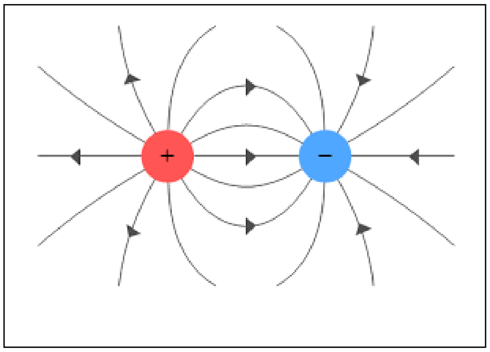
## Electric Field

The space surrounding an electric charge within which it is capable of exerting a force on another electric charge is called field.

An **electric field** is generated by electrically charged particles and time-varying magnetic fields.

Figure show an electric field produced by a positive and negative charge.




## Electric Field Strength or Intensity

The electric field strength or electric field intensity E at a point is expressed in magnitude and direction by the force per unit charge experienced by a small positive test charge $q_o$ placed at that point.

Mathematically, the electric field strength or electric field intensity Eat the point is defined as

$$E = \frac{F}{q_0}.$$

The value of $q_o$ should be so small that it should not disturb the electric field. In this case, the above equation can be written as

$$E = \lim_{q_0 \to 0} \frac{F}{q_o}$$

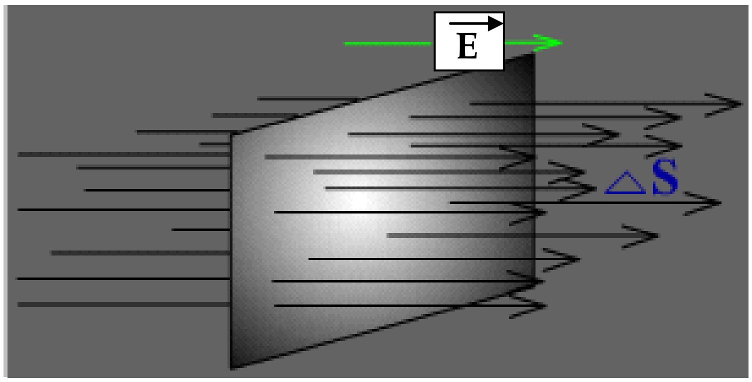
## Electric Flux

**Electric flux** is the rate of flow of the electric field through a given area. Electric flux is proportional to the number of electric field lines going through a virtual surface. If the electric field is uniform, the electric flux passing through a surface of vector area **S** is

$$\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}} = ES\cos\theta,$$

Where, E is the electric field, E is its magnitude, S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux".**




## Gauss's Law

Gauss's law states that the flux of the electric field E through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the total charge enclosed by the surface.

Mathematically

$$\Phi_E = \frac{1}{\varepsilon_o} q$$

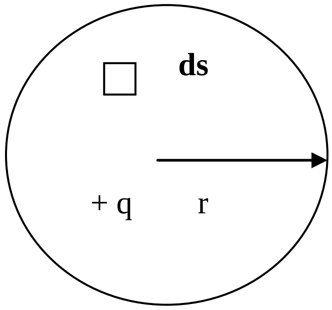



Fig: spherical surface of radius r surrounding a point charge q.

$\Phi_{E} . \varepsilon_{o} = q$ ……………………… (1)

This can be written by using integral form

$\Phi_{E} = \oint \vec{E} . \vec{ds}$

Therefore,

$\varepsilon_{o} . \oint E \ ds = q$

This is the mathematical expression of Gauss's Law in integral form. Where, **ds** is very small surface.

## Coulomb's law from Gauss's law

Let us consider a spherical surface of radius of r, centered on a point charge q, from Gauss's law

$\varepsilon_{o} . \oint \vec{E} \ \vec{ds} = q$ …………………….(1)

In figure, both **E** and **ds** at any point on the Gaussian surface are directed radially outward. The angle between them is zero. Therefore,

$\vec{E} . \vec{ds} = E. ds . \cos\theta = E . ds . \cos 0^{o} = E \ ds$

Then from equation no. 1

$\varepsilon_{o} . \oint E \ ds = q$ …………………….(2)

E is constant for all points in the surface

$\varepsilon_{o} . E \oint ds = q$ ……………………. (3)

Here, the integral is simple and the area of the sphere, therefore

$\varepsilon_{o} . E \ (4 \pi r^{2}) = q$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Let us put a second point charge $\mathbf{q_o}$ at the point at which **E** is calculated. The magnitude of the force that acts on it is

$$F = q_o E \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

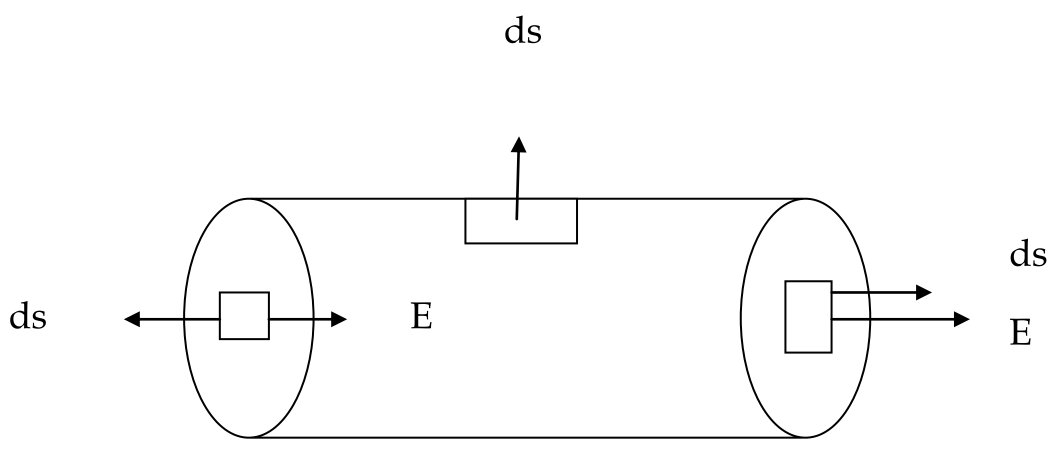
Combining equation number (4) and (5)

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q\,q_o}{r^2}$$

This is precisely coulomb's law. Thus we have deduced Coulomb's law from Gauss's law.

## Application of Gauss's law

- ✓ **Calculate the electric flux for a cylindrical surface immersed in a uniform electric field. The field being parallel to the cylindrical axis.**



The flux can be written as the sum of three terms. The left cylindrical cap, the right cap and the surface.

Thus for left cap the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 180^0 = E \oint ds = - E S$

Where, $S = \pi r^2$

Similarly for the right cap the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 0^0 = E \oint ds = - E S$

Finally, for the cylindrical wall, the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 90^0 = 0$

Thus flux for the closed cylindrical surface is

$\Phi_E = -ES + ES + 0 = 0$

## Electric Potential or Electric Field Potential or Electrostatic Potential

The electric potential at a point in an electric field is the work required to bring unit positive electric charge from infinity to the point.

Suppose, we have two points A and B in an electric field. We have a test charge $q_o$ from B to A. If the work done by the agent moving the charge $W_{AB}$, then

Electric potential difference

$V_A - V_B = \frac{W_{AB}}{q_0}$ ...........................(1)

A B

**$V_A$** **$V_B$**

Usually, point B is considered to be at infinite distance. In this case $V_B$ is assumed to be zero.

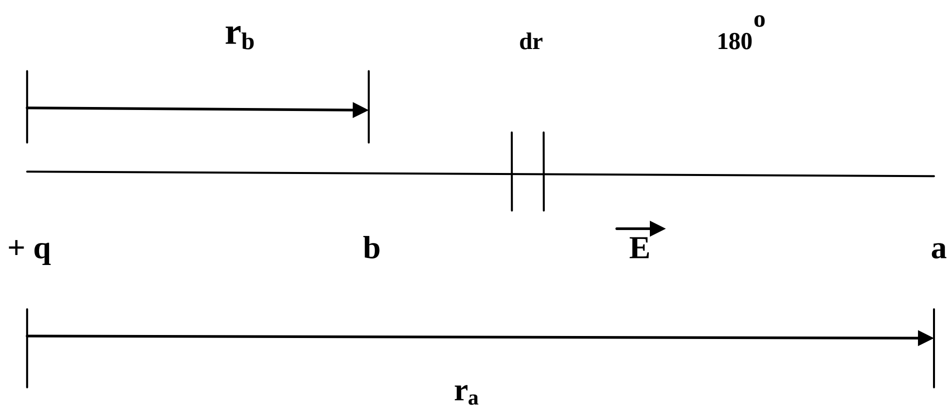
Putting $V_B = 0$ and $V_A = V$, Equation number 1 becomes

$$V = \frac{w}{q_o}$$

This equation gives the general representation of electric potential.

## Potential Due to a point charge

Let us consider, two points a and b in an electrostatic field of a single isolated point charge +q .




If a unit positive charge 'q' moves from 'a' to 'b' without acceleration, then the potential difference between 'a' and 'b' is given as

$$V_b - V_a = \int \vec{E} \cdot \vec{dr} = \int E\, dr \cos 180^o$$

But, $\vec{E} \cdot \vec{dr} = E\, dr \cos 180^o = - E\, dr$

$V_b - V_a = -\int E \,.\, dr$ ………………………….. (1)

From equation (1)

$$V_b - V_a = -\int_{r_a}^{r_b} E\,dr \qquad \text{but } E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\therefore \quad V_b - V_a = -\frac{1}{4\pi\epsilon_0} q \int_{r_a}^{r_b} \frac{1}{r^2}\,dr$$

$$V_b - V_a = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

If the point 'a' is at infinity, then

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{q}{r_b} \quad \text{………………..} (1)$$

Choosing reference point a to be infinitely distance

$r_a \rightarrow \propto$ and $V_a = 0$

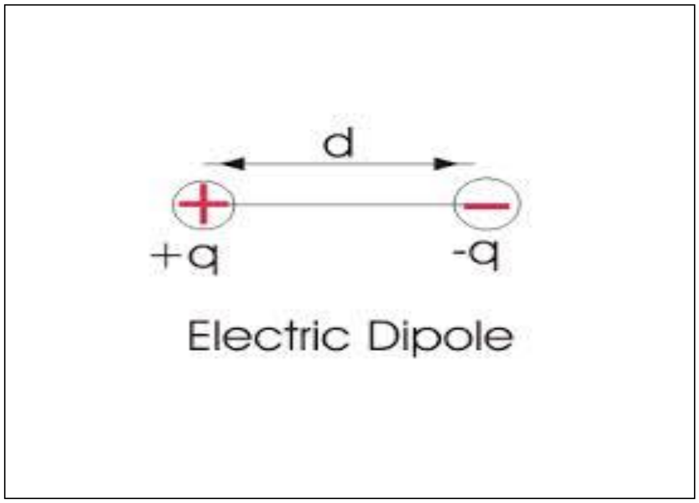
Then equation number (1)

$$V_b = V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

From the above, it is evident that for a given charge 'q', potential depends only on 'r'. Therefore, if the charge is positive, potential is positive and if the charge is negative, potential is negative.

## Electric Dipole

If two equal and opposite charges are placed at a short distance, the formation is called electric dipole.




## Electric potential at a point due to a dipole

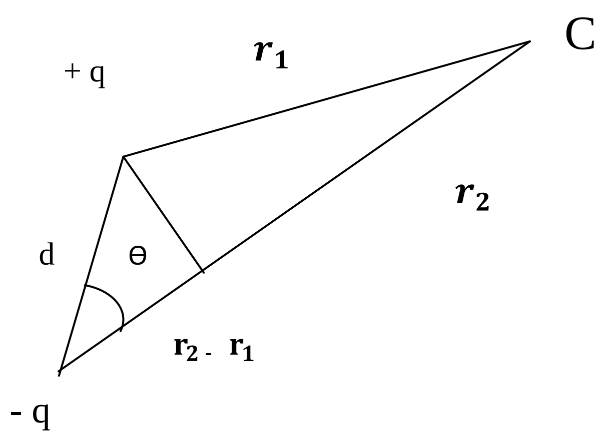
Let two equal and opposite charges +q and –q are placed at a short distance d. P is a point which is at a distance $r_1$ from +q distance $r_2$ from –q charge. We need to find the electric potential at P due to the dipole.

From the expression of electric potential at a point of distance r from a charge q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

So, the electric potential at p due to +q charge

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r1}$$ and for – q charge




$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{r2}\right)$$

So, mutual potential at P is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r1} - \frac{q}{r2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\ q\ \frac{r_2 - r_1}{r_1 r_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\ q.\ \frac{d\cos\theta}{r_1 r_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r_1 r_2}$$

If $r_1$ and $r_2$ >> d, and $r_1.r_1 === r^2$

**From Figure**

$$\cos\theta = \frac{r_2 - r_1}{d}$$

Then

$$r_2 - r_1 = d\cos\theta$$

**Electric Dipole Moment,**

**p = q × d**

Then

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

This is an expression for electric potential at a point due to a dipole.

## Electric Potential Energy

We define the electric potential energy of a system of point charges as the work required assembling this system of charges by bringing them close together, as in the system from an infinite distance.

However, we assume that initial kinetic energy of the charges is zero at infinity; they are at rest at infinity. Thus, an external work done against the forces between the charges and this external work done is stored in the system as the electrical potential energy of the configuration (or arrangement) of the charges. As the forces between the charges are of two types, attractive for opposite charges and repulsive for similar charges. Therefore, the work done will be positive in the case of like charges and this work done will be negative in the case of unlike (or dissimilar) charges so that, for similar charges potential energy is positive and for dissimilar it is negative.