ILP Formulation

Basic Equations/Notations:

Let x_i be a node and e_{ij} be an edge (incident with nodes x_i and x_j) in the original input network.

$$\forall i \ 0 \le x_i \le 1 \tag{1}$$

if x_i is a feasible location (as computed from the map data) for a base station, then x_i can be 1

$$\forall i, j \ 0 \le e_{ij} \le 1 \tag{2}$$

if node x_i and x_j are in line-of-sight (as computed from the map data) with each other, then e_{ij} can be 1.

The backbone-graph is a subgraph of the above graph and is denoted by node set $\{y_i\}$ and edge-set $\{b_{ij}\}$. From the subgraph property:

$$\forall i \ 0 \le y_i \le x_i \tag{3}$$

$$\forall i, j \ 0 \le b_{ij} \le e_{ij} \tag{4}$$

To enforce symmetry:

$$\forall i, j \ e_{ij} = e_{ji} \tag{5}$$

$$\forall i, j \ b_{ij} = b_{ji} \tag{6}$$

To enforce edge-incidence:

$$\forall i, j \ e_{ij} \le \frac{1}{2}(x_i + x_j) \tag{7}$$

$$\forall i, j \ b_{ij} \le \frac{1}{2}(y_i + y_j) \tag{8}$$

Requirements:

a) Each area is covered by at least one node in the backbone, here T_{ij} is 1 if target j is covered by a node i:

$$\forall \text{ target } j, \sum_{i \mid T_{ij}=1} y_i \ge 1 \tag{9}$$

or in variable notations:

$$\forall \text{ target } j, \sum_{i} (T_{ij} \times y_i) \ge 1$$

b) The backbone must be a connected graph with all the sink/gateway nodes in W and all the source nodes in T. To enforce connectivity, we set up a flow problem as follows:

we add two nodes, a super-source node s and a supersink node t and make them part of the backbone node. We also add an edge from the supersource to each source node and an edge from each sink node to the supersink:

$$\forall i \; 0 \leq e_{si} \leq x_i \tag{10}$$

$$\forall i \; 0 \leq b_{si} \leq y_i \tag{11}$$

$$\forall w \in W \;\; e_{wt} = 1 \tag{12}$$

Note that we do *not* define variables e_{st}, e_{is}, e_{ti} and the corresponding b_{st}, b_{is}, b_{ti}, and thats fine!

There is a nonzero flow from s to each source (adding the correct upper bound too): node:

$$\forall i, \;\; N y_i \geq f_{si} \geq \frac{y_i}{N}, \text{ where N is a very large number} \tag{13}$$

All the flow must go through the backbone edges:

$$\forall i, j \, : \; f_{ij} \leq b_{ij} \tag{14}$$

The total flow from s must equal to that of to t:

$$\sum_i f_{si} = \sum_j f_{jt} \tag{15}$$

Also the flows must be conserved (included s and t in the summation):

$$\forall j \, : \; \sum_{i \cup \{s\}} f_{ij} = \sum_{i \cup \{t\}} f_{ji} \tag{16}$$

Constraints:

a) The number of node in the network graph is bounded by a given parameter:

$$\sum_i x_i \leq n_{max} \tag{17}$$

b) The node degree (except for the **gateway nodes**) of the backbone graph is also bounded by a given parameter:

$$\forall i \mid i \notin W \;\; \sum_j b_{ij} \leq d_{max} \tag{18}$$

Objective

Maximize the total flow from the supersource to the supersink in the network graph. Let the flow in link e_{ij} be g_{ij}. The flow constraint equations can be formulated as follows:
The total flow from s must equal to that of to t:

$$\sum_i g_{si} = \sum_j g_{jt} \tag{19}$$

Also the flows must be conserved (included s, t in the summation):

$$\forall j \;\; \sum_{i \cup \{s\}} g_{ij} = \sum_{i \cup \{t\}} g_{ji} \tag{20}$$

The link capacity must not be exceeded:

$$\forall i, j \; 0 \leq \;\; g_{ij} \leq e_{ij} \tag{21}$$

Adding upper bounds to links connecting to s and t, so that the links connecting s to non-selected nodes have zero flow and similarly for t. Technically, the below equations are not really needed, since the flow conversation at internal nodes will ensure this for ILP – but, in the LP relaxation, the below equations will give us a tighter bound.

$$\forall i \;\; Nx_i \geq f_{si}, \text{ where N is a very large number} \tag{22}$$

$$\forall i \;\; Nx_i \geq f_{it}, \text{ where N is a very large number} \tag{23}$$

The objective function is:

$$\mathbf{max} : \sum_i g_{si} \tag{24}$$