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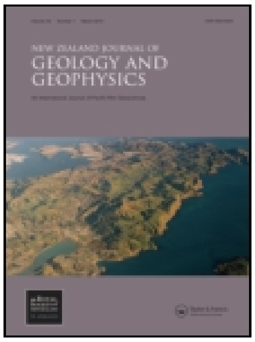

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HEAT FLOW AT ARRIVAL HEIGHTS, ROSS ISLAND, ANTARCTICA

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ABSTRACT

A study of temperature measurements taken over an interval of 2·4 years in a 7·6 m deep hole on Hut Point Peninsula (Ross Island) has shown that the vertical heat flux is 164 ± 60 mW/m^2 (4 × 10^{-6} cal/cm^2 s) which is about 2·5 times greater than the world average heat flux. The result has been obtained by fitting simultaneously all data to a series of harmonic functions using a least-squares method of curve fitting. The thermal effect of Quaternary basaltic extrusions which cover the peninsula is too small to explain the observed flux.

INTRODUCTION

The heat flux at the surface of the earth is caused by the conductive transfer of heat generated mostly by the decay of radioactive minerals in the earth's crust. If non-conductive heat transfer and the influence of near surface heat sources are small, the heat flux Φ can be computed from the vertical temperature gradient $a = \Delta T/\Delta z$ and the thermal conductivity λ of the rocks in which the gradient has been measured, since $\Phi = \lambda\, a$.

Only a few measurements of the natural heat flux have been published for Antarctica. This is, without doubt, related to various problems one faces in making such investigations on this continent. Temperatures in the ice over most of the interior, for example, are disturbed by movement of the ice sheet which often causes melting at the ice-rock interface and upsets the vertical temperature gradient due to the natural heat flux. Undisturbed temperature gradients, however, can be measured in holes drilled into the permanently frozen ground in the ice-free coastal strips where heat flux studies can also be related to particular geological problems. Such holes can be shallow since lateral transport of heat by meteoric water does not occur. Because of this, heat flux studies in permafrost ground are less difficult than those on land in warmer climates where the movement of ground water usually upsets the normal temperature gradient.

In this paper a heat flow study is described, based on results obtained from the analysis of temperature measurements made between 1966 and 1968, in a shallow drillhole in ice-free ground on Ross Island, Antarctica. The study was made to check whether an unusually high heat flux at Scott Base reported by Robertson & Macdonald (1962) might occur elsewhere under the island and be connected with a cooling intrusion at shallow depth.

Ross Island is a volcanic island which has been active presumably since the Tertiary, although its present activity is restricted to fumaroles on top of the main peak, Mt Erebus. The volcanic activity, however, was much greater about 0·5 to 1 m.y. ago as indicated by the presence of many volcanic cones and lava flows in the south-western part of the island on Hut Point Peninsula (Wellman 1964; Cole *et al.* 1971; Dry Valley Drilling Project 1973).

SITING OF THE HOLE AT ARRIVAL HEIGHTS

The first attempt to measure the heat flux on Hut Point Peninsula was made by Robertson & Macdonald (1962). Their analysis of temperature measurements made in a 4·6-m-deep hole at Scott Base (Fig. 1) over a period of nearly one year implied that the flux was about 1·7 W/m^2 (40 $\times$ 10^{-6} cal/cm^2 s), i.e., about 27 times the accepted figure of the world average heat flow (Lee & Uyeda 1965; von Herzen & Lee 1969). Robertson and Macdonald inferred from their result the presence of a hot intrusive body beneath the region.

Unusually high temperature gradients have also been found on the western side of McMurdo Sound in Lake Vanda (Wilson & Wellman 1962) and in Lake Bonney (Shirtcliffe 1964). These gradients, however, are apparently not of geothermal origin but are caused by solar heating of the lakes. Temperature measurements by Nichols & Ball (1964) in a 14·5-m-deep hole at Marble Point were unsuitable for determining the heat flux.

Hence, when we proposed a heat flux study on Hut Point Peninsula in 1965, we did not know whether high heat flow occurred throughout the region. Since it was possible that the ground temperatures at Scott Base were affected by the sea, the hole for our study was sited on a small plateau near Arrival Heights (Fig. 1) about 1·1 km away from the shore.

Heat Transfer from the Sea

In Antarctica the temperatures in a drillhole near the sea are affected by a local steady-state heat flow transferring heat from the sea to the land (Fig. 2a) and probably also by the infiltration of sea water (Fig. 2c). The model shown in Fig. 2b can be used to estimate the magnitude of the heat flux from sea to land. In this model the bottom of the sea, represented by half plane $\theta = \pi$, is kept at the constant temperature T_1 while the land, represented by the half plane $\theta = 0$, is at temperature T_2. Assuming that transient effects such as fluctuations of sea-level are small, the steady-state solution for the temperature T at point P is given by:

$$T(r, \theta) = T_2 + (T_1 - T_2)\,\theta/\pi \tag{1}$$

(Mackie 1965, p. 91). The vertical temperature gradient at the land surface at Q is then:

$$\partial T/\partial z = \partial T/r\partial\theta = (T_1 - T_2)/r\pi, \text{ (for } \theta = 0) \tag{2}$$

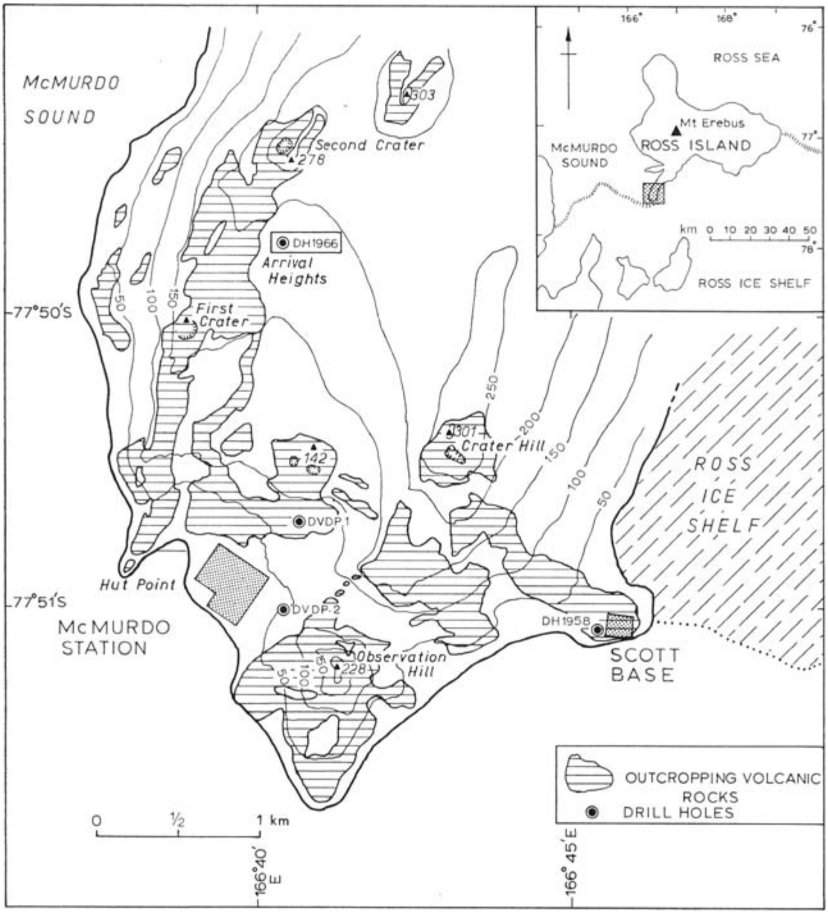

FIG. 1—Map of Hut Point Peninsula showing the location of the drillhole at Arrival Heights (DH1966) in which the heat flux has been determined. The location of the hole at Scott Base (DH1958, Robertson & Macdonald 1962) is also shown as well as the location of the two deep wells (DVDP-1 and DVDP-2) of the Dry Valley Drilling Project. Spot heights and contours give the elevation in metres (the exact location of the front of the Ross Ice Shelf is uncertain). The location of Hut Point Peninsula in relation to Ross Island is shown by the shaded area in the inset.

In the vicinity of Ross Island, the temperature of the sea T_1 is close to the freezing point of sea water, viz., $-1{\cdot}9°$C (Gilmour 1963), whereas the average surface temperature of the land T_2 is about $-19°$C (see Table 1). For the hole at Arrival Heights (Fig. 1) equation (2) gives, with $r =$ 1100 m, a gradient of $4{\cdot}9 \times 10^{-3}$ °C/m which is only about one-tenth of that caused by the world average flux of 63 mW/m² ($1{\cdot}5 \times 10^{-6}$ cal/cm² s) in basaltic rocks with a thermal conductivity of 1·7 Wm/m² °C, a value measured on a core from this hole (see Table 2). Hence, the influence of the heat flux shown in Fig. 2a is small at Arrival Heights and will be neglected in the analysis given later in this paper. The effect is much greater in the drill hole at Scott Base (Fig. 1) where, with $r = 60$ m, the vertical gradient is $9{\cdot}1 \times 10^{-2}$ °C/m, although this is still significantly smaller than that of 0·75°C/m observed by Robertson & Macdonald (1962).

On shore in the immediate vicinity of the sea the temperature gradient can be greater if some infiltration of sea water into the permafrost ground takes place (Fig. 2c). The lateral extent of the infiltration is probably limited. The gradient $\Delta T/\Delta z$ in a hole at a distance h above such an infiltration is $(T_1 - T_2)/h$. For the drillhole at Scott Base in which Robertson & Macdonald took their measurements the elevation of the surface above sea level is 17 m. If h is given this value a gradient of 1·0°C/m is obtained which is of the same order of magnitude as the observed gradient of 0·75 °C/m. The difference might be due to the limited width of the infiltration zone which might not extend further than about halfway between the shore and the hole or it might indicate that the top of the infiltration zone is below sea level beneath the hole. If this interpretation holds it can be assumed that the temperatures in the hole at Arrival Heights are not affected by such near-shore infiltration.

TEMPERATURE MEASUREMENTS IN THE HOLE AT ARRIVAL HEIGHTS

The hole was drilled in January 1966 at 77° 49·8′ S; 166° 40·5′ E on a plateau at an elevation of 180 m above sea level (Fig. 1). The planned depth of the hole was 50 m, the limit of the small drilling rig which was available for the project. However, owing to a series of breakdowns the drilling operations ceased at a depth of 7·6 m. Because of the strong influence of surface temperature variations it was not possible to determine the geothermal flux from a single set of temperature measurements and it became necessary to make sets of temperature measurements over a longer period.

After completion of drilling at the end of January 1966, the hole was left for three months to allow the temperature disturbances caused by the drilling to be dissipated. Six thermocouples of copper-constantan were then installed in the hole at depths of 1·52, 2·74, 3·96, 5·18, 6·40 and 7·47 m, and the temperatures were read at approximately monthly intervals between 1966 April 14 and 1968 September 10. The accuracy of the measurements was about ±0·2°C (Hochstein & Risk 1967). The data are plotted against time in Fig. 3.

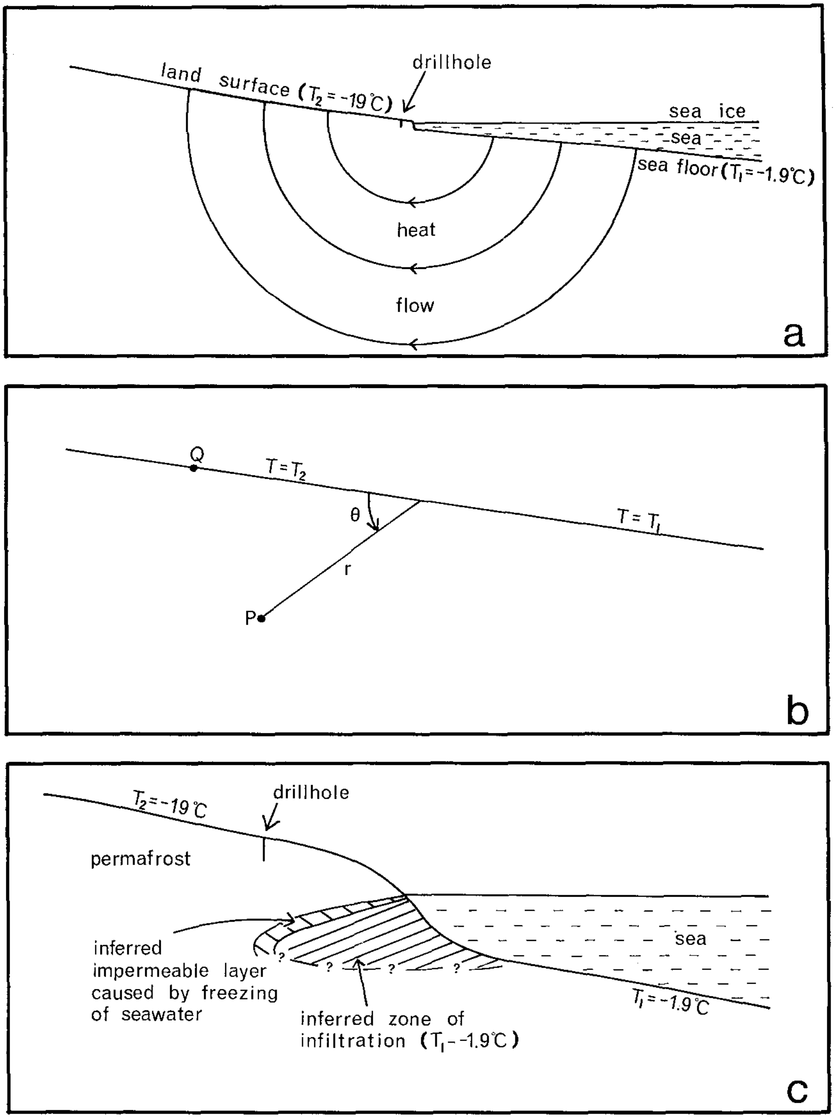

FIG. 2—Heat transfer from sea to land near the shore on Ross Island. a—Model illustrating the mechanism of a steady state heat transfer. b—Simplified model used for computing the vertical temperature gradient at point Q. c—Model illustrating heat transfer due to a limited infiltration of sea water into the land

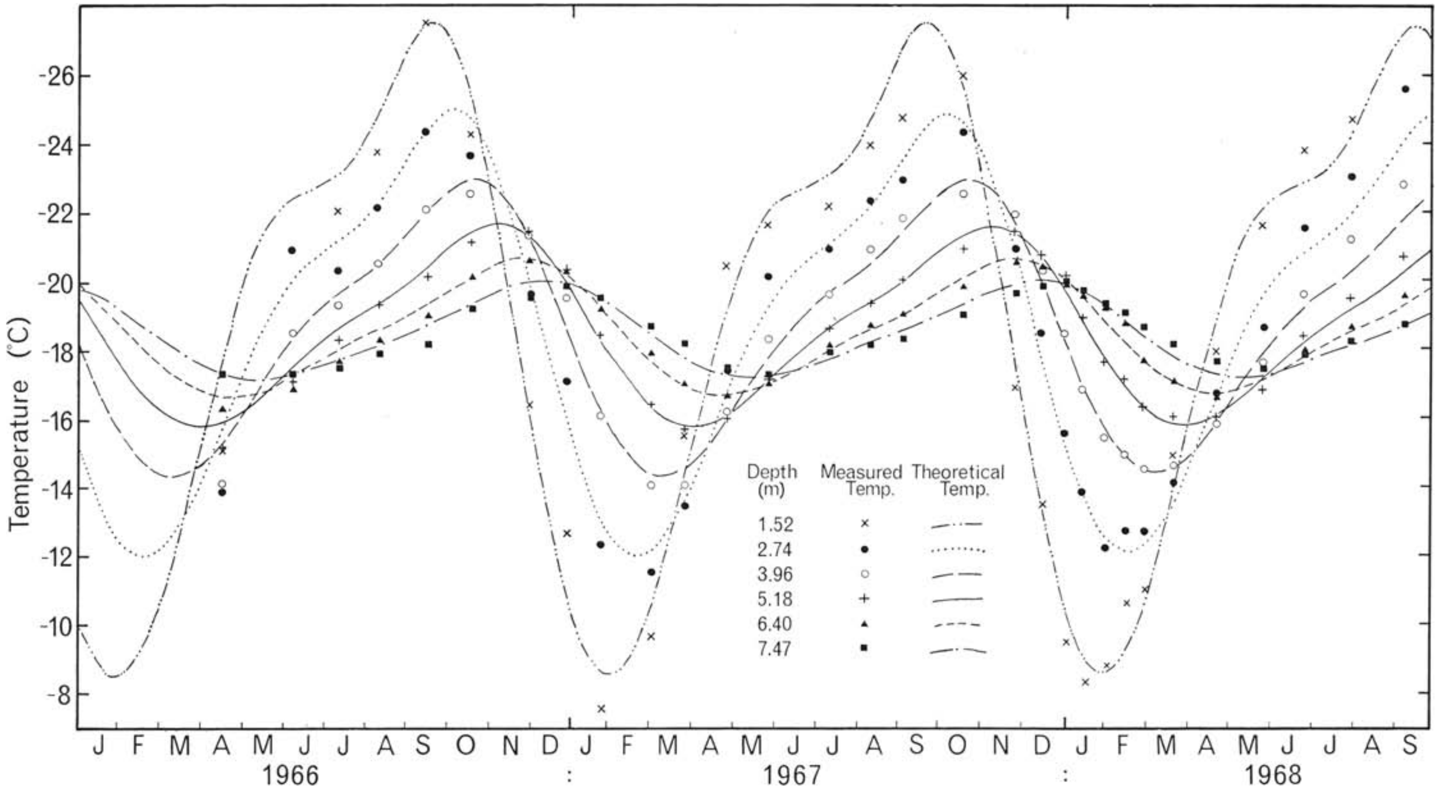

FIG. 3—Plot of measured and best fit theoretical temperatures against time for the hole at Arrival Heights between 1966 and 1968. The temperatures observed at different depths are shown by point symbols, the best fit theoretical temperatures at these depths are shown by solid and broken lines and were obtained from solution 1 (Table 1).

The analysis of long term temperature data in shallow holes in terms of an average temperature gradient and local heat flux has, apparently, seldom been attempted. It will be shown in the following that, for a shallow hole in permafrost ground, such an analysis gives meaningful results, especially if it is combined with an analysis of the resulting error.

ANALYSIS OF TEMPERATURES IN THE ARRIVAL HEIGHTS HOLE

Heating during summer and cooling during winter causes the near-surface ground temperatures on Ross Island to fluctuate with a peak to peak amplitude of more than 25°c. As can be seen from Fig. 3, the wave form is not exactly sinusoidal since there is a rather rapid warming from October until January followed by a more gradual cooling during the rest of the year. Assuming that the temperature variations are the same, year after year, the temperature at the surface ($z = 0$) can be expressed by a Fourier series of the form:

$$T(0, t) = T_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \varepsilon_n) \tag{3}$$

where T_0 is the average surface temperature, t the time (in days), A_n and ε_n the amplitude and the phase respectively of the n^{th} Fourier component, and $\omega = 2\pi/365{\cdot}25$ rad/day is the fundamental radian frequency, corresponding to a period of one year.

The temperature $T(z, t)$ at depth z (Carslaw & Jaeger 1959) is given by:

$$T(z, t) = az + T_0 + \sum_{n=1}^{\infty} A_n \exp(-kz\sqrt{n}) \cos(n\omega t + \varepsilon_n - kz\sqrt{n}) \tag{4}$$

where $a = \Delta T/\Delta z$ is the vertical temperature gradient, $k = (\omega/2\alpha)^{\frac{1}{2}}$ and α the thermal diffusivity of the rocks. If the density ρ and the specific heat capacity c are known, the thermal conductivity λ can be calculated from the relation $\lambda = \rho\alpha c$. Hence the vertical heat flux can be obtained.

Methods of Analysis Used Elsewhere

Several methods have been used to obtain numerical values for a, k and α from the analysis of long-term temperature observations in shallow drill holes.

One of the simplest methods is described, for example, in Ingersoll *et al.* (1955), in which the assumptions are made that k is constant and that all the Fourier components in equation (4) can be neglected with the exception of the fundamental component ($n = 1$). The temperature decreases then with depth z as $\exp(-zk)$ and the temperature extrema show a phase lag at depth z of zk from each of which k can be computed. The effect of the annual temperature cycle at depth z can be subtracted using equation (4) and the average temperature gradient a and hence the heat flux Φ determined.

We tried this method, but the values of k obtained from the decay of the temperature with depth were significantly different from those obtained from the analysis of the phase lag. The reason for this is that the annual temperature variations at Arrival Heights cannot be approximated by a single sinusoidal component (see Fig. 3).

Another method is the graphical method after Schmidt, also cited in Ingersoll *et al.* (1955) which was used by Robertson & Macdonald (1962) in their analysis. This method is applicable if the initial temperature distribution with depth as well as any subsequent changes of the temperatures at the surface are known. The temperature at any depth can then be calculated as a function of time. For the inverse problem, however, a trial and error approach must be used which often does not give satisfactory results. In addition, any estimation of the resultant error is difficult, a problem common to most graphical methods.

A more refined method has been described by Lettau (1951) and has been used for the analysis of daily temperature measurements in soils (Lettau 1954; Stearns 1969) and annual temperature variations in ice sheets (Weller 1967) although the changes of the parameters k and α were the object of these studies, rather than the derivation of the geothermal flux Φ. In this method the temperature variations at a certain depth are analysed in terms of a Fourier series and the amplitudes and phases of each of the Fourier components as well as the average temperature are obtained. Similar, but separate analyses are made of the temperature variations at other depths from which the temperature gradient and thus the heat flux Φ can be derived.

Method of Analysis Used Here

Since the temperature variations at the different depths are dependent on each other we went one step further than Lettau (1951; 1954) and analysed all data at all depths simultaneously. A least squares fitting procedure was used to obtain the best estimates of the various parameters in equation (4). The procedure is described in detail in the Appendix. In addition, the variance of the best estimates as well as the covariances and the coefficients of correlation between the parameters were computed.

For the analysis it was assumed that: (1) k is independent of depth, temperature and time; (2) temperature variations occurs only in the z-direction; (3) the surface temperatures have a fundamental period of one-year; (4) heat is transferred only by conduction.

Assumptions (1) and (3) do not hold exactly. The thermal diffusivity α, for example, is not constant with depth, although changes in thermal constants are, in general, quite small in solid rocks (Clark 1966). It can also be assumed that variations in k are random and that, in bulk, the basaltic rocks at Arrival Heights have a constant value of k. To check assumption (3), the monthly mean air temperatures for 1966 to 1968 at Scott Base were compared. It was found that the mean air temperature for a given month can vary from year to year by as much as $\pm 4°$C. However, the variation of the monthly temperatures can be approximated by a short-period Fourier component the amplitude of which is rapidly attenuated with depth and will not affect the temperatures at greater depths in the hole. Assumptions (2) and (4) are fulfilled since the hole is located on a plateau and stands in permafrost ground which prohibits other modes of heat transfer. Hence, the violations of the assumptions used in our analysis are, on the whole, not too serious.

A total of 178 temperature measurements made over an interval of 2·4 years were fitted to equation (4) by applying the procedure described in the Appendix and using an Elliott 503 computer. Several constraints had to be applied to the fitting process. It was found, for example, that the summation in equation (4) had to be truncated at $n = 6$, since A_n and ε_n could not be determined uniquely for $n \geqslant 6$ and therefore only solutions with 3, 4, and 5 Fourier components were computed. A variety of solutions were also obtained by using subsets of the data to check whether better estimates of the various parameters could be found from data measured over intervals of one and two years rather than the arbitrary interval of 2·4 years. Besides the fundamental period of one year, periods of two and three years were also tried.

DISCUSSION OF THE SOLUTIONS

Some of the solutions thus obtained are listed in Table 1. For solution 1 the entire set of data, i.e., interval of 2·4 years, was used and three Fourier components were analysed. A surprising result is that the temperature gradient a of this solution is about 20% smaller than that of a similar analysis for which data measured during an interval of 2 years were used (solution 4). Hence, the additional portion of data used produces significant changes in both a and T_0, which implies that the method is sensitive to the choice of a particular subset of data. To check this, various sets of temperatures recorded during different intervals of about one year each were analysed. Solution 6 is listed as an example showing extreme values for a and T_0 which can be obtained from the analysis of such intervals. The changes in a and T_0 are likely to have been caused by short-period changes in near-surface temperature variations which differ from one year to another. The problem, however, can be overcome in part by using, for the analysis, temperature sets observed over long intervals since it can be assumed that the irregular temperature variations will tend to cancel. Hence, solutions 1 and 4 are acceptable for further discussion whereas solution 6 can be rejected.

For solutions 2, 3, and 4, data from the two-year interval 1966 October 18 until 1968 September 10 were analysed and the number of the Fourier components was restricted to 5, 4, and 3 respectively. A comparison of the parameters a, T_0, and k of these solutions listed in Table 1 shows that the inclusion of the higher order components has a very small effect on the magnitude and the standard deviation of these parameters. Each of the solutions is, therefore, acceptable and is in turn preferable to solution 1 where the extra weighting of the winter temperatures between 1966 April 4 and 1966 October 17 has adversely affected the fit as indicated by the greater root mean square deviation S.

TABLE 1—Solutions for the theoretical parameters obtained by using *various* subsets of the *total* set of *data* and various values for N

	Solution No.	1	2	3	4	5	6
	No. of Fourier components	3	5	4	3	3	4
	Data included for analysis	14·4·66–10·9·68 all depths (all data)	18·10·66–10·9·68 all depths	18·10·66–10·9·68 all depths	18·10·66–10·9·68 all depths	18·10·66–10·9·68 all depths except $z = 1{\cdot}52$ m	17·10·67–10·9·68 all depths
Temperature gradient	a (°C/m)	$0{\cdot}0581 \pm 0{\cdot}023$	$0{\cdot}0726 \pm 0{\cdot}023$	$0{\cdot}0718 \pm 0{\cdot}023$	$0{\cdot}0710 \pm 0{\cdot}024$	$0{\cdot}0797 \pm 0{\cdot}019$	$0{\cdot}1258 \pm 0{\cdot}020$
Surface temperature	T_o (°C)	$-18{\cdot}96 \pm 0{\cdot}11$	$-19{\cdot}11 \pm 0{\cdot}11$	$-19{\cdot}11 \pm 0{\cdot}12$	$-19{\cdot}10 \pm 0{\cdot}12$	$-19{\cdot}15 \pm 0{\cdot}10$	$-19{\cdot}45 \pm 0{\cdot}10$
k $(= \sqrt{\omega/2\alpha})$	k (m^{-1})	$0{\cdot}310 \pm 0{\cdot}007$	$0{\cdot}314 \pm 0{\cdot}007$	$0{\cdot}314 \pm 0{\cdot}007$	$0{\cdot}313 \pm 0{\cdot}007$	$0{\cdot}322 \pm 0{\cdot}007$	$0{\cdot}320 \pm 0{\cdot}006$
Amplitudes of Fourier components*	A_1 (°C)	$13{\cdot}29 \pm 0{\cdot}30$	$13{\cdot}45 \pm 0{\cdot}29$	$13{\cdot}41 \pm 0{\cdot}29$	$13{\cdot}33 \pm 0{\cdot}30$	$14{\cdot}05 \pm 0{\cdot}39$	$14{\cdot}55 \pm 0{\cdot}26$
	A_2 (°C)	$5{\cdot}48 \pm 0{\cdot}29$	$5{\cdot}60 \pm 0{\cdot}28$	$5{\cdot}59 \pm 0{\cdot}28$	$5{\cdot}59 \pm 0{\cdot}29$	$5{\cdot}32 \pm 0{\cdot}33$	$5{\cdot}21 \pm 0{\cdot}26$
	A_3 (°C)	$1{\cdot}01 \pm 0{\cdot}33$	$0{\cdot}65 \pm 0{\cdot}33$	$0{\cdot}54 \pm 0{\cdot}33$	$0{\cdot}35 \pm 0{\cdot}33$	$0{\cdot}50 \pm 0{\cdot}40$	$1{\cdot}17 \pm 0{\cdot}29$
	A_4 (°C)	—	$1{\cdot}27 \pm 0{\cdot}41$	$1{\cdot}13 \pm 0{\cdot}40$	—	—	$1{\cdot}67 \pm 0{\cdot}34$
	A_5 (°C)	—	$0{\cdot}90 \pm 0{\cdot}45$	—	—	—	—
Date of maximum temperature*	ε_1 (day)	Jan 13 ± 1	Jan 11 ± 1	Jan 11 ± 1	Jan 12 ± 1	Jan 9 ± 2	Jan 12 ± 1
	ε_2 (day)	Dec 28 ± 2	Dec 26 ± 2	Dec 26 ± 2	Dec 27 ± 2	Dec 28 ± 2	Dec 18 ± 1
	ε_3 (day)	Mar 16 ± 6	Mar 10 ± 9	Mar 12 ± 12	Mar 13 ± 18	Jan 19 ± 16	Mar 22 ± 5
	ε_4 (day)	—	Jan 19 ± 4	Jan 18 ± 5	—	—	Jan 10 ± 3
	ε_5 (day)	—	Dec 25 ± 6	—	—	—	—
Thermal diffusivity	α (10^{-6}m^2/s)	$1{\cdot}04 \pm 0{\cdot}05$	$1{\cdot}01 \pm 0{\cdot}05$	$1{\cdot}01 \pm 0{\cdot}05$	$1{\cdot}02 \pm 0{\cdot}05$	$0{\cdot}958 \pm 0{\cdot}05$	$0{\cdot}97 \pm 0{\cdot}05$
Heat flux	Φ (mW/m^2)	130 ± 60	157 ± 60	155 ± 60	155 ± 60	164 ± 60	263 ± 60
Root mean square deviation	S (°C)	$0{\cdot}621$	$0{\cdot}555$	$0{\cdot}564$	$0{\cdot}579$	$0{\cdot}350$	$0{\cdot}371$

*The periods of the Fourier components with parameters (A_1, ε_1), (A_2, ε_2), (A_3, ε_3), (A_4, ε_4), (A_5, ε_5) are respectively, 1, 1/2, 1/3, 1/4, 1/5 year.

The influence of irregular temperature variations with short periods was further restricted in solution 5 by omitting temperatures observed at the depth $z = 1{\cdot}52$ m from the analysis, since these variations are attentuated more rapidly with depth than, for example, the fundamental annual variation. Otherwise the same data and the same number of components ($n = 3$) were used as for solution 4. It can be seen from Table 1 that the values of the parameters a, T_0 and k and their standard deviations are about the same for both solutions but the root mean square deviation S of the differences between theoretical and observed temperatures of solution 5 is significantly smaller than that of solution 4. Hence, solution 5 gives a better fit and is chosen as the best of all solutions discussed.

The best fit parameters of solution 5 imply that the temperature gradient at Arrival Heights is $0{\cdot}0797 \pm 0{\cdot}019$°c/m and the average thermal diffusivity of the volcanic rocks $0{\cdot}958 \pm 0{\cdot}05 \times 10^{-6}$ m²/s. Using the dry density and the heat capacity measured on a core (Table 2), the geothermal heat flux becomes 164 ± 60 mW/m² (4×10^{-6} cal/cm²s) which is about 2·5 times the world average heat flux.

TABLE 2—Measured physical properties of a core sample of basalt from Arrival Heights (7·1 to 7·6 m in depth)

Property	Value	Unit
Dry density	2·85	Mg/m³
Wet density	2·90	Mg/m³
Particle density	3·00	Mg/m³
Porosity	0·05	—
Thermal heat capacity per unit mass	754	J/kg °c
Thermal diffusivity	0·88	10^{-6} m²/s
Thermal conductivity	1·73	W/m °c

An independent check of the computed thermal diffusivity α is given by comparing it with that obtained from measurements of various thermal constants of a 0·5 m long basalt core from the bottom of the hole (Table 2). It was found that the computed best fit diffusivity ($\alpha = 0{\cdot}96 \times 10^{-6}$m²/s), which is representative of the rock in bulk, is similar to that measured on a core from the bottom ($\alpha = 0{\cdot}88 \times 10^{-6}$m²/s). The difference might be caused, in part, by lenses of ice which occurred in the hole and which would tend to increase the average diffusivity since α of ice is $1{\cdot}15 \times 10^{-6}$m²/s at 0°c (Pounder 1965). The similarity of computed and observed diffusivities supports the assumption made previously that k, and hence α, are approximately constant with depth.

The procedure used for the data analysis also allowed that estimates of the correlation coefficients between the various parameters could be determined. It was found that some of these parameters are not independent of each other. As an example, the matrix of the correlation coefficients for solution 1 is given in Table 3 which shows that there is a significant correlation between the temperature gradient a and the surface temperature

TABLE 3—Matrix of correlation coefficients obtained for solution 1

	a	T_o	k	A_1	A_2	A_3	ε_1	ε_2	ε_3
a	1·00	−0·91	0·04	0·06	0·12	0·08	0·00	−0·04	−0·07
T_o	−0·91	1·00	−0·01	−0·05	−0·13	−0·10	0·02	0·04	0·08
k	0·04	−0·01	1·00	0·77	0·37	0·12	0·78	0·38	0·10
A_1	0·06	−0·05	0·77	1·00	0·31	0·15	0·57	0·28	0·07
A_2	0·12	−0·13	0·37	0·31	1·00	0·08	0·22	0·13	−0·01
A_3	0·08	−0·10	0·12	0·15	0·08	1·00	0·09	0·02	0·01
ε_1	0·00	0·02	0·78	0·57	0·22	0·09	1·00	0·26	0·15
ε_2	−0·04	0·04	0·38	0·28	0·13	0·02	0·26	1·00	0·10
ε_3	−0·07	0·08	0·10	0·07	−0·01	0·01	0·15	0·10	1·00

T_o. The same holds for the parameters k and A_1 and ε_1. The accuracy with which a, α, and hence Φ can be determined is therefore limited even if measurements over a longer time interval were available.

DISCUSSION OF THE HEAT FLUX AT HUT POINT PENINSULA

The heat flux of 164 ± 60 mW/m^2 at Arrival Heights is an order of magnitude smaller than that given by Robertson & Macdonald (1962) for Scott Base. It can be inferred, however, that their value is not representative for the heat flux coming from greater depths but is disturbed by some heat transfer from the sea which has been discussed in the beginning of this paper. The heat flux at Arrival Heights is of the same order of magnitude as that obtained in 1967 by Cousins and Christoffel from measurements on the sea floor in McMurdo Sound near Ross Island (D. A. Christoffel, pers. comm.). Our temperature gradient of 0·08°C/m is also of the same order of magnitude as that implied by conditions in the lower halves of two deep wells (DVDP1 and 2) which were drilled in 1972 on Hut Point Peninsula (Fig. 1). Although no reliable temperatures have been measured so far in these wells, the presence of ice lenses at a depth of about 165 m in hole DVDP2 (Dry Valley Drilling Project 1973) implies temperatures below the freezing point at this depth from which a temperature gradient $a<0{\cdot}115$°C/m can be inferred.

Assuming that the heat flux of 164 mW/m^2 at Arrival Heights is representative for the flux elsewhere under Hut Point Peninsula, the origin of this flux remains to be discussed. The value is higher than the average flux of 90 m W/m^2 observed in areas with recent volcanism (Lee & Uyeda 1965) and it is possible that it contains a residual flux which is a relict of the Quaternary volcanism on the peninsula.

The drill site at Arrival Heights is only about 0·6 km distant from a volcanic cone (see Fig. 1). Assuming that the volcanic rocks at Arrival Heights were emplaced in the form of a coherent lava sheet about 4×10^5 years ago—the time of the most recent activity on the peninsula (R. L. Armstrong, pers. comm.)—an order of magnitude estimate can be given for the present-day temperature gradient of the sheet. For this we can use an equation given by Carslaw & Jaeger (1959, p. 62) which describes the temperature field of a two-dimensional sheet of thickness d as a function of time t. For the asymptotic case when $t >> d^2/4\alpha$, i.e., a long time after the eruption, this equation can be simplified to give the following expression for the temperature gradient at the surface of the sheet:

$$\partial T/\partial z|_{z=0} \simeq T_0 d^2/4(\pi\alpha^3 t^3)^{\frac{1}{2}} \tag{5}$$

where T_0 is the initial temperature of the sheet at $t = 0$. Using realistic data: $d < 150$ m, $T_0 < 1700°$C (allowing for the latent heat of the lava), $\alpha = 1 \times 10^{-6}$ m²/s and $t > 4 \times 10^5$ years, the residual gradient is less than $0{\cdot}1 \times 10^{-3}$°C/m which is insignificant compared with the observed gradient of about 80×10^{-3}°C/m. The thermal effects of any basaltic intrusive feeder pipes or dykes is presumably also insignificant because such features are usually quite narrow.

Since the observed heat flux cannot be explained by the Quaternary volcanism of the area, either recent climatic changes or heat sources in in deeper parts of the crust (magmatic bodies, concentrations of radioactive minerals) are left as the likely causes.

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REFERENCES

ANDERSSEN, R. S. 1969: On the solution of certain overdetermined systems of linear equations that arise in geophysics. *Journal of Geophysical Research 74*: 1045–51.

CARSLAW, H. S.; JAEGER, J. C. 1959: "Conduction of heat in solids". 2nd ed. Clarendon Press, Oxford. 510 p.

CLARK, S. P. (Ed.) 1966: "Handbook of physical constants". *Geological Society of America Memoir 97.*

COLE, J. W.; KYLE, P. R.; NEALL, V. E. 1971: Contributions to Quaternary geology of Cape Crozier, White Island and Hut Point Peninsula, McMurdo Sound Region, Antarctica. *N.Z. Journal of Geology and Geophysics 14* (3): 528–46.

DRY VALLEY DRILLING PROJECT (DVDP) 1973: Bulletin No. 2, Northern Illinois University.

GILMOUR, A. E. 1963: Hydrological heat and mass transport across the boundary of the ice shelf in McMurdo Sound, Antarctica. *N.Z. Journal of Geology and Geophysics 6* (3): 402–22.

HOCHSTEIN, M. P.; RISK, G. F. 1967: Geophysical measurements on the McMurdo Ice Shelf, Antarctica, during 1965–66. *Geophysics Division Report No. 47.* DSIR, Wellington. 51 p.

INGERSOLL, L. R.; ZOBEL, O. J.; INGERSOLL, A. C. 1955: "Heat Conduction". Thames and Hudson, London. 325 p.

LEE, W. H. K.; UYEDA, S. 1965: Review of heat flow data. Pp. 87–190 *in* LEE, W. H. K. (Ed.): "Terrestrial Heat Flow." *Geophysical Monograph 8*, American Geophysical Union, Washington D.C.

LETTAU, H. 1951: Theory of surface-temperature and heat transfer oscillations near a level ground surface. *Transactions, American Geophysical Union 32*: 189–200.

——— 1954: Improved models of thermal diffusion in the soil. *Transactions, American Geophysical Union 35*: 121–32.

MACKIE, A. G. 1965: "Boundary value problems". Oliver and Boyd, London. 252 p.

NICHOLS, R. L.; BALL, D. G. 1964: Soil temperatures, Marble Point, McMurdo Sound, Antarctica. *Journal of Glaciology 5 (39)*: 357–9.

POUNDER, E. R. 1965: "The Physics of Ice". Pergamon Press, Oxford, 151 p.

ROBERTSON, E. I.; MACDONALD, W. J. P. 1962: Electrical resistivity and ground temperature at Scott Base, Antarctica. *N.Z. Journal of Geology and Geophysics 5 (5)*: 797–809.

SHIRTCLIFFE, T. G. L. 1964: Lake Bonney, Antarctica. Cause of elevated temperatures. *Journal of Geophysical Research 69 (24)*: 5257–68.

STEARNS, C. R. 1969: Application of Lettau's theoretical model of thermal diffusion to soil profiles of temperature and heat flux. *Journal of Geophysical Research 74 (2)*: 532–41.

VON HERZEN, R. P.; LEE, W. H. K. 1969: Heat flow in oceanic regions. Pp. 89–95 *in* HART, P. J. (Ed.): "The Earth's Crust and Upper Mantle." *Geophysical Monograph 13*, American Geophysical Union, Washington D.C.

WELLER, G. 1967: The effect of absorbed solar radiation on the thermal diffusion in Antarctic fresh-water ice and sea ice. *Journal of Glaciology 6 (48)*: 859–78.

WELLMAN, H. W. 1964: Later geological history of Hut Point Peninsula, Antarctica. *Transactions of the Royal Society of N.Z. (Geology) 2 (10)*: 147–54.

WILSON, A. T.; WELLMAN, H. W. 1962: Lake Vanda: An antarctic lake. *Nature 196 (4860)*: 1171–3.

APPENDIX

CURVE FITTING PROCEDURE

The experimental data are a set of M temperature measurements and the corresponding depths and times at which the measurements were made. These are denoted as $T_{(\mathrm{obs}),\,m}$, z_m and t_m respectively, where m is an integer which takes the values 1 to M. We require to determine the best fitting values (using a least squares criterion to be explained later) for the parameters: a, T_0, k, A_1, ε_1, A_2, ε_2, . . . A_N, ε_N, which are defined in the text and which, for simplicity, are renamed here as: p_1, p_2, p_3, ... , p_I respectively, where $I = 2N + 3$ is the total number of parameters to be determined. Using this notation, equation (4) can be written in the following form:

$$T_{(\mathrm{theo}),\,m} = p_1 z_m + p_2 + \sum_{n=1}^{N} p_{2n+2} \exp(-p_3 z_m \sqrt{n}) \cos(n\omega t_m + p_{2n+3} - p_3 z_m \sqrt{n}) \qquad \text{(A1)}$$

where $T_{(\mathrm{theo}),\,m}$ is the theoretical temperature corresponding to the m^{th} observation.

Let us assume that we know approximate values for the required parameters which are denoted by: $p_1^{(1)}$, $p_2^{(1)}$, . . . $p_I^{(1)}$. The superscripted number indicates the order of iteration being considered, in this case the first order. Using these values in equation (A1), we can calculate the corresponding initial theoretical temperatures, denoted $\overset{(1)}{T}_{(\text{theo}),\, m}$. Let us now define a set of initial residuals, $r_m^{(1)}$, to be the differences between the theoretical and corresponding measured temperatures, viz:

$$r_m^{(1)} = T_{(\text{obs}),\, m} - \overset{(1)}{T}_{(\text{theo}),\, m} \tag{A2}$$

The magnitudes of the residuals indicate the goodness of fit between the observed and theoretical temperatures. We require to find a set of corrections for the required parameters, say: $\Delta p_1^{(1)}$, $\Delta p_2^{(1)}$, . . . $\Delta p_I^{(1)}$, such that when a second set of temperatures, denoted $\overset{(2)}{T}_{(\text{theo}),\, m}$ is calculated using the parameters $p_i^{(2)} = p_i^{(1)} + \Delta p_i^{(1)}$, ($i = 1$ to I), the sum of the squares of the corresponding residuals $\sum\limits_{m=1}^{M} (r_m^{(2)})^2$ becomes a minimum. Since the problem is non linear, several iterations are required before the sum of squares of the residuals reaches a minimum value.

It remains to explain how the increments $\Delta p_i^{(n)}$ are obtained for the arbitrary n^{th} order of iteration. For a particular value of m, it follows from elementary calculus that a small change in theoretical temperature, between the n^{th} and $(n+1)^{\text{th}}$ iteration is related to the corresponding increments $\Delta p_i^{(n)}$ ($i = 1$ to I) in the following way:

$$\overset{(n+1)}{T}_{(\text{theo}),\, m} \simeq \overset{(n)}{T}_{(\text{theo}),\, m} + \sum_{i=1}^{I} \frac{\partial \overset{(n)}{T}_{(\text{theo}),\, m}}{\partial p_i^{(n)}} \Delta p_i^{(n)} \tag{A3}$$

Using equation (A1) the partial differentials in equation (A3) can be expressed as follows:

$$\partial \overset{(n)}{T}_{(\text{theo}),\, m} / \partial p_1 = z_m$$

$$\text{(A4)}$$

$$\partial \overset{(n)}{T}_{(\text{theo}),\, m} / \partial p_2 = 1$$

$$\partial \overset{(n)}{T}_{(\text{theo}),m} / \partial p_3 = - \sum_{n=1}^{N} \{p_{2n+2}\, z_m \sqrt{n} \exp(-p_3 z_m \sqrt{n}) \times$$

$$[\cos (n\omega t + p_{2n+3} - p_3 z_m \sqrt{n}) - \sin (n\omega t + p_{2n+3} - p_3 z_m \sqrt{n})]\}$$

$$\partial \overset{(n)}{T}_{(\text{theo}),\, m} / \partial p_{2n+2} = \exp(-p_3 z_m \sqrt{n}) \cos (n\omega t + p_{2n+3} - p_3 z_m \sqrt{n}),\ (n > 3)$$

$$\partial \overset{(n)}{T}_{(\text{theo}),\, m} / \partial p_{2n+3} = - p_{2n+2} \exp(-p_3 z_m \sqrt{n})$$
$$\sin (n\omega t + p_{2n+3} - p_3 z_m \sqrt{n}),\ (n > 4)$$

Equations (A2) and (A3) can be combined to obtain:

$$r_m^{(n)} \simeq \sum_{i=1}^{I} \frac{\partial T^{(n)}_{(\text{theo}),\, m}}{\partial p_i^{(n)}} \Delta p_i^{(n)} + r_m^{(n+1)} \qquad \text{(A5)}$$

which is true for all M values of m.

These M equations can be expressed in matrix form as follows:

$$[r^{(n)}] \simeq [\partial T^{(n)}/\partial p]\ [\Delta p^{(n)}] + [r^{(n+1)}] \qquad \text{(A6)}$$

where $[r^{(n)}]$ and $[r^{(n+1)}]$ are $M \times 1$ matrices, $[\Delta p^{(n)}]$ is a $I \times 1$ matrix, and $[\partial T^{(n)}/\partial p]$ is a $M \times I$ matrix. The expressions enclosed within the parentheses are indicative of the elements of the matrices.

It can be shown that the required solution, for which $\sum_{m=1}^{M} (r_m^{(n+1)})^2$ is a minimum, is:

$$[\Delta p^{(n)}] = \left[[\partial T^{(n)}/\partial p]^T \quad [\partial T^{(n)}/\partial p] \right]^{-1} [\partial T^{(n)}/\partial p]^T \quad [r^{(n)}] \qquad \text{(A7)}$$

where the superscripts "T" and "-1" indicate matrix transposition and inversion, respectively. Equation (A7) was evaluated by using a method which has been described by Anderssen (1969) and which also facilitates the computation of the variance and the covariance of the parameters $\Delta p_1^{(n)}$, $\Delta p_2^{(n)}$ . . . $\Delta p_I^{(n)}$. In this procedure use is made of the least-squares method of curve fitting, but it differs from the usual least-squares approach in that a Gram-Schmidt orthogonalisation process is used to form orthogonal combinations of the parameters being determined. This has the advantage of overcoming certain difficulties that sometimes arise in the process of matrix inversion.

In practice, it was found that the iteration process converged after about 10 to 20 steps and that the final solution was independent of the particular values assigned initially to the required parameters.