Chapter 1

Introduction to algorithm design

n/a

Chapter 2

Algorithm analysis

2-10

a) $f(n) = (n^2 - n)/2, \ g(n) = 6n$.

Is $f(n) = O(g(n))$? If so, there is c such that $f(n) \leq cg(n)$ for sufficiently large n.

$$\frac{1}{2}\left(n^2 - n\right) \leq 6n \ \rightarrow \ n^2 - n \leq 12n \ \rightarrow \ n(n-1) \leq 12n$$

Suppose there is such a c, then

$$n(n-1) \leq 12cn \ \rightarrow \ n - 1 \leq 12c$$

Clearly we can always find n such that this inequality won't hold, so $f(n) \neq O(g(n))$.

Is $g(n) = O(f(n))$? If so, there is c such that $g(n) \leq cf(n)$ for sufficiently large n.

$$6n \leq \frac{1}{2}\left(n^2 - n\right) \ \rightarrow \ 12n \leq n^2 - n = n(n-1) \ \rightarrow \ 12 \leq n - 1 \ \rightarrow \ 13 \leq n.$$

So with $c = 1$ the inequality will hold for $n_0 \geq 13$, and $g(n) = O(f(n))$.

b) $f(n) = n + 2\sqrt{n}, \ g(n) = n^2$.

$f(n) = O(g(n)) \Leftrightarrow f(n) \leq cg(n)$ for sufficiently large n.

$$n + s\sqrt{n} \leq cn^2, \text{ with } c = 1, n = 4,$$
$$4 + 2\sqrt{4} = 8 \leq 4^2 = 16 \text{ so } f(n) = O(g(n)).$$