Mahendra's

YOUR SUCCESS IS OUR SUCCESS

QUANTITATIVE APTITUDE

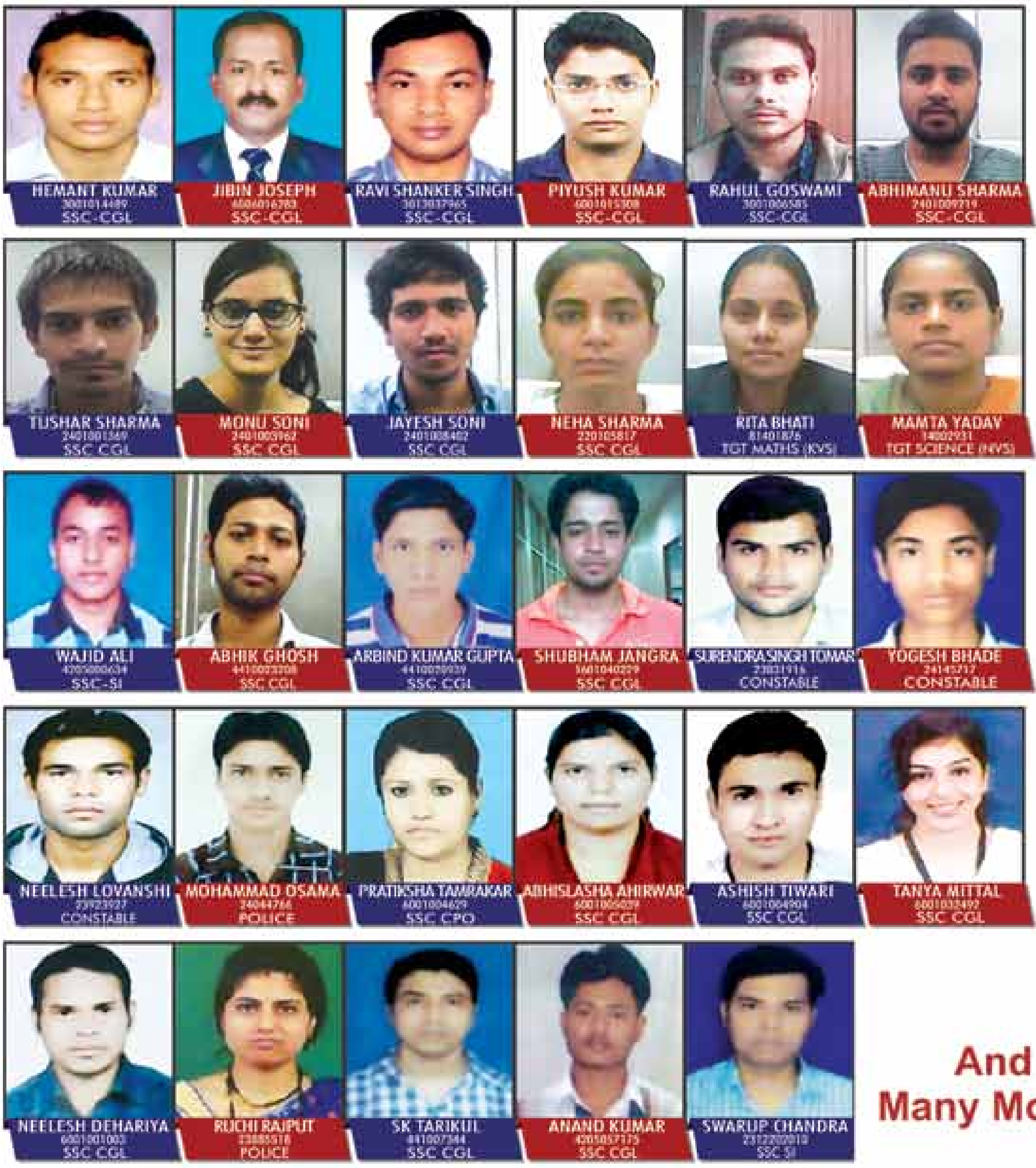
FOR CGL, CHSL, CPO, STENOGRAPHER, MULTI-TASKING etc.

(Based on Latest Exam Pattern)

Mahendra's

YOUR SUCCESS IS OUR SUCCESS

GLIMPSE OF OUR SELECTED CANDIDATES



And Many More..

www.mahendras.org

Toll Free (9 A.M. - 6 P.M.): 1800-103-5225

Quantitative Aptitude

SALIENT FEATURES:

- Chapterwise Discussion
- Topicwise Rules and Examples
- Exercise based on previous papers
- Easy and comprehensible language

Mahendra Publication Pvt. Ltd.
New Delhi

Mahendra Publication Pvt. Ltd.

Registered Office

Mahendra Publication Pvt. Ltd., 103, Pragatideep Building, Plot No. 08, Laxminagar,
District Centre, New Delhi - 110092, TIN-09350038898, w.e.f. 12-06-2014

Branch Office

Mahendra Publication Pvt. Ltd. E-42,43,44, Sector-7, Noida (U.P.)

For queries regarding promotion, distribution & advertisement, contact:-
E-mail-sales@mahendras.org
Ph.: 09208037962

Owned, printed & published by N.K. Jain

103, Pragatideep Building, Plot No. 08, Laxminagar, District Centre, New Delhi - 110092

Writer

Mahendra Publication Team

Please send your suggestions and grievances to:-

Mahendra Publication Pvt. Ltd., CP-9, Vijayant Khand, Gomti Nagar Lucknow - 226010
E-mail:info@mahendras.org

PREFACE

Change is the law of life, and those who look only to the past or present are certain to miss the future.

John F. Kennedy

This gives us immense pleasure, to present you the latest edition of this book. We thank you from the depth of our hearts, for the love and affection given by you just from the beginning. Dear friends, change is the law of nature. We must learn to tolerate the blows of time with patience and learn not only to endure, but also to expect, welcome, and enjoy both the joys as well as the sorrows of life. We must do what we can to “get success.” We have to operate with the information and skills that are necessary for winning.

Today’s era is governed by technology. The technology has increased the pace of changing the world we see day by day, and so the pattern of examination and criteria of selection has also changed. As we are aware that the interview is a part of the various examinations and the written/objective exams are going to be tougher than earlier.

Every choice you make — including the thoughts you have — has consequences. When you start choosing the right behaviour and thoughts — which will take a lot of discipline — you’ll get the right outcomes.

Be aware of the factors that influence the way you see the world so that you can deal with them and react against them. You are your own most important resource for making your life work. Success is a moving target that must be tracked and continuously pursued.

In this context, we have completely updated this book keeping in mind the forthcoming examination pattern. This edition caters to need of Mathematics paper that is asked in most of the examinations conducted by SSC for various posts. Every chapter in this book describes the concept with the help of various examples and at the end gives miscellaneous examples to clear the concepts. Examples have been solved with standard as well as short methods where required. At the end of chapter exercise also gives to solve So we have now given a complete focus on the concept building through this book.

Practice makes a man perfect. You will get a lot of questions on our websites for practice in the form of e-books for free, which will be continuously updated to match the latest trends.

If this book will help students in getting selection in examination we will be highly satisfied.

Any correction and changes related to the material contained in this book shall always be welcomed and we shall endeavour to incorporate them in our upcoming issues.

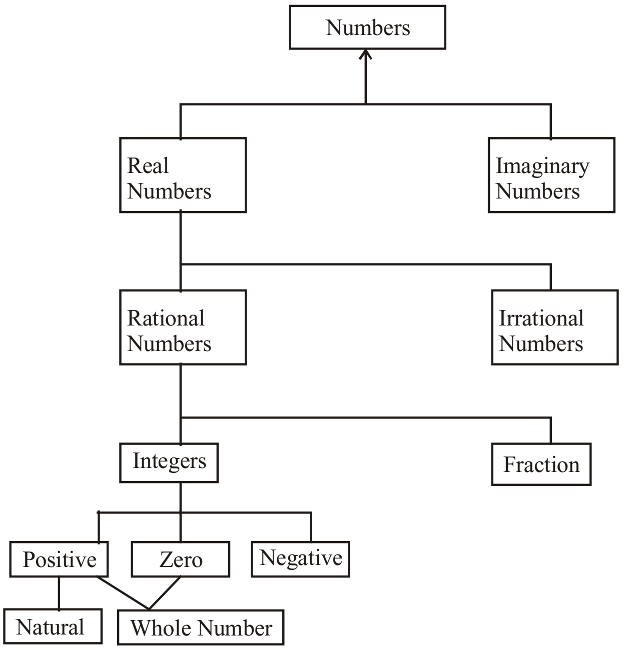
Research Team

CONTENTS

CHAPTER-1
NUMBER SYSTEM

Scan the QR code to get video of this chapter.

Types of numbers



(i) **Natural Numbers :** Counting numbers are called natural numbers. Thus 1, 2, 3, 4,5, 6,.......etc. are all natural numbers.

(ii) **Whole Numbers :** All counting numbers together with zero form the set of whole numbers.

Note :

(a) 0 is a whole number which is not a natural number

(b) Every natural number is a whole number.

Thus 0, 1, 2, 3, 4, 5, 6,........are whole numbers.

(iii) **Integers :** When negative value of natural number are included with the whole number these number are known as integers.

(a) **Positive Integers :** All integers greater than 0 are known as positive integers.

{1, 2, 3, 4, 5............}

(b) **Negative Integers :** All the integers less than 0 is known as negative integers.

{-1, -2, -3, -4......}

(c) **Non-Negative Integers :** 0 and all the positive integers are known as non negative integers.

{0, 1, 2, 3, 4, 5........}

(d) **Non-Positive Integers :** 0 and all the negative integers are known as non positive integers.

{0, -1, -2, -3, 4,.........}

(iv) **Even Numbers :** A natural number which is divisible by 2 is called an even number.

Thus 2, 4, 6, 8, 10........etc. are all even numbers.

(v) **Odd Numbers :** A natural number which is not divisible by 2 is called an odd number.

Thus 1, 3, 5, 7, 9.......etc. are odd numbers.

(vi) **Prime Numbers :** A natural number which has two distinct factor i.e. 1 and number itself only is called a prime number.

(vii) **Composite number :** Natural number which has more than two distinct factors are called composite numbers eg. 6, 8, 10, 12, 14.

Note :

1. 1 is neither prime nor composite.

2. 2 is the only even number which is a prime number.

(viii) **Co-Prime Numbers :** Two natural numbers a and b are said to be co-prime, if their H.C.F is 1.

e.g. (2, 3), (4, 5), (7, 9), (8, 11), (9, 14)

(ix) **Rational Numbers :** A number that can be expressed in the form of p/q is called a rational number where p and q are integer number and q $\neq$ 0.

2/1, 3/5, 7/9, 8/9, 13/15 etc.

(x) **Irrational Numbers :** A number that can not be expressed in the form of p/q is called a rational number where p and q are integer number and q $\neq$ 0.

$\pi, \sqrt{2}, \sqrt{3}, \sqrt{7}$ etc.

(xi) **Real Numbers :** Real Numbers include both rational and irrational number.

7/9, $\sqrt{2}, \pi$

Ex. The sum of a two-digit number and the number obtained by reversing its digits is a perfect square number. How many such numbers exist ?

(A) 6 **(B)** 5

(C) 8 **(D)** 4

Sol.(C) Let the number be- 10x + y

So, Sum = 10x + y + 10y + x = 11(x+y)

Sum of the numbers should be divisible by 11 and should be a perfect square at the same time that means:

$11^2 = 121$

Now = $\frac{121}{11}$ = 11 sum of digits

numbers are-

29 92

38 83

47 74

56 65

Total 8 numbers

Ex. In a two-digit number, it is known that its unit digit exceeds its tens digit by 2 and the product of the given number, and the sum of its digits is equal to 144, then the number is-

(A) 42 **(B)** 24

(C) 13 **(D)** 35

Sol.(B) The possible numbers of such types are

13 13×4 = 52 (wrong)

24 24×6 = 144 (Ans. 24)

35 35×8 = 280 (wrong)

Traditional Method:

x = unit digit

y = tenth digit

x - y = 2

(x+10y) (x+y) = 144

put x = y + 2

(2+y+10y) (2+2y) = 144

(11y+2) (2y+2) = 144

(11y+2) (y+2) = 72

$11y^2 + 2y + 11y + 2 = 72$

By solving y = 2, $\frac{-35}{11}$

Then x = 4

Then number = 24

Division Algorithm

Divident = Divisor × Quotient+ Reaminder

Ex. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, then find the dividend.

(A) 5388 **(B)** 5343

(C) 5336 **(D)** 5391

Sol.(C) Given, divisor = 5 × remainder

= 5 × 46 = 230

Also, 10 × Quotient = 230

Quotient = 23

We know that,

Divident = (Divisor × Quotient) + Remainder

= (230 × 23) + 46 = 5290 + 46 = 5336

Test of Divisibility

(i) **Divisibility by 2 :** A number is divisible by 2, if its unit digit is 0, 2, 4, 6, 8 as-

64892 is divisible by 2, while 64895 is not divisible by 2.

(ii) **Divisibility by 3 :** A number is divisible by 3 only when the sum of its digits is divisible by 3.

(a) Consider the number 587421. Sum of its digits is 27, which is divisible by 3. So, 587421 is divisible by 3.

(b) Consider the number 689453. Sum of its digits is 35, which is not divisible by 3. So, 689453 is not divisible by 3.

(iii) **Divisibility by 4 :** A number is divisible by 4, if the last two digits (unit and tens places) of the number are zeros or a multiple of 4.

(a) 5249376 is divisible by 4, since 76 is divisible by 4.

(b) 633214 is not divisible by 4, since 14 is not divisible by 4.

(iv) **Divisibility by 5 :** A number is divisible by 5 if its unit digit is 5 or 0.

(a) 328695 is divisible by 5.

(b) 947312 is not divisible by 5.

(v) **Divisibility by 6 :** A number is divisible by 6 if it is divisible by both 2 and 3.

Note : If an number is divisible by p and q then it will be also divisible by this L.C.M.

(a) 974562 is divisible by 2 as well as 3. So, it is divisible by 6.

(b) 975416 is not divisible by 6, since sum of its digits is 32, which is not divisible by 3.

(vi) **Divisibility by 7 :** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

(a) Check if 133 is divisible by 7 ?

Step :I -

133 = 13 – 3×2 = 13 – 6 = 7

Since, 7 is divisible by 7. So, the given number 133 is divisible by 7.

(b) Check if 1071 is divisible by 7 ?

Step :I -

1071 = 107 – 1×2 = 107 – 2 = 105

Step : II -

105 = 10 – 5×2 = 10 – 10 = 0

Since, 0 is divisible by 7. Hence the given number 1071 is divisible by 7.

(vii) **Divisibility by 8 :** A number is divisible by 8 only when the number formed by its last 3 digits is divisible by 8.

(a) 6754120⇒120 is divisible by 8, since 6754120 is divisible by 8.

(b) 5943246⇒246 is not divisible by 8, since 5943246 is not divisible by 8.

(viii) **Divisibility by 9 :** A number is divisible by 9 if the sum of its digits is divisible by 9.

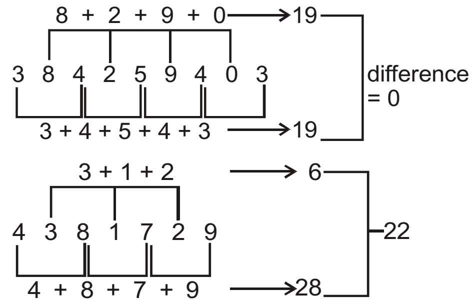
(a) 594324 is divisible by 9, since the sum of its digits is 27, which is divisible by 9.

(b) 3714529 is not divisible by 9, since the sum of its digits is 31, which is not divisible by 9.

(ix) **Divisibility by 10 :** A number is divisible by 10 only when its unit digit is 0.

234780 is divisible by 10 while 234785 is not divisible by 10.

(X) **Divisibility by 11** ⇒ A number is divisible by 11 if the difference of the sum of its digits at odd place and the sum of its digits at even place is either '0' or multiple of 11.



(XI) **Divisibility by 12** ⇒ A number which is divisible by 4 and 3, is also divisible by 12.

2244 is divisible by both 3 and 4. Therefore it is divisible by 12 also.

EXAMPLES

Ex. What is the value of M and N respectively if M39048458N is divisible by 8 and 11; where M and N are integer values from 1 to 8.

(A) 4, 6 **(B)** 6, 8

(C) 6, 4 **(D)** 6, 7

Sol.(C) Here, the last three digits 58N should be divisible by 8, then N = 4

Difference of the sum of alternate digits should be divisible by 11 or should be 0.

(M+9+4+4+8) - (3+0+8+5+N) = (M+5)

should be divisible by 11

So M = 6

Ex. How many numbers between 400 and 800 are divisible by 4,5 and 6.

(A) 7 **(B)** 8

(C) 6 **(D)** 5

Sol.(A) LCM of 4,5 and 6 = 60

Quotient on dividing 800 by 60 = 13

Quotient on dividing 400 by 60 = 6

Ans : 13 - 6 = 7

Concept of Remainder

(a) When a_1, a_2, a_3 .... are divided by 'd', they give respective remainders R_1, R_2, R_3,..... and when (a_1+a_2+a_3+.......) is divided by 'd' then remainder can be obtained by dividing (R_1+R_2+R_3+.....) by 'd'.

(b) When a_1, a_2, a_3 .... are divided by 'd', they give respective remainders R_1, R_2, R_3,....... and when ($a_1 \times a_2 \times a_3 \times$ ....) is divided by 'd' then remainder can be obtained by dividing ($R_1 \times R_2 \times R_3 \times$ .....) by 'd'.

(c) When, 'a' is divided by 'd' the remainder is R and when 'a_1' and 'a_2' are divided by 'd' the remainder is R_1 and R_2. The remainder R will be equal to the difference of remainder R_1 and R_2 if $a_1 - a_2 = a$.

(d) $\frac{(a+1)^n}{a}$ gives always remainder 1.

(e) $\frac{a^n}{(a+1)}$ gives remainder 1, when n is even and gives remainder 'a' when n is odd.

Concept of Negative Remainder

For fast calculation we can use concept of negative remainder. Suppose a number p is divided by another number q and

p = qx–a

it means a is negative remainder.

If 15 is divided by 4 it gives a remainder of 3 or a negative remainder of 1.

Use : Find the remainder in $\frac{15^{2019}}{4}$.

If we divide 15 by 4 we get a negative remainder of 1.

Again, $(-1)^{odd}$ will give (-1).

But, to get final answer we will add 4 in this.

Required answer = –1 + 4 = 3

Ex. Find the remainder when 678 + 687 +6879 + 6890 is divided by 17.

(A) 3 **(B)** 4
(C) 5 **(D)** 6

Sol.(B) The individual remainders of 678, 687, 6879, 6890 when divided by 17 are 15, 7, 11, 5 respectively.

Hence, the remainder

=(15+7+11+5 = 38) ÷ 17 = 4.

Ex. Find the remainder when 7^7 is divided by 4.

(A) 1 **(B)** 2
(C) 3 **(D)** 0

Sol.(C) $\frac{7^7}{4} = \frac{7\times7\times7\times7\times7\times7\times7}{4}$

remainder of $\frac{7}{4}$ = 3

$= \frac{3\times3\times3\times3\times3\times3\times3}{4} = \frac{3^7}{4}$

$= \frac{a^n}{a+1}$ gives remainder a when n is odd

So, remainder = 3

Ex. The remainder when 3^{21} is divided by 5 is:

(A) 1 **(B)** 2
(C) 3 **(D)** 0

Sol.(C) $3^{21} = (3^4)^5 \times 3 = 81^5 \times 3$

$= \frac{81}{5}$ = remainder = 1

81^5 remainder $(1)^5$

$\frac{3}{5}$ remainder 3

1 × 3 = 3

Ex. 9^6-1 when divided by 8 would leave a remainder of:

(A) 1 **(B)** 2
(C) 3 **(D)** 0

Sol.(D) $\frac{9}{8}$ remainder = 1

$(1)^6$ remainder = 1

1 - 1 = 0

Ex. A number when divided by 119, leaves 19 as a remainder. If it is divided by 17. It will leave a remainder :

(A) 1 **(B)** 2
(C) 3 **(D)** 4

Sol.(B) Since 119 is divisible by 17, so we can divide 19 by 17 then remainder will be 2

and 2 is the answer.

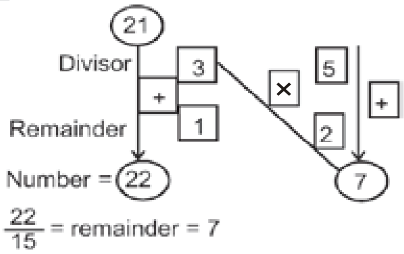
Successive Division

If the quotient in a division is further used as dividend for the next divisor and again the latest obtained divisor is used as quotient for another divisor and so on then it is called **Successive Division.**

Ex. A certain number when successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number is divided by 15?

(A) 4 **(B)** 6 **(C)** 5 **(D)** 7

Sol.(D)

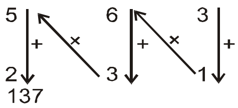


Ex. When a number is successively divided by 5,6 and 3, the respective remainders are 2,3 and 1. What will be the remainder if the order of divisors is being reversed.

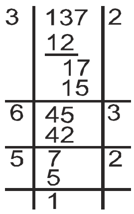
(A) 2, 3, 2 **(B)** 2, 2, 3

(C) 3, 3, 2 **(D)** 3, 2, 2

Sol.(A)



⇒ Now divisors are 3,6 and 5



Ans: 2, 3, 2

NUMBER OF ZEROs IN N !

$\frac{N}{5}+\frac{N}{5^2}+\frac{N}{5^3}+$..........

Numerator should be greater than or equal to denominator.

Ex. How many zeros are there in 100 ! ?

(A) 22 **(B)** 23

(C) 24 **(D)** 25

Sol.(C) $\frac{100}{5}+\frac{100}{5^2}$ = 20+4 = 24

Ex. How many zeros are there in 47 ! ?

(A) 8 **(B)** 10

(C) 12 **(D)** 11

Sol.(B) $\frac{47}{5}+\frac{47}{25}$ = 9+1 = 10

Note : While solving we take only integer value and ignore the decimal part.

CONCEPT OF UNIT PLACE

⇒ The last digit (or unit digits) of 0, 1, 5 and 6 are always the same irrespective of their powers raised on them.

Cyclic order -

Unit place	Unit place	
$2^1 = 2$	$3^1 =$	3
$2^2 = 4$	$3^2 =$	9
$2^3 = 8$	$3^3 =$	7
$2^4 = 6$	$3^4 =$	1
$2^5 = 2$	$3^5 =$	3

⇒ If the last digit (or unit digits) is even and the power is divisible by 4 then unit place is 6.

⇒ If the last digit (or unit digits) is odd and the power is divisible by 4 then unit place is 1, accept for digit 5.

Ex. Find the unit in the product

6892 × 3568 × 7239 × 3624 = ?

(A) 4 **(B)** 7

(C) 6 **(D)** 5

Sol.(C) Product of unit digits = 2 × 8 × 9 × 4

= 576

= 6 (unit digit)

Ex. What is the unit digit in $(3683)^{169}$

(A) 1 **(B)** 2

(C) 4 **(D)** 3

Sol.(D) We will take the last two digits in the power 169 i.e. only 69. Divide 69 by 4, get the remainder

$3^{169/4} = 3^1 = 3$

Ex. What is the unit digit in

$7^{78} \times 4^{93} \times 6^{41}$

(A) 6 **(B)** 4

(C) 2 **(D)** 3

Sol.(A) $7^2 \times 4^1 \times 6$

= 49 × 4 × 6 = 6

Number of factors

As we know prime numbers have only two factors i.e. 1 and number itself. So 1 and number will be always a factor of number.

Number of factors are total numbers which divide any given number.

For example 12 is divisible by 1, 2, 3, 4, 6, 12. So total number of for 12 will be 6. But if the number is large it becomes very dificult to calculate number of factors manually. There is a formula to calculate number of factors.

$N = a^p \times b^q \times c^r$.............(Where a, b, c...... are prime numbers)

Total number of factors =(p+1)(q+1)(r+1).....

Number of odd factors = (q+1)×(r+1).

Number of even factors = Total number of factor - number of odd factors.

Sum of factors = $(a^0+a^1+a^2.........a^p)(b^0+b^1+b^2.........b^q)(c^0+c^1+c^2.........c^r)...$

Sum of odd factors = $(a^0)(b^0+b^1+b^2.........b^q)(c^0+c^1+c^2.........c^r)...$

Sum of even factors = $(a^0+a^1+a^2.........a^p)(b^0+b^1+b^2.........b^q)(c^0+c^1+c^2.........c^r)...$

Ex. Find the total number of factors, odd factors and even factors of $168 = 2^1\times3^4$

Total number of factors = (1+1)(4+1)

= 10

Total number of odd factors = (4+1) = 5

Total number of even factors = (10-5)

= 5

Ex. Find the sum of the factors of 240?

Sum of factors = $(2^0+2^1+2^2+2^3+2^4)(3^0+3^1)(5^0+5^1)$

= 31×4×6= 744

Decimal and Recurring Decimal

If in a decimal fraction, a digit or a set of digits is repeated then such a number is known as a recurring decimal.

Ex. I. $\frac{1}{3}$ = .333.......... = $0.\overline{3}$

II. $\frac{1}{7}$ = .142857142857.........= $0.\overline{142857}$

Pure recurring decimal : A decimal fraction in which all the digits after the decimal point are repeated is called a pure recurring decimal.

Ex. 0.5353 ....= $0.\overline{53}$

Mixed recurring decimal : A decimal fraction in which some figures after the decimal point do not repeat and some figures repeat is called a mixed recurring decimal.

Ex. 0.177--------- = $0.1\overline{7}$

To convert a pure recurring decimal to fraction

Rule : Write the repeated figure or only one in the numerator without decimal point and take as many nines in the denominator as is the number of repeating digits.

Express the following as fraction

(i) $0.\overline{7} \Rightarrow \frac{7}{9}$ (ii) $0.\overline{41} \Rightarrow \frac{41}{99}$

(iii) $0.\overline{057} \Rightarrow \frac{57}{999}$

To convert a mixed recurring decimal to fraction

Rule : In the numerator, take the difference between the numerator formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as is the number of followed as many zeros of non-repeating digits.

Express the following as fraction :

(i) $0.1\overline{7} \Rightarrow \frac{17-1}{90} = \frac{8}{45}$

(ii) $0.12\overline{54} = \frac{1254-12}{9900} = \frac{1242}{9900} = \frac{69}{550}$

(iii) $2.63\overline{4} = 2.63\overline{4} = 2+\left(\frac{634-63}{900}\right)$

$2+\frac{571}{900} = \frac{2371}{900}$

Ex. Given that 0.111........= $\frac{1}{9}$. then 0.444.... is euqal to:

(A) $\frac{1}{90}$ **(B)** $\frac{2}{45}$

(C) $\frac{1}{99}$ **(D)** $\frac{4}{9}$

Sol.(D) 0.444....

$= 0.\overline{4} = \frac{4}{9}$

Ex. $\left(0.\overline{11}+0.\overline{22}\right)\times3$ is equal to

Sol. $\left(\frac{1}{9}+\frac{2}{9}\right)\times3,\ \frac{3}{9}\times3 = 1$

Ex. $1.\overline{2}\times0.\overline{03}$

Sol. $1\frac{2}{9}\times\frac{3}{99} = \frac{11}{9}\times\frac{1}{33} = \frac{1}{27}$

27) 100 (0.037
 81
 190
 182
 1

$= 0.\overline{037}$

Some Important Facts

1. Square of every even number is an even number while square of every odd number is an odd number.
2. A number obtained by squaring a number does not have 2, 3, 7 or 8 at its unit place.
3. Sum of first n natural numbers = $\frac{n(n+1)}{2}$
4. Sum of first n odd numbers = n^2
5. Sum of first n even numbers = n (n+1)
6. Sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
7. Sum of cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

EXAMPLES

Ex.1. Which of the following will divide- $3^{25} + 3^{26} + 3^{27} + 3^{28}$

(A) 2, 4, 6, 8

(B) 3, 5, 7, 9

(C) 3, 4, 5, 6

(D) All of the above

Sol.(C) $3^{25} + 3^{26} + 3^{27} + 3^{28}$

$\Rightarrow 3^{25}\left(1+3+3^2+3^3\right)$

$= 3^{25}(1+3+9+27) \Rightarrow 3^{25}(40)$

divided by = 3, 4, 5, 6

Ex.2. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}$

(A) 5/6 **(B)** 6/7

(C) 7/8 **(D)** 8/9

Sol.(C) $1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}$

$1-\frac{1}{8}=\frac{7}{8}$

Ex.3. $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\frac{1}{10\times13}+\frac{1}{13\times16}+\frac{1}{16\times19}$

(A) 6/23 **(B)** 5/19

(C) 6/19 **(D)** 7/23

Sol.(C) $\frac{1}{3}\left[1-\frac{1}{4}+\frac{1}{4}-\frac{1}{7}+.........+\frac{1}{16}-\frac{1}{19}\right]$

$\frac{1}{3}\left[1-\frac{1}{19}\right]$

$=\frac{1}{3}\times\frac{18}{19}=\frac{6}{19}$

QUICK TRICK :

Difference of the number = 4 - 1 = 3

Sum = $\frac{1}{\text{Difference}}$ [first number- last number]

$=\frac{1}{3}\left[1-\frac{1}{19}\right]=\frac{1}{3}\times\frac{18}{19}=\frac{6}{19}$

Ex.4. Find the sum of

$\frac{1}{2^2-1}+\frac{1}{4^2-1}+\frac{1}{6^2-1}.........\frac{1}{20^2-1}$

(A) $\frac{20}{21}$ **(B)** $\frac{15}{20}$

(C) $\frac{18}{21}$ **(D)** $\frac{10}{21}$

Sol.(D) $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+.........\frac{1}{19\times21}$

$\frac{1}{2}\left[\frac{1}{1}-\frac{1}{21}\right] \Rightarrow \frac{1}{2}\left[\frac{20}{21}\right]=\frac{10}{21}$

Ex.5. Let X, Y and Z be three distinct integers, X and Y are odd and positive, and Z is even and positive. Which one of the following statements cannot be true?

(A) $(X - Z)^2Y$ is even **(B)** $(X - Z)Y^2$ is odd

(C) $(X - Z)Y$ is odd **(D)** $(X - Y)^2Z$ is even

Sol.(A) X is odd and Z is even so, X - Z will be odd, and Y is odd.

Therefore, $(X - Z)^2$ will be odd and $(X - Z)^2 Y$ will be odd

Ans: A is not true.

Ex.6. If both 11^2 and 3^2 are factors of the number $a\times 4^3 \times 6 \times 13^{11}$, which is the smallest possible value of 'a'?

(A) 121 **(B)** 605

(C) 363 **(D)** 37

Sol.(C) 4,6 or 13 are not divisible by 11, Hence 'a' should include 11^2.

Now $\frac{4^3 \times 6 \times 13^{11}}{3^2}$

$= \frac{4 \times 4 \times 4 \times 3 \times 2 \times 13^{11}}{3 \times 3}$

That means 'a' will include 3 also. Minimum value of 'a' = $11^2 \times 3 = 363$

Ex.7. There is a number consisting of two digits, the digit in the units place is twice of that in the tens place and if two is subtracted from sum of the digits then it is equal to 1/6th of the number. The number is:

(A) 24 **(B)** 36

(C) 42 **(D)** 48

Sol.(A) Number = 2x + 10x

sum of the digits

$x + 2x - 2 = \frac{2x + 10x}{6}$

$3x - 2 = \frac{12x}{6}$

18x - 12 = 12x

6x = 12

x = 2

So, number = 24

Ex.8. A number consists of two digits such that the digits in the ten's place is less by 2 than the digit in the unit place. Three times the number added to 6/7 times the number obtained by reversing the digits equals 108. The sum of digits of the number is:

(A) 8 **(B)** 9

(C) 6 **(D)** 7

Sol.(C) Unit digit = x

Tens digit = (x - 2)

Number = (x+10x - 20)

on reversing the digits = (x-2+10x)

$3(x+10x-20) + \frac{6}{7}(x-2+10x) = 108$

$3x + 30x - 60 + \frac{6x}{7} - \frac{12}{7} + \frac{60x}{7} = 108$

21x+210x - 420 + 6x - 12 + 60x = 756

$x = \frac{1188}{237} = 4$

x = 4

x - 2 = 2

4+2 = 6

Ex.9. In a three-digit number, the digit at the hundred's place is two times the digit at the unit's place and the sum of the digits is 18. If the digits are reversed, the number is reduced by 396. The difference between hundred's and ten's digit of the number is:

(A) 2 **(B)** 3

(C) 5 **(D)** 7

Sol.(A) Let unit's digit = x and ten's digit = y

100^{th} digit = 2x

then number

100 (2x) + 10y + x = 201x+10y ......(1)

2x + y + x = 18

3x + y = 18 ....(2)

When the digits are reversed, number

100 (x) + 10y + 2x = 102x + 10y .... (3)

201x + 10y - 102x - 10y = 396

Solving the equations

99x = 396

x = 4

y = 18 - 12 = 6

Number = 864

Required difference

= 2x - y = 2 × 4 - 6 = 2.

Ex.10. The expression $2^{6n} - 4^{2n}$, where n is a natural number is always divisible by-

(A) 15 **(B)** 18

(C) 36 **(D)** 48

Sol.(D) $2^{6n} - 4^{2n}$,

$= (2^6)^n - (4^2)^n = 64^n - 16^n$

64-16 is always divisible by 48.

Ex.11. The value of λ for which the expression $x^3+x^2-5x+\lambda$ will be divisible by (x-2) is:

(A) 2 **(B)** -2

(C) -3 **(D)** 4

Sol.(B) (x-2) is a factor of polynomial

$P(x) = x^3+x^2 - 5x + \lambda$

$P(2) = 0$

(i.e., on putting x =2)

$2^3+2^2 - 5\times2+\lambda=0$

$8+4-10+\lambda = 0$

$\lambda+2 = 0$

$\lambda= -2$

Ex.12. The greatest common division of $3^{3^{333}}+1$ and $3^{3^{334}}+1$ is-

(A) 2 **(B)** 3

(C) 4 **(D)** 8

Sol.(A) $3^1 = 3 + 1$ = even number

$3^2 = 9 + 1$ = even number

$3^3 = 27 + 1$ = even number

$3^4 = 81 + 1$ = even number

So both the numbers are divisible by 2

Ex.13. If a and b are two odd positive integers, which of the following integers is $(a^4 - b^4)$ always divisible?

(A) 3 **(B)** 6

(C) 8 **(D)** 12

Sol.(C) $(a^2)^2 - (b^2)^2$

$= (a^2+ b^2) (a^2 - b^2)$

$= (a^2+ b^2) (a + b) (a - b)$

= put different values of a and b

Let a = 3

b = 1

$10\times4\times2 = 80$

divisible by 8

Ex.14. The solution of inequality $12x - 61 \le 6$ is

(A) $x \le \frac{67}{12}$ **(B)** $0 \le x \le \frac{67}{12}$

(C) $-\frac{67}{12} \le x \le 6$ **(D)** $-\frac{67}{12} \le x \le 0$

Sol.(A) $12x \le 6+61$

$x \le \frac{67}{12}$

Ex.15. If the sum of five consecutive integers is S, then the largest of those integers in terms of S is-

(A) $\frac{S-10}{5}$ **(B)** $\frac{S+4}{5}$

(C) $\frac{S+5}{4}$ **(D)** $\frac{S+10}{5}$

Sol.(D) Integers are in AP

Average of an AP is middle term i.e.

$\frac{S}{5}$ = middle term, The greatest term

$\frac{S}{5}+2 = \frac{S+10}{5}$

Ex.16. The number 323 has

(A) Three prime factors

(B) Five prime factors

(C) Two prime factors

(D) No prime factors

Sol.(C) $17 \times 19 = 323$

two prime factors

Ex.17. $a\times b = a+b + \frac{a}{b}$

then the value of 12×4 is :

(A) 20 **(B)** 21

(C) 48 **(D)** 19

Sol.(D) $12 + 4 + \frac{12}{4}$

$12 + 4 + 3 = 19$

Ex.18. To get the ratio p : q. (for $p \neq q$)

one has to add a number to each term of the ratio x : y, the number is:

(A) $\frac{px-qy}{p-q}$ **(B)** $\frac{qx-py}{p-q}$

(C) $\frac{px-qy}{p-q}$ **(D)** $\frac{py-qx}{p-q}$

Sol.(B) $\frac{x+a}{y+a} = \frac{p}{q}$

$q (x+a) = p (y+a)$

$qx+qa = py+pa$

$qa-pa = py-qx$

$a(q-p) = py - qx$

$a = \frac{py-qx}{q-p} = \frac{qx-py}{p-q}$

EXCERCISE

Q.1. The unit digit in ($7^{95} - 3^{58}$) is :

(A) 0 **(B)** 4

(C) 6 **(D)** 7

Q.2. If the unit digit in (549×56×28*×684) be 8, then what digit will be there in place of * ?

(A) 6 **(B)** 3

(C) 4 **(D)** 5

Q.3. If 67^{67} is divided by 68 the remainder is -

(A) 67 **(B)** 1

(C) 68 **(D)** 135

Q.4. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42},..........+\frac{1}{n(n+1)}=?$

(A) $\frac{1}{n}$ **(B)** $\frac{1}{n+1}$

(C) $\frac{n}{n+1}$ **(D)** $\frac{2(n-1)}{n}$

Q.5. $\frac{1}{1\times7}+\frac{1}{7\times13}+\frac{1}{13\times19}+\frac{1}{19\times25}$

(A) $\frac{1}{7}$ **(B)** $\frac{1}{19}$

(C) $\frac{4}{25}$ **(D)** $\frac{1}{19\times25}$

Q.6. In a question on division with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is :

(A) 0 **(B)** 12

(C) 13 **(D)** 20

Q.7. On dividing a number by 68, we get 269 as dividend and 0 as remainder. On dividing the same number by 67 what will be the remainder ?

(A) 0 **(B)** 1

(C) 2 **(D)** 3

Q.8. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and 15 as remainder. What is the smaller number ?

(A) 240 **(B)** 270

(C) 295 **(D)** 360

Q.9. The number of prime factors in the expression$(6)^{10}\times(7)^{17}\times(11)^{27}$is:

(A) 54 **(B)** 24

(C) 37 **(D)** 64

Q.10. Find the remainder when $(51)^{138}$ is divided by 7.

(A) 2 **(B)** 1

(C) 4 **(D)** 3

Q.11. Find the sum of the following

$$\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}+\frac{1}{132}\right)$$

(A) $\frac{11}{12}$ **(B)** $\frac{13}{15}$

(C) $\frac{219}{213}$ **(D)** $\frac{87}{89}$

Q.12. The sum $11^2+12^2+......21^2=?$

(A) 2926 **(B)** 3017

(C) 3215 **(D)** 3311

Q.13. In four consecutive prime numbers that are in ascending order, the product of the first three is 385 and that of the last three is 1001. The largest given prime number is -

(A) 11 **(B)** 13

(C) 17 **(D)** 19

Q.14. A boy was asked to multiply a number by 12.By mistake he multiplied the number by 21 and got his answer 63 more than the correct answer.The number to be multiplied was -

(A) 7 **(B)** 9

(C) 8 **(D)** 12

Q.15. A number was divided successively in order by 4, 5 and 6. The remainders were respectively 2,3 and 4. The number is -

(A) 214 **(B)** 318

(C) 216 **(D)** 224

Q.16. Find the difference between the local value and face value of 7 in the numerical 867351.

(A) 7000 **(B)** 6993

(C) 7344 **(D)** 7007

Q.17. The sum of five consecutive even numbers is equal to 170. What is the sum of the second largest number amongst them and the square of the smallest number amongst them together?

(A) 940 **(B)** 932

(C) 934 **(D)** 936

Q.18. There are two numbers, 6 times the square of second number is 540 more than the square of first number. If the respective ratio between first number and second number is 3: 2, what is the value of second number?

(A) 10 **(B)** 12
(C) 16 **(D)** 8

Q.19. The sum obtained on taking consecutive three of the four natural numbers is 180, 197, 208 and 222 respectively. What is the value of twice the sum of the first and last number?

(A) 320 **(B)** 322
(C) 326 **(D)** 324

Q.20. The smallest number of five digits exactly divisible by 476 is

(A) 47600 **(B)** 10000
(C) 10476 **(D)** 10472

Q.21. The least number of five digits which has 123 as a factor is

(A) 10037 **(B)** 10086
(C) 10081 **(D)** 10063

Q.22. When 335 is added to 5A7, the result is 8B2. 8B2 is divisible by 3. What is the largest possible value of A ?

(A) 8 **(B)** 2
(C) 1 **(D)** 4

Q.23. Which one of the following will completely divide $5^{71} + 5^{72} + 5^{73}$?

(A) 150 **(B)** 160
(C) 155 **(D)** 30

Q.24. If 146! is divisible by 5^n , then find the maximum value of n.

(A) 34 **(B)** 35
(C) 36 **(D)** 37

Q.25. Find the number of zeros in the product: $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$.

(A) 8 **(B)** 9
(C) 12 **(D)** 13

Q.26. Find the number of zero in $\left(\frac{140!}{50!}\right)$

(A) 20 **(B)** 22
(C) 24 **(D)** 16

Q.27. Find the last digit of the number $N = 1^3 + 2^3 + 3^3 \ldots\ldots\ldots\ldots\ldots + 99^3$.

(A) 0 **(B)** 1
(C) 2 **(D)** 5

Q.28. Find the total number of factors of 888888.

(A) 6 **(B)** 64
(C) 32 **(D)** 128

Q.29. What will be the remainder, when $11^{12^{13}}$ is divided by 9?

(A) 1 **(B)** 8
(C) 7 **(D)** 2

Q.30. What is the remainder when $6!^{4!} + 4!^{6!}$ is divided by 10?

(A) 0 **(B)** 2
(C) 4 **(D)** 6

Q.31. How many prime numbers exist in the factors of the product $67 \times 35^3 \times 11^{10}$?

(A) 20 **(B)** 27
(C) 30 **(D)** 23

Q.32. Find the number of composite factors of $N = 420$.

(A) 15 **(B)** 18
(C) 21 **(D)** 19

Q.33. What is the remainder obtained when $(1421 \times 1423 \times 1425)$ is divided 12?

(A) 7 **(B)** 3
(C) 5 **(D)** 6

Q.34. What is the remainder when $(16^3 + 17^3 + 18^3 + 19^3)$ is divided by 70?

(A) 69 **(B)** 14
(C) 0 **(D)** 1

Q.35. Find the units digit of $23478^{2345} \times 3483^{4857}$.

(A) 2 **(B)** 4
(C) 9 **(D)** 8

EXPLANATION

Q.1.(B) The unit digit is in

$7^{4\times23+3} - 3^{4\times14+2}$

$7^3 - 3^2 = 343 - 9 = 4$

Q.2.(B) 549×56×28*×684

9×6×*×4 = 8

6×* = 8

6×3 = 8

Ans. 3

Q.3.(A) If 67^{67} is divided by 68. The remainder is

$$\frac{a^n}{a+1}$$

a = 67

a+1 = 68

n =67 = Odd number

Note = When the power of Number is odd and the number is $\left(\frac{a^n}{a+1}\right)$ so always get Remainder a

So, Remainder is 67.

Q.4.(C) $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}........\frac{1}{n(n+1)}$

$1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+.......+\frac{1}{n}-\frac{1}{(n+1)}$

$=1-\frac{1}{n+1}=\frac{n}{n+1}$

Quick Trick :

$$\frac{1}{\text{Difference of No.}}\left[\text{First No. - Last No.}\right]$$

$$\frac{1}{1}\left[1-\frac{1}{(n+1)},\right]=\frac{n+1-1}{n+1}=\frac{n}{n+1}$$

Q.5.(C) $\frac{1}{1\times7}+\frac{1}{7\times13}+\frac{1}{13\times19}+\frac{1}{19\times25}$

$\frac{1}{6}\left[1-\frac{1}{25}\right]=\frac{4}{25}$

Q.6.(D) Number = (35×12) = 420

Correct quotient = 420 ÷ 21 = 20

Q.7.(B) Number = 269 × 68 + 0 = 18292

$$\begin{array}{l} 67\,)\,18292\,(273 \\ \quad\;\; \underline{134} \\ \quad\quad\; 489 \\ \quad\quad\; \underline{469} \\ \quad\quad\;\;\; 202 \\ \quad\quad\;\;\; \underline{201} \\ \quad\quad\;\;\;\;\;\; \underline{1} \end{array} \qquad \begin{array}{r} 269 \\ \underline{\times68} \\ 2152 \\ \underline{1614} \\ \underline{18292} \end{array}$$

⇒ Required remainder = 1

Q.8.(B) Let the smaller number be x. Then larger number = (x+1365)

x+1365=6x+15⇒ 5x=1350⇒ x = 270

Smaller number = 270.

Q.9.(D) $6^{10} \times 7^{17} \times 11^{27}$

Prime factor = 2 × 3 × 7 × 11

Required answer = 10+10+17+27 = 64

Q.10.(B) ∴ 51 = 49 + 2

and $2^{138} = (2^3)^{46} = 8^{46} = (7 + 1)^{48}$

∴ Remainder = 1

Q.11.(A) $S=\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}+\frac{1}{132}\right)$

$$\because \frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{(n-1)} \Rightarrow S$$

$$=\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+......\right.$$

$$\left.+\left(\frac{1}{10}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{12}\right)\right] \Rightarrow S=1-\frac{11}{12}$$

$$=\frac{11}{12}$$

Q.12.(A) $1^2 + 2^2 + 3^2 ....+ n^2$

$$=\frac{n(n+1)(2n+1)}{6}$$

$\therefore 11^2 + 12^2 + ...21^2$

$= (1^2 + 2^2 + 3^2 ....+ 21^2) - (1^2 + 2^2 ....+ 10^2)$

$$=\frac{21(21+1)(42+1)}{6}-\frac{10(10+1)(20+1)}{6}$$

= 3311 - 385 = 2926

Q.13.(B) Let the four consecutive prime numbers be a, b, c and d where a < b < c < d.

$\Rightarrow$ abc = 385, bcd = 1001

$\therefore$ HCF = bc = 77

bcd = 1001

$d = \frac{bcd}{bc} = \frac{1001}{77} = 13$

Q.14.(A) Let required number be x

21x-12x = 63

$9x = 63, \quad x = \frac{63}{9} = 7$

Q.15.(A) Required number

= 2 + (4 x 3) + (4 × 5 × 4) + 4 × 5 × 6

= 2 + 12 + 80 + 120 = 214

Q.16.(B) 867351

Local Value of 7 is = 7000

Face Value of 7 is = 7

Difference : 7000 – 7 = 6993

Q.17.(D) According to the question

x + x + 2 + x + 4 + x + 6 + x + 8 = 170

5x = 1270 – 20 = 150

x = 30

Required value = 36 + 900 = 936

Q.18.(B) Let first number be 3x and second number be 2x.

According to the question,

$6(2x)^2 = (3x)^2 + 540$

$24x^2 = 9x^2 + 540$

$24x^2 - 9x^2 = 540$

$16x^2 = 540$

x = 36, x = 6

Second number= 12

Q.19.(B) Let four natural numbers are a, b, c and d respectively.

a + b + c = 180 ... (I)

b + c + d = 197 ... (II)

c + d + a = 208 ... (III)

d + a + b = 222 ... (IV)

On adding all the four equations, we get

a + b + c + d = 807/3 = 269 ... (V)

d = 269 - 180 = 89

a = 269 - 197 = 72

Twice the sum of first and last number

= 2(89 + 72) = 322

Q.20.(D) 1000 ÷ 476 remainder = 4

Hence number = 10000 +(476-4) = 10472

Q.21.(B) 10000 ÷ 123 remainder = 37

Hence number = 10000 +(123- 37) = 10086

Q.22.(D)

	3	3	5
+	5	A	7
	8	B	2

8B2 is divisible by 3 hence sum of (8 + B + 2) is divisible by 3

So possible value of B = 2 , 5,8,

4 + A = 8

A = 4

Q.23.(C) $5^{71}(1 + 5 + 5^2)$

$5^{71}(31)$

Hence it is divisible by 31 or 31×5, or 31×5^2 or 31×5^3-------31×5^{71}

to maximum power of 71

Q.24.(B) In 146!, number of 5s would be 29. Also number of 5^2 would be 5. The number of 5^3 would be 1.

Hence the maximum value of n would be 29 + 5 + 1 = 35.

Q.25.(B) 5 × 10 × 25 × 40 × 50 × 55 × 65 × 125 × 80.

$= 5^{13} \times 2^9$

Hence number of zeroes = 9

Q.26.(B) $\frac{140}{5} + \frac{140}{25} + \frac{140}{125} = 28 + 5 + 1 = 34$

$\frac{50}{5} + \frac{50}{25} = 10 + 2 = 12$

Hence number of zeroes in $\left(\frac{140!}{50!}\right)$

=34 -12 = 22

Q.27.(A) $1^3 + 2^3 + 3^3$ ……………. $+ 99^3$

$= \left(\frac{99 \times 100}{2}\right)^2$ last digit is 0

Q.28.(D) The number 888888 can be written as $2^3 \times 3 \times 7 \times 11 \times 13 \times 37$.

Applying the formula of total number of factors, we get the factors as -

(3 + 1)(1 + 1)(1 + 1)(1 + 1)(1 + 1) (1 + 1)

= 4 × 2 × 2 × 2 × 2 × 2 = 128.

Q.29.(A) $(11)^{12^{13}} \div 9$ remainder $= (2)^{12^{13}} \div 9$

$= \left(2^3\right)^{4^{13}} \div 9$ remainder =1

Q.30.(D) $6!^{4!} \div 10$ remainder = 0

$4!^{6!} \div 10$ remainder = 6

Hence resultant = 6

Q.31.(C) $6^7 \times 35^3 \times 11^{10} = (2 \times 3)^7 \times (5 \times 7)^3 \times 11^{10} = 2^7 \times 3^7 \times 5^3 \times 7^3 \times 11^{10}$

∴ No. of prime factors = addition of powers of prime nos. = 7 + 7 + 3 + 3 + 10 = 30

Q.32.(D) $420 = 2^2 \times 3^1 \times 5^1 \times 7^1$

Number of prime factors = 4 (namely 2, 3, 5, 7).

Total number of factors = (2 + 1) (1 + 1) (1 + 1) (1 + 1) = 3 × 2 × 2 × 2 = 24

Therefore, the total number of composite factors

= total number of factors − prime factors − 1

= 24 − 4 − 1 = 19.

Q.33.(B) Remainder of 1421/12 = 5

Remainder of 1423/12 = 7

Remainder of 1425/12 = 9

Remainder (1421 × 1423 × 1425)/12

= Remainder (5 × 7 × 9)/12

= Remainder (5 × 63)/12

= Remainder (5 × 3)/12 = 3

Q.34.(C) $\frac{21700}{70} = 310 \times 70 + 0$

So, remainder will be 0.

Q.35.(B) $8^{4n+1} + 3^{4n+1} = 8 \times 3 = 24$

Notes

CHAPTER-2
HCF AND LCM

Scan the QR code to get video of this chapter.

Factors and Multiples

If 'a' divides 'b' exactly we say that 'a' is a factor of 'b' and also we say that 'b' is a multiple of 'a'.

Thus, 5 and 7 are factor of 35 etc.

Factor of 42 $\Rightarrow$ 7×6, 14×3, 42× 1

Factor of 30 $\Rightarrow$ 1 ×30, 5×6, 10×3, 15× 2

H.C.F (Highest Common Factor)

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

Example $\Rightarrow$ 18, 24, 42

$18 = \underline{2} \times 3 \times \underline{3}$
$24 = 2 \times \underline{2} \times 2 \times \underline{3}$
$42 = \underline{2} \times \underline{3} \times 7$
H.C.F $\Rightarrow$ The common factor of the number is = 6

METHODS OF FINDING H.C.F.

1. Division Method :

We have to find the H.C.F of two given numbers, divide the larger number by the smaller number. Now, divide the divisor by the remainder. Repeat this process till the remainder is zero.

The last divisor is the required H.C.F.

Ex. **48, 114**

Sol.

```
48) 114 (2
     96
     18) 48(2
         36
         12) 18(1
             12
              6)12(2
                12
                ××
H.C.F = 6
```

2. Prime Factors method :

Two or more than two, express each one of the given numbers as the product of prime factors. The product of common prime factors with least powers gives H.C.F.
Example = 570, 1197
Factor 570 = 2 × 3 × 5 × 19
1197 = 3 × 3 × 7 × 19
Common factor = 3 × 19 = 57
H.C.F = 57
Example Find the H.C.F of 714 and 374.
714 = 2 × 3 × 7 × 17
374 = 2 × 11 × 17
H.C.F. = 2 × 17 = 34

L.C.M (Lowest Common Multiple)

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. Division Method :

We divide the numbers from the prime number untill all the remainders are 1.

Ex. 12, 18, 36, 48 Find the L.C.M

Sol.

2	12, 18, 36, 48
2	6, 9, 18, 24
2	3, 9, 9, 12
2	3, 9, 9, 6
3	3, 9, 9, 3
3	1, 3, 3, 1
	1, 1, 1, 1

L.C.M = 2 × 2×2×2×3×3 = 144

2. Prime factors method :

Find the prime factors of all the given numbers and take maximum power of each factor and multiply them.

Ex. Find the L.C.M. of 24, 12 and 40

Sol. 24 = 2 × 2 × 2 × 3
40 = 2 × 2 × 2 × 5
12 = 2 × 2 × 3
Now, L.C.M. = 2 × 2 × 2 × 3 × 5 = 120

Ex- Find the L.C.M of $2^3 \times 3^6 \times 5$, $2^5 \times 3^2 \times 5^2 \times 7^4$, $2^4 \times 3^5 \times 7^2$

Sol. L.C.M. = $2^5 \times 3^6 \times 5^2 \times 7^4$

3. Product of two numbers

= L.C.M. × H.C.F
Number 1 × Number 2
= H.C.F. × L.C.M.

4. L.C.M and H.C.F. of Fractions :

$$\text{L.C.M.} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

$$\text{H.C.F} = \frac{\text{H.C.F of Numerators}}{\text{L.C.M. of Denominators}}$$

Ex- Find L.C.M. of $\frac{2}{3},\frac{4}{9},\frac{5}{6}$

Sol. L.C.M. = $\frac{\text{L.C.M}(2,4,5)}{\text{H.C.F}(3,9,6)} = \frac{20}{3}$

Ex- Find L.C.M. of $\frac{24}{17}, \frac{32}{7}, \frac{48}{13}$

Sol. $\frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$

$$\frac{\text{L.C.M}(24,32,48)}{\text{H.C.F}(17,7,13)} = \frac{96}{1} = 96$$

Ex. Find the H.C.F. of $\frac{3}{4},\frac{6}{25},\frac{12}{35},\frac{3}{5}$

Sol. H.C.F = $\frac{\text{H.C.F}(3,6,12,3)}{\text{L.C.M}(4,25,35,5)} = \frac{3}{700}$

Ex. Find the H.C.F of $\frac{1.2}{5},\frac{0.8}{7},\frac{0.2}{3.5},\frac{0.7}{14}$

Sol. $\frac{\text{H.C.F}(1.2,0.8,0.2,0.7)}{\text{L.C.M}(5,7,3.5,14)} = \frac{0.1}{70} = \frac{1}{700}$

Ex. Find the H.C.F. of 72, 108, 360.

Sol.

```
108) 360 (3
     324
     36) 108(3
         108
         ××
              36) 72(2
                  72
                  ××
```

H.C.F. = 36

Ex. Find the H.C.F. of 513, 783, and 1107.

Sol.

```
513) 783 (1
     513
     270) 513 (1
          270
          243)270(1
              243
               27)243(9    27) 1107(41
                  243          1107
                  ×××          ××××
```

H.C.F = 27

Ex. Find the H.C.F. of 42, 63, 140

Sol.

```
42) 63 (1
    42
    21) 42 (2
        42
        ××
          21) 140 (6
              126
               14) 21 (1
                   14
                    7) 14 (1
                       14
                       ××
```

H.C.F = 7

H.C.F. with remainder :

Different remainder :

Ex. Find the greatest number which when divided by 37 and 58, it leaves remainder 2 and 3 respectively.

Sol. H.C.F. of (37 – 2) and (58 – 3)

= 35 and 55 H.C.F. is 5.

So, greatest number is 5.

Same remainder :

Ex. Find the greatest number with which when divided by 60 and 98, it leaves remainder 3 in each case.

Sol. H.C.F. of (60 – 3) and (98 – 3)

= 57 and 95 H.C.F. is 19.

So, greatest number is 19.

Same remainder but unknown:

Ex. Find the greatest possible number by which when we divide the numbers 38, 66 and 80 the remainders are same.

Sol. H.C.F. of (66 – 38), (80 – 66) and (80 – 38)

= 28, 14 and 42

H.C.F. is 14. So, greatest number is 14.

L.C.M. with remainder :

When the remainders are different for different divisions, but the respective difference between the divisor and remainder remains constant.

Ex. What is the least possible number which when divided by 18, 35 or 42 it leaves 2, 19, 26 as the remainders respectively.

Sol. (18 – 2) = (35 – 19) = (42 – 26) = 16

Hence the least possible number

= (LCM of 18, 35 and 42) – 16

= 630 – 16 = 614

When divisor are different and remainder is same

Ex. What is the least possible number which when divided by 24, 32 and 42 in each case, it leaves the remainder 5?

Sol. So, required number is

(L.C.M. of 24, 32 and 42) + 5

= 672 + 5 = 677

When neither the divisor is same nor respective difference between divisor and the remainder remains constant.

Ex. What is the possible number which when divided by 13 leaves the remainder 3 and when it is divided by 5, it leaves the remainder 2.

Sol. Let the required number be N then it can be expressed as follows.

N = 13k + 3................(1)

and N = 5l + 2 .... (2)

where k and l are the quotients that belong to the set of integers.

Thus 5l+2 = 13k +3

$\Rightarrow$ 5l – 13 k = 1 $\Rightarrow$ 5l = 13k +1

$\Rightarrow$ $l = \frac{13k+1}{5}$

puting k = 3, then l = 8.

So, least number is 42.

Important Point

I Number × II Number = H.C.F × L.C.M

EXAMPLES

Ex.1. Find the H.C.F of a the reciprocal of fractions $\frac{3}{4}$, $\frac{9}{10}$ and $\frac{15}{16}$?

(A) $\frac{3}{80}$ **(B)** $\frac{80}{3}$

(C) $\frac{2}{15}$ **(D)** $\frac{2}{15}$

Sol.(C) Reciprocals of fractions $\frac{3}{4}\frac{9}{10}$ and $\frac{15}{16}$ $= \frac{4}{3}, \frac{10}{9}, \frac{16}{15}$

H.C.F of Fraction $= \frac{\text{H.C.F of 4,10,16}}{\text{L.C.M. of 3,9,15}} = \frac{2}{15}$

Ex.2. Two numbers are in ratio 7 : 8. If their H.C.F is 11 then find the largest number?

(A) 77 **(B) 88**

(C) 11 **(D) 99**

Sol.(B) Find the numbers. Let the required numbers be 7x and 8x

Then, their H.C.F. is x, So, x = 11

The numbers are 7x = 7 × 11 = 77

8x = 8 × 11 = 88

The largest numbers is 88

Ex.3. The H.C.F of two number is 96 and their L.C.M. is 1296. If one number is 864, the other is-

(A) 132 **(B)** 134

(C) 135 **(D)** 144

Sol.(D) Number 1 × Number 2 = H.C.F × L.C.M.

864 × number 2 = 96×1296

Second number = 144

Ex.4. Find the largest number of four digits exactly divisible by 12, 15, 18,27 and 54

(A) 9999 **(B)** 9720

(C) 9830 **(D)** 9980

Sol.(B) The largest number of four digits = 9999

Required number must be divisible by L.C.M. of 12, 15, 18, 27, 54 = 540

On dividing 9999 by 540, we get

279 as remainder.

Required number = 9999 - 279 = 9720

Ex.5. Find the smallest number of four digits exactly divisible by 12, 24, 36, 54

(A) 1000 **(B)** 1010

(C) 1080 **(D)** 1096

Sol.(C) The smallest number of four digits is 1000

Required number must be divisible by their L.C.M

L.C.M. of 12, 24, 36 and 54 $\Rightarrow$ 216

1000 is divided by 216, remainder is 136

Required answer = 1000 + (216 - 136) = 1080

Ex.6. The HCF of two three digit numbers is 23 and the other two factors of LCM are 13 and 23. The larger of the two numbers is-

(A) 529 **(B)** 576

(C) 434 **(D)** 422

Sol.(A) Let the number be 23x and 23 y

when x and y are co-prime

$\therefore$ LCM = 23xy = 23×13×23

x = 13 and y = 23

So, Larger number= 23×y = 23×23 = 529

Ex.7. The least number when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder is-

(A) 1677 **(B)** 1683
(C) 2523 **(D)** 3363

Sol.(B) The LCM of 5, 6, 7 and 8 = 840

Required number =840k+3

Which is exactly divisble by 9 for some value of k.

Now, 840k+3 = 93×9k+(3k+3)

Required number= 840×2+3 = 1683

Ex.8. The greatest number of four digits which when divided by 12, 16 and 24 leaves remainder 2, 6 and 14 respectively. It is-

(A) 9974 **(B)** 9970
(C) 9807 **(D)** 9998

Sol.(A) (12-2) = (16-6) = (24-14) = 10

Now, LCM of 12, 16 and 24 = 48

The greatest four digits number exactly divisible by 48 is 9984

Required least number =9984-10 = 9974

Ex.9. The traffic lights at three different road crossings change after 24 seconds, 36 seconds and 54 seconds respectively. If they all change simultaneously at 10:15:00AM then at what time will they again change simultaneously ?

(A) 10:16:54 AM **(B)** 10:18:36AM
(C) 10:17:62 AM **(D)** 10:22:12 AM

Sol.(B) L.C.M of 24, 36 and 54 second is 216

Required time = 10:15:00+3 Minute 36 Second = 10:18 : 36 AM

Ex.10. Six bells are tolling together at an intervall time of 2, 4, 6, 8, 10, and 12 second. So find how many time they will toll together in 2 hours ?

(A) 120 **(B)** 60
(C) 80 **(D)** 61

Sol.(D) L.C.M. of 2, 4, 6, 8, 10 and 12 is 120 sec.

$\Rightarrow \frac{120}{60} = 2\text{min}$

In every 2 minutes, they will toll together, So, in 120 min they will toll

$\Rightarrow \frac{120}{2} = 60 + 1$ = 61 times

Ex.11. 84 Maths books, 90 Physics books and 120 Chemistry books have to be stacked topicwise. How many books will be there in each stock so that each stock will have the same height too ?

(A) 12 **(B)** 18
(C) 6 **(D)** 21

Sol.(C) For taking same height= H.C.F of 84, 90,12 =6

Ex.12. What is the least number of square tiles required to pave the floor of a room 15 m, 17cm long and 9m 2 cm broad ?

(A) 840 **(B)** 841
(C) 820 **(D)** 814

Sol.(D) Length 15 m. 17 cm = 1517 cm.

Breadth = 9 m. 2 cm. = 902 cm.

Area of Floor = 1517×902

The number of square tiles will be least when the size of each tiles is max.

Side of each tile should be H.C.F of

= 1517, 902 = 41

Number of tiles = $\frac{1517 \times 902}{41 \times 41} = 814$

Ex.13. If L.C.M. of 2 numbers is twice of larger number and difference between the smallest number and H.C.F. is 8, So find the smallest number ?

(A) 16 **(B)** 8
(C) 4 **(D)** 24

Sol.(A) Larger number = x

Smaller number = y

L = 2x, y - H = 8

y - 8 = H, x×y = L×H

x×y = 2x×(y-8)

y = 2y - 16

y = 16

Ex.14. Three friends started to run in a circular path of 12 km. long from a certain point and in same direction, at a certain time. If they are running at the rate of 3 km/hr, 7 km/hr. and 13 km/hr, then find the time when they will meet each other.

(A) 8 hr. **(B)** 10 hr.
(C) 12 hr. **(D)** 14 hr.

Sol.(C) Time = $\frac{\text{Distance}}{\text{Speed}}$

Time taken by each to complete 1 round

= $\frac{12}{3}, \frac{12}{7}, \frac{12}{13}$ hr.

L.C.M. $\frac{12}{3}, \frac{12}{7}, \frac{12}{13}$ = 12 hr.

EXCERCISE

Q.1. The least number which when divided by 4, 6, 8 and 9 leaves zero remainder in each case and when divided by 13 leaves a remainder of 7, is-

(A) 144 **(B)** 72

(C) 36 **(D)** 85

Q.2. A is a set of those positive integers such that when these are divided by 2, 3, 4, 5 and 6 leaves the remainders 1, 2, 3, 4 and 5, respectively. How many integers between 0 and 100 belong to the set A?

(A) No integer **(B)** One

(C) Two **(D)** Three

Q.3. There are two numbers p and q such that their HCF is 1.

Which of the Following statements are correct?

I. Both p and q may be prime.

II. One number may be prime and theOther composite.

III. Both the numbers may be composite.

(A) I and II **(B)** II and III

(C) I and III **(D)** I, II and III

Q.4. Two pipes of lengths 1.5 m and 1.2 m are to be cut into equal pieces without leaving any extra length of pipes. The greatest length of the pipe pieces of same size, which can be cut from these two lengths, will be

(A) 0.13 m **(B)** 0.4 m

(C) 0.3 m **(D)** 0.41 m

Q.5. The difference of two numbers is 1/9 of their sum and their sum is 45. Find the LCM of these numbers?

(A) 225 **(B)** 100

(C) 150 **(D)** 200

Q.6. The sum of two numbers is 1056 and their HCF is 66, find the number of such pairs.

(A) 6 **(B)** 2

(C) 4 **(D)** 8

Q.7. The sum of HCF and LCM of two numbers is 403 and their LCM is 12 times their HCF. If one number is 93, then find the other number

(A) 115 **(B)** 122

(C) 124 **(D)** 138

Q.8. The LCM of two numbers is 495 and their HCF is 5. If sum of the numbers is 100, then find the difference of the numbers.

(A) 10 **(B)** 46

(C) 70 **(D)** 90

Q.9. The LCM of two numbers is 20 times of their HCF and (LCM + HCF) = 2520. If one number is 480, then what will be the triple of another number?

(A) 1200 **(B)** 1500

(C) 2100 **(D)** 1800

Q.10. Find the side of the largest possible square slabs which can be paved on the floor of a room 2 m 50 cm long and 1 m 50 cm broad. Also, find the number of such slabs to pave the floor.

(A) 25, 20 **(B)** 30, 15

(C) 50, 15 **(D)** 55, 10

Q.11. Find the largest number which divides 1305, 4665 and 6905 leaving same remainder in each case. Also, find the common remainder.

(A) 1210, 158 **(B)** 1120, 158

(C) 1120, 185 **(D)** 1210, 185

Q.12. There are four numbers. The HCF of each pair is 5 and the LCM of all the four numbers is 2310. What is the product of four numbers?

(A) 288750 **(B)** 288570

(C) 828570 **(D)** 288650

Q.13. Find the greatest number of three digits which when divided by 6, 9 and 12 leaves 3 as remainder in each case.

(A) 975 **(B)** 996

(C) 903 **(D)** 939

Q.14. Find the greatest 4-digit number, which when divided by 12, 18, 21 and 28, leaves a remainder 3 in each case.

(A) 9831 **(B)** 9913

(C) 9940 **(D)** 9911

Q.15. The least number of four digits which is divisible by each one of the numbers 12. 18, 21 and 28, is

(A) 1008 **(B)** 1006

(C) 1090 **(D)** 1080

Q.16. Three electronic devices make a beep after every 48 s, 72 s and 108s, respectively. They beeped together at 10 am. The time when they will next make a beep together at the earliest is

(A) 10 : 07 : 12 **(B)** 10 : 07 : 24

(C) 10 : 07 : 36 **(D)** 10 : 07 : 48

Q.17. Find the HCF of $4x^3y$, $12x^2y^2$ and $6x^2y^3$.

(A) x^2y **(B)** xy^2

(C) $2xy^2$ **(D)** $2x^2y$

Q.18. Amit, Ajay and Altaf begin to jog around a circular stadium. They complete their revolutions in 42 s, 56 s and 63 s, respectively. After how many seconds will they be together at the starting point?

(A) 366 **(B)** 252

(C) 504 **(D)** 604

Q.19. A General can draw up his soldiers in the rows of 10, 15 or 18 soldiers and he can also draw them up in the form of a solid square. Find the least number of soldiers with the General.

(A) 100 **(B)** 3600

(C) 900 **(D)** 90

Q.20. The least number which should be added to 2497, so that the sum is exactly divisible by 5, 6, 4 and 3, is

(A) 3 **(B)** 13

(C) 23 **(D)** 33

Q.21. What is the least number which when increased by 9, is divisible by each one of 24, 32, 36 and 54?

(A) 855 **(B)** 890

(C) 756 **(D)** 895

Q.22. The HCF of three numbers is 23. If they are in the ratio of 1: 2 : 3 then find the numbers.

(A) 69, 15, 22 **(B)** 23, 46, 69

(C) 25, 31, 41 **(D)** 23, 21, 35

Q.23. Three numbers are in the ratio of 3 : 4 : 5 and their LCM is 1200. Find the HCF of the numbers.

(A) 40 **(B)** 30

(C) 80 **(D)** 20

Q.24. The HCF and LCM of two numbers m and n are 6 and 210 respectively. If m + n = 72 then $\frac{1}{m}+\frac{1}{n}$ = ?

(A) $\frac{1}{35}$ **(B)** $\frac{3}{35}$

(C) $\frac{5}{37}$ **(D)** $\frac{2}{35}$

Q.25. First number is 20% more than second number. If their HCF is 3. Find their LCM.

(A) 60 **(B)** 30

(C) 45 **(D)** 90

Notes

EXPLANATION

Q.1.(B) LCM of 4,6, 8 and 9 is 72.

Hence required number = 72

Q.2.(B) k = 2-1 = 3 - 2 = 4 - 3 = 5 - 4 = 6-5 = 1

Required number= LCM of (2,3,4,5,6)-k

=60-1=59

Q.3.(D) Let two prime numbers be 2 and 3 then their HCF=1

Hence, Statement I is true.

Let p=5 and q=6, then their HCF is also 1.

Let p=8 and q=9

Q.4.(C) To find the greatest length of same pipe pieces, we have to find the HCF of 1.5 m and 1.2 m

Required length of pipe pieces=0.3m

Q.5.(B) let the numbers be x and y.

According to the question,

x+y=45-------------------(i)

Difference of two numbers

x-y=5 ------------------(ii)

on solving both equation we get

x = 25, y = 20

Now, LCM of 25 and 20 = 100

Q.6.(C) Let the numbers be 66a and 66b, where a and b are coprimes.

According to the question,

66a+66b=1056

66(a+b)=1056

Possible values of a and b are

(a=1, b=15), (a=3,b=13)

(a=5, b=11), (a=7,b=9)

Hence the possible number of pairs is 4.

Q.7.(C) LCM+HCF = 403

LCM = 12 HCF

Hence HCF = $\frac{403}{13}$ = 31

We know

Ist × IInd number = LCM× HCF

93 × IInd number = 31 ×132

IInd number =$\frac{31\times372}{93}$ = 124

Q.8.(A) a+b = 100

a × b = 495 × 5

= 2475

$(a-b)^2 = (a+b)^2 - 4ab$

= 10000-9900 = 100

a-b = 10

Q.9.(D) LCM+HCF = 2520

LCM = 20 HCF

Hence HCF = $\frac{2520}{21}$ = 120

We know

Ist × IInd number = LCM × HCF

480 × IInd number = 2400 ×120

IInd number =$\frac{2400\times120}{480}$ = 600

Hence required number = 600 × 3

= 1800

Q.10.(C) HCF of 250 and 150 is 50

Hence required number of slabs

= $\frac{250\times150}{50\times50}$ = 15

Q.11.(C) Given, x=1305, y=4665 and z=6905.

x ~ y = 3360

y ~ z = 2240

z ~ y = 5600

Required number HCF of 3360, 2240, and 5600 = 1120

Hence the common remainder is 185.

Q.12.(A) Given, HCF=5, LCM=2310 and n=4

Required product= $(HCF)^{n-1}$ × LCM

= $(5)^{4-1} \times 2310 = 5^3 \times 2310$

= 125 × 2310 = 288750

Q.13.(A) LCM of 6, 9 and 12 = 36

Hence required number = 36 k + 3

Where k is = 1, 2, 3,....

999 ÷ 36 remainder = 27

Hence greatest 3-digit number

= 999 - 27 + 3 = 975

Q.14.(A) LCM of 12, 18, 21 and 28 = 252

Hence required number = 252 k + 3

Where k is = 1, 2, 3,....

9999 ÷ 252 remainder = 171

Hence greatest 4-digit number

= 9999 - 171 + 3 = 9831

Q.15.(A) LCM of 12, 18, 21 and 28 = 252

Hence required number = 252 k

Where k is = 1, 2, 3,....

When we divide 1000 by 252, remainder is 244.

Hence smallest 4-digit number

= 1000 + (252- 244) = 1008

Q.16.(A) First we have to find out the LCM of 48 72 and 108 s.

LCM of 48,72 and 108 s.

=432 s

=7 min 12 s

Time of beep together

=10:00+07:12

=10:07:12

Q.17.(D) HCF of 4, 12 and 6 = 2

LCM of $4x^3y$, $12x^2y^2$ and $6x^2y^3$ = x^2y

So, required answer = $2x^2y$

Q.18.(C) Required time= LCM of 42, 56 and 63 s

LCM of 42,56 and 63 is 504 s.

Q.19.(C) LCM of 10,15 and 18 is 90

To make it perfect square, we multiplay it with 10

Required number of soldiers

= 90×10 = 900

Q.20.(C) LCM of 5,6,4 and 3 =60

Required number =60- (Remainder of 2497÷60) =23

Q.21.(A) Required least number

= LCM of (24, 32, 36, 54)-9

= 864 - 9 = 855

Q.22.(B) Let the numbers be x, 2x and 3x in which x = HCF

Given that , HCF = x = 23

So the numbers are 23, 46, and 69

Q.23.(D) Let the numbers be 3x, 4x and 5x in which x = HCF = x and LCM = 60x

Given, LCM = 1200

Now, 60x = 1200

x=20

Q.24.(D) Given HCF=6 and LCM=210

m×n = 6 × 210

= 1260

$\frac{1}{m}+\frac{1}{n}=\frac{m+n}{mn}$

$=\frac{72}{1260}=\frac{2}{35}$

Q.25.(D) First number = 6×3 = 18

Second number = 5×3 = 15

LCM of 18 and 15 = 90

Notes

CHAPTER-3
SURDS AND INDICES

Scan the QR code to get video of this chapter.

Indices

Let n be a positive integer and a be a real number, then :

$a^n = a \times a \times a \times ....\times a$ (n factors)

a^n is called "n^{th} power of 'a' or a raised to the power 'n' where, 'a' is called the base and 'n' is called index or exponent of the power a^n.

Laws of Indices-

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $(a \times b)^n = a^n \times b^n$

(v) $a^{-n} = \frac{1}{a^n}, \frac{1}{a^{-n}} = a^n$

(vi) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(vii) $a^0 = 1$

Ex. $(2^{-2})^2$ is equal to-

(A) -8 **(B)** 1/16 **(C)** 16 **(D)** 1/8

Sol.(B) $\left(2^{-2}\right)^2 = \left(\frac{1}{2^2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Ex. $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$ is equal to

(A) 5 **(B)** 19 **(C)** 25 **(D)** 95

Sol.(B) $\frac{5^n \times 5^3 - 6 \times 5^n \times 5}{9 \times 5^n - 4 \times 5^n}$

$= \frac{125 \times 5^n - 30 \times 5^n}{9 \times 5^n - 4 \times 5^n} = \frac{95 \times 5^n}{5 \times 5^n} = 19$

Ex. If $2^{x+3} = 32$ then find the value of 3^{x+1} ?

(A) 27 **(B)** 81

(C) 72 **(D)** 9

Sol.(A) $2^{(x+3)} = 32 \Rightarrow 2^5$

$x+3 = 5$

$x = 5-3 = 2$

So, $3^{2+1} = 3^3 \Rightarrow 27$

Ex. $\frac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$

(A) 0.14 **(B)** 1.4 **(C)** 1 **(D)** $1.\overline{4}$

Sol.(D) $\frac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$

$= \frac{2.5 \times 0.12 + 1}{0.3 \times 3} = \frac{1.3}{0.9} = 1.444.... = 1.\overline{4}$

Ex. If $5^x = 3^y = 15^{-z}$ then $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is equal to-

(A) 0 **(B)** 1

(C) $\frac{3}{2}$ **(D)** $-\frac{1}{2}$

Sol.(A) Let $5^x = 3^y = 15^{-z} = K$

$K^{\frac{1}{x}} = 5$.............(i)

$K^{\frac{1}{y}} = 3$.............(ii)

$K^{-\frac{1}{z}} = 15$.............(iii)

from equation (i) and (ii)

$5 \times 3 = K^{\frac{1}{x}} \times K^{\frac{1}{y}} \Rightarrow 15 = K^{\frac{1}{x}+\frac{1}{y}}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Ex. $(x^{b+c})^{(b-c)} . (x^{c+a})^{(c-a)} . (x^{a+b})^{(a-b)} = ?$

(A) 1 **(B)** x^{abc}

(C) x^{a+b+c} **(D)** $x^{ab+bc+ca}$

Sol.(A) $x^{(b+c)(b-c)} . x^{(c+a)(c-a)} . x^{(a+b)(a-b)}$

$= x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)} = x^0 = 1.$

Surd :

If 'a' is a rational number and 'n' is a positive integer and $a^{\frac{1}{n}} = \sqrt[n]{a}$ is **irrational**, then $\sqrt[n]{a}$ is called a "surd of order n" or "n^{th} root of a". For the surd $\sqrt[n]{a}$, n is called the surd-index or the order of the surd and 'a' is called the redicand. The symbol '$\sqrt{\ }$' is called the surd sign or radical.

Ex. $\sqrt{7}$ is a surd of order 2 or square root of 7

$\sqrt[3]{11}$ is a surd of order 3 or cube root of 11.

Laws of Surds -

(i) $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

(ii) $\left(\sqrt[n]{a}\right)^n = a$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

(iv) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

(v) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(vi) $\left(\sqrt{a}+\sqrt{b}\right)^2$ =a+b+$2\sqrt{ab}$

(vii) $\left(\sqrt{a}-\sqrt{b}\right)^2$ = a+b-$2\sqrt{ab}$

(viii) $\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$ = (a-b), a >b

Comparison of surds :

(i) If two surds are of the same order, then the one whose redicand is larger, is the larger of the two.

Ex. $\sqrt[3]{17} > \sqrt[3]{15}, \sqrt{8} > \sqrt{7}, \sqrt[13]{9} > \sqrt[13]{5}$ etc.

(ii) If two surds are of distinct order, we change them into the surds of the same order.

This order is the L.C.M. of the orders of the given surds.

Ex. Which is larger $\sqrt{7}$ or $\sqrt[3]{5}$

Sol. Given surds are of order 2 and 3 respectively whose L.C.M. is 6.

Convert each into a surd of order 6, as shown below :

$$\sqrt{7} = 7^{\frac{1}{2}} = 7^{\frac{1}{2}\times\frac{3}{3}} = 7^{\frac{3}{6}} = 343^{\frac{1}{6}}$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{1}{3}\times\frac{2}{2}} = 5^{\frac{2}{6}} = 25^{\frac{1}{6}}$$

Clearly $343^{\frac{1}{6}} > 25^{\frac{1}{6}}$ so, $\sqrt{7} > \sqrt[3]{5}$

Important Formula

(i) $y = \sqrt{x+\sqrt{x+\sqrt{x+}}}$...... **and**

If x = n(n+1)

then y = (n+1)

(ii) If $y = \sqrt{x-\sqrt{x-\sqrt{x-\sqrt{x-\sqrt{x}}}}}$.........

If x= n (n+1)

then y = n

Square root of surds

$$\sqrt{a+\sqrt{b}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2-b^2}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b^2}}{2}}\right)$$

$$\sqrt{a-\sqrt{b}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2-b^2}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b^2}}{2}}\right)$$

SOME IMPORTANT TRICKS

(i) If $x = \frac{4\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

then $\frac{x+2\sqrt{a}}{x-2\sqrt{a}} + \frac{x+2\sqrt{b}}{x-2\sqrt{b}} = 2$

(ii) $x = \frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

then $\frac{x+\sqrt{a}}{x-\sqrt{a}} + \frac{x+\sqrt{b}}{x-\sqrt{b}}$ =2

EXAMPLES

Ex.1. Find the square root of $(6-\sqrt{35})$.

(A) $\pm\left(\sqrt{\frac{7}{2}}+\sqrt{\frac{5}{2}}\right)$ **(B)** $\pm\left(\sqrt{\frac{7}{2}}-\sqrt{\frac{5}{2}}\right)$

(C) $\pm\left(\sqrt{\frac{5}{2}}-\sqrt{\frac{7}{2}}\right)$ **(D)** $\pm\left(\sqrt{\frac{5}{2}}-\frac{7}{2}\right)$

Sol.(B) $\sqrt{6-\sqrt{35}} = \pm\left(\sqrt{\frac{6+\sqrt{6^2-35}}{2}} - \sqrt{\frac{6-\sqrt{6^2-35}}{2}}\right)$

$= \pm\left(\sqrt{\frac{6+1}{2}} - \sqrt{\frac{6-1}{2}}\right) = \pm\left(\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\right)$

Ex.2. Find the minimum value of $\left(\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}\right)$.

(A) 3 **(B) 9**

(C) $\frac{1}{9}$ **(D)** $\frac{1}{\sqrt{3}}$

Sol.(D) $\sqrt{26-15\sqrt{3}}=\sqrt{\frac{52-30\sqrt{3}}{2}}$

$=\sqrt{\frac{\left(3\sqrt{3}-5\right)^2}{2}}=\frac{3\sqrt{3}-5}{\sqrt{2}}$

$\sqrt{38+5\sqrt{3}}=\sqrt{\frac{76+10\sqrt{3}}{2}}$

$=\sqrt{\frac{\left(5\sqrt{3}+1\right)^2}{2}}=\frac{5\sqrt{3}+1}{\sqrt{2}}$

$=\left(\frac{\frac{3\sqrt{3}-5}{\sqrt{2}}}{5\sqrt{2}-\frac{5\sqrt{3}+1}{\sqrt{2}}}\right)$

$=\left(\frac{3\sqrt{3}-5}{9-5\sqrt{3}}\right)=\left(\frac{3\sqrt{3}-5}{\sqrt{3}\left(3\sqrt{3}-5\right)}\right)=\frac{1}{\sqrt{3}}$

Ex.3. If a = $\sqrt{2}+1$ and b = $\sqrt{2}-1$, then

$\frac{1}{a+1}+\frac{1}{b+1}$.

(A) 9 **(B) 3** **(C) 1** **(D) 2**

Sol.(C) $\frac{1}{a+1}=\frac{1}{\sqrt{2}+1+1}=\frac{1}{\sqrt{2}+2}$

$=\frac{\left(2-\sqrt{2}\right)}{\left(\sqrt{2}+2\right)\left(2-\sqrt{2}\right)}=\frac{\left(2-\sqrt{2}\right)}{2}$

Similarly, $\frac{1}{b+1}=\frac{1}{\left(\sqrt{2}-1+1\right)}=\frac{\sqrt{2}}{2}$

$\frac{1}{a+1}+\frac{1}{b+1}=\frac{\left(2-\sqrt{2}\right)}{2}+\frac{\sqrt{2}}{2}=1$

Ex.4. $\left\{\left(\sqrt[n]{x^2}\right)^{\frac{n}{2}}\right\}^2=?$

(A) $\frac{1}{x^2}$ **(B) x** **(C)** $x^{\frac{n}{2}}$ **(D)** x^2

Sol.(D) $x^{\frac{2}{n}\times\frac{n}{2}\times 2}=x^2$

Ex.5. If $\left(\sqrt{3}\right)^5\times 9^2=3^n\times\left(\sqrt{3}\right)^3$

then find the value of n ?

(A) 5 **(B)** 2 **(C)** 3 **(D)** 4

Sol.(A) $\left(\sqrt{3}\right)^5\times 9^2=3^n\times\left(\sqrt{3}\right)^3$

$3^{\frac{5}{2}}\times 3^4=3^n\times 3^{\frac{2}{3}}$

$3^n=\frac{3^{\frac{5}{2}}\times 3^4}{3^{\frac{2}{3}}}=3\times 3^4=3^5$

n = 5

Ex6. $\sqrt[3]{\frac{0.000729}{0.085184}}=?$

(A) $\frac{24}{44}$ **(B)** $\frac{9}{44}$ **(C)** $\frac{44}{9}$ **(D)** $\frac{27}{42}$

Sol.(B) $\sqrt[3]{\frac{0.000729}{0.085184}}=\sqrt[3]{\frac{729}{85184}}$

$=\sqrt[3]{\frac{9\times 9\times 9}{44\times 44\times 44}}=\frac{9}{44}$

Ex.7. The value of $\sqrt{\frac{\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{3}+\sqrt{2}\right)}{5+\sqrt{24}}}$ is

(A) $\sqrt{6}-\sqrt{2}$ **(B)** $\sqrt{6}+\sqrt{2}$

(C) $\sqrt{6}-2$ **(D)** $2-\sqrt{6}$

Sol.(C) $\sqrt{\frac{\sqrt{36}+\sqrt{24}-\sqrt{24}-\sqrt{16}}{5+\sqrt{24}}}$

$=\sqrt{\frac{6-4}{5+2\sqrt{6}}}=\sqrt{2\left(5-2\sqrt{6}\right)}=\sqrt{2\left(\sqrt{3}-\sqrt{2}\right)^2}$

$=\sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$

$=\left(\sqrt{6}-2\right)$

Ex.8. The value of $\left(3+2\sqrt{2}\right)^{-3}+\left(3-2\sqrt{2}\right)^{-3}$ is

(A) 198 **(B)** 180

(C) 108 **(D)** 189

Sol.(A) $\frac{1}{\left(3+2\sqrt{2}\right)^3}+\frac{1}{\left(3-2\sqrt{2}\right)^3}$

$=\left(3-2\sqrt{2}\right)^3+\left(3+2\sqrt{2}\right)^3$

$(a+b)^3+(a-b)^3=2a^3+6ab^2$

$=2(3)^3+6\times 3\times\left(2\sqrt{2}\right)^2=54+18\times 8=198$

Ex.9. $\dfrac{(x-\sqrt{24})(\sqrt{75}+\sqrt{50})}{\sqrt{75}-\sqrt{50}}=1$ then the value of x is

(A) $\sqrt{5}$ **(B)** 5 **(C)** $2\sqrt{3}$ **(D)** $2\sqrt{5}$

Sol.(B) $\dfrac{(x-\sqrt{24})(\sqrt{75}+\sqrt{50})}{\sqrt{75}-\sqrt{50}}=1$

$\dfrac{(x-2\sqrt{6})(5\sqrt{3}+5\sqrt{2})}{(5\sqrt{3}-5\sqrt{2})}=1$

$\left(\dfrac{(x-2\sqrt{6})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}\right)=1$

$(x-2\sqrt{6})=\dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$(x-2\sqrt{6})=\dfrac{3+2-2\sqrt{3\times 2}}{1}$

$x-2\sqrt{6}=5-2\sqrt{6}$, x = 5

Ex.10. $\dfrac{1}{\sqrt{2}+1}+\dfrac{1}{\sqrt{3}+\sqrt{2}}+..........+\dfrac{1}{\sqrt{100}+\sqrt{99}}$ is

(A) 1 **(B)** 9 **(C)** $\sqrt{99}$ **(D)** $\sqrt{99}-1$

Sol.(B) $\dfrac{1}{(\sqrt{2}+1)}\times\dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)}=\dfrac{\sqrt{2}-\sqrt{1}}{1}$

$\dfrac{1}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\dfrac{\sqrt{3}-\sqrt{2}}{1}$

$\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+........\sqrt{100}-\sqrt{99}$

$\sqrt{100}-\sqrt{1}=10-1=9$

Ex.11. The smallest number among $\sqrt[6]{12},\sqrt[3]{4},\sqrt[4]{5},\sqrt{3}$

(A) $\sqrt[6]{12}$ **(B)** $\sqrt[3]{4}$ **(C)** $\sqrt{3}$ **(D)** $\sqrt[4]{5}$

Sol.(D) $\sqrt[6]{12},\sqrt[3]{4},\sqrt[4]{5},\sqrt{3}$

$\Rightarrow \sqrt[6\times 2]{12^2},\sqrt[3\times 4]{4^4},\sqrt[4\times 3]{5^3},\sqrt[2\times 6]{3^6}$

$\Rightarrow \sqrt[12]{144},\sqrt[12]{256},\sqrt[12]{125},\sqrt[12]{729}$

So, $\Rightarrow \sqrt[12]{125}<\sqrt[12]{144}<\sqrt[12]{256}<\sqrt[12]{729}$

Smallest $\Rightarrow \sqrt[12]{125}$, So, $\sqrt[4]{5}$

Ex.12. The largest number among $\sqrt{2}$, $\sqrt[3]{3},\sqrt[4]{4}$ is

(A) $\sqrt{2}$ **(B)** $\sqrt[3]{3}$

(C) $\sqrt[4]{4}$ **(D)** None of these

Sol.(B) $\sqrt{2},\sqrt[3]{3},\sqrt[4]{4}$, $\sqrt[2\times 6]{2^6},\sqrt[3\times 4]{3^4},\sqrt[4\times 3]{4^3}$

$=\sqrt[12]{64},\sqrt[12]{81},\sqrt[12]{64}=\sqrt[12]{81}>\sqrt[12]{64}$

So, largest number = $\sqrt[12]{81}=\sqrt[3]{3}$

Ex.13. The smallest number of

$\sqrt{8}+\sqrt{5},\sqrt{7}+\sqrt{6},\sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ is-

(A) $\sqrt{8}+\sqrt{5}$ **(B)** $\sqrt{7}+\sqrt{6}$

(C) $\sqrt{10}+\sqrt{3}$ **(D)** $\sqrt{11}+\sqrt{2}$

Sol.(D) Squaring all the numbers

$(\sqrt{8}+\sqrt{5})^2=8+5+2\sqrt{40}\Rightarrow 13+2\sqrt{40}$

$(\sqrt{7}+\sqrt{6})^2=7+6+2\sqrt{42}=13+2\sqrt{42}$

$(\sqrt{10}+\sqrt{3})^2=10+3+2\sqrt{30}=13+2\sqrt{30}$

$(\sqrt{11}+\sqrt{2})^2=11+2+2\sqrt{22}=13+2\sqrt{22}$

$(13+2\sqrt{22})<(13+2\sqrt{30})<(13+2\sqrt{40})<(13+2\sqrt{42})$

So, smallest number is $\sqrt{11}+\sqrt{2}$

Ex.14. The greatest number among

$\sqrt{7}-\sqrt{5},\sqrt{5}-\sqrt{3},\sqrt{9}-\sqrt{7},\sqrt{11}-\sqrt{9}$ is

(A) $\sqrt{7}-\sqrt{5}$ **(B)** $\sqrt{5}-\sqrt{3}$

(C) $\sqrt{9}-\sqrt{7}$ **(D)** $\sqrt{11}-\sqrt{9}$

Sol.(B) Rationalising the given numbers

$\sqrt{7}-\sqrt{5}=\dfrac{2}{(\sqrt{7}+\sqrt{5})}$

$\sqrt{5}-\sqrt{3}=\dfrac{2}{(\sqrt{5}+\sqrt{3})}$

$(\sqrt{9}-\sqrt{7})=\dfrac{2}{(\sqrt{9}+\sqrt{7})}$

$(\sqrt{11}-\sqrt{9})=\dfrac{2}{(\sqrt{11}+\sqrt{9})}$

$\dfrac{2}{(\sqrt{5}+\sqrt{3})}>\dfrac{2}{(\sqrt{7}+\sqrt{5})}>\dfrac{2}{(\sqrt{7}+\sqrt{9})}<\dfrac{2}{(\sqrt{11}+\sqrt{9})}$

Greatest number is $=\sqrt{5}-\sqrt{3}$

EXERCISE

Q.1. If $2x^{1/3}+2x^{-1/3} = 5$, then $x^{1/3}$ is equal to-

(A) 1 or -1 **(B)** 2 or $\frac{1}{2}$

(C) 8 or $\frac{1}{8}$ **(D)** 3 or $\frac{1}{3}$

Q.2. If $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{1/3} \div \left(\frac{p^5q^{-3}}{p^{-2}q^3}\right)^{1/3} = p^aq^b$, then the value of a + b, where p and q are different positive primes, is

(A) 0 **(B)** 1

(C) 2 **(D)** None of these

Q.3. If $\frac{2^{n+4} - 2.2^n}{2.2^{n+3}} + 2^{-3} = x$ then the value of x is-

(A) $-2^{n+1}+\frac{1}{8}$ **(B)** 1

(C) 2^{n+1} **(D)** $\frac{n}{8} - 2^n$

Q.4. $\frac{1}{\sqrt{1331}} \times \sqrt[3]{121} \div 11^{2.5} = \frac{1}{121^x}$. Find the value of x.

(A) $\frac{10}{3}$ **(B)** $\frac{5}{3}$

(C) 2 **(D)** $\frac{5}{2}$

Q.5. $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ is equal

(A) 4 **(B)** 0

(C) $\sqrt{2}$ **(D)** $3\sqrt{6}$

Q.6. The value of $3 + \frac{1}{3} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3}$ is

(A) $3+\sqrt{3}$ **(B)** 3

(C) 1 **(D)** 0

Q.7. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then the value of x is

(A) 42 **(B)** 15/8

(C) 17/8 **(D)** 39

Q.8. Find the value of m-n, if

$$\frac{9^n \times 3^2 \times \left(\frac{-n}{3^2}\right)^{-2} - (27)^n}{3^3 m \times 2^3} = \frac{1}{27}$$

(A) 1 **(B)** -2

(C) -1 **(D)** 2

Q.9. If $a = \frac{\sqrt{3}}{2}$, then $\sqrt{1+a} + \sqrt{1-1} = ?$

(A) $(2-\sqrt{3})$ **(B)** $(2+\sqrt{3})$

(C) $\frac{\sqrt{3}}{2}$ **(D)** $\sqrt{3}$

Q.10. If $m = 7-4\sqrt{3}$, then $\left(\sqrt{m} + \frac{1}{\sqrt{m}}\right) = ?$

(A) 8 **(B)** 3 **(C)** 4 **(D)** 9

Q.11. If $2^p+3^q = 17$ and $2^{p+2} - 3^{q+1} = 5$, then find the values of p and q.

(A) -2, 3 **(B)** 2, -3

(C) 3, 2 **(D)** 2, 3

Q.12. Find the value of $\left(\frac{a^p}{a^q}\right)^{p+q-r} \times \left(\frac{a^r}{a^p}\right)^{r+p-q} \times \left(\frac{a^q}{a^r}\right)q + r - p$

(A) a^{pqr} **(B)** a^{p+q+r}

(C) $a^{pq+qr+pr}$ **(D)** 1

Q.13. The values of x which astisfy the equation $5^{1+x}+5^{1-x} = 26$ are

(A) (1, 1) **(B)** (3, 0)

(C) (2, 0) **(D)** (1, -1)

Q.14. If $x = \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$ and $y = \frac{1}{x}$, then the value of $3x^2-5xy + 3y^2$ is

(A) 1771 **(B)** 1177

(C) 1717 **(D)** 1171

Q.15. If $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, then $bx^2 - ax + b$ is equal to (given that, $b \neq 0$)

(A) 0 **(B)** 1 **(C)** ab **(D)** 2ab

Q.16. $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$ is equal to-

(A) 366 **(B)** 355 **(C)** 305 **(D)** 300

Q.17. If $x^2 = y + z$, $y^2 = z + x$ and $z^2 = x + y$, then what is the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$?

(A) -1 **(B)** 1 **(C)** 2 **(D)** 4

Q.18. If $2^a = 3^b = 6^{-c}$, then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = ?$

(A) $\frac{7}{32}$ **(B)** 0 **(C)** $\frac{7}{16}$ **(D)** $\frac{7}{48}$

Q.19. If $9^x3^y = 2187$ and $2^{3x}\,2^{2y}$-$4^{xy} = 0$, then what can be the value of (x+y) ?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7

Q.20. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$ $t > 0$, $t \neq 1$, then what is the relation between x and y?

(A) $y^x = x^{1/y}$ **(B)** $x^{1/y} = y^{1/x}$

(C) $x^y = y^x$ **(D)** $x^y = y^{1/x}$

EXPLANATION

Q.1.(B) Given that, $2x^{\frac{1}{3}} + 2x^{-\frac{1}{3}} = 5$

Let $x^{\frac{1}{3}} = m$ then $2m + \frac{2}{m} = 5$

$\Rightarrow$ $2m^2 - 5m + 2 = 0$

$\Rightarrow$ (2m-1)(m-2)=0

$\therefore$ $m = \frac{1}{2}$ or m=2 $\Rightarrow$ $x^{\frac{1}{3}}$=2 or $\frac{1}{2}$

Q.2.(D) $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{3}} \div \left(\frac{p^5q^{-3}}{p^{-2}q^3}\right)^{\frac{1}{3}} = p^aq^b$

$\Rightarrow$ $\left(p^{-4}q^4\right)^{\frac{1}{3}} \div (p^7q^{-6})^{\frac{1}{3}} = p^aq^b$

$\Rightarrow$ $p^{\frac{-4}{3}}q^{\frac{4}{3}} \div p^{\frac{7}{3}}q^{\frac{-6}{3}} = p^aq^b$

$\Rightarrow$ $p^{\left(\frac{-4}{3}-\frac{7}{3}\right)}q^{\left(\frac{4}{3}+\frac{6}{3}\right)} = p^aq^b$

$\therefore$ $a = \frac{-11}{3}$ and $b = \frac{10}{3}$

Now, a+b $= \frac{-11}{3} + \frac{10}{3} = -\frac{1}{3}$

Q.3.(B) $x = \frac{2^{n+4} - 2.2^n}{2.2^{n+3}} + 2^{-3}$

$= \frac{2^{n+4} - 2^{n+1}}{2^{n+4}} + 2^{-3} = \frac{2^{n+1}(2^3 - 1)}{2^{n+1}.2^3} + \frac{1}{2^3}$

$\frac{8-1}{2^3} + \frac{1}{2^3} = \frac{7}{8} + \frac{1}{8} = 1$

Q.4.(B) $(11)^{\frac{-3}{2}+\frac{2}{3}-\frac{5}{2}} = 11^{-2x}$

$-2x = \frac{-9+4-15}{6}$

-2x = $\frac{-20}{6}$

$x = \frac{5}{3}$

Q.5.(B) $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(6-\sqrt{3})} - \frac{4\sqrt{3}\sqrt{6}-\sqrt{2}}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$

$+ \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

[by rationalisation]

$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}^2-\sqrt{3}^2)} - \frac{4\sqrt{3}\sqrt{6}-\sqrt{2}}{(\sqrt{6}^2-\sqrt{2}^2)} + \frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{(\sqrt{3})^2-(\sqrt{2})^2}$

[(a+b)(a-b) $= a^2 - b^2$]

$= \frac{(3\sqrt{12}-3\sqrt{6})}{3} - \frac{(4\sqrt{18}-4\sqrt{6})}{4} + \frac{\sqrt{18}-\sqrt{12}}{1}$

$= \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6} + \sqrt{18} - \sqrt{12} = 0$

Q.6.(B) $3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3}$

$3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}}{(3+\sqrt{3})(\sqrt{3}-3)} + \frac{3+\sqrt{3}}{(\sqrt{3}-3)(3+\sqrt{3})}$

$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}}{9-3} + \frac{\sqrt{3}+3}{3-9}$

$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}}{6} + \frac{\sqrt{3}+3}{6}$

$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}-3}{6}$

$= 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3$

Q.7.(C) Given $3^{x+y} = 81$

$\Rightarrow$ $3^{x+y} = 3^4$

$\Rightarrow$ $x + y = 4$.........(i)

and $81^{x-y} = 3$

$\Rightarrow$ $(3^4)^{x-y} = 3 \Rightarrow 3^{4x-4y} = 3^1$

$\Rightarrow$ 4x-4y=1....(ii)

On multiplying by 4 in eq. (i) then adding Eqs. (i) and (ii) we get

4x+4y=16

4x-4y=1

8x=17

$\Rightarrow \ x=\frac{17}{8}$

Q.8.(A) $\frac{9^n\times 3^2\times(3^{-n/2})-(27)^n}{3^{3m}\times 2^3}=\frac{1}{27}$

$\Rightarrow \ \frac{3^{2n}\times 3^2\times 3^n-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}$

$\Rightarrow \ \frac{3^{n+2}-3^{3n}}{3^{3m}\times 8}=\frac{1}{27}$

$\Rightarrow \ \frac{3^{3n}-3^2-1}{3^{3m}\times 8}=\frac{1}{27}$

$\Rightarrow \frac{3^{3n}-8}{3^{3m}\times 8}=\frac{1}{27}\Rightarrow(3^3)^{n-m}=3^{-3}$

$\Rightarrow \ n-m=-1 \text{ or } m-n=1$

Q.9.(D) $(\sqrt{1+a}+\sqrt{1-a})^2$

$=(1+a)+(1-a)+2\sqrt{1-a^2}$

$=2(1+\sqrt{1-a^2})=2\left(1+\sqrt{1-\frac{3}{4}}\right)$

$2\left(1+\frac{1}{2}\right)=2\times\frac{3}{2}=3$

$\therefore \ (\sqrt{1+a}+\sqrt{1-a})=\sqrt{3}$

Q.10.(C) Given, $m=7-4\sqrt{3}$

$\therefore \ \frac{1}{m}=\frac{1}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}=\frac{7+4\sqrt{3}}{49-48}$

$=7+\sqrt{3}$

Now, $m+\frac{1}{m}=7-4\sqrt{3}+7+4\sqrt{3}=14$

$\Rightarrow \ m+\frac{1}{m}+2=14+2=16$

Now, $\left(\sqrt{m}+\frac{1}{\sqrt{m}}\right)^2=m+\frac{1}{m}+2$

$\left(\sqrt{m}+\frac{1}{\sqrt{m}}\right)^2=4^2\Rightarrow\left(\sqrt{m}+\frac{1}{\sqrt{m}}\right)=4$

Q.11.(C) Given, $2^p+3^q=17$ ....(i)

and

$2^{p+2}+3^{q+1}=5 \text{ or } 4.2^p-3.3^q=5$....(ii)

On multiplying Eq.(i) by 3 and adding it with Eq. (ii) we get

$3.2^p+3.3^q=51$

$4.2^p-3.3^q=5$

$\Rightarrow \quad 5y^2-26y+5=0$

$\Rightarrow \quad 5y^2-25y-y+5=0$

$\Rightarrow \quad 5y(y-5)-1(y-5)=0$

$\Rightarrow \quad (y-5)(5y-1)=0$

$\Rightarrow$ y=5, $\frac{1}{5}$

$\Rightarrow \ 5^x=5 \text{ or } 5^{-1}$

$\Rightarrow$ x=1 of -1

$7.2^p=56$

$\Rightarrow 2p=8=2^3\Rightarrow p=3$

On putting the value of P in Eq(i) we, get

$2^3+3^q=17 \quad 3^q=9$

p=3 and q=2

Q.12.(D) $\left(\frac{a^p}{a^q}\right)^{p+q-r}\times\left(\frac{a^r}{a^p}\right)^{(r+p-q)}\times\left(\frac{a^q}{a^r}\right)^{(q+r-p)}$

$=a^{(p-q)(p+q-r)}\times(a^{r-p})^{r+p-q}\times(a^{q-r})^{q+r-p}$

$=a^{p^2-q^2-rp+rq+r^2-p^2}\times a^{r^2-p^2-qr+pq}\times a^{q^2-r^2-pq+pr}$

$= a^{p^2-q^2-rp+rq+r^2-p^2-qr+pq+q^2-r^2-pq+pr} = a^0 = 1$

Q.13.(A) We have, $5^{1+x} + 5^{1-x} = 26$

$\Rightarrow 5.5^x + 5.5^{-x} = 26$

$\Rightarrow 5.5^x + \frac{5}{5^x} = 26$

Let $5^x = y$

$\therefore 5y + \frac{5}{y} = 26$

$y + \frac{1}{y} = \frac{26}{5} = 5\frac{1}{5}$

$5y^2 - 26y + 5 = 0$

$y = 5$

$5^x = 5$

$x = 0$

$y = 5^{-1}$

$5^x = 5^{-1}$

$x = -1$

Hence, x = (1, -1)

Q.14.(C) Given, $x = \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

and $y = \frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}$

$\therefore \left[(a+b)^2 = a^2 + b^2 + 2ab \right]$

$= \frac{\left(\sqrt{13}+\sqrt{11}\right)^2 \left(\sqrt{13}-\sqrt{11}\right)^2}{(\sqrt{13})^2 - (\sqrt{11})^2}$

$= \frac{2\left(\sqrt{13})^2 + (\sqrt{11}\right)^2 \left(\sqrt{13}-\sqrt{11}\right)^2}{13-11} = 13 + 11 = 24$

$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

and $xy = \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} \times \frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}} = 1$

$3x^2 - 5xy + 3y^2 = 3(x+y)^2 - 11xy = 3(24)^2 - 11 = 1717$

Q.15.(A) Given, $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

by rationalisation, we get

$x = \frac{\sqrt{(a+2b)^2} + \sqrt{(a-2b)^2} + 2\sqrt{(a+2b)(a-2b)}}{\sqrt{(a+2b)^2 - \left(\sqrt{a-2b}\right)^2}}$

$\Rightarrow x \frac{a+2b+a-2b+2\left(\sqrt{a^2-4b^2}\right)}{4b}$

$\Rightarrow x = \frac{a+\sqrt{a^2-4b^2}}{2b}$

$\therefore bx^2 - ax + b$

$= b\left(\frac{a+\sqrt{a^2-4b^2}}{2b}\right)^2 - a\left(\frac{a+\sqrt{a^2-4b^2}}{2b}\right) + b$

$= b\left(\frac{a^2+a^2-4b^2+2a\sqrt{a^2-4b^2}}{4b^2}\right) - \left(\frac{a^2+a\sqrt{a^2-4b^2}}{2b}\right) + b$

$= \left(\frac{2a^2-4b^2+2a\sqrt{a^2-4b^2}-2a^2-2a\sqrt{a^2-4b^2}+4b^2}{4b}\right) = 0$

Q.16.(D) $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$

$\frac{36+49+64+81+100}{\left[\sqrt{2^2+(\sqrt{3})^2+2.2\sqrt{3}} - \sqrt{(\sqrt{3})^2+1^2+2.1.\sqrt{3}}\right]}$

$2 \times 3 = 6$

$= \frac{300}{\left[2+\sqrt{3}-\sqrt{3}-1\right]} = 300$

Q.17.(B) Given, $x^2 = y + z \Rightarrow x = \frac{y+z}{x}$ ....(i)

$y^2 = z + x \Rightarrow y = \frac{z+x}{y}$ .....(ii)

$z^2 = x + y \Rightarrow z = \frac{x+y}{z}$ .....(iii)

Now, $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$= \frac{1}{\frac{y+z}{x}+1} + \frac{1}{\frac{z+x}{y}+1} + \frac{1}{\frac{x+y}{z}+1}$

$$= \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

Q.18.(B) Let $2^a = 3^b = 6^{-c} = K$

$\therefore y^x = x^y$

We know that, $2 \times 3 = 6$

$\therefore \quad K^{1/a} \times K^{1/b} = K^{1-c}$

$\Rightarrow \quad K^{\left(\frac{1}{a}+\frac{1}{b}\right)} = k^{\frac{-1}{c}}$

On comparing we get

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

Q.19.(C) We have,

$9x.3y = 2187$

$\Rightarrow \quad (3^2)^x.3y = 2187$

$\Rightarrow \quad 3^{2x+y} = 3^7$

$\Rightarrow \quad 2x + y = 7.$ ....(i)

Again,

$2^{3x}.2^{2y} - 4^{xy} = 0$

$\Rightarrow \quad 2^{3x}.2^{2y} = 4^{xy}$

$\Rightarrow \quad 2^{3x+2y} = (2^2)^{xy}$

$\Rightarrow \quad$ 3x+2y=2xy.....(ii)

From eqs. (i) and (ii)

$3x + 2(7-2x) = 2x(7-2x)$

$3x + 14 - 4x = 14 - 4x^2$

$\Rightarrow \quad 4x^2 - 15x + 14 = 0$

$\Rightarrow \quad (x-2)(4x-7) = 0$

$\Rightarrow \quad x = 2, \frac{7}{4}$

$\therefore$ y=3, $\frac{7}{2}$

$\therefore \quad$ x+y=5 or $\frac{21}{4}$

Q.20.(C) We have,

$x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}, t > 0, t \neq 1$

Now, $y = \left(t^{\frac{1}{t-1}}\right)^t = x^t$ ....(i)

Again $\quad \frac{y}{x} = \frac{t^{\frac{t}{t-1}}}{t^{\frac{1}{t-1}}} = t^{\frac{t}{t-1}-\frac{1}{t-1}} = t^{\frac{t-1}{t-1}} = t$

$\Rightarrow \frac{y}{x} = t$ -------(ii)

From eqs. (i) and (ii), we get $y = x^{y/x}$

$\Rightarrow \quad y^x = x^y$

Notes

CHAPTER-4
ALGEBRAIC IDENTITIES

Scan the QR code to get video of this chapter.

Polynomials

A polynomial in x is an expression obtained by taking powers of x, multiplying them by constants, and adding them. It can be written in the form

$c_0x^n + c_1x^{n-1} + c_2x^{n-2} + \cdots + c_{n-1}x + c_n$

where n is an integer 0, and $c_0, c_1, \ldots, c_n$ are constants.

Ex. $2x^6 + 5x^5 + x^3 + \frac{1}{2}x^2 - 6x - 10$

Note that $x^{\frac{1}{2}} + 3$ and $4x^3 + \frac{3}{x^2}$ are **not** polynomials.

The constants $c_0, \ldots, c_n$ are called the **coefficients.**

The **constant** term is c_n.

The **leading term** is the term involving the highest power of x, here c_0x^n.

The degree is the power of x in the leading term.

A degree 0 polynomial is just a **constant**, e.g., 2 is (a) constant

A degree 1 polynomial is called **linear**, e.g., 3x + 2 is linear

A degree 2 polynomial is called quadratic, e.g., $x^2 + 2x + 1$ is quadratic

A degree 3 polynomial is called cubic, e.g., $y^3 + 7y - 2$ is a cubic in y.

Polynomial division

Similar to long division for ordinary numbers.

Ex. To divide $2x^3 + 10x^2 - 3x + 1$ by x + 3.

Sol.

$$
\begin{array}{l}
x+3)\ 2x^3+10x^2-3x+1\ (2x^2+4x-15 \\
\quad\ \ \underline{-2x^3 \pm 6x} \\
\qquad\quad 4x^2-3x \\
\qquad\ \ \underline{-4x^2 \pm 12x} \\
\qquad\qquad\quad -15x+1 \\
\qquad\qquad\quad \underline{\mp 15x \mp 45} \\
\qquad\qquad\qquad\ 46
\end{array}
$$

The quotient is $2x^2 + 4x - 15$ and the remainder is 46.

polynomial = divisor · quotient + remainder

$2x^3 + 10x^2 - 3x + 1 = (x + 3)(2x^2 + 4x - 15) + 46$

Note. If the remainder is zero, we say that the polynomial is exactly divisible by the divisor, or that the divisor is a factor of the polynomial.

Remainder Theorem

When a polynomial f(x) is divided by (x −a), then the remainder is f(a).

Ex. The remainder when $f(x) = 2x^5 - 3x^2 + 1$ is divided by x + 2 is f(−2)

$= (-2)^5 -3(-2)^2 +1 = -32-12+1 = -43$. Check this yourself by doing the long division.

Proof. If the quotient is q(x) and the remainder is r, then

f(x) = (x − a).q(x) + r

Substituting x = a, gives f(a) = (a − a) · q(a) + r = r.

Factor Theorem

If (x − a) is a factor of a polynomial f(x), then f(a) = 0. Conversely, if f(a) = 0, then (x − a) is a factor of f(x).

Ex. Is x − 1 is a factor of $f(x) = x^5 - 1$?

Since f(1) = 0, the Factor Theorem tells us it must be. In fact $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$.

Proof. If x − a is a factor of f(x) then f(x) = (x − a) · g(x) for some polynomial g(x).

Therefore f(a) = (a − a) · g(a) = 0.

If f(a) = 0 then by the Remainder Theorem, the remainder on dividing f(x) by x − a is 0. Therefore f(x) = (x − a) · q(x) where q(x) is the quotient.

Proof. If x − a is a factor of f(x) then f(x) = (x − a) · g(x) for some polynomial g(x).

Therefore f(a) = (a − a) · g(a) = 0.

Factorizing polynomials

It is often useful to write a polynomial as a product of polynomials of lower degree.

Ex. $f(x) = x^3 - 2x^2 - x + 2$

can be factorized as $(x^2 - 1)(x - 2)$ or as $(x - 1)(x + 1)(x - 2)$.

Factorizing helps find the roots since, **if a product of numbers is zero, then one of them must be zero.**

Therefore if $f(x) = 0$ then $x - 1 = 0$, or $x + 1 = 0$ or $x - 2 = 0$. Therefore $x = 1, -1$ or 2.

Some well-known factorizations:

(i) Difference of squares:

$x^2 - a^2 = (x - a)(x + a)$

for example, $x^2 - 9 = (x + 3)(x - 3)$.

(ii) Perfect square:

$x^2 \pm 2ax + a^2 = (x \pm a)^2$

for example $x^2 - 10x + 25 = (x - 5)^2$.

(iii) If there is no constant term, then x is a common factor. For example,

$x^2 + 4x = x(x + 4)$.

Basic Formule

(i) $(a+b)^2 = a^2+b^2+2ab$

(ii) $(a-b)^2 = a^2+b^2-2ab$

(iii) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

(iv) $(a+b)^2 - (a-b)^2 = 4ab$

(v) $a^2-b^2 = (a-b)(a+b)$

(vi) $(a+b)^3 = a^3+b^3+3ab(a+b)$

(vii) $(a-b)^3 = a^3-b^3-3ab(a-b)$

(viii) $a^3+b^3 = (a+b)(a^2-ab+b^2)$

(ix) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

(x) $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

(xi) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

If $a+b+c = 0$

$\Rightarrow a^3+b^3+c^3 = 3abc$

$a^2+b^2+c^2-ab-bc-ca$

$$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

SOME IMPORTANT TRICKS:

Trick:1 If $x + \frac{1}{x} = A$

then $x^2 + \frac{1}{x^2} = A^2 - 2$

$$x^3 + \frac{1}{x^3} = A^3 - 3A$$

$$x - \frac{1}{x} = \sqrt{A^2 - 4}$$

If $x + \frac{1}{x} = A$........(i)

Squaring on both sides

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = A^2$$

$$x^2 + \frac{1}{x^2} = A^2 - 2$$

$$x + \frac{1}{x} = A$$

On cubing both sides

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = A^3$$

$$x^3 + \frac{1}{x^3} + 3A = A^3$$

$$x^3 + \frac{1}{x^3} = A^3 - 3A$$

$$x + \frac{1}{x} = A$$

Let $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$

$$x - \frac{1}{x} = \sqrt{A^2 - 4}$$

Ex. $x + \frac{1}{x} = 3$ then $x^2 + \frac{1}{x^2} = ?$ $x^3 + \frac{1}{x^3} = ?$ $x - \frac{1}{x} = ?$

$$x + \frac{1}{x} = 3 = A$$

$$x^2 + \frac{1}{x^2} = A^2 - 2 = 3^2 - 2 = 7$$

$$x^3 + \frac{1}{x^3} = A^3 - 3A = 3^3 - 3 \times 3 = 18$$

$$x - \frac{1}{x} = \sqrt{A^2 - 4} = \sqrt{3^2 - 4} = \sqrt{5}$$

Trick: 2 If $x - \frac{1}{x} = A$ then

$$x^2+\frac{1}{x^2}=A^2+2 \quad x^3-\frac{1}{x^3}=A^3+3A$$

$$x+\frac{1}{x}=\sqrt{A^2+4}$$

$$\Rightarrow x-\frac{1}{x}=A.........(i)$$

Squaring on both sides

$$x^2+\frac{1}{x^2}-2x\times\frac{1}{x}=A^2$$

$$x^2+\frac{1}{x^2}=A^2+2$$

$$\Rightarrow x-\frac{1}{x}=A$$

cube on both sides

$$x^3-\frac{1}{x^3}-3x\times\frac{1}{x}\left(x-\frac{1}{x}\right)=A^3$$

$$x^3-\frac{1}{x^3}=A^3+3A$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=\left(x-\frac{1}{x}\right)^2+4$$

$$x+\frac{1}{x}=\sqrt{A^2+4}$$

Trick: 3 If $x^n+\frac{1}{x^n}$=A then , $x^{\frac{n}{2}}+\frac{1}{x^{\frac{n}{2}}}=\sqrt{A+2}$

$$x^{\frac{n}{2}}-\frac{1}{x^{\frac{n}{2}}}=\sqrt{A-2}$$

$$x^2+\frac{1}{x^2}=A$$

$$x+\frac{1}{x}=\sqrt{A+2} \quad x-\frac{1}{x}=\sqrt{A-2}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$x+\frac{1}{x}=\sqrt{A+2}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$x-\frac{1}{x}=\sqrt{A-2}$$

Trick: 4 $x+\frac{1}{x}=A$

$$x^3+\frac{1}{x^3}=\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)-\left(x+\frac{1}{x}\right)$$

$$x^5+\frac{1}{x^5}=\left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)$$

$$x^7+\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$x^9+\frac{1}{x^9}=\left(x^5+\frac{1}{x^5}\right)\left(x^4+\frac{1}{x^4}\right)-\left(x+\frac{1}{x}\right)$$

Trick: 5 $x+\frac{1}{x}=A$

$$x^2+\frac{1}{x^2}=A^2-2=K$$

$$x^3+\frac{1}{x^3}=A^3-3A=L$$

$$x^4+\frac{1}{x^4}=K^2-2=M$$

$$x^6+\frac{1}{x^6}=L^2-2=N$$

$$x^8+\frac{1}{x^8}=M^2-2=O$$

$$x^{12}+\frac{1}{x^{12}}=N^2-2$$

$$x^{16}+\frac{1}{x^{16}}=O^2-2$$

EXAMPLES

Ex-1. If $x+\frac{1}{x}=2$ then value of $x^{57}+\frac{1}{x^{57}}$

(A) 0 **(B)** 1

(C) 2 **(D)** -2

Sol.(C) $x+\frac{1}{x}=2$

So, $x=1$

So, $x^{57}+\frac{1}{x^{57}}=(1)^{57}+\frac{1}{(1)^{57}}=2$

Ex-2. If $x^2+\frac{1}{x^2}=7$ then find the value of

$x^3 + \frac{1}{x^3}$

(A) 19 **(B)** 28

(C) 38 **(D)** 18

Sol.(D) $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2\times x \times \frac{1}{x} = 7+2 = 9$

$\left(x+\frac{1}{x}\right) = \sqrt{9} = 3$

cubing both sides

$\left(x+\frac{1}{x}\right)^3 = 3^3$

$x^3 + \frac{1}{x^3} + 3\times x \times \frac{1}{x}\left(x+\frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\times 3 = 27$

$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 = 18$

QUICK TRICK :

$x^2 + \frac{1}{x^2} = 7$

$\therefore x + \frac{1}{x} = \sqrt{7+2} = 3$

$\Rightarrow x^3 + \frac{1}{x^3} = 3^3 - 3\times 3 = 27 - 9 = 18$

Ex-3. If $x = 3+2\sqrt{2}$ then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is

(A) 1 **(B)** 2

(C) $2\sqrt{2}$ **(D)** $\sqrt{2}$

Sol.(B) $x = 3+2\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$= 3+2\sqrt{2}+3-2\sqrt{2}-2 = 4$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

Ex-4. If $x + \frac{1}{x} = 6$ then find the value of

$\frac{3x}{2x^2 - 5x + 2}$

(A) $\frac{2}{3}$ **(B)** 0

(C) 1 **(D)** $\frac{3}{7}$

Sol.(D) $\frac{3x}{2x^2-5x+2} = \frac{3x}{x\left[\left(2x+\frac{2}{x}\right)-5\right]} = \frac{3}{2\left(x+\frac{1}{x}\right)-5} = \frac{3}{2\times 6 - 5}$

$= \frac{3}{7}$

Ex-5. $x + \frac{1}{x} = 1$ then value of $x^{12} + x^9 + x^6 + x^3 + 1$

(A) -2 **(B)** -1 **(C)** 1 **(D) 0**

Sol.(C) $x + \frac{1}{x} = 1$

$x^2 - x + 1 = 0$

$(x^{12}+x^9)+(x^6+x^3)+1$

$= x^9(x^3+1) + x^3(x^3+1) + 1$

$= x^9(x+1)(x^2-x+1) + x^3(x+1)(x^2-x+1) + 1 = 1$

Ex-6. If $m + \frac{1}{m-2} = 4$ then find the value of

$(m-2)^2 + \frac{1}{(m-2)^2}$

(A) -2 **(B)** 0

(C) 2 **(D) 4**

Sol.(C) $m + \frac{1}{m-2} = 4$

$(m-2) + \frac{1}{m-2} = 2 \Rightarrow (m-2)^2 + \frac{1}{(m-2)^2}$

$= \left[(m-2) + \frac{1}{(m-2)}\right]^2 - 2 = 2^2 - 2 = 2$

Ex-7. If $x^4 + \frac{1}{x^4} = 119$ and x>1 then $x^3 - \frac{1}{x^3} = ?$

(A) 54 **(B)** 18

(C) 72 **(D)** 36

Sol.(D) $x^4+\frac{1}{x^4}=119$

$\Rightarrow (x^2)^2+\left(\frac{1}{x^2}\right)^2+2x^2.\frac{1}{x^2}=119+2$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=121 \Rightarrow x^2+\frac{1}{x^2}=11$

$\Rightarrow x^2+\frac{1}{x^2}-2x^2.\frac{1}{x^2}=11-2$

$\Rightarrow \left(x-\frac{1}{x}\right)^2=9 \Rightarrow x-\frac{1}{x}=3$

Now, $x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3x\times\frac{1}{x}\left(x-\frac{1}{x}\right)$

$=3^3+3\times3$

$x^3-\frac{1}{x^3}=36$

QUICK TRICK

$x^4+\frac{1}{x^4}=119$

$x^2+\frac{1}{x^2}=\sqrt{119+2}=11$

$x-\frac{1}{x}=\sqrt{11-2}=3$

$x^3-\frac{1}{x^3}=3^3+3\times3=36$

Ex-8. If $x=\frac{4ab}{a+b}$ then find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$

(A) a **(B)** b

(C) 2ab **(D)** 2

Sol.(D) $x=\frac{4ab}{a+b} \Rightarrow \frac{x}{2a}=\frac{2b}{a+b}$

$\frac{x+2a}{x-2a}=\frac{3b+a}{b-a}$

[By componendo and dividendo]

Similarly, $\frac{x}{2b}=\frac{3a+b}{a-b}$

$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$

$=\frac{3b+a}{b-a}-\frac{3a+b}{b-a}-\frac{3a+b}{b-a}=\frac{3b+a-3a-b}{b-a}$

$=\frac{2b-2a}{b-a}=\frac{2(b-a)}{b-a}=2$

Componendo and Dividendo

If $\frac{a}{b}=\frac{x}{y}$ then $\frac{a+b}{a-b}=\frac{x+y}{x-y}$

Ex-9. If $a^2=b+c$, $b^2=c+a$ and $c^2=a+b$ then find the value of $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$

(A) abc **(B)** $a^2b^2c^2$

(C) 1 **(D)** 0

Sol.(C) $a^2=b+c$ $b^2=a+c$ $c^2=a+b$

$=\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$

$=\frac{a}{a+a^2}+\frac{b}{b+b^2}+\frac{c}{c+c^2}$

$=\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}$

$=\frac{a+b+c}{a+b+c}=1$

Ex-10. If $x=2-2^{1/3}+2^{2/3}$ then find the value of x^3-6x^2+18x

(A) 22 **(B)** 33

(C) 40 **(D)** 45

Sol.(A) $x=2-2^{1/3}+2^{2/3}$

$x-2=2^{2/3}-2^{1/3}$

On cubing both the sides

$x^3-8-6x^2+12x=4-2-6(x-2)$

$x^3-6x^2+12x+6x=4-2+12+8$

$x^3-6x^2+18x=22$

EXERCISE

Q.1. If $x = 2 + \sqrt{3}$, then (x^2+x^{-2}) equal to ?

(A) 12 **(B)** 13
(C) 14 **(D)** 15

Q.2. If $a^x = b^y = c^z$ and abc = 1, then what is xy+yz+zx equal to?

(A) xyz **(B)** x+y+z
(C) 0 **(D)** 1

Q.3. If pq+qr+rp = 0, then what is the value of $\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$ equal to?

(A) 0 **(B)** 1
(C) -1 **(D)** 3

Q.4. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$ then find the value of z?

(A) $\frac{7}{48}$ **(B)** $\frac{3}{48}$
(C) $\frac{6}{49}$ **(D)** $\frac{7}{50}$

Q.5. What is the value of k that (2x-1) may be a factor of $4x^4 -(k-1)x^3 + kx^2 - 6x + 1$?

(A) 8 **(B)** 9
(C) 12 **(D)** 13

Q.6. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}, x \neq 0$ then the value of $x^2 + \frac{1}{x^2}$ is-

(A) $2\frac{1}{3}$ **(B)** $2\frac{2}{3}$
(C) $2\frac{4}{9}$ **(D)** $2\frac{5}{9}$

Q.7. If a + b + c = 0 the value of $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$ is-

(A) 2 **(B)** 3
(C) 4 **(D)** 5

Q.8. If x + y = 2z then tha value of $\frac{x}{x-z}+\frac{z}{y-z}$ is-

(A) 3 **(B)** 4
(C) 2 **(D)** 1

Q.9. If $x+\frac{1}{x}=a$, then what is $x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}$ equal to?

(A) $a^3 + a^2$
(B) $a^3 + a^2 - 5a$
(C) $a^3 + a^2 - 3a - 2$
(D) $a^3 + a^2 - 4a - 2$

Q.10. If $x^3 + 5x^2 + 10$ K leaves remainder $-2x$ when divided by $x^2 + 2$, then what is the value of K?

(A) – 2 **(B)** – 1
(C) 1 **(D)** 2

Q.11. If $a^x = b$, $b^y = c$ and xyz = 1, then what is c^z equal to?

(A) a **(B)** b
(C) ab **(D)** $\frac{a}{b}$

Q.12. If 2x+y = 15, 2y+z = 25 and 2z+x = 26, what is the value of z?

(A) 11 **(B)** 7
(C) 9 **(D)** 12

Q.13. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$, then the value of $\frac{a+b+c}{c}$ is-

(A) 7 **(B)** 2
(C) $\frac{1}{2}$ **(D)** $\frac{1}{7}$

Q.14. If $z+\frac{1}{z}=\sqrt{3}$, then find the value of $z^3+\left(\frac{1}{z}\right)^3$.

(A) 0 **(B)** 1
(C) -1 **(D)** 9

Q.15. If p+q=2, find the value of p^3+q^3+6pq.

(A) 0 **(B)** 1
(C) 5 **(D)** 8

Q.16. If 2a-3b=5, then find the value of $8a^3-27b^3-90ab$-

(A) 5 **(B)** 25
(C) 125 **(D)** 625

Q.17. If $\frac{5x-7y+10}{1}=\frac{3x+2y+1}{8}+\frac{11x+4y-10}{9}$ then what is x + y equal to ?

(A) 1 **(B)** 2
(C) 3 **(D)** -3

Q.18. If a + b + c = 6, $a^2 + b^2 + c^2 = 26$, then what is ab + bc + ca equal to ?

(A) 0 **(B)** 2
(C) 4 **(D)** 5

Q.19. If $3x^3 - 2x^2y - 13xy^2 + 10y^3$ is divided by x - 2y ,then what is the remainder ?

(A) 0 **(B)** x
(C) y + 5 **(D)** x - 3

Q.20. If x + y + z = 0, then what is

$\frac{xyz}{(x+y)(y+z)(z+x)}$ equal to ?

$[x \neq -y, y \neq -z, z \neq -x]$

(A) -1 **(B)** 1

(C) xy + yz + zx **(D)** None of the above

Q.21. If $a^2 + b^2 + 2b + 4a + 5 = 0$,then the value of $\frac{a-b}{a+b}$ is-

(A) 3 **(B)** -3

(C) $\frac{1}{3}$ **(D)** $-\frac{1}{3}$

Q.22. If $x = a^{\frac{1}{3}} - a^{-\frac{1}{3}}$ then what is x^3+3x equal to ?

(A) 0 **(B)** $a-\frac{1}{a}$

(C) $a+\frac{1}{a}$ **(D)** $a^3+\frac{1}{a^3}$

Q.23. If $3^{(x-1)} + 3^{(x+1)} = 30$, then what is the value of $3^{(x+2)} + 3^x$?

(A) 30 **(B)** 90

(C) 60 **(D)** 81

Q.24. If $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$ then what is the value of x ?

(A) 0 **(B)** 1

(C) a+b+c **(D)** abc

Q.25. If $3a - \frac{3}{a} + 5 = 0$, the value of $\left(a^3 - \frac{1}{a^3} + 2\right)$ is

(A) 8 **(B)** $-\frac{157}{27}$

(C) $-\frac{200}{27}$ **(D)** $-\frac{206}{27}$

Q.26. If $x = \frac{\sqrt{5}}{3}$ and $\sqrt{5} = 2.23$, $\sqrt{6} = 2.44$, then the value of $\sqrt{1+x} + \sqrt{1-x}$ will be

(A) 1.82 **(B)** 2.82

(C) 0.23 **(D)** 0.44

Q.27. If x is real and $x^4 + \frac{1}{x^4} = 322$, then the value of $\left(x-\frac{1}{x}\right)$ is

(A) ± 4 **(B)** ± 9

(C) ± 3 **(D)** ± 2

Q.28. If $a = \sqrt{3}+1$, $b = \sqrt{3} - 1$, then the value of $\frac{1}{a+1}$ +b+1 is

(A) 9 **(B)** 3

(C) 1 **(D)** 2

Q.29. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then $x^2 - 4x + 2 = ?$

(A) 0 **(B)** 1

(C) -1 **(D)** 2

Q.30. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{41}+y^{45}$ is -

(A) 0 **(B)** 1

(C) -1 **(D)** 2

Q.31. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is

(A) 1 **(B)** 2

(C) $2\sqrt{2}$ **(D)** $3\sqrt{3}$

Q.32. If a+b+c=0 then $\frac{a^2+b^2+c^2}{b^2-ca}$ is

(A) -2 **(B)** 1

(C) 0 **(D)** 2

Q.33. If $t^2 - 4t + 1 = 0$, then the value of $\left(t^3 + \frac{1}{t^3}\right)$ is

(A) 44 **(B)** 48

(C) 52 **(D)** 64

Q.34. If $\frac{x}{(b-c)(b+bc-2a)} = \frac{y}{(c-a)(c+ca-2b)}$

$\frac{z}{(c-b)(a+b-2c)}$, then the value of

(x+y+z) is :

(A) a+b+c **(B)** $a^2+b^2+c^2$

(C) 0 **(D)** indeterminate

Q.35. If $a^2+b^2+c^2 = 2(a-b-c)-3$, then the value of 2a -3b+4c is

(A) 3 **(B)** 1

(C) 2 **(D)** 4

Q.36. If $\sqrt{4x-9} + \sqrt{4x+9} = 5 + \sqrt{7}$, then the value of x is

(A) 5 **(B)** 7

(C) 3 **(D)** 4

Q.37. If $f(x) = ax^4 - 4x^2 + ax - 3$, then f (b) – f (–b) will equal

(A) 2ab **(B)** $2ab^4 - 8b^2 - 6$

(C) $-2ab^4 + 8b^2 - 6$ **(D)** $2ab^4 - 8b^2 + 2ab - 6$

Q.38. If a and b are the roots of the equation $x^2 + x - 13 = 0$, what is the value of $a^2 + b^2 - ab$?

(A) – 13 **(B)** 20
(C) 40 **(D)** – 20

Q.39. If $R = \frac{x^3+1}{x+2}$ and $S = \frac{x^2-x+1}{x+1}$ then find the value of R ÷ S.

(A) $\frac{x+1}{x+2}$ **(B)** $\frac{(x+1)^2}{x+2}$

(C) $\frac{(x+1)}{(x+2)^2}$ **(D)** $\frac{(x-1)}{(x+2)}$

Q.40. If $\frac{a}{a^2-2a+1} = \frac{1}{3}$, then the value of $a^3 + \frac{1}{a^3}$ is:

(A) 81 **(B)** 110
(C) 125 **(D)** 27

Q.41. If $p + \frac{1}{p} = 2$, then the value of $\frac{5p^2+5}{3p^2+9p+3}$

(A) $\frac{2}{3}$ **(B)** 0

(C) $\frac{1}{3}$ **(D)** 1

Q.42. If $m^2 - 2\sqrt{10}\,m + 1 = 0$, then what is the value of $\left(m^2 - \frac{1}{m^2}\right)$?

(A) $8\sqrt{10}$ **(B)** $12\sqrt{10}$
(C) $6\sqrt{10}$ **(D)** $10\sqrt{10}$

Q.43. If $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$, then find the value of x.

(A) $\frac{263}{20}$ **(B)** $\frac{187}{39}$
(C) $\frac{327}{57}$ **(D)** $\frac{427}{39}$

Q.44. If x = 3qp – qr – rp, y = 3qr – pq – pr, z = 3pr – pq – qr, find the value of $\frac{x+y+z}{pqr}$.

(A) pqr **(B)** pq + qr + rp
(C) $r^{-1} + p^{-1} + q^{-1}$ **(D)** p + q + r

Q.45. If $3p + 7 = p^2 + N = 7p + 5$, what is the value of p?

(A) $\frac{1}{2}$ **(B)** $8\frac{1}{4}$

(C) $8\frac{1}{2}$ **(D)** $5\frac{1}{2}$

Q.46. If xy (x + y) = 1, then $\frac{1}{x^3y^3} - x^3 - y^3$

(A) 0 **(B)** 1
(C) 3 **(D)** – 3

Q.47. If $xya = (3x + y)^2 - (3x - y)^2$, then the value of a is
(A) 12 **(B)** 6
(C) 0 **(D)** 18

Notes

EXPLANATION

Q.1.(C) x= $2+\sqrt{3}$

$\Rightarrow x^2=4+3+4\sqrt{3}=7+4\sqrt{3}$

$\therefore x^2+\frac{1}{x^2}=\left(7+4\sqrt{3}+\frac{1}{7+4\sqrt{3}}\right)$

$=\left(7+4\sqrt{3}+7-4\sqrt{3}\right)$

= 14

Q.2.(C) $a^x = b^y = c^z$ and abc=1

Let $a^x = b^y = c^z = k$

$\Rightarrow a=(k)^{\frac{1}{x}}, b=(k)^{\frac{1}{y}}, c=(k)^{\frac{1}{z}}$

$\Rightarrow abc=k^{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$

$\Rightarrow 1=k^{\left(\frac{yz+zx+xy}{xyz}\right)}, \Rightarrow k^0=k^{\left(\frac{xy+yz+zx}{xyz}\right)}$

$\Rightarrow$ xy+yz+zx=0

Q.3.(B) pq+qr+rp=0

$\therefore \frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$

$=\frac{p^2}{p^2+pq+rp}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$

$=\frac{p+q+r}{p+q+r}=1$

Q.4.(A) $2^x = 4^y = 8^z = 2^x = 2^{2y} = 2^{3z}$

x = 2y = 3z

and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$

$\frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z}=\frac{24}{7}$

$\frac{3}{6z}=\frac{24}{7}$

Therefore $z=\frac{3}{6}\times\frac{7}{24}=\frac{7}{48}$

Q.5.(D) $4x^4-(k-1)x^3+kx^2-6x+1$

$\therefore 4\left(\frac{1}{2}\right)^4-(k-1)\left(\frac{1}{2}\right)^3+k\left(\frac{1}{2}\right)^2-6\times\frac{1}{2}+1=0$

$\Rightarrow \frac{2-k+1+2k-16}{8}=0$

$\Rightarrow$ k = 13

Q.6.(C) $3x-2=\frac{3}{x}$

$3x-\frac{3}{x}=2$

Dividing both sides by 3

$x-\frac{1}{x}=\frac{2}{3}$

on squaring both side.

$x^2+\frac{1}{x^2}-2=\frac{4}{9}$

$x^2+\frac{1}{x^2}=\frac{4}{9}+2=x^2+2\frac{4}{9}$

Q.7.(B) $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$

$\frac{a^3+b^3+c^3}{abc}=\frac{3abc}{abc}=3$

Q.8.(D) x + y = 2z

x = 2z - y

x- z = 2z - y - z = z - y

$\frac{x}{x-z}+\frac{z}{y-z}=\frac{x}{x-z}-\frac{z}{z-y}$

$=\frac{x}{x-z}-\frac{z}{x-z}=\frac{x-z}{x-z}=1$

Q.9.(C) $x+\frac{1}{x}=a \Rightarrow x^2+\frac{1}{x^2}+2=a^2$

$\Rightarrow a^2-x+\frac{1}{x}=a^2-2$

$\therefore x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}$

$=x^3+\frac{1}{x^3}+x^2+\frac{1}{x^2}$

$=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)+\left(x^2+\frac{1}{x^2}\right)$

$= a(a^2-2-1)+a^2-2$

$= a^3-3a+a^2-2 = a^3+a^2-3a-2$

Q.10.(C) $x^3 + 5x^2 + 10$ k

Put, $x^2 = -2$

$(-2x) + 5(-2) + 10k = -2x$

$\Rightarrow -2x-10+10k=-2x \Rightarrow k=1$

Q.11.(A) $a^x = b, b^y = c \Rightarrow b=c^{\frac{1}{y}}$

$xyz = 1$

$xy = \frac{1}{z}$

$\Rightarrow a^{xy} = b^y = c \Rightarrow a^{xy} = c$

$\Rightarrow a^{\frac{1}{z}} = c$

$\Rightarrow c^z = a$

Q.12.(A) $2x + y = 15$ _______(I)

$2y + z = 25$ _______ (II)

$2z + x = 26$ ________ (III)

$\Rightarrow 2(25 - 2y) + x = 26$

$\Rightarrow 50 - 4y + x = 26$

$\Rightarrow x - 4y = -24$ ______ (IV)

From (I) and (IV),

$y = 7$

$x = 4$

$\Rightarrow z = 11$

Q.13.(B) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k,$ say

$\Rightarrow a = 3k, b = 4k, c = 7k$

$\Rightarrow \frac{a+b+c}{c} = \frac{14}{7} = 2$

Q.14.(A) $Z^3 + \left(\frac{1}{z}\right)^3 = \left(z + \frac{1}{z}\right)^3 - 3\left(z + \frac{1}{z}\right)$

$= \left(\sqrt{3}\right)^3 - 3 \times \sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0$

Q.15.(D) $p+q = 2$

$(p+q)^3 = 8$

$\Rightarrow p^3+q^3+3pq(p+q)=8$

$\Rightarrow p^3+q^3+3pq\times2=8$

$\Rightarrow p^3+q^3+6pq=8$

Q.16.(C) $(2a-3b)=5$

$\therefore (2a-3b)^3=5^3$

$\Rightarrow 8a^3-27b^3-3\times2a\times3b(2a-3b)$

$= 125$

$\Rightarrow 8a^3-27b^3-18ab\times5 = 125$

Q.17.(C) $\frac{5x-7y+10}{1} = \frac{3x+2y+1}{8} = \frac{11x+4y-10}{9}$

$\Rightarrow 40x - 56y + 80 = 3x + 2y + 1$

$\Rightarrow 37x - 58y - 79 = 0$

$\Rightarrow 37x - 58y = 79$ ....(I)

and

$27x + 18y + 9 = 88x + 32y - 80$

$\Rightarrow 61x + 14y = 89$.........(II)

(I) and (II)

$x = 1$

$y = 2$

$\therefore x + y = 1 + 2 = 3$

Q.18.(D) $a + b + c = 6$

$a^2 + b^2 + c^2 = 26$

$\therefore (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

$\Rightarrow ab + bc + ca = \frac{36-26}{2}$

$= 5$

Q.19.(A)

$$
\begin{array}{r|l}
x-2y & 3x^3 - 2x^2y - 13xy^2 + 10y^3 \;\big(\; 3x^2 + 4xy - 5y^2 \\
 & \underline{\overset{-}{3x^3} \overset{+}{-} 6x^2y} \\
 & 4x^2y - 13xy^2 \\
 & \underline{\overset{-}{4x^2y} \overset{+}{-} 8xy^2} \\
 & -5xy^2 + 10y^3 \\
 & \underline{\overset{+}{-5xy^2} \overset{-}{+} 10y^3} \\
 & 0
\end{array}
$$

$\therefore$ Remainder = 0

Q.20.(A) $x + y + z = 0$

$\therefore \frac{xyz}{(x+y)(y+z)(z+x)} = \frac{xyz}{(-z)(-x)(-y)} = -1$

Q.21.(C) $a^2 + b^2 + 2b + 4a + 5 = 0$

$\Rightarrow a^2 + 4a + 4 + b^2 + 2b + 1 = 0$

$\Rightarrow (a+2)^2 + (b-1)^2 = 0$

$\therefore a = -2, b = -1$

$\therefore \frac{a-b}{a+b} = \frac{-2+1}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$

Q.22.(B) $x = a^{\frac{1}{3}} - \frac{1}{a^{\frac{1}{3}}}$

$x^3 + 3x = \left(a^{\frac{1}{3}} - \frac{1}{a^{\frac{1}{3}}}\right)^3 + 3\left(a^{\frac{1}{3}} - \frac{1}{a^{\frac{1}{3}}}\right)$

$= \left(a^{\frac{1}{3}}\right)^3 - \left(\frac{1}{a^{\frac{1}{3}}}\right)^3$

$= a - \frac{1}{a}$

Q.23.(B) $3^{(x-1)} + 3^{(x+1)} = 30$

$\frac{3^x}{3} + 3^x \times 3 = 30$

$3^x + 9 \times 3^x = 90$

$3^x (1+9) = 90$

$3^x = 9$

$3^{(x+2)} + 3x = 3^x.3^2 + {}^{3x}$

$= 9 (9+1)$

$= 90$

Q.24.(C) $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$

$\frac{x-(a+b+c)}{b+c} + \frac{x-(a+b+c)}{c+a} + \frac{x-(a+b+c)}{a+b} = 0$

$\Rightarrow [x-(a+b+c)]\left[\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right]$

$= 0$

$\Rightarrow x - (a+b+c) = 0 \Rightarrow x = a + b + c$

Q.25.(D) $3a - \frac{3}{a} + 5 = 0$

$\therefore a - \frac{1}{a} = -\frac{5}{3}$

Cubing both side,

$\therefore \left(a - \frac{1}{a}\right)^3 = \left(-\frac{5}{3}\right)^3$

$\Rightarrow a^3 - \frac{1}{a^3} - 3 \times - \frac{5}{3} = -\frac{125}{27} \Rightarrow a^3 - \frac{1}{a^3} - 3 \times - \frac{5}{3} = -\frac{125}{27}$

$\Rightarrow a^3 - \frac{1}{a^3} = -\frac{260}{27}$

$\therefore a^3 - \frac{1}{a^3} + 2 = -\frac{260}{27} + 2 = \frac{-260+54}{27} = -\frac{206}{27}$

Q.26.(A) $x = \frac{\sqrt{5}}{3}$

$\Rightarrow \sqrt{1+x} = \sqrt{1 + \frac{\sqrt{5}}{3}} = \sqrt{\frac{3+\sqrt{5}}{3}} = \sqrt{\frac{6+2\sqrt{5}}{6}}$

$= \sqrt{\frac{(\sqrt{5}+1)^2}{6}} = \frac{\sqrt{5}+1}{\sqrt{6}}$

and $\sqrt{1-x} = \frac{\sqrt{5}-1}{\sqrt{6}}$

$\therefore \sqrt{1+x} + \sqrt{1-x} = \frac{\sqrt{5}+1}{\sqrt{6}} + \frac{\sqrt{5}-1}{\sqrt{6}} = \frac{2\sqrt{5}}{\sqrt{6}}$

$= \frac{2 \times 2.23}{2.44} = \frac{4.46}{2.44} = 1.82$

Q.27.(A) $x^4 + \frac{1}{x^4} = 322$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 324$, $\Rightarrow x^2 + \frac{1}{x^2} = 18$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 18$, $\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16$

$x - \frac{1}{x} = \pm 4$

Q.28.(D) $a = \sqrt{3} + 1$

$\Rightarrow a+1 = \sqrt{3} + 2$

$\Rightarrow \frac{1}{a+1} = \frac{1}{2+\sqrt{3}} = \frac{(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = 2 - \sqrt{3}$

and $b = \sqrt{3} - 1$

$\Rightarrow b+1 = \sqrt{3}$

$\therefore \frac{1}{a+1} + b + 1 = 2 - \sqrt{3} + \sqrt{3} = 2$

Q.29.(B) $x^2 - 4x + 2$

$= (x^2 - 4x + 4) - 2 = (x-2)^2 - 2 = \left[\frac{\sqrt{3}+1}{\sqrt{3}-1} - 2\right]^2 - 2$

$= \left(\frac{\sqrt{3}+1-2\sqrt{3}+2}{\sqrt{3}-1}\right)^2 - 2 = \left(\frac{3-\sqrt{3}-1}{\sqrt{3}-1}\right)^2 - 2$

$= \left(\frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}-1)}\right)^2 - 2 = (\sqrt{3})^2 - 2 = 1$

Q.30.(C) $x^2 + y^2 + 2x + 1 = 0$

$\Rightarrow x^2 + 2x + 1 + y^2 = 0 \Rightarrow (x+1)^2 + y^2 = 0$

$x + 1 = 0 \Rightarrow x = -1$

and $y = 0 \Rightarrow x^{41} + y^{45} = (-1)^{41} + (0)^{45} = -1$

Q.31.(B) $x = 3 + 2\sqrt{2}$

$\Rightarrow \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$\Rightarrow \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$

$\Rightarrow \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$\Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

Q.32.(D) a+b+c=0

$(a+c)^2 = (-b)^2$

$a^2+b^2+c^2=2(b^2-ca)$

$\frac{a^2+b^2+c^2}{b^2-ca}=2$

Q.33.(C) $t^2 - 4t + 1 = 0$

$\Rightarrow t^3 + 1 = 4t$

$\Rightarrow \frac{t^2+1}{t} = 4 \Rightarrow t + \frac{1}{t} = 4$

$\left(t+\frac{1}{t}\right)^3 = 64 \Rightarrow t^3 + \frac{1}{t^3} + 3\left(t + \frac{1}{t} = 64\right)$

$t^3 + \frac{1}{t^3} = 52$

Q.34.(C) Let $\frac{x}{(b-c)(b+bc-2a)} = \frac{y}{(c-a)(c+ca-2b)}$

$= \frac{z}{(c-b)(a+b-2c)} = k$

Then,

x = k(b-c)(b+c-2a), y = k9c-a)(c+a)

x+y+z = k[(b-c)(b+c-2a)+(c-a)(c+a-2b) +(a+b-2c)]

$= k[(b^2-c^2+c^2+a^2a^2) - 2\{a(a(b-c)+b(c-a)+(c-b)] = 0$

Q.35.(B) $a^2+b^2+c^2 = 2(a-b-c)-3$

$\Rightarrow (a^2-2a+1)+(b^2+2b+1) +(c^2+2c+1) = 0$

$\Rightarrow (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$

a-1 = 0, b+1= 0, c+1 = 0

$\Rightarrow$ a = 1, b = -1, c = -1

Now, 2a -3b+4c = 2(1) - 3(-1)+4(-1) = 1

Q.36.(D) by option (d), putting x = 4 we get

$\sqrt{4x-9} + \sqrt{4x+9}$

$= \sqrt{4x4-9} + \sqrt{4x \times +9}$

$= \sqrt{7} + \sqrt{25} = \sqrt{7} + 5$

Q.37.(A) $(ab^4 - 4b^2 + ab-3) - (ab^4 - 4b^2 - ab-3)$

= 2ab

Q.38.(C) $\alpha+\beta = -1$

$\alpha\beta = -13$

Now, $(\alpha+\beta)^2 = \alpha^2+\beta^2 + 2\alpha\beta$

$1 = \alpha^2+ \beta^2-26$

$\alpha^2+ \beta^2 = 27$

$\alpha^2+ \beta^2-\alpha\beta= 27+13 = 40$

Q.39.(B) $R = \frac{x^3+1}{x+2}, S = \frac{x^2-x+1}{x+1}$

$R \div S = \frac{x^3+1}{x+2} \div \frac{x^2-x+1}{x+1}$

$= \frac{x^3+1}{x+2} \times \frac{(x+1)}{x^2-x+1}$

$= \frac{(x+1)^2}{(x+2)}$

Q.40.(B) $\frac{a}{a^2-2a+1} = \frac{1}{3}$

On dividing by 'a' in the numerator and denominator, we get

$\frac{1}{a-2+\frac{1}{a}} = \frac{1}{3}$

$a + \frac{1}{a} - 2 = 3$

$a + \frac{1}{a} = 5$

Taking cube on both the sides, we get

$a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right)$

= 125 – 3 × 5 =125 – 15 = 110

Q.41.(A) $\frac{5p^2+5}{3p^2+9p+3} = \frac{5p+\frac{5}{p}}{3p+9+\frac{3}{p}}$

$= \frac{5\left(p+\frac{1}{p}\right)}{3\left(p+\frac{1}{p}\right)+9}$

$= \frac{5\times 2}{3\times 2+9} = \frac{2}{3}$

Q.42.(B) $m^2 - 2\sqrt{10}\,m + 1 = 0$

On dividing above equation by m, we get

$m - 2\sqrt{10} + \frac{1}{m} = 0$

$$m+\frac{1}{m}=2\sqrt{10}$$

We know that

$$\left(m+\frac{1}{m}\right)^2-\left(m-\frac{1}{m}\right)^2=4$$

$$\left(2\sqrt{10}\right)^2-\left(m-\frac{1}{m}\right)^2=4$$

$$40 - \left(m-\frac{1}{m}\right)^2=4$$

$$\left(m-\frac{1}{m}\right)^2=40-4=36$$

$$m-\frac{1}{m}=\sqrt{36}=6$$

$$m^2-\frac{1}{m^2}=6\times 2\sqrt{10}=12\sqrt{10}$$

Q.43.(A) $\dfrac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\dfrac{5}{2}$

$$\Rightarrow \frac{\left(\sqrt{x+4}+\sqrt{x-10}\right)+\left(\sqrt{x+4}-\sqrt{x-10}\right)}{\left(\sqrt{x+4}+\sqrt{x-10}\right)-\left(\sqrt{x+4}-\sqrt{x-10}\right)}=\frac{5+2}{5-2}$$

$$\frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{7}{3}$$

On squaring, $\Rightarrow \dfrac{x+4}{x-10}=\dfrac{49}{9}$

$9x + 36 = 49x - 490$

$49x - 9x = 36 + 490$

$40x = 526$

$$\therefore\ x=\frac{526}{40}=\frac{263}{20}$$

Q.44.(C) $x + y + z = 3qp - qr - rp + 3qr - pq - pr + 3pr - pq - qr$

$= 3qp + 3qr + 3pr - 2qr - 2rp - 2pq$

$= pq + qr + rp$

Now,

$$\frac{x+y+z}{pqr}=\frac{pq+qr+rp}{pqr}$$

$= r^{-1} + p^{-1} + q^{-1}$

Q.45.(B) $3p + 7 = 7p + 5$

$4p = 2$

$$p=\frac{1}{2}$$

And $7p + 5 = p^2 + N$

$$\frac{7}{2}+5=\left(\frac{1}{2}\right)^2+N$$

$$N=\frac{17}{2}-\frac{1}{4}$$

$$N=\frac{33}{4}=8\frac{1}{4}$$

Q.46.(C) $xy\,(x + y) = 1 \Rightarrow \dfrac{1}{xy}=x+y$

ℜℜℜ $\dfrac{1}{x^3y^3}$ x^3 y^3 $(x\quad y)^3$ x^3 y^3

$= x^3 + y^3 + 3xy\,(x + y) - x^3 - y^3)$

$= 3 \times 1 = 3$

Q.47.(A) $xya = (3x + y)^2 - (3x - y)^2$

$xya = 9x^2 + y^2 + 6xy - 9x^2 - y^2 + 6xy$

$xya = 12\,xy,\ a = 12$

Notes

CHAPTER-5
PERCENTAGE

Scan the QR code to get video of this chapter.

Introduction: Percentage is a fraction whose denominator is always 100. x percentage is represented by x%.

I. To express x% as a fraction :

We know x% = $\frac{x}{100}$.

Thus 20% = $\frac{20}{100}$ ....(means 20 parts out of 100 parts)

= $\frac{1}{5}$ ....(means 1 part out of 5 parts)

and 20% = $\frac{20}{100} = \frac{1}{5}$

II. To express $\frac{x}{y}$ as a percentage:

We know that $\frac{x}{y} = \left(\frac{x}{y} \times 100\right)\%$

Ex. Change in %

Thus $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$

and 0.8 = $\frac{8}{10} = \left(\frac{8}{10} \times 100\right)\% = 80\%$

III. Remember it :

1 = 100%

$\frac{1}{2} = 50\%$ $\quad$ $\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{1}{4} = 25\%$ $\quad$ $\frac{1}{5} = 20\%$

$\frac{1}{6} = 16\frac{2}{3}\%$ $\quad$ $\frac{1}{7} = 14\frac{2}{7}\%$

$\frac{1}{8} = 12\frac{1}{2}\%$ $\quad$ $\frac{1}{9} = 11\frac{1}{9}\%$

$\frac{1}{10} = 10\%$ $\quad$ $\frac{1}{11} = 9\frac{1}{11}\%$

$\frac{1}{12} = 8\frac{1}{3}\%$ $\quad$ $\frac{1}{13} = 7\frac{9}{13}\%$

IV. Comparison between two values x and y.

(i) If x is compared to y then we always assume that y is always equal to 100%

(ii) When any question asks y is what percent of x then x is always written in the denominator

Ex. If x is 80% of y, what percent of x is y?

(A) 75% **(B)** 80%
(C) 100% **(D)** 125%

Sol.(D) **Ist Method :** $y = \frac{100 \times 100}{80} \times x$

$\therefore$ y = 125% of x

IInd Method : Let y is 100 then x = 80

Required % = $\frac{100}{80} \times 100$ = 125%

Ex. The Income of Ram and Shyam are Rs. 4000 and Rs. 6000 respectively then Shyam's income is how much% more than Ram ?

(A) 25% **(B)** 50%
(C) 66.66% **(D)** 75%

Sol.(B) More% = $\frac{6000 - 4000}{4000} \times 100$ = 50%

V. If A is R% more than B, then B is less than A by

$\left[\frac{R}{(100+R)} \times 100\right]\%$

If A is R% less than B, then B is more than A by

$\left[\frac{R}{(100-R)} \times 100\right]\%$

Ex. If the income of A is 40% less than that of B, How much percent B's income is more than that of A's?

(A) 60% **(B)** 40%
(C) 50% **(D)** 66.66%

Sol.(D) More% = $\frac{40}{60} \times 100$ = 66.66%

VI. If the price of a commodity increases by R%, then % reduction in the consumption so as not to increase the expenditure is-

$$\left[\frac{R}{(100+R)}\times 100\right]\%$$

If the price of a commodity decreases by R%, then the % increase in consumption so as not to decrease the expenditure is-

$$\left[\frac{R}{(100-R)}\times 100\right]\%$$

Ex. If the price of petrol be raised by 20%, then the percentage by which a car owner must reduce its consumption so as not to increase the expenditure on petrol is:

(A) $16\frac{1}{3}\%$ **(B)** $16\frac{2}{3}\%$

(C) $15\frac{2}{3}\%$ **(D)** $15\frac{1}{3}\%$

Sol.(B) Reduction on consumption = $\frac{20}{120}\times 100$

$= 16\frac{2}{3}\%$

VII. **Result on population : (a)** Let the population of a town be P and it increases at the rate of R% per annum, then :

1. Population after n years = $P\left(1+\frac{R}{100}\right)^n$

2. Population n years ago = $\frac{P}{\left(1+\frac{R}{100}\right)^n}$

Result on Population : (b) Let the population of a town be P now and it decreases at the rate of R% per annum, then:

1. Population after n years = $P\left(1-\frac{R}{100}\right)^n$

2. Population n years ago = $\frac{P}{\left(1-\frac{R}{100}\right)^n}$

Note : These formulae are also used for the depreciation value of machine.

VIII. Net % change = + x + y + $\frac{xy}{100}$

'+' means increment

In case, when x is negative and y is negative then

Net % change = – x – y + $\frac{xy}{100}$.

Observation :

I. If 20% candidates failed in an exam then observations are

→ 80% represent passed in exam

→100% represent total appeared in exam

→ (80% - 20%) = 60% represent difference between passed and failed candidates in exam.

Difference 60% < 20% (Failed), 80% (Passed) > 100% Total

II. If a number is increased by 25% then observations are

→ 100% represent the old number.

→ 125% represent the new number.

i.e., $\boxed{100\%} \xrightarrow{25\%\uparrow} \boxed{125\%}$

EXAMPLES

Ex-1. If the difference between 62% of a number and $\frac{3}{5}$th of that number is 36. What is the number?

(A) 1800 **(B)** 1600

(C) 2000 **(D)** 1500

Sol.(A) Let the number be x.

Then x × 62% - $x\times\frac{3}{5} = 36$

x × 62% - x × 60% = 36........(60% = 3/5)

x × 2% = 36

$x\times\frac{2}{100} = 36$

x = $\frac{36\times 100}{2} = 1800$

QUICK TRICK :

$\therefore \frac{3}{5} = 60\%$

∴ 62% - 60% = 36

2% = 36

100% = 1800

Ex-2. A man spends 40% on food, 20% on house rent, 12% on travel and 10% on education. After all these expenditures he saved Rs. 7200. Find the amount spent on travel ?

(A) Rs. 9000 **(B)** Rs.1080

(C) Rs. 4800 **(D)** Rs. 900

Sol.(C) **Ist Method :**

Let the total income be x. Then

Total expenditure

= x × (40% + 20% + 12% + 10%)

= x × 82%

Total savings = x - x × 82%

= x × 18%

Then x × 18% = 7200

x = $\frac{7200}{18} \times 100 = 40{,}000$

Expenditure on travel = 12%

x × 12% = $40000 \times \frac{12}{100}$ = Rs. 4800

IInd Method :

Total income = 100% → represent total

Total Exp → Food → 40%, House Rent → 20%, Travel → 12%, Education → 10% $\}$ 82%

Savings =100% - 82% = 18%

Expenditure on Travel = $\frac{7200}{18} \times 12$ = Rs. 4800

Ex-3. If the length of a rectangle is increased by 20% and breadth is decreased by 10%. Find the percentage change in the area of that rectangle ?

(A) 10% increase **(B)** 10% decrease

(C) 8% increase **(D)** 8% decrease

Sol.(C) We know that

Net % change = x + y + $\frac{x \times y}{100}$

Let x = 20% ....(we put x = +20)

and y = 10%............ (we put y = -10)

Now, net % change = +20 -10 + $\frac{(+20) \times (-10)}{100}$

= +10-2 = +8

Increase % = 8%

Ex-4. What should be added to 15% of 160 so that the sum may be equal to 25% of 240?

(A) 24 **(B)** 84

(C) 60 **(D)** 36

Sol.(D) 160×15% + ? = 240 × 25%

? = 60 - 24 = 36

Ex-5. In an exam, one candidate got 23% and failed by 36 marks while another candidate got 35% and got 48 marks more than pass marks then maximum marks are ?

(A) 500 **(B)** 700

(C) 800 **(D)** 480

Sol.(B) 35% - 23% = 48 + 36

12% = 84

Max. Marks = $\frac{84}{12} \times 100$

= 700

Ex-6. In a village 30% of the population is literate. If the total population of the village is 6600, then the number of illiterate persons is-

(A) 1980 **(B)** 4620

(C) 2200 **(D)** 3280

Sol.(B) Illiterate person = 6600 × 70%

= 4620

Ex-7. Out of his total income, Mr. Kapur spends 20% on house rent and 70% of the rest on house hold expense. If he saves Rs. 1800, what is his total income?

(A) Rs. 7800 **(B)** Rs. 7000

(C) Rs. 8000 **(D)** Rs. 7500

Sol.(D) Total percentage expenditure

$= \left(20 + \frac{80 \times 70}{100}\right)\% = 76\%$

$x \times \frac{24}{100} = 1800$

$x = \frac{1800}{24} \times 100$

= Rs. 7500

Ex-8. Two numbers are less than a third number by 30% and 37% respectively. The percentage by which second number is less than the first is:-

(A) 10% **(B)** 7%

(D) 4% **(D)** 3%

Sol.(A) Third number = 100

First number = 70

Second number = 63

$\therefore$ Required percentage

$= \frac{7}{70} \times 100 = 10\%$

Ex-9. If the price of sugar is reduced by 20% it enables the man to buy 4 kg more sugar for Rs. 250. Find the reduced rate of sugar per kg?

(A) Rs. 10 **(B)** Rs. 8.50

(C) Rs. 11.50 **(D)** Rs. 12.50

Sol. (D) Reduced Rate = $\frac{250 \times 20}{100 \times 4}$ = Rs.12.50

Ex-10. A man spends 75% of his income. If the income increased by 20% and expenditure also increased by 10% then by how much percent his saving increase?

(A) 40% **(B)** 30%

(C) 50% **(D)** 25%

Sol.(C) Let man's income = Rs. 100

Expenditure = Rs. 75

Saving = Rs. 25

New income = Rs. 120

New expenditure = 75× 110 = 82.5

Saving = 120 - 82.5 = 37.5

Increase saving% = $\frac{12.5}{25} \times 100$ = 50%

Ex-11. A number is first decreased by 20%. Then decreased number is increased by 20% the resulting number is less than the original by 20.Then the original number is-

(A) 200 **(B)** 400

(C) 500 **(D)** 600

Sol.(C) Effective percentage

$= (-20+20 - \frac{20 \times 20}{100}) = -4\%$

If the number be x, then

4% of x = 20

$x = \frac{20 \times 100}{4} = 500$

Ex-12. The value of Machine depreciates every year by 10%. If its present value is Rs. 50000, then the value of the Machine after 2 years

(A) Rs. 40,050 **(B)** Rs. 45,000

(C) Rs. 40,005 **(D)** Rs. 40,500

Sol.(D) $50000\left(1 - \frac{10}{100}\right)^2$

$= 50000 \times \frac{9 \times 9}{100} = 40500$

Ex-13. Fresh fruit contains 68% water and dry fruit contains 20% water. How much dry fruit can be obtained from 100 kg fresh fruit?

(A) 32 kg **(B)** 40 kg

(C) 52 kg **(D)** 80 kg

Sol.(B) 100 kg fresh fruit containts 68 kg. water, 32 kg. fruit

Fruit 32 kg. = 80% of dry fruit

Dry fruit = $\frac{32}{80} \times 100$ = 40 kg.

Ex-14. In 5 litre mixture of water and sugar, there is 20% sugar. 2 litre mixture was taken out of it and replaced by 2 litre water then in new mixture what is the percentage of sugar?

(A) 12.5% **(B)** 11.5%

(C) 12% **(D)** 13%

Sol.(C) Remaining mixture = 5 - 2 = 3 litre

Now remaining sugar in mixture

= 3 × 20% = 0.6

Quantity of resulting mixture after adding 2 litre water = 5

% of sugar in resulting mixture

$= \frac{0.6}{5} \times 100 = 12\%$

Ex-15. In 6 litre solution of sugar and water 4% is sugar and remaining is water. One litre water evaporates then in remaining solution the percentage of sugar is

(A) 4.2% **(B)** 4.9%

(C) 4.8% **(D)** 5%

Sol.(C) Quantity of sugar = $\frac{6 \times 4}{100}$ = 0.24 litre

Remaining solution = 6-1 = 5 litre

% of sugar in remaining solution

$= \frac{0.24}{5} \times 100 = 4.8\%$

Ex-16. A seller got 5% commision on selling for Rs. 10000 and above he got 4% commission . One day he deposited Rs. 31100 in company after deduction his commission then on that day what was the sale amount ?

(A) 32500 **(B)** 33500
(C) 32000 **(D)** 3300

Sol.(A) Total sale = 10000 + 100x

After deducting commision

9500 + 96x = 31100

100x = 22500

Total = 35500

Ex-17. In production cost of a motor, three types of expenses include cost of raw material, expenses on labours and extra expenses. In a year the respective ratio of expenses on these are 4 : 3 : 2. In the next year cost of raw material increased by 10%, labour charge increased by 8% and extra expenses reduced by 5% then by how much percent will cost of motor be increased?

(A) 4% **(B)** 6%
(C) 7% **(D)** 8%

Sol.(B) Let total expenditure be 90

According to question

Material = 40

Labour = 30

Extra = 20

Net increase or decrease

= 40 × 10% + 30 × 8% - 20 × 5% = 5.4

% increase = $\frac{5.4}{90} \times 100$ =6%

Ex-18. Total salary of A and B are Rs. 1500. A spends 90% while B spends 80% of their salary. If ratio of their savings is 3 : 4. Then what are the salaries of A and B?

(A) Rs. 900, Rs. 600
(B) Rs.600, Rs. 900
(C) Rs.800, Rs. 600
(D) Rs. 700, Rs. 900

Sol.(A) Let A's salary = 100 A

B's salary = 100 B

Savings, $\frac{10A}{20B} = \frac{3}{4}$

$\frac{A}{B} = \frac{3}{2}$

A's income is = $\frac{3}{5} \times 1500 = \text{Rs.}900$

B's income is = $\frac{2}{5} \times 1500 = \text{Rs.}600$

Ex-19. In an examination max marks were 500. A got 10% less than B and B got 25% more than C and C got 20% less than D. If A got 360 marks then how much percent D got in this examination?

(A) 90% **(B)** 80%
(C) 50% **(D)** 60%

Sol.(B) A = 360

and B = $\frac{360 \times 100}{90} = 400$

C = $\frac{400}{125} \times 100 = 320$

D = $\frac{320}{80} \times 100 = 400$

So, marks of D in %=$\frac{400}{500} \times 100 = 80\%$

Ex-20. In an exam paper there were five questions, 5% students solved all questions 5% students did not solve any question and 25% of remaining solved a single question, 18% solved four questions. If $24\frac{1}{2}\%$ students solved 2 questions and 200 students solved three questions then find total number of students?

(A) 800 **(B)** 600
(C) 900 **(D)** 500

Sol.(A) Let total students = 100

Number of students who attempted all the questions =5

Number of students who attempted no question = 5

Number of students who attempted only one question = 90×25% = 22.5

Number of students who attempted four questions = 18

Number of students who attempted two questions = 24.5

Number of students who attempted three questions =100- (5+5+22.5+18+24.5)= 25

25 = 200

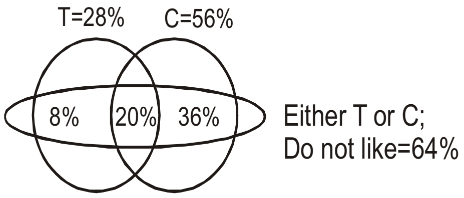
∴ 100 = 200×4 = 800

Ex.21. In a factory, 72% people like tea, while 44% like coffee. If 20% like neither tea nor coffee, find the no. of people who like either coffee and tea but not both, if 108 people like both coffee and tea?

(A) 108 **(B)** 124

(C) 132 **(D)** 144

Sol.(C) According to question,



Hence % of people like Both = 100% - 64%= 36%

Hence Required Number $= \frac{108}{36} \times (8+36) = 132$

Ex-22. In a competitive exam 15% of the applicants were found ineligible and 62% of the eligible applicants were male. If the number of eligible female applicants was 11628. How many applicants were there (both eligible and ineligible)?

(A) 35000 **(B)** 36000

(C) 38000 **(D)** 40000

Sol.(B) Let total candidates = 100

Ineligible candidates = 15

Eligible candidates = 100 - 15 = 85

Eligible male candidates $= \frac{85 \times 62}{100}$

Eligible female candidates $= \frac{85 \times 38}{100} = 11628$

Then total candidates $= \frac{11628}{85 \times \frac{38}{100}} \times 100$

= 36000

Ex-23. If the income tax is increased by 19%, the net income is reduced by 1%, find the rate of income tax.

(A) 4% **(B)** 5%

(C) 6% **(D)** 7.2%

Sol.(B) Let the total income be Rs.100 and income tax rate be x%.

Total tax = 100 × x%

According to question

Net Income = (100 - x)

19% of x = 1% of (100 - x)

19 x = 100 - x

20 x = 100,

x = 5%

Ex-24. The ratio of number of boys and girls in a school is 3:2. If 20% of boys and 30% of girls are scholarship holders, then the percentage of students, who do not get scholarship?

(A) 50% **(B)** 80%

(C) 60% **(D)** 76%

Sol.(D) Let the total students be 100.

Boys $= 100 \times \frac{3}{5} = 60$

Girls $= 100 \times \frac{2}{5} = 40$

Scholarship holder students

= (60×20% + 40×30%) = 24

Number of students who did not get scholarship = 100 - 24 = 76

∴ 76%

Ex-25. The salary of Amit is $\frac{3}{4}$th of the salary of Bablu. While the salary of Shyam is $28\frac{4}{7}$% of the salary of Bablu. If the difference of the salary of Amit and Shyam is Rs. 19500, find $16\frac{2}{3}$% of the difference of the salary of Bablu and Shyam?

(A) 5000 **(B)** 3500

(C) 5500 **(D)** 4000

Sol.(A) Let Bablu's salary = 100

Amit's salary = 75

Shyam's salary $= \frac{200}{7}$

Ratio of their salary = 21 : 28 : 8

So, According to question,

13 = 19500

$20 = 19500 \times \frac{20}{13}$ = Rs.30,000

Required answer = $30000 \times 16\frac{2}{3}\%$ = Rs.5,000

EXERCISE

Q.1. When numerator of a fraction is increased by 40% and denominator of a fraction is increased by 60%, resultant fraction becomes $\frac{5}{8}$. Find the original (old) fraction.

(A) $\frac{7}{2}$ **(B)** $\frac{2}{7}$

(C) $\frac{5}{7}$ **(D)** $\frac{7}{8}$

Q.2. A man spends 60% of his salary. If his salary is increased by 30% then his expanditure also increased by 20%. Find the increase / decrease% of his savings.

(A) 45% **(B)** 50%

(C) 60% **(D)** 75%

Q.3. If the product of 15% and 10% of a number is 250. Find the sum of 15% and 10% of that number.

(A) 41.60 **(B)** 41.65

(C) 41.66 **(D)** Can't be determined

Q.4. 6% of income of Ram is equal to 18% of income of Shyam. 10% of income of Shyam is equal to 30% of income of Mohan. Find the total income of Ram, Shyam and Mohan?

(A) 2000 **(B)** 4000

(C) 6000 **(D)** Can't be determined

Q.5. If the price of petrol is increased by 30%, by how much percent a car owner must reduce his consumption in order to maintain the same budget ?

(A) 21% **(B)** $21\frac{1}{3}$%

(C) $23\frac{1}{13}$% **(D)** 33%

Q.6. A's height is 40% less than that of B, how much percent B's height is more than that of A ?

(A) $33\frac{1}{3}$% **(B)** 40%

(C) 60% **(D)** $66\frac{2}{3}$%

Q.7. Two numbers are more than a third number by 12 $\frac{1}{2}$% and 25% respectively. The first number as percentage of the second number is :

(A) 50 % **(B)** 60%

(C) 75% **(D)** 90%

Q.8. Due to reduction by 10% in the price of sugar, a person is able to buy 2 kg more for Rs. 540. Find the original rate Rs./kg of sugar ?

(A) Rs. 48 per kg **(B)** Rs. 30 per kg

(C) Rs. 24 per kg **(D)** Rs. 20 per kg

Q.9. In a factory 60% of the workers are above 30 years and of these 75% are males and the rest are females. If there are 1350 male workers above 30 years, the total number of workers is the factory is

(A) 3000 **(B)** 2000

(C) 1800 **(D)** 1500

Q.10. If an electricity bill is paid before due date, one gets a reduction of 4% on the amount of the bill. By paying the bill before due date a person got a reduction of Rs. 13. The amount of his electricity bill was

(A) Rs. 125 **(B)** Rs. 225

(C) Rs. 325 **(D)** Rs. 425

Q.11. Amit scored 189 marks in Science, 156 marks in Hindi and 72 marks in Mathematics. The maximum marks of Science are 210, Hindi is 180 and Math-metics is 110. What percent of marks did Amit score overall?

(A) 82.8 **(B)** 84.4

(C) 83.4 **(D)** 84.2

Q.12. If the length of a rectangular field is increased by 20% and the breadth is reduced by 20%, the area of rectangle will be 192 m^2. What is the area of the original rectangle?

(A) 184 m^2. **(B)** 196 m^2.

(C) 200 m^2. **(D)** 225 m^2.

Q.13. In an examination, a student who gets 20% of the maximum marks fails by 5 marks. Another student who scores 30% of the maximum marks gets 20 marks more than the pass marks. The necessary percentage required for passing is

(A) 32% **(B)** 23%

(C) 22% **(D)** 20%

Q.14. The sum of 55% of a number and 40% of the same number is 180.5. What is 90% of that number?

(A) 152 **(B)** 190

(C) 171 **(D)** 166

Q.15. In an election survey, 30% people promised to vote candidate A and remaining promised to vote for candidate B. If on the day of election x% of people who promised to vote for A, voted for B and 40% of people who promised to vote for B voted against him and in the end B lost by 10 votes. What is value of x, if total 250 votes were?

(A) 20 **(B)** 30

(C) 50 **(D)** 70

Q.16. Mr. Ram Niwas Singh spent 20% of his monthly income on food and 15% on children's education. 40% of the remaining, he spent on entertainment and transport together and 30% on the medical. He is left with an amount of Rs.8775 after all these expenditures. What is Mr. Ram Niwas Singh's monthly income?

(A) Rs.40000 **(B)** Rs.35000

(C) Rs.42000 **(D)** Rs.45000

Q.17. In an examination A got 25% more marks than B. B got 10% less marks than C and C got 25% more marks than D. Then if D has got 64% marks out of total marks (i.e. 500), then what is the marks obtained by A.

(A) 405 **(B)** 450

(C) 360 **(D)** 400

Q.18. A gave 30% of his money to B. B gave $\frac{2}{3}$rd of what he received to C. C gave $\frac{5}{8}$ of the money she received from B to D. D is now having Rs.1025. How much money did A have initially?

(A) Rs.6200 **(B)** Rs.8000

(C) Rs.6000 **(D)** Rs.8200

Q.19. In an examination, every candidate must take either Mathematics or Science and may take both if he so chooses, 83.6% of the total number of candidates took Mathematics, 31.2% took Science. If 370 candidates took both, find the number of candidates.

(A) 3200 **(B)** 2800

(C) 3000 **(D)** 2500

Q.20. In an examination 70% students passed in Physics, 80% students passed in Chemistry, 75% students passed in Maths and 85% students passed in Biology, and x% failed in all the four subjects. What is the least value of x?

(A) 10 **(B)** 12

(C) 15 **(D)** 18

Q.21. In an election between two candidates, 75% of the voters cast their votes, out of which 2% votes were declared invalid. A candidate got 12348 votes which were 75% of the total valid votes. The total number of voters enrolled in that election was

(A)18800 **(B)** 22000

(C) 22800 **(D)** 22400

Q.22. In an examination in which maximum marks are 500. A got 10% less than B, B got 25% more than C and C got 20% less than D. If A got 360 marks what percentage of marks was obtained by D?

(A) 75% **(B)** 80%

(C) 78% **(D)** 82%

Q.23. Out of a total population of 1500 people in a village, men increased by 10% and women by 15%. Now the total population becomes 1690 in a year. The men population in the village was originally.

(A) 700 **(B)** 800

(C) 1200 **(D)** 900

Q.24. Raman's expenditure and savings are in the ratio 6:1. If his salary is increased by 25% and saving by 15%, the percentage increase in his expenditure.

(A) $26\frac{2}{3}\%$ **(B)** $28\frac{3}{4}\%$

(C) 27% **(D)** $29\frac{1}{2}\%$

Notes

EXPLANATION

Q.1.(C) $\frac{140}{160} = \frac{5}{8}$

$(?) = \frac{5}{8} \times \frac{160}{140} = \frac{5}{7}$

Q.2.(A)

salary	expenditure	saving
100	60	40
130 (+30%)	72 (+20%of 60) =12	58 (+18)

% increase in savings = $\frac{18}{40} \times 100$
= 45%

Q.3.(C) 15% ×10% = 250
150% = 250
25% = $\frac{250}{150} \times 25$
= 41.66

Q.4.(D) 6% of Ram = 18% of income of Shyam
Ram = 3 × Shyam's income - (I)
10% of Shyam = 30% of Mohan
Shyam = 3 × Mohan -(II)
From I and II there is no result.
Hence CND

Q.5.(C) $\frac{30}{130} \times 100 = \frac{300}{13}$

= $23\frac{1}{13}$ % less

Q.6.(D) Ratio of height of A and B is -
A : B = 60 : 100
Then B's height is more than that of A

$= \frac{40}{60} \times 100 = 66\frac{2}{3}\%$

Q.7.(D) I : II : III = $112\frac{1}{2}$: 125 : 100

= 9 : 10 : 8

∴ Required % = $\frac{9}{10} \times 100$
= 90%

Q.8.(B) 10% of 540 (due to reduction)
Rs. 54 = 2 kg
Rs. 27 = 1 kg new rates (reduced rate/kg)
Original Price = $\frac{27 \times 100}{90}$ = Rs 30

Q.9.(A) Let the total number of workers in factory be x.

$\therefore\ x \times \frac{60}{100} \times \frac{75}{100} = 1350$

$x = \frac{1350 \times 100 \times 100}{60 \times 75} = 3000$

Q.10.(C) Let the amount of the bill be Rs. x.

$\therefore \frac{4x}{100} = 13$

$\Rightarrow x = \frac{1300}{4} = \text{Rs. } 325$

Q.11.(C) Total percentage of marks

$= \frac{(189 + 156 + 72)}{(210 + 180 + 110)} \times 100 = \frac{417}{500} \times 100$

= 83.4%

Q.12.(C) Change in area = $\left[20 - 20 - \frac{20 \times 20}{100}\right]\%$
= – 4%
∴ 4% area is decreased.
Let x be the original area.
$\frac{96x}{100} = 192 \Rightarrow x = 200\ m^2$.

Q.13.(C) According to the question,
20% + 5 = 30% – 20
10% = 20 + 5 = 25
100% = 250
Passing marks
= 75 – 20 = 55

Required percentage = $\frac{55}{250} \times 100$
= 22%

Q.14.(C) (55% + 40%) = 180.5
95% = 180.5

90% = $\frac{180.5}{95} \times 90$ = 171

Q.15.(A) A = 75 – 20% of (75) + 40% of (175)
= 130
B = 175 + 20% of (75) – 40% of (175)
= 120

Q.16.(D) Let his monthly income be Rs.10000
Amount spent on Food = Rs.2000
Amount spent on Children's education
= Rs.1500
Amount spent on Entertainment and transport
= Rs.2600
Amount spent on Medical = Rs.1950
Remaining = 10000 – (2000 + 1500 + 2600 + 1950)
1950 ratio = 8775
1 ratio = 4.5
100 ratio = 45000

Q.17.(B) D got marks = $\frac{64}{100} \times 500$ = 320

C's marks = $\frac{320}{100}\times125$ = 400

B's marks = $\frac{400}{100}\times90$ = 360

A's marks = $\frac{360}{100}\times125$ = 450

Q.18.(D) A = $1025\times\frac{8}{5}\times\frac{3}{2}\times\frac{100}{30}$ = 8200

Q.19.(D) Let total number of candidates be x
Then, 83.6% of x + 31.2% of x – 370 = x
114.8% of x – 370 = x
14.8% of x = 370
x = 2500

Q.20.(C) Failed in Physics/= 100 - 70 = 30%
Failed in Chemistry = 100 - 80 = 20%
Failed in Mathematics 100 - 75 = 25%
Failed in Biology= 100 - 85 = 15%
The lowest failed % of students is in Biology = 15%
The least value of x is 15%

Q.21.(D) Let number of candidates = 100
Number of voters cast their vote = 75
Invalid votes = 75 – 2% of 75 = 73.5
According to the question,
55.125 ratio = 12348
1 ratio = 224
100 ratio = 22400

Q.22.(B) Let marks obtained by D = x
C's marks = 0.80x
B's marks = 1.25 (0.80x)
A's marks = (0.90) (1.25) (0.80x)
= 0.9x
But 0.9x = 360 ⇒ x = 400
% of marks obtained by D

= $\frac{400}{500}\times100$ = 80%

Q.23.(A) Let no. of women be x
then men = 1500 – x

$$\frac{(1500-x)110}{100}+\frac{x\times115}{100}=1690$$

165000 – 110x + 115x = 169000
5x = 4000
x = 800
No. of men = 1500 – 800 = 700

Q.24.(A) Let Raman's salary be Rs.100
Then increased salary = Rs.125
According to question,
6x + x = 100

x = Rs. $\frac{100}{7}$

Increase in savings = $\frac{100}{7}\times\frac{15}{100}$

= Rs. $\frac{15}{7}$

Savings = $\frac{100}{7}+\frac{15}{7}$ = Rs. $\frac{115}{7}$

New expenditure = $125-\frac{115}{7}$ = Rs. $\frac{760}{7}$

% increase = $\frac{\frac{760}{7}-\frac{600}{7}}{\frac{600}{7}}\times100$

= $\frac{160}{600}\times100$ = $26\frac{2}{3}$ %

Notes

CHAPTER-6
PROFIT AND LOSS

Scan the QR code to get video of this chapter.

I. **Cost price :** The price at which an article is purchased is called its Cost Price and denoted by CP

If an article is purchased for some amount and there is some additional expense on transportation, labour cost commission or repair charges etc, then it would be added in cost price.

II. **Selling Price :** The price at which an article is sold, is called its selling price, denoted by SP

III. **Profit or Gain :** If S.P. is greater than CP, then seller is said to have a profit or gain.

Profit or Gain = SP-CP

Profit % = $\frac{\text{Profit}}{\text{CP}} \times 100$

IV. **Loss :** If SP is less than CP then seller is said to have a loss.

Loss = CP-SP

Loss % = $\frac{\text{Loss}}{\text{CP}} \times 100$

V. Profit and loss are always counted on CP

VI. CP is always 100% in the case of profit and loss.

Observation:

I. In the case of profit or loss, the relation is generated between CP and SP

If 20% profit accrued on an article, it means

→100% represent cost price (CP)

→120% represent selling price (S.P.)

→20% represent profit

i.e. Profit 20% → 100% (CP)
→ 120% (SP)

If 10% loss is accrued on an article, it means :

→ 100% → represent cost price (CP)

→ 90% → represent selling price (S.P.)

→ 10% → represent loss i.e.,

i.e. Loss 10% → 100% (CP)
→ 90% (SP)

Note :

(i) When in any question, cost price, profit %, or loss % is given

$$SP = CP \times \frac{(100 \pm \text{Profit\% or Loss\%})}{100}$$

For profit we take '+'

For loss we take '-'

(ii) When in any question, SP, profit% or loss % is given then cost price

$$CP = SP \times \frac{100}{(100 \pm \text{Profit\% or Loss\%})}$$

For profit we take '+'

For loss we take '-'

Ex. On selling an article for Rs. 651 there is a loss of 7%, the cost price of that article is ?

(A) Rs. 744 **(B)** Rs. 751

(C) Rs. 793 **(D)** Rs. 700

Sol.(D) CP = $\frac{651}{93} \times 100 = \text{Rs.}700$

Ex. A man bought an old type writer for Rs. 1200 and spent Rs. 200 on its repair. He sold it for 1680. His profit percentage is-

(A) 20% **(B)** 10%

(C) 8% **(D)** 16%

Sol.(A) Total cost = 1200+200 = 1400

Profit = 1680-1400 = 280

Profit % = $\frac{280}{1400} \times 100$ =20%

VII. **Marked Price :** MRP of an article is known as Marked Price or labelled price or listed price and denoted by MP.

Discount :

The reduction made on the marked price of an article is called discount.

Discount = MP-SP

Discount percent $= \frac{\text{Discount}}{\text{MP}} \times 100$

When marked price and discount percentage is given then

SP = $\text{M.P.} \times \frac{(100 - \text{Discount\%})}{100}$

Successive Discount :

If two or more discount allowed one after the another then such discount are known as successive discount.

If two discount allowed then

Net Percent Discount = $x + y - \frac{xy}{100}$

IMPORTANT POINT

Relation between CP and MP

$$\text{MP} = \frac{\text{CP} \times (100 \pm \text{Profit\% / Loss\%})}{(100 - \text{Discount\%})}$$

For profit we take '+'

For Loss we take '-'

Ex. A fan is listed at Rs. 150 and a discount of 20% is given. Then the selling price is-

(A) Rs. 180 **(B)** Rs. 150

(C) Rs. 120 **(D)** Rs. 110

Sol.(C) SP = $150 \times \frac{(100 - 20)}{100}$ = Rs. 120

Ex. While selling to the retailer, a company allows 30% discount on the marked price of its products. If the retailer sells those product at marked price his profit % will be :

(A) 30% **(B)** $42\frac{1}{7}\%$

(C) 40% **(D)** $42\frac{6}{7}\%$

Sol.(D) If the MP of the product be Rs. 100 then

CP to the retailer = Rs. 70

SP of the retailer = Rs. 100

$\therefore$ Gain percent = $\frac{30}{70} \times 100 = 42\frac{6}{7}\%$

Ex. Find the selling price of an article a shopkeeper allows two successive discount of 5% each on the marked price of Rs. 80.

(A) Rs. 70.20 **(B)** Rs. 70.10

(C) Rs. 72.00 **(D)** Rs. 72.20

Sol.(D) Net discount = 5+5 $- \frac{5 \times 5}{100} = 9.75\%$

SP = $80 \times \frac{(100 - 9.75)}{100} = 72.20$

II Method

20×20 = 400

19×19 = 361

SP = $\frac{80}{400} \times 361$ =Rs. 72.20

EXAMPLE

Ex.1 A salesman expects a gain of 13% on his cost price. If in a month his sale was Rs. 7,91000, what was his profit ?

(A) Rs. 85659 **(B)** Rs. 88300

(C) Rs. 91000 **(D)** Rs. 97786

Sol.(C) CP $= \frac{791000}{(100 + 13)} \times 100 = 700000$

Profit =791000-700000 = Rs. 91000

QUICK TRICK :

Profit $= \frac{791000}{113} \times 13$ = 91000

Ex.2. If there is a profit of 20% on the cost price of an article, the percentage of profit calculated on its selling price will be-

(A) 24% **(B)** $16\frac{2}{3}\%$

(C) $8\frac{1}{3}\%$ **(D)** 20%

Sol.(B) If the cost price Rs. 100, then

Selling price = Rs. 120 and gain= Rs. 20

Required percent = $\frac{20}{120} \times 100 = 16\frac{2}{3}\%$

Ex.3. The cost price of 40 articles is equal to the the selling price of 25 articles. Find the gain percent.

(A) 65% **(B)** 60%

(C) 15% **(D)** 75%

Sol.(B) Let CP of 40 articles is x then

SP of 25 articles is x

CP of 1 article $= \frac{x}{40}$

SP of 1 article $= \frac{x}{25}$

Profit $= \frac{x}{25} - \frac{x}{40}$

$= \frac{8x - 5x}{200} = \frac{3x}{200}$

Profit % $= \frac{\frac{3x}{200}}{\frac{x}{40}} \times 100 = 60\%$

QUICK TRICK :

Gain Percent = $\frac{40 - 25}{25} \times 100 = 60\%$

Ex.4. A man sold 20 apples for Rs. 100 and gained 20%. How many apples did he buy for Rs. 100 ?

(A) 20 **(B)** 22

(C) 24 **(D)** 25

Sol.(C) SP of 20 Apples = Rs. 100

SP of 1 Apple = Rs. 5

CP of 1 Apple = $\frac{5}{120} \times 100$

$= \text{Rs.}\frac{25}{6}$

So, he purchase for Rs. 100

$= \frac{100}{25} \times 6 = 24$

QUICK TRICK :

When cost price and selling price of certain articles are same then

Purchased article

$= \frac{\text{No. of articles sold}}{100} \times (100 + \text{Profit\%})$

$= \frac{20}{100} \times 120 = 24$

Ex.5. Two toys are sold Rs. 504 each. One toy brings the dealer a gain of 12% and the other at a loss of 4%. The gain or loss percent by selling both articles.

Sol.(A) $CP_1 = \frac{504}{112} \times 100 = \text{Rs.}450$

$CP_2 = \frac{504}{96} \times 100 = \text{Rs.}525$

Cost price of both toys = 450+525

= Rs. 975

Selling price of both toys = 2×504

= Rs.1008

Profit = 1008-975 = Rs. 33

Profit % $= \frac{33}{975} \times 100 = 3\frac{5}{13}\%$

QUICK TRICK :

When two articles are sold at same price. First at P % profit and another at L % loss

then net profit or loss percent

$$\frac{100(P - L) - 2 \times P \times L}{(100 + P) + (100 - L)}$$

If we get resultant positive then profit.

If we get resultant negative then loss

$$\frac{100(12 - 4) - 2 \times 12 \times 4}{(100 + 12) + (100 - 4)}$$

$= \frac{800 - 96}{112 + 96} = 3\frac{5}{13}\%$

Or

$\frac{2ab}{a + b}$ where a and b are the selling price (in percentage)

$\frac{2 \times 112 \times 96}{112 + 96} = 103\frac{5}{13}$

So, profit is= $3\frac{5}{13}\%$

Ex-6. A dishonest shopkeeper sells goods at its cost price but uses a weight of 960 gm for one kg weight. Find his gain percent.

(A) $5\frac{1}{6}\%$ **(B)** $7\frac{5}{6}\%$

(C) $4\frac{1}{6}\%$ **(D)** $8\frac{5}{6}\%$

Sol.(C) The CP of shopkeeper = 960 gm

The SP of shopkeeper

= 1000 gm (1 kg = 1000 gm)

The profit of shopkeeper

= 1000–960 = 40 gm

% profit of shopkeeper

$= \frac{\text{Profit of shopkeeper}}{\text{CP of shopkeeper}} \times 100$

$\therefore$ % P = $\frac{40}{960} \times 100 = 4\frac{1}{6}\%$

Ex.7. A person sold an article at a profit of 13% if he had sold it for Rs. 25 more, he would have gained 18%. What is the cost price?

(A) Rs. 100 **(B)** Rs. 500

(C) Rs. 208 **(D)** Rs. 1000

Sol.(B) Let the CP of an article be x Rs. then

113% of x + 25 = 118% of x

⇒ 118% of x - 113% of x = 25

⇒ 5% of x = 25

$x = \frac{25}{5} \times 100$ = Rs. 500

QUICK TRICK :

$CP = \frac{25}{(18-13)} \times 100$ = Rs. 500

Ex-8. A person bought articles at 7 for Rs. 10. How many for Rs. 10 must to sell to gain 40% ?

(A) 6 **(B)** 5

(C) 4 **(D)** 3

Sol.(B) CP of 7 articles = Rs. 10

SP of 7 articles = 140% of Rs. 10

= Rs. 14

For Rs. 14 articles sold = 7

For Rs. 10 article sold = $\frac{7}{14} \times 10 = 5$

QUICK TRICK :

40% gain, mean 100% / 140%

∴ Article sold in Rs.10= $7 \times \frac{100}{140} = 5$

Ex-9. A vendor bought bananas at 2 for a rupee and sold them 5 for a rupee. Then find his gain or loss percent ?

(A) 20% profit **(B)** 20% loss

(C) 30% profit **(D)** 60% loss

Sol.(D) Suppose, number of bananas bought = LCM of 2 and 5 = 10

CP = Rs. $\left(\frac{1}{2} \times 10\right)$ = Rs. 5

SP= Rs. $\left(\frac{1}{5} \times 10\right)$ = Rs. 2

Since CP > SP

Hence,

% Loss = $\frac{5-2}{5} \times 100 = 60\%$

QUICK TRICK :

2 Bananas 1 Rs.

5 Bananas 1 Rs.

$L\% = \frac{5-2}{5} \times 100$

$= 60\%$

Ex.10. An article was sold at profit of 12%. If the cost price would be 10% less and selling price would be Rs. 5.75 more, there would be profit of 30% then at what price it should be sold to make profit of 20% ?

(A) Rs. 115 **(B)** Rs. 120

(C) Rs. 138 **(D)** Rs. 215

Sol.(C) CP of article = x

$SP = \frac{112x}{100}$

New CP $= \frac{9x}{10}$

$SP = \frac{9x}{10} \times \frac{130}{100} = \frac{117}{100}x$

$\frac{117x}{100} - \frac{112x}{100} = 5.75$

Required SP = $\frac{115 \times 120}{100}$

= Rs. 138

IInd Method : By difference method

$\left(\frac{90 \times 130}{100}\right) = 117\%$

112% ~117% = 5.75

5% = 5.75

120% = Rs. 138

Ex.11. A man bought two goats for Rs. 1008, sold one at a loss of 20% and other at a profit of 44%. If each goat was sold at same price, then cost price of the goat which was sold at loss-

(A) Rs. 648 **(B)** Rs. 568

(C) Rs. 360 **(D)** Rs. 440

Sol.(A) If x and y be cost price of two goats, then

80% of x = 144% of y

x : y = 9 : 5

Cost of first goat $= \frac{9}{14} \times 1008$

= Rs. 648

Ex.12. Dinesh purchased a land for Rs. 48000 and sold $\frac{2}{5}$ part at a loss of 6%. On how much % of profit should he sold the remaining part so that he made 10% profit on whole ?

(A) $15\frac{2}{3}\%$ **(B)** $18\frac{2}{3}\%$

(C) $16\frac{2}{3}\%$ **(D)** $20\frac{2}{3}\%$

Sol.(D) Total S.P. $= \frac{48000 \times 110}{100}$

= Rs. 52800

S.P. of $\frac{2}{5}$ Part

$= \frac{2}{5} \times 48000 \times \frac{94}{100} = 18048$

CP of $\frac{3}{5}$ part $= \frac{3}{5} \times 48000$

= Rs. 28800

SP of $\frac{3}{5}$ part = 52800-18048

= Rs. 34752

Proft = 34752-28800= Rs. 5952

Profit % $= \left(\frac{5952}{28800} \times 100\right) = 20\frac{2}{3}\%$

QUICK TRICK :

$-\frac{2}{5} \times 6 + \frac{3}{5} \times x = 10 \times 1$

-12+3x = 50

3x = 62

x = $20\frac{2}{3}\%$

Ex.13. The difference between the discount on Rs. 500 at the rate of 40% to the two consecutive discount of 36% and 4%-

(A) 0 **(B)** Rs. 2

(C) Rs. 7.20 **(D)** Rs. 1.93

Sol.(C) Difference

$= 500\left[40 - \left\{(36+4) - \frac{36 \times 4}{100}\right\}\right]\%$=Rs. 7.20.

Ex.14. After allowing a discount of 10% on any article a shopkeeper earns 10% profit, then mark price is how much % more than cost price ?

(A) $22\frac{2}{9}\%$ **(B)** $11\frac{1}{9}\%$

(C) $9\frac{1}{11}\%$ **(D)** 20%

Sol.(A) Let CP = x

SP = $\frac{x \times 110}{100}$

MP = $\frac{x}{90} \times 110 = \frac{11x}{9}$

Required % $= \frac{\frac{2x}{9}}{x} \times 100 = 22\frac{2}{9}\%$

QUICK TRICK :

MP is more than CP $= \frac{P\% + D\%}{100 - D\%} \times 100$

$= \frac{10+10}{100-10} \times 100 = 22\frac{2}{9}\%$

Ex.15. A shopkeeper make mark price of an article 20% more than cost price after allowing discount he earns 10% profit, then discount percent is-

(A) $16\frac{2}{3}\%$ **(B)** 15%

(C) $8\frac{1}{3}\%$ **(D)** 12%

Sol.(C) Let CP = x

MRP $= x \times \frac{120}{100} = \frac{6x}{5}$

$SP = \frac{x \times 110}{100} = \frac{11x}{10}$

Discount = $\frac{6x}{5} - \frac{11x}{10} = \frac{5x}{50} = \frac{x}{10}$

Discount % = $\frac{\frac{x}{10}}{\frac{6x}{5}} \times 100 = 8\frac{1}{3}\%$

QUICK TRICK :

Discount % $= \frac{(20-10)}{(100+20)} \times 100 = 8\frac{1}{3}\%$

Ex.16. A bakery bakes cake with the expectation that it will earn a profit of 40% by selling each cake at marked price. But during the delivery to showroom 16% of the cakes were completely damaged and hence could not be sold. 24% of the cakes were slightly damaged and hence could be sold at 80% of the cost price. The remaining 60% of the cakes were sold at marked price. What is the percentage profit in the whole consignment?

(A) 2.5% **(B)** 3.2%
(C) 8.2% **(D)** 8.2%

Sol.(B) Let the CP = 100

SP of 24% of cake = 80% of 24

=19.2

S.P. of 60% of cake = 140% of 60 = Rs 84

Total SP. = 19.2 + 84

=103.2

Profit% = 3.2%

Ex.17. A dealer sold an article at 30% profit. If he increases the CP of article by Rs 10 and decreases the SP by Rs 11, the percentage profit decreases to 15%. Find the actual cost price of article.

(A) Rs. 140 **(B)** Rs. 130
(C) Rs. 120 **(D)** Rs. 150

Sol.(D) Let CP of article = x

SP of article = x × $\frac{130}{100} = 1.3$ x

According to question

(x+10) × $\frac{115}{100}$ = (1.3x - 11)

1.15x + 11.5 = (1.3x - 11)

= 0.15x = 22.5

x = $\frac{2250}{15}$ = Rs. 150

Ex.18. By selling an article at 80% of its marked price, a merchant makes a profit of 12%. What will be the % profit made by the merchant if he sells the article at 95% of its marked price?

(A) 30% **(B)** 32%
(C) 33% **(D)** 35%

Sol.(C) Let CP of Article =x

According to question

SP of article = x×112% = 1.12x

MRP of article = $\frac{1.12x}{80} \times 100 = 1.4x$

New SP = 1.4x × 95%

= 1.33x

Profit = 1.33x - x = 0.33x

Profit % = $\frac{0.33x}{x} \times 100$

= 33%

Ex.19. A man losses Rs. 20 by selling some toys at the rate of Rs. 3 per piece and gains Rs. 30, if he sells them at Rs. 3.25 per piece. The no. of pieces sold by him are?

(A) 100 **(B)** 300
(C) 400 **(D)** 200

Sol.(D) Let no. of purchased Toys = T

According to question,

T × 3 + 20 = CP of toys .... (i)

T × 3.25 - 30 = CP of toys ..... (ii)

From eq.(i) and (ii)

T ×3 +20 = T × 3.25 - 30

T × 0.25 = 50

T = 200

Ex.20. Cost price of 12 apples is equal to the selling price of 9 apples and the discount on 10 apples is equal to the profit on 5 apples. What is the difference between the profit percent and discount percent ?

(A) 22% **(B)** 30%
(C) 22.22% **(D)** 22.5%

Sol.(C) CP : SP = 3 : 4

P% = 1/3 × 100 = 33.33%

Profit is double that of discount

Discount : Profit = 1 : 2

Since,

CP : SP : MP = 3 : 4 : 4.5

D% = 1/9 × 100 = 11.11%

difference = 33.33 - 11.11 = 22.22%

EXERCISE

Q.1. When a plot is sold for Rs. 18,700, the owner loses 15%. At what price it must be sold in order to gain 15% ?

(A) Rs. 21,000 **(B)** Rs. 22,500

(C) Rs. 25,300 **(D)** Rs.25,800

Q.2. If selling price is doubled, the profit triples. Find the profit percent :

(A) $66\frac{2}{3}\%$ **(B)** 100%

(C) $105\frac{1}{3}\%$ **(D)** 120%

Q.3. The C.P. of an article is 60% of the S.P. The percent that the S.P. is of C.P is :

(A) 250% **(B)** 240%

(C) $166\frac{2}{3}\%$ **(D)** 40%

Q.4. If books bought at prices ranging from Rs. 200 to Rs. 325 are sold at prices ranging from Rs. 300 to Rs. 450, what is the greatest possible profit that might be made in selling eight books?

(A) Rs. 400

(B) Rs. 600

(C) Cannot be determined

(D) Rs. 2000

Q.5. A shopkeeper purchased 70 kg of potatoes for Rs. 420 and sold the whole at the rate of Rs. 6.50 per kg. What will be his gain percent ?

(A) $4\frac{1}{6}\%$ **(B)** $6\frac{1}{4}\%$

(C) $8\frac{1}{3}\%$ **(D)** 20%

Q.6. A sells an article which costs him Rs. 500 to B at a profit of 20%. B then sells it to C, making a profit of 10% on the price he paid to A. How much does C pay B ?

(A) Rs. 472 **(B)** Rs. 476

(C) Rs. 528 **(D)** None of these

Q.7. The percentage profit earned by selling an article for Rs. 1920 is equal to the percentage loss incurred by selling the same article for Rs. 1280. At what price should the article be sold to make 25% profit ?

(A) Rs. 2000 **(B)** Rs. 2200

(C) Rs. 2400 **(D)** Data inadequate

Q.8. On an order of 5 dozen boxes of a consumer product, a retailer receives an extra dozen free. This is equivalent to allowing him a discount of :

(A) 15% **(B)** $16\frac{1}{6}\%$

(C) $16\frac{2}{3}\%$ **(D)** 20%

Q.9. A man sold 18 coat for Rs. 16,800, gaining thereby the cost price of 3 coats. The cost price of a coat is :

(A) Rs. 650 **(B)** Rs. 700

(C) Rs. 750 **(D)** Rs. 800

Q.10. A man buys 2 dozen bananas at Rs. 16 per dozen. After selling 18 bananas at the rate of Rs. 12 per dozen, the shopkeeper reduced the rate at Rs. 4/dozen The percent loss is :

(A) 25.2% **(B)** 32.4%

(C) 36.5% **(D)** 37.5%

Q.11. Oranges are bought at the rate of 10 for Rs. 25 and sold them at the rate of 9 for Rs. 25. The profit is :

(A) $9\frac{1}{11}\%$ **(B)** 10%

(C) $11\frac{1}{9}\%$ **(D)** $12\frac{1}{2}\%$

Q.12. In a certain store, the profit is 320% of the cost price. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit?

(A) 30% **(B)** 70%

(C) 100% **(D)** 250%

Q.13. The cost price of 20 articles is same as the selling price of x articles. If the profit is 25%, then the value of x is :

(A) 15 **(B)** 16

(C) 18 **(D)** 25

Q.14. The listed price of a fan is Rs.150, a customer purchase of Rs. 108 after two successive discount. If discount is 20% then find second discount?

(A) 8% **(B)** 12%

(C) 15% **(D)** 10%

Q.15. A shopkeeper allows 4% discount on his marked price. If the cost price of an article is Rs. 100 and he has to make a profit of 20%, then his marked price must be

(A) Rs. 96 **(B)** Rs. 120

(C) Rs. 125 **(D)** Rs. 130

Q.16. A bought a car for Rs. 80,000 and sold it to B at a loss of 3%. If B sold it to C for Rs. 85360, profit percent for B is

(A) 8% **(B)** 10%

(C) 6% **(D)** 9%

Q.17. The cost price of an article is Rs.42000. The shopkeeper offers a discount of 20% on it and loses at a rate half of the rate of discount rate. The marked price of the article is

(A) Rs.42000 **(B)** Rs.47250

(C) Rs.37800 **(D)** Rs.40500

Q.18. The profit obtained on selling an article at Rs.3240 is equal to $\frac{1}{6}$of the loss incurred on selling the same article at Rs.1469. What is the selling price of the article when it is sold for 45% profit?

(A) Rs.4333.15 **(B)** Rs.4331.25

(C) Rs.4221.15 **(D)** Rs.4331.15

Q.19. An article whose marked price is Rs.900 is sold at a profit of 62% when there is a discount offered of 20%, 10% and 5% by a shopkeeper. Find the selling price when the article is sold at a profit of 75%.

(A) Rs.651.6 **(B)** Rs.655

(C) Rs.651 **(D)** Rs.665

Q.20. The cost price of two articles are equal. One article is sold at 16% profit and the other article is sold at 8% loss. What is the overall percentage profit?

(A) 4% **(B)** 10%

(C) 12% **(D)** 8%

Q.21. A profit of 20% is made on goods when a discount of 10% is given on the marked price. What profit percent will be made when a discount of 20% is given on the marked price?

(1) $6\frac{2}{3}\%$ (2) $7\frac{2}{3}\%$

(3) $3\frac{1}{4}\%$ (4) $7\frac{4}{5}\%$

Q.22. The market price of a T.V. is Rs 4800. The shopkeeper allows a discount of 10% and gains 8%. If no discount is allowed, his gain percent would be-

(A) 18% **(B)** 18.5%

(C) 20% **(D)** 25%

Q.23. A shopkeeper allows two successive discounts on an article whose marked price is Rs. 150 and selling price is Rs. 105. What is first discount if second discount is 12.5%?

(A) 25% **(B)** 17.5%

(C) 20% **(D)** 16.67%

Q.24. A trader has 50 articles, a part of which he sells at 14% profit and the rest at 6% loss. On the whole his loss is 4%. Find the ratio between quantities sold at 14% profit and that at 6% loss?

(A) 4: 1 **(B)** 1: 9

(C) 3: 7 **(D)** 2: 3

Q.25. The difference (in rupees) in discounts between two successive discounts of 8% each and a single discount of 16% on Rs.2400 is-

(A) Rs.13.56 **(B)** Rs.15.36

(C) Rs.17.52 **(D)** Rs.15.2

Q.26. An article is marked at Rs.24000. A trader bought it at successive discounts of 25% and 10% respectively. He spent Rs.1800 on its transportation to his shop and then sold the article for Rs.20000. What is the trader's profit in the whole transaction?

(A) $16\frac{2}{3}\%$ **(B)** 14%

(C) 10% **(D)** $11\frac{1}{9}\%$

Q.27. While selling a watch, a shopkeeper gives a discount of 5%. If he gives a discount of 7%, he earns Rs.15 less as profit, the marked price of the watch is -

(A) Rs. 697.50 **(B)** Rs. 712.50

(C) Rs. 787.50 **(D)** Rs. 750

Q.28. Dipika found that she had made a loss of 10% while selling her smartphone. She also found that had she sold it for Rs.50 more, she would have made a profit of 5%. The initial loss was what percentage of the profit earned.

(A) 75% **(B)** 85%

(C) 100% **(D)** 200%

Q.29. The cost price of item B is Rs.660/- more than the cost price of item A. Item B was sold at a profit of 20% and item A was sold at a loss of 30%. If the respective ratio of selling prices of items A and B is 35: 93, what is the cost price of item B?

(A) Rs.1860 **(B)** Rs.1520

(C) Rs.1430 **(D)** Rs.1200

Q.30. A book seller sells a book at a profit of 10%. If he had bought it at 4% less and sold it for Rs. 6 more, he would have gained $18\frac{3}{4}$%. The cost price of the book is-

(A) Rs.130 **(B)** Rs.140

(C) Rs.150 **(D)** Rs.160

Q.31. A dealer sold a radio at a loss of 2.5%. Had he sold it for Rs. 120 more, he would have gained 7.5%. In order to gain 12.5% after a 25% discount, the marked price should be:

(A) 1750 **(B)** 1800

(C) 1857.75 **(D)** 1925

Q.32. The cost price of article B is 20% more than that of article A. Articles A and B were marked up by 50% and 25% respectively. Article A was sold at a discount of 4% and article B was sold at a discount of 0.5%. If the selling price of article A was Rs.28.35 more than the selling price of article B, what was the cost price of article A?

(A) Rs.600 **(B)** Rs.400

(C) Rs.500 **(D)** Rs.540

Q.33. Vansh sold two articles together for Rs.152000 thereby making a profit of 25% on the first article and 10% on the second article. By selling them together for Rs.153500 he would have made a profit of 10% on the first article and 25% on the second article. Find the cost price of the first article.

(A) Rs. 60,000 **(B)** Rs. 70,000

(C) Rs. 65,000 **(D)** Rs. 75,000

Q.34. A shopkeeper allows a discount of 5% on an article. If he allows a discount of 7%, he earns Rs.45 less in the profit. What is the selling price when there is a profit of 33(1/3) % made on marked price.

(A) Rs. 750 **(B)** Rs. 1025

(C) Rs. 1000 **(D)** Rs. 1250

Q.35. A shopkeeper buys 40 bicycles and marks them at 25% above the cost price. He allows a discount of 10% on the marked price for cash sales, and 5% for credit sales. If one – fourth of the stock is sold for credit and the rest for cash, and if the total profit be Rs.37125, what is the cost price of a bicycle?

(A) Rs. 7000 **(B)** Rs. 6500

(C) Rs. 6200 **(D)** Rs. 6600

Q.36. Kritika bought two article for Rs.54000. He sold first at a gain of 20% and the second at a loss of 10%, thereby gaining 2% on the whole. Find the cost of the first article.

(A) Rs.18000 **(B)** Rs.21600

(C) Rs.27000 **(D)** Rs.30600

Q.37. A shopkeeper bought an article marked at Rs. 22800 at two successive discounts of 15% and 20%. He spent Rs.1064 on transportation and sold the article for Rs. 20710. Then, what was the profit percentage of the shopkeeper?

(A) 25% **(B)** 30%

(C) 35% **(D)** 20%

Q.38. What is the ratio between 60% discount on the price of a certain commodity and 20%, 25% and 15% of three successive discounts on that commodity?

(A) 60: 59 **(B)** 60: 49

(C) 60: 51 **(D)** 49: 59

Q.39. The cost price of an article is Rs.360. If it is increased by 25% and then again by $33\frac{1}{3}$% and then sold at a price of Rs.810. What is the profit percentage on increased price?

(A) 36 **(B)** 35

(C) 40 **(D)** 25

Q.40. A person bought two articles for Rs.3200 and sold the first at a profit of 10% and the second at 20% profit. If he sold the first at 20% profit and the second at 10% profit, he would get Rs.5 more. The ratio of the cost price of the two articles was

(A) 63: 57 **(B)** 65: 63

(C) 53: 57 **(D)** 75: 63

EXPLANATION

Q.1.(C) $SP = \frac{18700}{85} \times 115 = 25300$

Hence, S.P. = Rs. 25,300

Q.2.(B) Let C.P. be Rs. x and S.P. be Rs. y

Then, $3(y - x) = (2y - x) \Rightarrow y = 2x$.

Profit% = $\left(\frac{x}{x} \times 100\right)$%= 100%

Q.3.(C) C.P. = $\frac{60}{100} \times$ S.P. $\Rightarrow$ S.P. = $\frac{5}{3}$ C.P.

$= \left(\frac{5}{3} \times 100\right)$% of C.P. = $166\frac{2}{3}$% of C.P.

Q.4.(D) Least C.P. = Rs. (200×8)

= Rs. 1600. Greatest

S.P. = (450×8) = Rs. 3600)

Required profit = Rs. (3600-1600)

= Rs. 2000

Q.5.(C) C.P. of 1 kg = Rs. $\left(\frac{420}{70}\right)$ = Rs. 6

S.P. of 1 kg = Rs. 6.50

$\therefore$ Gain% = $\left(\frac{0.50}{6} \times 100\right)\% = \frac{25}{3} = 8\frac{1}{3}\%$

Q.6.(D) C.P. for B = 120% of Rs. 500

= Rs. $\left(\frac{120}{100} \times 500\right)$ = Rs. 600

C.P. for C = 110% of Rs. 600

= Rs. $\left(\frac{110}{100} \times 600\right)$ = Rs. 660

Q.7.(A) Let C.P. be Rs. x.

Then, $\frac{1920 - x}{x} \times 100 = \frac{x - 1280}{x} \times 100$

$\Rightarrow$ 1920 - x = x - 1280

$\Rightarrow$ 2x = 3200 $\Rightarrow$ x = 1600

$\therefore$ Required S.P. = 125% of Rs. 1600

$= \left(\frac{125}{100} \times 1600\right)$ = Rs. 2000

Q.8.(C) Clearly, the retailer get 1 dozen out of 6 dozens free.

$\therefore$ Equivalent discount = $\left(\frac{1}{6} \times 100\right)$%

$= 16\frac{2}{3}$%

Q.9.(D) (S.P. of 18 coats) - (C.P. of 18 coats)

= (C.P. of 3 coats)

$\Rightarrow$ C.P. of 21 coats =S.P. of 18 coats

= Rs.16800

$\Rightarrow$ C.P. of 1 coat = Rs. $\left(\frac{16800}{21}\right)$ = Rs.800

Q.10.(D) C.P. = Rs. (16 × 2) = S.P.

= Rs. (12×1.5+4×0.5) = Rs. (18 + 2)

= Rs. 20

Loss% = $\left(\frac{12}{32} \times 100\right)$% = 37.5%

Q.11.(C) Suppose, number of oranges bought

= L.C.M. of 9 and and 10 = 90.

C.P. of 90 oranges = Rs. $\left(\frac{25}{10} \times 90\right)$

= Rs.225

S.P. of 90 oranges = Rs. $\left(\frac{25}{9} \times 90\right)$

= Rs. 250

Profit% = $\left(\frac{25}{225} \times 100\right)\% = \frac{100}{9}\%$

$= 11\frac{1}{9}$%

Q.12.(B) Let C.P. = Rs. 100. Then, Profit = Rs. 320,

S.P. = Rs. 420.

New C.P. = 125% of Rs. 100 = Rs. 125,

New S.P. = Rs. 420.

Profit = Rs. (420-125) = Rs. 295

Required percentage = $\left(\frac{295}{420} \times 100\right)$%

$= \frac{1475}{21}\% \approx 70\%$

Q.13.(B) 20 C.P. = x S.P.

C.P. : S.P. = x : 20

$25 = \frac{20 - x}{x} \times 100$

x = 80 – 4x

5x = 80

x = 16

Q.14.(D) MP = Rs. 150

150 ×80% = 120 Rs. ⇒ 120 - 108 = 12

∴ IInd discount = $\frac{12\times100}{120}=10\%$

Q.15.(C) Let M.P. = Rs. x

∴ 96% of x = 120

$x=\frac{120}{96}\times100$

x = Rs. 125

Q.16.(B) S.P. of A = 80,000 – $\frac{80000\times3}{100}$

= Rs.77600

C.P. of B = 77600 Rs.

Profit % for B = $\frac{85360-77600}{77600}\times100$

= $\frac{7760}{77600}\times100=10\%$

Q.17.(B) Marked price of the article

= $42000\times\frac{100}{80}\times\frac{90}{100}$ = Rs.47250

Q.18.(D) Let cost price of the article

According to the question,

(3240 – x) = $\frac{1}{6}$(x – 1469)

19440 – 6x = x – 1469

7x = 20909

x = 2987

Required selling price = Rs.4331.15

Q.19.(D) Selling price when there is a discount of 20%, 10% and 5%

= 900 × $\frac{80}{100}\times\frac{90}{100}\times\frac{95}{100}$ = Rs. 615.6

Cost price = 615.6 ×$\frac{100}{162}$ = Rs.380

Required selling price = 380 ×$\frac{175}{100}$

= Rs. 665

Q.20.(A) Let cost price of each article be Rs.100

S.P.1 = Rs. 116

S.P.2 = Rs. 92

Total S.P. = Rs. 116 + Rs. 92

= Rs. 208

Total C.P. = Rs. 100 + Rs. 100

= Rs. 200

Overall profit = $\frac{208-200}{200}\times100=4\%$

Q.21.(A) Let M.P. = 100

S.P. = 90

C.P. = $90\times\frac{100}{120}=75$

New S.P. = 80

New P% = $\frac{5}{75}\times100=6\frac{2}{3}\%$

Q.22.(C) S.P. of T.V. = $\frac{4800\times90}{100}=4320$

$8=\frac{4320-CP}{CP}\times100$

8 CP = 432000 – 100CP

CP = Rs.4000

According to the question

Gain % = $\frac{4800-4000}{4000}\times100=20\%$

Q.23.(C) Let first discount be x%

105 = 150 × $\left(\frac{100-12.5}{100}\right)\left(\frac{100-x}{100}\right)$

105000 = 15 × 87.5 × (100 – x)

80 = 100 – x

x = 100 – 80 = 20%

Q.24.(B) By using method of alligation

14		–6
	–4	
2		18

Required ratio = 1: 9

Q.25.(B) Two successive discounts 8% each

$= \left(8+8-\frac{8\times8}{100}\right)$% single discount

= 15.36% single discount

$\therefore$ Required difference

$= \frac{2400\times15.36}{100} \sim \frac{2400\times16}{100}$

$= \frac{36864}{100} \sim \frac{38400}{100}$

$= \frac{1536}{100}$ = Rs.15.36

Q.26.(C) Price of the article after discount

$= 24000 - 24000 \times \left(25+10-\frac{25\times10}{100}\right)\%$

= 24000 – 24000 × 32.5% = 24000 – 7800

= 16200

Total cost price = 16200 + 1800 = 18000

Profit %= $\frac{20000-18000}{20000}\times100\,\%$

= 10%

Q.27.(D) Let the marked price be Rs. x

95% of x – 93% of x =15

$\Rightarrow$ 2% of x = 15

$\Rightarrow x = \frac{15\times100}{2}$ = Rs. 750

Q.28.(D) Required % = $\frac{10}{5}\times100$ = 200%

Q.29.(A) Let the CP of item A be x

CP of item B is x + 1200.

$$\frac{\left(\frac{70}{100}\times x\right)}{\left\{\frac{120}{100}\times(x+660)\right\}} = \frac{35}{93}$$

x = 1200

C.P. of item B is = 1200 + 660 = Rs.1860.

Q.30.(C) Let the C.P. of the book = Rs.x

Initial S.P. = Rs.$\frac{110x}{100}$ = 1.1 x

New C.P. = Rs.0.96

New S.P.= Rs.$\left(100+\frac{75}{4}\right)$% of Rs.0.96x

$= \frac{475}{400}\times0.96x$ = Rs.1.14x

1.14 x – 1.1 x = 6

0.04x = 6

$x = \frac{6}{0.04}$ = Rs.150

Q.31.(B) 120 = 10% of the CP (2.5% loss to 7.5% profit)

or CP = Rs. 1200

To gain 12.5%, SP = 1200 × 1.125

= Rs. 1350

Rs. 1350 = 25% discounted of Marked Price

or marked Price = 1350 ÷ 0.75

= Rs. 1800

Q.32.(D)

Article – A	Article – B
C.P. – 100	120
M.P. – 150	150
S.P. – 144	149.25

According to the question,

(149.25 – 144) ratio = 28.35

5.25 ratio = 28.35

100 ratio = 540

Q.33.(A) Let the C.P. of Ist and second article be x and y respectively

1.25x + 1.10y = 152000 ------ (1)

1.10x + 1.25y = 153500 ------ (2)

On solving (1) and (2), we get

x = 60000, y = 70000

Q.34.(C) If the marked price be Rs.x, then

(7 - 5)% of x = 15

2% of x = 15

x = 750

Profit when 33(1/3) % profit is made on marked price = Rs.250

Required selling price = Rs.750 + 250

= Rs.1000

Q.35.(D) Let C.P. of one bicycle be x

S.P. for cash sale = $1.25x \times \frac{90}{100} = 1.125x$

S.P. for credit sale

$= 1.25x \times \frac{95}{100} = 1.1875x$

Number of bicycles sold for credit

= 40/4 = 10

Number of bicycles sold for cash = 30

Total S.P.

=[1.125x × 30 + 1.1875x×10] = 45.625x

Profit = (45.625 – 40) x = 5.625x

But, actual profit = Rs.37125

5.625x = 37125

x = 6600

Q.36.(B) Let the cost of first and second article be Rs.x and Rs.y respectively.

x + y = 54000 … (I)

120x + 90y = 5508000

4x + 3y = 183600 … (II)

On solving (I) and (II), we get

y = 32400, x = 21600

Q.37.(A) Marked price of the article = Rs. 22800

Cost price of the article

$= \left(22800 - 22800 \times \frac{15}{100}\right) \times \frac{80}{100} = \text{Rs.}15504$

Actual cost price = 15504 + 1064

= Rs.16568

Profit % = $\frac{(20710 - 16568)}{16568} \times 100 = 25\%$

Q.38.(B) Let the price of the commodity be Rs. 100.

Total discount at 60% = Rs. 60

For 3 successive discounts,

First discount = $\frac{100 \times 20}{100}$ = Rs. 20

Second discount = $\frac{(100 - 20) \times 25}{100}$

= Rs. 20

Third discount = $\frac{(100 - 20 - 20) \times 15}{100}$

= Rs. 9

So, Required ratio = $\frac{60}{20 + 20 + 9}$

= 60: 49

Q.39.(B) Profit % = $\frac{810 - 360 \times \frac{125}{100} \times \frac{4}{3}}{360 \times \frac{125}{100} \times \frac{4}{3}} \times 100$

= 35%

Q.40.(B) Let the cost price of both the articles be Rs. x and Rs. y respectively.

According to the question,

x + y = 3200 … (I)

$\left(\frac{120}{100}x + \frac{110}{100}y\right) - \left(\frac{110}{100}x + \frac{120}{100}y\right) = 5$

x – y = 50 … (II)

On solving (I) and (II), we get

x = 1625, y = 1575

Required ratio = 1625: 1575

= 65: 63

Notes

CHAPTER-7
SIMPLE INTEREST AND COMPOUND INTEREST

Scan the QR code to get video of this chapter.

Principal: When you first deposit money in a saving account or When you borrow some money from another person, bank, or any financial institute that amount is known as principal.

Interest: The extra amount paid by the borrower to the lender for the use of amount lent is called Interest.

Simple Interest: When we borrow some amount from another person for a certain period after some time we have to pay some extra money to him this extra money is called the Interest. This interest is always same for same time period at the same rate of interest. So it is known as simple interest denoted as S.I.

FORMULA :

$$S.I = \frac{P \times R \times T}{100}$$

Where P $\rightarrow$ Principal

R $\rightarrow$ Rate percent per annum

T $\rightarrow$ Number of years

When S.I. is added to principal. It changes to amount.

P + S.I. = A

S.I. = A - P

Case I : If S.I, R and T are known,

$$P = \frac{S.I \times 100}{R \times T}$$

Case II : If S.I., P and T, are known,

$$R = \frac{S.I \times 100}{P \times T}$$

Case III : When S.I. P, R are known

$$T = \frac{S.I \times 100}{P \times R}$$

EXAMPLES

Ex.1. Find S.I on Rs. 5000 at the rate of interest 5% p.a. for 5 years.

(A) Rs. 1000 **(B)** Rs. 1500
(C) Rs. 1250 **(D)** Rs. 2700

Sol.(C) S.I. $= \frac{PRT}{100} = \frac{5000 \times 5 \times 5}{100} = Rs.1250$

QUICK TRICK :

$100 \xrightarrow{1year} 5$

$100 \xrightarrow{5year} 25$

$5000 \rightarrow 25 \times 50 = 1250$

Ex.2. On a certain sum of money at 6% p.a. simple interest is Rs. 324 for 3 years. Find sum of money

(A) Rs. 1200 **(B)** Rs. 1500
(C) Rs. 1800 **(D)** Rs. 2400

Sol.(C) $P = \frac{SI \times 100}{RT}$

$= \frac{324 \times 100}{6 \times 3}$ = Rs. 1800

QUICK TRICK :

$100 \xrightarrow{1year} 6$

$100 \xrightarrow{3year} 18$

$\times 18 \quad \times 18$

$1800 \longrightarrow 324$

Ex.3. At what rate of interest per annum, S.I. on Rs. 1000 will be Rs. 343 in 7 years?

(A) 15% **(B)** 4.9%
(C) 6% **(D)** 7%

Sol.(B) S.I. = PRT/100

$343 = \frac{1000 \times R \times 7}{100}$, R = 4.9%

QUICK TRICK :

$1000 \xrightarrow{7year} 343$

$1000 \xrightarrow{1year} 49$

$100 \rightarrow 4.9$

Ex.4. A certain sum of money becomes 5/2 times of itself in 5 years. Find the rate of interest ?

(A) 30% **(B)** 25% **(C)** 20% **(D)** 50%

Sol.(A) $(n-1)=\frac{rt}{100} \Rightarrow \left(\frac{5}{2}-1\right)=\frac{r\times 5}{100}$

$\frac{3}{2}=\frac{r}{20}$

$r = 30\%$

Ex.5. Ram lent a sum of money to Rahul at 5% p.a. if at the end of time duration Rahul paid 6/5 of money to Ram. Find the time duration?

(A) 5 years **(B)** 3 years

(C) 6 years **(D)** 4 years

Sol.(D) $(n-1)=\frac{rt}{100} \Rightarrow \left(\frac{6}{5}-1\right)=\frac{5\times t}{100}$

$\frac{1}{5}=\frac{t}{20}$, t = 4 years

Ex.6. A sum of money is lent at simple interest. After 5 years its S.I. becomes 2/5 of pricipal. Find the rate of interest?

(A) 8% **(B)** 7%

(C) 6% **(D)** 5%

Sol.(A) $n=\frac{rt}{100} \Rightarrow \frac{2}{5}=\frac{r\times 5}{100}$, r = 8%

Ex.7. A sum of money amounts to Rs. 6400 after 3 years and Rs. 7200 after 5 years at same rate of interest. What is rate p.c.p.a.?

(A) 7% **(B)** $6\frac{1}{3}\%$

(C) $7\frac{9}{13}\%$ **(D)** $8\frac{9}{13}\%$

Sol.(C) SI for 1 year $=\frac{7200-6400}{5-3}$ = Rs. 400

SI = 1200 (for 3 years)

P = 6400 - 1200

P = Rs. 5200

R = $\frac{S.I.\times 100}{PT}$, R = $\frac{400\times 100}{5200}=\frac{100}{13}$

R = $7\frac{9}{13}\%$

QUICK TRICK :

A = P + I

Let 1 year S.I. = x

7200 = P + 5x

6400 = P + 3x

800 = 2x

400 =x

1200 = 3x

P = 6400 - 1200

P = 5200

R = $\frac{400\times 100}{5200}=7\frac{9}{13}\%$

Ex.8. The difference of simple interest on a certain sum of money for 4 years and 6 years is Rs. 1125, if the rate of interest is 5% p.a., find sum of money.

(A) Rs. 11250 **(B)** Rs. 12500

(C) Rs. 13500 **(D)** Rs. 14000

Sol.(A) $P=\frac{(SI_1-SI_2)\times 100}{R(T_1-T_2)}$

$P=\frac{1125\times 100}{5\times 2}$

P = Rs. 11250

QUICK TRICK :

T = 2 years R = 5%

SI = RT%

10% = Rs. 1125

100% = $\frac{1125}{10}\times 100$ = Rs. 11250

Ex.9. Ram lent some money to Mohan at 6% p.a. for 5 years and same money to Gaurav at the same rate of interest for 3 years. If Ram gains Rs. 1920 as total interest from Mohan and Gaurav.Find the money lent to each?

(A) Rs. 5000 **(B)** Rs. 6000

(C) Rs. 8000 **(D)** Rs. 4000

Sol.(D) $P=\frac{(SI_1+SI_2)\times 100}{R(T_1+T_2)}$

P = $\frac{1920\times 100}{6\times 8}$

P = Rs. 4000

QUICK TRICK:

T = 8 years, R = 6%

SI = RT%

48% = 1920

1% = 40

100% = Rs. 4000

Ex.10. Tushar lent Rs. 9000 in two parts to Danish and Deepak one at 5% p.a. and another at 8% p.a. respectively. After 2 years he gained a combined interest rate of 7% p.a. Find the money lent at 8% p.a.

(A) Rs. 7000 **(B)** Rs. 6000

(C) Rs. 8000 **(D)** Rs. 5000

Sol.(B) Let sum at 8% p.a. is x

$$\frac{x\times8\times2}{100}+\frac{(9000-x)5\times2}{100}=\frac{9000\times7\times2}{100}$$

6x = 36000, x = Rs. 6000

QUICK TRICK :

$$\frac{P_1}{P_2}=\frac{8-7}{7-5}=\frac{1}{2},\quad P_2=\frac{2}{3}\times9000=\text{Rs. }6000$$

Ex.11. A man borrowed Rs. 12000 from a bank at a condition that rate of interest for first 4 years will be 6%, for next 3 years will be 8% and for the period beyond 7 years will be 9%. If he paid total amount at the end of 10 years, then how much interest would be pay?

(A) Rs. 5000 **(B)** Rs. 9000

(C) Rs. 7000 **(D)** Rs. 6500

Sol.(B) $SI=\frac{P(r_1t_1+r_2t_2+r_3t_3)}{100}$

S.I. = $\frac{12000}{100}$ [4×6+3×8+3×9]

= 120 × 75 = Rs. 9000

Ex.12. A sum of money becomes 4 times in 30 years at a certain rate of simple interest. In how many years it will become 6 times of itself at the same rate of interest?

(A) 90 years **(B)** 50 years

(C) 60 years **(D)** 70 years

Sol.(B) $\frac{n_1-1}{T_1}=\frac{n_2-1}{T_2}$

$$\begin{pmatrix} n_1, n_2 = \text{No. of times} \\ T_1, T_2 = \text{Number of years} \end{pmatrix}$$

$$\frac{4-1}{30}=\frac{6-1}{t_2}$$

t_2 = 50 years

Ex.13. Gaurav lent some money at 6% p.a. if rate of interest is 8% p.a. then Gaurav will earn Rs. 420 extra per year. Find money lent by Gaurav?

(A) 21000 **(B)** 24000

(C) 27000 **(D)** 30000

Sol.(A) $P=\frac{420\times100}{2\times1}=Rs.21000$

INSTALLMENT : A sum of money due as one of several equal payments for something, spread over an agreed period of time.

Equal Installment = $\frac{100A}{100T+RT\frac{(T-1)}{2}}$

Where A is a amount T is a time and R is the rate of interest.

Ex.14. What equal installment of annual payment will discharge a debt which is due as Rs. 848 at the end of 4 years at 4% per annum simple interest?

(A) Rs. 100 **(B)** Rs. 200

(C) Rs. 300 **(D)** Rs. 400

Sol.(B) Installment

$$=\frac{100\times848}{100\times4+4\times4\left(\frac{4-1}{2}\right)}=\frac{100\times848}{424}=\text{Rs. }200$$

COMPOUND INTEREST

As we discussed the S.I. is same in all the years but in the case of compound Interest it is different in each and every year. e.g.

Suppose we take 100 Rs. at the rate of 10% compound interest. After one year we are not able to pay amount. So, here C.I. is imposed on amount at the rate of Ist year. ie. 10% is imposed on 110 and so on......

So. C.I. is equal to S.I. in first year

but after 1 year C.I. > S.I.

BASIC FORMULA :

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$C.I. = P\left[\left(1+\frac{r}{100}\right)^n-1\right]$$

A → Amount

P → Principal

r → Rate of Interest

n → Number of years

NOTE

Type	**Rate**	**Time**
Yearly	R	n
Half-yearly	R/2	2n
Quarterly	R/4	4n

EXAMPLES

Ex.1. Find C.I. on Rs. 16000 at the rate 5% p.c.p.a. for 2 years, compounded annually?

(A) Rs. 1764 **(B)** Rs. 1640

(C) Rs. 1840 **(D)** Rs. 1600

Sol.(B) By formula, C.I. = A - P

$$A = 16000 \times \left(1+\frac{5}{100}\right)^2 = 16000 \times \frac{441}{400}$$

A = 17640

CI= 17640 - 16000

C.I. = 1640

QUICK TRICK :

$$\begin{array}{lll} \text{1 year} & 800 & \\ \text{2 year} & 800+ & \xrightarrow{5\%} 40 \end{array}$$

CI = 1640

Ex.2. Find C.I. on Rs. 1200 at 10% p.a. for 3 years, compounded yearly

(A) Rs. 1597.2 **(B)** Rs. 397.2

(C) Rs. 597.2 **(D)** Rs. 672.2

Sol.(B) $A = 1200\left(1+\frac{10}{100}\right)^3 = 1200 \times \frac{1331}{1000}$

A = 1597.2

CI = 1597.2 – 1200, CI = 397.2

QUICK TRICK :

$$\begin{array}{lll} 1200 \xrightarrow{10\%} & 120 & \\ \xrightarrow{10\%} & 120 + 12 & \\ \xrightarrow{10\%} & 120 + 12 + 12 + 1.2 & \end{array}$$

CI = Rs. 397.2

Ex.3. Find C.I. on Rs. 16000 at 10% p.a. for 4 year, compounded annually?

(A) Rs. 7425.6 **(B)** Rs. 6425.2

(C) Rs. 5235.2 **(D)** Rs. 8395.2

Sol.(A) $A = 16000\left(1+\frac{10}{100}\right)^4 = 16000 \times \frac{14641}{10000}$

A = 23425.6

CI = 23425.6-16000

CI. = 7425.6

QUICK TRICK :

4 years, 4641

$$\frac{16000 \times 10}{100} = 1600 \times 4 = 6400$$

$$\frac{1600 \times 10}{100} = 160 \times 6 = 960$$

$$\frac{160 \times 10}{100} = 16 \times 4 = 64$$

$$\frac{16 \times 10}{100} = 1.6 \times 1 = 1.6$$

CI = 7425.6

Ex.4. Find C.I. on Rs. 2000 at 10% p.a. for $1\frac{1}{2}$ years, compounded half yearly?

(A) Rs. 320 **(B)** Rs. 315.25

(C) Rs. 415.25 **(D)** Rs. 300

Sol.(B) By formula

$$A = 2000\left(1+\frac{10}{200}\right)^3 = 2000 \times \frac{9261}{8000}$$

A = 2315.25

CI = 2315.25 - 2000 = Rs. 315.25

Ex.5. Find C.I. on Rs. 15625 at 16% p.a. for 9 months compounded quarterly?

(A) Rs. 1950 **(B)** Rs. 1951

(C) Rs. 1952 **(D)** Rs. 1953

Sol.(B) By formula, 9 months = 3/4 year

$$A = 15625\left(1+\frac{16}{400}\right)^{\frac{3}{4}\times 4}$$

$$A = 15625 \times \frac{17576}{15625}$$

A = Rs. 17576

CI = 17576 - 15625

CI = Rs. 1951

QUICK TRICK :

$\frac{R}{4} = 4\%$,4T=36 months = 3 years

3 years - 331

$$\frac{15625 \times 4}{100} = 625 \times 3 = 1875$$

$$\frac{625 \times 4}{100} = 25 \times 3 = 75$$

$\frac{25\times4}{100}=1\times1=1$, CI = Rs. 1951

Ex.6. Find the C.I. on Rs. 6000 at 6% p.a. for $1\frac{1}{2}$ years compounded annually?

(A) Rs. 495.60 **(B)** Rs. 550.80

(C) Rs. 500 **(D)** Rs. 610

Sol.(B) If $t=n\frac{x}{y}$

$$A = P\left(1+\frac{R}{100}\right)^n\left(1+\frac{R}{100}\times\frac{x}{y}\right)$$

$$A = 6000\left(1+\frac{6}{100}\right)^1\left(1+\frac{6}{100}\times\frac{1}{2}\right)$$

$$A = 6000\times\frac{106}{100}\times\frac{103}{100}$$

A = 6550.8

C.I. = Rs. 550.8

Ex.7. A sum of money become Rs. 5290 in 2 years and Rs. 6083 in 3 years at a certain rate of compound interest find rate of interest p.a.?

(A) 8% **(B)** 12%

(C) 15% **(D)** 17%

Sol.(C) $\frac{6083}{5290}=\left(1+\frac{R}{100}\right)^{3-2}$

$\frac{6083}{5290}=1+\frac{R}{100}$, $\frac{793}{5290}=\frac{R}{100}$

R $\approx$ 15%

Ex.8. A certain sum of money becomes 3 times in 10 years. In how many years it becomes 9 times at the same rate of compounded interest?

(A) 30 years **(B)** 40 years

(C) 20 years **(D)** 50 years

Sol.(C) Let Principal (P) and Rate of Interest (r)

According to the question,

$3P=P\left(1+\frac{r}{100}\right)^{10}$ ....(i)

$9P=P\left(1+\frac{r}{100}\right)^n$, $3^2=\left(1+\frac{r}{100}\right)^n$

From eq. (i)

$\left(1+\frac{r}{100}\right)^{2\times10}=\left(1+\frac{r}{100}\right)^n$, n = 20 years

QUICK TRICK :

3--------------→10

9=3²----------→10×2

Ex.9. A certain sum of money become 4 times in 10 years. In how many years it becomes 8 times at the same rate of compound interest?

(A) 15 years **(B)** 25 years

(C) 30 years **(D)** 20 years

Sol.(A) Let Principal (P) and Rate of Interest (r)

According to the question,

$4P=P\left(1+\frac{r}{100}\right)^{10}$

$2^2=\left(1+\frac{r}{100}\right)^{10}$

$2=\left(1+\frac{r}{100}\right)^5$ ....(i)

$8P=P\left(1+\frac{r}{100}\right)^n$, $2^3=\left(1+\frac{r}{100}\right)^n$

From eq. (i)

$\left(1+\frac{r}{100}\right)^{3\times5}=\left(1+\frac{r}{100}\right)^n$, n = 15 years

QUICK TRICK :

4 = 2²---------------→10

2----------→10/2 = 5

8 =2³----------→5×3 = 15 years

Ex.10. The difference between SI and CI in 2 years is Rs. 6.25 at 5% p.a., find the principal?

(A) Rs. 2000 **(B)** Rs. 2500

(C) Rs. 3000 **(D)** Rs. 2750

Sol.(B) $P=\frac{D\times100^2}{R^2}=\frac{6.25\times100\times100}{25}$

=Rs. 2500

Ex.11. The difference between SI and CI in 3 years is Rs. 124 at 10% p.a., find the principal?

(A) Rs. 6500 **(B)** Rs. 45000

(C) Rs. 6000 **(D)** Rs. 4000

Sol.(D) $P=\frac{D\times100^3}{R^2(300+R)}=\frac{124\times100\times100\times100}{100\times310}$

=Rs. 4000

Ex.12. Simple interest and compound interest on certain sum for two years is Rs. 600 and Rs. 640 respectively. Find the principal?

(A) Rs. 2500 **(B)** Rs. 2750

(C) Rs. 3000 **(D)** Rs. 2250

Sol.(D) $P = \frac{(\text{1year S.I.})^2}{\text{difference}} = \frac{(300)^2}{40}$ =Rs.2250

OR

300

300 + 40

$R = \frac{40}{300} \times 100 = \frac{40}{3}\%$

$\frac{40}{3}\%$ =1 year SI = 300

1% = $\frac{900}{40}$

$100\% = \frac{900}{40} \times 100$, P = Rs. 2250

Ex.13. If the SI on a certain sum of money at 4% p.a. for 2 years is Rs. 125. What would be the compound interest if it is compounded annually at same rate of interest for the same period?

(A) Rs. 125 **(B)** Rs. 127.50

(C) Rs. 130 **(D)** Rs. 122

Sol.(B) SI = 2 × 4 = 8%

CI = $2 \times 4 + \frac{4^2}{100}$ = 8.16%

CI = $\frac{125}{8} \times 8.16$ = Rs. 127.50

OR

1st year - 62.5 4%

2nd year - 62.5 + 2.5

CI = Rs. 127.50

Ex.14. If the SI on a certain sum of money at 4% p.a. for 3 years is Rs. 1875. What would be the CI at the same rate for the same time?

(A) Rs. 1951 **(B)** Rs. 2015

(C) Rs. 2250 **(D)** Rs. 1900

Sol.(A) $\frac{CI}{SI} = \frac{r^2}{30000} + \frac{r}{100} + 1$

$\frac{CI}{1875} = \frac{16}{30000} + \frac{4}{100} + 1$

CI = 1875 $\left(\frac{16 + 1200 + 30000}{30000}\right)$

CI = Rs. 1951

OR

1st year	625
2nd year	625 + 25
3rd year	625 + 25 + 25 + 1

CI = Rs. 1951

Ex.15. A certain sum of money become Rs. 4500 at a certain rate of compound interest in 4 years and Rs. 7500 in 8 years. Find sum of money?

(A) Rs. 2300 **(B)** Rs. 2900

(C) Rs. 2500 **(D)** Rs. 2700

Sol.(D) $P = \frac{A_1^2}{A_2} = \frac{(4500)^2}{7500}$ = Rs. 2700

Ex.16. A builder borrows Rs. 2550 to be paid back with compound interest at the rate of 4% p.a. by the end of 2 years in two equal yearly installments. How much will each installment be?

(A) Rs. 1520 **(B)** Rs. 1352

(C) Rs. 1452 **(D)** Rs. 1552

Sol.(B)

$$P = x\left[\frac{1}{\left(1+\frac{R}{100}\right)} + \frac{1}{\left(1+\frac{R}{100}\right)^2}\right]$$

$$2550 = x\left[\frac{1}{\left(1+\frac{4}{100}\right)} + \frac{1}{\left(1+\frac{4}{100}\right)^2}\right]$$

$$2550 = x\left[\frac{25}{26} + \left(\frac{25}{26}\right)^2\right]$$

$x = 2550 \times \frac{26}{25} \times \frac{26}{51}$, x = Rs. 1352

II Method by formula

$$\frac{A \times \frac{R}{100}}{1 - \left(\frac{100}{100+R}\right)^n} = \frac{\frac{2550 \times 4}{100}}{1 - \left[\frac{100}{104}\right]^2}$$

$$= \frac{2550 \times 1}{\frac{51}{676}} = 1352$$

EXERCISE

Q.1. Rs. 800 becomes Rs. 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years?

(A) Rs. 1020.80 **(B)** Rs. 1025
(C) Rs. 1052 **(D)** Data inadequate

Q.2. A sum invested at 5% simple interest per annum increase to Rs. 504 in 4 years. The same amount at 10% simple interest per annum in $2\frac{1}{2}$ years will grow to-:

(A) Rs. 420 **(B)** Rs. 450
(C) Rs. 525 **(D)** Rs. 550

Q.3. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond eight years. If the total interest paid by him at the end of eleven years is Rs. 8160, how much money did he borrow?

(A) Rs. 8000 **(B)** Rs. 10,000
(C) Rs. 12,000 **(D)** Data inadequate

Q.4. The simple interest on Rs. 1820 from March 9, 2003 to May 21, 2003 at $7\frac{1}{2}$% rate will be :

(A) Rs. 22.50 **(B)** Rs. 27.30
(C) Rs. 28.80 **(D)** Rs. 29

Q.5. A person borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}$% p.a. for 2 years. Find his gain in the transaction per year.

(A) Rs. 112.50 **(B)** Rs. 125
(C) Rs. 150 **(D)** Rs. 167.50

Q.6. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he chargs an interest of 10%, the effective rate of interest becomes :

(A) 10% **(B)** 10.25%
(C) 10.5% **(D)** 8.5

Q.7. A sum of money lent out at simple interest amounts to Rs. 720 after 2 years and to Rs. 1020 after a further period of 5 years. The sum is :

(A) Rs. 500 **(B)** Rs. 600
(C) Rs. 700 **(D)** Rs. 710

Q.8. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years ?

(A) 6% **(B)** 6.5%
(C) 7% **(D)** 7.5%

Q.9. The principal that amounts to Rs. 4913 in 3 years at $6\frac{1}{4}$% per annum compounded interest compounded annually, is :

(A) Rs. 3096 **(B)** Rs. 4076
(C) Rs. 4085 **(D)** Rs. 4096

Q.10. Sum of money invested at compound interest amounts to Rs. 4624 in 2 years and Rs. 4913 in 3 years. The sum of money is-

(A) Rs. 4086 **(B)** Rs. 4260
(C) Rs. 4335 **(D)** Rs. 4096

Q.11. Durgesh borrowed an amount of Rs. 15,000 at the simple interest rate of 12 p.c.p.a. and another amount at the simple interest rate of 15 p.c. p.a. for a period of two years each. He paid amount of Rs. 9000 as total interest. What is the total amount borrowed?

(A) Rs. 18,000 **(B)** Rs. 32000
(C) Rs. 35000 **(D)** Rs. 33000

Q.12. The S.I. occured on a sum of money at the rate of interest 5% per annum for two years is Rs. 410. The compound interest is the same as S.I. as occured on other sum of money at the same rate of interest and the same time. Find the difference between the two sum of money (principal) ?

(A) Rs. 80 **(B)** Rs. 90
(C) Rs. 150 **(D)** Rs. 100

Q.13. The compound interest earned by Kanishk on a certain amount at the end of two years at the rate of 8 p.c.p.a. was Rs.1414.40. What was the total amount that Neetu got back at the end of two years in the form of principal plus interest earned?

(A) Rs. 9414.40 **(B)** Rs. 9914.40
(C) Rs. 9014.40 **(D)** Rs. 8914.40

Q.14. The difference between compound interest compounded every 6 months and simple interest after 2 years is 1025.6. The rate of interest is 20 percent. Find the sum

(A) Rs.12000 **(B)** Rs.14000
(C) Rs.16000 **(D)** Rs.18000

Q.15. An amount at 5 % compound interest for the first 2 years, followed by simple interest of 10% for the subsequent 3 years becomes Rs.25798.5. Find the initial amount.

(A) 12000 (B) 14000
(C) 16000 (D) 18000

Q.16. What will be the amount of the compound interest on a certain sum for three years, if simple interest on the same sum for the same time period at the rate of 10% p.a. is 1500?

(A) Rs.1654 **(B)** Rs.1655
(C) Rs.1600 **(D)** Rs.1555

Q.17. The compound interest on Rs.10240 at $6\frac{1}{4}\%$ per annum for 2 years 73 days is

(A) Rs.1464.5 **(B)** Rs.1500
(C) Rs.1565.5 **(D)** Rs.181.5

Q.18. A person took a loan of Rs.15000 for 3 years, at 5% per annum compound interest. He repaid Rs.5250 in each of the first 2 years. The amount he should pay at the end of 3rd year to clear all his debts is

(A) Rs.6063.75 **(B)** Rs.6381.25
(C) Rs..6588.75 **(D)** Rs..6863.75

Q.19. Maneesh invests some money in three different schemes for 4 years, 8 years and 12 years at 10%, 15% and 20% simple interest respectively. At the completion of each scheme, he gets the same interest. The ratio of his investments is

(A) 6: 2: 1 **(B)** 5: 2: 1
(C) 5: 2: 3 **(D)** 5: 2: 7

Q.20. If the simple interest on a certain sum of money be Rs.303.60 for 3 years at 4% per annum. What will be the compound interest on the same sum of money and same time and rate?

(A) Rs.310.80 **(B)** Rs.421.70
(C) Rs.315.90 **(D)** Rs.350.70

Q.21. If the simple interest on a certain sum for 3 years at 8% per annum is half of the compound interest on Rs.4000 for 2 years at 10% per annum, what is the sum?

(A) Rs.1250 **(B)** Rs.1750
(C) Rs.2000 **(D)** Rs.1500

Q.22. Mukund lends 40% of a certain sum at 15% p.a., 50% of rest sum at 10% p.a. and the rest at 18% p.a. rate of interest. What would be the rate of interest, if the interest is calculated on the whole sum?

(A) 13.4 % p.a. **(B)** 14.33% p.a.
(C) 14.4% p.a. **(D)** 13.33% p.a.

Q.23. A man lent out Rs.21600 partly at 12% and partly at 14% simple interest. His total income after 1.5 years was Rs.4050. Find the ratio of the sum lent at 12% and 14% simple interest respectively.

(A) 5: 1 **(B)** 4: 1
(C) 3: 1 **(D)** 4: 5

Q.24. Monali deposited a certain sum of money in a company at 12% per annum simple interest for 4 years and deposited equal amount in fixed deposit in a bank for 5 years at 15% per annum simple interest. If the difference in the interest from two sources is Rs.2160, then the sum deposited in each case is-

(A) Rs.8000 **(B)** Rs.7000
(C) Rs.6000 **(D)** Rs.8500

Q.25. The difference between the simple interest on a certain sum at the rate of 10% per annum for 2 years and compound interest which is compounded every 6 months is Rs. 124.05. What is the principal?

(A) Rs. 4500 **(B)** Rs. 5800
(C) Rs. 8000 **(D)** Rs. 12000

Notes

EXPLANATION

Q.1.(C) S.I. = Rs. (956 - 800) = Rs. 156

Rate = $\left(\frac{100 \times 156}{800 \times 3}\right)\% = 6\frac{1}{2}\%$

New rate = $\left(6\frac{1}{2} + 4\right)\% = 10\frac{1}{2}\%$

New S.I. = Rs. $\left(800 \times \frac{21}{2} \times \frac{3}{100}\right)$

= Rs. 252

$\therefore$ New amount = Rs. (800 + 252)

= Rs. 1052

Q.2.(C) Let the sum be Rs. x. Then, S.I. = Rs. (504-x)

$\therefore \left(\frac{x \times 5 \times 4}{100}\right)$ = 504 -x $\Leftrightarrow$ 20x

= 50400 - 100x $\Leftrightarrow$ 120x = 50400 $\Leftrightarrow$ x = 420

Now, P = Rs. 420, R = 10%, T = $\frac{5}{2}$ years

S.I. = Rs. $\left(\frac{420 \times 10}{100} \times \frac{5}{2}\right)$ = Rs. 105

$\therefore$ Amount = Rs. (420 + 105) = Rs. 525

Q.3.(A) Let the sum be Rs. x, Then,

$\left(\frac{x \times 6 \times 3}{100}\right) + \left(\frac{x \times 9 \times 5}{100}\right) + \left(\frac{x \times 13 \times 3}{100}\right) = 8160$

$\Leftrightarrow$ 18x + 45x + 39x = (8160 ×100)

$\Leftrightarrow$ 102x = 816000 $\Leftrightarrow$ x = Rs. 8000

Q.4.(B) Time = (22 + 30 + 21) days = 73 days

$\Rightarrow \frac{73}{365} = \frac{1}{5}$ year.

$\therefore$ S.I. = Rs. $\left(1820 \times \frac{15}{2} \times \frac{1}{5} \times \frac{1}{100}\right)$

= Rs. 27.30

Q.5.(A) Gain in 2 yrs.

= Rs. $\left[\left(5000 \times \frac{25}{4} \times \frac{2}{100}\right) - \left(\frac{5000 \times 4 \times 2}{100}\right)\right]$

= Rs. (625 - 400) = Rs. 225

$\therefore$ Gain in 1 year = Rs. $\left(\frac{225}{2}\right)$

= Rs. 112.50

Q.6.(B) Let the sum be Rs. 100 Then,

S.I. for first 6 months

= Rs. $\left(\frac{100 \times 10 \times 1}{100 \times 2}\right)$ =Rs. 5

S.I. for last 6 months

= Rs. $\left(\frac{105 \times 10 \times 1}{100 \times 2}\right)$ = Rs. 5.25

So, amount at the end of 1 year

= Rs. (100 + 5 + 5.25) = Rs. 110.25

$\therefore$ Effective rate = (110.25 - 100) = 10.25%

Q.7.(B) S.I. for 5 years = Rs. (1020 - 720) = Rs. 300

S.I. for 2 years = Rs. $\left(\frac{300}{5} \times 2\right)$ =Rs. 120

$\therefore$ Principal = Rs. (720 - 120) = Rs. 600

Q.8.(A) Let the rate be R% p.a. Then,

$1200 \times \left(1 + \frac{R}{100}\right)^2 = 1348.32$

$\Leftrightarrow \left(1 + \frac{R}{100}\right)^2 = \frac{134832}{120000} = \frac{11236}{10000}$

$\left(1 + \frac{R}{100}\right)^2 = \left(\frac{106}{100}\right)^2$ or $1 + \frac{R}{100} = \frac{106}{100}$ or

R = 6%

Q.9.(D) Principal = Rs. $\left[\frac{4913}{\left(1 + \frac{25}{4 \times 100}\right)^3}\right]$

=Rs. $\left(4913 \times \frac{16}{17} \times \frac{16}{17} \times \frac{16}{17}\right)$ =Rs. 4096

Q.10.(D) S.I. on Rs. 4624 for 1 year = Rs. (4913 - 4624)

= Rs. 289

$\therefore$ Rate = $\left(\frac{100 \times 289}{4624 \times 1}\right)\% = 6\frac{1}{4}\%$

Now, $x\left(1 + \frac{25}{4 \times 100}\right)^2 = 4624$

or $x \times \frac{17}{16} \times \frac{17}{16} = 4624$

$\therefore x = \left(4624 \times \frac{16}{17} \times \frac{16}{17}\right)$ = Rs. 4096

Q.11.(D) Let the amount borrowed at 15% p.a. is x

$15000 x \frac{2x12}{100} + \frac{15 \times x \times 2}{100} = 9000$

$$3600+\frac{3x}{10}=9000$$

$$\frac{3x}{10}=5400$$

x = 18000

$\therefore$ Total amount Borrowed is

= 15000 + 18000 = Rs. 33000

Q.12.(D) From S.I.

$$P=\frac{410\times100}{5\times2}=\text{Rs. }4100$$

From C.I.

$$P=\frac{410}{\frac{5}{50}+\left(\frac{5}{100}\right)^2}$$

$$=\frac{410}{\frac{1}{10}+\frac{1}{400}}=\frac{410}{\frac{41}{400}}$$

$$=\frac{410\times400}{41}=\text{Rs. }4000$$

$\therefore$ Difference = 4100 –4000 = Rs. 100

Q.13.(B) Let amount be Rs.100

Compound interest for the first year = Rs.8

Compound interest for the second year

= Rs.8 + Rs.0.64

Total compound interest = Rs.16.64

According to the question,

16.64 = 1414.40

1 = 85

100 = 8500

Total amount = 8500 + 1414.40 = 9914.40

Q.14.(C) $1025.6=P\left[\left(1+\frac{10}{100}\right)^4-1-\frac{10\times4}{100}\right]$

$$1025.6=P\left[\left(\frac{11}{10}\right)^4-1-\frac{2}{5}\right]$$

$$1025.6=P\left[\frac{14641-10000-4000}{10000}\right]$$

$$P=\frac{1025.6\times10000}{641}=\text{Rs.}16000$$

Q.15.(D) Let the initial amount be x

Amount with CI for 2 years

= x × 1.05 ×1.05

Amount with SI for 3 years on this

= x × 1.05 ×1.05 (1+ 3 × 0.1)

= x × 1.05 × 1.05 × 1.3

Given:

x × 1.05 × 1.05 × 1.3 = 25798.5

or x = 25798.5 ÷ (1.05 × 1.05 × 1.3)

= Rs.18000

Q.16.(B) $P=\frac{1500\times100}{10\times3}=5000$

$$\text{C.I.}=5000\left[\left(1+\frac{10}{100}\right)^3-1\right]$$

$$=5000\left[\left(\frac{11}{10}\right)^3-1\right]$$

$$=5000\times\frac{331}{1000}=\text{Rs.}1655$$

Q.17.(A) C.I.

$$=\left[10240\times\left(1+\frac{25}{4\times100}\right)^2\left(1+\frac{\frac{1}{5}\times\frac{25}{4}}{100}\right)\right]-10240$$

$$=\left[10240\times\frac{17}{16}\times\frac{17}{16}\times\frac{81}{80}\right]-10240$$

= 11704.5 – 10240 = 1464.5

Q.18.(A) Amount for first year = $15000\left(1+\frac{5}{100}\right)$

= Rs.15750

Amount for second year

$$=10500\left(1+\frac{5}{100}\right)=\text{Rs.}11025$$

Amount for second year

$$=5775\left(1+\frac{5}{100}\right)=\text{Rs.}6063.75$$

Q.19.(A) Let Principal = x_1, x_2 and x_3

$x_1 \times 4 \times 10 = x_2 \times 8 \times 15 = x_3 \times 12 \times 20$

$x_1 : x_2 = 3 : 1; x_2 : x_3 = 2 : 1$

$x_1 : x_2 : x_3 = 6 : 2 : 1$

Q.20.(C) $P = \frac{303.60 \times 100}{3 \times 4}$ = Rs. 2530

Required C.I. = $2530 \times \left[\left(1+\frac{4}{100}\right)^3 - 1\right]$

= Rs. 315.90

Q.21.(B) According to the question,

$\frac{P \times 3 \times 8}{100} = \frac{1}{2} \times 4000 \times \left[\left(1+\frac{10}{100}\right)^2 - 1\right]$

P = 1750

Q.22.(C) $R = \frac{40 \times 15 + 30 \times 10 + 30 \times 18}{100}$ = 14.4%

Q.23.(C) Let amount given on 12% rate of interest be Rs.x.

According to the question,

$\frac{x \times 12 \times 3}{200} + \frac{(21600 - x) \times 14 \times 3}{200} = 4050$

36x + 21600 × 14 × 3 – 42x

= 4050 × 200

– 6x = 810000 – 907200

x = 16200

Ratio between amounts

= 16200: (21600 – 16200)

= 16200: 5400

= 3: 1

Q.24.(A) $\frac{x \times 15 \times 5}{100} - \frac{x \times 12 \times 4}{100} = 2160$

$\frac{75x}{100} - \frac{48x}{100} = 2160$

$\frac{27x}{100} = 2160$

$x = \frac{2160 \times 100}{27} = 8000$

Q.25.(C) Let the principal be Rs.P

C.I. = $P\left[\left(1+\frac{10}{100 \times 2}\right)^{2 \times 2} - 1\right]$

$= P\left[\left(\frac{21}{20}\right)^4 - 1\right]$

$= P\left[\left(\frac{194481}{160000}\right) - 1\right] = \frac{34481}{16000}P$

S.I. = $\frac{P \times 10 \times 2}{100} = \frac{P}{5}$

According to the question

$\frac{34481}{160000}P - \frac{P}{5} = 124.05$

$\frac{(34481 - 32000)}{160000}P = 124.05$

$\frac{2481}{160000}P = 124.05$

$P = \frac{124.05 \times 160000}{2481}$

P = 8000

CHAPTER-8
AVERAGE

Scan the QR code to get video of this chapter.

The Concept of average is equal to the distribution of the overall value among all the things or persons present there. So the formula for finding the average is as follows :

$$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

IMPORTANT FACTS

- Average of first n consecutive natural numbers $= \frac{n+1}{2}$
- Average of first n consecutive even natural numbers = n+1
- Average of first n consecutive odd natural numbers = n
- Average of square of first n consecutive natural numbers $= \frac{(n+1)(2n+1)}{6}$

EXAMPLES

Ex.1. If the average of m numbers is n^2 and that of n numbers is m^2, then the average of (m + n).

(A) (m+n) **(B)** mn

(C) m/n **(D)** m-n

Sol.(B) According to question,

(Sum of m numbers) = $m \times n^2 = mn^2$

(Sum of n numbers) = $n \times m^2 = nm^2$

So, average of (m + n numbers)

$$= \frac{\text{Sum of numbers}}{\text{Total Numbers}}$$

$$= \frac{mn^2 + nm^2}{m+n}$$

$$= \frac{mn\,(m+n)}{(m+n)} = mn$$

Ex.2. The average of three numbers is 28. The first number is half of the second number and the third number is twice the second. Find the third number.

(A) 18 **(B)** 24

(C) 36 **(D)** 48

Sol.(D) Let the first number be = x

So the second number = 2x

Third number = 4x

Total = x +2x + 4x = 28 × 3

7x = 28 × 3

x = 12

Third number = 4x = 4 × 12 = 48

Ex.3. Out of three numbers, the first is twice the second number and half of third number. If the average of all the three numbers is 56. The difference between first and third number is:

(A) 12 **(B)** 20

(C) 24 **(D)** 48

Sol.(D) Let the first number = 2x

Second number = x

Third number = 4x

Total of three numbers= 2x + x + 4x= 56×3

7x = 56 × 3

x = 8 × 3 = 24

Difference between first and third number

4x - 2x = 2x = 2×24 = 48

Ex.4. The mean temperature from Monday to Wednesday was 37^0C and that from Tuesday to Thursday was 34^0C. If the temperature for Thursday was 4/5 of that on Monday, then what was the temperature on Thursday?

(A) 36.5^0 **(B)** 36^0

(C) 35.5^0 **(D)** 45^0

Sol.(B) Let the temperature on Monday = 5a

Temperature on Thursday = 4a

According to question

Monday+Tuesday + Wednesday

= 37 × 3............(i)

Tuesday+Wednesday+Thursday

=34×3 ....(ii)

From equation (i) and (ii) = a = 3 × 3 = 9

So (Temperature of Thursday) = 4a = 4 × 9

= 36^0

Ex.5. There were 24 students in a class. One of them who was 18 years old left the class and his place was filled up by a new student. If the average of the class was therefore lowered by 1 month the age of new student is:

(A) 14 **(B)** 15

(C) 16 **(D)** 17

Sol.(C) Let the previous average be = x

Let the age of new student = y

So,

$24 \times x - 18 + y = 24\left(x - \frac{1}{12}\right)$

$24x - 18 + y = 24x - 24\times\frac{1}{12}$

$-18 + y = 2$

y = 16 years

OR

$18 - \frac{24\times1}{12}$ = 16 years

Ex.6. The average of 3^{30}, 3^{60}, 3^{90} is:

(A) 3^{60} **(B)** $3^{30}+3^{60}+3^{90}$

(C) $3^{10}+3^{20}+3^{30}$ **(D)** $3^{29}+3^{59}+3^{89}$

Sol.(D) $\frac{\text{Sum of all terms}}{\text{Number of terms}} = \frac{3^{30}+3^{60}+3^{90}}{3}$

$= \frac{3(3^{29}+3^{59}+3^{89})}{3} = 3^{29}+3^{59}+3^{89}$

Ex.7. The average of six numbers is 3.95. The average of two of them is 3.4 while the average of the other two is 3.85, the average of the remaining two numbers is:

(A) 4.5 **(B)** 4.7

(C) 4.6 **(D)** 4.8

Sol.(C) $= \frac{3.95\times6-(3.4\times2+3.85\times2)}{2}$

$= \frac{23.70 - (6.80 + 7.70)}{2}$

$= \frac{9.2}{2} = 4.6$

Ex.8. The average of the largest and smallest 3 digit number formed by 0,2 and 4 would be.

(A) 213 **(B)** 303

(C) 312 **(D)** 222

Sol.(C) The largest and smallest number formed by 0,2,4 are-

420 and 204

Average = $\frac{204+420}{2} = 312$

Ex.9. The arithmetic mean of 100 observations is 24. 6 is added to each of the observations and then each of them is multiplied by 2.5. Find the new arithmetic mean.

(A) 30 **(B)** 75

(C) 35 **(D)** 60

Sol.(B) If any calculation is done on every number. Average also shows same change.

So, new average = (24 + 6) × 2.5 = 75

Ex.10. The average of first three numbers is double of the fourth number. If the average of all the four numbers is 12, find the fourth number.

(A) 16 **(B)** $\frac{48}{7}$

(C) 20 **(D)** $\frac{18}{7}$

Sol.(B) Let the average of 3 numbers be = x

so the fourth number is = $\frac{x}{2}$

According to question,

Total of three numbers + Fourth number

$= 3x + \frac{x}{2}$, $12 \times 4 = \frac{7x}{2}$, $x = \frac{96}{7}$

Fourth number $= \frac{96}{7}\times\frac{1}{2} = \frac{48}{7}$

Ex.11. Out of 9 persons, 8 persons spent Rs.30 each for their meals. The 9th person spent Rs. 20 more than the average expenditure of all the 9 persons, the total money spent by all of them was-

(A) Rs.260 **(B)** Rs. 290

(C) Rs.292.50 **(D)** Rs.400.50

Sol.(C) Let the average of nine persons be x.

So, total = 9 × x = 8×30 + x + 20

$9x = 240 + x + 20$

$8x = 260$

$x = \frac{260}{8} = \frac{65}{2}$

$= \frac{65}{2} \times 9 = 292.50$ Rs.

QUICK TRICK

Since 9th person spent 20 Rs. more

Share of each person = $30 + \frac{20}{8} = 32.5$

Total expenditure = 292.5

Ex.12. In a school there are 600 students the average age of boys is 12 years and that of girls is 11 years. If the average of the students in the school is 11 years and 9 months, then the number of girls in the school is:

(A) 450 **(B)** 150

(C) 250 **(D)** 350

Sol.(B) Let the number of boys = x

So the girls = 600 - x

According to question,

$12 \times x + 11\ (600 - x)$

$= 600 \times 11$ years 9 months

$= 600 \times 11\frac{9}{12}$

$12x + 6600 - 11x = 600 \times \frac{47}{4}$

$x = 7050 - 6600$

$x = 450$

Girls = 600 - 450 = 150

Ex.13. The average marks obtained by 8 students in a certain examination is 51. The average marks obtained by another 9 students is 68. What will be the average marks obtained by all the students?

(A) 59 **(B)** 60

(C) 59.5 **(D)** 60.5

Sol.(B) Total marks of first 8 students = $8 \times 51 = 408$

Total marks of another 9 students = $9 \times 68 = 612$

Average marks of total students = $\frac{408 + 612}{17}$

$= \frac{1020}{17} = 60$

Ex.14. What is the average of square of the natural numbers from 1 to 41 ?

(A) 20.5 **(B)** 581

(C) 703 **(D)** 307

Sol.(B) Average of square of the first n natural numbers

$= \frac{(n+1)(2n+1)}{6} = \frac{(41+1)(2 \times 41+1)}{6}$

$= \frac{42 \times 83}{6} = \frac{3486}{6} = 581$

Ex.15. Find the average of cubes of natural number from 1 to 27.

(A) 5292 **(B)** 5392

(C) 5492 **(D)** 5592

Sol.(A) Cubes of first 'n'natural numbers

$= \frac{n(n+1)^2}{4}$

$= \frac{27 \times (27+1)^2}{4} = \frac{27 \times 28 \times 28}{4}$

$= \frac{21168}{4} = 5292$

Ex.16. The production of a company for three successive years has increased by 10%, 20% and 40% respectively. What is the average percentage increase in production?

(A) 30% **(B)** 22.72%

(C) 25% **(D)** 30.5%

Sol.(B) If the average percentage increase be x%

$100+x = \sqrt[3]{110 \times 120 \times 140}$

$100+x = \sqrt[3]{1848000}$

100+x = 122.72 (approx upto two decimal places)

x = 22.72%

Ex.17. A train covers 50% of the journey at 30 km/hr, 25% of the journey at 25 km/hr. and the remaining at 20 km/hr. Find the average speed of the train during the entire journey.

(A) $25\frac{25}{47}$ km/hr. **(B)** $23\frac{27}{57}$ km/hr.

(C) 30 km/hr. **(D)** $24\frac{27}{47}$ km/hr.

Sol.(A) The average speed

$$= \left[\frac{100}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}}\right] = \left[\frac{100}{\frac{50}{30} + \frac{25}{25} + \frac{25}{20}}\right]$$

[Here A = 50, B = 25 and C = 25]

$= \frac{100}{47/12} = \frac{1200}{47} = 25\frac{25}{47}$ km/hr.

Ex.18. There is a sequence of 11 consecutive odd numbers. If the average of first 7 numbers is x, then find the average of all the integers.

(A) x+3 **(B)** x+4

(C) x+5 **(D)** x+7

Sol.(B) Average of 7 consecutive numbers is 4th number so, 4th number is x

Average of 11 consecutive numbers is 6th number so the 6th number is = x + 4

Ex.19. If the mean of a, b, c is m and ab + bc + ca = 0 then the mean of a^2, b^2, c^2 is-

(A) $2\ m^2$ **(B)** $3m^2$

(C) $6m^2$ **(D)** $9m^2$

Sol.(B) Average of a, b, c is m

Total = $\frac{a+b+c}{3}$ =m

a + b + c = 3 m, on squaring both side

$a^2 + b^2 + c^2 + 2\ (ab+bc+ca) = (3m)^2 = 9m^2$

$a^2 + b^2 + c^2 = 9\ m^2$

So the average of a^2, b^2, c^2 is.

$\frac{a^2+b^2+c^2}{3} = \frac{9m^2}{3} = 3m^2$

Ex.20. Average of 10 positive numbers is x, if each number increases by 10% then x.

(A) Remains unchanged

(B) is increased by 10%

(C) may decreased by 10%

(D) may either increase or decrease

Sol.(B) When every number increases by some value then average also increases by same value so x is increased by 10%.

Ex.21. A batsman has a certain average of runs in 12 innings. In the 13th inning he scores 96 runs, thereby increasing his average by 5 runs. What is his average after the 13th inning?

(A) 64 **(B)** 48

(C) 36 **(D)** 72

Sol.(C) Let the average of 12 innings be x

So, $\frac{12 \times x + 96}{13} = x + 5$

12x + 96 = 13 x + 65

x = 31

the new average is x + 5

31 + 5 = 36

Ex.22. a, b, c, d, e are five consecutive odd numbers, their average is-

(A) 5 (a + 4)

(B) a + b + c + d + e

(C) a + 4

(D) 5

Sol.(C) For odd number

The average = Ist Step + (n-1)

Where n is number of step

a+(5-1) = a+4

Ex.23. The average of 9 numbers is M and the average of first three numbers is P. If the average of remaining numbers is N, then-

(A) M = N + P

(B) 2 M = N + P

(C) 3 M = 2 N + P

(D) 3 M = 2 P + N

Sol.(C) According to question,

Sum of 9 numbers = Sum of first 3 numbers + Sum of remaining 6 numbers

9 × M = 3×P + 6×N

9 M = 3 (P + 2N)

3 M = P + 2N

Ex.24. The average of any 5 consecutive odd natural numbers is K, if two more such numbers just next to the previous 5 numbers are added then the new average becomes-

(A) $\frac{2}{7}$ (K + 1) **(B)** 2 K - 3

(C) 2 K + 1 **(D)** K + 2

Sol.(D) There are 5 consecutive numbers so average is 3rd number.

If 2 more are added then there will be 7 numbers.

Now average is 4th number that is K+2

EXERCISE

Q.1. The average of all odd numbers up to 100 is :

(A) 50 **(B)** 45

(C) 67 **(D)** 82

Q.2. The average of four consecutive even numbers is 27. The largest of these numbers is :

(A) 31 **(B)** 30

(C) 24 **(D)** 35

Q.3. The average height of 30 boys out of 50 students of a class is 160 cm. If the average height of the remaining boys is 165 cm, the average height of the whole class (in cm) is :

(A) 162 **(B)** 214

(C) 245 **(D)** 352

Q.4. A class has two sections, in one of the section there are 40 students with an average of 14.5 years and the average age of the students of the class is 14.2 years. If there are 32 students in the other section, its average age is :

(A) 11.32 years **(B)** 13.825 years

(C) 14.23 years **(D)** 9.21 years

Q.5. The average marks of a student in eight subjects is 87. After re-evaluation in two subjects the marks were changed to 138 from 98 and 132 from 108. In remaining subjects the marks remained unchanged. What are the new average marks?

(A) 97 **(B)** 95

(C) 98 **(D)** 94

Q.6. A cricketer has a certain average for 10 innings. In the eleventh inning, he scored 108 runs, thereby increases his average by 6 runs. His new average is-

(A) 48 runs **(B)** 52 runs

(C) 55 runs **(D)** 60 runs

Q.7. The average salary of all the workers in a workshop is Rs 8000. The average salary of 7 technicians is Rs. 12000 and the average salary of the rest is Rs. 6000. The total number of workers in the workshop is:

(A) 20 **(B)** 21

(C) 22 **(D)** 23

Q.8. Mukesh has twice as much money as Sohan and Sohan has 50% more money than what Pankaj has. If the average money with them is Rs.110, then Mukesh has:

(A) Rs. 123 **(B)** Rs. 134

(C) Rs. 167 **(D)** Rs. 180

Q.9. A motorist travels to a place 150 km away at an average speed of 50 km per hour and returns at 30 km per hour. His average speed for the whole journey in km per hour is : (km./hr.)

(A) 42.3 **(B)** 37.5

(C) 13.23 **(D)** 43.22

Q.10. The average weight of 8 men increases by 1.5 kg when one of the man whose weight is 65 kg is replaced by a new man. The weight of the new man is :

(A) 88 kg. **(B)** 77 kg.

(C) 63 kg. **(D)** 54 kg.

Q.11. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is same today. The present age of the baby is :

(A) 2 years **(B)** 6 years

(C) 8 years **(D)** 5 years

Q.12. Three Maths classes X,Y and Z, take an algebra test. The average score of class X is 83.The average score of class Y is 76. The average score of class Z is 85. The average score of class X and Y is 79 and average score of class Y and Z is 81. What is the average score of classes X,Y and Z?

(A) 81.5 **(B)** 80.5

(C) 83 **(D)** 78

Q.13. The average weight of a group of 75 girls was calculated as 47 kgs. It was later discovered that the weight of one of the girls was read as 45 kgs., whereas her actual weight was 25 kgs. What is the actual average weight of the group of 75 girls? (Rounded off to two digits after decimal)

(A) 46.73 kgs. **(B)** 46.64 kgs.

(C) 45.96 kgs. **(D)** 45.48 kgs.

Q.14. The average weight of a political party decreases by 1 kg., when a new politician joined the party, whose weight is $\frac{1}{4}$ of the existing politicians and the total weight of the new politician is 209 kg. What is the new average weight of all the politicians if it is known that in any case the number of politicians always must be greater than 50 but less than 100.

(A) 15 kg. **(B)** 16 kg.

(C) 18 kg. **(D)** 19 kg.

Q.15. The traffic lights at three different road crossing change after 24 seconds, 36 seconds and 54 seconds respectively. If they all change simultaneously at 10

: 00 AM, then at what time will they again change simultaneously?

(A) 10 : 16 : 54 AM **(B)** 10 : 18 : 36 AM

(C) 10 : 17 : 02 AM **(D)** 10 : 22 : 12 AM

Q.16. The average Mathematics marks of two Sections A and B of Class IX in the annual examination is 74. The average marks of Section A is 77.5 and that of Section B is 70. The ratio of the number of students of Sections A and B is

(A) 7: 8 **(B)** 7: 5

(C) 8 : 7 **(D)** 8 : 5

Q.17. The average value of property of Sweta, Anushika and Anjali is Rs.130 lakhs. The Property of Sweta is 20 lakhs greater than the property value of Anushika and Anjali's property value is 50 lakhs greater than the Sweta's property value. The value of property of Anjali is

(A) Rs.120 lakhs **(B)** Rs.170 lakhs

(C) Rs.150 lakhs **(D)** Rs.100 lakhs

Q.18. The average of eight numbers is 99. Among them, the greatest number exceeds the next greatest number by 18. If both these numbers are excluded, then average of the remaining 6 numbers becomes 87, then what is the greatest number?

(A) 162 **(B)** 144

(C) 187 **(D)** 126

Q.19. If A scored p runs, B scored q runs and C scored r runs, then p: q = q: r = 3: 2. If total runs scored by A, B and C is 342, then what will be the average of the runs scored by A and B?

(A) 110 **(B)** 125

(C) 130 **(D)** 135

Q.20. The average wages of 500 workers was found to be Rs. 200. Later on it was discovered that the wages of two workers were misread as 180 and 20 instead of 80 and 220. The correct average wages is

(A) Rs. 200.10 **(B)** Rs. 200.20

(C) Rs. 200.50 **(D)** Rs. 201.00

Q.21. The average of four numbers is 122.5. Of the four numbers, the first is 1.5 times the second, the second is 1/3 rd of the third, and the third is 2 times the fourth number. Then what is the smallest of all those numbers?

(A) 35 **(B)** 105

(C) 210 **(D)** 70

Q.22. The average of 10 numbers is 17.3. If the average of five numbers is 14.2 and the average of last six numbers is 19.4. What is the fifth number?

(A) 14.4 **(B)** 12

(C) 13.8 **(D)** 17.8

Q.23. The average monthly income of a person in a certain family of 5 members is Rs.16675. What will be approximate monthly average income of a person in the same family if the income of one person increased by Rs.12300 per year?

(A) Rs.16700 **(B)** Rs.17880

(C) Rs.16800 **(D)** Rs.16880

Q.24. A man purchased 7 bags of rice at the rate of Rs. 800 each, 8 bags of rice at Rs. 1000 each and 5 bags of rice at the rate of Rs. 1200 each. What is the average cost of one bag of rice?

(A) Rs.1000 **(B)** Rs.980

(C) Rs.1120 **(D)** Rs.1050

Q.25. The average marks of some students in a class is 43. When 4 students are included the average becomes 42.5 and the marks of those 4 students are 42, 36.5, 39 and 42.5 in the class.

(A) 15 **(B)** 20

(C) 25 **(D)** 35

Notes

EXPLANATION

Q.1.(A) The average of all odd numbers up to 100 is 1+3 + 5 + 7 + 9 + ... + 99

Therefore here Tn = 99

Sum for A.P = $\frac{\text{First term + last term}}{2}$

$= \frac{99+1}{2}$ =100/2 = 50

Q.2.(B) $\frac{x+x+2+x+4+x+6}{4} = 27$

$\frac{4x+12}{4} = 27$

4x + 12 = 27 × 4

4x + 12 = 108

4x = 108 - 12 = 96

x = 96/4 = 24

Therefore the largest number is x + 6

= 24 + 6 = 30

Q.3.(A) Given the average of 30 boys is 160 cm.

Total = 50

∴The remaining (50 - 30 = 20)

∴The average of 20 boys is 165 cm

∴ The required average

$= \frac{(30\times160)+(20\times165)}{50}$

$= \frac{4800+3300}{50}$ = 8100/50 = 162 cm

Q.4.(B) Given, Average of 40 students = 14.5 years

(40 + 32) = 72 students average 14.2 years

∴ The total age of 32 students

= (72 ×14.2 - 40×14.5) years

= (1022.4 - 580) years

= 442.4 years

∴ The average = 442.4/32 = 13.825 years

Q.5.(B) After re-evaluation difference of marks in one subject = 138 – 98 = 40

and difference of marks in other subject

= 132 – 108 = 24

∴ Total difference = 40 + 24 = 64

∴ New average = 87+$\frac{64}{8}$ = 95

Q.6.(A) Let the average for 10 innings be x

Then $\frac{10x+108}{11}$ = x + 6

10x + 108 = 11x + 66

108 - 66 = - 10x +11x

x = 42

∴The new average = x +6

= 42 + 6

= 48 runs

Q.7.(B) Let the total number of workers in the workshop = x

Given Average of all workers = 8000

Sum = 8000x

Average of 7 members = 12000

Sum = 7× 12000 = 84000

Average of remaining workers = 6000

Sum = 6000 × (x–7)

= (6000x–42000)

Total number of workers

8000x = 84000 + 6000x – 42000

2000x = 42000

x =21

Q.8.(D) Suppose Pankaj has Rs. x Then, Sohan has 150% of Rs. x.

Then, Sohan has 150% of Rx. x

$= \left(\frac{150}{100}\times x\right) = \text{Rs.}\left(\frac{3x}{2}\right)$

Then Mukesh has Rs. 3x

⇒ x + 3x/2 + 3x = (110 × 3)

⇒ 2x + 3x + 6x = 660

= 11x = 660

⇒ x = 660/11 = 60

∴Mukesh has Rs. 3 × 60 = Rs. 180

By ratio

M : S = 2 : 1)× 3

S : P = 3 : 2)× 1

M : S : P = 6 : 3 : 2

Average money = 110

Total = 110 × 3

= 330

Money of Mukesh = $\frac{330 \times 6}{11}$ = Rs. 180

Q.9.(B) Given x = 50, y = 30

Average Speed = $\frac{2xy}{x+y}$ km/hr

= $\frac{2 \times 50 \times 30}{50+30}$ km/hr

= 3000/80

= 37.5 km/hr

Q.10.(B) Total weight increased by = (8×1.5) kg

= 12 kg

$\therefore$ The weight of new man = (65 + 12) kg

= 77 kg

Q.11.(A) Total age of 5 members, 3 years ago

= (17×5) = 85 years

Total age of 5 members, now

= [85 + (3×5)]

= 85 + 15 = 100 years

Total age of 6 members now = (17×6)

= 102 years

$\therefore$ The age of the baby = (102-100)

= 2 years.

Q.12.(A) Let the number of students in classes X,Y and Z be a,b, and c respectively then total score of

X = 83 a, Y =76b and Z = 85c and

$\frac{83a+76b}{a+b} = 79 \Rightarrow 4a = 3b$

$\frac{76b+85c}{b+c} = 81 \Rightarrow 4c = 5b$

$b = \frac{4}{3}a,\ c = \frac{5}{3}a$

Average score of X,Y,Z

$= \frac{83a+76b+85c}{a+b+c} = \frac{978}{12} = 81.5$

Q.13.(A) Actual average weight of 75 girls

$= 47 - \left(\frac{45-25}{75}\right)$

$= 47 - \left(\frac{20}{75}\right)$

$= 47 - 0.27$

= 46.73

Q.14.(B) Let there are n politicians (initially) in the party and their average weight be x kg. then

$nx + 209 = \frac{5}{4}n(x-1) \quad \Rightarrow \frac{n}{4}(x-5) = 209$

$\Rightarrow x = \frac{209 \times 4}{n} + 5$

$\Rightarrow x = \frac{4 \times 11 \times 19}{n} + 5$

$\therefore$ Posible value of n = 19 × 4 = 76

$\therefore$ x = 16

Average weight of all politicians = 16 kg.

Q.15.(B) LCM of 24, 36 and 54 = 216 seconds

= 3 minutes 36 seconds

$\therefore$ Required time = 10 : 15 : 00 + 3 minutes 36 seconds

= 10 : 18 : 36 AM.

Q.16.(C) If number of students in section A = x and in section B = y

$74 = \frac{77.5 \times x + y \times 70}{x+y}$

$\Rightarrow$ 74x + 74y = 77.5x + 70y

$\Rightarrow$ 77.5x – 74x = 74y - 70y

$\Rightarrow$ 3.5x = 4y

$\Rightarrow \frac{x}{y} = \frac{4}{3.5} = \frac{8}{7}$

Q.17.(B) Property value of Anushika is Rs.x

130 × 3 = x + x + 20 + x + 20 + 50

390 = 3x + 90

3x = 300

x = 100

Anjali = 100 + 20 + 50 = 170 lakhs

Q.18.(B) Let the greatest number is x.

According to the question,

8 × 99 = x + x – 18 + 6 × 87

2x = 792 + 18 – 522

x = 144

Hence, the greatest number = 144

Q.19.(D) p : q = 3 : 2 = 9 : 6

q : r = 3 : 2 = 6 : 4

p : q : r = 9 : 6 : 4

Average of A's and B's runs

$$= \frac{1}{2} \times \frac{15}{19} \times 342 = \frac{270}{2} = 135$$

Q.20.(B) Correct average wage

$$= \frac{100000 - 180 - 20 + 220}{500}$$

$$= \frac{100100}{500} = \text{Rs. } 200.20$$

Q.21.(D) First = 1.5x

Second = x

Third = 3x

Fourth = 1.5x

Average = 122.5

$$\frac{(1.5x + x + 3x + 1.5x)}{4} = 122.5$$

x = 70

Smallest number = 70

Q.22.(A) Required fifth number = 5 × 14.2 + 6 × 19.4 – 17.3 × 10

= 71 + 116.4 – 173= 187.4 – 173 = 14.4

Q.23.(D) Total monthly income of family of 5 members = 16675 × 5 = 83375

According to the question,

Monthly increase $= \frac{12300}{12}$ = Rs.1025

Total monthly income = 83375 + 1025

= 84400

Increased average monthly income

$$= \frac{84400}{5} = 16880$$

Q.24.(B) Required average

$$= \frac{7 \times 800 + 8 \times 1000 + 5 \times 1200}{7 + 8 + 5}$$

$$= \frac{5600 + 8000 + 6000}{20}$$

= Rs.980

Q.25.(B) 43x + 42 + 36.5 + 39 + 42.5

= 42.5(x + 4)

0.5x = 170 – 160

0.5x = 10

x = 20

Notes

CHAPTER-9
RATIO AND PROPORTION

Scan the QR code to get video of this chapter.

Ratio

Ratio is the comparison of two quantities by division. The ratio of a to b is written as

$a : b = \frac{a}{b} = a \div b$

Types of Ratio:

(i) Compounded Ratio: When two or more than two ratios are multiplied with each other, then it is called as Compounded Ratio.

Ex. $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{48}{105}$ is the compound ratio of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$.

So $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} .... = \frac{k}{l}$

(ii) Duplicate Ratio: Square of any ratio is known as Duplicate Ratio.

Duplicate Ratio of a : b = $a^2 : b^2$.

Ex. Duplicate ratio of 7 : 5 = 49 : 25

(iii) Triplicate Ratio: Cube of any ratio is known as Triplicate Ratio.

Triplicate Ratio of a : b = $a^3 : b^3$.

Ex. Triplicate ratio of 9 : 2 = 729 : 8

(iv) Sub Duplicate Ratio: Square root of any ratio is known as Sub Duplicate Ratio:

Sub Duplicate Ratio of a : b = $a^{1/2} : b^{1/2}$.

Ex. Sub duplicate ratio of 9 : 16 = 3 : 4

(v) Sub Triplicate Ratio: Cube root of any ratio is known as Sub Triplicate Ratio:

Sub Triplicate Ratio of a : b = $a^{1/3} : b^{1/3}$.

Ex. Sub triplicate ratio of 27 : 125 = 3 : 5

Properties of Ratio :

1. In a ratio, two quantities are compared, so the quantities must be of the same kind.
2. The ratio of two quantities determines how many times are quantity is contained by the other.

Comparison of Ratios

Let a : b and c : d be two ratios, then

a : b > c : d if ad > bc

ie. $\frac{a}{b} > \frac{c}{d}$ if ad > bc

Similarly

a : b < c : d if ad < bc

a : b = c : d if ad = bc

Two Important Results

If $\frac{a}{b} > 1$, it is implied that a > b

Ist term > 2nd term

and $\frac{a}{b} < 1 \Rightarrow a < b$

IMPORTANT FACTS

(a) If A : B = x : y and B : C = p : q then

(1) A : C = x × p : y × q

(2) A : B : C = (x : y) ×p : y × q

= x × p : y × p : y × q

or

A : B = x : y

B : C = p : q

x×p:y×p:y×q

(b) If A : B = x : y, B : C = p : q and C : D = m : n, then

(1) A : D = x × p × m : y × q × n

(2) A : B : C : D = xpm : ypm : yqm : yqn

or

A : B : C : D

x : y

p : q

m : n

xpm:ypm:yqm:yqn

Ex. If A : B = 3 : 4 and B : C = 8 : 9 then A:B:C is

(A) 8 : 6 : 9 **(B)** 9 : 8 : 6

(C) 6 : 8 : 9 **(D)** 3 : 32 : 9

Sol.(C) A : B = 3 : 4 × 2

B : C = 8 : 9

A : B : C = 6 : 8 : 9

QUICK TRICK

A : B = 3 : 4

B : C = 8 : 9

A : B : C = 24 : 32 : 36

A : B : C = 6 : 8 : 9

Ex. If a : b = $\frac{2}{9}:\frac{1}{3}$, b : c = $\frac{2}{7}:\frac{5}{14}$ and

d : c = $\frac{7}{10}:\frac{3}{5}$ then a:b:c:d is-

(A) 4:6:7:9 **(B)** 16:24:30:35

(C) 8:12:15:7 **(D)** 30:35:24:16

Sol.(B) a : b = $\frac{2}{9}:\frac{1}{3}$ = 2:3

b : c = $\frac{2}{7}:\frac{5}{14}$ = 4:5

d : c = $\frac{7}{10}:\frac{3}{5}$ = 7:6

⇒ c : d = 6 : 7

a : b = 2 : 3

b : c = 4 : 5

c : d = 6 : 7

a:b:c:d = 2×4×6 : 3×4×6 : 3×5×6 :3×5×7

= 16 : 24 : 30 : 35

Proportion : A proportion is an expression which states that given two ratios are equal.

eg. $\frac{3}{12}=\frac{1}{4}$ is a proportion.

It can also be expressed as

3 : 12 = 1 : 4 or 3 : 12 : : 1 : 4

Each quantity in proportion is called a term proportional. The first and the last terms are called the extremes whereas the second and the third terms are called middle term.

TYPE OF PROPORTION

(i) Third Proportion :- If a : b :: b : c, then c is called the 3rd proportional to a and b. c will be calculated as below a : b :: b : c or a : b = b : c

⇒ a ×c = b × b

$\therefore c=\frac{b^2}{a}$

Ex. Find the third proportion of 0.36 and 0.48.

(A) 0.64 **(B)** 0.1728

(C) $24\sqrt{0.0003}$ **(D)** 0.1828

Sol.(A) Third proportion = $\frac{0.48\times0.48}{0.36}=0.64$

(ii) Fourth Proportion: If a : b : : c : d, then d is called the 4th proportion to a, b and c. d will be calculated as below:

a : b :: c : d or a : b = c:d

⇒ a×d = b×c $\therefore d=\frac{bc}{a}$

(iii) Mean Proportion: If a and b then mean proportion is $\sqrt{ab}$

If mean proportion between a and b is A then :

a:A :: A:b

⇒ A^2 = ab

or

a ×b = A ×A

∴ A = $\sqrt{ab}$

(iv) Continued Proportion : Three quantities a, b, c of same kind are said to be in continued proportion, when a : b = b : c

* The middle number b is said to be a mean proportional to two extreme numbers a and c

So, in such case of continued proportion.

b^2 = ac

(middle number)^2 = First number × Last number

Relation Among the Quantities More than two

Given a : b = x : y

b : c = m : n

the these three quantites are related as,

$$\begin{aligned} a : b &= x : y \\ b : c &= m : n \end{aligned}$$

$$a : b : c = xm : my : yn$$

$$a:c = xm : yn$$

Useful Results on Proportion

If four quantities a, b, c and d are said to be in proportion if and only if

$$a : b = c : d$$

$$a \times d = b \times c$$

⇒ product of extrems = product of means

Componendo and **Dividendo:**

If a : b is equal to c : d

$$\frac{a}{b} = \frac{c}{d}$$

i) Componendo Rule - $\frac{a+b}{b} = \frac{c+d}{d}$

ii) Dividendo Rule - $\frac{a-b}{b} = \frac{c-d}{d}$

iii) Componendo and **Dividendo Rule -**

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

EXAMPLES

Ex.1 If x : y = 3 : 1 then $x^3 - y^3 : x^3 + y^3$ = ?

(A) 13 : 14 **(B)** 14 : 13

(C) 10 : 11 **(D)** 11 : 10

Sol.(A) $\frac{x}{y} = \frac{3}{1} \Rightarrow \frac{x^3}{y^3} = \frac{27}{1}$

$$\Rightarrow \frac{x^3 - y^3}{x^3 + y^3} = \frac{27-1}{27+1}$$

[By componendo and dividendo]

$$= \frac{26}{28} = \frac{13}{14} = 13:14$$

or Direct put x = 3, y = 1

Ex.2. If a : b = c : d = e : f = 1 : 2 then (pa+qc + re) : (pb + qd +rf) is equal to

(A) p:(q+r) **(B)** (p+q) :r

(C) 2 : 3 **(D)** 1 : 2

Sol.(D) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{2}$

$$\Rightarrow \frac{pa}{pb} = \frac{qc}{qd} = \frac{re}{rf} = \frac{1}{2}$$

$$\Rightarrow \frac{pa+qc+re}{pb+qd+rf} = \frac{1}{2} \text{ or } 1:2$$

Ex.3. Divide Rs. 3200 among P, Q, R, in the ratio 5 : 2 : 9. Find the amount received by Q.

(A) Rs. 1000 **(B)** Rs. 400

(C) Rs. 500 **(D)** Rs. 1800

Sol.(B) Amount received by Q

$$= \frac{\text{Its related ratio term}}{\text{Sum of ratio terms}} \times \text{Total amount}$$

$$= \frac{2}{5+2+9} \times 3200 = \text{Rs. } 400$$

Ex.4. Find the mean proportional between 9 and 16.

(A) 3/4 **(B)** 256/9

(C) 12 **(D)** 4/3

Sol.(C) Required mean proportional

$$= \sqrt{9 \times 16} = 12$$

Ex.5. If 3, x, 27 are in continued proportion, then find the value of x.

(A) 9 **(B)** 81

(C) 27 **(D)** 54

Sol.(A) Since 3, x, 27 are in continued proportion.

$x^2 = 3 \times 27$

$x^2 = \sqrt{81}$

x = 9

Ex.6. The ratio between two numbers is 12 : 13. If each number is reduced by 20; the ratio becomes 2 : 3. Find the square of the second number ?

(A) 26 **(B)** 36

(C) 576 **(D)** 676

Sol.(D) Let two numbers are 12x and 13 x

$$\frac{12x-20}{13x-20} = \frac{2}{3} \Rightarrow x = 2$$

Numbers are = 24, 26

So, required number = $26^2 = 676$

Ex.7. A person distributes his pens among four friends A, B, C and D in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$. What is the minimum number of pens that the person should have ?

(A) 20 **(B)** 15

(C) 12 **(D)** 57

Sol.(D) LCM of 3, 4, 5 and 6 is 60.

Pens are distributed in Ratio

A : B : C : D

$\frac{1}{3}\times 60 : \frac{1}{4}\times 60 : \frac{1}{5}\times 60 : \frac{1}{6}\times 60$

i.e. 20 : 15 : 12 : 10

Total number of pens

= 20 x + 15x + 12x + 10x = 57x

For minimum number of pens x = 1

The person should have atleast 57 pens.

Ex.8. An amount of money is to be divided between P, Q and R in the ratio of 2 : 5 : 7 respectively. If the total of P's and R's share is Rs. 800 more than Q's share. What will be P's share in it ?

(A) Rs.1400 **(B)** Rs. 1000

(C) Rs. 400 **(D)** Rs. 500

Sol.(C) $\frac{2x}{14}+\frac{7x}{14}-\frac{5x}{14}=800$

$\frac{4}{14}x = 800 \Rightarrow$ x = 2800

P's share is = $2800\times\frac{2}{14}=400$

Ex.9. The ratio between the length and the breadth of a rectangular field is 5 : 4 respectively. If the perimeter of that field is 360 metres, then what is the breadth of that field in metres ?

(A) 100 **(B)** 60

(C) 80 **(D)** 90

Sol.(C) Perimeter = 2 (5 + 4) = 18

Mean Value of 18 = 360

Breadth = $\frac{360}{18}\times 4$ = 80 metres

Ex.10. A bag contains 50 P, 25 P and 10P coins are in the ratio 5 : 9 : 4. The value of the coins amounts to Rs. 206. Find the number of 50 paise coins.

(A) 100 **(B)** 200

(C) 360 **(D)** 160

Sol.(B) Let the number of 50P, 25P and 10P coins be 5x, 9x and 4x respectively.

$\frac{5x}{2}+\frac{9x}{4}+\frac{4x}{10}=206$

50x + 45x + 8x = 4120

103x = 4120

x = 40

No. of 50 P coins = 5 ×40 = 200

Ex.11. A mixture contains alcohol and water in the ratio of 4 : 3. If 5 litres of water is added to the mixture the ratio becomes 4 : 5. Find the quantities of alcohol in the given mixture ?

(A) 20 lit. **(B)** 10 lit.

(C) 5 lit. **(D)** 8 lit.

Sol.(B) Let the quantity of alcohol and water be 4x litres and 3x litres respectively.

$\frac{4x}{3x+5}=\frac{4}{5}\Rightarrow$ 8x = 20

x = 2.5

Quantity of alcohol = 4 × 2.5 = 10 litres

Or

4 : 3

↓

4 : 5

2 = 5

4 = 10

Ex.12. A varies directly proportional to B and inversely proportional to C. A is 12 when B is 6 and C is 2. What is the value of A when B is 12 and C is 3?

(A) 16 **(B)** 12

(C) 10 **(D)** 15

Sol.(A) A $\propto$ B and $A \propto \frac{1}{C}$

$\Rightarrow A \propto \frac{B}{C} \Rightarrow A = K\frac{B}{C}$

When A = 12, B =6 C = 2 then

$12 = K\times\frac{6}{2}$, K = 4

Again A = $K \times \frac{B}{C} = 4 \times \frac{12}{3} = 16$

A = 16

Ex.13. The students in three classes are in the ratio of 2 : 3 : 4. If 40 students are added in each class, the ratio becomes 4 : 5 : 6. Find the total number of students in all the three classes is

(A) 180 **(B)** 153

(C) 90 **(D)** 270

Sol.(A) 2x + 40 = 4y - (i)

3x + 40 = 5y - (ii)

4x + 40 = 6y -(iii)

(i) and (ii) x =20

Total number of students

= 2x + 3x + 4x = 9x

= 9×20 = 180

Or

2 : 3 : 4

↓ ↓ ↓

4 : 5 : 6

2 = 40

9 = 180

Ex.14. Divide 721 between P, Q and R such that P gets $1\frac{1}{3}$ times as much as R and R gets $3\frac{1}{4}$ times as much as Q. Find the Q's share.

(A) 364 **(B)** 273

(C) 16 **(D)** 84

Sol.(D) P : R = 4/3 : 1 = 4 : 3

P : R = 4 : 3

R : Q = 13 : 4

P : Q : R = 52 : 12 : 39

Q's share = $\frac{721}{52+12+39} \times 12 = 84$

Ex.15. In a class ratio between boys and girls is 3 : 5, if 5 boys and 5 girls leave then ratio will become 1 : 2 then total number of students were-

(A) 24 **(B)** 32

(C) 40 **(D)** 48

Sol.(C) Let total number of boys 3x and girls 5x

$\therefore \frac{3x-5}{5x-5} = \frac{1}{2}$

$\Rightarrow 6x - 10 = 5x - 5$

$\Rightarrow$ x = 5

$\therefore$3x + 5x = 8x = 40

Ex.16. The income of A,B,C are in the ratio 7 : 9 : 12 and their expenditure are in the ratio 8 : 9 : 15. If A saves 1/4 of his income then find the ratio of savings of A,B,C

(A) 56 : 99 : 69 **(B)** 99 : 56 : 69

(C) 99 : 69 : 56 **(D)** 69 : 56 : 99

Sol.(A) Let income of A,B,C are 7x, 9x,12x

and their expenditure are 8y, 9y, 15y

Then saving = (7x-8y),(9x-9y), (12x-15y)

Now, $7x - 8y = \frac{7x}{4} \Rightarrow 21x = 32y \Rightarrow \frac{x}{y} = \frac{32}{21}$

$$\therefore \frac{7x-8y}{9x-9y} = \frac{7\left(\frac{x}{y}\right)-8}{9\left(\frac{x}{y}\right)-9} = \left\{\frac{7\times\frac{32}{21}-8}{9\times\frac{32}{21}-9}\right\} = \frac{8}{3}\times\frac{7}{33} = \frac{56}{99}$$

and

$$\therefore \frac{9x-9y}{12x-15y} = \frac{9\left(\frac{x}{y}\right)-9}{12\left(\frac{x}{y}\right)-15} = \left\{\frac{9\times\frac{32}{21}-9}{12\times\frac{32}{21}-15}\right\} = \frac{33}{23} = \frac{99}{69}$$

Saving ratio = 56 : 99 : 69

EXERCISE

Q.1. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount. How much amount does B have ?

(A) Rs. 460 **(B)** Rs.484

(C) Rs. 550 **(D)** Rs. 664

Q.2. Seats for Maths, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats ?

(A) 2 : 3 : 4 **(B)** 6 : 7 : 8

(C) 6 : 8 : 9 **(D)** None of these

Q.3. Ratio of the earnings of A and B is 4 : 7. If the earnings of A is increased by 50% and those of B decreased by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earning ?

(A) Rs. 21,000 **(B)** Rs. 26,000

(C) Rs. 28,000 **(D)** Data inadequate

Q.4. What least number must be subtracted from each of the numbers 14, 17 , 34 and 42 so that the remainders may be proportional ?

(A) 0 **(B)** 1

(C) 2 **(D)** 7

Q.5. The fourth proportional to 5,8,15 is :

(A) 18 **(B)** 24

(C) 19 **(D)** 20

Q.6. If A : B = 5 : 7, B : C = 6 : 11 then A : B : C is

(A) 55 : 77 : 66 **(B)** 30 : 42 : 77

(C) 35 : 49 : 42 **(D)** 55 : 42 : 96

Q.7. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B is

(A) 4: 3 **(B)** 3: 4

(C) 1: 1 **(D)** 2: 3

Q.8. Amount of Rs. 735 was divided between A,B and C. If each of them had received 25 less, their shares would have been in the ratio of 1 : 3 : 2. The money received by C was :

(A) Rs. 195 **(B)** Rs. 200

(C) Rs. 220 **(D)** Rs. 245

Q.9. The speeds of three cars are in the ratio 5 : 4 : 6. The ratio between the time taken by them to travel the same distance is :

(A) 5 : 4 : 6 **(B)** 6 : 4 : 5

(C) 10 : 12 : 15 **(D)** 12 : 15 : 10

Q.10. The electricity bill of a certain establishment is partly fixed and partly varies as the number of units of electricity consumed. When in a certain month 540 units are consumed, the bill is Rs. 1800. In another month 620 units are consumed and the bill is Rs. 2040. In yet another month 500 units are consumed. The bill for that month would be :

(A) Rs. 1560 **(B)** Rs. 1680

(C) Rs. 1840 **(D)** Rs. 1950

Q.11. The ratio of the incomes of A and B is 5 : 4 and the ratio of their expenditures is 3 : 2. If at the end of the year, each saves Rs. 1600, then the income of A is :

(A) Rs. 3400 **(B)** Rs. 3600

(C) Rs. 4000 **(D)** Rs. 4400

Q.12. If $5x^2 - 13xy + 6y^2 = 0$ then x:y is

(A) 2 :1 **(B)** 3 : 5

(C) 5 : 3 and 1 : 2 **(D)** (3 : 5) and (2 : 1)

Q.13. Mr. X invested a certain amount in Debit and Equity funds in the ratio of 4 : 5 respectively.At the end of one year,he earned a total dividend of 30% on his investment.After one year he reinvested the amount including dividend in the ratio of 6 : 7 in Debit and Equity funds.If the amount invested in Equity funds was Rs 94500, what was the original amount invested in Equity Funds?

(A) Rs 75000 **(B)** Rs 81000

(C) Rs 60000 **(D)** Rs 65000

Q.14. If $P : Q = \frac{3}{5} : \frac{5}{7}, Q : R = \frac{3}{4} : \frac{2}{5}$, then what is P: Q : R equal to ?

(A) $\frac{3}{5} : \frac{5}{7} : \frac{2}{5}$ **(B)** $\frac{9}{20} : \frac{15}{28} : \frac{2}{7}$

(C) $\frac{3}{5} : \frac{3}{4} : \frac{2}{5}$ **(D)** $\frac{3}{5} : \frac{5}{7} : \frac{3}{4}$

Q.15. If a : b = c : d = e : f = 1 : 3 then (pa +qc + re): (pb+qd+rf) is equal to -

(A) p: (q+r) **(B)** (p+q) :r

(C) 1:3 **(D)** 3:1

Q.16. The ratio of the monthly salaries of P and Q is in the ratio 15 : 16 and that of Q and R is in the ratio 17 : 18. Find the monthly income of R if the total of their monthly salary is Rs.1,87,450.

(A) Rs.66240 **(B)** Rs.72100

(C) Rs.62200 **(D)** Rs.65800

Q.17. A sum of Rs 300 is divided among P, Q and R in such a way that Q gets Rs 30 more than P and R gets Rs 60 more than Q. The ratio of their share is -

(A) 2 : 3 : 5 **(B)** 3 : 2 : 5

(C) 2 : 5 : 3 **(D)** 5 : 2 : 3

Q.18. Divide Rs.63145 between A, B, C, D such that A: B = 3: 4, B: C = 5: 6, C: D = 11: 9, then difference between the average of shares of A and D together and B and C together is -

(A) Rs. 3579.5 **(B)** Rs. 3795.5

(C) Rs. 3759.5 **(D)** Rs. 3779.5

Q.19. The ratio of monthly income of K, P and S is 147: 172: 123 respectively. K's per day income in the first month of new financial year is Rs.980. What is P's annual income, if it is given that salary for all months is same as the salary of April month?

(A) Rs.318500 **(B)** Rs.52500

(C) Rs.412800 **(D)** Rs.22400

Q.20. Per hour income of two persons Vimal and Sunil is given. 6 times the square of income of Sunil is 588 less than 3 times the square of income of Vimal. If the ratio between the income of Vimal and Sunil is 3: 2, what is the value of per hour income of Sunil?

(A) 14 **(B)** 42

(C) 28 **(D)** 24

Q.21. If the incomes of N and S are in the ratio of 3: 4 and their expenditures in the ratio of 4: 5. Find the ratio of their savings, given that S saves one third of her income.

(A) 3: 20 **(B)** 17: 20

(C) 13: 20 **(D)** 13: 17

Q.22. A's monthly income is three times B's monthly income. B's monthly income is fifteen percent more than C's monthly income. C's annual income is Rs.384000. What is A's annual income?

(A) Rs.110400 **(B)** Rs.1117800

(C) Rs.36800 **(D)** Rs.52200

Q.23. The sum of two positive numbers is 20% of the sum of their squares and 25% of the difference of their squares. If the numbers are x and y then, $[(x^2 + y^2): 5y^2]$ is equal to

(A) 4: 1 **(B)** 3: 2

(C) 2: 1 **(D)** 5: 2

Q.24. The wages of laborers in a factory has increased in the ratio 22: 25 and their number decreased in the ratio 15: 11. What was the original wage bill of the factory if the present bill is Rs.103125?

(A) Rs.127350 **(B)** Rs.122100

(C) Rs.123750 **(D)** Rs.132750

Q.25. What is the mean proportional between $\left(15+10\sqrt{2}\right)$ and $\left(27-18\sqrt{2}\right)$ is -

(A) 4 **(B)** $14\sqrt{7}$

(C) $3\sqrt{5}$ **(D)** $5\sqrt{3}$

Q. 26. A, B, C and D divided Rs.44352 among themselves. A took $\frac{3}{8}$th of money, N took $\frac{1}{6}$th of the remaining amount and rest was divided among C and D in the ratio of 3: 4. How much did C get as his share?

(A) Rs.9600 **(B)** Rs.10600

(C) Rs.10300 **(D)** Rs.9900

Q.27. Rs. 1310 is to be divided among A, B and C such that 40 more of $\frac{2}{5}$ of A, 20 more of $\frac{2}{7}$ of B, 10 more of $\frac{9}{17}$ of C all are equal. What is the share of B?

(A) Rs. 559 **(B)** Rs. 375

(C) Rs. 595 **(D)** Rs. 569

Q.28. The numbers p, q, r are respectively proportional to 2, 3, 5 and the sum of p, q and r is 80. If the number r is given by the equation r = mp – 8, then m is

(A) 6 **(B)** 3/2

(C) 5/2 **(D)** 3

EXPLANATION

Q.1.(B) $\frac{4}{15}A = \frac{2}{5}B$

$A = \left(\frac{2}{5} \times \frac{15}{4}\right)B$

$A = \frac{3}{2}B$

$\frac{A}{B} = \frac{3}{2}$ =A : B = 3 : 2

B's share = Rs. $\left(1210 \times \frac{2}{5}\right)$ =Rs. 484

Q.2.(A) Let nos. be 5x, 7x and 8 x respective ly.

Ratio of increased seats are

(140% of 5x) : (150% of 7x) : (175% of 8x)

7x : $\frac{21}{2}$x : 14x

Required Ratio = 7x + $\frac{21x}{2}$ +14x

= 14x + 21x + 28x

= 2 : 3 : 4

Q.3.(D) Let the original earnings of A and B be Rs. 4x and Rs. 7x

New earnings of A = 150% of Rs. 4x

= Rs. $\left(\frac{150}{100} \times 4x\right)$ = Rs. 6x.

New earning of B = 75% of Rs. 7x

=Rs. $\left(\frac{75}{100} \times 7x\right)$ = Rs. $\frac{21x}{4}$

$\therefore 6x : \frac{21x}{4} = 8 : 7 \Leftrightarrow \frac{6x \times 4}{21x} = \frac{8}{7}$

This does not give x.

So, the given data is inadequate.

Q.4.(C) Let the required number be x.

Then, (14-x) : (17-x) :: (34-x) : (42-x)

$\therefore \frac{14-x}{17-x} = \frac{34-x}{42-x}$

$\Leftrightarrow (14-x)(42-x) = (17-x)(34-x)$

$\Leftrightarrow x^2 - 56x + 588 = x^2 - 51x + 578$

$\Leftrightarrow 5x = 10 \Leftrightarrow$

$\therefore$ Required number = 2

Q.5.(B) Let the fourth proportional to 5,8, 15 be x.

Then, 5 : 8 :: 15 : x $\Leftrightarrow$ 5x = (8×15)

$\Leftrightarrow x = \frac{(8 \times 15)}{5}$ =24

Q.6.(B) A : B = 5 : 7

B : C = 6 : 11

A : B : C = 30 : 42 : 77

Q.7.(A) 5% of A + 4% of B = $\frac{2}{3}$(6% of A + 8% of B)

15% of A + 12% of B = 12% of A + 16% of B

[15 – 12] % of A = [16 – 12] % of B

3% of A = 4% of B

A: B = 4: 3

Q.8.(D) Remainder = Rs. [735-(25×3)] = Rs. 660

Money received by C

= Rs. $\left[\left(660 \times \frac{2}{6}\right) + 25\right]$ = Rs. 225

Q.9.(D) Ratio of time taken = $\frac{1}{5}:\frac{1}{4}:\frac{1}{6}$ = 12 : 15 :10

Q.10.(B) Let the fixed amount be Rs. x and the cost of each unit be Rs. y then,

540y + x = 1800 ...(i) and 620 y + x

= 2040 ....(II)

On subtracting (i) from (ii) we get 80y =240

$\Leftrightarrow$ putting y = 3 in (i) we get.

540×3+x = 1800 $\Leftrightarrow$ x = (1800-1620) = 180

Fixed charges = Rs. 180, charge per unit

= Rs. 3

Total charges for consuming 500 units

= Rs. (180 + 500 × 3) = Rs. 1680

Q.11.(C) Let the income of A and B be Rs. 5x and Rs.4xrespectively and let their expenditures be Rs. 3y and Rs. 2y respectively.

Then, 5x - 3y = 1600 .. (i) and 4x-2y

= 1600

on multiplying (i) by 2, (ii) by 3 and subtracting . We get : 2x = 1600

$\Leftrightarrow$ x = 800

A's income = Rs. 5x = Rs. (5 × 800)

= Rs. 4000

Q.12.(D) $5x^2 - 13xy + 6y^2 = 0$

$5x^2 - 10xy - 3xy + 6y^2 = 0$

$5x(x-2y) - 3y(x-2y) = 0$

$(x-2y)(5x-3y) = 0$

$x - 2y = 0 \Rightarrow x = 2y$

$5x - 3y = 0 \Rightarrow 5x = 3y$

$\frac{x}{y} = \frac{2}{1}$ and $\frac{x}{y} = \frac{3}{5}$

$\therefore (x : y) = (2 : 1)$ and $(3 : 5)$

Q.13.(A) Let his investment in Debit and Equity funds be 4x and 5x.

After one year

$4x \times \frac{130}{100} + 5x \times \frac{130}{100}$

= 11.7x

amount reinvested in equity funds

$= 11.7x \times \frac{7}{13}$

$\Rightarrow 11.7x \times \frac{7}{13} = 94500$

$\Rightarrow 6.3x = 94500$

$\Rightarrow x = 15000$

Original amount invested in Equity fund

= 5 × 15000

= Rs 75000

Q.14.(B) $P:Q = \left(\frac{3}{5} : \frac{5}{7}\right) \times \frac{3}{4}$

$Q:R = \left(\frac{3}{4} : \frac{2}{5}\right) \times \frac{5}{7}$

$P : Q : R = \frac{9}{20} : \frac{15}{28} : \frac{10}{35}$

$= \frac{9}{20} : \frac{15}{28} : \frac{2}{7}$

Q.15.(C) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{3}$

$\Rightarrow \frac{pa + qc + re}{pb + qd + rf} = \frac{\frac{pb}{3} + \frac{qd}{3} + \frac{rf}{3}}{pb + qd + rf}$

$\Rightarrow \frac{1}{3}\left(\frac{pb + qd + rf}{pb + qd + rf}\right) = 1:3$

Q.16.(A) P : Q = 15 : 16
Q : R = 17 : 18
P : Q : R = 15 × 17 : 16 × 17 : 18 × 16
= 255 : 272 : 288

R's salary $= \frac{280}{(255 + 272 + 288)} \times 187450 = 66240$

Q.17.(A) Let P gets Rs.x.

x + x + 30 + x + 90 = 300
3x = 300 – 120

$x = \frac{180}{3}$

x = 60
Required ratio of their share
= 60 : 90 : 150 = 2 : 3 : 5

Q.18.(C) A: B = 3: 4, B: C = 5: 6, C: D = 11: 9
A: B: C = 15: 20: 24, C: D = 11: 9
A: B: C: D = 165: 220: 264: 216
(A + D)'s share

$= \frac{(165 + 216)}{(165 + 220 + 264 + 216)} \times 63145$

= 27813
(B + C)'s share

$= \frac{(220 + 264)}{(165 + 220 + 264 + 216)} \times 63145 = 35332$

Required difference$= \frac{35332}{2} - \frac{27813}{2}$

= 17666 – 13906.5 = Rs.3759.5

Q.19.(C) K's monthly salary = 980 × 30 = Rs.29400

147 ratio = 29400

172 ratio = 34400

P's monthly salary = Rs.34400

P's annual salary = 34400 × 12 = Rs.412800

Q.20.(C) Let Vimal's per hour income is Rs.3x and Sunil's per hour income is Rs.2x

According to the question,

$6(2x)^2 = 3(3x)^2 - 588$

$24x^2 = 27x^2 - 588$

$13x^2 = 588$

$x^2 = 196$

$x = 14$

Sunil's per hour income= $2 \times 14 = 28$

Q.21.(C) According to the question,

$4x - 5y = \frac{1}{3}(4x)$

$12x - 15y = 4x$

$8x = 15y$

$\frac{x}{y} = \frac{15}{8}$

Now, required ratio

$= \frac{3x - 4y}{4x - 5y} = \frac{3\frac{x}{y} - 4}{4\frac{x}{y} - 5}$

$= \frac{3 \times \frac{15}{8} - 4}{4 \times \frac{15}{8} - 5} = \frac{13}{20}$

Q.22.(B) B's monthly income

$= \frac{324000}{12} \times \frac{115}{100} = \text{Rs.}31050$

A's annual income= $3 \times 31050 \times 12 = \text{Rs.}1117800$

Q.23.(C) According to the question

$x + y = (x^2 + y^2) \times \frac{1}{5}$

Again, $x + y = (x^2 + y^2) \times \frac{1}{4}$

$\frac{x^2 + y^2}{5} = \frac{x^2 - y^2}{4}$

$5x^2 - 5y^2 = 4x^2 + 4y^2$

$5x^2 - 4x^2 = 5y^2 + 4y^2$

$x^2 = 9y^2$

$x = 3y$

$\frac{x^2 + y^2}{5y^2} = \frac{x^2 + \frac{x^2}{9}}{\frac{5x^2}{9}} = \frac{10x^2}{5x^2} = 2$

$[(x^2 + y^2) : 5y^2] = 2 : 1$

Q.24.(C) Ratio of increase of wages= 22: 25

Ratio of decrease of number of laborers = 15: 11

Compound ratio of wages of laborers

$= 22 \times 15 : 25 \times 11 = 330 : 275$

Final bill= Rs.103125

275 ratio= 103125

330 ratio= Rs.123750

Q.25.(C) $\sqrt{\left(15+10\sqrt{2}\right)\left(27-18\sqrt{2}\right)}$

$= \sqrt{(15\times27) - \left(15\times18\sqrt{2}\right) + \left(27\times10\sqrt{2}\right) - \left(10\sqrt{2}\times18\sqrt{2}\right)}$

$= \sqrt{405 - 270\sqrt{2} + 270\sqrt{2} - 360}$

$= \sqrt{405 - 360} = \sqrt{45} = 3\sqrt{5}$

Q.26.(D) A's share = $44352 \times \frac{3}{8} = 16632$

B's share= $(44352 - 16632) \times \frac{1}{6} = 4620$

C's share= $27720 \times \frac{3}{7} = 9900$

Q.27.(C) Given $\frac{2A}{5} + 40 = \frac{2B}{7} + 20 = \frac{9C}{17} + 10 = k$ (let)

$A = \frac{5(k-40)}{2}, B = \frac{7(k-20)}{2}, C = \frac{17(k-10)}{9}$

$\frac{5(k-40)}{2} + \frac{7(k-20)}{2} + \frac{17(k-10)}{9} = 1310$

Solve the above, k = 190

Share of B= Rs. 595

Q.28.(D) p: q: r = 2: 3: 5

Hence, p = 2x, q = 3x and r = 5x

p + q + r = 80

2x + 3x + 5x = 80

10x = 80, x = 8

p = 16, q = 24 and r = 40

From the given equation

$40 = m \times 16 - 8$

m = 3

CHAPTER-10
PROBLEMS ON AGES

Scan the QR code to get video of this chapter.

INTRODUCTION

Problems based on ages are generally asked in most of the competitive examinations. To solve these problems, the knowledge of linear equations is essential. In such problems, there may be three situations:

(i) Age some years ago

Ex. Harsha is 40 years old Ritu is 60 years old. How many years ago was the ratio of their ages of 3 : 5 ?

(A) 10 years **(B)** 20 years

(C) 37 years **(D)** 5 years

Sol.(A) $\frac{40-x}{60-x}=\frac{3}{5}$

x = 10

(ii) Present age

Ex. The ratio of the present age of Rahul and Rashmi is 2 : 1. The ratio of their age after 30 years will be 7 : 6. What is the present age of Rahul

(A) 6 years **(B)** 10 years

(C) 12 years **(D)** 20 years

Sol.(C) Let the present age of Rahul and Rashmi be 2x and x years respectively.

After 30 years,

$\frac{2x+30}{x+30}=\frac{7}{6}$

12x+180 = 7x+210

12x-7x = 210-180

5x = $30 \Rightarrow x=\frac{30}{5}=6$

$\therefore$ Rahul's present age

= 2x = 2×6 = 12 years

(iii) Age some years hence

Ex. Jayesh is as younger to Amit as he is older to Prashant. If the sum of the ages of Amit and Prashant is 48 years. What is the age of Jayesh in years ?

(A) 22 years **(B)** 24 years

(C) 25 years **(D)** 28 years

Sol.(B) Let the age of Jayesh = x

Let the age of Amit = y

Let the age of Prashant = z

y–x = x–z

2x = y+z = 48

Age of Jayesh (x) = 24 years

Ex. Neeraj is as younger to Gopal as he is older to Deepak. If the sum of the ages of Gopal and Deepak is 58 years. What is Neeraj's age ?

(A) 29 years **(B)** 28 years

(C) 31 years **(D)** 32 years

Sol.(A) G – N = N–D

G+D = 2N

58 = 2N, N = 29 years

Important Points :

The difference between the age of two person always be constant.

EXAMPLES

Ex.1. The ratio of Laxmi's age to the age of her mother is 3 : 11. The difference between their age is 24 years. The ratio of their ages after 3 years will be:

(A) 1 : 3 **(B)** 2 :5

(C) 3 : 1 **(D)** 5 : 2

Sol.(A) Let Laxmi's present age is 3x

Mother's age is 11x

Difference = 11x-3x = 24 years

x = 3

Required ratio = (3×3+3) : (11×3+3)

= 1 : 3

Ex.2. A man's age is 3 years more than three times the age of his son. If after three years his age will be 10 years more than twice of the age of his son. Find the present age of his son ?

(A) 10 years **(B)** 12 years

(C) 33 years **(D)** 30 years

Sol.(A) Let Son's present age = x

Then father's age = (3x+3)

According to question,

(3x+3) + 3 = 2 (x+3) + 10

x = 10

Son's present age = 10 years

Ex.3. Ram's present age is three times of his son and two fifth of his father's age. If their average age is 46 years then find the difference between Ram and his father's age?

(A) 36 years **(B)** 90 years

(C) 54 years **(D)** 64 years

Sol.(C) Let Ram's son present age = x

Ram's age =3x

Ram's father age = $\frac{15x}{2}$

Average age = $\frac{x+3x+7.5x}{3} = 46$

$\frac{23x}{6} = 46 \Rightarrow x = 12$

So, difference = 90-36 = 54 years

Ex.4. A man's present age is 2/5th of his mother's age. After 8 years his age will be half of his mother's age. Find the present age of mother?

(A) 40 years **(B)** 32 years

(C) 35 years **(D)** 45 years

Sol.(A) Let man's present age =x

Mother's present age = $\frac{5x}{2}$

According to question,

$$\frac{x+8}{\frac{5x}{2}+8} = \frac{1}{2}$$

x = 16

Mother's age = $16 \times \frac{5}{2}$ = 40 years

QUICK TRICK :

$S^0 : M^0 = 2 : 5 \Rightarrow S^0 : M^0 = 2 : 5$

$S^{+8} : M^{+8} = 1 : 2 \Rightarrow S^{+8} : M^{+8} = 3 : 6$

1 ratio = 8 years

5 ratio = 8 × 5 = 40 years

Ex.5. A couple has one son and one daughter. Father's age is four times of his son and daughter's age is one third of her mother's age. If wife is 6 years younger than husband and sister is 3 years elder than her brother then find the present age of mother?

(A) 54 years **(B)** 62 years

(C) 58 years **(D)** 60 years

Sol.(A) Let age of son = x

So, age of father = 4x

So, age of sister = x+3

So, age of mother = 3(x+3)

According to question

4x - 3(x+3)= 6

x = 15

Mother's age = 3(15+3) = 54 years

Ex.6. The age of Samir and Tanuj are in the ratio 8:15, after 9 years the ratio of their ages will be 11:18. What is the difference between their ages?

(A) 9 years **(B)** 23 years

(C) 20 years **(D)** 21 years

Sol.(D) Let Samir's present age = 8x

And Tanuj's age = 15x

According to question,

$$\frac{8x+9}{15x+9} = \frac{11}{18}$$

21x = 63

x = 3

Difference of their ages = 21 years

QUICK TRICK :

	S		T
D →	8	:	15
9 ↓			
	11	:	18

3 ratio = 9

Difference in ratio = 7

Difference of the ages = 7×3 = 21

Ex.7. The ratio of the ages of Ajay and Vijay is 3:4, after 5 years the new ratio of their ages will be 4:5. What is the age of Vijay at present?

(A) 20 years **(B)** 30 years

(C) 10 years **(D)** 40 years

Sol.(A) Let Ajay's present age =3x

Vijay's present age = 4x

According to the question,

$$\frac{3x+5}{4x+5}=\frac{4}{5}$$

15x + 25 = 16x+20

x = 5

Vijay's present age

= 4x ⇒4×5 = 20 years

QUICK TRICK :

A V

P → 3 : 4

5 ↓

4 : 5

1 ⇒ 5 years

4 ⇒ 20 years

Ex.8. The sum of age of father and his son is 100 years. Now, five years ago their ages were in the ratio of 2 : 1. The ratio of the ages of father and son after 10 years will be:

(A) 5 :3 **(B)** 3 : 5

(C) 4 : 5 **(D)** 1 : 5

Sol.(A) Let 5 years ago, son's age = x

And father's age = 2x

According to the question,

x+5+2x+5 = 100

3x = 90

x = 30

Son's present age = 30+5 = 35

Father's present age = 30×2+5 = 65

Required ratio = (65+10) : (35+10)

= 5 : 3

Ex.9. The ratio of age of Father and his son 10 years hence will be 5 : 3, while 10 years ago it was 3 : 1. The ratio of the age of the son to that of his father at present-

(A) 3 : 1 **(B)** 2 : 1

(C) 1 : 4 **(D)** 1 : 2

Sol.(D) Let after 10 years, son's age will be 3x

And father's age = 5x

According to question,

$$\frac{5x-20}{3x-20}=\frac{3}{1}$$

4x = 60-20= 40

x = 10

Son's present age = 3×10-10 = 20

Father's age = 5×10-10 = 40

Required ratio = 20 : 40 = 1 : 2

QUICK TRICK :

F S

3 : 1

2 diff.

2 → 20

5 : 3 1 → 10

Required ratio = 20 : 40 = 1 : 2

Ex.10. Three years ago average age of A and B was 18 years, C joined them now the average become 22 years. How old is C now?

(A) 24 years **(B)** 29 years

(C) 32 years **(D)** 33 years

Sol.(A) Total sum of the age of A and B at present

A + B = 36+6 = 42........(i)

Sum of the present age of A, B and C

= 22×3 = 66 years..........(ii)

Subtracting eq. (ii) from (i) we get

C = 66-42 = 24 years

Ex.11. Fifteen years hence a man will be just 4 times as old as he was 15 years ago. His present age is:

(A) 20 years **(B)** 25 years

(C) 22 years **(D)** 35 years

Sol.(B) Let man's present age is x.

According to question,

(x - 15)4 = x+15

3x =75

x = 25 years

Ex.12. The ratio of present age of two brothers is 1:2 and 5 years back the ratio was 1 : 3 what will be the ratio of their ages after 5 years.

(A) 3 : 5 **(B)** 4 : 5

(C) 2 : 5 **(D)** 1 : 5

Sol.(A) Let the brothers present age be x, 2x respectively.

According to the question,

$$\frac{x-5}{2x-5}=\frac{1}{3}$$

x = 10

Required ratio = (10+5) : (20+5)

= 15 : 25 = 3 : 5

Ex.13. The difference between the present age of Anil and Sudhir is 6 years. The ratio between their ages after 4 years will be 3 : 4. What can be the present age of Sudhir?

(A) 15 years **(B)** 18 years

(C) 20 years **(D)** 23 years

Sol.(C) Let after 4 years Anil's age will be 3x and

Suresh's age will be 4x

According to question,

4x-3x = 6

x = 6

Sudheer’s present age = 6×4-4 =20

Ex.14. The average age of woman and her daughter is 21 years. And their ages are in the ratio of 5 : 1. Then find the ratio of their ages after 5 years ?

(A) 10 : 3 **(B)** 3 : 10

(C) 5 : 7 **(D)** 8 : 7

Sol.(A) Total age of mother and daughter

= 21 × 2 = 42

M = $\frac{5}{6}\times 42$ = 35 years

D = 42 - 35 = 7 years

So, required ratio after 5 years

= (35 + 5) : (7 + 5)

= 40 : 12

= 10 : 3

Ex.15. The ratio of the age of two persons is 4 : 7 and one of them is 30 years more than the other. Then sum of their ages-

(A) 100 years **(B)** 110 years

(C) 120 years **(D)** 130 years

Sol.(B) A : B = 4 : 7

7x - 4x = 30

x = 10

So, sum = 4x + 7x

= 11x

= 110 years

QUICK TRICK :

$$\frac{30}{(7-4)}\times(7+4)$$

⇒ 110 years

Ex.16. The ratio of the age of Sumit and Prakash is 2 : 3. Sumit is 6 year younger than Prakash. Then find the ratio of their ages after 6 years ?

(A) 4 : 3 **(B)** 2 : 3

(C) 3 : 4 **(D)** 3 : 2

Sol.(C) Sumit : Prakash = 2 : 3

3x - 2x = 6

x = 6 years

Required ratio = (12+6) : (18+6)

= 3 : 4

QUICK TRICK :

S : P

2 : 3

1 = 6

12 18

+6 +6

18 : 24 = 3 : 4

Ex.17. 4 years ago the ratio of the age of A and B was 2 : 3 and after 4 years their age will be 5 : 7. Then find the present age of A?

(A) 52 years **(B)** 36 years

(C) 44 years **(D)** 16 years

Sol.(B) Let 4 years ago A and B was 2x and 3x respectively.

So,

After 4 years, $\frac{2x+8}{3x+8}=\frac{5}{7}$

14x + 56 = 15x + 40

x = 16

Present age of A = 2x + 4

= 2 × 16 + 4 = 36 years

QUICK TRICK :

A : B

2 : 3 (- 4 years)

5 : 7 (+4 years)

4 : 6 (- 4 years)

1 : 1 = 8

So,

32	48
+4	+4
36	52

Ex.18. The ratio of the age of P and Q is 8 : 5. After 4 years the ratio of their age will be 4 : 3. Find the ratio of age of P after 7 years and Q's present age?

(A) 4 : 5 **(B)** 3 : 1

(C) 8 : 5 **(D)** 5 : 8

Sol.(B) According to the question,

P : Q= 8 : 5

$\frac{8x+4}{5x+4}=\frac{4}{3}$

24x + 12 = 20x + 16

24x - 20x = 4

4x = 4

x = 1

Present age of A and B is = 8, 5

Required ratio = (8+7) : (5)

= 15 : 5

= 3 : 1

QUICK TRICK :

P : Q

8 : 5

4 : 3

12 : 9

4 : 4 = 4

1 = 1

8	5
+7	
15 :	5
3 :	1

Ex.19. The ratio of the present age of P and Q is 11 : 7. And the ratio of P's age two years ago and Q's age after two years will be 7 : 5. Then find the ratio of P's age after 7 years and Q's age 7 years ago ?

(A) 51 : 21 **(B)** 7 : 3

(C) 17 : 7 **(D)** 15 : 7

Sol.(C) P : Q = 11 : 7

Let P = 11x, Q=7x

According to the question,

$\frac{11x-2}{7x+2}=\frac{7}{5}$

55x - 10 = 49x + 14

6x = 24

x = 4

Required ratio

$=\frac{44+7}{28-7}=\frac{51}{21}=17:7$

Ex.20. Father's age is 3 years more than 3 times the age of his son. If father's age is 10 years more than twice the age of son. Find the present age of his son?

(A) 5 years **(B)** 7 years

(C) 8 years **(D)** 10 years

Sol.(B) Let son's present age x

Father's age = 3x+3

According to the question,

(3x+3)-2x = 10

x = 7

Ex.21. The sum of the ages of A and B is 7 times their difference at present. After 5 years sum of their ages is 9 times their difference. Find the present age of the older member?

(A) 10 years **(B)** 20 years

(C) 35 years **(D)** 25 years

Sol.(B) According to question

A+B = 7 (A-B)

6A - 8B = 0

3A - 4B = 0

$A + B + 10 = 9A - 9B$

$8A - 10B = 10$

On solving,

A = 20 years

QUICK TRICK :

Let A>B $(A^0 + B^0) = 7 (A^0 - B^0)$

$8B^0 = 6A^0$

$A^0 : B^0 = 4 : 3$

$(A^{+5} + B^{+5}) = 9 (A^{+5} - B^{+5})$

$10B^{+5} = 8A^{+5}$

$A^{+5} : B^{+5} = 5 : 4$

$A^0 = \frac{4 \times 5 \times 1}{1}$ = 20 years

Ex.22. Father's age is 2 times the age of his elder son. After 10 years father will be 3 times the age of his younger son. If the difference between the ages of his two sons is 15 years, find the present age of the father.

(A) 50 years **(B)** 55 years

(C) 60 years **(D)** 70 years

Sol.(A) Let younger son's present age is x

So elder son's present age = x+15

Father's age = $(x+15)\times 2$

$= 2x+30$

According to question,

$(x+10)3 = 2x+40$

$x = 10$

Father's present age = 50 years

Ex.23. 10 years ago, Neha's mother is four times the age of Neha. 10 years hence Neha's mother will be 2 times of Neha's age. Find the present age of Neha.

(A) 20 years **(B)** 10 years

(C) 30 years **(D)** 15 years

Sol.(A) Let Neha's age 10 year's ago x

Mother's age = 4x

According to question,

$(x+20)2 = 4x+20$

$x = 10$

Neha's present age = 10+10 = 20 years

QUICK TRICK :

$M^{-10} : N^{-10} = 4 : 1$

$M^{+10} : N^{+10} = (2 : 1)\times 3 = 6 : 3$

$N^{-10} = \frac{20}{2}$ = 10 years

$\therefore N^0 = 10+10 = 20$ years

Ex.24. The ratio of the ages of Ram and Rahim 10 years ago was 1 : 3. The ratio of their age five years hence will be 2 : 3. Then the ratio of their present age is-

(A) 1 : 2 **(B)** 3 : 5

(C) 3 : 4 **(D)** 2 : 5

Sol.(B) Let the age of Ram and Rahim 10 years ago be x and 3x years respectively.

After 5 years from now,

$\frac{x+15}{3x+15} = \frac{2}{3}$

$\Rightarrow 6x+30 = 3x+45$

$\Rightarrow 3x = 45-30 = 15$

$\Rightarrow x = 5$

$\therefore$ Ratio of their present age

$= (x+10) : (3x+10)$

$= 15 : 25 = 3 : 5$

Ex.25. The ratio of the ages of a father that of his son is 5 : 2. If the product of their ages is 1000 years then the father's age (in years) after 10 years will be :

(A) 50 years **(B)** 60 years

(C) 80 years **(D)** 100 years

Sol.(B) Let father's age be 5x years.

Son's age = 2x years

$\therefore 5x\times 2x = 1000$

$x^2=100 \Rightarrow x = 10$

$\therefore$ Father's age after 10 years

$= 5x+10 = 5\times 10+10 = 60$ years

Ex.26. The ratio of the present age of Puneet and Appu is 2 : 3. After 3 years the ratio of their age will be 3 : 4. The present age of Puneet is :

(A) 3 years **(B)** 6 years

(C) 9 years **(D)** 4 years

Sol.(B) Let the present ages of Puneet and Appu be 2x and 3x years respectively.

After 3 years,

$\frac{2x+3}{3x+3} = \frac{3}{4}$

$\Rightarrow$ 9x+9 = 8x+12

$\Rightarrow$ x = 3

$\therefore$ Present age of Puneet

= 2x = 2×3 = 6 years

Ex.27. The present ages of two persons are 36 and 50 years respectively. If after n years the ratio of their ages will be 3 : 4, then the value of n is :

(A) 4 years **(B)** 7 years

(C) 6 years **(D)** 3 years

Sol.(C) $\frac{36+n}{50+n}=\frac{3}{4}$

$\Rightarrow$ 144+4n = 150+3n

$\Rightarrow$ 4n-3n = 150-144

$\Rightarrow$ n = 6

Ex.28. 16 years ago my grandfather was 9 times older than me. He will be 3 times of my age 8 years from now. Eight years ago, the ratio of my age to that of my grandfather was-

(A) 3 : 8 **(B)** 2 : 5

(C) 1 : 2 **(D)** 1 : 5

Sol.(D) 16 years ago,

My age= x years

My grandfather's age = 9x years

After 8 years from the present,

9x+16+8 = 3(x+8+16)

$\Rightarrow$ 9x+24 = 3x+24 +48

$\Rightarrow$ 9x+24 = 3x+72

$\Rightarrow$ 9x-3x = 72-24 $\Rightarrow$ 6x = 48

$\Rightarrow x=\frac{48}{6}=8$

Required ratio 8 years ago,

=(x+8) : (9x+8)

=(8+8) : (9×8+8)= 16 : 80 = 1 : 5

Ex.29. Rajan got married 8 years ago. His present age is six-fifth times his age at the time of his marriage. Rajan's sister was 10 years younger to him at the time of his marriage. The present age of Rajan's sister is :

(A) 32 years **(B)** 36 years

(C) 38 years **(D)** 40 years

Sol.(C) Let the Rajan's age be x at the time of marriage.

His present age = x+8

According ot question,

$\frac{6}{5}x=x+8$

$\therefore$ x = 40

Sister's age at the time of marriage

= 40–10 = 30

$\therefore$ Sister's present age = 30+8=38 years.

Ex.30. The ratio of the ages of the husband and the wife five years ago was 11 : 9 where as at the same time the ratio of the ages of the husband and his son was 5 : 1. Five years hence the ratio of the ages of the husband and the wife became 13 : 11. What is the sum of the present ages of all the three persons of the family ?

(A) 128 years **(B)** 127 years

(C) 126 years **(D)** Data inadequate

Sol.(C) Let the age of husband 5 years ago was x , age of wife 5 years ago is y and age of son 5 years ago is z.

$\frac{x}{y}=\frac{11}{9}$ ....(I)

$\frac{x}{z}=\frac{5}{1}$ ....(II)

After 5 years from the present,

According to question,

$\frac{x+5+5}{y+5+5}=\frac{13}{11}$

11x + 110 = 13y + 130 ------- III

$x=\frac{11}{9}y$ ....**(from I)**

Putting the value of x in eq^n III

$11\times\frac{11}{9}y+110=13y+130$

y = 45

Age of husband 5 years ago was

$=45\times\frac{11}{9}=55$ years

and age of son was $55\times\frac{1}{5}=11$ years

Sum of present age of family is

= 50 + 60 + 16 = 126 years

EXERCISE

Q.1. The ratio between the present ages of P and Q is 5 : 7 respectively. If the difference between Q's present age and P's age after 6 years is 2, what is the total of P's and Q's present ages ?

(A) 48 years **(B)** 52 years
(C) 56 years **(D)** Cannot be determined

Q.2. The total of the ages of Jayant, Prem and Saransh is 93 years. Ten years ago, the ratio of their ages was 2:3:4. What is the present age of Saransh ?

(A) 24 years **(B)** 32 years
(C) 34 years **(D)** 38 years

Q.3. Tanya's grandfather was 3 times older to her 8 years ago. He would be 2 times of her age 7 years from now. Four years ago, what was the ratio of Tanya's age to that of her grandfather ?

(A) 11 : 3 **(B)** 11 : 5
(C) 13 : 5 **(D)** 5 : 13

Q.4. Sneh's age is $\frac{2}{5}$th of her father's age. Sneh's father's age will be twice of Vimal's age after 10 years. If Vimal's eighth birthday was celebrated two years before then what is Sneh's present age ?

(A) $6\frac{2}{3}$ years **(B)** 24 years
(C) 30 years **(D)** 12 years

Q.5. If 6 years are subtracted from the present age of Gagan and the remainder is divided by 18, then the present age of his grandson Anoop is obtained. If Anoop is 2 years younger to Madan whose age is 5 years, then what is Gagan's present age ?

(A) 48 years **(B)** 60 years
(C) 84 years **(D)** 96 years

Q.6. The respective ratio between the present age of Manisha and Deepali is 5 : x. Manisha is 9 years younger than Parineeta.Parineeta's age after 9 years will be 33 years. The difference between Deepali's and Manisha's age is same as the present age of Parineeta, what will come in place of x ?

(A) 23 **(B)** 39
(C) 15 **(D)** 13

Q.7. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further next 8 years, how many times would he be of Ronit's age ?

(A) 2 times **(B)** $2\frac{1}{2}$ times
(C) $2\frac{3}{4}$ times **(D)** 3 times

Q.8. My brother is 3 years elder to me. My father was 28 years of age when my sister was born while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then, what was the age of my father and mother respectively when my brother was born ?

(A) 32 yrs, 23 yrs **(B)** 32 yrs, 29 yrs
(C) 35 yrs, 29 yrs **(D)** 35 yrs, 33 yrs

Q.9. A person was asked to state his age in years. His reply was, "Take my age three years hence, multiply it by 3 and then subtract three times my age three years ago and you will know how old I am." What was the age of the person ?

(A) 18 years **(B)** 20 years
(C) 24 years **(D)** 32 years

Q.10. The present ages of a father is 3 years more than 3 times the age of his son. Three years hence father's age will be 10 years more than twice the age of the son. Find the present age of the father.

(A) 32 years **(B)** 33 years
(C) 34 years **(D)** 36 years

Q.11. Anushika told her cousin Mahi that 5 years earlier, she was seven times as old as her. After 15 years, she will be thrice as old as her. Find the sum of their present ages.

(A) 75 years **(B)** 60 years
(C) 80 years **(D)** 90 years

Q.12. A man's age is 125% of what it was 10 years ago, but $83\frac{1}{3}\%$ of what it will be after 10 years. After how many years his age will become 2^6?

(A) 12 years **(B)** 16 years
(C) 15 years **(D)** 14 years

Q.13. The ratio between the ages of A and B is 7: 9 and the ratio between the ages of B and C is 8: 7. If after 7 years the age of C is 2 years less than the present age of B. Then find the age of A after 6 years?

(A) 56 years **(B)** 62 years
(C) 64 years **(D)** 66 years

Q.14. The age of a man is three times the sum of the ages of his two sons. Five years hence, his age will be double of the sum of the ages of his sons. The father's present age is:

(A) 40 years **(B)** 45 years

(C) 50 years (S) 55 years

Q.15. The respective ratio between present age of Aanchal and Bhavna is 3: 11. Bhavna is 12 years younger to Chhavi. Chhavi's age after 7 years will be 85 years. What is the present age of Aanchal's father who is 25 years older than Aanchal?

(A) 43 years **(B)** 67 years

(C) 45 years **(D)** 69 years

Q.16. Father's age is 1 year more than five times the son's age. After 3 years, father's age will be 2 less than four times the son's age. What is father's present age?

(A) 31 years **(B)** 35 years

(C) 33 years **(D)** 37 years

Q.17. Komal's age after 17 years will be 6 years less than twice her present age. The respective ratio between Preeti's present age and Anushika's present age is 21: 19. If Anushika's age 31 years hence will be same as three times Komal's present age, what will be Preeti's age 4 years hence?

(A) 44 years **(B)** 45 years

(C) 46 years **(D)** 48 years

Q.18. A is 4 years younger than B. B is 12 years younger than C. 4 years hence the ratio between C and A is 9: 5. Find the age of B.

(A) 20 **(B)** 16

(C) 24 **(D)** 28

Q.19. The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find the son's present age.

(A) 5 years **(B)** 3 years

(C) 4 years **(D)** 6 years

Q.20. 7 years ago, Varun's age was five times the square of Swati's age. 3 years hence, Swati's age will be two-fifth of Varun's age. Find Swati's present age

(A) 10 years **(B)** 27 years

(C) 7 years **(D)** 9 years

Q.21. Naimish's present age is 2/7th of his father's present age. Naimish's brother is three years older to Naimish. The respective ratio between present ages of Naimish's father and Naimish's brother is 14: 5. What is the present age of Naimish?

(A) 12 **(B)** 13

(C) 19 **(D)** 23

Q.22. My grandmother was 9 times as old as me, 16 years ago. She would be 3 times of my age, 8 years from now. Eight years ago, what was the ratio of my age to that of my grandmother?

(A) 3: 8 **(B)** 1: 5

(C) 1: 2 **(D)** 2: 5

Q.23. Jayant was 3 times as old as to Puneet 8 years ago. He would be 2 times of his age 8 years from now. Four years ago, what was the ratio of Jayant's age to that of Puneet's age?

(A) 11: 3 **(B)** 13: 5

(C) 11: 5 **(D)** 5: 13

Q.24. Total age of Ram and Shyam is 15 years more than that of Shyam and Mohan. While the average age of Ram, Shyam and Mohan is equal to that of average age of Ram and Shyam then what is the difference between the ages of Ram and Shyam?

(A) 15 years **(B)** 30 years

(C) 10 years **(D)** 40 years

Q.25. 5 years ago, the average age of Vijayshree, Vaishnavi and Unnati was the square of the third prime number. 7 years ago, the average age of Vaishnavi and Unnati was 20% less than the average age of the three girls five years ago. Find the age of Vijayshree?

(A) 36 years **(B)** 40 years

(C) 45 years **(D)** 54 years

Notes

EXPLANATION

Q.1.(A) Let the present ages of P and Q be 5x years and 7x years respectively.

Then, 7x - (5x + 6) = 2

2x = 8, x = 4.

$\therefore$ Required sum = 5x + 7x = 12x = 48 years.

Q.2.(D) Let the ages of Jayant, Prem and Saransh 10 years ago be 2x, 3x and 4x years respectively.

Then, (2x+10)+(3x+10)+ (4x + 10) = 93

9x = 63

x = 7

$\therefore$ Saransh's present age = (4x + 10)

= 38 years.

Q.3.(D) Let 8 years ago, Tanya's age = x years then 8 years ago Tanya's grand father's age = 3 x years

8 years hence Tanya's age = x+16

8 years hence Tanya's grand father's age

= 3x + 16

According to question

3x+16 = 2(x+16)

$\Rightarrow$ x = 16

4 years ago Tanya's age = 16+4

= 20 years

4 years ago Tanya's grand father's age

= 16×3+4

= 52 years

$\therefore$ Required ratio = $\frac{20}{52}$ = 5 : 13

Q.4.(D) Vimal's age after 10 years = (8 + 2 + 10) years

= 20 years.

Sneh's father's age after 10 years

= 2 × 20 = 40 years.

Sneh's father's present age = 40-10 = 30 years.

Sneh's age = $\frac{2}{5}\times 30$ years = 12 years.

Q.5.(B) Anoop's age = (5 - 2) years = 3 years.

Let Gagan's age be x years.

Then, x - 6 = 54, x = 60.

Q.6.(D) Present age of Parneeta

= 33 - 9 = 24

Age of Manisha = 24 - 9 = 15

$\because$ Manisha-Deepali= 24

$\therefore$ Age of Deepali = 15 + 24 = 39

$\frac{\text{Manisha}}{\text{Deepali}} = \frac{5}{x}$

$\Rightarrow \frac{15}{39} = \frac{5}{x}$

$\therefore$ x = 13

Q.7.(A) Let Ronit's present age be x years. Then, father's present age = (x + 3x) years

= 4x years.

$\therefore$ (4x + 8) = $\frac{5}{2}(x+8)$

8x +16 = $5x$ + 40, $3x$ = 24, x = 8.

Hence, required ratio = $\frac{4\times 8+16}{24}$ = 2 times

Q.8.(A) Clearly, my brother was born 3 years before I was born and 4 years after my sister was born.

So, father's age when brother was born

= (28 + 4) years = 32 years

Mother's age when brother was born

= (26 - 3) years = 23 years.

Q.9.(A) Let the present age of the person be x years.

Then, 3 (x + 3) - 3 (x – 3) = x

(3x + 9) - (3x - 9) = x $\Leftrightarrow$ x = 18.

Q.10.(B) Let the son's present age be x years.

Then, father's present age = (3x+3) years

$\therefore$ (3x+3+3) = 2(x+3)+10

(3x+6) = 2x+16

x = 10

Hence father's present age = (3x+3)

= (3×10+3) = 33 years

Q.11.(D) Let Anushika present age be y years.

and Mahi present age be x years.

According to question,

y – 5 = 7 (x–5)

7x – y = 30 ----------- (1)

Again, (y+15) = 3 (x+15)

y – 3x = 30 -------- (2)

From (1) and (2)

x = 15, y = 75

Required sum = 75+15 = 90 years.

Q.12.(D) Let his present age be x years

According to the question,

x = 125% of (x – 10)

4x = 5x – 50

x = 50 years

Required number of years = 64 – 50

= 14 years

Q.13.(B) A : B = 7 : 9

B : C = 8 : 7

A : B : C = 56 : 72 : 63

According to the question,

63x + 7 = 72x - 2

9x = 9

x = 1

A's age after 6 years = 56 × 1 + 6

= 62 years

Q.14.(B) Let the age of father = x years

x = 3 (y + z)

x + 5 = 2 (y + z + 10)

$$x + 5 = 2\left(\frac{x}{3} + 10\right)$$

3x + 15 = 2x + 60

x = 45 years

Q.15.(A) A: B = 3: 11

C – B = 12 years … (I)

Current age of Chhavi = 85 – 7

= 78 years

B = 78 – 12 = 66 years

Aanchal's father's age = 3 × 6 + 25

= 43 years

Q.16.(A) F = 5S + 1

F – 5S = 1 .... (I)

(F + 3) = 4(S + 3) – 2

F – 4S = 7 .... (II)

From eq. (I) and (II)

F = 31 years

Q.17.(C) Let present age of Komal = x years

Given, x + 17 = 2x – 6

x = 23

Present age of Komal = 23 years

Now, let present age of Preeti and Anushika are 21k and 19k.

Then, 19k + 31 = 3x

19k + 31 = 3 × 23

19k = 69 – 31

19k = 38

k = 2

Present age of Preeti after 4 years = 21 × 2 + 4

= 46 years

Q.18.(A) A = B – 4

B = C – 12

C – A = 16

4 ratio = 16 years

1 ratio = 4 years

$A^{+4} = 20$

B = 20 – 4 + 4 = 20 years

Q.19.(C) Let the present age of son be x years

$2x^2 + 8 = 3 (x + 8) + 4$

$2x^2 + 8 = 3x + 24 + 4$

$2x^2 – 3x – 20 = 0$

$$x = 4 \text{ or } x = -\frac{5}{2}$$

Son's present age = 4 years

Q.20.(D) Let the present age of Varun and Swati be x years and y years respectively.

Then,

$5(y – 7)^2 = x – 7$

$5y^2 – 70y + 252 = x$ .... (i)

$$\text{and, } (y + 3) = \frac{2}{5}(x+3)$$

5y + 15 = 2x + 6

$$x = \frac{(5y+9)}{2} \text{ ....(ii)}$$

from (i) and (ii)

$5y^2 - 70y + 252 = \frac{(5y+9)}{2}$

$10y^2 - 145y + 495 = 0$

$2y^2 - 29y + 99 = 0$

$y = 9, y = \frac{11}{2}$

Q.21.(A) Naimish's father present age = x

Naimish's age = $\frac{2}{7}x$

Naimish's brother age = $\frac{2}{7}x + 3$

$$\frac{x}{\frac{2}{7}x+3} = \frac{14}{5}$$

$5x = 4x + 42$

$x = 42$

Naimish's age = 12 years

Q.22.(B) 16 years ago,

Let my age = x years

and my grandmother's age = 9x years

8 years from now,

$3(x + 24) = (9x + 24)$

$3x + 72 = 9x + 24$

$6x = 48$

$x = 8$

Required ratio = (8 + 8): (72 + 8)

= 16: 80 = 1: 5

Q.23.(B) According to the question

$(J - 8) = 3(P - 8)$

$J - 8 = 3P - 24$

$3P - J = 16$ … (i)

and, $(J + 8) = 2(P + 8)$

$J + 8 = 2P + 16$

$2P - J = -8$ … (ii)

On solving (i) and (ii), we get

P = 24, J = 56

Required ratio = (56 – 4): (24 – 4)

= 52: 20 = 13: 5

Q.24.(B) $R + S = S + M + 15$

$R - M = 15$ _______ (I)

and $\frac{R+S+M}{3} = \frac{R+S}{2}$

or, $3R + 3S = 2R + 2S + 2M$

$M = \frac{R+S}{2}$ (II)

Putting (II) in (I)

$$\left[R - \left(\frac{R+S}{2}\right)\right] = 15$$

$R - S = 30$,

Q.25.(A) 5 years ago

Vijayshree + Vaishnavi + Unnati

$= (5)^2 \times 3 + 15 = 25 \times 3 + 15 = 90$ years

7 years ago

Vaishnavi + Unnati = 80% of $25 \times 2 + 14$

$= 20 \times 2 + 14 = 54$ years

Age of Vijayshree = 90 – 54 = 36 years

Notes

CHAPTER-11
PARTNERSHIP

Scan the QR code to get video of this chapter.

Partnership : When two or more than two persons run a business jointly they are called partners and the deal is known as partnership.

Ratio of Division of Gains

(1) When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.

Ex. A, B and C started a business by investing Rs. 120000 Rs. 135000 and Rs. 150000 respectively. Find the share of A, out of an annual profit of Rs. 56,700.

(A) Rs. 15800 **(B)** Rs. 14800

(C) Rs. 17800 **(D)** Rs. 16800

Sol.(D) Ratio of shares of A, B and C

= Ratio of their investment

A : B : C = 120000 : 135000 : 150000

= 8 : 9 : 10

A's share= Rs. $\left[56700 \times \frac{8}{27}\right]$ = Rs. 16800

Ex. A, B, C invested Rs.26000, Rs. 34000 and Rs.10000 in a business. At the end of the year, they earn a profit of Rs.3500, B's share of profit is:

(A) 1500 **(B)** 1700

(C) 1800 **(D)** 2000

Sol.(B) The ratio between profit of A, B and C

= Ratio of their investment

= $C_A : C_B : C_C$

= 26 : 34 : 10

= 13 : 17 : 5

B's share = $3500 \times \frac{17}{35} = 1700$

(2) When investments are for different time period, then equivalent capitals are calculated for a unit of time by taking (Capital × number of units of time). Now, gain or loss is divided in the ratio of their capital.

Ex. A, B, C enter into a partnership. A invests some money at the begining, B invests double the amount after 6 months and C invest thrice the amount after 8 months. If the annual profit be Rs.27000, C's share is:

(A) Rs. 18000 **(B)** Rs. 15000

(C) Rs. 12000 **(D)** Rs. 9000

Sol.(D) Let invested amount of A = x

According to question,

Invested amount of B = 2x

Invested amount C = 3x

then ratio profit = x×12 : 2x×6 : 3x × 4 = 1 : 1 : 1

Share of C's = $\frac{27000}{3} \times 1$ = Rs. 9000

Ex. Three students A, B and C hired a computer for a month. A runs 27 disc for 19 days, B runs 21 for 17 days and C runs 24 for 23 days. If at the end of the month, the rent amounts to Rs. 23700, how much amount to be paid by C?

(A) Rs. 8200 **(B)** Rs. 8000

(C) Rs. 9000 **(D)** Rs. 9200

Sol.(D) A's disc×days : B's disc×days : C's disc× days

= A : B : C

= 27 × 19 : 21 × 17 : 24 × 23

= 171 : 119 : 184

Payment for rent by C = $\frac{184}{474} \times 23700$

= Rs. 9200

EXAMPLES

Ex.1. A,B,C invested Rs.50000 for a business. A gave 4000 more than B and B invest Rs.5000 more than C. Out of total profit of Rs.35000 A receive ?

(A) Rs. 14700 **(B)** Rs. 14500

(C) Rs. 15500 **(D)** Rs. 17400

Sol.(A) Let invested amount of C = x

According to question,

A + B + C = 50000

x+9000 + x+5000 + x = Rs. 50000

$3x + 14000 = 50000$

$3x = 36000$

$x = 12000$

Ratio of profit of A, B and C

21000 : 17000 : 12000

21 : 17 : 12

Share of A = $\frac{35000}{50} \times 21$ = Rs. 14700

Ex.2. A,B,C entered into partnership and their capital are in the proportion $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$. A withdraw half of his capital at the end of 4 months. Out of a total annual profit of Rs.8470, A's share is?

(A) Rs. 2200 **(B)** Rs. 2600

(C) Rs. 2800 **(D)** Rs. 3000

Sol.(C) Ratio of profit of A, B and C

$= \left(\frac{x}{3}\times 4 + \frac{x}{6}\times 8\right) : \left(\frac{x}{4}\times 12\right) : \left(\frac{x}{5}\times 12\right)$

$= \frac{8x}{3} : 3x : \frac{12x}{5}$ = 40 : 45 : 36

Share of A = $\frac{8470}{121} \times 40$ = Rs. 2800

Ex.3. Ram, Rahim and Robert started a partnership business investing Rs.30,000, Rs.50,000 and Rs.40,000 respectively. If they made an annual profit of Rs.18504, then find the share of Rahim.

(A) Rs. 6610 **(B)** Rs. 7610

(C) Rs. 7710 **(D)** Rs. 5510

Sol.(C) Ratio of Profit of

Ram : Rahim : Robert

30000 50000 40000

3 : 5 : 4

Rahim = $\frac{18504}{12} \times 5$ = 1542 × 5

Rahim = Rs. 7710

Ex.4. Ram, Shyam and Kamal started a business in partnership. The ratio of their capital is 3:4:7. If their annual profit be Rs.21000, what will be Kamal's share in profit?

(A) Rs. 12500 **(B)** Rs. 15000

(C) Rs. 10500 **(D)** Rs. 13200

Sol.(C) Ratio of profit of Ram, Shyam and Kamal

= 3: 4 :7

Share of Kamal = $\frac{7}{14} \times 21000$ = 10500

Ex.5. A, B ,C started a business each investing Rs.20000. After 5 months, A withdraw Rs.5000, B Rs.4000 and C invested Rs.6000 more. At the end of the year a total profit of Rs.69900 was recorded. What is the share of B?

(A) Rs. 20200 **(B)** Rs. 21400

(C) Rs. 28200 **(D)** Rs. 21200

Sol.(D) Ratio of profit of A, B and C

(20000×5 + 15000×7) : (20000×5+16000×7) : (20000×5+26000×7)

205 : 212 : 282

Share of B = $\frac{69900}{699} \times 212$ = Rs. 21200

Ex.6. A,B,C invest their amount in 3:4:5. After 6 months C withdraw half of his money, if the amount invested by A is Rs.27000 and a total profit of Rs. 86000 occurs at the end of year, find the difference between the share of A and C?

(A) Rs. 3000 **(B)** Rs. 5000

(C) Rs. 4000 **(D)** Rs. 6000

Sol.(D) Given invested amount of A = 27000

According to question,

$3x = 27000$, $x = 9000$

Then amount of B = $4x$ = 36000

Amount of C $5x$ = 45000

Ratio of profit of A, B and C

A : B : C

27000 ×12 : 36000 × 12 : 45000 × 6+ 22500 × 6

324000 : 432000 : 405000

54000 : 72000 : 67500

108 : 144 : 135 or 12 : 16 : 15

Difference between the share of A and C.

$= \frac{86000}{43} \times 3$ = Rs. 6000,

Ex.7. A start a business by Rs.3500. After 5 months, B enter into the partnership if at the end of year profit is divided between them in 2:3, find the amount invested by B?

(A) Rs. 8000 **(B)** Rs. 8500

(C) Rs. 9000 **(D)** Rs. 7500

Sol.(C) Let amount invested of B = x

Then A : B

$3500\times12 : x\times7 \Rightarrow \frac{500\times12}{x} = \frac{2}{3}$

x = 3×6×500

Amount invested by B = Rs. 9000

Ex.8. Three partners start a business by investing Rs. 2000, Rs. 2500 and Rs. 1000. If a total profit of Rs.880 occurs find the share of third partner in the profit?

(A) Rs. 400 **(B)** Rs. 350

(C) Rs. 180 **(D)** Rs. 160

Sol.(D) Ratio of profit of Ist, IInd and IIIrd partner

= 2000 : 2500 : 1000 = 4 : 5 : 2

$\therefore$ Share of IIIrd partner = $\frac{880}{11}\times2$

= Rs. 160

Ex.9. A begins a business with Rs.4500 and was joined B with Rs.5400. If the profit at the end of the year was divided in the ratio 2 : 1. B join the business after?

(A) 5 months **(B)** 7 months

(C) 12 months **(D)** 6 months

Sol.(B) Ratio of profit of A and B = A : B

4500×12 : 5400×x= 2 : 1

x = 5

B join after (12 - 5) = 7 months

Ex.10. A and B are partners in a business A contributes, $\frac{1}{4}$ of capital for 15 months, B received $\frac{2}{3}$ of the profit find how long B's money was used.

(A) 12 months **(B)** 6 months

(C) 10 months **(D)** 8 months

Sol.(C) Ratio of profit of A and B = $\frac{1}{4}\times15 : \frac{3}{4}\times x$

$= \frac{1}{3} : \frac{2}{3}$

x = 10 months

Ex.11. A,B and C enter into a partnership and their shares are in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his shares by 50%. If the total profit at the end of one year be Rs.21,600 then find the share of B.

(A) Rs. 2500 **(B)** Rs. 3200

(C) Rs. 3600 **(D)** Rs. 4000

Sol.(D) Ratio of capital for A : B : C

$= \frac{7}{2} : \frac{4}{3} : \frac{6}{5}$ = 105 : 40 : 36

= 210 : 80 : 72

Ratio of their profit

= 210×4+315×8 : 80×12 : 72×12

3360	:	960	:	864
560	:	160	:	144
35	:	10	:	9

Share of B = $\frac{21600}{54}\times10$ = Rs. 4000

Ex.12. P, Q and R started a business by investing Rs. 200000, Rs. 250000, Rs. 400000 respectively. They decide to receive 10% interest on their capitals and the balance of the profit to be divided equally. If they got Rs. 205000 as the annual profit, find the share of P including the interest ?

(A) 60000 **(B)** 40000

(C) 50000 **(D)** 70000

Sol.(A) 10% of interest on the capitals of A, B and C respectively

= 20000, 25000, 40000

Now, the total of interest = 20000 + 25000 +40000 = 85000

Remaining of the profit after interest on capitals

= 205000 - 85000 = 120000

But the remaining of the profit has to be divided equally

So, P's share of profit = 120000/3

= 40000

Total profit = Interest on capital + share of principal after interest on capital

= 20000 + 40000 = Rs. 60000

Ex.13. A, B, C rent a pasture. A put 10 cows for 7 months, B put 12 cows for 5 months and C puts 15 cows for 3 months for grazing. If the rent of the pasture is Rs.17500. How much must C pay as his share of rent?

(A) Rs. 4500 **(B)** Rs. 5000

(C) Rs. 5500 **(D)** Rs. 6000

Sol.(A) A : B : C = 10×7 : 12×5 : 15×3

= 70 : 60 : 45= 14 : 12 : 9

C's rent = 17500 × $\frac{9}{35}$ = 4500

Ex.14. A, B,C started a business by investing Rs.20000, Rs.25000, Rs.40000 respectively. They decided to receive 10% interest on their capital and balance of the profit to be divided equally. If they get Rs.20500 as annual profit, find the share of C including the interest?

(A) Rs. 4000 **(B)** Rs. 8000

(C) Rs. 6000 **(D)** Rs. 12000

Sol.(B) Now total interest = 2000 + 2500 + 4000 = 8500

Remaining of the profit = 20500 - 8500

= Rs. 12000

But the remaining of the profit has to be divided equally

C's share = 12000 × $\frac{1}{3}$ = Rs. 4000

Total C received = 4000 + 4000 = Rs. 8000

Ex-15. A invested Rs. 76000 in a business. After few months, B joined him with Rs. 57000. At the end of the year, the total profit was divided between them in the ratio of 2 : 1. After how many months did B join ?

(A) 8 **(B)** 4

(C) 6 **(D)** 12

Sol.(B) Suppose B joined after x months. Then B's money was invested for (12 -x) months

$$\frac{76000 \times 12}{57000 \times (12 - x)} = \frac{2}{1}$$

912000 = 114000 (12-x)

114 (12-x) = 912

x = 4, Hence, B joined after 4 months.

Ex-16. A and B together invested Rs. 12000 in a business. At the end of the year, out of a total profit of Rs. 1800. A's share was Rs. 750. Find the investment of A ?

(A) 6000 **(B)** 5500

(C) 5200 **(D)** 5000

Sol.(D) $\frac{\text{A's investment}}{\text{B's investment}} = \frac{\text{Profit share of A}}{\text{Profit share of B}}$

$\frac{\text{A's investment}}{\text{B's investment}} = \frac{750}{1800 - 750} = \frac{750}{1050} = \frac{5}{7}$

Investment of A = $\frac{5}{(5+7)} \times 12000$ = Rs. 5000

Ex-17. Rs. 1290 is divided between A, B and C so that A's share is $1\frac{1}{2}$ times of B's and B's share is $1\frac{3}{4}$ times of C what is C's share ?

(A) Rs. 370 **(B)** Rs. 240

(C) Rs. 420 **(D)** Rs. 340

Sol.(B) A : B = $1\frac{1}{2} : 1$ = 3 : 2

B : C = $1\frac{3}{4} : 1$ = 7 : 4

A : B : C = 21 : 14 : 8

Share of C = $\frac{8}{21 + 14 + 8} \times 1290$ = Rs. 240

Ex-18. In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for whole time. Find A's share of the total profit of Rs. 6900.

(A) Rs. 300 **(B)** Rs. 600

(C) Rs. 900 **(D)** Rs. 100

Sol.(A) Capital of C = $1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}$

Let the total time be 12 months.

A's profit : B's profit : C's prifit = A : B : C

$= \frac{1}{6} \times \left(\frac{1}{6} \times 12\right) : \frac{1}{3} \times \left[\frac{1}{3} \times 12\right] : \frac{1}{2} \times 12$

$= \frac{1}{3} : \frac{4}{3} : 6$ = 1 : 4 : 18

Share of A = $\frac{1}{1 + 4 + 18} \times 6900$ = Rs. 300

Ex.19. A,B,C started a business investing capital Rs.12000, 15000 and 18000 respectively. A gets 25% of total annual income and remaining profit is distributed among A, B and C. At end of the year if A gets Rs. 1000 less than B and C together. Find the total profit ?

(A) Rs. 12000 **(B)** Rs. 15000

(C) Rs. 10000 **(D)** Rs. 18000

Sol.(C) Total Profit = 100%

Capital A : B : C = 12000 : 15000 : 18000

= 4 : 5 : 6

Profit A : B : C = 4 : 5 : 6

A's Profit (25% + 75%× $\frac{4}{15}$) = 45%

B's Profit (75% × $\frac{5}{15}$) = 25%

C's Profit $(75\% \times \frac{6}{15}) = 30\%$

(B+C)'s profit = (25% + 30%) = 55%

A = 45%

Difference = 10%

10% = 1000, 100% = 10000

Ex.20. A,B and C hired a house for 1 year in Rs.13825. They lived together for four months after that C left the house. After next five months B also left the house. Find the share of C in total rent ?

(A) Rs. 2212 **(B)** Rs. 2012

(C) Rs. 2112 **(D)** Rs. 2022

Sol.(A) A = 12 Months

$\therefore$ A : B : C = 12 : 9 : 4

B = 9 Months C = 4 Months

C's rent $= \frac{4}{25} \times 13825 =$ Rs. 2212

Ex.21. Amit and Deepak were partners in a family. Amit invested one-third of total capital for 8 months. After that he withdrew 50% of his capital. At the end of 2 years Amit received 2/7 of total profit. For how many months Deepak invested his capital?

(A) 34 **(B)** 24 **(C)** 12 **(D)** 20

Sol.(D) Ratio of capitals of Amit and Deepak = 1 : 2

Hence required time

$$\frac{1 \times 8 + \frac{1}{2} \times 16}{2 \times x} = \frac{2}{5}, \quad x = 20 \text{ Months}$$

So Deepak invested for 20 months

Ex.22. A,B and C are partner in a company. A gets 1/3 of total profit and B gets 1/4 of total profit. At the end of the year C gets Rs. 5000 of profit. Find the profit of A ?

(A) Rs. 5000 **(B)** Rs. 6000

(C) Rs. 4000 **(D)** Rs. 3000

Sol.(C) Share of "C" $= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$

So $\frac{5}{12}$ of profit = 5000

Total profit $= 5000 \times \frac{12}{5} =$ Rs. 12000

$\therefore$ A's share $= \frac{1}{3} \times 12000 =$ Rs. 4000

Ex.23. A and B started a business investing capital amount in the ratio 5 : 6. After 8 months A withdrew his capital. They recieved the profit in the ratio 5:9. Find the time taken by B in business ?

(A) 10 **(B)** 8 **(C)** 6 **(D)** 12

Sol.(D) Let B invest for y months

$$\frac{5 \times 8}{6 \times y} = \frac{5}{9},$$

y = 12 months

Ex.24. Some amount distributed among A, B and C. A received 3/16 of total amount and B received 1/4 of total amount. If C gets Rs. 81 then B gets amount is-

(A) Rs. 36 **(B)** Rs. 46 **(C)** Rs. 56 **(D)** Rs. 26

Sol.(A) C's part $= 1 - \frac{3}{16} - \frac{1}{4}$

$= \frac{16 - 3 - 4}{16} = \frac{9}{16}$

$\frac{9}{16}$ Part = Rs.81

so total amount $= \frac{81 \times 16}{9}$ = Rs. 144

B's Part $= \frac{1}{4} \times 144$ = Rs. 36

Ex.25. A and B started a business investing capital Rs. 36000 and Rs. 42000 for equal time. After 4 months C join them investing certain amount. At the end of 1 year C gets Rs. 8000 profit of total profit Rs. 39200. Find the capital invested by C ?

(A) Rs. 26659 **(B)** Rs. 26687

(C) Rs. 26689 **(D)** Rs. 30000

Sol.(D) Let C investment = x × 1000

A : B : C

36000×12 : 42000×12 : x×1000×8

36 × 12 : 42 × 12 : x × 8

432 : 504 : 8x

Profit share of A and B = 39200 - 8000 = 31200

$$\frac{936}{8x} = \frac{31200}{8000}$$

$$\frac{936}{x} = \frac{312}{10}$$

$$x = \frac{936 \times 10}{312} = 30$$

So C's investment = 30×1000 = Rs. 30,000

EXERCISE

Q.1. Aman started a business investing Rs. 70,000. Rakhi joined him after six months with an amount of Rs. 1,05,000 and Sagar joined them with Rs. 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Aman, Rakhi and Sagar respectively, 3 years after started the business ?

(A) 7 : 6 : 10 **(B)** 12 : 15 : 16
(C) 42 : 45: 56 **(D)** Cannot be determined

Q.2. A, B and C enter into a partnership. They invest Rs. 40,000, Rs. 80,000 and Rs. 1,20,000 respectively. At the end of the first year, B withdraws Rs. 40,000, while at the end of the second year, C withdraws Rs. 80,000. In what ratio will the profit be shared at the end of 3 years ?

(A) 2 : 3 : 5 **(B)** 3 : 4 : 7
(C) 4 : 5 : 9 **(D)** 4 : 5 : 7

Q.3. Shekhar started a business investing Rs. 25,000 in 1999. In 2000, he invested an additional amount of Rs. 10,000 and Rajeev joined him with an amount of Rs. 35,000. In 2001, Shekhar invested another additional amount of Rs. 10,000 and Jatin joined them with an amount of Rs. 35,000. What will be Rajeev's share in the profit of Rs. 1,50,000 earned at the end of 3 years from the start of the business in 1999 ?

(A) Rs. 45,000 **(B)** Rs. 50,000
(C) Rs. 70,000 **(D)** Rs. 75,000

Q.4. A and B entered into a partnership investing Rs. 16,000 and Rs. 12,000 respectively. After 3 months, A withdrew Rs. 5000 while B invested Rs. 5000 more. After 3 more months, C joins the business with a capital of Rs. 21,000. The share of B exceeds that of C, out of a total profit of Rs. 26,400 after one year by :

(A) Rs. 2400 **(B)** Rs. 3000
(C) Rs. 3600 **(D)** None of these

Q.5. A and B start a business with investments of Rs. 5000 and Rs. 4500 respectively. After 4 months, A takes out half of his capital. After two more months, B takes out one- third of his capital while C joins them with a capital of Rs. 7000. At the end of a year, they earn a profit of Rs. 5080. Find the share of each member in the profit.

(A) A - Rs. 1400, B - Rs. 1900, C - Rs. 1780
(B) A - Rs. 1600, B - Rs. 1800, C - Rs. 1680
(C) A - Rs. 1800, B - Rs. 1500, C - Rs. 1780
(D) A - Rs. 1680, B - Rs. 1600, C - Rs. 1800

Q.6. A, B, C subscribe Rs. 50,000 for a business. A subscribes Rs. 4000 more than B and B subscribes Rs. 5000 more than C. Out of a total profit of Rs. 20,000 A receives :

(A) Rs. 8400 **(B)** Rs. 11,900
(C) Rs. 13,600 **(D)** Rs. 14,700

Q.7. Three partners A, B, C start a business. Twice A's capital is equal to thrice B's capital and B's capital is four times C's capital. Out of a total profit of Rs. 16,500 at the end of the year, B's share is :

(A) Rs. 4000 **(B)** Rs. 8000
(C) Rs. 7500 **(D)** Rs. 6000

Q.8. A, B and C rent a pasture. A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing. If the rent of the pasture is Rs. 175, how much must C pay as his share of rent ?

(A) Rs. 45 **(B)** Rs. 50
(C) Rs. 55 **(D)** Rs. 60

Q.9. In a business, A and C invested amounts in the ratio of 2 : 1, whereas the ratio between amounts invested by A and B was 3 : 2. If Rs. 1,57,300 was their profit, how much amount did B receive?

(A) Rs. 24,200 **(B)** Rs. 36,300
(C) Rs. 48,400 **(D)** Rs. 72,600

Q.10. A and B started a business by investing Rs. 350000 and Rs. 140000 respectively. A gets 20% of the yearly profit for managing the business. There after the profit is divided in the ratio of the capital. If A receives totally Rs. 38000 more than B at the end of year, then the profit is-

(A) Rs 105000 **(B)** Rs 70000
(C) Rs 128000 **(D)** Rs 280000

Q.11. Ajay, Vikas and Manav enter into a partnership. Vikas invests some amount at the beginning. Vikas invests double the amount after 6 months and Manav invests thrice the amount invested by Ajay after 8 months. They earn a profit of Rs. 45,000 at the end of the year. What is Manav's share in the profit?

(A) Rs. 25,000 **(B)** Rs. 15,000
(C) Rs. 12,000 **(D)** Rs. 9,000

Q.12. A and B started a bussiness with initial investment in the ratio 5:6.If after one year their profits were in the ratio 1:2 and the period for A's investment was 6 months,B invested the money for?

(A) 6 months **(B)** 4 months

(C) 10 months **(D)** 16 months

Q.13. In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for the whole time. Out of a profit of Rs. 4600, B's share is :

(A) Rs. 650 **(B)** Rs. 800

(C) Rs. 960 **(D)** Rs. 1000

Q.14. A and B start a business jointly. A invests Rs. 16,000 for 8 months and remains B in the business for 4 months. Out of total profit, B claims $\frac{2}{7}$ of the profit. How much money was contributed by B ?

(A) Rs. 10,500 **(B)** Rs. 11,900

(C) Rs. 12,800 **(D)** Rs. 13,600

Q.15. Two friends P and Q started a business investing in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of Q's. At the end of the year, 20% profit was earned which was equal to Rs. 98,000. What was the amount invested by R?

(A) Rs. 1,05,000 **(B)** Rs. 1,75,000

(C) Rs. 2,10,000 **(D)** Data inadequate

Q.16. Arun started a business investing Rs. 45,000. Three months after he started. Promod joined him with a capital of Rs. 60,000. If at the end of the year, the total profit in the business is Rs. 26,000. What would be Pramod's share in it ?

(A) Rs. 12,000 **(B)** Rs. 14,000

(C) Rs. 13,000 **(D)** Rs. 16,000

Q.17. A started a business investing Rs.35,000. After six months B joined him with a capital of Rs. 60,000. At the end of the year the total profit was Rs. 26,000. What will be the difference between the share of profits of A and B ?

(A) Rs. 4,000 **(B)** Rs. 2,000

(C) Rs. 1,500 **(D)** Rs.3,000

Q.18. Rajesh, Ranjan and Jitendra invested Rs. 5000, Rs.10000 and Rs. 15000 respectively to start a business. Rajesh left the business after one year. If at the end of two years, they earned a profit of Rs.12001, What is Ranjan's share in the profit? (approx)

(A) Rs. 4634 **(B)** Rs. 3464

(C) Rs. 3446 **(D)** Rs. 4364

Q.19. Three persons Vikas, Rakesh and Sunil invested Rs.2000, 3000 and 4000 respectively in a business. They had a profit of Rs. 2700 at the end of the year. Find the difference between shares of Vikas and Sunil ?

(A) Rs.1200 **(B)** Rs. 600

(C) Rs. 750 **(D)** Rs. 700

Q.20. A and B started a business investing the money in ratio 4: 6, after 6 months B withdraw his investment and C joins him with twice the amount of B. At the end of the year total profit is 3315. Find share of C.

(A) 1530 **(B)** 1535

(C) 1695 **(D)** 1655

Q.21. Two persons Praveen and Suresh invested in a business with 115000 and 75000 rupees respectively. They agree that 40% of the profit should be divided equally among them and rest is divided between them according to their investment. If Praveen got 500 rupee more than Suresh, then the total profit is.

(A) Rs.3599.33 **(B)** Rs.3699.33

(C) Rs.3958.33 **(D)** Rs.3999.33

Q.22. Anil and Ruhi started a business by investing Rs.2000 and Rs.2800 respectively. After 8 months, Anil added Rs.600 and Ruhi added Rs.400. At the same time Teena joined them with Rs.4200. Find the share of Teena if they get a profit of Rs.34, 300 after a year.

(A) Rs.7350 **(B)** Rs.7530

(C) Rs.7550 **(D)** Rs.7500

Q.23. A and B entered into a partnership with Rs. 7000 and Rs. 6000 respectively. After 3 months, A withdrew 2/7 of his investment but after another 3 months, he put back 3/5 of what he had withdrawn. The profit of the end of the year is Rs. 7260. How much of this should A received?

(A) Rs. 3360 **(B)** Rs. 3660

(C) Rs. 6330 **(D)** Rs. 6630

Q.24. P and Q entered into a partnership investing Rs.16000 and Rs.12000 respectively. After 3 months P withdrew Rs.5000 while Q invested Rs.5000 more. After 3 more months R joins the business with a capital of Rs.21000. The share of Q exceeds that of R, out of a total profit of Rs.26488 after one year by?

(A) Rs.3162 **(B)** Rs.3126

(C) Rs.3216 **(D)** Rs.3612

EXPLANATION

Q.1.(B) Aman : Rakhi : Sagar = (70000 × 36) : (105000 × 30) : (140000 × 24) = 12 : 15 : 16.

Q.2.(B) A : B : C = (40000 × 36) : (80000 × 12 + 40000 × 24) : (120000 × 24 + 40000 × 12)
= 144 : 192 : 336 = 3 : 4 : 7.

Q.3.(B) Shekhar : Rajeev : Jatin
= (25000 × 12 + 35000 × 12 + 45000 × 12) : (35000 × 24) : (35000 × 12)
= 1260000 : 840000 : 420000 = 3 : 2 : 1

∴ Rajeev's share = Rs. $\left(150000 \times \frac{2}{6}\right)$
= Rs. 50000.

Q.4.(C) A : B : C = (16000 × 3 + 11000 × 9) : (12000 × 3 + 17000 × 9) : (21000 × 6)
= 147 : 189 : 126 = 7 : 9 : 6.
Difference of B's and C's shares
= Rs. $\left(26400 \times \frac{9}{22} - 26400 \times \frac{1}{22}\right)$
= Rs. 3600.

Q.5.(B) A : B : C = (5000 × 4 + 2500 × 8) : (4500 × 6 + 3000 × 6) : (7000 × 6) = 40000 : 45000 : 42000
= 40 : 45 : 42.

∴ A's share = Rs. $\left(5080 \times \frac{40}{127}\right)$
= Rs. 1600;

B's share = Rs. $\left(5080 \times \frac{45}{127}\right)$
= Rs. 1800;

C's share = Rs. $\left(5080 \times \frac{42}{127}\right)$
= Rs. 1680.

Q.6.(A) Let C = x. Then, B = x + 5000 and A = x + 5000 + 4000 = x + 9000.
So, x + x + 5000 + x + 9000 = 50000 ⇔ 3x = 36000 ⇔ x = 12000.
A : B : C = 21000 : 17000 : 12000 = 21 : 17 : 12.

∴ A's share = Rs. $20000 \times \frac{21}{50}$
= Rs. 8,400.

Q.7.(D) Let C = x. Then, B = 4x and 2A = 3 × 4x = 12x or A = 6x.
∴ A : B : C = 6x : 4x : x = 6 : 4 : 1.

So, B's capital = Rs. $\left(16500 \times \frac{4}{11}\right)$
= Rs. 6000.

Q.8.(A) A : B : C = 10 × 7 : 12 × 5 : 15 × 3 = 70 : 60 : 45 = 14 : 12 : 9.

∴ C's rent = Rs. $\left(175 \times \frac{9}{35}\right)$ = Rs. 45.

Q.9.(C) A : B = 3 : 2 ⇔ B : A = 2 : 3 = 4 : 6 and A : C = 2:1 = 6:3.
So, B : A : C = 4 : 6 : 3 or A : B : C = 6 : 4 : 3.

∴ B's share = Rs. $\left(157300 \times \frac{4}{13}\right)$
= Rs. 48400.

Q.10.(B) Ratio of profit = 350000 : 140000 = 5 : 2
If the total profit is x,
then A's share

$$= \frac{5}{7} \times \frac{4x}{5} + \frac{x}{5} = \frac{4x}{7} + \frac{x}{5} = \frac{20x + 7x}{35} = \frac{27x}{35}$$

and B's share = $\frac{2}{7} \times \frac{4x}{5} = \frac{8x}{35}$

$$\Rightarrow \frac{27x}{35} - \frac{8x}{35} = 38000$$

$$\frac{19x}{35} = 38000$$

∴ x = Rs. 70000

Q.11.(B) Let Ajay invests Rs. x
Amount invested by Vikas = 2x
Amount invested by Manav = 3x
Ratio of profit
x × 12 : 2x × 6 : 3x ×4
i.e. 1 : 1 : 1
∴ Manav's share = Rs. 15,000

Q.12.(C) Let investment of A and B respectively be 5x and 6x and period of B's investment be y months.

$\therefore \frac{5x \times 6}{6x \times y} = \frac{1}{2}$

⇒ y = 10 months

Q.13.(B) Suppose A invests Rs. $\frac{x}{6}$ for $\frac{y}{6}$ months.

Then, B invests Rs. $\frac{x}{3}$ for $\frac{y}{3}$ months.

C invests $\left[x - \left(\frac{x}{6} + \frac{x}{3}\right)\right]$ i.e., Rs. $\frac{x}{2}$ for y months.

∴ A : B : C

$= \left(\frac{x}{6}\times\frac{y}{6}\right):\left(\frac{x}{3}\times\frac{y}{3}\right):\left(\frac{x}{2}\times y\right) = \frac{1}{36}:\frac{1}{9}:\frac{1}{2}$

= 1 : 4 : 18.

Hence, B's share = Rs. $\left(4600\times\frac{4}{23}\right)$
= Rs. 800.

Q.14.(C) Let the total profit be Rs. x. Then, B

$= \frac{2x}{7}$ and A $= \left(x-\frac{2x}{7}\right) = \frac{5x}{7}$

So, A : B $= \frac{5x}{7}:\frac{2x}{7}$ = 5 : 2.

Let B's capital be Rs. y. Then, $\frac{16000\times 8}{y\times 4} = \frac{5}{2}$

$y = \left(\frac{16000\times 8\times 2}{5\times 4}\right) = 12800$

Q.15.(C) Let the total profit be Rs. z.

Then, 20% of z = 98000

$z = \left(\frac{98000\times 100}{20}\right) = 490000$

Let the capitals of P, Q and R be Rs. 5x, Rs. 6x and Rs. 6x respectively. Then,

(5x × 12) + (6x × 12) + (6x × 6)

= 490000 × 12

⇔ 168x = 490000×12

$\Leftrightarrow x = \left(\frac{490000\times 12}{168}\right) = 35000$

∴ R's investment = 6x = Rs.(6 × 35000)

= Rs. 210000.

Q.16.(C) Profit sharing ratio of Arun and Promod

Arun	: Promod
45000 × 12	: 60,000 × 9
540	: 540
1	: 1

Share of promod in profit Rs 26000 is

$= \frac{1}{2}\times 26000$

= Rs 13000/-

Q.17.(B) Ratio between equalent capitals

= 35000 ×12 : 60000 ×6

= 7 : 6

Required difference = $\frac{1}{13}\times 26000$ = 2000

Q.18.(D)

Profit sharing Ratio of Rajesh, Ranjan and Jitendra

Rajesh	: Ranjan	: Jitendra
5000	: 10,000	: 15000
× 1	: × 2	: × 2
5000 :	: 20,000	: 30,000
5 :	: 20	: 30
1 :	: 4	: 6

Share of Ranjan in profit of Rs 12000

= 12001 × $\frac{4}{11}$ = 4364

Q.19.(B) Profit ratio of Vikas, Rakesh and Sunil

= 2 × 12 : 3 × 12 : 4 × 12

= 2 : 3 : 4

Total profit (2 + 3 + 4) = 2700

then difference in profit of Vikas and Sunil (4 - 2)

$= \frac{2700\times 2}{9}$ = Rs. 600

Q.20.(A) $A:B:C = 4\times 12:6\times 6:12\times 6 = 4:3:6$

$\frac{3315}{4+3+6}\times 6 = 1530$

Q.21.(C) Let total profit be Rs.x

According to the question,

$\left[\frac{23}{38}-\frac{15}{38}\right]\times\left(\frac{60}{100}\right)\times x = 500$

x = 3958.33

Q.22.(A) Ratio of equivalent capitals

= (2000 × 8 + 2600 × 4): (2800 × 8 + 3200 × 4): 4200 × 4

= (16000 + 10400): (22400 + 12800): 4200 × 4

= 26400: 35200: 16800 = 33: 44: 21

Teena's profit share = $\frac{21}{98}\times 34300$ = Rs.7350

Q.23.(B) Investment by A : Investment by B

= [7000×3+(7000–2000)×3+(5000+1200)×6] : 6000 × 12

= 73200 : 72000 = 61 : 60

Profit of A = $\frac{61}{121}\times 7260$ = 3660

Q.24.(D) Ratio between equivalent capitals

= (16000 × 3 + 11000 × 9): (12000 × 3 + 17000 × 9): 21000 × 6

= 147000: 189000: 126000

= 49: 63: 42

Q's share– R's share

$= \frac{63-42}{63+42+49}\times 26488 = 3612$

CHAPTER-12
MIXTURE ALLIGATION

Scan the QR code to get video of this chapter.

GENERAL RULES

I. Alligation

It enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

II. Mean price

The cost price of a unit quantity of the mixture is called the mean price.

III. Basic Formula

If two ingredients are A and B are mixed of price x and y respectively, then the price of resultant mixture is M mean price. Then ratio are in which in ingredients are mixed is given by rule of alligation

$$R = \frac{M-y}{x-M}$$

The above formula can be respented as

We present as :

Ingredient A   Ingredient B
x   y
mean price M
(M-y) : (x-M)

Thus the required ratio is $R = \frac{M-y}{x-M} = \frac{y-M}{M-x}$

Ex. If the average age of students in a class is 16.0 years the average age of boys is 16.2 years and that of girls is 15.8 years. What is the ratio of the no. of boys to the girls ?

(A) 2 : 3   **(B)** 1 : 1
(C) 1 : 2   **(D)** 3 : 1

Sol.(B)

Boys   Girls
16.2   15.8
16.0
(16.0–15.8)   (16.2–16.0)
0.2 : 0.2
1 : 1

The ratio of Boys and Girls is 1 : 1

Ex. A man has 100 kg of rice. A part of which he sells at 10% profit and remaining at 5% loss. He gains 7% on whole. Find the quantity sold at 10% profit?

(A) 80 kg   **(B)** 85 kg
(C) 60 kg   **(D)** 70 kg

Sol.(A)

Ist part   IInd part
+10%   –5%
+7%
12%   3%
12 : 3
4 : 1

Quantity sold at 10% profit = $100\times\frac{4}{5}$ = 80 kg

Ex. The average salary of staff of company is Rs. 1200. If the average. salary of officers is Rs. 4600 and that of non-officers is Rs. 1100. If the number. of officers are 15. Find the number of non-officers?

(A) 410   **(B)** 570
(C) 510   **(D)** 360

Sol.(C)

Officers   Non-officers
4600   1100
1200
100   3400
1 : 34

No . of non-officers = 15×34 = 510

IV. Mixture of more than two elements

These question may seen a little tricky at first, but it is similar concept applied repeatedly in order to calculate final ratio of ingredients when mixture contains more than two ingredients.

1. Take two ingredients in such a way that first ingredient is lower than the mean value and the other one is HIGHER than mean value.

2. Calculate the ratio of ingredients.

3. Repeat for all possible pairs.

4. Final ratio is the ratio obtained from step 2 (if an ingredients is common in the ratios, add value for this particular ingredient)

Ex. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety of the ratio 1 :1 : 2 If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be :

(A) Rs. 169.50 **(B)** Rs. 170

(C) Rs.175.50 **(D)** Rs. 180

Sol.(C) Since first and second varieties are mixed in equal proportions, so their average price

= Rs. $\left(\frac{126+135}{2}\right)$ =Rs. 130.50

So, the mixture is formed by mixing two varieties, one at Rs. 130.50 per kg and the other at say, Rs. x per kg in the ratio 2 : 2, i.e., 1 : 1. We have to find x.

By the rule of alligation, we have :

130.50 x

153

2 2

1 : 1

Hence, $\frac{153-130.5}{x-153}=\frac{1}{1}$

x = 175.50 kg.

Ex. In what ratio must a person mix three kind of tea each of which has a price of 70,80 and 120 rupees perkg in such a way that the mixture costs him 100 rupees/kg

(A) 2 : 2 : 5 **(B)** 3 : 2 : 5

(C) 2 : 3 : 5 **(D)** 3 : 1 : 2

Sol.(A) Here the price of tea are 70, 80 and 120 and mean price is 100.

Here price lower than mean are 70 and 80 and price higher than the mean is 120. Thus possible pair are (70,120) (80,120)

For the first pair

I III

70 120

100

20 30

Similarly

II pair III pair

80 120

100

20 20

Thus find ratio

= I : II : III

= 20 : 20 : (30 + 20) = 2 : 2 : 5

V. Mixture containing 4 ingredients

Ex. In what ratio a shop owner mixes 4 types of rice worth Rs. 95, Rs. 60, Rs. 90 and Rs. 50 perkg so that he can make the mixture of these rice worth Rs. 80 perkg.

(A) 4 : 4 : 5 : 1 **(B)** 13 : 10 : 15 : 20

(C) 4 : 3 : 6 : 2 **(D)** 3 : 1 : 2 : 5

Sol.(A)

I II

95 60

80

20 15

4 : 3

II III

60 90

80

10 20

1 : 2

III IV

90 50

80

30 10

3 : 1

I : II : III : IV

= 4 : (3+1) : (2+3) : (1)

= 4 : 4 : 5 : 1

VI. Concept of replacement

Suppose a container contains a - units of liquid from which b -units are taken out and replaced by water. After k-operations, the quantity of pure liquid

$$= \left[a\left(1-\frac{b}{a}\right)^k\right] \text{ units.}$$

Ex. A container contains 50 kg of milk. From this container 5 kg of milk was taken out and replaced with water. This process was further repeated two times. How much milk is now contained by the container?

(A) 48.25 lit **(B)** 39.58 lit

(C) 38.56 lit **(D)** 36.45 lit

Sol.(D) Amount of milk left = $50\left(1-\frac{5}{50}\right)^3$

$= 50\left(1-\frac{1}{10}\right)^3$

$= 50\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}$ = 36.45 lit

Ex. 8 litres are drawn from a cask full of milk and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 :65. How much milk did the cask hold originally ?

(A) 18 litres **(B)** 24 litres

(C) 32 litres **(D)** 42 litres

Sol.(B) Let initial quantity of milk in a cask = x lit

So, After n process quantity of remaining milk

$$= \left[x\left(1-\frac{8}{x}\right)^4\right] \text{ lit.}$$

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81} \Rightarrow \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4 \Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$$

⇒3x - 24 = 2x⇒x = 24 lit.

EXAMPLES

Ex.1. In what ratio must a grocer mix two varieties of pulses costing Rs. 18 and 25/kg respectively so as to get mixture of Rs. 20.5 per/kg

(A) 3 : 5 **(B)** 5 : 9

(C) 9 : 5 **(D)** 7 : 3

Sol.(C) By alligation rule

I II
18 25
20.5
4.5 2.5

∴ I : II = 9 : 5

Ex.2. How many kilogram of sugar costing Rs. 9 per kg must be mixed with Rs. 27 per kg of sugar costing Rs. 7 per kg so that there may be a profit of 10% by selling the mixture at Rs. 9.24 per kg

(A) 67 kg **(B)** 63 kg

(C) 27 kg **(D)** 73 kg

Sol.(B) By alligation rule

C.P. of mixture = $\frac{9.24}{110}\times 100$

= Rs. 8.4 per kg

I III
9 7
8.4
1.4 0.6

7 : 3

weight of sugar costing Rs. 9

$= \frac{27}{3}\times 7$ = 63 kg

Ex.3. A merchant has 1200 kg of sugar part of which he sells at 10% profit and the rest at 18% profit. He gains 15% on the whole. The quantity sold at 10% profit?

(A) 450 kg. **(B)** 650 kg.

(C) 550 kg. **(D)** 750 kg.

Sol.(A) **By alligation rule**

I II
10% 18%
15%
3 5

Quantity sold at 10%

$= \frac{1200}{8} \times 3 = 450$ kg

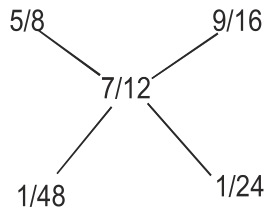
Ex.4. Milk and water are mixed in a vessel A in the ratio 5 : 3 and in vessel B in ratio 9 : 7. In what ratio should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 7 : 5

(A) 3 : 5 **(B)** 2 : 1

(C) 1 : 2 **(D)** 4 : 1

Sol.(C) **By alligation rule**

Milk in vessel I Milk in vessel II



Required ratio = 1 : 2

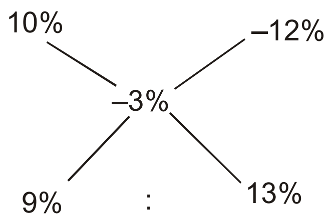
Ex.5. A merchant has 66 kg of suger. A part of this he sells at a gain of 10% and the remaining at a loss of 12%. On the total he loses 3%. What is the quantity sold at a loss of 12% ?

(A) 42 kg. **(B)** 39 kg.

(C) 52 kg. **(D)** 40 kg.

Sol.(B) **By alligation rule :**

Ist Part **11nd part**



ratio = 9 : 13

Quantity of sugar sold at 12% loss is,

$= \frac{66}{22} \times 13 = 39$ kg.

Ex.6. In what ratio water be mixed with milk so as to gain 25% by selling at c.p. ?

(A) 1 : 4 **(B)** 4 : 1

(C) 5 : 1 **(D)** 1 : 5

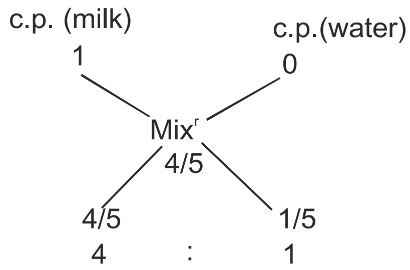
Sol.(B) By alligation

Let c.p. of pure milk =1

then s.p. of mixture =1

% profit = 25%

$(c.p.)_{mix} = \frac{1}{125} \times 100 = \frac{4}{5}$



Ratio between milk and water to gain 25% profit,

= 4 : 1

QUICK TRICK

In this type of questions we considered always milk 100 litre and water is equal to litre which is given of profit percent. That means milk = 100 litre

water 25 litre

Hence required ratio of milk and water.

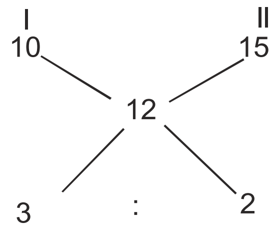
= 4 : 1

Ex.7. A goldsmith has some quantity of of gold. One of 10 carats and another of 15 carats purity. In what proportion should he mix both to make and an alloy of 12 carats purity?

(A) 4 : 2 **(B)** 5 : 3

(C) 2 : 3 **(D)** 3 : 2

Sol.(D) **By alligation**



= 3 : 2

Ex.8. Deepak purchased 30 kg of Rice at the rate of Rs. 17.50/kg and another 30 kg Rice at a certain rate. He mixed the two and sold the entire quantity at the rate of Rs. 18.60/kg and made 20% overall profit. At what price per kg. did he purchase the another 30 kg rice ?

(A) Rs. 12.50 **(B)** Rs. 11.50

(C) Rs. 10.50 **(D)** Rs. 13.50

Sol.(D) CP of mixture $= \frac{18.60}{120} \times 100 = 15.50$ Rs.

By alligation rule

I $\qquad\qquad$ II

17.50 $\qquad\qquad$ x

15.50

(15.50-x) $\qquad\qquad$ (17.50-15.50)

Ratio between Ist and IInd part = 1 : 1

then

$$\frac{17.50-15.50}{15.50-x}=\frac{1}{1}$$

$$\frac{2}{15.50-x}=\frac{1}{1}$$

x = 13.50 Rs.

Ex.9. If goods be purchased for Rupees 750 and $\frac{1}{3}$ part of goods be sold at the loss of 10%. What % of profit should be taken on the remaining goods so as to gain 20% on whole ?

(A) 30% $\qquad$ **(B)** 35/2%

(C) 35% $\qquad$ **(D)** 40%

Sol.(C) **By alligation rule**

1/3 : 2/3 $\Rightarrow$ 1 : 2

I $\qquad\qquad$ II

–10% $\qquad\qquad$ +x%

+20%

(x-20) $\qquad\qquad$ (20+10)

then $\frac{x-20}{20+10}=\frac{1}{2}$

2x–40 = 30

x = 35%

Ex.10. A mixture of 150 liters of wine and water contains 20% water. How much more water should be added so that water becomes 25% of the new mixture

(A) 15 lit. $\qquad$ **(B)** 18 lit.

(C) 10 lit. $\qquad$ **(D)** 17 lit.

Sol.(C) By alligation rule

in this type of question we apply alligation rule in two different ways

First when we take concentration of wine.

20% $\qquad\qquad$ 100%

25%

75 : 5

15 : 1

or we take concentration of water

80% $\qquad\qquad$ 0%

75%

75 : 5

15 : 1

quantity of added water = $\frac{150}{15}\times 1$=10 lit.

Ex.11. Two varieties of wheat A and B costing Rs. 18 per kg and Rs. 10 per kg were mixed in the ratio 1 : 3. If 5 kg of the mixture is sold at 25% profit, Find the profit made?

(A) Rs. 20 $\qquad$ **(B)** Rs.15

(C) Rs. 12 $\qquad$ **(D)** Rs. 14

Sol.(B) **By alligation rule**

18 $\qquad\qquad$ 10

x

x-10 $\qquad\qquad$ 18-x

$$\frac{x-10}{18-x}=\frac{1}{3}$$

On solving x = 12

Cost 5 kg = 5 × 12 = Rs. 60

Profit = $60\times\frac{25}{100}$=Rs.15

Ex.12. Two vessels A and B contain milk and water in the ratio 4 : 3 and 2 : 3. The ratio in which these mixtures be mixed to form a new mixture containing half milk and half water?

(A) 5 : 7 $\qquad$ **(B)** 7 : 5

(C) 5 : 3 $\qquad$ **(D)** 7 : 6

Sol.(B) **By alligation rule**

4/7 2/5

1/2

1/10 : 1/14

7 : 5

Ex.13. The average weekly salary per-head of the entire staff of a factory consisting of supervisors and the laboures is Rs. 60. The avarage salary per head of the supervisors is Rs. 400 and that of laboures is Rs. 56 given the number of supervisor is 12. Find the no. of laboures in the factory.

(A) 1020 **(B)** 1220

(C) 1120 **(D)** 1320

Sol.(A) **By alligation rule**

Average salary of labourers 56 Average salary of Supervisor 400

60

340 : 4

85 : 1

No of labourers in the factory

= 85 × 12 = 1020 labourers

Ex.14. A man having Rs. 8400 lent a part of it at 8% simple interest and the remaining at 20/3%. His total income of the 1.5 years was Rs. 882. Find the sum lent at 8%

(A) Rs. 2000 **(B)** Rs. 2100

(C) Rs. 1800 **(D)** Rs. 2300

Sol.(B) $R = \frac{100 \times 882 \times 2}{8400 \times 3} = 7\%$

By alligation rule

I 8% II 20/3%

7%

1/3 : 1

1 : 3

money lent at 8% = $\frac{8400}{4} \times 1 = \text{Rs. } 2100$

Ex.15. A man travelled a distance of 80 km in 7 hours partly on foot at the rate of 8 km/hr and partly on bicycle at 16 km/hr. Find the distance travelled on foot.

(A) 28 km. **(B)** 32 km.

(C) 30 km. **(D)** 35 km.

Sol.(B) **By alligation rule**

Average distance travelled in one hour

= 80/7 km

Distance (By foot) 8 Distance (By bicycle) 16

80/7

32/7 : 24/7

4 : 3

In this case we find ratio of time

Thus out of 7 hours in all he took 4 hour to travel on foot.

Distance covered on foot in 4 hours

= 4 × 8 = 32 km

Ex.16. A sum of Rs. 41 was divided among 50 boys and girls. Each boy gets 90 paise and a girl get 65 paise. Find the ratio of boys and girls.

(A) 17 : 8 **(B)** 15 : 8

(C) 18 : 8 **(D)** 14 : 8

Sol.(A) **By alligation rule**

Boys 90 Girls 65

82

17 : 8

Ratio of boys and girls = 17 : 8

Ex.17. One quality of wheat at Rs. 9.30 per kg is mixed with another quality at a certain rate in the ratio 8 : 7. If the mixture so formed be worth Rs. 10 per kg, what is the rate per kg of the second quality of wheat ?

(A) Rs. 10.30 **(B)** Rs. 10.60

(C) Rs. 10.80 **(D)** Rs. 11

Sol.(C) Let the rate of the second quality be Rs. x per kg.

By the rule of alligation, we have :

C.P. of 1 kg. wheat of 1st kind
930 paise

C.P. of 1 kg. wheat of 2nd kind
100 x paise

Mean Price
1000 P

(100x - 1000) P   70 P

$\therefore \frac{100x-1000}{70} = \frac{8}{7}$ = 700x - 7000 = 560

700x = 7560 $\Rightarrow$ x = Rs. 10.80

Ex.18. A grocer buys mustard oil at the rate of Rs. 34 a litre and vegetable oil at the rate of Rs. 30 a litre. He mixes the two in the ratio of 2 : 1 and sells the mixture at the rate of Rs. 36 a litre. What approximate profit percentage does he earn ?

(A) 10% **(B)** 8%

(C) 12% **(D)** 16%

Sol.(A) **BY ALLIGATION METHOD**

C.P of mixture = x Rs.

Mustard oil   Vegetable Oil
34   30

x

2   1

$\frac{34-x}{x-30} = \frac{1}{2}$

x = Rs. 32.67

Profit = 36 - 32.67 = 3.33

Profit Percent= $\frac{3.33}{32.67} \times 100$

= 10% (Approx.)

Ex.19. A man lent out Rs. 9600 partly at 12% and remaining at 14% simple interest. His total income after 1.5 years was Rs. 1800. Find the ratio of the sum lent at 12% and 14% simple interest respectively?

(A) 5 : 1 **(B)** 4 : 1

(C) 3 : 1 **(D)** Data inadequate

Sol.(C) Interest - on Rs.9600

for 1 year

= $\frac{1800}{1.5} \times 1$ = 1200

Average Interest Rate= $\frac{1200}{7600} \times 100\%$

= 12.5%

By Alligation

12%   14%

12.5%

1.5   0.5

Ratio = 3:1

Ex.20. A jar full of whisky contains 40% of alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :

(A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$

(C) $\frac{2}{3}$ **(D)** $\frac{3}{5}$

Sol.(C) Using the method of alligation.

40%   19%

26%

7 : 14

Required ratio = 7 : 14 = 1 : 2

$\therefore$ Required quantity = $\frac{2}{3}$

Ex.21. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk, the second contains 50% water. How much milk should be mixed from each of the containers so as to get 12 litres of mixture such that the ratio of milk to water is 5 : 3?

(A) 4 litres, 4 litres **(B)** 6 litres, 6 litres

(C) 5 litres, 7 litres **(D)** 7 litres, 5 litres

Sol.(B) Taking percentage of water and applying alligation rule

25   50

37.5

12.5   12.5

= 1 : 1

So to get 12 litre mixture we need to mix 6 litre from each solution.

Ex.22. A class has two sections, in one of which there are 40 students with an average of 14.5 years and the average of the class is 14.2 years. If there be 32 students in the other section, their average age is:

(A) 11 .32 years **(B)** 13.825 years

(C) 14.23 years **(D)** 9.21 years

Sol.(B) Given average age of 40 students = 14.5 years

(40 + 32) = average age of 72 students= 14.2 years

$\therefore$ The total age of 32 students

= (72 ×14.2 - 40×14.5) years

= (1022.4 - 580) years

= 442.4 years

$\therefore$ The average = 442.4/32 = 13.825 years

By alligation rule

I II
14.5 x
14.2
(14.2-x) : (14.5-14.2)

According to question

$$\frac{14.2-x}{14.5-14.2}=\frac{40}{32}$$

x = 13.825

Ex.23. Gold is 19 times as heavy as water and copper is 9 times as heavy as water. In what ratio should these be mixed to get an alloy 15 times as heavy as water ?

(A) 1 : 1 **(B)** 2 : 3

(C) 1 : 2 **(D)** 3 : 2

Sol.(D)

I Alloy II Alloy
19 9
15
6 4

Required ratio = 3 : 2

Ex.24. The total population of a city is 15000. The number of males and females increases by 10% and 15% respectively and consequently the population of the village becomes 16800. What was the number of males in the village ?

(A) 4000 **(B)** 6000

(C) 9000 **(D)** 5000

Sol.(C) Total % increased population

$$=\frac{16800-15000}{15000}\times 100$$

$$=\frac{1800}{15000}\times 100=12\%$$

By alligation method

Male Female
+10 +15
+12
3 : 2

⇒Male population before increment

Male = $\frac{15000}{5}\times 3$ = 9000 male

EXERCISE

Q.1. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 per kg. and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 per kg ?

(A) 3 : 7 **(B)** 5 : 7

(C) 7 : 3 **(D)** 7 : 5

Q.2. Find the ratio in which rice at. Rs. 7.20 a kg be mixed with rice at Rs.5.70 a kg to produce a mixture worth Rs. 6.30 a kg.

(A) 1 : 3 **(B)** 2 : 3

(C) 3 : 4 **(D)** 4 : 5

Q.3. In what ratio must. tea at Rs. 62 per kg be mixed with tea at Rs. 72 per kg so that the worth of mixture becomes Rs. 64.50 per kg ?

(A) 3 : 1 **(B)** 3 : 2

(C) 4 : 3 **(D)** 5 :3

Q.4. In what ratio must water be mixed with milk costing Rs.24 per litre to obtain a mixture worth Rs. 8 per litre ?

(A) 1 : 2 **(B)** 2 : 1

(C) 2 : 3 **(D)** 3 : 2

Q.5. The cost of Type 1 rice is Rs. 15 per kg. and Type 2 rice is Rs.20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3 then the price per kg of the mixed variety of rice is :

(A) Rs. 18 **(B)** Rs. 18.50

(C) Rs.19 **(D)** Rs. 19.50

Q.6. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10% ?

(A) 3 : 2 **(B)** 3 : 4

(C) 3 : 5 **(D)** 4 : 5

Q.7. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg ?

(A) 36 kg **(B)** 42 kg

(C) 54 kg **(D)** 63 kg

Q.8. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price ?

(A) 1 : 6 **(B)** 6 : 1

(C) 2 : 3 **(D)** 4 : 3

Q.9. A dishonest milkman claims to sell his milk at cost price but he mixes it with water and thereby gains 25% . The percentage of water in the mixture is :

(A) 4% **(B)** $16\frac{1}{4}\%$

(C) 28% **(D)** 20%

Q.10. Two vessels A and B contain spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these two be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5 ?

(A) 4 : 3 **(B)** 3 : 4

(C) 5 : 6 **(D)** 7 : 9

Q.11. Two vessels A and B contain milk and water mixed in the ratio 8 : 5 and 5 : 2 respectively. The ratio in which these two mixtures be mixed to get a new mixture containing $69\frac{3}{13}\%$ milk, is :

(A) 2 : 7 **(B)** 3 : 5

(C) 5 : 2 **(D)** 5 : 7

Q.12. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk, the second contains 50% water. How much milk should be mixed from each of the containers so as to get 12 litres of mixture such that the ratio of water to milk is 5 : 3?

(A) 4 litres, 4 litres **(B)** 6 litres, 6 litres

(C) 5 litres, 7 litres **(D)** 7 litres, 5 litres

Q.13. One quality of wheat at Rs. 9.30 per kg is mixed with another quality at a certain rate in the ratio 8 : 7. If the mixture so formed be worth Rs. 10 per kg, what is the rate per kg of the second quality of wheat ?

(A) Rs. 10.30 **(B)** Rs. 10.60

(C) Rs. 10.80 **(D)** Rs. 11

Q.14. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains

14% on the whole. The quantity sold at 18% profit is -

(A) 400 kg **(B)** 560 kg.

(C) 600 kg **(D)** 640 kg

Q.15. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$

(C) $\frac{2}{5}$ **(D)** $\frac{3}{5}$

Q.16. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container ?

(A) 26.34 litres **(B)** 27.36 litres

(C) 28 litres **(D)** 29.16 litres

Q.17. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 :65. How much wine did the cask hold originally?

(A) 18 litres **(B)** 24 litres

(C) 32 litres **(D)** 42 litres

Q.18. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

(A) 10 **(B)** 20

(C) 21 **(D)** 25

Q.19. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

(A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$

(C) $\frac{1}{5}$ **(D)** $\frac{1}{7}$

Q.20. Rs. 1000 is lent out in two parts, one at 6% simple interest and the other at 8% simple interest. The yearly income is Rs. 75. The sum lent at 8% is :

(A) Rs. 250 **(B)** Rs. 500

(C) Rs. 750 **(D)** Rs. 600

Q.21. A marchant has 50 kg. of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is :

(A) 20 kg. **(B)** 30 kg

(C) 15 kg. **(D)** 35 kg

Q.22. A mixture of 20 kg. of spirit and water contains 10% water. How much water must be added to this mixture to raise the percentage of water to 25% ?

(A) 4 kg **(B)** 5 kg

(C) 8 kg **(D)** 30 kg

Q.23. Kantilal mixes 80 kg. of sugar worth of Rs. 6.75 per kg. with 120 kg. worth of Rs. 8 per kg.At what rate should be sell the mixture to gain 20%?

(A) Rs. 7.50 **(B)** Rs. 9

(C) Rs. 8.20 **(D)** Rs. 8.85

Q.24. A man lent out Rs. 9600 partly at 12% and partly at 14% simple interest. His total income after 1.5 years was Rs. 1800. Find the ratio of the sum lent at 12% and 14% simple interest respectively?

(A) 5 : 1 **(B)** 4 : 1

(C) 3 : 1 **(D)** Data inadequate

Q.25. A grocer buys mustard oil at the rate of Rs. 34 a litre and repeseed oil at the rate of Rs. 30 a litre. He mixes the two in the ratio of 2 : 1 and sells the mixture at the rate of Rs. 36 a litre. What approximate profit percentage does he earn ?

(A) 10% **(B)** 8%

(C) 12% **(D)** 16%

Q.26. Madhulika deposited two parts of a sum of Rs. 25,000 in different banks at the rates of 15% per annum and 18% per annum respectively. In one year she got Rs. 4050 as the total interest. What was the amount deposited at the rate of 15% per annum ?

(A) Rs. 10,000 **(B)** Rs. 15,000

(C) Rs. 9,000 **(D)** Data inadequate

Q.27. By mixing two varieties of tea casting Rs. 65 and Rs. 115 per kg. and selling the mixture at the rate of Rs. 100 kg., a seller made a profit of 15%. In what ratio did he mix the two varieties ?

(A) 101 : 129 **(B)** 100 : 113

(C) 129 : 101 **(D)** 129:100

Q.28. In two mixture the ratio of milk and water are 3 : 4 and 5 : 8. If 14 lit. of first mixture and 26 lit. of second mixture are mixed, then find the ratio of milk and water in the new mixture ?

(A) 3 : 5 **(B)** 2 : 3

(C) 2 : 9 **(D)** 3 : 7

Q.29. A container contains 120 kg. milk. From this container, 12 kg. of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now there in the container?

(A) 87.48 kg. **(B)** 92.44 kg.

(C) 60.22 kg. **(D)** 83 kg.

Q.30. Two equal vessels are filled with the mixtures of water and milk in the ratio of 3: 4 and 5 : 3 respectively. If the mixtures are poured into a third vessel, the ratio of water and milk in the third vessel will be

(A) 15 :12 **(B)** 53 : 59

(C) 20 : 9 **(D)** 59 : 53

Q.31. In two types of brass ratio of copper and zinc are 8: 3 and 15: 7 respectively. The ratio in which these two types of brass should be mixed, so that the ratio of copper and zinc in this new type of brass become 5: 2 is-

(A) 5: 3 **(B)** 5: 2

(C) 7: 4 **(D)** 7: 3

Q.32. At southern sphere water is 75%. If on earth $\frac{5}{8}$ part is water. Find the ratio of land and water on northern sphere.

(A) 3 : 2 **(B)** 2 : 3

(C) 2 : 1 **(D)** 1 : 1

Q.33. A person has three varities of milk (a, b and c) whose cost prices are Rs. 48 per litre, Rs. 80 per litre and Rs. 90 per litre. In what ratio he should mix these three to get a mixture which on selling Rs. 72 per litre gives him a profit of 20%?

(A) 7 : 5 : 3 **(B)** 6 : 9 : 4

(C) 10 : 3 : 2 **(D)** 10 : 4 : 5

Notes

EXPLANATION

Q.1.(C) **By the rule of alligation:**

CP of 1st kind — CP of 2nd kind

15 — 20

16.50

3.50 — 1.50

$\therefore$ Required rate=3.50 :1.50 =35 : 15 = 7:3

Q.2.(B) By the rule of alligation:

CP of 1st kind — CP of 2nd kind

720 — 570

630

60 — 90

$\therefore$ Required rate = 60 : 90 = 2 : 3.

Q.3.(A) By the rule of alligation:

CP of 1st kind — CP of 2nd kind

6200 — 7200

6450

750 — 250

$\therefore$ Required rate = 750 : 250 = 3 : 1.

Q.4.(B) By the rule of alligation:

C.P. of 1 litre of water — C.P. of 1 litre of milk

0 — 24

8

16 — 8

$\therefore$ Ratio of water to milk = 16 : 8 = 2 : 1.

Q.5.(A) Let the price of the mixed variety be Rs. x per kg:

By the rule of alligation, we have :

CP of 1st kind — CP of 2nd kind

15 — 20

x

(20-x) — (x-15)

$\therefore \frac{(20-x)}{(x-15)} = \frac{2}{3}$

$\Rightarrow$ 60 - 3x = 2x - 30

$\Rightarrow$ 5x = 90 $\Rightarrow$ x = 18.

So, price of the mixture is Rs. 18 per kg.

Q.6.(A) S.P. of 1 kg of the mixture = Rs. 68.20, Gain= 10%.

C.P. of 1 kg of the mixture = Rs. $\left(\frac{100}{110} \times 68.20\right)$

=Rs. 62

By the rule of alligation, we have

CP of 1st kind — CP of 2nd kind

60 — 65

62

3 — 2

$\therefore$ Required ratio = 3 : 2

Q.7.(D) S.P. fo 1 kg of mixture = Rs. 9.24, Gain = 10%

$\therefore$ C.P. of 1 kg of mixture = Rs. $\left(\frac{100}{110} \times 9.24\right)$

=Rs. 8.40

By the rule fo alligation, we have :

CP of 1st kind — CP of 2nd kind

9 — 7

8.40

1.40 — 0.60

$\therefore$ Ratio of quantities of 1st and 2nd kind

= 14 : 6 = 7 : 3.

Let x kg of sugar of 1st kind be mixed with 27 kg of 2nd kind.

Then, 7 : 3 = x : 27 or x= $\left(\frac{7 \times 27}{3}\right)$

= 63 kg

Q.8.(A) Let C.P. of 1 litre milk be Re 1.

S.P. of 1 litre of mixture = Re. 1, Gain

= $\frac{50}{3}$%.

$\therefore$ C.P. of 1 litre of mixture = Rs. $\left(100 \times \frac{3}{350} \times 1\right)$

=Rs. $\frac{6}{7}$.

By the rule of alligation, we have :

C.P. of 1 litre of water — C.P. of 1 litre of milk

0 — 1

$\frac{6}{7}$

$\frac{1}{7}$ — $\frac{6}{7}$

$\therefore$ Ratio of water and milk = $\frac{1}{7}:\frac{6}{7}$ = 1 : 6.

Q.9.(D) Let C.P. of 1 litre be Rs 1.

Then, S.P. of 1 litre of mixture = Rs. 1, Gain

= 25%

C.P. of 1 litre mixture = Rs. $\left(\frac{100}{125}\times 1\right)$=Rs. $\frac{4}{5}$

CP of 1 litre milk Rs. 1 — CP of 1 litre of water 0

Mean: Rs. $\frac{4}{5}$

$\frac{4}{5}$ — $\frac{1}{5}$

Ratio of milk to water = $\frac{4}{5}:\frac{1}{5}$ = 4 : 1

Hence, percentage of water in the mixture

$=\left(\frac{1}{5}\times 100\right)\%$ = 20%

Q.10.(D) Let the C.P. of spirit be Rs. 1 per litre.

Spirit in 1 litre mixture of A = $\frac{5}{7}$ litre, C.P. of 1 litre mixture in A = Rs $\frac{5}{7}$.

Spirit in 1 litre mixture of B = $\frac{7}{13}$ litre, C.P. of 1 litre mixture in B = Rs. $\frac{7}{13}$.

Spirit in 1 litre mixture of C = $\frac{8}{13}$ litre

Mean price = Rs. $\frac{8}{13}$

By the rule of alligation, we have :

CP of 1 litre mixture in A $\left(\frac{5}{7}\right)$ — CP of 1 litre mixture in B $\left(\frac{7}{13}\right)$

Mean: $\left(\frac{8}{13}\right)$

$\left(\frac{1}{13}\right)$ — $\left(\frac{9}{91}\right)$

∴ Required Ratio = $\frac{1}{13}:\frac{9}{91}$
= 7 : 9

Q.11.(A) Let cost of 1 litre milk be Rs. 1.

Milk in 1 litre mixture in A = $\frac{8}{13}$ litre, C.P. of 1 litre mixture in A = Rs. $\frac{8}{13}$

Milk in 1 litre mixture in B = $\frac{5}{7}$ litre, C.P. of 1 litre mixture in B = Rs. $\frac{5}{7}$

Milk in 1 litre of final mixture

$=\left(\frac{900}{13}\times\frac{1}{100}\times 1\right)=\frac{9}{13}$ litre, Mean price = Re. 1

By the rule of alligation, we have:

CP of 1 litre mixture in A $\frac{8}{13}$ — CP of 1 litre mixture in B $\frac{5}{7}$

Mean: $\frac{9}{13}$

$\frac{2}{91}$ — $\frac{1}{13}$

∴ Required ratio = $\frac{2}{91}:\frac{1}{13}$ = 2 : 7

Q.12.(B) Let cost of 1 litre milk be Rs 1.

Milk in 1 litre mix. in 1st can = $\frac{3}{4}$ litre, C.P. of 1 litre mix in 1st can

Milk in 1 litre mix. in 2nd can = $\frac{1}{2}$ litre, C.P. of 1 litre mix. in 2nd can

Milk in 1 litre of final mix = $\frac{5}{8}$ litre, Mean price = Rs. $\frac{5}{8}$

By the rule of alligation, we have

CP of 1st kind $\frac{3}{4}$ — CP of 2nd kind $\frac{1}{2}$

Mean: $\frac{5}{8}$

$\frac{1}{8}$ — $\frac{1}{8}$

$\therefore$ Ratio of two mixtures = $\frac{1}{8} : \frac{1}{8}$ = 1 : 1

So, quantity of mixture taken from each can

$= \left(\frac{1}{2} \times 12\right)$ = 6 litres.

Q.13.(C) Let the rate of the second quality be Rs. x per kg.
By the rule of alligation, we have :

CP of 1st kind   CP of 2nd kind

930   100x

1000

100x -1000   70

$\therefore \frac{100x - 1000}{70} = \frac{8}{7}$ = 700x - 7000 = 560

$\Rightarrow$ 700x = 7560 $\Rightarrow$ x = Rs. 10.80

Q.14.(C) By the rule of alligation, we have

Profit on 1st part   Profit on 2nd part

8%   18%

14%

4%   6%

Ratio of 1st and 2nd parts = 4 : 6 = 2. 3.

$\therefore$ Quantity of 2nd kind = $\left(\frac{3}{5} \times 1000\right)$ kg
= 600 kg.

Q.15.(B) By the rule of alligation, we have :

Strength of 1st jar   Strength of 2nd jar

40%   19%

26%

7   14

So, ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2.

$\therefore$ Required replaced quantity = $\frac{2}{3}$

Q.16.(D) Amount of milk left after 3 operations

$= \left[40\left(1 - \frac{4}{10}\right)^3\right]$ litres

$= \left(40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right)$ = 29.16 litres.

Q.17.(B) Let the quantity of the wine in the cask originally be x litres.
Then, quantity of wine left in cask after 4 operations = $\left[x\left(1 - \frac{8}{x}\right)^4\right]$ litres.

$\therefore \frac{x \times \left(1 - \frac{8}{x}\right)^4}{x} = \frac{16}{81} \Rightarrow \left(1 - \frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4 \Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$

$\Rightarrow$ 3x - 24 = 2x $\Rightarrow$ x = 24

Q.18.(C) Suppose the can initially contains 7x and 5x litres of mixtures A and B respectively.
Quantity of A in mixture left

$= \left(7x - \frac{7}{12} \times 9\right)$ litres = $\left(7x - \frac{21}{4}\right)$ litres.

Quantity of B in mixture left = $\left(5x - \frac{5}{12} \times 9\right)$ litres = $\left(5x - \frac{15}{4}\right)$ litres.

$\therefore \frac{\left(7x - \frac{21}{4}\right)}{\left(5x - \frac{15}{4}\right) + 9} = \frac{7}{9} \Rightarrow \frac{28x - 21}{20x + 21} = \frac{7}{9}$

$\Rightarrow$ (252x-189)
= 140x + 147
$\Rightarrow$ 112x = 336 $\Rightarrow$ x =3
So, the can contained 21 litres of A.

Q.19.(C) Suppose the vessel initially contains 8 litres of liquid.
Let x litres of this liquid be replaced with water.
Quantity of water in new mixture

$= \left(3 - \frac{3x}{8} + x\right)$ litres.

Quantity of syrup in new mixture= $\left(5 - \frac{5x}{8}\right)$ litres.

$\therefore \left(3 - \frac{3x}{8} + x\right) = \left(5 - \frac{5x}{8}\right) \Rightarrow$ 5x + 24

= 40 - 5x $\Rightarrow$ 10x = 16 $\Rightarrow$ x = $\frac{8}{5}$

So, part of the mixture replaced

$= \left(\frac{8}{5} \times \frac{1}{8}\right) = \frac{1}{5}$.

Q.20.(C) Total interest = Rs. 75.

Average rate = $\left(\frac{100 \times 75}{1000 \times 1}\right)\% = 7\frac{1}{2}\%$

$\therefore$ (Sum at 6%) : (Sum at 8%) = $\frac{1}{2} : \frac{3}{2}$ = 1 : 3.

Money of 8% interest rate

= $1000 \times \frac{3}{4}$

= Rs. 750

Q.21.(B) 1st part profit (8%)   2nd part profit (18%)

Mean profit (14%)

4   6

Ratio of 1st and 2nd part = 4 : 6 = 2 : 3.

$\therefore$ Quantity sold at 18% = $\left(50 \times \frac{3}{5}\right)$ kg.

= 30 kg.

Q.22.(A) In first mixture:

water = $\left(\frac{10}{100} \times 20\right)$ kg. and spirit

= 18 kg.

In second mixture :

75 kg. spirit is contained in a mix. of 100 kg.

$\therefore$ 18 kg spirit is contained in a mix. of

= $\left(\frac{100}{75} \times 18\right)$ = 24 kg.

So, water to be added = (24 - 20) kg.

= 4 kg.

Q.23.(B) By the rule of alligation

CP of 1st kind   CP of 2nd kind

6.75   8

x

8-x   x-6.75

$\frac{8-x}{x-6.75} = \frac{80}{120} = \frac{2}{3}$

24 - 3x = 2x - 13.50

5x = 37.50

x = 7.5

Required answer = 120% of 7.5

=7.5 + 1.5 = Rs. 9 per kg.

Q.24.(C) Interest - on Rs.9600

for 1 year

= $\frac{1800}{1.5} \times 1$ = 1200

Average Interest Rate= $\frac{1200}{7600} \times 100\%$

= 12.5%

By Allegation

12%   14%

12.5%

3   1

Ratio = 3:1

Q.25.(A) By Alligation Method

C.P of mixture = x Rs.

Mustard oil   Rapeseed Oil

34   30

x

2   1

$\frac{34-x}{x-30} = \frac{1}{2}$

x = 32.67 Rs.

Profit = 36 - 32.67 = 3.33

Profit Percent= $\frac{3.33}{32.67} \times 100$ = 10% (Approx.)

Alternate Method :

Let the profit percentage be x

$\frac{2 \times 34 + 1 \times 30}{3} \times \frac{100 + x\%}{100} = 36$

$\frac{68 + 30}{3} \times \frac{100 + x\%}{100} = 36$

$100 + x\% = \frac{36 \times 100 \times 3}{98}$

100 + x% = 110.20

x% = 110.20 - 100 = 10.20 or 10

Q.26.(B) Rs. 15000/-

Interest at 15 % =3750 Rs.

Interest at 18 % = 4500 Rs.

Average Inetrest = 4050 Rs.

By Alligation Method

3750   4500

4050

450   300

Ratio 3:2

Sum invested on 15% = 25000 × $\frac{3}{5}$ = 15000 Rs.

Q.27.(C) Average Cost Price = 100 x 100 /115

By Allegation method

I II

65 115

$\frac{2000}{23}$

$\frac{645}{23}$: $\frac{505}{23}$

= 645 : 505 = 129 : 101

Q.28.(B) By the rule of alligation

$\frac{3}{7}$ $\frac{5}{13}$

x

$x-\frac{5}{13}$ $\frac{3}{7}-x$

$$\frac{x-\frac{15}{13}}{\frac{3}{7}-x}=\frac{14}{26}=\frac{7}{13}$$

$$\frac{13x-5}{3-7x}\times\frac{7}{13}=\frac{7}{13}$$

13x - 5 = 3 -7 x, 20x = 8

$x=\frac{2}{5}$, Required answer = 2 : 3

Q.29.(A) Out of x kg., say y kg. is taken out each time and replaced by water.

Then, after n operations, quantity of milk

$$=\left\{x\left(1-\frac{y}{x}\right)^{n}\right\}\text{ kg., }\Rightarrow\left\{120\left(1-\frac{12}{120}\right)^{3}\right\}$$

$$=120\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}=87.48\text{ kg.}$$

Q.30.(D) Taking part of water and applying alligation rule

$\frac{3}{7}$ $\frac{25}{8}$

x

$\frac{5}{8}-x$ $x-\frac{3}{7}$

According to questions

$$\frac{5}{8}-x=x-\frac{3}{7}$$

$$2x=\frac{5}{8}+\frac{3}{7}$$

$$x=\frac{35+24}{56}\times21$$

$x=\frac{59}{112}$, Required answer = 59 : 53

Q.31.(B) Quantity of copper in first type of brass = (8/11)

Quantity of copper in first type of brass = (15/22)

Quantity of copper in mixture = (8/11)

Now, using method of allegation

$\frac{8}{11}$ $\frac{15}{22}$

$\frac{5}{7}$

$\frac{5}{154}$ $\frac{1}{77}$

Required ratio = 5: 2

Q.32.(D) Southern Northern

75% x

62.5

62.5-x 12.5

As both spheres are equal

62.5-x = 12.5

x = 62.5 - 12.5 = 50%, Required answer = 1 : 1

Q.33.(C) CP of mixture = $\frac{72\times100}{120}=60$

By the rule of alligation

a b

48 80

60

20 12

= 5 : 3

a c

48 90

60

30 12 = 5 : 2

a : b : c = (5+5) : 3 : 2 = 10 : 3 : 2

CHAPTER-13
TIME AND WORK

Scan the QR code to get video of this chapter.

In most of the problems on time and work, one of the following basic parameters is to be calculated :

(a) Time : Time needed by one or more than one person to complete a job or time for which a person(s) actually worked on the assigned job.

(b) Alone time : Time needed by single person to complete a job.

(c) Work : The amount of total work (assigned) or the part of total work actually done.

BASIC CONCEPTS

(1) Total amount of a complete job (or assigned job) = 1, always, unless specified.

(2) If any person 'M' completes a job alone in t days, then alone time for 'M' = t

(3) 1 day's work by any person

$= \left(\frac{1}{\text{alone time}}\right)^{th} \times$ part of total work

(4) The reciprocal of 1 day's work gives the alone time i.e.

Alone time $\propto \frac{1}{1 \text{ day's work}}$

(5) When more than one person working on the same piece of work, then their combined 1 day's work = sum of 1 day's work by each person. i.e., If A, B and C are three persons working on a job, then (A+B+C)'s 1 day's work = A's 1 day's work + B's 1 day's work + C's 1 day's work.

(6) It is the application of concept (4) for more than one person.

The reciprocal of combined 1 day's work gives the time for completion by the person working together.

i.e., time for completion

$= \frac{1}{\text{combined 1 day's work}}$

It implies that.

If three persons, say, A, B and C are working together on a job, then

Time for completion by them

$= \frac{1}{(A+B+C)\text{'s 1 day's work}}$

IMPORTANT FORMULAS:

(i) If A can do a piece of work in X days and B can do the same work in Y days, then both of them working together will do the same work in

$\left(\frac{XY}{X+Y}\right)$ days

(ii) A,B,C while working alone can complete a work X,Y and Z days respectively then they will together complete the work in

$\frac{XYZ}{XY+YZ+ZX}$ days

Ex. A can do a piece of work in 6 days and B in 9 days. How many days will both take together to complete the work?

(A) 7.5 days **(B)** 5.4 days

(C) 3.6 days **(D)** 3 days

Sol.(C) **METHOD I:**

Here A = 6 and B = 9

They together complete work in 1 days

$= 1/6 + 1/9 = \frac{3+2}{18} = \frac{5}{18}$

They together complete the whole work $= \frac{18}{5}$ = 3.6 days

METHOD II (L.C.M METHOD) :

3 w/d ← A — +6 d

2 w/d ← B — +9 d

18 w

5 w/d ← A+B

They together complete the whole work $= \frac{18}{5}$

= 3.6 days

FORMULA METHOD:

They together complete the whole work

$= \frac{9 \times 6}{9+6}$ = 3.6 day

Ex. If man A complete a piece of work in 12 days and the same work is completed by B and C in 15 days and 20 days respectively. In how many days the work is completed if they work together ?

(A) 4 days **(B)** 8 days

(C) 5 days **(D)** 6 days

Sol.(C) **METHOD I :**

Work/ day of A,B and C = $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}$

$=\frac{5+4+3}{60}=\frac{1}{5}$

Total days = 5 days

METHOD II (L.C.M METHOD) :

5w/d ← A — 12 days
4w/d ← B — 15 days > L.C.M.= 60 work
3w/d ← C — 20 days

Total work/day by (A+B+C) = 12 work

Total work is done by (A+B+C) = $\frac{60}{12}$ =5 days

FORMULA METHOD :

They together complete the whole work

$=\frac{xyz}{xy+yz+zx}$

$=\frac{12\times15\times20}{12\times15+15\times20+20\times12}=5$ days

2. Two person A and B working together, can complete a piece of work in X days. If A alone can complete the work in Y days, then B alone will complete the work in $\left(\frac{XY}{X-Y}\right)$days

Ex. A and B together can do a piece of work in 5 days and A alone can do it in 8 days. B alone can do the same piece of work in (in days)

(A) $11\frac{1}{3}$ **(B)** $12\frac{3}{5}$ **(C)** $13\frac{1}{3}$ **(D)** $16\frac{4}{5}$

Sol.(C) **METHOD I :**

Here (A+B) = 5 days, A = 8 days

Part of work B can finish in one day $=\frac{1}{5}-\frac{1}{8}$

$=\left(\frac{8-5}{40}\right)$part

B finish the whole work = $\frac{40}{3}$ days

$=13\frac{1}{3}$ days

METHOD II. (L.C.M METHOD) :

8 w/d ← A+B- 5 day
5 w/d ← A - 8 day > 40 (L.C.M) Total work

3 w/d ← B
(One day work)

B finish the whole work = $\frac{40}{3}=13\frac{1}{3}$ days

FORMULA METHOD :

by formula

$=\left(\frac{XY}{X-Y}\right)=\frac{8\times5}{8-5}=13\frac{1}{3}$ days

3. If A and B working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days then.

A, B and C working together will finish the job in $\frac{2XYZ}{XY+YZ+ZX}$ days

Ex. A and B can complete a piece of work in 8 days B and C can do it in days 12 days, C and A can do it in 8 days, A B and C together can complete it in-

(A) 4 days **(B)** 5 days

(C) 6 days **(D)** 7 days

Sol.(C) Work done by A and B in one day = $\frac{1}{8}$

work done by B and C in one day= $\frac{1}{12}$

work done by C and A in 1 day = $\frac{1}{8}$

work done by A,B and C together in 1 day

$=\frac{1}{2}\left(\frac{1}{8}+\frac{1}{12}+\frac{1}{8}\right)=\frac{1}{6}$

A,B and C together complete the whole job =6 days

METHOD II (L.C.M METHOD) :

3 w/d ← (A+B) → 8 d
2 w/d ← (B+C) → 12 d > 24 work
3 w/d ← (C+A) → 8 d

8 w/d ← 2(A+B+C)
4 w/d ← (A+B+C)
one day work

Time taken by A, B and C together to complete the work = 24/4 = 6 days

FORMULA METHOD:

A + B and C can do finish the whole work

$$= \frac{2\times8\times12\times8}{8\times12+12\times8+8\times8} = \frac{2\times8\times12\times8}{96+96+64} = 6 \text{ day}$$

Ex. A can complete a piece of work in 20 days and B can complete the same work in 25 days. If they start the work together but After 5 days A left the work. In how many days the total work would be finished?

(A) $13\frac{3}{4}$ days **(B)** $18\frac{3}{4}$ days

(C) 11 days **(D)** 16 days

Sol.(B) **METHOD I:**

Work/day of A and B

$$= \frac{1}{20} + \frac{1}{25} = \frac{9}{100}$$

A and B's 5 day's work = $\frac{5\times9}{100}$ = 9/20

Remaining work = 1 - 9/20 = 11/20

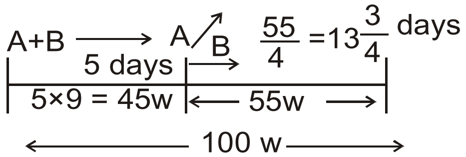
Since A left after 5 days hence remaining work done by B = 11/20

$$\frac{11/20}{\frac{1}{25}} = \frac{11}{20}\times\frac{25}{1} = 13\frac{3}{4} \text{ days}$$

So, total days = 5 + $13\frac{3}{4}$ = $18\frac{3}{4}$ days

METHOD II:

5w/d ← A — 20 days
4w/d ← B — 25 days } 100 work
9w/d ← A+B —



Total Time = $5+13\frac{3}{4} = 18\frac{3}{4}$ days

Ex. A and B can complete a piece of work in 24 days and 36 days respectively. They start the work but 3 days before completion of work, A left. In how many days will the total work be completed.

(A) $15\frac{1}{5}$ days **(B)** $13\frac{1}{5}$ days

(C) $16\frac{1}{5}$ days **(D)** $12\frac{1}{5}$ days

Sol.(C) **METHOD I :**

A's 1 day's work = 1/24

B's 1 days' work = 1/36

∵ A left before 3 days

∴ For last 3 days B's work = $\frac{3}{36} = \frac{1}{12}$

Remaining work = $1 - \frac{1}{12} = \frac{11}{12}$

Hence remaing work done by both so total days

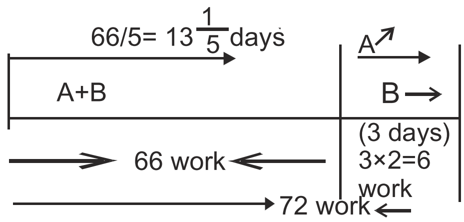
$$= \frac{\frac{11}{12}}{\frac{1}{24}+\frac{1}{36}} = \frac{\frac{11}{12}}{\frac{5}{72}} = 13\frac{1}{5}$$

So total days = $13\frac{1}{5}$ +3 = $16\frac{1}{5}$ days

METHOD II:

3 w/d ← A–24 day
2 w/d ← B–36 day } 72 (L.C.M) Total work

5 w/d ← A+B
(One day work)



Total days So total days = $13\frac{1}{5}$ +3 = $16\frac{1}{5}$ days

Ex. A can complete a piece of work in 9 days and B is 12 days respectively. If they work for a day alternately, In how many days the work would be finished, If A begins the work?

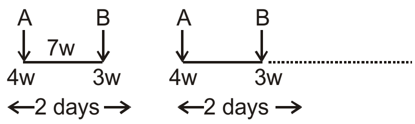
(A) $5\frac{1}{4}$ days **(B)** $17\frac{8}{9}$ days

(C) $10\frac{1}{4}$ days **(D)** 20 days

Sol.(C)

4w/d ← A - 9 days
3w/d ← B - 12 days } LCM = 36 work

Now,



7 w → 2 days

5 × 7w → 2 days ×5

35w → 10 days

Remaining work = 36 -35 = 1

Work will be done by A (because A starts the work)

$\therefore$ 1w $\rightarrow \frac{1}{4}$ days----------- (done by A)

$\therefore$ Total time= $\left(10+\frac{1}{4}\right)$ days = $10\frac{1}{4}$ days

4. Work efficiency of any worker is inversely proportional to time taken by him.

i.e, work efficiency $\alpha \frac{1}{\text{Time}}$

Ex. A is twice as good work man as B and together they can finish a piece of work in 18 days. In how many days A alone will finish the work ?

(A) 6 days **(B)** 12 days

(C) 54 days **(D)** 27 days

Sol.(D) A : B = 2 : 1

Time taken by A and B = 18 days

work done by A and B = 18 × 3 = 54 work

A will do alone = $\frac{54}{2}$ = 27 days

Ex. If the ratio of work efficiency of A and B is 6 : 5 and that of B and C is 6 : 5. If A can complete the piece of work in 2 days, In How many days the same work can be completed by B alone ?

(A) $2\frac{2}{5}$ days **(B)** $3\frac{1}{5}$ days

(C) $5\frac{4}{5}$ days **(D)** $5\frac{22}{25}$ days

Sol.(A) A : B = [6 : 5] × 6 = 36 : 30

B : C = [6 : 5] × 5 = 30 : 25

A : B : C = 36 : 30 : 25

time taken by A = 2 days

work done by A = 2 × 36 = 72 work

time taken by B = $\frac{72}{30} = 2\frac{2}{5}$ days

5. **MDH Formula:**

(i) More men can do more work

(ii) More work means more times required to do work

(iii) More men can do same work in less time

(iv) If M_1 can do W_1 work in D_1 days working H_1 hr/day for Rs. R_1 and M_2 man can do W_2 work in D_2 days working H_2 hr/day for Rs. R_2 then

Formula is given as:

$$\frac{M_1D_1H_1}{W_1R_1} = \frac{M_2D_2H_2}{W_2R_2}$$

M = no. of person

D = no. of days

H = no. of hours

W = work

R = Wadges (in Rs.)

Ex. If 10 men complete half work in 12 days when they work 8 hours per day, In how many days 18 men complete the full work when they work 6 hours/day?

(A) $17\frac{8}{9}$ days **(B)** $3\frac{1}{5}$ days

(C) $5\frac{4}{5}$ days **(D)** $17\frac{7}{9}$ days

Sol.(D) By formula :

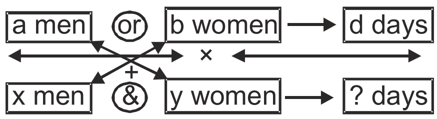
$$\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$$

$$\frac{10\times12\times8}{1/2} = \frac{18\times D_2\times6}{1}$$

$10 \times 12 \times 8 \times 2 = 18 \times D_2 \times 6$

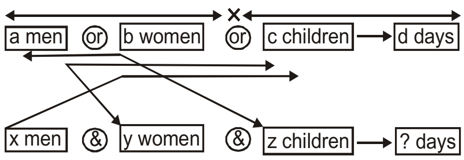
$D_2 = \frac{160}{9} = 17\frac{7}{9}$ days.

6.(i) If a men or b women can do a piece of work in d days then x men and y women together finish the whole work-



$$D = \frac{a\times b\times d}{xb+ya}$$

(ii) If a men or b women or c children can do a piece of work in d days then x men, y women and z children together finish the whole work-



$$D = \frac{a\times b\times c\times d}{xbc+yac+zab}$$

Ex. 3 Men or 4 Women can do a piece of work in 43 days. In how many days 7 men and 5 women can do the same work ?

(A) 12 days **(B)** $3\frac{1}{5}$ days

(C) $5\frac{4}{5}$ days **(D)** $17\frac{7}{9}$ days

Sol.(A) 3m or 4w = 43 days

7m + 5w = ?

From formula

$M_1 D_1 = M_2 D_2$

$3\text{ m} \times 43 = (7\text{m} + 5\text{w}) \times D_2$

$129\text{ m} = (7\text{m} + \frac{15}{4}\text{m}) \times D_2$

$129\text{ m} = \frac{43\text{m}}{4} \times D_2 \Rightarrow D_2 = 12$ days

QUICK TRICK :

$$D = \frac{a \times b \times d}{xb + ya}$$

$$D = \frac{3 \times 4 \times 43}{28 + 15} = 12 \text{ days}$$

Ex. 1 man or 2 boys or 3 girls can do a piece of work in 88 days. In how many days one man, one boy and one girl can do the same work ?

(A) 38 days **(B)** 48days

(C) 44 days **(D)** 58 days

Sol.(B) 1m = 2b = 3g = 88 days

(1m + 1b + 1g) = ?

By formula

$m_1 d_1 = m_2 d_2$

$3g \times 88 = (1m + 1b + 1g) \times d_2$

$3g \times 88 = (3g + \frac{3}{2}g + 1g)\, d_2$

$3g \times 88 = \left(\frac{6g + 3g + 2g}{2}\right) d_2$

$3 \times 88 = \frac{11}{2} \times d_2$

$d_2 = 48$ days

QUICK TRICK :

$$D = \frac{a \times b \times c \times d}{xbc + yac + zab}$$

$$D = \frac{1 \times 2 \times 3 \times 88}{6 + 2 + 3} = 48 \text{ days}$$

EXAMPLES

Ex.1. A can complete $\frac{1}{3}$ of total work in 12 days. B can complete $\frac{1}{4}$ of total work in 6 days. C can complete $\frac{1}{5}$ of total work in 8 days. In how days can A,B and C complete the half of the work together.

(A) $6\frac{25}{31}$ **(B)** $5\frac{5}{17}$ **(C)** $11\frac{19}{31}$ **(D)** $6\frac{19}{31}$

Sol.(B) A's $\frac{1}{3}$ of total work = 12 days

A's total work = 36 days

A's 1 days work = $\frac{1}{36}$ w/d

B's $\frac{1}{4}$ of total work = 6 days

B's total work = 24 days

B's 1 days work = $\frac{1}{24}$ w/d

C's $\frac{1}{5}$ of total work = 8 days

C's total work = 40 days

C's 1 days work = $\frac{1}{40}$ w/d

Hence 1 day work of A,B and C together

$= \frac{1}{36} + \frac{1}{24} + \frac{1}{40} = \frac{10 + 15 + 9}{360} = \frac{34}{360}$

Hence total no. of days by A, B and C together

$= \frac{360}{34}$ days $= 10\frac{10}{17}$ days

Hence time taken by A, B and C to do $\frac{1}{2}$ work

$= 5\frac{5}{17}$ days.

METHOD II. By L.C.M. Method:

10 w/d ← A 36 d
15 w/d ← B 24 d > 360 work
9 w/d ← C 40 d

31 w/d ← A+B+C

Time taken by (A,B and C) $\Rightarrow \frac{360}{34} = 10\frac{10}{17}$ days.

Time taken by A,B and C to do half work

$= 5\frac{5}{17}$ days.

FORMULA METHOD :

$$\frac{xyz}{(xy+yz+zx)} \times \frac{1}{2}$$

$$= \frac{1}{2} \times \left(\frac{36 \times 24 \times 40}{36 \times 24 + 24 \times 40 + 40 \times 36} \right)$$

$= 5\frac{5}{17}$ days.

Ex.2. A particular job can be completed by a team of 10 men in 12 days. The same job can be completed by a team of 10 women in 6 days. How many days are needed to complete the job if the both team work together.

(A) 2 days **(B)** 3 days

(C) 4 days **(D)** 5 days

Sol.(C) **METHOD I:**

According to question.

10 men's one day's work = $\frac{1}{12}$

$\therefore$ 1 man one day's work = $\frac{1}{12 \times 10} = \frac{1}{120}$

Similarly,

1 woman one day's work = $\frac{1}{6 \times 10} = \frac{1}{60}$

$\therefore$ (1m + 1w)'s one day's work = $\frac{1}{120} + \frac{1}{60}$

$= \frac{1}{40}$

$\therefore$ (10m + 10w)'s one day's work = $\frac{10}{40}$

Hence time taken by 10 men and 10 women

= 4 days

METHOD II. By L.C.M. Method:

$$\left.\begin{array}{l} 1\text{ w/d} \leftarrow 10\text{ m} — 12\text{ days} \\ 2\text{ w/d} \leftarrow 10\text{ w} — 6\text{ days} \end{array}\right\rangle 12\text{ work}$$

$$3\text{ w/d} \longleftarrow (10\text{ m} + 10\text{w})$$

No. of days $\Rightarrow \frac{12}{3} = 4$ days

Ex-3. Three men A,B and C working together can do a job in 6 hours less time than A alone, in 1 hour less time than B alone and in one half time needed by C when working alone. Then A and B together can do the job in.

(A) 3/4 hours **(B)** 5/4 hours

(C) 4/3 hours **(D)** 1/4 hours

Sol.(C) Let A,B and C together do the work in x hours. According to question

$$\frac{1}{x+6} + \frac{1}{x+1} + \frac{1}{2x} = \frac{1}{x}$$

$$\frac{1}{x+6} = \frac{1}{x} - \frac{1}{2x} - \frac{1}{x+1}$$

$$\frac{1}{x+6} = \frac{1-x}{2x^2+2x}$$

$2x^2 + 2x = x + 6 - x^2 - 6x$

$3x^2 + 7x - 6 = 0$

Solving this equation we get x = $\frac{2}{3}$

So time taken of A = 6+$\frac{2}{3}$

So time taken of B = 1+$\frac{2}{3}$

then time taken by Short Trick

A and B = $\frac{xy}{x+y} = \frac{\frac{20}{3} \times \frac{5}{3}}{\frac{20}{3} + \frac{5}{3}}$

$= \frac{\frac{100}{9}}{\frac{25}{3}} = \frac{100}{9} \times \frac{3}{25} = \frac{4}{3}$ hours.

Ex.4. 10 boys and 9 men can complete a work in 18 days. 12 boys and 6 men can complete the same work in 20 days. Find the ratio of efficiency of 1 boy and 1 man.

(A) 10:7 **(B)** 7:10 **(C)** 3:7 **(D)** 10:3

Sol.(B) By Formula

$m_1d_1 = m_2d_2$

(10B + 9M) × 18 = (12B + 6M) × 20

180B + 162M = 240B + 120M

42M = 60B

21M = 30B

7M = 10B

B:M = 7:10

Ex.5. 5 men and 8 boys can complete a work in 8 days. 6 men and 4 boys can complete the same work in 12 days. In how many days can 8 boys complete the same work.

(A) $10\frac{1}{2}$ days **(B)** $8\frac{1}{2}$ days

(C) $5\frac{3}{4}$ days **(D)** $6\frac{1}{2}$ days

Sol.(A) According to question : $m_1d_1 = m_2d_2$

$(5m + 8b) \times 8 = (6m + 4b) \times 12$

$40m + 64b = 72m + 48b$

$16b = 32m$

$1b = 2m$

Again,

$(5m + 8b)\times 8 = 8b \times d_2$

$(5m + 16m)\times 8 = 16b \times d_2$

$\frac{21\times 8}{16} = d_2$

$d_2 = 10\frac{1}{2}$ days.

Ex.6. A is twice efficient than B and thrice efficient than C. If B finishes a work in 12 days. In how many days will they complete the same work together.

(A) $3\frac{3}{11}$ days **(B)** $4\frac{3}{7}$ days

(C) $5\frac{2}{7}$ days **(D)** $6\frac{3}{7}$ days

Sol.(A) **METHOD I:**

Ratio of efficiency of A,B and C

Time ratio among A,B and C = 1 : 2 : 3

So let A can complete a work = x days

B can complete a work = 2x days

C can complete a work = 3x days

According to question:

$B = 2x \Rightarrow 2x = 12$

$x = 6$

So, A can complete a work = 6 days

B can complete a work = 12 days

C can complete a work = 18 days

Hence,

1 day work of A,B and C= $\frac{1}{6}+\frac{1}{12}+\frac{1}{18}=\frac{11}{36}$

So time taken by A,B and C= $\frac{36}{11} = 3\frac{3}{11}$ days

METHOD II:

$W_A : W_B = (2 : 1) \times 3 = 6 : 3$

$W_A : W_C = (3 : 1) \times 2 = 6 : 2$

$W_A : W_B : W_C = 6 : 3 : 2$

Total work of B = 3 × 12 = 36 work

Time taken by A, B and C

$= \frac{36}{11} = 3\frac{3}{11}$ days

Ex.7. A,B and C can complete a piece of work in 10 days, 15 days, and 20 days. A start the work but after 2 days. A replaced by B after next 3 days B replaced by C. If total wages of A,B and C is 2000. Find the wages of A.

(A) Rs. 500 **(B)** Rs. 400

(C) Rs. 300 **(D)** Rs. 600

Sol.(B) **METHOD I:**

$C = 1 - \frac{2}{10} - \frac{3}{15} = \frac{3}{5}$

Ratio of wages of A,B and C

$= \frac{2}{10} : \frac{3}{15} : \frac{3}{5} = 1 : 1 : 3$

According to question:

$x + x + 3x = 2000$, $5x = 2000$, $x = 400$

So wages of A = 400

METHOD II:

$$\left.\begin{array}{l} 6\ w/d \leftarrow A - 10d \\ 4\ w/d \leftarrow B - 15d \\ 3\ w/d \leftarrow C - 20d \end{array}\right\rangle 60 \text{ work}$$

A's work = 6 × 2 = 12

B's work = 4 × 3 = 12

C's work = 36 (Remaining)

So, Ratio of wages of A,B and C

= 12 : 12 : 36 = 1 : 1 : 3

Shared A in wages = $\frac{2000}{5}\times 1 = 400$

Ex.8. A certain number of persons were assigned to complete a piece of work in 55 days. If there were 6 persons more, the work could be finished in 11 days less. How many persons were originally there?

(A) 20 **(B)** 24 **(C)** 26 **(D)** 28

Sol.(B) Let in starting x persons were assigned to do a piece of work.

Then, According to formula:

$m_1d_1 = m_2d_2$

$x \times 55 = (x + 6)\times 44$

$x \times 5 = (x + 6)\times 4$

$5x = 4x + 24$

$x = 24$

Ex.9. A,B and C can complete a piece of work in 10, 12 and 15 days respectively. They started the work together but A left the work before 5 days of its completion. B also left the work 2 days after A left. In how days was the work completed.

(A) 4 days **(B)** 5 days

(C) 7 days **(D)** 8 days

Sol.(C) **METHOD I:**

Let the work be completed in x days.

According to question:

$$\frac{x-5}{10}+\frac{x-3}{12}+\frac{x}{15}=1$$

$$\frac{6x-30+5x-15+4x}{60}=1$$

15x - 45 = 60

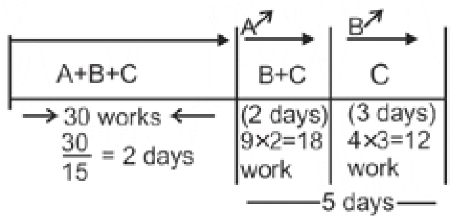
15x = 105

x = 7

METHOD II:

6 w/d A → 10 d
5 w/d B → 12 d
4 w/d C → 15 d
} 60 work

15 w/d ← A+B+C



Total days = (A+B+C) 5 days + (B+C) 5 days + C's alone days = 2 + 2 + 3 = 7 days

Ex.10. 40 men can complete a piece of work in 40 days. They started the work together but at the end of each 10th day, 5 men left the job, the work would have completed in:

(A) $56\frac{2}{3}$ days **(B)** $53\frac{1}{3}$ days

(C) 52 days **(D)** 50 days

Sol.(A) Total work = 40 × 40 = 1600

Ist 10 days work = 40×10 = 400

Next 10 days work = 35×10 = 350

Next 10 days work = 30×10 = 300

Next 10 days work = 25×10 = 250

Next 10 days work = 20×10 = 200

In 50 days ⇒ work = 1500

Remaining work = 1600 - 1500 = 100

Time for Remaining work

$$=\frac{100}{15}=6\frac{2}{3}\text{ days}$$

Total days = 50 + $6\frac{2}{3}$ = $56\frac{2}{3}$ days.

Ex.11. A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by BandC on every third day?

(A) 18 days **(B)** 25 days

(C) 15 days **(D)** 12 days

Sol.(C) (A+B+C)'s 1 day's work = $\frac{1}{20}+\frac{1}{30}+\frac{1}{60}$

$$=\frac{3+2+1}{60}=\frac{1}{10}$$

A's 2 days's work = $\frac{2}{20}=\frac{1}{10}$

Work done in first 3 days

$$=\frac{1}{10}+\frac{1}{10}=\frac{2}{20}=\frac{1}{5}$$

[A's work for 2 days + (A+B+C) work on 3rd day]

5 × 3 = 15

Hence the work will be finished in 15 days.

Ex.12. 16 women take 12 days to complete a piece of work which can be completed by 12 men in 8 days. 16 men started working and after 3 days 10 men left and 4 women joined them. How many days will it take to complete the remaining work?

(A) 4 days **(B)** 6 days

(C) 8 days **(D)** 10 days

Sol.(B) By formula:

$m_1d_1 = m_2d_2$

16w×12 = 12m×8

2w = 1m

Again

16w × 12 = 16m × 3 + (6m + 4w) ×d_2

8m × 12 = 16m × 3 + (6m + 2m) × d_2

96m - 48m = 8m × d_2

$$\frac{48}{8}=d_2$$

d_2 = 6 days

Ex.13. If 6 men and 8 boys can do a piece of work in 10 days and 26 men and 48 boys can do the same in 2 days, then time taken by 15 men and 20 boys to do the same type of work will be:

(A) 5 days **(B)** 4 days

(C) 6 days **(D)** 7 days

Sol.(B) By formula:

$m_1d_1 = m_2d_2$

(6m + 8b) × 10 = (26m + 48b)×2

60m + 80b = 52m + 96b

8m = 16b

1m = 2b

Again,

(6m + 8b) × 10 = (15m + 20b)×d_2

(6m + 4m) × 10 = (15m + 10m)×d_2

$\frac{100}{25} = d_2 \Rightarrow 4$ days

Ex.14. A wall of 100 metres can be built by 7 men or 10 women in 10 days. How many days will 14 men and 20 women take to build a wall of 600 metres?

(A) 15 days **(B)** 20 days

(C) 25 days **(D)** 30 days

Sol.(A) Given 7m or 10w $\Rightarrow$ 7m = 10w

By formula :

$m_1d_1 = m_2d_2$

$$\frac{7m \times 10}{100} = \frac{(14m + 20w) \times d_2}{600}$$

420m = (14m + 14m) × d_2

$\frac{420}{28} = d_2$, $15 = d_2$

Ex.15. If the ratio of work done by (x-1) men in (x+1) days to that of work done by (x+2) men in (x-1) days is 9:10 then the value of x is equal to:

(A) 5 **(B)** 6

(C) 7 **(D)** 8

Sol.(D) According to question:

$$\frac{m_1d_1}{m_2d_2} = \frac{9}{10}$$

$$\frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{9}{10}$$

10x + 10 = 9x+18, x = 8

Ex.16. One man, 3 women and 4 boys can do a piece of work in 96 hours, 2 men and 8 boys can do it in 80 hours 2 men and 3 women can do it in 120 hours, 5 men and 12 boys can do it in:

(A) $39\frac{1}{11}$ hr. **(B)** $42\frac{7}{11}$ hr.

(C) $43\frac{7}{11}$ hr. **(D)** 44 hr.

Sol.(C) **METHOD I :**

1 hours work of 1 man and 4 boys = $\frac{1}{160}$

1 hour work of 1 man 3 women and 4 boys = $\frac{1}{96}$

1 hour work of 3 women= $\frac{1}{96} - \frac{1}{160} = \frac{1}{240}$

1 hour work of 2 men= $\frac{1}{120} - \frac{1}{240} = \frac{1}{240}$

1 hours work of 4 boys = $\frac{1}{160} - \frac{1}{480} = \frac{1}{240}$

∴ 2 men = 3 women = 4 boys

∴ 2 men + 8 boys = 12 boys

5 men + 12 boys = 22 boys

$m_1h_1 = m_2h_2$

12×80 = 22×h_2

$h_2 = \frac{12 \times 80}{22} = 43\frac{7}{11}$ hr.

METHOD II :

5 w/h ← 1m+3w+4b → 96 h

6 w/h ← 2m+8b → 80 h

4 w/h ← 2m+3w → 120 h

480 work

5 m + 12b $\Rightarrow$?

Again

10 w/h $\Leftarrow$ 2m + 6w + 8b --------(i)

6 w/h $\Leftarrow$ 2m + 8b ---------- (ii)

(ii)–(i)

4 w/h $\Leftarrow$ 6w

2 w/h $\Leftarrow$ 3w

Hence, work/hr of 2m = 2w/h

So, 5m = 5 w/h

and w/h of 8b = 4 w/h

Now w/h of 5m + 12b $\Rightarrow$ 11 w/h

No of hr = $\frac{480}{11} = 43\frac{7}{11}$ hr.

Ex.17. 5 persons can prepare an admission list in 8 days working 7 hours a day. If 2 persons join then so as to complete the work in 4 days, they need to work per day for.

(A) 10 hours **(B)** 9 hours

(C) 12 hours **(D)** 8 hours

Sol.(A) By formula:

$m_1d_1 = m_2d_2$

$5m \times 8 \times 7 = 7m \times 4 \times h_2 \Rightarrow h_2 = 10$ hours

Ex.18. Ganga and Saraswati, working separately can mow a field in 8 and 12 hour respectively. If they work in stretches of one hour alternately and Ganga beginning at 9 am, when will the mowing be completed?

(A) 5:30 pm **(B)** 6:30 pm

(C) 7:30 pm **(D)** 6:00 pm

Sol.(B) Part of the field mowed by Ganga and Saraswati in first 2 hour

$= \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}$

Part of the field mowed in first 8 hours

$= \frac{5 \times 4}{24} = \frac{5}{6}$

Remaining work = $1 - \frac{5}{6} = \frac{1}{6}$

Now it is the turn of Ganga,

Part of work done by Ganga in1 hour $= \frac{1}{8}$

Remaining work = $\frac{1}{6} - \frac{1}{8} = \frac{4-3}{24} = \frac{1}{24}$

Now time taken by Saraswati in completing this part of work $= \frac{1}{24} \times 12 = \frac{1}{2}$ hours

Hence the mowing will be completed at 6:30 p.m.

Ex.19. A contractor undertakes to make a road in 40 days and employs 25 men. After 24 days, he finds that only one third of the road is made. How many extra men should be employ so that he is able to complete the work 4 days earlier.

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75

Sol.(D) By formula-

$$\frac{m_1d_1}{w_1} = \frac{m_2d_2}{w_2}$$

$$\frac{25m \times 24}{\frac{1}{3}} = \frac{(25+x) \times m \times 12}{\frac{2}{3}}$$

$1800m = 25 \times 18 + 18x$ m

$1800 - 450 = 18x$

$x = \frac{1350}{18} = 75$ person.

Ex.20. If 4 men or 6 women can do a piece of work in 12 days working 7 hours a day, how many days will it take to complete a work twice as large with 10 men and 3 women working together 8 hours a day?

(A) 6 days **(B)** 7 days

(C) 8 days **(D)** 10 days

Sol.(B). By formula given (4m = 6w)

$m_1d_1h_1 = m_2d_2h_2$

$$\frac{4m \times 12 \times 7}{1} = \frac{(10m + 3w) \times d_2 \times 8}{2}$$

$84 \times 4m = (10m + 2m) \times 4 \times d_2$

$7 = d_2$

Ex.21. 6 men can complete a piece of work in 12 days, 8 women can complete the same piece of work in 18 days where as 18 children can complete the piece of work in 10 days. 4 men, 12 women, and 20 children work together for 2 days. If only men were to complete the remaining work in 1 day. How many more men would be required totally?

(A) 28 Men **(B)** 32 Men **(C)** 30 Men **(D)** 40 Men

Sol.(B) By formula:

$m_1d_1 = m_2d_2 \pm m_3d_3$

$6m \times 12 = 8w \times 18 = 18c \times 10$

$72m = 144w = 180c$

$2m = 4w = 5c$

Again

$6m \times 12 = (4m + 12w + 20c) \times 2 + (4+x)\, m \times 1$

$72m = (4m + 6m + 8m) \times 2 + (4+x)\, m \times 1$

$72m = 36m + 4m + xm, \quad x = 32$

Ex.22. A sum of Rs.2500 was paid for a work which A can do in 32 days, B in 20 days, BandC in 12 days, and D in 24 days. How many did C receive if all the four work together?

(A) Rs.$666\frac{2}{3}$ **(B)** Rs.$533\frac{1}{3}$

(C) Rs.$524\frac{5}{7}$ **(D)** Rs.$233\frac{1}{3}$

Sol.(B) C's 1 day's work = $\frac{1}{12} - \frac{1}{20} = \frac{1}{30}$

Ratio of their efficiency

$= \frac{1}{32} : \frac{1}{20} : \frac{1}{30} : \frac{1}{24} = 15 : 24 : 16 : 20$

C's share $= \frac{16}{24 + 24 + 16 + 20} \times 2500 = \frac{1600}{3}$

$= \text{Rs. } 533\frac{1}{3}$

Ex.23. If a man 30% less efficient than woman and take 30 days to complete the work. In how many days can both complete the same work.

(A) 23 days **(B)** 21 days **(C)** 24 days **(D)** 18 days

Sol.(B) **METHOD : I**

Ratio of work efficiency

$W_M : W_W = 70 : 100 = 7 : 10$

Ratio of time M and W = 10 : 7

that means m takes time = 10x

women takes time = 7x

So according to question

10x = 30, x = 3

Men's days = 30

Men's 1 day work = $\frac{1}{30}$

Women's days = 21

Women's 1 day work = $\frac{1}{21}$

Hence total days = 21 days

METHOD : II

$W_M : W_W = 70 : 100 = 7 : 10$

Work done of men = 7 × 30 = 210 work

Time taken by women = $\frac{210}{10}$ = 21 days

Ex.24. A can do a pice of work in 10 days and B in 20 days. If they together work on it for 5 days, then the fraction of the work that is left is-

(A) $\frac{3}{4}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{20}$

Sol.(B) **METHOD : I**

A's one day work = $\frac{1}{10}$

B's one day work = $\frac{1}{20}$

(A+B)'s one day's work = $\frac{1}{10} + \frac{1}{20} = \frac{2+1}{20} = \frac{3}{2}$

(A+B) days work = $\frac{3}{20} \times 5 = \frac{3}{4}$ work

Remaining work = $1 - \frac{3}{4} = \frac{1}{4}$

METHOD : II

2 w/d ← A - 10 d
1 w/d ← B - 20 d > 20 work
3 w/d ← A+B

5 days →
| A + B
3 × 5 = 15
work | 5 work |
→ 20 work →

Remaining part of work = $\frac{5}{20} = \frac{1}{4}$

Ex.25. A,B and C can do a piece of work in 24, 30 and 40 days respectively. They begain the work together but C left 4 days before completion of the work. In how many days was the work done alone?

(A) 11 days **(B)** 12 days
(C) 13 days **(D)** 14 days

Sol.(A) **METHOD : I**

A's 1 day work = $\frac{1}{24}$

B's 1 day work = $\frac{1}{30}$

C's 1 day work = $\frac{1}{40}$

For last 4 days (A+B)'s work = $4\left(\frac{1}{24} + \frac{1}{30}\right)$

$= 4 \times \frac{(5+4)}{20} = \frac{9}{30}$

Remaining work done = $1 - \frac{9}{30} = \frac{21}{30}$

Time taken by all = $\dfrac{\frac{21}{30}}{\frac{1}{24} + \frac{1}{30} + \frac{1}{40}}$

$= \dfrac{\frac{7}{10}}{\frac{5+4+3}{120}} = 7 \text{ days}$

Total time = 4 + 7 = 11 days

METHOD : II

5 w/d ← A 24 d
4 w/d ← B 30 d > 120 work
3 w/d ← C 40 d

12 w/d ← A+B+C

| ← A+B+C → | C ↗ → A+B → |
| Remaining work
= 120-36 = 84
Time Taken by
A+B+C = $\frac{84}{12}$ = 7 days | → 4 days →
9×4= 36 work |

Total days = 7 + 4 = 11 days

EXERCISE

Q.1. A, B and C can complete a piece of work in 24, 6 and 12 days respectively. They will complete the same work in :

(A) 3 days **(B)** $\frac{7}{24}$ days

(C) $3\frac{3}{7}$ days **(D)** 4 days

Q.2. A can lay railway track between two given stations in 16 days and B can do the same job in 12 days. With the help of C, they did the job in 4 days only. Then C alone can do the job in :

(A) $9\frac{1}{5}$ days **(B)** $9\frac{2}{5}$ days

(C) $9\frac{3}{5}$ days **(D)** 10 days

Q.3. A takes twice as much time as B and thrice as much time as C to finish a piece of work. Working together, they can finish the work in 2 days. A can do the work alone in :

(A) 4 days **(B)** 6 days

(C) 8 days **(D)** 12 days

Q.4. Rohan and Sohan are working on an assignment. Rohan takes 6 hours to type 32 pages on a computer, while Sohan takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages ?

(A) 7 hours 30 minutes **(B)** 8 hours

(C) 8 hours 15 minutes **(D)** 8 hours 25 minutes

Q.5. Two workers A and B are engaged to do a work. A working alone takes 8 hours more to complete the job than if both worked together. If B worked alone, he would need $4\frac{1}{2}$ hours more to complete the job than they both working together. What time would they take to do the work together ?

(A) 4 hours **(B)** 5 hours

(C) 6 hours **(D)** 7 hours

Q.6. P can complete a work in 12 days working 8 hours a day. Q can complete the same work in 8 days working 10 hours a day. If both P and Q work together, working 8 hours a day, in how many days can they complete the work ?

(A) $5\frac{5}{11}$ **(B)** $5\frac{6}{11}$

(C) $6\frac{5}{11}$ **(D)** $6\frac{6}{11}$

Q.7. A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it ?

(A) 8 hours **(B)** 10 hours

(C) 12 hours **(D)** 24 hours

Q.8. Rakhi can do a piece of work in 20 days. Tanya is 25% more efficient than Rakhi. The number of days taken by Tanya to do the same piece of work is :

(A) 15 **(B)** 16

(C) 18 **(D)** 25

Q.9. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete the work, how much time shall B take to do it ?

(A) 30 days **(B)** 35 days

(C) 40 days **(D)** 20 days

Q.10. Ram and Mohan can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days Mohan had to leave and Sohan alone completed the remaining work. The whole work was completed in :

(A) 8 days **(B)** 10 days

(C) 12 days **(D)** 15 days

Q.11. A can finish a work in 24 days, B in 9 days and C in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in :

(A) 5 days **(B)** 6 days

(C) 10 days **(D)** $10\frac{1}{2}$ days

Q.12. Machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print the same number of books in 12 hours. All the machines are started at 9 a.m. while machine P is closed at 11 a.m. and the remaining two machines complete the work. Approximately at what time will the work be finished ?

(A) 11:30 a.m. **(B)** 12 noon

(C) 12:30 p.m. **(D)** 1 p.m.

Q.13. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last ?

(A) 6 days **(B)** 10 days
(C) 15 days **(D)** 20 days

Q.14. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job?

(A) 40 **(B)** 50
(C) 54 **(D)** 60

Q.15. If 5 men or 9 women can do a piece of work in 19 days. 3 men and 6 women will do the same work in how many days ?

(A) 10 days **(B)** 12 days
(C) 13 days **(D)** 15 days

Q.16. If 1 man or 2 women or 3 boys can do a piece of work in 44 days, then the same work will be done by 1 man, 1 woman and 1 boy in how many days:

(A) 21 days **(B)** 24 days
(C) 26 days **(D)** 33 days

Q.17. P can complete $\frac{1}{4}$ of a work in 10 days, Q can complete 40% of the same work in 15 days, R, $\frac{1}{3}$ of the work in 13 days and S, $\frac{1}{6}$ of the work in 7 days. Who will be able to complete the work first?

(A) P **(B)** Q
(C) R **(D)** S

Q.18. A is four times more efficient than B, C takes twice as many days as B to do a piece of work.If A can finish a job in 12 days. In how many days can A, B and C working together do the same job?

(A) $6\frac{2}{3}$ **(B)** $7\frac{2}{11}$
(C) $6\frac{1}{3}$ **(D)** $8\frac{8}{11}$

Q.19. 4 boys and 5 girls can do a piece of work in 10 days. 6 boys and 6 girls can do the same work in 7 days. In how many days can 2 boys and 7 girls complete the same works working together?

(A) 14 **(B)** 15
(C) 18 **(D)** 21

Q.20. A is 50% as efficient as B.C does half of the work done by A and B together. If C alone does the work in 20 days, then A, B and C together can do the work in

(A) $5\frac{2}{3}$ days **(B)** $6\frac{2}{3}$ days
(C) 6 days **(D)** 7 days

Q.21. Amit and Sujit together can complete an assignment of data entry in 5 days. Sujit's speed is 80% of Amit's speed and the total key deprssions in the assignment are 576000. What is Amit's speed in key depressions per hour if they work for 8 hours a day?

(A) 4800 **(B)** 6400
(C) 8000 **(D)** 7200

Q.22. A and B together can do $\frac{11}{19}$ of a work. In the same time B and C together can do $\frac{14}{19}$ of the same work. The ratio of work done by A, B and C is –

(A) 3: 4: 5 **(B)** 4: 5: 7
(C) 5: 7: 8 **(D)** 5: 6: 8

Q.23. If 10 men or 20 women or 40 children can do a piece of work in 7 days, then 5 men, 5 women and 5 children together can do half of the work in

(A) 6 days **(B)** 4 days
(C) 5 days **(D)** 8 days

Q.24. 15 men or 24 women or 36 boys can do a piece of work in 12 days, working 8 hours a day, how many men must be associated with 12 women and 6 boys to do another piece of work $2\frac{1}{4}$ times as great in 30 days working 6 hours a day?

(A) 12 **(B)** 15
(C) 10 **(D)** 8

Q.25. P and Q can finish a work in 20, 15 days respectively. Q starts the work and leaves it after 5 days. The number of days in which P can complete the work is.

(A) $7\frac{1}{3}$ days **(B)** $13\frac{1}{2}$ days
(C) $13\frac{1}{3}$ days **(D)** $15\frac{1}{3}$ days

EXPLANATION

Q.1.(C) Formula

(A+B +C)'s work together

$= \frac{A \times B \times C}{(AB + BC + CA)}$

(A+B +C)'s 1 day's work

$= \left(\frac{1}{24} + \frac{1}{6} + \frac{1}{12}\right) = \frac{7}{24}$

So, A, B and C together will complete the job in $\frac{24}{7} = 3\frac{3}{7}$ days.

Q.2.(C) (A+B+C)'s 1 day's work = $\frac{1}{4}$, A's 1 day's work

$= \frac{1}{16}$' B's 1 day work = $\frac{1}{12}$

$\therefore$ C's 1 day's work = $\frac{1}{4} - \left(\frac{1}{16} + \frac{1}{12}\right)$

$= \left(\frac{1}{4} - \frac{7}{48}\right) = \frac{5}{48}$

So, C alone can do the work in $\frac{48}{5} = 9\frac{3}{5}$ days.

Q.3.(D) Suppose A, B and C take x, $\frac{x}{2}$ and $\frac{x}{3}$ day respectively to finish the work

Then , $\left(\frac{1}{x} + \frac{2}{x} + \frac{3}{x}\right) = \frac{1}{2} \Rightarrow \frac{6}{x} = \frac{1}{2} \Rightarrow$ x = 12

So, A takes 12 days to finish the work.

Q.4.(C) Number of pages typed by Rohan in 1 hour

$= \frac{32}{6} = \frac{16}{3}$

Number of pages typed by Sohan in 1 hour

$= \frac{40}{5} = 8$

Number of pages typed by both in 1 hour

$= \left(\frac{16}{3} + 8\right) = \frac{40}{3}$

$\therefore$ Time taken by both to type 110 pages

$= \left(110 \times \frac{3}{40}\right)$ hrs = $8\frac{1}{4}$ hrs = 8 hrs 15 min.

Q.5.(C) Let A and B together take x hours to complete the work. Then,

A alone takes (x+8) hrs and B alone takes $\left(x + \frac{9}{2}\right)$ hrs to complete the work

$\frac{1}{(x+8)} + \frac{1}{\left(x + \frac{9}{2}\right)}$

$= \frac{1}{x} \Rightarrow \frac{1}{(x+8)} + \frac{2}{(2x+9)} = \frac{1}{x}$

$\Rightarrow$ x(4x+25) = (x+8)

$\Rightarrow 2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow$ x = 6 hours

Q.6.(A) P can complete the work in (12 × 8) hrs. = 96 hrs.

Q can complete the work in (8 × 10) hrs. = 80 hrs.

$\therefore$ P's 1 hour's work = $\frac{1}{96}$ and Q's 1 hour's work $= \frac{1}{80}$.

(P + Q)'s 1 hour's work

$= \left(\frac{1}{96} + \frac{1}{80}\right) = \frac{11}{480}$.

So, both P and Q will finish the work in $\left(\frac{480}{11}\right)$ hrs.

$\therefore$ Number of days of 8 hours each

$= \left(\frac{480}{11} \times \frac{1}{8}\right) = \frac{60}{11}$ days $= 5\frac{5}{11}$ days

Q.7.(C) A's 1 hour's work = $\frac{1}{4}$'(B+C)'s 1 hour's work

$= \frac{1}{3}$' (A+C)'s 1 hour's work = $\frac{1}{2}$.

(A+B+C)'s 1 hour's work = $\left(\frac{1}{4} + \frac{1}{3}\right) = \frac{7}{12}$.

B's 1 hour's work = $\left(\frac{7}{12} - \frac{1}{2}\right) = \frac{1}{12}$

$\therefore$ B alone will take 12 hours to do the work.

Q.8.(B) Ratio of times taken by Rakhi and Tanya = 125 : 100 = 5 : 4

Suppose Tanya takes x days to do the work.

5 : 4 :: 20 : x $\Rightarrow$ x = $\left(\frac{4 \times 20}{5}\right)$

$\Rightarrow$ x = 16 days

Hence, Tanya takes 16 days to complete the work.

Q.9.(A) Suppose B takes x days to do the work.

$\therefore$ A takes $\left(2\times\frac{3}{4}x\right)=\frac{3x}{2}$ days to do it

(A + B)'s 1 day's work = $\frac{1}{18}$

$\therefore \frac{1}{x}+\frac{2}{3x}=\frac{1}{18}$ or x = 30

So, B can do the same work in 30 days.

Q.10.(C) Ram and Mohan's 1 day's work

$=\left(\frac{1}{15}+\frac{1}{10}\right)=\frac{1}{6}$

Work done by Ram and Mohan in 2 days

$=\left(\frac{1}{6}\times2\right)=\frac{1}{3}$.Remaining work = $\left(1-\frac{1}{3}\right)$

Now, $\frac{1}{15}$ work is done by Ram in 1 day.

$\therefore \frac{2}{3}$ work will be done by Ram in $\left(15\times\frac{2}{3}\right)$

= 10 days

Hence, total time taken = (10+2) = 12 days.

Q.11.(C) (B + C)'s 1 day's work = $\left(\frac{1}{9}+\frac{1}{12}\right)=\frac{7}{36}$

Work done by B and C in 3 days

$=\left(\frac{7}{36}\times3\right)=\frac{7}{12}$

Remaining work = $\left(1-\frac{7}{12}\right)=\frac{5}{12}$

Now, $\frac{1}{24}$ work is done by A in 1 day.

So, $\frac{5}{12}$ work is done by A in $\left(24\times\frac{5}{12}\right)$

=10 days.

Q.12.(D) (P +Q +R)'s 1 hours' work

$=\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{12}\right)=\frac{37}{120}$

Work done by P,Q and R in 2 hours

$=\left(\frac{37}{120}\times2\right)=\frac{37}{60}$

Remaining work = $\left(1-\frac{37}{60}\right)=\frac{23}{60}$.

(Q+R)'s 1 hour's work = $\left(\frac{1}{10}+\frac{1}{12}\right)=\frac{11}{60}$.

Now, $\frac{11}{60}$ work is done by Q and R in 1 hour.

So, $\frac{23}{60}$ work will be done by Q and R in

$\left(\frac{60}{11}\times\frac{23}{60}\right)=\frac{23}{11}$ hours ≈ 2 hours.

So, the work will be finished approximately in 2 hours after 11 a.m., i.e., around 1 pm.

Q.13.(B) Work done by X in 4 days

$=\left(\frac{1}{20}\times4\right)=\frac{1}{5}$. Remaining work

$=\left(1-\frac{1}{5}\right)=\frac{4}{5}$. (X+Y)'s 1 day work

$=\left(\frac{1}{20}+\frac{1}{12}\right)=\frac{8}{60}=\frac{2}{15}$.

So, $\frac{2}{15}$ work is done by X and Y in 1 day.

So, $\frac{4}{5}$ work is done by X and Y in $\left(\frac{15}{2}\times\frac{4}{5}\right)$

= 6 days

Hence, total time taken = (6 +4) days

= 10 days.

Q.14.(D) (A +B)'s 20 day's work = $\left(\frac{1}{30}\times20\right)=\frac{2}{3}$.

Remaining work = $\left(1-\frac{2}{3}\right)=\frac{1}{3}$.

Now, $\frac{1}{3}$ work is done by A in 20 days.

Whole work will be done by A in (20 × 3)

= 60 days.

Q.15.(D) 5 m or 9 w ⇒ 19 days ⇒ 5 m = 9 m = 19 days

3m and 6 w ⇒ ? ⇒ 3m + 6w ⇒ ?

Formula -

$m_1d_1 = m_2 d_2$

$5m \times 19 = (3m + 6w)\times d_2$

$5m \times 19 = (3m+\frac{5}{9}\times 6)\times d_2$

$95m = \left(\frac{27m+30m}{9}\right)\times d_2$

$95m \times 9 = 57m \times d_2$

$\frac{95\times 9}{57} = d_2 \Rightarrow 15$ days

Q.16.(B) Work done by 1 man in 1 day= $\frac{1}{44}$

Work done by 1 woman in 1 day = $\frac{1}{88}$

Work done by 1 boy in 1 day = $\frac{1}{132}$

⇒ Work done by 1 man,1 woman and 1 boy in i days $= \frac{1}{44}+\frac{1}{88}+\frac{1}{132}=\frac{1}{24}$

⇒ Time reqd. to complete the work in 24 days.

Q.17.(B) P = 10×4

= 40 days

$Q = \frac{15\times 5}{2} = \frac{75}{2}$ days

R = 13×3 = 39 days

S = 7×6 = 42 days

Q.18.(D) A's one day's job = $\frac{1}{12}$

B's one day's job = $\frac{1}{48}$

C's one day's job = $\frac{1}{96}$

∴ (A+B+C)'s 1 day's job = $\frac{1}{12}+\frac{1}{48}+\frac{1}{96} = \frac{11}{96}$

∴ Required days = $\frac{96}{11} = 8\frac{8}{11}$ days

Q.19.(A) 4B + 5G = 10

⇒ 40B + 50G = 1 (I)

6B + 6G = 7

⇒ 42B + 42G = 1 (II)

On comparing

40B + 50G = 42B + 42G

⇒ 2B = 8G

⇒ 1B = 4G

⇒ (42 × 4 + 42) G = 1

and 2B +7G = 15 G

$\therefore 15G = \frac{210}{15} = 14$ days

Q.20.(B) If B can do the workin x days the A alone will do the work in 2x days

$= \frac{1}{x}+\frac{1}{2x} = \frac{2+1}{2x} = \frac{3}{2x}$

∴ C's one day's work

$= \frac{3}{4x}$

$\therefore \frac{3}{4x} = \frac{1}{20}$

⇒ 4x = 3 × 20

$\Rightarrow x = \frac{3\times 20}{4} = 15$

∴(A+B+C)'s one day's work

$= \frac{1}{2x}+\frac{1}{x}+\frac{3}{4x} = \frac{1}{30}+\frac{1}{15}+\frac{1}{20}$

$= \frac{2+4+3}{60} = \frac{9}{60} = \frac{3}{20}$

∴ Reqd.answer $= \frac{20}{3} = 6\frac{2}{3}$ days

Q.21.(C) Ratio between work done by Amit and Sujet

= 100 : 80 = 5 : 4

Total work done by Amit = $576000\times\frac{5}{9}$

= 320000

Total working hours of Amit = 8×5 = 40 hours

∴ Amit's speed = $\frac{320000}{40}$ = 8000

Q.22.(C) Work done by (A + B + C = 1

Work done by (A + B = $\frac{11}{19}$

Work done by (B + C) = $\frac{14}{19}$

Work done by A = 1 – $\frac{14}{19} = \frac{11}{19}$

Work done by C = 1 – $\frac{11}{19} = \frac{8}{19}$

Work done by B/ = $\frac{14}{19} - \frac{8}{19} = \frac{6}{19}$

Required ratio = $\frac{5}{19} : \frac{6}{19} : \frac{8}{19}$ = 5: 6: 8

Q.23.(D) 1 M = 2 W = 4 C

5 M + 5 W + 5 C = 5 M + $\frac{5}{2}$ M + $\frac{5}{4}$ M

= $\frac{20+10+5}{4}$ M = $\frac{35}{4}$ M

According to the question,

20 × 7 = $\frac{35}{4}$ × D × 2

D = 8 days

Q.24.(D) 15 M = 24 W = 36 B
5M = 8 W = 12 B

12 W + 6 B = $\frac{15}{2}$ M + $\frac{5}{2}$ M = 10 M

According to the question,

$$\frac{15\times8\times1\times12}{1} = \frac{(10+x)\times30\times6\times1}{2\frac{1}{4}}$$

10 + x = 18
x = 8

Q.25.(C) P's one day work = $\frac{1}{20}$

Q's one day work = $\frac{1}{15}$

(P + Q)'s one day work

= $\frac{1}{20} + \frac{1}{15} = \frac{3+4}{60} = \frac{7}{60}$

(P + Q)'s five days work/(P + Q)

= $\frac{7}{60} \times 5 = \frac{7}{12}$

Remaining work = $1 - \frac{7}{12} = \frac{5}{12}$

$\frac{5}{12}$ work done by P

Total time taken by P to complete the work

= 5 + 20 × $\frac{5}{12}$

= 5 + $\frac{25}{3} = \frac{40}{3} = 13\frac{1}{3}$ days

Notes

CHAPTER-14
PIPE AND CISTERN

Scan the QR code to get video of this chapter.

Pipe : Pipes are connected to a tank or cistern and are used to fill or empty the tank, they are of two types.

Inlet : A pipe connected with a tank or cistern that fills it is known as inlet, means nature of pipe is positive.

Outlet : A pipe connected with a tank or cistern emptying it is known as outlet, means nature of pipe is negative.

IMPORTANT POINTS

(1) Pipe and cisterns is more similar to Time and Work.

(2) If an Inlet can completely fill the empty tank in x hours, the part of the tank filled in 1 hour = 1/x.

(3) If an outlet can empty the full tank in 1 hours, the part of the tank empty in 1 hours = 1/y.

(4) If both inlet and outlet are open, net part of the tank filled in 1 hours = $\frac{1}{x}-\frac{1}{y}$.

(5) If a pipe A alone can fill the tank in x hours and pipe B can fill or empty the tank in y hours. If both pipe working simultaneously then tank to fill or empty by $\left(\frac{xy}{x \pm y}\right)$ hours.

For filling pipe, we take '+' sign.

For empty or drain pipe we take '-' sign.

Ex. Two pipes A and B can fill a tank in 20 min. and 30 min. respectively. If both the pipes are opened simultaneously. How much time will be taken to fill the tank ?

(A) 12 min. **(B)** 16 min.
(C) 20 min. **(D)** 30 min.

Sol.(A) **METHOD I:**

Here A = 20 min, B = 30 min

∴ Part of the system filled by A and B in 1 min. =

$$\frac{1}{20}+\frac{1}{30}=\frac{5}{60}=\frac{1}{12}$$

Both Pipe A and B together fill the tank in 12 min.

METHOD II:

+ 3 l/m ⟵ A — +20m
+ 2 l/m ⟵ B — +30m   60 l
+ 5 l/m ⟵ A+B

time taken by (A + B) = $\frac{60}{5}$ = 12 min

METHOD III: BY FORMULA-

Both Pipe A and B together fill the tank in

$$=\frac{20\times30}{(20+30)}=\frac{600}{50}$$

= 12 min.

Ex. If pipe 'A' can fill a tank in 8 hr. and pipe 'B' can empty a tank in 16 hr. When both pipes are opened simultaneously, How much time will be taken to fill the tank ?

(A) 8 hr. **(B)** 16 hr.
(C) 10 hr. **(D)** 24 hr.

Sol.(B) **METHOD I:**

Here A = 8 hour, B = 16 hour

Part of cistern fill by A and B in 1 hour

$$=\frac{1}{8}-\frac{1}{16}=\frac{2-1}{16}=\frac{1}{16}$$

Both pipe fill the tank = 16 hours

METHOD II:

+ 2 l/h ⟵ A = +8 h
- 1 l/h ⟵ B = -16h   16 l
+ 1 l/h ⟵ A+B

time taken when A and B both are opened,

A + B = $\frac{16}{1}$ = 16 hour.

METHOD III: BY FORMULA-

Both pipe fill the tank = $\frac{16\times8}{16-8}$ = 16 hours

(6) Three pipes A,B,C can fill the tank in X,Y and Z hours respectively.If all three pipes opened simultaneously the time taken to fill the cisterns is given by -

$$\frac{X\times Y\times Z}{XY+YZ+ZX}$$

Ex. Three taps A,B,C, can fill an over tank in 4,6 and 12 hour respectively. How long would the these three taps take to fill the tank if all of them are opened together?

(A) 2 hr. **(B)** 4 hr.

(C) 3 hr. **(D)** 5 hr.

Sol.(A) **METHOD I:**

Here A = 4 hour, B = 6 hour and C = 12 h

All together file tank in 1 hour

$= \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$

$= \frac{3+2+1}{12} = \frac{1}{2}$ Part,

All fill the tank in 2 hours.

METHOD II:

+ 3 l/h ⟵ A — +4h
+ 2 l/h ⟵ B — +6h
+ 1 l/h ⟵ C — +12h
(12 l)

+6 l/h ⟵ A + B + C

All fill the tank = $\frac{12}{6}$ hours = 2 hours

METHOD III: BY FORMULA-

All together fill the tank

$= \frac{4 \times 6 \times 12}{4 \times 6 + 6 \times 12 + 12 \times 4} = \frac{4 \times 6 \times 12}{24 + 72 + 48}$

$= \frac{4 \times 6 \times 12}{144}$ = 2 hours

(7) Two pipes A and B can fill the tank in X,Y respectively. There is also and outlet pipe C. If all three pipe opened simultaneously tank will full in Z hours the time taken by C to empty the full tank is given by -

$$\frac{X \times Y \times Z}{XZ + YZ - XY}$$

Ex. Two taps A and B, can fill an over tank in 30 min and 60 min respectively. There is a third exhaust pipe C at the bottom of the tank. If all taps are opened together then tank will be full in 45 minutes. In what time can exhaust tap C empty the cistern when tank is completly full?

(A) 30 min. **(B)** 36 min.

(C) 45 min. **(D)** 75 min.

Sol.(B) **METHOD I:**

Here A = 30 min, B = 60 min and A+B-C = 45 min.

C can empty the tank in one minute

$= \frac{1}{30} + \frac{1}{60} - \frac{1}{45}$

$= \frac{6+3-4}{180}$

$= \frac{1}{36}$ Part.

All fill the tank in 36 minutes.

METHOD II:

+ 6 l/m ⟵ A — +30m
+ 3 l/m ⟵ B — +60m
+ 4 l/m ⟵ A+B-C — +45m
(180 l)

-5 l/m ⟵ C

C can empty the tank= $\frac{180}{5}$ min.=36min

METHOD III: BY FORMULA-

C can empty the tank

$= \frac{30 \times 60 \times 45}{30 \times 45 + 60 \times 45 - 30 \times 60}$

$= \frac{81000}{1350 + 2700 - 1800}$ = 36 min.

(8) One fill pipe A is k times as fast as the other fill pipe B.

(a) If B can fill a cistern in x hrs., then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{x}{k+1}\right)$ hrs.

(b) If A can fill a cistern in y hrs., then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{k}{k+1}\right)y$ hrs.

(9) One fill pipe A is k times fast and takes x mins less time than the other fill pipe B, then

(a) the time taken to fill a cistern, if both the pipes are opened together is $\frac{kx}{(k^2-1)}$ mins.

(b) A will fill the cistern in $\left(\frac{x}{k-1}\right)$ mins.

(c) B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ mins.

Ex. One fill pipe A is 9 times faster than second fill pipe B. If B can fill a cistern in 40 mins, then find the time when the cistern will be full if both fill pipes are opened together.

(A) 5 min. **(B)** 4 min.

(C) 8 min. **(D)** 6 min.

Sol.(B) Here k = 9 and x = 40

$\therefore$ Cistern will be full in = $\left(\dfrac{x}{k+1}\right)$ mins

= $\left(\dfrac{40}{9+1}\right)$ mins or 4 mins.

Ex. One fill pipe A is 3 times as fast as second fill pipe B. If A can fill a cistern in 16 mins, then find the time when the cistern will be full if both fill pipes are opened together.

(A) 15 min. **(B)** 16 min.

(C) 12 min. **(D)** 10 min.

Sol.(C) Here k = 3 and x = 16

$\therefore$ Cistern will be full in = $\left(\dfrac{x}{k+1}\right)$y mins

= $\left(\dfrac{3}{3+1}\right)$ 16 = 12 minute.

Ex. One fill pipe A is 3 times as fast as second fill pipe B. If they together can fill the tank in 36 min, then in how much time slower pipe can fill the tank.

(A) 81 min. **(B)** 108 min.

(C) 144 min. **(D)** 192 min.

Sol.(C) **METHOD I:**

Let time taken by faster pipe be x min then slower pipe take = 3x

$\therefore \dfrac{1}{x}+\dfrac{1}{3x}=\dfrac{1}{36}$

x = $\dfrac{36\times 4}{3}$ = 48 minutes

The time taken by slower pipe to fill the tank

= 3×48 = 144 mins.

METHOD II: BY FORMULA-

Both pipes fill the tank

= $\left(\dfrac{x}{k+1}\right)$

Here both the pipes fill the tank = 36 mins

k = 3, so slower pipe x fill the tank

= 36 × (3 + 1) = 144 mins

Ex. One fill pipe A is 5 times as fast as second fill pipe B and take 32 min less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

(A) $\dfrac{20}{3}$ min. **(B)** $\dfrac{10}{3}$ min.

(C) 9 min. **(D)** $\dfrac{40}{3}$ min.

Sol.(A) Here k = 5 and x = 32.

$\therefore$ Cistern will be full in = $\left(\dfrac{kx}{(k^2-1)}\right)$min.

= $\left(\dfrac{5\times 32}{5^2-1}\right)=\dfrac{20}{3}$ min.

EXAMPLES

Ex.1. If A and B two pipes can fill a tank in 15 hour when 'A' pipe can fill a tank in 6 hr. alone then in how much time will be taken to fill/empty the tank when pipes 'B' open alone ?

(A) 15 hr. **(B)** 16 hr.

(C) 12 hr. **(D)** 10 hr.

Sol.(D) **METHOD I:**

Here A+B = 15 hrs, A = 6 hrs,

B can empty/fill tank in 1 hrs = $\dfrac{1}{15}-\dfrac{1}{6}$

= $\dfrac{2-5}{30}=-\dfrac{1}{10}$

B can empty the tank in 10 hrs.

METHOD II:

2 l/h $\longleftarrow$ A+B = +15h
5 l/h $\longleftarrow$ A = +6h 30 l
- 3 l/h $\longleftarrow$ B

B = $\dfrac{30}{-3}$ = - 10 hour.

i.e. B can empty the tank alone in 10 h.

Ex.2. To fill a cistern, pipes A, B and C takes 20, 15 and 12 minutes respectively. The time in minutes that the three pipes together will take to fill a cistern is ?

(A) 8 min. **(B)** 7 min.

(C) 5 min. **(D)** 10 min.

Sol.(C) **METHOD I:**

Here A= 20 min, B= 15 min, C= 12 min

All pipe can fill the tank in 1 min

$\Rightarrow \dfrac{1}{20}+\dfrac{1}{15}+\dfrac{1}{12}=\dfrac{3+4+5}{60}=\dfrac{1}{5}$

So, all pipes fill the tank = 5 mins

METHOD II:

+ 3 l/m $\longleftarrow$ A — +20m
+ 4 l/m $\longleftarrow$ B — +15m 60 l
+ 5 l/m $\longleftarrow$ C — +12m
+ 12 l/m $\longleftarrow$ A + B + C

Time taken by A, B and C to fill the cistern

$A + B + C = \frac{60}{12} = 5$ min

Ex.3. Pipe A can fill a tank in 20 min while pipe B alone can fill it in 10 min and pipe C can empty the full tank in 30 min. If all three pipes are opened simultaneously then how much time taken to fill the tank?

(A) 8 min **(B)** 16 min

(C) $7\frac{6}{7}$ min **(D)** $8\frac{4}{7}$ min

Sol.(D) **METHOD I:**

A= 20 mins, B= 10 mins, C= 30 mins

Part of the tank filled by all the pipe 1 mins

$$\Rightarrow \frac{1}{20} + \frac{1}{10} - \frac{1}{30} = \frac{3+6-2}{60} = \frac{7}{60}$$

So, all pipes fill the tank = $8\frac{4}{7}$ mins.

METHOD II:

+ 3 l/m ← A — +20m
+ 6 l/m ← B — +10m } 60 l
– 2 l/m ← C — –30m

+ 7 l/m ← A + B + C

Time taken by A+B+C = $\frac{60}{7} = 8\frac{4}{7}$ hours.

Ex.4. Two pipes A and B are opened together to fill a tank. Both the pipes fill the tank in time 't'. If A and B takes 4 min. and 64 min. more seperately than the time taken by A and B together. Find the value of t.

(A) 18 min **(B)** 16 min

(C) 12 min **(D)** 15 min

Sol.(B) Time taken by pipe A = (t+4) min.

Time taken by pipe B = (t+64) min.

$$\therefore \frac{1}{(t+4)} + \frac{1}{(t+64)} = \frac{1}{t}$$

On solving the equation

t = 16 min.

QUICK TRICK :

In this case time taken by both pipes to fill the tank (t) = $\sqrt{ab}$

t = $\sqrt{4 \times 64}$ = 2 × 8 = 16 min

Ex.5. An outlet pipe can empty a cistern in 6 hours. In what time will the pipe empty two-third part of the cistern?

(A) 4 hours **(B)** 6 hours

(C) 8 hours **(D)** 2 hours

Sol.(A) Time taken by pipe to empty the tank

=6 h

Time tgaken by pipe to empty 2/3 part

$= 6 \times \frac{2}{3}$ = 4 hours

Ex.6. Pipe A can fill a tank in 60 min and pipe B in 75 min, pipe C empty the tank. If all the three pipes are opened together the tank is completely full in 50 min. In what time pipe C alone can empty the full tank.

(A) 140 min **(B)** 120 min

(C) 60 min **(D)** 100 min

Sol.(D) **METHOD I:**

According to question,

$$\frac{1}{60} + \frac{1}{75} - \frac{1}{x} = \frac{1}{50}$$

$$\frac{1}{60} + \frac{1}{75} - \frac{1}{50} = \frac{1}{x}$$

$$\frac{3}{300} = \frac{1}{x}$$

x = 100 min.

METHOD II:

+ 5 l/m ← A — +60m
+ 4 l/m ← B — +75m } 300 l
+ 6 l/m ← A+B–C — +50m

– 3 l/m ← C

Time taken by C alone to empty

$= \frac{300}{3}$ =100 mins.

Ex.7. Capacity of tap B is 80% more than that of A. If both the taps are opened simultaneously, then take 45 hours to fill the tank. How long will B take to fill the tank alone?

(A) 70 hours **(B)** 80 hours

(C) 60 hours **(D)** 50 hours

Sol.(A) Eff. of A : B = 5 : 9

Total parts to be filled = (5+9)×45

= 14×45

Time taken by B = $\frac{14 \times 45}{9}$ = 70 hours

Ex.8. Pipe A can fill a tank 3 times as fast as pipe B and takes 28 min less than pipe B to fill the tank. If both the pipes are opened simultaneously then find the time taken to fill the tank.

(A) $\frac{28}{3}$ min **(B)** 60 min

(C) $\frac{21}{2}$ min **(D)** 21 min

Sol.(C) Eff. of pipes A : B

3 : 1

Ratio of time 1 : 3

2 parts ....28 min

1 part........14 min

Time taken by A = 14 min

Efficiency of A = 3

Total work = 14 × 3

Time taken by both the pipes = $\frac{14 \times 3}{4}$

= $\frac{21}{2}$ min.

Ex.9. A can fill a tank in 24 mins and B can fill the same tank in 32 mins. Both the pipes are opened together, after how many minutes pipe B should be closed so that the tank is completely full in 18 min?

(A) 8 min **(B)** 12 min

(C) 18 min **(D)** 6 min

Sol.(A) **METHOD I:**

Part filled by pipe A in 1 min.= $\frac{1}{24}$

Part filled by pipe A in 18 min

$=\frac{1}{24} \times 18 = \frac{3}{4}$ Part

Remaining Part

= 1 - $\frac{3}{4}$ Part = $\frac{1}{4}$ Part

Time taken by pipe B in filling whole tank

=32 minute

Time taken by pipe B in filling $\frac{1}{4}$ part

= $32 \times \frac{1}{4}$ min. = 8 min.

Pipe B is closed after 8 min.

METHOD II:

+ 4 l/m ⟵ A — +24m
+ 3 l/m ⟵ B — +32m
96 l

Let after x min pipe B is closed then according to question

7x+4 (18–x) = 96

7x+72–4x=96

3x=24, x=8 min

Ex.10. Pipe A, B and C together can fill a tank in 6 hours. After working for 2 hours, pipe C is closed and the remaining tank is now filled by A and B together in 8 hours. Find the time in which the tank can be filled by C alone.

(A) 24 hours **(B)** 12 hours

(C) 8 hours **(D)** 16 hours

Sol.(B) **METHOD I:**

Time taken by A + B in filling $\frac{2}{3}$ part

= 8 hours

Time taken by A + B in filling whole part

= $8 \times \frac{2}{3}$ = 12 hours

Part filled by C in 1 hour

$\Rightarrow \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$ Part

Time taken to fill the whole tank = 12 hours

METHOD II:

A + B fill $\frac{2}{3}$ part in 8 hours

A + B can fill full part in $8 \times \frac{3}{2}$ = 12 hours.

Now,

+ 2 l/m ⟵ A+B+C — +6m
+ 1 l/m ⟵ A+B — +12m
12 l

+1 l/m ⟵ C

hence time takQ R by C alone= $\frac{12}{1}$ =12 hrs.

Ex.11. Pipe P can fill a tank in 12 min and another pipe Q can fill it in 15 min. But, the 3rd pipe R can fill empty in 6 min. The first two pipe P and R are kept open for $\frac{1}{3}$ time of the time in which Q can fill and then 3rd pipe R is also opened. In what time is the tank will full/empty?

(A) 60 min **(B)** 45 min

(C) 36 min **(D)** 90 min

Sol.(B) **METHOD I:**

Part filled by P+Q in 5 min.

$= (\frac{1}{12} + \frac{1}{15}) \times 5 = \frac{45}{60}$

Part filled by all the three pipes working together

$= (\frac{1}{12} + \frac{1}{15} - \frac{1}{10}) = -\frac{1}{60}$

Time taken to empty $= \frac{45}{60}$ part

$\Rightarrow \frac{45}{60} \times 60 = 45$ min.

METHOD II:

+ 5 l/m ⟵ P — +12m
+ 4 l/m ⟵ R — +15m
–10 l/m ⟵ C — –6m
} 60 l

P+R | P+R+C
- 5 min-

Part filled by P+R in 5 min = 9×5

= 45 part

Now P+R+C will work together = 5+4-10

= -1

Now the tank will be emptied

Time taken to empty the tank$= \frac{45}{1}$ =45 min

Ex.12. A tank has a leak which would empty it in 8 hours. A tap is turned on which admits 3 Ltr a min into the tank and it is now emptied in 12 hours. How many litres the tank can hold.

(A) 180 litres **(B)** 7200 litres

(C) 4320 litres **(D)** 2400 litres

Sol.(C) **METHOD I:**

Part filled by filling pipe in 1 hour

$= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ part

$\frac{1}{24}$ part ....3×60 Ltrs.

1 Part .... 3×60×24 =4320 Ltrs

METHOD II:

Let the Leakage pipe be A

Let the other tap pipe be B

– 3 l/h ⟵ A — –8h
– 2 l/h ⟵ A+B — –12h
} 24 l
+1 l/h ⟵ B

Time taken by B to fill the tank is equal to 24/1 = 24 hr.

hence, volume of the tank = 24 × 60 × 3

= 4320 ltrs

Ex.13. Two pipes can fill a tank in 20 and 24 min respectively and a waste pipe empty 17 gallons per minute. All the three pipes working together can fill the tank in 15 min. Find the capacity of the tank.

(A) 180 gallon **(B)** 5100 gallon

(C) 680 gallon **(D)** 240 gallon

Sol.(C) **METHOD I:**

Part emptied by waste pipe in 1 min.

$\frac{1}{15} - (\frac{1}{20} + \frac{1}{24}) = \frac{1}{15} - \frac{11}{120} = -\frac{1}{40}$

$\frac{1}{40}$ Part of volume = 17 gallon

Whole part of volume = 17×40

= 680 gallon.

METHOD II:

+ 6 l/m ⟵ A — +20m
+ 5 l/m ⟵ B — +24m
+ 8 l/m ⟵ A+B–C — +15m
} 120 l
– 3 l/m ⟵ C

Time taken by C to empty the tank

$= \frac{120}{3} = 40$ min.

In 1 min it empties 17 gallons

In 40 min it empties 17×40 gallons

= 680 gallons.

Ex.14. A cistern has three pipes A,B and C. Pipes A and B can fill it in 3 and 4 hrs. respectively, while pipe C can empty the completely full cistern in 1 hour. If the pipes are opened at 2:00 pm, 3 pm and 4 pm, respectively, at what time the cistern will be empty?

(A) 6 : 12 p.m. **(B)** 6 : 12 a.m.

(C) 7 : 12 p.m. **(D)** 6 : 00 p.m.

Sol.(A) **METHOD I:**

All the three pipes will run simultaneously at 4 P.M.

Part filled by pipe A in two hours from 2 p.m. to 4 p.m. $= \frac{1}{3} \times 2 = \frac{2}{3}$ Part

Part filled by pipe B in 1 hour from 3 p.m. to 4 p.m. $= \frac{1}{4}$ Part

Part filled by both the pipes together till 4 p.m.

$= \frac{2}{3} + \frac{1}{4} = \frac{11}{12}$ Part

Working together all the three pipes

$= \frac{1}{3} + \frac{1}{4} - 1 = -\frac{5}{12}$ Part empty

Time taken in emptying $\frac{5}{12}$ Part = 1 hour

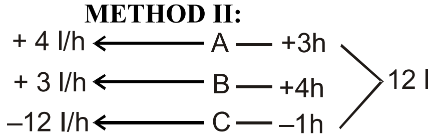
Time taken in emptying whole part $= \frac{12}{5}$ hrs

Time taken to emptying $\frac{11}{12}$ part $= \frac{12}{5} \times \frac{11}{12}$ hrs

$\frac{11}{5}$ hrs = 2 hrs 12 mins.

Tank will be empty at 6 : 12 p.m.

METHOD II:



A works from 2.00 pm to 4 pm ..... (2 hrs)

B works from 3.00 pm to 4 pm .....(1 hr)

Work done of A

= 4×2 = 8, work done of B = 3×1 = 3

Hence, total part fill = 11

At 4.00 pm all the three pipes work together

=4+3-12 = -5

Tank will be empty now, time taken

$= \frac{11}{5} = 2\frac{1}{5}$ =2 hrs. 12 min

The tank will be empty at 6 : 12 pm

Ex.15. A,B, C are three pipes attached to a cistern A and B can fill it in 20 and 30 min respectively, while C can empty it in 15 min. If A,B and C are be kept open successively for 1 min each, how soon will the cistern be filled?

(A) 1 hours 40 min **(B)** 2 hours 47 min

(C) 2 hours **(D)** 2 hours 30 min

Sol.(B) **METHOD I:**

All the pipes when kept open successively for 1 minutes each the part filled

$= \frac{1}{20} + \frac{1}{30} - \frac{1}{15} = \frac{1}{60}$ Part

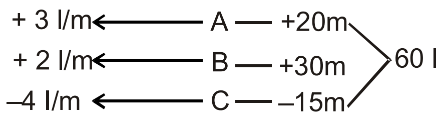
In the next 165 minutes (3×55), $\frac{55}{60}$ Part of the tank will be full.

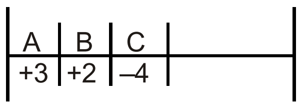
Remaining part $= \frac{5}{60}$ part

In the next 2 minutes pipe A and B will fill remaining $= \frac{1}{20} + \frac{1}{30} = \frac{5}{60}$ part

Total time = 165+2 = 167 min. = 2 hr. 47 min.

METHOD II:





In 3 mins 1 litre filled

In (3×55) 165 min = 55 litre filled.

Remaining 5 litre filled by A and B in next 2 min

Total tank is full in 165+2 =167 min.

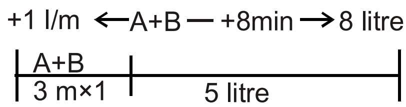
= 2 hr. 47 min

Ex.16. A water tub can be filled by two taps in 8 min. One tap is closed after 3 min, the other tap fills the remaining tub in 15 min. How much time the faster tap will take to fill the tub.

(A) 24 min **(B)** 12 min

(C) 30 min **(D)** 18 min

Sol.(B) A+B = 8 min



Since, after 3 min pipe A is closed hence remaining 5 litre is filled by B in 15 min

So efficiency of B $= \frac{5}{15} = \frac{1}{3}$

Then efficiency of A (faster)$= 1 - \frac{1}{3} = \frac{2}{3}$

Time taken by faster pipe $= \frac{8}{\frac{2}{3}} = 12$ min

Ex.17. Tap A,B and C are attached with a tank and velocity of water coming through them are

42 lt/hr., 56 lt/hr. C 48 lt/hr. respectively. A and B are inlets and C is outlet. If all the taps are opened simultaneously, tank is filled in 16 hour. What is the capacity of the tank?

(A) 80 liter **(B)** 800 liter

(C) 60 liter **(D)** 600 liter

Sol.(B) Quality of water admitted by A in 1 hr

= 42 L

Quality of water admitted by B in 1 hr

= 56 L

Quality of water admitted by C in 1 hr

= 48 L

Total water admitted = (42+56-48) = 50 L

Tank is completed full in 16 hours

Total water admitted in 16 hours = 16×50

= 800 Litre (capacity of tank)

Ex.18. There are three pipes connected with a tank. The first pipe can fill $\frac{1}{2}$ part of the tank in 1 hour the second pipe fill $\frac{1}{3}$ part of tank in 1 hour. Third pipe is connected to empty the tank. All the pipes working together fills $\frac{7}{12}$ part in 1 hour, then in what time third pipe will empty the tank.

(A) 6 hours **(B)** 8 hours

(C) 4 hours **(D)** 10 hours

Sol.(C) **METHOD I:**

Let the third pipe be C

According to question-

$$\frac{1}{2}+\frac{1}{3}-\frac{1}{C}=\frac{7}{12}$$

$$\frac{1}{2}+\frac{1}{3}+\frac{7}{12}=\frac{1}{C}$$

$$\frac{1}{C}=\frac{1}{4}$$

Pipe C will take 4 hours to empty the full tank.

METHOD II:

$$\left.\begin{array}{lcl} +6\ l/h & \longleftarrow A — & +2h \\ +4\ l/h & \longleftarrow B — & +3h \\ +7\ l/h & \longleftarrow A+B–C — & +12/7h \end{array}\right\rangle 12\ l$$

$$\begin{array}{lc} –3\ l/h & C \end{array}$$

Time taken to empty the tank $=\frac{12}{3}=4$ hrs.

Ex.19. Three pipes A, B and C can fill a tank in 12,15 and 20 min respectively. When the tank is empty, all the three pipes are opened. What part of the water from pipe C will be there in the tank after 3 mins.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{1}{6}$ **(D)** $\frac{2}{3}$

Sol.(A) **METHOD I:**

Water filled by pipe C

$$=\frac{\text{Water filled by C in 3 min.}}{\text{Water filled by all the pipes in 3 min}}$$

$$=\frac{3\times\frac{1}{20}}{3\times\left(\frac{1}{12}+\frac{1}{15}+\frac{1}{20}\right)}=\frac{1}{4}$$

The water filled by C in 3 minutes $=\frac{1}{4}$ Part

METHOD II:

$$\left.\begin{array}{lcl} +5\ l/m & \longleftarrow A — & +12m \\ +4\ l/m & \longleftarrow B — & +15m \\ +3\ l/m & \longleftarrow C — & +20m \end{array}\right\rangle 60\ l$$

$$\frac{\text{Part of C in 3 min}}{\text{Total in 3 min}}=\frac{3\times3}{12\times3}=\frac{1}{4}$$

Ex.20. A boy and girl together fill a cistern with water : The boy pours 4 litres of water every 3 minutes and the girls pours 3 litres every 4 minutes. How much time will it take to fill 100 litres of water in the cistern ?

(A) 36 minutes **(B)** 42 minutes

(C) 48 minutes **(D)** 44 minutes

Sol.(C) Water filled by the boy and girls in 1 minute

$$=\frac{4}{3}+\frac{3}{4}=\frac{16+9}{12}=\frac{25}{12}\text{ litres}$$

Time taken to fill 100 litres

$$=\frac{100}{25}\times12=48\text{ minutes}$$

EXERCISE

Q.1. A tap can fill a tank in 25 minutes and another can empty it in 50 minutes. Find in how many minutes the tank will be filled up or emptied ?

(A) Tank is filled up in 50 minutes

(B) Tank is emptied in 25 minutes

(C) Tank is filled up in 25 minutes

(D) Tank is filled up in 30 minutes

Q.2. Two pipes can fill a tank in 10 hours and 12 hours respectively. While a third pipe emptied the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled ?

(A) 7 hours 30 min

(B) 6 hours 40 min

(C) 8 hours 30 min

(D) 9 hours 30 min

Q.3. Two pipes A and B can fill a cistern in 24 minutes and 30 minutes, respectively. There is also an outlet C. If all the three pipes are opened together, the cistern is filled in 20 minutes. How much time will be taken by *C* to empty the full cistern?

(A) 30 min **(B)** 40 min

(C) 45 min **(D)** 50 min

Q.4. A cistern is normally filled in 8 hours but takes 2 hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in-

(A) 35 hours **(B)** 45 hours

(C) 40 hours **(D)** 30 hours

Q.5. A cistern has a leak which would empty in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern and it is now emptied in 12 hours. The cistern can hold

(A) 6840 litres **(B)** 7860 litres

(C) 8640 litres **(D)** 1000 litres

Q.6. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir ?

(A) 35 hours **(B)** 20 hours

(C) 40 hours **(D)** 32 hours

Q.7. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

(A) 20 hours **(B)** 25 hours

(C) 35 hours **(D)** None of these

Q.8. There are two taps to fill a tank while a third to empty it. When the third tap is closed, they can fill the tank in 10 minutes and 12 minutes respectively. If all the three taps be opened, the tank is filled in 15 minutes. If the first two taps are closed, in what time can the third tap empty the tank when it is full?

(A) 7 min **(B)** 9 min and 32 sec

(C) 8 min and 34 sec **(D)** 6 min.

Q.9. Two pipes A and B can separately fill a cistern in 15 minutes and 18 minutes respectively, while a third pipe C can empty it in 6 minutes. Two pipes A and B are kept open for 6 minutes in the beginning and then the third pipe is also opened. In what time will the cistern be emptied?

(A) $16\frac{1}{2}$ min **(B)** 15 min

(C) $15\frac{1}{2}$ min **(D)** 16 min

Q.10. A water tank is $\frac{2}{5}$ th full. Pipe *A* can fill the tank in 10 minutes and pipe *B* can empty it in 6 minutes. If both the pipes are opened, how long will it take to empty or fill the tank completely?

(A) 6 minutes to empty

(B) 6 minutes to fill

(C) 9 minutes to empty

(D) 9 minutes to fill

Q.11. A pump can fill a tank with water in 2 hours. Because of a leak in the tank it was taking $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water off the tank in

(A) 8 hours **(B)** 7 hours

(C) $4\frac{1}{3}$ hours **(D)** 14 hours

Q.12. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe at the bottom of the cistern to empty it., If all the three pipes are simultaneously opened, then the cistern is filled in 50 minutes. In how much time can the third pipe alone empty the cistern?

(A) 110 minutes **(B)** 100 minutes

(C) 120 minutes **(D)** 90 minutes

Q.13. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What, is the total time taken to fill the tank completely?

(A) 4 hours

(B) 4 hours 15 minutes

(C) 3 hours 15 minutes

(D) 3 hours 45 minutes

Q.14. Two pipes A and B can fill a tank in 6 hours and 4 hours respectively. If they are opened on alternate hours and if pipe A is opened first, in how many hours, the tank shall be full ?

(A) 4 hrs **(B)** $4\frac{1}{2}$ hrs

(C) 5 hrs **(D)** $5\frac{1}{2}$ hrs

Q.15. Three taps A, B and C can fill a tank in 12,15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be filled in :

(A) 6 hrs **(B)** $6\frac{2}{3}$ hrs

(C) 5 hrs **(D)** 7 hrs

Q.16. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:

(A) 60 gallons **(B)** 100 gallons

(C) 120 gallons **(D)** 180 gallons

Q.17. Pipes P and Q can fill a tank in 10 and 12 hours respectively and R can empty it in 6 hours. If all the three are opened at 7 a.m., at what time will one-fourth of the tank be filled?

(A) 10 a.m. **(B)** 10 p.m.

(C) 11 p.m. **(D)** 11 a.m.

Q.18. Tap A can fill a tank in 20 hours, B in 25 hours but tap C can empty a full tank in 30 hours. Starting with A, followed by B and C each tap opens alternatively for one hour period till the tank gets filled up completely. In how many hour the tank will be filled up completely?

(A) $51\frac{11}{15}$ **(B)** $52\frac{2}{3}$

(C) $24\frac{4}{11}$ **(D)** None of these

Q.19. If two pipes function simultaneously, a tank is filled in 12 hours. One pipe fills the tank 10 hours faster than the other. How many hours does the faster pipe alone take to fill the tank?

(A) 20 **(B)** 22

(C) 25 **(D)** 30

Q.20. Pipe P can fill a certain tank in 18 hours and pipe Q can fill the same tank in in 15 hours. Q operated for 10 hours and then turned off. In how many hours, P alone can fill the remaining tank?

(A) 6 hours **(B)** $5\frac{1}{2}$ hours

(C) 5 hours **(D)**8 hours

Notes

EXPLANATION

Q.1.(A) Here, X = 25 and Y = 50

Part of the tank filled or emptied in 1 minute

$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50}$

Sign is positive, therefore the tank will be filled.

$\therefore$ Total time taken to fill the tank

= 50 minutes.

Q.2.(A) Here, X = 10, Y = 12 and Z = 20

$\therefore$ The tank will be filled in

$= \left(\frac{X \times Y \times -Z}{XY - YZ - ZX}\right)$ hour-s

$= \left(\frac{10 \times 12 \times -20}{10 \times 12 - 12 \times 20 - 20 \times 10}\right)$ hours

$= \left(\frac{15}{2}\right)$ hours or, 7 hours 30 minutes.

Q.3.(B) Here, X = 24, Y = 30 and Z = 20

$\therefore$ The total time taken by C to empty the full cistern

$= \left(\frac{XYZ}{XZ + YZ - XY}\right)$ minutes

$= \left(\frac{24 \times 30 \times 20}{24 \times 20 + 30 \times 20 - 24 \times 30}\right)$ minutes

= 40 minutes

Q.4.(C) Here, X = 8, Y = 8 + 2 = 10

$\therefore$ The leak will empty the cistern in

$= \left(\frac{XY}{Y - X}\right)$ hours

$= \left(\frac{8 \times 10}{10 - 8}\right)$ hours or, 40 hours.

Q.5.(C) Here, X = 8, Y = 6 × 60 = 360 and Z = 12

$\therefore$ The capacity of the cistern is

$= \left(\frac{XYZ}{Z - X}\right)$ litres

$= \left(\frac{8 \times 360 \times 12}{12 - 8}\right)$ litres = 8640 litres.

Q.6.(B) Let one pipe take x hours to fill the reservoir.

Then, another pipe takes (x - 10) hours.

$\therefore \frac{1}{x} + \frac{1}{x - 10} = \frac{1}{12} \Rightarrow$ (2x- 10)12 = x (x -10)

$24x - 120 = x^2 - 10x$

$x^2 - 34x + 120 = 0$

$x^2 - 30x - 4x + 120 = 0$

$(x - 30)(x - 4) = 0$

x = 30, 4

Q.7.(C) Let A alone take x hours to fill the tank.

Then pipe B's efficiency = $\frac{x}{2}$

Pipe C's efficiency = $\frac{x}{4}$

According to question, $\frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$

$\Rightarrow \frac{7}{x} = \frac{1}{5} \Rightarrow x = 35$ hrs.

Q.8.(C) Part emptied by the third pipe in 1 min.

$= \left(\frac{1}{10} + \frac{1}{12}\right) - \frac{1}{15} = \frac{7}{60}$

So, the full tank will be emptied by third pipe $\left(\frac{60}{7}\right)$ minutes = 8 minutes 34 seconds

Q.9.(A) Pipe (A+B)'s 6 minutes job

$= 6 \times \left(\frac{1}{15} + \frac{1}{18}\right) = \frac{11}{15}$

Net work done by the three pipes (A+B+C)

1 minute = $\left(\frac{1}{15} + \frac{1}{18}\right) - \frac{1}{6} = \frac{-4}{90} = \frac{-2}{45}$

Net $\frac{2}{45}$ part of the tank is emptied by the pipe in 1 minute.

Net $\frac{11}{15}$ part of the tank is emptied by the pipe in

$\frac{45}{2} \times \frac{11}{15} = \frac{33}{2}$ minutes

$= 16\frac{1}{2}$ minutes

Q.10.(A) Pipe A in 1 minute fills $\frac{1}{10}$ part and pipe B in 1 minute empties $\frac{1}{6}$ part

$\therefore$ Pipe (A +B) in 1 minute fill $\frac{1}{10}-\frac{1}{6}=\frac{-1}{15}$ part

$\therefore$ $\frac{1}{15}$ part gets emptied in 1 minute

$\therefore$ $\frac{2}{5}$ part gets emptied in $15\times\frac{2}{5}$ minute= 6 minutes

Q.11.(D) Required time $\frac{2\times\frac{7}{3}}{\frac{7}{3}-2}$ = 14 hours

Q.12.(B) Let the third pipe empty the cistern in K minutes

i.e. $\frac{1}{60}+\frac{1}{75}-\frac{1}{K}=\frac{1}{50}$

or $\frac{1}{60}+\frac{1}{75}-\frac{1}{50}=\frac{1}{K}$

K = 100 minutes

Q.13.(D) Half of the tank is filled in $\frac{1}{2}\times 6$ = 3 hours.

Now, we have four taps and each tap can fill the tank in 6 hours.

When all the four taps are opened, then they can fill

$\frac{1}{2}$ of the tank in $\frac{6}{4}\times\frac{1}{2}=\frac{3}{4}\times 60$ hours

= 45 minutes

$\therefore$ Total time = 3 hours 45 minutes

Q.14.(C) A's work in 1 hour = $\frac{1}{6}$, B's work in 1 hour = $\frac{1}{4}$.

(A+B)'s 2 hour's work when opened alternately

$=\left(\frac{1}{6}+\frac{1}{4}\right)=\frac{5}{12}$.

(A+B)'s 4 hour's work when opened alternately

$=\frac{10}{12}=\frac{5}{6}$.

Remaining part = $\left(1-\frac{5}{6}\right)=\frac{1}{6}$.

Now, it is A's turn and $\frac{1}{6}$ part is filled by A in 1 hour.

$\therefore$ Total time taken to fill the tank = (4+1) hrs.

= 5 hrs.

Q.15.(D) (A + B)'s 1 hour's work

$=\left(\frac{1}{12}+\frac{1}{15}\right)=\frac{9}{60}=\frac{3}{20}$

(A+C)'s 1 hour's work

$=\left(\frac{1}{12}+\frac{1}{20}\right)=\frac{8}{60}=\frac{2}{15}$.

Part filled in 2 hrs. = $\left(\frac{3}{20}+\frac{2}{15}\right)=\frac{17}{60}$,

Part filled in 6 hrs. = $\left(3\times\frac{17}{60}\right)=\frac{17}{20}$

Remaining part = $\left(1-\frac{17}{20}\right)=\frac{3}{20}$.

Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.

$\therefore$ Total time taken to fill the tank = (6+1) hrs.

= 7 hours

Q.16.(C) Work done by the waste pipe in 1 minute

$=\frac{1}{15}-\left(\frac{1}{20}+\frac{1}{24}\right)=\left(\frac{1}{15}-\frac{11}{120}\right)=-\frac{1}{40}$

(–sign means emptying)

Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3 ×40) gallons = 120 gallons.

Q.17.(B)

P—10		6
Q—12	60	5
R—6		–10

P+Q+R = 1

Part to be filled = $\frac{1}{4} \times 60 = 15$

Required time = $\frac{15}{1}$ = 15 h

Required answer = 7 : 00 + 15 : 00 = 22 : 00 hrs.

= 10 pm

Q.18.(A) Efficiency of A = 5%

Efficiency of B = 4%

Efficiency of C = 3.33%

It means in every 3 consecutive hours taps A, B and C can fill 5.66% (= 5 + 4 - 3.33)

Therefore in 51 hours (= 3 × 17) taps A, B and C acan fill 96.33% (= 5.66 × 17)

The remaining part i.e. 3.66% (= 100-96.33) can be filled up by A in $\frac{11}{15}$ hours $\left(=\frac{3.66}{5}\right)$, since it is now A's turn.

Hence, the total time required = $51 + \frac{11}{15} = 51\frac{11}{15}$

Q.19.(A) A --------- x ---------- 12(x+10)

B --------- (x + 10) ---------- 12x

A + B ---- 12 ----- x(x + 10)

L.C.M. of x, (x + 10) and 12 = 12x (x + 10)
$12x + 120 + 12x = x^2 + 10x$

x = + 20, – 6

Q.20.(A) P …………… 18

(5 parts of tank per hour)

(L.C.M. of 18 and 15 = 90)

Q …………… 15

(6 parts of tank per hour)

According to question,

D × 5 + 6 × 10 = 90

D × 5 = 90 – 60 = 30

D = 6 hours

Notes

CHAPTER-15
SPEED, TIME AND DISTANCE

Scan the QR code to get video of this chapter.

Speed

The distance covered in a unit time interval is known as speed. It is obtained by dividing the distance covered by an object by the time it takes to cover that distance.

$$\text{Speed (S)} = \frac{\text{Distance Travelled}}{\text{Time Taken}}$$

Distance (d) = Speed ×Time

$$\text{Time (t)} = \frac{\text{Distance}}{\text{Speed}}$$

Unit conversion :

1. **km/hr to m/sec conversion:**

$$\text{A km/hr} = \left(A \times \frac{5}{18}\right) \text{m/sec.}$$

2. **m/sec to km/hr conversion:**

$$\text{A m/sec} = \left(A \times \frac{18}{5}\right) \text{km/hr.}$$

Ex. A plane is moving with the speed of 180 km/hr. Its speed in metre per second is-

(A) 5 m/s **(B)** 40 m/s
(C) 30 m/s **(D)** 50 m/s

Sol.(D) $180 \times \frac{5}{18} = 50$ m/s

IMPORTANT POINTS :

1. If the time taken is constant, the distance travelled is directly proportional to the speed i.e., more speed more the distance travelled in the same time.

$\boxed{S \propto d}$ → **at constant time**

2. If the speed is constant, the distance travelled is directly proportional to the time taken, i.e. more the distance travelled, more the time taken at the same speed.

$\boxed{d \propto t}$ → **at constant speed**

3. If the distance travelled is constant, the speed is inversly proportional to the time taken, i.e. more speed, less the time taken for the same distance travelled.

$\boxed{S \propto \frac{1}{t}}$ → **at constant distance**

Ex. In covering a certain distance the speed of A and B are in the ratio of 3 : 4. A takes 30 minute more than B to reach the destination. The time taken by A to reach the destination is-

(A) 1 hour **(B)** $1\frac{1}{2}$ hour
(C) 2 hour **(D)** $2\frac{1}{2}$ hour

Sol.(C) Let the distance of destination be D km

Let the speed of A= 3x km/hr.

The speed of B = 4x km/hr.

According to the question

$$\frac{D}{3x} - \frac{D}{4x} = 30 = \frac{1}{2}, \frac{D}{12x} = \frac{1}{2}, \frac{D}{x} = 6$$

Time taken by A = $\frac{D}{3x} = \frac{6}{3}$ = 2 hours.

QUICK TRICK :

Speed ratio = 3 : 4
Time ratio = 4 : 3
1 = 30 min
4 = 120 min =2 hours

Ex. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by the person is:

(A) 50 km **(B)** 40 km
(C) 100 km **(D)** 60 km

Sol.(A) Let the actual distance travelled be x km.

Then, $\frac{x}{10} = \frac{x+20}{14} \Rightarrow 14x = 10x + 200$

$\Rightarrow 4x = 200 \Rightarrow x = 50$ km.

QUICK TRICK :

$(14 - 10) = \frac{20}{T}$

T = 5 hrs.

Distance = 10 × 5 = 50 km

Ex. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

(A) 12 min **(B)** 10 min

(C) 6 min **(D)** 9 min

Sol.(B) Due to stoppages, it covers 9 km less.

Time taken to cover 9 km

$= \left(\frac{9}{54}\times 60\right)$ min = 10 min

Average Speed

The average speed of an object is a measure of the distance covered by that object in a set period of time.

Average Speed = $\frac{\text{Total Distance}}{\text{Total Time Taken}}$

(a) If A covers a distance d_1 km at S_1 km/hr and then d_2 km at S_2 km/hr. then the average speed during whole journey is given by

Average Speed $= \frac{S_1S_2(d_1+d_2)}{S_1d_2+S_2d_1}$

(b) A person goes certain distance (A to B) at speed of S_1 and return (B to A) at speed of S_2 km/hr. If the takes T hrs in all, the

Average speed $= \frac{2S_1\times S_2}{S_1+S_2}$

Distance a between A and B is

$t\left(\frac{S_1\times S_2}{S_1+S_2}\right)$

(c) If a person travelled three equal distance by three different speed S_1, S_2 and S_3 then

Average speed $= \frac{S_1\times S_2\times S_3}{S_1S_2+S_2S_3+S_3S_1}$

Ex. A man goes from place A to B at a speed of 12 km/hr. and return at the speed of 18 km/hr. The average speed for the whole journey is-

(A) $14\frac{2}{5}$ km/hr. **(B)** 15 km/hr

(C) $15\frac{1}{2}$ km/hr. **(D)** 16 km/hr.

Sol.(A) Average Speed = $\frac{2\times 12\times 18}{(12+18)}$

$= \frac{2\times 12\times 18}{30}$

$= 14\frac{2}{5}$ km/hr.

Ex. One-third of a certain journey is covered at the rate of 25 km/hr. one fourth at the rate of 30 km/hr. and the rest at 50 km/hr. The average speed for whole journey.

(A) 35 km/hr **(B)** $33\frac{1}{3}$ km/hr

(C) 30 km/hr. **(D)** $37\frac{1}{12}$ km/hr

Sol.(B) Let total journey = x km

So, according question

Total time = $\frac{x}{3\times 25}+\frac{x}{4\times 30}+\frac{5x}{12\times 50}$

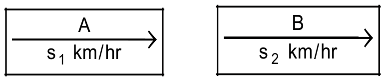
$= \frac{3x}{100}$

Average Speed = $\frac{x}{\frac{3x}{100}}$

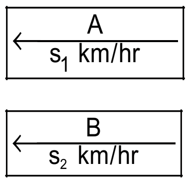
$= 33\frac{1}{3}$ km/hr.

Relative Speed

I. If two objects are traveling in the same direction at s_1 km/hr. and s_2 km/hr respectively such that $s_1 > s_2$ then s_1-s_2 is called is relative speed.



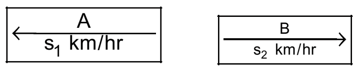
Relative Speed = s_1-s_2 ($s_1 > s_2$)



Relative Speed = s_1-s_2 ($s_1 > s_2$)

II. If two objects are travelling in the opposite direction at s_1 km/hr and s_2 km/hr, respectively, then s_1 +s_2 is called their relative speed.

Relative Speed = s_1+s_2



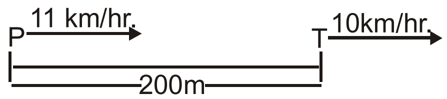
Relative Speed = s_1+s_2

Ex. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km/hr and 11 km/hr. respectively. What is the distance between them after 6 minutes.

(A) 100 m **(B)** 190 m

(C) 200 m **(D)** 150 m

Sol.(A)



Relative speed of police = 11-10 = 1 km/hr.

$= \frac{5}{18}$ m/sec

$\therefore$ Distance decreased in 6 minutes

$= \frac{5}{18} \times 6 \times 60$ = 100 m

$\therefore$ Distance remained between them

= 200-100 = 100 m

IMPORTANT POINTS

I. If the new speed is $\frac{a}{b}$ of the original speed, then the change in time taken to cover the same distance is given by

Change in time $= \left(\frac{b}{a} - 1\right) \times \text{original time}$

II. If a man travels at the speed of x km/hr. reach t_1 hr. late and if he travels at the speed of y km/hr. reach t_2 hr. early the travel distance.

$$\frac{xy}{y-x}(t_1 + t_2)\text{km}$$

III. If two persons A and B start at the same time from two points P and Q towards each other and after crossing they take T_1 and T_2 hrs in reaching Q and P respectively, then

$$\frac{\text{A's Speed}}{\text{B's Speed}} = \sqrt{\frac{T_2}{T_1}}$$

Ex. If a man travels $\frac{3}{4}$ th of his original speed then he reaches 20 minute late. Find the actual time taken by him :

(A) 45 min **(B)** 60 min

(C) 75 min **(D)** 120 min

Sol.(B) Let the actual speed x km/hr and the actual time t

$$\frac{1}{\frac{3x}{4}} - \frac{1}{x} = \frac{20}{60}$$

$$\frac{4}{3x} - \frac{1}{x} = \frac{1}{3}$$

x = 1 km/hr.

Actual time $= \frac{1}{1}$ = 1 hr. = 60 min.

QUICK TRICK :

Actual time $= \frac{3}{(4-3)} \times 20$

= 60 min.

Ex. Ritu goes at the speed of 4 km/hr. reaches her school 6 minutes late and next day she goes at the speed of 5 km/hr. reaches her school 6 minutes earlier, find the distance between her home and school ?

(A) 5 km **(B)** 4 km

(C) 6 km **(D)** 3 km

Sol.(B) Let Distance = x

So, $\frac{x}{4} - \frac{x}{5} = \frac{12}{60}$

$$\frac{x}{20} = \frac{1}{5}$$

x= 4 km

QUICK TRICK :

Distance $= \frac{4 \times 5}{1} \times \frac{12}{60}$ = 4 km

Race

General Instruction

(i) Any contest of speed in running, driving, riding or rowing is called a **Race.**

(ii) The point from where a race begins is called the **Starting Point.**

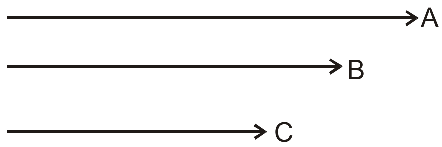
(iii) The point where the race finishes is called the **Finishing Point or Winning Post**

(iv) The person who first reaches the finishing point is called the **Winner**

Its an application of speed, time and Distance there are two types of question.

(1) **Time fixed-** The win or loss depends on distance.

(2) **Distance fixed-** The win or loss depends on the basis of time.



Ex. In a km race, A beats B by 56 meters or 7 seconds. Find A's time over the course.

(A) 1 min **(B)** 1 min 58 sec
(C) 1 min 50 sec **(D)** 1 min 52 sec

Sol.(B)

1000 meter
B-56m-A
-7 sec

B's time over the course

$= \left(\frac{7}{56} \times 1000\right)$ sec

= 125 seconds

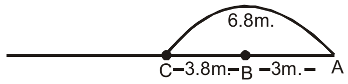
A's time over the course = (125 - 7) sec

= 1 min 58 seconds

Ex. In a km race A beats B by 30 meter, in 500 meter race B beats C by 20 meter so in 100 meter race A beats C by

(A) 9.7 meter **(B)** 3.8 meter
(C) 6.8 meter **(D)** 3 meter

Sol.(C)



Since, in a km race A beats B by - 30 meter

In a 100 meter race A beats B by-

$\frac{30}{1000} \times 100 = 3$ meter

In 500 meter race B beats C by 20 meter

In 97 meter race B beats C by $= \frac{20}{500} \times 97 = 3.8$

So, A beats C by = 3 + 3.8 = 6.8 meter.

Ex. In a race of 600 m., A can beat B by 60 m and in a race of 500 m., B can beat C by 50. By how many m. will A beat C in a race of 400 m.?

(A) 99 meter **(B)** 76 meter
(C) 136 meter **(D)** 164 meter

Sol.(B) Clearly, if A runs 600 metres, B runs = 540 m

$\therefore$ If A runs 400 m., B runs

$= \left(\frac{540}{600} \times 400\right) = 360$ metres

Similarly, when B runs 500 m., C runs = 450 m.

$\therefore$ When B runs 360 m., C runs

$= \left(\frac{450}{500} \times 360\right) m = 324$ m. Hence a beat C in a 400 metre race = 400–324 = 76

EXAMPLES

Ex.1. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is:

(A) 1 hour **(B)** 2 hours
(C) 3 hours **(D)** 4 hours

Sol.(A) Let the duration of the flight be x hours.

than $= \frac{600}{x} - \frac{600}{x + \left(\frac{1}{2}\right)} = 200$

x(2x + 1) = 3

$2x^2 + x - 3 = 0$

(2x + 3)(x - 1) = 0

x = 1, –3 hence during of flight x=1

Ex.2. In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is:

(A) 5 kmph **(B)** 6 kmph
(C) 6.25 kmph **(D)** 7.5 kmph

Sol.(A) Let Abhay's speed be x km/hr.

$\frac{30}{x} - \frac{30}{2x} = 3$

6x = 30

x = 5 km/hr.

Ex.3. Mohan is travelling on his cycle and has calculated to reach point A at 2 PM If he travels at 10 kmph, he will reach there at 12 noon. If he travels at 15 kmph. At what speed must he travel to reach A at 1 PM?

(A) 8 kmph **(B)** 11 kmph
(C) 12 kmph **(D)** 14 kmph

Sol.(C) Let the distance travelled by x km.

Then $\frac{x}{10} - \frac{x}{15} = 2$

3x - 2x = 60

x = 60 km.

Time taken to travel 60 km at 10 kmph.

$= \frac{60}{10}$ hrs= 6 hrs.

So, Mohan started 6 hours before 2 PM i.e., at 8 AM

Required speed = 60/5 kmph.= 12 kmph.

QUICK TRICK :

Required speed= $\frac{2\times10\times15}{10+15}$ = 12 kmph.

Ex.4. It takes eight hours for a 600 km journey, If 120 km is done by train and the rest by car. It takes 20 minutes more, If 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the cars is:

(A) 2 : 3 **(B)** 3 : 2

(C) 3 : 4 **(D)** 4 : 3

Sol.(C) Let the speed of the train be x km/hr and that of the car be y km/hr.

$\frac{120}{x}+\frac{480}{y}=8$ ------------ (i)

$\frac{200}{x}+\frac{400}{y}=\frac{25}{3}$ ------------ (ii)

Solving (i) and (ii), we get: x = 60, y = 80.

Ratio of speeds = 60 : 80 = 3 : 4.

Ex.5. A man covered a certain distance at some speed. Had he moved 3 kmph faster, he would have taken 40 minutes less. If he had moved 2 kmph slower, he would have taken 40 minutes more. The distance (in km) is:

(A) 35 **(B)** 37

(C) 40 **(D)** 42

Sol.(C) Let the distance = d km and speed s km/hr.

then, $\frac{d}{s}-\frac{d}{s+3}=\frac{40}{60}$ ....(i)

$\frac{d}{s-2}-\frac{d}{s}=\frac{40}{60}$ ....(ii)

d = 40 km

QUICK TRICK :

Actual speed= $\frac{2\times2\times3}{3-2}$ =12 km/hr.

Distance $=\frac{15\times10}{15-10}\times\frac{80}{60}$ = 40 km.

Ex.6. A and B start simultaneously in the same direction with speeds of 6 km/hr and 12 km/hr. A car from behind passes them in 9 and 10 seconds respectively. What is the speed of the car?

(A) 22 km/hr **(B)** 33 km/hr

(C) 66 km/hr **(D)** 44 km/hr

Sol.(C) A can travel distance 9 second

$= 6\times\frac{5}{18}\times9 = 15$ metre.

B can travel distance 10 second

$= 12\times\frac{5}{18}\times10 = 100$ metre.

Hence in 1 second Car can travel = $\frac{55}{3}$ metre.

Hence Car speed = $\frac{55}{3}\times\frac{18}{5}$ = 66 km./hr.

Ex.7. Raj travelled from his house to his office at 10 km/hr and returned at rate of 9 km/hr. Rohit travelled through the same path both ways at 12 km/hr and took 10 minutes less than Raj. What is the distance between Raj's house and his office?

(A) 3000m **(B)** 7500m

(C) 3750m **(D)** 7000m

Sol.(C) Let the distance between office and house = x km.

According to question

$\left(\frac{x}{10}+\frac{x}{9}\right)-\left(\frac{x}{12}+\frac{x}{12}\right)=\frac{10}{60}$

$\Rightarrow \frac{4x}{90}=\frac{1}{6}$

x = $\frac{30}{8}$ km= 3750 m.

Ex.8. Mayur left for his school at 6 am by foot and walked at the rate of 2 kmph. After reaching his school he found it was closed and immediately turned back and started walking back to his home at 3 kmph. If he reached his home at 9 am, find the distance between his school and home.

(A) 2.5 km **(B)** 7.2 km

(C) 7 km **(D)** 3.6 km

Sol.(D) Let distance between house and school = x km

According to question

$\left(\frac{x}{2}+\frac{x}{3}\right)=3$

$\frac{5x}{6}=3$

x = 3.6 km.

QUICK TRICK :

Distance between house and school

$= \frac{2\times3}{(2+3)}\times3$ = 3.6 km

Ex.9. Two bikes start at the same time from two destinations 300 km apart and travel towards each other. If they cross each other at a distance of 130 km from one of the destination, what is the ratio of their speeds?

(A) 13:17 **(B)** 17:13

(C) 13:30 **(D)** 7 : 3

Sol.(A) Distance travelled by one bike is 130 km and that by the other bike is 170 km.

Ratio of their speed $= \frac{130}{T} : \frac{170}{T}$

= 13 : 17

Ex.10. A squirrel can climb 5 metres of a pole in one minute but slips 2 metres in the next minute. How much time will the squirrel take to climb 26 metres?

(A) 14 minutes **(B)** 12 minutes

(C) 15 minutes **(D)** 18 minutes

Sol.(C) According to question in first minute squirrel climbs = 5 m

In next minute she slips = 2 metres

So, net climbs in two minute = 5-2 = 3 m

The squirrel climbs in last minute = 5 m

So, rest 21 km. climbs $= \frac{21}{3}$ = 7×2 min

= 14 min

Total time required to climbs= 14+1= 15 min.

Ex.11. Two guns are fired from the same place at an interval of 6 min. A person approaching the Place observe that 5 min-52 sec. have elapsed between the hearing of the sound of the two guns. If the velocity of the sound is 330 m/sec. The man was approaching that place at what speed?

(A) 25 km/hr. **(B)** 27 km/hr.

(C) 28 km/hr. **(D)** 30 km/hr.

Sol.(B) Difference of time = 6 min - 5 min 52 sec = 8 sec.

Distance covered by man = Distance covered by sound = 330×8 sec.

Speed of man $= \frac{2640}{5 \text{ min } 52 \text{ sec}}$

$= \frac{2640}{352}\times\frac{18}{5}$ = 27 km/hr.

Ex.12. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

(A) 21 min **(B)** 19 min

(C) 17 min **(D)** 28 min

Sol.(B) Let the time taken to going on foot = x hr. and by riding = y hr. Then,

According question

x+y = 37 min....................(i)

2x = 55

$x = \frac{55}{2}$ min

$y = \frac{74-55}{2} = \frac{19}{2}$ min

∴ If riding both sides then time taken

$= \frac{19}{2}\times2$ = 19 min.

Ex.13. A thief steals a car at 1.30 pm and drives its at 40 km/hr. The theft is discovered at 2 pm and the owner sets off in another car at 50 km/hr. He will overtake the thief at.

(A) 5 Pm **(B)** 4 Pm

(C) 4.30 Pm **(D)** 6 Pm

Sol.(B) Distance covered by the thief in half an hour

$= \frac{1}{2}\times40$ = 20 km.

Relative speed of car owner 50-40 = 10 km

Required time $= \frac{\text{Difference of Distance}}{\text{Relative Speed}} = \frac{20}{10}$

= 2 hr.

Thief overtaken at = 2+2 = 4 pm

Ex.14. A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

(A) 360m **(B)** 750m

(C) 305m **(D)** 400m

Sol.(D) Relative speed of the policeman = (10-8) km/hr

=2 km/hr.

Time taken by police man to cover 100m : (100 x 1/2)/1000 hr = 1/20hr.

In 1/20 hrs, the thief covers a distance of

$= \frac{8 \times 1}{20} = \frac{2}{5}$ km = 2/5 km = 400 m

Ex.15. A cab on its way to Bangalore overtakes an auto also going in the same direction at 11 am. The cab reaches Bangalore at 1.30 pm and starts the return journey after taking a break of 1 hour. On its way back it meets the auto at 3 pm. At what time will the auto reach Bangalore?

(A) 3 p.m. **(B)** 3.30 p.m.

(C) 5 p.m **(D)** 4 p.m.

Sol.(D) Let distance to the bangalore be 100 km

$\therefore$ Speed of cab = $\frac{100}{2.5} = 40$ km/hr.

$\therefore$ Distance cover in $\frac{1}{2}$ hr. = 20 km

Distance cover by auto = 100-20= 80 km

Speed of auto $= \frac{80}{4}$ = 20 km/hr.

Time taken by auto = $\frac{100}{20} = 5$ hrs.

Required time = 4 pm.

Ex.16. A train leaves Delhi at 6 AM and reaches Jay Nagar at 10 AM. Another train leaves Jay Nagar at 8 AM and reaches delhi at 11:30 AM. At what time do the two train cross one another eachother ?

(A) 8 : 48 AM **(B)** 8 : 56 AM

(C) 6:56 AM **(D)** 9 : 15 AM

Sol.(A) Let distance between Delhi and Jay Nagar d km.

So, speed of train A = $\frac{d}{4}$ km/hr.

Speed of Train B = $\frac{d}{3.5}$ km/hr.

Train A travel distance in 2 hr.

$\frac{d}{4} \times 2 = \frac{d}{2}$ km.

Relative Speed = $\frac{d}{4} + \frac{d}{3.5} = \frac{15\,d}{28}$

They meet $= \frac{\frac{d}{2}}{\frac{15\,d}{28}} = 56$ min

$\therefore$ So, that = 8 : 56 AM

QUICK TRICK :

They meet

$= 6 + \frac{4(11.30-6)}{(4+3.5)} = 8:56$ AM

Ex.17. A man had to cover a distance of 60 km. However he started 6 min. later than his scheduled time and raced at a speed 1 km/hr. higher than his original speed and reached on time. Find his original speed.

(A) 25 km/hr. **(B)** 20 km/hr.

(C) 24 km/hr. **(D)** 30 km/hr.

Sol.(C) Let Man's original speed = S km/hr.

According question

$\frac{60}{S} - \frac{60}{S+1} = \frac{6}{60}$

$S^2+S-600=0$

+25-24 → -25+24

Original Speed (S) = 24 km/hr.

Ex.18. The distance between 2 stations A and B is 220 km. Two cars start simultaneously from A and B in opposite direction and the distance between them after 2 hours is 30 km. The faster car speed is 5 km/hr. more than slower car. Find the speed of slower car ?

(A) 45 km/hr. **(B)** 50 km/hr.

(C) 55 km/hr. **(D)** 60 km/hr.

Sol.(A) Let Speed of slower car = x km/hr.

Speed of faster car = x+5 km/hr.

According to question

$\frac{190}{(x+x+5)} = 2$ hr.

x = 45 km/hr.

Ex.19. The ratio between the speed of truck, car and train is 3 : 8 : 9. The car moved uniformly and covered a distance of 1040 km. in 13 hr. What is the average speed of truck and the train together ?

(A) 60 km/hr. **(B)** 55 km/hr.

(C) 150 km/hr. **(D)** 75 km/hr.

Sol.(A) $S_{car} = \frac{1040}{13} = 80$ km / hr

$S_{truck} : S_{car} : S_{train} = 3 : 8 : 9$

Speed of car (8) = 80 km/hr.

Speed of truck (3) = $\frac{80}{8} \times 3$ = 30km/hr.

Speed of train (9) = $\frac{80}{8} \times 9$ = 90 km/hr.

Average Speed of truck and train

$= \frac{30+90}{2} = \frac{120}{2} = 60\text{km / hr.}$

Ex.20. A bike during a fog passes a man who was walking at a speed of 3 km/hr. in the same direction. He could see the bike for 4 min and it was visible to him upto a distance of 100 m. Find the speed of the bike ?

(A) 1.5 km/hr **(B)** 3 km/hr

(C) 4.5 km/hr **(D)** 6 km/hr

Sol.(C) x km/hr.

3 k/hr.

(x-3) = $\frac{100\text{m.}}{(4 \times 60)\text{sec.}} \times \frac{18}{5}$

(x-3) = 1.5

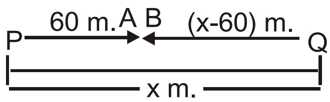
x = 4.5 km/hr.

Ex.21. Two persons A and B start simultaneously from P and Q towards Q and P respectively. A meets B at a distance of 60 meter from P. After A reaches Q and B reaches P, they turn around and start walking in the opposite direction. Now B meets A at a distance of 40 meter from Q. Find the distance between P and Q.

(A) 100 m **(B)** 120 m

(C) 140 m **(D)** 160 m

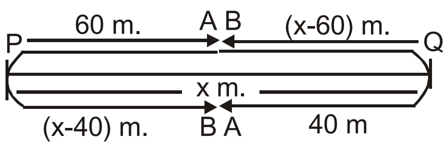
Sol.(C) According to first condition



$\frac{60}{S_A} = \frac{x-60}{S_B}$

$\frac{S_A}{S_B} = \frac{60}{x-60}$ ....(i)

According to second condition



$\frac{x-60+40}{S_A} = \frac{60+x-40}{S_B}$

$\frac{S_A}{S_B} = \frac{x-20}{x+20}$.............(ii)

On solving both

$\frac{x-20}{x+20} = \frac{60}{x-60}$

x = 140 m

Ex.22. A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{10}$. He goes 10 km less in that time. Find his speed per hour.

(A) $3\frac{1}{3}$ km/hr **(B)** $4\frac{1}{3}$ km/hr

(C) $3\frac{2}{3}$ km/hr **(D)** $3\frac{1}{4}$ km/hr

Sol.(A) $30 \times s - 30 \times \frac{9}{10}s = 10$

$\Rightarrow$ 3s = 10

$\Rightarrow$ $s = \frac{10}{3} = 3\frac{1}{3}$ km/hr.

Ex.23. A man covers the journey from a station A to B at a uniform speed of 36 km/hr. and returnes to A with a uniform speed of 45 km/hr. His average speed for the whole journey is -

(A) 40 km/hr. **(B)** 40.5 km/hr.

(C) 41 km/hr. **(D)** 42 km/hr.

Sol.(A) $\frac{2 \times 36 \times 45}{81}$ = 40 km/hr.

Ex.24. A starts from a place at 7 am with a speed of 10 km/hr. B starts from the same place at 9 am with a speed of 15 km/hr.When will B meet A?

(A) 2 p.m. **(B)** 1 p.m.

(C) 3 p.m. **(D)** 12 noon

Sol.(B) Distance covered by A in 2 hours = 20 km

$\therefore$ Required time taken= $\frac{20}{15-10}$ = 4 hr

time = 9.00 AM+4 hours = 1.00 PM

Ex.25. Mr. Kumar drives to work at an average speed of 48 km/hr.The time taken to cover the first 60% of the distance is 10 min more than the time taken to covers the remaining distance.How far is his office?

(A) 30 km **(B)** 40 km.

(C) 45 km. **(D)** 48 km.

Sol.(B) Let distance be x km

$$\frac{\frac{x \times 60}{100}}{48} - \frac{\frac{x \times 40}{100}}{48} = \frac{10}{60}$$

$$\Rightarrow \frac{0.6x - 0.4x}{48} = \frac{1}{6}$$

$\Rightarrow$ x

= 40 km

Ex.26. In one kilometre race, A beats B by 64 metres or 8 seconds. Find A's time over the course.

(A) 119 second **(B)** 117 second

(C) 121 second **(D)** 120 second

Sol.(B) Time = $\frac{8}{64} \times 936$ = 117 seconds

Ex.27. A runs $2\frac{1}{3}$ times as fast as B. If A gives B a start of 80 metres, how far must be the winning post, so that the race ends in a dead heat ?

(A) 40 metre **(B)** 50 metre

(C) 30 metre **(D)** 140 metre

Sol.(D) Let the distance be L m. and speed be v m/sec.

$$\frac{3L}{7} = L - 80$$

$80 = L - \frac{3L}{7}$, L = 140 m.

Ex.28. A can run a km in 3 min 10 sec and B in 3 min 20 sec. By what distance can A beat B ?

(A) 50 metre **(B)** 40 metre

(C) 30 metre **(D)** 60 metre

Sol.(A) Time of A = 190 seconds

Time of B = 200 seconds

or A beats B by 10 seconds.

Distance covered by B in 10 seconds.

$$\frac{1000}{200} \times 10$$

= 50 metre

A will beat B by 50 metre.

Ex.29. A can run 100 metres in 45 seconds and B in 50 seconds. A will beat B by-

(A) $20\frac{3}{4}$ metre **(B)** 12 metre

(C) 8 metre **(D)** 10 metre

Sol.(D) Distance = $\frac{100}{50} \times 5$

= 10 metre

Ex.30. In a 500 metres race, B gives A a start of 160 metres. The ratio of the speeds of A and B is 2 : 3. B beat to A is-

(A) $6\frac{2}{3}$ metre **(B)** 8 metre

(C) $8\frac{1}{3}$ metre **(D)** $6\frac{2}{3}$ metre

Sol.(A) Let after t seconds B reaches 500 metres then A reaches x metres

$$\frac{x - 160}{500} = \frac{2}{3}$$

x = $493\frac{1}{3}$ metre

B beats A by = 500- $493\frac{1}{3}$

= $6\frac{2}{3}$ metre

EXERCISE

Q.1. Robert is travelling on his cycle and has calculated to reach point A at 2 pm., if he travels at 10 km./hr., he will reach at 12 noon if he travels at 15 km./hr. At what speed must he travel to reach A at 1 pm.?

(A) 8 km./hr. **(B)** 11 km./hr.

(C) 12 km./hr. **(D)** 14 km./hr.

Q.2. A father and his son start a point A with speeds of 16 km./hr. and 24 km./hr. respectively, and reach another point B. If his son starts 60 minutes after his father at A and reaches B 60 minutes before his father, what is the distance between A and B?

(A) 90 km. **(B)** 72 km.

(C) 96 km. **(D)** None of the above

Q.3. A man has to be at a certain place at a certain time. The finds that he shall be 20 minutes late if he walk at 3 km./hr. speed and 10 minutes earlier if he walks at a speed of 4 km./hr. The distance he has to walk is-

(A) 24 km. **(B)** 12.5 km.

(C) 10 km. **(D)** 6 km.

Q.4. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minute?

(A) 100 m. **(B)** 150 m.

(C) 75 m. **(D)** 250 m

Q.5. From a point on circular track 5 km long A, B and C started running in a same direction at same time with speed of $2\frac{1}{2}$ km per hours, 3 km per hours, 2 km per hours respectively than on a starting point all three will meet again after -

(A) 30 hours **(B)** 6 hours

(C) 10 hours **(D)** 15 hours

Q.6. A, B and C start at the same time in the same direction to run around a circular field. A completes a round in 252 sec. B in 308 sec and C in 198 sec, after how much time will they meet again at the starting point?

(A) 60 min **(B)** 59 min 10 sec.

(C) 54 min **(D)** 46 min 12 sec.

Q.7. A car travels along the four sides of a square at speeds V, 2V, 3V, 4V respectively. If U is the average speed of the car in its travel around the square then which one of the following is correct ?

(A) U = 2.25V **(B)** U = 3V

(C) V < U < 2V **(D)** 3V < U < 4V

Q.8. Sound of any word to travel in air at about 1100 feet per second. A man hears the axe striking the tree, $\frac{11}{5}$ seconds after he sees it strike the tree. How far is the man from the wood chopper ?

(A) 2197 ft **(B)** 2420 ft

(C) 25 ft **(D)** 2629 ft

Q.9. An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long it take to reach its destination 600 km from the starting point ?

(A) 6 hrs 21 min **(B)** 6 hrs 24 min.

(C) 6 hrs 27 min **(D)** 6 hrs 30 min

Q.10. The speed of a car increases by 2 kms/hr. after every one hour. If the distance covered in the first one hour was 35 km, what was the total distance travelled in 12 hours ?

(A) 456 kms **(B)** 482 kms

(C) 552 kms **(D)** 556 kms

Q.11. A person has to cover a distance of 60 km in 45 minutes. If he covers one half of distance in two-thirds of the total time, to cover the remaining distance in the remaining time, his speed (km/hr) must be:

(A) 60 **(B)** 80

(C) 120 **(D)** 150

Q.12. A can complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/ hr and second half at the rate of 24 km/hr. Find the total journey

(A) 220 km **(B)** 224 km

(C) 230 km **(D)** 234 km

Q.13. A person travels equal distances with speeds of 3 km/hr, 4 km/hr and 5 km and takes a total time of 47 minutes. The total distance (in km) is :

(A) 2 **(B)** 3

(C) 4 **(D)** 5

Q.14. A is faster than B. A and B each walk 24 km. The sum of their speeds is 7 km and the sum of times taken by them is 14 hours. Then, A's speed is equal to:

(A) 3 km/ hr **(B)** 4 km/ hr

(C) 5 km/ hr **(D)** 6 km/ hr

Q.15. A car driver travels from the plains to the hill station, which are 200 km apart at an average speed of 40 km/ hr. In the return trip, he covers the same distance at an average speed of 20 km/ hr. The average speed of the car over the entire distance of 400 km is :

(A) 25 km/ hr **(B)** 26.67 km/ hr

(C) 28.56 km/ hr **(D)** 30 km/ hr

Q.16. The average speed of a train in the onward journey is 25 % more than that in the return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is :

(A) 45 km/hr **(B)** 47.5 km/hr

(C) 52 km/hr **(D)** 56.25 km/hr

Q.17. I started on my bicycle at 7 a.m. to reach a certain place. After going a certain distance, by bicycle went out of order. Consequently, I rested for 35 minutes and came back to my house walking all the way. I reached my house at 1 p.m. If my cycling speed is 10 kmph and my walking speed is 1 kmph, then on my bicycle I covered a distance of:

(A) $4\frac{61}{66}$ km **(B)** $13\frac{4}{9}$ km

(C) $14\frac{3}{8}$ km **(D)** $15\frac{10}{21}$ km

Q.18. A,B and C are on a trip by car.A drives during the first hour at an average speed 50 km/hr. B drives during the next 2 hours at an average speed of 48 km/hr. A and C drives during the next 3 hours at an average speed of 52 km./hr. and they reached the destination in 6 hours. Their mean speed is :

(A) 50 km/hr **(B)** $50\frac{1}{3}$ km/hr

(C) 51 km/hr$\frac{1}{3}$ **(D)** 52 km/hr

Q.19. A boy rides his bicycle 10 km at an average speed of 12 km/hr and again travels 12 km at an average speed of 10 km/hr. His average speed for the entire trip is approximately.

(A) 10.4 km/hr **(B)** 10.8 km/hr

(C) 11 km/hr **(D)** 12.2 km/hr

Q.20. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the one third distance with speed of 20 km/hr. and the last one third distance with speed of 60 km/hr the average speed of the car for the whole journey is:

(A) 18 km/hr **(B)** 24 km/hr

(C) 30 km/hr **(D)** 36 km/hr

Q.21. A motorist covers a distance of 39 km in 45 minutes by moving at a speed for the first 15 mintes, then moving at double the speed for the next 20 m. the again moving at his original speed for the rest of the journey. Then find the speed (in km/hr).

(A) 31.2 **(B)** 36

(C) 40 **(D)** None of these

Q.22. Starting from his house one day, a student walks at a speed of $2\frac{1}{2}$ kmph and reached his school 6 minutes late. Next day he increased his speed by 1kmph and reached school 6 minutes early. How far is the school from his house ?

(A) 1 km **(B)** $2\frac{1}{2}$ km

(C) $1\frac{3}{4}$ km **(D)** 2 km

Q.23. If a train runs at 40 kmph, it reaches its destination late by 11 minutes but if it runs at 50 kmph, it is late by 5 minutes only. The correct time for the train to its journey is :

(A) 13 min. **(B)** 15 min.

(C) 19 min. **(D)** 21 min.

Q.24. A car travels from P to Q at a constant speed. If its speed were increased by 10 km/hr, it would have taken one hour less to cover the distance. It would have taken further 45 minutes lesser if the speed was further increased by 10 km/hr . What is the distance between the two cities ?

(A) 420 km **(B)** 540 km

(C) 600 km **(D)** 650 km

Q.25. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour ?

(A) 9 **(B)** 10

(C) 12 **(D)** 20

Q.26. In a race of 100 mtr. A's speed is 5 k/hr. In the start of race B is forward by 8 mtr. to A. Still he reaches on destination before 8 sec. Find speed of B.

(A) 4.14 km/hr. **(B)** 5 km/hr.

(C) 5.56 km/hr. **(D)** 6 km/hr.

Q.27. A, B and C are three contestants taking a part of in a km. race. If in the start A gives to pass of B by 40 meters forward and A to C by 64 mtr. forward. How many meter's gives forward by D to C ?

(A) 36 meters **(B)** 24 meters

(C) 25 meters **(D)** 30 meters

Q.28. In a kilometer race, A beat B by 30 second and B beats C by 15 seconds. If A beats C by 180 metres, the time taken by A to run 1 kilometer is-

(A) 250 seconds **(B)** 205 seconds

(C) 200 seconds **(D)** 210 seconds

Q.29. A runs $\frac{7}{4}$ times as fast as B. If A gives to pass of B by 300 metre forward. How many far away must the wining post if both A and B have to end the race at same time ?

(A) 1400 metres **(B)** 700 metres

(C) 350 metres **(D)** 210 metres

Q.30. In a km. race A, B and C are three participants. A can give B a start of 50 m and C a start of 69 m. How many metres start can B give to C?

(A) 19 m **(B)** 20 m

(C) 18 m **(D)** 21 m

Q.31. In one kilometre race, A beats B by 64 metres or 8 seconds. Find A's time over the course.

(A) 119 second **(B)** 117 second

(C) 121 second **(D)** 120 second

Q.32. A runs $2\frac{1}{3}$ as fast as B. If A gives B a start of 80 metres, how far must be the winning post, so that the race ends in a dead heat ?

(A) 40 metre **(B)** 50 metre

(C) 30 metre **(D)** 140 metre

Q.33. A can run a km in 3 min 10 sec and B in 3 min 20 sec. By what distance can A beat B ?

(A) 50 metre **(B)** 40 metre

(C) 30 metre **(D)** 60 metre

Q.34. A can run 100 metres in 45 seconds and B in 50 seconds. A will beat B by-

(A) $20\frac{3}{4}$ metre **(B)** 12 metre

(C) 8 metre **(D)** 10 metre

Q.35. In a 500 metres race, B gives A a start of 160 metres. The ratio of the speeds of A and B is 2 : 3. B beat to A is-

(A) $6\frac{2}{3}$ metre **(B)** 8 metre

(C) $8\frac{1}{3}$ metre **(D)** $6\frac{2}{3}$ metre

Q.36. Pramod takes 4 hours 40 minutes in going to some place on foot and returns on a vehicle. Had he travelled both the sides on foot, he would have taken 2 hours more. How much time will he take in travelling both sides on a vehicle?

(A) 2 hours 10 minutes

(B) 2 hours 30 minutes

(C) 2 hours

(D) 2 hours 40 minutes

Q.37. Ranbir after travelling 42 km, found that if he travelled 2.5 km an hour, he would take 5 hr. less, he actually travelled at a rate of

(A) 7 km/hr. **(B)** 10 km/hr.

(C) 5 km/hr. **(D)** 6 km/hr.

Q.38. A man travels equal distances of his journey at 40, 30 and 15 km/hr. respectively. Find his average speed for whole journey.

(A) 25 km/hr. **(B)** 24 km/hr.

(C) 30 km/hr. **(D)** 20 km/hr.

Q.39. Aditi walks half of the journey at 4 km/h, by cycle does one third of journey at 12 km/h and rides the remaining journey in a horse cart at 9 km/h, thus completing the whole journey in 14 hours 48 minutes. The length of the journey is

(A) 77.6 km. **(B)** 57.6 km.

(C) 58.8 km. **(D)** 86.4 km.

Q.40. Ravi travels 300 km. partly by train and partly by car. He takes 4 hr. to reach, if he travels 60 km. by train and rest by car. He will take 10 minutes more if he were to travel 100 km. by train and rest by car. The speed of the train is:

(A) 50 kmph **(B)** 60 kmph

(C) 100 kmph **(D)** 120 kmph

EXPLANATION

Q.1.(C) Let the distance travelled by x km.

$\Rightarrow \frac{x}{10} - \frac{x}{15} = 2$

$\Rightarrow 3x - 2x = 60$

x = 60 km.

Time taken to traved 60 km. at 10 km./hr.

$= \frac{60}{10}$

= 6 hours

$\therefore$ Required speed

$= \frac{60}{5}$

= 12 km./hr.

Q.2.(C) Let distacne between A and B = x km.

$\frac{x}{16} - \frac{x}{24} = 2 \Rightarrow \frac{3x-2x}{48} = 2 \Rightarrow x = 96$ km.

Q.3.(D) Let distance = x km.

$\therefore \frac{x}{3} - \frac{20}{60} = \frac{x}{4} + \frac{10}{60}$

$\Rightarrow \frac{x}{3} - \frac{x}{4} = \frac{10}{60} + \frac{20}{60}$

$\Rightarrow$ x = 6 km. (distance)

Q.4.(A) Relative speed of police

= 11 - 10 = 1 kmph = $\frac{5}{18}$ m / sec

$\therefore$ distance covered in 6 minute

$\therefore \frac{5}{18} \times 6 \times 60$

Distance remained

= 200-100 = 100 m

Q.5.(C) A makes one complete round in = $\frac{5}{\frac{5}{2}}$

= 2 hours.

B makes one complete round in

$= \frac{5}{3}$ hours

C makes one complete round in

$= \frac{5}{2}$ hours

Hence the required time

= LCM of 2, $\frac{5}{3}, \frac{5}{2}$

$= \frac{\text{LCM of } 2,5,5}{\text{HCF of } 3,2}$

$= \frac{10}{1}$ hours.

Q.6.(D) L.C.M. of 252, 308, 198

= 2772 second

= 46 min 12sec.

Q.7.(C) Average speed = $\frac{\text{Total Distance}}{\text{Total Time}}$

$u = \frac{a+a+a+a}{\frac{a}{v} + \frac{a}{2v} + \frac{a}{3v} + \frac{a}{4v}} = \frac{48v}{25}$

u = 1.92v

$\therefore$ v < u < 2v

Q.8.(B) Distance = $\left(1100 \times \frac{11}{5}\right)$ feet = 2420 feet.

Q.9.(A) Time taken to cover 600 km.

$= \left(\frac{600}{100}\right)$ hrs = 6 hrs.

Number of stoppages = $\frac{600}{75}$ -1 = 7.

Total time of stoppage = (3 × 7) min

= 21 min.

Hence total time taken

= 6 hrs 21 min.

Q.10.(C) Total distance travelled in 12 hours

= (35+37+39+....up to 12 terms)

This is an A.P. with first term, a = 35, number of terms, n = 12, common difference, D = 2

Required distance = $\frac{12}{2}$ [2×35+(12-1)]

= 6 (70+22) = 552 km.

Q.11.(C) 45 min ..... 60 km

30 min ..... 30 km

Difference = 30 km

15 min ..... 30 km

60 min ..... 120 km

Required speed = 120 km/hr.

Q.12.(B) Let the total distance be x km. Then.

$$\frac{\frac{1}{2}x}{21}+\frac{\frac{1}{2}x}{24}=10 \Rightarrow \frac{x}{21}+\frac{x}{24}=20$$

$\Rightarrow$ 15x = 168 ×20 $\Rightarrow$ $x=\left(\frac{168\times 20}{15}\right)$

= 224 km.

Q.13.(B) Let the total distance be 3x km.

Then, $\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=\frac{47}{60} \Rightarrow \frac{47x}{60}=\frac{47}{60}$

$\Rightarrow$ x = 1, Total distance = (3×1) km = 3 km.

Q.14.(A) Let A's speed = x km/hr. Then, B's speed = (7-x) km/hr.

So, $\frac{24}{x}+\frac{24}{7-x}=14$

24 (7-x) +24x = 14x (7-x)

$14x^2 - 98x + 168 = 0$

$x^2 - 7x + 12$

(x-3) (x-4) = 0

x = 3 or x = 4

Since, A is faster than B, so A's speed

= 4 km/hr and B's speed = 3 km/hr.

Q.15.(B) Average speed

$=\left(\frac{2\times 40\times 20}{40+60}\right)\text{km / hr}=\left(\frac{80}{3}\right)$ km/hr

= 26.67 km/hr.

Q.16.(D) Let the speed in return journey be x km/hr.

Then, speed in onward journey

$=\frac{125}{100}x=\left(\frac{5}{4}x\right)$ km/hr.

Average speed = $\left(\frac{2\times\frac{5}{4}x\times x}{\frac{5}{4}x+x}\right)$ km/hr

$=\frac{10x}{9}$ km/hr. $\therefore$ $\left(800\times\frac{9}{10x}\right)=16$

$x=\left(\frac{800\times 9}{16\times 10}\right)$=45

So, speed in onward journey = $\left(\frac{5}{4}\times 45\right)$ km/hr = 56.25 km/hr.

Q.17.(A) Time taken = 5 hrs 25 min. = $\frac{65}{12}$hrs.

Let the required distance be x km.

Then, $\frac{x}{10}+\frac{x}{1}=\frac{65}{12}$

$= 11x=\frac{650}{12} \Leftrightarrow x=\frac{325}{66}=4\frac{61}{66}$ km.

Q.18.(C) Total distance travelled

= (50×1+48×2+52×3)km = 302 km.

Total time taken = 6 hrs.

Mean speed = $\left(\frac{302}{6}\right)$ km/hr = $50\frac{1}{3}$ km/hr.

Q.19.(B) Total distance travelled = (10+12)km/hr = 22 km/hr.

Total time taken = $\left(\frac{10}{12}+\frac{12}{10}\right)$ hrs.= $\frac{61}{30}$ hrs.

Average speed

$=\left(22\times\frac{30}{61}\right)$ km/hr. =10.8 km/hr.

Q.20.(A) Let the whole distance travelled be x km and the average speed of the car for the whole journey be y km/hr.

Then, $\frac{(x/3)}{10}+\frac{(x/3)}{20}+\frac{(x/3)}{60}=\frac{x}{y}$

$=\frac{x}{30}+\frac{x}{60}+\frac{x}{180}=\frac{x}{y}$

$\frac{1}{18}y=1, Y=18\text{km / hr.}$

Q.21.(B) $x\times\frac{15}{60}+2x\times\frac{20}{60}+x\times\frac{10}{60}=39$

$=\frac{x}{4}+\frac{2x}{3}+\frac{x}{6}=39$

= 3x+8x+2x=468

x = 36.

Q.22.(C) Let the distance be x km.

Difference in timings = 12 min = $\frac{12}{60}$ hr.

$=\frac{1}{5}$hr.

$\therefore \frac{2x}{5} - \frac{2x}{7} = \frac{1}{5}$ = 14x - 10x = 7

x = $1\frac{3}{4}$ km.

Q.23.(C) Let the correct time to complete the journey be x min.

Distance covered in (x +11) min. at 40 kmph.

= Distance covered in (x+5) min. at

$\frac{(x+11)}{60} \times 40 = \frac{(x+5)}{60} \times 50$

= x = 19 min.

Q.24.(A) Let distance = x km and usual rate = y kmph. Then,

$\frac{x}{y} - \frac{x}{y+10}$ = 1 or y (y+10)= 10x

and, $\frac{x}{y} - \frac{x}{y+20} = \frac{7}{4}$ or y(y+20)= $\frac{80x}{7}$

On dividing (i)by (ii), we get y, = 60.

Substituting y = 60 in (i) , we get : x

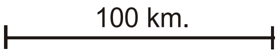
= 420 km.

Q.25.(B) Due to stoppage, it covers 9 km less.

Then, taken to cover 9 km = $\left(\frac{9}{54} \times 60\right)$ min.

= 10 min.

Q.26.(A)



A's speed = $5 \times \frac{5}{18}$

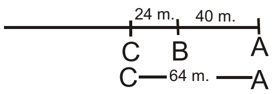
A takes time = $\frac{100}{25} \times 18$ = 72 second

B takes = 72+8 = 80 second

and B complete (100 – 8) = 92 meter

Speed = $\frac{92}{80} \times \frac{18}{5}$ = 4.14 km./hr.

Q.27.(C)



B and C 24 meters in 960 meters

In 960 meters B can give starts to C = 24 meters

In 1000 meters _____ = $\frac{24}{960} \times 100$

= 25 meters

Q.28.(B) A beat B by 30 seconds and B beat C by 15 seconds.

A beat C by 30+15 = 45 seconds.

Also A beat C by 180 meters

Hence C covered 180 metres in 45 seconds.

Speed of C = $\frac{180}{45}$ = 4 m./sec.

Time taken by C to cover 1000 metre

= $\frac{1000}{4}$ = 250 seconds

Time taken by A to cover 1000 metre

= 250 – 45

= 205 seconds

Q.29.(B)

	A:B
Speed	7:4
Time of A = Time of B	
Distance	4:7

Now, 7x – 4x = 300

3x = 300, x = 100, 7x = 700

Thus the winning post be 700 metre away from the starting point.

Q.30.(B) While A covers 1000 m, B covers (100-50) = 950 m., C covers (1000 - 69) = 921 m.

When B covers 1000 m., C will cover

$\frac{931}{950} \times 1000 = 980$

$\therefore$ starts = 20 m.

Q.31.(B) Time = $\frac{8}{64} \times 936$ = 117 seconds

Q.32.(D) Let the distance be L m. and speed be v m/sec.

$\frac{3L}{7} = L - 80$

$80 = L - \frac{3L}{7}$, L = 140 m.

Q.33.(A) Time of A = 190 seconds

Time of B = 200 seconds

or A beats B by 10 seconds.

Distance covered by B in 10 seconds.

$\frac{1000}{200}\times 10$

= 50 metre

A will beat B by 50 metre.

Q.34.(D) Distance = $\frac{100}{50}\times 5$

= 10 metre

Q.35.(A) Let after t seconds B reaches 500 metres then A reaches x metres

$\frac{x-160}{500}=\frac{2}{3}$

x = $493\frac{1}{3}$ metre

B beats A by - 500-$493\frac{1}{3}$

= $6\frac{2}{3}$ metre

Q.36.(D) Time taken in travelling both sides on vehicle

= 4 hours 40 minute – 2 hour

= 2 hours 40 minutes

Q.37.(A) Let his actual speed be x km/hr.

According to the question,

$\frac{42}{x}-\frac{42}{(x+5)}=2.5$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

x = 7, – 12

Hence speed = 7 km/hr.

Q.38.(B) Required average speed

$=\frac{3\times 40\times 30\times 15}{40\times 30+40\times 15+30\times 15}$

= 24 km/hr.

Q.39.(D) Let total distance be x km.

According to the question,

$\frac{x}{2\times 4}+\frac{x}{3\times 12}+\frac{x}{6\times 9}=14+\frac{48}{60}$

$\frac{x}{8}+\frac{x}{36}+\frac{x}{54}=14\frac{4}{5}$

$\frac{27x+6x+4x}{216}=\frac{74}{5}$

x = $\frac{216\times 74}{37\times 5}$ = 86.4 km.

Q.40.(B) Let the speed of train be x km/hr. and speed of car be y km/hr.

$\frac{60}{x}+\frac{240}{y}=4$

$\Rightarrow \frac{1}{x}+\frac{4}{y}=\frac{1}{15}$......(I)

$\frac{100}{x}+\frac{200}{y}=\frac{25}{6}$

$\frac{1}{x}+\frac{2}{y}=\frac{1}{24}$..........(II)

From (I) and (II), we get

y = 80 kmph, x = 60 kmph

Notes

CHAPTER-16
PROBLEM ON TRAINS

Scan the QR code to get video of this chapter.

This type of questions are based on the concept of time, distance and speed.

1. Object I:

Those object that cross another object is considered as object I (primarly trains are considered as Ist object but sometimes person also considered as 1st object when person crosses another object like stationary train or platform)

Ex. train, bus, person etc.

2. Object II.

Those object which are crossed by object I train, platform, over bridge, tunnel, person, pole tree, wall etc.

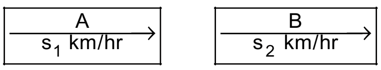
General Rule

Concept of speed

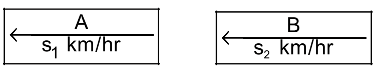
I. When one object is stationary and another is still then only speed of moving object is considered as S_1 =x (let) (S_2 =0)

(A) When both objects are moving in same direction

In this case relative speed is difference between speed of both objects that means relative speed = $S_1 \sim S_2$



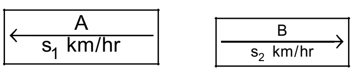
Relative Speed = s_1-s_2 ($s_1 > s_2$)



(B) When both object are moving in opposite direction.

In this case relative speed is sum of both speed of both object means relative speed = $S_1 + S_2$

Relative Speed = s_1+s_2



II. Concept of distance:

Distance is always added to both object with irrespective of direction but in some exception are there, when any vertical object is taken then distance of that object is considered as zero.

Ex. Length of pole, person, tree is zero.

III. Concept of Time :

Time is always constant (The time which is taken by object I to cross object II)

SOME IMPORTANT POINT

1. When a moving train crosses a standing person i.e.

$$S_1 = \frac{L_1}{T}$$

Ex. Find the time taken by a train 180m long, running with the speed of 72 km/hr to cross an electrical pole-

(A) 7 sec **(B)** 6 sec

(C) 9 sec **(D)** 5 sec

Sol.(C) $S = \frac{L}{T}$ (because pole has no breadth)

$T = \frac{L}{S} = \frac{180m}{\frac{72 \times 5}{18} m/s} = \frac{180}{20}$

= 9 sec

2. When a moving train crosses a moving person in the same direction

$S_1 - S_2 = \frac{L_1}{T}$ and $L_2 = 0$

Ex. A train 100m long is running at a speed of 70 km/h. In what time will it pass a man who is running at a speed of 10 km/h in the same direction ?

(A) 7 sec **(B)** 6 sec

(C) 9 sec **(D)** 5 sec

Sol.(B) $S_1 - S_2 = \frac{L_1}{T}$

$T = \frac{L_1}{S_1 - S_2} = \frac{100}{(70-10)km/h}$

$= \frac{100}{60 \times \frac{5}{18}}$ m/s

$= \frac{100 \times 18}{60 \times 5}$

= 6 sec

3. When a moving train crosses a moving persons in the opposite direction

$$S_1 + S_2 = \frac{L_1}{T}$$

Ex. A train 270 metres long is moving at a speed of 25 km/hr. It will cross a man coming from the opposite direction at a speed of 2 km./hr. in:

(A) 36 seconds **(B)** 32 seconds

(C) 28 seconds **(D)** 24 seconds

Sol.(A) According to question

$$S_1 + S_2 = \frac{L_1}{T}$$

$$(25+2) \times \frac{5}{18} = \frac{270}{T}$$

T = 36 sec.

4. When a moving train crosses a platform

$$S_1 = \frac{L_1 + L_2}{T},\ S_2 = 0$$

Ex. A train 140 m long is running at 60 km/h. In how much time will it pass a platform 260m long?

(A) 22 sec **(B)** 12 sec

(C) 25 sec **(D)** 24 sec

Sol.(D) $S = \frac{L_1 + L_2}{t}$ $t = \frac{L_1 + L_2}{s}$

$$= \frac{140+260}{60 \times \frac{5}{18}}$$

$$= \frac{400 \times 18}{300} = 24 \text{ sec}$$

5. When a moving train crosses another stationary train?

$$S_1 = \frac{L_1 + L_2}{T}$$

$S_2 = 0$

Ex. A train 160 m long taken 30 sec. in crossing a stationary train 440 m long. The speed of the train is ?

(A) 12 m/s **(B)** 18 m/s

(C) 16 m/s **(D)** 20 m/s

Sol.(B) $S = \frac{L_1 + L_2}{t} = \frac{160+440}{30} = \frac{600}{30}$

= 20 m/s

6. When a moving train crosses another moving train along same direction?

$$S_1 - S_2 = \frac{L_1 + L_2}{T}$$

Ex. Two trains travelling in the same direction at 40 km/hr. and 22 km/hr. completely pass one another in 1 minute. If the length of the first train is 125 metres, the length of the second train is:

(A) 125 metres **(B)** 150 metres

(C) 175 metres **(D)** None of these

Sol.(C) According to question

$$S_1 - S_2 = \frac{L_1 + L_2}{T}$$

$$(40-22) \times \frac{5}{18} = \frac{125 + L_2}{60}$$

$5 \times 60 = 125 + L_2$

$L_2 = 175$ metres.

7. When a moving train crosses another moving train coming from opposite direction?

$$S_1 + S_2 = \frac{L_1 + L_2}{T}$$

Ex. Two trains are running in opposite directions with the same speed. If the length of each train is 135 metres and they cross each other in 18 seconds, the speed of each train is :

(A) 104 km/hr. **(B)** 27 km/hr.

(C) 54 km/hr. **(D)** 100 km/hr.

Sol.(B) Let the speed of each train be (x) m/sec.

Then, $\frac{135+135}{x+x} = 18$

$\therefore\ x = \frac{15}{2}$ m/sec. $= \left(\frac{15}{2} \times \frac{18}{5}\right)$ km/hr.

= 27 km/hr.

8. When a moving train crosses a person sitting in another train?

$$S_1 \pm S_2 = \frac{L_1}{T}$$

$L_2 = 0$

$S_1 \pm S_2$ (+ when both running in opposite direction, – when both running same direction)

Ex. A goods train of 50m. long moving with speed 54 km/h. A man is sitting in a passenger train which is moving with 18 km/hr. with same direction. In how many second the goods train crosses the man sitting in a passanger train ?

(A) 6 seconds **(B)** 2 seconds

(C) 8 seconds **(D)** 5 seconds

Sol.(D) Let the length of the goods train is L.

$S_1 - S_2 = \frac{L}{t}$ (because man has no breadth)

$\frac{5}{18}(54 - 18) = \frac{50m}{t}$

$\frac{5}{18} \times 36 = \frac{50}{t}$

t = 5 sec

9. When a moving train crosses two different object.

Ex. A train is running at a speed of 25 km/h takes 18 sec to pass a railway platform and it takes 13.5 sec to pass a man who is running at a speed of 5 km/h in the same direction. Find the Length of platform ?

(A) 75 m **(B)** 50 m

(C) 100 m **(D)** 125 m

Sol.(B) Let the length of the train is L.

and the length of the platform is P.

$S_1 = \frac{L+P}{t}$

$\frac{5}{18} \times 25 = \frac{L+P}{18}$

L + P = 125 ---- (I)

Again

$S_1 - S_2 = \frac{L}{T}$

$\frac{5}{18} \times (25 - 5) = \frac{L}{13.5}$

$L = \frac{5 \times 20 \times 13.5}{18}$

Length of train L = 75 m

Length of platform = 125 - 75

= 50 m.

Ex. A train is travelling at 36 km/hr. completely crosses another train having half its length and travelling in the opposite direction at 54 km/hr.

in 12 seconds. If it also passes a railway platform in $1\frac{1}{2}$ minutes, the length of the platform is:

(A) 750 metres **(B)** 700 metres

(C) 620 metres **(D)** 560 metres

Sol.(B) Relative speed = (36+54)km/hr. = 90 km/hr.

$= \left(90 \times \frac{5}{18}\right)$ m/sec. = 25 m/sec.

let', 'l' metre be the length of slower train.

Now, $\left(\frac{l + \frac{l}{2}}{25}\right) = 12$ or l = 200 m

Also, speed of slower train

$= \left(36 \times \frac{5}{18}\right)$ m/sec. = 10m/sec.

Now, if x metre be the length of platform, then

$\frac{x+200}{10} = 90$ or x = 700 metres.

10. When two trains start from two points X and Y towards each other at the same time and after crossing they take p and q second to reach Y and X respectively then ratio their

Speed = $\sqrt{q} : \sqrt{p}$

Ex. Two trains X and Y start from stations A and B towards B and A respectively. After passing each other, they take 4 hours 48 minutes and 3 hours 20 minutes to reach B and A respec tively. If train X is moving at 40 km/hr., the speed of train Y is:- .

(A) 60km/hr.

(B) 54km/hr.

(C) Can’t be determined

(D) 48 km/hr.

Sol.(D) $\frac{X\text{'s rate}}{Y\text{'s rate}} = \sqrt{\frac{\text{Time taken by Y to reach A}}{\text{Time taken by X to reach B}}}$

$\Rightarrow \frac{40}{Y\text{'s rate}} = \sqrt{\left(\frac{10}{3} \times \frac{5}{24}\right)} = \frac{5}{6}$

$\therefore$ Y’s rate $= \left(\frac{40 \times 6}{5}\right)$ km/hr. = 48 km/hr.

EXAMPLES

Ex.1. A train is travelling at the rate of 45 km/hr. How many seconds it will take to cover a distance of $\frac{4}{5}$ km?

(A) 45 sec **(B)** 42 sec

(C) 46 sec **(D)** 64 sec

Sol.(D) Here S_1=45 km/hr.

$D = \frac{4}{5}$ km.

So,

$$S_1 = \frac{D}{T}$$

$$45 \times \frac{5}{18} = \frac{4 \times 1000}{5 \times T}$$

$\Rightarrow$ T = 64 sec.

Ex.2. A train is running at the speed of 75 km/hr. crosses a signal pole in 12 sec. What is length of train.

(A) 200 Mtrs **(B)** 400 Mtrs

(C) 250 Mtrs **(D)** 300 Mtrs

Sol.(C) Here S_1=75 km/hr.

T = 12 sec.

So,

$$S_1 = \frac{L_1}{T}$$

$$75 \times \frac{5}{18} = \frac{L_1}{12}$$

$$L_1 = \frac{75 \times 5 \times 12}{18} = 250 \text{ Mtrs.}$$

Ex.3. Two train running in opposite directions cross a man standing on the platform in 36 second and 24 seconds and they cross each other in 26 seconds. Find the ratio of their speeds.

(A) 1 : 5 **(B)** 5 : 1 **(C)** 3 : 1 **(D)** 4 : 1

Sol.(A) Let the speed of two trains be a m/s and b m/s respectively.

then length of Ist train = 36a mtrs.

Length of IInd train = 24b mtrs.

According to question

$$\frac{36a + 24b}{a + b} = 26$$

36a + 24 b = 26a + 26b

10a = 2b

a : b = 2 : 10 = 1 : 5

QUICK TRICK :

By alligation :

36 24

26

2 10

1 : 5

Ex.4. Train A and Train B travel the same distance at speed of 9 km/hr and 10 km/hr respectively. If A takes 36 minutes more than B, the distance travelled by each is:

(A) 48 km. **(B)** 54 km.

(C) 60 km. **(D)** 66 km.

Sol.(B) $T_A - T_B = \frac{36 \text{ min}}{60}$

$$\frac{D}{9} - \frac{D}{10} = \frac{3}{5} \text{ hr.}$$

$$\frac{10D - 9D}{90} = \frac{3}{5}$$

D = 54 km.

QUICK TRICK :

$$D = \frac{S_1 S_2}{S_1 - S_2} \times T$$

$$\frac{10 \times 9}{10 - 9} \times \frac{3}{5} = 54 \text{ km.}$$

Ex.5. The distance between two cities A and B is 330 km. A train starts from A at 8 AM and travel towards B at 60 km/hr. another train starts from B at 9 a.m. and travels towards A at 75 km/hr. At what time do they meet.

(A) 10 AM **(B)** 10.30 AM

(C) 11 AM **(D)** 11.30 AM

Sol.(C) Distance travelled by first train in 1 hours =60×1 = 60 km

$\vdash\!\longrightarrow$ 330 km $\longleftarrow\!\dashv$

Therefore distance between two train at 9 am

= 330 - 60 = 270 km

Now time of meeting of two trains.

$$T = \frac{D}{S_1 + S_2} = \frac{270}{135} = 2 \text{ hr.}$$

Therefore both trains will meet at = 9 + 2 = 11am

Ex.6. Two trains of equal length, running in opposite directions passing a pole in 18 and 12 sec. The trains will cross each other in.

(A) 14.4 sec. **(B)** 15.4 sec.

(C) 18 sec. **(D)** 20.2 sec.

Sol.(A) Let the length of each train be x metre

Speed of first train = $\frac{x}{18}$ m/s

Speed of second train = $\frac{x}{12}$ m/s

Time taken by both trains to cross each other

$$= \frac{2x}{\frac{x}{18}+\frac{x}{12}} = \frac{2x \times 36}{5x} = 14.4 \text{ sec.}$$

QUICK TRICK :

$$S_1 + S_2 = \frac{L_1 + L_2}{T}$$

$$\frac{L_1}{T_1} + \frac{L_2}{T_2} = \frac{L_1 + L_1}{T}$$

$$\frac{L_1}{18} + \frac{L_1}{12} = \frac{2L_1}{T}$$

$$\frac{2L_1 + 3L_1}{36} = \frac{2L_1}{T}$$

$$\frac{5L_1}{36} = \frac{2L_1}{T}$$

$$T = \frac{72}{5} = 14.4 \text{ sec}$$

Ex.7. Without stoppage a train travels a certain distance with an average speed of 60 km/hr and with stoppage it travel the same distance with an average speed of 40 km/hr. How many minutes/hour does the train stop during the journey.

(A) 30 min. **(B)** 40 min.
(C) 20 min. **(D)** 80 min.

Sol.(C) Stoppage per min.

$$= \frac{\text{difference of distance}}{\text{speed without stoppage}} \times 60$$

$$= \frac{60-40}{60} \times 60 = 20 \text{ m/h}$$

Ex.8. Two trains start from Lucknow to Jabalpur at 6 am and 6.45 am at the speed of 100 km/hr and 136 km/hr respectively. How many km from Lucknow will they meet to each other.

(A) 283 km. **(B)** 287 km.
(C) 290 km. **(D)** 272 km.

Sol.(A) Let distance between Lucknow and Jabalpur = d

Then according to question

$T_1 - T_2 = 45$ min

$$\frac{d}{100} - \frac{d}{136} = \frac{45}{60} = \frac{3}{4}$$

$$\frac{d}{25} - \frac{d}{34} = 3$$

d = 283.33 ≈ 283 km.

QUICK TRICK :

$$d = \frac{S_1 S_2}{S_1 - S_2} \times T$$

$$\frac{100 \times 136}{36} \times \frac{3}{4} = 283.33 \approx 283$$

Ex.9. Two stations A and B are 390 km apart. A train starts from A at 10 am and travels towards B at 65 km/hr another train starts from B at 11 am and travels towards A at 35 km/hr. At what time do they meet?

(A) 3:15 pm **(B)** 2:15 pm
(C) 1:15 pm **(D)** 12:00 noon

Sol.(B) Let Ist train meet after 't' time:

then according to question

65 t + 35 (t - 1) = 390 km

65 t + 35 t - 35 = 390 km

100 t = 425, t = $4\frac{1}{4}$ hr.

i.e. meeting time = 10 + 4 h 15 min

= 2 : 15 pm

Ex.10. A train when moves at an average speed of 40 km/hr reaches on time, when speed reduces to 35 km/hr, reaches 15 minute late. Find the distance of Journey?

(A) 60 km. **(B)** 70 km.
(C) 75 km. **(D)** 68 km.

Sol.(B) Given - S = $\frac{D}{T}$

$40 = \frac{D}{T}$ .... (I)

$35 = \frac{D}{T + 15/60}$ .... (II)

From eq. (I) and (II)

40 T = 35 (T+ $\frac{1}{4}$)

8T = 7T + 7/4

T = $\frac{7}{4}$

Here

D = S × T = 40×$\frac{7}{4}$ = 70 km.

QUICK TRICK :

Ratio of speed = 40 : 35

= 8 : 7

Time taken ratio = 7 : 8

Difference of time = 15 min

Actual time taken = 15 × 7 = 105 min

Actual distance = $40 \times \frac{105}{60}$ = 70 km.

Ex.11. A jogger running at 12 km/hr along side a railway track in 240 meters ahead of the engine of a 120 meters long train running at 50 km/hr in the same direction. In how much time will the train pass the Joggers.

(A) 36 Sec. **(B)** 34 Sec.

(C) 80 Sec. **(D)** 52 Sec.

Sol.(B) According to question

$S_1 - S_2 = \frac{L_1 + L_2}{T}$

$(50 - 12) = \frac{240+120}{T}$

$38 \times \frac{5}{18} = \frac{360}{T}$

$T = \frac{72 \times 9}{19}$ = 34.10 sec.

Ex.12. Two trains start from a certain place on two parallel tracks in the same direction. The speed of the trains are 45 km/hr and 40 km/hr respectively. The distance between the two trains after 45 minutes will be-

(A) 2.5 km **(B)** 2.75 km

(C) 3.75 km **(D)** 3.25 km

Sol.(C) Relative speed = 45 - 40 = 5 km/hr

Required distance = $5 \times \frac{45}{60}$ = 3.75 km/hr

Ex.13. A train is moving at a speed of 80 km/hr and covers a certain distance in 4.5 hours. The speed of the train to cover the same distance in 4 hours.

(A) 100 km/h **(B)** 50 km/h

(C) 90 km/h **(D)** 75 km/h

Sol.(C) D = S × T

= 80 × 4.5 = 360 km

Required speed = $\frac{D}{h} = \frac{360}{4}$

= 90 km/h.

QUICK TRICK :

Ratio of time = 9 : 8

Ratio of speed = 8 : 9

Since, 8 unit = 80 km/hr.

9 unit = 90 km/hr.

Ex.14. A express train is running at speed of 90 km/hr. It crosses a bridge whose length is twice that of train in 36 sec. What is the length of bridge in meter?

(A) 550 meter **(B)** 450 meter

(C) 600 meter **(D)** 500 meter

Sol.(C) According to question

$S_1 = \frac{L_1 + L_2}{T}$

$90 \times \frac{5}{18} = \frac{x+2x}{36}$, 25 × 36 = 3x

x = 25 × 12 = 300 m

Length of Bridge = 2 × 300 = 600 meter

Ex.15. A train of speed 40 km/hr is moving in the same direction of a horse moving at the speed of 25 km/hr in 48 sec. Find the length of train?

(A) 400 meter **(B)** 500 meter

(C) 200 meter **(D)** 300 meter

Sol.(C) Accoording to question

$S_1 - S_2 = \frac{L_1 + L_2}{T}$

$(40 - 25) \times \frac{5}{18} = \frac{L_1}{48}$, $15 \times \frac{5}{18} = \frac{L_1}{48}$

L_1 = 200 meter

Ex.16. Two train start at the same time from Delhi and Kanpur towards each other at the rate of 75 km/hr and 65 km/hr respectively. When they meet, it is found that one train has travelled 10 km more than other. Find the distance between Delhi and Kanpur.

(A) 150 Km **(B)** 140 Km

(C) 100 Km **(D)** 120 Km

Sol.(B) Since speed are directly proportion. Hence ratio of speed = ratio of distance

$S_1 : S_2 = D_1 : D_2$

= 75 : 65

= 15 : 13

According to the question

15x - 13x = 10

2x = 10

x = 5

Hence total distance b/w Delhi and Kanpur

$15x + 13x = 28x = 28 \times 5 = 140$ km.

QUICK TRICK:

Distance b/w Delhi and Kanpur

$= d\left(\frac{S_1 + S_2}{S_1 - S_2}\right)$

$= 10\left(\frac{75 + 65}{75 - 65}\right)$

= 140 km.

Ex.17. A goods train starts running from a place at 1 p.m. at the rate of 18 km/hr. Another goods train starts from the same place at 3 pm in the same direction and overtakes the first train at 9 p.m. The speed of the second train in km/hr is-

(A) 24 **(B)** 30

(C) 15 **(D)** 18

Sol.(A) Distance covered by the first goods train in 8 hours = Distance covered by the second goods train in 6 hr.

$18 \times 8 = 6 \times x$

$x = \frac{18 \times 8}{6} = 24$ km/hr.

Ex.18. The ratio of length of two trains is 5 : 3 and the ratio of their speed is 6 : 5. The ratio of time taken by them to cross a pole.

(A) 18 : 25 **(B)** 25 : 18

(C) 17 : 25 **(D)** 25 : 17

Sol.(B) Given $L_1 : L_2 = 5 : 3$

$S_1 : S_2 = 6 : 5$

Hence $T_1 : T_2 = \frac{L_1}{S_1} : \frac{L_2}{S_2} = \frac{5}{6} : \frac{3}{5} = 25 : 18$

Ex.19. A train runs from Howrah to Bandel at an average speed of 20 km/hr. and returned at an average speed of 30 km/hr. The average speed of the train in the whole journey ?

(A) 20 km/hr. **(B)** 22.5 km/hr.

(C) 24 km/hr. **(D)** 25 km/hr.

Sol.(C) Let distance between Howarah and Bandel = D

Average Speed

$= \frac{\text{Total Distance}}{\text{Total time}} = \frac{D + D}{\frac{D}{20} + \frac{D}{30}}$

$= \frac{2D}{5D} \times 60 = 24$ km/hr.

QUICK TRICK :

Average Speed $= \frac{2xy}{x + y} = \frac{2 \times 20 \times 30}{20 + 30}$

= 24 km/hr.

Ex.20. A train 150 meter long passes a pole in 15 seconds and another train of the same length travelling in the opposite direction in 12 seconds. Speed of the second train is-

(A) 45 km/hr **(B)** 48 km/hr

(C) 52 km/hr **(D)** 54 km/hr

Sol.(D) According to question

$S_1 = \frac{150}{15} = 10$ m/s

Again,

$S_1 + S_2 = \frac{L_1 + L_2}{T}$

$10 + x = \frac{150 + 150}{12}$

$120 + 12x = 300$

$12x = 180$

$x = \frac{180}{12} \times \frac{18}{5} = 54$ km/hr.

EXERCISE

Q.1. A train 280 metres long is moving at a speed of 60 km/hr.The time taken by the train to cross a platform 220 metres long is:

(A) 20 seconds **(B)** 25 seconds

(C) 40 seconds **(D)** None of these

Q.2. A train running at the rate of 36 km/hr. passes a standing man in 8 seconds. The length of the train is:

(A) 45 metres **(B)** 28 metres

(C) 80 metres **(D)** 48 metres

Q.3. A train crosses a pole in 15 seconds and speeds passed a platform of 100 metres in 30 second Its length (in metres) is:

(A) 200 **(B)** 100

(C) 50 **(D)** Data inadequate

Q.4. A train 120 metres long travels at 60 km/hr. A man is running at 6 km/h. in the same direction in which the train is going. The train will cross man in:

(A) 6 seconds **(B)** 7 seconds

(C) $6\frac{2}{3}$ seconds **(D)** 8 seconds

Q.5. A train of length 150 metres, takes 10 seconds to pass over another train 100 metres long coming from the opposite direction. If the speed of the first train be 30 km/hr., the speed of the second train is:

(A) 54 km/hr. **(B)** 60 km/hr.

(C) 72 km/hr. **(D)** 36 km/hr.

Q.6. Two trains X and Y start from stations A and B towards B and A respectively. After passing each other, they take 4 hours 48 minutes and 3 hours 20 minutes to reach B and A respec tively. If train X is moving at 40 km/hr., the speed of train Y is : - .

(A) 60km/hr.

(B) 54km/hr.

(C) Can't be determined

(D) 48 km/hr.

Q.7. A train travelling at 36 km/hr. completely crosses another train having half its length and travelling in the opposite direction at 54 km/hr. in 12 seconds. If it also passes a railway plat form in $1\frac{1}{2}$ minutes, the length of the platform is:

(A) 750 metres **(B)** 700 metres

(C) 620 metres **(D)** 560 metres

Q.8. Two stations A and B are 110 kms apart on a straight line. One train starts fromA at 7A.M. and travels towards B at 20 km/hr. speed. Another train starts from B at 8 A.M. and travels towards A at 25 km/hr. speed. At what time will they meet ?

(A) 9 A.M. **(B)** 10 A.M.

(C) 11A.M. **(D)** 12 A.M.

Q.9. Two trains of lengths 130m and 110m run on parallel tracks.When running in the same directions the faster train crosses the slower are in 20 seconds.When running in opposite directions at speeds same as their earlier speeds, they pass each other completely in 12 seconds, find the speed of slower train.

(A) 4 m/sec **(B)** 8 m/sec

(C) 12 m/sec **(D)** 14 m/sec

Q.10. The circumference of wheel of train is $3\frac{3}{4}$ meter and makes 4 revolutions in 2 seconds. What is the speed of train ?

(A) 27 km./hr. **(B)** $\frac{15}{2}$ km./hr.

(C) 30 km./hr. **(D)** 12 km./hr.

Q.11. Train A crosses a pole in 25 sec and another train B crosses a pole in 1 minute and 15 seconds. Length of train A is half length B. What is the respective ratio between the speed of train A and Train B ?

(A) 3 : 2 **(B)** 3 : 4

(C) 4 : 3 **(D)** 2 : 5

Q.12. A 450 m long passenger train passes a goods train 300 m long travelling in opposite direction. If the speeds of the passenger train and goods train be 117 km/hr. and 63 km/hr., how long will it take to cross each other?

(A) 75 seconds **(B)** 30 seconds

(C) 45 seconds **(D)** 60 seconds

Q.13. If a train runs at 30 km/h, it reaches its destination late by 1 hour but if it runs at 45 km/h, it reaches 1 hour early. Find the time taken by it to complete its journey if it travels at speed of 25 Km/h.

(A) $7\frac{1}{3}$ hrs. **(B)** $8\frac{1}{2}$ hrs.

(C) $7\frac{1}{5}$ hrs. **(D)** $6\frac{1}{4}$ hrs.

Q.14. Two trains which are 150 m long each are moving in opposite directions. They cross each other in 12 seconds. If one train is moving two and a half times as fast the other train, then find the speed of the faster train.

(A) 125 km/hr. **(B)** 132 km/hr.

(C) 168 km/hr. **(D)** 172 km/hr.

Q.15. A train travelling at N km/hr. crosses another train half of its length and travelling in opposite direction at 84 km/hr. in 24 seconds. It also passes a railway platform in 45 seconds. What is the length of the railway platform, if N is $114\frac{2}{7}$% of the speed of the other train?

(A) 200 m **(B)** 300 m.

(C) 350 m. **(D)** 400 m.

Q.16. A train travels at a certain average speed for a distance of 63 km. and then travels a distance of 72 km. at an average speed of 6 km./hr. more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train in km./hr.?

(A) 24 hr. **(B)** 33 km/hr.

(C) 42 km/hr. **(D)** 66 km/hr.

Q.17. Two trains left from a station at the same time. First train is moving towards east direction and second train is moving towards north. If first train is moving 5 km/hr. faster than the second train. If both are at a distance of 50 km after 2 hours, then sum of the speeds of the trains will be

(A) 25 km/hr **(B)** 35 km/hr.

(C) 45 km/hr. **(D)** 55 km/hr.

Q.18. If the speed of the train be 90 kmph and it takes 6 seconds to pass a platform. Next it takes 5 seconds to pass a cyclist walking at the rate of 12 kmph in the same direction. What is the length of the platform? (In meters)

(A) 41.33 meters **(B)** 42.66 meters

(C) 42.33 meters **(D)** 41.66 meters

Q.19. A train travels 288 km at a uniform speed. If the speed had been 4 km/hr. more, it would have taken 1 hour less for the same journey, find the speed of the train.

(A) 30 kmph **(B)** 36 kmph

(C) 32 kmph **(D)** 34 kmph

Notes

EXPLANATION

Q.1.(D) Speed of train = $\left(60\times\frac{5}{18}\right)$ m/sec.= $\frac{50}{3}$ m/sec.

Time taken by the train to cover (220 + 280) metres

= $\left(\frac{500\times3}{50}\right)$ sec.= 30 sec.

Q.2.(C) Speed of train = $\left(36\times\frac{5}{18}\right)$ metres/sec.

= 10 m/sec.

Length of train = (speed × time) = 80 metres.

Q.3.(B) Let the length of train be x metres. Then,

$\frac{x}{15}=\frac{x+100}{30}\Rightarrow$ 30x = 15 x + 1500

$\Rightarrow$ x = 100.

Q.4.(D) Speed of train relative to man

= (60-6) km/hr. = $\left(54\times\frac{5}{18}\right)$ m/sec.= 15 m/sec.

Time taken by the train to cross the man

= $\left(\frac{120}{15}\right)$ sec. = 8 seconds.

Q.5.(B) Let the speed of second train be x km/hr.

Then, relative speed = (30 +x) km/hr.

$\therefore$ Time taken to cover (150 + 100) metres at (30 +x) km/hr. = 10 sec.

$\therefore \frac{250}{(30+x)\times\frac{5}{18}}$

$=10\Rightarrow \frac{250\times18}{150+5x}=10\Rightarrow x=60$ km/hr.

Q.6.(D) $\frac{\text{X's rate}}{\text{Y's rate}}=\sqrt{\frac{\text{Time taken by Y to reach A}}{\text{Time taken by X to reach B}}}$

or $\frac{40}{\text{Y's rate}}=\sqrt{\left(\frac{10}{3}\times\frac{5}{24}\right)}=\frac{5}{6}$

$\therefore$ Y's rate $\left(\frac{40\times6}{5}\right)$ km/hr. = 48 km/hr.

Q.7.(B) Relative speed = (36+54)km/hr. = 90 km/hr.

= $\left(90\times\frac{5}{18}\right)$ m/sec. = 25 m/sec.

= let l metre be the length of slower train.

Now, $\left(\frac{l+\frac{l}{2}}{25}\right)$=12 or l = 200 m

Also, speed of slower train

= $\left(36\times\frac{5}{18}\right)$ m/sec. = 10m/sec.

Now, if x metre be the length of platform, then

$\frac{x+200}{10}$=90 or x = 700 metres.

Q.8.(B) Suppose they meet x hours after 7 A.M.

Distance travelled by train from A in x hours.

= (20x) km.

Distance travelled by train from B in (x-1) hours, = 25 (x-1) km.

$\therefore$ 20x +25 (x-1) = 110 or x = 3 hours. So, the trains meet at 10 A.M.

Q.9.(A) Speed of the slower train

$=\left(\frac{l_1+l_2}{2}\right)\left(\frac{t_1-t_2}{t_1t_2}\right)=\left(\frac{130+110}{2}\right)\left(\frac{20-12}{20\times12}\right)$

$=\frac{120\times8}{240}=$ 4m/sec

Q.10.(A) Distance covered after four revolutions

$=\frac{15}{4}\times4=$ 15 m

$\therefore$ Speed = $\frac{15}{2}$ m./sec.

$=\frac{15}{2}\times\frac{18}{5}$ = 27 km./hr.

Q.11.(A) Ratio = $\frac{x}{25}:\frac{2x}{75}$

$=\frac{x}{21}\times75:\frac{2}{75}\times75$

= 3:2

Q.12.(A) Required time = $\frac{450+300}{(117+63)\times\frac{5}{18}}$

= 75 seconds

Q.13.(C) Let distance be d

$\frac{d}{30}-\frac{d}{45}=2$

$d=180$

New time to complete the journey

$$\frac{180}{25}=7\frac{1}{5} \text{ hours}$$

Q.14.(D) Let speed of slower train = x m./s., then speed of faster train = 2.5x m./s.

Their relative speed becomes = x + 2.5x = 3.5x

So, (150 + 150) = 3.5x × 12

x = 50/7

So, speed of faster train = 2.5 × (50/7)

= (125/7) m./s.= (125/7) × (18/5) km/hr.

Q.15.(D) According to the question,

$$\frac{L_1+\frac{L_1}{2}}{(96+84)\times\frac{5}{18}}=24$$

$L_1 = 800$

and, $$\frac{L_1+L_2}{(96)\times\frac{5}{18}}=45$$

$800 + L_2 = 1200$

$L_2 = 400$ m.

Q.16.(C) Let original speed of train be x km/hr.

$$\frac{63}{x}+\frac{72}{x+6}=3$$

$$\Rightarrow \frac{63(x+6)+72\times x}{(x^2+6x)}=3$$

$21x + 126 + 24x = x^2 + 6x$

$x^2 – 39x – 126 = 0$

$x^2 – 42x + 3x – 126 = 0$

$(x – 42) (x + 3) = 0$

x = 42 km/hr.

Q.17.(B) Let speed of the first train be (x + 5) km/hr. and speed of the second train be x km/hr. Distance moved by first train in 2 hours AB

= (x + 5) × 2 = (2x + 10) km.

Distance moved by second train in 2 hours AC

= x × 2 = 2x km

BC = 50 km.

Using Pythagoras theorem

$AB^2 + AC^2 = BC^2$

$[(2x + 10)]^2 + (2x)^2 = 50^2$

$4x^2 + 100 + 40x + 4x^2 = 2500$

$8x^2 + 40x - 2400 = 0$

Solving above equation, we get

x = 15 km/hr.

Speed of the first train = 15 km/hr.

Speed of the second train = 15 + 5

= 20 km/hr.

Sum of the speeds of the trains

= 20 + 15 = 35 km/hr.

Q.18.(D) Relative speed of the train with respect to the cyclist = (90 – 12) = 78 km/hr. Length of the train = Relative speed × time

$$= 78\times\frac{5}{18}\times5=\frac{1950}{18} \text{ meter}$$

and length of the train + length of the platform = speed× time

$$\frac{1950}{18}+x=\left(90\times\frac{5}{18}\right)\times6$$

$$x=150-\frac{1950}{18}$$

$$x=\frac{2700-1950}{18}=\frac{750}{18}$$

x = 41.66 meters

Q.19.(C) Let the speed of the train be x km/hr

$$\frac{288}{x}-\frac{288}{x+4}=1$$

$x^2+4x-1152=0$

x+36x-32x-1152=0

(x+36)(x-36)=0

x=-36, x = 32

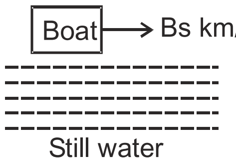
Speed of train = 32 km/hr.

CHAPTER-17
BOAT AND STREAM

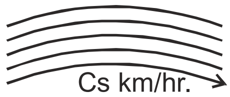
Scan the QR code to get video of this chapter.

Concept of Boat and Stream Depends on Concept of Speed, Time and Distance mainly it has 4 facts.

(1) **Still water :** If the speed of the water in the river or pond is zero, it is called still water. It is denoted as Bs.



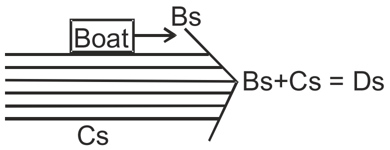
(2) **Stream:** If the water of the river is flowing at a constant speed it is called as stream. It is denoted as Cs.



(3) **Down stream speed:** If a boat or swimer goes along the stream is called down stream speed.

Down stream speed = speed of the boat + speed of the current

Ds = Bs + Cs



Ex. If the speed of the boatman in still water is 8 km/hr and speed of the water (stream) is 2 km/hr then speed of the boatman in downstream.

(A) 6 km/hr **(B)** 4 km/hr
(C) 10 km/hr **(D)** 12 km/hr

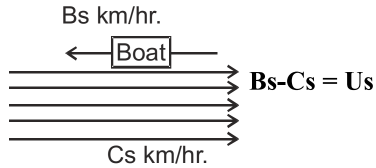
Sol.(C) **Since Ds = Bs + Cs**

Hence Ds = 8 + 2 = 10 km/hr.

(4) **Upstream speed:** If a boat or a swimer moves against the stream it is called upstream speed.

Upstream speed = speed of the boat - speed of the current

Us = Bs - Cs



Ex. If the speed of the boatman in still water is 18km/hr and speed of the water (stream) is 12 km/hr then the speed of the boatman in upstream.

(A) 6 km/hr **(B)** 8 km/hr
(C) 10 km/hr **(D)** 4 km/hr

Sol.(A) Us = Bs - Cs

Hence Us = 18-12 = 6 km/h

IMPORTANT FORMULAS

(1) Speed of the boat in still water

$$= \frac{\text{down stream speed + upstream speed}}{2}$$

$$Bs = \frac{D_s + U_s}{2}$$

Ex. A man can swim 30km/hr upstream and 34 km/hr downstream. Find the speed of man in still water-

(A) 38 km/hr **(B)** 32 km/hr
(C) 36 km/hr **(D)** 40 km/hr

Sol.(B) Speed of man in still water

$$(Ms) = \frac{Ds + Us}{2} = \frac{30 + 34}{2}$$

= 32 km/h

(2) Speed of the current

$$= \frac{\text{down stream speed - upstream speed}}{2}$$

$$Cs = \frac{D_s - U_s}{2}$$

Ex. A man can go 22km/hr upstream and 40 km/hr downstream. Find the speed of current.

(A) 8 km/hr **(B)** 6 km/hr
(C) 9 km/hr **(D)** 10 km/hr

Sol. (C) Speed of current Cs= $\frac{D_s - U_s}{2}$

$$\frac{40 - 22}{2} = 9 \text{ km/hr}$$

EXAMPLE

Ex.1. A man rows 42 km upstream in 7 hours and a distance of 48 km in 4 hours down stream then the speed of the boat in still water. (in km/hr)

(A) 5 **(B) 4**

(C) 6 **(D) 9**

Sol.(D) Speed of the boat in still water

$(Bs) = \frac{D_s + U_s}{2}$

$= \frac{48/4 + 42/7}{2}$

$= \frac{12+6}{2} = 9$ k/h

Ex.2. A man rows at a speed fo 10 km/h in still water to a certain distance upstream and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey (in km/hr)

(A) 4 **(B)** 6

(C) 10 **(D)** 8.4

Sol.(D) If a boat rows the same distance upstream and down stream then

Average speed = $\frac{D_s \times U_s}{B_s}$

$\frac{14 \times 6}{10}$ = 8.4 k/h

Ex.3. A man can row 8 km/hr in still water if the river is running at 3 km/hr it takes 6 hrs more in upstream than to go downstream for the same distance. Find the distance (in km)

(A) 55 **(B)** 60

(C) 44 **(D)** 50

Sol.(A) According to question

$t_1 - t_2 = 6$h

$\frac{D}{U_s} - \frac{D}{D_s} = 6$

$\frac{D}{5} - \frac{D}{11} = 6$

$\frac{11D - 5D}{55} = 6$

D = 55 km

QUICK TRICK :

The required distance $= \frac{\left(B_s^{\,2} - C_s^{\,2}\right)t}{2C_s}$

$= \frac{(64-9)\times 6}{2\times 3}$ = 55 km.

Ex.4. A man can row at the rate of 6 km/hr in still water if the time taken to row a certain distance upstream is 3 times as much as to row the same distance downstream. Find the speed of the current. (in km/hr)

(A) 2 **(B) 4**

(C) 5 **(D) 3**

Sol.(D) According to question

$t_1 = 1/3\ t_2$

$\frac{D}{Bs + Cs} = \frac{1}{3}\frac{D}{Bs - Cs}$

$\frac{1}{6 + Cs} = \frac{1}{3(6 - Cs)}$

18 - 3Cs = 6 +Cs

12 = 4Cs

Cs = 3 km/hr

QUICK TRICK :

Speed of the boat $= \frac{(n+1)}{(n-1)} \times$speed of the current

$6 = \frac{(3+1)}{(3-1)} \times Cs$

$Cs = \frac{6\times 2}{4} = 3$ k/h.

Ex.5. A man goes upstream 6 km in 15 m and returns to the same point in 12 m. Find the speed of the current.

(A) 1 km/hr **(B) 2** km/hr

(C) 3 km/hr **(D) 4** km/hr

Sol.(C) Down stream speed of boat $= \frac{6}{12} \times 60 = 30$ k/h

Upstream speed of boat$= \frac{6}{15} \times 60 = 24$ k/h

Speed of the current = $\frac{Ds - Us}{2}$

Speed of the current = $\frac{30 - 24}{2} = \frac{6}{2}$

= 3 k/h.

Ex.6. If a man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream be y km/hr, then the speed of the man in still water.

(A) $y\frac{(t_1 + t_2)}{(t_2 - t_1)}$ **(B)** $y\frac{(t_1 - t_2)}{(t_2 + t_1)}$

(C) $y\dfrac{(t_1-t_2)}{(t_2-t_1)}$ **(D)** None of these

Sol.(A) Let the speed of the man in still water be - x

Ds = x + y

Us = x - y

So, according to question

$(x+y)\,t_1 = (x-y)\,t_2$

$t_1 x + t_1 y = t_2 x - y$

$x(t_2 - t_1) = y(t_1 + t_2)$

$x = y\dfrac{(t_1+t_2)}{(t_2-t_1)}$

Ex.7. A boat covers a certain distance down stream in 6 hrs but takes 8 hrs to return upstream to the starting point. If the speed of the stream is 4 km/hr. find the speed of the boat. (in km/hr.)

(A) 30 km/hr **(B) 28** km/hr

(C) 25 km/hr **(D) 27** km/hr

Sol.(B) Given Ds = $\dfrac{D}{6}$ .... (1)

Us = $\dfrac{D}{8}$ .... (2)

From eq. (1) and (2)

$\dfrac{Ds}{Us}=\dfrac{8}{6}$

$\dfrac{Bs+Cs}{Bs-Cs}=\dfrac{8}{6}$

$\dfrac{Bs+4}{Bs-4}=\dfrac{4}{3}$

3Bs + 12 = 4Bs - 16

28 km/hr.

QUICK TRICK :

$Bs=\dfrac{t_{Ds}+t_{Us}}{2}\times Cs$

Bs = $\dfrac{14}{2}\times 4 = 28$ km/hr

Ex.8. A motor boat whose speed is 12 km/hr in still water goes 36 km downstream and come back in a total time of 6 hr 24 min. the speed of the current is.

(A) 3 km/hr **(B) 2** km/hr

(C) 1 km/hr **(D) 4** km/hr

Sol.(A) Let the speed of the current = x

$\dfrac{36}{x+12}+\dfrac{36}{x-12}$ = 6h.24 min = $6\dfrac{24}{60}=\dfrac{32}{5}$

144 - x^2 = $\dfrac{24\times36\times5}{32}=135$

$x^2 = 9$

x = 3 km/hr

Ex.9. A boy can swim in still water at a speed of 10 km/hr if the speed of the current would have been 2 km/hr. then the boy could swim 60 km down stream in—

(A) 6 hrs **(B)** 5 hrs

(C) 7.5 hrs **(D)** None of these

Sol.(B) Ds =Bs + Cs = 10 + 2 = 12 km/hr

required time = $\dfrac{60}{12}=5$ h

Ex.10. Speed of the boat in still water is 40 km/h if boat travels 80 km along the stream in 1 hrs 20 m. then the time taken by it to cover the the same distance against the stream will be

(A) 2 hr **(B)** 3 hr

(C) 4 hr **(D)** 5 hr

Sol.(C) Given Bs = 40 km/h

Ds = $\dfrac{80}{4}\times3$=60 km/hr

Since Ds = Bs + Cs

60 = 40 + Cs

Cs = 20

Required time = $\dfrac{D}{Us}=\dfrac{D}{Bs-Cs}$

= $\dfrac{80}{40-20}=\dfrac{80}{20}$ = 4h

Ex.11. A man can row 6 km/hr in still water if the speed of current is 2 km/hr it takes 3 hrs more in upstream than in the downstream of the same distance. The distance is (km)

(A) 12 **(B)** 20

(C) 18 **(D)** 24

Sol.(D) downstream = (Bs + Cc) = 6 + 2 = 8 km/h

Upstream speed = (Bs - Cs) = 6 - 2 = 4 km/h

Let the distance be x

According to question

$$\frac{x}{4}-\frac{x}{8}=3$$

$$\frac{x}{8}=3 \; x = 24 \text{ km}$$

QUICK TRICK :

Required distance

$$=\frac{\left(Bs^2-Cs^2\right)t}{2Cs}=\frac{(36-4)3}{4}$$

8 × 3 = 24 km

Ex.12. Speed of a boat is 5 km/hr in still water and the speed of the stream is 3 km/hr if the boat takes 5 hrs to go to a place and come back. The distance of the place is: (in km.)

(A) 4 **(B)** 6

(C) 8 **(D)** 10

Sol.(C) Let the distance be x

According to question

$$\frac{x}{Ds}+\frac{x}{Us}=5$$

$$\frac{x}{8}+\frac{x}{2}=5$$

$$\frac{5x}{8}=5$$

x = 8 km

Ex.13. Two boats A and B start towards each other from two places 108 km apart. Speed of the boat A and B in still water are 12 km/h and 15 km/h respectively. If A proceeds downstream and B up stream. They will meet in the time of-

(A) 6 hr. **(B)** 8 hr.

(C) 4 hr. **(D)** 2 hr.

Sol.(C) Let the speed of the current be x

and if A goes downstream and B goes upstream

So, Speed of A = 12 + x

Speed of B = 15 -x

Hence realtive speed of both boats = 12 + x + 15 - x = 27

So time taken = $\frac{108}{27}=4$ h

Ex.14. A man can row 30 km down stream and returns to that point in total 8 hrs. If the speed of the boat in still water is four times the speed of the current then the speed of the current is:

(A) 4 km/hr. **(B)** 2 km/hr.

(C) 6 km/hr. **(D)** 8 km/hr.

Sol.(B) Let the speed of the current be x

speed of boat in still water = 4x

$$\frac{30}{Ds}+\frac{30}{Us}=8$$

$$\frac{30}{5x}+\frac{30}{3x}=8$$

$$\frac{90+150}{15x}=8$$

240 = 120x

x = 2 km/hr.

Ex.15. The speed of boat in still water is 8 km/h. The boat goes 6 km and return to the starting point in 2 hours. Find the speed of the stream ?

(A) 1 km/hr **(B)** 3 km/hr

(C) 4 km/hr **(D)** 2 km/hr

Sol.(C) Let the speed of stream = x km/hr.

According to question,

$T_1 + T_2 = 2$ hr.

$$\frac{6}{8+x}+\frac{6}{8-x}=2$$

$$\frac{6(8-x)+6(8+x)}{8^2-x^2}=2$$

$$\frac{48-6x+48+6x}{64-x^2}=2$$

$96 = 128 - 2x^2$

$32 = 2x^2$

$x^2 = 16$

x = 4 km/h

Ex.16. A boat covers 12 km upstream and 18 km downstream in 3 hrs, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hrs. What is the speed of the current.

(A) 1 kmph **(B)** 3 kmph

(C) 4 kmph **(D)** 2 kmph

Sol.(D) Let the speed of boat in still water be x km/h and that of current be y km/h then

Then, $\frac{12}{x-y}+\frac{18}{x+y}=3$

$$\frac{36}{x-y}+\frac{24}{x+y}=\frac{13}{2}$$

Let the x + y = a

x - y = b

So, $\frac{12}{b}+\frac{18}{a}=3$ .... (i)

$\frac{36}{b}+\frac{24}{a}=\frac{13}{2}$ .... (ii)

Solving eq. (i) and (ii) we get

a = 12 (downstream) km/hr.

b = 8 (upstream) km/hr.

Speed of current $\frac{12-8}{2}=2$ km/hr

Ex.17. A boat man rows 2 km in 10 min along the stream and 8 km in 1 hr against the stream. The speed of the stream is- (in km/hr.)

(A) 1 **(B)** 2

(C) 3 **(D)** 4

Sol.(B) Downstream speed = 12 km/hr

Upstream speed = 8 km/hr

Speed of the current = $\frac{Ds-Us}{2}$

$=\frac{12-8}{2}=4/2=2$ km/hr.

Ex.18. The difference between downstream speed and upstream is 3km/hr and the total time taken during upstream and down stream is 3 hrs. What is the downstream speed if the down stream and upstream distance are 3 km each? (in km.)

(A) 2.3 **(B)** 4.3

(C) 6.3 **(D)** 8.3

Sol.(B) Let the upstream speed = x

downstream speed = x + 3

$\therefore \frac{3}{x}+\frac{3}{x+3}=3$

$x^2 + x - 3 = 0$

$x=1-\frac{\sqrt{13}}{2}=\frac{-1+3.6}{2}=1.3$

Ds = 3 + 1.3 = 4.3 km

Ex.19. A man row three fourth of a km against the stream in $11\frac{1}{4}$ min and returns in $7\frac{1}{2}$ min.Find the speed of the man in still water.

(A) 8 kmph **(B)** 5 kmph

(C) 4 kmph **(D)** 3 kmph

Sol.(B) Downstream speed= $\frac{\frac{3}{4}}{\frac{45}{4}}\times 60=4$ km/hr.

Upstream speed = $\frac{\frac{3}{4}}{\frac{15}{2}}\times 60=6$ km/hr.

Speed of the man in still water

$=\frac{4+6}{2}=\frac{10}{2}=5$ km/hr.

Ex.20. A boat running in down stream covers a distance of 30 km in 2 hr. while coming back the boat takes 6 hr. to cover the same distance. If the speed of the current is half that of the boat. What is the speed of boat in km/hr. ?

(A) 10 **(B)** 15

(C) 18 **(D)** 20

Sol.(A) Let speed of the boat be Bs km/h and that of current is Cs km/h.

$\frac{30}{Bs+Cs}=2 \Rightarrow$ Bs + Cs = 15 (i)

$\frac{30}{Bs-Cs}=6 \Rightarrow$ Bs - Cs = 5 (ii)

from (i) and (ii)

2Bs = 20

Bs = 10 km/h.

EXERCISE

Q.1. If the speed of a swimmer in still water is 9 km/h. Find the downstream speed of the swimmer, when the river is flowing with the speed of 6 km/h.

(A) 15 km/h **(B)** 18 km/h

(C) 3 km/h **(D)** 12 km/h

Q.2. A boat goes 48 km downstream in 20 h. It takes 4 h more to cover the same distance against the stream. What is the speed of the boat in still water?

(A) 2.2 km/h **(B)** 2 km/h

(C) 4 km/h **(D)** 4.2 km/h

Q.3. A boatman rows 1 km in 5 min along the stream and 6 km in 1 h against the stream. The speed of the stream is

(A) 3 km/h **(B)** 6 km/h

(C) 10 km/h **(D)** 12 km/h

Q.4. A person can row downstream 20 km in 2 h and upstream 4 km in 2 h. What is the speed of the current?

(A) 2 km/h **(B)** 2.5 km/h

(C) 3 km/h **(D)** 4 km/h

Q.5. A man rows 12 km in 5 h against the stream, the speed of current being 4 km/h. What time will be taken by him to row 15 km with the stream?

(A) 1h $26\frac{7}{13}$ min **(B)** 1h $25\frac{7}{13}$ min

(C) 1h $24\frac{7}{13}$ min **(D)** 1h $27\frac{7}{13}$ min

Q.6. A boat can travel with a speed of 16 km/h in still water. If the rate of stream is 5 km/h, then what is the time taken by the boat to cover distance of 84 km downstream?

(A) 4 h **(B)** 5 h

(C) 6 h **(D)** 8h

Q.7. A man can row against the three-fourth of a kilometre in 15 min and returns same distance in 10 min, then ratio of his speed to that of current is

(A) 3:5 **(B)** 5 : 3

(C) 1: 5 **(D)** 5:1

Q.8. The speed of the current is 5 km/hr. A motorboat goes 10 km/hr. upstream and back again to the starting point in 50 min. The speed (in km/hr.) of the motorboat in still water is

(A) 20 **(B)** 26

(C) 25 **(D)**28

Q.9. Speed of motorboat in still water is 45 km/h. If the motorboat travels 80 km along the stream in 1 h 20 min. then the time taken by it to cover the same distance against the stream will be.

(A) 4 h 20 min **(B)** 3 h 40

(C) 2 h 40 min **(D)** 2 h 55

Q.10. A boat has to travel upstream 20 km distance from point X of a river to point Y. The total time taken by boat in travelling from point X to Y and Y to X is 41 min 40 s. What is the speed of the boat?

(A) 66 km/h **(B)** 72 km/h

(C) 48 km/h **(D)** cannot be determined

Q.11. Pawan can row 24 km/h in still water.When the river is running at 4.8 km/h, it takes him 1 h to row to a place and to come back. How far is the place?

(A)11.52 km **(B)** 14 km

(C)12.52 km **(D)** 15 km

Q.12. A man rows in still water with a speed of 6.5 km/h to go to a certain place and to Come back. Find his average speed for the whole journey, if the river is flowing with a speed of 2.5 km/h.

(A) 6.65 km/h **(B)** 6.75 km/h

(C) 5.53 km/h **(D)** 5 km/h

Q.13. A boat's speed in still water is 5 km/h, while river is flowing with a speed of 2 km/h and time taken to cover a certain distance upstream is 2 h more than time taken to cover the same distance downstream. Find the distance.

(A) 10.5 km **(B)** 11 km

(C) 10.9 km **(D)** 15 km

Q.14. A sailor sails a distance of 48 km along the flow of a river in 8 h. If it takes 12 h to return the same distance, then the speed of the flow of the river is

(A) 0.5 km/h **(B)** 1 km/h

(C) 1.5 km/h **(D)** 2 km/h

Q.15. A steamer goes downstream from one port to another in 4 h. It covers the same distance upstream in 5 h. If the speed of the stream is 2 km/h, then find the distance between the two ports.

(A) 50 km **(B)** 60 km

(C) 70 km **(D)** 80 km

Q.16. A boat covers a distance of 4.8 kms in downstream in 8 minutes. The same boat while running upstream at same speed covers the same distance in 9 minutes. What is the speed of the boat?

(A) 2.4 kmph **(B)** 34 kmph

(C) 2 kmph **(D)** 32 kmph

Q.17. A boat covers 24 km upstream and 28 km downstream in 6 hours, while it cover 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hours. Find the speed of the boat in still water and speed of the stream.

(A) 6 km/hr. **(B)** 8 km/hr

(C) 10 km/hr. **(D)** 12 km/hr.

Q.18. A man goes 30 km. downstream and returns back in total 8 hours. If speed of boat in still water is four times the speed of the current then speed of current is

(A) 1 km/hr. **(B)** 2 km/hr.

(C) 3 km/hr. **(D)** 4 km/hr.

Q.19. A man can row 5 km/hr. in still water. If the speed of the stream is 1.5 km/hr., he takes one hour when he travels upstream to a place and back again to the starting point. How far is the place from the starting point?

(A) 2.5 km **(B)** 6.5 km

(C) 3.5 km **(D)** 2.275 km

Q.20. Twice the speed downstream is equal to thrice the speed upstream, the ratio of speed in still water to the speed of the current is

(A) 1: 5 **(B)** 5: 1

(C) 1: 3 **(D)** 2: 3

Q.21. A boat takes total time of 3 hours to cover a distance of 18 km downstream and 12 km upstream. While the same boat takes a total time of $6\frac{1}{2}$ hours to cover a distance of 36 km upstream and 24 km downstream. Then what is the speed of the boat?

(A) 8 km/hr. **(B)** 10 km/hr.

(C) 12 km/hr. **(D)** 15 km/hr.

Q.22. A person can row 7.5 km an hour in still water and he finds that it takes him twice as long to row up as to row down the river. Find the speed of the stream.

(A) 2 km/hr **(B)** 3 km/hr.

(C) 3.5 km/hr. **(D)** 2.5 km/hr.

Q.23. The speed of a motor boat is that of the current of water as 36:5. The boat goes along with the current in 5 hours 10 minutes. It will come back in -

(A) 5 hours 50 minutes

(B) 6 hours

(C) 6 hours 50 minutes

(D) 12 hours 10 minutes

Notes

EXPLANATION

Q.1.(A) Given, swimmer's speed in still water, x=9 km/h

Rate of stream, y=6 km/h

∴ Speed of downstream=x+y=9+6=15 km/h

Q.2.(A) Speed downstream= $\frac{48}{20} = 2.4$ km/h

and speed upstream= $\frac{48}{24} = 2$ km/h

∴ Speed of boat in still water

$= \frac{1}{2}(2.4 + 2) = \frac{4.4}{2}$ = 2.2 km/hr.

Q.3.(A) Downstream speed $= \frac{1}{5/60} 12 \text{ km/h}$

and upstream speed $= \frac{6}{1} = 6 \text{ km/h}$

∴ Speed of the stream $= \frac{12-6}{2} = \frac{6}{2} \text{ km/h}$

= 3 km/h

Q.4.(D) Let speed of person=x km/h

and speed of current=y km/h

Distance=20 km in downstream and 4 km in upstream

Accordint to the question,

$= \frac{20}{x+y} = 2 \Rightarrow x + y = 10$ ....(i)

$= \frac{4}{x-y} = 2 \Rightarrow x - y = 2$ ....(ii)

From Eqs.(i) and (ii) we get x=6 and y=4

Hence the speed of current is 4 km/h

Q.5.(A) Let the speed of current is 4 km/h

Then the speed of upstream=(x-4) km/h

Accordint to the question,

$= \frac{12}{x-y} = 5 \Rightarrow 12 = 5x - 20 \Rightarrow \frac{32}{5} \text{ km/h}$

∴ Time taken to row 15 km with the stream

$= \frac{15}{\frac{32}{5} + 4} = \frac{15 \times 5}{52} = \frac{75}{52} \text{ h}$

$= 1\frac{23}{52} \times 60 = 1 \text{ h } 26\frac{7}{13} \text{ min}$

Q.6.(A) $t = \frac{84}{(16+5)}$ = 4 hr.

Q.7.(D) Let the speed of man and current be x km/h and y km/h, respectively.

The speed upstream=(x-y) km/h

and speed downstream=(x+y) km/h

According to the question,

$\frac{3 \times 60}{4 \times 15} = x - y \Rightarrow x - y = 3$ ....(i)

and $\frac{3}{4} \times \frac{60}{10} = x + y \Rightarrow x + y = \frac{9}{2}$ ....(ii)

On adding Eqs. (i) and (ii) we get

$2x = 3 + \frac{9}{2} \Rightarrow 2x = \frac{6+9}{2} \Rightarrow x = \frac{15}{4}$

On putting the value of x in Eq.(ii), we get

$\frac{15}{4} + y = \frac{9}{2} - \frac{15}{4} = \frac{18-15}{4}$

$\Rightarrow \quad y = \frac{3}{4}$

∴ Speed of man, $x = \frac{15}{4}$ km/h

and speed of current, $y = \frac{3}{4}$ km/h

Hence, required ratio $= \frac{15}{4} : \frac{3}{4} = 5 : 1$

Q.8.(C) Let the speed of boat be x km/h

Given, speed of current=5 km/h

∴ Upstream speed of boat=(x-5) km/h

and downstrean speed of boat=(x+5) km/h

According to the question,

$\frac{10}{x-5} + \frac{10}{x+5} = \frac{50}{60}$

$\Rightarrow 10\left(\frac{x+5+x-5}{x^2-25}\right) = \frac{5}{6}$

$\Rightarrow 12 \times 2x = x^2 - 25$

$\Rightarrow x^2 - 24x - 25 = 0$

$\Rightarrow x^2 - 25x + x - 25 = 0$

$\Rightarrow (x-25)(x+1) = 0$

∴ x=25 $[\because x \neq -1]$

So, the speed of motorboat in still water is 25 km/h

Q.9.(C) Let speed of stream be x km/h

Given, speed of motorboat in still water

=45 km/h

$\therefore$ x=60-45=15 km/h

$\therefore$ Speed of boat against stream

=45-15=30 km/h

hence, required time $= \frac{\text{Distance}}{\text{Speed}} = \frac{80}{30}$ h

$$= \frac{8}{3} \times 60 = 160 \text{ min} = 2 \text{ h } 40 \text{ min}$$

Q.10.(D) Let x be the speed of the boat and y be the speed of the current.

Then, speed upstream=(x-y) km/h

and downstream=(x+y) km/h

According to the question,

$$\frac{20}{x-y} + \frac{20}{x+y} \Rightarrow \frac{41}{60} + \frac{40}{3600} = \frac{25}{36}\text{h}$$

In this equation, ther are two variables but only one equation. So the value of x cannot be determined.

Q.11.(A) Pawan's speed upstream

=24-4.8=19.2 km/h

Let the required distance be x.

According to the question,

$$\frac{x}{28.8} + \frac{x}{19.2} = 1 \Rightarrow \frac{1 = 9.2x + 28.8x}{552.96} = 1$$

$\Rightarrow$ 19.2x+28.8x=552.96

$\Rightarrow$ 48x=552.96

$$\therefore \quad x = \frac{552.96}{48} = 11.52 \text{ km}$$

QUICK TRICK:

$\because$ x= speed of Pawan in still water=24 km/h

y=Speedof river=4.8 km/h and T=1h

According to the formula,

Required distance $= \frac{T(x^2 - y^2)}{2x}$

$$= \frac{1[(24)^2 - (4.8)^2]}{2 \times 24} = \frac{576 - 23.04}{2 \times 24}$$

$$= \frac{552.96}{48} = 11.52 \text{ km}$$

Q.12.(C) Man's speed upstream=6.5-2.5

= 4 km/h

Man's sped downstream=6.5+2.5

=9 km/h

Let the distance in one direction be x km.

Then, time taken in upstream $= \frac{x}{4}$

and time taken in downstream $= \frac{x}{9}$

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{\frac{x}{4} + \frac{x}{9}}$$

$$\frac{2x \times 36}{9x + 4x} = 5.53 \text{ km/h}$$

QUICK TRICK:

Here, a=6.5 km/h and b=2.5 km/h]

$\therefore$ Average speed $= \frac{(a+b)(a-b)}{a}$

$$= \frac{(6.5+2.5)(6.5-2.5)}{6.5} = \frac{9 \times 4}{6.5} = \frac{36}{6.5}$$

=5.53 km/h

Q.13.(A) let the total distance be x km.

According to the question,

$$= \frac{x}{3} - \frac{x}{7} = 2 \text{ km} \Rightarrow 7x - 3x = 21 \times 2$$

$$x = \frac{21 \times 2}{4} = 10.5 \text{ km}$$

QUICK TRICK:

Here, a=5 km/h, b=2 km/h and T= 2h

Required distance $= \left(\frac{a^2 - b^2}{2b}\right) \times t$

$$\left(\frac{5^2 - 2^2}{2 \times 2}\right) \times 2 = \frac{25-4}{2} = \frac{21}{2} = 10.5 \text{ km}$$

Q.14.(B) Let the rate of sailing of sailer be x km/h and speedof the flow of water be y km/h.

Downstream speed (x+y) $= \frac{48}{8}$

$\Rightarrow$ x+y=6...............(i)

and upstream speed (x-y) $= \frac{48}{12}$

$\Rightarrow$ x-y=4...................(ii)

On solving eq.(i) and (ii) we get

x=5 km/h and y=1 km/h

Hence, speed of the flow of river is 1 km/h

QUICK TRICK:

$$\text{Speed of stream} = \frac{1}{2}\left(\frac{l}{t_1} - \frac{l}{t_2}\right)$$

[by technique 6(ii)]

$$= \frac{1}{2}\left(\frac{48}{8} - \frac{48}{12}\right) = \frac{1}{2}(6-4) = \frac{2}{2} = 1\,\text{km/h}$$

Q.15.(D) Let the distance between the two prots be x km.

Then, speed downstream $= \frac{x}{4}$ km/h

and speed upstream $= \frac{x}{5}$ km/h

$\therefore$ Speed of the stream

$$= \frac{1}{2}(\text{speed downstream} - \text{Speed upstream}))$$

$$\Rightarrow 2 = \frac{1}{2}\left(\frac{x}{4} - \frac{x}{5}\right) \Rightarrow 2 = \frac{1}{2}\left(\frac{5x-4x}{20}\right)$$

$$\Rightarrow \frac{x}{40} = 2 \Rightarrow x = 80 \text{ km}$$

QUICK TRICK:

Here, $t_1 = 4$ and $t_2 = 5h$

$\therefore$ Speed of stream $= \frac{l}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$

$$\Rightarrow 2 = \frac{l}{2}\left(\frac{1}{4} - \frac{1}{5}\right) \Rightarrow 2 = \frac{l}{2}\left(\frac{1}{20}\right)$$

$\therefore$ l=80 km

Q.16.(D) According to the question

$$\frac{4.8}{(x+y)} = \frac{8}{60} \text{ .... (i)}$$

$$\frac{4.8}{(x-y)} = \frac{9}{60} \text{ .... (ii)}$$

From (I) and (II), we get

x = 32 kmph = 2 kmph

Q.17.(C) $\frac{24}{u} + \frac{28}{v} = 6$ .... (i)

$\frac{30}{u} + \frac{21}{v} = \frac{13}{2}$ .... (ii)

From (I) and (II), we get,

u = 6, v = 14

Speed of the boat = 10 km/hr.

Q.18.(B) Let the speed of the current is x kmph. Then speed of the boat in still water = 4x kmph

According to the question

$$\frac{30}{4x+x} + \frac{30}{4x-x} = 8$$

$$\frac{30}{5x} + \frac{30}{3x} = 8$$

$$\frac{16}{x} = 8$$

x = 2

Q.19.(D) According to the question

$$\frac{D}{(5+1.5)} + \frac{D}{(5-1.5)} = 1$$

3.5D + 6.5D = 22.75

D = 2.275 km

Q.20.(B) Let speed of the boat = x km/hr.

and speed of the current = y km/hr.

Downstream speed = (x + y) km/hr.

Upstream speed = (x + y) km/hr.

According to the question

2(x + y) = 3(x – y)

2x + 2y = 3x – 3y

x = 5y

x: y = 5: 1

Q.21.(B) Let the speed of the boat down the stream and up the stream be x and y km/hr. respectively

According to the question,

$$\frac{12}{y}+\frac{18}{x}=3 \Rightarrow \frac{4}{y}+\frac{6}{x}=1....(I)$$

and

$$\frac{36}{y}+\frac{24}{x}=\frac{13}{2} \Rightarrow 4\left(\frac{9}{y}+\frac{6}{x}\right)=\frac{13}{2}$$

$$\Rightarrow \frac{9}{y}+\frac{6}{x}=\frac{13}{8}....(II)$$

On solving above two equations, we get

y = 8, x = 12

Speed of the boat = [(12+8)/2] = 20/2

= 10 km/hr.

Q.22.(D) Let the speed of the stream be x kmph

Speed of the upstream = $\frac{15}{2}-x$

Speed of downstream = $\frac{15}{2}+x$

Then, $2\left(\frac{15}{2}-x\right)=\frac{15}{2}+x$

$3x = \frac{15}{2}$

x = 2.5 km/hr.

Q.23.(A) Let the speed of motor boat be 36 km./hr. and speed of current 5x km./hr.

Distance

= 5 hours 10 minute × (36x + 5x)

$= \frac{41x \times 31}{6}$ km.

Rate of upstream

= 36x – 5x = 31 x

Time taken = $\frac{41x \times \frac{31}{6}}{31x} = \frac{41}{6}$ hours

= 6 hours 50 minutes

Notes

CHAPTER-18
MENSURATION-I

Scan the QR code to get video of this chapter.

Mensuration

Mensuration is the branch of Mathematics which deals with the study of different geometrical shapes, their area and volume. In the broadest sense, it is all about the process of measurement. It is based on the use of algebraic equations and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. While the measurement results obtained by the use of mensuration are usually considered very accurate. There are two types of geometric shapes :

1. 2D :

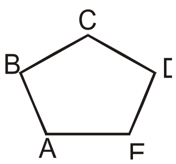
Those figures have two dimension is called 2D figures. like- square, rectangle, parallelogram, rhombus, triangle, trapezoid, circle etc.

2. 3D :

Those figures have three dimension length, breadth and height are called three dimension. like- cube, rectangular prism (cuboid), cylinder, cone, sphere, hemisphere, prism, pyramid etc.

Polygon :

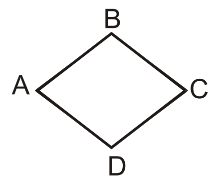
A polygon is a closed, plane figure bounded by 'n' straight lines ($n \geq 3$). Each of the n line segments forming the polygon is called its sides.



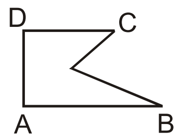
Type of polygon	No. Of Side
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

Regular Ploygon :- A polygon that has all sides and all interior angles equal.

Convex Polygon :- A convex polygon is defined as a polygon with all its interior angle less than 180°.



Concave Polygon :- A polygon that has one or more interior angle greater than 180°.



Properties of polygon :

(n = number of sides in polygon)

1. Sum of interior and exterior angle is 180°.

2. Sum of interior angle of polygon is $(n-2)\times180^\circ$.

3. Sum of the exterior angle of polygon is 360°.

4. Each interior angle of polygon is

$$\frac{(n-2)\times180}{n}$$

5. Each exterior angle of polygon

$$=\frac{360^0}{n}$$

6. Number of diagonals of a polygon

$$=\frac{n(n-3)}{2}$$

Ex. Each interior angle of a regular polygon is 144°. Find the number of sides of the polygon ?

(A) 6 **(B)** 8

(C) 10 **(D)** 12

Sol.(C) Let the number of side of the polygon be 'n' then

$$\Rightarrow \frac{(n-2)}{n} \times 180^\circ = 144^\circ$$

$$\Rightarrow \frac{(n-2) \times 5}{n} = 4$$

$\Rightarrow$ 4n = 5n - 10

$\Rightarrow$ n = 10

Ex. If the sum of the interior angles of regular polygon be 1080°. Find the number of sides of the polygon?

(A) 6 **(B)** 8

(C) 10 **(D)** 12

Sol.(B) We know that-

(n-2) × 180° = 1080°

$\Rightarrow$ (n-2) = 6

$\Rightarrow$ n = 8

Ex. The number of sides in two regular polygon are in the ratio 5 : 4 and the difference between each interior angle of the polygon is 6°, then the number of sides are-

(A) 5, 4 **(B)** 10, 8

(C) 15, 12 **(D)** 10, 15

Sol.(C) Let the number of side are 5x and 4 x respectively.

$$\frac{(5x-2) \times 180^\circ}{5x} - \frac{(4x-2)}{4x} \times 180^\circ = 6^\circ$$

$$\Rightarrow 180^\circ \left[\frac{(5x-2)}{5x} - \frac{(4x-2)}{4x}\right] = 6^\circ$$

$$\Rightarrow 30\left[\frac{4(5x-2)-5(4x-2)}{20x}\right] = 1$$

$$\Rightarrow 30[20x - 8 - 20x + 10] = 20x$$

$\Rightarrow$ 30 × 2 = 20x

$\Rightarrow$ x = 3

$\Rightarrow$ Number of sides are -

5x $\Rightarrow$ 5 × 3 = 15

4x $\Rightarrow$ 4 × 3 = 12

Ex. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between their interior angles is 2 : 3. The number of sides of these polygon.

(A) 7, 14 **(B)** 4, 8

(C) 5, 10 **(D)** 6, 12

Sol.(B) Let the number of side of regular polygon be x and 2x then

$$\frac{\text{interior angle of 1st polygon}}{\text{interior angle of 2nd polygon}} = \frac{2}{3}$$

$$\Rightarrow \frac{\frac{(x-2) \times 180}{x}}{\frac{(2x-2) \times 180^\circ}{2x}} = \frac{2}{3}$$

$$\Rightarrow \frac{(x-2)}{x} \times \frac{2x}{(2x-2)} = \frac{2}{3}$$

$$\Rightarrow \frac{(x-2)}{(x-1)} = \frac{2}{3}$$

$\Rightarrow$ 3x - 6 = 2x - 2

$\Rightarrow$ x = 4

Number of sides = 4, 8

Ex. If each interior angle is double of each exterior angle of a regular polygon with n sides. Find the value of n.

(A) 5 **(B)** 10

(C) 6 **(D)** 8

Sol.(C) According to question

$$\frac{(n-2) \times 180}{n} = 2 \times \frac{360}{n}$$

2n - 4 = 8

2n = 12

n = 6

Ex. If the sum of the interior angles of a regular polygon measures 1440°. How many sides does the polygon have

(A) 10 **(B)** 8

(C) 12 **(D)** 9

Sol.(A) According to question

Sum of all interior angle

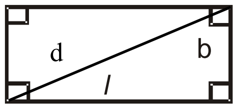
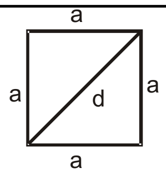
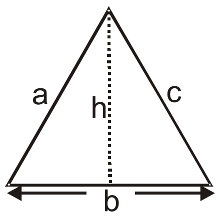
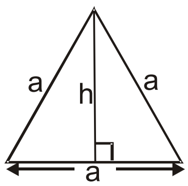
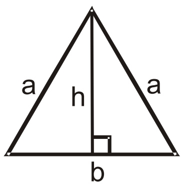
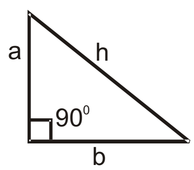
(n-2) × 180 = 1440

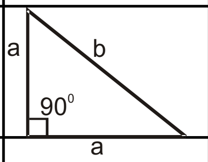
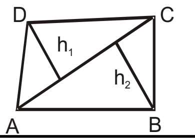
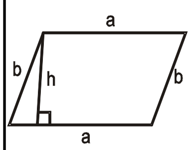
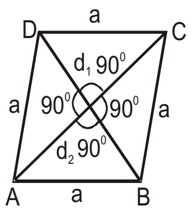
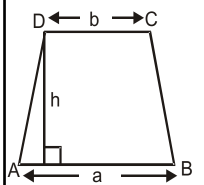
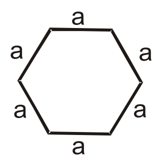
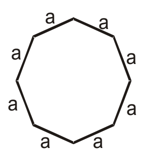
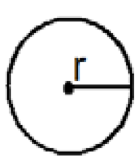
n-2 = $\frac{1440}{180} = 8$

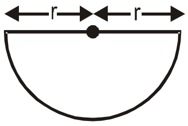
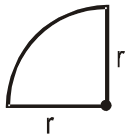
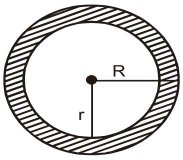
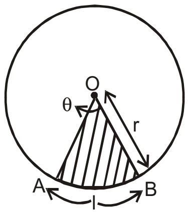
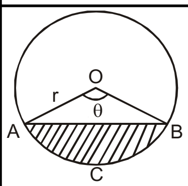
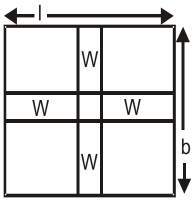
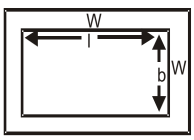
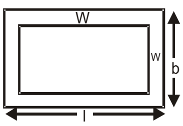
n=10

[Plane Figures]

Plane figures are flat two dimenstional shape. A plane figure can be made of straight lines, curved lines, or both straight and curved lines. **Ex.** Square, Rectangle and Triangle etc.
In Mathematics there are many plane figures which is listed below along with formulas.

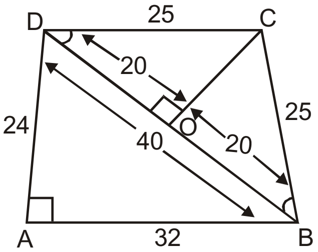
S. NO.	Name	Figure	Nomenclature	Area	Perimeter
1.	Rectangle		$l \rightarrow$ length $b \rightarrow$ breadth $d \rightarrow$ diagonal $d = \sqrt{l^2 + b^2}$	$l \times b = lb$	$2l+2b = 2(l+b)$
2.	Square		$a \rightarrow$ side $d \rightarrow$ diagonal $d = a\sqrt{2}$	(i) $a \times a = a^2$ (ii) $\frac{d^2}{2}$	$a+a+a+a = 4a$
3.	Triangle (Scalene)		a, b and c are three sides of triangle and s is semiperimeter, where $s = \left(\frac{a+b+c}{2}\right)$ b is the base and h is the altitutde of triangle h = height or altitude	(i) $\frac{1}{2} \times b \times h$ (ii) $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)	$a+b+c = 2s$
4.	Equilateral triangle		$a \rightarrow$ equal sides $h \rightarrow$ height or altitude $h = \frac{\sqrt{3}}{2}a$	(i) $\frac{1}{2} \times a \times h$ (ii) $\frac{\sqrt{3}}{4}a^2$	3a
5.	Isosceles triangle		$a \rightarrow$ equal sides $b \rightarrow$ base $h \rightarrow$ height or altitude $h = \frac{\sqrt{4a^2 - b^2}}{2}$	(i) $\frac{1}{2} \times b \times h$ (ii) $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$	2a+b
6.	Right angled triangle		$b \rightarrow$ base $a \rightarrow$ altitude/height h = hypotenuse $h = \sqrt{a^2 + b^2}$	$\frac{1}{2} \times b \times a$	b+h+a

S. NO.	Name	Figure	Nomenclature	Area	Perimeter
7.	Isosceles right angled triangle		a → equal sides b → other side $b = a\sqrt{2}$	$\frac{1}{2}a^2 = \frac{b^2}{4}$	2a + b
8.	Quadrilateral		AC is the diagonal and h_1, h_2 are the altitudes on AC from the vertices D and B respectively.	$\frac{1}{2} \times AC \times (h_1 + h_2)$	AB+BC+CD+AD
9.	Parallelogram		a and b are side adjacent to each other. h→ distance between the parallel sides.	a×h	2(a+b)
10.	Rhombus		a- length of each side of rhombus d_1 and d_2 are the diagonals $d_1 \rightarrow BD$ $d_2 \rightarrow AC$ $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$	$\frac{1}{2} \times d_1 \times d_2$	4a or $2\sqrt{d_1^2 + d_2^2}$
11.	Trapezium		a and b are parallel sides to each other and h is the perpendicular distance between parallel sides.	$\left(\frac{a+b}{2}\right) \times h$	AB+BC+CD+AD
12.	Regular hexagon		a → length of each side	$\frac{3\sqrt{3}}{2}a^2$	6a
13.	Regular octagon		a → length of each side	$2a^2(1+\sqrt{2})$	8a
14.	Circle		r → radius of the circle	πr^2	$2\pi r$ (called as circumference)

S. NO.	Name	Figure	Nomenclature	Area	Perimeter
15.	Semicircle		r → radius of the circle	$\frac{1}{2}\pi r^2$	$r(\pi+2) = \frac{36}{7}r$
16.	Quadrant		r → radius	$1/4\ \pi r^2$	$\frac{1}{2}\pi r + 2r = \frac{25}{7}r$
17.	Ring or circlular path (shaded region)		R→ outer radius r →inner radius	$\pi(R^2 - r^2)$	(outer) →$2\pi R$ (inner) →$2\pi r$
18.	Sector of a circle		O→ centre of the circle r → radius l→ length of the arc θ→ angle of the sector $l \to 2\pi r\left(\frac{\theta}{360^\circ}\right)$	$\pi r^2\left(\frac{\theta}{360^\circ}\right)$	l+2r
19.	Segment of a circle		θ → angle of the sector r → radius AB → chord ACB → arc of the circle	Area of segment ACB (minor segment) $= r^2\left(\frac{\pi\theta}{360^\circ} - \frac{\sin\theta}{2}\right)$	$2r\left[\frac{\pi\theta}{360^\circ} + \sin\left(\frac{\theta}{2}\right)\right]$
20.	Pathways running parallel inside to sides of a rectanlge		l → length b → breadth w → width of the path (road)	(l+b – w)w	2(l+b) - 4w
21.	Outer path		l → length b → breadth w → width of the path	(l+b+2w) 2w	(inner) →2 (l+b) (outer)→2 (l+b+4w)
22.	Inner path		l → length b → breadth w → width of the path	(l+b–2w) 2w	(outer)→2(l+b) (inner)→2(l+b-4w)

Examples

Ex.1. What is the area of the plot (in m^2)?



(A) 768 **(B)** 534

(C) 696 **(D)** 684

Sol.(D) The figure given above can be seen as

Area of the plot

$\Rightarrow$ Area of Δ ABD + Area of Δ BCD

$$\Rightarrow \frac{1}{2}\times AD\times AB+\left(\frac{1}{2}\times BD\times OC\right)$$

$$\Rightarrow \frac{1}{2}\times 24\times 32+\frac{1}{2}\times 40\times 15$$

$\Rightarrow$ 684 m^2

Ex.2. A lawn is in the form of a rectangle having its sides in the ratio 2:3 The area of the lawn is 6 hectares. Find the length and breadth of the lawn.

(A) 100 and 150 m **(B)** 200 and 300 m

(C) 300 and 200 m **(D)** 400 and 300 m

Sol.(B) Ratio of sides of lawn = 2 : 3

Lets, take sides are = 2x and 3x

Area = 2x × 3x

6 hectares = $6x^2$

$6x^2 = 60000\ m^2$

$x^2 = 10000\ m^2$

x = 100 m

So, sides are 200 m and 300 m

Ex.3. The length of a rectangle is twice of its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

(A) 40 cm **(B)** 100 cm

(C) 150 cm **(D)** 200 cm

Sol.(A) Lets, take breadth of the rectangle is x cm.

So, length is = 2x cm

area = x × 2x = $2x^2$

Now, according to question

$(2x-5)(x+5) = 2x^2+75$

$2x^2 + 5x - 25 = 2x^2 + 75$

5x = 100

x = 20 cm.

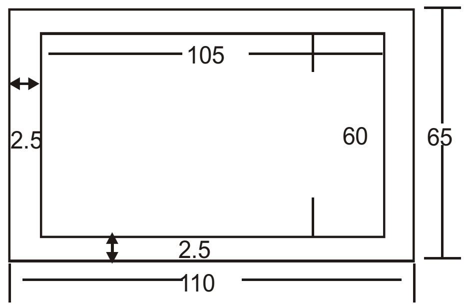
So, length = 2x = 40 cm

Ex.4. A rectangular grassy plot 110 m × 65 m has a gravel path 2.5m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

(A) Rs. 850 **(B)** Rs. 680

(C) Rs. 720 **(D)** Rs. 770

Sol.(B)



Length and breadth of plot with paths is

= 110 m and 65 m

Length and breadth of plot without path

= (110-5) = 105 m and

(65 - 5) = 60 m

Area of plot with path = 110×65 = 7150 m^2

Area of plot without path = 105 × 60 = 6300 m^2

So, area of the path = 7150 - 6300 = 850 m^2

Cost of gravelling = 850 × 0.8 = Rs. 680

Ex.5. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of these two squares.

(A) 20 cm **(B)** 24 cm

(C) 30 cm **(D)** 50 cm

Sol.(B) Perimeters of 1st square = 40 cm

$4a_1 = 40$

$a_1 = 10$

side is 10 cm

Perimeters of 2nd square = 32 cm

$4a_2 = 32$

$a_2 = 8$

Side is 8 cm.

Area of 3rd square = difference of the area of 1st and 2nd square

$= 10^2 - 8^2$

$= 100 - 64$

$= 36$

So, $a_2^2 = 36$

$a_2 = 6$

So, side of 3rd square is 6 cm.

Perimeters = 4 × side = 24 cm.

Ex.6. The diagonals of two squares are in the ratio of 2:5. Find the ratio of their areas.

(A) 2 : 5 **(B)** 3 : 5

(C) 4 : 25 **(D)** 4 : 15

Sol.(C) Ratio of diagonals = 2 : 5

So, take diagonals as 2x and 5x

Area = $\frac{(\text{diagonal})^2}{2}$

Ratio of area $= \frac{(d_2)^2}{2} : \frac{(d_2)^2}{2}$

$= (2x)^2 : (5x)^2 = 4 : 25$

Ex.7. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

(A) 72 cm² **(B)** 84 cm²

(C) 36 cm² **(D)** 92 cm²

Sol.(B) Sides of triangle = 13 cm, 14 cm, 15 cm

Half perimeter (s)= $\frac{13+14+15}{2} = 21$ cm

Area

$= \sqrt{21 \times (21-13) \times (21-14) \times (21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6} = 84$ cm²

Ex.8. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base.

(A) 900 m. **(B)** 700 m.

(C) 1000 m. **(D)** 600 m.

Sol.(A) Lets, take altitude = x

base = 3x

Area of field = $\frac{333.18}{24.68} = 13.5$ hectare

= 135000 m²

Area = 135000

$\frac{1}{2} \times 3x \times x = 135000$

$x^2 = \frac{135000 \times 2}{3}$

$x^2 = 90000$

x = 300 m.

Since height is = 300 m

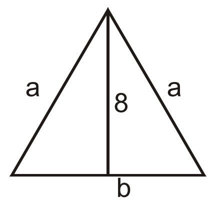
Hence base is = 900 m

Ex.9. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.

(A) 42 cm² **(B)** 46 cm²

(C) 48 cm² **(D)** 50 cm²

Sol.(C)



2a + b = 32 ......(i)

Using pythagorean theorem

$a^2 = 8^2 + \left(\frac{b}{2}\right)^2$

$a^2 = 64 + \frac{b^2}{4}$ ....(ii)

By solving equation (i) and (ii)

b = 12

Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 12 \times 8 = 48$ cm²

Ex.10. In two triangles, the ratio of the areas is 4:3 and the ratio of their heights is 3:4. Find the ratio of their bases.

(A) 16 : 9 **(B)** 9 : 16

(C) 3 : 4 **(D)** 4 : 3

Sol.(A) Ratio of areas = 4 : 3

Ratio of heights = 3 : 4

$\frac{\text{area of 1st triangle}}{\text{area of 2nd triangle}} = \frac{4}{3}$

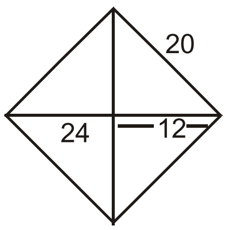
$\frac{\frac{1}{2} \times b_1 \times 3}{\frac{1}{2} \times b_2 \times 4} = \frac{4}{3}, \quad \frac{b_1}{b_2} = \frac{16}{9}$

$b_1 : b_2 = 16 : 9$

Ex.11. Find the area of a rhombus one side of which measures 20cm and one diagonal 24cm.

(A) 768 **(B)** 384 **(C)** 484 **(D)** 192

Sol.(B)



Half diagonal = $\frac{24}{2} = 12$ cm

Other digonals is = $2 \times \sqrt{20^2 - 12^2}$

$= 2 \times \sqrt{256} = 16 \times 2 = 32$

Area = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 24 \times 32 = 384 \text{ cm}^2$

Ex.12. The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm² find the smaller length of the parallel sides.

(A) 23 cm **(B)** 27 cm

(C) 24 cm **(D)** 26 cm

Sol.(A) Lets the parallel sides area a and b

Difference = (a - b) = 4..........(i)

Area = 475 cm^2

$\frac{1}{2}(a+b) \times h = 475$

$\frac{1}{2}(a+b) \times 19 = 475$

a + b = 50.............(ii)

By solving equation (i) and (ii)

a = 27

b = 23

Hence length of smaller parallel side is= 23 cm

Ex.13. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

(A) Rs. 5808 **(B)** Rs. 4803

(C) Rs. 6201 **(D)** Rs. 4812

Sol.(A) Area = 13.86 hectares = 138600 m^2

$\frac{22}{7} \times r^2 = 138600$

$r^2 = \frac{138600 \times 7}{22}$

$r^2 = 44100$, r = 210 m

Circumference of circular field= $2 \times \frac{22}{7} \times 210$

= 1320 m

So, cost of fencing =1320×4.4 = Rs. 5808

Ex.14. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel made in order to keep a speed of 66 kmph?

(A) 250 **(B)** 350

(C) 100 **(D)** 200

Sol.(A) Diameter of the wheel = 140 cm.

So, radius of that wheel = 70 cm.

Circumference of wheel = $2 \times \frac{22}{7} \times 70 = 440$ cm.

In one revolution is can cover 440 cm. or 4.4 m

Speed is = 66 km/hr $\times \frac{5}{18} = \frac{55}{3} m/s$

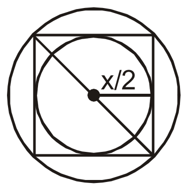
So, no of revolution is = $\frac{55}{3 \times 4.4} \times 60 = 250$

Ex.15. Find the ratio of the areas of the incircle and circumcircle of a square.

(A) 1 :2 **(B)** 2 : 1

(C) $\sqrt{2} : 1$ **(D)** $1 : \sqrt{2}$

Sol.(A)



Side of square = x unit

So, radius of incircle = $\frac{x}{2}$ unit

Diagonal of square=diameter of circumcircle

$\sqrt{2}\, x = d$

radius $= \frac{x\sqrt{2}}{2}$

ratio of area of incircle and circumcircle

$= \pi \times \left(\frac{x}{2}\right)^2 : \pi \times \left(\frac{x\sqrt{2}}{2}\right)^2$

$= \pi \times \frac{x^2}{4} : \pi \times \frac{2x^2}{4} = 1 : 2$

Note : In this type of problem the ratio of radius of incricle and circumcircle is always $= 1:\sqrt{2}$

The ratio of area of incircle and circumcircle is always = 1 : 2

Ex.16. The area of a rectangle is thrice that of a square. If the length of the rectangle is 40 cm and its breadth is $\frac{3}{2}$ times that of the side of the square, then the side of the square is :

(A) 15 cm **(B)** 20 cm
(C) 30 cm **(D)** 60cm.

Sol.(B) Area of rectangle = 3 × area of square

l × b = 3 × a^2

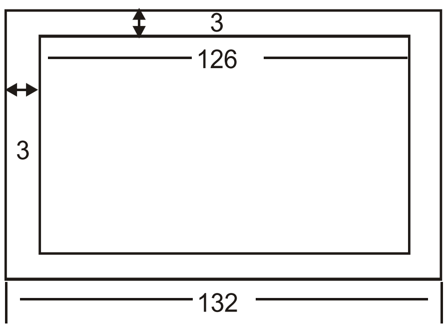
40 × 1.5 a = 3 × a^2

a = 20 cm.

Ex.17. The cost of fencing a square field at the rate of Rs. 20 per metre is Rs. 10,080. How much will it cost to lay a three metre wide pavement along the fencing outside the field at the rate of Rs. 50 per sq. metre?

(A) Rs. 37,350 **(B)** Rs. 73,800
(C) Rs. 77,400 **(D)** Rs. 76,400

Sol.(C)



Perimeter of sqaure field $= \frac{10080}{20} = 504$ m

Side of sqaure field = $\frac{504}{4}$ = 126 m

So, area of field = 126 × 126

Area of field with pavement part

= 132 × 132

Area of pavement part

= (132 × 132) - (126×126)= 1548 m^2

Hence, cost = 1548 × 50

= Rs. 77400

Ex.18. A man walked diagonally across a square. Approximately, what was the percent saved by not walking along the edges?

(A) 20 **(B)** 24
(C) 30 **(D)** 33

Sol.(C) Diagonal of square = $\sqrt{2}a$ = 1.4 a (approx)

% saved = $\frac{2a - 1.4a}{2a} \times 100$ = 30%

Ex.19. A cow is tethered in the middle of a field with a 14 feet long rope. If the cow grazes 100sq. feet. per day, then approximately what time will be taken by the cow to graze the whole field?

(A) 2 days **(B)** 6 day
(C) 18 days **(D)** 24 days

Sol.(B) Radius of grazed field = 14 ft.

Area = $\frac{22}{7} \times 14 \times 14$ = 616 ft^2

One day cow grazes 100 ft^2

Number of days = $\frac{616}{100}$ (approx)

= 6 days.

Ex.20. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, Then its area will be :

(A) 3520 cm^2 **(B)** 6400 cm^2
(C) 7744 cm^2 **(D)** 8800 cm^2

Sol.(C) Circumference of circle = $2 \times \frac{22}{7} \times 56$ = 352 cm

Length of wire = 352 cm.

Side of square = $\frac{352}{4} = 88$ cm

Then area = $(88)^2$ = 7744 cm^2

Ex.21. The areas of two circular fields are in the ratio 16:49 If the radius of the latter is 14m, then what is the radius of the former?

(A) 4 m **(B)** 8 m
(C) 18 m **(D)** 32 m

Sol.(B) $\frac{\text{area of 1st circle}}{\text{area of 2nd circle}} = \frac{16}{49}$

$\frac{\pi \times r^2}{\pi \times 14^2} = \frac{16}{49}$

$r^2 = \frac{16 \times 14 \times 14}{49}$

r = 8 m.

Notes : If radius ratio = $r_1 : r_2$

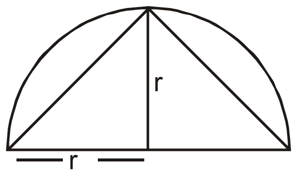
Then area ratio = $r_1^2 : r_2^2$

Ex.22. The area of the largest triangle that can be inscribed in a semi- circle of radius r, is :

(A) r^2 **(B)** $2r^2$

(C) r^3 **(D)** $2r^3$

Sol.(A)



Base of triangle = 2r

height = r

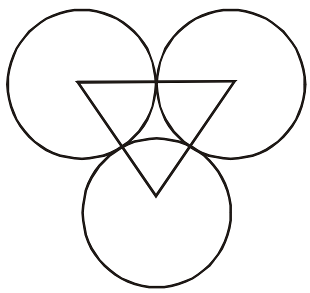
area = $\frac{1}{2} \times 2r \times r$ = r^2

Ex.23. Three circles of radius 3.5 cm are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is:

(A) 1.967 cm^2 **(B)** 1.975 cm^2

(C) 19.67 cm^2 **(D)** 21.21 cm^2

Sol.(A)



Side of triangle = 3.5×2 = 7 cm

Area of triangle = $\frac{\sqrt{3}}{4} \times 7 \times 7$ = 21.217 cm^2

According to figure since angle of one sector is 60^0

So, angle of 3 sector is = 180^o

That means half circle be made

So, area of half circle = $\frac{22}{7} \times \frac{3.5 \times 3.5}{2}$

= 19.25

Hence, area of enclosed area

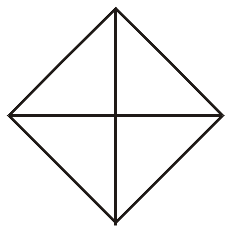
= 21.217 - 19.25 = 1.967 cm^2

Note : Area of enclosed area = area of equilateral triangle - area of semicircle.

Ex.24. In a rhombus, the length of the two diagonals are 40 and 30 metres respectivety. Find its perimeter.

(A) 150 **(B)** 200 **(C)** 100 **(D)** 300

Sol.(C)



Two diagonals d_1 = 40 m

d_2 = 30 m

Side of rhombus = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

$= \sqrt{(20)^2 + (15)^2}$

$= \sqrt{400 + 225}$

$= \sqrt{625}$ = 25

Perimter of rhombus = 4×side

= 4 × 25 = 100 m

Ex.25. In a parallelogram, the lengths of adjacent sides are 11 and 13 metres respectively. if the length of one diagonal is 16 metres, find the length of other diagonal.

(A) 18 m. **(B)** 22 m.

(C) 14 m. **(D)** 12 m.

Sol.(A) Length of one side a = 13 m

Length of other side b = 11 m

One diagonal d_1 = 16 m

Other diagonal d_2 = ?

In a parallelogram

$d_1^2 + d_2^2 = 2 (a^2+b^2)$

or, $(16)^2 + d_2^2 = 2 (13^2 + 11^2)$

$d_2^2 = 2 (169+121) - 256$

$d_2^2 = 580 - 256$

$d_2^2 = 324$

d_2 = 18 m

Ex.26. The sides of a rectangle are in the ratio 5:3 and its area is 1,500 square metre. The cost of fencing the rectangular field at the rate of Rs 2.5 per metre is

(A) Rs. 400 **(B)** Rs 200

(C) Rs. 300 **(D)** Rs. 500

Sol.(A) Let sides = 5x and 3x

area = 1500

$5x \times 3x = 1500$

$x^2 = 100$

x = 10

Sides are = 50 m and 30 m

Perimeter of field = 2 (50+30) = 160 m

Cost of fencing = 160 × 2.5 = Rs. 400

Ex.27. If the length of a rectangle is increased by 10% and the breadth decreased by 20%, a square of area 484 m^2 is obtained. The area of the rectangle in square metres is-

(A) 484 **(B)** 242

(C) 400 **(D)** 550

Sol.(D) Percentage change in area

$= +10 - 20 - \frac{10 \times 20}{100} = -12\%$

This means area is decreased by 12%

Hence new area = 100 - 12 = 88%

88% = 484

1% = 5.5

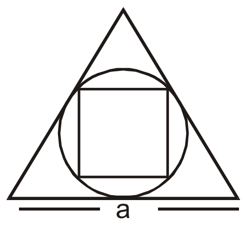
100% = 550 m^2

Ex.28. A square is inscribed in a circle which is inscribed in an equilateral. If one side of the triangle is 'a', find the area of the square ?

(A) $\frac{a^2}{6}$ **(B)** $\frac{a^2}{3}$

(C) $\frac{a^2}{2}$ **(D)** $\frac{a^2}{4}$

Sol.(A)



Radius of incircle of an equilateral triangle

$= \frac{a}{2\sqrt{3}}$

Diameter of cirlce = diagonal of square

$2 \times \frac{a}{2\sqrt{3}} = d$

$d = \frac{a}{\sqrt{3}}$

Area of square = $\frac{d^2}{2}$

$= \frac{a^2}{3 \times 2} = \frac{a^2}{6}$

Ex.29. There are two regular polygons with the number of sides in the ratio of 4 : 5 and the interior angles in the ratio of 25 : 26 .Find the number of sides in the first polygon ?

(A) 20 **(B)** 16 **(C)** 12 **(D)** 8

Sol.(C) Each interior angle of polygon

$= \frac{(2n-4) \times 90}{n}$

Lets, take number of sides 4x and 5x.

$= \frac{\frac{(2 \times 4x - 4)}{4x} \times 90}{\frac{(2 \times 5x - 4)}{5x} \times 90} = \frac{25}{26}$

x = 3

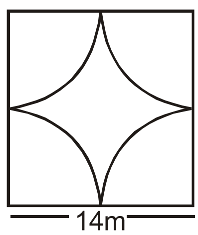
Number of sides of Ist polygon= 4 × 3= 12.

Ex.30. Four horses are tied on the four corners of a square field of 14 m length so that each horse.can touch just the other two horses. They were able to graze in the area accessible to them for 11 day. For how many days is the ungrazed area sufficient for them?

(A) 16 days **(B)** 14 days

(C) 11 days **(D)** 3 days

Sol.(D)



Area of square = 14 × 14 = 196 m^2

Area of grazed part = $\frac{22}{7} \times 7 \times 7$ = 154 m^2

Remaining part = 196 - 154 = 42 m^2

154 m^2 = graze in 11 days

1 days horse can graze = 14 m^2

ungrazed area is suffcience for

$= \frac{42}{14} = 3$ days

EXERCISE

Q.1. Find the area of a regular octagon whose side 10 cm.

(A) 824.40 cm^2 **(B)** 284.80 cm^2

(C) 482.80 cm^2 **(D)** 428.00 cm^2

Q.2. In the triangle ABC, AB = 2 cm, BC = 3 cm and AC = 4 cm. D is the middle point of AC. If a square is constructed on the side BD, what is the area of the square?

(A) 4.5 cm^2 **(B)** 2.5 cm^2

(C) 6.25 cm^2 **(D)** None of these

Q.3. Four circles of radius 1 cm are placed in such a way on a plane paper such that each touches each other. Find the area of the space left in between the four circles?

(A) 1.89 cm^2 **(B)** 2.62 cm^2

(C) 0.86 cm^2 **(D)** 0.23 cm^2

Q.4. A square and an equilateral triangle are drawn on the same base.The ratio of their area is?

(A) 2 : 1 **(B)** 1 : 1

(C) $\sqrt{3} : 4$ **(D)** $4 : \sqrt{3}$

Q.5. Diameter of a wheel is 3m. The wheel revolves 28 times in a minute. To cover 5.280km distance, the wheel will take

$\left(\text{take } \pi = \frac{22}{7}\right)$

(A) 10 min. **(B)** 20 min.

(C) 30 min. **(D)** 40 min.

Q.6. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid points of the sides of the triangle is

(A) 6 **(B)** 3

(C) $\frac{3}{2}$ **(D)** $\frac{3}{4}$

Q.7. If the area of a circle is equal to the area of a square with side $2\sqrt{\pi}$ units, what is the diameter of the circle?

(A) 1 unit **(B)** 2 units

(C) 4 units **(D)** 8 units

Q.8. Each side of an equilateral triangle is equal to the radius of a circle whose area is 308cm^2 The Area of the a equilateral tringle is?

(A) 42.4 cm^2 **(B)** 45.3 cm^2

(C) 46.5 cm^2 **(D)** 47.2 cm^2

Q.9. A square park has each side 80 cm, At each corner of the park there is flower bed in the form of a quadrant of radius 7 cm, The area of the remaining part of the park is?

(A) 6300 cm^2 **(B)** 6455 cm^2

(C) 6246 cm^2 **(D)** 6350 cm^2

Q.10. A lawn in the form of a rectangle is one and half as long again as it is broad. The area of the lawn is $\frac{2}{3}$ hectares. The length of the lawn is:

(A) 100 metres **(B)** $33\frac{1}{3}$ metres

(C) $66\frac{2}{3}$ metres **(D)** $\left(\frac{100}{\sqrt{3}}\right)$ metres

Q.11. The dimensions of the floor of recangular hall is 4m ×3m. The floor of the hall is to be tile fully with 8cm × 6 cm rectangular tiles without breaking tiles to smaller sizes. The number of tiles required is:

(A) 4800 **(B)** 2600

(C) 2500 **(D)** 2400

Q.12. The length of a rectangle is twice its breadth. If its length is decrease by 5 cm and breadth is increased by 5 cm, the area of rectangle is increased by 75 cm^2. therefore, the length of the rectangle is :

(A) 24 cm. **(B)** 30 cm.

(C) 40 cm. **(D)** None of these

Q.13. A 5m wide lawn is cultivated all along the outside of rectangular plot measuring 80m × 40m . The total area of the lawn is :

(A) 1200 m^2 **(B)** 1300 m^2

(C) 1350 m^2 **(D)** 4800 m^2

Q.14. The length of plot of land is 4 times its breadth. A playground measuring 1200 m^2 occupies one third of the total area of the plot in metres. What is the length of plot ?

(A) 90 m. **(B)** 80 m.

(C) 120 m. **(D)** None of these

Q.15. The length and breadth of a playground are 36m and 21m respectively. Flagstaffs are required to be fixed on all along the boundary at a distance 3m apart, the number of flagstaffs will be:

(A) 37 **(B)** 38

(C) 39 **(D)** 40

Q.16. A room 5m × 4m is to be carpeted leaving a margin of 25cm from each wall. if the cost of the carpet is Rs. 80 per m^2, the cost of carpeting the room will be :

(A) Rs.1440 **(B)** Rs. 1260

(C) Rs. 1228 **(D)** Rs.1192

Q.17. A rectangle is having 15 cm as its length and 150 cm^2 as its area then area is increased to $1\frac{1}{3}$ times the original area by increasing only its length, its new perimeter is :

(A) 50 cm **(B)** 60 cm

(C) 70 cm **(D)** 80 cm

Q.18. A man drives 4 km. distance to go around a rectangular park. If the area of the rectangle is 0.75 sq. km, the difference between the length and the breadth of the rectangle is :

(A) 10.25 km **(B)** 0.5 km

(C) 1 km **(D)** 2.75 km

Q.19. The length and breadth of a recangular piece of land are in the ratio of 5 : 3 The owner spent Rs. 3000 for surrounding it from all the sides at the rate of Rs. 7.50 per metre. The difference between length and breadth is:

(A) 50 m **(B)** 100 m

(C) 200 m **(D)** 150 m

Q.20. A rectangular lawn 60m by 40m has two roads, each 5m wide, running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 60p. per m^2 is:

(A) Rs. 300 **(B)** Rs. 285

(C) Rs.250 **(D)** Rs.265

Q.21. The sides of a rectangular park are in the ratio 3:2 and its area is 3750m^2 .The cost of fencing it at 50 paise per metre is :

(A) Rs. 312.50 **(B)** Rs.375

(C) Rs.187.50 **(D)** Rs. 125

Q.22. The length of square is increased by 40% while breadth is decreased by 40% .The ratio of area of the resulting rectangle. So formed to that of the original square is :

(A) 25 : 21 **(B)** 21 : 25

(C) 16 : 15 **(D)** 15 :16

Q.23. The ratio of areas of two squares, one having double its diagonal than the other is :

(A) 2 : 1 **(B)** 3 :1

(C) 3 : 2 **(D)** 4 :1

Q.24. The area of rectangle is thrice that of square.Length of the rectangle is 40 cm and breadth of the rectangle is $\frac{3}{2}$ times that of the side of the square. The side of the square in cms is :

(A) 60 **(B)** 20

(C) 30 **(D)** 15

Q.25. The length and breadth of room are 1075 cm and 825 cm, respectively. The floor is to be paved with square tiles of the largest possible size. The size of each tile is ---

(A) 25 cm × 25 cm **(B)** 50 cm × 50 cm

(C) 20 cm × 20 cm **(D)** 30 cm × 30 cm

Q.26. Area of four walls of a room is 77m^2. The length and breadth of the room are 11.5 m and 3.5 m respectively. The height of the room is :

(A) 7.7 m **(B)** 2.5 m

(C) 6.77 m **(D)** 5.4 m

Q.27. The cost of papering four walls of a room is Rs. 48. Each one of length, breadth and height of another room as double that of the room. The cost of papering the walls of this new room is :

(A) Rs.96 **(B)** Rs.192

(C) Rs.384 **(D)** Rs.288

Q.28. The perimeters of both, a square and a rectangle are each equal to 48 m and the difference between their areas is 4 m^2. The breadth of the rectangle is :

(A) 10 m **(B)** 12 m

(C) 14 m **(D)** 20 m

Q.29. The area of a parallelogram is 72 cm^2 and its altitude is twice the corresponding base.Then the length of the base is :

(A) 3 cm **(B)** 6 cm

(C) 12 cm **(D)** 8 cm

Q.30. A tin sheet is in the form of rhombus whose side is 5 cm and one of its diagonals is 8 cm. Then the cost of painting the sheet at the rate of Rs. 3.50 per cm^2 on both of its sides is :

(A) Rs.84 **(B)** Rs. 140

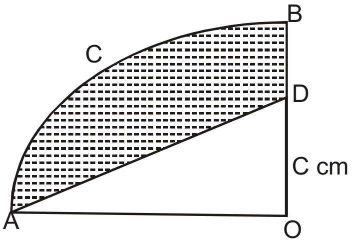
(C) Rs.168 **(D)** Rs. 68

Q.31. The altitude of an equilateral trangle of area $4\sqrt{3}$ cm^2 is :

(A) 6 cm **(B)** $\frac{1}{2}$ cms

(C) $\frac{\sqrt{3}}{4}$ cms **(D)** 3 cms 2 cm

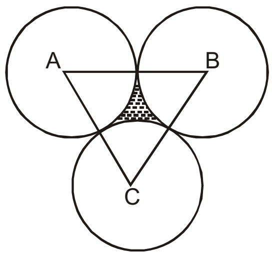
Q.32. In the adjoining figure, AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion.



(A) 35 cm^2 **(B)** 7.875 cm^2

(C) 9.625 cm^2 **(D)** 6.125 cm^2

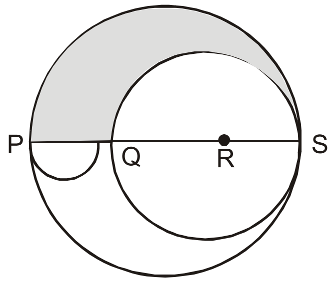
Q.33. Find the area of the shaded region if the radius of each of the circles is 1 cm.



(A) $2 - \frac{\pi}{3}$ **(B)** $\sqrt{3} - \pi$

(C) $\sqrt{3} - \frac{\pi}{2}$ **(D)** $\sqrt{3} - \pi/4$

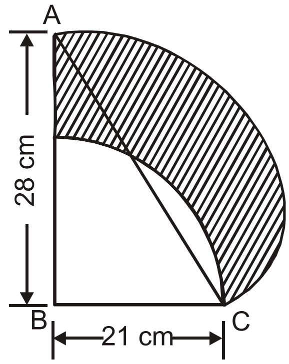
Q.34. PQRS is the diametre of a circle of radius 6 cm. The lengths PQ, QE and RS are equal. Semi-circles are drawn with PQ and QS as diametres as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the unshaded region.



(A) 1 : 2 **(B)** 25 : 121

(C) 5 : 18 **(D)** 5 : 13

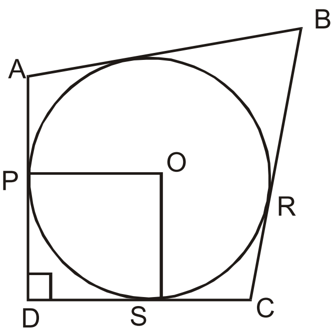
Q.35. In the figure, ABC is a right angled triangle with $\angle B = 90^0 = 21$ cm and AB = 28 cm. With AC as diametre of a semicircle and with BC as radius, a quarter cirlce is drawn. Find the area of the shaded portion correct to two decimal places.



(A) 429.6 cm^2 **(B)** 857.50 cm^2

(C) 214.37 cm^2 **(D)** 371.56 cm^2

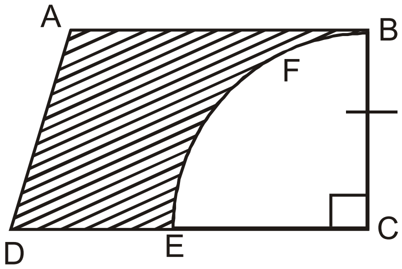
Q.36. In the adjoining figure, a circle is inscribed in the quadrilateral ABCD. Given that BC = 38 cm, AB = 27 cm and DC = 25 cm, and that AD is perpendicualr to DC. Find the maximum limit of the radius and the are of the circle.



(A) 10 cm; 226 cm^2 **(B)** 14 cm; 616 cm^2

(C) 14 cm; 216 cm^2 **(D)** 28 cm; 616 cm^2

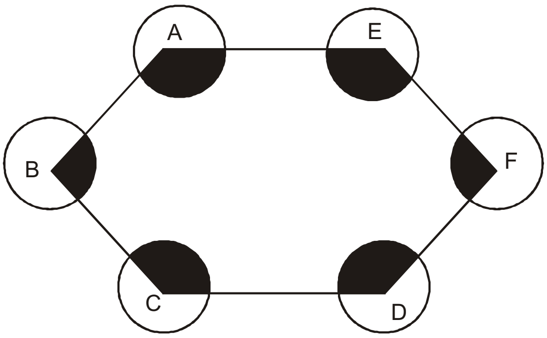
Q.37. From a piece of cardboard, in the shape of a trapezium ABCD and AB || DC and $\angle BCD = 90^0$, a quarter circle (BFEC) with C as its centre is removed Given AB = BC = 3.5 cm and DE = 2 cm, calcualte the area of the remaining piece of the cardboard.



(A) 3.325 cm^2 **(B)** 3.125 cm^2

(C) 6.125 cm^2 **(D)** 12.25 cm^2

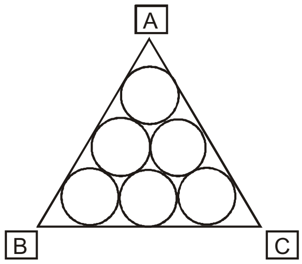
Q.38. Find the sum of the areas of the shaded sectors given that ABCDFE is any hexagon and all the circles are of same radius r with different vertices of the hexagon as their centres as shown in the figure.



(A) πr^2 **(B)** $2\pi r^2$

(C) $5\pi r^2/4$ **(B)** $3\pi r^2/2$

Q.39. The diagram shows six equal circles inscribed in equilateral triangle ABC. The circles touch externally among temselves and also touch the sides of the triangle. If the radius of each circle is R, area of the triangle is



(A) $(6+\pi\sqrt{3})R^1$ **(B)** $9R^2$

(C) $R^2(12+7\sqrt{3})$ **(D)** $R^2(9+6\sqrt{3})$

Q.40. If the sides of the triangle are in the ratio 4: 5: 6 and the in radius of the triangle is 6 cm, then the altitude of the triangle corresponding to the largest side as base is

(A) 15 cm. **(B)** 12 cm.

(C) 20 cm. **(D)** 16 cm.

Q.41. If the three sides of a triangle be $2n + 1$, $2n^2 + 2n + 1$ and $2n(n + 1)$, then the triangle is

(A) Equilateral **(B)** Right angled

(C) Isosceles **(D)** Obtuse angled

Q.42. A room is 36m long, 12m wide and 20m height. It has five windows, each 4.5m×4m, one door 9.5m×60m and one fire chimney 4m×4.5m. Find the expenditure of papering its walls at the rate of Rs.70 per meter, if the width of paper is 12m?

(A) Rs.3300 **(B)** Rs.4300

(C) Rs.5300 **(D)** Rs.6300

Notes

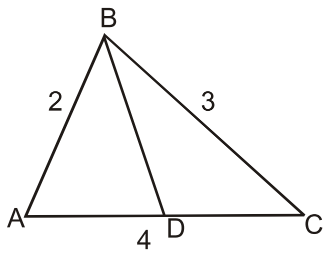
EXPLANATION

Q.1.(C) Area of a regular octagon

$= a^2\left(2+2\sqrt{2}\right)$ cm^2

$= 10^2\left(2+2\sqrt{2}\right) = 482.80$ cm^2

Q.2.(B)



$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{4^2+2^2-3^2}{2\times4\times2} = \dfrac{11}{16}$

InABD

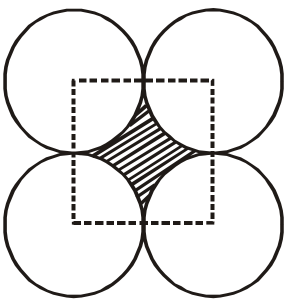
BD=a, b=2, d=2

$\cos A = \dfrac{b^2+d^2-a^2}{2bd}$

$\dfrac{11}{16} = \dfrac{2^2+2^2-a^2}{2\times2\times2}$

$a^2 = \dfrac{5}{2} = 2.5$ cm^2

Q.3.(C)



Shaded area = area of the square of 2 cm - area of the four quardant of circle of radius 1 cm.

$= 2^2 - 4\times\dfrac{1}{4}\times\pi\, 1^2$

$= (4 - \pi) = (4 - 3.14) = 0.86$ cm^2

Q.4.(D) $\dfrac{\text{Area of square}}{\text{Area of triangle}} = \dfrac{x^2}{\frac{\sqrt{3}}{4}x^2} = 4 : \sqrt{3}$

Q.5.(B) r = 1.5 m.

$\therefore 2\pi r = 9.42$

In 1 minute

$= 9.42 \times 28$

= 263.76 m. = .264 km

$\therefore$ time taken to cover 5.280 km

$= \dfrac{5.280}{.264} = 20$ minute

Q.6.(C) $s = \dfrac{a+b+c}{2} = \dfrac{3+4+5}{2} = 6$

Area of bigger triangle

$= \sqrt{6\times3\times2\times1}$

= 6 cm^2

$\therefore$ Area of smaller triangle

$= \dfrac{6}{4} = \dfrac{3}{2}$ cm^2

Q.7.(C) $\pi r^2 = (2\sqrt{\pi})^2 \Rightarrow \pi r^2 = 4\pi \Rightarrow r = 2$

$\therefore$ Diameter = 2×2 = 4 unit

Q.8.(A) Area of a circle $= \pi r^2$

$\pi r^2 = 308$

$\dfrac{22}{7}\times r^2 = 308$

$r^2 = \dfrac{308}{22}\times7$

$r^2 = \dfrac{28\times7}{2}$

$r^2 = 14\times7$

$r = 7\sqrt{2}$

Area of a equilteral triangle

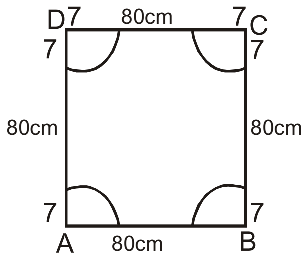
$\dfrac{\sqrt{3}}{4}\times a^2$

$\dfrac{\sqrt{3}}{4}\times\left(7\sqrt{2}\right)^2$

$\dfrac{\sqrt{3}\times49\times2}{4} = \dfrac{49\sqrt{3}}{2} = \dfrac{49\times1.732}{2}$

$= 49\times866 = 42.434$ cm^2

Q.9.(C)



Area of the square = 80 × 80

Area of the corner of circle part

$= \pi r^2 = \pi \times (7)^2$ cm^2

Hence Remaining Area = $80 \times 80 - \pi \times (7)^2$

$= 80 \times 80 - \frac{22}{7} \times (7)^2 = 6400 - 22 \times 7$

$= 6400 - 154 = 6246$ cm^2

Q.10.(A) Let length = l m. and breadth = b m.

According to question -

$l = \frac{3}{2}b$

$2l = 3b$

$lb = \frac{2}{3} \times 10000$

$l \times \frac{2l}{3} = \frac{2}{3} \times 10000$

$l = 100m$

Q.11.(C) Area of the floor = (400×300) cm^2

Area of one tile = (8×6) cm^2

Number of tiles $= \frac{400 \times 300}{8 \times 6} = 2500$

Q.12.(D) Then breadth = x cm and length = 2x cm.

Then (2x-5)(x+5) -x×2x=75

$2x^2+5x-25-2x^2 = 75$ or $5x = 50$ or $x = 10$.
Hence, length = 20 cm.

Q.13.(B) Area of the lawn = 2 × 5 (80+40+10)

= 1300 m^2

Q.14.(C) $x \times 4x = 3600 \Rightarrow x = 30$

Length = (4×30) m = 120 m.

Q.15.(B) Number of flagstaffs

$= \left[\left(\frac{36}{3}+1\right)+\left(\frac{36}{3}+1\right)+\left(\frac{21}{3}-1\right)+\left(\frac{21}{3}-1\right)\right]$

= 38.

Q.16.(B) Area of the carpet = (4.5 ×3.5) m^2.

Cost of the carpet = Rs. (80×4.5×3.5)

= Rs. 1260.

Q.17.(B) Original length = 15cm and breadth = $\frac{150}{15}$ = 10cm.

New area = $\left(150 \times \frac{4}{3}\right)$ cm^2 = 200 cm^2

New length

$= \frac{\text{New area}}{\text{Original breadth}} = \frac{200}{10}$ cm = 20 cm

New perimeter = 2(20+10) cm = 60 cm.

Q.18.(C) 2(x+y) = 4 x+y = 2 and xy = 0.75 = $\frac{3}{4}$.

$(x-y)^2 = (x+y)^2 - 4xy = \left(4 - 4 \times \frac{3}{4}\right) = 1$

So , (x-y) = 1 km.

Q.19.(A) Perimeter of the field = $\frac{3000}{7.50}$ = 400m .

$2(5x+3x) = 400 \Rightarrow x = 25$.

So, length = 125 m and breadth =75.

Difference between length and breadth

=(125-75) m = 50 m.

Q.20.(B) Area of cross roads

60 × 5 + 40 × 5 - 5 × 5 = 475 m^2

= Rs. $\left(475 \times \frac{60}{100}\right)$ = Rs. 285.

Q.21.(D) $3x \times 2x = 3750$ $x^2 = 625$ $x = 25$.

Length = 75 m and breadth = 50m

Perimeter = [2×(75+50)] m = 250 m

Cost of fencing = Rs. $\left(250 \times \frac{1}{2}\right)$ = Rs. 125

Q.22.(B) Let each side of the square be 100 m.

New area =(140×60) m^2

$\frac{\text{New area}}{\text{Original area}} = \frac{140 \times 60}{100 \times 100} = \frac{21}{25}$.

Q.23.(D) Let their diagonals be 2d and d respectively.

Ratio of their areas $= \frac{\frac{1}{2} \times (2d)^2}{\frac{1}{2} \times d^2} = \frac{4}{1}$

Q.24.(B) Length of the rectangle = 40 cm.

Let the side of the square be x cm.

Then, breadth of the rectangle $= \frac{3}{2}x$ cm.

$40 \times \frac{3}{2}x = 3x^2 \Rightarrow x = 20.$

Q.25.(A) H.C.F. of 1075 and 825 is 25.

size of each tile = 25 cm ×25 cm.

Q.26.(B) $2(115+35) \times h = 77 \Rightarrow h = \frac{77}{2\times 15} = 2.5$m.

Q.27.(B) Cost of papering $[2(l+b)\times h]m^2$=Rs.48

Cost of papering.

$= [\,2\,(2l + 2b) \times 2h\,]\ m^2$

i,e $4\,[2\,(\,l + b)) \times h]$

= 48 × 8 = 192 Rs.

Q.28.(A) Let the length of rectangle = x metres and its breadth = y m.

Also, let the side of the square be z metres.

Then $2(x+y) = 4z = 48 \Rightarrow x + y = 24$ and $z = 12$

Also, $z^2 - xy = 4 \Rightarrow xy = z^2 - 4 = 144 - 4 = 140.$

So, $(x-y)^2 = (x+y)^2 - 4xy = 576 - 560 = 16.$

x-y = 4 and x+y = 24. so, 2y = 20 or y=10 m

Q.29.(B) Let base = x cm and altitude = 2x cm.

$x \times 2x = 72 \Rightarrow x^2 = 36$ or x = 6 cm

Hence, base = 6 cm

Q.30.(C) Let another diagonal = 2x cm.

Then, $x^2 + 4^2 = 5^2 \Rightarrow x = \sqrt{(25-16)}$

= 3 cm.

Area of rombous $= \frac{1}{2}\times 8\times 6 = 24\ \ cm^2$

Area of both the sides

$= 2 \times 24\ cm^2 = 48\ cm^2.$

So, cost of painting the sheet

= Rs.(3.50×48)=Rs. 168.

Q.31.(A) Area $= \frac{\sqrt{3}}{4}\times 4^2 \times 3 = 12\sqrt{3}$

h = 6 cm.

Q.32.(D) Area of shaded portion= Area of quadrant-Area of triangle

$\Rightarrow \frac{\pi r^2}{4} - \frac{1}{2}\times 3.5\times 2 \equiv \frac{3.14\times(3.5)^2}{4} - 3.5$

$\Rightarrow 6.125\ cm^2$

Q.33.(C) ABC is an equilateral triangle with sides= C cm

Area of shaded portion= Area of equilateral triangle-Area of 3 quadrant

$\Rightarrow$ i.e $\frac{\sqrt{3}}{4}a^2 - 3\left(\pi r^2 \frac{\theta}{360^0}\right); \theta = 60^0$

($\therefore$ A, B, C is an equilateral triangle)

$\Rightarrow \frac{\sqrt{3}}{4}\times 2^2 - 3\left(3.14\times 1\times\frac{60}{360}\right)$

$\Rightarrow \sqrt{3} - \frac{3.14}{2} = \sqrt{3} - \frac{\pi}{2}$

Q.34.(D) PQ=QR=RS= $\frac{12}{3} = 4$ cm

Area of unshaded region $\Rightarrow \frac{\pi 6^2}{2} - \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi + 8\pi \Rightarrow 26\pi$

Area of shaded region

$\Rightarrow 18\pi - 8\pi = 10\pi$

Ratio $= \frac{10\pi}{26\pi} \Rightarrow \frac{5}{13} \Rightarrow 5:13$

Q.35.(A) Area of shaded portion= Area of ADC- Area of sector DC+ Area of ΔADB- sector BED

$\Rightarrow$ Area of ADC $= \pi\times(1.75)^2\times\frac{1}{2} = 481\,cm^2$

$\frac{\angle DBC}{\angle ABC} = \frac{21}{28} \Rightarrow \angle DBC = 67.5$ and $\angle DBA = 22.5$

$\Rightarrow$ Area of sector DC

$= \left(\pi\times 21^2\times\frac{67.5}{360}\right) - \left(\frac{1}{2}\times 21^2\times\sin 67.5\right) = 56\ cm^2$

$\Rightarrow$ Area of ADE

$= \left(\frac{1}{2}\times 28\times 21\right) - \left(204 + \frac{1}{2}\times 21^2\times\sin 22.5\right) = 5.6\ cm^2$

Thus are of shaded portion

=480-56+5.6=429.6 cm^2

Q.36.(D) Go through the option

Only option (d) is correct

Q.37.(C) Area of remaining cardboard = Area of trapezium- Area of quadrant

$\Rightarrow$ Area of trapezium

$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times (AB + DC) \times BC$

$\Rightarrow 4.5 \times 3.5 = 15.75 \text{ cm}^2$

Area of quadrant $= \frac{\pi r^2}{4} \Rightarrow \frac{3.14 \times 3.5 \times 3.5}{4}$ =9.6

$\Rightarrow$ Area of remaining cardboard $= 15.7 - 9.6 = 6.1 \text{ cm}^2$

Q.38.(B) Sum of interior angles of a hexagon=120^0

6 sector with the same radius r=2 full circle of same radius

So area of shaded region $\Rightarrow 2\pi r^2$

Q.39.(C)

Q.40.(A) Let the three sides of the triangle be 4x,

5x and 6x respectively.
Area of the triangle

$= \sqrt{\frac{15x}{2}\left(\frac{15x}{2} - 4x\right)\left(\frac{15x}{2} - 5x\right)\left(\frac{15x}{2} - 6x\right)}$

$= \sqrt{\frac{15x}{2}\left(\frac{7x}{2}\right)\left(\frac{5x}{2}\right)\left(\frac{3x}{2}\right)} = \frac{15}{4}\sqrt{7}x^2$

In radius of the triangle $= \frac{\frac{15}{4}\sqrt{7}x^2}{\frac{15}{2}x} = \frac{\sqrt{7}}{2}x$

$6 = \frac{\sqrt{7}}{2}x$

$x = \frac{12}{\sqrt{7}}$

Largest side $= 6 \times \frac{12}{\sqrt{7}} = \frac{72}{\sqrt{7}}$

We know that,

$\frac{1}{2} \times \frac{72}{\sqrt{7}} \times h = \frac{15}{4}\sqrt{7} \times \left(\frac{12}{\sqrt{7}}\right)^2$

h = 15 cm.

Q.41.(B) $(2n^2 + 2n + 1)^2$

$= 4n^4 + 4n^2 + 1 + 8n^3 + 4n + 4n^2$

$= 4n^4 + 8n^2 + 8n^3 + 4n + 1$

$(2n+1)^2 = 4n^2 + 1 + 4n$

$(2n^2 + 2n) = 4n^4 + 4n^2 + 8n^3$

$(2n^2 + 2n + 1)^2 = (2n + 1)^2 + (2n^2 + 2n)^2$

Triangle is a right angled triangle

Q.42.(D) Area of the wall = 2 × (l + b) × h

Area of the wall = 2× (36 + 12) × 20

= 1920 m^2

Total area of (window + door + chimney)

= (5×4.5×4+9.5×60+4×4.5)

= 840 m^2

Net area for papering = 1920 – 840

= 1080 m^2

Length of required paper = 1080/12

= 90 m

Hence the cost of papering = 90×70

= Rs.6300

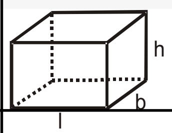
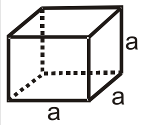
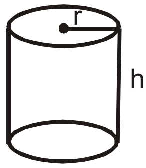
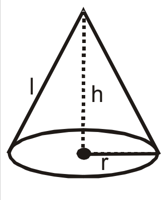
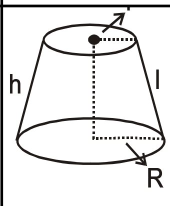
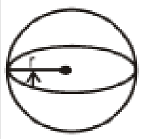
CHAPTER-19
MENSURATION-II
[SOLID FIGURES]

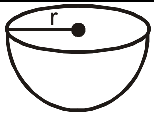
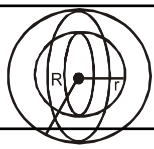
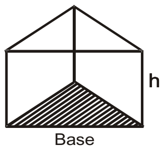
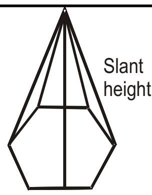
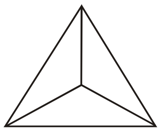
Scan the QR code to get video of this chapter.

Solid Figures : Solid figures are three dimensional objects what this mean is that solid figures have a length a width and a height (depth).

Ex. Computer, Laptop, phone etc.

In Mathematics there are many solid figures which is listed below alongwith formulas.

S. NO.	Name	Figure	Nomenclature	Volume	Curved/ Lateral surface area	Total surface area
1.	Cuboid		$l \rightarrow$ length $b \rightarrow$ breadth $h \rightarrow$ height	lbh	2(l+b)h	2(lb+bh+hl)
2.	Cube		$a \rightarrow$ edge/side	a^3	$4a^2$	$6a^2$
3.	Right circular cylinder		$r \rightarrow$ radius of base $h \rightarrow$ height of the cylinder	$\pi r^2 h$	$2\pi rh$	$2\pi r(h+r)$
4.	Right circular cone		$r \rightarrow$ radius $h \rightarrow$ height $l \rightarrow$ slant height $l = \sqrt{r^2+h^2}$	$\frac{1}{3}\pi r^2 h$	πrl	$\pi r(l+r)$
5.	Frustum of a cone			$\frac{\pi}{3}h(r^2+Rr+R^2)$	$\pi(r+R)l$	lateral surface area + $\pi[R^2+r^2]$
6.	Sphere		$r \rightarrow$ radius	$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$

S. NO.	Name	Figure	Namenclature	Volume	Curved/ Lateral surface area	Total surface area
7.	Hemisphere		r→ radius	$\frac{2}{3}\pi r^3$	$4\pi r^2$	$3\pi r^2$ $^4\pi[R^3 + r^3]$
8.	Spherical shell		r→inner radius R→outer radius	$\frac{4}{3}\pi[R^3 - r^3]$	$2\pi r^2$	
9.	Right triangular prism			area of base × height	perimeter of base × height	lateral surface area + 2(area of base)
10.	Right pyramid			$\frac{1}{3}$×area of base × height	$\frac{1}{2}$×perimeter of base × slant height	lateral surface area + area of base
11.	Tetrahedron		$(h) = \frac{1}{3} \times \sqrt{6} \times \text{side}$ $r = \frac{1}{12} \times \sqrt{6} \times \text{side}$ $R = \frac{1}{4} \times \sqrt{6} \times \text{side}$ (Where h height, r inradius R circumradius)	$\frac{1}{12} \times \sqrt{2} \times (\text{side})^3$		$\sqrt{3}(\text{side})^2$

EXAMPLES

Ex.1. A square of side 2 m is cut from the each corner of a rectangular sheet and remaining sheet is converted into an open box. Find the inner volume of the box.

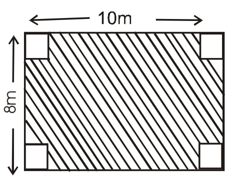
(A) 48 m^3 **(B)** 24 m^3 **(C)** 96 m^3 **(D)** 16m^3

Sol.(A) It is clear from given figure that a rectangular sheet of length 10m and 8m wide is folded and a box is formed, then

length of box = 10-4=6m

breadth of box = 8-4=4m

height of box = 2 m



Volume of box = length x breadth x height

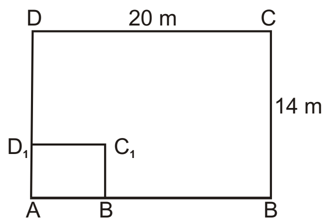
= 6m x 4m x 2m = 48m^3.

Ex.2. A rectangular field is 20 m long and 14 m broad. From its one corner a pit of 6m long, 3m wide and 2.5 m deep is dug and earth so obtained is spread evenly in the remaining part of the field. Find the raise in the level of field.

(A) 17.15 cm **(B)** 17.20 cm

(C) 17.18 cm **(D)** 15.18 cm

Sol.(C) Let ABCD is a rectangular field. $AB_1C_1D_1$is a pit in its one corner.



Volume of earth on digging

= 6m x 3m x 2.5m

= $45m^3$

Let raise in the level of field be 'h' m.

Area of field = l x b= 20m x 14m

= $280m^2$

Area of part of field in which pit is dug

= l x b

= 6m x 3m

= $18m^2$

Area of remaining part of field

= $280m^2 - 18m^2$

= $262m^2$

Volume of earth = Area of base x raise in the level

$45m^3 = 262\ m^2 \times h$

= $\frac{45m^3}{262m^2} = h$

= 0.1718m = h

= 17.18cm = h

Thus, raise in the level of field is 17.18 cm.

Ex.3. The length, breadth and height of a room are 7m, 6.5m and 4m respectively. It has a door 3m x 1.4m and three windows each of 2m x 1m. Find the cost of colouring the walls of room at the rate of Rs. 5.25 per square metre.

(A) Rs. 500.50 **(B)** Rs. 513.50

(C) Rs. 715.50 **(D)** Rs. 315.50

Sol.(B) Area to be coloured = Area of four walls -(Area of one door +Area of three windows)

Area of 4 walls of room = 2(L+B) x H

= 2(7m + 6.5) x 4m

= 2 x13.5m x 4m = $108m^2$

Area of door = 3m x 1.4m = $4.2m^2$

Area of three windows = 3 x 2m x 1m

= $6m^2$

Area of four walls of room to be coloured

= $[108- (4.2+6)]m^2 = 97.8m^2$

Cost of colouring the four walls of room

= Rs. 5.25 x 97.8 = Rs. 513.50

Ex.4. Area of bases of two solid cuboids of equal heights are $160cm^2$ and $230cm^2$. if volume of second cuboid is $2070cm^3$, then find the volume of first cuboid.

(A) $1440\ cm^3$ **(B)** $1240\ cm^3$

(C) $1240\ cm^3$ **(D)** $1456\ cm^3$

Sol.(A) Let height of cuboid be h.

$\therefore$ Volume of IInd cuboid = area of base × height

$\therefore$ 2070= 230 × h

$\therefore \frac{2070}{230} = h$

9 = h

Hence. volume of Ist cuboid

= Area of base × height

= $160\ cm^2 \times 9cm$

= $1440\ cm^3$

Ex.5. Edges of three metallic cubes are 2, 12 and 16 decimetres. The three cubes are melted and a new cube is formed. Then find the diagonal of new cube is-

(A) 324 dcm **(B)** $18\sqrt{3}$ dcm

(C) 18 dcm **(D)** $6\sqrt{3}$ dcm

Sol.(B) Volume of New cube = Sum of volume of three smaller cubes

= $2^3 + 12^3 + 16^3$

= 8 + 1728 + 4096

= 5832

Edge of new cube = $\sqrt[3]{5832}$

= 18 dm.

Diagonal = $a\sqrt{3} = 18\sqrt{3}$ dcm

Ex.6. It rained 10 cm in a rectangular field of 60 m long and 50 m broad. Find the water collected in the field in litres?

(A) 3000 liter **(B)** 300000 liter

(C) 30 liter **(D)** 300 liter

Sol.(B) Volume of water = l × b × h

= 60 × 50 × 0.1

= $300\ m^3$

So, volume of water (in litres) = 300×1000

= 300000 litres

Ex.7. Find the volume of a box whose area adjacent surfaces are $40m^2$,$20m^2$ and $8m^2$ respectively.

(A) $68\ m^3$ **(B)** $1600\ m^3$

(C) $4552\ m^3$ **(D)** $80\ m^3$

Sol.(D) Volume of box

$= \sqrt{\text{product of area of adjacent surfaces}}$

$= \sqrt{40 \times 20 \times 8}$

$= \sqrt{6400}$

$= 80\ m^3$

Ex.8. A rectangular water tank 1m long, 0.6m wide and 0.5m deep is filled through a tap whose area of cross section is 5 cm^2. Find the time taken to fill it completely if rate of flow of water is 25 metre per minute.

(A) 20 min **(B)** 24 min
(C) 30 min **(D)** 36 min

Sol.(B) Volume of tank = l × b × h = 1 × .6 × .5

= 0.30 m^3

Volume of water flown per min = 0.0005×25

= 0.0125 m^3

Time to fill the tank $= \frac{0.3000}{1.25} = 24$ min

Ex.9. The area of base of a right circular cylinder is 154 cm^2 and height is 10 cm. Find its volume.

(A) 1540 cm^3 **(B)** 616 cm^3
(C) 340 cm^3 **(D)** 840 cm^3

Sol.(A) Given, Area of base =154 cm^2 and height = 10cm.

We know, Volume of right circular cylinder = $\pi r^2 h$

= area of base × height

= 154 × 10 = 1540 cm^3.

Ex.10. The radii of two right circular cylinders are in the ratio 2:3 and their heights are in the ratio 5:4. Find the ratio of their curved surface areas-

(A) 6 :5 **(B)** 5 : 6
(C) 10 : 6 **(D)** 6 : 10

Sol.(B) Let the radii of the two cylinders be 2r and 3r respectively and their heights be 5h and 4h

$$\frac{\text{Curved surface area of 1st cylinder}(S_1)}{\text{Curved surface area of 2st cylinder}(S_2)} = \frac{2\pi \text{ x } 2r \text{ x } 5h}{2\pi \text{ x } 3r \text{ x } 4h}$$

i.e, $\frac{S_1}{S_2} = \frac{5}{6}$

or $S_1 : S_2 = 5:6$

Ex.11. The ratio of radii of two cylinders is 2:3 and the ratio of their heights is 5:4. Find the ratio of their volumes.

(A) 9 :5 **(B)** 5 : 6
(C) 10 : 6 **(D)** 5 : 9

Sol.(D) Let the radii of the two cylinders be 2r and 3r respectively and their heights be 5h and 4h

$$\frac{\text{Volume of 1st cylinder}(V_1)}{\text{Volume of 2st cylinder}(V_2)} = \frac{\pi \text{ x}(2r)^2 \text{ x } 5h}{\pi \text{ x}(3r)^2 \text{ x } 4h}$$

i.e, $\frac{V_1}{V_2} = \frac{5}{9}$

or $V_1 : V_2 = 5:9$

Ex.12. The area of the base of a right circular cylinder is 25π cm^2. Find the diameter of the base.

(A) 5 cm **(B)** 10 cm
(C) 4 cm **(D)** 8 cm

Sol.(B) Given, Base area of cylinder =25π

Let the radius of the base = r

Now, area of base of cylinde = πr^2

$25\pi = \pi r^2$

$r^2 = 25$

$r = \sqrt{25}$ =5cm.

Hence, the diameter of base of cylinder =2×5 = 10 cm.

Ex.13. The outer and inner diameters of a hollow cylindrical pipe are 10 cm and 6 cm. If its lenght be 21 cm. Find the total surface area.

(A) 154π cm^2. **(B)** 368π cm^2.
(C) 169π cm^2. **(D)** 472π cm^2.

Sol.(B) Given length of pipe = 21cm, $r_1 = \frac{10}{2} = 5$ cm and $r_2 = \frac{6}{2} = 3$ cm.

∴ Total surface area of pipe

$= 2\pi r_1 h + 2\pi r_2 h + 2\pi(r_1^2 - r_2^2)$

$= 2\pi(r_1 + r_2)h + 2\pi(r_1 + r_2)(r_1 - r_2)$

$= 2\pi(r_1 + r_2)[h + r_1 - r_2]$

$= 2\pi[5 + 3][21 + 5 - 3]$

$= 2\pi \text{x} 8 \text{x} 23$

$= 368\pi$ cm^2.

Ex.14. A roller 2 m long and 0.7 m in diameter makes 60 revolutions in a minute on a road. The area rolled by it is:

(A) 264 m^2 **(B)** 132 sq.cm
(C) 66 m^2. **(D)** none of these.

Sol.(A) Area of rolled by roller

$= 2\pi rh \times 60$

$= 2 \times \frac{22}{7} \times \frac{0.7}{2} \times 2 \times 60$

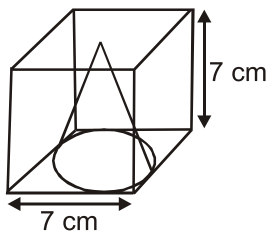
= 44 × 6 = 264 sq. meter

Ex.15. A greatest right circular cone is formed by cutting a wooden cubical piece of 7cm side. Find its volume.

(A) 98.3 cm^3 **(B)** 90.3 cm^3

(C) 89.83 cm^3. **(D)** 61.3 cm^3

Sol.(C) largest cone is cut from a cube of edge 7cm.



$\therefore$ Height and diameter of cone will be 7 cm each.

i.e, h= 7 cm, r = $\frac{7}{2}$cm

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7$ = 89.83 cm^3.

Ex.16. A conical military tent is 5 metres high and the diameter of the base is 24 metres. Find the cost of canvas used in making this tent at the rate of Rs. 14 per square metre.

(A) Rs. 6864 **(B)** Rs. 6468

(C) Rs. 6564 **(D)** Rs. 6360

Sol.(A) **Given,**

Height of tent (h)= 5m

Diameter of base= 24m

Radius of base (r) = 12m.

Slant height of tent (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144}$

$= \sqrt{169} = 13$cm.

Area of the slant surface = πrl

$= \frac{22}{7}$ x 12 x13 = $\frac{3432}{7}$ m^2.

$\therefore$ Cost of canvas= $\frac{3432}{7}$ x Rs. 14

= Rs. 6864.

Ex.17. There are two right circular cones of equal height. The radius of base of one cone is half of the radius of the other . Find the ratio of their volumes.

(A) 2 :1 **(B)** 4 :1

(C) 1 :4 **(D)** 1 : 2

Sol.(C) here heights of each cone are equal, let the height of each cone be 'h' and radii of the cones be $r_1$and r_2 respectively.

Given that $r_1 = \frac{1}{2} r_2$

i.e., $\frac{r_1}{r_2} = \frac{1}{2}$

Now, (V_1) volume of Ist cone $= \frac{1}{3}\pi r_1^2 h$

and (V_2) volume of IInd cone = $\frac{1}{3}\pi r_2^2 h$

$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

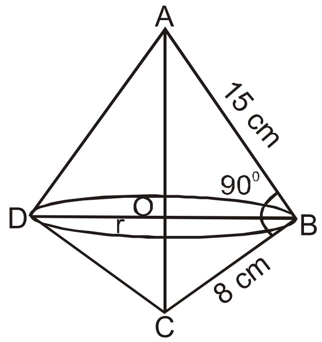
Hence' ratio their volume = 1: 4

Ex.18. The sides of a right angled triangle are 15 cm and 8 cm. If the triangle is revolved round the hypotenuse, find the volume.

(A) $\frac{5800}{17}\pi$ cm^3 **(B)** $\frac{6800}{17}\pi$ cm^3

(C) $\frac{7800}{17}\pi$ cm^3 **(D)** $\frac{4800}{17}\pi$ cm^3

Sol.(D) Let ABC be the right angled triangle in which BC = 8 cm and AB = 15 Cm.



By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2 = (15)^2 + (8)^2$

= 225 + 64 =289.

AC $= \sqrt{289}$ =17 cm.

Area of Δ ABC= $\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 17 \times OB$

Also, the area of Δ ABC = $\frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 15 \times 8 = 60$ sq. cm.

$\therefore \frac{1}{2} \times 17 \times OB = 60$

or OB $= \frac{60 \times 2}{17} = \frac{120}{17}$ Cm.

Volume of the solid = Volume of cone (A, BD) + volume of cone (C,BD)

$= \frac{1}{3}\pi\ OB^2.\ AO + \frac{1}{3}\pi\ OB^2.CO$

$= \frac{1}{3}\pi\ OB^2(AO + CO) = \frac{1}{3}\pi\ OB^2\ .\ AC$

$= \frac{1}{3}\pi \times \left(\frac{120}{17}\right)^2 \times 17 = \frac{4800}{17}\pi\ cm^3.$

Ex.19. There are two cones. The curved surface area of one is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

(A) 4 : 1 **(B)** 2 : 1

(C) 1 : 2 **(D)** 1 : 4

Sol.(A) Given

$(C.S.A.)_1 : (C.S.A.)_2 = 2 : 1$

$\frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{2}{1}$

$\frac{r_1 \times l}{r_2 \times 2} = \frac{2}{1}$

$r_1 : r_2 = 4 : 1$

Ex.20. A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl ?

(A) 50 **(B)** 36

(C) 54 **(D)** 16

Sol.(C) Internal radius of hemispherical bowl= 9 cm

Volume of hemispherical bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} x \frac{22}{7} x (9)^3 cm^3.$

Also, given height of bottle = 4 cm

= 1.5 cm

Radius of bottle $= \frac{2}{3}$ cm = 1.5 cm

Volume of bottle

$\pi r^2 h = \frac{22}{7} x (1.5)^2\ x\ 4\ cm^3$

Number of botteles required

$= \frac{\text{volume of liquid in the bottel}}{\text{volume of 1 bottle}}$

$= \frac{\frac{2}{3} x \frac{22}{7} x (9)^3}{\frac{22}{7} x (1.5)^2 x 4} = \frac{486}{9} = 54.$

Hence, required number of bottles is 54.

Ex.21. The diameter of a copper sphere is 6 cm. The sphere is melted and is drawn into a long wire of uniform circular cross- section. If the length of the wire is 36 cm, find its radius. (Take π=3.14).

(A) 1 cm **(B)** 3 cm

(C) 2 cm **(D)** 4 cm

Sol.(A) Volume of sphere = Volume of long wire

$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\frac{4}{3} \times 3^3 = r_2^2 \times 36$

$4 \times 9 = r_2^2 \times 36$

$r_2^2 = 1$

$r_2 = 1$ cm

Ex.22. The largest sphere is curved out of a cube of side 7cm. Find the volume of the sphere.(Take π=3.14).

(A) $49\sqrt{3}$ cm³ **(B)** 343 cm³

(C) $539\sqrt{3}$ cm³ **(D)** $7\sqrt{3}$ cm³

Sol.(C) Volume of largest sphere $= \frac{4}{3}\pi r^3$

Here radius of sphere

$= \frac{1}{2} \times$ diagonal of cube

$= \frac{1}{2} \times 7\sqrt{3} = \frac{7\sqrt{3}}{2}$

Hence volume $= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7\sqrt{3}}{2}\right)^3$

$= 11 \times 49\sqrt{3} = 539\sqrt{3}\ cm^3$

Ex.23. A prism has the base a right angled triangle whose side adjacent to the right angles are 10 cm and 12 cm long. The height of the prism is 20 cm. The density of the material of the prism is- 6gm/cubic cm. The weight of the prism is-

(A) 6.4 kg **(B)** 7.2 kg

(C) 3.4 kg **(D)** 4.8 kg

Sol.(B) Area of the base of prism

$= \frac{1}{2} \times 10 \times 12 = 60$ cm^2

Volume of prism = Area of base × height

= 60×20 = 1200 cm^3

Weight of prism = volume × density

$= \frac{1200 \times 6}{1000} = 7.2$ kg

Ex.24. The base of right prism is an equilateral triangle. If the lateral is surface area and volume is 120 cm^2, $40\sqrt{3}$ cm^3 respectively. then the side of base of the prism is-

(A) 4 cm **(B)** 5 cm

(C) 7 cm **(D)** 40 cm

Sol.(A) Lateral surface area of prism

120 = 3 × side × height

side × height = $\frac{120}{3} = 40$ ......(i)

Volume of prism = Area of base × height

$40\sqrt{3} = \frac{\sqrt{3}}{4} \times (\text{side})^2 \times h$

$(\text{side})^2 \times h = 160$ cm^3.........(ii)

from equation (i) and (ii)

side = $\frac{160}{40} = 4$ cm.

Ex.25. If each edge of a regular tetrahedran is 3 cm, then the volume is-

(A) $\frac{9\sqrt{2}}{4}$ cm^3 **(B)** $27\sqrt{3}$ cm^3

(C) $\frac{4}{9}\sqrt{2}$ cm^3 **(D)** $9\sqrt{3}$ cm^3

Sol.(A) Area of the tetrahedran

$= \frac{1}{3} \times \text{Area of base} \times \text{height}$

Area of base $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 3 \times 3$

$= \frac{9\sqrt{3}}{4} \text{cm}^2$

Now length of the perpendicular in the equilateral triangle

$= \sqrt{3^2 - \left(\frac{3}{2}\right)^2}$

$= \sqrt{9 - \frac{9}{4}} = \frac{3\sqrt{3}}{2}$ cm.

height $= \sqrt{\left(\frac{3\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{27}{4} - \frac{3}{4}}$

= 6 cm

Required volume $= \frac{1}{3} \times \frac{9\sqrt{3}}{4} \times \sqrt{6}$

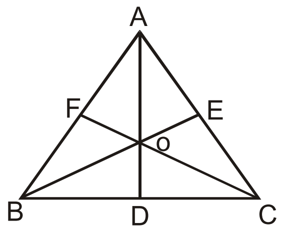
$= \frac{9\sqrt{2}}{4}$ cm^3

Ex.26. The perimeter of the triangular base of a right prism is 15 cm. and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm^3 then the height of the prism is-

(A) 6 cm **(B)** 7.5 cm

(C) 10 cm **(D)** 12 cm

Sol.(D)



Radius of the incircle = OE = OD = OF

= 3 cm.

Area of triangular base

$= \frac{1}{2} AB \times OF + \frac{1}{2} \times BC \times OD + \frac{1}{2} AC \times OE$

$= \frac{1}{2} \times 3 \times (AB + BC + AC)$

$= \frac{1}{2} \times 3 \times 15 = \frac{45}{2}$ cm^2

Volume of the prism

270 = Area of base × height

$270 = \frac{45}{2} \times \text{height}$

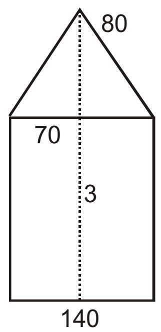
Height $= \frac{270 \times 2}{45} = 12$ cm

Ex.27. A circus tent is cylindrical to a height of 3 m and conical above it. If the diameter of the base is 140 m and slant height of the conical portion is 80 m. The length of the canvas 2 m wide required to make the tent is

(A) 9260 m **(B)** 9460 m

(C) 9560 m **(D)** 9660 m

Sol.(B)



Total curved surface area of tent

$= \pi rl + 2\pi rh$

$\frac{22}{7} \times 70 \times 80 + 2 \times \frac{22}{7} \times 70 \times 3$

$(22 \times 10 \times 80 + 22 \times 10 \times 6)$

Let required length be x m.

$\Rightarrow x \times 2 = 22 \times 10\ (80 + 6)$

$\Rightarrow x = 11 \times 10 \times 86 = 9460$ m

Ex.28. Find the curved surface area of a cone whose height and radius of base are 4 cm and 3 cm respectively.

(A) 12π cm^2 **(B)** 14π cm^2

(C) 21π cm^2 **(D)** 15π cm^2

Sol.(D) Here, h=4 cm and r =3 cm.

$\therefore l = \sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$cm.

Now, curved surface area of cone = πrl

$= \pi \times 3 \times 5$

$= 15\pi$ cm^2.

Ex.29. A right pyramid 6 m height has a square base of which the diagonal is $\sqrt{1152}$ m. Volume of the pyramid is-

(A) 144 m^3 **(B)** 288 m^3

(C) 576 m^3 **(D)** 1152 m^3

Sol.(D) Area of the base of pyramid

$= \frac{1}{2} \times (\text{diagonal})^2$

$= \frac{1}{2} \times 1152 = 576$

Volume of pyramid

$= \frac{1}{3} \times$ area of base $\times$ height

$= \frac{1}{3} \times 576 \times 6 = 1152$ m^3

Ex.30. The height of the right pyramid whose area of base is 30 m^2 and the volume is 500 m^3, is-

(A) 50 m **(B)** 60 m

(C) 40 m **(D)** 20 m

Sol.(A) Volume of the pyramid

$\frac{1}{3} \times$ area of base $\times$ height

$\Rightarrow 500 = \frac{1}{3} \times 30 \times h$

$h = \frac{500}{10} = 50$ m.

Notes

EXERCISE

Q.1. If the height of a prism is 10 cm. The base is a right angle triangle with sides of length 5 cm and 12 cm, then find the volume of the prism?
(A) 300 cm^2 **(B)** 240 cm^2
(C) 280 cm^2 **(D)** 320 cm^2

Q.2. A right angle triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. What is the volume of the solid so obtained?
(A) 50 π cm^2 **(B)** 100 π cm^2
(C) 125 π cm^2 **(D)** 150 π cm^2

Q.3. A cylindrical can of internal diameter 24 cm contains water. A solid sphere of radius 6 cm is completely immersed in water in the cylinder. The water level increases by-
(A) 0.25 cm **(B)** 0.5 cm
(C) 2 cm **(D)** 3 cm

Q.4. Three cubes of metal whose edges are in the ratio 3:4:5 are melted down in to a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edge of smallest of the three given cubes.
(A) 3 cm **(B)** 6 cm
(C) 8 cm **(D)** 5 cm

Q.5. The volume of metallic cylindrical pipe is 748 cm^3. Its length is 14 cm and its external radius is 9 cm. Find its thickness.
(A) 0.5 cm **(B)** 1 cm
(C) 1.5 cm **(D)** 1.25 cm

Q.6. A cardboard sheet in the form of a circular sector of radius 30 cm. and central angle 144^o is folded to make a cone. What is the radius of the cone?
(A) 12 cm. **(B)** 18 cm.
(C) 21 cm. **(D)** None of the above

Q.7. A large solid metallic cylinder whose radius and height are equal to each other is to be melted and 48 identical solid balls are to be recast from the liquid metal so formed. What is th ratio of the radius of a ball to the radius of the cylinder?
(A) 1:16 **(B)** 1:12
(C) 1:8 **(D)** 1:4

Q.8. Water flows in to a tank 15m×10 m through a rectangular pipe 2m×1.5 m at 20 km/hr.In how much time will the water level in the tank rise by 3m.
(A) 18 sec **(B)** 21 sec
(C) 24 sec **(D)** 27 sec

Q.9. The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with a sheet of some metal at Rs. 8 and Rs. 10 per square metre is Rs. 234. What are the length, breadth and height of the box respectively?
(A) 2m, 3m, 4m **(B)** 3m, 4.5m, 6m
(C) 4m, 6m, 8m **(D)** 5m, 7.5m, 10m

Q.10. If C_1 is a right circular cone with base radius r_1 cm and height h_1 cm, and C_2 is a right circular cylinder with base radius r_2 cm and height h_2 cm, and if $r_1 : r_2$ = 1: n (where n is a positive integer) and their volumes are equal, then which one of the following is correct?
(A) $h_1 = 3nh_2$ **(B)** $h_1 = 3n^2 h_2$
(C) $h_1 = 3h_2$ **(D)** $h_1 = n^2 h_2$

Q.11. Half of a large cylindrical tank open at the top is filled with water and identical heavy spherical balls are to be dropped into the tank without spilling water out. If the radius and the height of the tank are equal and each is four times the radius of a ball, what is the maximum number of balls that can be dropped?
(A) 12 **(B)** 24
(C) 36 **(D)** 48

Q.12. The total surface area of a solid right circular cylinder is twice that of a solid sphere. If they have the same radii, the ratio of the volume of the cylinder to that of the the sphere is given by.
(A) 9:4 **(B)** 2:1
(C) 3:1 **(D)** 4:9

Q.13. The curved surface area of a cylinderical pillar is 264 m^2. and its volume is 924 m^3. The ratio of its diameter to height is -
(A) 3:7 **(B)** 7:3
(C) 6:7 **(D)** 7:6

Q.14. The radius of the base of a conical tent is 16 meters. If $427\frac{3}{7}$ m^2. canvas is required to construct the tent. The slant height of the tent is-
(A) 17 m. **(B)** 15 m.
(C) 19 m. **(D)** 8.5 m.

Q.15. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m. 25 cm. area of the wet surface is:
(A) 49 m^2 **(B)** 50 m^2
(C) 53.5 m^2 **(D)** 55 m^2

Q.16. The volume of a rectangular block of stone is 10368 dm^3. Its dimensions are in the ratio of 3 : 2 : 1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be :
(A) Rs. 31.50 **(B)** Rs. 31.68
(C) Rs. 63 **(D)** Rs. 63.36

Q.17. The length, height and width of a wall is 6 m., 5 m and 50 m. respectively. Find the number of bricks whose dimension are 25 cm. × 12.5 cm. × 7.5 cm. If 5 % part of wall is filled by mortar.
(A) 3040 **(B)** 5740
(C) 6080 **(D)** 8120

Q.18. A rectangular water tank is 80 m x 40 m. Water flows into it through a pipe 40 cm sq. at a speed of 10 km/hr. By how much, the water level will rise in the tank in half an hour?

(A) $\frac{3}{2}$ cm **(B)** $\frac{4}{9}$ cm

(C) $\frac{5}{8}$ cm **(D)** None of these

Q.19. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls, the volume of the hall is :

(A) 720 m^3. **(B)** 900 m^3.

(C) 1200 m^3. **(D)** 1800 m^3.

Q.20. The sum of the length, breadth and depth of a cuboid is 19 cm and its one diagonal is $5\sqrt{5}$ cm. Its surface area is :

(A) 125 cm^2 **(B)** 236 cm^2

(C) 361 cm^2 **(D)** 486 cm^2

Q.21. A metallic sheet is of rectangular shape with dimensions 48 m x 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m^3) is :

(A) 4830 **(B)** 5120

(C) 6420 **(D)** 8960

Q.22. If the areas of the three adjacent faces of a cuboidal box are 120 cm^2, 72 cm^2 and 60 cm^2 respectively, then find the volume of the box.

(A) 720 cm^3 **(B)** 864 cm^3

(C) 7200 cm^3 **(D)** $(72)^2$ cm^3

Q.23. The height of a right circular cylinder is 14 cm and its curved surface area is 704 sq. cm. Then its volume is :

(A) 1408 cm^3 **(B)** 2816 cm^3

(C) 5632 cm^3 **(D)** 9856 cm^3

Q.24. Three spheres are kept inside a cone, as given in the figure. Spheres are touching both the slant sides of the cone and the adjacent spheres. If the radius of the 1st sphere and the 3rd sphere are 5 units and 20 units respectively, find the radius of the 2nd sphere.

(A) 5 unit **(B)** 10 unit

(C) 15 unit **(D)** 12.5 unit

Q.25. A cube is inscribed in a hemisphere of radius R such that four of its vertices lies on the base of hemisphere the other four touch the hemispherical surface of the sphere. Find the volume of cube.

(A) $0.25R^3$ **(B)** $0.67\sqrt{\frac{2}{3}}R^3$

(C) $0.5\sqrt{\frac{2}{3}}R^3$ **(D)** $0.67R^3$

Q.26. The biggest possible cube is taken out of a right solid cylinder of radius 15 cm and height 20 cm, respectively. What will be the volume (in cm^3) of the cube?

(A) $375\sqrt{2}$ **(B)** 375

(C) 8000 **(D)** 500

Q.27. A right prism has a square base 4cm and the height is 9cm . The prism is cut in three parts of equal heights by two planes to its base What is the ratio of the volume of the top, middle and the bottom part respectively?

(A) 1:8:27 **(B)** 1:7:19

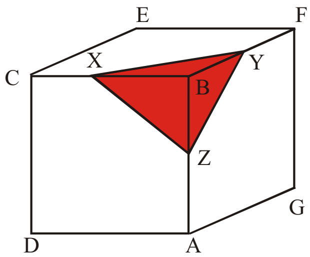
(C) 4:25:36 **(D)** 1:8:9

Q.28. A radius base of a hollow cone is 8cm and its height is 15 cm. A sphere of largest radius is put inside the cone. What is the ratio of radius of base of cone to the radius of sphere?

(A) 3 : 5 **(B)** 5 : 3

(C) 2 : 3 **(D)** 3 : 2

Q.29. A right triangular pyramid XYZB is cut from cube as shown in figure. The side of cube is 16 cm. X, Y and Z are mid points of the edges of the cube. What is the total surface area (in cm^2) of the pyramid?



(A) 48[(√3) + 1] **(B)** 24[4 + (√3)]

(C) 28[6 + (√3)] **(D)** 32[3 + (√3)]

Q.30. A plane divides a right circular cone into two parts if equal volume. If the plane is parallel to the base, then the ratio, in which the height of the cone is divided, is:-

(A) $1:\left(\sqrt{3}-1\right)$ **(B)** $1:\left(1-\sqrt[3]{2}\right)$

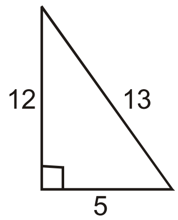
(C) $1:\left(\sqrt[3]{2}-1\right)$ **(D)** $\sqrt{3}:\left(\sqrt[3]{2}-1\right)$

EXPLANATION

Q.1.(A) Area of base triangle = $\frac{1}{2} \times 5 \times 12 = 30\ cm^2$

$\therefore$ Volume of prism = 30×10 = 300 cm^3

Q.2.(B)



Required volume = $\frac{1}{3}\pi 5^2 \times 12$

= $100\pi\ cm^3$

Q.3.(C) Let water level increase by x cm

$\therefore$ Volume of cylindrical can

= $\pi 12^2 \times x = 144x\pi$

Volume of sphere = $\frac{4}{3}\pi 6^3 = 288$

$\therefore 144x\pi = 288\pi$

$\Rightarrow$ x = 2 cm

Q.4.(B) Diagonal of the new cube = $12\sqrt{3}$

edge of new cube = 12 cm

volume = $12^3 = 1728\ cm^3$

$\Rightarrow (3x)^3 + (4x)^3 + (5x)^3 = 1728$

$\Rightarrow 216x^3 = 1728 \Rightarrow x^3 = 8$

$\therefore$ x = 2 cm

Edge of the smallest cube = 2×3 = 6 cm

Q.5.(B) $\pi(R^2 - r^2)\,l = 748$

$\Rightarrow \frac{22}{7}(9^2 - r^2) \times 14 = 748$

$\Rightarrow 9^2 - r^2 = 748 \times \frac{7}{22} \times \frac{1}{14}$

$\Rightarrow r^2 = 81 - 17 = 64$

r = 8

$\therefore$ Thickness of pipe = R - r = 9 - 8 = 1 cm

Q.6.(A) Length of arc = $\frac{2\pi r \times 144^\circ}{360^\circ}$

= $\frac{2\pi r \times 144^\circ}{360^\circ} = \frac{528}{7}$ cm.

Radius of cone = $\frac{528 \times 7}{2 \times 22 \times 7}$ = 12 cm.

Q.7.(D) Volume of cylinder = $\pi r_1^2 h = \pi r_1^3$

Volume of solid ball = $\frac{4}{3}\pi r_2^3$

According to question,

$\pi r_1^3 = \frac{4}{3}\pi r_2^3 \times 48 \Rightarrow r_1^3 = \frac{4}{3} r_2^3 \times 48$

$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{4 \times 4 \times 4}{1} \Rightarrow \frac{r_1}{r_2} = \frac{4}{1} \Rightarrow \frac{r_2}{r_1} = \frac{1}{4}$

Q.8.(D) $\therefore$ Level = 3m

$\therefore$ Volume = 15 × 10 × 3 = 450 m^3

speed of water = $20 \times \frac{1000}{3600} = \frac{50}{9}$ m/sec

$\Rightarrow$ Volume flow rate of water = $(2 \times 1.5) \times \frac{50}{9}$

= $\frac{50}{3}\ m^3/sec$

Time = $\frac{450}{\frac{50}{3}}$ = 27 sec.

Q.9.(B) Let the dimension of a rectangular box be 2x,3x, 4x

Total surface area of box

= 2(2x×3x+3x×4x+4x×2x)

= $2(6x^2 + 12x^2 + 8x^2) = 52x^2$

$\therefore 52x^2 \times 8 \sim 52x^2 \times 10 = 234$

$\Rightarrow x^2 = \frac{234}{104} \Rightarrow x = 1.5$

$\therefore$ dimensions are = 2 × 1.5, 3 × 1.5, 4 × 1.5

= 3,4.5,6

Q.10.(B) $\frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2 \Rightarrow r_1^2 h_1 = 3 r_2^2 h_2$

$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{3h_2}{h_2} \Rightarrow \frac{1}{n^2} = \frac{3h_2}{h_1} \Rightarrow \frac{1}{n^2} = \frac{3h_2}{h_1}$

$\Rightarrow h_1 = 3n^2 h_2$

Q.11.(B) Let number of balls

$\pi r^2 h = \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 \times n \Rightarrow n = 24$

Q.12.(C) $2\pi rh = 4\pi r^2 \times 2$

$\Rightarrow$ h = 4 r

$\therefore \dfrac{\pi r^2 h}{\frac{4}{3}\pi r^3} = \dfrac{3\times r^2\times 4r}{4r^3} = \dfrac{3}{1}$

Q.13.(B) $\dfrac{2\pi rh}{\pi r^2 h} = \dfrac{264}{924}$

$\Rightarrow r = \dfrac{924\times 2}{264}$

r = 7

$\therefore$ h = $\dfrac{264\times 7}{22\times 2\times 7}$

h = 6

$\therefore$ Required ratio = $\dfrac{2\times 7}{6}$ = 7:3

Q.14.(D) $\pi rl = \dfrac{22}{7}\times 16\times l$

$\Rightarrow 427\dfrac{3}{7} = \dfrac{22}{7}\times 16\times l$

$\Rightarrow$ 2992 = 352 × l

$\Rightarrow$ l = 8.5 m.

Q.15.(A) Area of the wet surface

= [2(lb+bh+lh)–lb]=2(bh+lh)+lb

= [2(4×1.25+6×1.25)+6×4]m² = 49 m².

Q.16.(D) Let the dimensions be 3x, 2x and x respectively. The,

$3x\times 2x\times x = 10368 \Rightarrow x^3 = \left(\dfrac{10368}{6}\right)$

= 1728 $\Rightarrow$ x =12.

So, the dimensions of the block are 36 dm, 24 dm, and 12 dm.

Surface area

= [2(36×24+24×12+36×12)]dm²

= [2×144(6+2+3)] dm² = 3168 dm².

$\therefore$ Cost of polishing = Rs. $\left(\dfrac{2\times 3168}{100}\right)$

= Rs. 63.36.

Q.17.(C) Volume of the bricks = 95% of volume of wall

$= \left(\dfrac{95}{100}\times 600\times 500\times 50\right)$ cm³.

Volume of 1 brick = (25×12.5×7.5) cm³.

$\therefore$ Number of bricks

$= \left(\dfrac{95}{100}\times \dfrac{600\times 500\times 50}{25\times 12.5\times 7.5}\right) = 6080$

Q.18.(C) Length of water column flown in 1 min.

$= \left(\dfrac{10\times 1000}{60}\right)m = \dfrac{500}{3}m.$

Volume flown per minute

$\left(\dfrac{500}{3}\times \dfrac{40}{100\times 100}\right)m^3 = \dfrac{2}{3}m^3$

Volume flown in half an hour

$= \left(\dfrac{2}{3}\times 30\right)m^3 = 20m^3$

$\therefore$ Rise in water level

$= \left(\dfrac{20}{40\times 80}\right)m = \left(\dfrac{1}{160}\times 100\right)cm = \dfrac{5}{8}cm.$

Q.19.(C) 2(15×12)=2h(15+12)

or h $= \dfrac{180}{27}m = \dfrac{20}{3}m.$

$\therefore$ Volume $= \left(15\times 12\times \dfrac{20}{3}\right)m^3 = 1200m^3$

Q.20.(B) (l+b+h) =19 and $\sqrt{l^2+b^2+h^2} = 5\sqrt{5}$ and so (l² + b²+h²) = 125.

Now, (l+b+h)² = 19² $\Rightarrow$ (I²+b²+h²)+2(lb+bh+lh) = 361

$\Rightarrow$ 2 (Ib+bh+Ih) = (361–125)=236.

$\therefore$ Surface area = 236 cm².

Q.21.(B) Clearly, l =(48–16)m = 32m, b = (36–16)m=20m, h=8m

$\therefore$ Volume of the box = (32×20×8)m³ = 5120 m³.

Q.22.(A) Let the length, breadth and height of the box be l, b and h respectively, Then

Volume = lbh

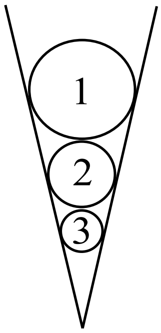
$= \sqrt{(lbh)^2} = \sqrt{lb\times bh\times lh} = \sqrt{120\times 72\times 60} = 720$ cm³.

Q.23.(B) $\dfrac{2\pi rh}{h} = \dfrac{704}{14} \Rightarrow 2\pi r = \dfrac{704}{14}$

$\therefore r = \left(\dfrac{704}{14}\times \dfrac{1}{2}\times \dfrac{7}{22}\right) = 8\,cm.$

$\therefore$ Volume $= \left(\dfrac{22}{7}\times 8\times 8\times 14\right)$ cm³= 2816 cm³.

Q.24.(B)

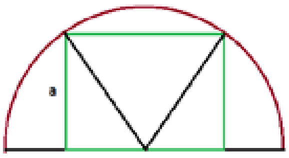


$$\frac{r_1}{r_2}=\frac{r_2}{r_3}$$

$$\frac{5}{r_2}=\frac{r_2}{20}$$

r_2 = 10unit

Q.25.(B)



Diagonal of cube = $a\sqrt{2}$

$$R^2=a^2+\frac{a^2}{2}$$

$$a=R\sqrt{\frac{2}{3}}$$

Volume of Cube = $0.67\sqrt{\frac{2}{3}}R^3$

Q.26.(C) a = h = 20cm

V = 8000

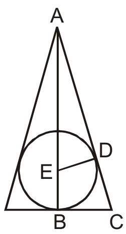
Q.27.(B) As we know from similarity

$a_1:a_2:a_3=h_1:h_2:h_3$ =1:2:3

Hence required answer

$=1^3 : (2^3-1^3) : (3^3-(1^3+2^3))$ =1:7:19

Q.28.(B)



Here from similarity

Required ratio= $\frac{r}{15-r}=\frac{8}{17}$

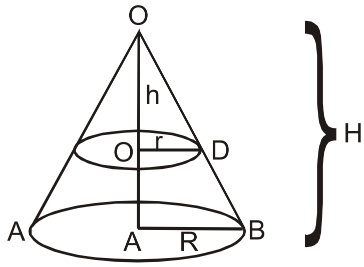
$$r=\frac{120}{25}=\frac{24}{5}$$

Q.29.(D) TSA = 3 (Area of right angle triangle) + Area of equilateral triangle

$$=3\left(\frac{1}{2}\times8\times8\right)+\frac{\sqrt{3}}{4}\times\left(8\sqrt{2}\right)^2$$

$$=3\times32+32\sqrt{3}=32\left(3+\sqrt{3}\right)$$

Q.30.(C)



$$\frac{h}{r}=\frac{H}{R}\rightarrow r=\frac{hR}{H}$$

$$\frac{1}{3}\pi r^2h=\frac{1}{3}\pi R^2H-\frac{1}{3}\pi r^2h$$

$$R^2H=2r^2h$$

$$R^2H=2\left(\frac{h^2R^2}{H^2}\right).h$$

$$H^3=2h^3$$

$$h=\frac{1}{\sqrt[3]{2}}.H$$

Required ratio=h:(H-h)

$$\frac{H}{\sqrt[3]{2}}:H-\frac{H}{\sqrt[3]{2}}=1:\left(\sqrt[3]{2}-1\right)$$

CHAPTER-20
GEOMETRY

Scan the QR code to get video of this chapter.

Geometry

Geometry has a long and rich history. The term 'Geometry' is the English equivalent of the Greek word 'Geometron'.Geo' means Earth and 'metron' means Measurement. According to historians, the geometrical ideas shaped up in ancient times, probably due to the need in art, architecture and measurement. These include occasions when the boundaries of cultivated lands had to be marked without giving room for complaints. Construction of magnificent palaces, temples, lakes, dams and cities, art and architecture propped up these ideas. Even today geometrical ideas are reflected in all forms of art, measurements, architecture, engineering, cloth designing etc. You observe and use different objects like boxes, tables, books, the tiffin box you carry to your school for lunch, the ball with which you play and so on. All such objects have different shapes. The ruler which you use, the pencil with which you write are straight. The pictures of a bangle, the one rupee coin or a ball appear round.

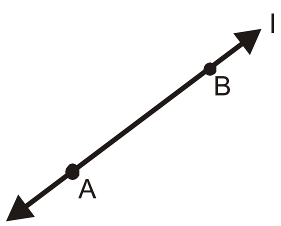
"Geometry is the branch of Mathematics concerned with the properties and relations of points, lines, triangle, quadrilateral and circle ."

Point

A point determines a location. It is usually denoted by a capital letter.

Line

A line segment corresponds to the shortest distance between two points. The line segment joinning points A and B is denoted by $\overline{AB}$

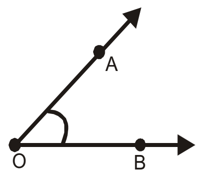


Angle

An angle is made up of two rays starting from a common end point.

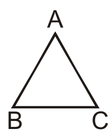
Two rays OA and OB make ∠AOB(or also called ∠BOA).

On the angle, the interior of the angle and the exterior of the angle.



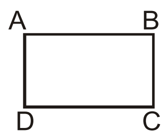
Triangle

A triangle is a three sided polygon.



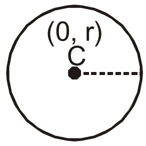
Quadrilateral

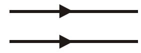
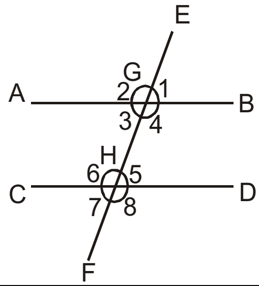
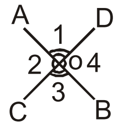
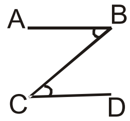
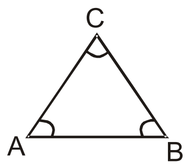
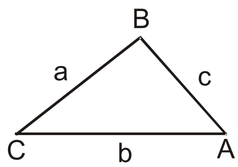
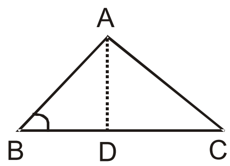
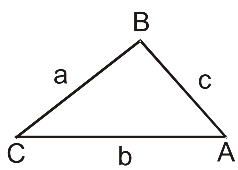
A quadrilateral is a four-sided polygon. (It should be named cyclically.)

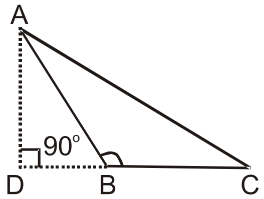
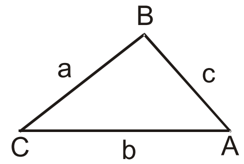
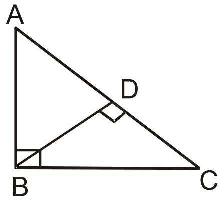
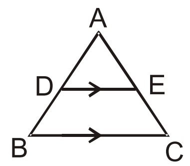
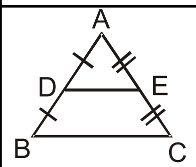


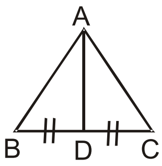
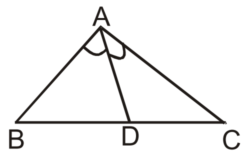
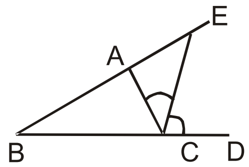
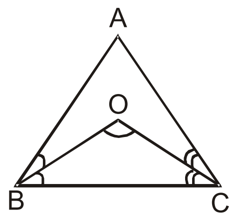
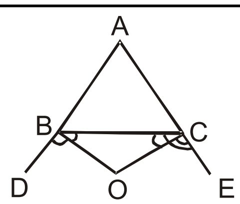
Circle

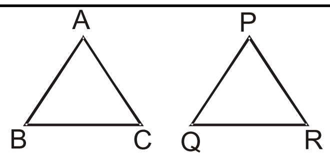
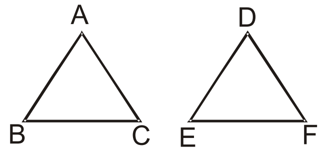
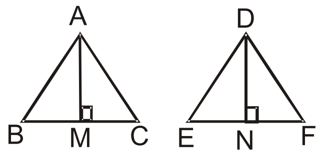
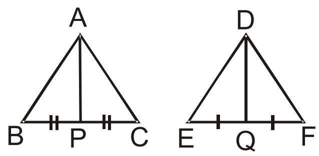
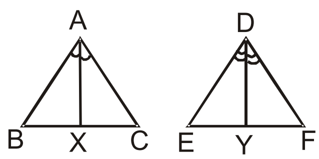
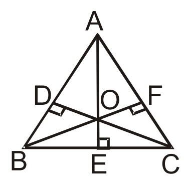
A circle is the path of a point moving at the same distance from a fixed point. The fixed point is the centre, the fixed distance is the radius and the distance around the circle is the circumference.

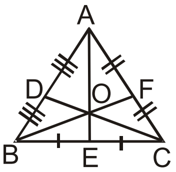
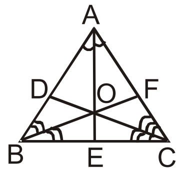
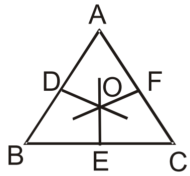
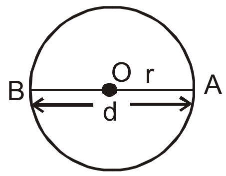


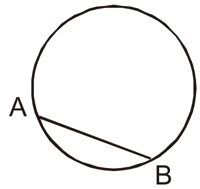
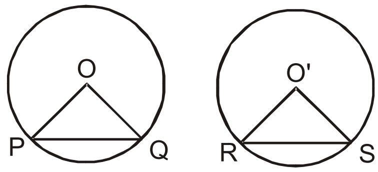
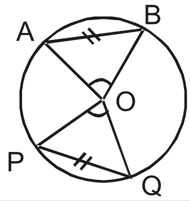
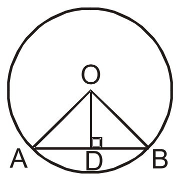
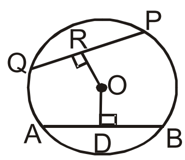
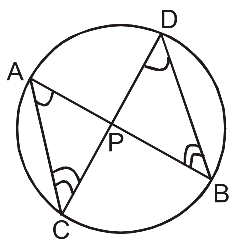
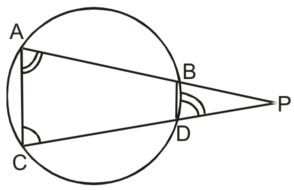
Figures/Theorems	Property	Diagram
Parallel lines	Two lines, lying in a plane and having no common interesecting point are called parallel lines. The distance between two parallel lines is constant.	
Corresponding angles	When two lines are intersected by a transversal, then they form four pairs of corresponding angles. (a) $\angle AGE = \angle CHG \Rightarrow (\angle 2 = \angle 6)$ (b) $\angle AGH = \angle CHF \Rightarrow (\angle 3 = \angle 7)$ (c) $\angle EGB = \angle GHD \Rightarrow (\angle 1 = \angle 5)$ (d) $\angle BGH = \angle DHF \Rightarrow (\angle 4 = \angle 8)$	
Vertically opposite angles	Vertically oppoiste angles are equals. $\angle 1 = \angle 3$ $\angle 2 = \angle 4$	
Alternate angles	$\angle ABC$ and $\angle BCD$ are alternate angle. $\angle ABC = \angle BCD$	
Triangle	A three sided closed plane figure which is formed by joining the three non-colinear point, is called as a triangle.	
Acute Angle Triangle	Each of the angle of a triangle is less than 90^o(if 'c' is the largest side) then, $c^2 < (a^2 + b^2)$	
Acute Angle Theorem	In an acute angle ΔABC, AD is perpendicular dropped on the opposite side of $\angle A$, then $AC^2 = AB^2 + BC^2 - 2BD.BC$	
Obtuse Angle Triangle	One of the angle of a triangle is more than 90^o. (if 'c' is the largest side) then, $c^2 > (a^2 + b^2)$	

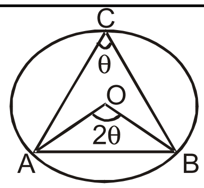
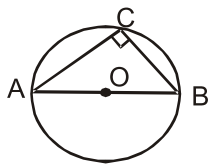
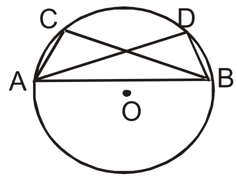
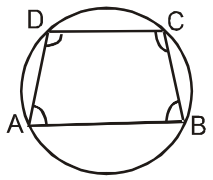
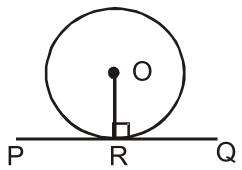
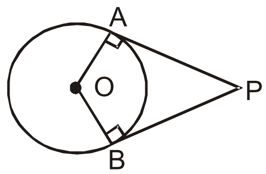
Obtuse Angle Theorem	In an obtuse angle ΔABC, AD is perpendicular dropped on BC. BC is produced to D to meet AD, Then $AC^2 = AB^2 + BC^2 + 2BD.\,BC$	
Right Angle Triangle	One of the angle of a triangle is equal to 90°. (if 'c' is the largest side) then, $c^2 = a^2 + b^2$	
Right Angle Theorem	In a right angle ΔABC, $\angle B = 90^\circ$, and AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle vertex B, then, (i) $BD = \frac{AB \times BC}{AC}$ (ii) $AD = \frac{AB^2}{AC}$ (iii) $CD = \frac{BC^2}{AC}$ (iv) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$	
Basic proportionality theorem (BPT) or Thales theorem	Any line parallel to one side of a triangle divides the other two sides proportionally. So, if DE is drawn parallel to BC, it would divides sides AB and AC proportionally i.e. $\frac{AD}{DB} = \frac{AE}{EC}$ or $\frac{AD}{AB} = \frac{AE}{AC}$	
Mid-point theorem	If the mid-point of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. i.e., if	

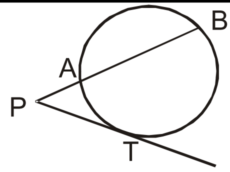
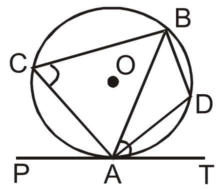
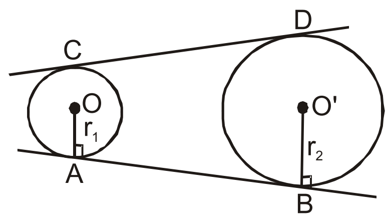
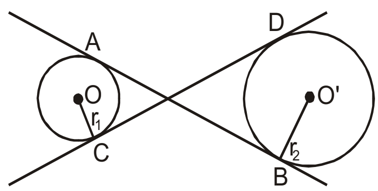
	AD = BD and AE = CE then DE‖ BC.	
Apollonius theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e., $AB^2 + AC^2 = 2(AD^2+BD^2)$	 BD = CD AD is the median
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$	
Exterior angle bisector theorem	**(i)** In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$	
	(ii) In a ΔABC, if the bisector of ∠B and ∠C meet at O then, $\angle BOC = 90^o + \frac{1}{2}\angle A$	
	(iii) In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectors of ∠DBC and ∠ECB intersect at O, then, $\angle BOC = 90^o - \frac{1}{2}\angle A$	

Similarity of triangle	**AAA similarity** Angle, Angle, Angle similarity If two angles are equal then third angle becomes equal automatically. $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}$ Every congurence triangle are similar.	
Area of two similar triangles	$\Delta ABC \sim \Delta DEF$, then, (i) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$ (ii)If, AM and DM are perpendicular from vertices. $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AM}{DN}\right)^2$ (iii) If AP and DQ are median then, $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AP}{DQ}\right)^2$ (iv) If, AX and DY is angle bisector of $\angle A$ and $\angle D$, then $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AX}{DY}\right)^2$ (v) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}\right)^2$	   
Center of triangle **(i) Orthocenter**	The point of intersection of the three altitudes of the triangle is called as the orthocentre. $\angle BOC = 180 - \angle A$ $\angle COA = 180 - \angle B$ $\angle AOB = 180 - \angle C$	

(ii) Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2:1 (vertex : base) $\frac{AO}{OE} = \frac{CO}{OD} = \frac{BO}{OF} = \frac{2}{1}$	
(iii) Incenter	The point of intersection of the angle bisectors of a triangle is called the incentre. Incentre O is the always equidistant from all three sides i.e., the perpendicular distance between the sides and incentre is always same for all the three sides. Inradius $\left[r = \frac{\Delta}{S}\right]$ Where, Δ is a area of triangle and S is a semiperimeter.	
(iv) Circumcenter	The point of intersection of the perpendicular bisector of the sides of a triangle is called the circumcentre. OA = OB = OC = (circum radius) Circumcentre O is always equidistant from all the three vertices A, B and C. Circumradius $\left[R = \frac{abc}{4\Delta}\right]$ Where, a, b, c are the side of a triangle ABC and Δ is area of triangle.	
Circle	The fixed point is called the centre. In the given diagram 'O' is the centre of the circle. AO = Radius of circle AB = Diameter	

Chord	A line segment whose end points lie on the circle. In the given diagram AB is a chord.	
	(i) In a circle (or in congruent circles) equal chords are made by equal arcs. $\overset{\frown}{PQ} = \overset{\frown}{RS}$ PQ = RS	
	(ii) Equal arcs (or chords) subtend equal angles at the centre. $\overset{\frown}{PQ} = \overset{\frown}{AB}$ (or PQ = AB) $\angle POQ = \angle AOB$	
	(iii) The perpendicular from the centre of a circle to a chord bisect the chord. AD = BD	
	(iv) Equal chords of a circle are equidistant from the centre. AB = PQ OD = OR	
	(v) If two chords of a circle intersects internally then, PA × PB = PC × PD	 Case-I Intersects Internally
	(vi)If two chords a circle intersect externally then, PA × PB = PC ×PD	 Case-II Intersects externally PA × PB = PC ×PD

	(vii) The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. $\angle AOB = 2\angle ACB$	
Semi-Circle	Angle in a semi-circle is a right angle. $\angle ACB = 90^o$	
Segment Theorem	Angle in the same segment of a circle are equal. $\angle ACB = \angle ADB$	
Cyclic Quadrilateral	A quadrilateral whose all the four vertices lie on the circle. The sum of pair of opposite angles of a cyclic quadrilateral is 180^o. $\angle DAB + \angle BCD = 180^o$ $\angle ABC + \angle CDA = 180^o$ (Inverse of this theorem is also ture.)	
Tangent	A line segement which has one common point with the circumference of a circle is called as a Tangent of circle. (i) A tangent at any point of a circle is perpendicular to the radius through the point of contact. (ii) The length of two tangent drawn from the external point to a circle are equal. PA = PB	 

Secant	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then, $PA.PB = (PT)^2$	
Alternate segment theorem	If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segements. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	
Direct Common Tangent	If two circle O, O' and radii r_1, r_2. AB and CD are two direct common tangents. DCT (AB = CD) = $\sqrt{(\text{Distance between centre})^2 - (\text{Difference of radii})^2}$ $= \sqrt{(OO')^2 - (r_1 - r_2)^2}$	
Transverse Common Tangent	If two circle O, O' and radii r_1, r_2. AB and CD are two transverse common tangents. TCT (AB = CD) = $\sqrt{(\text{Distance between centre})^2 - (\text{Sum of radii})^2}$ $= \sqrt{(OO')^2 - (r_1 + r_2)^2}$	

EXAMPLES

Ex.1. If the measure of three angles of a triangle are in the ratio of 2 : 3 : 5, then the triangle is:

(A) equilateral **(B)** isosceles

(C) Obtuse angled **(D)** right angled

Sol.(D) Let the angles of a triangle are 2x, 3x and 5x respectively.

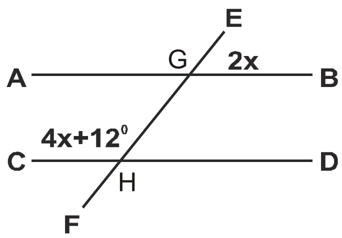
$2x + 3x + 5x = 180^0$

$x = 18^0$

angles are 36^0, 54^0 and 90^0

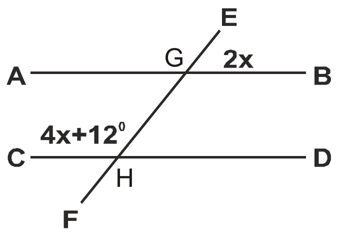
Thus the triangle will be Right angle Triangle

Ex.2. In the given figure, AB || CD and EF intersects them. Then the value of x is:



(A) 14^0 **(B)** 28^0 **(C)** 24^0 **(D)** 18^0

Sol.(B)



$\angle CHG = \angle HGB$ (Alternate interior)

$2x + 4x + 12^0 = 180^0$

$6x = 168^0$

$x = 28^0$

Ex.3. If the three angles of a triangle are:

$\left(x+15^0\right), \left(\frac{6x}{5}+6^0\right) \text{and} \left(\frac{2x}{3}+30^0\right)$

then the triangle is:

(A) scalene **(B)** isocsceles

(C) right angled **(D)** equilateral

Sol.(D) $x + 15^0 + \frac{6x}{5} + 6^0 + \frac{2x}{3} + 30^0 = 180^0$

$\frac{15x + 18x + 10x}{15} = 180^0 - 51^0 = 129^0$

$\Rightarrow x = 45^0$

So, $x + 15^0 = 45^0 + 15^0 = 60^0$

$\frac{6x}{5} + 6^0 = \frac{6 \times 45}{5} + 6^0 = 60^0$

$\frac{2x}{3} + 30^0 = \frac{2 \times 45}{3} + 30^0 = 60^0$

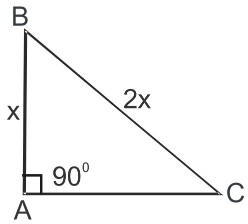
Each of the 3 angles are 60^0 thus it is equilateral triangle

Ex.4. In triangle ABC, $\angle BAC = 90^0$ if $AB = \frac{1}{2}$ BC then measures of $\angle ACB$:

(A) 30^0 **(B)** 45^0

(C) 60^0 **(D)** 50^0

Sol.(A)



$\sin C = \frac{AB}{BC} \Rightarrow \frac{x}{2x} = \frac{1}{2}$

$\sin C = \sin 30^0$

$\angle C = 30^0$

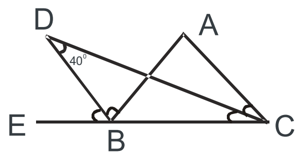
$\angle ACB = 30^0$

Ex.5. ABC is a triangle. The bisectors of the internal $\angle C$ and external $\angle B$ intersect at D. If $\angle BDC = 40^0$, then $\angle A$?

(A) 40^0 **(B)** 80^0

(C) 90^0 **(D)** 20^0

Sol.(B)



In ΔABC

Exterior $\angle ABE = \angle A + \angle C$ .... (I)

Exterior $\angle DBE = \angle D + \angle DCB$ ..... (II)

Given $\angle ABE = 2\ \angle DBE$

$\angle A + \angle C = 2\ (\angle D + \angle DCB)$..From I and II

$\angle A + \angle C = 2\ (40^0 + \angle DCB)$

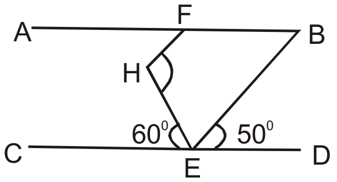
$\angle A + \angle C = 80^0 + 2\angle DCB$

($\angle DCB$ is the angle bisector of $\angle ACB$)

$\angle A + \angle C = 80^0 + \angle C$

$\angle A = 80^0$

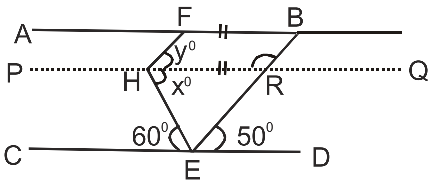
Ex.6.



In the figure AB is parallel to CD and BE is parallel to FH. What is ∠FHE equal to?

(A) 110^0 **(B)** 120^0 **(C)** 125^0 **(D)** 130^0

Sol.(A)



Drawn PQ parallel to AB and CD

$\angle EHQ = \angle CEH$ (Alternate angle)

$x = 60^0$

$\angle HRE = \angle DER$

$= 50^0$ (Alternate angle)

Again $\angle HRE + \angle HRB = 180^0$ (Linear pair of angle)

$50^0 + \angle HRB = 180^0$

$\angle HRB = 130^0$

Again, BE II FH

$\angle y + \angle HRB = 180^0$ (Interior opposite angle)

$\angle y = 50^0$

$\therefore \angle FHE = x + y = 60^0 + 50^0 = 110^0$

Ex.7. In right angle triangle ABC, 'D' is point on side BC in which AB is hypotenuse of triangle. Therefore,

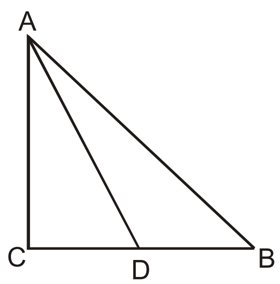
(A) $AB^2 + BD^2 = AC^2 + BC^2$

(B) $AB^2 + AD^2 = AC^2 + 2CD^2$

(C) $AB^2 + CD^2 = BC^2 + AD^2$

(D) $AC^2 + BD^2 = AB^2 + BC^2$

Sol.(C)



$AB^2 = AC^2 + CB^2$ .... (i)

$AD^2 = AC^2 + CD^2$ .... (ii)

$AC^2 = AD^2 - CD^2$

Put the value of AC^2 in eq. (i) we get,

$AB^2 = AC^2 + CB^2$

$AB^2 = AD^2 - CD^2 + CB^2$

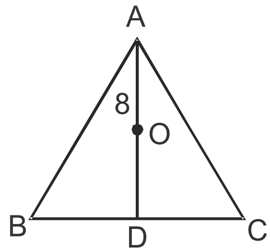
$AB^2 + CD^2 = BC^2 + AD^2$

Ex.8. AD is the median of ΔABC and 'O' is the centroid and AO = 8 cm. Find the length of OD = ?

(A) 2cm **(B)** 3cm

(C) 4cm **(D)** 5cm

Sol.(C)



According to the question:

O is the centroid, and O divide AD in the ratio 2 : 1

then, AO : OD = 2 : 1

then $\frac{AO}{OD} = \frac{2}{1} \Rightarrow \frac{8}{X}$

$\frac{8}{x} = \frac{2}{1}$

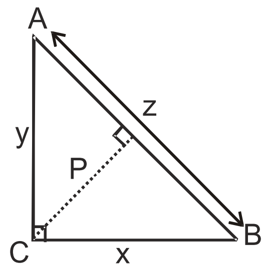
OD = x = 4cm

Ex.9. In a right angled ΔABC ∠C = 90^0, and 'P' is perpendicular distance on AB from 'C'. x,y and z are the lengths of sides. BC, CA and AB respectively then:

(A) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{2}{p^2}$ **(B)** $\frac{1}{x^2}-\frac{1}{y^2}=\frac{1}{p^2}$

(C) $\frac{1}{p^2}+\frac{1}{x^2}=\frac{1}{z^2}$ **(D)** $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{p^2}$

Sol.(D)



Area of Δ ABC = $\frac{1}{2}x \times y$

$= \frac{1}{2} \times z \times P$

$xy = z \times P$

$P = \frac{xy}{\sqrt{x^2+y^2}}$

$P^2 = \frac{x^2y^2}{x^2+y^2}$

$\frac{1}{P^2} = \frac{x^2+y^2}{x^2y^2}$

$\frac{1}{P^2} = \frac{x^2}{x^2y^2} + \frac{y^2}{x^2y^2}$

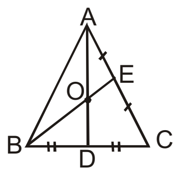
$\frac{1}{P^2} = \frac{1}{x^2} + \frac{1}{y^2}$

Ex.10. In a triangle ABC there are two medians AD and BE intersect right angled at 'O'. If AD is 18 cm and BE is 12 cm, then find BD?

(A) 10 cm **(B)** 8 cm

(C) 6 cm **(D)** 4 cm

Sol.(A)



As we know that every median divides in the ratio of 2:1.

OD = 6cm, BO=8cm

In ΔBOD -

$OB^2+OD^2=BD^2$

$8^2+6^2=BD^2$

$100 = BD^2$

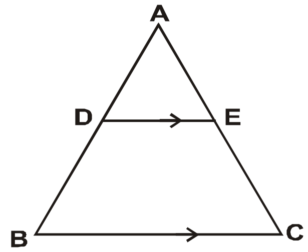
BD=10cm

Ex.11. If in a triangle ABC, 'D' and 'E' are points on the sides AB and AC such that, DE is parallel to BC and $\frac{AD}{BD} = \frac{3}{5}$. If AC=4 cm, then AE is:

(A) 1.5 cm **(B)** 2.0 cm

(C) 1.8 cm **(D)** 2.4 cm

Sol.(A)



If DE II BC then $\Delta ADE \sim \Delta ABC$

If $\frac{AD}{BD} = \frac{3}{5}$ then $\frac{AD}{AB} = \frac{3}{8}$

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{3}{8} = \frac{AE}{4}$

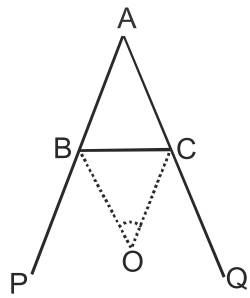
AE = 1.5 cm

Ex.12. In ΔABC external angle bisector of $\angle$ ABC and $\angle$ ACB meet at 'O'. If $\angle$ BAC = 80^0 then $\angle$ BOC=?

(A) 50^0 **(B)** 40^0

(C) 40^0 **(D)** 60^0

Sol.(A)



We know that

$\angle BOC = 90^0 - \frac{\angle A}{2}$

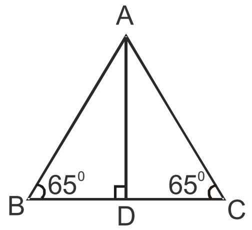
$= 90^0 - \frac{80^0}{2} = 90^0 - 40^0 = 50^0$

Ex.13. In an isosceles triangle ABC such that AB = AC and $\angle ACB = 65^0$. AD is the median to the base BC, then $\angle$BAD is-

(A) 30^0 **(B)** 35^0

(C) 25^0 **(D)** 90^0

Sol.(C)



In triangle ABC

AB = AC

$\angle ABC = \angle ACB = 65^0$

In Δ ABD

$\angle ABD + \angle ADB + \angle BAD = 180^0$

$65^0 + 90^0 + \angle BAD = 180^0$

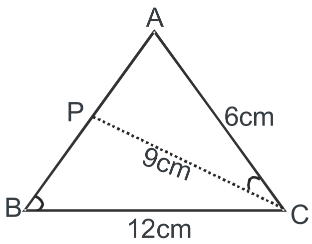
$\angle BAD = 180^0 - 155^0 = 25^0$

Ex.14. In a triangle ABC, P is a point on AB such that $\angle ACP = \angle ABC$. If AC=6cm, CP=9cm and BC=12cm then AP=?

(A) 4 cm **(B)** 4.5 cm

(C) 3.5 cm **(D)** 5 cm

Sol.(B)



In ΔAPC and ΔABC

$\angle ACP = \angle ABC$

Then $\Delta APC \sim \Delta ABC$

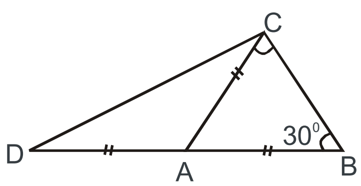
$$\therefore \frac{AP}{AC} = \frac{PC}{BC} = \frac{AP}{6} = \frac{9}{12}$$

$$AP = \frac{9 \times 6}{12} = \frac{18}{4} = 4.5cm$$

Ex.15. ABC is an isosceles triangle with AB = AC. The side BA is produced to D such that AB=AD if $\angle ABC = 30^0$. Then $\angle BCD$ is equal to:

(A) 30^0 **(B)** 45^0 **(C)** 60^0 **(D)** 90^0

Sol.(D)



In the triangle ABC

AB=AC, then

$\angle ABC = \angle ACB = 30^0$

$\angle BAC = 180^0 - (30^0 + 30^0) = 180^0 - 60^0 = 120^0$

In the triangle ACD

AD=AC

then, $\angle ADC = \angle ACD$ .... (I)

Exterior $\angle BAC = \angle ACD + \angle ADC$

$120^0 = \angle ACD + \angle ACD$ .... (from (i)

$2\angle ACD = 120^0$

$\angle ACD = 60^0$

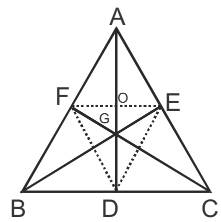
$\angle BCD = \angle BCA + \angle ACD = 60^0 + 30^0 = 90^0$

Ex.16. In a triangle ABC. Median AD, BE and CF meet at G. If the area of Δ ABC is 72cm², then find the area of Δ DGE.

(A) 36cm² **(B)** 12cm²

(C) 6cm² **(D)** 24cm²

Sol.(C)



FE || BC, FD|| AC, ED || AB

$\therefore \Delta ABC \sim \Delta AFE \sim \Delta BFD \sim \Delta DEC \sim \Delta DEF$

$$\frac{AB}{FB} = \frac{2}{1}, \quad \frac{\Delta ABC}{\Delta BFD} = \frac{\Delta ABC}{\Delta DFE} = \frac{4}{1}$$

Similarly, EF, BE, AC are medians on ΔFED

$$\Delta DGE = \frac{1}{3}\Delta DFE$$

$$= \frac{1}{3} \times \frac{1}{4} \times \Delta ABC = \frac{1}{12}\Delta ABC$$

Area of $\Delta DGE = \frac{1}{12} \times 72 = 6$ cm²

QUICK TRICK

We Know that

$$\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ DGE}} = \frac{12}{1}$$

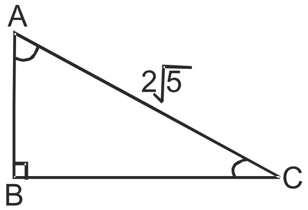
$$\frac{72}{\text{Area of } \Delta \text{ DGE}} = \frac{12}{1}$$

Area of Δ DGE = 6cm²

Ex.17. In a triangle ABC $\angle B=90^0$, AC= $2\sqrt{5}$ cm, if (AB-BC) = 2cm, then the value of ($\cos^2A-\cos^2C$) is:

(A) $\frac{3}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{5}$

Sol.(A)



In ABC

$AC^2 = AB^2+BC^2$

(AB - BC = 2)

BC = x

AB = x + 2

$20 = x^2 + 4 + 4x + x^2$

$20 = 2x^2 + 4 + 4x$

$x^2 + 2x - 8 = 0$

$(x+4)(x - 2) = 0$

x = 2, - 4

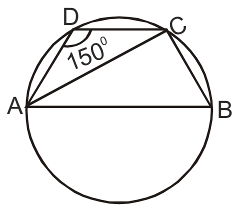
BC = 2 cm

AB = 2 + BC = 2 + 2 = 4 cm

$\cos^2 A - \cos^2 C = \left(\frac{AB}{AC}\right)^2 - \left(\frac{BC}{AC}\right)^2 = \frac{AB^2 - BC^2}{AC^2}$

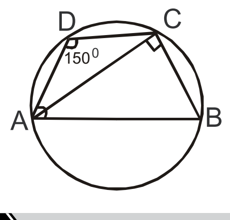
$= \frac{(4)^2-(2)^2}{20} = \frac{12}{20} = \frac{3}{5}$

Ex.18. In the given figure ABCD is a cyclic quadrilateral and AB is the diameter. $\angle ADC=150^0$ then find $\angle BAC$.



(A) 60^0 **(B)** 40^0 **(C)** 50^0 **(D)** 30^0

Sol.(A)



In cyclic quadilateral ABCD

$\angle D + \angle B = 180^0$ (opposite angles of quadilateral)

$\angle B = 180^0 - 150^0$

$\angle B = 30^0$

Now in ΔABC

$\angle BAC + \angle ABC + \angle ACB = 180^0$

$\angle BAC + 30^0 + 90^0 = 180^0$

$\Rightarrow \angle ACB = 90^0$ (Angle in semicircle)

$\Rightarrow \angle BAC = 180^0 - 120^0$

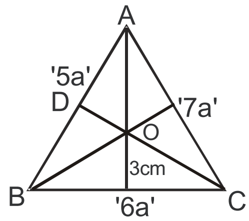
$\angle BAC = 60^0$

Ex.19. In a triangle ABC sides are in the ratio 5 : 6 : 7, if the inradius of the Δ is 3cm then the value of altitude of the triangle corresponding to the smallest side as the base.

(A) 6cm **(B)** 8cm

(C) 10.8cm **(D)** 8.5cm

Sol.(C)



Let the height CD = h cm

Area of ΔABC = area of ΔAOC + area of ΔBOC + area of ΔAOB

$\frac{1}{2} \times h \times 5a = \frac{1}{2} \times 6a \times 3 + \frac{1}{2} \times 7a \times 3 + \frac{1}{2} \times 5a \times 3$

5 h = 18 + 21 + 15

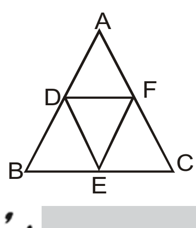
$h = \frac{54}{5}$, h = 10.8 cm

Ex.20. In an equilateral triangle ABC, D,E and F are the mid points of sides AB, BC, CA respectively, if these three points D,E and F are joined, then find the ratio between the area of ΔDEF and area of ΔABC?

(A) 1 : 4 **(B)** 1 : 2

(C) 3 : 2 **(D)** 1 : 8

Sol.(A)



When we join all the mid point it divides the triangle in 4 equal part.

area of $\Delta DEF = \frac{1}{4}$ area of ΔABC

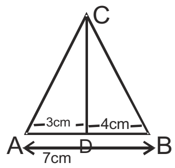
$$\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC} = \frac{1}{4}$$

Ex.21. ABC is a triangle with base AB. CD is a perpendicular on AB such that AB = 7cm and DB = 4cm. What is the ratio of the area of ΔADC to ΔABC?

(A) 5 : 2 **(B)** 9 : 49

(C) 3 : 7 **(D)** 4 : 5

Sol.(C)



Let the perpendicular height be h

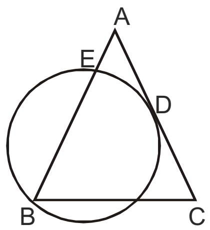
$$\text{Ratio} = \frac{\Delta ADC}{\Delta ABC} = \frac{\frac{1}{2}3xh}{\frac{1}{2}x7xh} = 3:7$$

Ex.22. ABC is an isosceles triangle in which AB = AC. 'D' is mid point of AC. A circle touches 'D' which passes through 'B' cuts AB at 'E' then find the ratio AE : AB-

(A) 1:3 **(B)** 1:2

(C) 1:4 **(D)** 1:6

Sol.(C)



AB = AC

AD is tangent and AB is secant

Let the AD = x

$AD^2 = AE \times AB$

$x^2 = AE \times AC = AE \times 2x$

$$AE = \frac{x^2}{2x} = \frac{x}{2}$$

$$\frac{AE}{AB} = \frac{\frac{x}{2}}{2x} = \frac{x}{4x} = \frac{1}{4}$$

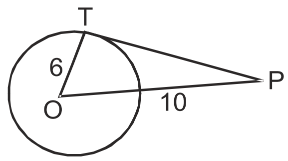
AE : AB = 1 : 4

Ex.23. A tangent is drawn to a circle of radius 6 cm from a point outside at a distance of 10 cm from the centre of the circle. The length of the tangent will be-

(A) 8 cm **(B)** 5 cm

(C) 4 cm **(D)** 7 cm

Sol.(A)



apply Pythagoras

$OT^2 + TP^2 = OP^2$

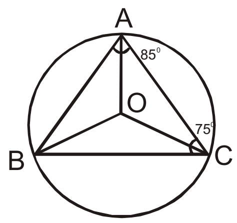
$PT = \sqrt{10^2 - 6^2} = 8$

Ex.24. 'O' is the circumcenter of ΔABC if $\angle BAC = 85^0$ $\angle BCA = 75^0$, then $\angle OAC$ is equal to-

(A) 40^0 **(B)** 70^0

(C) 60^0 **(D)** 50^0

Sol.(B)



In ΔABC

$\therefore \angle ABC = 180^0 - (85^0 + 75^0) = 20^0$

and $\angle AOC = 2\angle ABC = 40^0$

OA = OC (radii)

$$\therefore \angle OAC = \angle OCA = \left(\frac{180^0 - 40^0}{2}\right)$$

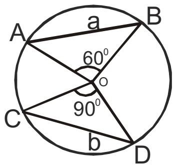
$= 70^0$

Ex.25. Two chords of length a unit and b unit of a circle make angles 60^0 and 90^0 at the centre of a circle respectively, then the correct relation is

(A) $b = a\sqrt{3}$ **(B)** $b = a\sqrt{2}$

(C) $b^2 = a\sqrt{2}$ **(D)** $b^2 = 3\sqrt{a}$

Sol.(B)



ΔABO is equilateral ($\therefore \angle A = \angle B = 60^0$)

a = r

ΔCOD is Right angled triangle

then $b = \sqrt{2}\, r$ ($\therefore CD^2 = OC^2 + OD^2$)

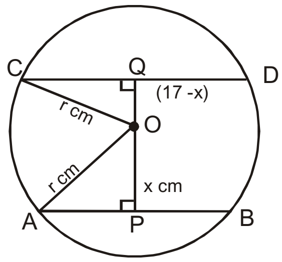
$b = a\sqrt{2}$

Ex.26. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on opposite sides of the centre and the distance between them is 17 cm, what is the radius of the circel?

(A) 14 cm **(B)** 10 cm

(C) 13 cm **(D)** 15 cm

Sol.(C)



In right Δs OAP and OQC, we have

$OA^2 = AP^2 + OP^2$ and $OC^2 = OQ^2 + CQ^2$

$r^2 = x^2 + 5^2$ and $r^2 = (17 - x)^2 + 12^2$

$\Rightarrow r^2 = x^2 + 5^2$ .... (i)

$\Rightarrow r^2 = 289 + x^2 - 34x + 144$ .... (ii)

From I and II

$\Rightarrow 34x = 408 \Rightarrow x = 12$ cm

$\therefore r^2 = 12^2 + 5^2 = 169$

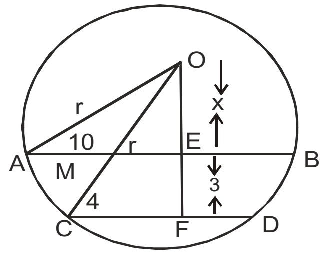
$\therefore r = 13$ cm

Ex.27. AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords of the circle are on the same side of the centre and the distance between them is 3 cm, then the diameter of the circle is

(A) $\sqrt{21}$ cm **(B)** $\sqrt{29}$ cm

(C) $2\sqrt{21}$ cm **(D)** $2\sqrt{29}$ cm

Sol.(D)



EF = 3 cm Let OE = x

AE = 5 cm CF = 2, OF = x +3

$r^2 = OE^2 + AE^2 = x^2 + 5^2$

$r^2 = OF^2 + CF^2 = (x+3)^2 + 2^2$

$x^2 + 5^2 = (x+3)^2 + 2^2$

x = 2

$r = \sqrt{5^2 + 2^2} = \sqrt{29}$

$D = 2r = 2\sqrt{29}$ cm

QUICK TRICK :

When chords on same side and the half of the difference of the chords is equal to the distance between the chords

then chords apply the pythogorus theorem with diameter.

If $\Rightarrow \frac{10}{2} - \frac{4}{2} = 3$

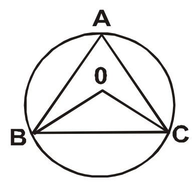
then $D = \sqrt{10^2 + 4^2} = 2\sqrt{29}$ cm

Ex.28. If O is the circumcenter of a triangle ABC lying inside the triangle, then $\angle OBC + \angle BAC$ is equal to.

(A) 110^0 **(B)** 90^0

(C) 120^0 **(D)** 60^0

Sol.(B)



$\angle OBC = \angle OCB = \theta$

$\because \angle BOC = 180^0 - 2\theta$

$\because \angle BAC = 1/2 \times \angle BOC$

$= 1/2 \times (180^0 - 2\theta) = 90^0 - \theta$

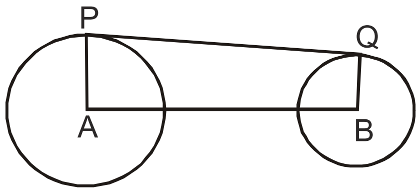
$\because \angle BAC + \angle OBC = 90^0 - \theta + \theta = 90^0$

Ex.29. A and B are centers of two circles of radii 11cm and 6 cm, respectively. PQ is a direct common tangent to the circle. If $\overline{AB}$ = 13 cm, then length of $\overline{PQ}$ will be.

(A) 13 cm **(B)** 12 cm

(C) 8.5 cm **(D)** 17 cm

Sol.(B)



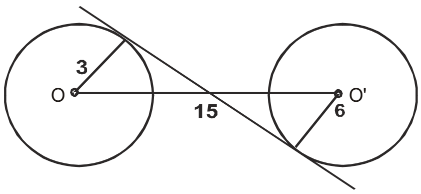
$= \sqrt{13^2 - (11-6)^2}$ = 12 cm

Ex.30. The distance between the centers of two circles of radii 6 cm and 3 cm is 15 cm. The length of the transverse common tangent to the circles is:

(A) $7\sqrt{6}$ cm **(B)** 18 cm

(C) $6\sqrt{6}$ cm **(D)** 12 cm

Sol.(D)



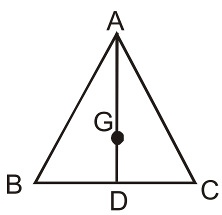
$OO' = \sqrt{15^2 - (3+6)^2}$ = 12 cm

Ex.31. G is the centroid of the equilateral ΔABC. If AB = 10 cm then length of AG is

(A) $\frac{5\sqrt{3}}{3}$ cm **(B)** $\frac{10\sqrt{3}}{3}$ cm

(C) $5\sqrt{3}$ cm **(D)** $10\sqrt{3}$ cm

Sol.(B)



AB = 10 cm

BD = 5 cm

ADB = 90^0

$\therefore AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{10^2 - 5^2} = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$ cm

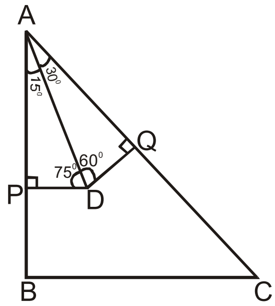
$AG = \frac{2}{3}AD = \frac{2}{3}\times 5\sqrt{3} = \frac{10\sqrt{3}}{3}$ cm

Ex.32. An isosceles triangle ABC is right angled at B.D is a point inside the triangle ABC. P and Q are the feet of the perpendiculars drawn from D on the side AB and AC respectively of ΔABC. If AP = a cm, AQ = b cm and ∠PAD = 15^0, sin 75^0 =

(A) $\frac{2b}{\sqrt{3}a}$ **(B)** $\frac{a}{2b}$

(C) $\frac{\sqrt{3}a}{2b}$ **(D)** $\frac{2a}{\sqrt{3}b}$

Sol.(C)



From ΔAQD

$\sin 60^0 = \frac{AQ}{AD}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{b}{AD} \quad \Rightarrow AD = \frac{2b}{\sqrt{3}}$

From ΔAPD

$\sin 75^0 = \frac{AP}{AD} = \frac{a}{\frac{2b}{\sqrt{3}}} = \frac{\sqrt{3}a}{2b}$

Ex.33. If the measures of the sides of triangle are $(x^2 - 1)$, (x^2+1) and 2x cm, then the triangle would be

(A) equilateral **(B)** acute -angled

(C) isosceles **(D)** right-angled

Sol.(D) $(2x)^2 + (x^2 - 1)^2$

$= 4x^2 + x^4 - 2x^2 + 1$

$= x^4 + 2x^2 + 1 = (x^2 + 1)^2$

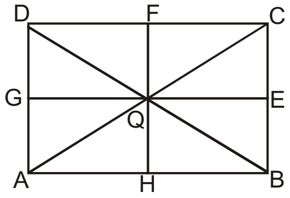
It is a right-angled triangle

Ex.34. Q is a point in the interior of a rectangle ABCD. If QA = 3 cm, QB = 4 cm and QC = 5 cm, then the length of QD (in cm) is

(A) $3\sqrt{2}$ **(B)** $5\sqrt{2}$

(C) $\sqrt{34}$ **(D)** $\sqrt{41}$

Sol.(A)



Using pythagoras theorem,

$QD^2 + QB^2 = QA^2 = QC^2$

$\Rightarrow QD^2 + 16 = 9 + 25$

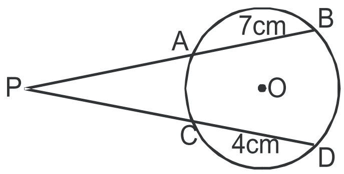
$\Rightarrow QD^2 = 34 - 16 = 18$

$\Rightarrow QD = \sqrt{18} = 3\sqrt{2}$ cm

Ex.35. Chords AB and CD of a circle intersect externally at P. If AB = 7 cm, CD = 4 cm and PD = 10 cm, then the length of PB is

(A) 15 cm **(B)** 12 cm
(C) 16 cm **(D)** 20 cm

Sol.(B)



PA × PB = PC × PD

$\Rightarrow x \times (x+7) = (10 - 4) \times 10$

$\Rightarrow x^2 + 7x = 60$

$\Rightarrow x^2 + 7x - 60 = 0$

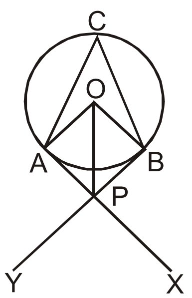
$\Rightarrow x = -12$ and 5

$\therefore$ PB = (PA + AB) = 5 + 7 = 12 cm

Ex.36. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65^0 at any point C on the circumference of the circle. If P is the point of intersection of the two lines, then the measure of $\angle APO$ is

(A) 25^0 **(B)** 65^0 **(C)** 90^0 **(D)** 40^0

Sol.(A)



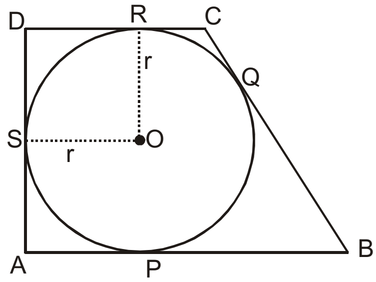
$\angle ACB = 65^0$

$\angle AOB = 2 \times 65^0 = 130^0$

$\angle OAP = 90^0$, $\angle AOP = 65^0$

$\therefore \angle APO = 180^0 - 90^0 - 65^0 = 25^0$

Ex.37. In the given figure, ABCD is a quadrilateral in which $\angle ADC = 90^0$. A circle C (O,r) touches the sides AB, BC, CD and DA at P,Q,R,S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find the value of r.



(A) 14 cm **(B)** 15 cm **(C)** 10 cm **(D)** 16 cm

Sol.(A) OR = OS, OR $\perp$ DR and OS $\perp$ DS

$\therefore$ ORDS is a square

Also, BP = BQ, CQ = CR and DR = DS

$\therefore$ BQ = BP = 27 cm

$\Rightarrow$ BC - CQ = 27 cm

$\Rightarrow$ 38 - CQ = 27

$\Rightarrow$ CQ = 11 cm

$\Rightarrow$ CR = 11 cm

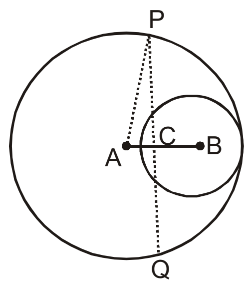
$\Rightarrow$ CD - DR = 11 cm

$\Rightarrow$ 25 - DR = 11 cm

$\Rightarrow$ DR = 14 cm

$\Rightarrow$ r = 14 cm

Ex.38. In the given figure, two circle with centres A and B of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q, find the length of PQ



(A) $4\sqrt{6}$ cm **(B)** $\sqrt{24}$ cm
(C) $8\sqrt{3}$ cm **(D)** $4\sqrt{3}$ cm

Sol.(A) If two circles touch internally, then distance their centres is equal to the difference of their radii.

$\therefore$ AB = (5 - 3) = 2 cm

Also, the common chord PQ is the perpendicular bisector of AB

$\therefore$ AC = BC = 1 cm

In right angled triangle ACP

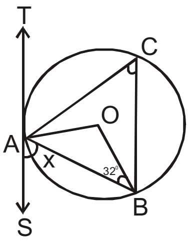
$AP^2 = AC^2 + CP^2$

$\Rightarrow 25 - 1 = CP^2$

$\therefore$ CP = $\sqrt{24}$ cm

Hence, PQ = 2CP = $2 \times 2\sqrt{6} = 4\sqrt{6}$ cm

Ex.39. In the given figure, TAS is a tangent to the circle at the point A. If $\angle OBA = 32^0$, what is the value of x?



(A) 64^0 **(B)** 40^0 **(C)** 58^0 **(D)** 50^0

Sol.(C) OA = OB $\Rightarrow \angle OAB = \angle OBA = 32^0$

$\therefore \angle OAB + \angle OBA = 32^0 + 32^0 = 64^0$

$\therefore \angle AOB = 180^0 - 64^0 = 116^0$

$\Rightarrow 2\angle ACB = \angle AOB$

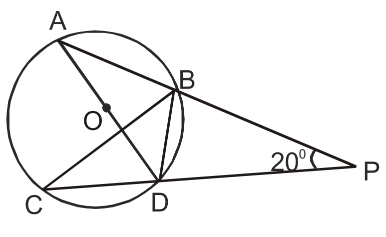
$\angle ACB = 58^0$

Also, $\angle ACB = \angle BAS$

(angles in alternate segments)

$\therefore \angle BAS = x = 58^0$

Ex.40. PBA and PDC are two secants. AD is the diameter of the circle with centre at 'O' $\angle A = 40^0$, $\angle P = 20^0$. Find the measure of $\angle DBC$?



(A) 30^0 **(B)** 50^0

(C) 45^0 **(D)** 450

Sol.(A) In ΔADP

Ext. $\angle ADC = \angle A + \angle P$

$= 40^0 + 20^0 = 60^0$

$\angle ABC = \angle ADC = 60^0$

(Angle on same segments)

since AD is diameter

$\Rightarrow \angle ABD = 90^0$

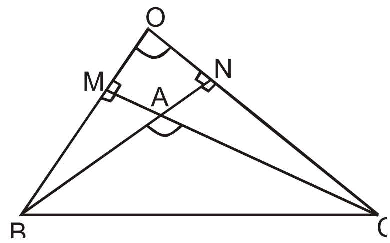
$\angle DBC = \angle ABD - \angle ABC$

$= 90^0 - 60^0 = 30^0$

Ex.41. In an obtuse - angles triangle ABC, $\angle A$ is the obtuse angle and O is the orthocentre. If $\angle BOC = 54^0$, then $\angle BAC$ is

(A) 108^0 **(B)** 126^0 **(C)** 136^0 **(D)** 116^0

Sol.(B)



From figure, $\angle BOC = 54^0$

$\therefore$ In ΔABC

$\angle MAN = 360^0 - (90^0 + 90^0 + \angle BOC)$

$= 360^0 - (180^0 + 54^0) = 126^0$

$\therefore \angle BAC = \angle MAN = 126^0$

Ex.42. If the ratio of area of two similar triangles is 9 : 16, then the ratio of their corresponding sides is

(A) 3 : 5 **(B)** 3 : 4 **(C)** 4 : 5 **(D)** 4 : 3

Sol.(B) In similar triangles

$(\text{Ratio of their corresponding sides})^2$

= Ratio of areas of two similar triangles.

Ratio of their corresponding sides $= \sqrt{\frac{9}{16}} = 3:4$

Ex.43. The tangent PT and chord of circle externally intersect point at P and PT = 2 AP, AB = 18 cm. Then find the length of PT

(A) 6 cm **(B)** 12 cm **(C)** 15 cm **(D)** 9 cm

Sol.(B) PT = 2AP

$AP = \frac{PT}{2}$

$BP = \left(\frac{PT}{2} - 18\right)$

We know that

$PT^2 = PA \times PB$

$\Rightarrow PT^2 = \frac{PT}{2} \times \left(\frac{PT}{2} - 18\right)$

$\Rightarrow 4PT^2 = PT^2 - 36PT$

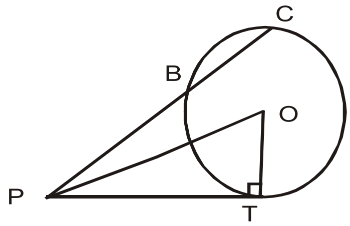
$\Rightarrow 3PT^2 = 36PT$

$\Rightarrow PT = 12$ cm

Ex.44. PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O. PBC is sectant line and PB = 5 cm, then what is the length of the chord BC?

(A) 7.8 cm **(B)** 8.0 cm **(C)** 8.4 cm **(D)** 9.0 cm

Sol.(A)



$\therefore PT = \sqrt{10^2 - 6^2}$

$\Rightarrow PT = 8$

$\therefore PT^2 = PB \times PC$

$\Rightarrow 64 = 5(5 + BC)$

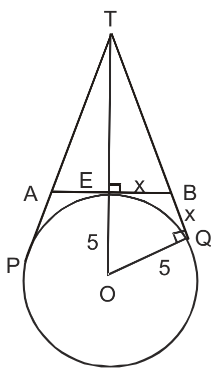
$\Rightarrow 64 = 25 + 5BC$

$\Rightarrow BC = 7.8$ cm

Ex.45. A circular ring with centre O is kept in the vertical position by two weightless thin string TP and TQ attached to the ring at P and Q. The line OT meets the ring at E whereas a tangential string at E meets TP and TQ at A and B respectively. If the radius of the ring is 5 cm. and OT = 13 cm., then what is the length of AB?

(A) $\frac{10}{3}$ cm. **(B)** $\frac{20}{3}$ cm. **(C)** 10 cm. **(D)** $\frac{40}{3}$ cm.

Sol.(B)



OT = 13 cm.

$TQ = \sqrt{13^2 - 5^2} = 12$ cm

$TB = TQ - BQ \quad \therefore (BQ = EB)$

$TB = 12 - x$

$EB = x$

$\therefore (12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow 24x = 80$

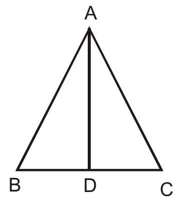
$\Rightarrow x = 3\frac{1}{3}$

$\therefore AB = 2 \times \frac{10}{3} = \frac{20}{3}$ cm.

Ex.46. In ΔABC, AD is drawn perpendicular from A on BC. If $AD^2 = BD. CD$, then $\angle BAC$ is

(A) 30^o **(B)** 45^o **(C)** 60^o **(D)** 90^o

Sol.(D)



In ΔABD and ΔADC

$AB^2 = AD^2 + BD^2$

$AC^2 = AD^2 + DC^2$

$\therefore AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$= 2BD \times CD + BD^2 + DC^2$

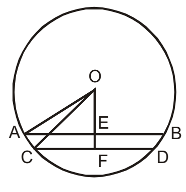
$\Rightarrow AB^2 + AC^2 = (BD+CD)^2 = BC^2$

$\therefore \angle BAC = 90^o$

Ex.47. If a circle with radius of 10 cm. has two parallel chords 16 cm. and 12 cm. and they are on the same side of the centre of the circle, then the distance between the two parallel chords is

(A) 2 cm. **(B)** 3 cm. **(C)** 5 cm. **(D)** 8 cm.

Sol.(A)



In ΔOEA,

$OA^2 = AE^2 + OE^2$

$10^2 = AE^2 + OE^2$

$OE^2 = 8^2 + OE^2$

$OE^2 = 36$

OE = 6 and In ΔOFC,

$OC^2 = OF^2 + CF^2$

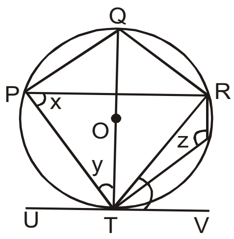
$10^2 = OF^2 + 6^2$

$OF^2 = 64$

$OF = 8$

Required distance (EF) = OF - OE = 8 – 6 = 2cm.

Ex.48.



In the figure given above, O is the centre of the circle. The line UTV is a tangent to the circle at T, $\angle VTR = 52^0$ and triangle PTR is an isosceles triangle such that TP = TR then what is the value of ($\angle x + \angle y + \angle z$)?

(A) 175^0 **(B)** 208^0 **(C)** 218^0 **(D)** 250^0

Sol.(C) $\angle VTR = 52^0$ (Given)

$\angle VTR = \angle TPR = 52^0$

$\Rightarrow x^0 = 52^0$

$\therefore$ TQ is a diameter, then $\angle TPQ = 90^0$

In cyclic quadrilateral TPQR

$\angle TPQ + \angle TRQ = 180^0$

$\therefore\ 90^0 + \angle TRQ = 180^0$

$\angle TRQ = 90^0$

$\therefore$ TP = TR (Given)

$\therefore TPR = \angle TRP = x^0 = 52^0$

$\therefore QRP = \angle TRQ - \angle TRP = 90^0 - 52^0$

$\angle QRP = 38^0$

Then, $\angle PTQ = \angle QRP$

(Angles on same segment), $y = 38^0$

Again, In cyclic quadrilateral TPRS

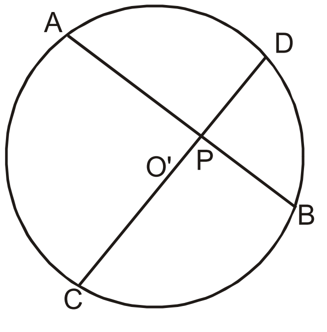
$\angle x + \angle z = 180^0$

$52^0 + \angle z = 180^0$

$\angle z = 128^0$

$\therefore\ x + y + z \Rightarrow 52^0 + 38^0 + 128^0 = 218^0$

Ex.49. In the figure given below, AB = 10 cm PB = 4 cm, CD = 11 cm and DP = x cm. What is the value of 'x' (PD < CP)



(A) 3 cm **(B)** 8 cm

(C) 6 cm **(D)** 4 cm

Sol.(A) We now that

$AP \times BP = CP \times DP$

$\Rightarrow (10 - 4) \times 4 = (11 - x) \times x$

$\Rightarrow 24 = 11x - x^2$

$\Rightarrow x^2 - 11 + 24 = 0$

$\Rightarrow x = 8, 3$

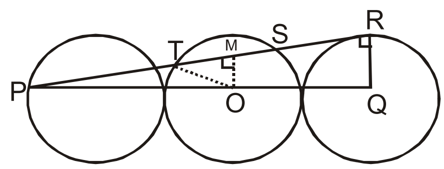
$\Rightarrow$ x = 3 cm (PD < CP)

Ex.50. Three circles each of radius 2 cm, are touching each other, PQ is the line passing through the centres of the three circles, with 'P' lying on one of the outmost circle and 'Q' being the centre of the other outmost circle. PR is tangent to the circle with centre 'Q' and cuts chord ST on the middle circle. Find the length of ST?

(A) 1.6 cm **(B)** 2 cm

(C) 3.2 cm **(D)** 4 cm

Sol.(C)



Let 'O' is the centre of middle circle and OM is perpendicular to ST, then 'M' will be the mid-point of ST.

Now ΔPOM and ΔPQR similar

$$\Rightarrow \frac{PO}{PQ} = \frac{OM}{QR}$$

$$\Rightarrow \frac{6}{10} = \frac{OM}{2} \text{ (Since radius of each circle is 2 cm)}$$

$$\Rightarrow OM = \frac{6}{5}$$

Now in ΔOTM

$OT^2 = OM^2 + MT^2$

$$(2)^2 = \left(\frac{6}{5}\right)^2 + MT^2$$

$$MT^2 = 4 - \frac{36}{25}$$

$$MT = \sqrt{\frac{64}{25}}$$

$$MT = \frac{8}{5}$$

$ST = 2 \times MT$

$$2 \times \frac{8}{5} = \frac{16}{5} = 3.2 \text{ cm}$$

EXERCISE

Q.1. PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O. PBC is sectant line and PB = 5 cm, then what is the length of the chord BC?

(A) 7.8 cm **(B)** 8.0 cm

(C) 8.4 cm **(D)** 9.0 cm

Q.2. ABC is a triangle in which AB = AC. Let BC be produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral ECDF thus formed. If $\angle$ ABC = 65^0 and $\angle$ EFD = 80^0, then what is $\angle$ FDC equal to ?

(A) 43^0 **(B)** 41^0

(C) 37^0 **(D)** 35^0

Q.3. The bisectors of the $\angle$ABC and $\angle$BCA of a triangle ABC meet in a point O. What is the angle at O facing the side BC?

(A) 90^0- (A/2) **(B)** 90^0+(A/2)

(C) 90^0- A **(D)** 90^0+ A

Q.4. Find the ratio of area of a square inscribed in a semicircle of radius r to the area of another square inscribed in the entire circle of radius r.

(A) 2:3 **(B)** 3:5

(C) 2:5 **(D)** 3:5

Q.5. In a triangle ABC, the measure of angle $\angle$BAC is 50^0 and external bisector of $\angle$ABC and $\angle$ACB meet at O. What is the measure of $\angle$BOC?

(A) 40^0 **(B)** 65^0

(C) 115^0 **(D)** 130^0

Q.6. ABCD is parallelogram. P is a point on BC such that PB : PC = 1 : 2. DP produced meets AB produced at Q. If the area of the triangle BPQ is 20 square units, what is the area of the triangle DCP?

(A) 20 square unit **(B)** 30 square unit

(C) 40 square unit **(D)** 80 square unit

Q.7. In the equilateral triangle ABC, the base BC is trisected at D and E. the line through D, parallel to AB, meets AC at F and the line through E parallel to AC meets AB at G. Let EG and DF intersect at H. What is the ratio of the sum of the area of parallelogram AGHF and the area of the triangle DHE to the area of the triangle ABC?

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$

(C) $\frac{1}{4}$ **(D)** $\frac{1}{6}$

Q.8. A circular ring with centre O is kept in the vertical position by two weightless thin string TP and TQ attached to the ring at P and Q. The line OT meets the ring at E whereas a tangential string at E meets TP and TQ at A and B respectively. If the radius of the ring is 5 cm. and OT = 13 cm., then what is the length of AB?

(A) $\frac{10}{3}$ cm. **(B)** $\frac{20}{3}$ cm.

(C) 10 cm. **(D)** $\frac{40}{3}$ cm.

Q.9. In a circle AB is diameter and points C and D are on the circumference such that $\angle$CAD = 30^o and $\angle$CBA = 70^o. What is the value of $\angle$ACD = ?

(A) 40^o **(B)** 50^o

(C) 30^o **(D)** 90^o

Q.10. In a trianlge ABC, the internal bisector of the angle A meets BC at D. If AB = 4, AC = 3 and $\angle$A = 60^o, then the length of AD is -

(A) $2\sqrt{3}$ **(B)** $\frac{15\sqrt{3}}{8}$

(C) $\frac{12\sqrt{3}}{7}$ **(D)** $\frac{6\sqrt{3}}{7}$

Q.11. If ABCDEF be a regular hexagon. Then what is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?

(A) 1:3 **(B)** 2:3

(C) 1:2 **(D)** 5:6

Q.12. If ABC is an equilateral triangle and AD is the altitude, then-

(A) $AB^2= 2AD^2$ **(B)** $AB^2= 3AD^2$

(C) $3AB^2= 4AD^2$ **(D)** $2AB^2= 3AD^2$

Q.13. The tangent at the vertex A to the circumcircle of ΔABC meets the side BC produced in T, then $\frac{TB}{TC}$ is equal to-

(A) $\frac{AC^2}{AB^2}$ **(B)** $\frac{AB^2}{AC^2}$

(C) $\frac{2AB^2}{3AC^2}$ **(D)** $\frac{AB+AC}{AC^2}$

Q.14. In a circle, O is the centre of the circle and A,B,C are points on the circle such that OA = 3 cm. AC = 3 cm and OM is perpendicular to AC. What is $\angle$

ABC equal to ?

(A) 60^0 **(B)** 45^0

(C) 45^0 **(D)** 30^0

Q.15. If a square ABCD is inscribed in a circle and AB = 4 cm., then the radius of circle is-

(A) 2 cm. **(B)** $2\sqrt{2}$ cm.

(C) 4 cm. **(D)** $4\sqrt{2}$ cm.

Q.16. Let O is the incentre of a triangle ΔABC and D be a point on the side BC of ΔABC, such that $OD \perp BC$. If $\angle BOD = 15^o$, then $\angle ABC$ = ?

(A) 75^o **(B)** 45^o

(C) 150^o **(D)** 90^o

Q.17. If G is the centroid and AD be a median with length 18 cm. of ΔABC, then the value of AG is

(A) 6 cm. **(B)** 10 cm.

(C) 12 cm. **(D)** 8 cm.

Q.18. In ΔABC, AD is drawn perpendicular from A on BC. If AD^2 = BD. CD, then $\angle BAC$ is

(A) 30^o **(B)** 45^o

(C) 60^o **(D)** 90^o

Q.19. ABC is triangle. The internal bisector of the angles $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at X, Y and Z respectively. If $\angle A = 60^o$, $\angle CZY = 40^o$, then BYZ will be

(A) 20^o **(B)** 35^o

(C) 45^o **(D)** 40^o

Q.20. If a circle with radius of 10 cm. has two parallel chords 16 cm. and 12 cm. and they are on the same side of the centre of the circle, then the distance between the two parallel chords is

(A) 2 cm. **(B)** 3 cm.

(C) 5 cm. **(D)** 8 cm.

Q.21. ABCD is a cyclic quadrilateral. Sides AB and DC, when produced meet at the point P and sides AD and BC, when produced meet at the point Q. If $\angle ADC = 85^o$ and $\angle BPC = 40^o$, then $\angle CQD$ is equal to

(A) 30^o **(B)** 40^o

(C) 55^o **(D)** 85^o

Q.22. If D is the mid point of BC of a triangle ABC and AD is the perpendicular to BC then -

(A) $3\ AC^2 = BC^2 - AB^2$

(B) $3\ BC^2 = AC^2 - 3\ AB^2$

(C) $BC^2 + AC^2 = 5\ AB^2$

(D) None of these

Q.23. Triangle ABC is right angled at A. AB = 3 units, AC = 4 units and AD is perpendicular to BC. What is the area of the triangle ADB?

(A) $\frac{5\sqrt{11}}{8}$ sq. unit **(B)** $\frac{54}{25}$ sq. units

(C) $\frac{72}{25}$ sq. units **(D)** $\frac{96}{25}$ sq. units

Q.24. If A and B are end points of diameter of a circle with centre at P, and C is a point on the circumference of the circle such that $\angle ABC = 35^o$, then what is $\angle PCA$?

(A) 25^o **(B)** 30^o

(C) 35^o **(D)** 55^o

Q.25. If AB is a line of length 3a, with M as midpoint. Semicircles are drawn on one side with AM, MB and AB as diameters. A circle with centre O and diameter 2r is drawn such that this circle touches all the three semicircles. What is the value of r?

(A) $\frac{2a}{3}$ **(B)** $\frac{a}{2}$

(C) $\frac{a}{3}$ **(D)** $\frac{a}{4}$

Q.26. The ratio of an interior angle to an exterior angle of a regular polygon 7 : 2. The number of sides of the polygon is-

(A) 6 **(B)** 9

(C) 7 **(D)** 8

Q.27. The ratio of the measure of an interior angles of a regular octagon to the measure of each of its exterior angles is-

(A) 1 : 2 **(B)** 1 : 3

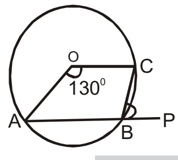
(C) 2 : 3 **(D)** 3: 1

Q.28. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ=10 cm, then AB = ?

(A) $6\frac{2}{3}$ cm **(B)** $\frac{10\sqrt{6}}{3}$ cm

(C) 15 cm **(D)** $66\frac{2}{3}$ cm

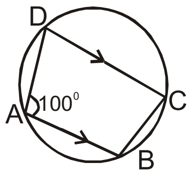
Q.29. If the given figure, O is the centre of a circle and arc ABC subtends an angle of 130^0 at O. AB is extended to P. Then, $\angle PBC$ = ?



(A) 75^0 **(B)** 70^0

(C) 65^0 **(D)** 80^0

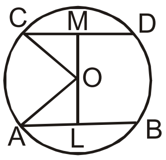
Q.30. In the given figure, ABCD is a cyclic quadrilateral in which AB || DC and $\angle$BAD = 100^0. Then, $\angle$ABC = ?



(A) 75^0 **(B)** 125^0

(C) 100^0 **(D)** 180^0

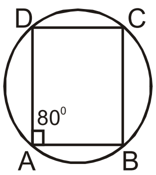
Q.31. AB and CD are two parallel chords on the opposite sides of the centre of the circle. If AB = 10 cm, CD = 24 cm and the radius of the circle is 13 cm, the distance between the chords is :



(A) 17 cm **(B)** 15 cm

(C) 16 cm **(D)** 18 cm

Q.32. In a cyclic quadrilateral ABCD, $\angle$A = 80^0, Then $\angle$C = ?



(A) 80^0 **(B)** 160^0

(C) 100^0 **(D)** 120^0

Q.33. In triangle ABC $\angle$A + $\angle$B = 65^0 and $\angle$B+ $\angle$C = 140^0 Then $\angle$B = ?

(A) 25^0 **(B)** 35^0

(C) 40^0 **(D)** 45^0

Q.34. The number of sides in two regular polygon are in the ratio 5 : 4 and the difference between each interior is of polygon is 6 then number of diagonals of regular polygon having larger sides.

(A) 54 **(B)** 60

(C) 90 **(D)** 70

Q.35. ΔABC and ΔDEF are similar and areas be respectively 81 cm^2. and 196 cm^2. If EF = 19.6 cm., BC is-

(A) 11.1 cm. **(B)** 12.6 cm.

(C) 14.8 cm. **(D)** 17 cm.

Q.36. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and the bigger circle at E. Point A is joined to D. The length of AD is -

(A) 20 cm **(B)** 19 cm

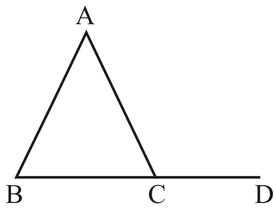
(C) 18 cm **(D)** 17 cm

Q.37. In a triangle DEF, points A, B and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. If $\angle$D = 40^o then what is the value of $\angle$ACB?

(A) 140^o **(B)** 70^o

(C) 100^o **(D)** 135^o

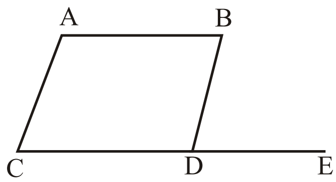
Q.38. In $\angle$ABC, $\angle$ACD = 100° and $\angle$ABC=40°. Find $\angle$BAC.



(A) 40° **(B)** 100°

(C) 80° **(D)** 60°

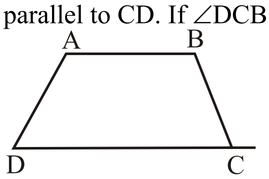
Q.39. In the given figure, ABCD is a parallelogram. Find 2$\angle$ABC-ADC if A = $\angle$135°.



(A) 50° **(B)** 60^C

(C) 45^C **(D)** 75^e

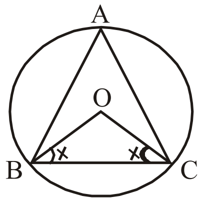
Q.40. ABCD is an isosceles trapezium with lines AB parallel to CD. If $\angle$DCB = 40°, $\angle$BAD equals:



(A) 40° **(B)** 80°

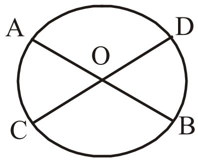
(C) 100° **(D)** 140°

Q.41. In the given figure, ΔABC is circumscribed by a circle having its centre at O. If OB and OC are the angle bisector of $\angle$ABC and $\angle$ACB, respectively, what is the value of $\angle$BAC?



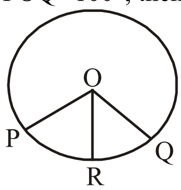
(A) 45° **(B)** 60°
(C) 75° **(D)** 50°

Q.42. In the given figure, AB and CD are two chord-circle intersecting at O. If AO = 4 cm, OB = 6 cm, and OC = 3 cm, then find OD.



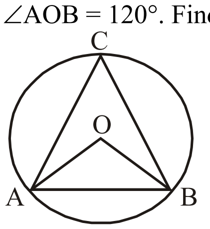
(A) 4 cm **(B)** 6 cm
(C) 8 cm **(D)** 10 cm

Q.43. In the given figure, if 2 ∠POR = 3 ∠ROQ and. Δ POQ=100°, then find ∠POR.



(A) 20° **(B)** 30°
(C) 45° **(D)** 60°

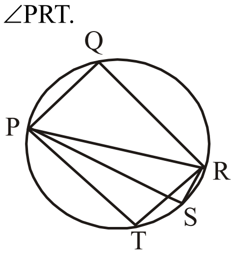
Q.44. In the above figure, O is the centre of the circle, and ∠AOB = 120°. Find ∠ACB.



(A) 60° **(B)** 30°
(C) 45° **(D)** 50°

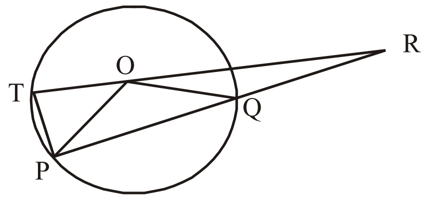
Q.45. PQRS is a cyclic quadrilateral. If PQR is an equilateral triangle, find ∠RSP.
(A) 120° **(B)** 60°
(C) 30° **(D)** 150°

Q.46. In the given figure, PT =TR, ∠PQR = 100°. Find ∠PRT.



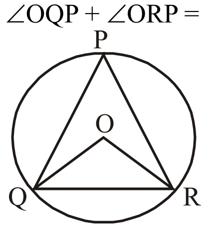
(A) 40° **(B)** 50°
(C) 60° **(D)** 70°

Q.47. In the given figure, O is the centre of the circle, OQ = QR and ∠QRO = 15°. Find ∠QOP.



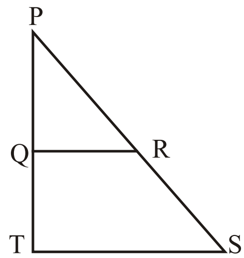
(A) 120° **(B)** 150°
(C) 135° **(D)** 105°

Q.48. In the given figure, O is the centre of the circle and ∠OQP + ∠ORP = 70°. Find ∠ORQ.



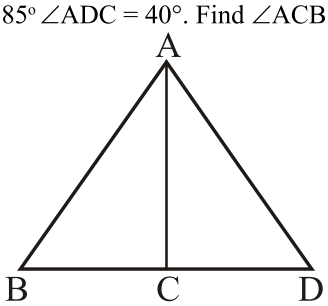
(A) 20° **(B)** 30°
(C) 40° **(D)** 50°

Q.49. In the given figure PQ × PS = PT × PR. If ∠PQR = ∠PST + 30° and ∠PTS = 100°, then find ∠PRQ.



(A) 100° **(B)** 70°
(C) 130° **(D)** 50°

Q.50. In the given figure, ∠CAB = 1/2 ∠ACD, ∠BAD = 85° ∠ADC = 40°. Find ∠ACB.

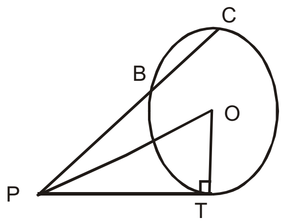


(A) 45° **(B)** 55°
(C) 70° **(D)** 60°

Q.51. How many different regular polygons can be formed with the interior angle exceeding the exterior angle and the sum of the interior angles not exceeding 180°?
(A) 7 **(B)** 8
(C) 6 **(D)** None of these

EXERCISE

Q.1.(A)



$\therefore$ PT= $\sqrt{10^2-6^2}$

$\Rightarrow$ PT=8

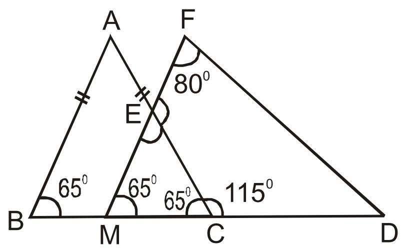
$\therefore$ PT^2=PB×PC

$\Rightarrow$ 64=5(5+BC)

$\Rightarrow$ 64=25+5BC

$\Rightarrow$ BC=7.8 cm

Q.2.(D)



$\because$ AB=AC

$\therefore$ $\angle$ABC=$\angle$ACB=65^0

$\therefore$ $\angle$ACD=180^0-65^0=115^0

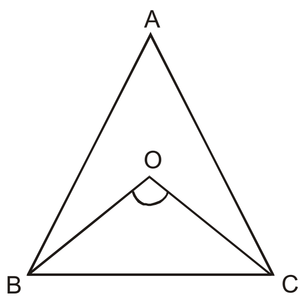
$\angle$EFD=80^0

$\therefore$ $\angle$MEC=180^0-130^0 =50^0

$\therefore$ $\angle$FEC=180^0-50^0 =130^0

$\therefore$ $\angle$FDC=360^0-115^0 -130^0-80^0=35^0

Q.3.(B)



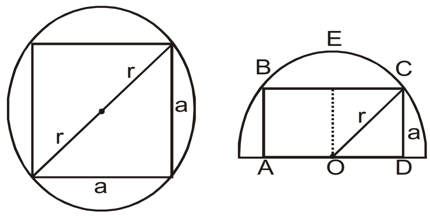
A+B+C=180^0

$$\Rightarrow \frac{B+C}{2} = 90 - \frac{A}{2}$$

$$\therefore \angle BOC = 180^0 - \left(90 - \frac{A}{2}\right)$$

$$\Rightarrow \angle BOC = 90 + \frac{A}{2}$$

Q.4.(C)



In circle of r radius for square of a side

$a^2 + a^2 = (2r)^2$

$a^2 = 2r^2$

and In semicircle of r radius for square of a side

In $\triangle$ OEC,

$(OE)^2 + (EC)^2 = (OC)^2$

$$\Rightarrow a^2 + \left(\frac{a}{2}\right)^2 = r^2 \Rightarrow \frac{5}{4}a^2 = r^2 \Rightarrow a^2 = \frac{4}{5}r^2$$

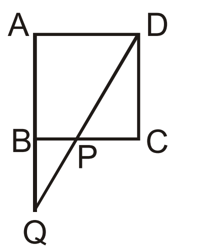
$$\text{Required ratio} = \frac{\frac{4}{5}r^2}{2r^2} = \frac{2}{5}$$

= 2 : 5

Q.5.(B) $\angle BOC = 90 - \frac{1}{2}\angle A$

$= 90 - \frac{1}{2} \times 50 = 65^0$

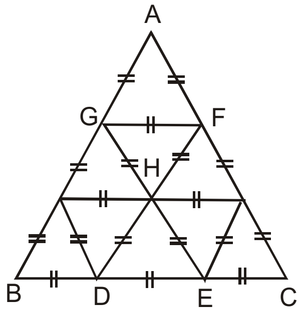
Q.6.(D)



$$\frac{\text{Area of } \Delta BPQ}{\text{Area of } \Delta DPC} = \frac{BP^2}{PC^2}$$

$$\Rightarrow \text{Area of } \Delta DCP = \frac{(2)^2 \times 20}{1^2} = 80 \text{ square unit}$$

Q.7.(B)



Let BC = x, $BD = \frac{x}{3}$

$\therefore$ Area of $\Delta ABC = \frac{\sqrt{3}}{4}x^2$

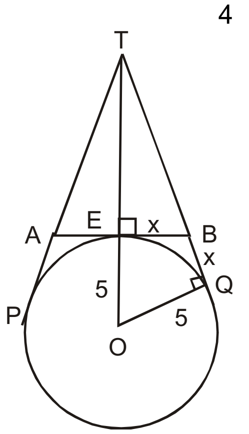
Area of parallelogram AGHF + Area of ΔDHE

$= 2\left(\frac{\sqrt{3}}{4}\times\frac{x^2}{9}\right)+\frac{\sqrt{3}}{4}\times\frac{x^2}{9}$

$= 3\times\frac{\sqrt{3}}{4}\times\frac{x^2}{9} = \frac{x^2\sqrt{3}}{12}$

Required ratio = $\frac{\frac{x^2\sqrt{3}}{12}}{\frac{x^2\sqrt{3}}{4}} = \frac{4}{12} = \frac{1}{3}$

Q.8.(B)



OT = 13 cm.

TQ = $\sqrt{13^2 - 5^2}$ = 12

TB = TQ – BQ $\because$ (BQ = EB)

TB = 12 – x

EB = x

$\because (12 - x)^2 = x^2 + 8^2$

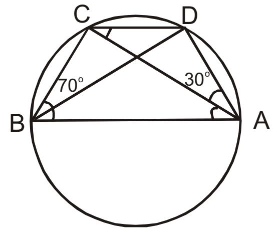
$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow 24x = 80$

$\Rightarrow x = 3\frac{1}{3}$

$\therefore$ AB = $2\times\frac{10}{3} = \frac{20}{3}$ cm.

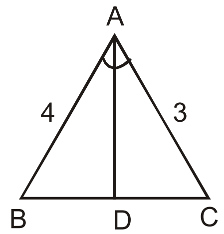
Q.9.(A)



$\angle CBA = 70^o$, $\angle ADC = 180^o - 70^o = 110^o$

$\angle ACD = 180^o - 110^o - 30^o = 40^o$

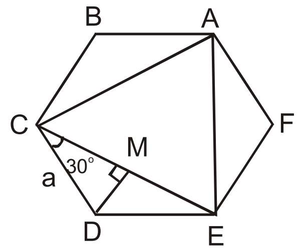
Q.10.(A)



$\frac{AD}{3} = \frac{4}{AD}$

$\Rightarrow AD = 2\sqrt{3}$

Q.11.(C)



In Δ CMD,

$\cos 30^o = \frac{CM}{a}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{2CM}{2a}$

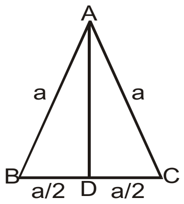
$CM = \frac{\sqrt{3}}{2}\times a$

$\Rightarrow CE = a\sqrt{3}$

$\therefore$ Requred ratio

$$= \frac{\frac{\sqrt{3}}{4}\left(a\sqrt{3}\right)^2}{\frac{3\sqrt{3}}{2}a^2} = \frac{\frac{3\sqrt{3}a^2}{4}}{\frac{3\sqrt{3}}{2}a^2} = 1:2$$

Q.12.(C)



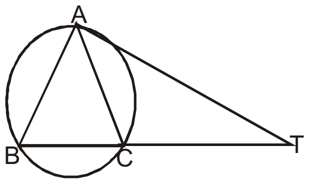
$AB^2 - BD^2 = AD^2$

$\therefore a^2 - \left(\frac{a}{2}\right)^2 = AD^2$

$2a^2 = 4AD^2$

$\Rightarrow 3AB^2 = 4AD^2$

Q.13.(B)



$TA^2 = TC \times TB$

$\frac{TB}{TC} = \frac{TA^2}{TC^2}$

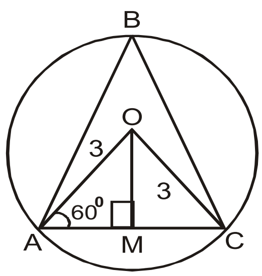
In two similar triangle TAC and ABC

$\frac{TA}{TB} = \frac{TC}{CA}$

$\frac{TA}{TC} = \frac{AB}{AC}$

$\frac{TB}{TC} = \frac{AB^2}{AC^2}$

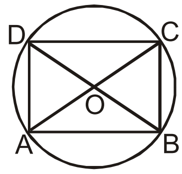
Q.14.(D)



$\therefore \angle AOC = 60^0$

$\therefore \angle ABC = 30^0$

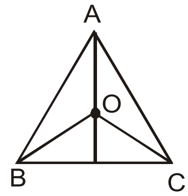
Q.15.(B)



Radius of circle = $\frac{1}{2}AC = \frac{1}{2}\sqrt{4^2 + 4^2}$

$= \frac{1}{2} \times 4\sqrt{2} = 2\sqrt{2}$ cm.

Q.16.(C)



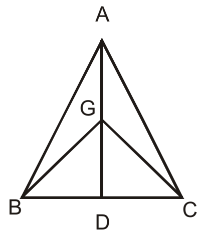
BO is the internal bisector of $\angle B$.

$\angle ODB = 90^o$, $\angle BOD = 15^o$

$\angle OBD = 180^o - 90^o - 15^o = 75^o$

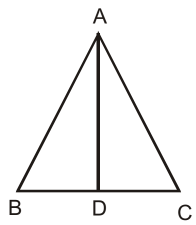
$\angle ABC = 2 \times 75^o = 150^o$

Q.17.(C)



$AG = \frac{2}{3} \times AD = \frac{2}{3} \times 18 = 12$ cm.

Q.18.(D)



In ΔABD and ΔADC

$AB^2 = AD^2 + BD^2$

$AC^2 = AD^2 + DC^2$

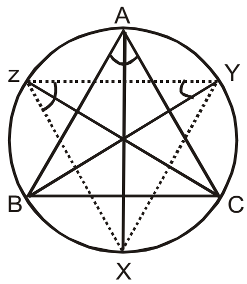
$\therefore AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$= 2BD \times CD + BD^2 + DC^2$

$\Rightarrow AB^2 + AC^2 = (BD+CD)^2 = BC^2$

$\therefore \angle BAC = 90^o$

Q.19.(A)



$\angle$ BYX = $\angle$ BAX

$\therefore \angle BYX = \frac{\angle A}{2} = 30^o$

$\angle$ BYZ = $\angle$ BCZ

$\angle BYZ = \frac{\angle C}{2}$

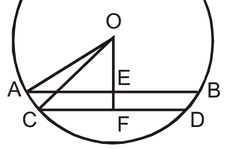
$\angle$ CBY = $\angle$ CZY = 40^o

$\therefore \angle B = 80^o$

$\therefore \angle C = 180^o - 80^o - 60^o = 40^o$

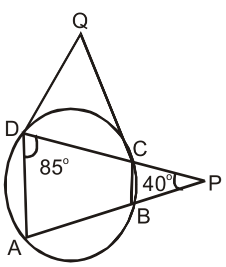
$\therefore \angle BYZ = \frac{40}{2} = 20^o$

Q.20.(A)



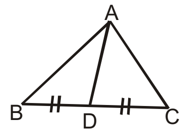
In ΔOEA,
$OA^2 = AE^2 + OE^2$
$10^2 = AE^2 + OE^2$
$OE^2 = 8^2 + OE^2$
$OE^2 = 36$
OE = 6
and In ΔOFC,
$OC^2 = OF^2 + CF^2$
$10^2 = OF^2 + 6^2$
$OF^2 = 64$
OF = 8
Required distance = 8 – 6 = 2cm.

Q.21.(A)



$\angle CBP = \angle ADC = 85^o$
$\angle BCP = 180^o - 40^o - 85^o = 55^o$
$\angle DCB = 180^o - 55^o = 125^o$
$\angle DCQ = 180^o - 125^o = 55^o$
$\angle QDC = 180^o - 85^o = 95^o$
$\therefore \angle DQC = 180^o - 55^o - 95^o = 30^o$

Q.22.(A)



$AB^2+AC^2 = 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]$

$\because AD^2 = CD^2 - AC^2$

$\Rightarrow AB^2+AC^2 = 2\left[CD^2 - AC^2 + \left(\frac{BC}{2}\right)^2\right]$

$\Rightarrow AB^2+AC^2 = 2\left[\left(\frac{BC}{2}\right)^2 - AC^2 + \left(\frac{BC}{2}\right)^2\right]$

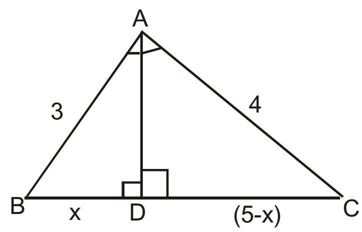
$\because$ BC = 2BD = 2 CD

$\Rightarrow AB^2+AC^2 = 2\left[2\times\frac{BC^2}{4} - AC\right]$

$\Rightarrow AB^2+AC^2 = BC^2-2AC^2$

$\Rightarrow 3AC^2= BC^2-AB^2$

Q.23.(B)



$\therefore (3)^2-(x^2) = 4^2 - (5-x)^2$

$\Rightarrow 9-x^2 = 16 - 25 - x^2 + 10x$

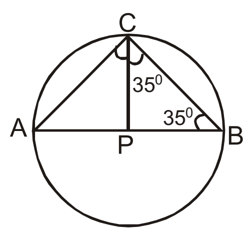
$\Rightarrow x = 1.8$

$\therefore AD = \sqrt{(3)^2 - (1.8)^2} = \sqrt{9 - 3.24}$

$AD = \sqrt{5.76} = 2.4$

$\therefore$ Area Δ ADB = $\frac{1}{2}\times 2.4\times 1.8 = \frac{54}{25}$ sq. unit

Q.24.(D)

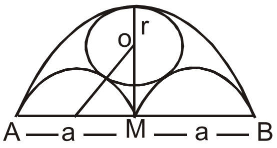


$\angle ACB = 90^0$

$\therefore \angle ACP + \angle BCP = 90^0$

$\therefore \angle ACP = 90^0 - 35^0 = 55^0$

Q.25.(B)



$OM = \frac{3a}{2} - r$

$\therefore \left(\frac{3a}{4}+r\right)^2 = \left(\frac{3a}{2}-r\right)^2 + \left(\frac{3a}{4}\right)^2$

$r = \frac{a}{2}$

Q.26.(B) Sum of interior angle and exterior angle = 180

$7x+2x = 180$

$x = \frac{180}{9} = 20$

So, interior angle $7\times20 = 140^0$

Exterior angle $=2\times20 = 40^0$

Number of sides $= \frac{360}{40} \Rightarrow$ **9**

Q.27.(D) Interior angle of a regular octagon

$= \frac{(2n-4\times90)}{n}$

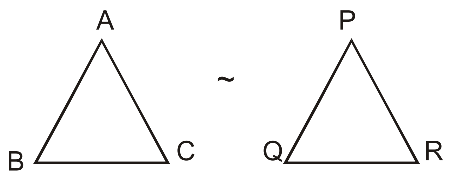
$= \frac{(2\times8-4)\times90}{8}$

$= \frac{12\times90}{8} = 135^0$

Exterior angle$= 180 - 135 = 45^0$

So, ratio = 135 : 45= 3 : 1

Q.28.(C)



$\Delta ABC \sim \Delta PQR$

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

$= \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$

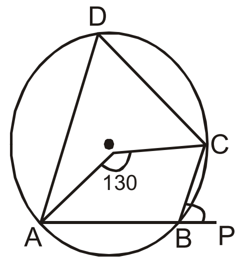
So, $\frac{AB}{PQ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$

$\frac{AB}{PQ} = \frac{36}{24}$

$\frac{AB}{10} = \frac{3}{2}$

$AB = \frac{3\times10}{2}$ = 15 cm.

Q.29.(C) Take a point D on the circumference of the circle



$\angle ACD = \frac{1}{2}\angle AOC$

$= \frac{1}{2}\times130 = 65^0$

So, In cyclic quardilateral ABCD

$\angle ADC+\angle ABC = 180^0$

$65^0 + \angle ABC = 180^0$

$\angle ABC = 180 - 65^0 = 115^0$

$\angle PBC = 180^0 - 115^0 = 65^0$

Q.30.(C) AB||CD

AD is transversal line

$\angle ADC + \angle DAB = 180^0$

$\angle ADC + 100^0 = 180^0$

$\angle ADC = 80^0$

Opposite angle of cyclic quadrilateral are supplimentry

So, $\angle ADC + \angle ABC = 180^0$

$80^0 + \angle ABC = 180^0$

$\angle ABC = 100^0$

Q.31.(A) Distance between AB and CD

is OM+OL = ML

$MO^2 = OC^2 - CM^2$

$= (13)^2 - \left(\frac{24}{2}\right)^2 = 169 - 144 = 25$

$OM = \sqrt{25}$ = 5 cm.

$OL^2 = OA^2 - AL^2$

Where AL = $\frac{1}{2} \times AB = \frac{10}{2} = 5$

OL $= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25} = \sqrt{144} = 12$

Distance ML = 12 + 5 = 17 cm.

Q.32.(C) Oppoiste angle of a cyclic quadrilateral are supplimentry

$\angle A + \angle C = 180^0$

$80 + \angle C = 1800 \Rightarrow \angle C = 100^0$

Q.33.(A) $(\angle B + \angle B) + (\angle B + \angle C) = (65+140) = 205^0$

$(\angle A + \angle B + \angle C) + \angle B = 205^0$

$180^0 + \angle B = 205 – 180 = 25^0$

Q.34.(C) According to the question

$$\frac{(5x-2)\times 180^0}{5x} - \frac{(4x-2)\times 180^0}{4x} = 6^0$$

$$\frac{20x - 8 - 20x + 10}{20x} = \frac{6}{180}$$

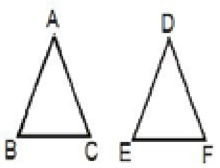
$$2 = \frac{2}{3}x, \qquad x = 3$$

Required number of sides are $= 5 \times 3, 4 \times 3 = 15, 12$

Number of diagonals in polygon with larger sides

$$= \frac{15(15-3)}{2} = \frac{15\times 12}{2} = 90$$

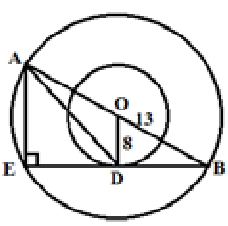
Q.35.(B)



$\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ DEF}} = \frac{(BC)^2}{(EF)^2} \Longrightarrow \frac{81}{196} = \frac{(BC)^2}{(EF)^2}$

$\Rightarrow \frac{9}{14} = \frac{BC}{19.6} \Rightarrow BC = \frac{9\times 19.6}{14} \Rightarrow BC = 12.6$ cm.

Q.36.(B)



$\angle$ AEB = 90^o (AB is a diameter)

In ΔODB,

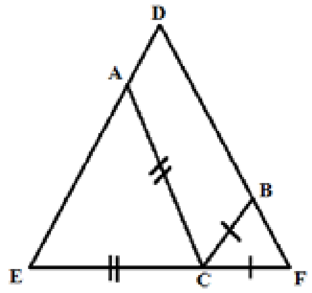
BD = $\sqrt{13^2 - 8^2} = \sqrt{105}$

BE = $2\sqrt{105}$

AE = $\sqrt{676 - 420}$ = 16 cm.

AD = $\sqrt{256 + 105}$ = 19 cm.

Q.37.(C)



EC = AC and CF = BC

$\angle$ CEA = $\angle$ CAE = θ (Let) and $\angle$ CFB = $\angle$ CBF = ϕ (Let)

In Δ DEF,

$\theta + \phi = 180^o - 40^o = 140^o$

Now, $\angle ACB = 180^o - (\angle ACE + \angle BCF)$

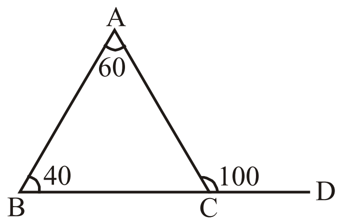
$= 180^o - (180^o - \theta - \theta + 180^o - \phi - \phi)$

$= 180^o - 360^o + 2\theta + 2\phi$

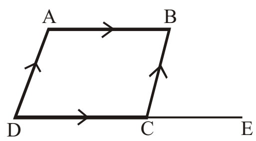
$= 2(\theta + \phi) - 180^o$

$= 2 \times 140^o - 180^o = 100^o$

Q.38.(D)



Q.39.(C)

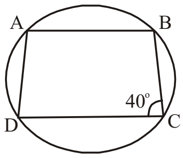


Since $\angle BAC = 135°$

$\angle ABD = \angle ADC = 45°$

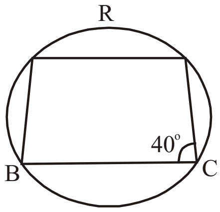
Hence, $2\angle ABC - \angle ADC = 45^0$

Q.40.(D)



Isosceles trapezium is always cyclic.
The sum of opposite angles of a cyclic quadrilateral is 180°.
$\therefore \angle BCD + \angle BAD = 180°$
$40° + \angle BAD = 180°$
$\angle BAD = 140°$

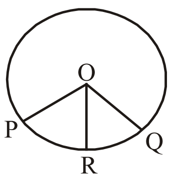
Q.41.(C)



6x= 180° *x = 30°*
$\angle BAC = 2x = 2 \times 30 = 60°$

Q.42.(C) $AO \times OB = OC \times OD$ $4 \times 6 = 3 \times OD$
OD = 8 cm

Q.43.(D)



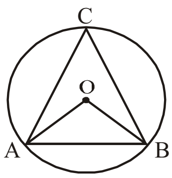
Let $\angle POR = x$

Therefore, $\angle ROQ = \frac{2}{3}x$

$x + \frac{2}{3}x = 100$ $\frac{5x}{3} = 100$

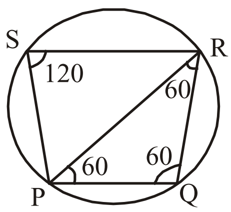
$x = 60°$, Therefore, $\angle POR = 60°$

Q.44.(A)

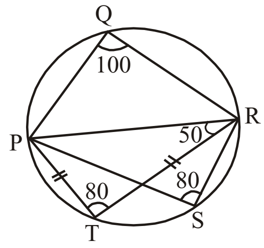


$\frac{1}{2}\angle AOB = \angle ACB$, $\angle ACB = 60°$

Q.45.(A)

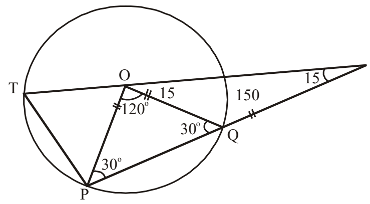


Q.46.(B)



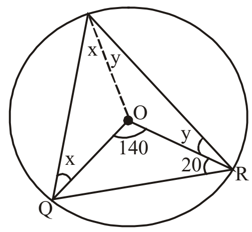
$\angle PRT = 50$

Q.47.(A)



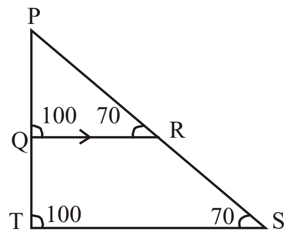
$\angle OQP = 30°$ Therefore, $\angle OQP = 120°$

Q.48.(A)



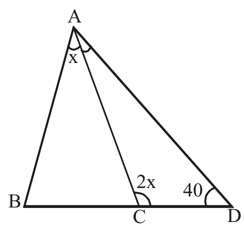
$\angle OQP + \angle ORP = 70°$
$x + y = 70°$
$\angle OPR = 70°$ $\angle ORQ = 20°$

Q.49.(B)



$PQ \times PS = PT \times PR$
QR II TS
$\angle PQR = \angle PST + 30°$
$\angle PST = 70°$
$\angle PRQ = 70°$

Q.50.(C)



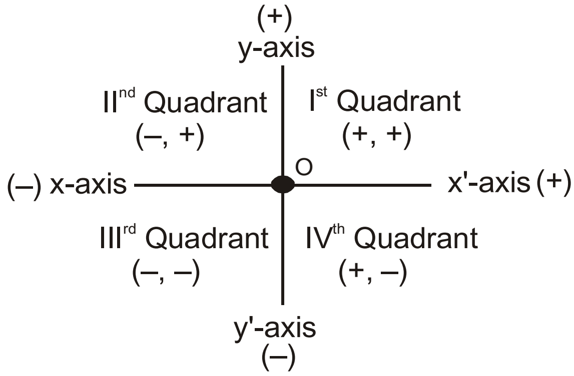
$80° - x + 2x + 40° = 180°$
$x = 55^0$
$\angle ACB = 70°$

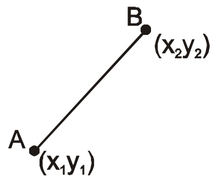
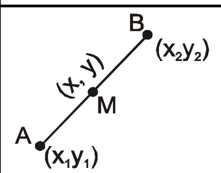
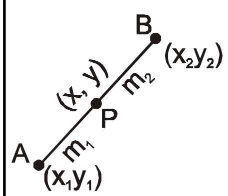
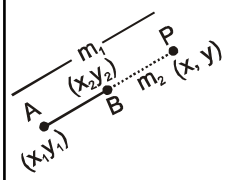
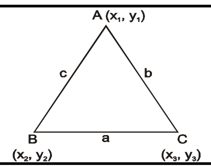
Q.51.(B) Only 8 such regular polygons are possible.

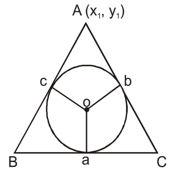
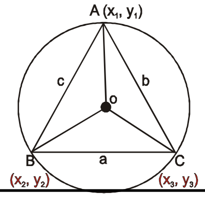
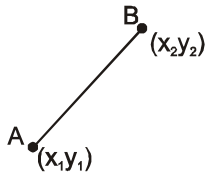
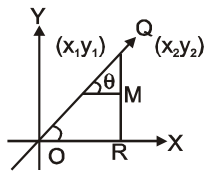
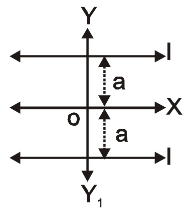
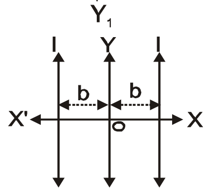
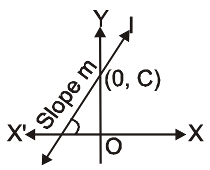
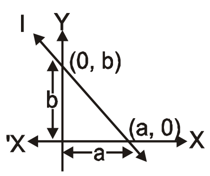
CHAPTER-21
CO-ORDINATE GEOMETRY

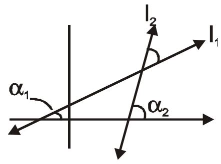
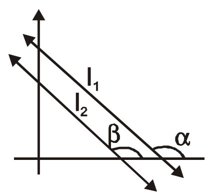
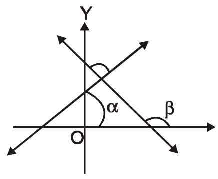
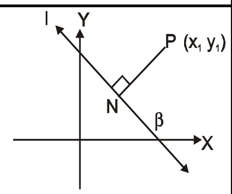
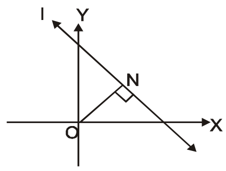
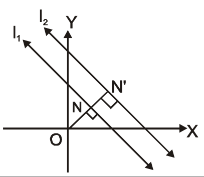
Scan the QR code to get video of this chapter.

The horizontal line is labelled 'X' and is called the x-axis. The vertical line is labelled 'Y' and is called y-axis. The point of intersection of the two axis O is the origin. We mark a number scale with the zero point at the origin on each of the axis. This set of axis forms a co-ordinate system. With the help of co-ordinate geometry we will discuss here the properties of different types of figures and its related formulas. Which is listed below.



S.N.	Name	Properties	Figure	Formula
1	Distance Between Two points	The distance between two points A (x_1, y_1) and B (x_2, y_2)		$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
2	Mid Point	The co-ordinate of the mid point $M_{(x, y)}$ of the line segment joining the two points A (x_1y_1) and B (x_2y_2).		$M_{(x,y)}=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
3	Section	**(A)** The co-ordinate of a point P (x, y) dividing the line segment joining the two point A, (x_1, y_1) and B (x_2, y_2) internally in the ratio of m_1 and m_2 **(B)** The co-ordinate of a point P (x, y) dividing the line segment joining the two points A, (x_1, y_1) and B (x_2, y_2) Externally in the ratio of m_1 and m_2	 	$P_{(x,y)}=\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2},\dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ $P_{(x,y)}=\left(\dfrac{m_1x_2-m_2x_1}{m_1-m_2},\dfrac{m_1y_2-m_2y_1}{m_1-m_2}\right)$
4	**Centre of triangle**	**(A) Centroid :** If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the co-ordinates of its centroid are-		$\text{Centroid}=\left(\dfrac{x_1+x_2+x_3}{3},\dfrac{y_1+y_2+y_3}{3}\right)$

S.N.	Name	Properties	Figure	Formula
4	**(B) Incenter :** **(C) Cicumcenter**	If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of a ΔABC and BC = a, CA = b and AB= c, then the co-ordinates of its incentre are If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of a ΔABC, then the co-ordinates of its circumcentre are	 	$\left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$ $\left(\dfrac{x_1\sin 2A+x_2\sin 2B+x_3\sin 2C}{\sin 2A+\sin 2B+\sin 2c}, \dfrac{y_1\sin 2A+y_2\sin 2B+y_3\sin 2C}{\sin 2A+\sin 2B+\sin 2c}\right)$
5.	Straight Line	A straight line is a curve such that every point on the line segment joining any two points on it lies on it.		$(y-y_1)=\left(\dfrac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$ General equation of line ax + by + c = 0
6.	Slope of Line	A line when co-ordinates of any two points on the line are given		Slope of line $m=\tan\theta=\left(\dfrac{y_2-y_1}{x_2-x_1}\right)$ Equation of line $(y - y_1) = m (x-x_1)$
7.	Horizontal and Vertical Line	**(A)** A horizontal line l is 'a' distance from the x-axis **(B)** A vertical line l is 'b' distance from Y axis	 	Line parallel to X axis then y = a or y = -a Line parallel to y axis then x = b or x = -b
8.	Slope Intercept form of line	A line l with slope m cuts the y axis is at a distance C from the orgin.		Equation of line y = mx + c
9.	Intercept form of line	A line l makes intercepts a and b in x axis and y axis at the point (a, 0) and (0, b)		Equation of line $\dfrac{x}{a}+\dfrac{y}{b}=1$

S.N.	Name	Properties	Figure	Formula
10	Angle between two lines	If two lines intersect then angle between two lines inclined of their slope		$m_1 = \tan\alpha_1$ $m_2 = \tan\alpha_2$ $\tan\theta = \frac{m_2 - m_1}{1 + m_1.m_2}$
11.	Condition for parallel lines	If two lines are parallel then their inclination are equal		$m_1 = m_2$ $\tan\alpha = \tan\beta$ $l_1 = a_1x + b_1y + c_1 = 0$ $l_2 = \tan\alpha\ x + b_2y + c_2 = 0$ For II $\frac{a_1}{a_2} = \frac{b_1}{b_2}$
12.	Condition for Perpendicular lines	If two line are perpendicular to each other if and only if their slopes are negative reciprocal of each other		$m_1 = \frac{-1}{m_2}$ OR $m_1m_2 = -1$ $a_1a_2 = b_1.b_2$
13.	Length of Perpendicular or Distance a point from a line	The length of perpendicular drawn from a given point (x_1, y_1) to a line ax + by +c = 0		$PN = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}}$
14.	Distance from origin	The length of perpendicular drawn from the origin to the line ax + by + c = 0 is given by		$ON = \frac{\lvert c \rvert}{\sqrt{a^2 + b^2}}$
15.	Distance between two parallel Lines:	Two lines are parallel, the distance between them will always be the same		$NN' = \frac{\lvert c_1 - c_2 \rvert}{\sqrt{a^2 + b^2}}$
16.	Area of Triangle	The cordinates of vertices of triangle ABC are (x_1, y_1), (x_2, y_2) and $(x_3, y_3,)$ respectively then the area of the triangle		$\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Ex. If we plot point A (7, 8) and B (–5, 3) on graph, then find the distance between AB.

(A) 12 **(B) 6**

(C) 7 **(D) 13**

Sol.(D) A = (7, 8) and B = (–5, 3)

d = $\sqrt{(7+5)^2+(8-3)^2}$ = $\sqrt{169}$ = 13

Ex. If the point (3, 4) lies on the equation 3y = ax + 7, then find the value of a?

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$

(C) $\frac{5}{3}$ **(D)** $\frac{1}{7}$

Sol.(C) 3y = ax + 7

(3, 4) lies on the equation putting

x = 3, y = 4

3×4 = a × 3 + 7

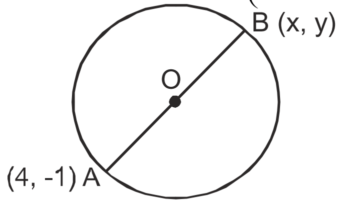
a = $\frac{5}{3}$

Ex. The co-ordinates of one end point of diameter of a circle are (4, – 1) and the co-ordinates of the centre of the cirlce are (1, – 3). Find the co-ordinates of the other end of the diameter.

(A) (–1, – 5) **(B)** (–2, – 5)

(C) (–3, – 5) **(D)** (–4, – 5)

Sol.(B) Centre of mid point = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$



1= $\frac{4+x}{2}$

x = – 2

$-3=\frac{-1+y}{2}$

y = –5

(–2, – 5)

Ex. Find the area of ΔABC of which vertices are A (1, 4), B (–2, 2) and C (3, 2).

(A) 9 unit² **(B)** 3 unit²

(C) 6 unit² **(D)** 5 unit²

Sol.(D) Area of ΔABC

$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[1(2-2)-2(2-4)+3(4-2)\right]$

= 5 unit square

Ex. Find the area of triangle formed by the lines y = x, x = 6 and y = 0.

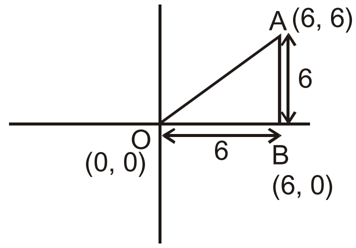
(A) 9 unit² **(B)** 3 unit²

(C) 18 unit² **(D)** 5 unit²

Sol.(C) y = x ....(I)

x = 6 .... (II)

y = 0 .... (III)



Area of triangle (OAB)

$=\frac{1}{2}\times OB\times AB$

$=\frac{1}{2}\times 6\times 6$

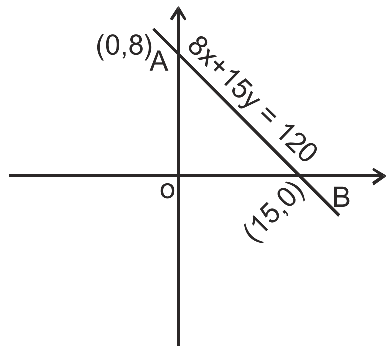
= 18 unit square

Ex. Find the length of the portion of the straight line 8x+15y= 120 intercepted between the axis.

(A) 15 unit **(B)** 14 unit

(C) 18 unit **(D)** 17 unit

Sol.(D)



8x+15y= 120

$$\frac{x}{15}+\frac{y}{8}=1$$

Compare it with the equation

$$\frac{x}{a}+\frac{y}{b}=1$$

a = 15, b = 8

Length of portion (AB) = $\sqrt{a^2+b^2}$

= $\sqrt{15^2+8^2}$ = 17 unit

EXAMPLES

Ex.1. P and Q are such points in XY plane whose co-ordinates are (2, 0) and (5, 4) respectively. What is the numerical value of area of circle whose radius is PQ.

(A) 14π **(B)** 25π

(C) 16π **(B)** 32π

Sol.(B) Distance between the point (x_1, y_1) and (x_2, y_2)

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Distance between the point (2, 0) and (5, 4)

$$PQ=\sqrt{(2-5)^2+(0-4)^2}$$

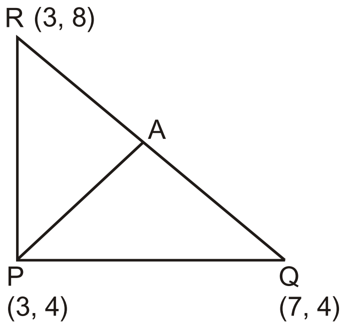
$$=\sqrt{9+16}$$

$$=\sqrt{25} = 5 \text{ unit}$$

Area of circle $=\pi r^2$

$$=\pi(5)^2$$
$$=25\pi$$

Ex.2. Co-ordinates of right angled triangle are P (3, 4), Q (7, 4) and R (3, 8) is right angled at point P. What are the co-ordinates of circumcentre of Δ PQR.



(A) (5, 6) **(B)** (3, 4)

(C) (7, 4) **(D)** (3, 8)

Sol.(A) Circumcentre of right angled triangle is mid point of hypotenuse.

Let mid point of hypotenuse = A

Co-ordinates of A $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$$=\left(\frac{7+3}{2},\frac{4+8}{2}\right)$$

= (5, 6)

Ex.3. At which point the graph of equation 25x + 75y = 225 and x = 9 meets ?

(A) (0, 9) **(B)** (9, 0)

(C) (3, 0) **(D)** (0, 3)

Sol.(B) Equation 25 x + 75 y = 225

Put x = 9

25 × 9 + 75y = 225

75y = 225-225

y = 0

So, graph meets at point (9, 0)

Ex.4. Find the point at which lines 2x+y = 5 and x+2y = 4 intersect each-other ?

(A) (1, 2) **(B)** (2, 1)

(C) (5/2, 0) **(D)** (0, 2)

Sol.(B) Given 2x+y = 5.........(i)

x+2y = 4..............(ii)

On solving both the equation

x = 2, y = 1

Ex.5. For what value of K equation Kx + 2y = 2 and 3x+y = 1 coincides ?

(A) 2 **(B)** 3

(C) 5 **(D)** 6

Sol.(D) Let two equations are $a_1x + b_1y+c$ = 0........(i)

And $a_2x + b_2y + c$ = 0.............(ii)

So, for coinside value

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{k}{3} = \frac{2}{1}$$

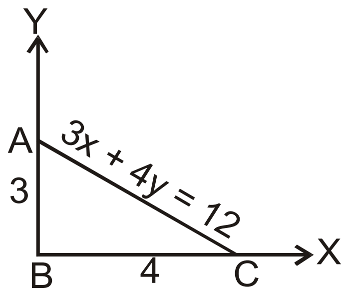
K = 3 × 2 = 6

Ex.6. What is the area of triangle (in square unit) made by line 3x + 4y = 12, x axis and y axis ?

(A) 8 **(B)** 4

(C) 12 **(D)** 6

Sol.(D)



Equation = 3x + 4y = 12

Figure made on x- axis

y = 0

3x = 12

x = 4

Figure made on y-axis

x = 0

4y = 12

y = 3

Area of triangle = $\frac{1}{2} \times$ base × height

$= \frac{1}{2} \times 4 \times 3$ = 6 unit square

Ex.7. The vertices of a triangle are (1, 5), (2, 4) and (-2, 6). Find the co-ordinates of circumcentre of the triangle.

(A) (2, 1) **(B)** $\left(\frac{1}{3}, 5\right)$

(C) (5, 5) **(D)** (-2, 1)

Sol.(D) Let the circumcentre of the triangle is (x, y), then OA = OB = OC⇒ radius

$\therefore$ $OA^2 = OB^2$

$\Rightarrow (x-1)^2 + (y-5)^2 = (x-2)^2 + (y-4)^2$

$\Rightarrow x^2 + 1 - 2x + y^2 + 25 - 10y$

$= x^2 + 4 - 4x + y^2 + 16 - 8y$

$\therefore$ 4x - 2x - 10y + 8y = 4 + 16 - 1 -25

2x - 2y = -6...........(i)

Now OA = OC ⇒ $OA^2 = OC^2$

$\therefore (x-1)^2 + (y-5)^2 = (x+2)^2 + (y-6)^2$

$\therefore x^2 + 1 - 2x + y^2 + 25 - 10y$

$= x^2 + 4 + 4x + y^2 + 36 - 12y$

$\therefore$ -2x - 10 y- 4x + 12y = 4 + 36 - 1 -25

$\therefore$ -6x + 2y = 14.............(ii)

Solving equation (i) and equation (ii)

We get x = -2 and y = 1

$\therefore$ Circumcentre of the triangle

(x, y) = (-2, 1)

Ex.8. The vertices of ΔABC are (1, 0), (5, 0) and (1, 3). Find the co-ordinates of incentre of ΔABC

(A) (-2, -1) **(B)** (-1, 2)

(C) (2, 1) **(D)** (-2, 3)

Sol.(C) In ΔABC, A = (1, 0), B = (5, 0), C = (1, 3)

$\therefore$ Length of the side AB

$= \sqrt{(1-5)^2 + (0-0)^2}$

$c = \sqrt{16+0} = \sqrt{16} = 4$ units.

Length of the side BC

$= \sqrt{(5-1)^2 + (0-3)^2}$

$a = \sqrt{16+9} = \sqrt{25} = 5$ units

Length of the side CA

$= \sqrt{(1-1)^2 + (3-0)^2}$

$b = \sqrt{0+9} = \sqrt{9} = 3$ units

$\therefore$ co-ordinates of incentre of the ΔABC

$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$= \left(\frac{5\times1+3\times5+4\times1}{5+3+4}, \frac{5\times0+3\times0+4\times3}{5+3+4}\right)$

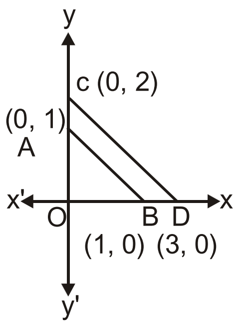
$= \left(\frac{5+15+4}{12}, \frac{0+0+12}{12}\right) = \left(\frac{24}{12}, \frac{12}{12}\right) = (2,1)$

Ex.9. The area bounded by the lines x = 0, y = 0, x+y = 1 and 2x + 3y = 6 (in square units) is-

(A) 2 **(B)** $2\frac{1}{3}$

(C) $2\frac{1}{2}$ **(D)** 3

Sol.(C)



x = 0 is the euqation of y-axis..

y = 0 is the equation of x-axis.

Putting x = 0 in x + y = 1, y = 1

Putting y = 0 in x + y = 1, x = 1

Putting x = 0 in 2x + 3y = 6

$3y = 6 \Rightarrow y = 2$

Putting y = 0 in 2x + 3y = 6

$2x = 6 \Rightarrow x = 3$

$\therefore$ OB = 1, OA = 1

OD = 3, OC = 2

Required area = $\Delta OCD - \Delta OAB$

$= \frac{1}{2}\times3\times2 - \frac{1}{2}\times1\times1$

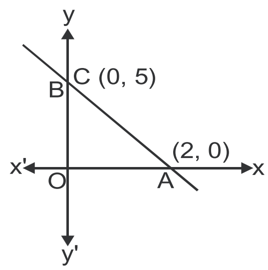
$= 3 - \frac{1}{2} = 2\frac{1}{2}$ units^2

Ex.10. The area of the triangle formed by the graph of 5x + 2y = 10, x-axis and y-axis (in sq. units) is-

(A) 4 **(B)** 12

(C) 5 **(D)** 8

Sol.(C)



x- axis y = 0, putting in equation 5x +2y

= 10

$5x = 10 \Rightarrow x = 2$

$\Rightarrow$ Co-ordinates of point of intersection on x-axis = (2, 0)

Putting x = 0 in the equation 5x+2y = 10

$2y = 10 \Rightarrow y = 5$

$\therefore$ Co-ordinates of point of intersection on y - axis = (0, 5)

$\therefore$ OA = 2

OB = 5

$\therefore$ Area of Δ OAB

$= \frac{1}{2}\times OA\times OB = \frac{1}{2}\times5\times2 = 5$ units^2

Ex.11. The area in sq. unit of the triangle formed by the graphs of x = 4, y = 3 and 3x + 4y = 12 is

(A) 12 **(B)** 8

(C) 10 **(D)** 6

Sol.(4) x = 4, a straight line parallel to y - axis.

y = 3, a straight line parallel to x-axis.

Putting x = 0 in 3x + 4y = 12

$3 \times 0 + 4y = 12$

$\Rightarrow 4y = 12 \Rightarrow y = \frac{12}{4} = 4$

$\therefore$ Point of intersection on y-axis

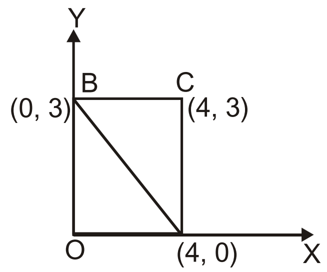
= (0, 4)

Again, putting y = 0 in 3x + 4y = 12

$3x + 4 \times 0 = 12$

$\Rightarrow 3x = 12 \Rightarrow x = \frac{12}{3} = 4$

$\therefore$ Point of intersection on x-axis = (4, 0)



Area of OACB = OA × OB

= 4 × 3 = 12 units^2

Area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 4 \times 3 = 6$ unit^2

Area of ΔABC = 12 -6 = 6 units^2

Ex.12. Area of the triangle formed by the graph of the straight lines x-y = 0, x+y = 2 and the x-axis is-

(A) 1 unit^2 **(B)** 2 unit^2

(C) 4 unit^2 **(D)** None of these

Sol.(A) On putting x = 0 in

x + y = 2.

$0 + y = 2 \Rightarrow y = 2$

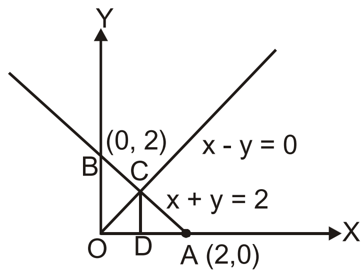
Point of intersection on y-axis = (0, 2)

Again, putting y = 0 in x + y = 2.

x = 2.

Point of intersection on x-axis = (2, 0)

x - y = 0 will pass through origin and be equally inclined to axis.



On putting x = y in x + y = 2

$2y = 2 \Rightarrow y = 1$

CD = 1

PA = 2

Area of $\Delta OAC = \frac{1}{2} \times OA \times CD$

$= \frac{1}{2} \times 2 \times 1 = 1$ units^2

Notes

EXERCISE

Q.1. What is the reflection of the point (-1, 3) in the line x = -4?

(A) (-7, -3) **(B)** (-7, 3)

(C) (7, -3) **(D)** (7, 3)

Q.2. The coordinates of the centroid of a triangle ABC are (1, -4). What are the coordinates of vertex C if coordinates of A and B are (3, -4) and (0, 5) respectively?

(A) (0, 13) **(B)** (0, 5)

(C) (0, -5) **(D)** (0, -13)

Q.3. ax + 5y = 8 has slope of $-\frac{4}{3}$. What is the value of a?

(A) $\frac{-20}{3}$ **(B)** $\frac{20}{3}$

(C) $\frac{18}{3}$ **(D)** None of these

Q.4. If there are four lines in a plane, then what cannot be the number of points of intersection of these lines?

(A) 0 **(B)** 5

(C) 4 **(D)** 7

Q.5. The point P(a, b) is first reflected in origin to P1 and P1 is reflected in Y-axis to (4, -3). What are the coordinates of point P?

(A) (4, 3) **(B)** (-4, 3)

(C) (3, 4) **(D)** (-3, 4)

Q.6. In what ratio is the segment joining (-1, 3)and (2, -4) divided by the Y-axis?

(A) 2 : 1 **(B)** 1 : 4

(C) 1 : 2 **(D)** 4 : 1

Q.7. What is the slope of the line 2x + 3y = 12?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$

(C) $-\frac{3}{2}$ **(D)** $-\frac{2}{3}$

Q.8. What is the reflection of the point (5, -2) in the line y = -1?

(A) (-7 , -2) **(B)** (-5 , 0)

(C) (5 , 0) **(D)** (-7 , 2)

Q.9. The coordinates of the centroid of a triangle ABC are (-1, 4). What are the coordinates of vertex C if coordinates of A and B are (-3, -1) and (3, 5) respectively?

(A) (-3 , 8) **(B)** (3 , 8)

(C) (-3 , -8) **(D)** (3 , -8)

Q.10. ax + 3y = 6 has slope equal to the line joining the points N (3, – 5) and A (5, 4). What is the value of a?

(A) – 13 **(B)** 13

(C) 13.5 **(D)** – 13.5

Q. 11. The length of the portion of the straight line 8x + 15y = 120 intercepted between the axes is –

(A) 14 units **(B)** 15 units

(C) 16 units **(D)** 17 units

Q.12. The equation of the line passing through the point (1, 1) and perpendicular to the line 3x + 4y – 5 = 0, is:

(A) 3x + 4y – 7 = 0 **(B)** 3x + 4y + k = 0

(C) 3x – 4y – 1 = 0 **(D)** 4x – 3y + 1 = 0

Q.13. If the point (x, y) is equidistant from the points (a + b, b – a) and (a–b, a + b) then bx =?

(A) a^2y **(B)** ay^2

(C) ay **(D)** $a^2 y^2$

Q.14. The ratio in which the line segment joining A (3, –5) and B (5, 4) is divided by x– axis is:

(A) 4: 5 **(B)** 5: 4

(C) 5: 7 **(D)** 6: 5

Q.15. The ratio in which the line segment joining P (– 3, 7) and Q (7, 5) is divided by y– axis is:

(A) 3: 7 **(B)** 4: 7

(C) 3: 5 **(D)** 4: 5

Q.16. The equation of a line passing through the point (5, 3) and parallel to the line 2x – 5y + 3 = 0, is:

(A) 2x – 5y – 7 = 0 **(B)** 2x – 5y + 5 = 0

(C) 2x – 2y + 5 = 0 **(D)** 2x – 5y = 0

Q.17. If the sum of the square of the distance of the point (x, y) from the point (a, 0) and (–a, 0) is $2b^2$, then:

(A) $x^2 + a^2 = b^2 + y^2$ **(B)** $x^2 + a^2 = 2b^2 - y^2$

(C) $x^2 - a^2 = b^2 + y^2$ **(D)** $x^2 + a^2 = b^2 - y^2$

Q.18. The angle between the lines represented by the equations 2y – 12 x – 9 = 0 and 3 y – x + 7 = 0, is:

(A) 30° **(B)** 45°

(C) 60° **(D)** $-\frac{4}{3}$

Q.19. The equation of a line with slope 5 and passing through the point (– 4, 1) is:

(A) $y = 5x + 21$ **(B)** $y = 5x - 21$

(C) $5y = x + 21$ **(D)** $5y = x - 21$

Q.20. The value of a so that the lines $x + 3y - 8 = 0$ and $ax + 12y + 5 = 0$ are parallel is:

(A) 0 **(B)** 1

(C) 4 **(D)** – 4

Q.21. The value of P for which the lines $3x + 8y + 9 = 0$ and $24x + py + 19 = 0$ are perpendicular is:

(A) –12 **(B)** – 9

(C) – 11 **(D)** 9

Q.22. The value of a so that line joining P (– 2, 5) and Q (0, – 7) and the line joining A (– 4, – 2) and B (8, a) are perpendicular to each other is:

(A) –1 **(B)** 5

(C) 1 **(D)** 0

Q.23. If P_1 and P_2 be perpendicular from the origin upon the straight lines $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$ respectively, then the value of $4P_1^{\,2} + P_2^{\,2}$ is:

(A) a^2 **(B)** $2a^2$

(C) $2a^2$ **(D)** $3a^2$

Q.24. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9?

(A) $x + 2y - 6 = 0$ but not $2x + y - 6 = 0$

(B) Neither $x + 2y - 6 = 0$ nor $2x + y - 6 = 0$

(C) $2x + y - 6 = 0$ but not $x + 2y - 6 = 0$

(D) $x + 2y - 6 = 0$ or $2x + y - 6 = 0$

Q.25. The co-ordinates of the point of intersection of the medians of a triangle with vertices P (0, 6), Q (5, 3) and R (7, 3) are:

(A) (4, 5) **(B)** (3, 4)

(C) (4, 4) **(D)** (5, 4)

Q.26. If the distance of the point P(x, y) from A (a, 0) is a + x, then y^2 =?

(A) $2ax$ **(B)** $4ax$

(C) $6ax$ **(D)** $8ax$

Notes

EXPLANATION

Q.1.(B) Reflection of point (x, y) in line x = a is (– x + 2a, y)

Now, Reflection of point (– 1, 3) in line x = – 4

= [1 + 2 (– 4), 3] = (– 7, 3)

Q.2.(A) (1, – 4)

$R^2H = 2\left(\frac{h^2R^2}{H^2}\right).h$

$= H^3 = 2h^3$

$h = \frac{1}{\sqrt[3]{2}}.H$

(x, y) = (0, 13)

Q.3.(B) $\frac{-4}{3} = \frac{-a}{5}$

$a = \frac{20}{3}$

Q.4.(A) Maximum Number of intersection lines by different points in plane

$= \frac{4 \times 3}{2} = 6$

Q.5.(A) P1 = (– a, – b)

Let P2 = (a, – b)

a = 4, b = 3

Q.6.(C) Let the ratio be k: 1

$\frac{2k - 1 \times (-1)}{k+1} = 0$

The abscissa of a point lying on Y-axis must be zero

2k + 1 = 0

$k = \frac{1}{2}$

Required ratio = 1: 2

Q.7.(D) $m = -\frac{2}{3}$

Q.8.(C) Reflection of point (x, y) in line y = a is

(x, – y + 2a)

Now, Reflection of point (5, – 2) in line y = – 1

= [(5, 2 + 2 (– 1)] = (5, 0)

Q.9.(A) $-1 = \frac{x - 3 + 3}{3}$, $4 = \frac{y - 1 + 5}{3}$

x = – 3, y = 8

Q.10.(D) $-\frac{a}{3} = \frac{4+5}{5-3}$

– 2a = 27

a = – 13.5

Q.11.(D) Point of intersection at x -axis = (x, 0)

8x + 15 y = 120

8x + 15 × 0 = 120

x = 15

Point of intersection = (15, 0)

Point of intersection at y-axis = (0, y)

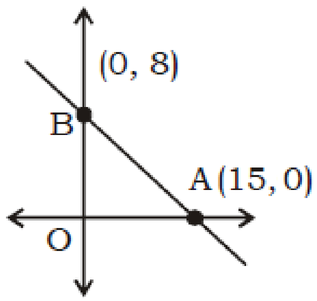
8x + 15 y = 120

0 + 15y = 120

y = 8

Point of intersection = (0, 8)

Required length = AB



$= \sqrt{(15-0)^2 + (0-8)^2} = \sqrt{225+64} = \sqrt{289}$

= 17 units

Q.12.(D) Slope of the given line $= \frac{-3}{4}$

Slope of the perpendicular to the given line $= \frac{4}{3}$

Equation of the line which passes through and having slope is $\frac{4}{3}$

$y - 1 = \frac{4}{3}(x - 1)$

3y – 4x + 1 = 0

Q.13.(C) Let (*x*, y), Q (a + b, b – a) and R (a – b, a + b) are given points.

PQ = PR.

$\sqrt{\{x-(a+b)\}^2 + \{y-(b-a)\}^2} = \sqrt{\{x-(a-b)\}^2 + \{y-(a+b)\}^2}$

$x^2 - 2x(a+b) + (a+b)^2 + y^2 - 2y(b-a) + (b+a)^2 = x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$

$ax + bx + by - ay = ax - bx + ay + by$

$2bx = 2ay$

$bx = ay$.

Q.14.(B) $\frac{4k - 5\times 1}{k+1} = 0$

Let the ratio be k: 1

The ordinate of a point lying on x-axis must be zero

$4k - 5 = 0$

$k = \frac{5}{4}$

Required ratio $= \frac{5}{4} : 1 = 5 : 4$

Q.15.(A) $\frac{7k - 3\times 1}{k+1} = 0$

Let the ratio be k: 1

The abscissa of a point lying on y-axis must be zero

$7k - 3 = 0$

$k = \frac{3}{7}$

Required ratio $= \frac{3}{7} : 1 = 3 : 7$

Q.16.(D) $2x - 5y + 3 = 0$

$y = \left(\frac{2}{5}\right)x + \frac{3}{5}$

its slope $m_1 = \frac{2}{5}$

Let the slope of line which is parallel to the given line is m_2.

$m_2 = m_1 = \frac{2}{5}$

Let the required equation be, y = x+ c

Since, it passes through (5, 3)

$3 = \frac{2}{5} \times 5 + c$

$c = 1$

Required equation is,

$y = \frac{2}{5}x + 1$ or $2x - 5y + 5 = 0$

Q.17.(D) Let A (x, y), P (a, 0) and Q (–a, 0), Then,

$AP^2 + AQ^2 = 2b^2$

$[(x-a)^2 + (y-0)^2] + [(x+a)^2 + (y-0)^2] = 2b^2$

$x^2 + a^2 - 2ax + y^2 + x^2 + a^2 + 2ax + y^2 = 2b^2$

$2(x^2 + a^2 + y^2) = 2b^2$

$x^2 + a^2 + y^2 = b^2$

$x^2 + a^2 = b^2 - y^2$

Q.18.(B) Slope of PQ

$m_1 = \frac{5-5}{4-3} = 0$

Slope of PR

$m_2 = \frac{6-5}{4-3} = 1$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{0-1}{1+0}\right| = 1$$

So, $\theta = 45^\circ$

Q.19.(A) Let the equation be $y = 5x + c$

Since it passes through (– 4, 1), we have

$1 = 5(-4) + c$

$c = 21$

so, its equation is

$y = 5x + 21$

Q.20.(D) $m_1 = m_2$

$\frac{-1}{3} = \frac{-a}{12}$

$a = 4$

Q.21.(D) $m_1 m_2 = -1$

$\frac{-3}{8} \times \frac{-24}{p} = -1$

$p = -9$

Q.22.(D) $m_1 = \text{Slope of PQ} = \frac{-7-5}{0+2} = \frac{-12}{2} = -6$

$m_2 = \text{Slope of AB} = \frac{a+2}{8+4} = \frac{a+2}{12}$

$m_1m_2 = -1$

$$-6\times\frac{a+2}{12}=-1$$

a + 2 = 2

a = 0

Q.23.(A) P_1 = length of perpendicular from (0, 0) on x sec θ + y cosec θ = a

P_1 = (0, 0) , x sec θ + y cosec θ = a

$$P_1 = \frac{a}{\sqrt{\sec^2\theta+\mathrm{cosec}^2\theta}} = \frac{a}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}$$

= a sin θ. cos θ

Similarly

$$P_2 = \frac{a\cos 2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}} = a\cos 2\theta$$

$4P_1^2 + P_2^2 = a^2(\sin^2 2\theta + \cos^2 2\theta) = a^2$

Q.24.(D) Let a and b are the intercepts on x and y axes respectively.

a + b = 9

b = 9 – a .... (i)

and the equation of the line is

$$\frac{x}{a}+\frac{y}{b}=1 \text{ .... (ii)}$$

From (i) and (ii)

$$\frac{x}{a}+\frac{y}{9-a}=1 \text{ .... (iii)}$$

This line also passes through the point (2, 2)

From (iii), $\frac{2}{a}+\frac{2}{9-a}=1$

On solving we get a = 6 or a = 3

If a = 6 then b = 9 – 6 = 3

Equation of the line is $\frac{x}{6}+\frac{y}{3}=1$

or x + 2y – 6 = 0

If a = 3 then b = 9 – 3 = 6

Equation of the line is $\frac{x}{3}+\frac{y}{6}=1$

or 2x + y – 6 = 0

Hence, required equation is

x + 2y – 6 = 0 or 2x + y – 6 = 0

Note : Solve this type of question with the help of given options.

Q.25.(C) Since, point of intersection of median is "centroid".

Co-ordinates of centroid

$$= \left(\frac{0+5+7}{3},\frac{6+3+3}{3}\right)$$

$$= \left(\frac{12}{3},\frac{12}{3}\right)$$

= (4, 4)

Q.26.(B) $\sqrt{(x-a)^2+(y-0)^2} = a+x$

$$(x-a)^2+(y)^2=(a+x)^2$$

$$y^2=(x+a)^2-(x-a)^2$$

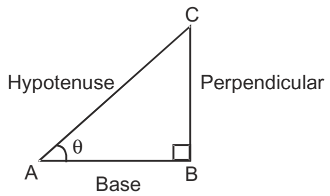
$y^2 = 4ax$

CHAPTER-22
TRIGONOMETRY

Scan the QR code to get video of this chapter.

Trigonometry

In trigonometry we study the ratio of the different sides of a right angled triangle.



According to Pythagorean theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

MEASUREMENT OF ANGLES

Degree measurement :

(Sexagonal system)

When we divide a right angle into 90 equal small parts one part is equal to one degree.

$1^o = 60'$ (minutes)

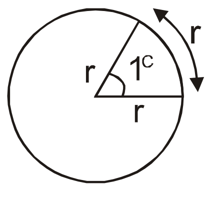
$1' = 60''$ (Seconds)

(Radian mesurement) :

(Circular System)

In a circle, angle subtended by an arc which is equal to the radius, is equal to one radian.

Central angle of a circle



$360^o = (2\pi)^C$

$180^o = \pi^c$

$1^o = \left(\frac{\pi}{180}\right)^C$

$1^C = \left(\frac{180}{\pi}\right)^o$

Ex. If the sum and difference of two angles are $\frac{22}{9}$ radian and 36^o respectively, then the value of smaller angle in degree if taking the value of π as $\frac{22}{7}$ is :

(A) 60^o **(B)** 48^o **(C)** 52^o **(D)** 56^o

Sol.(C) Let angles are x^o, and y^o

and $x^o > y^o$

$x^o + y^o = \left(\frac{22}{9}\times\frac{180}{22}\right)\times 7$

$x^o + y^o = 140$

$x^o - y^o = 36$

$2x^o = 176$

$x^o = 88^o$

$y^o = 52^o$

Ex. When a pendulum of length 50 cm oscillates, it produces an arc of 16 cm. The angle so formed in degree measure is (approx) :

(A) $18^o\ 25'$ **(B)** $18^o\ 35'$
(C) $18^o\ 20'$ **(D)** 18^o

Sol.(C) Angle (radian) $= \frac{\text{arc}}{\text{radius}}$

$\theta = \frac{16}{50} = \frac{8}{25}$ radian

$= \frac{8}{25}\times\frac{180^o}{\pi} = \frac{8}{25}\times\frac{180}{22}\times 7$

$= \frac{1008}{55}$

$= 18\frac{18^o}{55} = 18^o20'$ (approx)

Ex. The circular measures of an angle of an isoceles triangle is $\frac{5\pi}{9}$ circular measures of one of the other angles must be-

(A) $\frac{5\pi}{18}$ **(B)** $\frac{5\pi}{9}$

(C) $\frac{2\pi}{9}$ **(D)** $\frac{4\pi}{9}$

Sol.(C) $\frac{5\pi}{9}\times\frac{180}{\pi}=100$, $180 - 100 = \frac{80}{2}=40$

$40\times\frac{\pi}{180}=\frac{2\pi}{9}$

Ex. By decreasing 15° of each angle of a triangle, the ratios of their angles are 2 : 3 : 5. The radian measures of greatest angle is-

(A) $\frac{11\pi}{24}$ **(B)** $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$ **(D)** $\frac{5\pi}{24}$

Sol.(A) Total decrease = 15×3 = 45°

$180^0 - 45^o = 135^o$

So greatest angle = $\frac{135}{10}\times5+15$

$=\left(\frac{165}{2}\right)^0=\left(\frac{165^o}{2}\right)\times\frac{\pi}{180}=\frac{11\pi}{4}$

Ex. The angle between the hour hand and the minute hand at half past four.

(A) $\frac{\pi}{4}$ radian **(B)** $\frac{\pi}{6}$ radian

(C) $\frac{2\pi}{3}$ radian **(D)** $\frac{\pi}{3}$ radian

Sol.(A) Angle traced by the hour hand in 4 hours and 30 min $\left(\frac{9}{2}\text{hr.}\right)=\frac{360}{12}\times\frac{9}{2}=135^o$

Angle traced by the minute hand in 60 min = 360°

Angle traced by the minute hand in 30 min

$=\left(\frac{360}{60}\times30\right)^o=180^o$

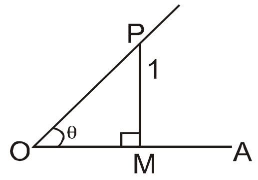
Thus the angle between two hands

= 180° -135° = 45°

$45\times\frac{\pi}{180^o}=\frac{\pi}{4}$

Trigonometrical Ratios

The ratio between different sides of right angle triangle w.r.t its accute angle are called trignometrical ratios.



The trigonometrical ratios, or functions, of angle A are defined as follows :

(i) $\sin\theta=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{PM}{OP}$

(ii) $\cos\theta=\frac{\text{Base}}{\text{Hypotenuse}}=\frac{OM}{OP}$

(iii) $\tan\theta=\frac{\text{Perpendicular}}{\text{Base}}=\frac{PM}{OM}$,

(iv) $\sec\theta=\frac{\text{Hypotenuse}}{\text{Base}}$

(v) $\text{cosec}\,\theta=\frac{\text{Hypotenuse}}{\text{Perpendicular}}$

(vi) $\cot\theta=\frac{\text{Base}}{\text{Perpendicular}}$

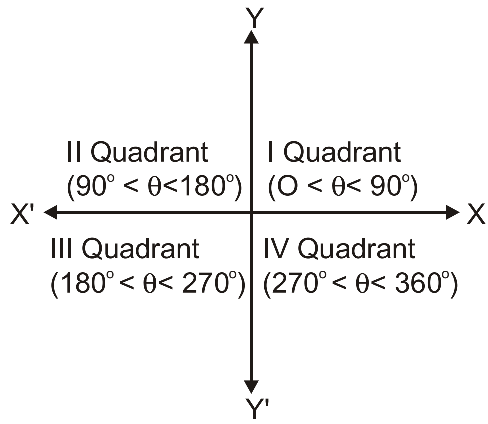
To remember

$\frac{PBP}{HHB}$

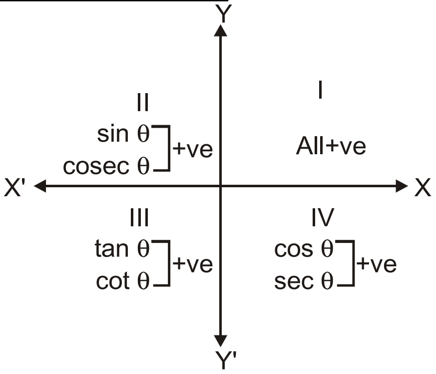
P — Perpendicular
B — Base
H — Hypotenuse

Quadrants

Let XOX' and YOY' be two mutually perpendicular lines. These lines divide the plane into four parts and each one of them is called a quadrant.



Signs of trigonometric ratios :



Always Remember

A S T C

I II III IV

A — All

S — sin θ

T — tan θ

C — cos θ

Tables of standard angles

Value of trigonometric ratios

θ	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
Cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	−1	$-\sqrt{3}$	∞
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	−2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	−1
Cosec θ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

Note : For (90 ± θ) and (270 ± θ)

Trigonometrical ratios change in following ways:

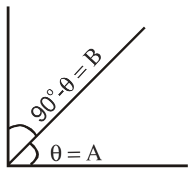
$\sin\theta \rightleftarrows \cos\theta$

$\tan\theta \rightleftarrows \cot\theta$

$\sec\theta \rightleftarrows \operatorname{cosec}\theta$

For other angles no change in trigonometrical ratios.

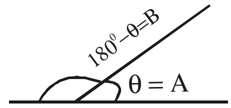
Complementary angles : When sum of the two angles is 90^0 then these angles called complementary angles.



A + B = 90°

Supplementry angles :

When of the sum of the two angles is 180^0 then these angles are supplementry angles.

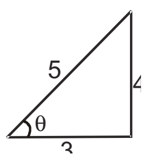


A + B = 180°

Ex. $\cos\theta = \frac{3}{5}$ then value of sin θ. sec θ. tan θ is-

(A) $\frac{19}{16}$ **(B)** $\frac{16}{9}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{3}$

Sol.(B) $\cos\theta = \frac{B}{H}$



$\sec\theta = \frac{H}{B} = \frac{5}{3}$

$\tan\theta = \frac{P}{B} = \frac{4}{3}$

$\sin\theta \cdot \sec\theta \cdot \tan\theta = \frac{4}{5}\times\frac{5}{3}\times\frac{4}{3} = \frac{16}{9}$

Ex. Find the value of

$\tan 4^\circ.\tan 43^\circ.\tan 47^\circ.\tan 86^\circ$

(A) 2 **(B)** $\frac{2}{3}$

(C) 1 **(D)** $\frac{1}{2}$

Sol.(C) ($\tan 4^\circ.\tan 86^\circ$) ($\tan 43^\circ.\tan 47^\circ$)

$= \tan 4^\circ.\cot(90-86)^\circ \tan 43^\circ.\cot(90-43)^\circ$

$= \tan 4^\circ.\cot 4^\circ.\tan 43^\circ.\cot 43^\circ = 1$

Ex. The measure of an angle whose supplement is three times as large as its complement, is-

(A) 45° **(B)** 75°

(C) 60° **(D)** 30°

Sol.(A) Let angle = x

Suplement = $(180^\circ - x)$

Complement = $(90^\circ - x)$

$(90^\circ - x)3 = (180^\circ - x)$

$270^\circ - 3x = 180^\circ - x$

$90^\circ = 2x$

$x = 45^\circ$

Ex. If two supplementary angles differ by 44° then one of the angles is-

(A) 78° **(B)** 72° **(C)** 112° **(D)** 65°

Sol.(C) Let two angles are x and y

$x + y = 180^\circ$

$x - y = 44^\circ$

$2x = 224^\circ$

$x = 112^\circ$

Ex. $\sin 17^\circ = \frac{x}{y}$ then the value of $\sec 17^\circ - \sin 73^\circ$ is-

(A) $\frac{y^2-x^2}{xy}$ **(B)** $\frac{x^2}{\sqrt{y^2-x^2}}$

(C) $\frac{x^2}{y\sqrt{y^2+x^2}}$ **(D)** $\frac{x^2}{y\sqrt{y^2-x^2}}$

Sol.(D) $\sec 17^\circ - \sin 73^\circ$

$= \sec 17^\circ - \cos 17^\circ$

$= \frac{1}{\cos 17^\circ} - \cos 17^\circ$

$= \frac{1-\cos^2 17^\circ}{\cos 17} = \frac{\sin^2 17}{\cos 17} = \frac{y^2}{\sqrt{1-\frac{x^2}{y^2}}}$

$= \frac{x^2}{y^2\sqrt{\frac{y^2-x^2}{y^2}}} = \frac{x^2}{y\sqrt{y^2-x^2}}$

Ex. The value of $\frac{\sin 39^\circ}{\cos 51^\circ} + 2\tan 11^\circ.\tan 31^\circ.\tan 45^\circ.\tan 59^\circ.\tan 79^\circ - 3(\sin^2 21 + \sin^2 69^\circ)$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

Sol.(A) $\Rightarrow \frac{\sin 39^\circ}{\cos 51^\circ} + 2\tan 11^\circ.\tan 79^\circ.\tan 31^\circ.\tan 59^\circ.\tan 45^\circ$

$-3(\sin^2 21^\circ + \sin^2 69^\circ)$

$= \frac{\sin 39^\circ}{\cos(90^\circ - 39^\circ)} + 2\tan 11^\circ.\tan(90^\circ - 11^\circ)$

$.\tan 31^\circ.\tan(90 - 59^\circ) \times$

$1 - 3\left[\sin^2 21^\circ + \sin^2(90-21)\right]$

$= 1 + 2 - 3 = 0$

Trigonometric Identities : Following are the basic trigonometrical identities.

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Some formulae :

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
6. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
7. $\sin 2A = 2\sin A \cos A$
8. $\cos 2A = \cos^2 A - \sin^2 A$
9. $\cos 2A = 1 - 2\sin^2 A$
10. $\cos 2A = 2\cos^2 A - 1$

Ex. The value of following is-

$\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o$

(A) $\frac{1}{2}$ **(B)** 1

(C) 2 **(D)** $-\frac{1}{2}$

Sol.(A) $\Rightarrow (\cos 24^o + \cos 204^o) + (\cos 55^o + \cos 125^o) + \cos 300^o$

$\cos 24^0 + \cos(180^0-24^0) + \cos 55^0 + \cos(180^0-55^0) + \cos(360^0-60^0)$

$= 0 + 0 + \cos 60^o = \frac{1}{2}$

Ex. The value of following is-

$3(\sin^4\theta + \cos^4\theta) + 2(\sin^6\theta + \cos^6\theta)$

$+12\sin^2\theta . \cos^2\theta$

(A) 3 **(B)** 0

(C) 5 **(D)** 2

Sol.(C) $3(1-2\sin^2\theta . \cos^2\theta) + 2(1-3\sin^2\theta . \cos^2\theta) + 12\sin^2\theta . \cos^2\theta$

$= 3 - 6\sin^2\theta . \cos^2\theta + 2 - 6\sin^2\theta . \cos^2\theta + 12\sin^2\theta . \cos^2\theta . = 5$

Ex. $\sec\theta + \tan\theta = 2+\sqrt{5}$ then the value of $\sin\theta$ —

(A) $\frac{2}{\sqrt{5}}$ **(B)** $\frac{\sqrt{3}}{2}$

(C) $\frac{4}{5}$ **(D)** $\frac{1}{\sqrt{5}}$

Sol.(A) $\sec\theta + \tan\theta = 2+\sqrt{5}$

Squaring both sides

$= \sec^2\theta + \tan^2\theta + 2\sec\theta . \tan\theta$

$= 4+5+4\sqrt{5}$

$= 1+\tan^2\theta + \tan^2\theta + 2\sec\theta . \tan\theta$

$= 9+4\sqrt{5}$

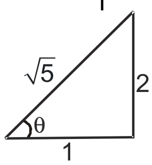
$= 2\tan^2\theta + 2\sec\theta . \tan\theta = 8+4\sqrt{5}$

$= \tan^2\theta + \sec\theta . \tan\theta = 4 + 2\sqrt{5}$

= tan θ **(tan** θ **+ sec** θ**)**

$= \tan\theta . (2+\sqrt{5}) = 2.(2+\sqrt{5})$

$\tan\theta = \frac{2}{1}$



$\sin\theta = \frac{2}{\sqrt{5}}$

Ex. The value of

$$(2\cos^2\theta - 1)\left(\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right)$$

(A) 4 **(B)** 1

(C) 3 **(D)** 2

Sol.(D) $(2\cos^2\theta - 1)\left(\frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{1-\tan^2\theta}\right)$

$= (2\cos^2\theta - 1)\left(\frac{2(1+\tan^2\theta)}{(1-\tan^2\theta)}\right)$

$= \cos 2\theta \frac{2\sec^2\theta}{\frac{(\cos^2\theta - \sin^2\theta)}{\cos^2\theta}}$

$= \cos 2\theta \frac{2}{\cos 2\theta} = 2$

Ex. If $(a^2-b^2)\sin\theta + 2ab\cos\theta = a^2+b^2$ then the value of $\tan\theta$ is

(A) $\frac{1}{2ab}(a^2+b^2)$ **(B)** $\frac{1}{2}(a^2-b^2)$

(C) $\frac{1}{2ab}(a^2-b^2)$ **(D)** $\frac{1}{2}(a^2+b^2)$

Sol.(C) Divide both side by $a^2 + b^2$

$$\frac{(a^2-b^2)}{(a^2+b^2)}\sin\theta + \frac{2ab\cos\theta}{(a^2+b^2)} = \frac{a^2+b^2}{a^2+b^2}$$

$$\frac{(a^2-b^2)}{(a^2+b^2)}\sin\theta + \frac{2\,ab\,\cos\theta}{(a^2+b^2)} = 1$$

Comparising from,

$\sin^2\theta + \cos^2\theta = 1$

$\sin\theta = \frac{(a^2-b^2)}{(a^2+b^2)}$ $\cos\theta = \frac{2ab}{a^2+b^2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a^2-b^2}{2ab} = \frac{a^2-b^2}{2ab}$

Ex. $\sin\theta + \cos\theta = \sqrt{2}$ then the value of θ

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ **(D)** $\frac{\pi}{2}$

Sol.(A) $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides

$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 2$

$1 + 2\sin\theta.\cos\theta = 2$

$2\sin\theta.\cos\theta = 1$

$\sin 2\theta = \sin\frac{\pi}{2}$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

Ex. If $3\sin^2\alpha + 7\cos^2\alpha = 4$

Then the value of $\tan\alpha$ is (where $0^o < \alpha < 90^o$)

(A) $\sqrt{2}$ **(B)** $\sqrt{5}$

(C) $\sqrt{3}$ **(D)** $\sqrt{16}$

Sol.(C) $3\sin^2\alpha + 7(1-\sin^2\alpha) = 4$

$\Rightarrow 3\sin^2\alpha + 7 - 7\sin^2\alpha = 4$

$\Rightarrow 7 - 4\sin^2\alpha = 4$

$\Rightarrow 4\sin^2\alpha = 3 \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$= \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{3}{4}} = \frac{1}{2}$

$\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \sqrt{3}$

Ex. If $a\cos^3\theta + 3a\cos\theta.\sin^2\theta = m$

$a\sin^3\theta + 3a\sin\theta.\cos^2\theta = n$, then

$(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = ?$

(A) $2a^{\frac{1}{3}}$ **(B)** $2a^{\frac{2}{3}}$

(C) $a^{\frac{2}{3}}$ **(D)** $a^{\frac{1}{3}}$

Sol.(B) $m+n = a\cos^3\theta + 3a\cos\theta.\sin^2\theta + a\sin^3\theta + 3a\sin\theta.\cos^2\theta$

$= a(\cos\theta + \sin\theta)^3$

Similarly,

$m - n = a(\cos\theta - \sin\theta)^3$

$\therefore \cos\theta + \sin\theta = \left(\frac{m+n}{a}\right)^{\frac{1}{3}}$ and

$\cos\theta - \sin\theta = \left(\frac{m-n}{a}\right)^{\frac{1}{3}}$

$= (\cos\theta + \sin\theta)^2 (\cos\theta - \sin\theta)^2$

$= 2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow (m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$

Maxima and Minima

Limit of the values of trignometric functions :

(i) $-1 \le \sin\theta \le 1$

(ii) $-1 \le \cos\theta \le 1$

(iii) $-\infty \le \tan\theta \le \infty$

(iv) $\sec\theta \ge 1$ or $\sec\theta \le -1$

(v) $\mathrm{cosec}\,\theta \le -1$ or $\mathrm{cosec}\,\theta \ge 1$

$0 \le \sin^2\theta \le 1$

$0 \le \cos^2\theta \le 1$

$\sin^2\theta \le 1$ sin θ 1≤ 1

- $\sin^{2n}\theta \pm \cos^{2n}\theta \le 1$

  Maximum value of $\sin^n\theta = \left(\frac{1}{2}\right)^n$

- Maximum and Minimum values of

  $a\sin\theta + b\cos\theta$

  Maximum value $= +\sqrt{a^2+b^2}$

  Minimum value $= -\sqrt{a^2+b^2}$

- Minimum value of

  $a\sec^2\theta + b\,\mathrm{cosec}^2\theta = \left(\sqrt{a}+\sqrt{b}\right)^2$

Ex. Find the max value $\sin^{113}\theta\cos^{113}\theta$

(A) 1 **(B)** $\left(\frac{3}{2}\right)^{113}$

(C) $\left(\frac{1}{4}\right)^{113}$ **(D)** $\left(\frac{1}{2}\right)^{113}$

Sol.(D) $\sin^{113}\theta.\cos^{113}\theta$

Maximum value $= \left(\frac{1}{2}\right)^{113}$

Ex. Find the minimum value of

$\sin^2\theta + \mathrm{cosec}^2\theta + \cos^2\theta + \sec^2\theta$

(A) 3 **(B)** 4

(C) 5 **(D)** 6

Sol.(C) $(\sin^2\theta + \cos^2\theta) + (\mathrm{cosec}^2\theta + \sec^2\theta)$

$= 1 + \left(\sqrt{1}+\sqrt{1}\right)^2$

$= 1+4 = 5$

Note : A.M. ≥ G.M.

Ex. If x + y + z = 12 then find max value (x-2) (y +1) (z-3)

(A) 13 **(B)** -4 **(C)** 2 **(D)** 27

Sol.(D) A.M. ≥ G.M.

$$\frac{(x-2)+(y+1)+(z-3)}{3} \geq \sqrt[3]{(x-2)(y+1)(z-3)}$$

$$\frac{x+y+z-4}{3} \geq \sqrt[3]{(x-2)(y+1)(z-3)}$$

$$\frac{13-4}{3} \geq \sqrt[3]{(x-2)(y+1)(z-3)}$$

27 ≥ (x-2) (y+1) (z-3)

27 ≥ (x-2) (y+1) (z-3)

Max value = 27

EXAMPLES

Ex.1. If $0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{2}$ and $\sin\theta = \cos\phi$. Then $\frac{\pi}{2}$ is the value of-

(A) $\frac{\theta}{2}+\frac{\phi}{2}$ **(B)** $\theta + \phi$

(C) $\phi - \theta$ **(D)** $\theta - \phi$

Sol.(B) $\sin\theta = \sin\left(\frac{\pi}{2}-\phi\right)$

$$\Rightarrow \theta = \frac{\pi}{2}-\phi$$

$$\Rightarrow \theta+\phi = \frac{\pi}{2}$$

Ex.2. If sin 2A = cos 3A, then

(A) A = 10° **(B)** A = 20°

(C) A = 30° **(D)** A = 18°

Sol.(D) sin 2A = cos 3A

Hence, 2A + 3 A = 90°

5A = 90° ⇒ A = 18°

Ex.3. The value of cos 20° + cos 40° + cos 60°+..........+ cos 160° +cos 180° is

(A) 1 **(B)** -1

(C) 0 **(D)** -½

Sol.(B) cos (180-θ) = - cos θ

∴ cos 160 = cos (180-20)

= - cos 20°

Similarly,

cos 140° = cos 140° = - cos 40°

cos 100° = - cos 80°

Given expression

(cos 20° + cos 160°) + (cos 40° + cos 140°) + (cos 60° +cos 120°) + (cos 80°+ cos 100°)+ cos 180°

= (cos 20° - cos 20°) + (cos 40°- cos 40°) + (cos 60°- cos 60°) + (cos 80° - cos 80°) +cos 180°

= cos 180° = -1

Ex.4. If $x \sin^3\theta + y \cos^3\theta = \sin\theta . \cos\theta$ and $x \sin\theta = y \cos\theta$, $\sin\theta \neq 0$, $\cos\theta \neq 0$ then $x^2 + y^2$ is

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{1}{2}$

(C) 1 **(D)** $\sqrt{2}$

Sol.(C) take θ = 45°

$$\therefore x\left(\frac{1}{\sqrt{2}}\right)^3 + y\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$$

or $x + y = \sqrt{2}$ ....(i)

and $\frac{x}{\sqrt{2}} = \frac{y}{\sqrt{2}}$

or

x = y ....(ii)

From (i) and (ii)

$x + x = \sqrt{2}$

$2x = \sqrt{2}$ $x = \frac{1}{\sqrt{2}}$

$\therefore y = \frac{1}{\sqrt{2}}$

$$\therefore x^2 + y^2 = \frac{1}{2}+\frac{1}{2} = 1$$

Ex.5. If cosec θ - sin θ = l and sec θ − cos θ = m then the value of $(l^2 \times m^2)(l^2+m^2+3)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2

Sol.(C) $(\text{cosec}\,\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2 \{(\text{cosec}\,\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2+3\}$

$$\left(\frac{1}{\sin\theta}-\sin\theta\right)^2\left(\frac{1}{\cos\theta}-\cos\theta\right)^2$$

$$=\left\{\left(\frac{1}{\sin\theta}-\sin\theta\right)+\left(\frac{1}{\cos\theta}-\cos\theta\right)^2+3\right\}$$

$$=\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)+\left(\frac{1-\cos^2\theta}{\cos\theta}\right)+3\right\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2+\left(\frac{\sin^2\theta}{\cos\theta}\right)^2+3\right\}$$

$$= \frac{\cos^4\theta}{\sin^2\theta}\times\frac{\sin^4\theta}{\cos^2\theta}\left\{\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3\right\}$$

$$= \cos^2\theta\times\sin^2\theta\left\{\frac{\cos^6\theta+\sin^6\theta+3\cos^2\theta.\sin^2\theta}{\cos^2\theta\ .\ \sin^2\theta}\right\}$$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta.\sin^2\theta$
$= \{(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta.\sin^2\theta\}$
$(\cos^2\theta + \sin^2\theta)\} + 3\cos^2\theta.\sin^2\theta$ sin $\{a^3+b^3$
$= (a+b)^2 - 3ab\ (a+b)\}$
$= 1-3\cos^2\theta.\sin^2\theta + 3\cos^2\theta.\sin^2\theta = 1$

Ex.6. If sin 2A = 2 sin A, then what is the value of A?

(A) 0^o **(B)** 60^o
(C) 30^o **(D)** 45^o

Sol.(A) sin 2 A = 2 sin A

$\Rightarrow$ 2 sin A cos A = 2 sin A

$\Rightarrow$ cos A = 1 $\Rightarrow$ A = 0

Ex.7. If $\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta} = 3$ and $0^o < \theta < 90^o$, then the value of θ is -

(A) 30^o **(B)** 60^o
(C) 45^o **(D)** None of these

Sol.(B) $\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta}=3$

$\Rightarrow\cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = 3\cot^2\theta$

$\Rightarrow 4\cos^2\theta = 3\times\frac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow\sin^2\theta = \frac{3}{4}$

$\Rightarrow\sin\theta = \frac{\sqrt{3}}{2} = \sin 60^o$

$\therefore\ \theta = 60^o$

Ex.8. If $3\sin^2\alpha + 7\cos^2\alpha = 4$, then the value of $\cot\alpha$ is (where $0^0 < \alpha < 90^0$)

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$ **(D)** 1

Sol.(B) $3\sin^2\alpha + 7\cos^2\alpha = 4$

$\Rightarrow 3\sin^2\alpha + 7(1-\sin^2\alpha) = 4$

$\Rightarrow -4\sin^2\alpha = -3$, $\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60^0$, $\therefore \cot\alpha^0 = \cot 60^0 = \frac{1}{\sqrt{3}}$

Ex.9. What is value of $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$?

(A) $\sin^2x$ **(B)** $\cot^2x$
(C) $\cos^2x$ **(D)** $\tan^2x$

Sol.(D) $\frac{1-\cos 2x}{1+\cos 2x} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2x$

Ex.10. $\frac{\cot^2 41^0}{\tan^2 49^0} - 2\frac{\sin^2 75^0}{\cos^2 15^0} = ?$

(A) 1 **(B)** -1 **(C)** 2 **(D)** 0

Sol.(B) $\frac{\cot^2 41^0}{\tan^2 49^0} - 2\frac{\sin^2 75}{\cos^2 15^0}$

$$= \frac{\cot^2(90^0-49^0)}{\tan^2 49^0} - \frac{2\sin^2(90^0-15^0)}{\cos^2 15^0}$$

$$\frac{\tan^2 49^0}{\tan^2 49^0} - \frac{2\cos^2 15^0}{\cos^2 15^0}$$

$= 1 - 2 = -1$

Ex.11. If $\tan\theta.\tan 2\theta = 1$, then the value of $\sin^2 2\theta + \tan^2 2\theta$ is-

(A) $1\frac{3}{4}$ **(B)** $2\frac{3}{4}$
(C) $3\frac{3}{4}$ **(D)** $5\frac{3}{4}$

Sol.(C) $\tan\theta.\tan 2\theta = 1$

$$\Rightarrow \tan\theta = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{3}\right)^2$$

$\Rightarrow \tan\theta = \tan(90^0 - 2\theta)$

$\Rightarrow \theta = 90^0 - 2\theta \Rightarrow \theta = 30^0$

$\therefore \sin^2 2\theta + \tan^2 2\theta$

$= \sin^2 60^0 + \tan^2 60^0$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{3}\right)^2 = \frac{3}{4} + 3$$

or $3\frac{3}{4}$

Ex.12. What is the value of $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$?

(A) 2 cosec θ **(B)** 2 tan θ
(C) 2 sec θ **(D)** 2 cot θ

Sol.(A) $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}$

$=\frac{2\sec\theta}{\tan\theta}=\frac{2}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

Ex.13. What is the value of
$\sec(90^0-A)-\cot A\cos(90^0-A)\tan(90^0-A)$?

(A) 0 **(B)** 1
(C) sinA **(D)** $\sin^2A$

Sol.(C) sec $(90^0$-A)-cotA cos $(90^0$-A) tan$(90^0$-A)
= cosecA - cot A sin A cot A

$=\frac{1}{\sin A}-\frac{\cos A}{\sin A}\times\sin A\times\frac{\cos A}{\sin A}$

$=\frac{1}{\sin A}-\frac{\cos^2 A}{\sin A}=\frac{1-\cos^2 A}{\sin A}=\frac{\sin^2 A}{\sin A}=\sin A$

Ex.14. The value of tan 4^o . tan 43^o.tan47^0 tan 86^o is -

(A) 1 **(B)** 0

(C) $\sqrt{3}$ **(D)** $\frac{1}{\sqrt{3}}$

Sol.(A) tan 4^0 .tan 43^0 .tan 47^0 .tan 86^0
= tan 4^0 .tan 43^0 .tan $(90^0 - 43^0)$ tan $(90^0 - 4^0)$
= tan 4^0 .cot 4^0 .tan 43^0 .cot 43^0=1

Ex.15. If $x+\left(\frac{1}{x}\right)=2\cos\alpha$, then what is the value of $x^2+\left(\frac{1}{x^2}\right)$?

(A) $4\cos^2\alpha$ **(B)** $4\cos^2\alpha-1$
(C) $2\cos 2\alpha$ **(D)** $\cos^2\alpha-\sin^2\alpha$

Sol.(C) $x+\frac{1}{x}=2\cos\alpha$
squaring both side

$x^2+\frac{1}{x^2}+2=4\cos^2\alpha$

$x^2+\frac{1}{x^2}=2(2\cos^2\alpha-1)$

$=2(2\cos^2\alpha-\sin^2\alpha-\cos^2\alpha)$

$=2(\cos^2\alpha-\sin^2\alpha)=2\cos 2\alpha$

Ex.16. If $x=\cos^2\theta$ and $\frac{1}{x}=\sin^2\theta$

What is the value of $x^3+\frac{1}{x^3}$

(A) $\frac{\cos^2\theta}{\sin\theta}$ **(B)** $\frac{1}{\cos\theta}$

(C) $\frac{-\sin^2 2\theta}{2}$ **(D)** $\frac{\cos^2\theta}{2}$

Sol.(C) $\left(x^3+\frac{1}{x^3}\right)=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

$=(\cos^2\theta+\sin^2\theta)(\cos^4\theta+\sin^4\theta-1)$

$=1\times(\cos^4\theta+\sin^4\theta-1)$

$(\cos^2\theta+\sin^2\theta)^2=\cos^4\theta+\sin^4\theta+2\cos^2\theta\sin^2\theta$

$1-2\cos^2\theta.\sin^2\theta=\cos^4\theta+\sin^4+$

$=1\times(1-2\cos^2\theta.\sin^2\theta-1)$

$=\frac{-2\cos\theta.\sin\theta\times2\cos\theta\times\sin\theta}{2}$

$-\frac{(\sin2\theta\times\sin2\theta)}{2}=-\frac{\sin^2 2\theta}{2}$

Ex.17. If $\frac{x+x\ \cos^2\theta-3\cos\theta+\frac{1}{x}}{\sin^2\theta}=x$

Find the value of x

(A) cos θ **(B)** sec θ
(C) sin θ **(D)** cosec θ

Sol.(B) $x+x\cos^2\theta-3\cos=+\frac{1}{x}=x\sin^2\theta$

$x^2+x^2\cos^2\theta-3x\cos\theta+1=x\sin^2\theta$

$x^2+x^2\cos^2\theta-3x\cos\theta+1=x^2(1-\cos^2\theta)$

$x^2\cos^2\theta-3x\cos\theta+1=x^2\cos^2\theta$

$x^2\cos^2\theta-3x\cos\theta+1=-x^2\cos^2\theta$

$2x^2\cos^2\theta-3x\cos\theta+1=0$

$2x^2\cos^2\theta-2x\cos\theta-x\cos\theta+1=0$

$2x\cos\theta(x.\cos-1)-1(x\cos-1)=0$

$(2x\cos\theta-1)(x\cos\theta-1)=0$

$x=\frac{1}{2\cos\theta}\quad x=\frac{1}{\cos\theta}$

$x=\sec\theta,\frac{\sec\theta}{2}$

Ex.18. If $x=\tan\theta$

what is the value of $x^3+\frac{1}{x^3}$

(A) secθ.cosecθ (sec²θ.cosec²θ–3)

(B) secθ.cosecθ

(C) tanθ

(D) cot θ

Sol.(A) $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

$x^3+\frac{1}{x^3}=(\tan\theta+\cot\theta)\left(\tan^2\theta+\frac{1}{\tan^2\theta}-1\right)$

$=\frac{1}{\sin.\cos}\times\left[\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}-1\right]$

$=\frac{1}{\sin\theta.\cos\theta}\times\left[\frac{\sin^4\theta+\cos^4\theta}{\cos^2\theta\sin^2\theta}-1\right]$

$=\frac{1}{\sin\theta.\cos\theta}\left[\frac{\sin^4\theta+\cos^4\theta-\cos^2.\sin^2\theta}{\cos^2\theta\sin^2\theta}\right]$

$=\frac{1}{\sin\theta.\cos\theta}\left[\frac{1-2\cos^2\theta.\sin^2\theta-\cos^2\theta\sin^2\theta}{\cos^2\theta.\sin^2\theta}\right]$

$=\frac{1-3\cos^2\theta.\sin^2\theta}{\sin^3\theta.\cos^3\theta}$

$=\sec^3\theta.\text{cosec}^3\theta-3\sec.\text{cosec}\theta$

$=\sec\theta.\cos\theta\ [\sec^2\theta\ .\text{cosec}^2\theta-3]$

Ex.19. If $x = a\cos^3\theta$, $y = b\sin^3\theta$, then $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=?$

(A) 1 **(B)** 2 **(C)** 0 **(D)** 4

Sol.(A) $x = a\cos^3\theta \Rightarrow \frac{x}{a}=\cos^3\theta$

$y = b\sin^3\theta \Rightarrow \frac{y}{b}=\sin^3\theta$

$\therefore \left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=(\cos^3\theta)^{\frac{2}{3}}+(\sin^3\theta)^{\frac{2}{3}}$

$=\cos^2\theta+\sin^2\theta=1$

Ex.20. The value of $\sin^2\theta\cos^2\theta\ (\sec^2\theta+\text{cosec}^2\theta)$ is -

(A) 2 **(B)** 4

(C) 1 **(D)** 3

Sol.(C) $\sin^2\theta\cos^2\theta\ (\sec^2\theta+\text{cosec}^2\theta)$

$=\sin^2\theta.\cos^2\theta\left(\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta.\sin^2\theta}\right)$

$=1$

Ex.21. If $\sec\theta-\text{cosec}\theta=0$ then the value of $(\sec\theta+\text{cosec}\theta)$ is

(A) $\sqrt{2}$ **(B)** $3\sqrt{3}$

(C) $2\sqrt{2}$ **(D)** $\frac{1}{\sqrt{2}}$

Sol.(C) $\sec\theta=\text{cosec}\theta$

$\frac{\sin\theta}{\cos\theta}=1$, $\tan\theta=1$, $\theta=45^0$

$\sec 45^0+\text{cosec}45^0=\sqrt{2}+\sqrt{2}=2\sqrt{2}$

Ex.22. Value of $(2\cos^2\theta-1)\left(\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right)$

(A) 1 **(B)** 0

(C) 2 **(D)** $\frac{\sqrt{3}}{2}$

Sol.(C) $(2\cos^2\theta-1)\left(\frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}\right)$

$=(2\cos^2\theta-1)\left(\frac{2(1+\tan^2\theta)}{1-\tan^2\theta}\right)$

$=(2\cos^2\theta-1)\left(2\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}\right)\right)$

$=(2\cos^2\theta-1)\left(\frac{2}{2\cos^2\theta-1}\right)=2$

Ex.23. If $\sin A+\text{cosec}A=3$ then $(\sin^2A+\text{cosec}^2A)=9$?

(A) 4 **(B)** 3 **(C)** 2 **(D)** 7

Sol.(D) $(\sin A+\text{cosec}A)^2=\sin^2A+\text{cosec}^2A+2\sin A.\text{Co-}$ $\text{sec}A$

$\sin^2A+\text{cosec}^2A=9-2=7$

Ex.24. If $x = a\sin\theta$ and $y = b\tan\theta$ then find the value of $\frac{a^2}{x^2}-\frac{b^2}{y^2}$

(A) -1 **(B)** 0

(C) 1 **(D)** 2

Sol.(C) $\frac{a^2}{x^2}-\frac{b^2}{y^2}=\frac{a^2}{a^2\sin^2\theta}-\frac{b^2}{b^2\tan^2\theta}$

$=\text{cosec}^2\theta-\cot^2\theta=1$

Ex.25. If $\sec 5A=\text{cosec}(A+36^0)$, where 5A is an acute angle, find the value of A.

(A) 5^0 **(B)** 7^0

(C) 9^0 **(D)** 13^0

Sol.(C) $\sec 5A=\text{cosec}(A+36^0)$

$\Rightarrow \sec5A=\sec[90^0-(A+36^0)]$

$\Rightarrow \sec 5A=\sec(54^0-A)$

$\Rightarrow 5A=54^0-A$

$\therefore A=9^0$

Ex.26. If $\tan^2\theta=1-a^2$, then the value of $\sec\theta+\tan^3\theta\ \text{cosec}\theta$ is-

(A) $(a^2-1)^{\frac{3}{2}}$ **(B)** $a^{\frac{3}{2}}$

(C) $(2-a)^{\frac{3}{2}}$ **(D)** $(2-a^2)^{\frac{3}{2}}$

Sol.(D) sec θ + $\tan^3\theta$ cosec θ

$$= \sec\theta\left(\frac{\sec\theta+\tan^3\theta\,\mathrm{cosec}\theta}{\sec\theta}\right)$$

$$= \sec\theta\left(1+\tan^3\theta\frac{\cos\theta}{\sin\theta}\right) = \sec\theta(1+\tan^3\theta\times\cot\theta)$$

$$= \sqrt{1+\tan^2\theta}\,(1+\tan^2\theta) = (1+\tan^2\theta)^{\frac{3}{2}}$$

$$= \left[1+(1-a^2)\right]^{\frac{3}{2}} = (2-a^2)^{\frac{3}{2}}$$

Ex.27. What is the value of (sec A - cos A)^2 + (cosec A - sin A)^2 - (cot A - tan A)^2

(A) 0 **(B)** -1 **(C)** 1 **(D)** 2

Sol.(B) put $\theta = 45^o$

(sec A - cos A)^2 + (cosec A - sin A)^2 - (cot A - tan A)^2

(sec 45 - cos 45)^2 + (cosec 45 - sin 45)^2 -(cot 45 - tan 45)^2

$$= \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2 - (1-1)^2$$

$$= \frac{1}{2}+\frac{1}{2} = 0$$

$$= \frac{2}{2} = 1$$

QUICK TRICK :

(sec A - cos A)^2 + (cosec A - sin A)^2 - (cot A + tan A)^2

= sec^2 A + cos^2 A - 2 sec A cos A + cosec^2 A + sin^2 A - 2 sin A cosec A - cot^2 - tan^2 A+2 tan A cot A

= (sin^2 A + cos^2 A) + (sec^2 A - tan^2 A) + (cosec^2 A - cot^2 A) = 1 + 1 + 1 - 2 + 2 - 2 = 1

Ex.28. If sec θ = $x+\frac{1}{4x}\left[0^\circ<\theta<90^\circ\right]$ then find the sec θ + tan θ

(A) x **(B)** 0

(C) 2x **(D)** 4x

Sol.(C) sec θ = $x+\frac{1}{4x}$

$$\sec^2\theta = x^2+\frac{1}{16x^2}+2x\times\frac{1}{4x}$$

$$1+\tan^2\theta = x^2+\frac{1}{16x^2}+\frac{1}{2}$$

$$\tan^2\theta = x^2+\frac{1}{16x^2}-\frac{1}{2}$$

$$\tan^2\theta = \left(x-\frac{1}{4x}\right)^2$$

$$\tan\theta = x-\frac{1}{4x}$$

$$\mathrm{sce}\theta+\tan\theta = x+\frac{1}{4x}+x-\frac{1}{4x} = 2x$$

Ex.29. Find the value of

$$\left[\frac{\cos^2 A(\sin A+\cos A)}{\mathrm{cosec}^2 A(\sin A-\cos A)}+\frac{\sin^2 A(\sin A-\cos A)}{\sec^2 A(\sin A+\cos A)}\right]\times\left(\sec^2 A-\mathrm{cosec}^2 A\right)\text{-}$$

(A) 1 **(B)** 2

(C) 0 **(D)** -2

Sol.(B) $\left[\frac{\sin^2 A\cos^2 A(\sin A+\cos A)}{(\sin A-\cos A)}+\frac{\sin^2 A\cos^2 A(\sin A-\cos A)}{\sin A+\cos A}\right]$

$$\times\left(\frac{1}{\cos^2 A}-\frac{1}{\sin^2 A}\right)$$

$$= \sin^2 A\cos^2 A\left[\frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{\sin^2 A-\cos^2 A}\right]$$

$$\times\frac{\sin^2 A-\cos^2 A}{\sin^2 A\cos^2 A}$$

= (sin A + cos A)^2 + (sin A - cos A)^2

= sin^2 A + cos^2 A + 2 sin A cos A + sin^{2}A+ cos^2 A - 2 sin A cos A

= 2 (sin^2 A + cos^2 A)

= 2×1 = 2

Ex.30. If (sinα +cosec α)^2 + (cos α + sec α)^2 = K+tan^2 α + cot^2 α then find the value of K is-

(A) 0 **(B)** 4

(C) 7 **(D)** 9

Sol.(C) Put $\alpha = 30^0$

$$\left(\frac{1}{2}+2\right)^2+\left(\frac{\sqrt{3}}{2}+\frac{2}{\sqrt{3}}\right)^2 = K+\frac{1}{3}+3$$

$$\frac{25}{4}+\frac{49}{12} = K+\frac{10}{3}$$

$\frac{75+49}{12} = K + \frac{10}{3}$

$\frac{124}{12} = K + \frac{10}{3}$

$\frac{31}{3} = K + \frac{10}{3}$

$K = \frac{31}{3} - \frac{10}{3}$

$= \frac{21}{3} = 7$

QUICK TRICK :

(sin α + cosec α)² + cos α + sec α)² = k + tan² α + cot² α

= sin² α +cosec² α + 2 sin α cosec α + cos² α + sec² α + 2 cos α sec α = k + tan² 2 + cot²}

= sin² α + cos² α + 4 + 1 + cot² α + 1 + tan² α

= k + tan² α + cot² α

= 1 + 4 + 1 + 1 = k

k = 7

Ex.31. Find the value of

$\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta(2\cos^2\theta - 1)}$

(A) 0 **(B)** 1

(C) -1 **(D)** 2

Sol.(B) $\sec^2\theta - \frac{\sin^2\theta(1-2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$

$\sec^2\theta - \frac{\tan^2\theta \times \cos 2\theta}{\cos 2\theta}$

sec² θ - tan² θ

= 1

Ex.32. Find the minimum value of-

16 cosec² θ + 25 sec² θ

(A) 81 **(B)** 41

(C) 82 **(D)** 90

Sol.(A) 16 cosec² θ + 25 sec² θ

= 16 (1+cot² θ) +25 (1+tan² θ)

= 41 + 16 cot² θ + 25 tan² θ ......(i)

Minimum value of $ax^2 + \frac{b}{x^2} = 2\sqrt{ab}$

Minimum value = $2\sqrt{16 \times 25} = 40$ + 41

= 81

Ex.33. Find the greatest value of $81^{\sin x}.27^{\cos x}$

(A) 3^5 **(B)** 3^4

(C) 3 **(D)** 3^3

Sol.(A) $81^{\sin x}.27^{\cos x} = 3^{4\sin x}.3^{3\cos x}$

$= 3^{4\sin x + 3\cos x}$

maximum value of 4 sinx + 3 cos x

$= \sqrt{4^2 + 3^2} = 5$

Greatest value of = $81^{\sin x}.21^{6\cos x} = 3^5$

Ex.34. Maximum value of sin $x + \sqrt{3}\cos x$ will be at

(A) x = 30° **(B)** x = 0°

(C) x = 45° **(D)** x = 60°

Sol.(A) Let y = (sin x $+\sqrt{3}$ cos x)

$= 2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$

= 2 sin (x + 60°)

y will be maximum when sin (x+60°) is maximum

i.e. sin (x + 60°) = 1 sin 90°

x = 30°

Ex.35. sin² 3 + sin² 6+............+sin² 87

(A) $29\frac{1}{2}$ **(B)** $14\frac{1}{2}$

(C) $14\frac{1}{\sqrt{2}}$ **(D)** 1

Sol.(B) sin² 3 + sin² 6 + ...sin² 87

Series 3 + 6 +...........87

n = $\frac{87-3}{3} + 1 = 29$

n = 29

↓

28 1 sin²45

$14 + \frac{1}{2}$ $\left(\frac{1}{\sqrt{2}}\right)^2$

$= 14\frac{1}{2}$

EXERCISE

Q.1. If $0 < x < 45^0$ and $45^0 < y < 90^0$, then which one of the following is correct?

(A) sin x = sin y **(B)** sin x < sin y

(C) sin x > sin y **(D)** sin x ≤ sin y

Q.2. If $\cot\theta = \frac{2xy}{x^2 - y^2}$, then cos θ equal to ?

(A) $\frac{x^2 - y^2}{x^2 + y^2}$ **(B)** $\frac{x^2 + y^2}{x^2 - y^2}$

(C) $\frac{2xy}{x^2 + y^2}$ **(D)** $\frac{2xy}{\sqrt{x^2 + y^2}}$

Q.3. If $3\cos^2 A + 7\sin^2 A = 4$, then find the value of cotA, given that A is an acute angle?

(A) 1 **(B)** 4

(C) 3 **(D)** $\sqrt{3}$

Q.4. If 3 sin x + 5 cos x = 5, then what is the value of (3 cos x – 5 sin x)?

(A) 0 **(B)** 2

(C) 3 **(D)** 5

Q.5. In a triangle ABC, right angled at C, find the value of tanA + tanB in terms of sides a,b,c.

(A) $\frac{a^2}{bc}$ **(B)** $\frac{a^2 b}{c}$

(C) $\frac{2a^2}{bc}$ **(D)** $\frac{c^2}{ab}$

Q.6. If tan θ + sin θ = m and tan θ – sin θ = n, then $m^2 - n^2$ is equal to -

(A) $4\sqrt{mn}$ **(B)** $\sqrt{mn}$

(C) $2\sqrt{mn}$ **(D)** $\frac{1}{mn}$

Q.7. Find the value of 22° 33’ in radian.

(A) $\frac{\pi^c}{8}$ **(B)** $\frac{\pi^c}{6}$

(C) $\frac{\pi^c}{4}$ **(D)** $\frac{\pi^c}{12}$

Q.8. If sin θ – cos Φ = 0, then which of the following is correct?

(A) θ + Φ = 2 π **(B)** θ + Φ = π

(C) $\theta + \Phi = \frac{\pi}{2}$ **(D)** $\theta - \Phi = \frac{\pi}{3}$

Q.9. If x and y real numbers, then equation $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible only when-

(A) x > y **(B)** x = y

(C) x = 2y **(D)** 2x = y

Q.10. If $(\alpha+\beta) = \frac{\pi}{2}$ and $(\beta+\gamma) = \alpha$ then which one of the following is correct ?

(A) 2 tan β + tan γ = tan α

(B) tan β + 2tan γ = tan α

(C) tan β + tan γ = tan α

(D) 2(tan β + tan γ) = tan α

Q.11. If A, B and C are angles of a trinangle such that tan A = 1, tan B = 2, then what is the value of tan C ?

(A) 0 **(B)** 1

(C) 2 **(D)** 3

Q.12. In a right-angled triangle ABC, AB = 2.5 cm., cos B = 0.5, ∠ACB = 90°. The length of side AC, in cm., is

(A) $5\sqrt{3}$ **(B)** $\frac{5}{2}\sqrt{3}$

(C) $\frac{5}{4}\sqrt{3}$ **(D)** $\frac{5}{16}\sqrt{3}$

Q.13. The maximum value of $(\sin x \ \cos x)$ is-

(A) 1 **(B)** $\frac{1}{2}$

(C) 2 **(D)** $\sqrt{2}$

Q.14. The maximum value of 8 sin θ + 6 cos θ is

(A) 6 **(B)** 8

(C) 10 **(D)** 14

Q.15. If tan 2θ. tan 4θ = 1, then the value of tan 3θ is

(A) 0 **(B)** 1

(C) $\frac{1}{2}$ **(D)** 2

Q.16. If cos θ + sec θ = $\sqrt{3}$, then the value of $\cos^3\theta + \sec^3\theta$ is

(A) 0 **(B)** 1

(C) – 1 **(D)** $\sqrt{3}$

Q.17. If 0° < θ < 90°, and $2\cos^2\theta + \sin\theta - 2 = 0$, then find the value of θ.

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q.18. If $\sin\theta = \frac{\sqrt{3}}{2}$, then the value of $(\operatorname{cosec}\theta + \cot\theta)$ =?

(A) 0 **(B)** 1

(C) $\sqrt{2}$ **(D)** $\sqrt{3}$

Q.19. The maximum value of (sin θ + cosθ) is

(A) 1 **(B)** $\sqrt{2}$

(C) 2 **(D)** $2\sqrt{2}$

Q.20. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = ?$

(A) $\frac{3}{4}$ **(B)** $\frac{-2}{3}$

(C) $\frac{2}{3}$ **(D)** $\frac{3}{2}$

Q.21. $\left(\sin^2 30^0 \cos^2 45^0 + 4\tan^2 30^0 + \frac{1}{2}\sin^2 90^0 - 2\cos^2 90^0\right) = ?$

(A) $\frac{15}{8}$ **(B)** $\frac{23}{12}$

(C) $\frac{47}{24}$ **(D)** $\frac{45}{24}$

Q.22. The minimum value of $\left(2\sin^2\theta + 3\cos^2\theta\right)$ is-

(A) 0 **(B)** 3

(C) 2 **(D)** 1

Q.23. The least value of $\left(4\sec^2\theta + 9\operatorname{cosec}^2\theta\right)$ is-

(A) 1 **(B)** 19

(C) 25 **(D)** 7

Q.24. The value of tan 1^0 tan 2^0 tan 3^0 .... tan 89^0 is-

(A) 1 **(B)** 0

(C) $\sqrt{3}$ **(D)** $\frac{1}{\sqrt{3}}$

Q.25. tan 35^0 tan 40^0 tan 45^0 tan 50^0 tan 55^0 = ?

(A) 0 **(B)** 1

(C) $\frac{2\sqrt{3}}{4}$ **(D)** None of these

Q.26. $(\cos 3\theta + \sin 3\theta)$ is maximum when is-

(A) 15^0 **(B)** 30^0

(C) 45^0 **(D)** 60^0

Q.27. If sin A : cos A = 4 : 7, then $\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A} = ?$

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$

(C) $\frac{3}{14}$ **(D)** $\frac{3}{2}$

Q.28. If 5 tan θ = 4, then $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right) = ?$

(A) $\frac{2}{5}$ **(B)** $\frac{1}{7}$

(C) $\frac{2}{7}$ **(D)** $\frac{5}{7}$

Q.29. The minimum value of $(\sin x + \cos x)$ is-

(A) 0 **(B)** 1

(C) -1 **(D)** $-\sqrt{2}$

Q.30. The maximum and minimum values of $(1 + \cos 2x)$ are-

(A) -1 and 1 **(B)** 1 and 2

(C) $-\frac{1}{2}$ and $\frac{1}{2}$ **(D)** 0 and 2

Q.31. $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = ?$

(A) 0 **(B)** 1

(C) 2 **(D)** 4 sin θ cos θ

Q.32. $\frac{\sin A + \sin B}{\cos A - \cos B} + \frac{\cos A + \cos B}{\sin A - \sin B} = ?$

(A) 0 **(B)** sin A cos B

(C) cos A cos B **(D)** tan A tan B

Q.33. The greatest value of $\sin^4\theta + \cos^4\theta$ is

(A)1 **(B)**1/2

(C) 0 **(D)** $\frac{\sqrt{3}}{2}$

Q.34. What is the minimum value of $\sin^2\theta + \cos^4\theta$

(A) 0 **(B)**3/4

(C) 2 **(D)** 1/4

Q.35. Find the maximum value of $\sin^{113}\theta - \cos^{113}\theta$

(A) $\left(\frac{3}{2}\right)^{113}$ **(B)** 1

(C) $\left(\frac{1}{4}\right)^{113}$ **(D)** $\left(\frac{1}{2}\right)^{113}$

Q.36. Find the minimum value of $16\operatorname{cosec}^2\theta + 25\sec^2\theta$

(A) 81 **(B)**41

(C)82 **(D)**90

Q.37. $\cos 3\theta + \sin 3\theta$ is maximum when θ is:

(A) 15^0 **(B)** 30^0
(C) 45^0 **(D)** 60^0

Q.38. What is the maximum value of $\sin^6\theta + \cos^6\theta$?

(A) 1/2 **(B)** 1/4
(C) 1 **(D)** None of these

Q.39. The greatest value of $81^{\sin x}.27^{\cos x}$ is

(A) 3^5 **(B)** 3^4
(C) 3 **(D)** 3^3

Q.40. Find minima or maxima of $7\tan\theta + 8\cot\theta$

(A) $7\sqrt{2}$ **(B)** $8\sqrt{7}$
(C) $2\sqrt{14}$ **(D)** $4\sqrt{14}$

Q.41. The least value of $2\sin 2\theta + 3\cos 2\theta$

(A) 1 **(B)** 2
(C) 3 **(D)** 5

Q.42. Find Min and Max values of $\sin 4x + 5$.

(A) 2, 6 **(B)** 4, 5
(C) -4, -5 **(D)** 4, 6

Q.43. Find the maximum value of the expression $(p^2 + 7p + 13)^{-1}$.

(A) 5/7 **(B)** 4/3
(C) 20 **(D)** 3/7

Q.44. What is the least value of $4\sec^2\theta + 9co\sec^2\theta$?

(A) 1 **(B)** 19
(C) 25 **(D)** 7

Q.45. If $\cot\theta + \operatorname{cosec}\theta = 3$ and θ is an acute angle, then what is the value of $\sin\theta$?

(A) $\frac{4}{5}$ **(B)** 1
(C) $\frac{3}{4}$ **(D)** $\frac{3}{5}$

Q.46. If $\tan\theta + \tan\psi = x$ and $\cot + \cot\psi = y$, then what is the value of $\cot(\theta + \psi)$?

(A) $\frac{1}{x} + \frac{1}{y}$ **(B)** $\frac{1}{x} - \frac{1}{y}$
(C) y **(D)** $x - y$

Q.47. If $2\sin\alpha + 15\cos^2\alpha = 7$, then find the value of $\cot\alpha$

(A) 3/4 **(B)** 4/5
(C) 2/5 **(D)** 3/5

Q.48. What is the value of $\tan\theta\cot\phi \times \left(\frac{\tan\phi - \cot\theta}{\tan\theta - \cot\phi}\right)$

(A) $\sin\theta$ **(B)** $\cos\phi$
(C) $\tan\theta\tan\phi$ **(D)** 1

Q.49. The expression $\frac{\tan 57^o + \cot 37^o}{\tan 33^o + \cot 53^o}$ is equal to

(A) $\tan 33^o \cot 53^o$ **(B)** $\tan 53^o \cot 37^o$
(C) $\tan 33^o \cot 57^o$ **(D)** $\tan 57^o \cot 37^o$

Q.50. If $0 \le \theta \le \frac{\pi}{2}$, $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the value of $x^2 + 4y^2$ is-

(A) 2 **(B)** 3
(C) 1 **(D)** 4

Q.51. If $\sec\alpha + \tan\alpha = 2 + \sqrt{5}$, then the value of $\sin\alpha + \cos\alpha$ is

(A) $4\sqrt{14}$ **(B)** $\sqrt{5}$
(C) $\frac{7}{\sqrt{5}}$ **(D)** $\frac{1}{\sqrt{5}}$

EXPLANATION

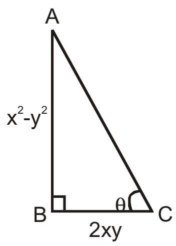
Q.1.(B) $0<x<45^0$ and $45^0<y<90^0$

$\therefore 0<x<45^0<y<90^0$

$\because \sin\theta$ increases 0 to 1

$\therefore \sin x < \sin y$

Q.2.(C)



$\cot\theta = \dfrac{2xy}{x^2-y^2}$

$\therefore AC = \sqrt{(x^2-y^2)^2+(2xy)^2}$

$AC = \sqrt{x^4+y^4-2x^2y^2+4x^2y^2}$

$AC = x^2+y^2$

$\therefore \cot\theta = \dfrac{2xy}{x^2+y^2}$

Q.3.(D) $3\cos^2 A + 7\sin^2 A = 4$

$\Rightarrow 3\cos^2 A + 3\sin^2 A + 4\sin^2 A = 4$

$\Rightarrow 3(\cos^2 A + \sin^2 A) + 4\sin^2 A = 4$

$\Rightarrow 4\sin^2 A = 1$

$\sin A = \dfrac{1}{2}$, $A = 30^0$

$\therefore \cot A = \sqrt{3}$

Q.4.(C) $3 \sin x + 5 \cos x = 5$

$\Rightarrow 9 \sin^2 x + 25 \cos^2 x + 30 \sin x . \cos x = 25$

$\Rightarrow 9 \sin^2 x + 25 \cos^2 x - 25 = -30 \sin x . \cos x$

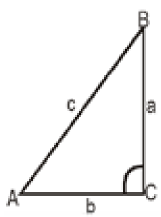
$(3 \cos x - 5 \sin x)^2 = 9 \cos^2 x + 25 \sin^2 x - 30 \sin x \cos x$

$= 9 \cos^2 x + 25 \sin^2 x + 9 \sin^2 x + 25 \cos^2 x - 25$

$= 34 - 25 = 9$

$\therefore 3 \cos x - 5 \sin x = 3$

Q.5.(D)



$\tan A = \dfrac{a}{b}$, $\tan B = \dfrac{b}{a}$

$\tan A + \tan B$

$= \dfrac{a}{b}+\dfrac{b}{a} = \dfrac{a^2+b^2}{ab} = \dfrac{c^2}{ab}$

Q.6.(A) $m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2 = \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta = 4\tan\theta\sin\theta$

$\therefore m^2 - n^2 = 4.\dfrac{\sin\theta}{\cos\theta}.\sin\theta = \dfrac{4\sin^2\theta}{\cos\theta}$

$\because mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \dfrac{\sin^4\theta}{\cos^2\theta}$

$\sqrt{mn} = \dfrac{\sin^2\theta}{\cos\theta}$

$\therefore m^2 - n^2 = 4\sqrt{mn}$

Q.7.(A) $180^o = \pi$

$\Rightarrow 22^o\ 30' = \left[22+\dfrac{1}{2}\right] = \left[\dfrac{45}{2}\right]^o$

$= \left[\dfrac{45}{2}\times\dfrac{\pi}{180}\right]^c = \dfrac{\pi^c}{8}$

Q.8.(C) $\sin\theta = \cos\phi$

$\Rightarrow \sin\theta = \sin\left(\dfrac{\pi}{2}-\phi\right)$

$\Rightarrow \theta = \left(\dfrac{\pi}{2}-\phi\right)$

$\Rightarrow \theta + \phi = \dfrac{\pi}{2}$

Q.9.(B) $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$

$\because \sec^2\theta \geq 1$

$\Rightarrow 4xy \geq (x+y)^2$

$0 \geq (x+y)^2 - 4xy$

$0 \geq (x-y)^2$

$(x-y)^2 \leq 0$

$(x-y)^2 < 0$

$(x-y) = 0$

$\Rightarrow x = y$

Q.10.(B) $\alpha+\beta = \frac{\pi}{2}, \beta+\gamma = \alpha$

$\tan(\alpha+\beta) = \tan\frac{\pi}{2}, \ \tan(\alpha-\beta) = \tan\gamma$

$\frac{\tan\gamma+\tan\beta}{1+\tan\gamma.\tan} = \frac{1}{o}, \frac{\tan\gamma-\tan\ \beta}{1-\tan\ \gamma.\tan\ \beta} = \tan\ \gamma$

$\tan\gamma.\ \tan\beta = -1$

From

$\tan\alpha - \tan\beta = 2\tan\gamma$

$\tan\alpha = 2\tan\gamma + \tan\beta$

Q.11.(B) $A+B+C = 180^0$

$\Rightarrow A+B = 180^0 - C$

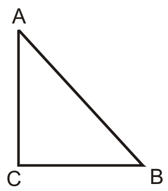
$\Rightarrow \frac{\tan A+\tan B}{1-\tan.\tan B} = \tan(180-C)$

$\Rightarrow \frac{1+2}{1+1\times2} = \cot C$

$\Rightarrow \frac{3}{3} = \cot\ C$

$\Rightarrow \tan C = 1$

Q.12.(C)



$ACB = 90^o$

$AB = 2.5$

$\cos B = 0.5 = \frac{1}{2}$

$\Rightarrow \sin B = \sqrt{1-\cos^2 B} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin B = \frac{AC}{AB}$

$\Rightarrow AC = AB\sin B = 2.5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$

Q.13.(B) $\sin 2x \leq 1 \Rightarrow \frac{1}{2}\sin 2x \leq \frac{1}{2}$.

$\therefore \sin x\cos x \leq \frac{1}{2}$ i.e. maximum value of $\sin x\cos x$ is $\frac{1}{2}$.

Q.14.(D) $\sqrt{8^2+6^2} = \sqrt{64+36} = 10$

Q.15.(B) $\tan 2\theta \tan 4\theta = 1$

$\Rightarrow \tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$

$\Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2}-4\theta\right)$

$\Rightarrow 2\theta = \frac{\pi}{2}-4\theta$

$\Rightarrow 6\theta = \frac{\pi}{2} \Rightarrow 3\theta = \frac{\pi}{4}$

$\Rightarrow \tan 3\theta = \tan\frac{\pi}{4} = 1$

Q.16.(A) $\cos\theta + \sec\theta = \sqrt{3}$

$\Rightarrow (\cos\theta+\sec\theta)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\cos\theta\sec\theta(\cos\theta+\sec\theta) = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta = 0$

Q.17.(A) $2\cos^2\theta + \sin\theta - 2 = 0$

$\Rightarrow 2(1-\sin^2\theta) + \sin\theta - 2 = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}$

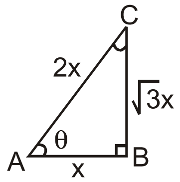
Q.18.(D) $\sin\theta = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}x}{2x} = \frac{BC}{AC}$

$AB^2 = (AC^2 - BC^{2)} = (4x^2 - 3x^2) = x^2$

$\therefore AB = x$, $(\operatorname{cosec}\theta + \cot\theta)$

$= \left(\frac{AC}{BC}+\frac{AB}{BC}\right) = \left(\frac{2x}{\sqrt{3}x}+\frac{x}{\sqrt{3}x}\right)$

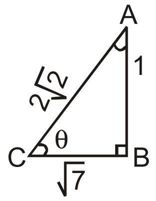
$= \left(\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right) = \frac{3}{\sqrt{3}} = \sqrt{3}$



Q.19.(B) Maximum value of sinθ+cosθ when $\theta = 45^0$

$\therefore \sin 45^0 + \cos 45^0 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

Q.20.(A)



$\tan\theta = \frac{1}{\sqrt{7}}$

In ΔABC

$AC^2 = AB^2 + BC^2 = 1^2 + \left(\sqrt{7}\right)^2$

$= 1+7 = 8$

$AC = 2\sqrt{2}$

$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$, sec θ

$= \frac{AC}{AB} = \frac{2\sqrt{2}}{\sqrt{7}}$

$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

$= \frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{48}{64} = \frac{3}{4}$

Q.21.(C) $\sin^2 30^0 \cos^2 45^0 + 4\tan^2 30^0 + \frac{1}{2}\sin^2 90^0 - 2\cos^2 90^0$.

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times \left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2} \times (1)^2 - 2 \times (0)^2$

$= \frac{1}{4} \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{1}{2} - 0 = \frac{1}{8} + \frac{4}{3} + \frac{1}{2}$

$= \frac{3+32+12}{24} = \frac{47}{24}$

Q.22.(B) $2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta = 2 + \cos^2\theta$

∴ Minimum value of cosθ = -1

∴ Required minimum value = 2 + 1 = 3

Q.23.(C) $4\sec^2\theta + 9\,\text{cosec}^2\theta$

$= 4(1+\tan^2\theta) + 9(1+\cot^2\theta)$

$= 13 + (4\tan^2\theta + 9\cot^2\theta)$

Now, AM ≥ GM

$\Rightarrow \frac{4\tan^2\theta + 9\cot^2\theta}{2} \geq \sqrt{4\tan^2\theta . 9\cot^2\theta}$

$\Rightarrow 4\tan^2\theta + 9\cot^2\theta \geq 2\sqrt{36} \geq 12$

∴ Minimum value of $4\sec^2\theta + 9\,\text{cosec}^2\theta$

= 13 + 12 = 25

Q.24.(A) $\tan 1^o \tan 2^o \tan 3^o$ ------ $\tan 89^o$

$= \tan 1^o \tan 2^o \tan 3^o$ ----- $\tan 45^o$

---- $\tan 87^o \tan 88^o \tan 89^o$

$= \tan 1^o \tan 2^o \tan 3^o$ ---- $\tan 45^o$

----- $\tan (90-2)^o \tan (90-1)^o$

$= \tan 1^o \tan 2^o \tan 3^o$ ---- $\tan 45^o$ ---- $\cot 2^o \cot 1^o$

= 1 ------------------ $(\tan\theta \times \cot\theta = 1)$

Q.25.(B) $\tan 35^0 \tan 40^0 \tan 45^0 \tan 50^0 \tan 55^0$

$= \tan 35^0 \tan 40^0 \times 1 \times \tan(90-40^0) \times \tan(90-35^0)$

$= \tan 35^0 \tan 40^0 \times \cot 40^0 \times \cot 35^0$

$= (\tan 35^0 \cot 35^0) \times (\tan 40^0 \times \cot 40^0)$

$= 1 \times 1$ {∴ tan θ into cot θ = 1} = 1

Q.26.(A) $\cos 3\theta + \sin 3\theta = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 3\theta + \frac{1}{\sqrt{2}}\sin 3\theta\right)$

$\sqrt{2}\,(\sin 45^0 \cos 3\theta + \cos 45^0 \sin 3\theta)$

$= \sqrt{2}\,\sin(45^0 + 3\theta)$

The maximum value occurs when $\sin(45^0+3\theta)$ = 1, i.e. $3\theta = 45^0$ or $\theta = 15^0$.

Q.27.(A) Let Sin A = 4x

cos A = 7x

$\therefore \frac{7\sin A - 3\cos A}{7\sin A + 2\cos A} = \frac{7 \times 4x - 3 \times 7x}{7 \times 4x + 2 \times 7x} = \frac{28x - 21x}{28x + 14x}$

$=\frac{7x}{42x}=\frac{1}{6}$

Q.28.(B) Given, $\tan\theta=\frac{4}{5}$

$=\frac{5\sin\theta-3\cos\theta}{5\sin\theta+3\cos\theta}=\frac{5\tan\theta-3}{5\tan\theta+3}$

= {divided by both numerator and denomerator}

$=\frac{5\times\frac{4}{5}-3}{5\times\frac{4}{5}+3}=\frac{4-3}{4+3}=\frac{1}{7}$

Q.29.(D) sinx+cosx =

$\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right]=\sqrt{2}\sin\left(45^0+x\right)$

Since the minimum value of (45^0 + x) is -1, so minimum value of (sinx+cosx) is $-\sqrt{2}$.

Q.30.(D) We know that cosθ ≥ 1 and cosθ ≤ 1.

∴ cos2x ≥ -1 ⇒ 1+cos2x ≥ 0.

cos2x ≤ 1 ⇒ 1+cos2x ≤ 2.

So, minimum value of 1+cos2x is 0 and the maximum value of (1+cos2x) is 2.

Q.31.(C) ∴ $(a–b)^2 + (a+b)^2 = 2(a^2+b^2)$

$(\cos\theta - \sin\theta)^2 + (\cos\theta+\sin\theta)^2 = 2 (\cos^2\theta+\sin^2\theta)$

= 2×1 = 2

Q.32.(A) $\frac{\sin A+\sin B}{\cos A-\cos B}+\frac{\cos A+\cos B}{\sin A-\sin B}$

$=\frac{\left(\sin^2 A-\sin^2 B\right)+\left(\cos^2 A-\cos^2 B\right)}{(\cos A-\cos B)(\sin A-\sin B)}$

$\frac{\left(\sin^2 A+\cos^2 A\right)-\left(\sin^2 B+\cos^2 B\right)}{(\cos A-\cos B)(\sin A-\sin B)}$

{∴ $\sin^2\theta+\cos^2\theta=1$}

$=\frac{1-1}{(\cos A-\cos B)(\sin A-\sin B)}=0$

Q.33.(A) $\sin^4\theta+\cos^4\theta=(\sin^2\theta)^2+(\cos^2\theta)^2$
$=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta.\cos^2\theta=1$

Q.34.(B) $x=\sin^2\theta+\cos^4\theta=1-\cos^2\theta+\cos^4\theta$

$1-\cos^2\theta.\sin^2\theta=\frac{3}{4}$

Q.35.(D) The maximum value of $\sin^n\theta.\cos^n\theta=\left(\frac{1}{2}\right)^n$

The maximum value of

$\sin^{113}\theta.\cos^{113}\theta=\left(\frac{1}{2}\right)^{113}$

Q.36.(A) Minimum value of

$(a\,\mathrm{cosec}^2\theta+b\sec^2\theta)=(\sqrt{a}+\sqrt{b})^2$

$16\,\mathrm{cosec}^2\theta+25\sec^2\theta=(\sqrt{16}+\sqrt{25})^2=81$

Q.37.(A) $\cos 3\theta+\sin 3\theta$

$=\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 3\theta+\frac{1}{\sqrt{2}}\sin 3\theta\right)$

$=\sqrt{2}(\sin 45^0.\cos 3\theta+\cos 45^0.\sin 3\theta)$

$\sqrt{2}\sin(45^0+3\theta)$

The maximum value occurs when

$\sin(45^0+3\theta)=1$

$\theta=15^0$

Q.38.(B) $x=\sin^6\theta+\cos^6\theta=(\sin^2\theta)^3+(\cos^2\theta)^3$

$1-3\sin^2\theta.\cos^2\theta=\frac{1}{4}$

Q.39.(A) $81^{\sin x}.27^{\cos x}=3^{4\sin x+3\cos x}$

$=3^5$

Q.40.(D) $7\tan\theta+8\cot\theta\geq 2\sqrt{7^2+8^2}\geq 4\sqrt{14}$

This means the minimum value of the expression 7 tan θ+8 cot θ is $4\sqrt{14}$.

Q.41.(B) = 2 sin2 θ + 3 (1- sin2 θ) ;(because sin2 θ + cos2 θ=1=>cos2 θ=1- sin2 θ)
= 2 sin2 θ + 3 – 3 sin2 θ
= 3 – sin2 θ
= 3 – (1) = 2
As sin2 θ is preceded by a negative sign therefore we have to take max. value of sin2 θ in order to get minimum value .
OR

= 2 sin2 θ + 3 cos2 θ

= 2 (1- cos2 θ) + 3 cos2 θ

= 2 – 2 cos2 θ + 3 cos2 θ

= 2 + cos2 θ

= 2 + 0 = 2

Q.42.(D) We know that, -1 ≤ Sin nx ≤ 1

= -1 ≤ Sin 4x ≤ 1

Adding 5 throughout, 4 ≤ Sin 4x +5 ≤ 6

Therefore, the minimum value is 4 and maximum value is 6

Q.43.(B) $x=(p^2+7p+13)^{-1}$

$$x=\frac{1}{(p^2+7p+13)}=\frac{1}{y}$$

At the minimum value of the expression y in the denominator, the given expression x reaches its maximum value.

So we can rephrase the problem as,

Find the minimum value of the expression $p^2+7p+13$

$y=p^2+7p+13$

$$=\left(p+\frac{7}{2}\right)^2-\frac{49}{4}+13=\left(p+\frac{7}{2}\right)^2+\frac{3}{4}$$

The expression y will reach its minimum value only when the square of sum term is zero. Thus the minimum value of y is, ¾

Finally then the desired maximum value of,

$$x=\frac{1}{y}=\frac{4}{3}$$

Q.44.(C) $4\sec^2\theta+9cosec^2\theta$

$=4(1+\tan^2\theta)+9(1+\cot^2\theta)=13+(4\tan^2\theta+9\cot^2\theta)$

$=13+2\sqrt{4\times 9}=25$

Q.45.(D) cot θ + cosec θ = 3

$$\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}=3$$

1 + cos θ = 3 sin θ

1 + cos² θ + 2 cos θ = 9(1 – cos2 θ)

10 cos² θ + 2 cos θ – 8 = 0

5 cos² θ + cos θ – 4 = 0

5 cos² θ + 5cos θ – 4 cos θ – 4 = 0

5 cos θ (cos θ + 1) – 4 (cos θ + 1) = 0

(5 cos θ – 4) (cos θ + 1) = 0

(5 cos θ – 4) = 0

$\cos\theta=\frac{4}{5}$

$$\sin\theta=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

Q.46.(B) $\cot(\theta+\psi)=\frac{1}{\tan(\theta+\psi)}=\frac{1-\tan\theta\,\tan\psi}{\tan\theta+\tan\psi}$

$$=\frac{1}{\tan\theta+\tan\psi}-\frac{\tan\theta\tan\psi}{\tan\theta+\tan\psi}$$

$$=\frac{1}{\tan\theta+\tan\psi}-\frac{1}{\left(\frac{1}{\tan\theta}+\frac{1}{\tan\psi}\right)}$$

$$=\frac{1}{\tan\theta+\tan\psi}-\frac{1}{\cot\theta+\cot\psi}=\frac{1}{x}-\frac{1}{y}$$

Q.47.(B) 2sinα + 15cos ²α = 7

2sinα + 15 - 15sin ²α = 7

15sin ²α − 2sinα − 8 = 0

15sin ²α + 10sinα − 12sinα − 8 = 0

5sinα (3sinα + 2) – 4 (3sinα + 2) = 0

sinα = 4/5, – 2/ 3

Q.48.(A) $\tan\theta\cot\phi\times\left(\frac{\tan\phi-\cot\theta}{\tan\theta-\cot\phi}\right)$

$$=\frac{\sin\theta}{\cos\theta}\frac{\cos\phi}{\sin\phi}\times\left(\frac{\frac{\sin\phi}{\cos\phi}-\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}-\frac{\cos\phi}{\sin\phi}}\right)$$

$$=\frac{\sin\theta}{\cos\theta}\frac{\cos\phi}{\sin\phi}\times\left(\frac{\frac{\sin\phi\sin\theta-\cos\theta\cos\phi}{\cos\phi\sin\theta}}{\frac{\sin\theta\sin\phi-\cos\theta\cos\phi}{\cos\theta\sin\phi}}\right)$$

$$=\frac{\sin\theta\cos\phi}{\cos\theta\sin\phi}\times\frac{\sin\theta\cos\phi}{\cos\theta\sin\phi}=1$$

Q.49.(D) $\frac{\tan 57^o+\cot 37^o}{\tan 33^o+\cot 53^o}=\frac{\frac{\sin 57^o}{\cos 57^o}+\frac{\cos 37^o}{\sin 37^o}}{\frac{\sin 33^o}{\cos 33^o}+\frac{\cos 53^o}{\sin 53^o}}$

$$=\frac{\frac{\sin 57^o\sin 37^o+\cos 57^o\cos 37^o}{\cos 57^o\sin 37^o}}{\frac{\sin 33^o\sin 53^o+\cos 33^o\cos 53^o}{\cos 33^o\sin 53^o}}$$

$$=\frac{\frac{\cos(57^o-37^o)}{\cos 57^o\sin 37^o}}{\frac{\cos(53^o-33^o)}{\cos 33^o\sin 53^o}}$$

$= \dfrac{\cos 33^o \sin 53^o}{\cos 57^o \sin 37^o} = \dfrac{\sin 57^o \cos 37^o}{\cos 57^o \sin 37^o}$

$= \tan 57^o \cot 37^o$

Q.50.(D) $2y \cos \theta = x \sin \theta$

$x \sec \theta = 2y \operatorname{cosec} \theta$

and, $2x \sec \theta - y \operatorname{cosec} \theta = 3$

$4y \operatorname{cosec} \theta - y \operatorname{cosec} \theta = 3$

$y = \sin \theta$

$x \sec \theta = 2y \operatorname{cosec} \theta$

$x \sec \theta = 2$

$x = 2 \cos \theta$

$x^2 + 4y^2 = 4 \cos^2 \theta + 4 \sin^2 \theta = 4$

Q.51.(A) $\sec \alpha + \tan \alpha = \sqrt{5} + 2$ … (I)

$\sec \alpha - \tan \alpha = \sqrt{5} - 2$ … (II)

On solving (I) and (II), we get

$\sec \alpha = \sqrt{5}$

$\tan \alpha = 2$

It means, $\cos \alpha$

$= \dfrac{1}{\sqrt{5}}$, $\sin \alpha = 2 \times \dfrac{1}{\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

$\sin \alpha + \cos \alpha = \dfrac{1}{\sqrt{5}} + \dfrac{2}{\sqrt{5}} = \dfrac{3}{\sqrt{5}}$

Notes

CHAPTER-23
HEIGHT AND DISTANCE

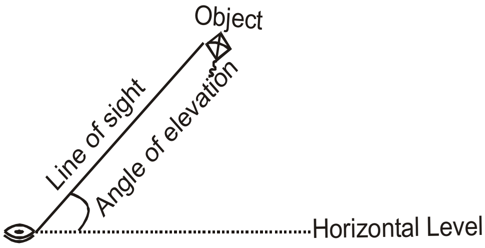
Scan the QR code to get video of this chapter.

Introduction

One of the important application of trigonometry is finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

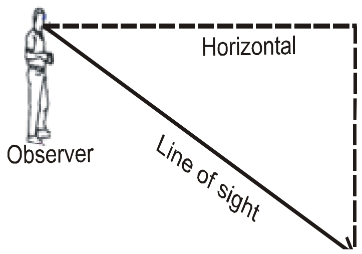
Angle of elevation

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.



Angle of depression

The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level.

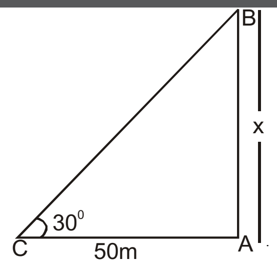


Ex. The angle of elevation of a tower from a distance 50m from its foot is 30^0. Then find the height of the tower ?

(A) $50\sqrt{3}\text{m}$ **(B)** $\frac{50}{\sqrt{3}}\text{m}$

(C) $25\sqrt{3}\text{m}$ **(D)** $\frac{100}{\sqrt{3}}\text{m}$

Sol.(B) Let AB be the tower and AC be the horizontal line such that AC = 50m and $\angle ACB = 30^0$



$$\tan 30^0 = \frac{AB}{AC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{50} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 50 \times \frac{1}{\sqrt{3}}\text{m} = \frac{50}{\sqrt{3}}\text{m}$$

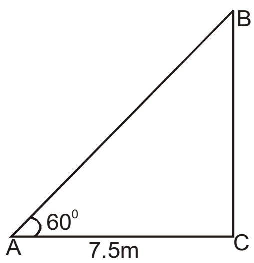
$\therefore$ Height of the tower $= \frac{50}{\sqrt{3}}\text{m.}$

Ex. The angle of elevation of a ladder leaning against a wall is 60^0 and the foot of the ladder is 7.5m away from the wall. The length of the ladder is-

(A) 15 m **(B)** 14.86 m

(C) 15.64 m **(D)** 15.8 m

Sol.(A) Let AB the ladder leaning against the wall CB.



Let AC be the horizontal such that AC = 7.5 m and $\angle CAB = 60^0$.

$\therefore \sec 60^0 = \frac{AB}{AC} = 2$

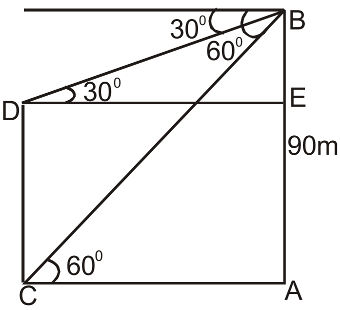
$$\frac{AB}{7.5} = 2 \Rightarrow AB = 15\text{m.}$$

$\therefore$ Length of the Ladder is 15 m.

Ex. From the top of a cliff which is 90 m high, the angles of depression of the top and bottom of a tower are observed to be 30^0 and 60^0 respectively. What is the height of the tower?

(A) 30 m **(B)** 45 m **(C)** 60 m **(D)** 75 m

Sol.(C)



Let AB be the cliff and CD be the tower. Draw DE II CA, Then, $\angle$ BDE = 30^0, $\angle$BCA = 60^0 and AB = 90 m.

From right angle Δ CAB we have

$$\frac{CA}{AB} = \cot 60^0$$

$$\frac{CA}{90} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{CA}{90} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow CA = \left(90 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 30\sqrt{3}m.$$

$\therefore$ DE = CA = $30\sqrt{3}m.$

From right angle Δ DEB, we have

$$\frac{BE}{DE} = \tan 30^0 = \frac{1}{\sqrt{3}} \Rightarrow \frac{BE}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BE = \left(30\sqrt{3} \times \frac{1}{\sqrt{3}}\right) = 30m.$$

$\therefore$ CD = AE =(AB-BE)=(90-30)m = 60m

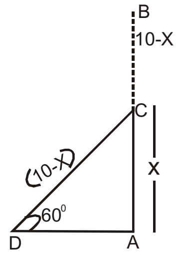
hence, the height of the tower is 60 m.

Ex. The height of a tree is 10 meter. It is bent by the wind in such a way that its top touches the ground and makes an angle of 60^0 with the ground. At what height from the bottom did the tree get bent? $\left(\sqrt{3} = 1.73\right)$

(A) 4.6 m **(B)** 4.8 m

(C) 5.2 m **(D)** 5.4 m

Sol.(A)



Let AB be the tree bent at the point C so that part CB takes the position CD. Then

CD = CB. Let AC = x metres. Then, CD =CB = (10-x)m and $\angle$ ADC = 60^0.

$$\frac{AC}{CD} = \sin 60^0 \Rightarrow \frac{x}{(10-x)} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 2x = 10\sqrt{3} - \sqrt{3}x$$

$$\Rightarrow \left(2+\sqrt{3}\right)x = 10\sqrt{3}$$

$$\Rightarrow x \Rightarrow \frac{10\sqrt{3}}{2+\sqrt{3}}$$

So, multiply numerator and denominator both by $2-\sqrt{3}$

$$= \frac{10\sqrt{3}}{\left(2+\sqrt{3}\right)} \times \frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)} = \left(20\sqrt{3}-30\right)m$$

= (20×1.73-30)m = 4.6m

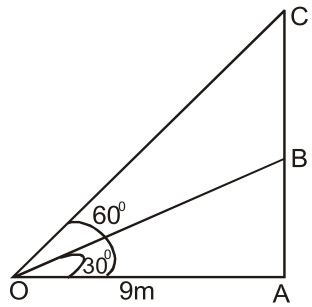
$\therefore$ Required height = 4.6 m.

Ex. On the horizontal plane, there is a vertical tower with a flagpole on it's top. At a point 9m away from the foot of the tower, the angles of elevation of the top and bottom of the flagpole are 60^0 and 30^0 respectively. The height of the flagpole is:

(A) $6\sqrt{3}m$ **(B)** $5\sqrt{3}$ m

(C) $6\sqrt{2}$ m **(D)** 4 m

Sol.(A) Let AB be the tower and BC be the flag pole and let O be the point of observation.



Then,

OA=9m, $\angle$ AOB = 30^0 and $\angle$ AOC = 60^0

From ΔAOB

$$\tan 30^0 = \frac{AB}{OA} = \frac{1}{\sqrt{3}} = \frac{AB}{9}$$

$$\Rightarrow AB = \left(9 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{.\sqrt{3}}\right) = 3\sqrt{3}m.$$

Now, From ΔAOC

$$\tan 60^0 = \frac{AC}{OA}$$

$$\sqrt{3} = \frac{AC}{9}$$

$$AC = 9\sqrt{3}.$$

$\therefore$ BC = (AC-AB)

$$= \left(9\sqrt{3} - 3\sqrt{3}\right)m = 6\sqrt{3}m$$

$\therefore$ Height of the flagpole is $6\sqrt{3}m$.

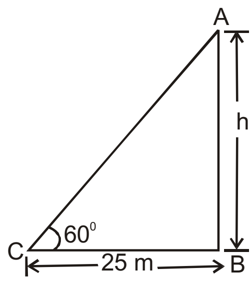
Examples

Ex.1. A vertical pole stands on the ground. From a point on the ground 25m away from the bottom of pole, the angle of elevation of its top found to be 60^0. Find the height of pole.

(A) 20 m **(B)** $20\sqrt{3}$ m

(C) 25 m **(D)** $25\sqrt{3}$ m

Sol.(D)



Let height of pole AB = (h) and angle of elevation $\theta = 60^0$

From right angled triangle ABC,

$$\tan 60^0 = \frac{AB}{BC} = \frac{h}{25}$$

$$\Rightarrow h = 25 \times \tan 60 = 25 \times \sqrt{3}$$

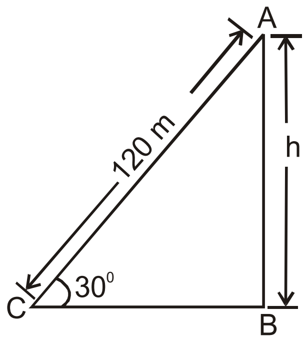
So, height of pole is $25\sqrt{3}$ m.

Ex.2. Length of thread of a kite is 120 m and its angle of elevation with any point on the earth is 30^0. Find the perpendicular height of kite from the earth.

(A) 30m **(B)** 60m

(C) 60m **(D)** 120m

Sol.(C)



Let the height of kite from ground is h meter.

Now, from right angle ΔABC

$$\sin 30^0 = \frac{h}{120}, \quad \frac{1}{2} = \frac{h}{120}$$

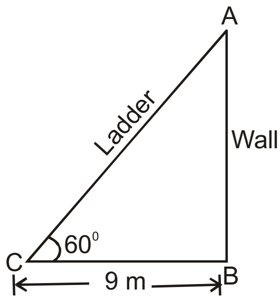
h = 60 m

Ex.3. The angle of elevation of a ladder leaning against a wall is 60^0 and foot of ladder is 9 m away from wall. Find the length of ladder.

(A) 17m **(B)** 18m

(C) 19m **(D)** 20m

Sol.(B)



In right angled ΔABC

$$\cos 60^0 = \frac{BC}{AC}$$

$$\Rightarrow \frac{9}{AC} = \frac{1}{2}$$

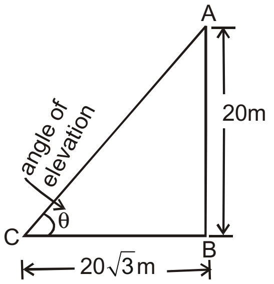
$$\Rightarrow AC = 9 \times 2 = 18 \text{ m}$$

Ex.4. The length of shadow of a vertical tower is $20\sqrt{3}$ m. If the height of tower is 20m, then angle of elevation of the sun is.

(A) 30^0 **(B)** 45^0

(C) 60^0 **(D)** 90^o

Sol.(A)



From right angle ΔABC

$\tan\theta = \frac{AB}{BC} \Rightarrow \frac{20}{20\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30°$

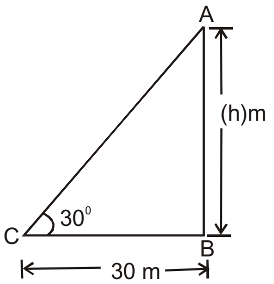
$\therefore \theta = 30°$

Ex.5. A tree got bend by wind, the top of tree touches the ground at an angle of 30^0 at a distance of 30 m from the root. What was original height of tree?

(A) $10\sqrt{3}$ m **(B)** $30\sqrt{3}$ m

(C) $20\sqrt{3}$ m **(D)** $40\sqrt{3}$ m

Sol.(B)



In right angle ΔABC

$\tan 30^0 = \frac{h}{30}$

$h = 30 \times \tan 30^0$

$h = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

So, $10\sqrt{3}$ m is height from bottom from which tree get bend

Now, from right angle ΔABC

$\cos 30^0 = \frac{30}{AC}$

$AC = \frac{30}{\cos 30^0} = \frac{30}{\sqrt{3}/2} = 20\sqrt{3}$

Now, original height of tree = BA + AC

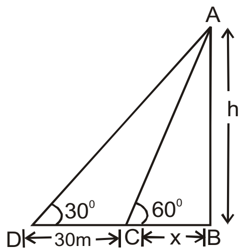
$= 10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3}$ m

Ex.6. A person standing on the bank of a river observes that angle of elevation of the top of the tree standing on the opposite bank is 60^0.When he moves 30 m away from bank, he founds the angle of elevation to be 30^0. Find the width of the river.

(A) 15 m **(B)** 30 m

(C) $15\sqrt{3}$ m **(D)** $30\sqrt{3}$ m

Sol.(A)



From ΔABC

$\tan 60^0 = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}\, x$ ..... (i)

From ΔABD

$\tan 30^0 = \frac{h}{x+30}$

$\Rightarrow \sqrt{3}\, h = x + 30$ (ii)

hence

$\sqrt{3} \times (\sqrt{3}\, x) = x + 30$

$3x = x + 30$

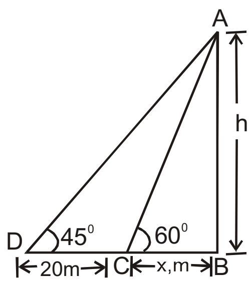
$\Rightarrow x = 15$ m

Ex.7. When the angle of elevation of changes from 60^0 to 45^o shadow of tower increases by 20 m . Then find the height of the tower?

(A) 10 m **(B)** 20 m

(C) 30 m **(D)** $10\left(3+\sqrt{3}\right)$m

Sol.(D)



In right angle ΔABD

$\tan 45^0 = \dfrac{h}{x+20}$

h =x + 20 ...... (i)

From ΔABC

$\tan 60^0 = \dfrac{h}{x}$

h = $\sqrt{3}$ x ..... (ii)

or x = $\dfrac{h}{\sqrt{3}}$ ..... (iii)

from eq. (iii) put value of x in eq (i)

$$h = \frac{h}{\sqrt{3}} + 20$$

$$h\left(1 - \frac{1}{\sqrt{3}}\right) = 20$$

$$\Rightarrow h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 20 \;\Rightarrow h = \frac{20\sqrt{3}}{\sqrt{3}-1} \text{ .... (iv)}$$

multiplying by congugate in eq (iv)

$$h = \frac{20\sqrt{3}\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - (1)^2} \Rightarrow h = 10\sqrt{3}\left(\sqrt{3}+1\right)$$

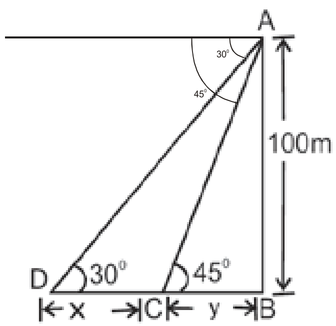
or h = $10\left(3+\sqrt{3}\right)$m

Ex.8. The angle of depression of the both banks of river from top of a temple are 30^0 and 45^0 respectively. If height of temple is 100 m. Then the width of river is?

(A) 100 m **(B)** $100\sqrt{3}$ m

(C) $100\left(\sqrt{3}-1\right)$ m **(D)** $100\left(\sqrt{3}+1\right)$ m

Sol.(C)



From right angle ΔABC

$\tan 45^0 = \dfrac{100}{y}$

$\Rightarrow$ y = 100 m ..... (i)

from right angle ΔABD

$\tan 30^0 = \dfrac{100}{x+y}$, $\dfrac{1}{\sqrt{3}} = \dfrac{100}{x+100}$

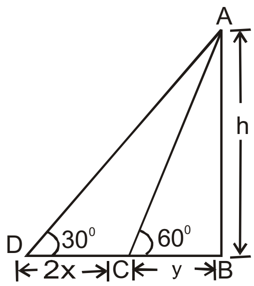
x = $100\left(\sqrt{3}-1\right)$m

Ex.9. A person walking towards a temple with a fixed speed. At any time he finds, the angle of elevation 30^0, with the top of temple. After two minutes it become 60^0. Find in how many minute he reaches the temple.

(A) 2 min **(B)** 1 min

(C) 3 min **(D)** 4 min

Sol.(B)



Let speed of man = x m/min

then distance cover in 2 min = (x×2)m

i.e. DC = 2xm

and height of temple let h.

Now, in ΔABC

$\tan 60^0 = \dfrac{h}{y}$

or $= \dfrac{h}{y} = \sqrt{3}$

h = $\sqrt{3}$ y .....(i)

Now, in ΔABD

$\tan 30^0 = \dfrac{h}{2x+y}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{2x+y}$

2x+y= $\sqrt{3}$ h

or 2x = $\sqrt{3}$ h-y ... (ii)

putting value of h from eq (i) in (ii)

2x = 3y - y

2x = 2y

x = y...........(iii)

Now, eq (iii) signifies that the man reaches the temple in 1 minute.

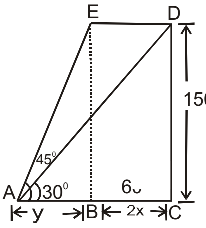
because xm/min is speed of man and it is equal to distance y.

Ex.10. An aeroplane flying parallel to earth at a height of 1500 m. An examiner finds it's angle of elevation 30^0. After 2 minutes it become 45^0. If plane is flying towards examiner then find it's speed ?

(A) 750 m/min **(B)** 750 ($\sqrt{3}$ -1) m/sec

(C) 750 m/sec **(D)** 750 ($\sqrt{3}$ -1) m/min

Sol.(D)



Let the speed of plane be x m/min

So, distance covered by plane in 2 min = (2x) m.

From right ΔABE

$\tan 45^0 = \dfrac{1500}{y}$

y = 1500 .... (i)

again from right ΔACD

$\tan 30^0 = \dfrac{1500}{2x+y}$

$1500\times\sqrt{3} = 2x+1500$

$2x = 1500(\sqrt{3}-1)$

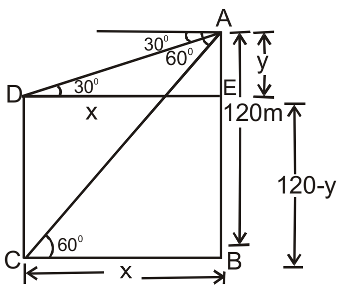
$x = \dfrac{1500(\sqrt{3}-1)}{2}$

Speed of aeroplane = 750($\sqrt{3}$ -1) m/min

Ex.11. From the top of the building 120 m high the angle of depression of top and bottom of a tower are observed to be 30^0 and 60^0. Find the height of tower.

(A) 80 m **(B)** 60 m **(C)** 20 m **(D)** 100 m

Sol.(A)



In right angle ΔABC

$\tan 60 = \dfrac{120}{x}$

$x = \dfrac{120}{\sqrt{3}}$ ...... (i)

Now, in right angle ΔAED

$\tan 30^0 = \dfrac{y}{x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{y\times\sqrt{3}}{120}$

or 120 = 3y ⇒y = 40 m

Therefore, height of tower is CD

and CD = 120 - y

i.e. CD = (120 - 40) m

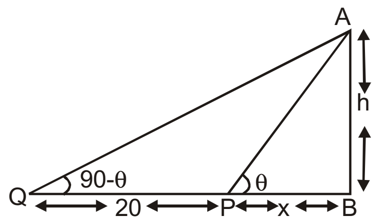
CD = 80 m, Height of tower = 80 m

Ex.12. From the top of the building which is $10\sqrt{3}$ m high. The distance between two point P and Q is 20 m. and angle of depression between two point is complementry. Then find the distance of building from point P (P is the nearest point of building)?

(A) $10\sqrt{3}$ m **(B)** $10(\sqrt{3}+1)$ m

(C) 10 m **(D)** $20(\sqrt{3}+1)$ m

Sol.(C)



In right angle Δ ABQ

$\tan(90-\theta) = \dfrac{10\sqrt{3}}{x+20}$

$\cot\theta = \dfrac{10\sqrt{3}}{x+20}$ ....(i)

Similarly in right angle ΔABP

$\tan\theta = \dfrac{AB}{BP} = \dfrac{10\sqrt{3}}{x}$ ....(ii)

Multiplying by eq. I to II

$1 = \dfrac{300}{x(x+20)}$

$x^2 + 20x - 300 = 0$

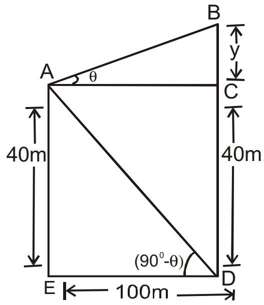
Hence x = 10 m.

Ex.13. The distance between house AE and tower BD is 100 m. When A joins with B and D a right angle triangle BAD forms. If height of house is 40 m what is the height of tower ?

(A) 250m **(B)** 290 m

(C) 300 m **(D)** 210 m

Sol.(B)



In right angle ΔBCA

$\tan\theta = \frac{y}{100}$ .... (i)

In right angle ΔAED

$\tan(90-\theta) = \frac{40}{100}$

$\cot\theta = \frac{40}{100}$

or $\tan\theta = \frac{40}{100}$ .... (ii)

putting value of tan θ from (ii) in eq (i)

we get, $\frac{100}{40} = \frac{y}{100} \Rightarrow y = 250$

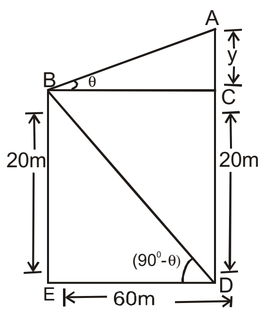
So, total height of tower = 250 + 40 = 290 m

Ex.14. The distance between building BE and tower AD is 60 m. When B joins with A and D a right angle triangle ABD forms. If height of building is 20 m what is the height of tower ?

(A) 180 m **(B)** 160 m

(C) 220 m **(D)** 200 m

Sol.(D)



In right angle ΔABC

$\tan\theta = \frac{y}{60}$ ..... (i)

In right angle ΔBED

$\tan(90^0 - \theta^0) = \frac{20}{60}$

$\cot\theta = \frac{1}{3}$

tan θ =3 ... (ii)

compare eq. (i) and (ii)

y = 180

So, total height of tower is (y +20) = 180 + 20

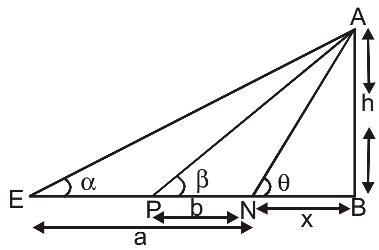
= 200 m

Ex.15. A tower on plane suface and tilted on the north side. On south side two points a and b units respectively away from it's base. If the angle of elevation from both points with top of tower are α and β respectively and angle of tower with surface is θ, then find the height of top of the tower.

(A) $\frac{\operatorname{cosec}\theta(a-b)}{\tan\alpha-\cot\beta}$ **(B)** $\frac{\operatorname{cosec}\theta(a-b)}{\tan\alpha-\tan\beta}$

(C) $\frac{\operatorname{cosec}\theta(a-b)}{\cot\alpha-\cot\beta}$ **(D)** $\frac{a-b}{\cot\alpha-\cot\beta}$

Sol.(C)



In right Δ ABN

$\tan\theta = \frac{h}{x}$

x = h cot θ

In right angle ΔAEB

$\tan\alpha = \frac{h}{a+x}$

a + x = h cot α

a + h cot θ = h cot α................(i)

Simlarily In right angle ΔAPB

b + h cot θ = h cot β...............(ii)

from eq. (i) and (ii) we get

$h = \frac{a-b}{\cot\alpha-\cot\beta}$

Hence in right ΔABN

cosec θ $= \dfrac{AN}{h}$

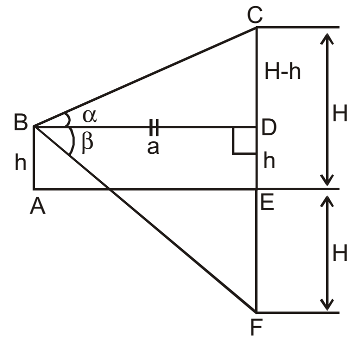
AN $= \dfrac{\text{cosec}\,\theta(a-b)}{\cot\alpha - \cot\beta}$

Ex.16. The angle of elevation of cloud from a point h metre high from the surface of a lake is α and angle of depression from the same point with the shadow of cloud in lake is β. Find height of cloud.

(A) $\dfrac{h\sin(\alpha+\beta)}{\sin\alpha+\sin\beta}$ **(B)** $\dfrac{h\sin(\alpha+\beta)}{\sin\beta-\sin\alpha}$

(C) $\dfrac{h\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ **(D)** $\dfrac{h\sin(\alpha+\beta)}{\cos(\beta-\alpha)}$

Sol.(C)



ΔBCD, tan α $= \dfrac{H-h}{a}$

$a = \dfrac{H-h}{\tan\alpha}$

$\tan\beta = \dfrac{H+h}{a}$

$\tan\beta = \dfrac{H+h}{H-h} \times \tan\alpha$

$H\tan\beta - h + \tan\beta = H\tan\alpha + h\tan\alpha$

$H = \dfrac{h(\tan\alpha+\tan\beta)}{\tan\beta-\tan\alpha}$

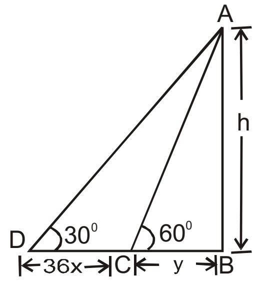
$H = \dfrac{h\left(\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}\right)}{\left(\dfrac{\sin\beta}{\cos\beta}-\dfrac{\sin\alpha}{\cos\alpha}\right)} = \dfrac{h(\sin\alpha+\beta)}{\sin(\beta-\alpha)}$

Ex.17. A person see a car coming towards him with an usual speed from top of the tower. If car takes 36 minute for changing angle of depression from 30^0 to 60^0. How soon it will reach the tower.

(A) 12 min **(B)** 16 min

(C) 10 min **(D)** 18 min

Sol.(D)



Let car travels x meteres in 1 min

Then distance covered by car to travel in 36 minute = (36x) m

Now in right ΔABC

tan $60^0 = \dfrac{h}{y}$

h = $\sqrt{3}$ y .... (i)

In right ΔABD, tan$30^0 = \dfrac{h}{36x+y}$

$\sqrt{3}$ h = 36x +y ... (ii)

putting value of h from eq (i) and (ii) eq we get

$\sqrt{3} \times \sqrt{3}$ y = 36x+y

3y -y = 36x

y = 18x

Now distance travelled to reach tower by car is 18x and speed of car is xm/min

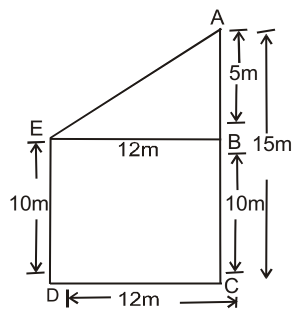
So time taken by car to reach tower is 18 minute

Ex.18. Two poles are standing perpendicular on the surface. If the distance between both is 12m and height of the poles are 10m and 15m respectively, then find the distance between the top of the two towers.

(A) 12 m **(B)** 13 m

(C) 25 m **(D)** 10 m

Sol.(B)



In right angle ΔABE

$(AB)^2 + (BE)^2 = (AE)^2$ .... (i)

Now, AB = AC - BC

AB = 15 - 10

AB = 5m

and

BE = CD

So, BE = 12m

putting values of AB and BE in eq. (i)

$(5)^2 + (12)^2 = AE^2$

$AE^2 = 25 + 144$

$AE = \sqrt{169}$

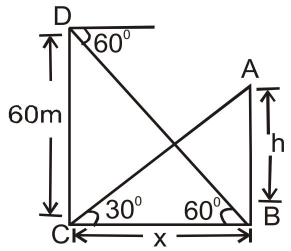
AE = 13 m

Ex.19. A tree and a tower both standing perpendicular. The angle of elevation from the base of the tower with the top of tree is 30^0 and angle of depression from the top of the tower with the base of tree is 60^0. If the height of tower is 60m,then find height of tree.

(A) 20 m **(B)** 22 m

(C) 28 m **(D)** 30 m

Sol.(A)



In right angle ΔABC

$\tan 30^0 = \frac{h}{x}$

$x = \sqrt{3}\,h$ .... (i)

In right angle ΔBCD

$\tan 60^0 = \frac{60}{x}$

$\sqrt{3} \times x = 60$ .... (ii)

putting value of x from eq (i) and (ii)

$\sqrt{3} \times \sqrt{3}h = 60$

3h = 60

h = 20

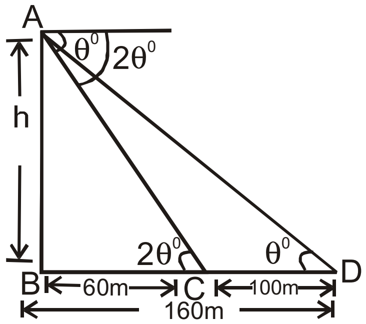
Therefore, height of tree is 20m

Ex.20. A horizontal tower standing on a plane, substends an angle on a point which is 160 m apart from its base. On moving 100 m towards it is found that angled doubles than the initial what is the length of the tower ?

(A) 40 m **(B)** 60 m

(C) 80 m **(D)** 50 m

Sol.(C)



In right angle ΔABD

$\tan\theta = \frac{h}{160}$

$h = 160 \times \tan\theta$ ... (i)

From right angle ΔABC

$\tan 2\theta = \frac{h}{60}$ ..... (ii)

putting value of h in eq (i) and (ii) we get,

$\tan 2\theta = \frac{160 \times \tan\theta}{60}$

$\therefore \left(\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{160\times\tan\theta}{60}$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{160\times\tan\theta}{60}$

or, $1-\tan^2\theta = \frac{3}{4} \Rightarrow \left[\tan\theta = \frac{1}{2}\right]$

Now, putting value of $\tan\theta$ in eq (i)

$h = 160 \times \frac{1}{2} = 80$ m

EXERCISE

Q.1. From a light house the angles of depression of two ships on apposite side of the light are observed to be 30^0 and 45^0. If the height of light house is 20 m, what is the distance between the ships?

(A) $\left(\sqrt{3}+1\right)20m.$ **(B)** $\left(\sqrt{3}-1\right)$ 20 m.

(C) $20\sqrt{3}$ m **(D)** $\left(1+\frac{1}{\sqrt{3}}\right)20$ m.

Q.2. From the top of a cliff 200 m. high, the angles of depression of the top and bottom of a tower are observed to be 30^o and 45^o respectively. What is the height of the tower?

(A) 400 m. **(B)** $400\sqrt{3}$ m.

(C) $\frac{400}{\sqrt{3}}$ m. **(D)** None of the above

Q.3. The angle of elevation of a cloud from a point 60 m. above a lake is 30^o and the angle of depression of the reflection of cloud in the lake is 60^o. Find the height of the cloud?

(A) 100 m. **(B)** 120 m.

(C) 115 m. **(D)** $100\sqrt{3}$ m.

Q.4. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45^0.If the angle of elevation of the top of complete pillar at the same point is to be 60^0, then the height of the incomplete pillar is to be increased by-

(A) $50\sqrt{2}$ m. **(B)** 100 m.

(C) $100\left(\sqrt{3}-1\right)$ m. **(D)** $100\left(\sqrt{3}+1\right)$ m.

Q.5. The angles of elevation of a tower from two points situated at distance 36m and 64 m from its base and in the same straight line with it are complementary . What is the height of the tower?

(A) 50 m. **(B)** 48 m.

(C) 25 m. **(D)** 24 m.

Q.6. Two poles of height a and b metres (b > a) are c metres apart. The height in metres of the intersection of the lines joining the top of each pole to the foot of the opposite poles is -

(A) $\frac{a+b}{ab}$ **(B)** $\frac{ab}{a+b}$

(C) $\frac{1}{2}+\frac{2}{b}$ **(D)** $\frac{1}{c}$

Q.7. An aeroplane flying horizontally 1 km. above the ground is observed at an elevation of 60^o. After 10 seconds its elevation is observed to be 30^o. The uniform speed of the aeroplane is -

(A) 240 km./hr. **(B)** $240\sqrt{2}$ km./hr.

(C) $240\sqrt{3}$ km./hr. **(D)** None of these

Q.8. From a light house the angles of depression of two ships on opposite sides of the light house one observed to be 30^0 and 45^0 If the height of lighthouse is h, what is the distance between the ships ?

(A) $\left(\sqrt{3}+1\right)h$ **(B)** $\left(\sqrt{3}-1\right)h$

(C) $\sqrt{3}h$ **(D)** $\left(1+\frac{1}{\sqrt{3}}\right)h$

Q.9. Two poles of height 6 meters and 11 meters stand on a plane ground. If the distance between their feet is 12 meters then find the difference in the distance between their tops:

(A) 12m **(B)** 5m

(C) 13m **(D)** 11m

Q.10. The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building if the height of the tower is 75 m. and the angle of elevation of the top of the tower from the bottom of the building is 60^o?

(A) 25 m. **(B)** 37.5 m.

(C) 50 m. **(D)** 60 m.

Q.11. The angle of elevation of the top of a building and the top of the chimney on the roof of the building form a point on the ground are 'x' and 45^o respectively. The height of building is h m. the height of the chimney, in m is -

(A) h cot x + h **(B)** h cot x – h

(C) h tan x – h **(D)** h tan x + h

Q.12. A pole 5 meter high is fixed on the top of a tower. The angle of elveation of the top of the pole observed from a point A on the ground is 60^0 and the angle of depression of the point A from the top of the tower is 45^0 .The height of the tower is -

$(\sqrt{3}=1.732)$

(A) 6.83 m. **(B)** 5.9 m.

(C) 7 m. **(D)** 7.66 m.

Q.13. A ladder 34 meter long is placed in a lane so as to reach a window 30 meter hight and on turning the ladder to other side of the lane and keeping its foot at the same place, it reaches a point 16 meter high. What is the width of the lane?

(A) 10 meter **(B)** 40 meter
(C) 46 meter **(D)** 50 meter

Q.14. Two towers of the same height stand on either side of road 60 meter wide. At a point on the road between the towers, the elevations of towers are 60^o and 30^o. Find the position of the point.

(A) 25 meter **(B)** 45 meter
(C) 30 meter **(D)** 55 meter

Q.15. Two poles of equal height are standing opposite to each other on either side of the road which is 100m wide. From a point between them on road angles of elevation of their top are 30^0 and 60^0 the height of each pole in meter is -

(A) $25\sqrt{3}$ **(B)** $20\sqrt{3}$
(C) $28\sqrt{3}$ **(D)** $30\sqrt{3}$

Q.16. The angle of elevation of an aeroplane from a point an the ground is 60^0. After 8 seconds flight the elevation changes to 30^0. If the aeroplane is flying at a height of $1600\sqrt{3}$ m, find the speed of the plane.

(A) 100 m/sec. **(B)** 200 m/sec
(C) 300 m/sec **(D)** 400 m/sec

Q.17. The angles of elevation of the top of an inaccessible tower from two points on the same straight line from the base of the tower are 30^0 and 60^0 respectively. If the points are separated at a distance of 100m the height of tower is : $\left(\sqrt{3} = 1.732\right)$

(A) 86.6m **(B)** 80.6m
(C) 82.6m **(D)** 84.6m

Q.18. The upper part of the broken tree (broken by the wind) makes an angle 30^o with the ground and the distance from the root to the point were the top of the tree meet the ground is 10 meter. What was the height of the tree? ($\sqrt{3}$ = 1.73)

(A) 17.3 meter **(B)** 16.5 meter
(C) 15 meter **(D)** 18 meter

Q.19. Two men are an opposite sides of a tower. They measure the angles of elevation of the top of the tower as 30^o and 45^o respectively. If the height of the tower is 60 meter, find the distance between the two men. (Here $\sqrt{3}$ = 1.73)

(A) 163.8 meter **(B)** 136.8 meter
(C) 152.5 meter **(D)** 125.4 meter

Q.20. A man standing in one corner of a square football field observe that the angle subtended by a pole in the corner just diagonally opposite to this corner is 60^0. When he retires 80 m. from the corner along the same straight line, he finds the angle to be 30^0. The length of the field in m. is-

(A) 40 **(B)** $20\sqrt{2}$
(C) 20 **(D)** $40\sqrt{2}$

Q.21. If the angle of elevation of top of a tower from two points distance a and b from the base and in the same straight line with it are comlpementary then the height of the tower is-

(A) ab **(B)** $\sqrt{ab}$
(C) $\frac{a}{b}$ **(D)** $\sqrt{\frac{a}{b}}$

Q.22. A telegraph post gets broken at a point against a storm and its top touches the ground at a distance 20 m. from the base of the post making, an anlge 30^o with the ground. What is the height of the post?

(A) $\frac{40}{\sqrt{3}}$ m. **(B)** $20\sqrt{3}$ m.
(C) $40\sqrt{3}$ m. **(D)** 30 m.

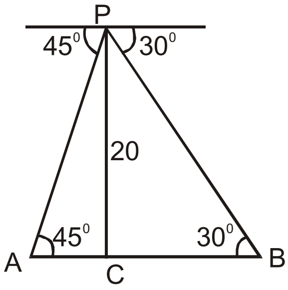
Q.23. The angle of elevation of an aeroplane from a point on the ground is 60^o. After 15 seconds flight, the elevation changes to 30^o. If the aeroplane is flying at a height of $1200\sqrt{3}$ m., find the speed of aeroplane.

(A) 180 m./sec. **(B)** 240 m./sec.
(C) 150 m./sec. **(D)** 160 m./sec.

Notes

EXPLANTION

Q.1.(A)



In Δ PCA

$\tan 45^0 = \dfrac{20}{AC}$

$\Rightarrow$ AC = 20 m.

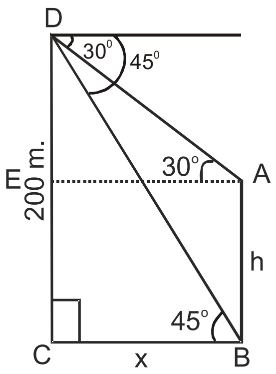
In Δ PCB

$\Rightarrow \tan 30^0 = \dfrac{20}{BC}$

$\Rightarrow$ BC = $20\sqrt{3}$

$\therefore$ AB = $(\sqrt{3} + 1)$ 20 m.

Q.2.(D)



In ΔDCB,

$\tan 45^o = \dfrac{200}{x} \Rightarrow x = 200$

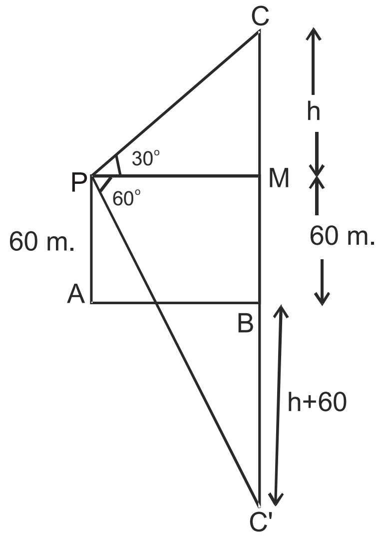
In ΔDEA,

$\therefore \tan 30^\circ = \dfrac{DE}{200} \Rightarrow DE = \dfrac{200}{\sqrt{3}}$

$\therefore$ AB = $200 - \dfrac{200}{\sqrt{3}}$

$= \dfrac{\sqrt{3}\times 200 - 200}{\sqrt{3}} = \dfrac{200\left(\sqrt{3}-1\right)}{\sqrt{3}}$ m.

Q.3.(B)



$\tan 30^o = \dfrac{h}{PM}$

$\Rightarrow PM = \sqrt{3}h$

$\tan 60^o = \dfrac{h+120}{PM}$

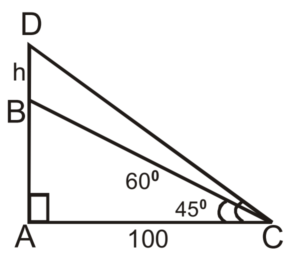
$\Rightarrow \sqrt{3} = \dfrac{h+120}{\sqrt{3}h}$

$\Rightarrow$ h = 60 m.

$\therefore$ Height of cloud

= 120 m.

Q.4.(C)



$\tan 45^0 = \dfrac{AB}{100}$

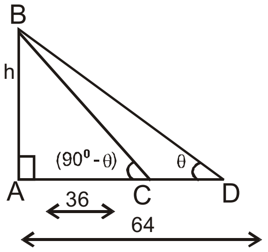
$\Rightarrow$ AB = 100 m.

$\tan 60^0 = \dfrac{100+h}{100}$

$\Rightarrow 100\sqrt{3} = 100 + h$

$\Rightarrow h = 100(\sqrt{3} - 1)$ m.

Q.5.(B)



$\tan(90^0 - \theta) = \dfrac{h}{36}$

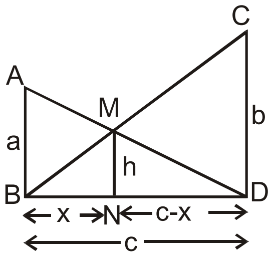
$\Rightarrow \cot\theta = \dfrac{h}{36}$

and $\cot\theta = \dfrac{64}{h}$

$\therefore \dfrac{h}{36} = \dfrac{64}{h}$

$\Rightarrow h = 48$ m.

Q.6.(B)



$\Delta BNM \sim \Delta BDC$

$\Rightarrow \dfrac{BN}{BD} = \dfrac{MN}{CD} = \dfrac{MB}{BC}$

$\Rightarrow \dfrac{x}{c} = \dfrac{h}{b} \Rightarrow x = \dfrac{ch}{b}$ __________(I)

$\Delta DNM \sim \Delta DBA$

$\dfrac{DN}{DB} = \dfrac{MN}{AB} = \dfrac{MD}{AD}$

$\Rightarrow \dfrac{c-x}{c} = \dfrac{h}{a}$

$\Rightarrow ac - ax = hc$

$\Rightarrow x = \dfrac{ac - ch}{a}$ __________(II)

From (I) and (II),

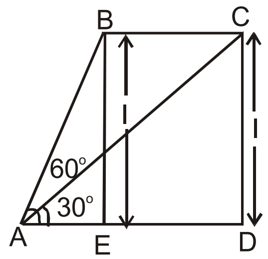
$\dfrac{ch}{b} = \dfrac{ac - ch}{a}$

$\Rightarrow a(ch) = b(ac - ch)$

$\Rightarrow ach + bch = abc$

$\Rightarrow h = \dfrac{ab}{a+b}$ metre

Q.7.(C)



BE = CD = 1 km. = 1000 m.

$\tan 60^o = \dfrac{BE}{AE} = \dfrac{1000}{AE}$

$\Rightarrow AE = 1000 \cot 60^o$

$= \dfrac{1000}{\sqrt{3}}$ m.

$\tan 30^o = \dfrac{DC}{AD} = \dfrac{1000}{AD}$

$\Rightarrow AD = 1000 \cot 30^o = 1000\sqrt{3}$

$\Rightarrow ED = AD - AE$

$= 1000\sqrt{3} - \dfrac{1000}{\sqrt{3}} = \dfrac{2000}{\sqrt{3}}$

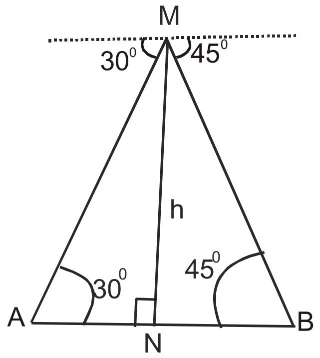
Distance travelled in 10 seconds = BC = DE

$= \dfrac{2000}{\sqrt{3}}$

$\therefore$ Speed $= \dfrac{\frac{2000}{\sqrt{3}}}{10} = \dfrac{2000 \times 60 \times 60}{10\sqrt{3} \times 1000}$

$= 240\sqrt{3}$ km./hr.

Q.8.(A)



$\tan 30^0 = \dfrac{h}{AN}$

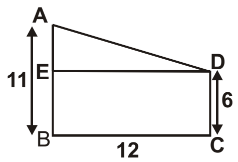
$\Rightarrow AN = \sqrt{3}h$

and $\tan 45^0 = \dfrac{h}{BN}$

$\Rightarrow BN = h$

$\therefore AN + BN = \sqrt{3}h + h = \left(\sqrt{3}+1\right)h$

Q.9.(C)



AB = 11 meter

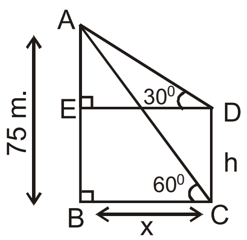
CD = 6 meter

$\therefore$ AE = 11 - 6 = 5 m.

BC = ED = 12 m.

$\therefore AD = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

Q.10.(C)

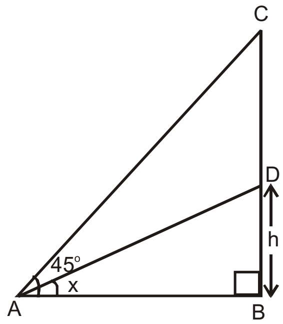


$\tan 60^0 = \dfrac{75}{x} \Rightarrow x = \dfrac{75}{\sqrt{3}} = 25\sqrt{3}$

and $\tan 30^0 = \dfrac{75-h}{x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{(75-h)}{25\sqrt{3}}$

$\Rightarrow 25 = 75 - h \Rightarrow h = 50$ m.

Q.11.(B)



$\tan 45^o = \dfrac{BC}{AB}$

$\Rightarrow h + CD = AB$

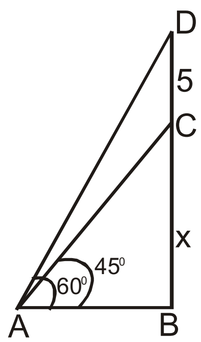
$\cot x = \dfrac{AB}{h}$

$\Rightarrow AB = h \cot x$

$\Rightarrow h + CD = h \cot x$

$\Rightarrow CD = h \cot x - h$

Q.12.(A)



$\dfrac{x+5}{\tan 60^0} = \dfrac{x}{\tan 45^0}$

$\Rightarrow \dfrac{x+5}{\sqrt{3}} = x$

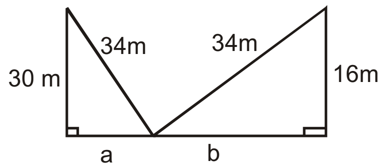
$\Rightarrow x + 5 = x\sqrt{3}$

$\Rightarrow x = \dfrac{5}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{5\sqrt{3}+5}{3-1}$

$= 5\dfrac{\left(\sqrt{3}+1\right)}{2} = 6.83$ m.

Q.13.(C)



$a^2 = 34^2 - 30^2$

$= 4 \times 64$

$a = 2\times 8$

$a = 16$

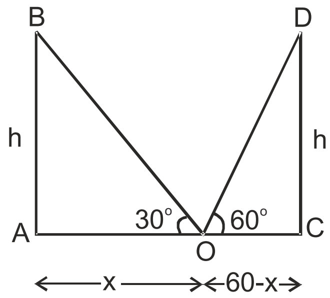
$b^2 = 34^2 - 16^2$

$= 50 \times 18$

$b = 30$

width of lane = 30+16 = 46 m.

Q.14.(B)



In ΔAOB,

$$\frac{AB}{AO} = \frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}} \quad \text{\_\_\_\_\_(I)}$$

In ΔCOD,

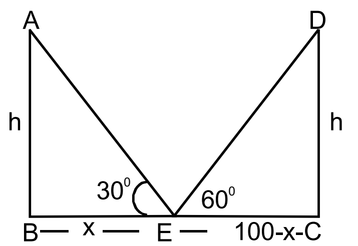
$$\frac{CD}{OC} = \frac{h}{(60-x)} = \tan 60° = \sqrt{3} \quad \text{\_\_\_\_\_(II)}$$

From equation (I) and (II)

$$\frac{x}{\sqrt{3}(60-x)} = \sqrt{3}$$

x = 45 meter

Q.15.(A)



AB = CD = h meter (Height of pole)

From ΔABE

$$\tan 30^0 = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$\Rightarrow \sqrt{3}$ h=x .... (I)

From ΔDEC

$$\tan 60^0 = \frac{h}{100 - x}$$

$$\sqrt{3} = \frac{h}{100-x}$$

$$\sqrt{3}(100-x) = h$$

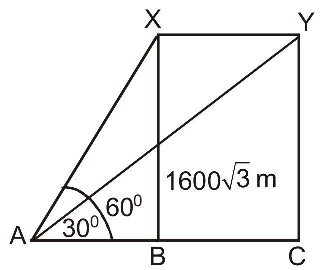
$100\sqrt{3} - \sqrt{3}h = x\sqrt{3}$

From equation (I)

$100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$

$h = 25\sqrt{3}$ meter

Q.16.(D)



X and Y are the point of plane

$\angle XAB = 60^0, \angle YAB = 30^0$

In ΔABX

$$\tan 60^0 = \frac{BX}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{1600\sqrt{3}}{AB}$$

$\therefore AB = 1600$

In ΔACY

$$\tan 30^0 = \frac{CY}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{1600\sqrt{3}}{AC}$$

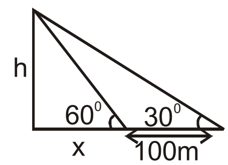
AC=1600 × 3 = 4800

$\Rightarrow PY = BC = AC - AB$

= 4800 - 1600 = 3200

Speed = $\frac{3200}{8}$ = 400 m/sec.

Q.17.(A)



$\tan 60^0 = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$

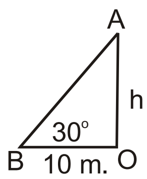
$\tan 30^0 = \frac{h}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{h}{\frac{h}{\sqrt{3}}+100}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h\sqrt{3}}{h+100\sqrt{3}}$

h = 86.60 m

Q.18.(A)



$\frac{h}{10} = \tan 30^o = \frac{1}{\sqrt{3}}$

$h = \frac{10}{\sqrt{3}}$

$\cos 30^o = \frac{10}{AB}$

$\frac{\sqrt{3}}{2} = \frac{10}{AB}$

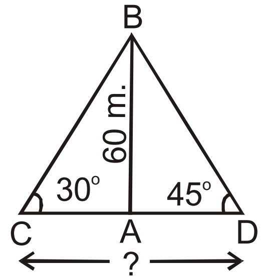
$AB = \frac{20}{\sqrt{3}}$

Height of the tree OA+AB

$= \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{30}{\sqrt{3}}$

$= 10\sqrt{3}$ = 10×1.73 = 17.3 meter

Q.19.(A)



Let AB be the tower and let C and D be the position of two men. Then $\angle ACB = 30^o$, $\angle ADB = 45^o$

AB = 60 meter

$\frac{AC}{AB} = \cot 30^o = \sqrt{3}$

$AC = 60\sqrt{3}$

$\frac{AD}{AB} = \cot 45^o = 1$

$\frac{AD}{60} = 1$

AD = 60 meter

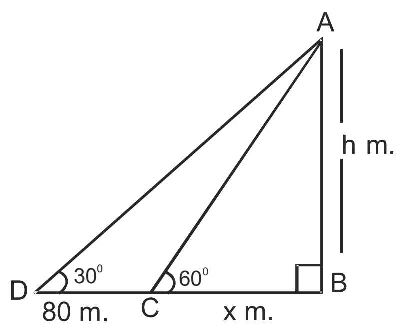
Distacne between the two men

= CD = AC + AD

$= (60\sqrt{3} + 60)$ meter $= 60(\sqrt{3}+1)$

= 60(1.73+1) = 60×2.73 = 163.8 meter

Q.20.(A)



In Δ ABC

$\tan 60^0 = \frac{AB}{BC}$

$\frac{h}{x} = \sqrt{3}$

$h = \sqrt{3}x$

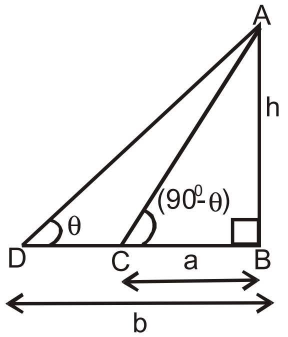
In Δ ABD

$\tan 30^0 = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+80}$

x = 40 m.

Q.21.(B)



In ΔABC,

$\tan(90 - \theta) = \frac{h}{a}$

$\Rightarrow \cot\theta = \frac{h}{a}$ ----------(i)

and

In ΔABD,

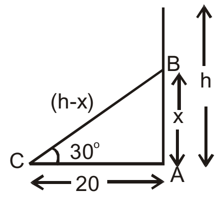
$\cot\theta = \frac{b}{h}$ ---------------(ii)

From (i) and (ii),

$\frac{h}{a} = \frac{b}{h}$

$\Rightarrow h = \sqrt{ab}$

Q.22.(B)



Let the height of the post be h m.

In Δ ABC,

$\tan 30^o = \frac{x}{20}$

$x = \frac{20}{\sqrt{3}}$

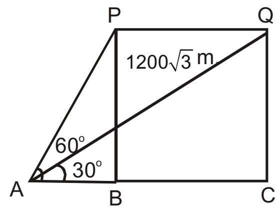
$\cos 30^o = \frac{20}{h-x}$

$\frac{\sqrt{3}}{2} = \frac{20}{h-x}$

$h - x = \frac{40}{\sqrt{3}}$

$\Rightarrow h = \frac{40}{\sqrt{3}} + \frac{20}{\sqrt{3}} = 20\sqrt{3}$ meter

Q.23.(D)



In Δ ABP,

$\tan 60^o = \frac{1200\sqrt{3}}{AB} \Rightarrow AB = 1200$ m.

In ΔACQ ,

$\tan 30^o = \frac{1200\sqrt{3}}{AC} \Rightarrow AC = 3600$ m.

$\therefore$ BC = 3600 – 1200 = 2400 m.

$\therefore$ Speed of aeroplane = $\frac{2400}{15}$ = 160 m./sec.

Notes

CHAPTER-24
DATA INTERPRETATION

Scan the QR code to get video of this chapter.

Data interpretation

Data interpretation is the act of transfering data with the objective of extracting useful information and facilating conclusions on the basis of the given data. depending upon the type of data and the questions, we might be required to apply certain statistical tools with various method to represent the given data.

There are different types of data interpretation.

(A) Tabulation

Tabulating is a way of processing information or data by putting it in a table. It refers to a table or chart with rows and column.

Ex.1-4. Based on the table given below which shows production of the number of scooters by a Company during first half of 2018

Type /Month	Jan	Feb	Mar.	April	May	June
X	25	25	18	40	20	15
Y	25	27	50	45	30	20
Z	25	27	15	25	30	20
T	25	26	25	0	30	35

Ex.1. In which month, did the company produce equal number of all types of scooters?

(A) May **(B)** June

(C) January **(D)** March

Sol.(C) January

Ex.2. The total number of scooters produced by the company, during the first half of 2018 is

(A) 623 **(B)** 197

(C) 90 **(D)** 143

Sol.(A) 623

Ex.3. In which two months, was the number of scooters produced by the company the same?

(A) January, March **(B)** January, May

(C) January, February **(D)** April, May

Sol.(D) April,May

Ex.4. In which month, was the production of all types of scooters the lowest?

(A) March **(B)** June

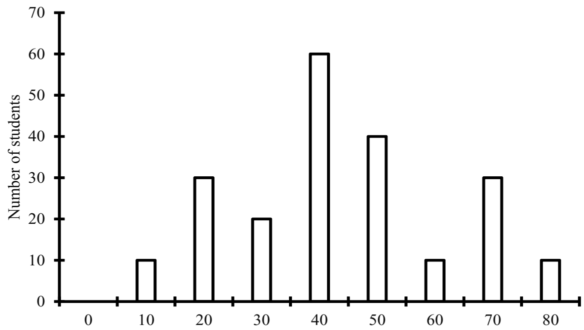
(C) January **(D)** February

Sol.(B) June

(B) Bar graphs

Bar graph is a chart that uses bars to show comparions between categories of data the bars can be either horizontal or vertical.

Ex.1-4. Study the following histogram and answer Four questions.



Ex.1. The total no. of students on whom this survey was made is.

(A) 200 **(B)** 210

(C) 190 **(D)** 220

Sol.(B) Total no. of student

$10 + 30 + 20 + 60 + 40 + 10 + 30 + 10$

$= 210$

Ex.2. The percentage of students securing marks less than 50 is.

(A) $47\frac{13}{21}\%$ **(B)** $27\frac{5}{7}\%$

(C) $23\frac{13}{21}\%$ **(D)** $28\frac{4}{7}\%$

Sol.(D) Percentage of students securing marks less then 50

$= \frac{60}{210} \times 100 = 28\frac{4}{7}\%$

Ex.3. The number of students securing marks in the range 90-100 is.

(A) 10 **(B)** 30

(C) 20 **(D)** 40

Sol.(A) 10

Ex.4. The range of marks obtained by maximum no. of students is

(A) 130-40 **(B)** 60-70

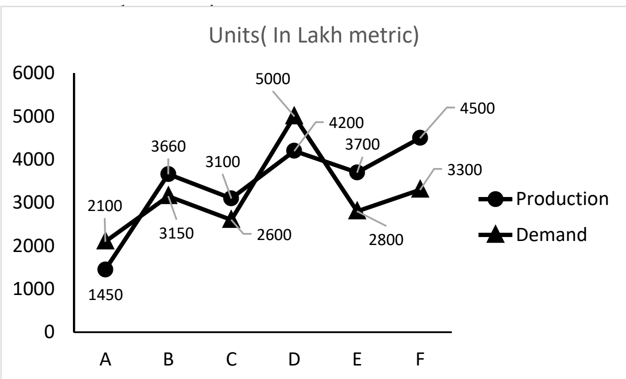
(C) 80-90 **(D)** 50-60

Sol.(D) 50 - 60

(C) Line graph

A graph that uses point connected by Lines to show how something changes in value (as time goes by)

Ex.1-4. The graph shows the demand and production of different companies. Study the graph and answer



Ex.1. What is the difference between the average demand and the average production of the companies (in lakh tonnes) approximately.

(A) 325 **(B)** 200

(C) 250 **(D)** 275

Sol.(D) Average demand = $\frac{18950}{6}$

Average production = $\frac{20610}{6}$

Difference = 275

Ex.2. What is the ratio of the number of companies having more demand than production to those having more production than demand?

(A) 3 : 2 **(B)** 1 : 2

(C) 2 : 3 **(D)** 2 : 1

Sol.(B) 1 : 2

Ex.3. The demand of company B is what percentage of the production of company F?

(A) 60% **(B)** 50%

(C) 70% **(D)** 80%

Sol.(C) Required percentage = $\frac{3150}{4500} \times 100$ = 70%

Ex.4. The production of company A is approximately what percent of the demand of company C?

(A) 65% **(B)** 50%

(C) 60% **(D)** 55%

Sol.(D) $\frac{1450}{2600} \times 100$ = 55%

(D) Pie chart

A pie chart is a circular chart divided into sectors each sector, shows the relative size of each value i.e. each represent a proportion of the whole.

Ex.1-5. Read it carefully and answer five questions.

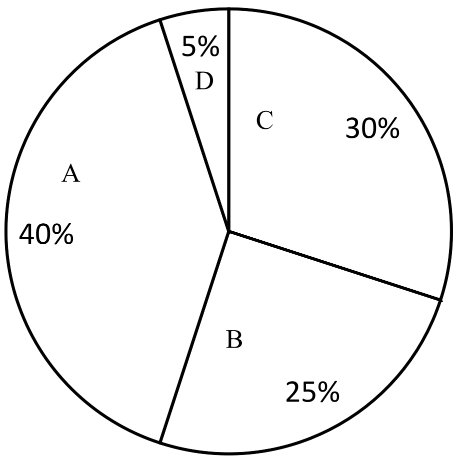
Total cost of production of a firm is Rs. 250 lakh, Fllowing pie-chart shows the percentage of costs of production in different purposes. Letter A, B, C, D represent the following:

A = Cost of raw materials

B = Cost of packing materials

C = Cost of labour

D = Maintenance cost



Ex.1. If the production increases to five times of the present cost, then the percentage increase of the cost is

(A) 50% **(B)** 400%

(C) 300% **(D)** 500%

Sol.(D) 500%

Ex.2. Cost of packing materials and raw materials together amounts to

(**A**) Rs.162.5 lakh (**B**) Rs.175 lakh

(**C**) Rs.137.5 lakh (**D**) Rs.87.5 lakh

Sol.(A) Packing material and raw material is

= 40%+25% = 65%

65% of 250 = 162.5

Ex.3. If the cost of production doubles in a period of 3 years, then the corresponding maintenance cost in rupees will be

(**A**) Rs.75 lakh (**B**) Rs.12.5 lakh

(**C**) Rs.25 lakh (**D**) Rs.125 lakh

Sol.(C) If cost become double then new cost

= 250 × 2 = 500 lakh

Cost of maintenance = 5% of 500 = 25 lakh

Ex.4. If the total maintenance cost increases from Rs. 12.5 lakh to Rs. 50 lakh, then the percentage increase of the maintenance cost is.

(**A**) 25% (**B**) 75%

(**C**) 400% (**D**) 300%

Sol.(D) 50 - 12.5 = 37.5

$$\frac{37.5}{12.5}\times 100 = 300\%$$

Ex.5. If the packing cost increase by 2%, then the new packing cost will be

(**A**) Rs. 62.5 lakh (**B**) Rs. 5 lakh

(**C**) Rs. 1.25 lakh (**D**) Rs. 63.75 lakh

Sol.(D) Packing of cost 25% of 250 = 62.5 lakhs

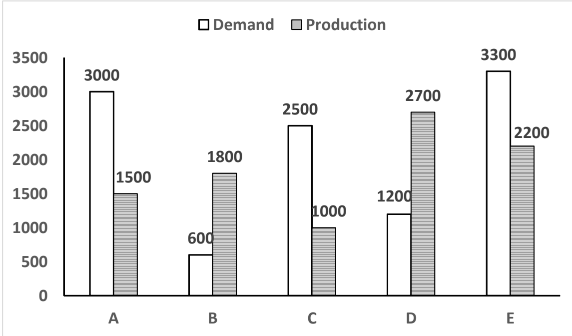
New cost = 102% of 62.5 = 63.75

Key Points:

1. First read the question and try to interpretate the questions as the topic says data interpretation.
2. Formulas and full concept of percentage average, approx, value and ratio etc.
3. Try to understand the question in simplest terms while solving D.I. This can be easily done by practicing more and more
4. Always keep in you mind that in the circular graph 100% = 360°
5. In, income- expenditure type of D.I.
   Expenditure is equivalent to C.P.
   and income is equivalent to S.P.
6. Don't feel confusing and no need to try and solve all the questions only focus on questions with you are more familiar.

Examples

Ex.1-5. The following chart represents Demand and Production for 5 companies A, B, C, D, E.



On the basis of the graph answer the questions.

Ex.1. If company A desires to meet the demand by purchasing surplus production of a company, then the most suitable company is:

(**A**) C (**B**) D

(**C**) E (**D**) B

Sol.(B) Company A has (3000-1500)

= 1500 less production

Company D has (2700-1200)

= 1500 more production

Ex.2. If x% of demand of company C equals demand of company B, then x equals :

(**A**) 24 (**B**) 20

(**C**) 60 (**D**) 4

Sol.(A) $\frac{600}{2500}\times 100 = 24\%$

Ex.3. If the production of company D is n times of the production of company A. Then n equals:

(**A**) 1.5 (**B**) 2.5

(**C**) 1.2 (**D**) 1.8

Sol.(D) $\frac{2700}{1500} = 1.8$

Ex.4. The difference between average demand and average production of the five companies taken together is:

(**A**) 400 (**B**) 280

(**C**) 130 (**D**) 620

Sol.(B) Total demand = 10600

Total production = 9200

Difference = 1400

Average difference = $\frac{1400}{5} = 280$

Ex.5. The ratio of the number of companies having more demand than production to those having more production than demand is:

(A) 4:1 **(B)** 2:2

(C) 3:2 **(D)** 2:3

Sol.(C) 3 : 2

Ex.6-9. The total expenditure of a company for a particular month is Rs.60000. The various heads of expenditure I to IV are indicated in a pie chart given below. These heads are:

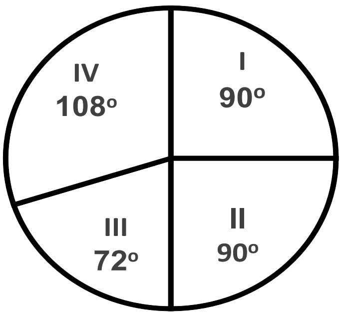
I. Raw materials

II. Conveyance

III. Electricity

IV. Overhead expenses

Study the pie chart and answer the questions.



Ex.6. Total expenditure on conveyance is:

(A) Rs. 12,000 **(B)** Rs. 15,000

(C) Rs. 20,000 **(D)** Rs. 10,000

Sol.(B) $\frac{90^\circ}{360^\circ} \times 60000 = 15000$

Ex.7. What percentage of total expenditure is on electricity?

(A) 23% **(B)** 25%

(C) 30% **(D)** 20%

Sol.(D) $\frac{72^\circ}{360^\circ} \times 100 = 20\%$

Ex.8. What is the amount spent on overhead expenses?

(A) Rs. 12,000 **(B)** Rs. 15,000

(C) Rs. 18,000 **(D)** Rs. 10,000

Sol.(C) $\frac{108^\circ}{360^\circ} \times 60000 = 18000$

Ex.9. What percentage of total expenditure is on raw materials?

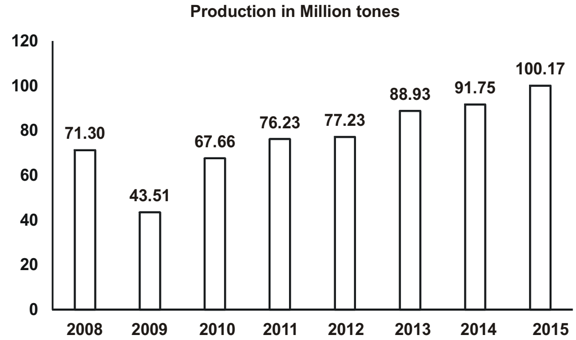
(A) 25% **(B)** 30%

(C) 60% **(D)** 23%

Sol.(A) $\frac{90^\circ}{360^\circ} \times 100 = 25\%$

Ex.10-13. The following table shows the world production of steel in 2008-2015. Study the table and answer the question.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Production (in Million Tons)	71.30	43.51	67.66	76.23	77.23	88.93	91.75	100.17



Ex.10. The ratio of production of steel in the year 2012 and 2013 to that of 2011 and 2015 is:

(A) 2005:2077 **(B)** 2205:2007

(C) 2077:2205 **(D)** 2205:2077

Sol.(C) 2077 : 2205

Ex.11. The average production of steel is:

(A) 76.09 **(B)** 77.10

(C) 75.13 **(D)** 74.07

Sol.(B) Total = 616.78

Average = $\frac{616.78}{8}$ = 77.0975 = 77.10 (approx).

Ex.12. The difference of the production of steel in the year 2011 and 2012 is x% of 2015. Then the value of x is approximately:

(A) 0.001% **(B)** 1%

(C) 0.1% **(D)** 0.01%

Sol.(B) Difference of 2011 and 2012

77.23 - 76.23 =1

$\Rightarrow x = \frac{1}{100.17} \times 100 = 1\%$

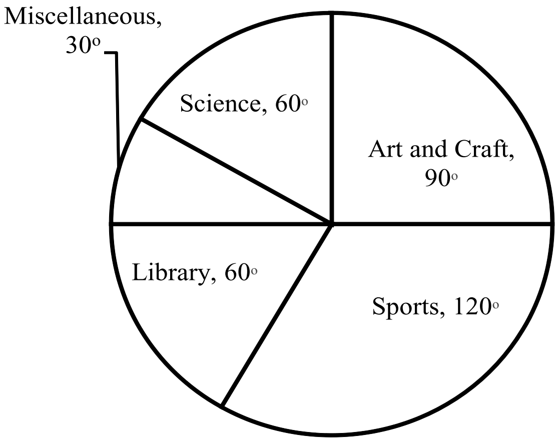
Ex.13. The number of years during which the company has its production less than the average production during 2011-2015 is approximately:

(A) 6 **(B)** 2

(C) 3 **(D)** 4

Sol.(D) 4

Ex.14-18. The pie chart shows how the school funds is spent under different heads in a certain school. Using the pie chart answer the questions.



Ex.14. Which head uses 25% of the funds?

(A) Sports **(B)** Art and Craft

(C) Library **(D)** Miscellaneous

Sol.(B) 25% $= 25 \times \frac{18}{5} = 90°$

Art and craft

Ex.15. Which head has the maximum expenditure?

(A) Art and Craft **(B)** Library

(C) Sports **(D)** Science

Sol.(C) Sports

Ex.16. What percentage of the total expenses is spent on library?

(A) 24% **(B)** 20%

(C) 16.6% **(D)** 24.3%

Sol.(C) $\frac{60°}{360°} \times 100$ = 16.6%

Ex.17. What is the ratio of expenditure on sports to that on art and craft?

(A) 1 : 4 **(B)** 4 : 3

(C) 2 : 1 **(D)** 1 : 1

Sol.(B) 120 : 90

= 4 : 3

Ex.18. Which heads have the same amount of expenditure?

(A) Science and Misc

(B) Misc and Library

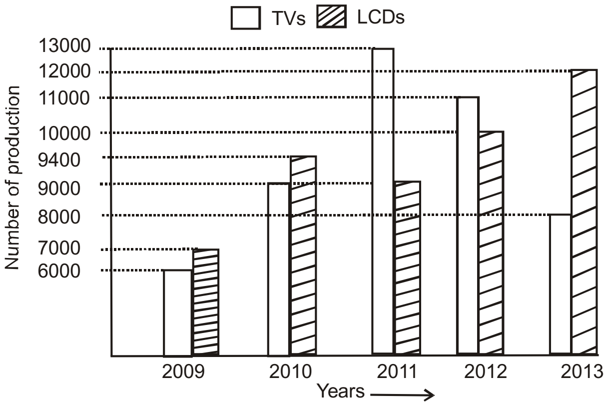
(C) Sports and science

(D) Library and science

Sol.(D) Library and Science

Ex.19-22. Study the following bar diagram carefully and answer the following four question.

The number of the production of electronic items (TVs and LCD) in a factory during the period from 2009 to 2013



Ex.19. The ratio of production of LCDs in the year 2011 and 2013

(A) 4 : 3 **(B)** 3 : 4

(C) 2 : 3 **(D)** 1 : 4

Sol.(B) 9000 : 12000

3 : 4

Ex.20. The difference between average of production of TVs and LCDs from 2009 to 2012 is

(A) 600 **(B)** 700

(C) 800 **(D)** 900

Sol.(D) Total Tv = 6000 + 9000 +13000 +11000

= 39000

Total LCD = 7000+9400+9000+10000

= 35400

Difference = 39000-35400

= 3600

Average difference = $\frac{3600}{4} = 900$

Ex.21. The ratio of production of TVs in the years 2009 and 2010 is

(A) 3 : 2 **(B)** 7 : 6

(C) 6 : 7 **(D)** 2 : 3

Sol.(D) 6000 : 9000 = 2 : 3

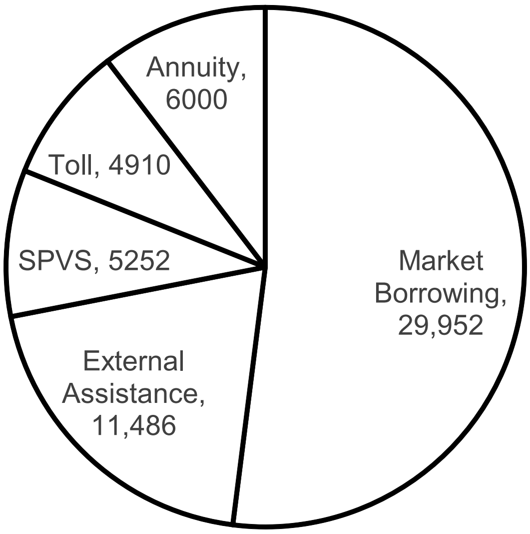
Ex.22. The total number of production of electronic items is maximum in the year.

(A) 2010 **(B)** 2011

(C) 2009 **(D)** 2013

Sol.(B) 2011

Ex.23-25. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the following Three questions:



Ex.23. The central angle corresponding to Market Borrowing is

(A) 52^0 **(B)** 192.4^0

(C) 187.2^0 **(D)** 137.8^0

Sol.(C) $\frac{29952}{57600}\times 360^\circ = 187.2^\circ$

Ex.24. If NHAI could receive a total of Rs. 9,695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowing to arrange for the shortage of funds?

(A) 4.5% **(B)** 8%

(C) 7.5% **(D)** 6%

Sol.(D) Shortage is = 11486 - 9695= 1791

% increase in borrowing

$= \frac{1791}{29952}\times 100 = 6\%$

Ex.25. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4,910 crores?

(A) Rs. 5,827 crores

(B) Rs. 5,455 crores

(C) Rs. 5, 316 crores

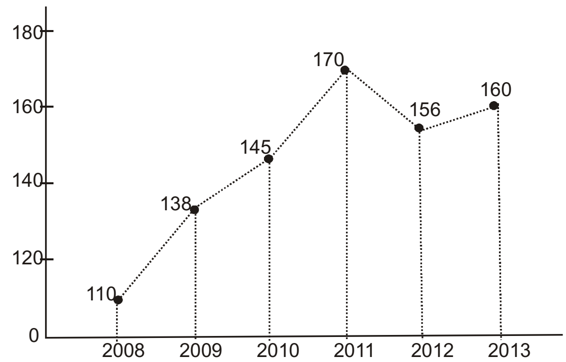
(D) Rs. 6,213 crores

Sol.(B) 90% = 4910

$1\% = \frac{4910}{90}$

$100\% = \frac{4910}{90}\times 100$ = 5455 (approx)

Ex.26-28. Study the following frequency polygon and answer the questions. Given a line graph showing the number of students passed in Higher Secondary Examination in a school over the years from 2008 to 2013.



Ex.26. The average of passed students in the years 2008, 2009, 2012 approximately is:

(A) 134.32 **(B)** 134.56

(C) 134.67 **(D)** 134.41

Sol.(C) Total is = 110 + 138 + 156 = 404

Average $= \frac{404}{3} = 134.67$

Ex.27. The increase in percentage of passed students from 2008 to 2011 approximately is

(A) 53.05% **(B)** 55%

(C) 54.5% **(D)** 50.5%

Sol.(C) Req. increase% $= \frac{170-110}{110}\times 100$ = 54.5%

Ex.28. The decrease in percentage of passed students from 2011 to 2012 approximately.

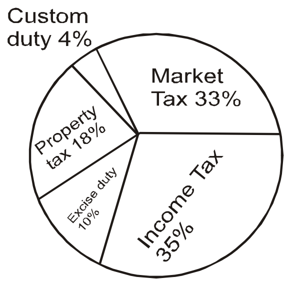
(A) 8.24% **(B)** 8.22%

(C) 8.27% **(D)** 8.25%

Sol.(A) Req. decrease % $= \frac{170-156}{170}\times 100$

= 8.24%

Ex.29-31. The income of a state under different heads is given in the following pie chart. Study the chart and answer the questions.



Ex.29. The central angle of the sector representing income tax is:

(A) 135^0 **(B)** 126^0

(C) 119^0 **(D)** 150^0

Sol.(B) Angle = $35 \times \frac{18}{5} = 126^0$

Ex.30. If the total income in a year be Rs.733 crores, then the income (in Rs. crores) from 'Income Tax' and 'Excise duty' is:

(A) Rs.329.85 **(B)** Rs.331.45

(C) Rs.331.50 **(D)** Rs.329.80

Sol.(A) Income from income tax and exice duty

$= 733 \times \frac{45}{100} =$ Rs. 329.85

Ex.31. If the income from market tax in a year be Rs.165 crores then the total income (in Rs. crores) from other sources is:

(A) 335 **(B)** 345 **(C)** 365 **(D)** 325

Sol.(A) Income from market tax = 33%

So, Income from other sources = 67%

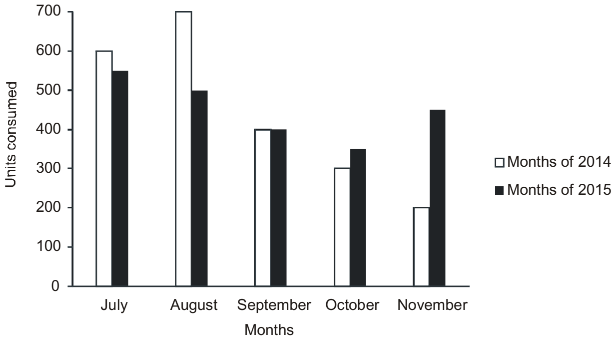
33% = 165 cr.

1% = 5 cr.

67% = 335 cr.

Ex.32-35. Study the following bar-diagram and answer the questions.

Electricity units consumed by a family in two consecutive years during July to November:



Ex.32. In how many months in 2014, the consumption of electric units was more than the average units consumption in that year?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 2

Sol.(D) Average is = $\frac{2200}{5} = 440$

So, 2 months

Ex.33. The total units consumption in the year 2015 during these 5 months, in respect of the same in the previous year has been:

(A) decreased by 2.27%

(B) increased by 2.22%

(C) found unaltered

(D) increased by 2.27%

Sol.(D) Total consumption in 2014 = 2200

Total consumption in 2015 = 2250

Increase % = $\frac{50}{2200} \times 100 = 2.27\%$

Ex.34. The maximum difference in the units consumption between these two years has been found in the month of:

(A) August **(B)** October

(C) November **(D)** July

Sol.(C) November

Ex.35. The average electric consumption by the family during these 5 months in 2015 is:

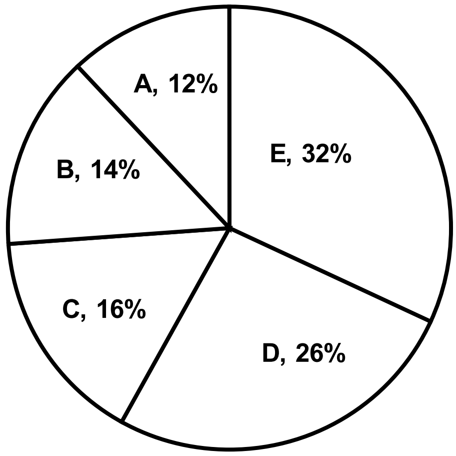
(A) 400 units **(B)** 440 units

(C) 450 units **(D)** 470 units

Sol.(C) Average = $\frac{2250}{5} = 450$

Ex.36-38. Study the following pie charts and table carefully and answer the questions.

Number of candidate appeared in a competitive examination from five state. (Total candidate, 460000)



State-wise percentage of qualified candidate

State	%Qualified over appeared from a state
A	52%
B	62%
C	48%
D	34%
E	42%

Ex.36. What is the number of candidates qualified from state E.

(A) 61824 **(B)** 61428

(C) 61284 **(D)** 61482

Sol.(A) Candidate appeared in an competitive examination from state E

$= \frac{32}{100} \times 460000 = 147200$

Number of condidate qualified from state E

$= \frac{42}{100} \times 14700 = 61824$

Ex.37. What is the total number of candidate qualified from state 'A' and 'B' together ?

(A) 68326 **(B)** 68263

(C) 68632 **(D)** 68362

Sol.(C) Number of candidate qualified from state A =

$55200 \times \frac{52}{100} = 28704$

Number of candidate qualified from state B =

$64400 \times \frac{62}{100} = 39928$

Total candidate = 28704 + 39928 = 68632

Ex.38. What is the percentage of candidate qualified from state A and D? (Approx)

(A) 30% **(B)** 40%

(C) 48% **(D)** 43%

Sol.(B) Number of candidate appread from state A and D

= 55200 + 119600 = 174800

Number of candidate qualified from state A and D = 28704 + 40664

= 69368

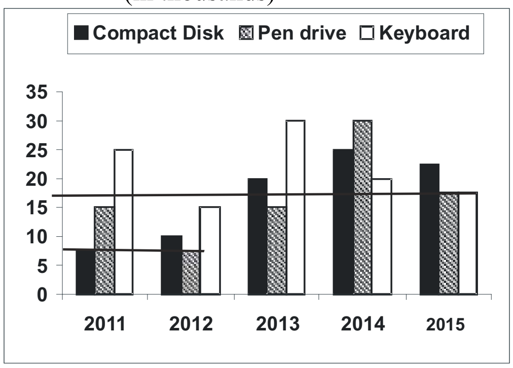
Percentage of candidate qualified from state A and

$D = \frac{69368 \times 100}{174800}$

$= 39.68 \approx 40\%$

Ex.39-40. Study the following graph which shows the production (in thousand of different cites, and answer the question)

(in thousands)



Ex.39. The total number of all products produced by the company in the year 2012 and 2014 together is

(A) 107500 **(B)** 105700

(C) 10750 **(D)** 1075

Sol.(A) Total production in 2012 = 32.5

Total production in 2014 = 75

Total production = 107.5 × 1000

= 107500

Ex.40. The average number of pendrives produced by the company overall the years together is

(A) 1700 **(B)** 170000

(C) 17000 **(D)** 85000

Sol.(C) Total production = 85 ×1000

$\text{Average} = \frac{85000}{5} = 17000$

EXERCISE

Q.1-5. The table given below shows the number of students who were absent and percentage of students who were present in the given two examinations from five different schools. The table also shows the percentage of students who were present in the Biology and Physics examination respectively.

School	Absent	Present (in %)	Biology (in %)	Physics (in %)
K	70000	65	32	68
L	81000	60	29	71
M	72000	40	30	70
N	44000	65	42	58
O	10800	55	25	75

Q.1. What is the difference between the number of students who were present in Physics and Biology examination from school N?

(A) 12450 **(B)** 10120

(C) 16520 **(D)** 24250

Q.2. Number of students who were present in Physics examination from school M is what percent of number of students who were absent from school M, L and O?

(A) 11 **(B)** 21

(C) 24 **(D)** 16

Q.3. What is the average of the number of the students who were present in Physics examination from school N, K and L?

(A) 67755 **(B)** 84632

(C) 74365 **(D)** 67894

Q.4. What are the total number of students who were present in the Biology examination from all the schools together?

(A) 193462 **(B)** 113015

(C) 126438 **(D)** 161738

Q.5. If the number of students who were present in the Physics examination from school A is 250% of the difference of the number of the students who were present in Physics and Biology examination, from school K, then what is the ratio of the number of students who were present from school L to number of students who were present in Physics examination from school A?

(A) 25:64 **(B)** 28:61

(C) 72:153 **(D)** 33:49

Q.6-10. The table given below shows the information about bats manufactured by 6 different companies. Each company manufactures only plastic and wooden bats. Each company labels these bats as Brand A or Brand B. The table shows the number of plastic bats as a percentage of total bats manufactured by each company. It also shows the ratio of wooden bats labeled A and B. Each company manufactured a total of 550000 bats.

Company	Plastic Bats	Brand A : Brand B
R	55%	21:7
S	70%	8 : 7
T	45%	6 : 19
U	75%	41 : 14
V	60%	7 : 15
W	40%	5 : 6

Q.6. What is the total number of wooden bats of brand A manufactured by company T?

(A) 23420 **(B)** 22990

(C) 68920 **(D)** 72600

Q.7. P = Sum of wooden bats of brand B manufactured by S and wooden bats of brand A manufactured by W.

Q = Difference of brand B wooden bats and brand A wooden bats manufactured by U.

What is the value P – Q?

(A) 675000 **(B)** 177700

(C) 159500 **(D)** 123500

Q.8. Taking all 6 companies together, how many wooden bats of Brand A have been produced?

(A) 691000 **(B)** 724000

(C) 683000 **(D)** 716000

Q.9. X = Average of plastic bats manufactured by V, U and T.

Y = Wooden bats of brand A manufactured by V.

What is the value X – Y?

(A) 197600 **(B)** 432890

(C) 260000 **(D)** 293300

Q.10. N= Wooden bats of brand B manufactured by U.

M = Total wooden bats manufactured by R and W together

What is the value of N/M?

(A) 0.043 **(B)** 0.061 **(C)** 0.125 **(D)** 0.087

Q.11-15. The given table shows the number (in percent) of employees working in different departments of an organization. The table also shows the ratio of males and females and the ratio of employees living in city Z and employees living in city Y. The total number of employees in the organization are 80000.

Department	Number of employee	M : F	City Z : City Y
A	10%	7:3	1 : 9
B	22%	13 : 9	3 : 19
C	12%	1 : 2	5 : 1
D	20%	3 : 2	1: 3
E	36%	8: 1	5 : 13

Q.11. How many employees of department A and C together are living in city Z?

(A) 9000 **(B)** 9200

(C) 8800 **(D)** 8200

Q.12. Male employees of department E is what percent of the employees living in city Z from department A?

(A) 1600 **(B)** 2400

(C) 3200 **(D)** 4200

Q.13. What is the ratio of male employee working in department B and D together to female employee working in department A and E together?

(A) 13 : 8 **(B)** 25 : 7

(C) 23 : 9 **(D)** 7 : 9

Q.14. On an average how many residents of city Y are working in each department?

(A) 11360 **(B)** 12420

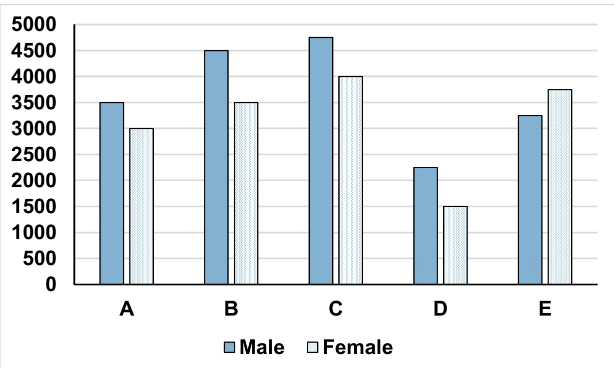
(C) 9130 **(D)** 10940

Q.15. What are the total number of employee in department A and E together?

(A) 29400 **(B)** 17600

(C) 46400 **(D)** 36800

Q.16-20. The following Bar-diagram shows total number of males and females in five different organisations. Study it carefully to answer the questions.



Q.16. What is the difference between the total number of females and the total number of males from all the organizations together?

(A) 2005 **(B)** 2050

(C) 2500 **(D)** 2055

Q.17. By how much percentage is the average number of females from all the organisations together is more than the number of males in organization 'D'?

(A) 30% **(B)** 38%

(C) 40% **(D)** 45%

Q.18. What is the ratio of the number of females from the organisations B and C to the number of males from the organisations D and E?

(A) 12 : 11 **(B)** 12 : 15

(C) 11 : 15 **(D)** 15 : 11

Q.19. Males from organisations A and B together form what percent of total number of males from organisations C, D and E together?

(A) 78.04% **(B)** 87.44%

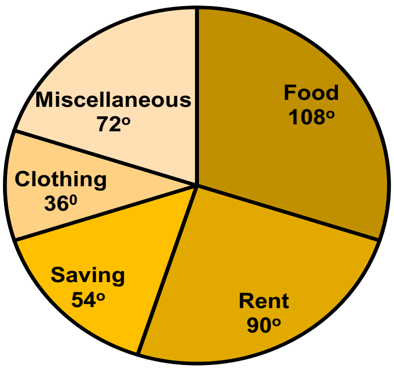
(C) 47.08% **(D)** 74.08%

Q.20. What is the ratio of average number of females from the organisations A, B and C to the average number of males from the organisations C, D and E?

(A) 42 : 41 **(B)** 41 : 42

(C) 40 : 41 **(D)** 41 : 40

Q.21-25. The following pit-chart shows the monthly expenditure of a family on various items. If the family spends Rs. 825 on clothing, answer the question



Q.21. What is the total monthly income of the faimly?

(A) Rs. 8025 **(B)** Rs. 8250

(C) Rs. 8520 **(D)** Rs. 8052

Q.22. What percent of the total income does the family save?

(A) 15% **(B)** 50%

(C) 20% **(D)** 25%

Q.23. What is the ratio of expanses on food and miscellaneous ?

(A) 3 :4 **(B)** 2 : 3

(C) 3: 2 **(D)** 2 :5

Q.24. What is the average of expense on clothing and rent?

(A) Rs. 1443.75 **(B)** Rs. 1314.57

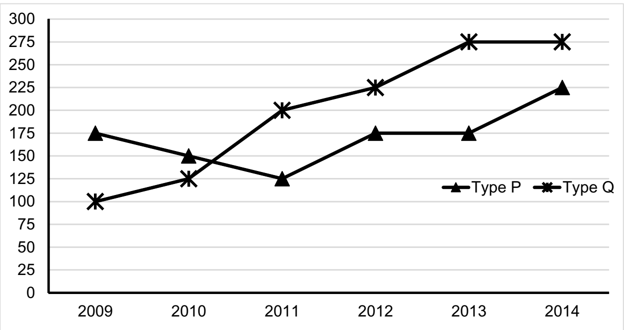
(C) Rs. 1574.34 **(D)** Rs. 1734.45

Q.25. The ratio or average of expenses on food, clothing and miscelaneous items to the average or expenses on savings and rent is -

(A) 3 : 2 **(B)** 1 : 3

(C) 2 : 1 **(D)** 1 : 1

Q.26-30. The following graph shows production (in thousands) of two types (P and Q) of vehicles by a factory over the years 2009 to 2014. Study the graph and answer the five questions:



Q.26. The total production of Type Q vehicles in the years 2009 and 2011 is what percent of total production of Type P vehicles in 2010 and 2014?

(A) 80 **(B)** 68.25

(C) 81.25 **(D)** 75

Q.27. The production of Type P vehicles in 2010 was approximately what percent of Type Q vehicles in 2014?

(A) 75 **(B)** 60

(C) 45 **(D)** 55

Q.28. Approximate percentage decrease in product on of Type P vehicles from 2010 to 2011 is

(A) 16.7 **(B)** 14.3

(C) 10.1 **(D)** 12.5

Q.29. The ratio of total production of Type Q vehicles to total production of Type P vehicles over the years is

(A) 8 : 5 **(B)** 48 : 41

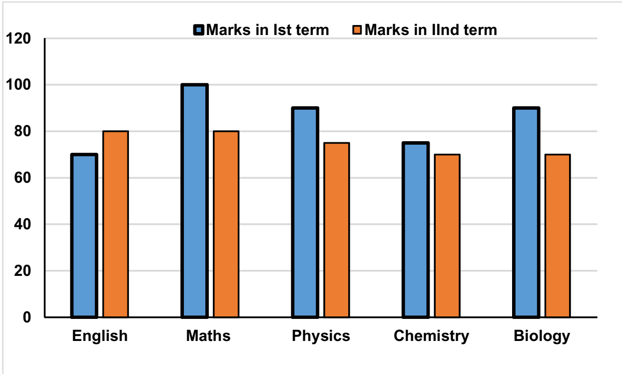
(C) 41 : 48 **(D)** 5 : 8

Q.30. In how many of the given years, was the production of Type Q vehicles of the company more than the average production of this type vehicles in the given years?

(A) 5 **(B)** 4

(C) 3 **(D)**2

Q.31-32. Study the bar diagram and answer



Q.31. Ratio of highest and lowest marks obtained in first term among all the subjects is

(A) 7:9 **(B)** 9:7

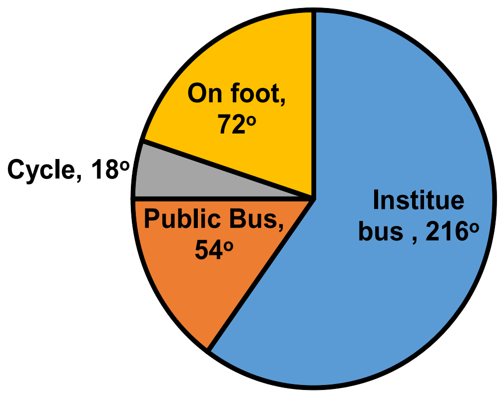
(C) 10:7 **(D)** 7:10

Q.32. Average marks obtained by the students for all subjects in second term is

(A) 65 **(B)** 73

(C) 62 **(D)**72

Q.33-35. In an Institution there are 800 students. Students use different modes of transport for going to the institution and return. The given pie diagram represents the requisite data. Study the diagram carefully and answer the questions.



Q.33. Number of students travel in public bus is

(A) 150 **(B)** 120

(C) 130 **(D)** 125

Q.34. Number of students who do not use institute bus is

(A) 330 **(B)** 350

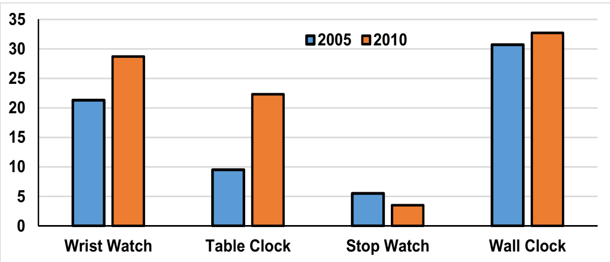
(C) 480 **(D)** 320

Q.35. Number of Students who go to institute on foot is--

(A) 160 **(B)** 170

(C) 120 **(D)** 106

Q.36-40. A watch company produces four different products. The sale of these products in lakhs during 2005 and 2010 are shown in the following bar diagram. "Study the graph and answer the question Nos. 36 to 40.



Q.36. The sales in percentage of wrist watch in 2010 more than the sales, of table clock in 2010 was nearly by

(A) 26.7% **(B)** 27.7%

(C) 28.7% **(D)** 21.7%

Q.37. The ratio of sales of stop watch in 2010 to the sale of table clock in 2005 is

(A) 6: 19 **(B)** 7:6

(C) 19:6 **(D)** 7: 19

Q.38. The sales of table clock in 2005 was less than the sales of wall clock in 2005 is nearly by

(A) 70.05% **(B)** 69.05%

(C) 68.05% **(D)** 62.05%

Q.39. During the period 2005-2010 the minimum rate of increase in sales is in the product of

(A) Wrist watch **(B)** Table clock

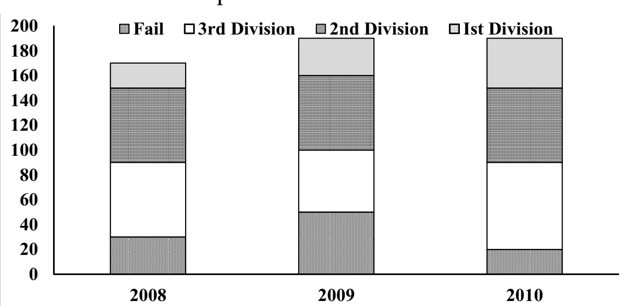
(C) Stop watch **(D)** Wall clock

Q.40. The sales have increased by nearly 135% from. 2005 to 2010 in the product of

(A) Table dock **(B)** Wrist watch

(C) Stop watch **(D)** Wall clock

Q.41-45 The bar graph given below depicts H.S. Students of a school for three years. Study the diagram and answer the questions:-



Q.41. The pass percentage in 1st division 2008 was-

(A) $14\frac{2}{17}\%$ **(B)** 32%

(C) $15\frac{5}{34}\%$ **(D)** $11\frac{13}{17}\%$

Q.42. The pass percentage in 2008 was-

(A) 67% **(B)** $83\frac{3}{26}\%$

(C) $79\frac{2}{13}\%$ **(D)** $82\frac{6}{17}\%$

Q.43. The percentage of the students passed in 2nd division in the year 2010 was-

(A) $30\frac{3}{17}\%$ **(B)** $33\frac{12}{17}\%$

(C) $28\frac{9}{34}\%$ **(D)** $31\frac{11}{17}\%$

Q.44. In 2009 number of students passed in second division were how much percent more than number of students failed in same year?

(A) 25% **(B)** 12.5%

(C) 20% **(D)** 16.67%

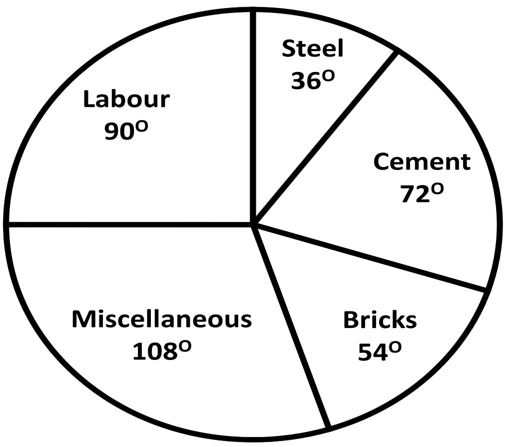
Q.45. In which year the school had the best result for H.S. in respect of percentage of pass candidates?

(A) 2008 **(B)** 2009

(C) 2010

(D) The percentage of pass student is same for all year.

Q.46-50: The following pie-chart shows the expenditure incurred on the construction of a house in a city. Study the chart and answer the following questions.



Q.46. The mean of the expenditure is on-

(A) Steel **(B)** Cement

(C) Bricks **(D)** Labour

Q.47. The ratio expenditure on Steel, Cement and Bricks is-

(A) 2 : 4 : 3 **(B)** 4 : 2 : 3

(C) 3 : 2 : 4 **(D)** 4 : 3 : 2

Q.48. The highest expenditure in percentage is-

(A) 40% **(B)** 30%

(C) 45% **(D)** 60%

Q.49. What part of expenditure on labour is in respect of total expenditure?

(A) 3/10 **(B)** 5/8

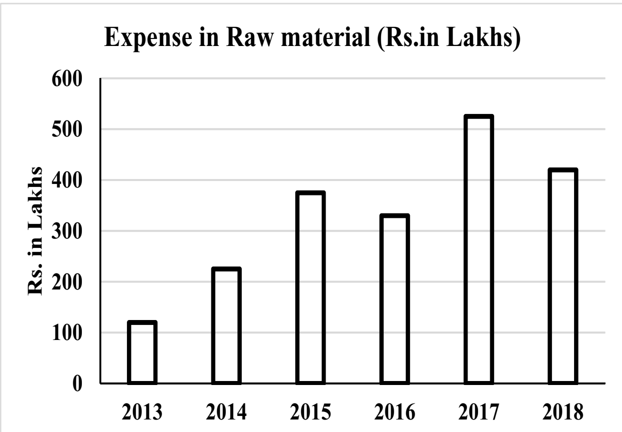
(C) 1/4 **(D)** 7/8

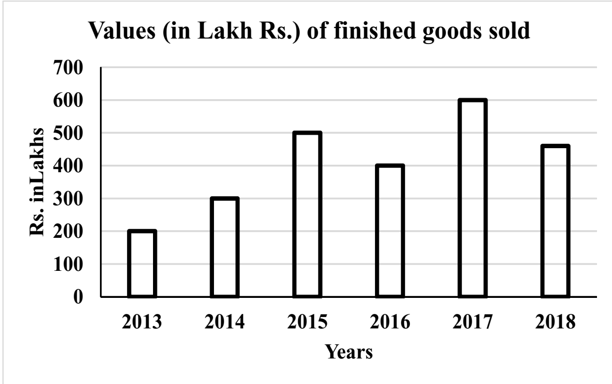
Q.50. Of the total expenditure the percentage of expenditure on steel and bricks together is-

(A) 90% **(B)** 20%

(C) 25% **(D)** 30%

Q.51-55. Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs.) invested by a Company in purchasing raw materials over the years and the other shows the values (in Lakh Rs.) of finished goods sold by the Company over the years.





Q.51. The maximum difference between the amount invested in Raw materials and value of sales of finished goods was during which year?

(A) 2013 **(B)** 2014

(C) 2015 **(D)** 2016

Q.52. The value of sales of finished goods in 2018 was approximately what percent of the sum of amount invested in Raw materials in the years 2015, 2016 and 2017?

(A) 33% **(B)** 37%

(C) 45% **(D)** 49%

Q.53. What was the difference between the average value of sales of finished goods during the given period and average amount invested in Raw materials during this given period ?

(A) Rs.62.5 Lakhs **(B)** Rs.68.5 Lakhs

(C) Rs.71.5 Lakhs **(D)** Rs.77.5 Lakhs

Q.54. In which year, the percentage change (compared to the previous year) in the investment on Raw materials is same as that in the value of sales of finished goods?

(A) 2013 **(B)** 2014

(C) 2015 **(D)** 2016

Q.55. In which year, there has been a maximum percentage increase in the amount invested in Raw materials as compared to the previous year?

(A) 2013 **(B)** 2014

(C) 2015 **(D)** 2016

EXPLANATION

Q.1-5.

School	Total present	Biology	Physics	difference
K	130000	41600	88400	46800
L	121500	35235	86265	51030
M	48000	14400	33600	19200
N	44000	18480	28600	10120
O	13200	3300	9900	6600

Q.1.(B) **Required answer= 10120**

Q.2.(B) Required answer

$=\frac{33600}{720000+81000+10800}\times100=\frac{33600}{163800}\times100\approx21\%$

Q.3.(A) Required answer

$=\frac{28600+88400+86265}{3}=67755$

Q.4.(B) Required answer

=41600+35235+14400+18480+3300 =113015

Q.5.(B) Required answer

=(41600+14400) : (88400+33600)

=56:122=28:61

Q.6-10.

Company	No of plastic bats	No of wooden bats	wooden bats of A	wooden bats of B
R	302500	247500	207900	39600
S	385000	165000	88000	77000
T	247500	302500	72600	229900
U	412500	137500	102500	35000
V	330000	220000	70000	150000
W	220000	330000	150000	180000

Q.6.(D) Required answer= 72600

Q.7.(C) P=77000+150000
Q=102500-35000
P-Q=227000-67500
Required answer= 159500

Q.8.(A) Required answer
= 207900+88000+72600+102500+70000
+150000=691000

Q.9.(C) Required answer=990000/3-70000
= 260000

Q.10.(B) Required answer=35000
(247500+330000) = 0.061

Q.11-15.

Department	Employees	Males	Females	City Z	City Y
A	8000	5600	2400	800	7200
B	17600	10400	7200	2400	15200
C	9600	3200	6400	8000	1600
D	16000	9600	6400	4000	12000
E	28800	25600	3200	8000	20800

Q.11.(C) Required answer=8000+800= 8800

Q.12.(C) Required answer= $\frac{25600}{800}\times100=3200\%$

Q.13.(B) Required answer=20000:5600=25:7

Q.14.(A) Required answer= 56800/5=11360

Q.15.(D) Required answer=8000+28800= 36800

Q.16-20.

	Male	Female	Difference
A	3500	3000	500
B	4500	3500	1000
C	4750	4000	750
D	2250	1500	750
E	3250	3750	500
Total	18250	15750	2500
Average	3650	3150	500

Q.16.(C)

Q.17.(C) Required % = $\frac{3150-2250}{2250}\times100=40\%$

Q.18.(D) 7500 : 5500 = 15 :11

Q.19.(A) Required % = $\frac{8000}{10250}\times100=78.04\%$

Q.20.(C) Required Ratio
= (3000+3500+4000)/3: (4750+2250+3250)/3
= 42 : 41

Q.21.(B) Total Income = $\frac{825}{36}\times 360 = 8250$

Q.22.(A) Percent of saving = $\frac{54}{360}\times 100 = 15\%$

Q.23.(C) 108 : 72 = 3 : 2

Q.24.(A) Rs. 1443.75

Q.25.(A)

Q.26.(A) Required answer = $\frac{100+200}{150+225}\times 100 = 80\%$

Q.27.(D) Required answer = $\frac{150}{275}\times 100 \approx 55\%$

Q.28.(A) Required answer = $\frac{150-125}{150}\times 100\% \approx 17\%$

Q.29.(B) Required answer

$= \frac{100+125+200+225+275+275}{175+150+125+175+175+225} = \frac{1200}{1025} = \frac{48}{41}$

Q.30.(C) Average production=1200/6=200

In 2012,2013 and 2014 the production is more than 200.

Q.31.(C) Required answer =100:70=10:7

Q.32.(D) Required answer

= (80+80+75+65+60)/5=72

Q.33.(B) $800\times\frac{54}{360} = 120$

Q.34.(A) $800\times\frac{144}{360} = 320$

Q.35.(A) $800\times\frac{72}{360} = 160$

Q.36-40.

	2005	2010	% Change
Wrist Watch	21.3	28.7	34.74178
Table Clock	9.5	22.3	134.7368
Stop Watch	5.5	3.5	36.3636
Wall Clock	30.7	32.7	6.514658

Q.36.(C) Required percent

$= \frac{28.7-22.3}{22.3}\times 100 = 28.69 \approx 28.7\%$

Q.37.(D) 3.5 : 9.5 = 7 : 19

Q.38.(B) Required percent

$= \frac{30.7-9.5}{30.7}\times 100 = 69.0553 \approx 69.05\%$

Q.39.(C)

Q.40.(A) Table Clock

Q.41.(A) Required answer $=\frac{20}{170}\times 100 = 14\frac{2}{17}\%$

Q.42.(D) Required answer $=\frac{140}{170}\times 100 = 82\frac{6}{17}\%$

Q.43.(D) Required answer $=\frac{60}{190}\times 100 = 31\frac{11}{17}\%$

Q.44.(C) Required answer = $\frac{60-50}{50}\times 100 = 20\%$

Q.45.(C) In 2010 percentage of failed students was minimum hence school had the best result for H.S. in 2010.

Q.46.(B) The mean of the expenditure is on cement

Q.47.(A) Required answer= 10:20:15=2:4:3

Q.48.(B) It is 30% on miscellaneous.

Q.49.(C) Required answer=90/360=1/4

Q.50.(C) Required answer=10+15=25%

Q.51.(C) In 2015 difference between the amount invested in Raw materials and value of sales of finished goods is maximum

Q.52.(B) Required answer=

$$\frac{460}{375+330+525}\times 100 \approx 37\%$$

Q.53.(D) Required answer

$$= \frac{200+300+500+400+600+460}{6} - \frac{120+225+375+330+525+420}{6}$$

= 77.5 lakhs

Q.54.(C) In year 2015 both are equals to 66.67%.

Q.55.(B) In year 2014 there is maximum percentage increase in the amount invested in Raw materials as compared to the previous year.