A Monte Carlo Model of Electromagnetic Showers

In doing this problems, you will need to know certain physical constants (particle lifetimes, radiation lengths of specific materials, etc). You can find everything you need on the Particle Data Group (PDG) web site:

http://www-pdg.lbl.gov/

In order to understand how their detectors respond, physicists use complex simulations. These simulations take as their input Monte Carlo generated events, propagate the particles in these events through a model of the detector and simulate the response. The output of the simulation is written in the same format as the real data and is used to understand the experiment's "acceptance" and resolution. Proper modeling of the detector response requires detailed understanding of the physics of particle interactions with matter. Today, most experiments use a simulation toolkit, called Geant4, to model this physics. In this problem, you will write your own Monte Carlo simulation of electromagnetic showers and use it to describe shower development in the CMS electromagnetic calorimeter (ECAL). A description of the CMS ECAL can be found at:

http://cms.web.cern.ch/news/electromagnetic-calorimeter

Write a Monte Carlo simulation that predicts the longitudinal development of an electromagnetic shower in the CMS ECAL. The final "answer" should be a plot that looks roughly like the black circles in Figure 33.20 of the PDG Review of the *Passage of Particles Through Matter*, but where the horizontal axis is the distance in cm from the front face of the calorimeter and the vertical axis is the average number of *charged* particles crossing a plane at that distance. To understand how the shower development depends on energy, make this plot for $E = 1$ GeV and $E = 10$ GeV electrons and compare the distance at which the maximum occurs. In order to keep the statistical uncertainties small, generate 1000 events at each energy.

You will need to make a number of simplifying assumptions in your model:

- Describe the calorimeter as a uniform crystal of lead tungstate, 23 cm deep. Assume electrons hit the front face of the crystal with fixed energy E and normal to the surface.

- Real EM showers develop in 3-dimensions, for this problem use a 1-dimensional model and ignore the transverse spreading of the shower.

- Electrons lose energy by bremsstrahlung. The mean distance over which a high energy electron loses all but $1/e$ of its energy is called the radiation length X_0. The bremsstrahlung spectrum of the emitted photons is peaked at low photon energy. While the true bremsstrahlung process is continuous, for this problem make the *unrealistic* approximation that the energy loss is a discrete process that occurs at random positions x along the electron trajectory. In other words, the probability of a discrete bremmstahlung occuring in distance dx is assumed to be
$$dP \equiv \frac{dN}{N} = -\frac{dx}{X_0}$$
To simplify the calculation, also make the *unrealistic* assumption that whenever the bremsstahlung occurs the energy is divided equally between the the outgoing electron and photon.

- When charged particles travel through matter, they lose energy via ionization. The distribution of energy loss per unit distance is a Landau distribution, with a mean that depends on the particle's velocity. Assume for this problem that the ionization energy loss per cm is constant, with the value for lead tungstate taken from the PDG *Atomic and nuclear properties of materials*. If a charged particle (e^+ or e^-) loses enough energy via ionization to stop in the crystal, then it will just be absorbed in the material. (In the real world, charged particles have a Bragg peak in their energy loss as they stop. We will ignore that effect here).

- Photons lose energy via Compton scattering, photo-nuclear interactions and pair production. Assume for this problem that pair production is the only process that matters and that the probability of pair production occuring distance between x and $x + dx$ is

$$dP = \frac{dx}{9/7X_0}$$

Also *unrealistically* assume that the e^+ and e^- produced always share the energy of the photon equally. Make the approximation that the electron is massless.