The Sphericity Tensor and the Discovery of the Gluon

Goals:

- Learn how to calculate the sphericity tensor and understand how to interpret its eigenvalues
- Reproduce, using simulated data, the analysis performed by Tasso to discover the gluon
- Develop a simple event display as an aid to visualizing what two jet and three jet events look like in e^+e^- annihilation events

The first experimental evidence for the existence of the gluon came from the analysis of data collected at the Petra accelerator at DESY. Several different analysis strategies were used by the four collaborations (see

https://arxiv.org/pdf/1012.2288.pdf

for a review). One such analysis, performed by the Tasso group, was based on studies of the sphericity tensor:

$$S_{\alpha\beta} = \frac{\sum_i p_{i\alpha} p_{i\beta}}{\sum_i \vec{p}_i^{\,2}}$$

where the sum is over all charged particles in the event (Tasso did not have good enough calorimetry to include neutrals in the analysis) and the α and β indices run from 1 to 3, representing the x, y and z components of the momentum vector. For each event, the Sphericity tensor can be diagonalized to obtain the principle axes $\hat{n}_1$ through $\hat{n}_3$ and eigenvalues Q_1 through Q_3. With the definition of S above, $Q_1 + Q_2 + Q_3 = 1$, so we only need two eigenvalues to specify each event. If the eigenvalues are ordered so that $Q_1 < Q_2 < Q_3$ then the sphericity is defined to be

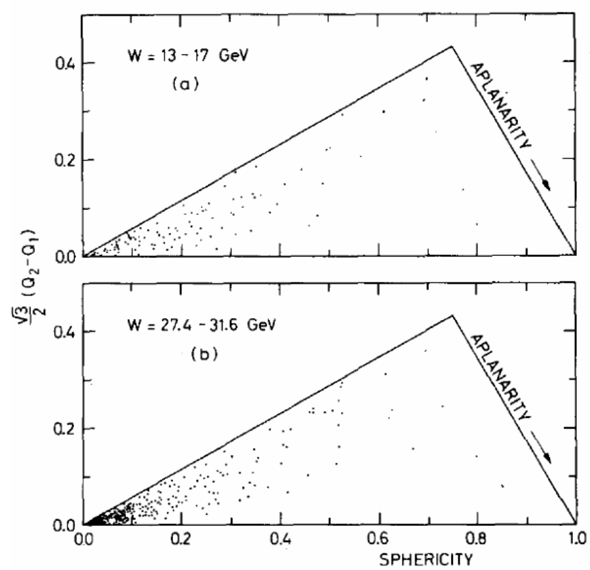
$$S \equiv \frac{3}{2}(1 - Q_3) = \frac{3}{2}(Q_1 + Q_2)$$

and the aplanarity is defined to be

$$A \equiv \frac{3}{2} Q_1$$

Here $0 < S < 1$ and $0 < A < 0.5$.

Note that the form of the sphericity tensor is the same as that of the moment of interia tensor (where the object's position is replaced by its momentum). We can therefore use our intuition from classical mechanics to interpret this tensor. Long, thin rods have one large eigenvalue and two small ones of rougly equal size, while a sphere has three eigenvalues of equal size. If we interpret the the process $e^+e^- \to hadrons$ at parton level as $e^+e^{\to}q\overline{q}$ we would expect most of the momentum to flow along the original $q\overline{q}$ axis with the momentum transverse to this direction limited by to a scale set by Λ_{QCD}. Thus, such events have S close to zero. The axis n_3 is a good estimate of the initial direction of the back-to-back q and *overlineq*. If instead, a single hard gluon is radiated so that the hard scattering process is $e^+e^{\to}q\overline{q}g$. The events would appear planar with only a small component of momentum outside the plane. Here is what Tasso observed:



Distribution of the sphericity and aplanarity of $e^+e^- \to hadron$ events measured by the TASSO collaboration in R. Brandelik et al., *Evidence for Planar Events in e+e Annihilation at High Energies*, Phys. Lett. B 86, 243 (1979).

You will now reproduce their analysis using simulated data created with the Pythia8 Monte Carlo generator. The file **Pythia8e+e-Toqqbar36GeV.dat** contains the charged particle information for 10000 events. The format of the file is specified in the metadata comments at the beginning of the file.

(a) For each event in the file, calculate the sphericity tensor and find its eigenvalues. Plot the data using the same x and y axes as the Tasso plot above.

(b) As discussed above, the momentum along the $\hat{n}_3$ axis should be larger than in the other directions. The momentum along the $\hat{n}_2$ direction should be small for 2-jet events, with a tail extending to larger values of momentum for 3-jet events where a gluon is radiated. The momentum along the $\hat{n}_1$ direction should be small unless more than one gluon is radiated. Since $\alpha_S \approx 0.12$ at Petra energies, the probability of multiple hard gluon radiation is small. Make histograms of the components of momentum along each of the three principle axes for the events you have analyzed. What do these plots show?

(c) When physicists observe a new phenomenon, they often like to display individual events to make sure they "look" they way we expect. We can display a single event by making a 3D plot with a vector representing the momentum of each charged particle. Make such four such displays, two for events with $S < 0.05$ and two for events with $S > 0.3$. Do they look the way you expect them to?