Critique of model evaluation by WA Dept of Ecology

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Error in assumptions

In the section titled "Uncertainty in Dissolved Oxygen Depletion Estimates" (p59), there is an error in the assumed relationship between the standard deviation of the predictions and root mean squared error (RMSE) of the predictions. Specifically, the document incorrectly suggests that

$$\text{Var}(\hat{x}) = \text{RMSE}(\hat{x})^2 \;\Rightarrow\; \text{SD}(\hat{x}) = \text{RMSE}(\hat{x}).$$

Specifically, the standard deviation of $\hat{x}$ is based upon differences between the predicted data points $\hat{x}_i$ and their mean $\bar{\hat{x}}$

$$\text{SD}(\hat{x}) = \sqrt{\frac{(\hat{x}_i - \bar{\hat{x}})^2}{N}},$$

whereas the RMSE of $\hat{x}$ is based upon differences between the predicted data points and their corresponding observed values x_i

$$\text{RMSE}(\hat{x}) = \sqrt{\frac{(\hat{x}_i - x_i)^2}{N}}.$$

Thus,

$$\sqrt{\frac{(\hat{x}_i - \bar{\hat{x}})^2}{N}} \neq \sqrt{\frac{(\hat{x}_i - x_i)^2}{N}}.$$

An example

Here is a simple example based up linear regression that shows how $\text{SD}(\hat{x})$ and $\text{RMSE}(\hat{x})$ are different. The model is

$$y_i = \alpha + \beta x_i + \epsilon_i$$

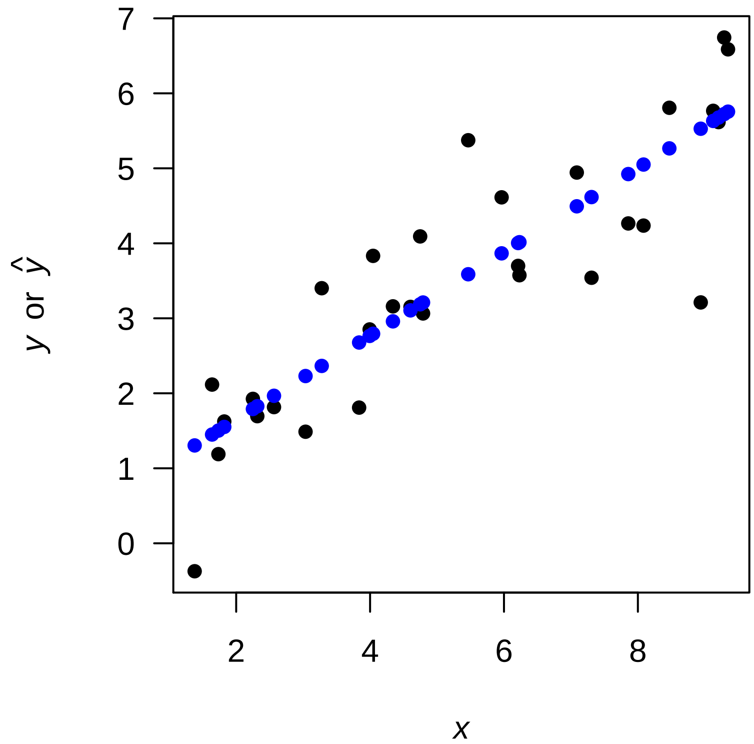
and the predictions are given by

$$\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i.$$

```r
## function to calculate RMSE
rmse <- function(fit, obs) {
  rmse <- sqrt(sum((fit - obs)^2)/length(fit))
  return(rmse)
}

## sample size
nn <- 30
## some values for the predictor x
xx <- runif(nn, 0, 10)
## intercept
alpha <- 1
## slope
beta <- 0.5
## random errors (mean = 0, SD = 1)
epsilon <- rnorm(nn)
## observed data
yy <- alpha + beta*xx + epsilon

## fit a regression model
mm <- lm(yy ~ xx)
## get predicted values
yhat <- predict(mm)
## plot observed and predicted
par(mai = c(1,1,1,1))
plot(xx, yy, pch = 16, las = 1,
     xlab = expression(italic(x)),
     ylab = expression(italic(y)~~or~~hat(italic(y))))
points(xx, yhat, pch = 16, col = "blue")
```




```
## calculate RMSE
round(rmse(yhat, yy), 2)
```

```
## [1] 0.85
```

```
## calculate SD
round(sd(yhat), 2)
```

```
## [1] 1.49
```