

# Assignment 2

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## Question 1

### Part a

`y, L = cbacbabab 0`

`y, L = eththch htt Twaaai , 4`

### Part b

```
reconstructB(y, L):
  B <-- new matrix
  for i in range( len(y) ):
    add y to B
    sort B by rows lexicographically

  return B
```

### Part c

```
reconstructx(y, L):
  B <-- reconstructB(y, L)

  x <-- B[L]

  return x
```

### Part d

Input	All Rotations	Sorted	y	L
ababbcbac	ababbcbac	ababbcbac	cbbacabab	0
-	babbcbaca	abbcbacab	-	-
-	abbcbacab	acababbcb	-	-
-	bbcbacaba	babbcbaca	-	-
-	bcbacabab	bacababbcb	-	-
-	cbacababb	bbcbacaba	-	-
-	bacababbcb	bcbacabab	-	-
-	acababbcb	cababbcb	-	-

Input	All Rotations	Sorted	y	L
-	cababbcba	cbacababb	-	-

y, L = cbbacabab, 0

Reconstruction:

a				
a				
a				
b				
b				
b				
b				
c				
c				
a	b			
a	b			
a	c			
b	a			
b	a			
b	b			
b	c			
c	a			
c	b			
a	b	a		
a	b	b		
a	c	a		
b	a	b		
b	a	c		
b	b	c		
b	c	b		
c	a	b		
c	b	a		
a	b	a	b	
a	b	b	c	
a	c	a	b	
b	a	b	b	
b	a	c	a	
b	b	c	b	
b	c	b	a	
c	a	b	a	
c	b	a	c	
a	b	a	b	b
a	b	b	c	b
a	c	a	b	a
b	a	b	b	c
b	a	c	a	b
b	b	c	b	a
b	c	b	a	c
c	a	b	a	b
c	b	a	c	a
a	b	a	b	b
a	b	b	c	b
a	c	a	b	a
b	a	b	b	c
b	a	c	a	b
b	b	c	b	a
b	c	b	a	c
c	a	b	a	b
c	b	a	c	a
a	b	a	b	b
a	b	b	c	b
a	c	a	b	a
b	a	b	b	c
b	a	c	a	b
b	b	c	b	a
b	c	b	a	c
c	a	b	a	b
c	b	a	c	a

```

c a b a b b c
c b a c a b a

a b a b b c b a
a b b c b a c a
a c a b a b b c
b a b b c b a c
b a c a b a b b
b b c b a c a b
b c b a c a b a
c a b a b b c b
c b a c a b a b

a b a b b c b a c
a b b c b a c a b
a c a b a b b c b
b a b b c b a c a
b a c a b a b b c
b b c b a c a b a
b c b a c a b a b
c a b a b b c b a
c b a c a b a b b

```

Knowing that  $L$  is 0, we can look at the 0th row in the final matrix at the bottom and find the original  $x$ , 'ababbcbac'

## Question 2

### Part a

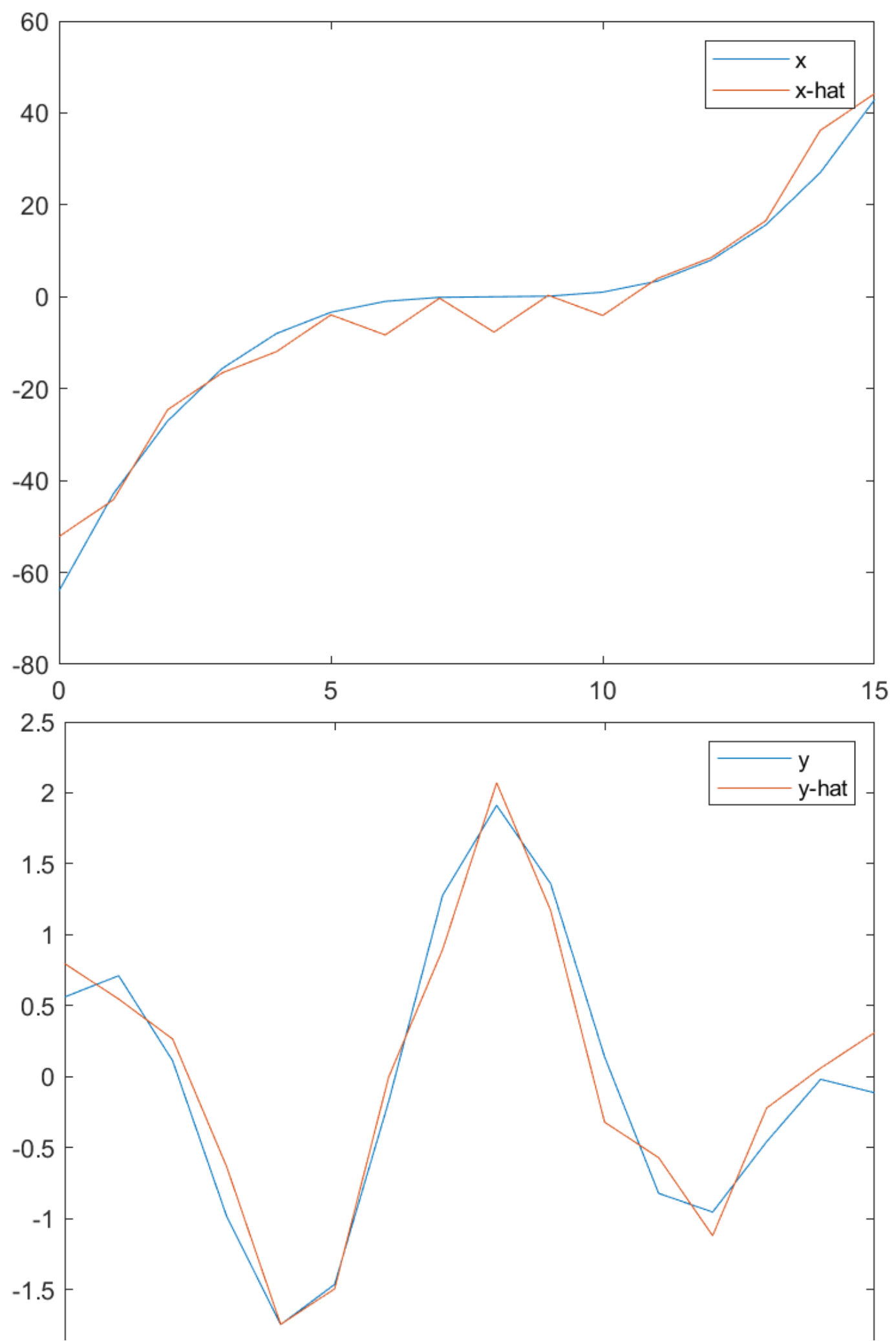
$X$	$\text{abs}(X)$	$X_{\text{hat}}$
-64.00000000000000 + 0.00000000000000i	64	0.00000000000000 + 0.00000000000000i
-64.00000000000000 - 123.616177165273i	139.201146751585	-64.00000000000000 - 123.616177165273i
-64.00000000000000 - 129.781745930520i	144.704186452135	-64.00000000000000 - 129.781745930520i
-64.00000000000000 - 88.5096491125439i	109.224347038678	-64.00000000000000 - 88.5096491125439i
-64.00000000000000 - 61.00000000000000i	88.4137998278549	0.00000000000000 + 0.00000000000000i
-64.00000000000000 - 41.3136896196506i	76.1762492512517	0.00000000000000 + 0.00000000000000i
-64.00000000000000 - 25.7817459305202i	68.9978146264496	0.00000000000000 + 0.00000000000000i
-64.00000000000000 - 12.4202176723800i	65.1940319893570	0.00000000000000 + 0.00000000000000i
-64.00000000000000 + 0.00000000000000i	64	0.00000000000000 + 0.00000000000000i

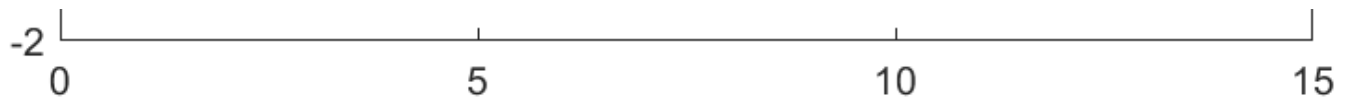
<b>X</b>	<b>abs(X)</b>	<b>X_hat</b>
-64.00000000000000 + 12.4202176723800i	65.1940319893570	0.000000000000000 + 0.000000000000000i
-64.00000000000000 + 25.7817459305202i	68.9978146264496	0.000000000000000 + 0.000000000000000i
-64.00000000000000 + 41.3136896196506i	76.1762492512517	0.000000000000000 + 0.000000000000000i
-64.00000000000000 + 61.00000000000000i	88.4137998278549	0.000000000000000 + 0.000000000000000i
-64.00000000000000 + 88.5096491125439i	109.224347038678	-64.00000000000000 + 88.5096491125439i
-64.00000000000000 + 129.781745930520i	144.704186452135	-64.00000000000000 + 129.781745930520i
-64.00000000000000 + 123.616177165273i	139.201146751585	-64.00000000000000 + 123.616177165273i
<b>Y</b>	<b>abs(Y)</b>	<b>Y_hat</b>
-0.672371817092243 + 0.000000000000000i	0.672371817092243	0.000000000000000 + 0.000000000000000i
-2.82044614666497 - 1.71736205248899i	3.30215821631346	-2.82044614666497 - 1.71736205248899i
10.1001352708745 + 1.16929927043018i	10.1675952552111	10.1001352708745 + 1.16929927043018i
-3.00390868182583 + 1.79548718923918i	3.49960592288198	-3.00390868182583 + 1.79548718923918i
-0.285120266061179 + 0.791046351673850i	0.840861401549212	0.000000000000000 + 0.000000000000000i
0.117560245577081 + 0.470415738793309i	0.484882850433585	0.000000000000000 + 0.000000000000000i
0.251076955046580 + 0.277348552317849i	0.374114764247910	0.000000000000000 + 0.000000000000000i
0.304205135549820 + 0.130202031658665i	0.330897769020779	0.000000000000000 + 0.000000000000000i
0.318935195078162 + 0.000000000000000i	0.318935195078162	0.000000000000000 + 0.000000000000000i
0.304205135549820 - 0.130202031658665i	0.330897769020779	0.000000000000000 + 0.000000000000000i

<b>Y</b>	<b>abs(Y)</b>	<b>Y_hat</b>
0.251076955046580 - 0.277348552317849i	0.374114764247910	0.000000000000000 + 0.000000000000000i
0.117560245577081 - 0.470415738793309i	0.484882850433585	0.000000000000000 + 0.000000000000000i
-0.285120266061179 - 0.791046351673850i	0.840861401549212	0.000000000000000 + 0.000000000000000i
-3.00390868182583 - 1.79548718923918i	3.49960592288198	-3.00390868182583 - 1.79548718923918i
10.1001352708745 - 1.16929927043018i	10.1675952552111	10.1001352708745 - 1.16929927043018i
-2.82044614666497 + 1.71736205248899i	3.30215821631346	-2.82044614666497 + 1.71736205248899i
<b>x</b>	<b>x_hat</b>	
-64	-24	
-42.8750000000000	-43.7153165735026	
-27	-34.9721695195455	
-15.6250000000000	-11.5266809833590	
-8	3.61168399340883	
-3.37500000000000	2.75656956972698	
-1	-2.52673303691548	
-0.125000000000000	0.132079019944577	
0	8	
0.125000000000000	9.45921991909269	
1	2.52673303691548	
3.37500000000000	-0.101998673081372	
8	12.3883160065912	
15.6250000000000	31.4995270846829	
27	34.9721695195455	
42.8750000000000	11.4966006364958	
<b>y</b>	<b>y_hat</b>	
0.560848025186123	0.534472555297965	
0.710921117351991	0.651875361516528	

<b>y</b>	<b>y_hat</b>
0.111575007354422	0.169283710889161
-0.984182560990524	-0.861610226458211
-1.74821079219786	-1.70162306407534
-1.46418183837943	-1.49229302141393
-0.176839039693525	-0.155473007394864
1.27775986032829	1.38399513875874
1.91149538702710	1.99056126242067
1.35974469626457	1.34029803455709
0.138513160937195	0.123041106718385
-0.822842762941691	-0.717153447936574
-0.955051908549474	-0.823410753643294
-0.458787552442807	-0.499880374659689
-0.0190481510710230	-0.136851810212681
-0.114084465275597	0.194768535636048

**Part b**



**Part c**

$$\text{mse}(x, \hat{x}) = 208.8$$

$$\text{mse}(y, \hat{y}) = 0.0115$$

**Part d**

$$\text{snr}(x, \hat{x}) = 1.794$$

$$\text{snr}(y, \hat{y}) = 0.501$$

**Question 3****Part a**

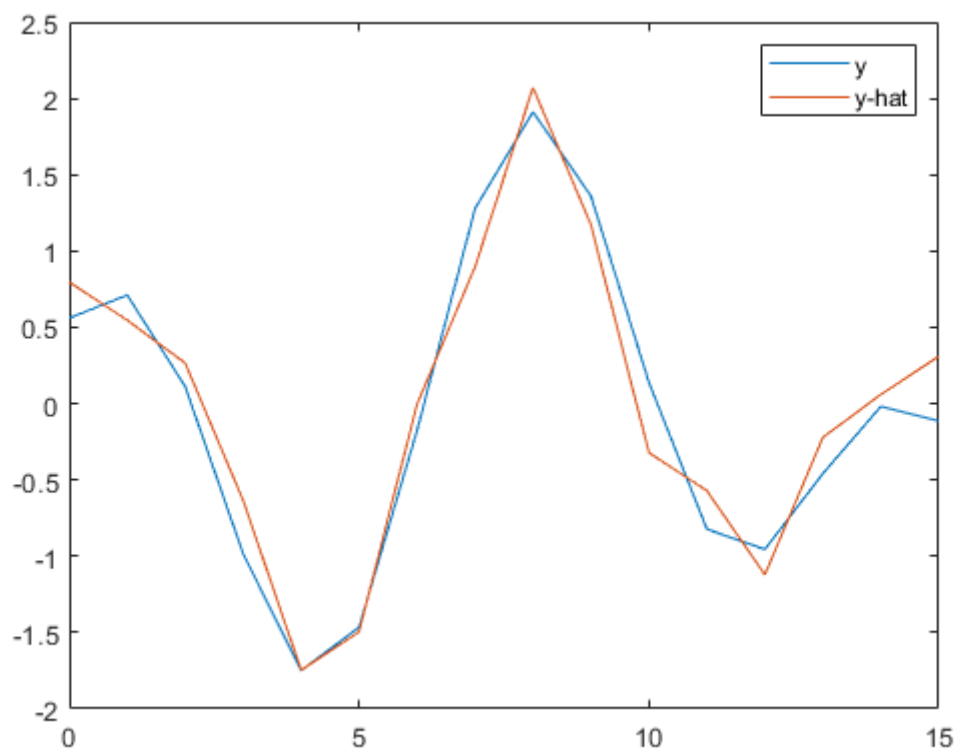
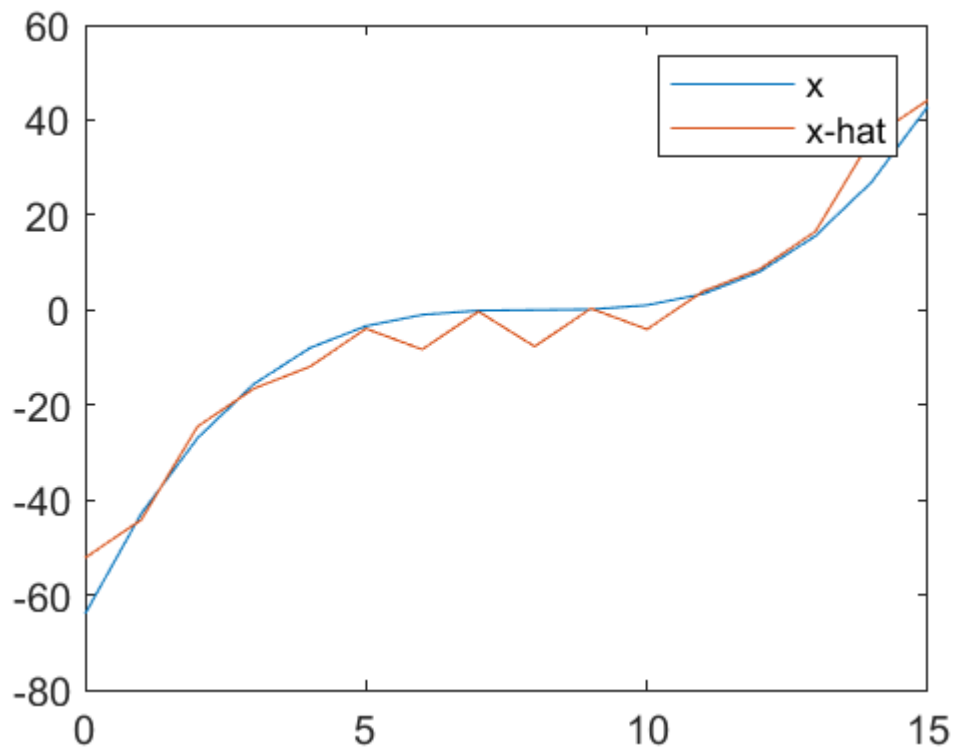
X	X_hat
-16.000000000000000	-16.000000000000000
-84.3925742766909	-84.3925742766909
-13.6662308878471	-13.6662308878471
-43.9646299590142	-43.9646299590142
-3.34566374608150	-3.34566374608150
-16.3338083634380	0
-1.42861336962544	0
-8.03325611408054	0
-0.7500000000000000	0
-4.46883615393278	0
-0.426179416775374	0
-2.55892619105595	0
-0.237769001671661	0
-1.34043281209945	0
-0.107555868735764	0
-0.418760225563996	0
Y	Y_hat
-0.168092954273061	-0.168092954273061



<b>Y</b>	<b>Y_hat</b>
-4.02175206550312e-16	-4.02175206550312e-16
-0.859563015680959	-0.859563015680959
1.75413162110037	1.75413162110037
3.14091062002642	3.14091062002642
-0.821049055044716	0
-1.23573204082214	0
3.95285218939724e-16	0
-0.269041654433757	0
-2.34161306779357e-16	0
-0.115195974496117	0
-1.17756934401283e-16	0
-0.0566228694781970	0
-7.85046229341888e-17	0
-0.0241663494448963	0
7.87159093625987e-17	0
<b>x</b>	<b>x_hat</b>
-64	-54.3998691788416
-42.8750000000000	-46.8835196321736
-27	-34.0694026587461
-15.6250000000000	-19.5870086752102
-8	-7.18466775870520
-3.37500000000000	0.540836557184180
-1	2.91903029454277
-0.125000000000000	1.23364640680428
0	-1.94150319017603
0.125000000000000	-3.78946664140808
1	-2.26676211692651
3.37500000000000	3.25558024003221
8	11.8874165374428
15.6250000000000	21.6059883902682

<b>x</b>	<b>x_hat</b>
27	29.9432958072591
42.8750000000000	34.7364056186537
<b>y</b>	<b>y_hat</b>
0.560848025186123	1.27933857119747
0.710921117351991	0.523691619485419
0.111575007354422	-0.575035775214032
-0.984182560990524	-1.41961127994718
-1.74821079219786	-1.55563359445319
-1.46418183837943	-0.915339651138115
-0.176839039693525	0.156218709980129
1.27775986032829	1.10195965879771
1.91149538702710	1.46201668627984
1.35974469626457	1.11502869155700
0.138513160937195	0.319046048686821
-0.822842762941691	-0.461735177499571
-0.955051908549474	-0.834910394486530
-0.458787552442807	-0.696612155073805
-0.0190481510710230	-0.263183396829538
-0.114084465275597	0.0923896215653293

## Part b

**Part c**

$\text{mse}(x, \hat{x}) = 22.67$

$\text{mse}(y, \hat{y}) = 0.1432$

**Part d**

$\text{snr}(x, \hat{x}) = 0.1626$

$\text{snr}(y, \hat{y}) = 0.6710$

## Question 4

### Part a

X	X_hat
-64	-64
-64	-64
-125	-125
-3	0
-226	-226
-6	0
-12	0
-12	0
-260	-260
-12	0
-24	0
-24	0
-48	0
-48	0
-96	-96
0	0

Y	Y_hat
-0.672371817092243	0
0.318935195078162	0
0.505926085612671	0
-1.07616661773503	0
6.64451595747060	6.64451595747060
4.59864698656283	4.59864698656283
11.6939666773281	11.6939666773281
-2.23470516951931	0
-2.75224862498878	-2.75224862498878
-1.90482195040049	0

<b>Y</b>	<b>Y_hat</b>
-4.84379959568833	-4.84379959568833
0.925645189120164	0
-1.62324915978154	0
0.769977673475807	0
1.22141361744444	0
-2.59809604390908	-2.59809604390908

<b>x</b>	<b>x_hat</b>
-64	-52.18750000000000
-42.87500000000000	-44.18750000000000
-27	-24.56250000000000
-15.62500000000000	-16.56250000000000
-8	-11.93750000000000
-3.37500000000000	-3.93750000000000
-1	-8.31250000000000
-0.12500000000000	-0.31250000000000
0	-7.68750000000000
0.12500000000000	0.31250000000000
1	-4.06250000000000
3.37500000000000	3.93750000000000
8	8.56250000000000
15.62500000000000	16.56250000000000
27	36.18750000000000
42.87500000000000	44.18750000000000

<b>y</b>	<b>y_hat</b>
0.560848025186123	0.796436584798458
0.710921117351991	0.546367716966739
0.111575007354422	0.264927705082124
-0.984182560990524	-0.634665173726865
-1.74821079219786	-1.74594261238310
-1.46418183837943	-1.49587374455138

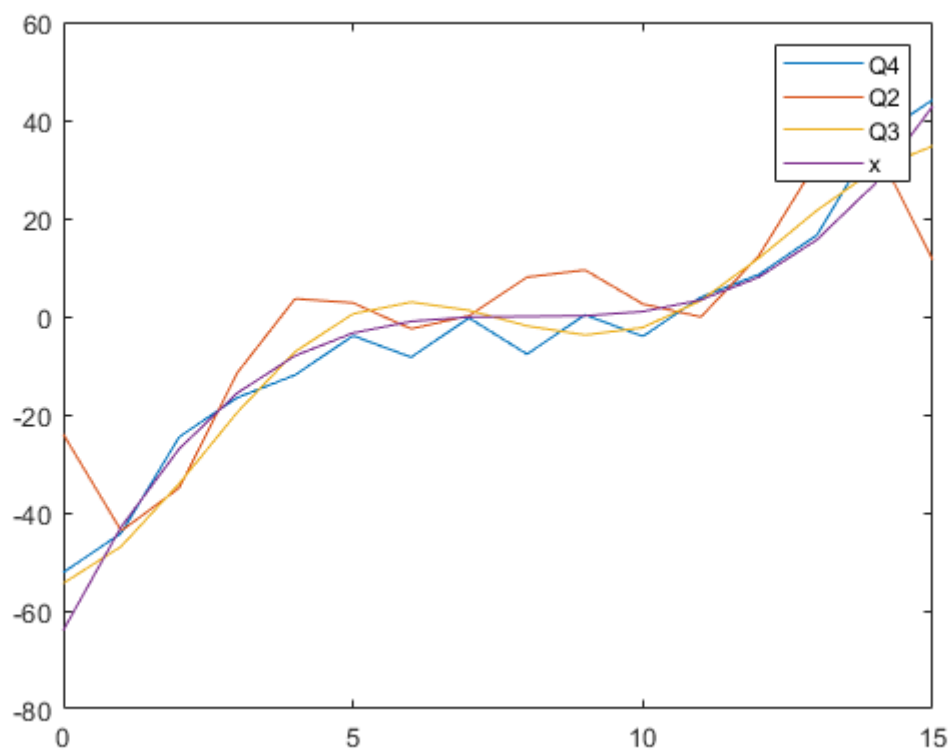
<b>y</b>	<b>y_hat</b>
-0.176839039693525	-0.00348383374468014
1.27775986032829	0.896109045064309
1.91149538702710	2.07070461787173
1.35974469626457	1.17111173906274
0.138513160937195	-0.321278171743955
-0.822842762941691	-0.571347039575674
-0.955051908549474	-1.12119859028709
-0.458787552442807	-0.221605711478104
-0.0190481510710230	0.0598343004065112
-0.114084465275597	0.309903168238230

### Part b

$\text{mse}(x, x_{\text{hat}}) = 24.363$

$\text{mse}(y, y_{\text{hat}}) = 0.0632$

### Part c



### Part d

MSE(x, x\_hat)

Q2

mse(x, x\_hat) = 208.8

mse(y, y\_hat) = 0.0115

Q3

mse(x, x\_hat) = 22.67

mse(y, y\_hat) = 0.1432

Q4

mse(x, x\_hat) = 24.363

mse(y, y\_hat) = 0.0632

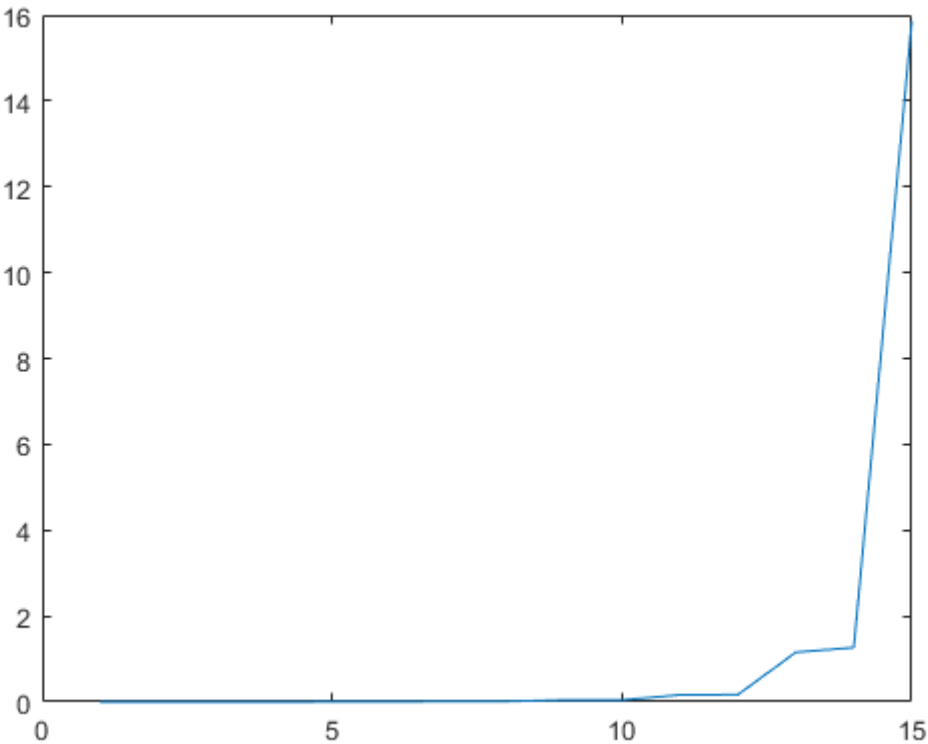
The DCT transform has the best mse(x, x\_hat) at 22.67

The Fourier transform has the best mse(y, y\_hat) at 0.0115

Question 5

Part a

0.1664	0.4741	0.9209	1.4773	2.0920	2.7083	3.2815	3.7931	4.2573	4.7257	5.2948	6.1348	7.5631	10.2324	15.8615
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**Part b**

We can see that as  $n$  grows larger, the fidelity of the decompressed data becomes better. The growth seems to be exponential. This makes sense as we would be better able to reconstruct the data with more degrees of freedom.