



GENERAL APTITUDE

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What is **Aptitude** ?

It is **your natural ability** to learn or excel in a certain area.

For example, you could have an **aptitude** for math and logic.

Key to **success**

1. Problem Recognition
2. Speed
3. Practice



Link for English Basics

- <https://www.myenglishpages.com/english/exercises.php>
- <https://www.grammarbank.com/>
- <https://www.really-learn-english.com/english-grammar-exercises.html>
- <https://www.really-learn-english.com/english-reading-comprehension-text-and-exercises.html>
- Practice Synonyms and Antonyms regularly.
- Read Idioms and Phrases.
- Book - Word Power Made Easy by Norman Lewis
- Book - English Grammar by Wren and Martin



Basic MATHS

- Tables at least from 1-25
 - Squares from 1-25
 - Prime numbers from 1-100
 - Divisibility rules for 1-20
- Methods for typical multiplications & divisions
 - Methods for finding HCF & LCM
 - Methods for finding squares & square roots

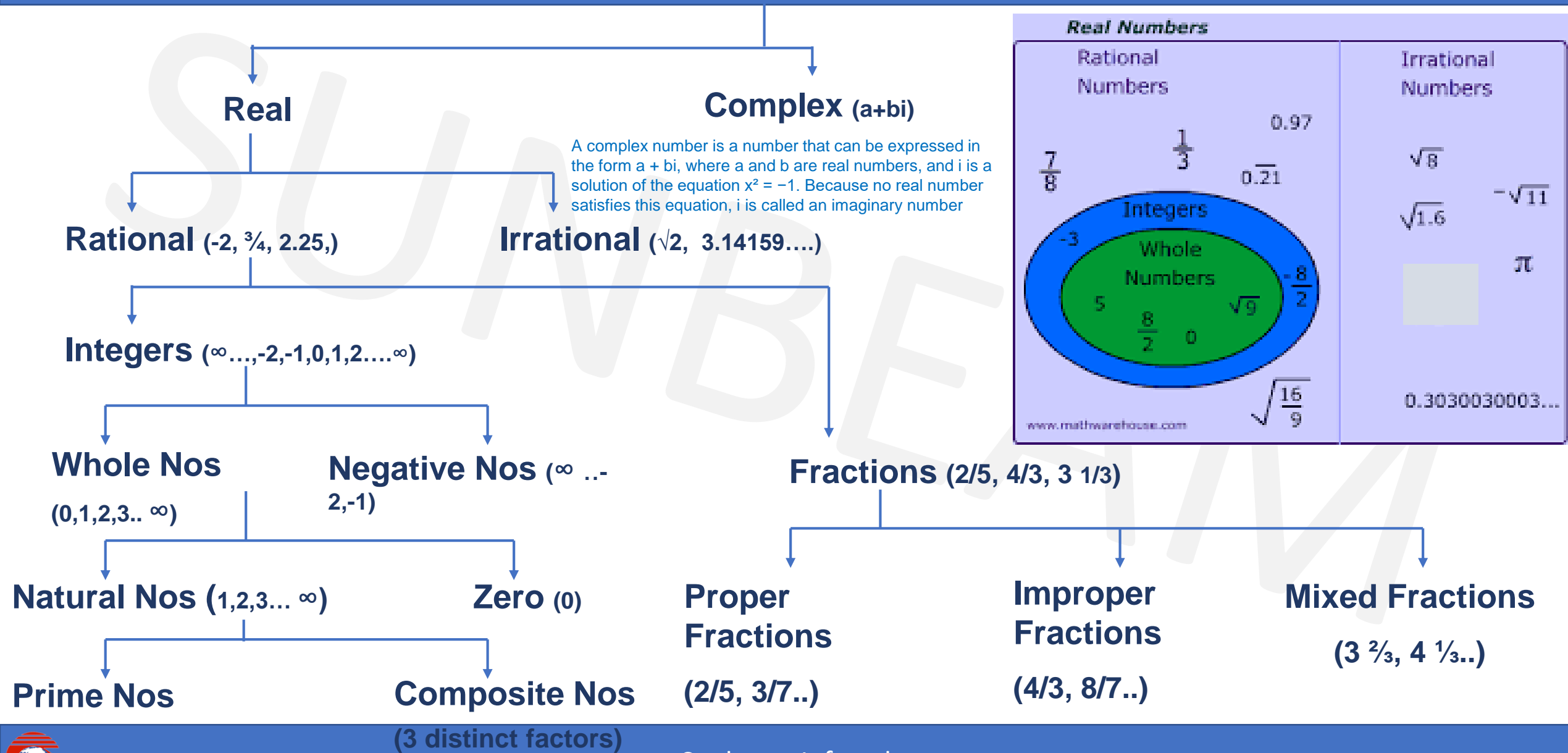


Topic Wise Test Plan

TEST NAME	TOPICS
APT I 1	Numbers + LCM + HCF + Ages + Averages
APT I 2	Percentages + Alligations & Mixtures + Profit & Loss
APT I 3	Time & Work + Pipes & Cisterns + Chain Rule
APT I 4	Time & Distance + Trains + Boats + Interest
APT I 5	Clock + Calendar + Probability + Permutation Combination



Numbers



What is the Difference Between Rational Numbers and Irrational Numbers?

Rational Numbers	Irrational Numbers
Numbers that can be expressed as a ratio of two numbers (p/q form) are termed as a rational number.	Numbers that cannot be expressed as a ratio of two numbers are termed as an irrational number.
Rational Number includes numbers, which are finite or are recurring in nature.	These consist of numbers, which are non-terminating and non-repeating in nature.
If a number is terminating number or repeating decimal, then it is rational. e.g: $1/2 = 0.5$	If a number is non-terminating and non-repeating decimal, then it is irrational. e.g: 0.31545673...
Example:- $1/2$, $3/4$, $11/2$, 0.45, 10, etc.	example:-Pi (π) = 3.14159...., Euler's Number (e) = (2.71828...), and $\sqrt{3}$, $\sqrt{2}$.



Basic MATHEMATICAL operations

- BODMAS

- B - Bracket () , { } , []
- O - Order
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction.



BASIC FORMULAE

- 1. $(a + b)^2 = a^2 + b^2 + 2ab$
- 2. $(a - b)^2 = a^2 + b^2 - 2ab$
- 3. $(a + b)^2 - (a - b)^2 = 4ab$
- 4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- 5. $(a^2 - b^2) = (a + b)(a - b)$
- 6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- 7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- 8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- 9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- 10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$



Basic NUMBER Representation

- Place Value : Units, Tens, Hundreds,
- Value of a 2 digit no. 'ab' where both a & b are natural numbers = $10(a) + b$
- The number with reversed digits will be 'ba' & the value of the number will be = $10(b) + a$



Numbers

Q. A number consists of two digits.
Sum of the digits is 9. If 63 is
subtracted from the number its
digits are interchanged. Find the
number.

- A. 72
- B. 90
- C. 63
- D. 81

Solution :

Let the tens digit be a & units digit be b

$$a + b = 9 \quad \text{.....(1)}$$

$$10a + b - 63 = 10b + a$$

$$9a - 9b = 63 \quad \text{.....(2)}$$

$$(1) \times 9$$

$$9a + 9b = 81 \quad \text{.....(3)}$$

$$(2) + (3)$$

$$18a = 144 \rightarrow a = 8, b = 1$$

$$\text{Number} = 81$$

Ans: D



Sum of Natural Numbers

- **Rule 1** : Sum of first n natural numbers = $\frac{n(n+1)}{2}$

e.g. sum of 1 to 74 = $74 \times (74+1)/2 = 2775$.

- **Rule 2** : Sum of first n odd numbers = n^2

e.g. sum of first seven odd numbers

= $(1+3+5+7+9+11+13) = 49 = 7^2$.

- **Rule 3** : Sum of first n even numbers = $n(n+1)$

e.g. sum of first 9 even numbers

= $(2+4+6+8+10+12+14+16+18) = 90$

= $9(9+1) = 9 \times 10 = 90$



Sum of Natural Numbers

- **Rule 4 :** Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

e.g. sum of squares of first 8 natural numbers

$$= (1 + 4 + 9 + 16 + 25 + 36 + 49 + 64) = 204$$

$$= 8 (8+1)(16+1) / 6 = 8 \times 9 \times 17 / 6 = 204$$

- **Rule 5 :** Sum of cubes of first n natural numbers = $[n(n+1)/ 2]^2$

e.g. sum of cubes of first 4 natural numbers

$$= (1 + 8 + 27 + 64) = 100$$

$$= [4 (4+1)/2]^2 = 100$$



DIVISION

- DIVISION by ZERO is NOT POSSIBLE
- If two numbers are divisible by a number then their sum & difference is also divisible by the number.
- E.g. For 63 is divisible by 9. 27 is also divisible by 9.
- So $63 + 27 = 90$ is also divisible by 9
- And $63 - 27 = 36$ is also divisible by 9



DIVISIBILITY RULES

- 2 : Unit place is even or zero(last digit should be divisible by 2)
- 3 : Sum of the digits is divisible by 3. e.g : 324
- 4 : Last 2 digits are divisible by 4 or last 2 digits are 0. e.g : 324
- 5 : Unit digit is 5 or 0
- 6 : **Divisible by co primes 2 & 3.** e.g : 324
- 8 : Number formed by last 3 digits is divisible by 8 or last 3 digits are 0.
e.g : 1088
- 9 : Sum of all digits is divisible by 9. e.g : 324
- 10: Units digit is 0.



DIVISIBILITY RULES

- **11** : Difference between sum of digits in odd & even places should either be zero or divisible by 11

e.g: 8283

e.g : 918071

- **12** : Divisible by co primes 3 & 4 e.g : 324
- **14** : Divisible by co primes 2 & 7
- **15** : Divisible by co primes 3 & 5
- **16** : No formed by last 4 digits divisible by 16/ last 4 digits 0.
- **18** : Divisible by co primes 2 & 9
- **20** : Units digit 0 & tens digit is even.



DIVISIBILITY RULES

- **7** : The difference between the two alternate groups taking 3 digits at a time should either be zero or multiple of 7.

eg- 550500006

eg- 7370356

- **13** : The difference between the two alternate groups taking 3 digits at a time should either be zero or multiple of 13.

eg- 200174



DIVISIBILITY RULES

- **17: A number is divisible by 17 if you multiply the last digit by 5 and subtract that from the rest. If that result is divisible by 17, then your number is divisible by 17.**
 - For example, for 986, then : $98 - (6 \times 5) = 68$.
 - Since, 68 is divisible by 17, then 986 is also divisible by 17.
 - Also, 876 is not divisible by 17 because $87 - (6 \times 5) = 57$ and 57 is not divisible by 17.
- **19: To determine if a number is divisible by 19, take the last digit and multiply it by 2. Then add that to the rest of the number. If the result is divisible by 19, then the number is divisible by 19.**
 - For example, 475 is divisible by 19 because $47 + (5 \times 2) = 57$, and 57 is divisible by 19.
 - But , 575 is not divisible by 19 because $57 + (5 \times 2) = 67$, and 67 is not divisible by 19.



PROPERTIES OF DIVISIBILITY

To find a number completely divisible by another :

A) Greatest 'n' digit number exactly divisible by a Number :

Method : By subtracting the remainder

e.g a) Greatest 3 digit number divisible by 13

Greatest 3 digit number = 999. $999/13$ gives remainder 11.

$999 - 11 = 988$ = Greatest 3 digit number divisible by 13

B) Least 'n' digit number exactly divisible by a Number :

Method : By adding the (divisor – remainder)

e.g b) Least 3 digit number divisible by 13

Least 3 digit number = 100. $100/13$ gives remainder 9

$100 + (13 - 9) = 104$ = Least 3 digit number divisible by 13



PROPERTIES OF DIVISIBILITY

Q. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is:

A.364724 B.365387 C.365737 D.366757 E. None of these

Soln-

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

$$\begin{aligned}\text{Required number} &= 999 \times 366 + 103 \\ &= (1000 - 1) \times 366 + 103 \\ &= 366000 - 366 + 103 \\ &= 365737\end{aligned}$$

Ans: C



PROPERTIES OF DIVISIBILITY

Q. A number when divided by 5 leaves 3 as remainder. If the square of the same number is divided by 5, the remainder obtained is :

A. 9

B. 4

C. 1

D. 3

Soln:

number when divided by 5 leaves a remainder 3

Let the given number = $5n + 3$ ---> using dividend = divisor quotient + remainder

Square of the number = $(5n + 3)^2$

$$= 25n^2 + 30n + 9 \rightarrow (a + b)^2 = a^2 + 2ab + b^2$$

$$= 5 \times 5n^2 + 5 \times 6n + 5 + 4$$

$$= 5(5n^2 + 6n + 1) + 4$$

Required remainder = 4

Ans: B



PRIME NUMBERS

- A number that is divisible only by itself and 1 (e.g. 2, 3, 5, 7, 11).
- There are **25** prime numbers between 1 - 100
- *1 is neither prime nor composite number.*
- **2 is the only prime number which is even.**
- A number having more than 2 factors is a composite number
- Find prime numbers between 101 and 200??
- There are **21** prime numbers between 101 - 200



Co-Prime

- When two numbers (they may not be prime) do not have any common factor other than one between them they are called co-prime or relatively prime.
- It is obvious that two prime numbers are always co-prime. e.g : 17 and 23
- Two composite numbers can also be co-prime. e.g: 16 & 25 do not have any common factor other than one.
- Similarly 84 and 65 do not have any common factor and hence are co-prime.



Prime Number

- **Sieve of Eratosthenes** is the fastest technique to find whether given number is prime or composite number.
- Let **p** be a given number and **n** be the smallest counting number such that $n^2 \geq p$.
- Ex: check 811 is prime or not. $29^2 > 811$.
- check if 811 is divisible by any prime number below 29 (2,3,5,7,11,13,17,19,23,29).
- none of the prime numbers divides 811.
- 811 is a prime number.



Prime Number

Q. Find whether 467 is prime or not

Step 1 : Sq root of 467 → Between 21 (441) and 22 (484)

Step 2 : 467 is not divisible by 2, 3, 5, 7, 11, 13, 17, 19. Next prime is 23 which exceeds the square limit.

Therefore 467 is prime.



Prime Number(Assignment)

Q. Which of the following is a prime number?

A. 303

B. 477

C. 113

D. None of these

Ans : C



Numbers(Assignment)

Which of the following is the output of $57 \times 57 + 43 \times 43 + 2 \times 57 \times 43$?

A. 10000

B. 5700

C. 4300

D. 1000

Ans : A



Numbers(Assignment)

Q. Which of the following is the output of 6894×99 ?

A. 685506

B. 682506

C. 683506

D. 684506

Ans: B



Numbers(Assignment)

Q. What is the unit digit in $584 \times 428 \times 667 \times 213$?

A. 2

B. 3

C. 4

D. 5

Ans: A



Numbers(Assignment)

Q. The sum of reciprocals of two consecutive numbers is $\frac{15}{56}$. The first number is

- A. 8 B. 7 C. 6 D. 15.

Ans : B



Divisibility (Assignment)

Q. What percentage of the numbers from 1 to 50 have squares ending in the digit 1?

A. 1 B. 10 C. 11 D. 20

Ans : D



Numbers(Assignment)

Q. If $64^2 - 36^2 = 20 \times A$, then $A = ?$

- A. 70 B. 120 C. 180 D. 140 E. None of these

Ans: D



Numbers(Assignment)

Q. On dividing a number by 19 the difference between quotient and remainder is 9. The number is?

A. 370

B. 371

C. 361

D. 352

Ans : B



Numbers(Assignment)

Q. $(112 \times 5^4) = ?$

A. 67000

B. 70000

C. 76500

D. 77200

E. None of these

Ans: B



Numbers(Assignment)

Q. Which of the following is a prime number?

A. 143

B. 289

C. 117

D. 359

Ans : D



HCF & LCM

HCF / GCF(Highest/Greatest Common Factor)

- HCF of two or more numbers is the greatest / largest / highest/biggest number which can divide those two or more numbers exactly.

Factors of 6 : 1, 2, 3, 6

Factors of 8 : 1, 2, 4, 8

Common 1 & 2 Highest & Common 2

• LCM(Least Common Multiple)

- The LCM of two or more numbers is the smallest / lowest / least number which is exactly divisible by those two or more numbers.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54,...

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64....

Common 24, 48, Lowest & common 24



HCF (Factorization method)

- Eg. HCF for 136, 144, 168

2	136	144	168
2	68	72	84
2	34	36	42
	17	18	21

↓
NO FURTHER COMMON FACTOR

So HCF = $2 \times 2 \times 2 = 8$

Note : HCF is always \leq the smallest of given numbers



HCF (Factorization method) - (Assignment)

- HCF of 54,72,126 (factorization method)

A. 21 B. 18 C. 36 D. 54

Ans : B



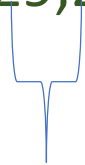
HCF (Difference Method)

- Find HCF of 203,319

Keep smaller here



- (203, 319)
- (116,203)
- (87,116)
- (29,87)
- (29,58)
- (29,29)



HCF = 29



HCF (Difference Method) - (Assignment)

- HCF of 161,253 (difference method)

A. 27 B. 18 C. 23 D. 17

Ans : C



HCF (Difference Method)

Q. Find HCF of 84,125

- (84,125)
 - (41,84)
 - (41,43)
 - (2,41)
 - (2,39)
- If nothing is common then $HCF = 1$ and numbers are said to be co prime numbers.



HCF & LCM

Q. Find the greatest number which can divide 284, 698 & 1618 leaving the same remainder 8 in each case?

- A. 36 B. 46 C. 56 D. 43.

Soln-

Remainder 8 \rightarrow (numbers – 8) would be exactly divisible.

$$\rightarrow 284 - 8 = 276$$

$$\rightarrow 698 - 8 = 690$$

$$\rightarrow 1618 - 8 = 1610$$

\rightarrow Greatest number dividing above 3 = HCF(276, 690, 1610) (difference method)

$$\rightarrow \text{HCF} = 46$$

Ans: B



HCF & LCM

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35 B. 46 C. 56 D. 43.

Soln:-

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 132 - 62 = 70$$

$$\rightarrow 237 - 132 = 105$$

$$\rightarrow 237 - 62 = 175$$

→ Greatest number dividing above 3 = HCF(70, 105, 175)

$$\rightarrow \text{HCF} = 35$$

Ans: A



HCF & LCM

Q. Find the largest number such that 43,65,108 are divisible by that number and we get the remainder as 1,2,3 respectively in each case?

- A. 21 B. 27 C. 42 D. 63

Soln:

→ (numbers – remainder) would be exactly divisible.

$$\rightarrow 43 - 1 = 42$$

$$\rightarrow 65 - 2 = 63$$

$$\rightarrow 108 - 3 = 105$$

$$\text{HCF}(42, 63, 105) = 21$$

Ans : A



HCF & LCM

Q. A teacher has 25 books, 73 pens & 97 erasers. She wants to distribute them equally to maximum number of students so that after distribution she has equal number of books, pens & erasers left. What is the maximum number of students for such a distribution?

A. 32

B. 21

C. 12

D. 24

Soln:-

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 73 - 25 = 48$$

$$\rightarrow 97 - 73 = 24$$

$$\rightarrow 97 - 25 = 72$$

→ Greatest number dividing above 3 = $\text{HCF}(72, 48, 24)$

$$\rightarrow \text{HCF} = 24$$

Ans: D



HCF & LCM(Assignment)

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35 B. 46 C. 56 D. 43.

Ans : A



HCF & LCM(Assignment)

Q. Find largest number such that if 45,68 and 113 are divided by that number we get the remainder as 1,2 and 3 respectively.

- A. 21 B. 22 C. 26 D. 24

Ans: B



HCF & LCM(Assignment)

Q. Find the greatest number which can divide 41, 131 & 77 leaving the same remainder in each case?

A. 28

B. 18

C. 36

D. 24

Ans : B



LCM

- Eg. LCM for 18, 28, 108, 105

2	18	28	108	105
2	9	14	54	105
3	9	7	27	105
3	3	7	9	35
3	1	7	3	35
5	1	7	1	35
7	1	7	1	7
Till all quotients are 1	1	1	1	1

So LCM = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

Note : LCM is always \geq the greatest of given nos



LCM(Assignment)

Q. LCM for 12,24,20

A. 210

B. 180

C. 120

D. 144

Ans : C



LCM (Assignment)

Q. Find LCM of 72,125

A. 9000 B. 1200 C. 1000 D. 800

Ans : A



Rules to Remember

- Product of two given numbers is equal to the product of their HCF & LCM

$$A \times B = \text{HCF}(A,B) \times \text{LCM}(A,B)$$

- If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of } a, b, c) + r \quad \text{e.g } 3,4,5 \text{ \& rem } 1$$



Q. Find LCM of 147 & 231

Soln:-

- As we know,
- **HCF X LCM = product**
- Find HCF by difference method
- Put in the formula,
- $21 \times \text{LCM} = (147 \times 231)$
- 1617

Q. Find LCM of 84 and 125

Soln:-

- As they are co-prime numbers the product is the LCM because $HCF = 1$ (for co-primes)
- $HCF \times LCM = \text{product}$
- $1 \times LCM = 84 \times 125$
- $LCM = 10500$



LCM

Q. Find the least number which when divided by 12,15,24 leaves a remainder of 5 in each case

- **Soln:**
- Find $\text{LCM}(12,15,24) = ?$

If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of a, b, c}) + r \quad \text{e.g 3,4,5 \& rem 1}$$

- $\text{LCM} = 120$
- In an LCM problem, if remainder is common then,
Result = LCM + common remainder
 $= 120 + 5 = 125$



Q. Find the smallest number which when divided by 20,36,45 leaves a remainder 15,31 and 40 respectively.

- **Soln:**
- Find LCM(20,36,45)
- In LCM problem , if difference is common(constant) then,
- **Result = LCM – Common difference**

• 20	36	45	} 5
• 15	31	40	

- Result = $180 - 5$
 $= 175$

Q. Four numbers are in the ratio of 10: 12 : 15 : 18. If their HCF is 3, then find their LCM.

A. 420

B. 540

C. 620

D. 680

Ans : B



Q. Find the least number which when divided by 5,6,7 and 8 leaves a reminder of 3 but when divided by 9 leaves no remainder.

A. 1677

B. 2523

C. 3363

D. 1683

Ans: D



HCF/LCM with Decimal point

- Find HCF of 1.08, 0.36 and 0.9

- **Soln:**

1. Convert each of the decimals into like decimals.

1.08, 0.36 and 0.90

2. Write each number without decimal point.

$\text{HCF}(108, 36, 90) = 18$

3. Put decimal point after the numbers which are in like decimals.

Here it is after 2 numbers(digits)

$\text{HCF}(1.\underline{08}, 0.\underline{36} \text{ and } 0.\underline{90}) = 0.\underline{18}$



HCF(Assignment)

Q. In a school of 437 boys & 342 girls it was decided to divide the girls & boys into separate classes. However it was required that each class consist of the same number of students. What would be the number of classrooms required?

A. 41 classrooms B. 14 classrooms C. 17 classrooms D. 26 classrooms

Ans : A

Same Class Size = HCF (Boys, Girls)

→ $\text{HCF}(437, 342) = 19$

→ Boys Classes = $437/19 = 23$

→ Girls Classes = $342/19 = 18$

→ Total Classes = $23 + 18 = 41$



LCM(Assignment)

Q. Find the least number which when divided by 12,15,40 leaves a remainder of 5 in each case

A. 120

B. 125

C. 130

D. 140

Ans : B



LCM(Assignment)

Q. If the product of two numbers is 324 and their HCF is 3, then their LCM will be = ?

A. 972 B. 327 C. 321 D. 108

Ans: D



LCM(Assignment)

Q. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

- A. 40 B. 80 C. 120 D. 200

Ans: A



LCM(Assignment)

Q. Find the least number which when divided by 16,18,20 and 25 leaves a reminder of 4 but when divided by 7 leaves no remainder.

A. 17004

B. 18000

C. 18002

D. 18004

Ans: D



HCF & LCM(Assignment)

Q. The HCF of two numbers is 8. Which one of the following can never be their LCM ?

- A. 24 B. 48 C. 56 D. 60

Ans: D

If $HCF = 8$ then LCM should have a factor of 8

Going by options 60 does not have a factor 8. So never be their LCM.



HCF & LCM(Assignment)

Q. The LCM of three different numbers is 120. Which of the following cannot be their HCF?

A. 8

B. 12

C. 24

D. 35

Ans: D



Numbers(Assignment)

Q. The number nearest to 43582 divisible by each of 25, 50 and 75 is ?

A. 43500

B. 43550

C. 43600

D. 43650

Ans: D



Numbers(Assignment)

Q. What is the smallest 5 digits number which is divisible by 12, 15, and 18?

A.10010

B. 10015

C.10020

D. 10080

Ans: D



Rules to Remember

- **Fractions :**

LCM = LCM of Numerators / HCF of Denominators

HCF = HCF of Numerators / LCM of Denominators

LCM of 25/12 & 35/18

LCM = 175/6

HCF of 25/12 & 35/18

HCF = 5/36



HCF & LCM Fractions(Assignment)

- Find HCF & LCM of $\frac{5}{9}$ and $\frac{25}{36}$
- Ans : HCF = $\frac{5}{36}$ and LCM = $\frac{25}{9}$



Properties of Square Numbers

- A square can't end with odd number of zeroes. The number of 0's of perfect square is always even and the non-zero part should also be a perfect square.

- A square can't end with 2, 3, 7 or 8.

1	2	3	4	5
6	7	8	9	0

- Square of **odd** no. is **odd** & **even** no. is **even**
- Whenever last digit of square is 6, then second last digit is always odd.
- Whenever last digit of square is 5, then second last digit is always 2.
- Whenever last digit of square is 1,4,9, then second last digit is always even.



Squares

Q. A man plants his orchard with 15876 trees & arranges them so that there are as many rows as there are trees in each row. How many rows does the orchard have?

- A. 124 B. 134 C. 126 D. 136

• Soln:-

- No of trees = No. of rows x no of trees/row
- $15876 = n \times n$
- $n = \sqrt{15876}$
- $n = \sqrt{9 \times 1764}$
- $= \sqrt{9 \times 9 \times 196}$
 $= ?$
 $= 9 \times 14$
 $= 126$

• **Ans C**



Squares(Assignment)

Q. Find a positive number x , such that the difference between the square of this number and 21 is the same as the product of 4 times the number?

- A. 9 B. 27 C. 7 D. 13

Ans : C



Progression

- Arithmetic Progression :

- If quantities increase or decrease by a common difference then they are said to be in AP e.g. 3, 5, 7, 9, 11,
- If a is first term, d is the common difference, l is the last term then
- General form : $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$
- n^{th} term $T_n = a + (n-1)d$, **$n = 1, 2, \dots$**
- Sum of first n terms $S_n = \frac{n}{2} [2a + (n-1)d]$
 $S_n = \frac{n}{2} (a + l)$



Progression

- Prove that the sum S_n of n terms of an Arithmetic Progress (A.P.) whose first term 'a' and common difference 'd' is
- $S = n/2[2a + (n - 1)d]$
- Or, $S = n/2[a + l]$, where $l = \text{last term} = a + (n - 1)d$
- **Proof:**
- $a, a+d, a+2d, a+3d, \dots, a(n-2)d, a(n-1)d$, as $l = \text{last term}$
- $a, a+d, a+2d, a+3d, \dots, l-d, l$
- $S = a + a+d + a+2d + a+3d + \dots + l-d + l \text{ -----1}$
- Writing equation 1 in reverse order(sum remains same even if we write in reverse order)
- $S = l + l-d + l-2d + l-3d + \dots + a+d + a \text{ -----2}$
- Adding equation 1 and 2
- $2S = (a + l) + (a + l) + (a + l) + \dots + (a + l) + (a + l)$
- So for n terms,
- $2S = n(a + l)$
- $S = \frac{n}{2} (a + l)$



Progression

Q. The sum of all two digit numbers divisible by 3 is

A. 550

B. 1550

C. 1665

D. 1680

Soln

Two digit numbers divisible by 3 are :

12, 15, 18, 21,, 96, 99.

This is an A.P. with $a = 12$, $d = 3$, $l = 99$

Let n be the number of terms.

Last term $= a + (n-1)d$

$$99 = 12 + (n-1) \times 3$$

$$3n = 90, \quad n = 30$$

$$\begin{aligned} \text{Sum} &= n/2 (a + l) = 30/2 \times (12 + 99) \\ &= 1665 \end{aligned}$$

Ans: C



Progression

Q. Find the sum of all natural numbers between 10 and 200 which are divisible by 7

A. 2835

B. 2865

C. 2678

D. 2646

Soln:

Two digit numbers divisible by 7 are :

14, 21, 28, 35,, , 196.

This is an A.P. with $a = 14$, $d = 7$, $l = 196$

Last term $= a + (n-1)d$

$$196 = 14 + (n-1) \times 7$$

$$196 - 14 = (n-1) \times 7$$

$$n-1 = 26$$

$$n = 27$$

$$\text{Sum} = n/2 (a + l)$$

$$= 27/2 \times (14 + 196)$$

$$= 27 \times 210 / 2$$

$$= 27 \times 105$$

$$= 2835$$

OR

$$n = \frac{\text{LastTerm} - \text{FirstTerm}}{d} + 1$$

Ans: A



Progression(Assignment)

Q. Find the sum of the series 3,8,13,18,,93

A. 912

B. 925

C. 998

D. 936

Ans : A



Progression

- Geometric Progression :
- If quantities increase or decrease by a constant factor then they are said to be in GP e.g. 4, 8, 16, 32,
- If a is first term, r is the common ratio, then
- General form : a, ar, ar², ar³,, arⁿ⁻¹
- nth term $T_n = ar^{(n-1)}$
- Sum of first n terms $S_n = \frac{a(r^n - 1)}{(r - 1)}$



Geometric Progression of n terms :

- To prove that the sum of first n terms of the Geometric Progression whose first term 'a' and common ratio 'r' is given by-

- $S = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$ ----- 1

- Multiply both sides of this equation by r

- $Sr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} + ar^n$ ----- 2

- - - - -

- Eq 2 - Eq 1

- $Sr - S = ar^n - a$

- $S(r - 1) = a(r^n - 1)$

- $S = \frac{a(r^n - 1)}{(r - 1)}$



Geometric Progression

Q. Find the 10th term of the series: 4, 16, 64, 256, 1024,

- A. 4^{10} B. 4^8 C. 4^9 D. 1022480

Soln:

The given series is in geometric progression

Where $a = 4$, $r = 4$

$$\begin{aligned}\text{So } T_{10} &= a \times r^{(10-1)} \\ &= 4 \times 4^{(10-1)} \\ &= 4^{10}\end{aligned}$$

Ans: A



Progression

- What is the difference between arithmetic progression and geometric progression?
- A sequence is a set of numbers, called terms, arranged in some particular order. An arithmetic sequence is a sequence with the difference between two consecutive terms constant. The difference is called the common difference. A geometric sequence is a sequence with the ratio between two consecutive terms constant.



Averages

- **Simple Average :**

- An average of a set of values is the sum of values divided by the total number of values.
- Average of 'n' values = (Sum of the 'n' values)/n
- This is also called as Arithmetic Mean.
- Average (A) = Sum (S)/ Number(n)
- $S = A \times n$

- **Weighted Average :**

- When all values whose average we want to find do not have uniform occurrences we calculate the weighted average.
- If values $y_1, y_2, y_3 \dots$ occur $w_1, w_2, w_3 \dots$ times then
- Weighted Avg =
$$\frac{(w_1y_1 + w_2y_2 + w_3y_3 + \dots)}{(w_1 + w_2 + w_3 \dots)}$$



Averages

Q. In a class of 50 students, 24 secured 60 in Physics, 16 secured 70 marks and the rest secured 80. What is the average score for Physics in the class?

A. 64.8 B. 65.4 C. 67.2 D. 66.7

Soln :-

Students	24	16	10.
Marks	60	70	80
Average	$= \frac{24 \times 60 + 16 \times 70 + 10 \times 80}{24 + 16 + 10}$ $= 3360/50$ $= 67.2$		

Ans : C



Averages

- Only For Consecutive Numbers -

- Whenever, we have consecutive numbers or consecutive odd numbers or consecutive even numbers, then always remember the middle number is the Average.

- Examples-

- A. 5,6,7,8,9 → Avg =7
- B. 5,6,7,8 → Avg =6.5
- C. 1,3,5,7,9 → Avg =5
- D. 21,23,25,27 → Avg =24



Averages

Q. The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

A. 48

B. 45

C. 43

D. 44

Ans: D



Averages

Q. The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

Solution 1:-

- Before teacher , total age of students = 22×21
- After teacher is added,

Total age of all students + Age of the teacher = 23×22

- Age of the teacher = $23 \times 22 - 22 \times 21$
= $22(23 - 21)$
= 22×2
= 44 years



Averages

- The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

- **Solution 2:-**

- New value = old avg + $(n \pm 1)(\text{diff})$

- Where, n = total no. of students

- New value = $21 + (22+1)(1)$
= $21 + 23$
= 44 years

+ if member added
- If member removed

difference = | Old avg – new avg |



Averages

Q. There are 50 students in a class. Their average weight is 45 kg. When one student leaves the class the average weight reduces by 100 g. What is the weight of the student who left the class ?

A. 45 kg.

B. 47.9 kg.

C. 49.9 kg.

D. 50.1 kg.

Soln:

$$\text{New value} = \text{old avg} + (n \mp 1)(\text{diff})$$

$$= 45 + (50 - 1)(0.1)$$

$$= 45 + 49(0.1)$$

$$= 45 + 4.9$$

$$= 49.9 \text{ kg}$$

$$(\text{ as we convert } 100\text{g into kg} = \frac{100}{1000} = 0.1 \text{ kg })$$

Ans: C



Averages

Q. There are 50 students in a class. Their average weight is 45 kg. When one student leaves the class the average weight reduces by 100 g. What is the weight of the student who left the class ?

- A. 45 kg. B. 47.9 kg. C. 49.9 kg. D. 50.1 kg.

Soln:

Total weight of 50 students = (45×50) kg = 2250 kg

Average weight of 49 students = $45\text{kg} - 100\text{g} = 44.9$ kg

So, total weight of 49 students = $(44.9 \times 49)\text{kg} = 2200.1\text{kg}$

Weight of the students who left the class = $2250 - 2200.1 = 49.9$ kg

Ans: C



Averages

Q. The average age of 16 men increases by 3 years when a person 27 years old is replaced by another. How old is the new person?

A. 75 B. 30 C. 48 D. 64

Soln:-

Number of men = 16

Let average age be a

→ Total age of 16 men = $16a$ (Old total)

New average = $a+3$

→ New total age of 16 men = $16(a+3) = 16a + 48$

New Total – Old Total = 48

→ Age of new man = $27 + 48 = 75$

Ans : A



Averages

Q. The average age of 16 men increases by 3 years when a person 27 years old is replaced by another. How old is the new person?

A. 75 B. 30 C. 48 D. 64

Soln:-

- Average of 16 men increases by 3 years means,
- total age increases by $16 \times 3 = 48$
- If the age of new person same as replaced person then there would have been no change in average.
- But average age of 16 men increased by 3 years
- So, total age of the person replacing another person = $27 + 48 = 75$ years

Ans : A



Averages

Q. The average age of 8 men is decreased by 2 years when two of them, whose ages are 22 and 28, are replaced by two new men.. What is the average age of two men?

A. 34years

B. 30years

C. 15years

D. 17years

Soln:

- Average of 8 men reduce by 2 years means total age reduces by 16 if two men leave.
- So, the total age of the new men replacing the old men = $22 + 28 - 16 = 34$
- \Rightarrow Average = $34/2 = 17$ years.

OR

- Total age decreased = $(8 * 2)$ years = 16 years.
- Sum of ages of two new men = $(22 + 28 - 16)$ years = 34 years
- Average age of two new men = $(34/2)$ years = 17 years.
- **Ans: D**



Averages

Q. The average age of students is 7 years and average age of 10 teachers is 50 years. If average age of group of all teachers and students is 8 years. Find the number of students?

A. 420

B. 250

C. 300

D. 270

Soln:

We know, Total = avg x n

	S	T
No.	z	10
Avg	7	50

(student + teacher) x avg = (student) x avg + (teacher) x avg

$(z + 10) \times 8 = (z) \times 7 + (10) \times 50$

$$8z + 80 = 7z + 500$$

Z = 420 students

Ans :A



Averages(Assignment)

Q. The average weight of 16 boys in a class is 50.25 kg and that of the remaining 8 boys is 45.15 kg. Find the average weights of all the boys in the class.

A. 47.55 kg

B. 48 kg

C. 48.55 kg

D. 49.25 kg

Ans: C

$$\begin{aligned}\text{Average} &= \frac{50.25 \times 16 + 45.15 \times 8}{16 + 8} \\ &= (804 + 361.2) / 24 \\ &= 1165.2 / 24 \\ &= 48.55\end{aligned}$$



Averages(Assignment)

Q. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

A. 20 years

B. 25 years

C. 30 years

D. 27 years

Ans : B



Averages(Assignment)

Q. The average marks of a class of 87 students is 56. When a new student was added and average becomes 56.5. Find marks of new student.

- A. 56 B. 44 C. 100 D. 90

Ans: C



Averages(Assignment)

Q. Find the average of first 97 natural numbers.

A. 47

B. 37

C. 48

D. 49

E. 49.5

Ans: D



Averages(Assignment)

Q. The average age of a class of 30 students is 9 years. When teacher's age is also added, the average becomes 10. What is the age of the teacher?

A. 41 years

B. 40 years

C. 39 years

D. 42 years

Ans: B



Averages(Assignment)

Q. The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is nearly:

A. 28.32

B. 29.68

C. 28.78

D. 29.27

Ans: B



Averages(Assignment)

Q. The average age 8 men is increased by 2 years when two of them whose ages are 21 years and 23 years are replaced by two new men . The average age of the two new men is?

A. 22 years

B. 24 years

C. 28 years

D. 30 years

Ans: D



Averages(Assignment)

Q. The average weight of the students of a class is 60 kg. If eight new students of average weight 64 kg join the class, the average weight of the entire class becomes 62 kg. How many students were there in the class initially ?

A. 8 students

B. 16 students

C. 10 students

D. 12 students

Ans: A



Averages(Assignment)

Q. The average of ten numbers is 8. If the average of first nine numbers is 7. Find the 10th number?

A. 17

B. 16

C. 15

D. 12

Ans: A



Averages(Assignment)

Q. The average marks obtained by 150 students is 30. If the average marks of passed candidates was 40 and that of failed candidates was 20. Find the number of candidates who passed the exam?

A. 25

B. 85

C. 75

D. 45

Ans: C



Averages(Assignment)

Q. The average expenditure of a man for the first five months is Rs. 3600 and for next seven months is Rs. 3900, if he saves Rs.8700 during the year, his average income per month is ?

A. Rs.4500

B. Rs.8500

C. Rs.7500

D. Rs.5400

Ans: A



Averages(Assignment)

Q. The average of first five multiples of 3 is:

A. 9

B. 10

C. 8

D. 11

Ans: A



Averages(Assignment)

Q. Find the average of first 100 positive numbers

A. 49.5

B. 50.5

C. 51

D. 100

Ans: B



Averages(Assignment)

Q. The average expenditure of a man for the first five months of a year is Rs. 5000 and for next seven months is Rs. 5400, if he saves Rs.2300 during the year, his average income per month is ?

A. Rs.5425

B. Rs.5446

C. Rs.5500

D. Rs.5600

Ans: A



