Neutrino Oscillations

Goals:

- Understand the phenomology of neutrino oscillations by studying the expression of the mixing probability as a function of distance from the neutrino production point.
- Learn how the best estimate of Δm^2 and its uncertainty can be obtained using a fit to experimental data.

Consider a system with two massive neutrinos,ν_e and ν_μ. An electron neutrino, ν_e, created at time $t = 0$ in the core of the Sun, is described as a linear combination of two mass eigenstates $\nu_1$and nu_2:

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

where θ is a two-generation mixing angle. Similarly, the muon neutrino would be defined as the orthogonal linear combination:

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

(a) Show that the probability of finding a muon neutrino $\nu_m u$ at a distance $L = t$ from the source is given by

$$P_{\nu_e\to\nu_\mu}(t) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

where $E \approx p >> m_1, m_2$ is the neutrino energy and $\Delta m^2 = m^2{}_1 - m^2{}_2$.

(b) With E measured in GeV, L in km,and in Δm^2 eV^2, show that the previous formula can be re-written as

$$P_{\nu_e\to\nu_\mu}(t) = \sin^2(2\theta)\sin^2\left(\frac{1.27\Delta m^2 L}{E}\right)$$

This formula applies to any two-flavor neutrino mixing.

(c) Super-Kamiokande (SuperK) experiment has analyzed the atmospheric (ν_e and $\nu_m u$) neutrino data and shown that they are consistent with two-flavor $\nu_\mu \to \nu_\tau$ oscillations with $\sin^2(2\theta) \sim 1$ and $\Delta m^2 \approx 10^{-3}$. They have also looked at the ratio of the number of ν_μ events observed in data to the Monte Carlo prediction. The muon neutrinos were detected through the charge-current reaction emitting a muon. The approximate results are shown below. The distance from the production point to the detector was varied by observing the azimuthal dependence of the neutrino flux. Here, Monte Carlo takes into account muon production in the atmosphere, energy-dependent efficiencies, etc, but assumes no oscillations. Thus, if neutrinos oscillate, the ratio of the yield in data to that in Monte Carlo should vary with distance according to the formula you derived above.

L/E_ν (km/GeV)	Data / Monte Carlo
0.7	1.00±0.20
20	1.00±0.08
70	0.90±0.08
200	0.83±0.15
700	0.65±0.15
2000	0.70±0.08
7000	0.63±0.06
20000	0.61±0.08

Make a plot of the results above with L/E_ν on the horizontal axis in log scale. At large L/E_ν, the muon neutrinos have presumably undergone numerous oscillationsand have averaged out to roughly half the initial rate. Using $\sin^2(2\theta) = 1$, make an estimate of Δm^2.