

# Presentation outline

- Feifan, 5 min: Introduction
  - Big picture
    - What is epidemiology?
    - Why is it important? (Real world usage of compartment models?)
  - Basic SIR model(Deterministic, Stochastic model)
    - Include Scheme, ODE
  - General concepts: Basic Reproduction Number, Herd Immunity Thres., ...
- Amy, 5 min: Model(Our own model)
  - Brief explanation of Parsons paper? (concept about epidemic outcomes with stochastic sim.: fizzle, burnout, persist)
  - Why vaccines matter? (current news reference?)
  - Introduce our version of SIR with vaccination event (Scheme, ODEs, probably the simplest one with constant vaccination rate)
- Jacob, 5 min: Results
  - Stochastic/Deterministic simulations
    - Visualizations
    - focus on defining epidemic outcome classification method with reasonable justification
  - Brief idea for future works
    - Extension of analytical work for estimating burnout probability in model with vaccination?
    - Maybe good to include different model definitions we discussed?
    - Just a word about waning immunity?

Background of our question : Get familiar with the question: How does vaccination strategy affect the disease transmit?

1. Why we care about this question
2. Find an example to prove the vaccination is important, bring the critical point (Use some data)
3. Eg: MMR without vaccination how it spreads in Texas to prove why vaccination is important

# Vaccination in Epidemic Modeling

Jacob Kang, Feifan Li, Amy Wang

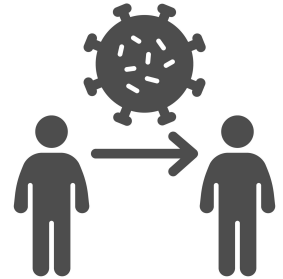


# Background Info

# What is Epidemiology?

Epidemiology is the study of disease spread in populations.

- Identify **patterns** and **causes** of diseases in groups of people.
- Provide **evidence** for public health decisions/policies.
- Develop strategies to **prevent**, **control**, and **eliminate** disease

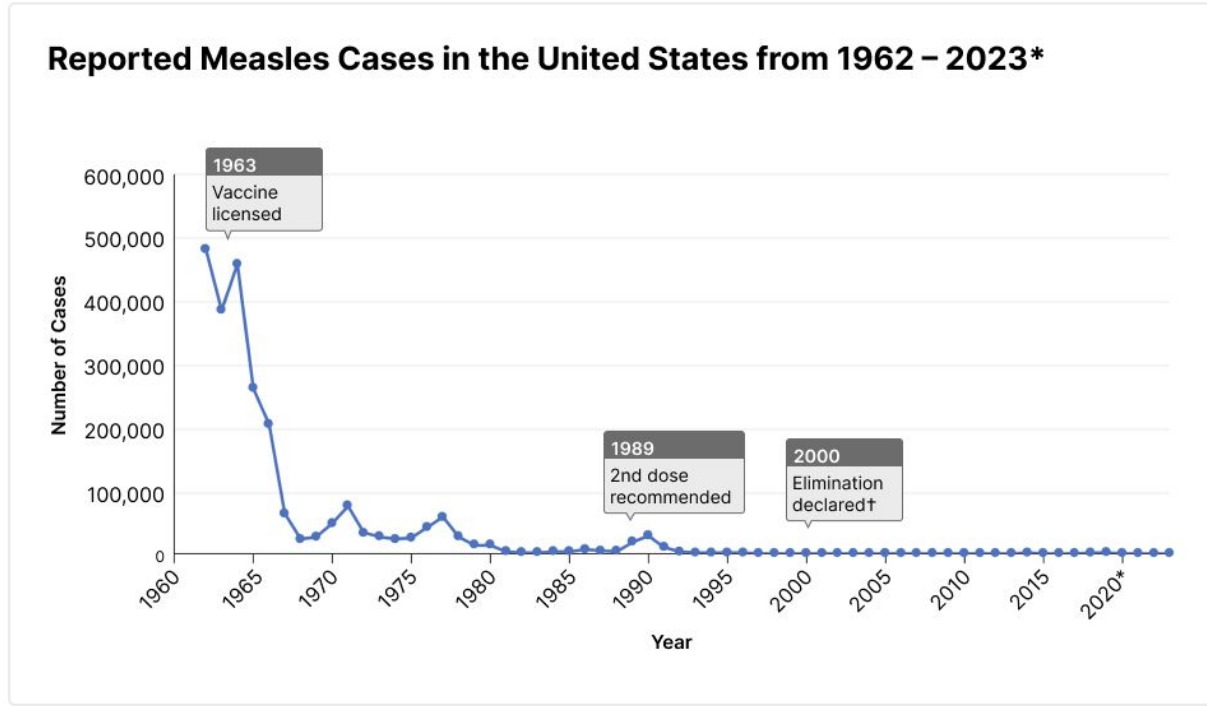


# Why do we care about vaccination?

Disease	Before vaccination	After vaccination
<b>Smallpox</b>	30%->50% death rate	1%-11% death rate
<b>COVID-19</b>	9.79% death rate	5.01% death rate
<b>Ebola</b>	40% death rate	Local outbreaks can be controlled with vaccines
<b>Measles</b>	2.6mil deaths per year	128k deaths per year

Source: State and country health department, C.D.C;The government of the Hong Kong Special Administrative Region; National Institute Of Health, N.I.H; World Health Organization, W.H.O

# Real life examples - Vaccination in current events

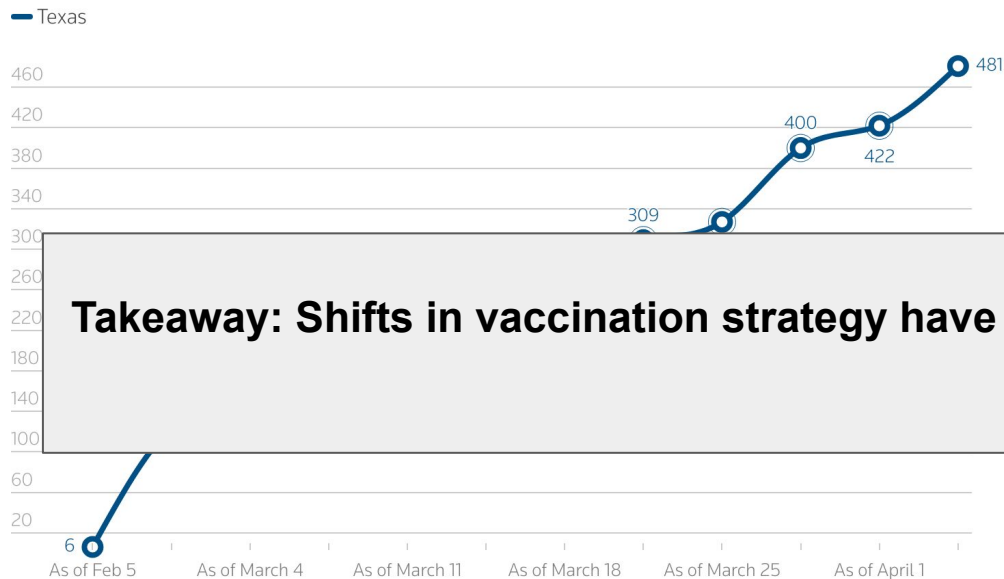


Source: State and country health department, C.D.C

## Real life examples - Vaccination in current events

## Measles cases reported in Texas amid ongoing outbreak

Measles cases in Texas jumped to 481 on April 4, from six cases in early Feb



By Bhanvi Satija • Source: Texas State Health Department

## Measles cases in West Texas and nearby states



Sources: State and county health departments; C.D.C.



# Herd Immunity

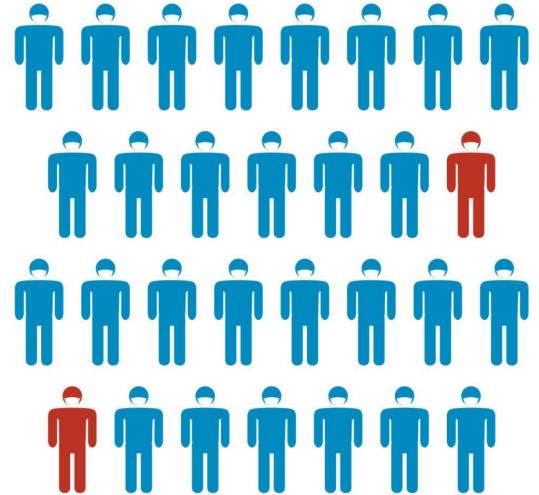
Sufficient fraction of the population  
immunized



Outbreak no longer possible

$$HerdImmunityThreshold(HIT) = 1 - \frac{1}{R_0}$$

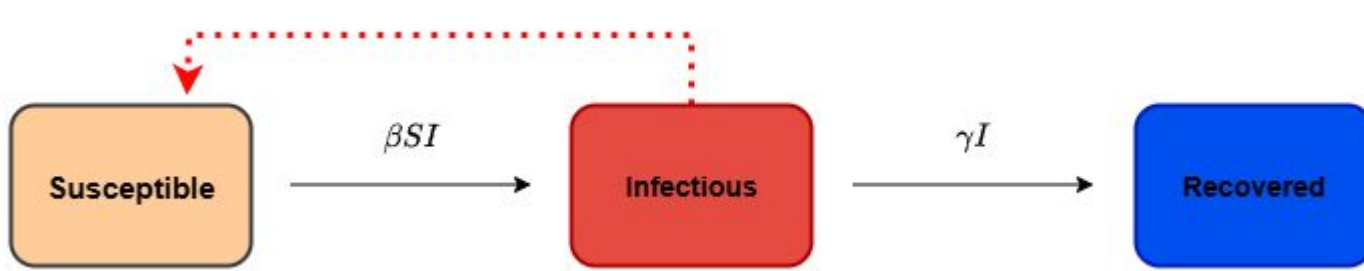
🦠 **HERD IMMUNITY** 🦠



How does vaccination strategy affect the epidemic progression?

What is the SIR model?

# SIR Model



$$\begin{aligned}\frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I\end{aligned}$$

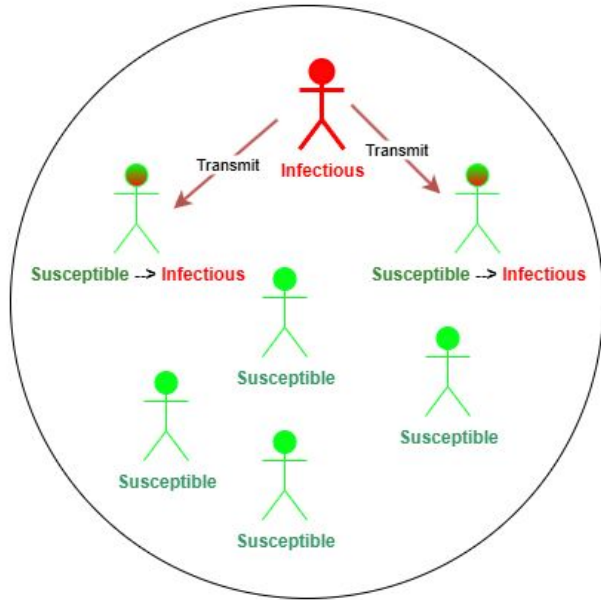
$\beta$ : Transmission Rate

$\gamma$ : Recovery Rate

# Basic Reproduction Number

$$R_0 = \frac{\beta}{\gamma}$$

The average number of new infections caused by a single infectious individual in an otherwise susceptible population.



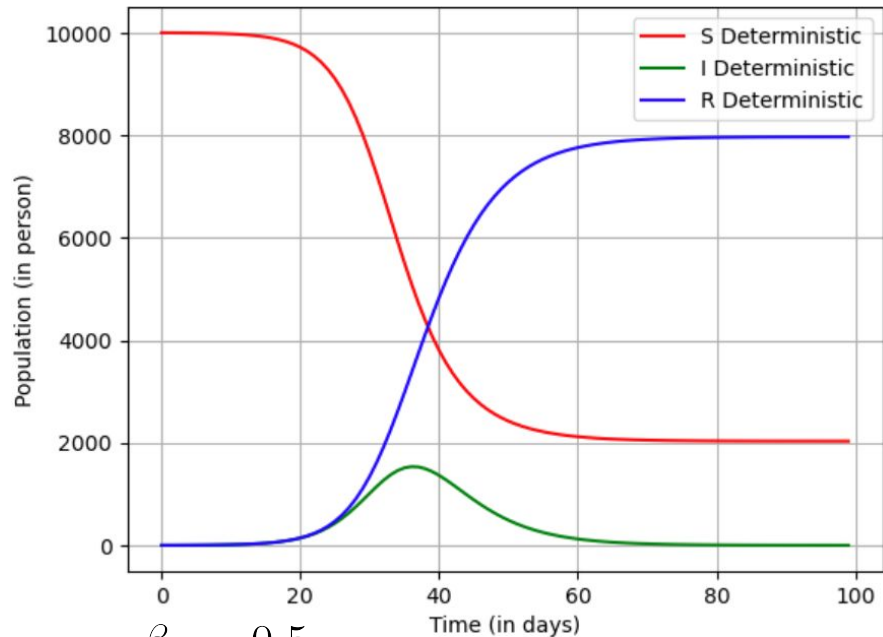
$$R_0 = 2$$

Indicates Strength / Transmissibility of the disease.

Two ways to model:  
Deterministic vs. Stochastic

# Varying values of $R_0$ , Deterministic Simulation

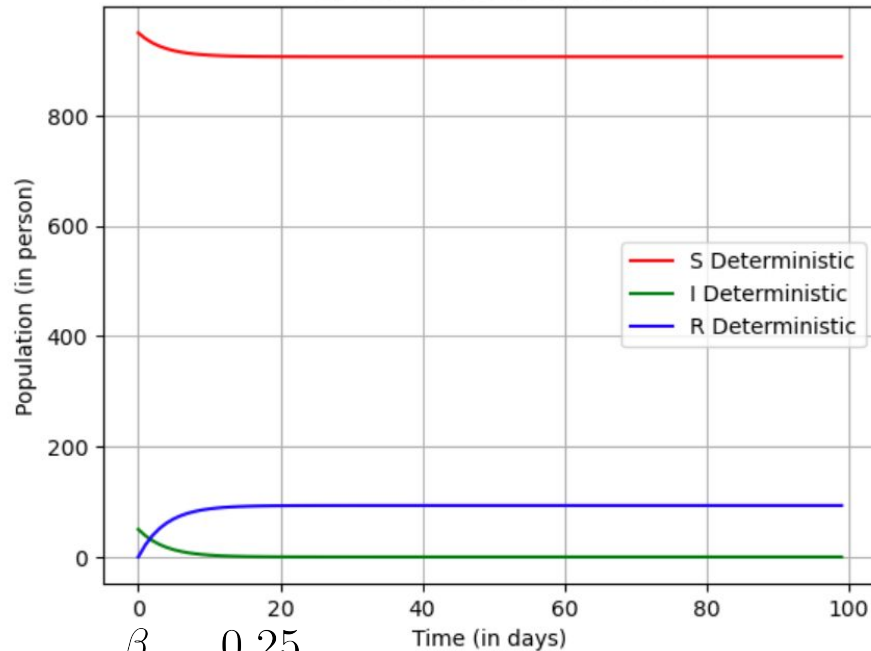
$$R_0 > 1$$



$$R_0 = \frac{\beta}{\gamma} = \frac{0.5}{0.25} = 2$$

$N = 10000$ ,  $I(0) = 1$ , Time Frame =  $[0, 100]$

$$R_0 < 1$$

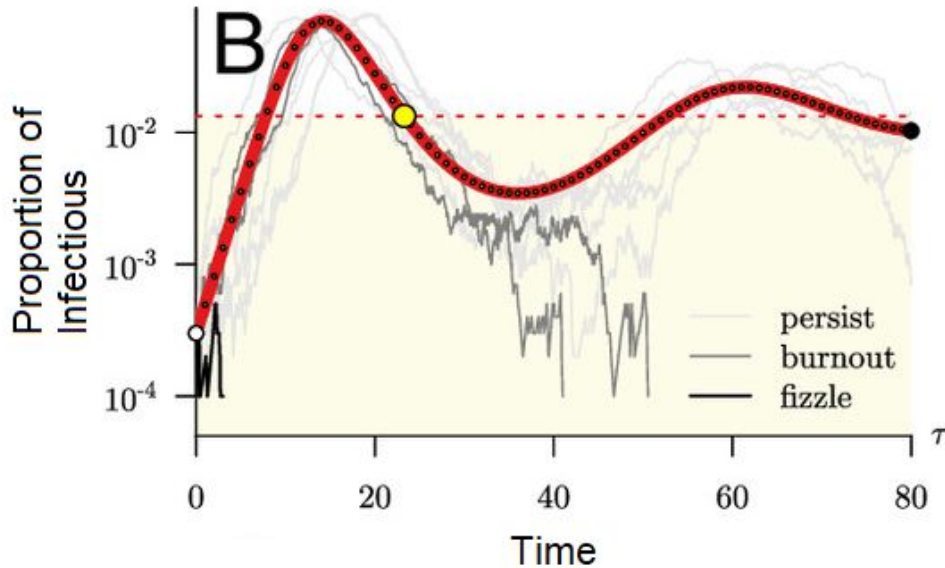


$$R_0 = \frac{\beta}{\gamma} = \frac{0.25}{0.5} = 0.5$$

$N = 1000$ ,  $I(0) = 50$ , Time Frame =  $[0, 100]$

# Stochastic Simulation

In 2024 paper [1] by Parsons et al., stochastic simulation of SIR model is utilized

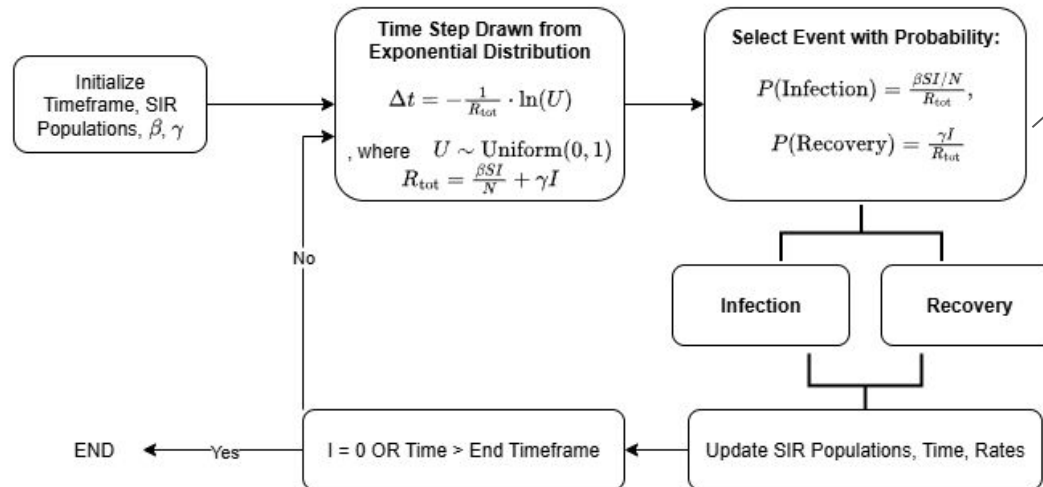




# Stochastic SIR model, Gillespie Algorithm

Time between events in poisson process follows Exponential Distribution

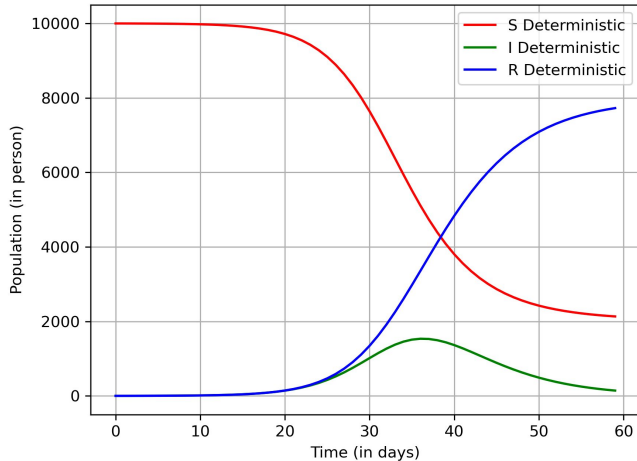
Event	Reaction	Rate
Infection	$S \rightarrow I$	$\frac{\beta SI}{N}$
Recovery	$I \rightarrow R$	$\gamma I$



Events in poisson process Randomly & Independently

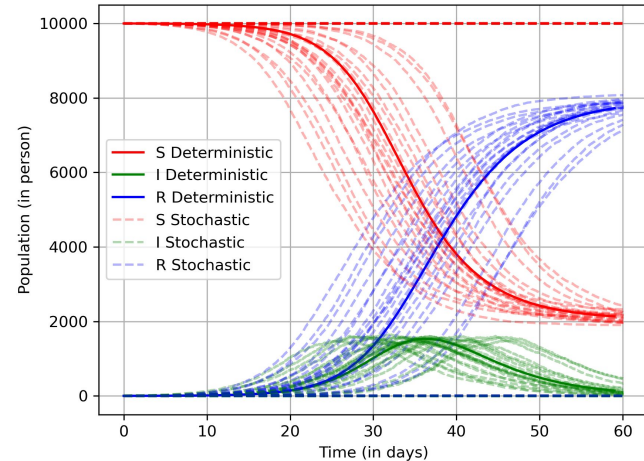
# Deterministic vs. Stochastic SIR Models

- **Deterministic:** Based solely on the ODEs we saw previously
- When given the same conditions, exhibit the same outcome each time.



$R_0=2$ ,  $N = 10000$ ,  $I(0) = 1$ , Time Frame =  $[0, 60]$

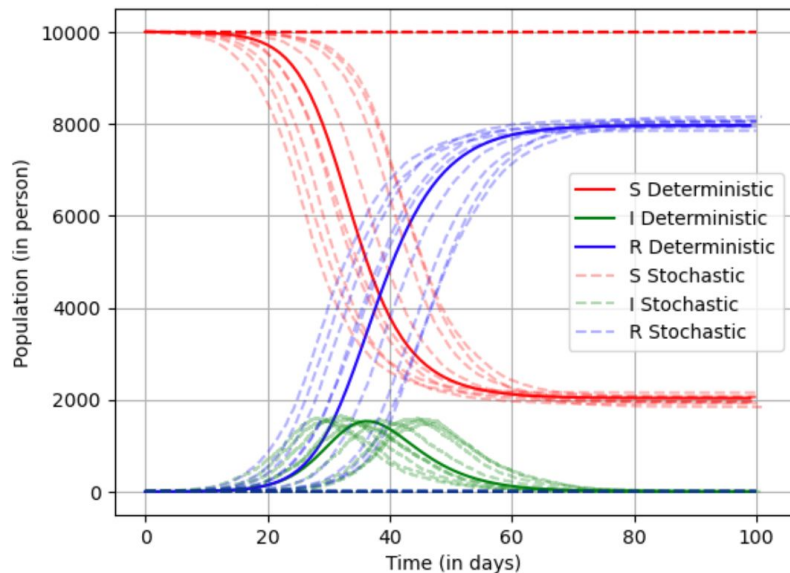
- **Stochastic:** Events are randomly determined, based on *Gillespie algorithm*
- When given the same conditions, exhibits various *outcomes* and behavior.



$R_0=2$ ,  $N = 10000$ ,  $I(0) = 1$ , Time Frame =  $[0, 60]$ , 50 Stochastic Runs

# Varying values of $R_0$ , Stochastic Simulation

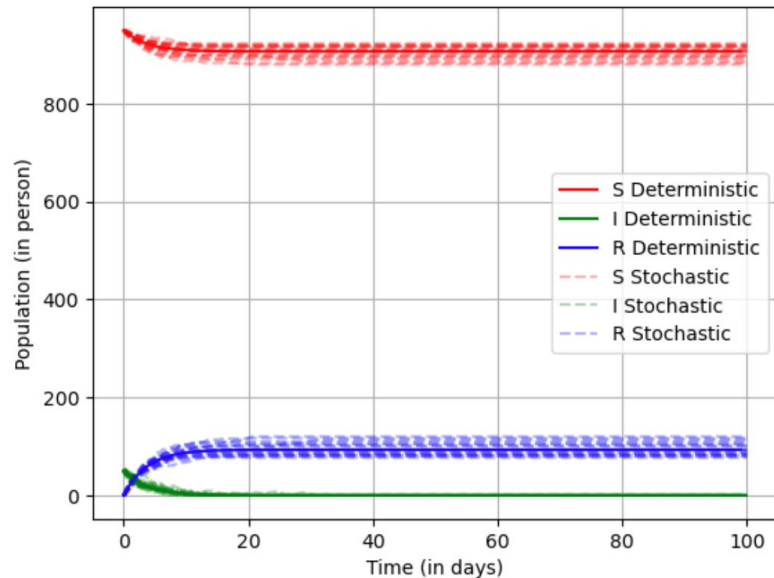
$$R_0 > 1$$



$$R_0 = \frac{\beta}{\gamma} = \frac{0.5}{0.25} = 2$$

$N = 10000$ ,  $I(0) = 1$ , Time Frame =  $[0, 100]$ , 20 Stochastic Simulation

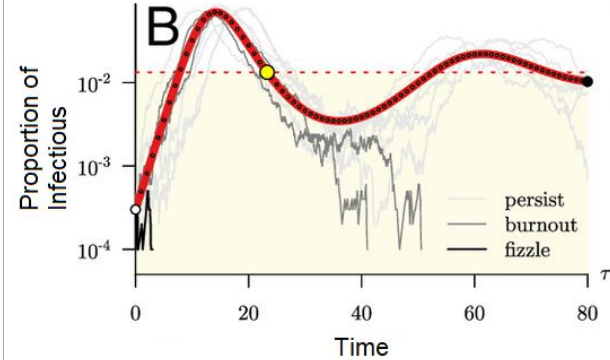
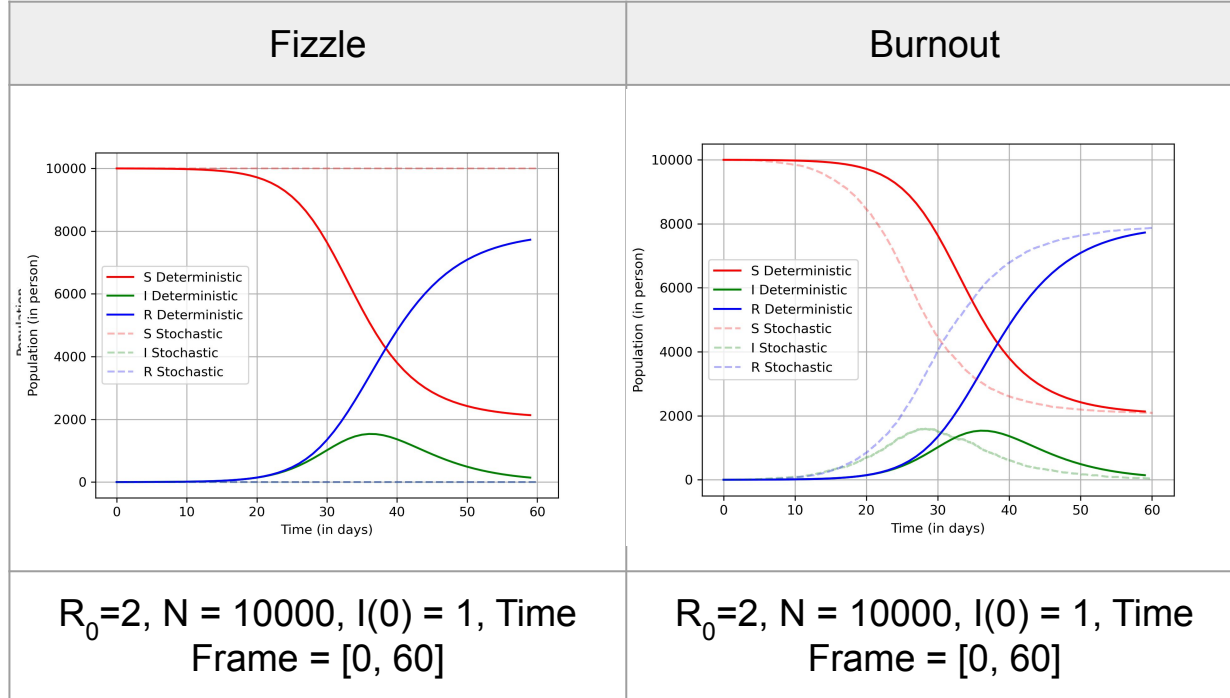
$$R_0 < 1$$



$$R_0 = \frac{\beta}{\gamma} = \frac{0.25}{0.5} = 0.5$$

$N = 1000$ ,  $I(0) = 50$ , Time Frame =  $[0, 100]$ , 20 Stochastic Simulation

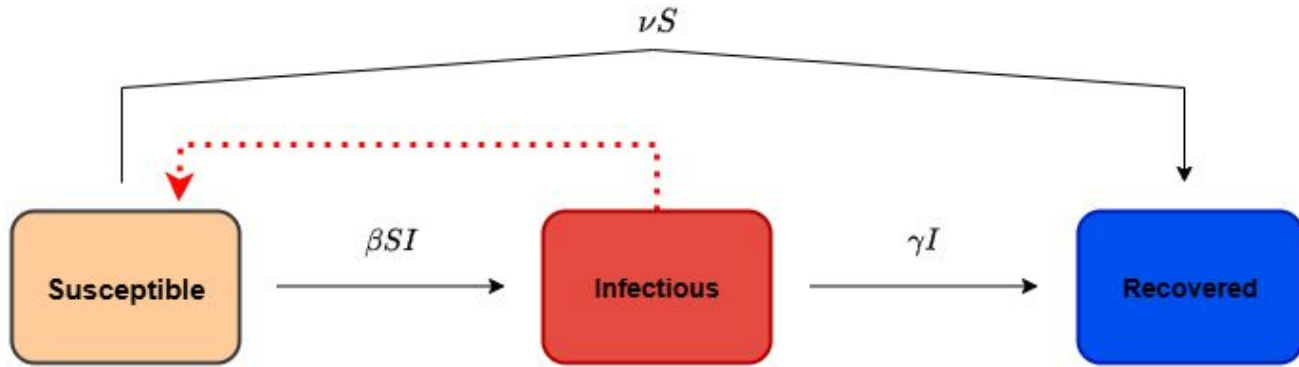
# Epidemic Outcomes in Stochastic Simulation



# Incorporating Vaccination



## SIR Model with **Vaccination** Event (SIRV)



$\nu$ : Vaccination Rate

Vaccination gives **Full Recovery**

$$\frac{dS}{dt} = -\beta SI - \nu S$$

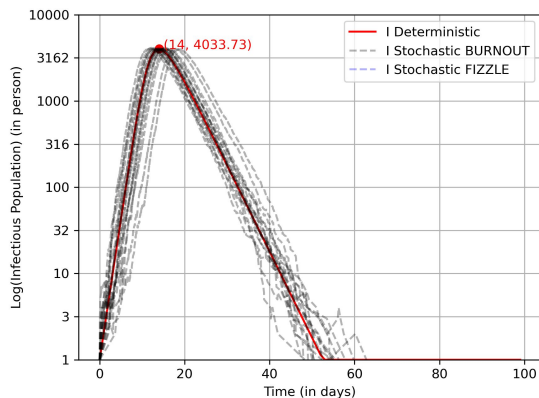
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I + \nu S$$

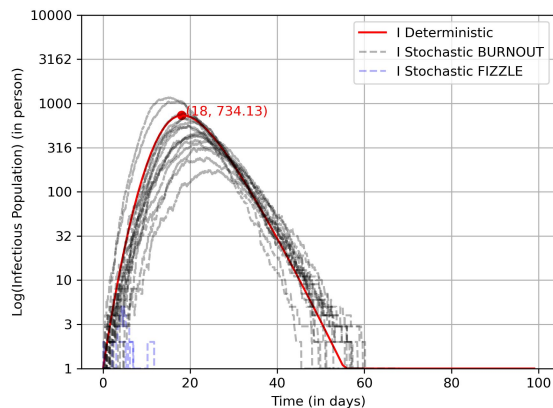
# Log-Scaled Infectious Trajectories

$$R_0 = \frac{\beta}{\gamma} = \frac{1}{0.25} = 4$$

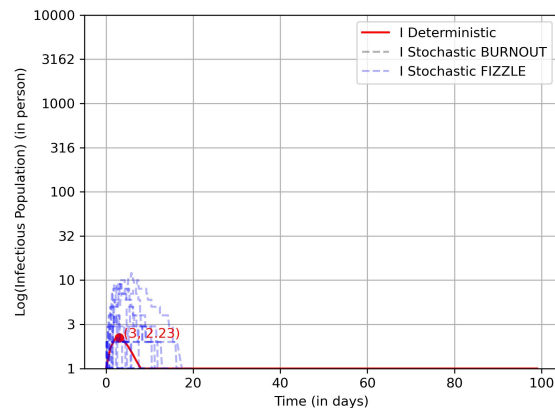
$$\nu = 0$$



$$\nu = 0.05$$



$$\nu = 0.5$$

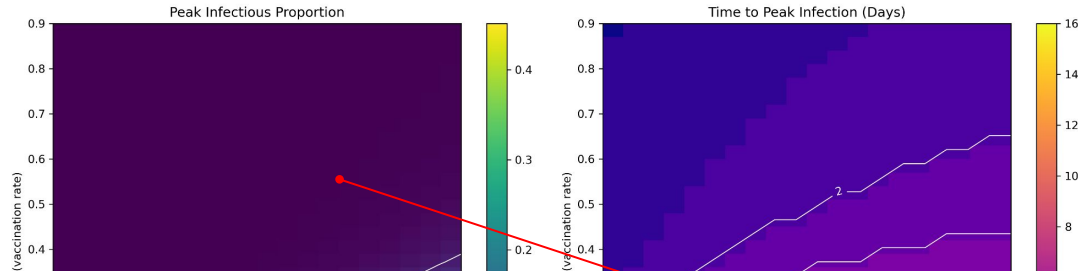


Increasing Vaccination Rate  $\rightarrow$

- Peak of Infectious trajectories SOONER
- Peak Infectious population INCREASE

Why? Higher  $\nu \rightarrow$  Faster Depletion of Susceptible pool  $\rightarrow$  Faster Increase in Infectious pool

# Peak Infectious Proportion / Time



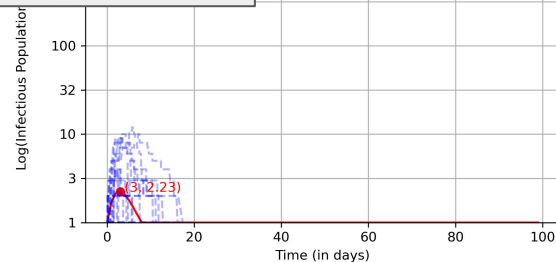
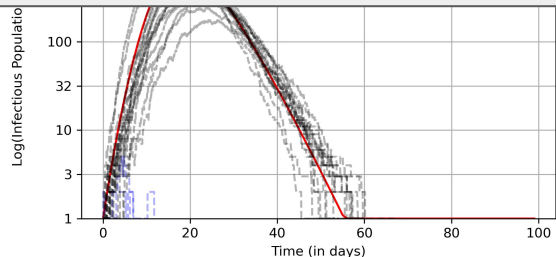
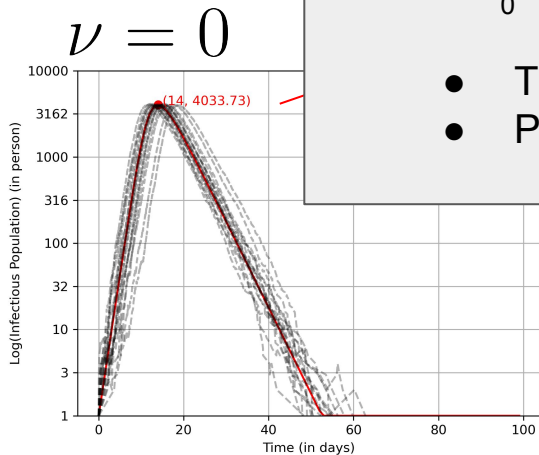
Increasing Vaccination Rate  $\rightarrow$  Decrease in Susceptible pool  $\rightarrow$

as  $R_0$  Increases,

- Time to Peak Infectious SOONER
- Peak Infectious population DECREASE

$$0 = \frac{\beta}{\gamma} = \frac{1}{0.25} = 4$$

$$\nu = 0.5$$



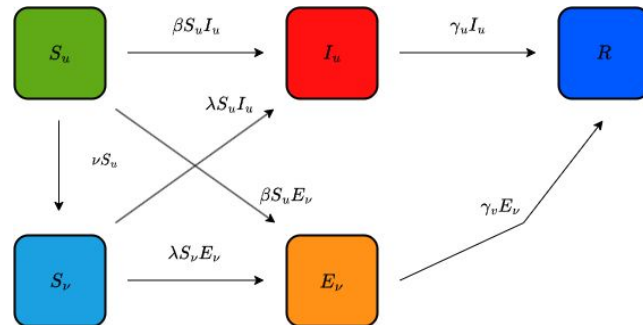
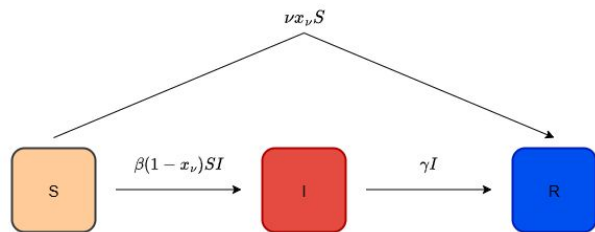


# Future Work

- We aim to assess the differential effects of time-dependent and time-independent vaccination strategies on epidemic dynamics, with an emphasis on their capacity to suppress transmission under varying epidemiological conditions.

$$\nu(t) = \begin{cases} ?, & t < t_\nu \\ ?, & t \geq t_\nu \end{cases}$$

- Future work will investigate the influence of alternative vaccination strategies on disease dynamics by extending the model to account for susceptible replenishment driven by waning immunity and the birth and death processes



# References

[1] Todd L. Parsons et al. “The probability of epidemic burnout in the stochastic SIR model with vital dynamics”. en. In: Proceedings of the National Academy of Sciences 121.5 (2024), e2313708120. ISSN: 0027-8424, 1091-6490. DOI: 10.1073/pnas.2313708120. URL: <https://pnas.org/doi/10.1073/pnas.2313708120>.

Thank you!