Trees

In linear data structure data is organized in sequential order and in non-linear data structure data is organized in random order. A tree is a very popular non-linear data structure used in a wide range of applications. A tree data structure can be defined as follows...

Tree is a non-linear data structure which organizes data in hierarchical structure and this is a recursive definition.

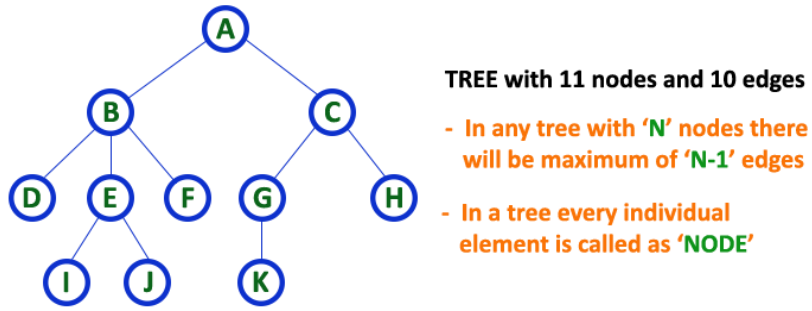
A tree data structure can also be defined as follows...

Tree data structure is a collection of data (Node) which is organized in hierarchical structure recursively

In tree data structure, every individual element is called as **Node**. Node in a tree data structure stores the actual data of that particular element and link to next element in hierarchical structure.

In a tree data structure, if we have **N** number of nodes then we can have a maximum of **N-1** number of links.

Example

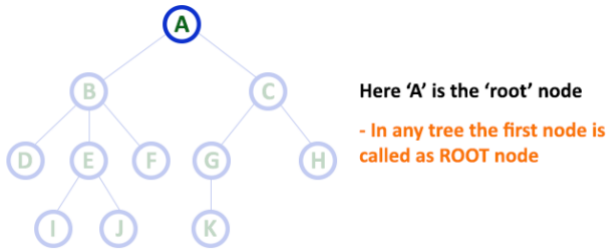


Terminology

In a tree data structure, we use the following terminology...

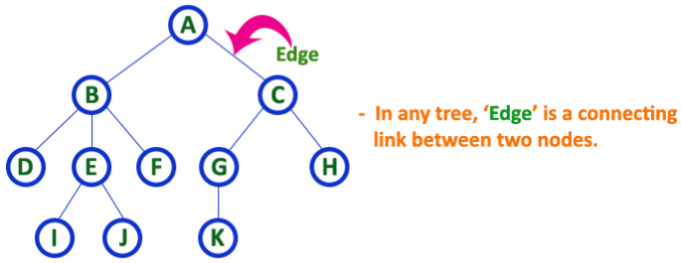
1. Root

In a tree data structure, the first node is called as **Root Node**. Every tree must have a root node. We can say that the root node is the origin of the tree data structure. In any tree, there must be only one root node. We never have multiple root nodes in a tree.



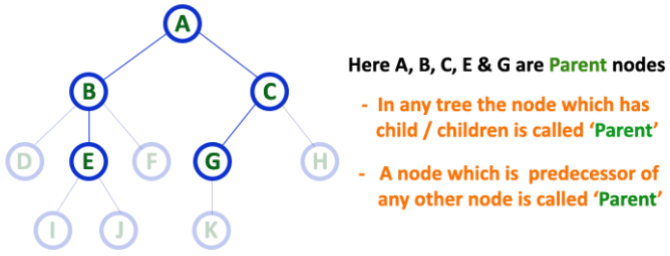
2. Edge

In a tree data structure, the connecting link between any two nodes is called as **EDGE**. In a tree with '**N**' number of nodes there will be a maximum of '**N-1**' number of edges.



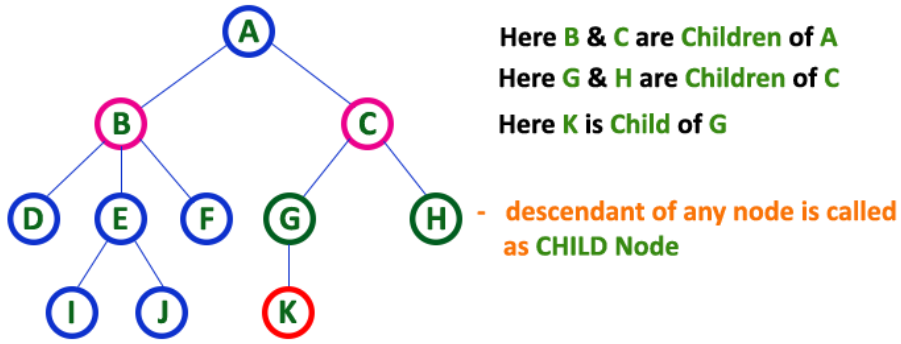
3. Parent

In a tree data structure, the node which is a predecessor of any node is called as **PARENT NODE**. In simple words, the node which has a branch from it to any other node is called a parent node. Parent node can also be defined as "**The node which has child / children**".



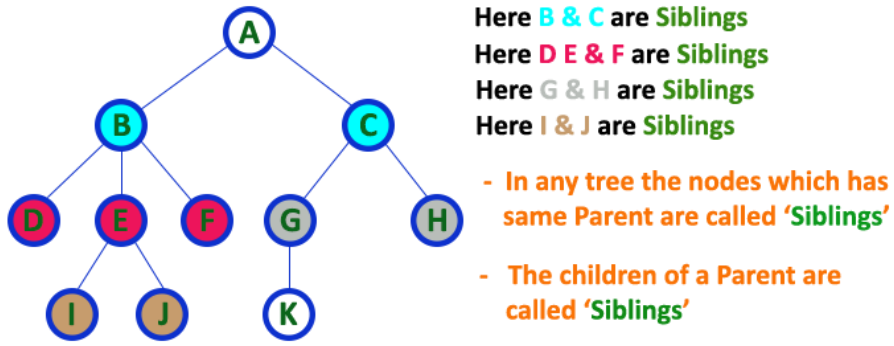
4. Child

In a tree data structure, the node which is descendant of any node is called as **CHILD Node**. In simple words, the node which has a link from its parent node is called as child node. In a tree, any parent node can have any number of child nodes. In a tree, all the nodes except root are child nodes.



5. Siblings

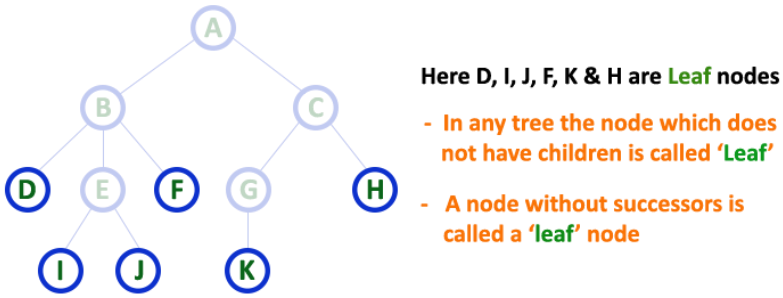
In a tree data structure, nodes which belong to same Parent are called as **SIBLINGS**. In simple words, the nodes with the same parent are called Sibling nodes.



6. Leaf

In a tree data structure, the node which does not have a child is called as **LEAF Node**. In simple words, a leaf is a node with no child.

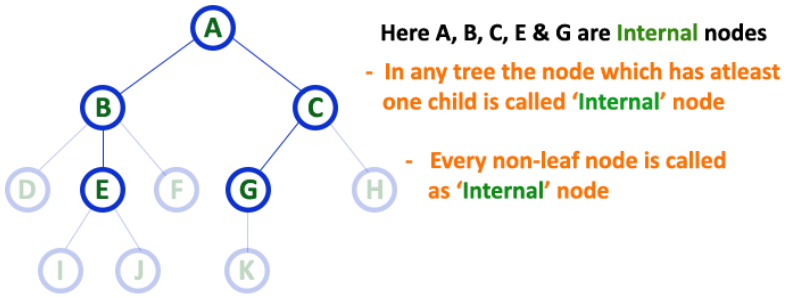
In a tree data structure, the leaf nodes are also called as **External Nodes**. External node is also a node with no child. In a tree, leaf node is also called as '**Terminal**' node.



7. Internal Nodes

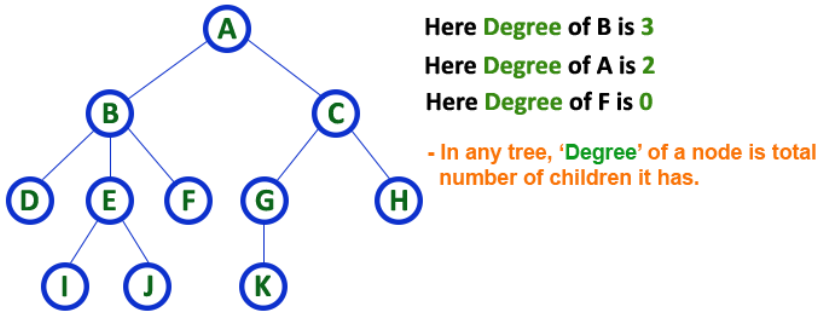
In a tree data structure, the node which has atleast one child is called as **INTERNAL Node**. In simple words, an internal node is a node with atleast one child.

In a tree data structure, nodes other than leaf nodes are called as **Internal Nodes**. **The root node is also said to be Internal Node** if the tree has more than one node. Internal nodes are also called as '**Non-Terminal**' nodes.



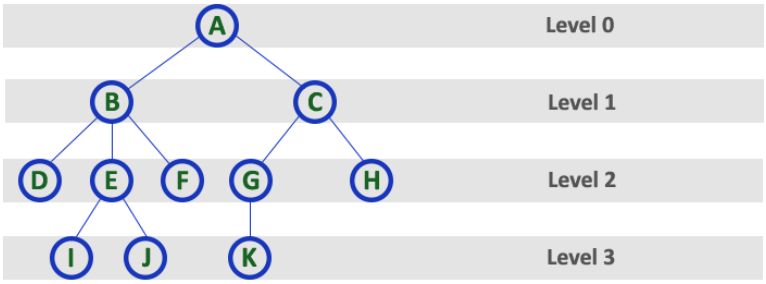
8. Degree

In a tree data structure, the total number of children of a node is called as **DEGREE** of that Node. In simple words, the Degree of a node is total number of children it has. The highest degree of a node among all the nodes in a tree is called as '**Degree of Tree**'



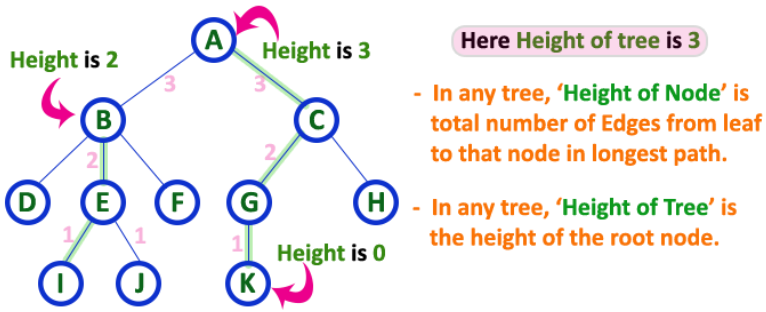
9. Level

In a tree data structure, the root node is said to be at Level 0 and the children of root node are at Level 1 and the children of the nodes which are at Level 1 will be at Level 2 and so on... In simple words, in a tree each step from top to bottom is called as a Level and the Level count starts with '0' and incremented by one at each level (Step).



10. Height

In a tree data structure, the total number of edges from leaf node to a particular node in the longest path is called as **HEIGHT** of that Node. In a tree, height of the root node is said to be **height of the tree**. In a tree, **height of all leaf nodes is '0'.**

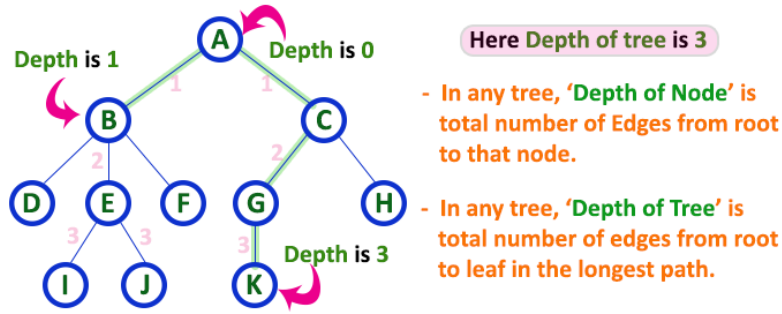


Here Height of tree is 3

- In any tree, 'Height of Node' is total number of Edges from leaf to that node in longest path.
- In any tree, 'Height of Tree' is the height of the root node.

11. Depth

In a tree data structure, the total number of egdes from root node to a particular node is called as **DEPTH** of that Node. In a tree, the total number of edges from root node to a leaf node in the longest path is said to be **Depth of the tree**. In simple words, the highest depth of any leaf node in a tree is said to be depth of that tree. In a tree, **depth of the root node is '0'.**

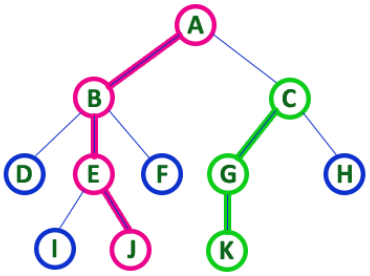


Here Depth of tree is 3

- In any tree, 'Depth of Node' is total number of Edges from root to that node.
- In any tree, 'Depth of Tree' is total number of edges from root to leaf in the longest path.

12. Path

In a tree data structure, the sequence of Nodes and Edges from one node to another node is called as **PATH** between that two Nodes. **Length of a Path** is total number of nodes in that path. In below example **the path A - B - E - J has length 4**.



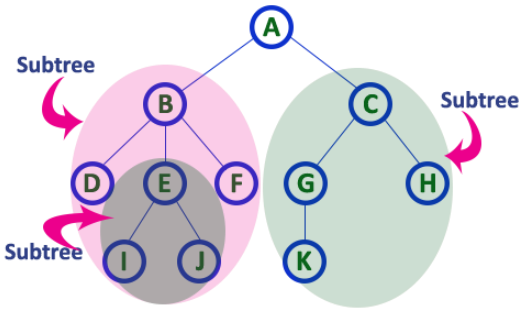
- In any tree, 'Path' is a sequence of nodes and edges between two nodes.

Here, 'Path' between A & J is
A - B - E - J

Here, 'Path' between C & K is
C - G - K

13. Sub Tree

In a tree data structure, each child from a node forms a subtree recursively. Every child node will form a subtree on its parent node.

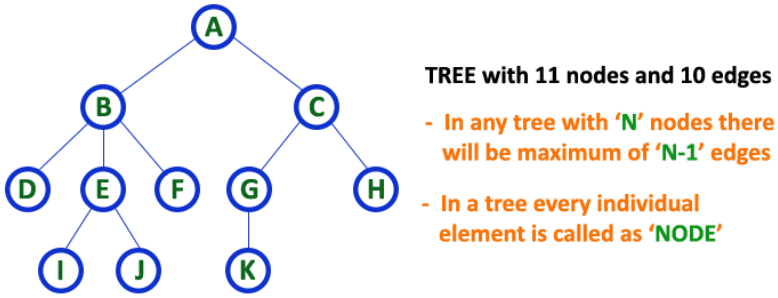


Tree Representations

A tree data structure can be represented in two methods. Those methods are as follows...

1. **List Representation**
2. **Left Child - Right Sibling Representation**

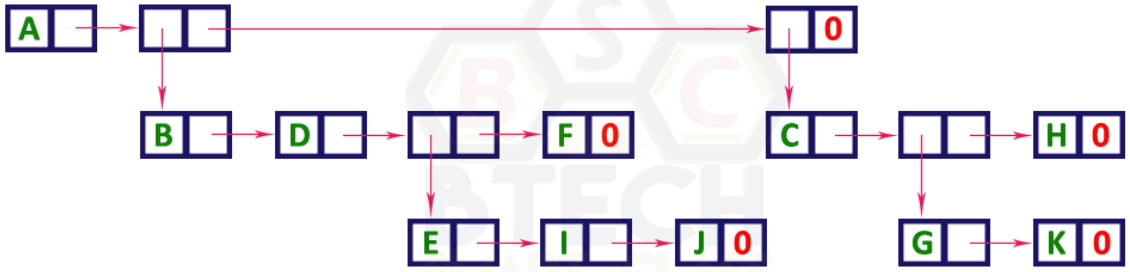
Consider the following tree...



1. List Representation

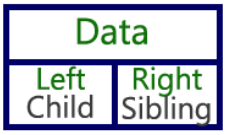
In this representation, we use two types of nodes one for representing the node with data called 'data node' and another for representing only references called 'reference node'. We start with a 'data node' from the root node in the tree. Then it is linked to an internal node through a 'reference node' which is further linked to any other node directly. This process repeats for all the nodes in the tree.

The above example tree can be represented using List representation as follows..



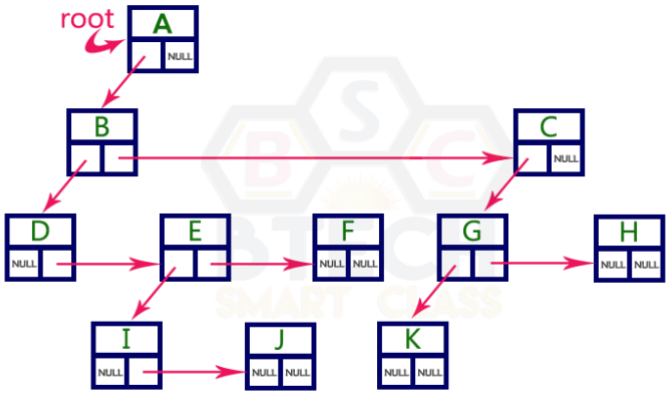
2. Left Child - Right Sibling Representation

In this representation, we use a list with one type of node which consists of three fields namely Data field, Left child reference field and Right sibling reference field. Data field stores the actual value of a node, left reference field stores the address of the left child and right reference field stores the address of the right sibling node. Graphical representation of that node is as follows...



In this representation, every node's data field stores the actual value of that node. If that node has left a child, then left reference field stores the address of that left child node otherwise stores NULL. If that node has the right sibling, then right reference field stores the address of right sibling node otherwise stores NULL.

The above example tree can be represented using Left Child - Right Sibling representation as follows...



Applications of Trees:

- Trees are used to store simple as well as complex data. Here simple means an integer value, character value and complex data means a structure or a record.
- Trees are often used for implementing other types of data structures like hash tables, sets, and maps.
- A self-balancing tree, Red-black tree is used in kernel scheduling, to preempt massively multiprocessor computer operating system use.
- Another variation of tree, B-trees are prominently used to store tree structures on disc. They are used to index a large number of records.
- B-trees are also used for secondary indexes in databases, where the index facilitates a select operation to answer some range criteria.
- Trees are an important data structure used for compiler construction.
- Trees are also used in database design.
- Trees are used in file system directories.
- Trees are also widely used for information storage and retrieval in symbol tables.

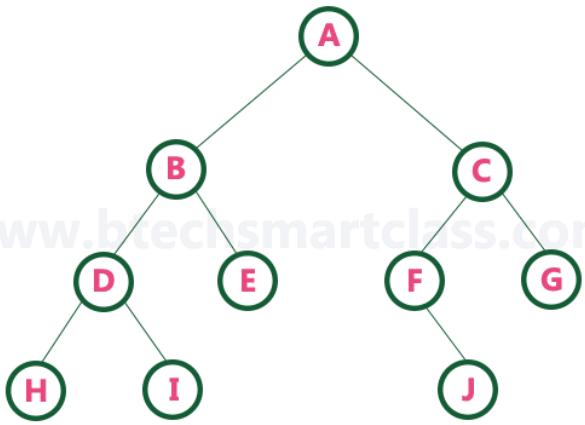
Binary Tree

In a normal tree, every node can have any number of children. A binary tree is a special type of tree data structure in which every node can have a **maximum of 2 children**. One is known as a left child and the other is known as right child.

A tree in which every node can have a maximum of two children is called Binary Tree.

In a binary tree, every node can have either 0 children or 1 child or 2 children but not more than 2 children.

Example



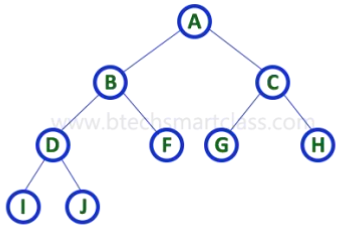
There are different types of binary trees and they are...

1. Strictly Binary Tree

In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none. That means every internal node must have exactly two children. A strictly Binary Tree can be defined as follows...

A binary tree in which every node has either two or zero number of children is called Strictly Binary Tree

Strictly binary tree is also called as **Full Binary Tree** or **Proper Binary Tree** or **2-Tree**

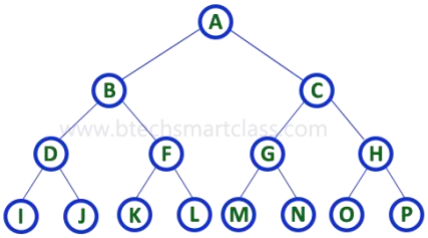


2. Complete Binary Tree

In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none and in complete binary tree all the nodes must have exactly two children and at every level of complete binary tree there must be 2^{level} number of nodes. For example at level 2 there must be $2^2 = 4$ nodes and at level 3 there must be $2^3 = 8$ nodes.

A binary tree in which every internal node has exactly two children and all leaf nodes are at same level is called Complete Binary Tree.

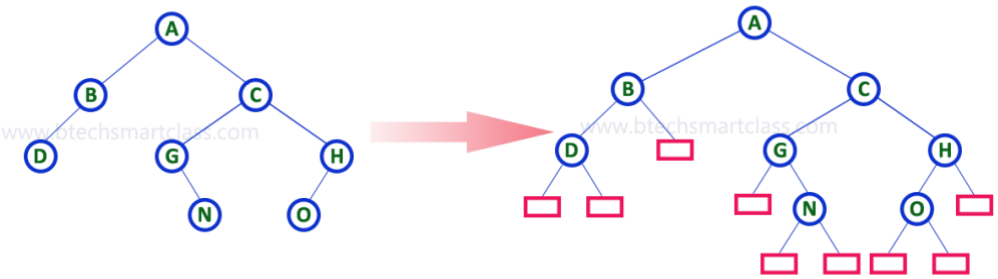
Complete binary tree is also called as **Perfect Binary Tree**



3. Extended Binary Tree

A binary tree can be converted into Full Binary tree by adding dummy nodes to existing nodes wherever required.

The full binary tree obtained by adding dummy nodes to a binary tree is called as Extended Binary Tree.



In above figure, a normal binary tree is converted into full binary tree by adding dummy nodes

Binary Tree Traversals

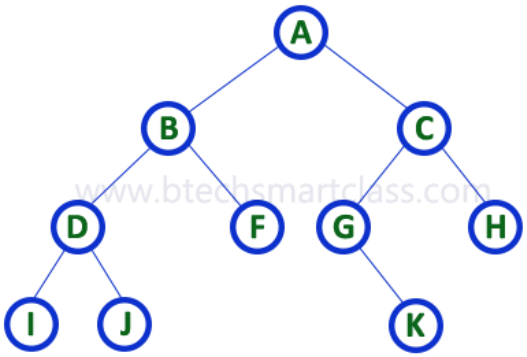
When we wanted to display a binary tree, we need to follow some order in which all the nodes of that binary tree must be displayed. In any binary tree, displaying order of nodes depends on the traversal method.

Displaying (or) visiting order of nodes in a binary tree is called as Binary Tree Traversal.

There are three types of binary tree traversals.

1. **In - Order Traversal**
2. **Pre - Order Traversal**
3. **Post - Order Traversal**

Consider the following binary tree...



1. In - Order Traversal (leftChild - root - rightChild)

In In-Order traversal, the root node is visited between the left child and right child. In this traversal, the left child node is visited first, then the root node is visited and later we go for visiting the right child node. This in-order traversal is applicable for every root node of all subtrees in the tree. This is performed recursively for all nodes in the tree.

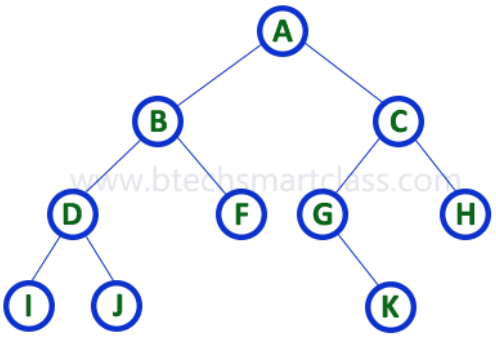
In the above example of a binary tree, first we try to visit left child of root node 'A', but A's left child 'B' is a root node for left subtree. so we try to visit its (B's) left child 'D' and again D is a root for subtree with nodes D, I and J. So we try to visit its left child 'I' and it is the leftmost child. So first we visit **'I'** then go for its root node **'D'** and later we visit D's right child **'J'**. With this we have completed the left part of node B. Then visit **'B'** and next B's right child **'F'** is visited.

With this we have completed left part of node A. Then visit root node **'A'**. With this we have completed left and root parts of node A. Then we go for the right part of the node A. In right of A again there is a subtree with root C. So go for left child of C and again it is a subtree with root G. But G does not have left part so we visit **'G'** and then visit G's right child K. With this we have completed the left part of node C. Then visit root node **'C'** and next visit C's right child **'H'** which is the rightmost child in the tree. So we stop the process.

That means here we have visited in the order of **I - D - J - B - F - A - G - K - C - H** using In-Order Traversal. **In-Order Traversal for above example of binary tree is**

I - D - J - B - F - A - G - K - C - H

2.Pre - Order Traversal (root - leftChild - rightChild)



In Pre-Order traversal, the root node is visited before the left child and right child nodes. In this traversal, the root node is visited first, then its left child and later its right child. This pre-order traversal is applicable for every root node of all subtrees in the tree.

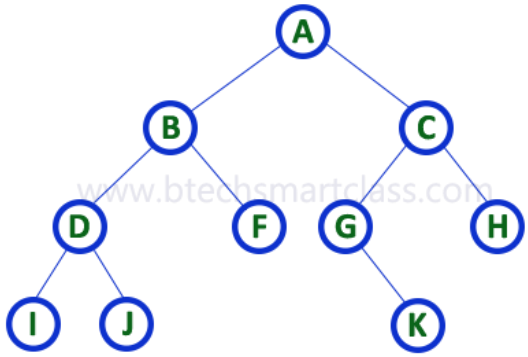
In the above example of binary tree, first we visit root node **'A'** then visit its left child **'B'** which is a root for D and F. So we visit B's left child **'D'** and again D is a root for I and J. So we visit D's left child **'I'** which is the leftmost child. So next we go for visiting D's right child **'J'**. With this we have completed root, left and right parts of node D and root, left parts of node B. Next visit B's right child **'F'**. With this we have completed root and left parts of node A. So we go for A's right child **'C'** which is a root node for G and H. After visiting C, we go for its left child **'G'** which is a root for node K. So next we visit left of G, but it does not have left child so we go for G's right child **'K'**. With this, we have completed node C's root and left parts. Next visit C's right child **'H'** which is the rightmost child in the tree. So we stop the process.

That means here we have visited in the order of **A-B-D-I-J-F-C-G-K-H** using Pre-Order Traversal.

Pre-Order Traversal for above example binary tree is

A - B - D - I - J - F - C - G - K - H

3. Post - Order Traversal (leftChild - rightChild - root)



In Post-Order traversal, the root node is visited after left child and right child. In this traversal, left child node is visited first, then its right child and then its root node. This is recursively performed until the right most node is visited.

Here we have visited in the order of **I - J - D - F - B - K - G - H - C - A** using Post-Order Traversal.

Post-Order Traversal for above example binary tree is

I - J - D - F - B - K - G - H - C – A

Heap Sort

Heap data structure is a specialized binary tree-based data structure. Heap is a binary tree with special characteristics. In a heap data structure, nodes are arranged based on their values. A heap data structure some times also called as Binary Heap.

There are two types of heap data structures and they are as follows...

1. **Max Heap**
2. **Min Heap**

Every heap data structure has the following properties...

Property #1 (Ordering): Nodes must be arranged in an order according to their values based on Max heap or Min heap.

Property #2 (Structural): All levels in a heap must be full except the last level and all nodes must be filled from left to right strictly.

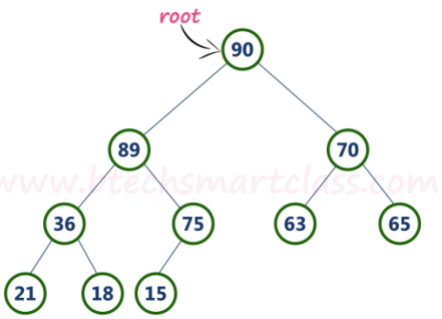
Max Heap

Max heap data structure is a specialized full binary tree data structure. In a max heap nodes are arranged based on node value.

Max heap is defined as follows...

Max heap is a specialized full binary tree in which every parent node contains greater or equal value than its child nodes.

Example



Above tree is satisfying both Ordering property and Structural property according to the Max Heap data structure.

Operations on Max Heap

The following operations are performed on a Max heap data structure...

1. **Finding Maximum**
2. **Insertion**
3. **Deletion**

Finding Maximum Value Operation in Max Heap

Finding the node which has maximum value in a max heap is very simple. In a max heap, the root node has the maximum value than all other nodes. So, directly we can display root node value as the maximum value in max heap.

Insertion Operation in Max Heap

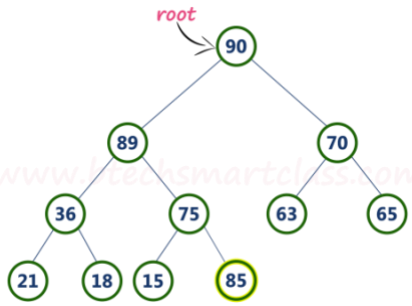
Insertion Operation in max heap is performed as follows...

- **Step 1 -** Insert the **newNode** as **last leaf** from left to right.
- **Step 2 -** Compare **newNode value** with its **Parent node**.
- **Step 3 -** If **newNode value is greater** than its parent, then **swap** both of them.
- **Step 4 -** Repeat step 2 and step 3 until newNode value is less than its parent node (or) newNode reaches to root.

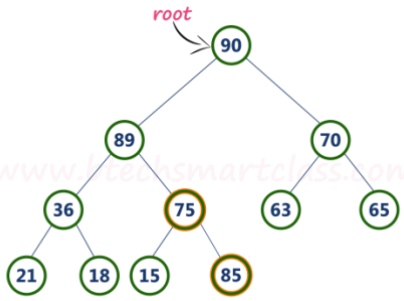
Example

Consider the above max heap. **Insert a new node with value 85.**

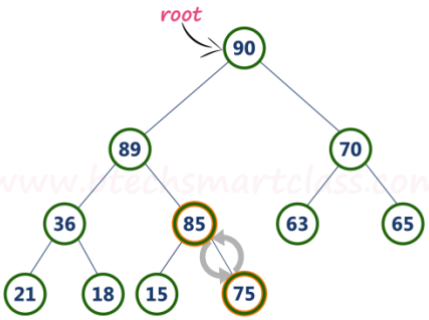
- **Step 1 -** Insert the **newNode** with value 85 as **last leaf** from left to right. That means newNode is added as a right child of node with value 75. After adding max heap is as follows...



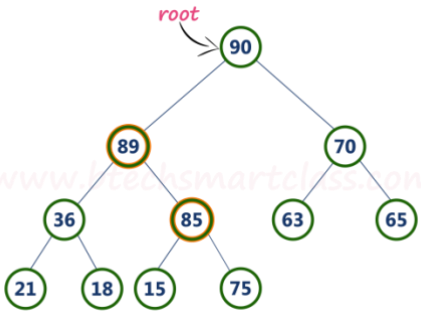
- **Step 2 -** Compare **newNode value (85)** with its **Parent node value (75)**. That means **85 > 75**



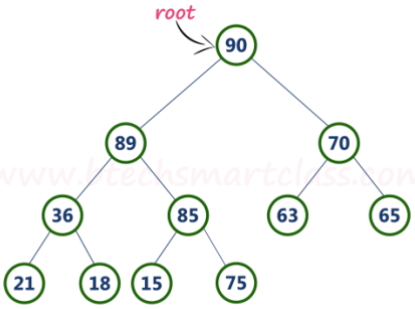
- **Step 3 -** Here **newNode value (85) is greater** than its **parent value (75)**, then **swap** both of them. After swapping, max heap is as follows...



Step 4 - Now, again compare newNode value (85) with its parent node value (89).



Here, newNode value (85) is smaller than its parent node value (89). So, we stop insertion process. Finally, max heap after insertion of a new node with value 85 is as follows...



Deletion Operation in Max Heap

n a max heap, deleting the last node is very simple as it does not disturb max heap properties.

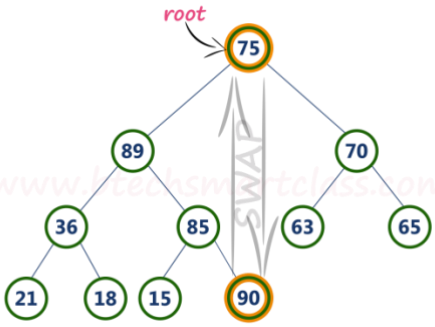
Deleting root node from a max heap is little difficult as it disturbs the max heap properties. We use the following steps to delete the root node from a max heap...

- **Step 1 - Swap** the **root** node with **last** node in max heap
- **Step 2 - Delete** last node.
- **Step 3 -** Now, compare **root value** with its **left child value**.
- **Step 4 -** If **root value is smaller** than its left child, then compare **left child** with its **right sibling**. Else goto **Step 6**
- **Step 5 -** If **left child value is larger** than its **right sibling**, then **swap root** with **left child** otherwise **swap root** with its **right child**.
- **Step 6 -** If **root value is larger** than its left child, then compare **root value** with its **right child** value.
- **Step 7 -** If **root value is smaller** than its **right child**, then **swap root** with **right child** otherwise **stop the process**.
- **Step 8 -** Repeat the same until root node fixes at its exact position.

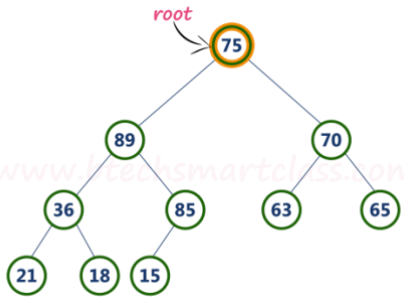
Example

Consider the above max heap. **Delete root node (90) from the max heap.**

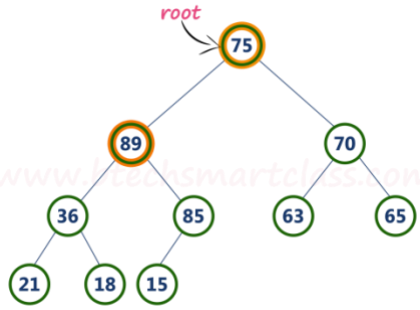
- **Step 1 - Swap** the **root node (90)** with **last node 75** in max heap. After swapping max heap is as follows...



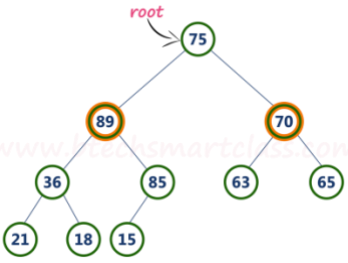
- **Step 2 - Delete** last node. Here the last node is 90. After deleting node with value 90 from heap, max heap is as follows...



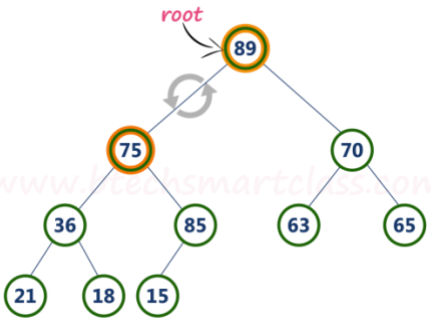
Step 3 - Compare **root node (75)** with its **left child (89)**.



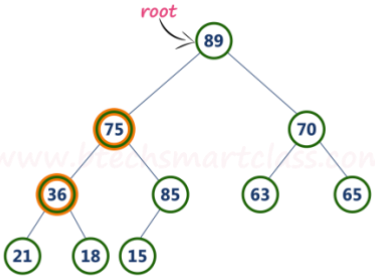
Here, **root value (75) is smaller** than its left child value (89). So, compare left child (89) with its right sibling (70).



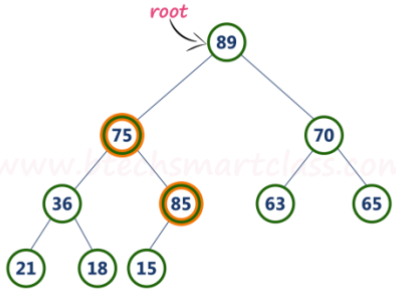
- **Step 4 -** Here, **left child value (89) is larger** than its **right sibling (70)**, So, **swap root (75)** with **left child (89)**.



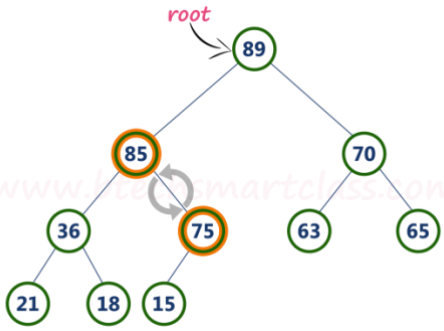
- **Step 5 -** Now, again compare **75** with its **left child (36)**.



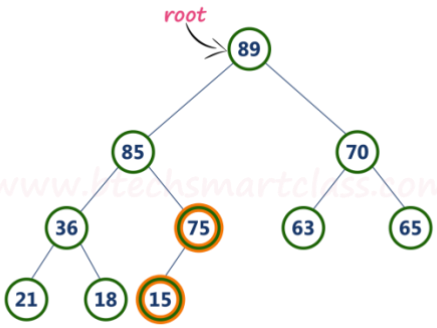
Here, node with value **75** is larger than its left child. So, we compare node **75** with its right child **85**.



- **Step 6 -** Here, node with value **75** is smaller than its **right child (85)**. So, we swap both of them. After swapping max heap is as follows...

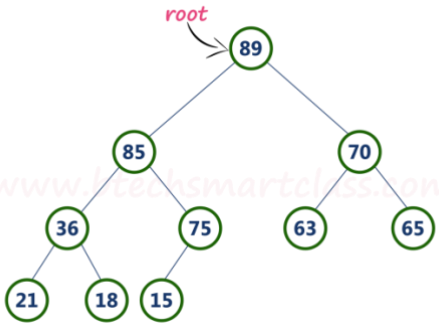


- **Step 7 -** Now, compare node with value **75** with its left child (**15**).



Here, node with value **75** is larger than its left child (**15**) and it does not have right child. So we stop the process.

Finally, max heap after deleting root node (**90**) is as follows...

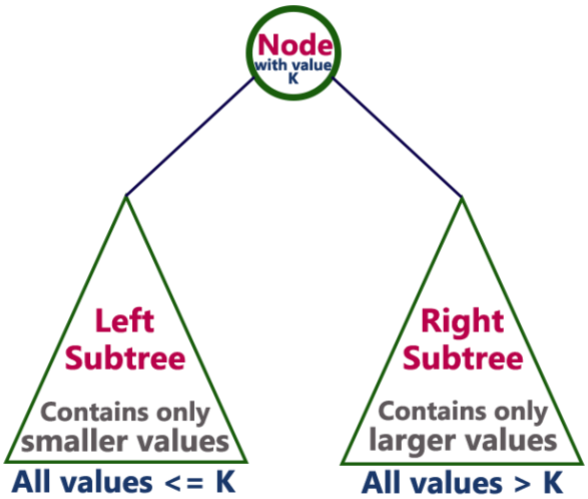


Binary Search Tree

To enhance the performance of binary tree, we use a special type of binary tree known as **Binary Search Tree**. Binary search tree mainly focuses on the search operation in a binary tree. Binary search tree can be defined as follows...

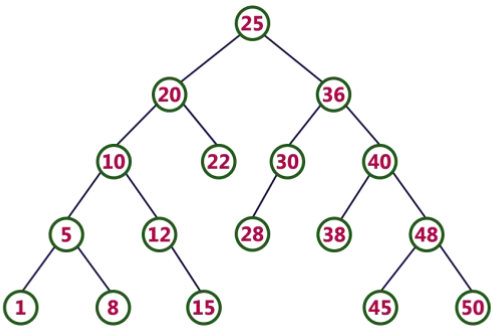
Binary Search Tree is a binary tree in which every node contains only smaller values in its left subtree and only larger values in its right subtree.

In a binary search tree, all the nodes in the left subtree of any node contains smaller values and all the nodes in the right subtree of any node contains larger values as shown in the following figure...



Example

The following tree is a Binary Search Tree. In this tree, left subtree of every node contains nodes with smaller values and right subtree of every node contains larger values.



Every binary search tree is a binary tree but every binary tree need not to be binary search tree.

Operations on a Binary Search Tree

The following operations are performed on a binary search tree...

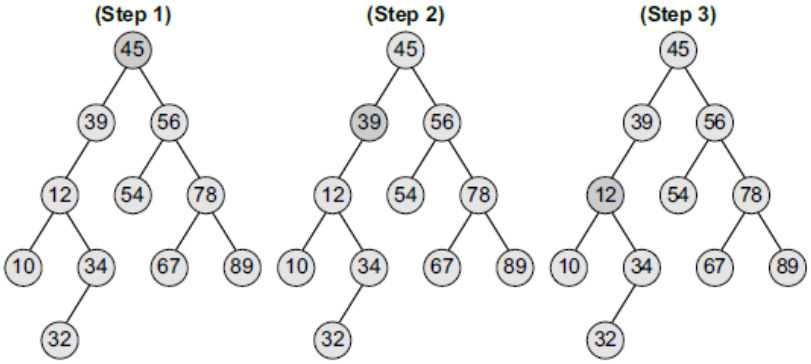
1. **Search**
2. **Insertion**
3. **Deletion**

Search Operation in BST

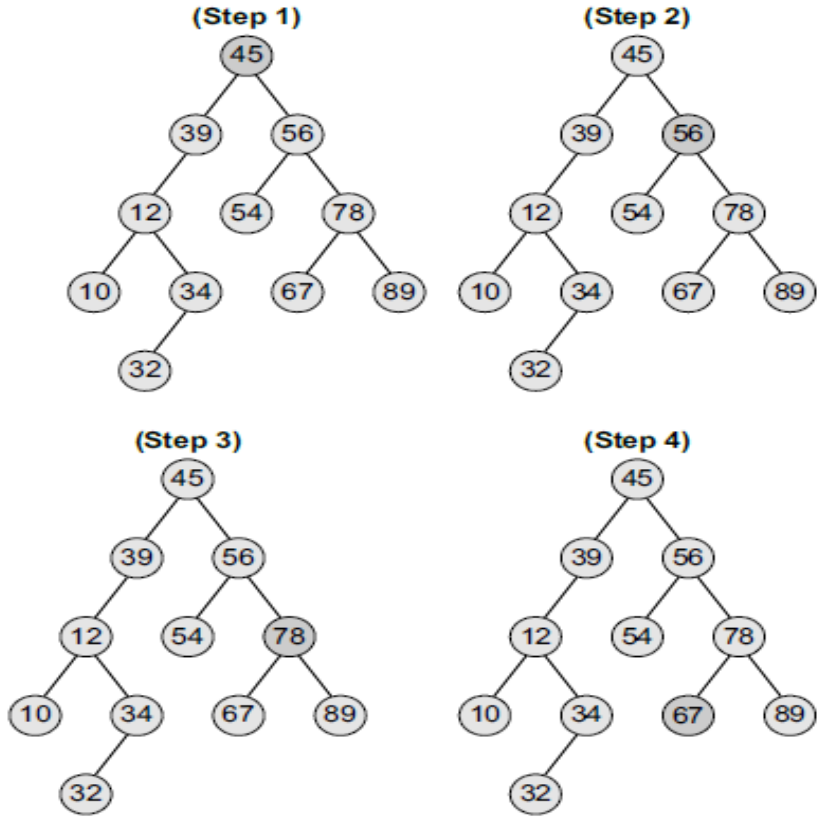
In a binary search tree, the search operation is performed with **O(log n)** time complexity. The search operation is performed as follows...

- **Step 1 -** Read the search element from the user.
- **Step 2 -** Compare the search element with the value of root node in the tree.
- **Step 3 -** If both are matched, then display "Given node is found!!!" and terminate the function

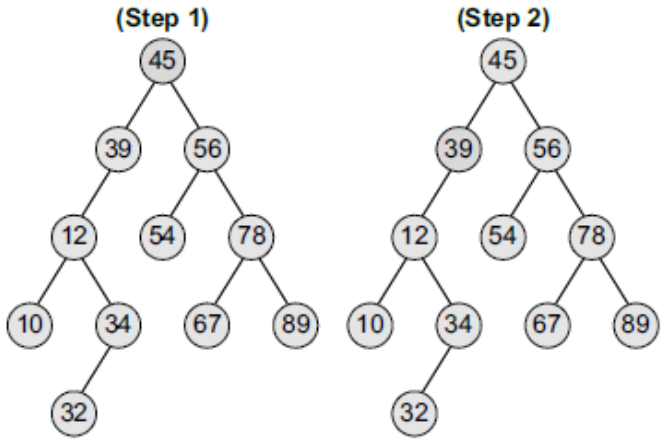
- Step 4 - If both are not matched, then check whether search element is smaller or **larger** than that node value.
- **Step 5 -** If search element is smaller, then continue the search process in left subtree.
- **Step 6-** If search element is larger, then continue the search process in right subtree.
- **Step 7 -** Repeat the same until we find the exact element or until the search element is compared with the leaf node
- **Step 8 -** If we reach to the node having the value equal to the search value then display "Element is found" and terminate the function.
- **Step 9 -** If we reach to the leaf node and if it is also not matched with the search element, then display "Element is not found" and terminate the function.



The above figure shows how a binary tree is searched to find a specific element. First, see how the tree will be traversed to find the node with value 12. The procedure to find the node with value 67 is illustrated in the below figure.



The procedure to find the node with value 40 is shown in the below figure. The search would terminate after reaching node 39 as it does not have any right child.



Insertion Operation in BST

In a binary search tree, the insertion operation is performed with **O(log n)** time complexity. In binary search tree, new node is always inserted as a leaf node. The insertion operation is performed as follows...

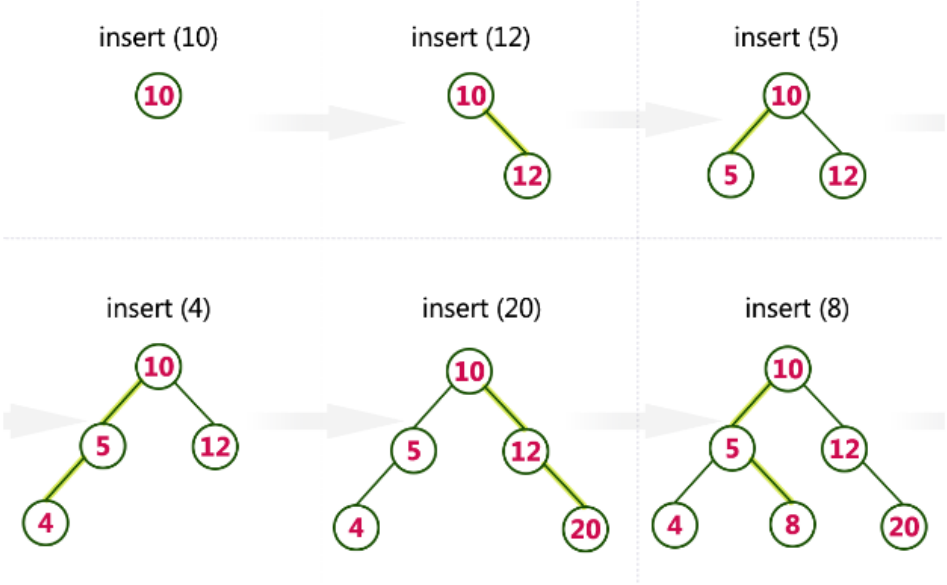
- **Step 1 -** Create a newNode with given value and set its **left** and **right** to **NULL**.
- **Step 2 -** Check whether tree is Empty.
- **Step 3 -** If the tree is **Empty**, then set **root** to **newNode**.
- **Step 4 -** If the tree is **Not Empty**, then check whether the value of newNode is **smaller** or **larger** than the node (here it is root node).
- **Step 5 -** If newNode is **smaller** than **or equal** to the node then move to its **left** child. If newNode is **larger** than the node then move to its **right** child.
- **Step 6-** Repeat the above steps until we reach to the **leaf** node (i.e., reaches to NULL).
- **Step 7 -** After reaching the leaf node, insert the newNode as **left child** if the newNode is **smaller or equal** to that leaf node or else insert it as **right child**.

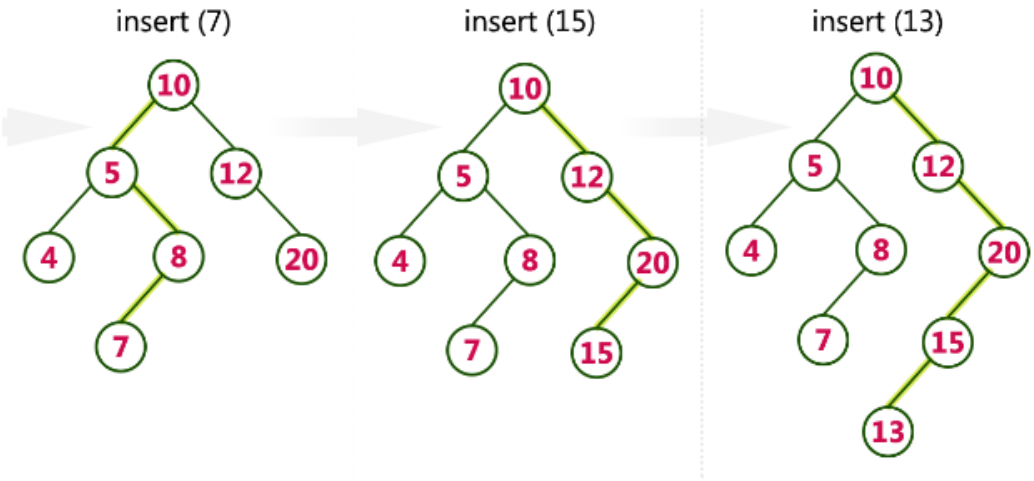
Example

Construct a Binary Search Tree by inserting the following sequence of numbers...

10,12,5,4,20,8,7,15 and 13

Above elements are inserted into a Binary Search Tree as follows...





Deletion Operation in BST

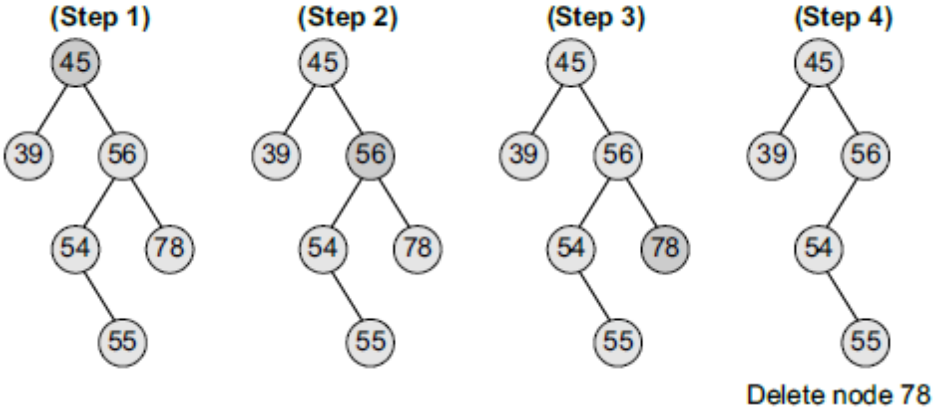
In a binary search tree, the deletion operation is performed with **O(log n)** time complexity. Deleting a node from Binary search tree includes following three cases...

- **Case 1: Deleting a Leaf node (A node with no children)**
- **Case 2: Deleting a node with one child**
- **Case 3: Deleting a node with two children**

Case 1: Deleting a leaf node

We use the following steps to delete a leaf node from BST...

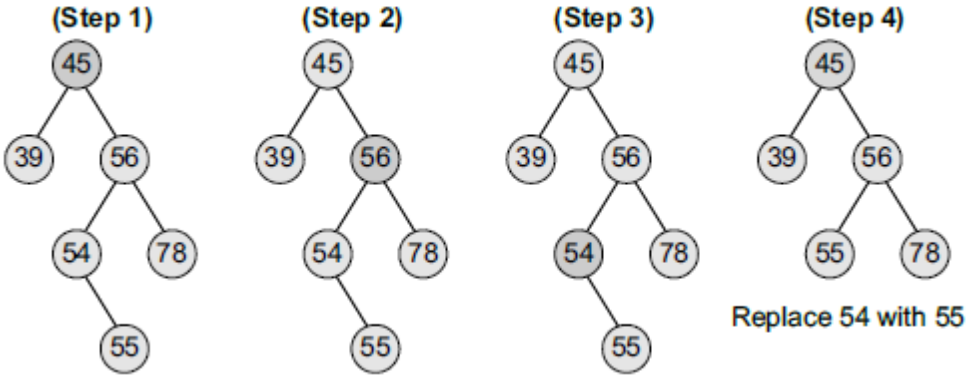
- **Step 1 - Find** the node to be deleted using **search operation**
- **Step 2 -** Delete the node using **free** function (If it is a leaf) and terminate the function.
- ***Case 1: DeletingNode that has No Children***
  The binary search tree given in below figure. If we have to delete node 78, we can simply remove this node without any issue. This is the simplest case of deletion.



Case 2: Deleting a node with one child

We use the following steps to delete a node with one child from BST...

- **Step 1 - Find** the node to be deleted using **search operation**
- **Step 2 -** If it has only one child then create a link between its parent node and child node.
- **Step 3 -** Delete the node using **free** function and terminate the function.
- The binary search tree shown in below figure and see how deletion of node 54 is handled.
-



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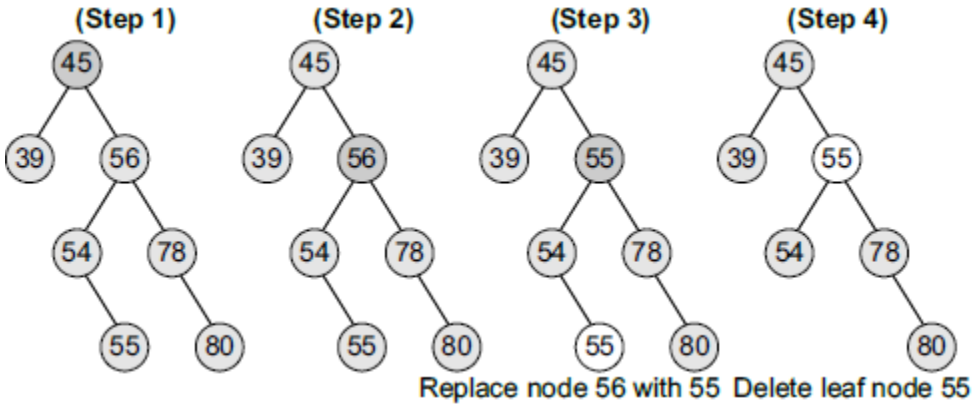
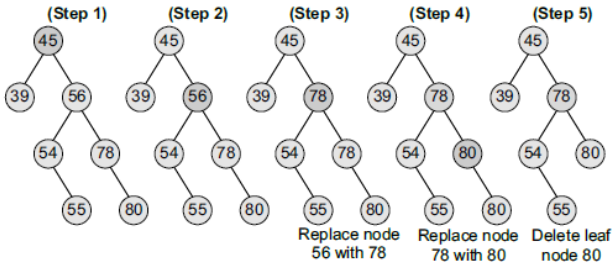
Case 3: Deleting a node with two children

We use the following steps to delete a node with two children from BST...

- **Step 1 - Find** the node to be deleted using **search operation**
- **Step 2 -** If it has two children, then find the **largest** node in its **left subtree** (OR) the **smallest** node in its **right subtree**.
- **Step 3 - Swap** both **deleting node** and node which is found in the above step.
- **Step 4 -** Then check whether deleting node came to **case 1** or **case 2** or else goto step 2
- **Step 5 -** If it comes to **case 1**, then delete using case 1 logic.
- **Step 6-** If it comes to **case 2**, then delete using case 2 logic.
- **Step 7 -** Repeat the same process until the node is deleted from the tree.
- The binary search tree given in below figure and see how deletion of node with value 56 is handled.



- This deletion could also be handled by replacing node 56 with its in-order successor, as shown in below figure.



AVL TREE:

AVL tree is a height balanced binary search tree. That means, an AVL tree is also a binary search tree but it is a balanced tree. A binary tree is said to be balanced if, the difference between the heights of left and right subtrees of every node in the tree is either -1, 0 or +1. In other words, a binary tree is said to be balanced if the height of left and right children of every node differ by either -1, 0 or +1. In an AVL tree, every node maintains an extra information known as **balance factor**. The AVL tree was introduced in the year 1962 by G.M. Adelson-Velsky and E.M. Landis.

An AVL tree is defined as follows...

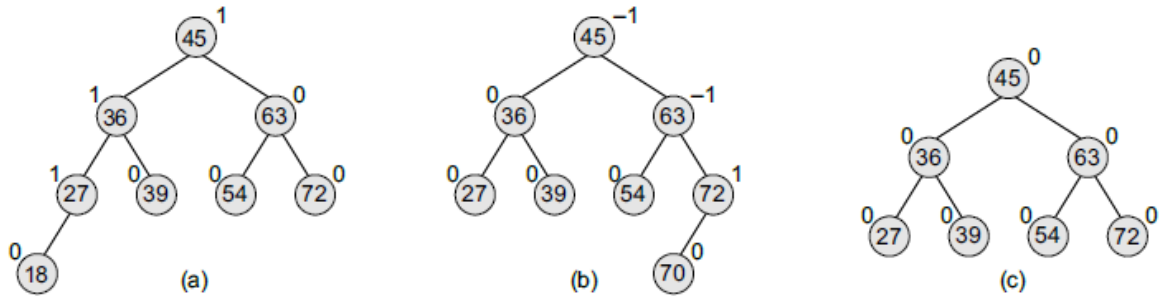
An AVL tree is a balanced binary search tree. In an AVL tree, balance factor of every node is either -1, 0 or +1.

Balance factor of a node is the difference between the heights of left and right subtrees of that node. The balance factor of a node is calculated either **height of left subtree - height of right subtree** (OR) **height of right subtree - height of left subtree**. In the following explanation, we calculate as follows...

Balance factor = heightOfLeftSubtree - heightOfRightSubtree

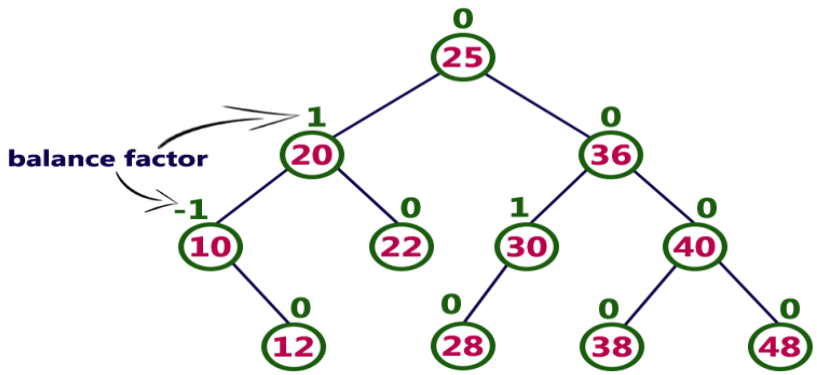
Example of AVL Tree:

1.



- If the balance factor of a node is 1, then it means that the left sub-tree of the tree is one level higher than that of the right sub-tree. Such a tree is therefore called as a *left-heavy tree* and is shown in above fig(a).
- If the balance factor of a node is 0, then it means that the height of the left sub-tree (longest path in the left sub-tree) is equal to the height of the right sub-tree and is shown in fig(c).
- If the balance factor of a node is –1, then it means that the left sub-tree of the tree is one level lower than that of the right sub-tree. Such a tree is therefore called as a *right-heavy tree* and is shown in above fig(b).

2. The below tree is a binary search tree and every node is satisfying balance factor condition. So this tree is said to be an AVL tree. Every AVL Tree is a binary search tree but every Binary Search Tree need not be AVL tree.

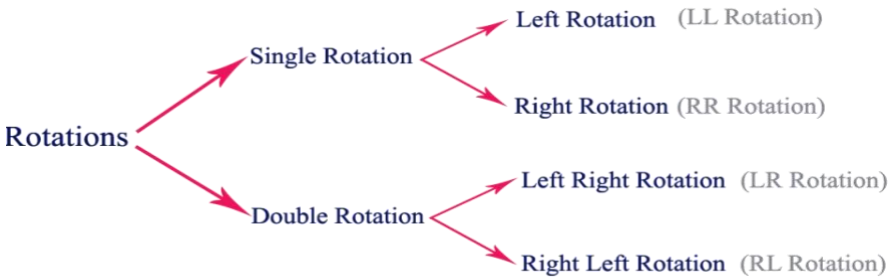


AVL Tree Rotations:

In AVL tree, after performing operations like insertion and deletion we need to check the **balance factor** of every node in the tree. If every node satisfies the balance factor condition then we conclude the operation otherwise we must make it balanced. Whenever the tree becomes imbalanced due to any operation we use **rotation** operations to make the tree balanced.

Rotation operations are used to make the tree balanced.

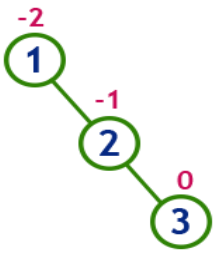
Rotation is the process of moving nodes either to left or to right to make the tree balanced. There are **four** rotations and they are classified into **two** types.



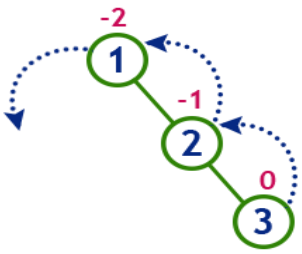
1.Single Left Rotation (LL Rotation):

In LL Rotation, every node moves one position to left from the current position. To understand LL Rotation, let us consider the following insertion operation in AVL Tree...

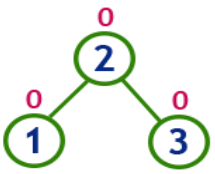
insert 1, 2 and 3



Tree is imbalanced



To make balanced we use LL Rotation which moves nodes one position to left

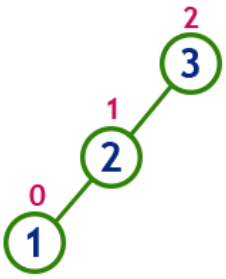


After LL Rotation Tree is Balanced

2. Single Right Rotation (RR Rotation):

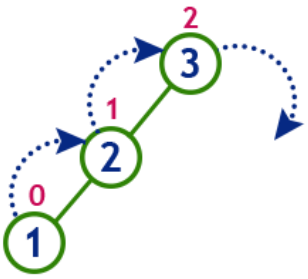
In RR Rotation, every node moves one position to right from the current position. To understand RR Rotation, let us consider the following insertion operation in AVL Tree...

insert 3, 2 and 1

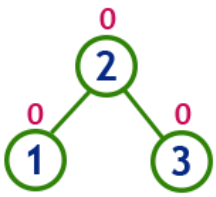


Tree is imbalanced

because node 3 has balance factor 2



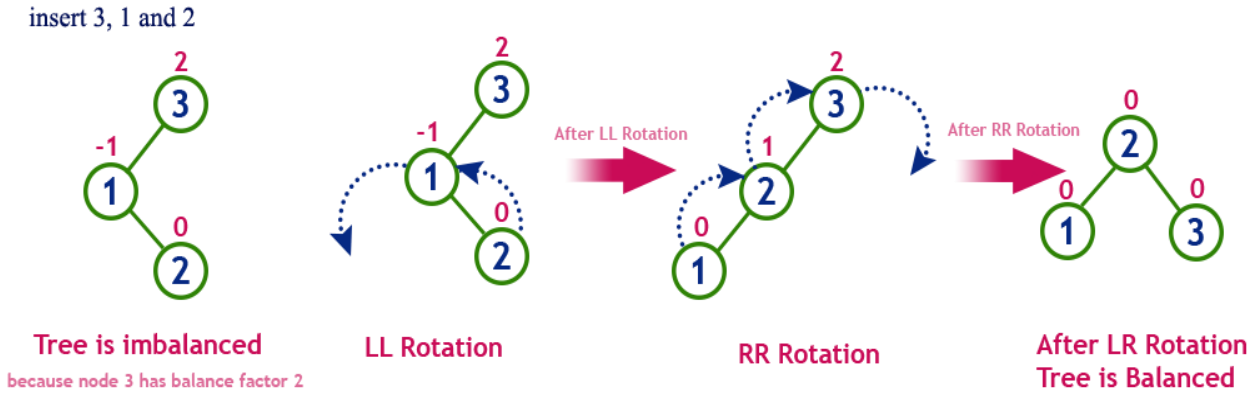
To make balanced we use RR Rotation which moves nodes one position to right



After RR Rotation Tree is Balanced

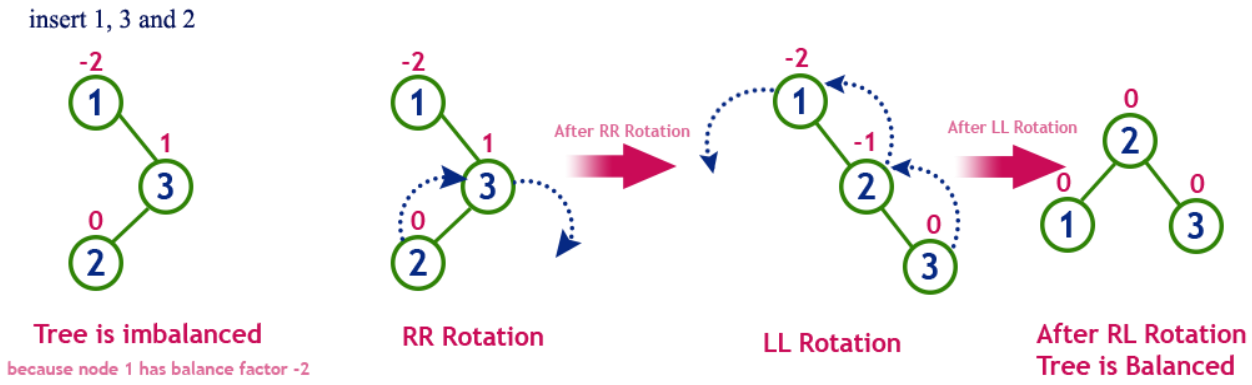
3. Left Right Rotation (LR Rotation):

The LR Rotation is sequence of single left rotation followed by single right rotation. In LR Rotation, at first every node moves one position to left and one position to right from the current position. To understand LR Rotation, let us consider the following insertion operation in AVL Tree...



4. Right Left Rotation (RL Rotation):

The RL Rotation is sequence of single right rotation followed by single left rotation. In RL Rotation, at first every node moves one position to right and one position to left from the current position. To understand RL Rotation, let us consider the following insertion operation in AVL Tree...



Operations on an AVL Tree:

The following operations are performed on AVL tree...

1.Search

2.Insertion

3.Deletion

1. Search Operation in AVL Tree:

In an AVL tree, the search operation is performed with **O(log n)** time complexity. The search operation in AVL tree is similar to search operation in Binary search tree. We use the following steps to search an element in AVL tree...

Step 1 - Read the search element from the user.

Step 2 - Compare the search element with the value of root node in the tree.

Step 3 - If both are matched, then display "Given node is found!!!" and terminate the function

Step 4 - If both are not matched, then check whether search element is smaller or larger than that node value.

Step 5 - If search element is smaller, then continue the search process in left subtree.

Step 6 - If search element is larger, then continue the search process in right subtree.

Step 7 - Repeat the same until we find the exact element or until the search element is compared with the leaf node.

Step 8 - If we reach to the node having the value equal to the search value, then display "Element is found" and terminate the function.

Step 9 - If we reach to the leaf node and if it is also not matched with the search element, then display "Element is not found" and terminate the function.

2. Insertion Operation in AVL Tree:

In an AVL tree, the insertion operation is performed with **O(log n)** time complexity. In AVL Tree, new node is always inserted as a leaf node. The insertion operation is performed as follows...

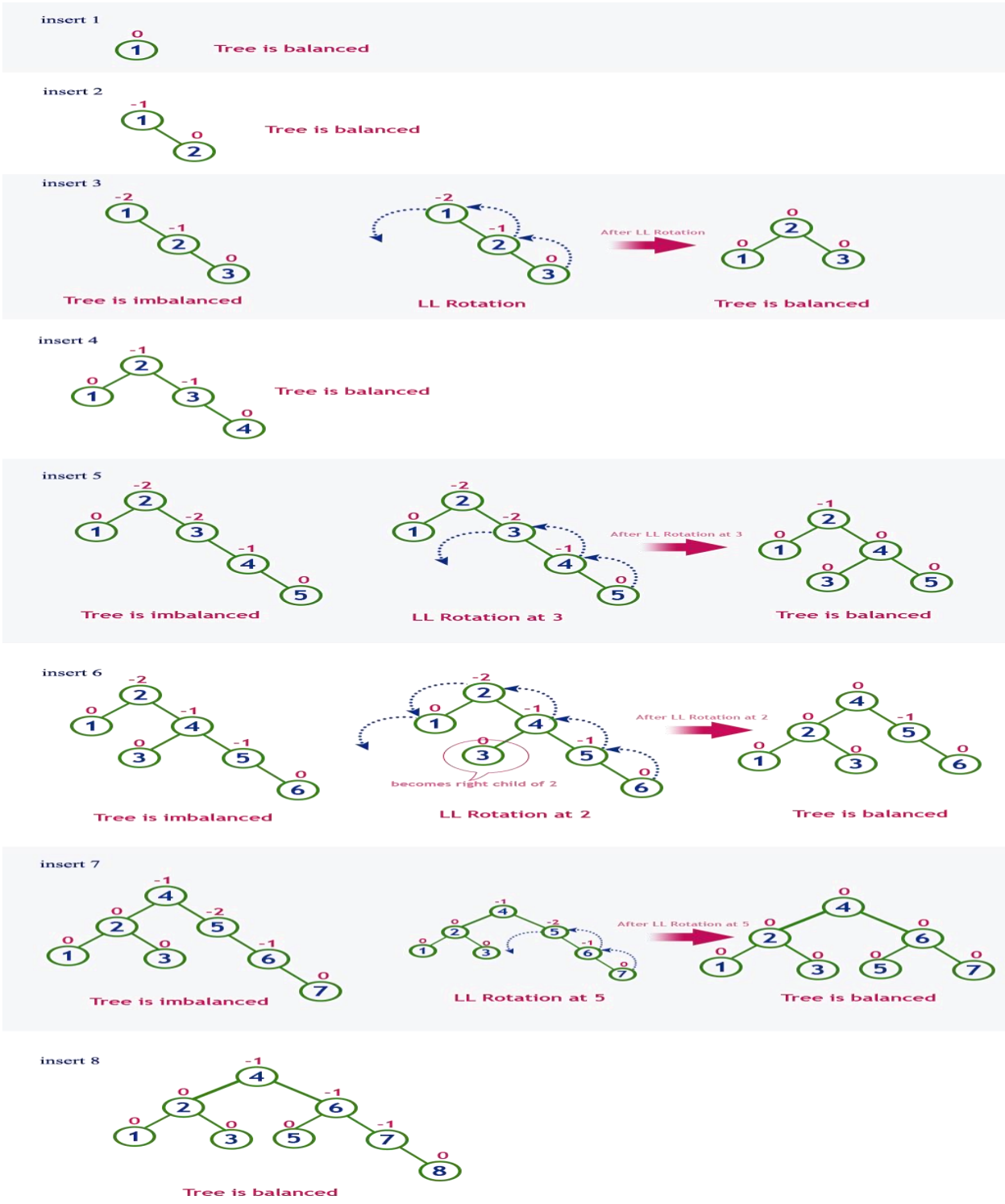
Step 1 - Insert the new element into the tree using Binary Search Tree insertion logic.

Step 2 - After insertion, check the **Balance Factor** of every node.

Step 3 - If the **Balance Factor** of every node is **0 or 1 or -1** then go for next operation.

Step 4 - If the **Balance Factor** of any node is other than **0 or 1 or -1** then that tree is said to be imbalanced. In this case, perform suitable **Rotation** to make it balanced and go for next operation.

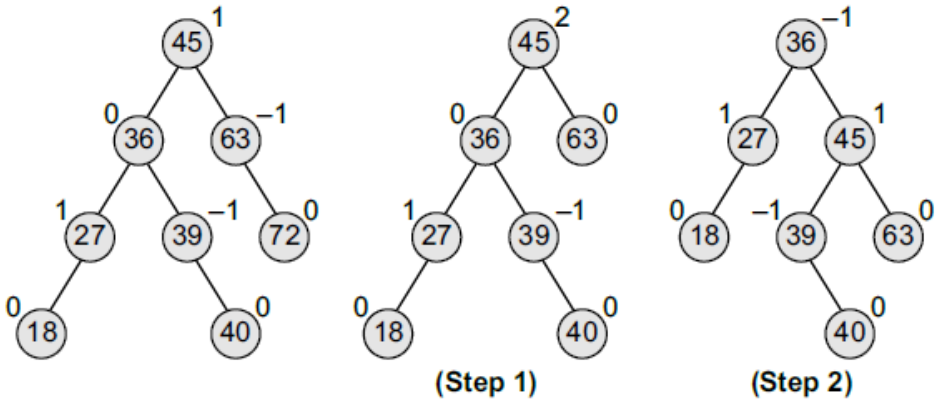
<u>Example:</u> Construct an AVL Tree by inserting numbers from 1 to 8.



3. Deletion Operation in AVL Tree:

The deletion operation in AVL Tree is similar to deletion operation in BST. But after every deletion operation we need to check with the Balance Factor condition. If the tree is balanced after deletion go for next operation otherwise perform suitable rotation to make the tree Balanced.

Consider the AVL tree given in below figure and delete 72 from it.



Expression Trees:

Expression tree is a binary tree, because all of the operations are binary. It is also possible for a node to have only one child, as is the case with the unary minus operator. The leaves of an expression tree are operands, such as constants or variable names, and the other (non leaf) nodes contain operators.

Once an expression tree is constructed we can traverse it in three ways:

- Inorder Traversal
- Preorder Traversal
- Postorder Traversal

An expression tree can be generated for the infix and postfix expressions.

An algorithm to convert a postfix expression into an expression tree is as follows:

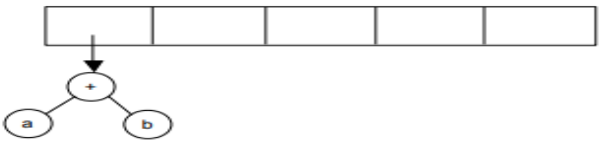
1. Read the expression one symbol at a time.

2. If the symbol is an operand, we create a one-node tree and push a pointer to it onto a stack.

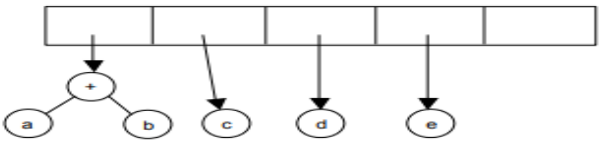
3. If the symbol is an operator, we pop pointers to two trees T1 and T2 from the stack (T1 is popped first) and form a new tree whose root is the operator and whose left and right children point to T2 and T1 respectively. A pointer to this new tree is then pushed onto the stack.

Example 1: Construct an expression tree for the postfix expression: a b + c d e + * *
The first two symbols are operands, so we create one-node trees and push pointers to them onto a stack

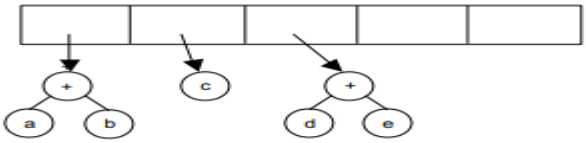
Next, a '+' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.



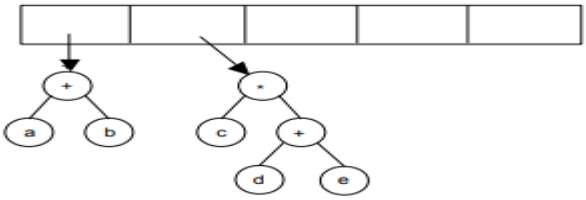
Next, c, d, and e are read, and for each one–node tree is created and a pointer to the corresponding tree is pushed onto the stack.



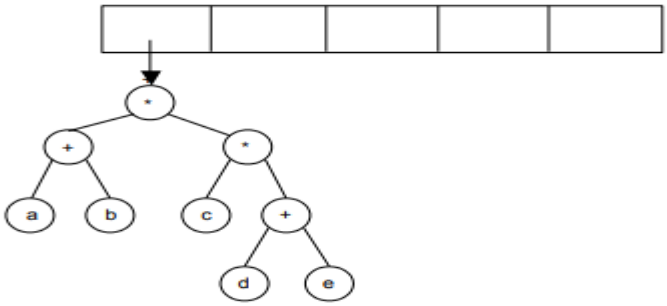
Now a '+' is read, so two trees are merged.



Continuing, a '*' is read, so we pop two tree pointers and form a new tree with a '*' as root.



Finally, the last symbol is read, two trees are merged, and a pointer to the final tree is left on the stack.



For the above tree:

Inorder form of the expression: a + b * c * d + e

Preorder form of the expression: * + a b * c + d e

Postorder form of the expression: a b + c d e + * *

Example 2:

Construct an expression tree for the arithmetic expression:

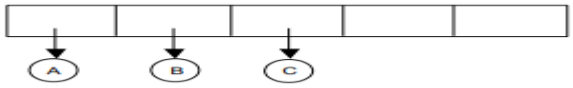
(A + B * C) – ((D * E + F) / G)

Solution:

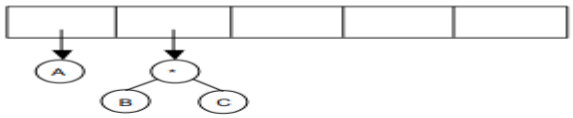
First convert the infix expression into postfix notation.

Postfix notation of the arithmetic expression is: A B C * + D E * F + G / -

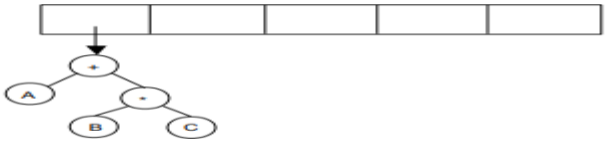
The first three symbols are operands, so we create one-node trees and pointers to three nodes pushed onto the stack.



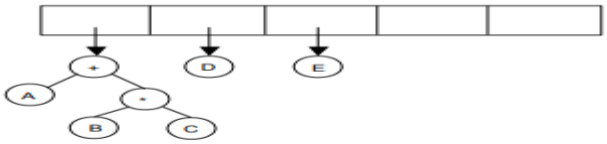
Next, a '*' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.



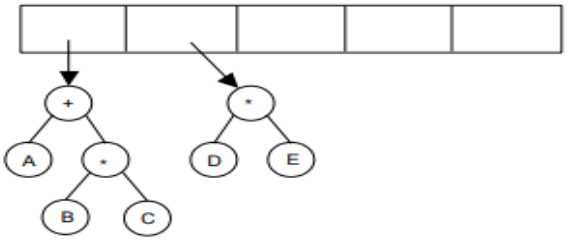
Next, a '+' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.



Next, D and E are read, and for each one–node tree is created and a pointer to the corresponding tree is pushed onto the stack.



Continuing, a '*' is read, so we pop two tree pointers and form a new tree with a '*' as root.



Proceeding similar to the previous steps, finally, when the last symbol is read, the expression tree is as follows: