Points

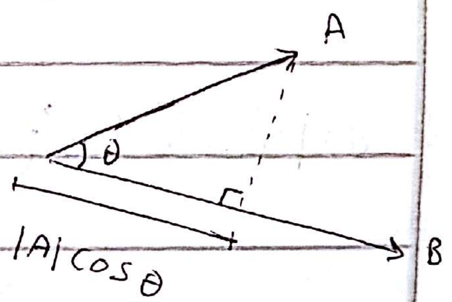
1) distance of 2 points (x_1, y_1), (x_2, y_2) : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

2) θ counter clockwise rotation $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

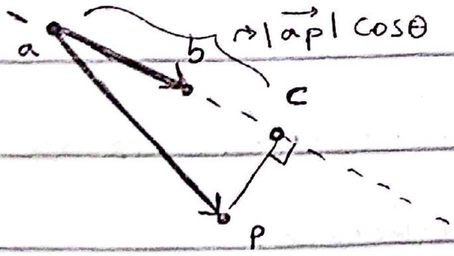
3) $\dfrac{\text{degree}}{180} = \dfrac{\text{radian}}{\pi}$

Vectors

1) $\vec{V_1} \cdot \vec{V_2} = x_1 \times x_2 + y_1 \times y_2 = |\vec{V_1}| \times |\vec{V_2}| \times \cos\theta$ زاویه بینشون



2) distance of a point to line



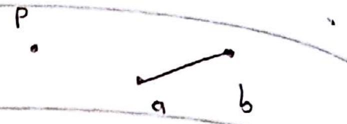
$\vec{ab} \cdot \vec{ap} = |\vec{ab}|\,|\vec{ap}| \cos\theta$

$|\vec{ap}| \cos\theta = \dfrac{\vec{ab} \cdot \vec{ap}}{|\vec{ab}|}$

$\vec{ac} = \dfrac{\vec{ab}}{|\vec{ab}|} \times \dfrac{\vec{ab} \cdot \vec{ap}}{|\vec{ab}|}$

$\boxed{c = a + \vec{ac}}$ the answer is distance of points p & c

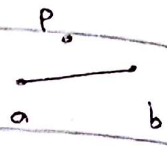
3) distance of a point to a line segment



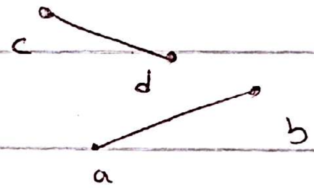
$u = \frac{\vec{ap} \cdot \vec{ab}}{|\vec{ab}|^2}$ if $u < 0$ dis(p, a)

if $u > 1$ dis(p, b)

else dis(p, c)



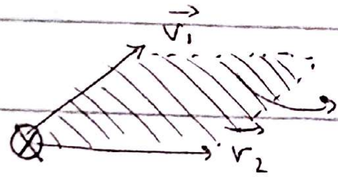
4) angle of 2 vectors $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} \rightarrow \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}\right) = \theta$



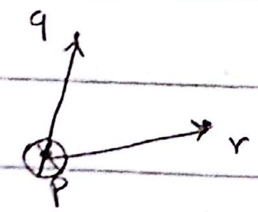
5) distance of 2 line segments

↳ min(dis(c, ab), dis(d, ab), dis(a, cd), dis(b, cd))

6) $\vec{V_1} \times \vec{V_2} = \vec{k}(V_1.x * V_2.y - V_1.y * V_2.x) = (|\vec{V_1}|\,|\vec{V_2}| \sin\theta)\,\vec{k}$



$S = |\vec{V_1} \times \vec{V_2}|$

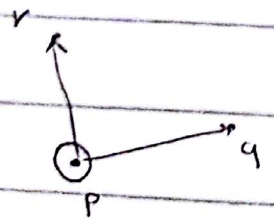


$\vec{pq} \times \vec{pr} < 0$

right turn: p → q → r

(clockwise)

right side of line pq

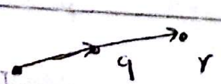


$\vec{pq} \times \vec{pr} > 0$

left turn: p → q → r

(counter clockwise)

left side of line pq



$\vec{pq} \times \vec{pr} = 0$

collinear

on line pq

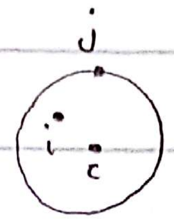
Circle

center = (a, b)

radius = r

equation: $(x-a)^2 + (y-b)^2 = r^2$

1) Status of a point



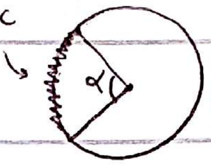
$dis(i, c) < r$ inside circle

$dis(j, c) = r$ on circle

$dis(k, c) > r$ outside circle

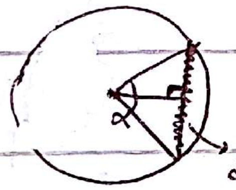
2) diameter $= 2r$ (longest chord), area $= \pi r^2$, primeter $= 2\pi r$

3) Arc (طول قطاع)



$$\boxed{arc = \frac{\alpha}{\cancel{2}\pi} \times \cancel{2}\pi r = \alpha r}$$

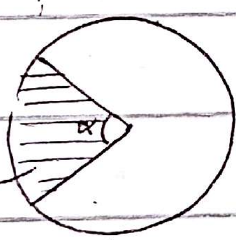
4) Chord (وتر)



$$\sin\left(\frac{\alpha}{2}\right) = \frac{d}{r} \rightarrow d = r\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow 2d = 2r\sin\left(\frac{\alpha}{2}\right)$$

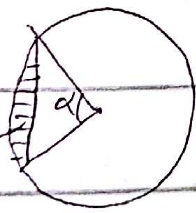
5) Sector (مساحت قطاع)



sector

$$\boxed{\frac{\alpha}{2\cancel{\pi}} \times \cancel{\pi} r^2 = \frac{\alpha}{2} r^2}$$

6) Segment



segment

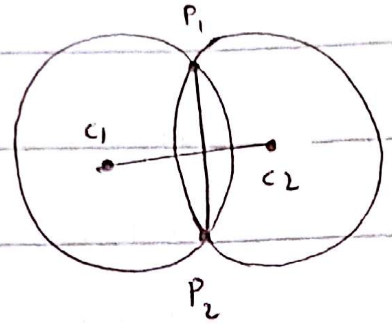
$$segment = \frac{\alpha}{2} r^2 - \frac{\sqrt{r^2 - d^2} \times \cancel{2}d}{\cancel{2}}$$

$$= \boxed{\frac{\alpha}{2} r^2 - d\sqrt{r^2 - d^2}}$$

7) بین اشکالی که محیط یکسان دارند دایره بیشترین مساحت را دارد

Subject: Date:

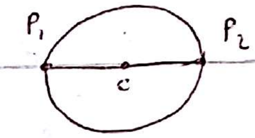
8) given 2 points & radius, find 2 circles which these points are on them.



$dis(P_1, P_2) = l$

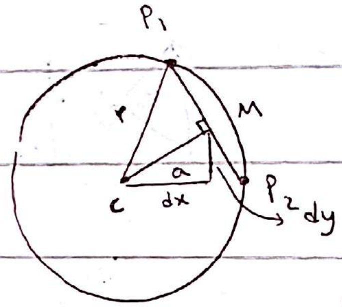
if $l > 2r$ no circles

if $l = 2r$ one circle



$c = \frac{P_1 + P_2}{2}$

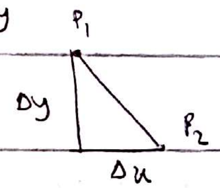
else 2 circles



$a = \sqrt{r^2 - \frac{l^2}{4}}$

$\boxed{M = \frac{P_1 + P_2}{2}}$

$c = (M.x + dx, M.y + dy)$



① $dx^2 + dy^2 = a^2$ $\quad \Delta x^2 + \Delta y^2 = l^2$, $\quad \frac{dx}{dy} = -\frac{\Delta y}{\Delta x}$ چون عمودند

$dx = -\frac{\Delta y}{\Delta x} dy \xrightarrow{①} \frac{\Delta y^2}{\Delta x^2} dy^2 + dy^2 = a^2 \longrightarrow dy^2 \left(\frac{\Delta y^2}{\Delta x^2} + 1\right) = a^2$ $\quad \times \Delta x^2$

$dy^2 * (\underbrace{\Delta y^2 + \Delta x^2}_{l^2}) = a^2 \Delta x^2$

$dy^2 = \frac{a^2 \Delta x^2}{l^2} \longrightarrow \boxed{dy = \frac{a \Delta x}{l}, \quad dx = -\frac{a \Delta y}{l}}$

$\boxed{c_1 = M.x - dx, M.y - dy}$

$\boxed{c_2 = M.x + dx, M.y + dy}$

9) Problem statement: given convex polygon, find the scale such that if we shrink it by that much then the shape that is distance r outwards will have same perimeter as the original polygon.

solution

the minimal perimeter is $2\pi r$, so make sure you check if the original polygon already has smaller perimeter, in which case it's impossible.

imagine walking along the polygon with a stick of length r pointing outwards. we will sweep out the length of each side, and then sweep out an arc of some angle at each corner.

well, if we go all the way around, then those arcs add up to a full circle because we've rotated our stick the full way. hence, if original perimeter is P, scale is s, then the resulting perimeter is $P*S + 2\pi r$. hence $S = (P - 2\pi r) / P$.