## Euler deconvolution of gravity data

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**Summary**

Euler deconvolution can be usefully applied to gravity data provided that thought is given to the correct Structural Index (SI).. For simple bodies, the gravity SI is one less than the magnetic SI. For more complex bodies (including the contact) The Euler method is at best an approximation.

**Introduction**

Euler deconvolution of both profile and gridded magnetic data (Thompson, 1982; Reid et al, 1990) has found wide application. It has been implemented by many organisations and individuals and is commercially available from several suppliers. Applications to gravity are fewer. Marson & Klingele (1993) applied it to gravity vertical gradients. Keating applied it to gravity point data. Zhang et al (2000) applied it to gravity tensor gradient data. But Euler deconvolution requires a well-founded understanding of a critical parameter, the Structural Index (SI), which characterizes the source geometry. In the magnetic case it varies from zero (contact of infinite depth extent) to 3 (point dipole). It can be thought of as the index in the field strength fall-off with distance.

The value of the SI is important, because use of the wrong value leads to the calculation of misleading depths. But there seems to be a problem in the gravity case. Stavrev (1997) gives an admirably complete and closely argued analysis of the problem, showing that the gravity SI for any given structure should simply be one less than the equivalent magnetic value. This is plausible, because the pseudogravity transformation, which converts the magnetic field of a source body to the gravity field that would be observed from the same body involves a pole reduction and a vertical integration. The integration should reduce the SI by 1.0.

A difficulty arises from this viewpoint. It leads to a value of –1.0 for the infinite contact. This is theoretically awkward because it implies an increase in the gravity field strength with distance from the source, reaching infinite value at infinity.

We discuss the question of gravity SIs below.

**Theory**

Strictly speaking, Euler deconvolution is only valid for homogeneous functions. A function $f(\mathbf{v})$ of a set of variables $\mathbf{v} = (v_1, v_2, ...)$ has a degree of homogeneity $n$, if

$$f(t\mathbf{v}) = t^n f(\mathbf{v}) \qquad (1)$$

where $t$ is a real number. If $f$ has a differential at $\mathbf{v}$, then

$$\mathbf{v}\, \nabla_{\mathbf{v}} f(\mathbf{v}) = nf(\mathbf{v}). \qquad (2)$$

This is Euler's equation, and Euler deconvolution relies on solving it in appropriate cases. A field f which may be expressed in the form

$$F = A/r^N \qquad (3)$$

will be homogeneous of degree –N. For convenience we define the Structural Index (SI) as N (= -n, i.e. the negative degree of homogeneity).

The least ambiguous way to arrive at the true SI for any case is to solve for SI in Euler's equation itself (2).

But it should be sufficient to establish the index n in the fall-off with distance relation (3).

We now apply equation (2) to determine the SI's of some simple model bodies, the results shown in Table 1 can be obtained.

**Table 1 Structural Index for gravity**

| Source | Gravity SI |
|---|---|
| Sphere | 2 |
| Horizontal cylinder | 1 |
| Fault (small step) | 0 |

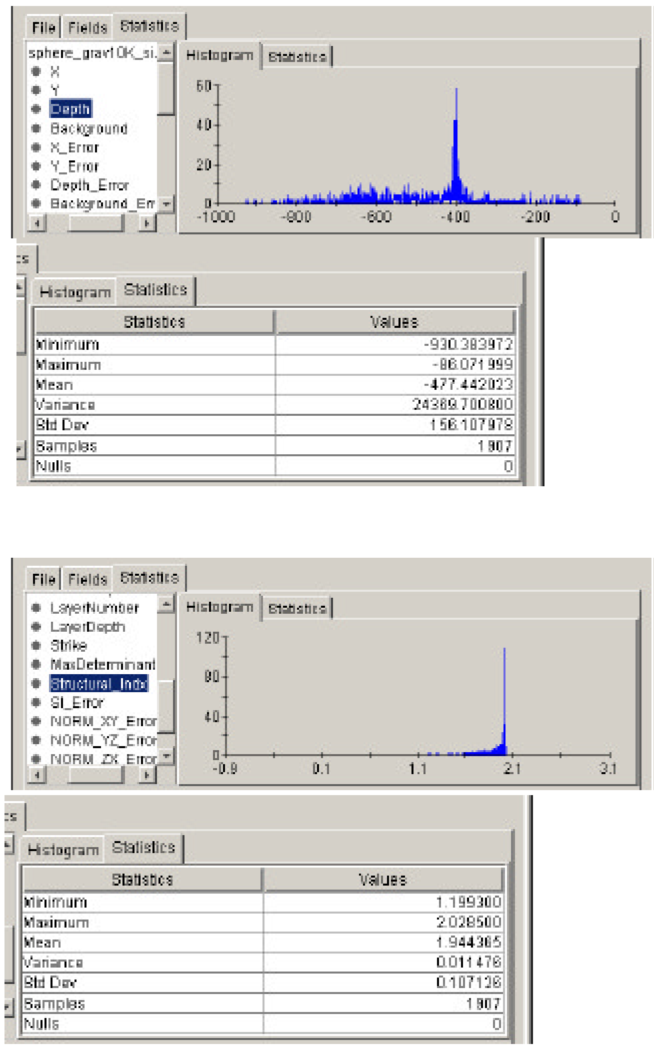
The small step case is derived by Reid et al. (1990), but their derivation contains some mathematic errors and should be disregarded. It has an SI of 1.0.

Other possible sources such as large steps give rise to fields which are are not strictly homogeneous and the application of Euler deconvolution to such cases is at best an approximation..

**Examples**

Recent extensions to the Euler method (Nabighian & Hansen 2001; Mushayandebvu et al., 2001, 2003) permit direct solution for some or all of SI, strike, dip and physical property (density or magnetization) contrast. Figure 1 shows the results of applying Euler deconvolution (solving for depth and SI) to the gravity field of a sphere whose depth to centre is 400m. Correct depth (400 m) and SI (2) are returned. Figure 2 shows the results from a contact. The returned depth to top is only approximate. The SI is returned as 0.5

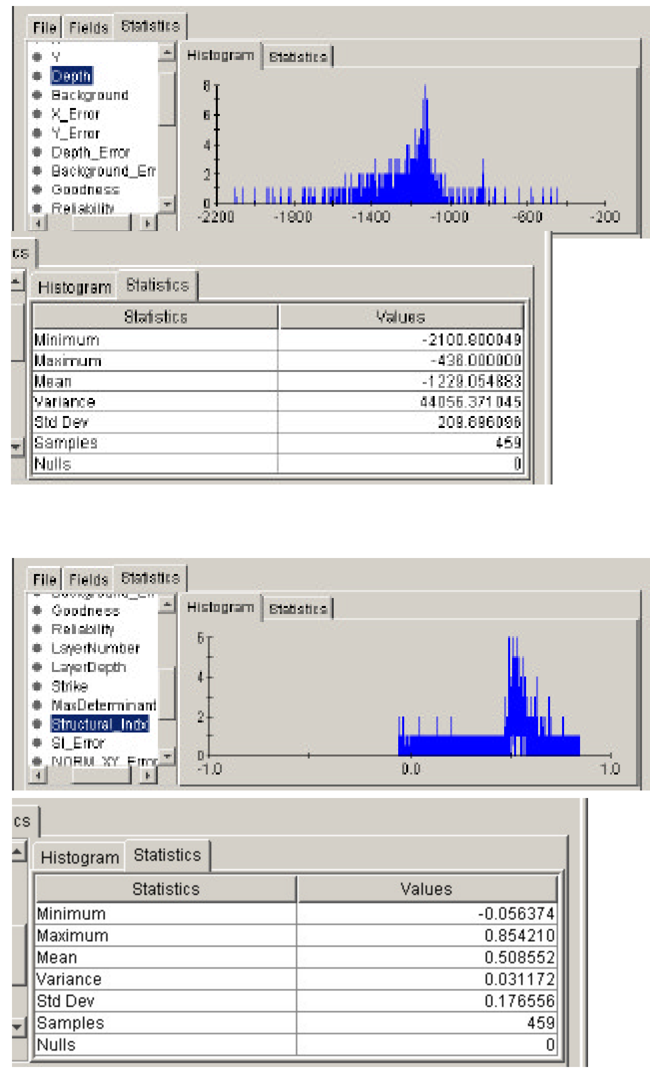
## Euler deconvolution of gravity data




| Statistics | Values |
|---|---|
| Minimum | -930.383972 |
| Maximum | -86.071999 |
| Mean | -477.442023 |
| Variance | 24369.700800 |
| Std Dev | 156.107978 |
| Samples | 1907 |
| Nulls | 0 |

| Statistics | Values |
|---|---|
| Minimum | 1.199300 |
| Maximum | 2.028500 |
| Mean | 1.944365 |
| Variance | 0.011476 |
| Std Dev | 0.107126 |
| Samples | 1907 |
| Nulls | 0 |

**Figure 1**. Euler deconvolution of gravity from a sphere at 400 m depth to centre. Depth and SI statistics.




| Statistics | Values |
|---|---|
| Minimum | -2100.800049 |
| Maximum | -438.000000 |
| Mean | -1229.054883 |
| Variance | 44056.371045 |
| Std Dev | 209.896096 |
| Samples | 459 |
| Nulls | 0 |

| Statistics | Values |
|---|---|
| Minimum | -0.056374 |
| Maximum | 0.854210 |
| Mean | 0.508552 |
| Variance | 0.031172 |
| Std Dev | 0.176556 |
| Samples | 459 |
| Nulls | 0 |

**Figure 2**. Euler deconvolution of gravity from a contact. Depth and SI statistics.

## Euler deconvolution of gravity data

Figure 3 shows the gravity of the Penola Trough, Otway Basin, offshore eastern South Australia. Euler deconvolution solved for source location and depth, SI, strike and the uncertainties in those parameters.

The linears displayed are thought to be normal faults at basement level (4 s TWT).

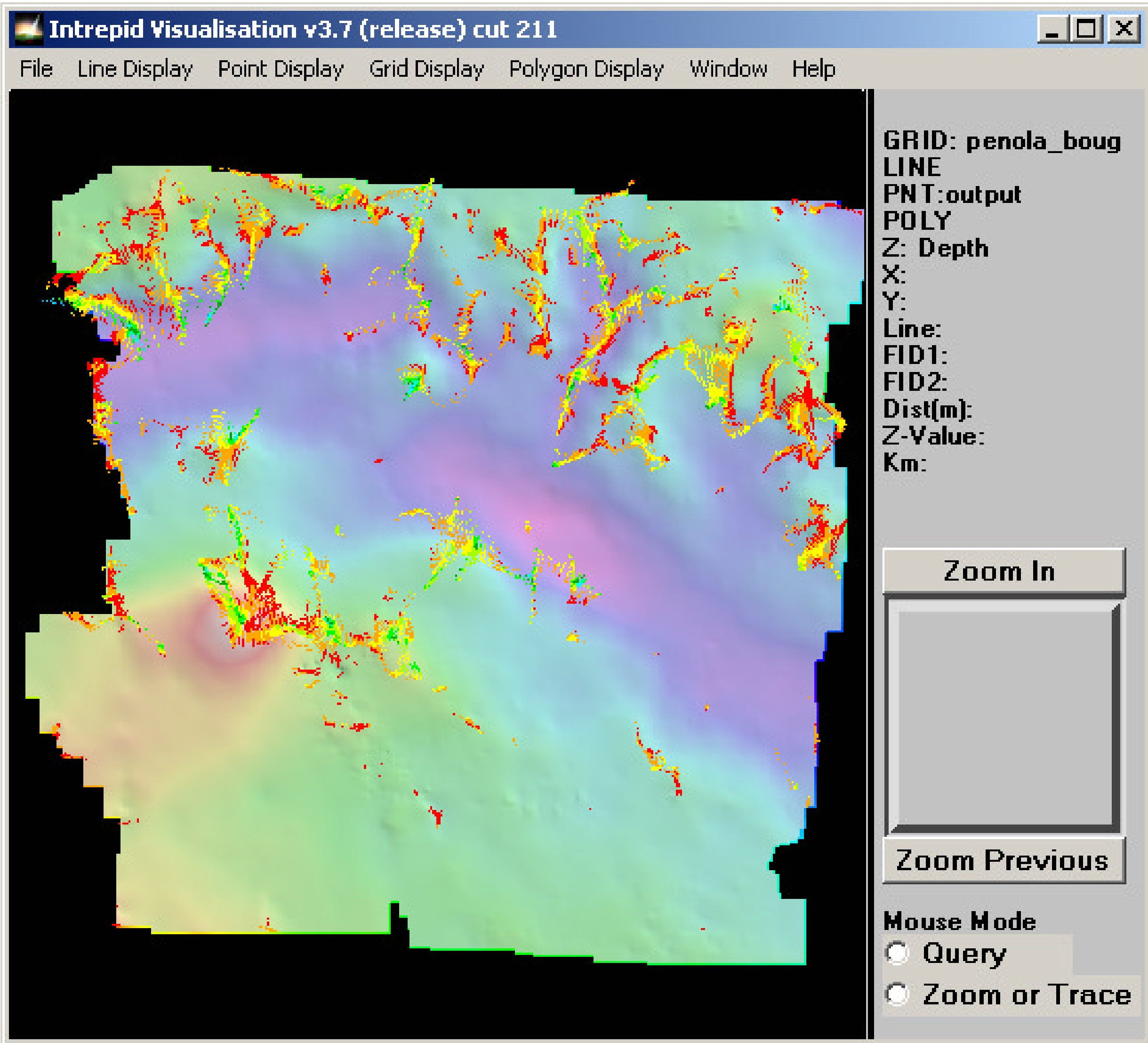
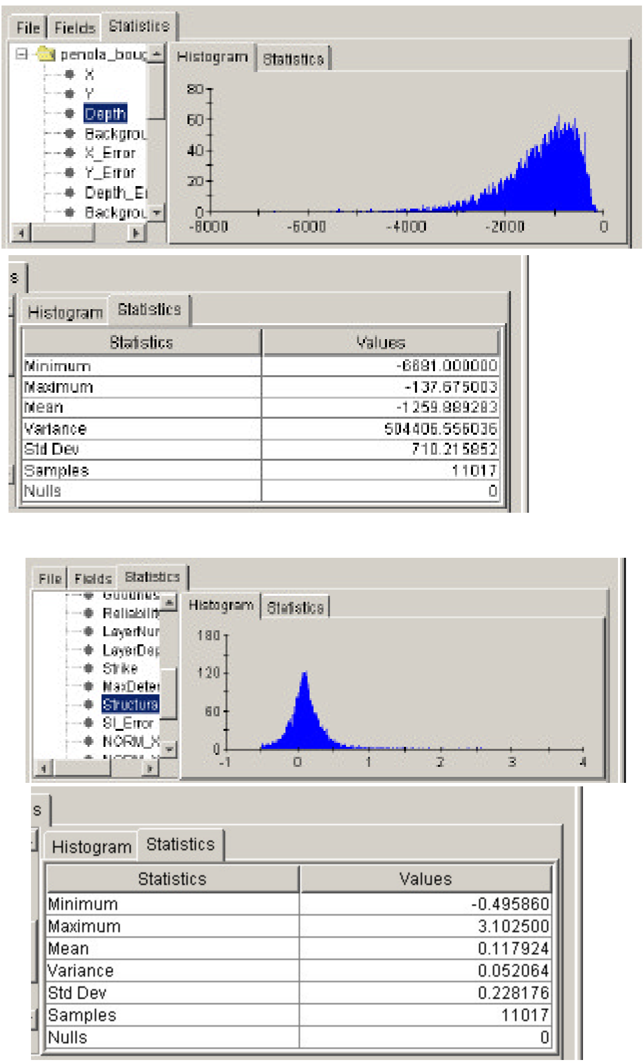



**Figure 3**. Euler deconvolution of gravity, Penola Trough, Otway Basin, offshore eastern S Australia. Area shown is 125 km NS and E-W, centred at about 140°40' E, 37°25'S. Depth range Red shallow, green deep. Strike is symbol orientation. Reliability is symbol size. Basement is at about 4 s TWT.

# Euler deconvolution of gravity data

Figure 4 shows the statistics on the Penola Trough solution depths and SIs. The ability to solve for SI is still new enough that we are still learning what they show us, especially with gravity data. Here the mean SI is about 0.1. We suggest that the SI tail below zero represents estimation uncertainty.




| Statistics | Values |
|---|---|
| Minimum | -6881.000000 |
| Maximum | -137.675003 |
| Mean | -1259.889283 |
| Variance | 504406.556036 |
| Std Dev | 710.215852 |
| Samples | 11017 |
| Nulls | 0 |



| Statistics | Values |
|---|---|
| Minimum | -0.495860 |
| Maximum | 3.102500 |
| Mean | 0.117924 |
| Variance | 0.052064 |
| Std Dev | 0.228176 |
| Samples | 11017 |
| Nulls | 0 |

**Figure 4**. Euler deconvolution statistics from Penola trough gravity data. Solution depth and structural index.

**Conclusions**

Euler deconvolution can be applied to gravity data. For simple bodies the SI is one less than the SI for the equivalent magnetic bodies. The contact of significant depth extent does not give rise to a homogeneous gravity field and the Euler method is therefore at best an approximation in that case. Geologically plausible results are obtained with real data. Work still requires to be done to understand SI values estimated by the process in these conditions.

**Acknowledgements**

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