EULDPH: A new technique for making computer-assisted depth estimates from magnetic data

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ABSTRACT

A method for rapidly making depth estimates from large amounts of magnetic data is described. The technique is based upon Euler's homogeneity relationship (hence, the acronym EULDPH) and differs from similar techniques which are currently available in that no basic geologic model is assumed. Therefore, EULDPH can be applied in a wider variety of geologic situations than can model-dependent techniques. The price paid for this increased flexibility is a heavier burden on the interpreter. Successful interpretation of EULDPH results is partially dependent upon the interpreter's intuitive understanding of the concept of the equivalent stratum and also partially dependent upon experience with model studies. The theoretical basis, the computational algorithm, and applications of EULDPH to model and real data are presented.

INTRODUCTION

The most universal application of magnetic data has been to determine the depth to the top of magnetic sources. For hydrocarbon exploration, this is usually equivalent to determining the thickness of the sedimentary section. For minerals exploration, depth estimates are usually used to determine the depth of ore bodies which contain magnetic minerals. Because of the importance of this application of magnetic data, there has been a sustained effort to develop improved techniques which yield accurate depth estimates from magnetic data, as evidenced by the continuous stream of papers in the geophysical literature beginning with Peters' (1949) classic paper.

This paper describes a technique developed for making depth estimates from magnetic data which uses a significantly different concept from most of the methods which have been published. Based upon Euler's homogeneity equation (hence the acronym EULDPH), the method reported herein derives a "structural index" in addition to producing depth estimates. Used together, the structural index and depth estimates can identify and make depth estimates for a wide variety of geologic structures such as faults, magnetic contacts, dikes, extrusives, etc.

EULDPH belongs to a class of techniques which are sometimes referred to as "automatic" depth estimate methods. These techniques are designed to provide computer-assisted analysis on large volumes of digital magnetic data, as opposed to methods utilizing rules of thumb, characteristic curves, or iterative or inverse curve matching, which are useful for more detailed analysis of isolated anomalies on very limited amounts of data. The so-called automatic techniques find application in the rapid analysis of large amounts of data; they usually operate directly on field data which have been digitally recorded.

Several computer-assisted methods of the automatic class have been reported in the literature. O'Brien (1972) reported on a technique which basically locates the vertices of a polygonal model. Koulomzine et al (1970) and Naudy (1971) described a method which utilizes vertical prism and thin plate models. The most successful technique is the method known as Werner deconvolution. This method was originally proposed by Werner (1953). He showed that the simplified equation for a thin two-dimensional (2-D) dike could be written in a form which is linear in the parameters of the dike. By choosing an appropriate number of points along a magnetic profile, a system of linear equations could be devised, the solution of which gives the position of the dike. Hartman et al (1971) extended the Werner technique to include analysis for a variety of magnetic discontinuities through the use of the vertical and horizontal derivatives of the total intensity of the magnetic field. Subsequent to these and other significant advancements at Aero Service, the Werner technique has found wide application to the analysis of magnetic data by oil and mining companies.

The potential advantage of EULDPH is that the method does not assume any particular geologic model. Thus, EULDPH can be applied and interpreted even when the geology cannot be properly represented by prisms or dikes. Moreover, EULDPH is the only technique among the published automatic techniques which theoretically can be applied directly to gridded, mapped magnetic data.

Throughout the following discussion, I emphasize that EULDPH is not fully automatic, i.e., it does not produce a geologic model from the given magnetic field. Procedures which attempt to do this make very restrictive assumptions and can be applied successfully only in specific geologic situations. EULDPH does not make any assumptions about the geologic model, and therefore, is applicable in a wider variety of situations than other techniques. The price paid for this increased flexibility is a heavier load upon the interpreter. In many cases, the interpretation of the EULDPH profiles is nearly unambiguous; in other cases, the

interpreter may be required to exercise his geologic knowledge of the area in order to choose between two or more possibilities pointed out by EULDPH.

As with all profile-oriented interpretation techniques, depth estimates inferred from the EULDPH profiles must be corrected for strike effects or otherwise weighted according to the position of the profile relative to the anomaly.

THEORY

In this section the theoretical basis for the EULDPH technique will be established, previous work related to the concept will be reviewed, and the details of the actual equations used will be presented.

Euler's equation

Consider any function of the three Cartesian coordinates x, y, and z denoted by $f(x, y, z)$. The plane of observation will be taken as the plane $z = 0$, and positive z is downward. By convention, then, the x-axis points to the north and the y-axis points to the east.

The function $f(x, y, z)$ is said to be *homogeneous of degree n* if

$$f(tx, ty, tz) = t^n f(x, y, z). \quad (1)$$

Moreover, it can be shown that if $f(x, y, z)$ is homogeneous of degree n, the following equation is satisfied:

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = nf. \quad (2)$$

This partial differential equation is known as Euler's homogeneity equation or simply as *Euler's equation*.

Suppose that $f(x, y, z)$ has the general functional form

$$f(x, y, z) = \frac{G}{r^N}, \quad (3)$$

where $r = (x^2 + y^2 + z^2)^{1/2}$, and $N = 1, 2, 3, \ldots$; G not dependent on x, y, and z. Clearly, equation (3) is homogeneous of order $n = -N$. Many simple point magnetic sources have the form of equation (3). The relationship between these simple sources and measured magnetic data will be discussed below.

The structural index

Consider a *point source* (point mass, magnetic dipole, etc.) located at the point x_0, y_0, z_0 relative to the plane of measurement. The total magnetic intensity will be of the form

$$\Delta T(x, y) = f[(x - x_0), (y - y_0), z_0]. \quad (4)$$

Euler's equation for the functional form (4) can be written as

$$(x - x_0) \frac{\partial \Delta T}{\partial x} + (y - y_0) \frac{\partial \Delta T}{\partial y} - z_0 \frac{\partial \Delta T}{\partial z} = -N \Delta T(x, y). \quad (5)$$

The gradients in the three Cartesian directions can be calculated using standard potential theory in the space or wavenumber domains. In some cases, the vertical gradient may have been measured and can be used directly in equation (5). Equation (5) could be used to analyze mapped magnetic data. However, for the present, attention will be focused upon applying this expression to profile or line-oriented data. In this case, the x-coordinate is a measure of the distance along the profile, and the y-coordinate can be set to zero along the entire profile.

If the transverse gradient $(\partial \Delta T/\partial y)$ in equation (5) is now assumed to be zero, the second term on the left is identically zero.

Table 1. Structural indices for simple models.

Simple model	N
Line of poles	1.0
Point pole	2.0
Line of dipoles	2.0
Point dipole	3.0

This is quite clearly the case if the anomaly is 2-D, i.e., if the anomaly is uniform transverse to the profile. Nearly all existing profile processes and interpretation aids make the 2-D assumption. In order to eliminate the transverse gradient term, however, we need only assume that the anomaly is symmetric transverse to the profile. This is a less restrictive assumption than that of two-dimensionality and provides some interpretational advantages. Slack et al (1967) applied similar reasoning in their discussion of applications of the measured vertical magnetic gradient. As they pointed out, the assumption of transverse symmetry, as opposed to the 2-D assumption, provides an advantage only when the vertical gradient is measured. In either case, equation (5) for profile data then reduces to the expression

$$(x - x_0) \frac{\partial \Delta T}{\partial x} - z_0 \frac{\partial \Delta T}{\partial z} = -N \Delta T(x).$$

Rearrangement of this expression yields

$$x_0 \frac{\partial \Delta T}{\partial x} + z_0 \frac{\partial \Delta T}{\partial z} = x \frac{\partial \Delta T}{\partial x} + N \Delta T(x). \quad (6)$$

The derivatives or gradients in equation (6) can be measured or, more commonly, calculated from the data. In the event that the vertical derivative $\partial \Delta T/\partial z$ is calculated from the observed total field ΔT, the 2-D assumption must be made. The only unknown quantities in equation (6) are x_0, z_0, and N. The coordinates (x_0, z_0) represent the depth and location along the profile of the point equivalent source, and N represents the type of source which best represents the anomaly. It is easily verified that various simple models have prescribed values of N (Smellie, 1956). Table 1 lists the structural indices for some simple point models [see, for example, Hood (1965)]. The structural index is also a measure of how "sharp" an anomaly is relative to its depth. Many geologic features have distinctive fall-off rates or structural indices. For example, a narrow, 2-D dike has a structural index of $N = 1$ at the magnetic pole, while a contact at the pole has a structural index of less than 0.5. The connection between the structural index N and real geologic anomalies forms the basis for EULDPH and has been discussed by Slack et al (1967), Thompson (1973), and Briener (1973), and will be discussed further below.

Implementation

Equation (6) can be solved exactly for the unknowns x_0, z_0, and N by evaluating the derivatives and total field values at three different x-coordinates along the profile. This results in three linear equations in three unknowns, which can in principle be solved if the determinant of the coefficients is not zero.

Application of equation (6) directly to observed data is not useful for three reasons:

Table 2. Structural indices and symbols used to represent the indices.

Structural index N	Symbol
0.5	□
1.0	0
1.5	*
2.0	△
3.0	+

(1) Most anomalies, even at the magnetic pole, prefer high structural indices, i.e., they are more dipolar in nature. However, the lower structural indices are better depth estimators (Thompson, 1973).

(2) The absolute level of the anomalous field ΔT is rarely known. Regional fields or dc offsets due to nearby anomalies are almost always present.

(3) On actual data, anomalies are seldom represented exactly by point sources.

These factors make the exact solution of equation (6) very unreliable and erratic. Methods have been developed to circumvent each of these three problems, and they are described in detail in the following three paragraphs.

The problem of forcing the method to give depths for low structural indices is solved by performing the analysis for a series of prescribed structural indices. At present, EULDPH uses five structural indices as given in Table 2. It is still possible to determine which structural index a given anomaly prefers by observing the grouping or clustering of the indices on the plotted output, as will be discussed below.

The problem of properly removing the bias from the observed data is solved in the following way. Assume the anomalous field is perturbed by a constant amount B in the window in which equation (6) is being evaluated. The observed quantity is

$$T(x) = \Delta T(x) + B, \qquad (7)$$

where B is constant in the coordinate x over the portion of the

profile where the analysis is being made. Solving equation (7) for ΔT, substituting into equation (6), and arranging terms yield

$$x_0 \frac{\partial T}{\partial x} + z_0 \frac{\partial T}{\partial z} + NB = x \frac{\partial T}{\partial x} + NT. \qquad (8)$$

Since actual anomalies are only approximated by simple models, the third major problem is solved by creating an overdetermined set of linear equations. If one evaluates equation (8) at four or more points within a window of a profile, an overdetermined set of equations results. At present an operator consisting of seven points is used. The seven equations in the three unknowns x_0, z_0, and B are then solved using a least-squares procedure.

The least-squares solution of the overdetermined set of equations also yields estimates of the standard deviation of the parameter z_0. This quantity σ_z is treated as an "error bar" on the depth estimate and forms the basis for an algorithm that determines whether or not a depth estimate is to be retained.

The results of the procedure consist of both printed and plotted data. The procedure is most easily described with reference to Figure 1. For a given index (say $N = 0.5$, for example), window length and window position along the profile seven data points are chosen, and solutions are obtained for x_0, z_0, σ_z, and B [See equation (8)]. If the acceptance criteria are met, the symbol for the index is plotted at the (x_0, z_0) coordinate, and the value for $\pm\sigma_z$ is indicated by a vertical error bar. It is easy to see that, through the convolution of several windows and the five different indices with the entire magnetic profile, a great many depth estimates are made. Depending upon the acceptance criteria, only those estimates which adequately reproduce the data within the window will be accepted for output.

The acceptance criterion actually used is empirically derived. Clearly, the acceptance criterion must depend upon the depth of the estimate, since deeper estimates are by nature less certain. Moreover, as mentioned above, most geologic structures are better represented by higher indices, but the lower indices are usually more directly related to the depth to the top of the structure. The actual value of the acceptance tolerance (TOL) will depend upon the quality of the data and must be adjusted by the interpreter. For standard high-resolution aeromagnetic data, a particular estimate (for a given structural index N and window length W) is accepted for plotting if the following inequality is satisfied:

$$TOL = 20 \leq \frac{z_0}{N\sigma_z}. \qquad (9)$$

INTERPRETATION

General considerations

The fundamental concepts of EULDPH are based on the idea of the equivalent stratum. Stated simply, the concept of the equivalent stratum is that the magnetic anomaly due to a causative body with uniform magnetization can be exactly duplicated by placing an appropriate distribution of magnetic poles on the surface of the causative body. Thus, a thin extrusive magnetic body would have positive poles induced on its upper surface and negative poles induced on its lower surface. At a distance, such a magnetic source would have dipolar characteristics. On the other hand, intrusive magnetic bodies which have their origins deep within the crust would have poles induced on their upper surface,
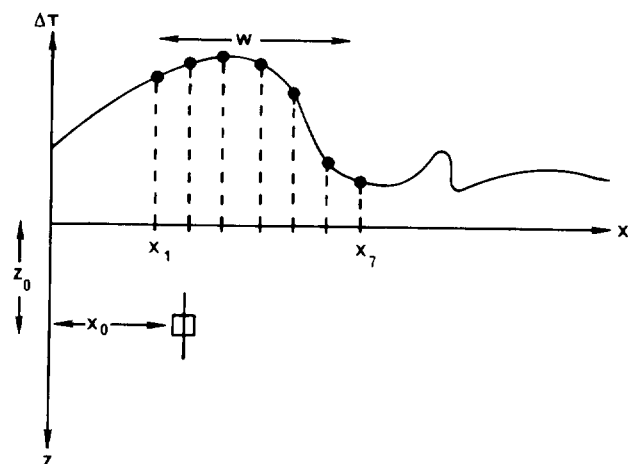

FIG. 1. Schematic representation of a single valid depth estimate.

but the corresponding negative poles would be so deeply buried that they would not contribute to the measured magnetic field. Such an intrusive body, according to this oversimplified reasoning, would exhibit polar behavior. The idea of using simple models to represent magnetic sources has been documented by Hood (1965), Smellie (1956), Thompson (1973), and Briener (1973), among others.

It is known from potential theory that there is no unique inverse solution to the magnetic problem. Therefore, the representation of the anomalous magnetic field as due to a subsurface distribution of simple magnetic models is also not unique. The EULDPH procedure does not attempt to construct magnetic models of the subsurface geology. It only helps the interpreter to construct these models by presenting him with a series of possibilities. The greatest strength of EULDPH, as compared to other techniques, is that the simple models (represented by the structural indices) are capable of resolving and sometimes identifying a wide variety of geologic situations. In other words, no *geologic* model is assumed in the performance of the EULDPH procedure.

Some typical magnetic structures

Figure 2 shows some 2-D magnetic models and their magnetic responses. The induced fields due to these models have been calculated at the magnetic pole (although this is not a restriction on the EULDPH technique) and have been chosen to illustrate the ability of the EULDPH technique to determine the type of causative body as well as its depth.

Figure 2a represents a magnetic dike intruded into less magnetic rocks. This feature prefers the higher structural indices as evidenced by the tighter clusters representing the 1.5 and 2.0 indices. However, the 0.5 index is clearly the best depth estimator, consistent with our intuitive understanding that a distribution of magnetic poles would be induced on the upper surface of the causative body. The width/depth (W/D) ratio of this dike is 2.0. As the W/D ratio of the dike changes, the EULDPH pattern will also change. For large W/D ratios, the procedure will respond as though the dike represents two magnetic contacts. For very narrow dikes (W/D < 0.5), the response (at the pole) will be best represented by a line of poles (N = 1). These results are entirely consistent with the discussions by Slack et al (1967) and Thompson (1973). As a rule of thumb, for intrusive-type bodies the best depth estimator is the 0.5 index.

Figure 2b represents the magnetic signal and EULDPH responses of a fault in basement rocks, with a depth/throw ratio of 5.0. In this case, the anomaly is best reproduced by the 1.0 index, as evidenced by the very tight clustering of that index. Typical of faulted structures, which have a slower fall-off rate than do intrusive bodies, the high structural indices are poorly grouped and erratic. In this example the 1.0 index is also clearly the best depth estimator.

Figure 2c represents a deep vertical magnetic contact. As predicted by Briener (1973), contacts have very low fall-off rates and are therefore poorly represented by high structural indices. This is evident from Figure 2c by the complete absence of the 1.5, 2.0, and 2.5 indices. Typical of contacts, the 0.5 index best represents the anomaly, as evidenced by the relatively good clustering of this index. An empirical rule of thumb has been devised to estimate the depth to the top of magnetic contacts: Subtract the distance between the 0.5 and 1.0 index clusters from the depth of the 0.5 index. The EULDPH patterns for magnetic contacts are quite distinct, and the rule can be applied to obtain depth estimates for magnetic contacts.

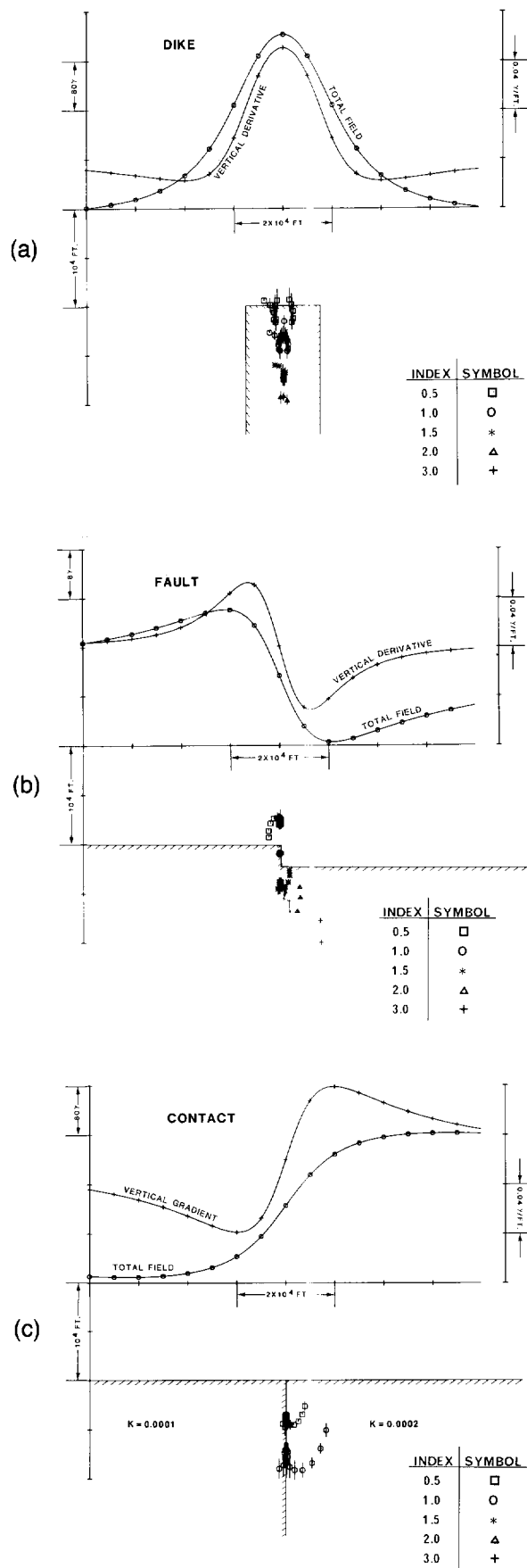The three examples of Figure 2 are not intended to be an

FIG. 2. (a) EULDPH response of a 2-D dike. (b) EULDPH response of a 2-D fault. (c) EULDPH response of a 2-D contact.
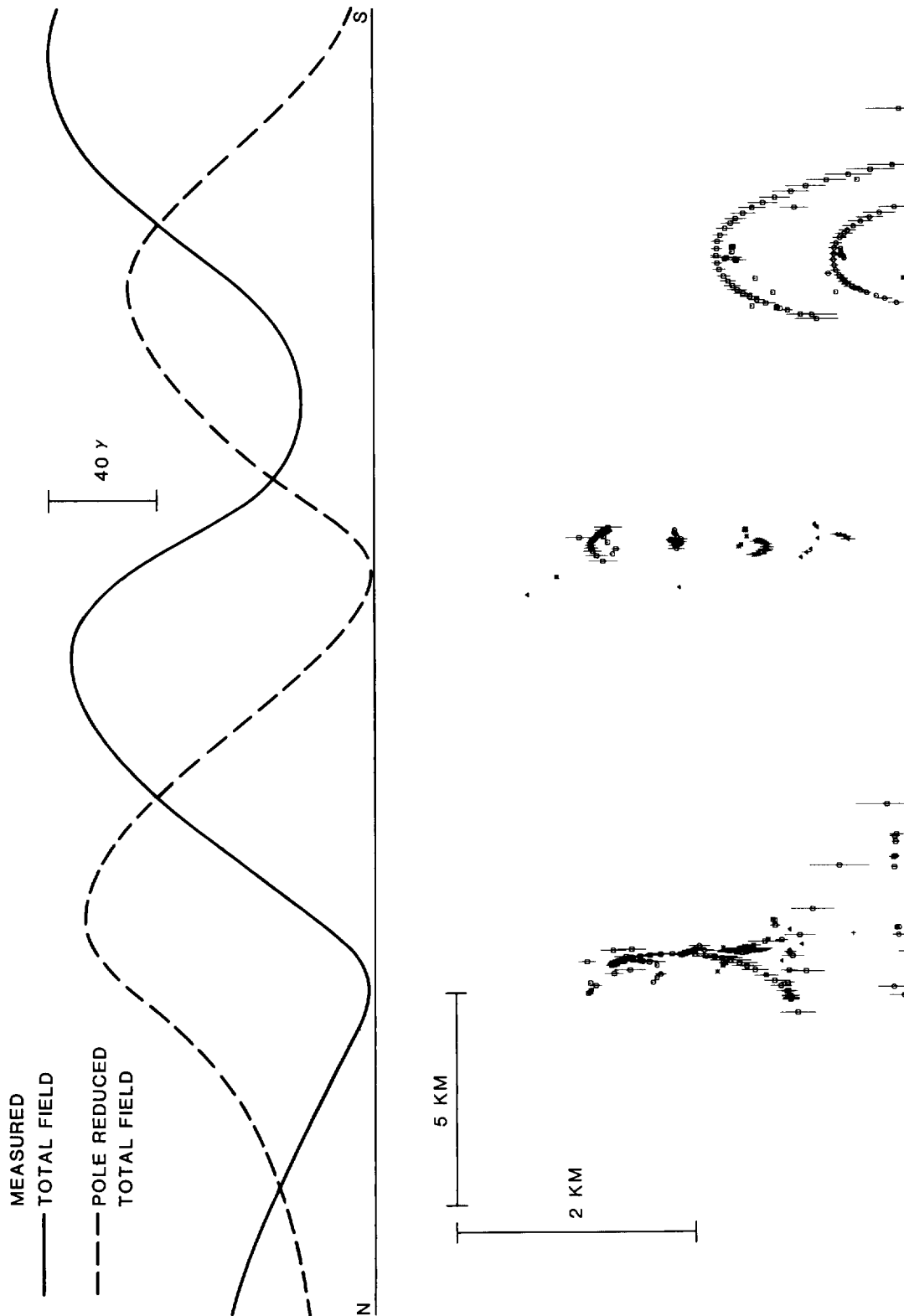
FIG. 3. A measured aeromagnetic profile and EULDPH response from West Africa.
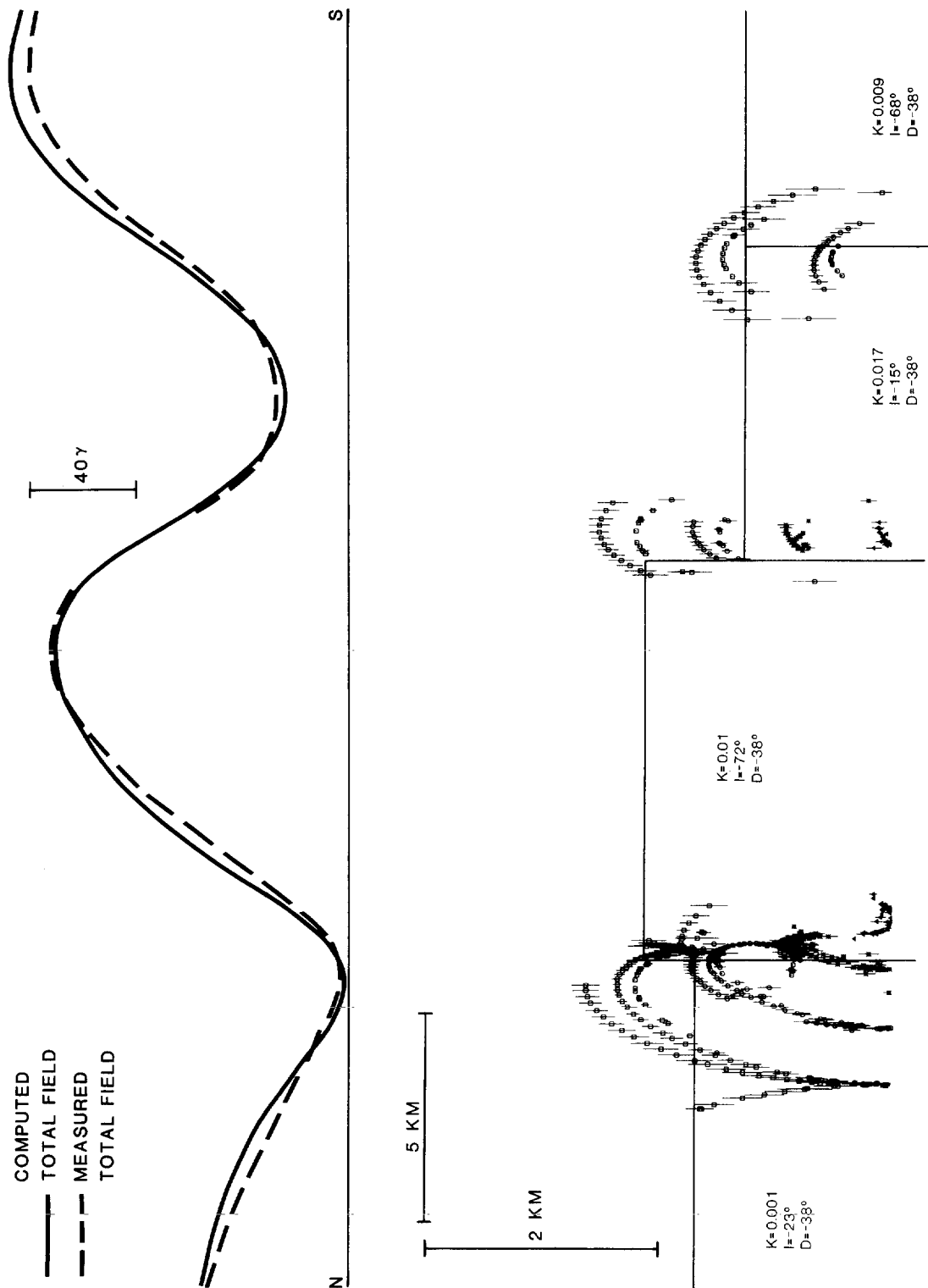
FIG. 4. Total field and EULDPH response of a magnetic model derived from Figure 3.

exhaustive catalogue of EULDPH responses. In fact, the responses of structures to the EULDPH technique vary slightly with the geometry of the body and with magnetic inclination.

EULDPH can be applied to data at any magnetic inclination. However, depth estimates are usually more accurate for data which have been reduced to the magnetic pole. Work previously reported by Thompson (1973) shows that, as the magnetic inclination decreases, simple models tend to overestimate the depths to causative bodies. Thus, if EULDPH is applied to data at lower magnetic latitudes as well as to the data which have been pole reduced, one would normally expect depth estimates to be slightly shallower and more accurate on the pole-reduced data. Any variations from this pattern would indicate that the assumptions of pole reduction have been violated, i.e., the structures are not 2-D and/or remanent magnetization is present.

An example from West Africa

Figure 3 shows a high-sensitivity aeromagnetic profile taken from the offshore region of western Africa. This profile covers the transition from oceanic crust to the south to continental crust to the north. For the purposes of clarity, the low-amplitude anomalies to the north are not shown on Figure 3. The magnetic inclination at this latitude is −23 degrees, and EULDPH "picks" shown on the figure have been made using the measured data. It is clear from the areal coverage that the magnetic sources can be considered 2-D for the purposes of interpretation.

Also shown on Figure 3 are the data which have been pole reduced. It is not possible, due to limitations of black and white printing, to show EULDPH picks on the pole-reduced data (the current algorithm plots the pole-reduced estimates in red). It is sufficient to note that the pole-reduced estimates show a pattern very similar to the picks on the measured data but are consistently deeper. Since it is known from the areal coverage that the anomaly is 2-D, the deeper pole-reduced estimates imply the possible existence of remanent magnetization.

A magnetic model was constructed which consists of a series of vertical prisms whose boundaries and depths were constrained by the EULDPH picks shown on Figure 3. The good grouping of the 1.0 index in the series of estimates in the middle of the profile suggests that there may be considerable structure associated with this group of estimates, while the group of estimates on the southern end of the profile is suggestive of a contact. By adjusting the remanent magnetizations of the model blocks, a satisfactory model was achieved, as shown in Figure 4.

The EULDPH response shown on Figure 4 was derived using the magnetic data calculated from the model at a magnetic inclination of −23 degrees. As with Figure 3, the pole-reduced EULDPH model response would normally be displayed in red. The constructed model shown in Figure 4 shows the less mag-

netic continental crust to the north and several more magnetic blocks to the south, representing a series of normally and reversely magnetized blocks. The magnetic inclinations shown on Figure 4 are the vector sum of induced and remanent magnetization in each block.

This example demonstrates that EULDPH analysis can provide the interpreter with information usually not available from conventional analysis.

CONCLUSIONS

EULDPH represents a significant departure, both in theory and practical utility, from conventional methods now available to the industry. As with all magnetic depth estimate procedures, EULDPH must be used by interpreters who are trained in its use. Considerable experience is necessary to implement the procedure properly.

The most important technical limitation of the EULDPH procedure is its sensitivity to interference among neighboring anomalies. When this occurs, erroneous picks sometimes group in a deceptive way.

REFERENCES

Briener, S., 1973, Applications manual for portable magnetometers: GeoMetrics, Sunnyvale, CA.

Hartman, R. R., Tesbey, D. J., and Friedberg, J. L., 1971, A system for rapid digital aeromagnetic interpretation: Geophysics, v. 36, p. 891–918.

Hood, P., 1965, Gradient measurements in aeromagnetic surveying: Geophysics, v. 30, p. 891–902.

Koulomzine, Th., Lamontagne, Y., and Nadeau, A., 1970, New methods for the direct interpretation of magnetic anomalies caused by inclined dikes of infinite length: Geophysics, v. 35, p. 812–830.

Naudy, H., 1971, Automatic determination of depth on aeromagnetic profiles: Geophysics, v. 36, p. 717–722.

O'Brien, D., 1972, CompuDepth, a new method for depth-to-basement computation: Presented at the 42nd Annual International SEG Meeting, November 27, in Anaheim.

Peters, L. J., 1949, The direct approach to magnetic interpretation and its practical application: Geophysics, v. 14, p. 290–320.

Slack, H. A., Lynch, V. M., and Langan, L., 1967, The geomagnetic gradiometer: Geophysics, v. 32, p. 877–892.

Smellie, D. W., 1956, Elementary approximations in aeromagnetic interpretation: Geophysics, v. 21, p. 1021–1040.

Thompson, D. T., 1973, Identification of magnetic source types using equivalent simple models: Presented at the 1973 Fall Annual AGU Meeting in San Francisco.

Werner, S., 1953, Interpretation of magnetic anomalies at sheet-like bodies: Sver. Geol. Undersok, ser. C. C. Arsbok 43, N:06.