

Bottom-Up Parsing - II

Lecture 8

Section 4.5, 4.6

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1 LR Parsing

2 The Parsing Tables

3 The Action Table

4 The Goto Table

5 Assignment

6 The Algorithm



Outline

1 LR Parsing

2 The Parsing Tables

3 The Action Table

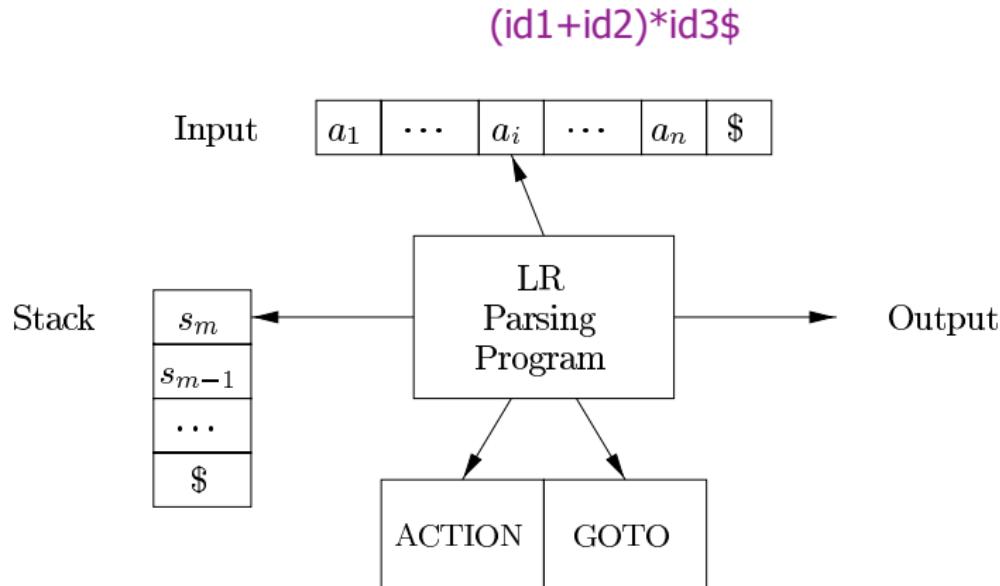
4 The Goto Table

5 Assignment

6 The Algorithm



LR Parsing - Model



Outline

- ① LR Parsing
- ② The Parsing Tables
- ③ The Action Table
- ④ The Goto Table
- ⑤ Assignment
- ⑥ The Algorithm



The LR(0) Parsing Tables

- There are two tables that we will construct.
- The **action table** contains **shift** and **reduce actions** to be taken upon processing terminals.
- The **goto table** contains changes of state upon matching productions.



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The Action Table

E → E + T (r6)

s6
r5

s5

r6

- The action table contains one row for each state in the PDA and one column for each terminal and EOF (\$).
- The entries are
 - Shift n .
 - Push the current token and move to state n .
 - Reduce n .
 - Pop the symbols of the right side of production n and then push the nonterminal of the left side.
 - Then change state according to the goto table.



Building the Action Table

$E \rightarrow E . + T$

$T \rightarrow T * F.$

- The action table is built from items of the form $[A \rightarrow \alpha \bullet a\beta]$ and $[A \rightarrow \alpha \bullet]$.
 - Items $[A \rightarrow \alpha \bullet a\beta]$ produce shift operations.
 - Items $[A \rightarrow \alpha \bullet]$ produce reduce operations.
- The goto table is built from items of the form $[A \rightarrow \alpha \bullet B\beta]$.

Building the Action Table

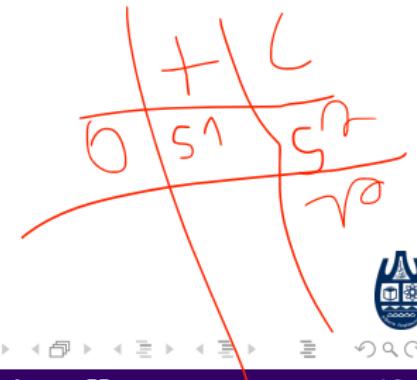
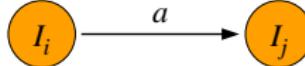
$$E \rightarrow E . + T$$

I_0

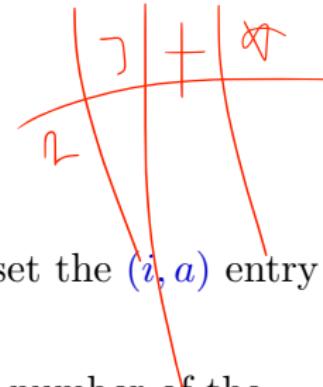
I_1



- If $[A \rightarrow \alpha \bullet a\beta]$ is in state I_i and the PDA transition on a is from state I_i to state I_j , then set the (i, a) entry to “shift j .”
- In the table, we write “ sj ,” where j is the number of the destination state.
- Thus, every transition on a terminal becomes a shift entry in the action table.



Building the Action Table



- If $[A \rightarrow \alpha \bullet] \in I_i$ and $[S' \rightarrow S \bullet] \notin I_i$, then set the (i, a) entry to “reduce $A \rightarrow \alpha$ ” for all a in $\text{FOLLOW}(A)$.
- In the table, we write “ $r k$,” where k is the number of the production $A \rightarrow \alpha$.



Building the Action Table



- If $[S' \rightarrow S \bullet]$ is in state I_i , then the $(i, \$)$ entry is “accept.”
- In the table, we write “acc.”
- All empty cells are labeled “error.”



Example

X

Example (FIRST and FOLLOW)

- To find the action table for our example, we need to know nullability, FIRST, and FOLLOW.

$$\begin{array}{l}
 E' \rightarrow E \\
 E \rightarrow E + T \mid T \\
 T \rightarrow T * F \mid F \\
 F \rightarrow (E) \mid \text{id}
 \end{array}$$

Q

+, *, (,), id



Example

Example (FIRST and FOLLOW)

	Nullable	FIRST	FOLLOW
E'	No	$\{(), \text{id}\}$	$\{\$\}$
E	No	$\{(), \text{id}\}$	$\{\$, +,)\}$
T	No	$\{(), \text{id}\}$	$\{\$, +,), *\}$
F	No	$\{(), \text{id}\}$	$\{\$, +,), *\}$



Example

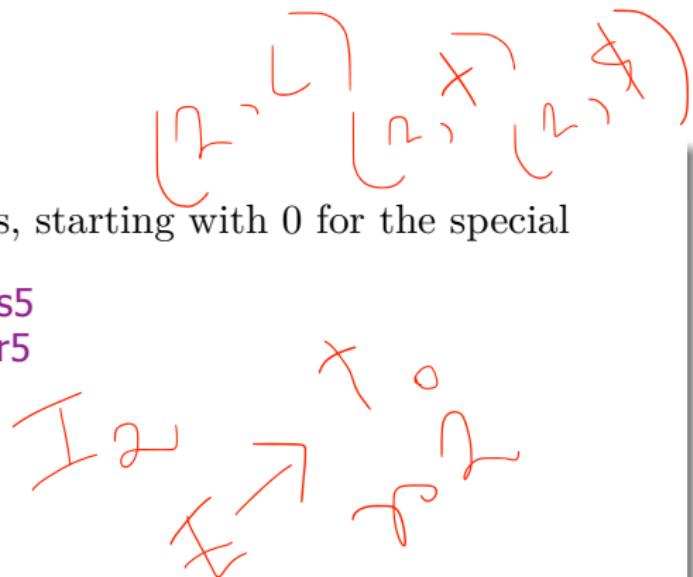
Example (The Action Table)

- Then number the productions, starting with 0 for the special production:

0. $E' \rightarrow E$
1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \text{id}$

706

s5
r5



Example $I_0 \rightarrow I_4$

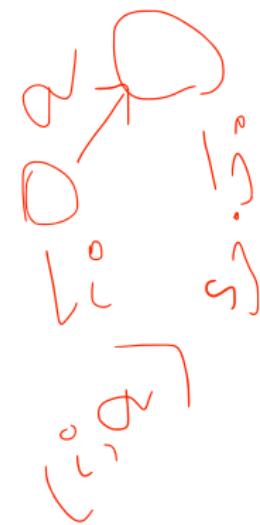
$E \rightarrow E^*$

Example (The Action Table)

i^*
 $I_0 \rightarrow I_5$

$I2$
 $E \rightarrow T$.
 $R \rightarrow T.*F$

	+	*	()	id	\$
0			s4		s5	
1	s6				acc	
2	r2	s7		r2		r2
3	r4	r4		r4		r4
4			s4		s5	
5	r6	r6		r6		r6
6			s4		s5	
7			s4		s5	
8	s6			s11		
9	r1	s7		r1		r1
10	r3	r3		r3		r3
11	r5	r5		r5		r5



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The Goto Table

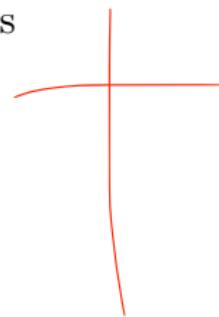
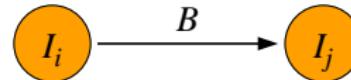
- The goto table has one row for each state in the PDA and one column for each nonterminal except S' .
- The entries are states of the PDA.



Building the Goto Table

$E \rightarrow E + T.$

- If $[A \rightarrow \alpha \bullet B\beta]$ is in state I_i and the PDA transition is



then set the (i, B) entry to “goto j .”

- In the goto table, we write “ j .”
- Thus, every transition on a nonterminal becomes an entry in the goto table.



Example

Example (The Goto Table)

	E	T	F
0	1	2	3
1			
2			
3			
4	8	2	3
5			
6		9	3
7			10
8			
9			
10			
11			

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Assignment

Homework

- Let the grammar be

$$\begin{aligned}S &\rightarrow (L) \mid \text{id} \\L &\rightarrow L , S \mid S\end{aligned}$$

- Build the action and goto tables.



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INPUT: An input string w and an LR- parsing table with functions **ACTION** and **GOTO** for a grammar G

OUTPUT: If w is in $L(G)$ the reduction steps of a bottom- up parse for w otherwise an error indication.

METHOD: Initially the parser has s_0 on its stack, where s_0 is the initial state and w in the input buffer.

Action[4,()] = s4
 Action[2,()) = r2
 on[4,()) = blank (ERROR)
 Action[2,\$] = Accept

```

let  $a$  be the first symbol of  $w\$$ ;           2. E—>E+T
while(1) { /* repeat forever */
    let  $s$  be the state on top of the stack;
    if ( ACTION[ $s, a$ ] = shift  $t$  ) {
        push  $t$  onto the stack;
        let  $a$  be the next input symbol;
    } else if ( ACTION[ $s, a$ ] = reduce  $A \rightarrow \beta$  ) {
        pop  $|\beta|$  symbols off the stack;
        let state  $t$  now be on top of the stack;
        push GOTO[ $t, A$ ] onto the stack;
        output the production  $A \rightarrow \beta$ ;
    } else if ( ACTION[ $s, a$ ] = accept ) break; /* parsing is done */
    else call error-recovery routine;
}
    
```



Example

Action[0, ()] = s4

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Stack	Input
0	(id + id) * id \$
0 (4	id + id) * id \$
0 (4 id 5	+ id) * id \$
0 (4 F 3	+ id) * id \$
0 (4 T 2	+ id) * id \$
0 (4 E 8	+ id) * id \$
0 (4 E 8 + 6	id) * id \$
0 (4 E 8 + 6 id 5) * id \$
0 (4 E 8 + 6 F 3) * id \$



Example

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Action[0, ()] = s4
 Action[4, id] = s5

Stack	Input
0	(id + id) * id \$
0 (4	id + id) * id \$
0 (4 id 5	+ id) * id \$
0 (4 F 3	+ id) * id \$
0 (4 T 2	+ id) * id \$
0 (4 E 8	+ id) * id \$
0 (4 E 8 + 6	id) * id \$
0 (4 E 8 + 6 id 5) * id \$
0 (4 E 8 + 6 F 3) * id \$



Example

0 (4 F

Example (Parse (id + id)* id)

Action[5,+] = r6

6. F → id

Stack	Input
0	(id + id) * id \$
0 (4	id + id) * id \$
0 (4 id 5	+ id) * id \$
0 (4 F 3	+ id) * id \$
0 (4 T 2	+ id) * id \$
0 (4 E 8	+ id) * id \$
0 (4 E 8 + 6	id) * id \$
0 (4 E 8 + 6 id 5) * id \$
0 (4 E 8 + 6 F 3) * id \$

Example

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Action[3,+]

Stack	Input
0	$(\text{id} + \text{id}) * \text{id} \$$
0 (4	$\text{id} + \text{id}) * \text{id} \$$
0 (4 id 5	$+ \text{id}) * \text{id} \$$
0 (4 F 3	$+ \text{id}) * \text{id} \$$
0 (4 T 2	$+ \text{id}) * \text{id} \$$
0 (4 E 8	$+ \text{id}) * \text{id} \$$
0 (4 E 8 + 6	$\text{id}) * \text{id} \$$
0 (4 E 8 + 6 id 5	$) * \text{id} \$$
0 (4 E 8 + 6 F 3	$) * \text{id} \$$



Example

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Stack	Input
0	$(\text{id} + \text{id}) * \text{id} \$$
0 (4	$\text{id} + \text{id}) * \text{id} \$$
0 (4 id 5	$+ \text{id}) * \text{id} \$$
0 (4 F 3	$+ \text{id}) * \text{id} \$$
0 (4 <u>T</u> 2	$+ \text{id}) * \text{id} \$$
0 (4 E 8	$+ \text{id}) * \text{id} \$$
0 (4 E 8 + 6	$\text{id}) * \text{id} \$$
0 (4 E 8 + 6 id 5	$) * \text{id} \$$
0 (4 E 8 + 6 F 3	$) * \text{id} \$$

Example

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Stack	Input
0	$(\text{id} + \text{id}) * \text{id} \$$
0 (4	$\text{id} + \text{id}) * \text{id} \$$
0 (4 id 5	$+ \text{id}) * \text{id} \$$
0 (4 F 3	$+ \text{id}) * \text{id} \$$
0 (4 T 2	$+ \text{id}) * \text{id} \$$
0 (4 E 8	$+ \text{id}) * \text{id} \$$
0 (4 E 8 + 6	$\text{id}) * \text{id} \$$
0 (4 E 8 + 6 id 5	$) * \text{id} \$$
0 (4 E 8 + 6 F 3	$) * \text{id} \$$

Example

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0 (4 id 5	$+ \text{id}) * \text{id} \$$
0 (4 F 3	$+ \text{id}) * \text{id} \$$
0 (4 T 2	$+ \text{id}) * \text{id} \$$
0 (4 E 8	$+ \text{id}) * \text{id} \$$
0 (4 E 8 + 6	$\text{id}) * \text{id} \$$
0 (4 E 8 + 6 id 5	$) * \text{id} \$$
0 (4 E 8 + 6 F 3	$) * \text{id} \$$



Example

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Stack	Input
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0 (4 E 8	$+ \text{id}) * \text{id} \$$
0 (4 E 8 + 6	$\text{id}) * \text{id} \$$
0 (4 E 8 + 6 id 5	$) * \text{id} \$$
0 (4 E 8 + 6 F 3	$) * \text{id} \$$



Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

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Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

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Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

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Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

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Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	ee

Example

Example (LR Parsing)

Stack	Input
0 (4 E 8 + 6 T 9) * id \$
0 (4 E 8) * id \$
0 (4 E 8) 11	* id \$
0 F 3	* id \$
0 T 2	* id \$
0 T 2 * 7	id \$
0 T 2 * 7 id 5	\$
0 T 2 * 7 F 10	\$
0 T 2	\$
0 E 1	\$
Accept	\$

- In practice, we need only push the state numbers onto the stack.
- There is no need to push the grammar symbols.
- Thus, the previous example can be streamlined.



Example

Example (Parse $(\text{id} + \text{id}) * \text{id}$)

Stack	Input
0	$(\text{id} + \text{id}) * \text{id} \$$
0 4	$\text{id} + \text{id}) * \text{id} \$$
0 4 5	$+ \text{id}) * \text{id} \$$
0 4 3	$+ \text{id}) * \text{id} \$$
0 4 2	$+ \text{id}) * \text{id} \$$
0 4 8	$+ \text{id}) * \text{id} \$$
0 4 8 6	$\text{id}) * \text{id} \$$
0 4 8 6 5	$) * \text{id} \$$
0 4 8 6 3	$) * \text{id} \$$

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0 4 3	$+ \text{id}) * \text{id} \$$
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0 4 8	$+ \text{id}) * \text{id} \$$
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Stack	Input
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Example

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Stack	Input
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0 4 8	$+ \text{id}) * \text{id} \$$
0 4 8 6	$\text{id}) * \text{id} \$$
0 4 8 6 5	$) * \text{id} \$$
0 4 8 6 3	$) * \text{id} \$$



Example

Example (LR Parsing)

Stack	Input
0 4 8 6 9) * id \$
0 4 8) * id \$
0 4 8 11	* id \$
0 3	* id \$
0 2	* id \$
0 2 7	id \$
0 2 7 5	\$
0 2 7 10	\$
0 2	\$
0 1	\$
Accept	\$

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Example

Example (A Simplified Grammar)

- We may simplify our grammar to

$$E \rightarrow E + E$$

$$E \rightarrow E * E$$

$$E \rightarrow (E)$$

$$E \rightarrow \text{id}$$

- In this form, the precedence rules for **+** and ***** are not implicit.
- They must be incorporated into the tables.

Assignment

Homework

- The grammar for if and if-else statements is

$$\begin{aligned} S \rightarrow & \text{if (} E \text{) } S \\ | & \text{if (} E \text{) } S \text{ else } S \\ | & \text{id = num} \\ E \rightarrow & \text{id == num} \end{aligned}$$

where S is a statement and E is an expression (boolean).

- Write the action and goto tables for this grammar.
- Find the shift/reduce conflict(s) and decide how to handle them.

(continued...)