Vaccination in Epidemic Modeling

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Introduction

In the case of COVID-19, large-scale vaccination campaigns were estimated to have **reduced mortality by more than 50%** in the United States, preventing millions of deaths worldwide [1].
Similarly, during the measles outbreak in Texas in January 2025, there have been 762 confirmed cases, 99 hospitalizations, and two fatalities, **all among unvaccinated people** [2].

And so, vaccination is effective in controlling the spread of infectious diseases, and absence of it results in a large outbreak.

Earlier studies conducted stochastic simulation of SIR model with vital dynamics, and observed epidemic outcomes: **burnout**, **fizzle**, and **persist** [3]. Our work explores the **effect of vaccination using the SIRV model in epidemic progression and outcomes**.

To understand how vaccination affects epidemic dynamics, it is important to consider the **basic reproduction number** (R_0), showing the **strength** of the infectious disease.

$$R_0 = \frac{\beta}{\gamma}$$

- The average number of new infections caused by a single infectious individual in an otherwise susceptible population.

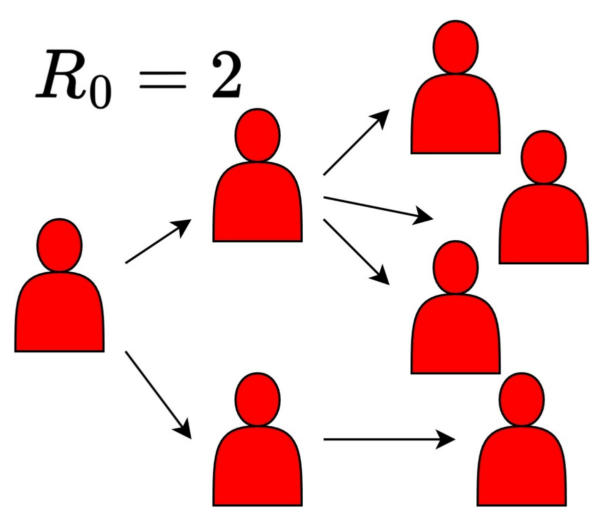


Figure 1. R_0 Visualization

Methods

We provided the SIRV model respecting the full protection vaccination is given to the susceptible population.

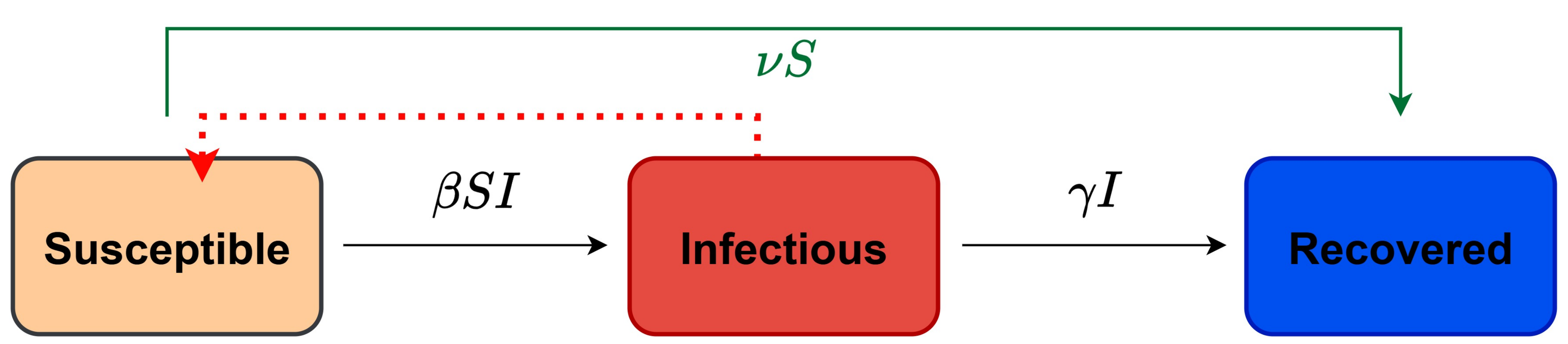


Figure 2. **SIRV** Model - Susceptible (S): likely to be infected, Infectious (I): is currently infected and can spread the disease, Recovered (R): recovered from the disease

$$\frac{dS}{dt} = -\beta SI - \upsilon S$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I + \upsilon S$$

β : Transmission Rate
γ : Recovery Rate
υ : Vaccination Rate

*S, I, R: the **proportion** of susceptible, infectious, recovered*

- **Deterministic Simulation** Figure 4a: is represented by a system of differential equations, producing a single, predictable trajectory for the epidemic under given parameters.
- **Stochastic Simulation** Figure 4b: incorporates randomness in infection and recovery events.
  - Gillespie Algorithm (Figure 3)
  - results in two epidemic outcomes (Figure 5a, 5b) [3]:
    - **Burnout**, a large outbreak occurs
    - **Fizzle**, disease fails to spread widely and dies quickly

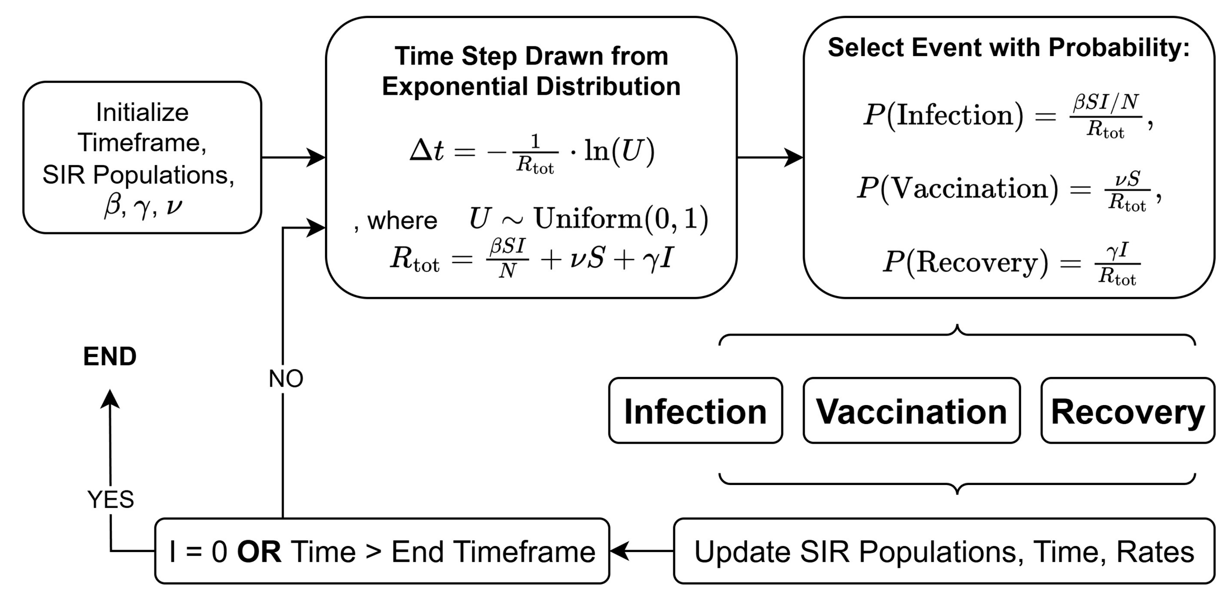
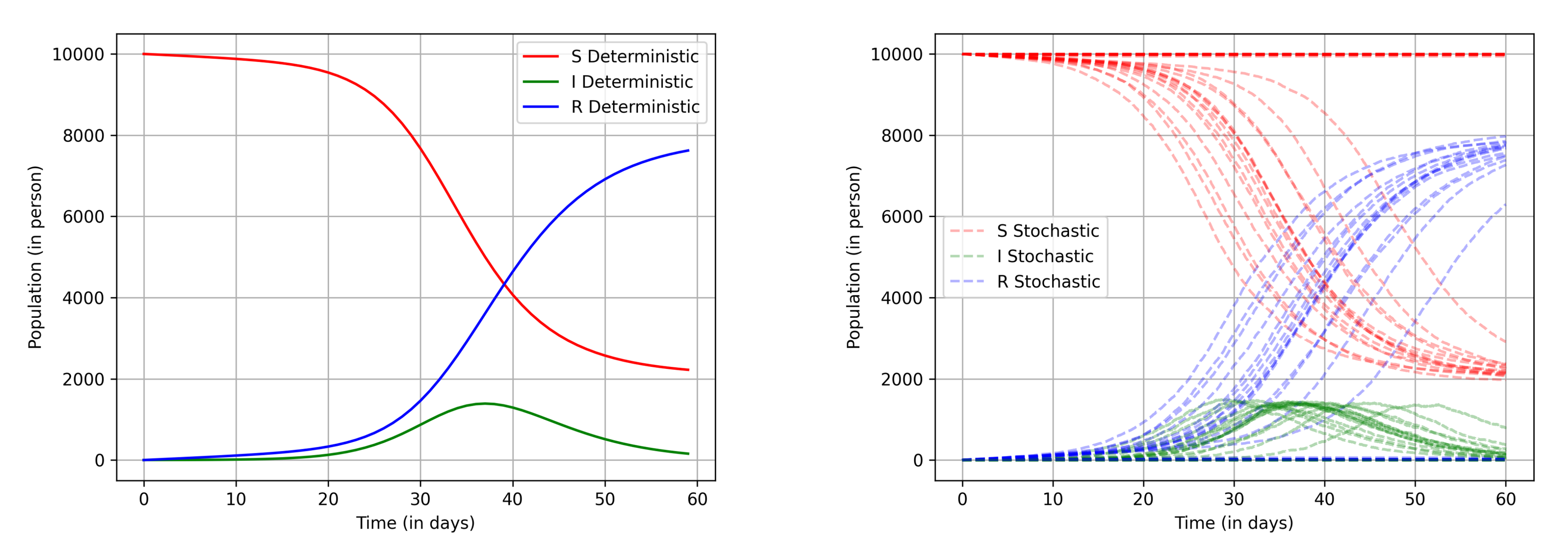


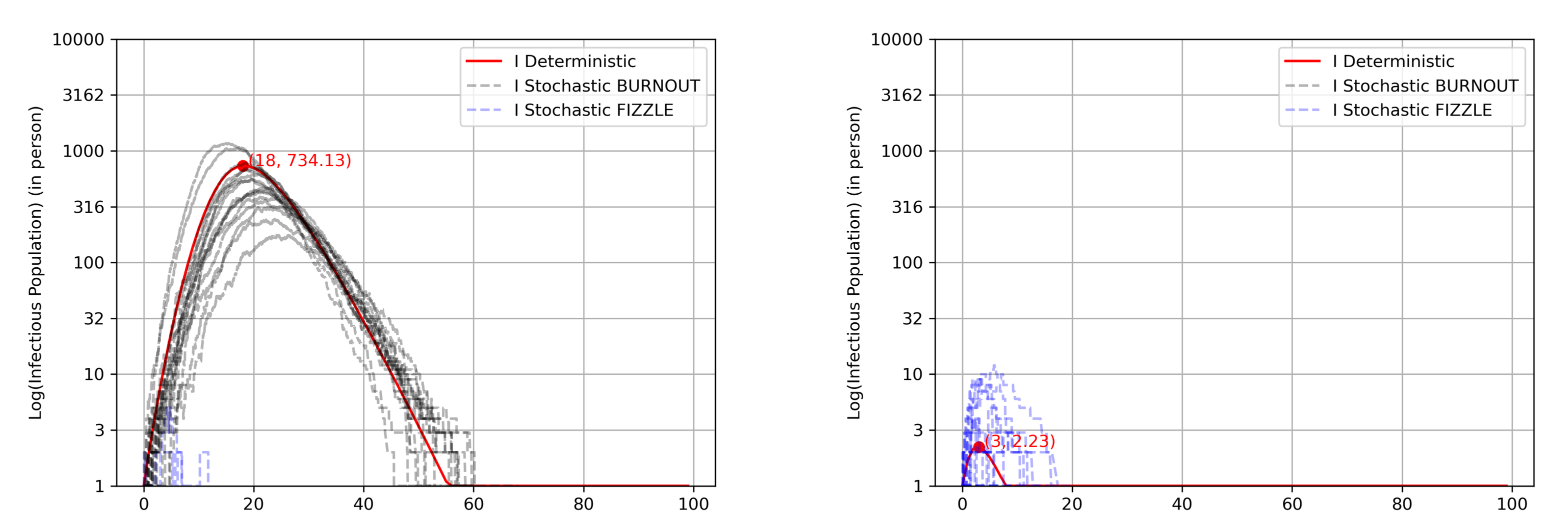
Figure 3. Gillespie Algorithm Flowchart



(a) Deterministic (b) Stochastic

Figure 4. Two Simulation Approaches

Results



(a) $\nu = 0.05$ (b) $\nu = 0.5$

Figure 5. Infectious Curve based on Deterministic and Stochastic (showing Fizzle and Burnout Cases) simulation for two Vaccination rates: 0.05, 0.5 when R_0 = 4

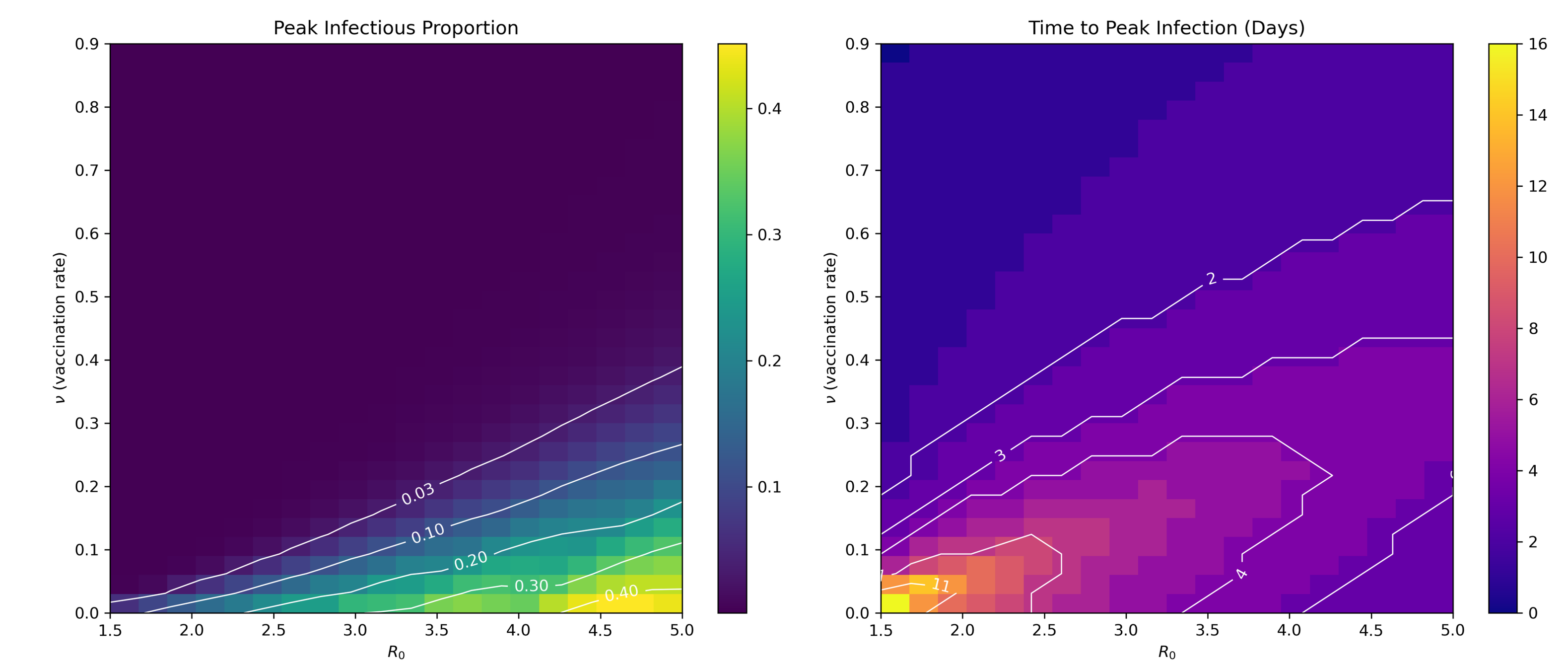


Figure 6. Peak Infection Proportion (left) and Peak Infection Time (right) for combinations of Vaccination rate (0.0-0.9) and R_0 (1.5-5.0)

Figure 5 shows **Fizzle increases** as υ increases

Figure 6 shows:

- **Peak Infectious Proportion Decreases**
  - as R_0 decreases and υ increases
- **Peak Infection Time gets Sooner**
  - as R_0 and υ increases
- a clear threshold, beyond which the marginal cost of vaccination decreases.

Discussion/Conclusion

- Stochastic simulation on SIRV model show that higher vaccination rates **lower peak infection** and **increases fizzle cases**.
- Results reveal a **threshold effect** where vaccination rate beyond a certain point has diminishing benefits.
- Vaccination plays a **critical role** in epidemic control, emphasizing the need for optimized strategies.

In the future, we plan to:

- Experiment with extended SIRV model that accounts for **susceptible replenishment** via waning immunity or birth & death process.
- Investigate the effects of **time-dependent vaccination strategies** on epidemic dynamics, especially regarding their ability to suppress disease transmission.

References

[1] Effect of COVID-19 vaccination on mortality by COVID-19 and on mortality by other causes, the Netherlands, January 2021–January 2022 — pmc.ncbi.nlm.nih.gov.

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[3] Todd L. Parsons, Benjamin M. Bolker, Jonathan Dushoff, and David J. D. Earn. The probability of epidemic burnout in the stochastic SIR model with vital dynamics. *Proceedings of the National Academy of Sciences*, 121(5):e2313708120, 2024.