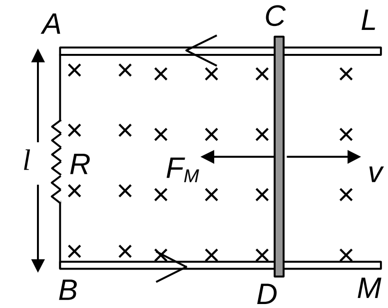
Illustration 9.

Two parallel wires *AL* and *BM* placed at a distance l are connected by a resistor R and placed in a magnetic field B which is perpendicular to the plane containing the wires. Another wire *CD* now connects the two wires perpendicularly and made to slide with velocity v. Calculate the work done per second needed to slide the wire *CD*. Neglect the resistance of all the wires.

Solution:

When a rod of length l moves in a magnetic field with velocity v as shown in figure, an emf Bvl will be induced in it. Due to this induced emf, a current

$I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$ will flow in the circuit as shown in figure. Due to this induced current, the wire will experience a force



$$F_M = BIl = \frac{B^2 l^2 v}{R}$$

which will oppose its motion. So to maintain the motion of the wire *CD* a force $F = F_M$ must be applied in the direction of motion, and so the work done per second, i.e., power needed to slide the wire,

$$P = \frac{dW}{dt} = Fv = F_M\, v = \frac{B^2 v^2 l^2}{R}$$

6. INDUCED ELECTRIC FIELD DUE TO A TIME VARYING MAGNETIC FIELD

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till $t = 0$ and then changes with time. An induced current starts in the loop at $t = 0$.

The free electrons were at rest till $t = 0$ (we are not interested in the random motion of the electrons). The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field. So we conclude that an electric field appears at time $t = 0$. This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is non-electrostatic and non-conservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field is $q\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is $E.d\vec{l}$. The emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

Using Faraday's law of induction,

$$\varepsilon = -\frac{d\phi}{dt}$$

or, $\oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt}$ ... (**6**)

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

Illustration 10.

A thin, non-conducting ring of mass *m*, carrying a charge *q*, can rotate freely about its axis. At the instant *t* = 0 the ring was at rest and no magnetic field was present. Then suddenly a magnetic field *B* was set perpendicular to the plane. Find the angular velocity acquired by the ring.

Solution:

Due to the sudden change of flux an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$\varepsilon \text{ (induced emf)} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}.d\vec{s}$$

Also $\varepsilon = \oint \vec{E}.d\vec{l}$ where E is the induced electric field.

$$\therefore \quad \oint \vec{E}.d\vec{l} = \frac{d}{dt}\int \vec{B}.d\vec{s} \Rightarrow E.2\pi r = \frac{d}{dt}(B\pi r^2)$$

$$\Rightarrow \quad E = -\frac{r}{2}\frac{dB}{dt}$$

Force experienced = $q\,|\vec{E}|$

torque experienced $\tau = (qE)\,r = \frac{qr^2}{2}\frac{dB}{dt}$

$\therefore$ angular impulse experienced

$$= \int \tau dt = \frac{qr^2}{2}\int \frac{dB}{dt}dt = qr^2\,\frac{B}{2}$$

Also angular impulse acquired = $I\omega$ where I is moment of inertia of the ring about its axis $= mr^2$

$\therefore\ mr^2\omega = qr^2B/2 \qquad \Rightarrow \omega = qB/2m$

7. SELF INDUCTANCE

When the current flows in a coil, it gives rise to a magnetic flux through the coil itself. When the strength of current changes, the flux also changes and an e.m.f. is induced in the coil. This e.m.f. is called self induced e.m.f. and the phenomenon is known as self induction.

The flux through the coil is proportional to current through it, i.e.,

$$\phi \propto i$$

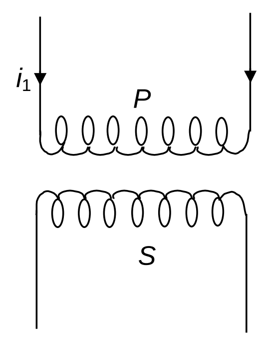
$$\therefore \qquad \varepsilon = -\frac{d\phi}{dt} \text{ i.e., } \varepsilon \propto \frac{di}{dt}$$

$$\therefore \qquad \varepsilon = -L\frac{di}{dt} \qquad \ldots (\mathbf{7})$$

Where L is a constant of proportionality and is called self-inductance or simply inductance of the coil. The unit of inductance is Henry. If the coil is wound over an iron core, the inductance is increased by a factor μ (permeability of iron).

8. MUTUAL INDUCTANCE

Consider two coils P and S placed close to each other as shown in the figure. When current passing through a coil increases or decreases, the magnetic flux linked with the other coil also changes and an induced e.m.f. is developed in it. This phenomenon is known as mutual induction. This coil in which current is passed is known as primary and the other in which e.m.f. is developed is called as secondary.



Let the current through the primary coil at any instant be i_1. Then the magnetic flux ϕ_2 in the secondary at any time will be proportional to i_1 i.e.,

$$\phi_2 \propto i_1$$

Therefore the induced e.m.f. in secondary when i_1 changes is given by

$$\varepsilon = -\frac{d\phi_2}{dt} \text{ i.e., } \varepsilon \propto -\frac{di_1}{dt}$$

$$\therefore \qquad \varepsilon = -M\frac{di_1}{dt} \qquad \ldots (\mathbf{8})$$

where M is the constant of proportionality and is known as mutual inductance of two coils. It is defined as the e.m.f. Induced in the secondary coil by unit rate of change of current in the primary coil. The unit of mutual inductance is henry (H).

8.1 MUTUAL INDUCTANCE OF A PAIR OF SOLENOIDS ONE SURROUNDING THE OTHER COIL

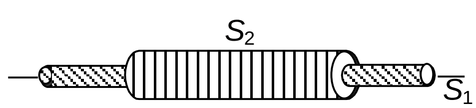


Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.

To calculate M between them, let us assume a current i_1 through the inner solenoid S_1

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0\left(\frac{N_1}{l_1}\right)i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B2} through the surrounding coil is, therefore,

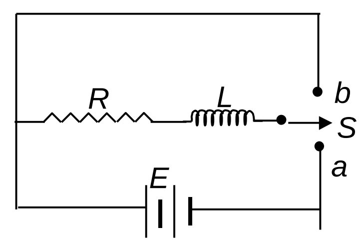
$$\phi_{B2} = B\,(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\pi R_1^2$$

Now, $$M = \frac{N_2 \phi_{B2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1},\ M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior.

9. GROWTH AND DECAY OF CURRENT IN L-R CURCUIT

Consider a circuit containing a resistance R, an inductance L, a two way key and a battery of e.m.f. E connected in series as shown in figure. When the switch S is connected to a, the current in the circuit grows from zero value. The inductor opposes the growth of the current this is due to the fact that when the current grows through inductance, a back e.m.f. is developed which opposes the growth of current in the circuit. So the rate of growth of current is reduced. During the growth of current in the circuit, let i be the current in the circuit at any instant t. Using Kirchoffs voltage low in the circuit we obtain



$$E - L\frac{di}{dt} = Ri$$

or $$E - Ri = \frac{Ldi}{dt}$$

or $$\frac{di}{E - Ri} = \frac{dt}{L}$$

Multiplying by $-R$ on both the sides, we get

$$\frac{-R\,di}{E - Ri} = \frac{-Rdt}{L}$$

Integrating the above equation, we have

$$\log_e (E - Ri) = -\frac{R}{L}t + A \qquad \ldots \text{(i)}$$

Where A is integration constant. The value of this constant can be obtained by applying the condition that current i is zero just at start i.e. at $t = 0$. Hence

$$\log_e E = 0 + A$$

or $$A = \log_e E \qquad \ldots \text{(ii)}$$

Substituting the value of A from equation (ii) in equation (i) we get

$$\log_e (E - Ri) = -\frac{R}{L}t + \log_e E$$

or $$\log_e\left(\frac{E-Ri}{E}\right)=-\frac{R}{L}t$$

or $$\left(\frac{E-Ri}{E}\right)=\exp.\left(-\frac{R}{L}t\right)$$

or $$1-\frac{Ri}{E}=\exp.\left(-\frac{R}{L}t\right)$$

or $$\frac{Ri}{E}=\left\{1-\exp.\left(-\frac{R}{L}t\right)\right\}$$

$\therefore$ $$i=\frac{E}{R}\left\{1-\exp.\left(-\frac{R}{L}t\right)\right\}$$

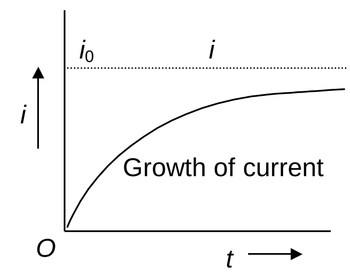
The maximum current in the circuit $i_0 = E/R$. So

$$i=i_0\left\{1-\exp.\left(-\frac{R}{L}t\right)\right\} \qquad \dots \textbf{(9)}$$

Equation (9) gives the current in the circuit at any instant t. It is obvious from equation (9) that $i = i_0$, when

$$\exp.\left(-\frac{R}{L}t\right) = 0 \text{ i.e., at } t = \infty$$

Hence the current never attains the value i_0 but it approaches it asymptotically. A graph between current and time is shown in figure.



We observe the following points

(i) When $t = (L/R)$ second, then

$$i = i_0\left\{1-\exp.\left(-\frac{R}{L}\times\frac{L}{R}\right)\right\}$$

$$= i_0\,\{1-\exp.(-1)\}=i_0\left(1-\frac{1}{e}\right)$$

$$= i_0\left(\frac{e-1}{e}\right)=i_0\left(\frac{2.718-1}{2.718}\right)$$

$$= 0.63\, i_0$$

Thus after an interval of (L/R) second, the current reaches to a value which is 63% of the maximum current. The value of (L/R) is known as time constant of the circuit and is represented by λ. Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 63% of its final value. In terms of λ,

$$i = i_0\,(1-e^{-t/\lambda})$$

(ii) The rate of growth of current (*di/dt*) is given by

$$\frac{di}{dt} = \frac{d}{dt}\left[i_0\left\{1 - \exp.\left(-\frac{R}{L}t\right)\right\}\right]$$

$$\Rightarrow \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\exp.\left(-\frac{R}{L}t\right) \qquad \ldots \textbf{(10)}$$

From equation (3), exp. $\left(-\frac{R}{L}t\right) = \frac{i_0 - i}{i_0}$

$$\therefore \qquad \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\left(\frac{i_0 - i}{i_0}\right) = \frac{R}{L}\left(i_0 - i\right) \qquad \ldots \textbf{(11)}$$

This shows that the rate of growth of the current decreases as i tends to i_0. For any other value of current, it depends upon the value of R/L. Thus greater is the value of time constant, smaller will be the rate of growth of current.

9.2 DECAY OF CURRENT

Let the circuit be disconnected from battery and switch S is thrown to point b in the figure. The current now begins to fall. In the absence of inductance, the current would have fallen from maximum i_0 to zero almost instantaneously. But due to the presence of inductance, which opposes the decay of current, the rate of decay of current is reduced.

Suppose during the decay of current, i be the value of current at any instant t. Using Kirchhoff's voltage law in the circuit we get

$$-L\frac{di}{dt} = Ri$$

or $$\frac{di}{dt} = -\frac{R}{L}i$$

Integrating this expression, we get

$$\log_e i = -\frac{R}{L}t + B$$ Where B is constant of integration.

The value of B can be obtained by applying the condition that when $t = 0$, $i = i_0$

$\therefore \qquad \log_e i_0 = B$

Substituting the value of B, we get

$$\log_e i = -\frac{R}{L}t + \log_e i_0$$

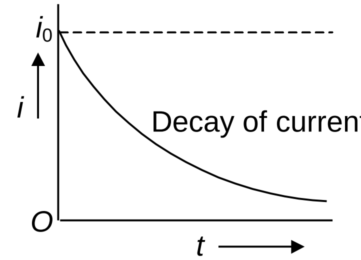
or $$\log_e \frac{i}{i_0} = -\frac{R}{L}t$$

or $$(i/i_0) = \exp.\left(-\frac{R}{L}t\right)$$

or $\quad i = i_0 \exp.\left(-\frac{R}{L}t\right) = i_0 \exp.(-t/\lambda) \quad \ldots$ **(12)**

where $\lambda = L/R$ = inductive time constant of the circuit.

It is obvious from equation that the current in the circuit decays exponentially as shown in figure.



We observe the following points

(i) After $t = L/R$ second, the current in the circuit is given by

$$i = i_0 \exp.\left(-\frac{R}{L}\times\frac{L}{R}\right) = i_0 \exp.(-1)$$

$$= (i_0/e) = i_0/2.718 = 0.37\, i_0$$

So after a time (L/R) second, the current reduces to 37% of the maximum current i_0. (L/R) is known as time constant λ. This is defined as the time during which the current decays to 37% of the maximum current during decay.

(ii) The rate of decay of current is given by

$$\frac{di}{dt} = \frac{d}{dt}\left\{i_0 \exp.\left(-\frac{R}{L}t\right)\right\}$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{R}{L} i_0 \exp.\left(-\frac{R}{L}t\right) = -\frac{R}{L} i \quad \ldots \textbf{(13)}$$

or $\quad -\frac{di}{dt} = \frac{R}{L} i$

This equation shows that when L is small, the rate of decay of current will be large i.e., the current will decay out more rapidly.

Illustration 11.

In the circuit diagram shown in figure. $R = 10\ \Omega$. $L = 5\ H$, $E = 20$ V , $i = 2$A. This current is decreasing at a rate of –1.0 A/s. Find V_{ab} at this instant.

Solution:

P.D. across inductor,

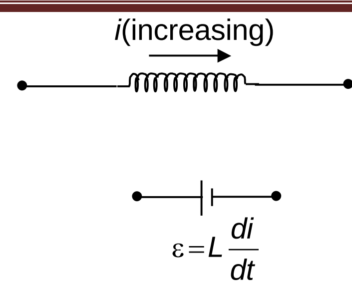
$$V_L = L\frac{di}{dt} = (5)(-1.0) = -5 \text{ volt}$$

Now, $V_a - iR - V_L - E = V_b$

$\therefore \quad V_{ab} = V_a - V_b = E + iR + V_L$

$= 20 + (2)(10) - 5 =$ **35 volt**

10. ENERGY STORED IN AN INDUCTOR



The energy of a capacitor is stored in the electric field between its plates. Similarly an inductor has the capability of storing energy in its magnetic field.

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power.

$$P = \frac{dW}{dt} = -\varepsilon i = -Li\frac{di}{dt}$$

As $dW = -dU$

or, $\dfrac{dW}{dt} = -\dfrac{dU}{dt}$

we have, $\dfrac{dU}{dt} = Li\dfrac{di}{dt}$

or, $dU = Lidi$

The total energy U supplied while the current increases from zero to a final value i is,

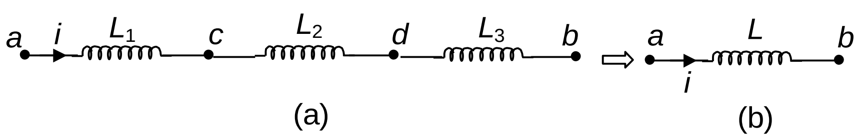
$$U = L\int_0^i idi = \frac{1}{2}Li^2$$

$$U = \frac{1}{2}Li^2 \qquad \ldots (\mathbf{14})$$

11. COMBINATION OF INDUCTANCES

11.1 SERIES COMBINATION

In series: If several inductances are connected in series and there are no interactions between them then their equivalent inductance is obtained by following method



Refer figure (a)

$$V_a - V_c = L_1\frac{di}{dt}$$

$$V_c - V_d = L_2\frac{di}{dt} \text{ and } V_d - V_b = L_3\frac{di}{dt}$$

adding all these equations, we have

$$V_a - V_b = (L_1 + L_2 + L_3)\frac{di}{dt} \qquad \ldots \text{(i)}$$

Refer figure (b)

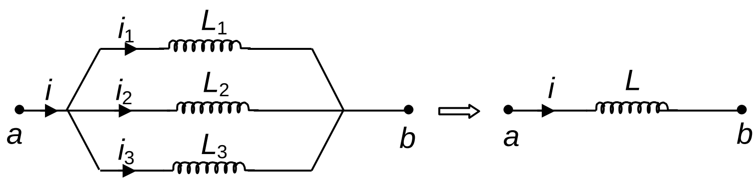
$$V_a - V_b = L\frac{di}{dt} \quad \ldots \text{(ii)}$$

Here L = equivalent inductance.

From equations (i) and (ii), we have $L = L_1 + L_2 + L_3$ … (**15**)

11.2 PARALLEL COMBINATION

If several inductors are connected in parallel combination such that there are no interactions between them, then their equivalent inductance is obtained by following method.



(a) (b)

Refer figure (a)

$$i = i_1 + i_2 + i_3$$

or $$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

or $$\frac{di}{dt} = \frac{V_a - V_b}{L_1} + \frac{V_a - V_b}{L_2} + \frac{V_a - V_b}{L_3} \quad \ldots \text{(i)}$$

Refer figure (b)

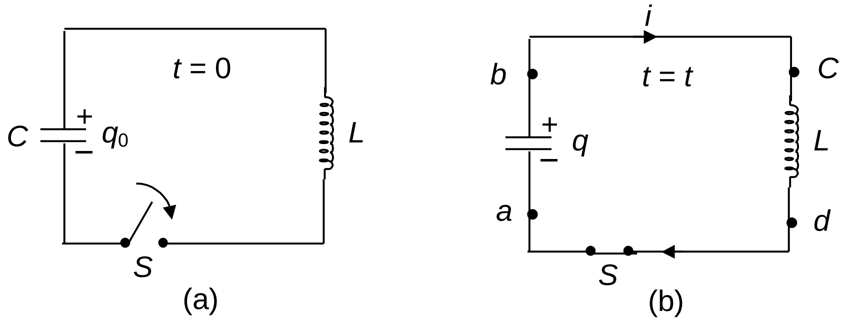
$$\frac{di}{dt} = \frac{V_a - V_b}{L} \quad \ldots \text{(ii)}$$

From equation (i) and (ii),

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \quad \ldots (\mathbf{16})$$

12. OSCILLATIONS IN L-C CIRCUIT

If a charged capacitor C is short-circuited though an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. Assume an idealized situation in which energy is not radiated away from the circuit. With these idealizations-zero resistance and no radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor' s magnetic field back and forth. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy.



Let us now derive an equation for the oscillations in an L-C circuit.

Refer figure (a): The capacitor is charged to a potential difference V such that charge on capacitor $q_0 = CV$

Here q_0 is the maximum charge on the capacitor. At time $t = 0$, it is connected to an inductor through a switch *S*. At time $t = 0$, switch S is closed.

Refer figure (b): When the switch is closed, the capacitor starts discharging. Let at time *t* charge on the capacitor is q ($< q_0$) and since, it is further decreasing there is a current *i* in the circuit in the direction shown in figure.

The potential difference across capacitor = potential difference across inductor, or

$$V_b - V_a = V_c - V_d$$

$$\therefore \quad \frac{q}{C} = L\left(\frac{di}{dt}\right) \qquad \ldots \text{(i)}$$

Now, as the charge is decreasing, $i = \left(\frac{-dq}{dt}\right)$

or $\quad \frac{di}{dt} = -\frac{d^2q}{dt^2}$

Substituting in equation (i), we get

$$\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$$

or $$\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q \qquad \ldots \text{(ii)}$$

This is the standard equation of simple harmonic motion $\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$

Here $$\omega = \frac{1}{\sqrt{LC}} \qquad \ldots \text{(iii)}$$

The general solution of equation (ii), is

$$q = q_0 \cos(\omega t \pm \phi) \qquad \ldots \textbf{(17)}$$

In our case $\phi = 0$ as $q = q_0$ at $t = 0$.

Thus, we can say that charge in the circuit oscillates with angular frequency given by equation (iii). Thus, In $L - C$ oscillations, q, i and $\frac{di}{dt}$ all oscillate harmonically with same angular frequency ω. But the phase difference between q and i or between i and $\frac{di}{dt}$ is $\frac{\pi}{2}$. Their amplitudes are q_0, $q_0\omega$ and $\omega^2 q_0$ respectively. So

$$q = q_0 \cos \omega t, \text{ then} \qquad \ldots \textbf{(18)}$$

$$i = \frac{dq}{dt} = -q_0\omega \sin\omega t \text{ and} \qquad \textbf{(19)}$$

$$\frac{di}{dt} = -q_0\omega^2 \cos\omega t \qquad \textbf{(20)}$$

Similarly potential energy across capacitor (U_C) and across inductor (U_L) also oscillate with double the frequency 2ω. The different graphs are shown in figure.

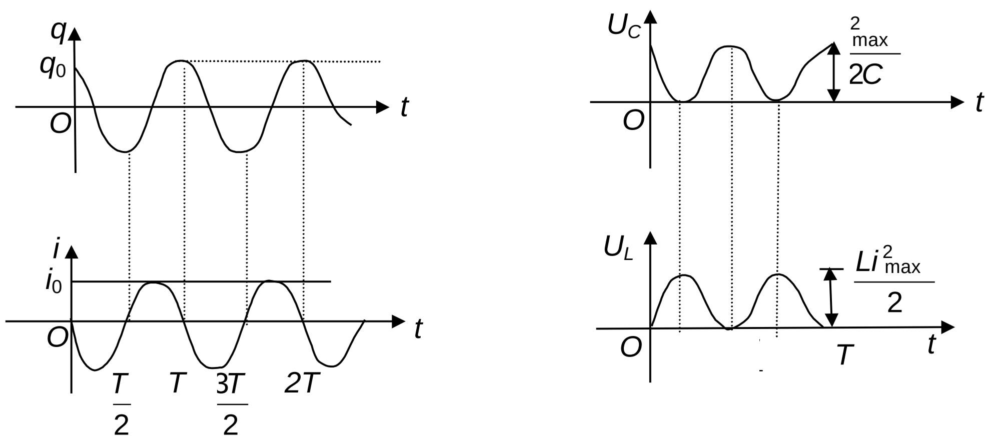


Illustration 12.

A capacitor of capacitance 25 μF is charged to 300 V. It is then connected across a 10 *mH* inductor. The resistance of the circuit is negligible.

(a) Find the frequency of oscillation of the circuit.

(b) Find the potential difference across capacitor and magnitude of circuit current 1.2 ms after the inductor and capacitor are connected.

(c) Find the magnetic energy and electric energy at $t = 0$ and $t = 1.2$ ms.

Solution:

(a) The frequency of oscillation of the circuit is,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Substituting the given values we have, $f = \dfrac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}}$

(b) Charge across the capacitor at time t will be,

$$q = q_0 \cos \omega t$$

and $\quad i = -q_0\omega\sin\omega t$

Here $q_0 = CV_0 = (25 \times 10^{-6})\ (300) = 7.5 \times 10^{-3}$ C

Now, charge in the capacitor after $t = 1.2 \times 10^{-3}$ s is,

$q = (7.5 \times 10^{-3}) \cos (2\pi \times 318.3)\ (1.2 \times 10^{-3})$ C $= 5.53 \times 10^{-3}$ C

$\therefore$ P.D. across capacitor, $V = \dfrac{|q|}{C} = \dfrac{5.53 \times 10^{-3}}{25 \times 10^{-6}} = 221.2$ volt

The magnitude of current in the circuit at t = 1.2×10^{-3} s is,

$|i| = q_0 \omega \sin\omega t$

$= (7.5 \times 10^{-3})\ (2\pi)\ (318.3) \sin (2\pi \times 318.3)\ (1.2 \times 10^{-3})$ A = 10.13 A

(c) At $t = 0$: current in the circuit is zero. Hence, $U_L = 0$

Charge on the capacitor is maximum

Hence, $U_C = \dfrac{1}{2}\dfrac{q_0^2}{C}$

or $U_C = \dfrac{1}{2} \times \dfrac{(7.5 \times 10^{-3})^2}{(25 \times 10^{-6})} = 1.125$ J

$\therefore$ Total energy $E = U_L + U_C = 1.125$ J

At **t = 1.2 ms**

$U_L = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(10 \times 10^{-3})(10.13)^2 = 0.513$ J

$U_C = E - U_L = 1.125 - 9.513 = 0.612$ J

Otherwise U_C can be calculated as,

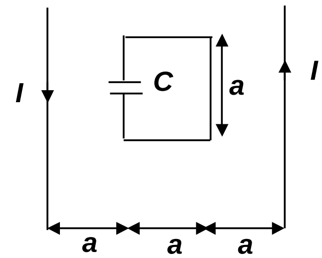
$U_C = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2} \times \dfrac{(5.33 \times 10^{-3})^2}{(25 \times 10^{-6})}$ **= 0.612 J**

Illustration 13.

Two long parallel wires carrying current I are separated by a distance 3a. There exists a square loop of side a with a capacitor of capacity C as shown in figure. The value of current varies with time as $I = I_0 \sin\omega t$

(a) Calculate maximum current in the square loop

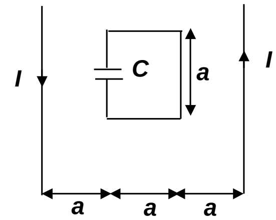
(b) Draw a graph between charge on plate of capacitor as a function of time



Solution:

(a) $I = I_0 \sin\omega t$

Flux linked with square loop



$$\int d\phi = \int_a^{2a} \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a-x}\right] a dx$$

$$= \frac{\mu_0 I a}{2\pi}\left[\ln x - \ln(3a-x)\right]_a^{2a}$$

$$\frac{\mu_0 I a}{2\pi}\left[\ln 2a - \ln a - \ln a + \ln 2a\right]$$

$$= \frac{\mu_0 I a}{2\pi} 2\ln 2 = \frac{\mu_0 I a}{\pi}\ln 2$$

Charge an capacitor = $\left|C\frac{d\phi}{dt}\right| = C\frac{\mu_0 a}{\pi}\ln 2\frac{dI}{dt}$

$$= \frac{\mu_0 C a}{\pi}\ln 2 (I_0\omega)\cos\omega t \quad \ldots \text{(i)}$$

$$= \frac{\mu_0 C(I_0\omega)a}{\pi}\ln 2 \cos\omega t \quad \ldots \text{(ii)}$$

Maximum current $I = \left|\frac{dq}{dt}\right|_{\max} = \frac{\mu_0 C I_0 \omega^2 a}{\pi}\ln 2$

(b) The graph for charge and time can be drawn from equation(i) as shown in figure.

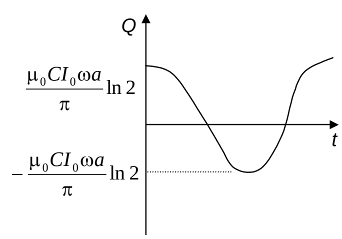
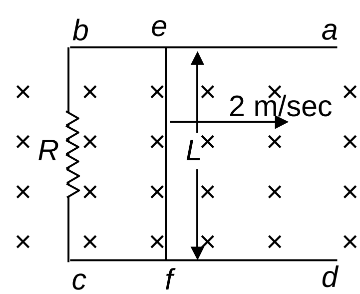


Illustration 14.

The Figure shows a conductor of length l = 0.5 m and resistance r = 0.5 ohm sliding without friction at a velocity v = 2 m/s over two conducting parallel rods ab and cd lying in a horizontal plane. A resistance R = 2.5 Ω connects the ends b and c. A vertical uniform magnetic field of induction B = 0.6 T exists over the region. Determine (i) the current in the circuit, (ii) the force in the direction of motion to be applied to the conductor for the latter to move with the velocity v and (iii) the thermal power dissipated by the circuit. Neglect the resistance of the guiding rods ab and cd.



Solution:

The conductor ef moves with a velocity v perpendicular to a uniform magnetic induction B and hence induces an e.m.f. E = Blv.

The resistance of the circuit = (R + r)

(i) Hence the current in the circuit $=\dfrac{Blv}{R+r}$

$$=\frac{(0.6)(0.5)(2)}{(2.5+0.5)}$$

$= 0.2$ A

(ii) The power spend in the system

$$F.V=\frac{(Blv)^2}{(R+r)}$$

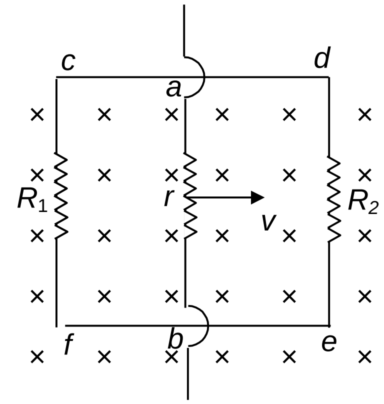
$$F=\frac{B^2l^2v}{R+r}=\frac{(0.6)^2}{3\times 2}=\mathbf{0.06N}$$

A force of 0.06 N is required to maintain the motion of the conductor

(ii) The power generated F.V = $\dfrac{(Blv)^2}{(R+r)}$

$= 0.06 \times 2 = 0.12$ W

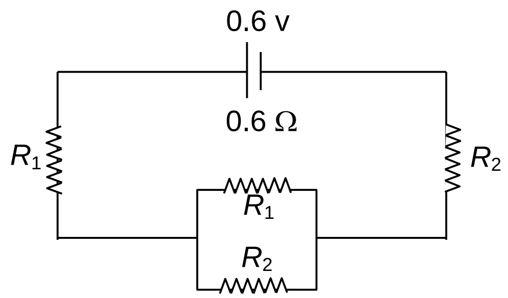
Illustration 15.



A conductor ab of length l = 0.4 m and with a resistance r = 0.6 Ω moves along conducting guides cd and ef with a uniform speed of v = 5 m/s normal to a uniform magnetic field of induction B = 0.3 T. The guides are short circuited with resistances $R_1 = 6\ \Omega$ and $R_2 = 4\ \Omega$ as shown.

Determine (i) the current through the conductor ab, (ii) the current through the resistors R_1 and R_2 and (iii) the mechanical power needed for the motion of the conductor ab. Ignore the resistance and the friction of the guides.

Solution:



The e.m.f. induced in the conductor ab

E = Blv = (0.3T) (0.4 m) (5m/s) = 0.6 volt

The conductor ab can now be taken as a voltage source with an internal resistance r =0.6 ohm while two resistors R_1 = 6 ohm and R_2 = 4 ohm are connected in parallel with it.

Hence the total resistance of the circuit $=\dfrac{6\times 4}{6+4}+0.6\Omega=3\Omega$

(i) The current through the conductor

$$ab=\frac{0.6V}{3\Omega}=\mathbf{0.2\mathring{A}}$$

(ii) The current through the resistor

$$R_1 = 0.2\times\frac{4}{10} = \mathbf{0.08Å}$$

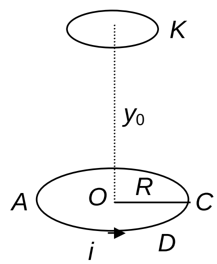
The current through the resistor $R_2 = 0.2Å - 0.08Å == \mathbf{0.12Å}$

(iii) The mechanical power needed for the motion of the conductor

$$ab = \frac{E}{R} = \frac{(0.6)^2}{3}\,watt = \mathbf{0.12W}$$

Illustration 15.

A coil ACD of N turns and radius R carries a current I amp and is placed on a horizontal table. K is a small conducting ring of radius r placed at a distance y_0 from the centre of and vertically above the coil ACD. Find and expression for the e.m.f. established when the ring K is allowed to fall freely. Express the e.m.f. in terms of speed.



Solution:

The field along the axis of the coil ACD is given by

$$\frac{\mu_0}{4\pi}\cdot\frac{2\pi nIR^2}{(R^2+y^2)^{3/2}}$$

Since the ring is small it may be assumed that the induction through it is uniform and is equal to that on the axis.

$\therefore$ the magnetic flux linked with it is

$$\phi = BA = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nIR^2}{(R^2+y^2)^{3/2}}\times\pi r^2 = \frac{\mu_0}{2}\cdot\frac{\pi nIR^2 r^2}{(R^2+y^2)^{3/2}}$$

Here y varies as

$y = y_0 - \frac{1}{2}gt^2$, where by $\frac{dy}{dt} = -gt = -v$ Where v is the instantaneous velocity of fall.

The induced e.m.f in the ring is given by

$$E = \frac{d\phi}{dt} = \frac{\mu_0}{2}\pi nIR^2 r^2\cdot\frac{d}{dt}(R^2+y^2)^{-3/2}$$

$$= -\frac{3}{4}\mu_0\pi nIR^2 r^2\cdot(R^2+y^2)^{-5/2}\cdot 2y\frac{dy}{dt}$$

$$= -\frac{3}{2}\cdot\frac{\mu_0\pi nIR^2 r^2}{(R^2+y^2)^{5/2}}\cdot y\cdot(-v)$$

$$= \frac{3}{2}\cdot\frac{\mu_0\pi R^2 r^2 nI}{(R^2+y^2)^{5/2}}\,yv$$

Illustration 16.

An infinitesimally small bar magnet of dipole moment $\vec{M}$ is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius a and of negligible self-inductance lies in the y-z plane with its centre at x = 0, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

Solution:

Field due to the bar magnet at distance x (near the loop)

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$\Rightarrow$ Flux linked with the loop : $\phi = BA = \pi a^2 \cdot \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$

Emf induced in the loop : $e = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi} \cdot \frac{6\pi Ma^2}{x^4} \cdot \frac{dx}{dt}$

$$= \frac{\mu_0}{4\pi} \cdot \frac{6\pi Ma^2}{x^4} v.$$

$\Rightarrow$ Induced current : $i = \frac{e}{R} = \frac{\mu_0}{4\pi} \cdot \frac{6\pi Ma^2}{Rx^4} .v.$

Let F = force opposing the motion of the magnet

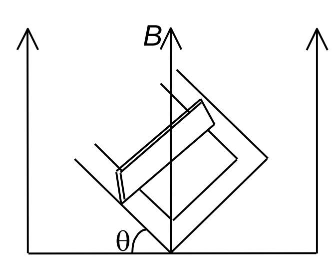
Power due to the opposing force = Heat dissipated in the coil per second

$$\Rightarrow \quad Fv = i^2R \Rightarrow F = \frac{i^2R}{v} = \left(\frac{\mu_0}{4\pi}\right)^2 \times \left(\frac{6\pi Ma^2}{Rx^4}\right)^2 xv^2 \times \frac{R}{v}$$

$$= \frac{9}{4}\left(\frac{\mu_0^2 M^2 a^4 v}{Rx^8}\right)$$

Illustration 17.

A square wire of length l, mass m and resistance R slides without friction down the parallel conducting wires of negligible resistance as shown in Figure.



The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and rails form a closed rectangular loop. The plane of the rails makes an angle θ with horizontal and a uniform vertical field of magnetic induction B exists throughout the region. Show that the wire acquires a steady state velocity of magnitude $v = \frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$

Solution:

Force down the plane = mg sin θ

At any instant if the velocity is v the induced e.m.f. = lB cos θ × v

Current in the loop = $\frac{lB\cos\theta v}{R}$

Force on the conductor in the horizontal direction

$$= \frac{Bl \times B\cos\theta}{R} \times v \times l$$

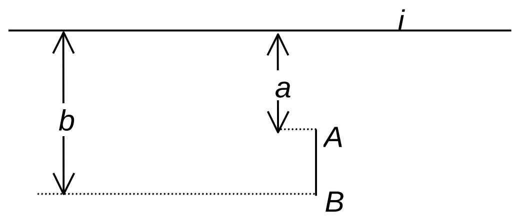
Component parallel to the incline = $\frac{B^2 l^2 \cos\theta}{R} \times v$

If v is constant $\frac{B^2 l^2 \cos^2\theta}{R} \times v$ = mg sin θ

$$\therefore \qquad \frac{mRg\sin\theta}{B^2 l^2 \cos^2\theta}$$

Illustration 18.

Figure shows a copper rod moving with velocity $\vec{v}$ parallel to a long straight wire carrying a current i. Calculate the induced e.m.f. in the rod assuming v = 5 m/s, i = 100 ampere, a = 10 cm, b = 20 cm.



Solution:

The induction at a point whose perpendicular distance from the rod is x is given by

$$B_{(x)} = \frac{\mu_o i}{2\pi x}$$

The e.m.f. induced on moving the rod $= \varepsilon = v\int_{x=a}^{x=b} B_{(x)} dx = v\int_a^b \frac{\mu_o i dx}{2\pi x}$

$$= \frac{\mu_o i v}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_o i v}{2\pi}\log_e\left(\frac{b}{a}\right) = \frac{4\pi\times10^{-7}\times100\times5}{2\pi}\log_e 20$$

$$= 10^{-4}\log_e 20 = 3 \times 10^{-4} \text{ V}$$

Illustration 19.

In an oscillating LC circuit, the energy is shared equally between the electric and magnetic fields. If L = 12 mH and C = 1.7 μF, how much time is needed for this condition to arise, assuming an initially fully charged capacitor.

Solution:

Total energy $U_E = \frac{1}{2}U_{E,\max}$

$$\Rightarrow \qquad \frac{q^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \Rightarrow q = \frac{Q}{\sqrt{2}}$$

Since, at t = 0, it is given that C has maximum charge, we have the solution to be

$$q = Q\cos\omega t \Rightarrow \frac{Q}{\sqrt{2}} = Q\cos\omega t$$

$$\Rightarrow \omega t = 45^\circ = \frac{\pi}{4}\text{ rad}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(12\times10^{-3})(1.7\times10^{-6})}} = 7\times10^3\text{ rad/s}$$

$$\therefore \text{ required } t = \frac{\omega t}{\omega} = \frac{(\pi/4)}{(7\times10^3)} = 1.12\times10^{-4}\text{ s}$$

$$\Rightarrow t = 110\ \mu\text{s}$$

Illustration 20.

The current in a coil of self inductance L = 2H is increasing according to the law $i = 2\sin t^2$. Find the amount of energy spent during the period when the current changes from 0 to 2 ampere.

Solution:

Let the current be 2 amp at $t = \tau$.

Then $2 = 2\sin\tau^2 \Rightarrow \tau = \sqrt{\frac{\pi}{2}}$.

When the instantaneous current is i, the self induced emf is $L\cdot\frac{di}{dt}$. If dq amount of charge is displaced in time dt then elementary work done $= L\left(\frac{di}{dt}\right)dq = L\cdot\frac{di}{dt}idt$ = Li di

$$W = \int_0^\tau Li\,di = \int_0^\tau L\,2\sin t^2\,d(2\sin t^2)$$

$$W = \int_0^\tau 8L\sin t^2\cos t^2\,t\,dt = 4L\int_0^\tau \sin 2t^2\,t\,dt$$

Let $\theta = 2t^2$ $d\theta = 4t\,dt$

$$\therefore \qquad \text{the integral} = 4L\int\frac{\sin\theta\,d\theta}{4}$$

$$= L(-\cos\theta)$$

$= -L\cos 2t^2$

$$W = -L\left[\cos 2t^2\right]_0^{\sqrt{\pi/2}}$$

$= 2\,L = 2 \times 2$

$= 4$ Joules

Illustration 21.

Calculate the self inductance per unit length of a current loop formed by joining the ends of two long parallel wires of radius r separated by a distance d between their axes, neglecting the end effects and magnetic flux within the wires.

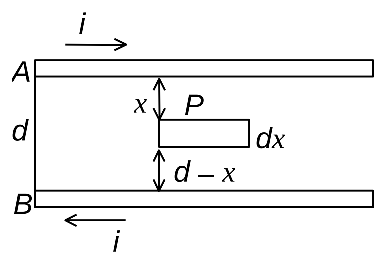
Solution:

Let A and B are the two long parallel wires as shown which are formed to form a current loop. The two wires would obviously be carrying equal currents in opposite direction.

Consider a space length l and thickness dx as shown in Figure.

The magnetic flux through the area l dx due to the currents in the two wires is $d\phi = Bl\,dx$

where B is the intensity of magnetic field due to current carrying wires at a distance x from A.



$$B = \frac{\mu_0}{4\pi}\cdot\frac{2i}{x} + \frac{\mu_0}{4\pi}\cdot\frac{2i}{d-x}$$

$$d\phi = Bl\,dx = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x}+\frac{1}{d-x}\right)l\,dx$$

$$\therefore \qquad \phi = \int_r^{d-r}\frac{\mu_0 i}{2\pi}\left(\frac{1}{x}+\frac{1}{d-x}\right)l\,dx$$

$$= \frac{\mu_0 il}{2\pi}\left[\log_e x - \log_e(d-x)\right]_r^{d-r} = \frac{\mu_0 il}{\pi}\log_e\frac{d-r}{r}$$

Magnetic flux linked with unit length of the loop $= \dfrac{\mu_0 i}{\pi}\log_e\dfrac{d-r}{r}$

But $\phi = Li$ where L is self-inductance.

$$\therefore \qquad L = \frac{\mu_0}{\pi}\log_e\frac{d-r}{r}$$