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Isostasy and Flexure of the Lithosphere

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The Development of the Concept of Isostasy

The keynote of isostasy is a working toward equilibrium. Isostasy is not a process which upsets equilibrium, but one which restores equilibrium.

(Chamberlin, 1932, p. 455)

1.1 Introduction

Isostasy is a term derived from the Greek words “iso” and “stasis” meaning “equal standing”. The term is used to describe a condition to which the Earth’s crust and mantle tend, in the absence of disturbing forces. In its simplest form, isostasy views that the lighter crust is floating on the dense underlying mantle. It is therefore an idealised state: a condition of rest and quiet. The transport of material over the Earth’s surface during the waxing and waning of ice sheets, sedimentation, and volcanism, however, disturbs isostasy and in some cases prevents equilibrium from being achieved. Seismic and gravity data suggest that the Earth’s outermost layers generally adjust to these disturbances.

One of the principal objectives of isostatic studies during the 20th century has been to determine the temporal and spatial scales over which these adjustments occur. This information provides constraints on the physical nature of the Earth’s outermost layers, thereby improving our understanding of what drives more complex geodynamical processes such as mountain building and sedimentary basin formation.

The term *isostasy* was first introduced in 1882, but there is evidence that questions concerning the equilibrium of the Earth’s crust were being posed as far back as the Renaissance. Isostasy played a central role in the development of geological thought and featured prominently in some of the great controversies of the late 19th and early 20th centuries, such as the contraction theory, continental drift and the permanence of the oceans and continents.

The discovery that the Earth’s crust might be in a state of *isostatic equilibrium* is one of the most fascinating stories in the history of the science. There were periods, for example, when it was accepted by one group of workers but rejected by another. There has also been considerable debate on which isostatic models best apply at a particular geological feature. These debates have led to some vigorous exchanges on

isostasy in the literature and, on occasions, to the development of “schools of thought”, which divided geophysicists and geologists, and North Americans and Europeans.

Today, isostasy still holds a central place in the Earth Sciences. This is true despite a considerable body of work that shows the Earth to be a dynamic planet that responds to loads over a wide range of spatial and temporal scales. Since isostasy is usually only concerned with how the crust and mantle adjusts to shifting loads of limited spatial and temporal dimensions, it is only a “snapshot” of the dynamical processes that are actually occurring. Nevertheless, it is an important snapshot. By comparing the observed adjustments with models based on flotation, differential cooling and bending plates we have learnt a considerable amount about the Earth, its rheology, its composition and its structure.

This introductory chapter outlines some of the key developments in the concept of isostasy. Particular emphasis is given to the development of the Airy and Pratt models of local isostasy. These models proved particularly useful to the geodesists since they helped them in the practical problems related to surveying. They were of less interest to geologists, who found the models difficult to incorporate into geological thought. The tussle between the geodesist and geologist is an intriguing one that helps set the scene for later chapters.

1.2 First Isostatic Ideas

Some of the first ideas about the equilibrium of the Earth’s outer layers originate with the engineer, artist and humanitarian, Leonardo da Vinci (1452–1519). Translations of da Vinci’s notebooks by E. MacCurdy (1928, 1956) show that he had given considerable thought to how Earth might respond to shifts in loads over its surface. For example, the following quote (Delaney, 1940) shows how da Vinci thought the removal of sediment from a mountain might cause it to rise:

That part of the surface of any heavy body will become more distant from the centre of its gravity which becomes of greater lightness. The earth therefore, the element by which the rivers carry away the slopes of mountains and bear them to the sea, is the place from which such gravity is removed; it will make itself lighter. . . . The summits of the mountains in course of time rise continually.

It was not until nearly 200 years later, however, when the first attempts were made to determine the Earth’s shape, that it became possible to determine the equilibrium state of the mountains.

In the early 18th century, there were two main schools of thought concerning the shape of the Earth: an English and a French one. The English school, led by I. Newton (1642–1727), considered the Earth to be flattened at the poles, while the French school, under J. Cassini (1677–1756), thought the Earth to be flattened at the equator. The Académie Royale des Sciences, under the direction of Louis XV, sponsored a team of scientists to travel to different parts of the Earth to measure the length of a meridian degree in order to resolve the controversy. The first team, led by Ch.-M. de La Condamine (1698–1758), made measurements in the region of the equator near Quito, Ecuador, while the second team, led by M. de Maupertius (1698–1759), made measurements in the region of the Arctic Circle near Tornio, Finland.

The techniques used by Condamine and Maupertius involved the measurement of the distance between two points of known position. The positions were determined astronomically by measuring the angle of elevation, Φ , between the pole star (Polaris) and the horizon, as indicated by level bubbles on an astrolabe. Level bubbles follow a surface, known as an *equipotential surface*, along which no component of gravity exists. The equipotential surface, which coincides with mean sea level, is the *geoid*, and so Φ is the angle between the pole star and the geoid. Because the direction of the pole star is normal to the equatorial plane, then it follows (e.g. Fig. 1.1) that Φ is also the angle between the normal to the geoid (i.e. the plumb-line direction) and the equatorial plane and, hence, the *astronomical latitude* at a point.

The distance between astronomical positions was determined by *triangulation*. In this technique, a network of triangles with vertices permanently marked on the Earth's surface are set up so that they connect the astronomical positions. One of the astronomical positions is then chosen as one vertex on the first triangle. If the length of one side of the triangle and its included angles are accurately measured, then it is possible to determine the distance between each vertex of the triangle. By extending the network of triangles to include the second astronomical position (Fig. 1.2), it is possible to estimate the *total* distance between astronomical positions from an accurate measurement of a length in the first triangle (which may be quite short) and the angles between vertices of all the other triangles.

Table 1.1 shows that the length of the meridian degree measured by Condamine was much smaller than that measured by Maupertius. Furthermore, the length of the meridian degree on the Arctic Circle was greater, by about 900 m, than the length determined previously near Paris; and the length near the equator was smaller, by a slightly larger amount. These results convinced both Condamine and Maupertius that the Earth was indeed flattened at the poles, as suggested by Newton. The flattening

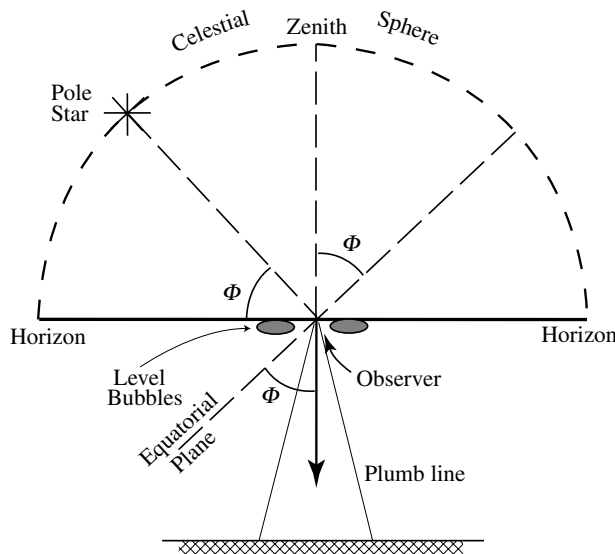


Fig. 1.1 The determination of the astronomical latitude, Φ , from observations of the pole star.

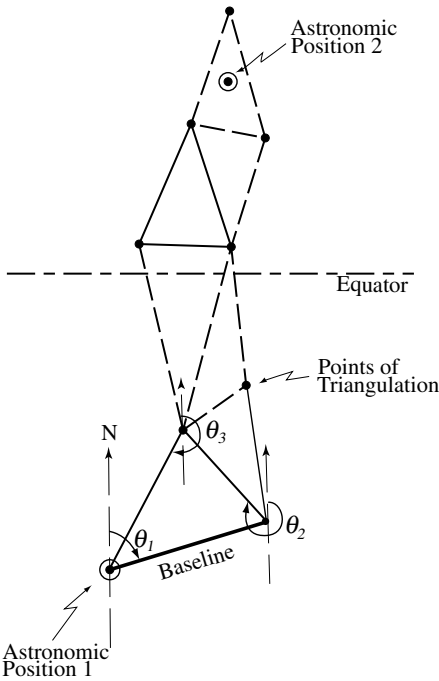


Fig. 1.2. The measurement of the distance between two points where astronomical latitude and longitude have been determined using triangulation.

was estimated to be about 1/216.8 (Fig. 1.3). The measurements of Condamine and Maupertius therefore solved the controversy of the overall shape of the Earth, although perhaps not quite in the way their sponsor, the Royal Court in Paris, had anticipated!

One member of Condamine’s party, P. Bouguer (1698–1758; Fig. 1.4), was not content, however, to let the matter rest there. Bouguer was puzzled by the consistency of the results because the measurements near the equator were obtained in the presence of much greater topographic relief than those near the Arctic Circle. He surmised (Bouguer, 1749) that the mass of the mountains in the vicinity of Quito was

Table 1.1. Summary of Measurements of the Length of a Meridian Degree According to the Surveys led by Condamine and Maupertius

Nearest Town/City	Approximate Latitude	Length of a Meridian Degree (Toise)	Difference from Paris (Toise)
Tornio	66° (Arctic Circle)	57,525	+342
Paris (L’observatoire de Paris)	48°50’	57,183	0
Quito	0° (Equator)	56,753	−430

Note: 1 Toise = 1.949 m.

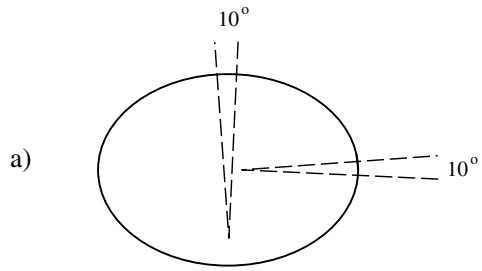


Fig. 1.3. The flattening of the Earth, f_e , can be approximated by an ellipse with a semi-major axis, a , and semi-minor axis, b .

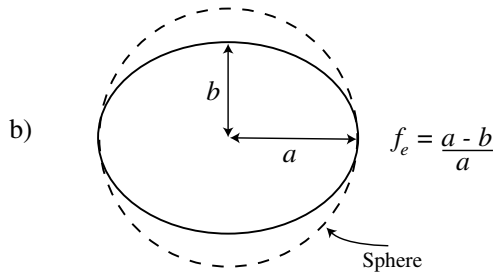


Fig. 1.4. Pierre Bouguer, the French mathematician and astronomer, who accompanied Charles-Marie de La Condamine during the expedition of 1735 to Quito, Ecuador to measure the length of a degree meridian at the equator. (Pastel de Perroneau, 18th century; Musée du Louvre; Photo: Giraudon.)



sufficiently large that it should have caused the local *plumb line* to be deflected by as much as 1'43" from the vertical. Such a deflection would introduce errors into the astronomical positions because the elevation of a distant star is measured on a table with level bubbles so that the measurement is made with respect to the local plumb-line direction (Fig. 1.1). The astronomical positions were apparently not in error, however, leading Bouguer to conclude that the attraction of the mountains in the vicinity of Quito "is much smaller than expected from the mass of matter represented in such mountains".

A few years later, the Italian astronomer and mathematician, R. G. Boscovich (1711–87) provided an explanation of the problem that puzzled Bouguer. He said (Boscovich, 1755):

The mountains, I think, are to be explained chiefly as due to the thermal expansion of material in depth, whereby the rock layers near the surface are lifted up. This uplifting does not mean the inflow or addition of material at depth, the void within the mountain compensates for the overlying mass.

This passage is the first to use the term *compensates*. Boscovich speculates that the mass excess of the mountain is compensated in some way by a mass deficiency at depth. Thus, the deflection of a plumb line near a mountain range may well be small, as Bouguer had suspected.

Little more appears to have been said on the matter for a further 100 years. The statements made by Boscovich on the compensation of mountains therefore had little impact with leading geologists of the time.

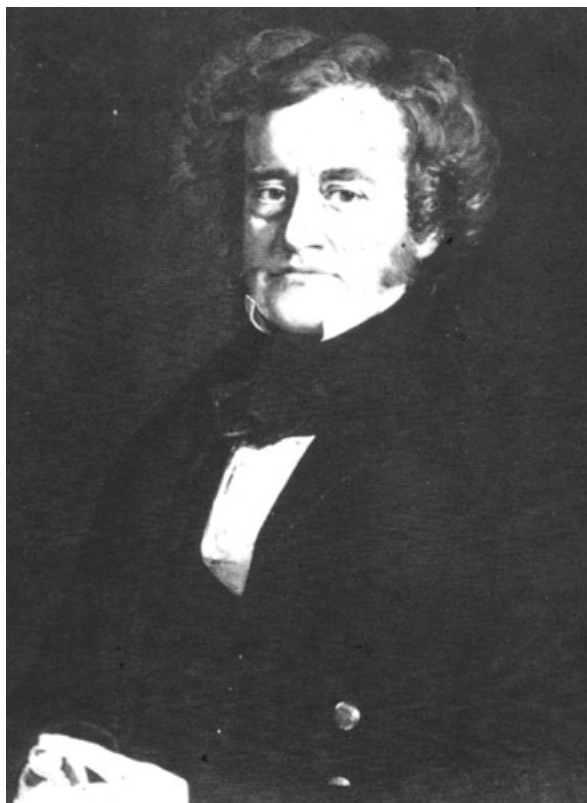
In the early 1800s geological thought in Europe was dominated by the *contraction theory*. According to this theory, the Earth's surface features were thought to have been the consequence of a gradual cooling of the Earth following its formation. Mountains were considered as regions that had not cooled as much as ocean regions. The theory had its origins in the work of G. Baron von Leibnitz (1646–1716) and R. Descartes (1596–1650). Baron J. Fourier (1768–1830) subsequently measured the temperature gradient at shallow depths in the Earth, concluding that it was in accord with the predictions of the contraction theory. Therefore Boscovich's statements on the thermal expansion of mountains may not have seemed, all that inconsistent with the theory.

The eminent British geologist C. Lyell (1797–1875), was sceptical about the contraction theory. By 1833 he had completed his widely acclaimed book, *Principles of Geology* (Lyell, 1832–33), in which he presents the view that the Earth's surface is continually subject to periods of rest and change. He disagreed strongly with theories of catastrophes to explain geological events and with ideas such as those put forward by L. Elie de Beaumont (1798–1874) in France and H. de la Beche (1796–1855) in England that geological processes, such as mountain building, were global events that occurred at similar times over widely separated regions. Lyell wrote:

It is preposterous to imagine that just because they had similar trends the Allegheny and Pyrenees mountain ranges could have been formed by the same catastrophic event.

Among Lyell's many influential friends was J. Herschel (Fig. 1.5), son of the astronomer, W. Herschel. In a letter addressed to Lyell, Herschel (1836) pointed out that he also disagreed with the contraction theory. In his opinion, the outer-

Fig. 1.5. John Frederick William Herschel, the son of the astronomer William Herschel who discovered the planet Uranus. Reproduced from the portrait on p. 161 of Robinson (1980) with permission of the Royal Society.



most layer or *crust* of the Earth was in some form of dynamic equilibrium with its underlying substratum or *sea of lava*. He wrote “the whole (crust) to float on a sea of lava.” According to Herschel, if the crust was loaded, say by sediments, it would sink, thereby causing the lava to flow out from beneath the load and into flanking regions (Fig. 1.6).

Herschel’s ideas on the equilibrium of the Earth’s crust might never have been published had it not been for C. Babbage (1790–1871), the mathematician and inventor of the computer, who was also friendly with Lyell. Babbage decided to include the letter that Lyell had received from Herschel in a treatise that he was writing. The treatise was prepared by Babbage, on his own initiative, as a “sequel” to the eight volumes of the “Bridgewater Treatise”, which had just been published using proceeds from Lord Bridgewater’s estate. Babbage felt compelled to publish a ninth treatise because, in his view, a prejudice was emerging that the pursuit of science was unfavourable to religion. In a letter to Lyell he pointed out that the ninth treatise would provide “an opportunity to illustrate some of the magnificent examples of creation”.

In the ninth treatise, Babbage included some observations (Babbage, 1847) he had made at the Temple of Serapis in Pozzuoli, Italy. Built towards the end of 200 A.D., the temple served as a spa (Fig. 1.7) for wealthy Romans. When Babbage visited the site in 1828, the temple’s three remaining columns showed a dark encrustation about 4 m above their base. Above the dark encrustation, 2.5 m of the column had been perfo-

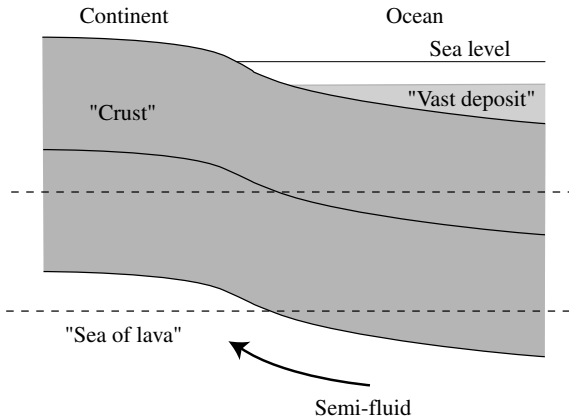


Fig. 1.6. The adjustment of the crust to a "vast deposit" by flow in the underlying "sea of lava". Reproduced from a figure in Herschel (1836) with permission of the Royal Society.

rated in all directions by a marine boring animal (*Modiola lithophaga*). This observation¹ suggested to Babbage that the temple had undergone a period of subsidence, followed by one of uplift. He attributed these movements of the crust to the action of heat, because of the location of the temple near the historically active volcano Vesuvius. Babbage considered that the heating caused the crust to expand and con-

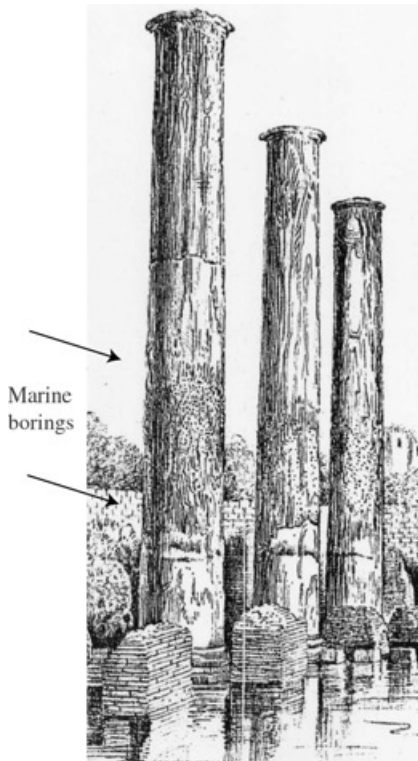


Fig. 1.7. The Temple of Serapis in Pozzuoli, Italy. The borings in the Roman-built columns were used by Babbage to infer that the crust had undergone subsidence followed by uplift. Reproduced from Babbage (1847) with permission of the Geological Society of London.

¹ H. Jeffreys, the British geophysicist, was later so intrigued by this observation that he suggested (Crittenden, 1967, p. 279) that a picture of the Temple of Serapis should hang in every Department of Geophysics as a reminder that movements of the Earth's crust are not simple and may involve both subsidence and uplift of the crust at the same locality.

tract locally and that these movements were in some way accommodated by movement in the underlying fluid lava. Thus, Herschel and Babbage agreed that the crust accommodated loads by lateral flow in a weak underlying substratum. While Lyell supported this view, it was strongly opposed by the supporters of the contraction theory, who believed that the subsidence and uplift was the consequence of thermal contraction and expansion on a global scale.

1.3 The Deflection of the Vertical in India

The next major development in understanding the equilibrium of the Earth's crust followed the measurement of the meridian degree length in India. The measurements were carried out in 1840–59 by G. Everest (1790–1866), who as Surveyor General of India was charged with mapping the country. Everest's measurement techniques differed from those of Condamine and Maupertius as he used astronomical as well as *geodetic* positions. Geodetic positions (Fig. 1.8) were computed at the vertices of each triangle by assuming the position at one vertex of the first triangle was known and then using the equation for the Earth's best-fitting reference ellipsoid to compute the other positions. (Thus, instead of regarding the flattening of the Earth as unknown, as the Condamine and Maupertius parties did, Everest assumed it was by now well enough known to compute geodetic positions at points in between the astronomical positions.)

At points where both astronomical and geodetic positions are determined it is possible to compare them. As Fig. 1.9 shows, astronomic and geodetic latitude is referenced to a common surface: the *equatorial plane*. The astronomic position is defined as an angle between the equatorial plane and the local plumb-line direction, whereas the geodetic position is defined as an angle between the equatorial plane and the local normal to the Earth's best-fitting ellipsoid. The plumb-line direction does not necessarily follow the local normal to the ellipsoid because of disturbing masses in the Earth. The amount that the plumb line is deflected from the local normal is known as the *deflection of the vertical*.

As part of his survey in India, Everest computed the geodetic position at a number of localities where he had already measured astronomic positions. He found that for two stations, Kaliana and Kalianpur, on the Ganges Plain, the latitude difference between them computed geodetically was $5.24''$ smaller than that determined astronomically. Everest (1857–59) concluded that the discrepancy was not in the astronomical position, which he considered to have been determined extremely accurately, but

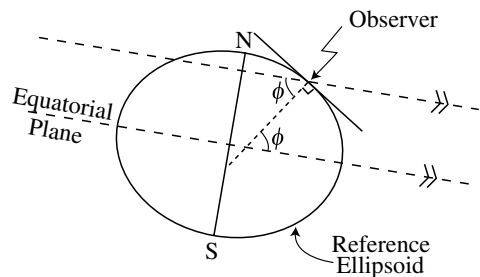


Fig. 1.8. The determination of the geodetic latitude, ϕ , from the theoretical formula for the Earth's best-fit reference ellipsoid.

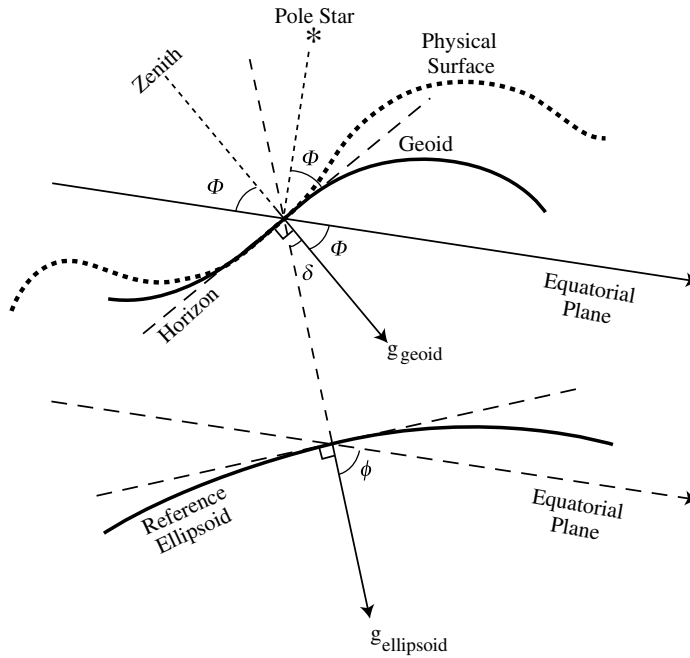


Fig. 1.9. The determination of the deflection of the vertical, δ , from the difference between the astronomical latitude, Φ , and the geodetic latitude, ϕ , measured at a point P on the Earth's surface.

in the geodetic position. Small closure errors in the triangulation surveys and an incorrect reference ellipsoid were, he thought, the most likely contributors.

But a Cambridge-trained mathematician, J. H. Pratt (1809–71: Fig. 1.10), who was the Archdeacon of Calcutta at the time, disagreed with Everest. He believed that the discrepancy was somehow due to the disturbing effects of the nearby Himalayas. The largest discrepancies in the survey of India were measured between Kalia and Kalia, which are located south of the foothills of the Himalayas. As pointed out earlier by Bouguer, the gravitational attraction of the mountains would locally perturb the direction of the plumb line, thereby introducing an error into the astronomical positions since these positions are referred to the local plumb-line direction.

In a paper read before the Royal Society on December 7th, 1854, Pratt presented the results of his calculations of the gravitational attraction due to the Himalaya and its hinterland at Kalia and Kalia. He estimated the attraction by dividing the mountains up into a number of “compartments”, computing the gravitational attraction of each compartment, and then summing the results. The problem was determining the topography of the Himalaya and its hinterland. Since the region was largely unexplored, Pratt had to rely on interviews with travellers who had returned from there!

Consider an elementary mass dm at M and a unit mass at P. The gravitational force of attraction, dF , between the masses is given by *Newton's inverse square law*:

$$dF = G \frac{dm}{d^2} \quad (1.1)$$

Fig. 1.10. John Henry Pratt 1 year after his appointment as Chaplain to the Bishop of Calcutta. Portrait courtesy of D. P. McKenzie of Cambridge University.



where G is the universal gravitational constant and d is the distance between the two masses. In the case that M and P are points on the surface of a spherical Earth, then it follows (Fig. 1.11) that the component of the attraction at a station P due to the elementary mass at M *in the direction of gravity* is given by

$$dg = \frac{G \, dm \, \sin(\xi)}{d^2}$$

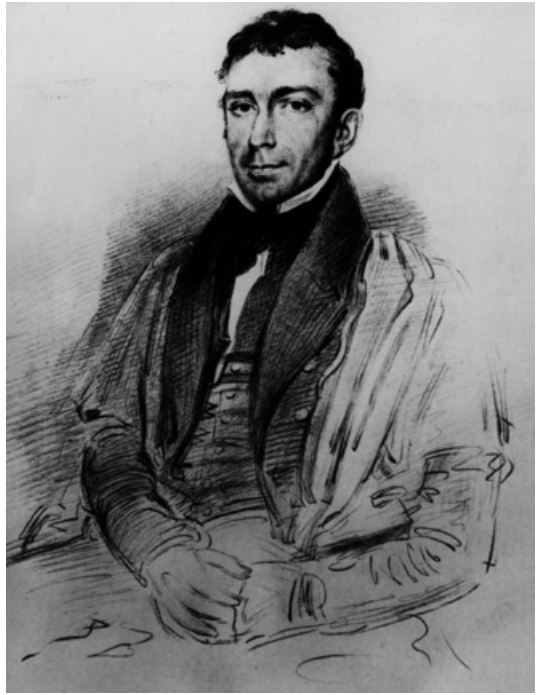
where

$$\xi = \frac{\theta_s}{2} \quad \text{and} \quad d = 2r_e \sin\left(\frac{\theta_s}{2}\right)$$

$$\therefore dg = G \, dm \frac{\sin\left(\frac{\theta_s}{2}\right)}{4r_e^2 \sin^2\left(\frac{\theta_s}{2}\right)} \quad (1.2)$$

Pratt used Eq. (1.2) to calculate the gravitational effect of the Himalayas at Kaliana and Kalianpur, and published the results in a 75-page long paper (Pratt, 1855). He found that the gravitational effect of the mountains is large enough to deflect the plumb line by $15.885''$, more than three times the observed value (Fig. 1.12). Pratt was satisfied, despite problems with not knowing the detailed topography of the Himalayas, that he had correctly computed the effect of the mountains at Kaliana and Kalianpur. He concluded his paper by saying that he did not understand the cause of the discrepancy and that the problem should be investigated further.

Fig. 1.13. George Biddell Airy, Professor of Astronomy at Cambridge University and Astronomer Royal. Portrait courtesy of D. P. McKenzie of Cambridge University.



the state of the Earth's crust lying upon the lava may be compared with perfect correctness to the state of a raft of timber floating upon water; in which, if we remark one log whose upper surface floats much higher than the upper surfaces of the others, we are certain that its lower surface lies deeper in the water than the lower surfaces of the others.

By using the analogy of icebergs, Airy suggested that an elevated region, such as a “*table-land*” would be underlain by a less dense region, such that there would be a substitution of “light crust” for “heavy lava”. The effect on the local direction of gravity, he viewed, would depend on two actions: the positive attraction of the elevated table-land and the negative attraction of the light crust. He argued that the reduction in attraction of the light crust would be equal to the increase in attraction of the heavy mass above, so that the *total* effect on the local direction of gravity would be small. This argument, which he was able to make in a 3-page-long paper, provided a simple explanation for the observations of Everest (e.g. Fig. 1.12).

Airy was cautious in applying his simple model, pointing out that it may not be entirely appropriate to consider *all* features on the Earth's surface as floating. In the case of a wide table-land (e.g. South Africa and Tibet), for example, Airy argued that it was unlikely to be supported in any way, other than by a lower density crust that protruded into the underlying “lava”. He argued that “*fissures*” or “*breakages*” would form at the edges of the table-land (Fig. 1.14). In the case of narrow features, such as Mt. Schiehallion² in Perthshire, however, Airy suggested that his model may not apply, and he implied that the crust might be sufficiently strong so as to be able to support this feature without breakage.

² Mt. Schiehallion was where the Rev. Maskelyne (1732–1811) used the deviation of the plumb line to estimate the mean density of the Earth.

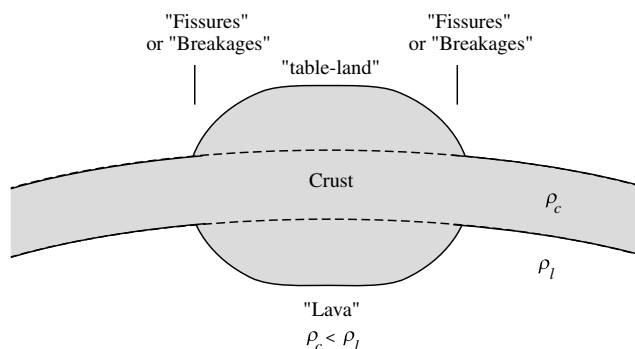


Fig. 1.14. Airy's hypothesis of a crust that "floats" upon "lava". Reproduced from Fig. 2 of Airy (1855) with permission of the Royal Society.

1.5 Isostasy According to Pratt

There is little doubt that Airy's paper must have come as a surprise to Pratt. Pratt was a mathematician and his approach to the problem was to carry out detailed, somewhat tedious, calculations. Airy, on the other hand, approached the problem without mathematical reasoning, using simple physical concepts. Furthermore, there was the difference in the amount of work carried out; Pratt's paper was 75 pages in length while Airy's contribution was only 3 pages!

In 1858, Pratt followed up Airy's suggestions on mass excess and deficit, proposing his own model for the equilibrium of the Earth's outer layers. He criticised Airy's model on three grounds (Pratt, 1859). First, the model was based on assumption of a thin crust. Second, the model assumed that the crust was less dense than the underlying lava. Third, the model was not in accord with the contraction theory of the Earth. The first objection was based on a suggestion by a colleague of Pratt's at Cambridge, Mr. Hopkins, that the Earth's crust was at least 150 km thick. The second and third objections, however, were clearly a result of Pratt's adherence to the contraction theory.

In 1864 and 1870 Pratt presented two further papers on the subject to the Royal Society. In particular, he expanded on his view, based on the contraction theory, that the depressions and elevations of the Earth's surface were the product of thermal contraction and expansion (Pratt, 1864, 1871). He stated that the

amount of matter in any vertical column drawn from the surface to a level surface below the crust is now and ever has been, approximately the same in every part of the Earth.

This statement implies (e.g. Fig. 1.15) that elevated regions were underlain by low-density rocks whereas depressed regions were underlain by high-density rocks. Unlike Airy, however, Pratt did not speculate on what may cause different portions of the Earth's outer layers to be colder or hotter than others. However, he recognised these regions as providing the *compensation* for surface depressions and elevations, a term that had not been used since Boscovich's study.

Despite Pratt's criticisms, Airy did not return to the discussion. The only reported occasion when Airy referred to the subject was in a lecture presented to the Cumberland Association for the Advancement of Literature and Science. A brief

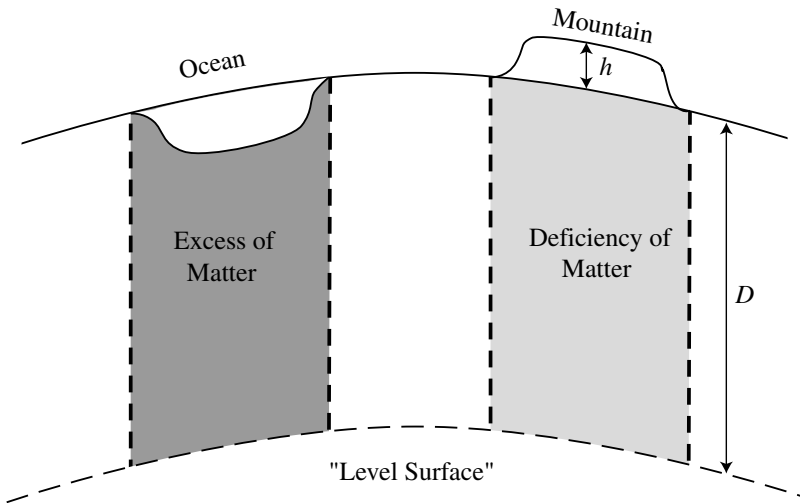


Fig. 1.15. Pratt's hypothesis that mountains are underlain by low-density regions while oceans are underlain by high-density regions. At some depth, known as the "level surface", masses are equal.

edited abstract of his lecture was published in *Nature* in 1878 (Anonymous, 1878). Apparently, Airy did not even refer to Pratt's model in his lecture; instead he simply repeated the model that he had outlined to the Royal Society in 1855.

1.6 Fisher and Dutton on Isostasy

It is curious that the dispute between Airy and Pratt was not really taken up by other workers at the time. The next development was not until 1881, when the Rev. O. Fisher published his book entitled *Physics of the Earth's Crust* (Fisher, 1881). The book pays little attention to the controversy between the Airy and Pratt models. Instead, its main purpose was to develop the arguments *against* the contraction theory, which was still dominating much of geological thought at the time.

Fisher, like Herschel and Airy before him, viewed the crust and underlying substratum as being in some form of equilibrium; the lighter crust floating on the denser substratum. He went on, however, to make an important new statement about the achievement of equilibrium. In his words (Fisher, 1881, p. 275, p. 278):

the crust must be in a condition of approximate hydrostatical equilibrium, such that any considerable addition of load will cause any region to sink, or any considerable amount deduced off an area will cause it to rise.

the crust analogous to the case of a broken-up area of ice, refrozen and floating upon water.

With these statements Fisher captured the essence of what would later be described as isostasy. According to Fisher, the crust obeyed *Archimedes' principle*. The weight of a floating block of crust of thickness B and density ρ_{block} that is floating in a fluid substratum of density ρ_{fluid} is then equal to the weight of the fluid displaced (Fig. 1.16):

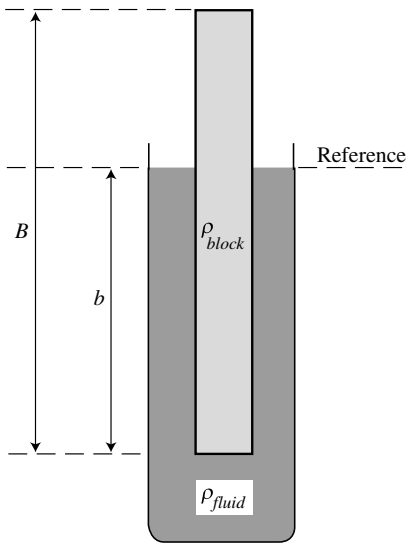


Fig. 1.16. Archimedes' principle, which states that the weight of a block that floats in a liquid is equal to the volume of liquid that is displaced.

$$\begin{aligned}
 B\rho_{block} &= b\rho_{fluid} \\
 \therefore \frac{b}{B} &= \frac{\rho_{block}}{\rho_{fluid}}
 \end{aligned}
 \tag{1.3}$$

where b is the height that the block is immersed in the fluid. Fisher used the analogy of a relatively light iceberg ($\rho_{ice} = 917 \text{ kg m}^{-3}$) that floats in denser seawater ($\rho_{seawater} = 1030 \text{ kg m}^{-3}$) to estimate that the part of a crustal block that would be immersed in the fluid substratum would be about 9/10 of its total thickness. Using Maskeyne's preferred value of $\rho_{crust} = 2750 \text{ kg m}^{-3}$ this implies $\rho_{substratum} = 3100 \text{ kg m}^{-3}$, a plausible result if, as was thought in Fisher's time, the material beneath the crust consisted of lava.

The American army officer and geologist C. E. Dutton (1841–1912) was captivated by Fisher's book, writing a complimentary review of it in 1882 (Dutton, 1882). Like Fisher, Dutton also opposed the contraction theory, especially since it was based on vertical rather than horizontal movements and therefore offered no explanation for folding in mountain belts. In Dutton's words (p. 127):

this hypothesis (interior contraction by secular cooling) is nothing but a delusion and a snare, and the quicker it is thrown aside and abandoned, the better it will be for geological science.

Dutton's review of Fisher's book was noteworthy in another respect. The review contained the first reference to the term *isostasy*. In Dutton's view, one of the "fundamental doctrines" of Fisher's book was the notion that the broader features of the Earth's surface were simply those that were due to its flotation. This idea, he thought, should form an important part of any true theory of the Earth's evolution. He wrote in a footnote (Dutton, 1882, p. 289):

In an unpublished paper I have used the terms *isostatic* and *isostasy* to express that condition of the terrestrial surface which follow from the flotation of the crust upon a

liquid or highly plastic substratum . . . isobaric would have been a preferable term, but it is preoccupied in hypsometry.

It took Dutton a further 7 years, however, to publish the paper that he referred to in the footnote. In this subsequent paper, Dutton (1889) pointed out that if the Earth was composed of homogeneous matter, its equilibrium figure would be a true spheroid of revolution; but if some parts were denser or lighter than others, its normal figure would no longer be spheroidal. Where the lighter material accumulated, he argued, there would be a tendency to bulge, and where the denser matter accumulated there would be a tendency to depress the surface. He wrote (1889, p. 53):

For this condition of equilibrium of figure, to which gravitation tends to reduce a planetary body, irrespective of whether it is homogeneous or not, I propose the name isostasy.

Thus, the term *isostasy* (note that it was now spelled with an “s” instead of a “c”) was born.

A major contribution of Dutton’s work was to point out the relevance of isostasy to geology. Isostasy, he argued, explained the subsidence of a large thickness of shallow-water sediments and the progressive uplifts of mountain belts. Subsidence and uplift were, in his view, a result of gravitation restoring isostasy to regions that were disturbed on the one hand by *sedimentation* and on the other by *erosion*. Dutton, however, also recognised limitations with isostasy, particularly in its inability to explain *regional* subsidence and uplift such as observed in submerged *oceanic plateaus* and uplifted *mountain platforms* that were once at sea level. Isostasy, as defined by Dutton, gives no explanation for such permanent changes of the level surface. On the contrary, he realised that isostasy is a working towards equilibrium that implies a certain degree of “protection” of the crust against “massive lowering and raising”.

1.7 The Figure of the Earth and Isostasy

By the end of the 19th century, isostasy was still just an idea. It could not be proved by geological observations although, as Dutton pointed out, certain geological “facts” were in support of isostasy. The proof eventually came not from geology but from geodesy.

The science of *geodesy* may be considered as comprising two parts: *geometrical geodesy* and *physical geodesy*. In geometrical geodesy, triangulation techniques are used together with astronomical observations to determine information on the Earth’s shape. Physical geodesy, on the other hand, uses the Earth’s gravity field to determine its shape. The results of geometrical geodesy have been of immense practical importance in surveying on land.

The task of surveying the United States during the late 1890s was in the hands of the United States Coast and Geodetic Survey.³ As a “frontier” at this time, the western United States was experiencing booming land sales. It was important therefore that a network of monuments be established whose positions were accurately known. By the end of the 19th century, the Survey had carried out triangulation surveys along the Atlantic, Gulf and Pacific coasts. In addition, it had measured the

³ This began as Survey of the Coast in 1878, and was renamed as the Coast Survey in 1836 and the Coast and Geodetic Survey in 1878. Its present-day name is the National Geodetic Survey.

astronomic positions at a large number of the triangulation stations. The triangulation of the country as a whole, however, was still not complete. When the triangulation surveys along the coasts were extended into the interior it was found that significant gaps, overlaps and offsets occurred. These differences had to be eliminated if a national network of stations was to be established.

In 1899, J. F. Hayford (1868–1925) took charge of the Computing Division of the Survey and immediately set about the task of adjusting the different triangulation surveys. Early on in the work, Hayford realised that the source of error between the various surveys lay in the astronomical positions that were used to “fix” the position of the monuments. The reason, he argued, was that these positions were referenced to the geoid which, in turn, was influenced by the gravitational effect of the local terrain. He therefore set about the task of correcting the astronomical positions for the gravitational effect of the local topography. The amount of work involved was enormous. This was because it was necessary to compute the effect of the topography above and below sea level, not only at the station, but also over an area within a radius of up to few hundred kilometres from the station point. In order to save time, Hayford constructed specially scaled templates for quickly reading topographic maps and making the corrections.

Hayford found that by including the topographic correction he substantially improved the misfit between the different triangulation surveys. However, he found that an even better fit could be obtained if it was assumed that the topography was in some form of isostatic equilibrium. The question was which of the models of compensation that were available at the time should be utilised, Airy’s or Pratt’s?

For reasons that are not entirely clear, Hayford chose Pratt’s model rather than Airy’s. Perhaps Hayford, like Pratt, supported the contraction theory. Alternatively, it may have been the convenience of computation (Jeffreys, 1926). In any event, Hayford’s choice was a popular one with his colleagues and it set a trend that was to last for more than three decades in which the “*American school*” of geodesy preferred the Pratt model for their isostatic studies.

Unfortunately, the Pratt model was not in any form that could be used in geodetic computations. Hayford therefore put the Pratt model in a more precise form. In setting up the model (Fig. 1.17), he assumed the following:

- Isostatic compensation is uniform.
- The compensating layer is located directly beneath the mountains and reaches to the depth of compensation, D_c , where equilibrium prevails.
- The density of the crust above sea level is the same as the density of the crust at the coast.
- The density of the crust below sea level varies laterally, being less under mountains than under oceans.
- The depth of compensation is everywhere equal.

We can then write for the pressure P_I at the base of a column of rock beneath a mountain:

$$P_I = \frac{\text{force}}{\text{unit area}} = \frac{\text{mass}}{\text{unit area}} g$$

$$\therefore P_I = D_c \rho_I g + h \rho_{sc} g$$

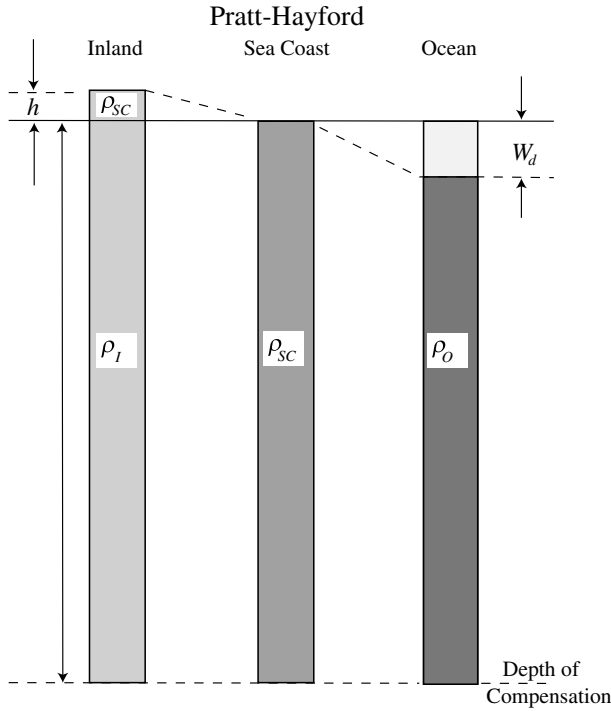


Fig. 1.17. The Pratt-Hayford model of isostatic compensation. Reproduced from Illustration 10 in Hayford (1909).

and for the pressure P_{sc} beneath the sea coast:

$$P_{sc} = D_c \rho_{sc} g$$

Hence, for isostatic equilibrium we have

$$\begin{aligned} D_c \rho_I g + h \rho_{sc} g &= D_c \rho_{sc} g \\ \therefore \rho_I &= \rho_{sc} \frac{(D_c - h)}{D_c} \end{aligned} \quad (1.4)$$

The density of a column of rock beneath the ocean is then given by

$$\rho_o = \frac{(\rho_{sc} D_c - \rho_w W_d)}{(D_c - W_d)} \quad (1.5)$$

If we assume $\rho_{sc} = 2670 \text{ kg m}^{-3}$ and $D_c = 113.7 \text{ km}$, then Eq. (1.4) shows that the mean density underlying a 3-km-high elevated region is about 3% less than under the sea coast. Similarly, with $\rho_w = 1030 \text{ kg m}^{-3}$ we find from Eq. (1.5) that the density beneath a 5-km-deep ocean is about 3% more.

The Pratt model was used by Hayford to correct the triangulation surveys of the entire United States. He found that the model successfully reduced the discrepancies between overlapping surveys (Hayford, 1909). Moreover, the model helped reduce the discrepancy between geodetic and astronomic positions. Hayford found, for example, that the discrepancy in the deflection of the vertical was on average reduced to about 10% of what it would have been if no isostatic model had been used.

With this success, it is little surprise that the Pratt model continued to be widely used not only in the United States, but also elsewhere, such as India (Burrard, 1920) and Asia (Bowie, 1926). Curiously, there were few attempts to use the model in Europe. As S. G. Burrard (1860–1943), the Surveyor-General of India, wrote (1920, p. 57):

The only countries so far that have calculated isostasy for their stations systematically are the United States and India. Europe has not touched the subject.

When in 1924 the Europeans did address the problem, however, it was not the Pratt model they turned to, but Airy's!

During the 1920s most geodetic work in Europe was being carried out in Finland. In 1924, while at the Superior Technical School in Helsinki, W. A. Heiskanen (1895–1971) considered the different types of isostatic models available at that time, choosing the Airy model (Heiskanen, 1931) ahead of Pratt's for his isostatic calculations. Like Hayford before him, he had first to put the model in a more precise form. In doing this, he assumed the following (Fig. 1.18):

- Isostatic compensation is uniform.
- The crust as a whole is floating in a "sima-layer" according to Archimedes' principle.
- The compensating masses lie directly beneath mountains and oceans.

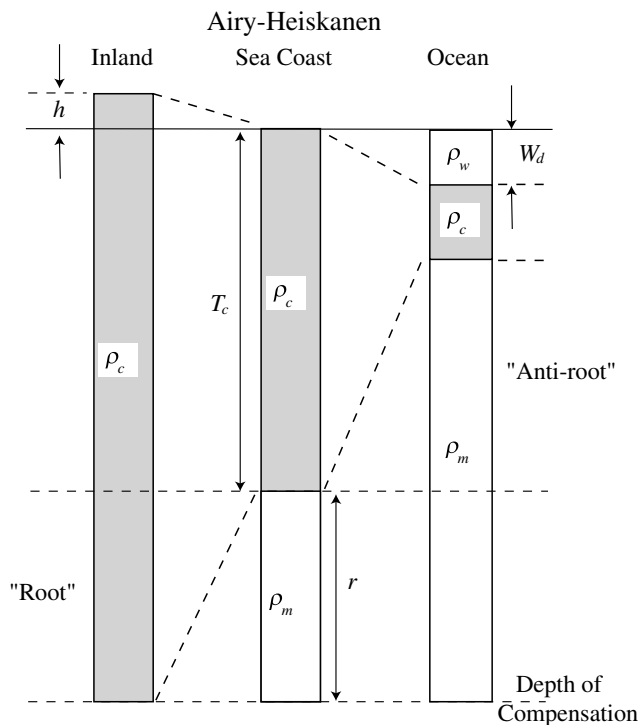


Fig. 1.18 The Airy-Heiskanen model of isostatic compensation.