Possible option for modeling juvenile salmon emigrants based on adult escapement and trapping data

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Background

This is a different way of approaching the modeling problem, which is based upon a more direct relationship between spawners, resulting offspring, and subsequent emigrants. There are no covariate effects in any of the relationships as written, but they could be added.

Definitions

Here are some definitions for model variables and parameters. They ignore location for now.

- S_y: number of observed female spawners in year y (data)
- N_y: number of true female spawners in year y (estimated state)
- f_y: product of fecundity and egg-to-juvenile survival in year y (estimated parameter)
- J_y: total number of juveniles produced from brood year y (estimated state)
- $E_{d,k,y}$: number of true emigrants on day d in season k and year y (estimated state)
- $C_{d,k,y}$: number of juvenile emigrants caught in a trap on day d in season k and year y (data)
- $M_{k,y}$: number of tagged juvenile emigrants released upstream in season k and year y (data)
- $R_{k,y}$: number of tagged juvenile emigrants recaptured in season k and year y (data)
- $p_{k,y}$: capture probability of juvenile emigrants in season k and year y (estimated parameter)

State models

Here I assume that the production of pre-emigrant juveniles is a linear function of the number of female spawners, such that

$$J_y \sim \text{Poisson}(f_y N_y) \tag{1}$$
$$\log(f_y) \sim \text{Normal}(\mu_f, \sigma_f) \tag{2}$$

or, if there is presumed overdispersion, then

$$J_y \sim \text{NegBin}(r, a) \tag{3}$$

$$r = \frac{(f_y N_y)^2}{\sigma_J - f_y N_y} \tag{4}$$

$$a = \frac{r}{r + f_y N_y} \tag{5}$$

$$\log(f_y) \sim \text{Normal}(\mu_f, \sigma_f) \tag{6}$$

The number of emigrants is then a function of the number of juveniles, such that

$$\log(E_{k,y}) \sim \text{Normal}(\log(g(J_{k,y})), \sigma_E) \tag{7}$$

$$E_{k,y} = \sum^{d} E_{d,k,y} \tag{8}$$

$$J_{k,y} = \begin{cases} J_{k,y}, & \text{if k} = 1 \\ J_{k-1,y} - E_{k-1,y}, & \text{if k} > 1 \end{cases} \tag{9}$$

where $g(J_{k,y})$ is a (potentially nonlinear) function mapping the number of juveniles onto the number of emigrants.

Observation models

The number of observed female spawners in year t is assumed to be lognormally distributed about the true number of female spawners in year t, such that

$$\log(S_y) \sim \text{Normal}(\log(N_y), \sigma_S) \tag{10}$$

The number of juvenile emigrants caught in a trap on day d in season k and year y is a fraction of the total number of true emigrants on day d in season k and year y, such that

$$C_{d,k,y} \sim \text{Binomial}(E_{d,k,y}, p_{k,y}) \tag{11}$$

$$p_{k,y} = \frac{R_{k,y}}{M_{k,y}} \tag{12}$$

The cutpoints for identifying the life history variants (or "seasons" k) could be easily estimated as a mixture of univariate Gaussian distributions based upon the observed emigrants ($C_{d,k,y}$) across all years combined, such that

$$C_{d,k} = \sum^{y} C_{d,k,y} \tag{13}$$

$$C_{d,k} \sim \text{Normal}(\mu_{z_k}, \sigma_{z_k}) \tag{14}$$

$$z_k \sim \text{Categorical}(\boldsymbol{\phi}) \tag{15}$$