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3DINVER.M: a MATLAB program to invert the gravity anomaly over a 3D horizontal density interface by Parker–Oldenburg's algorithm☆

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Abstract

A MATLAB source code 3DINVER.M is described to compute 3D geometry of a horizontal density interface from gridded gravity anomaly by Parker–Oldenburg iterative method. This procedure is based on a relationship between the Fourier transform of the gravity anomaly and the sum of the Fourier transform of the interface topography. Given the mean depth of the density interface and the density contrast between the two media, the three-dimensional geometry of the interface is iteratively calculated. The iterative process is terminated when either the RMS error between two successive approximations is lower than a pre-assigned value—used as convergence criterion, or until a pre-assigned maximum number of iterations is reached. A high-cut filter in the frequency domain has been incorporated to enhance the convergence in the iterative process. The algorithm is capable of handling large data sets requiring direct and inverse Fourier transforms effectively. The inversion of a gravity anomaly over Brittany (France) is presented to compute the Moho depth as a practical example.

Keywords: Three-dimensional gravity inversion; Density interface; Fourier transform; MATLAB

1. Introduction

The determination of the geometry of a three-dimensional density interface from the gravity anomaly is a classical problem arising from many geophysical studies. One such application is in mapping of crustal discontinuities, viz. Mohorovicic discontinuity from the corresponding gravity anomaly. Many techniques have been used in isolating the anomalies associated with these crustal discontinuities (e.g. Syberg, 1972; Chakraborty and Agarwal, 1992; Lefort and Agarwal, 2000). In these cases, one of the main purposes is to invert the filtered gravity anomaly in terms of the geometry of the interface.

Several authors have presented different algorithms to compute the geometry of a density interface related to a known gravity anomaly. Some of them (e.g. Cordell and Henderson, 1968; Dyrelius and Vogel, 1972; Bhaskara Rao and Rameshbabu, 1991, among others) use an approximation to the perturbing body by means of several rectangular prisms of constant density. The gravity effect for each prism is calculated and then, the

☆Code on server at http://www.iamg.org/CGEditor/index.htm.

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total gravitational field is determined by adding the effect of all prisms. Tsuboi (1983) gave a simple but efficient method based on equivalent stratum technique to compute 3D topography of a density interface. Some other algorithm (e.g. Oldenburg, 1974) is based on the rearrangement of the forward algorithm by Parker (1973). The Parker's scheme is based on the Fourier transform of the gravitational anomaly as a result of the sum of the Fourier transforms of the powers of the surface causing the anomaly. Oldenburg (1974) demonstrated that Parker's expression could be rearranged in order to determine the geometry of the density interface from the gravity anomaly. A computer program in FORTRAN language based on the Parker–Oldenburg method has been presented by Nagendra et al. (1996) for analysing two-dimensional gravity data. The aim of this work is to present a MATLAB function based program for three-dimensional extension of the Parker–Oldenburg's method in obtaining the geometry of the density interface related to the gravitational anomaly. This MATLAB function has the capability to manage large gravity data sets efficiently in the inversion process and emerges out as a useful tool in any gravity data analyses and tectonic interpretation.

2. Theory

The inversion procedure uses the equation described by Parker (1973) to calculate the gravity anomaly caused by an uneven, uniform layer of material by means of a series of Fourier transforms. This expression, in its one-dimensional form, is defined as

$$F(\Delta g) = -2\pi G\rho e^{(-kz_0)} \sum_{n=1}^{\infty} \frac{k^{n-1}}{n!} F[h^n(x)], \tag{1}$$

where $F(\Delta g)$ is the Fourier transform of the gravity anomaly, G is the gravitational constant, ρ is the density contrast across the interface, k is the wave number, $h(x)$ is the depth to the interface (positive downwards) and z_0 is the mean depth of the horizontal interface.

Oldenburg (1974) rearranged this equation to compute the depth to the undulating interface from the gravity anomaly profile by means of an iterative process and is given by

$$F[h(x)] = -\frac{F[\Delta g(x)]e^{(-kz_0)}}{2\pi G\rho} - \sum_{n=2}^{\infty} \frac{k^{n-1}}{n!} F[h^n(x)]. \tag{2}$$

This expression allows us to determine the topography of the interface density by means of an iterative inversion procedure. In this procedure we assume the mean depth of the interface, z_0, and the density contrast associated with two media, ρ. The gravity anomaly is first demeaned prior to the calculation of the Fourier transform. Then, the first term of Eq. (2) is computed by assigning $h(x) = 0$ (Oldenburg, 1974) and its inverse Fourier transform provides the first approximation of the topography interface, $h(x)$. This value of $h(x)$ is then used in Eq. (2) to evaluate a new estimate of $h(x)$. This process is continued until a reasonable solution is achieved.

Following Oldenburg (1974), the process is convergent if the depth to the interface is greater than zero and it does not intercept the topography. Further, the amplitude of the interface relief should be less than the mean depth of the interface.

As the inversion operation (Eq. (2)) is unstable at high frequencies, a high-cut filter, HCF(k) is included in the inversion procedure to ensure convergence of series. This filter is defined by

$$\mathrm{HCF}(k) = \frac{1}{2}\left[1 + \cos\left(\frac{k - 2\pi \mathrm{WH}}{2(\mathrm{SH} - \mathrm{WH})}\right)\right] \tag{3}$$

for $\mathrm{WH} < k < \mathrm{SH}$,
$\mathrm{HCF}(k) = 0$ for $k > \mathrm{SH}$,
and $\mathrm{HCF}(k) = 1$ for $k < \mathrm{WH}$
is used to restrict the high frequency contents in the Fourier spectrum of the observed gravity anomaly. The frequency, k can be expressed as $1/\lambda$, λ being the wavelength in kilometres.

The iterative process is terminated when a certain number of iterations has been accomplished or when the difference between two successive approximations to the topography is lower than a pre-assigned value as the convergence criteria.

Once the topographic relief is computed from the inversion procedure, it is desirable to compute the gravity anomaly produced by this computed topography. In general, this modelled anomaly must be very similar to the one used as input at the first step of the inversion process.

3. Description of the MATLAB function

The 3DINVER.M function takes advantage of some MATLAB routines used in computing direct and inverse two-dimensional Fourier transforms (FFT2 and IFFT2) and allows us to manage large arrays of data. Furthermore, the computer time required to perform all the computations is small leading to an efficient and effective inversion method. In addition to this, the function TUKEYWIN.m is also used. As this function is provided with the Signal Toolbox package in Matlab 6.0 (R12) or included with Matlab (R13), these versions are needed in order to use the 3DINVER.M code.

The algorithm first loads the gridded data values of the gravity anomaly and the parameters used in the process, viz. number of rows and columns, data spacing, density contrast, mean interface depth, convergence criterion, and roll off frequencies for the high-cut filter.

These input data are not fed through the monitor but defined directly into the MATLAB function by using an ASCII text editor. This is useful when several sets of computations have to be performed by varying only one or two parameters (e.g. density contrast or mean interface depth). Thus, it is not necessary to introduce all of the data at every run of the program. Gravity anomaly units are milligal, density contrast is in $g\,cm^{-3}$, and the mean interface depth and the grid spacing is in kilometre. The frequencies used in the high-cut filter are expressed as km^{-1}, and the convergence criterion is expressed as the RMS value between two successive topography approximations, also in kilometres. The gravity data are read and stored in the form of a one-dimensional array, covering sequentially the anomaly values along each of the rows parallel to the x-axis. This is the same format as an ASCII xyz.dat file derived from a gridded data file using SURFER Package, but without the x and y columns. The name of the input gravity data file, as well as the output surface depth and inverted gravity anomaly files are also introduced in the MATLAB function. The algorithm allows both squared and rectangular input grids, but the number of rows and columns must be an even number in this latter case. In order to avoid anomalies of different wavelength are averaged, a square grid input gravity data array is highly recommended.

In order to minimise edge effects, it is recommended to prepare the input gravity data before introducing into the MATLAB function by using a suitable technique. Several methods exist in the literature to achieve this (e.g. padding and windowing the original data sets). The MATLAB function uses a cosine taper window at the edges of the rectangular data set to reduce the values till zero is observed. The default window length of 10 percent of the matrix data length in each direction is found suitable for present inversion algorithm. Further, it is recommended to use an input gravity data set larger than the area of interest, or to extrapolate the gravity data beyond the limits of the area prior to the inversion procedure.

After reading and storing all data and parameters, the input gravity data are displayed. Then, the gravity data are demeaned. After this, the FFT routine implemented in MATLAB is used to compute a matrix with the amplitude spectrum displayed onto the monitor. A matrix with the frequencies corresponding to the amplitude spectrum is also constructed.

At this point, the iterative procedure starts. The first term of the series is calculated using the Parker's expression, and the resulting topography (in wave number domain) is filtered with the high-cut filter. Then, by applying the inverse FFT (also implemented in MATLAB), the topography is calculated in space domain. This first approach to the topography of the interface is used to compute the second term of the series (Eq. (2)). This second term is newly filtered and after applying the inverse FFT, the RMS between the new topography and the previous one is computed. If the RMS is lower than a pre-assigned value, i.e. the convergence criterion, the process is stopped. If not, the new topography is used to compute the third term of the series and so on, and the iterative procedure continues until the convergence criterion is reached or a maximum number of ten iterations have been accomplished.

Finally, the function displays three new graphics: the inverted topography, the gravity anomaly due to the inverted topography computed using the Parker's expression, and the difference between the input gravity data and the computed ones. The RMS value and the iteration at which the process has stopped are also displayed.

Two outputs one-column each ASCII files are then written; the first containing the depth to the density interface, and the second containing the gravity anomaly derived from the forward modelling after adding the mean of the original gravity anomaly. Knowing the number of rows and columns, as well as the minimum and maximum co-ordinate values of the data sets, these files are easily converted to a xyz.dat file or to a grid file (e.g. a SURFER ASCII grd file).

4. Application example

Fig. 1 is a subset of the gravity anomaly map associated to the crust-mantle boundary in Brittany, France (Lefort and Agarwal, 2000). A gravity anomaly low with a NW–SE trend has a total amplitude of about 40 mGal. The area has an extent of $300 \times 300\,km^2$. The square grid interval is 6 km. The parameters used for the inversion are: density contrast of $0.4\,g\,cm^{-3}$ (mantle density minus crust density) and a mean depth for the crust–mantle boundary of 30 km (Lefort and Agarwal, 2000). The original map area has been extended by using the maximum entropy method (Burg, 1978) to $666 \times 666\,km^2$ to avoid edge effects. Thus, the total number of columns and rows are 112×112. The filter cut-off parameters have been chosen as $WH = 0.01$ and $SH = 0.012$, as per frequency intervals determined for the Moho by Lefort and Agarwal (2000). Finally, the truncation window data length established for the cosine taper window is selected as 10% of the extended data length.

Fig. 2A shows the results obtained from the application of the 3DINVER.M function. Convergence of the iterative procedure has been achieved at the second iteration, with a RMS error of 0.0147 km (the convergence criterion had been established in 0.02 km). The geometry of the inverted interface shows a maximum depth of about 32.5 km, located at the NW, related to a

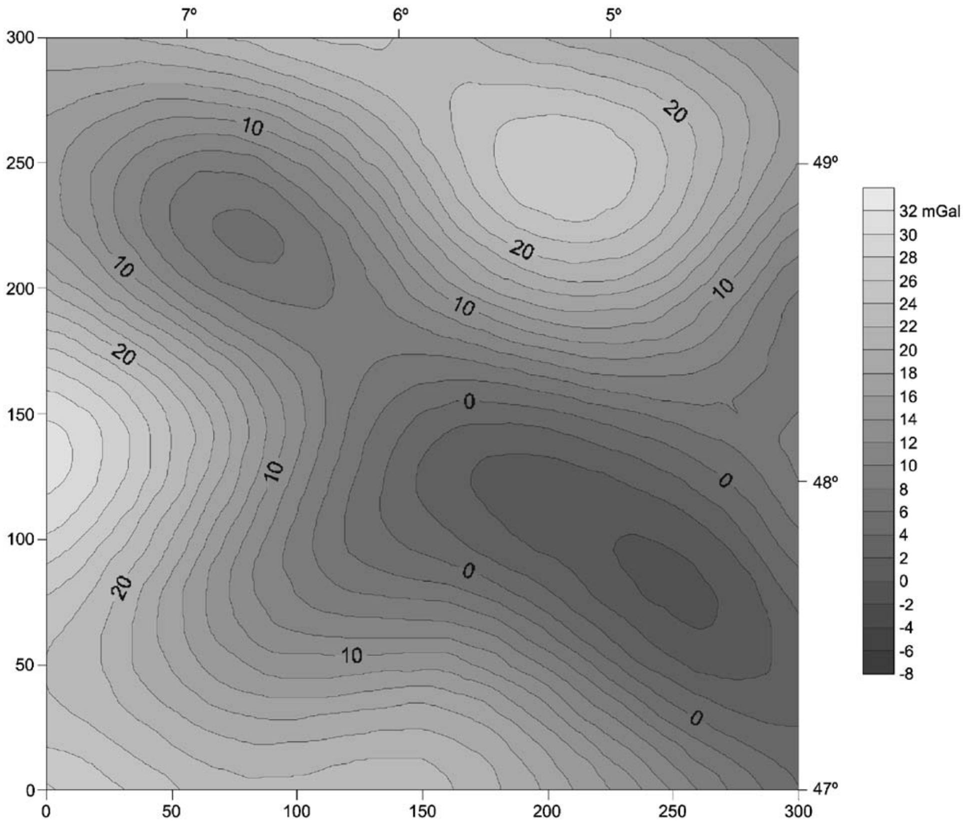

Fig. 1. Bouguer anomaly map attributed to Moho interface in Brittany (France). Contour interval 2 mGal. Coordinates are in kilometres and latitude and longitude in degrees. Modified from Lefort and Agarwal (2000).

NW–SE trend direction. Minimum depth is about 27.5 km, located at the NW corner of the area, so the total Moho amplitude variation is about 5 km for this zone. Fig. 2B shows the Moho depth map presented by Lefort and Agarwal (2000) for the same area derived from the inversion method described by Tsuboi (1983). As can be seen, the similarities between the Moho geometry and depth obtained from the two independent methods are remarkable, and only minor differences appear, mainly in the amplitude of the Moho geometry.

Two more pieces of information are provided by the MATLAB function. Fig. 3A shows the gravity anomaly map obtained from the application of Eq. (1) to the interface topography obtained, i.e. by means of the forward modelling algorithm. This map also reveals a good correlation with the original gravity input map (Fig. 1). The difference between the original and computed anomaly maps is shown in Fig. 3B. It can be observed that these differences are insignificant and are in the range of -0.25–$+1.5$ mGal.

Finally, in order to compare 3D modelled results with the ones obtained by using a $2+\frac{1}{2}D$ gravity modelling software (GM-SYS from Geosoft), two gravity profiles, marked P_1 and P_2 have been selected (Fig. 4). The locations of these profiles are marked on Fig. 2A. The first profile (P_1) cuts the Moho low trending NW–SE at a high angle, whereas the second one (P_2) is parallel to it. Fig. 4A shows the results obtained for the profile P_1. It can be observed from this figure that the Moho relief obtained from the GM–SYS software produces a gravity anomaly very similar in shape to the observed anomaly, but with slightly greater amplitude. However, the magnitude of error between both curves is only 2.7% of the anomaly amplitude. Fig. 4B illustrates the results obtained for the second profile P_2. Again, the Moho relief obtained generates a theoretical gravity anomaly that fits well with the observed one, but in this case, with a slightly lesser amplitude. The error magnitude between calculated and observed anomalies in this case is very small (about 1.07%). Differences in amplitude between both curves may be attributed to the improper selection of the SH and WH frequency parameters of the high-cut filter used during the inversion. It is suggested that the values of SH and WH may be selected from trial and error approach.

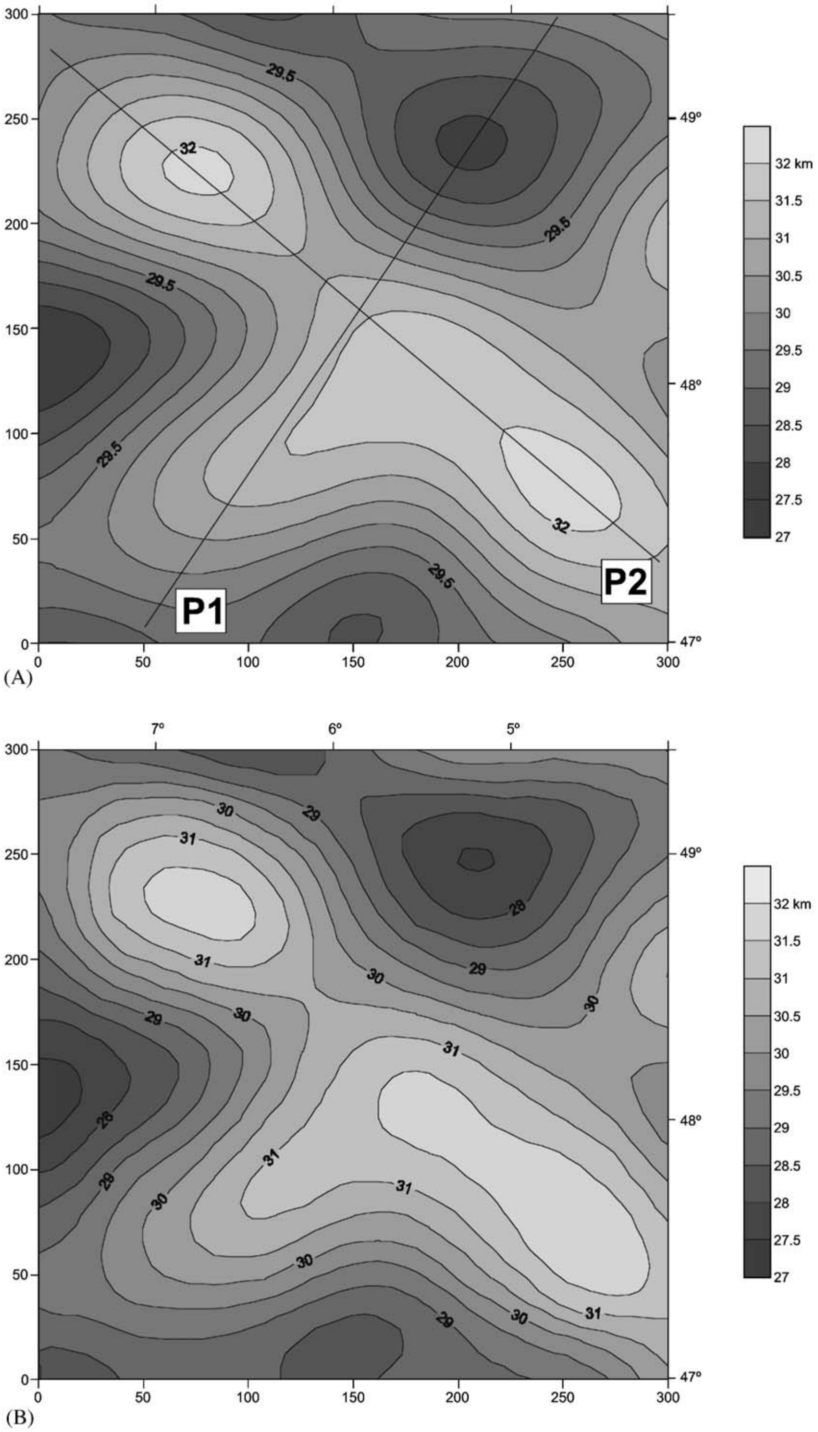

Fig. 2. (A) 3D Moho relief in Brittany (France) derived from application of 3DINVER.M function to gravity data of Fig. 1. Contour interval 0.5 km. Coordinates are in kilometres and latitude and longitude in degrees. P_1 and P_2 are locations of gravity profiles used for $2\frac{1}{2}D$ modelling. (B) 3D isobath map of Moho as determined by Lefort and Agarwal (2000) using method of Tsuboi (1983). Contour interval 0.5 km. Coordinates are kilometres and latitude and longitude in degrees. Modified from Lefort and Agarwal (2000).

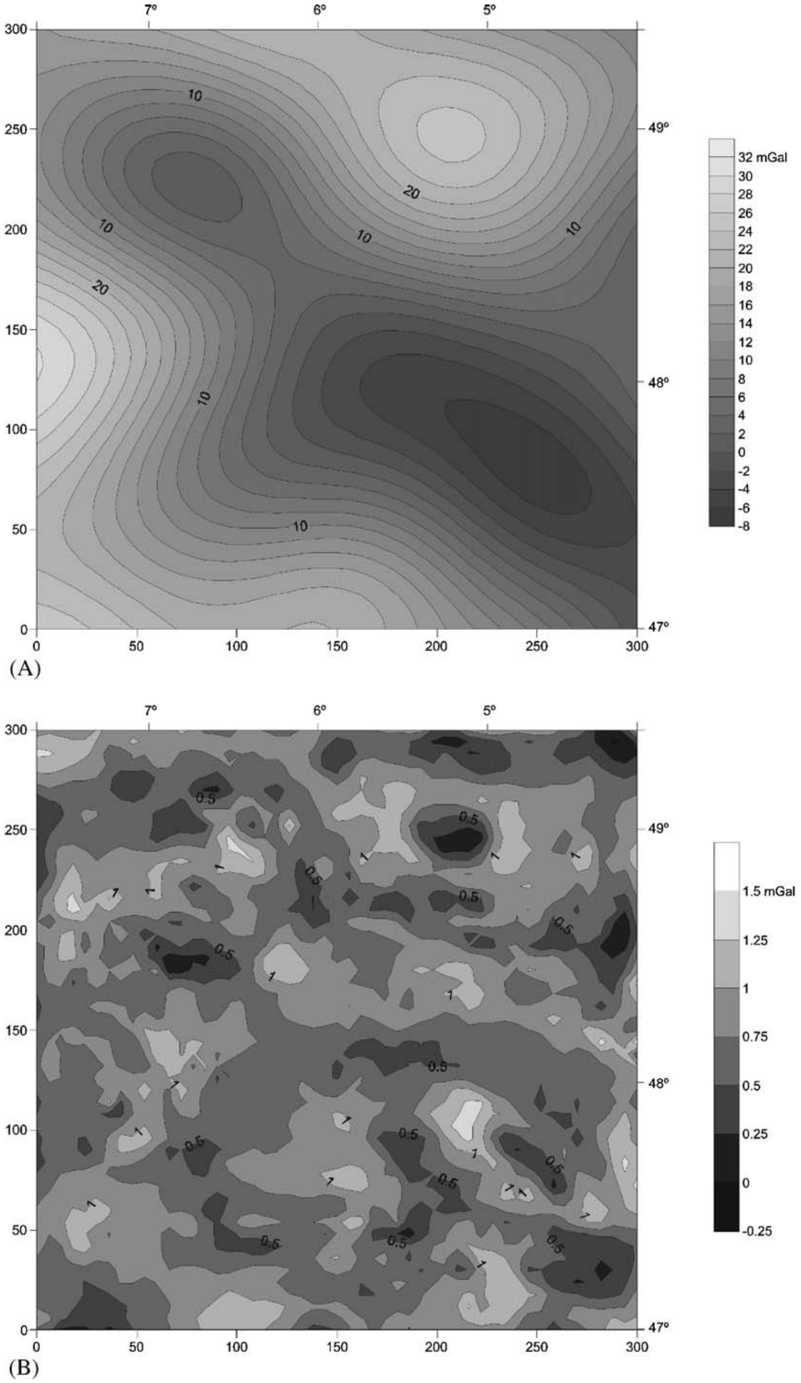

Fig. 3. (A) Gravity anomaly map obtained from 3D Moho relief map derived from present inversion procedure. Contour interval 2 mGal. Coordinates are kilometres and latitude and longitude in degrees. (B) Difference between gravity map of Fig. (A) and observed gravity map presented in Fig. 1. Contour interval 0.25 mGal. Coordinates are kilometres and latitude and longitude in degrees.

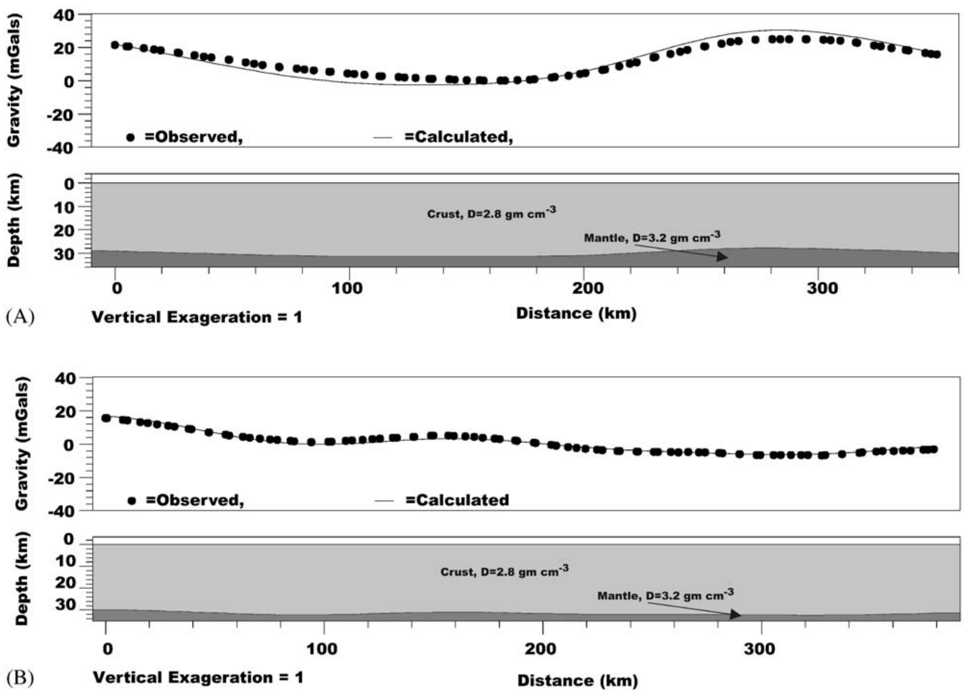

Fig. 4. $2\frac{1}{2}D$ gravity modelling results in two different orientations: (A) transverse to main Moho depression (profile P_1) and (B) parallel to it (profile P_2). Locations of both profiles are marked on Fig. 2A.

5. Conclusions

An inversion algorithm developed in MATLAB to compute three-dimensional geometry of a density interface using the Parker–Oldenburg method has been presented. The function, called 3DINVER.M, represents a useful tool when the density interface does not intercept the surface topography and when its mean depth is larger than its amplitude. The program is very efficient and can handle large data sets due to use of MATLAB functions FFT2 and IFFT2.

The program requires a previous knowledge of two parameters, viz. the mean depth and the density contrast of the interface. Using these values and the gravity anomaly due to the density interface as the input, an iterative procedure based on the rearrangement of Parker's formula proposed by Oldenburg (1974) computes the geometry of the interface until a user-defined convergence criterion, or a maximum number of iterations are satisfied. An output file with the depth to the interface is obtained, together with the RMS error value and the iteration at which the process has finished.

The comparison of the results obtained using the MATLAB function with other results obtained from a different inversion method (Tsuboi, 1983) or from a $2 + \frac{1}{2}D$ gravity modelling, has revealed that all methods provide a very similar geometry, with small differences related to the amplitude of the interface. The 3DINVER.M program can be obtained from the web site http://www.escet.urjc.es/~dgomez/, or from the IAMG server at http://www.iamg.org/CGEditor/index.htm, or from the corresponding author for use in many geophysical and tectonic problems.

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