# Examining the Effect of Various Vaccination Strategies on the Stochastic SIR model with Vital Dynamics

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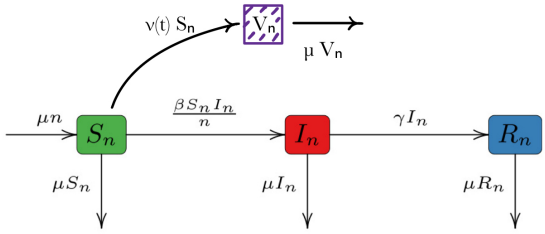
**Background Information and Literature Review:** The SIR model, seen in both deterministic and stochastic form, is used to model how infectious diseases spread through a population over time [5]. The two forms of the model corroborate each other, allowing us to examine epidemics through multiple lenses. Traditional deterministic SIR models suggest that varying parameters, such as reducing the pool of susceptibles, and thus the effective reproduction number, can lead to disease extinction [5]. Meanwhile, stochastic epidemic simulations of the SIR model, combined with vital dynamic birth-death processes, can exhibit varying outcomes such as fizzle, burnout, and persistence, as examined in the Parsons paper [4]. While this model is already useful, its applications can be furthered by modifying it to incorporate control factors of disease spread, such as vaccination.

**Statement of the Research Problem:** Vaccination is a critical tool to control infectious diseases, but its effectiveness depends on how it is implemented [2]. Parson's work [4] provides a solid foundation for modeling SIR dynamics stochastically with vital dynamics; our objective is to adapt this framework to examine the effect of vaccination within the epidemic. Using our vaccination-included version of the SIR model with vital dynamics [4], we will examine the effect of different vaccination strategies by varying vaccination parameters, such as timing and coverage.

**Planned Work:**

The Parsons paper [4] implemented the SIR model with vital dynamics; births and deaths within the population are included in the model. Recent studies [1] [3] have proposed the SIRV model that incorporates the vaccination compartment, and we adopt this extended framework in our study.




$$v(t) = \begin{cases} 0 & \text{if } t < t_v \\ \nu & \text{if } t \geq t_v \end{cases}$$

Figure 1: SIRV model with vital dynamics. Adapted and modified from [4].

Thus, with additional events of Vaccinated Death and Vaccination, with rate of $\mu V_n$ and $\nu(t)S_n$, respectively, the set of ODEs is as follows:

$$\begin{aligned}
\frac{dS_n}{dt} &= -\beta S_n I_n/n - \nu(t)S_n - \mu S_n + \mu n \\
\frac{dV_n}{dt} &= \nu(t)S_n - \mu V_n \\
\frac{dI_n}{dt} &= \beta S_n I_n - \gamma I_n - \mu I_n \\
\frac{dR_n}{dt} &= \gamma I_n - \mu R_n
\end{aligned}$$

Based on the above model, we aim to evaluate the effectiveness of vaccination by conducting Gillespie algorithm-based stochastic simulations. Through this simulation framework, we will estimate the probabilities of burnout and persistence, and analyze how these outcomes vary under different vaccination strategies. We aim to extend and implement the classification logic introduced in [4] which distinguishes epidemic outcomes: fizzle, burnout, and persistent, and to determine the number of simulation runs required to achieve certain accuracy.

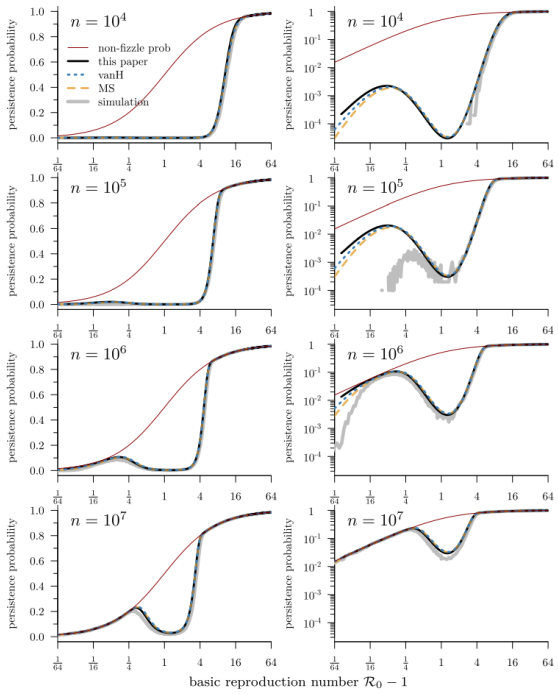



Figure 2: Persistence Probability Estimation. Adapted from [4].

Next, we will explore how key parameters introduced in our model, specifically the vaccination rate, $\nu(t)$ and the vaccination start time, $t_\nu$ influence epidemic dynamics. To efficiently explore this extended parameter space, we will construct a grid of simulations with number of bins and resolution, balancing computational cost with accuracy, given the long runtime of individual stochastic trajectories.

Figure 2 presents the baseline case in which no vaccination is assumed. We will therefore use it as a reference to assess the efficacy of different vaccination strategies, by comparing against this non-vaccinated benchmark.

If time permits, we plan to extend the current model by incorporating immunity acquisition process, representing the transition from vaccinated to recovered individuals, implementing a sigmoid-shaped vaccination rate function to model gradual vaccine rollout, and extend analytic framework for estimating persistence probability of the model with vaccination.

**Timeline: Week of July 13:** Finalize abstract; Reproduce key figures from [4]; Understand [4]'s code and method for calculating burnout probability; Start producing results, **Week of July 20:** Finalize results; Make visualization, interpretation, written materials; Prepare slides for presentation.

## References


[1] Zhilan Feng, Sherry Towers, and Yiding Yang. "Modeling the effects of vaccination and treatment on pandemic influenza". In: *The AAPS journal* 13.3 (2011), pp. 427–437.

[2] Laura Matrajt and Ira M Longini Jr. "Optimizing vaccine allocation at different points in time during an epidemic". In: *PloS one* 5.11 (2010), e13767.

[3] Paul Orovwuje and Stella Osaide. "Modeling the Control of the Spread of Infectious Diseases with Vaccination Strategy". In: *International Journal of Research and Innovation in Applied Science* X (Mar. 2025), pp. 262–283. DOI: 10.51584/IJRIAS.2025.10030019.

[4] Todd L. Parsons et al. "The probability of epidemic burnout in the stochastic SIR model with vital dynamics". en. In: *Proceedings of the National Academy of Sciences* 121.5 (2024), e2313708120. ISSN: 0027-8424, 1091-6490. DOI: 10.1073/pnas.2313708120. URL: https://pnas.org/doi/10.1073/pnas.2313708120.

[5] Joshua S. Weitz. *Quantitative Biosciences: Dynamics across Cells, Organisms, and Populations*. Princeton University Press, 2024. ISBN: 9780691181516.