



Gridding gravity data using an equivalent layer[☆]

G.R.J. Cooper*

Departments of Geophysics and Geology, University of the Witwatersrand, Johannesburg 2050, South Africa

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Abstract

Gravity data are routinely gridded in preparation for contouring or imaging, or to enable processing by a fast Fourier transform method. Frequently used gridding techniques such as $1/r^N$, minimum curvature, or kriging, interpolate data values with no regard to the nature of the data involved; they will produce the same interpolated values if either magnetic or gravity measurements with the same numerical values are used. Equivalent layer gridding however fits the known potential field data values using the geophysical response of a thin surface layer whose physical properties vary spatially. The response of the surface layer can then be calculated at any other points for the purpose of interpolation. The interpolated data values will then depend on the nature of the data being used. A program to perform such gridding (running under Windows'95 or '98) has been written and is available free via ftp from [ftp.cs.wits.ac.za](ftp://ftp.cs.wits.ac.za) in directory /pub/general/geophys. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Geophysical data are generally interpolated to produce a grid that can be imaged or contoured. There are many ways of doing this, such as the weighted average method (Franke, 1980), polynomial surface fitting (Davis, 1986, p. 405), kriging (Haas and Viallix, 1976) and others. The minimum curvature method (MCM) of Briggs (1974) is frequently applied to magnetic and gravity data. The equivalent layer gridding technique differs from all of these methods in that it would produce a different set of interpolated values if it were applied to gravity or magnetic data with the

same numerical values, since a different forward model would be used in each case.

The gravitational potential caused by a three-dimensional density distribution can be indistinguishable from that produced by a surface density distribution spread over any of its equipotential surfaces (Blakely, 1995, p.62). Fig. 1 shows a gravity anomaly measured along a profile that could be produced by two separate bodies at depth or by a thin surface layer. The model is two-dimensional i.e. the bodies have infinite strike length in and out of the paper and the profile cuts the bodies at 90°. If such an 'equivalent layer' is known, then the potential at any desired point can be calculated and hence interpolation of the data can be performed.

Mendonça and Silva (1995) compared the performance of the equivalent layer method (ELM) with that of the MCM and found it to be superior when used with magnetic data. They used an equivalent layer of

* Code available at <http://www.iapg.org/cgeditor/index.htm>

* Tel.: +27-11-716-31599; fax: +27-1-339-7367.

E-mail address: 006grc@cosmos.wits.ac.za (G.R.J. Cooper).

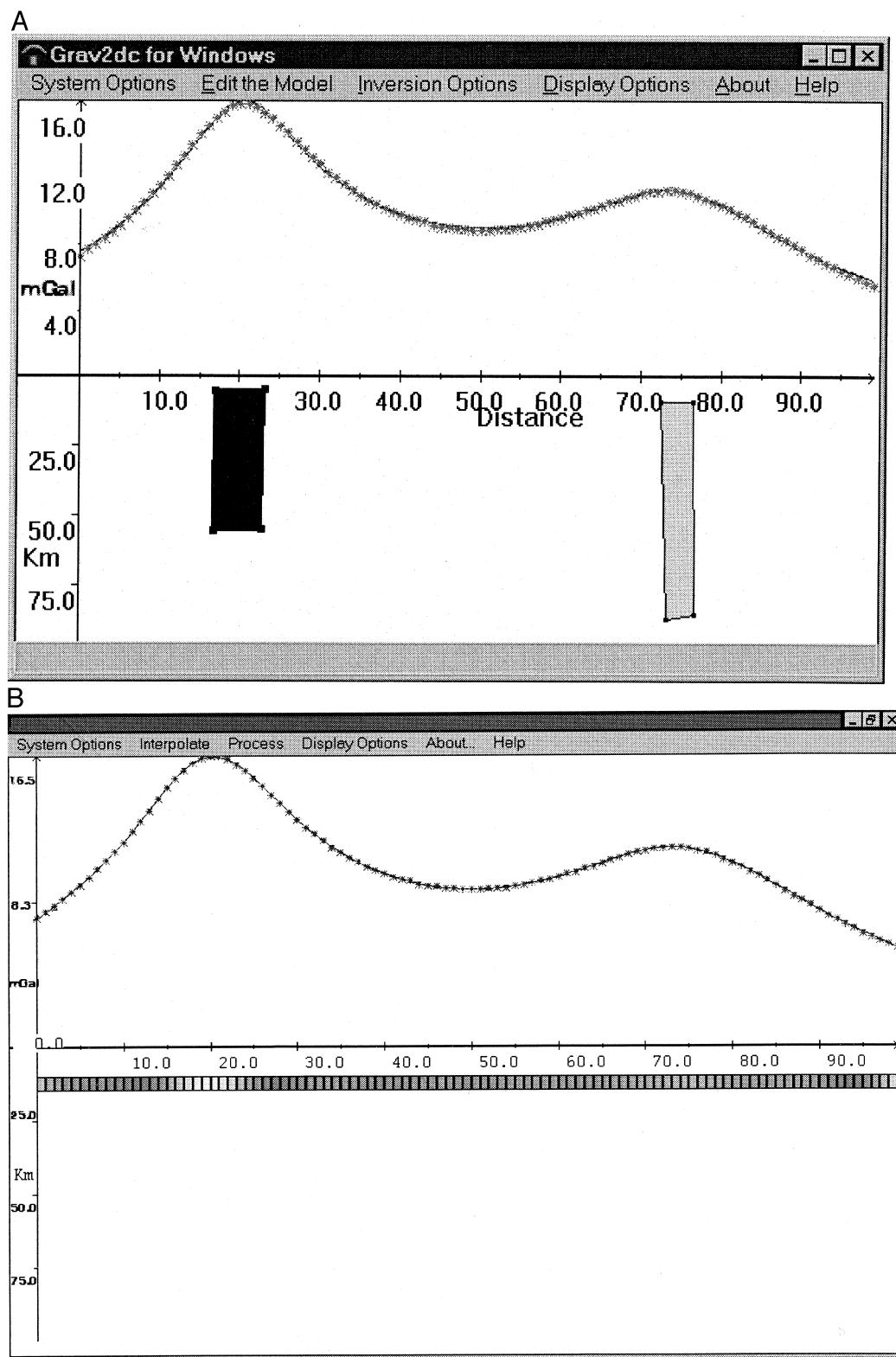


Fig. 1. Equivalent layer concept. (A) Gravitational anomaly produced by two bodies. (B) Gravitational anomaly produced by an equivalent layer.

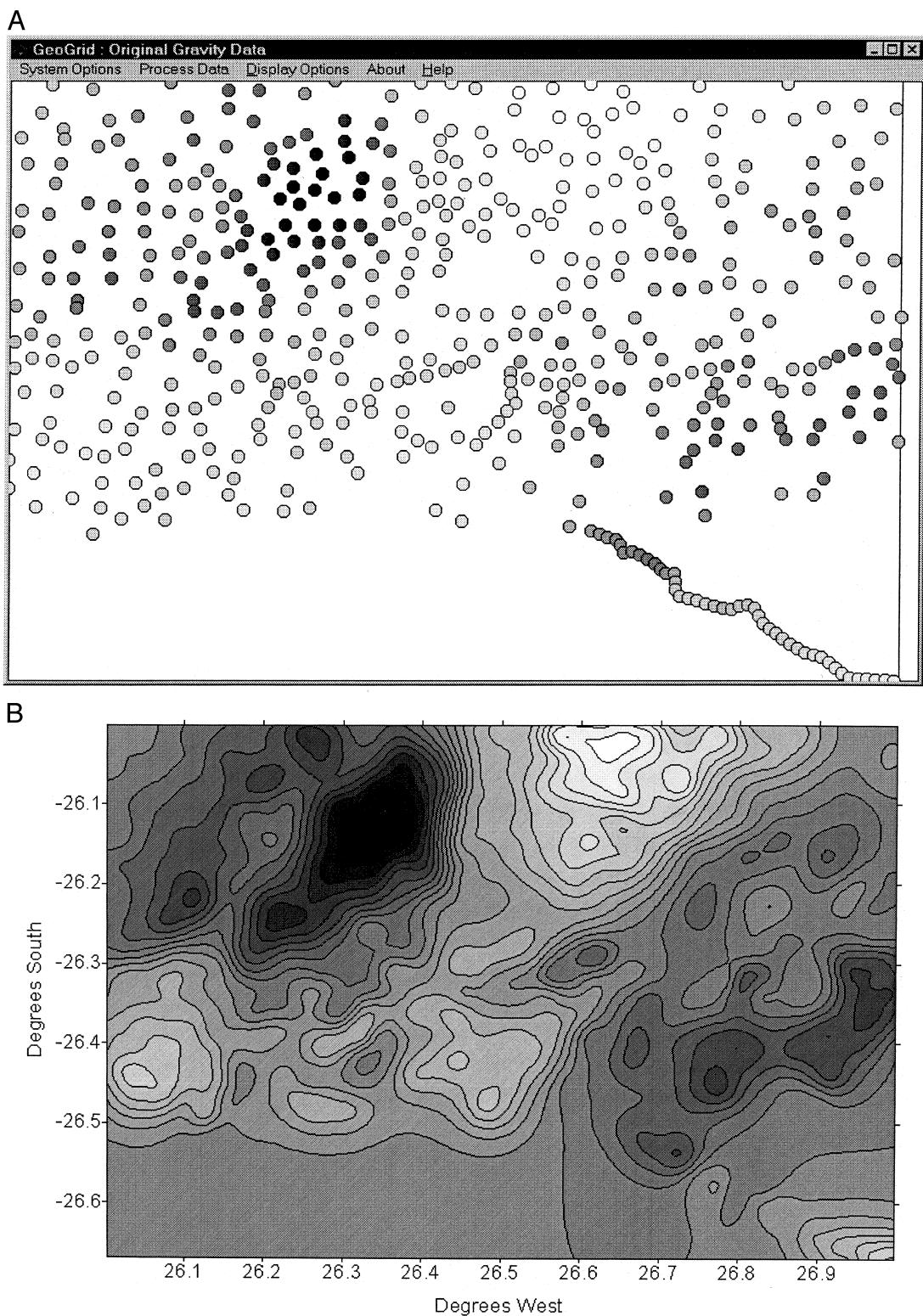


Fig. 2. (A) Original gravity data points. (B) Contour map of data in (A) produced using equivalent layer gridding.

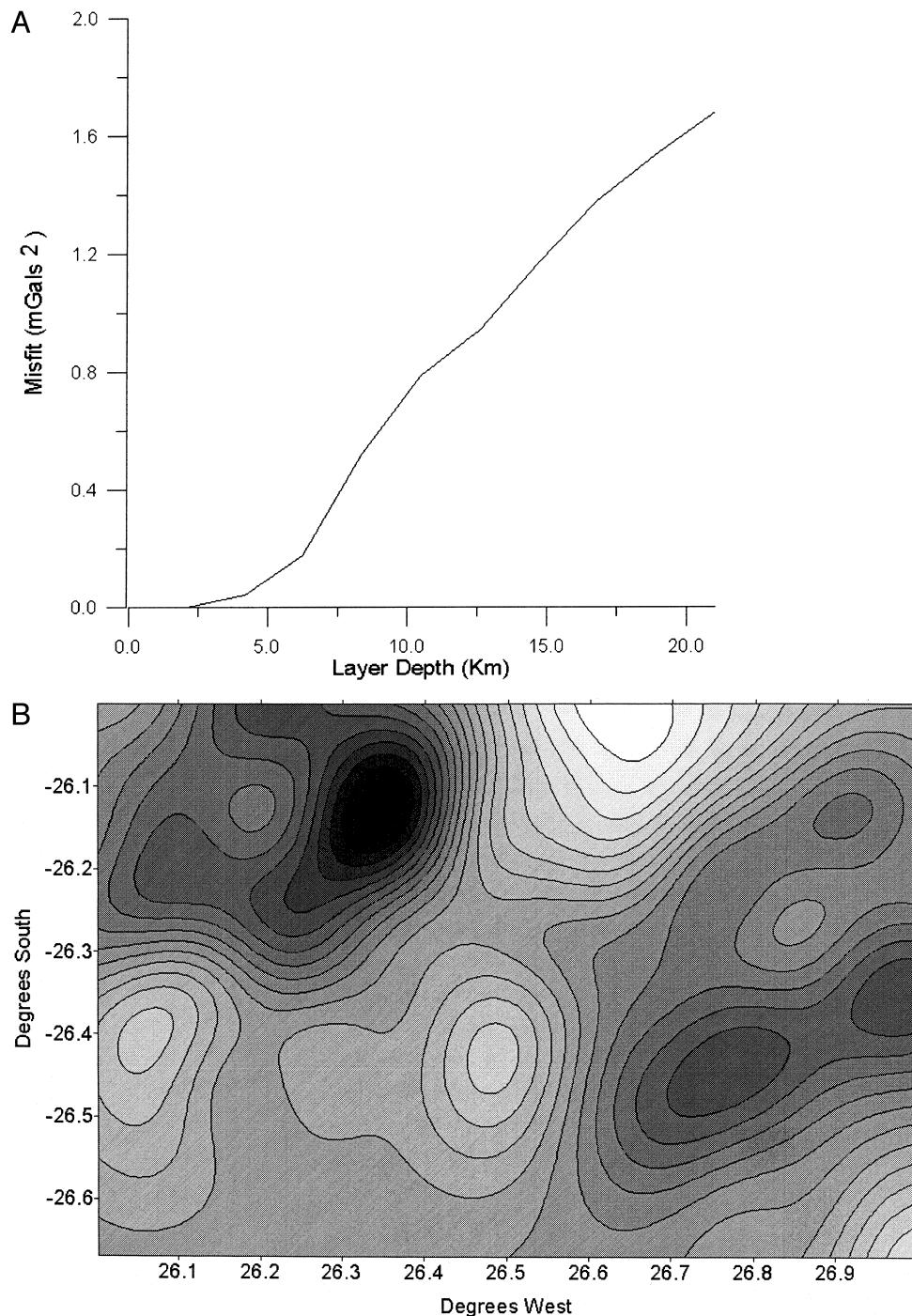


Fig. 3. (A) Misfit vs. depth for application of equivalent layer gridding method to data in Fig. 2A. (B) Gravity data in Fig. 2A gridded and smoothed by use of deep equivalent layer.

constant thickness consisting of many identical sources (greater than the number of observed data points) and inverted for the susceptibility contrast. The gridding program described here is for gravity data and approximates an equivalent layer by using spheres whose density contrast is inverted.

2. The forward model

The vertical gravitational attraction produced by a buried sphere is given by (Beck, 1981, p. 73)

$$\Delta g_v = \frac{GM\Delta z}{(\Delta x^2 + \Delta y^2 + \Delta z^2)^{\frac{3}{2}}} \quad (1)$$

where M is the mass of the sphere, G is the gravitational constant and Δx , Δy and Δz are the distances from the measurement point to the centre of the sphere in the x , y and z directions. Spheres were chosen to make up the equivalent layer since the anomaly is very fast to calculate.

Fig. 2A shows the locations at which measurements were taken for a sample gravity dataset (from South Africa) which displays some common sampling problems. In the upper portion of the map the sampling is fairly uniform, but in the lower portion measurements are only present at the lower right. This results in the large region at the lower left which is poorly sampled. The dataset thus presents a range of difficulties for any gridding algorithm. Fig. 2B shows the result of gridding the data using the ELM.

By default the depth of the equivalent layer (i.e. to the centre of the spheres) is set to three times the smallest distance between the observed data points, to avoid aliasing (Dampney, 1969), however it may be useful to increase this distance if two points lie extremely close together. The depth of the layer can be varied across the map, depending on the distance between the observed data points. The deeper the layer that is used, the smoother will be the resulting gravitational field. Fig. 3A shows the misfit between the observed data and the response from the equivalent layer as a function of layer depth, for the dataset shown in Fig. 2. It can be seen that the misfit (and hence the degree of smoothing) increases almost linearly with the depth of the layer. Fig. 3B shows the result of smoothing the data in Fig. 2 by using an equivalent layer 20 km deep (compared to 2 km deep for Fig. 2B).

The radii of the spheres used is initially set to half the closest distance between each sphere and its nearest neighbour, but (similarly to the depth of the layer) it can be useful to increase this distance if two observed points lie extremely close together (causing the default radius to be very small). The radii of the spheres can

also be made to vary across the map, being set to half the distance between each individual sphere and its nearest neighbour.

3. Inversion method

The density contrast of each sphere can be determined using least-squares inversion for the overdetermined situation (Menke 1989, p. 41) by:

$$dP = (A^T A + kI)^{-1} A^T e \quad (2)$$

where dP was the parameter (density contrast in this case) change matrix to the parameter matrix P , e was the misfit between the calculated anomaly and the data ($e_i = O_i - C_i$, where O_i and C_i are the i th observed and calculated gravity values respectively) and A was the matrix of partial differentials with respect to the parameters. I is the identity matrix and k is a scalar constant whose purpose is to damp the inversion (Menke, 1989, p.52). The anomaly amplitude of a cell scales linearly with the density contrast (i.e. for a linear system if $f_{in} \rightarrow f_{out}$, then $\alpha f_{in} \rightarrow \alpha f_{out}$ (Pippard, 1985, p. 40)), so only one inversion iteration would be necessary for the model to fit the data when $k = 0.0$. It was however found that better results were obtained when a small non-zero value of k was used, although that resulted in two inversion iterations being required to fit the data in most cases.

The main computational effort required in the gridding is in the matrix inversion required in Eq. (2), which was performed using singular value decomposition (SVD) (Press et al., 1992, p.51). If there are N_S spheres used then the matrix that must be inverted is $N_S \times N_S$, where N_S is normally also the number of observed data points, N_{obs} . Fortunately this does not need to be done every inversion iteration, since the same inverted matrix $(A^T A + kI)^{-1}$ can be used throughout. Sample times for gridding on a 300 MHz Pentium II were about 1s for 50 observed gravity points and about 15 minutes for 500 observed points.

While there is a maximum of one sphere per observed gravity point in the equivalent layer, this can be reduced if the gridding process is found to take too long. The percentage of the datapoints that are to have spheres associated with them can be specified and the program uses only that amount (though all of the observed gravity points are still used). The spheres that are to be removed must be selected carefully, since removal of a source from a region where the observed data points are sparsely situated could result in a poor fit to observed data being achieved. To prevent this occurring, the distances between each observed point and its nearest neighbour are calculated and then

sources are removed first from regions where the points are closest together.

Fig. 4 shows the effect on the misfit between the ELM response at the measurement points as a function of the percentage of the maximum number of spheres possible in the equivalent layer. As can be seen, in this case there is little point in using more than 50% of the total number of observed gravity points as sources, when the sources are selected in the manner described above. The time taken to grid the data rises rapidly as the number of spheres increases, with the time taken when 100% of the sources are used being 9.0 times that when only 50% of the sources were used.

The SVD method described above is very inefficient when it is to be used to solve large sets of linear equations, if the matrix to be inverted is sparse i.e. only a relatively small number of its elements are non-zero. Because the matrix A in Eq. (2) is sparse, with the elements of greatest value lying on or near the leading diagonal of the matrix, it lends itself well to a different approach. The biconjugate gradient method (BGM) uses information about the gradient of the function. Four sequences of vectors r_1 , r'_1 , p_1 and p'_1 are initially constructed, then new vectors r_k and p_k are iteratively generated from these using the recurrence relationships (Press et al., 1992, p.77);

$$\alpha_k = \frac{r'_k \times r_k}{p'_k \times A \times p_k},$$

$$r_{k+1} = r_k - \alpha_k A \times p_k,$$

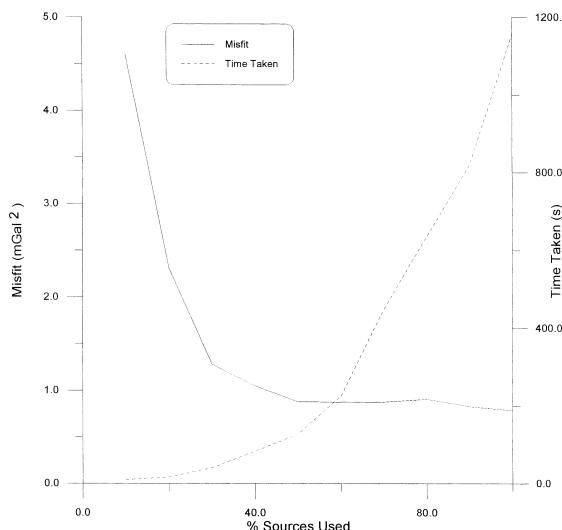


Fig. 4. Misfit and processing time against % sources used for the dataset in Fig. 2(a) using SVD inversion.

$$r'_{k+1} = r'_k - \alpha_k A^T \times p'_k,$$

$$\beta_k = \frac{r'_{k+1} \times r_{k+1}}{r'_k \times r_k},$$

$$p_{k+1} = r_k + \beta_k p_k,$$

$$p'_{k+1} = r'_k + \beta_k p'_k \quad (3)$$

where r_1 is the residual error in the fit between the calculated response of an initial starting model and the observed gravity data i.e. matrix e in Eq. (2) and r'_1 is initially set to r_1 . p'_1 is similarly set to p_1 .

Before the inversion can proceed it must be determined how sparse the matrix A actually is. A threshold value t is chosen (as a percentage of the maximum value of A) and all values of A less than this are set to zero and ignored. The larger t is, the more elements of A will be discarded and the faster the inversion will proceed. However if too many elements are discarded then a poor solution will result. A histogram of the number of elements of A (in classes of size 1% of the maximum magnitude of A) is produced, enabling the user to see how the values of the matrix are distributed. The BGM proved highly effective in reducing the time taken to grid the data; the dataset described above which took 15 min to grid using the brute force SVD approach took only 15 s with the BGM (all elements of A less than 5% of the maximum value were discarded in this case).

The SVD inversion method was retained in the program alongside the BGM since it is faster for small grids ($< 50 \times 50$ points) and because it is more stable. If the threshold value t was badly chosen, then the BGM approach was particularly likely to diverge from the optimum solution, whereas the SVD technique rarely had any such problems.

4. The GeoGrid program

The program that performed all the gravity gridding described in this paper is named GeoGrid and it runs under the Windows'95 (or '98) operating system. It was written using version 2.0 of Borland's Delphi compiler (Borland, 1996. Inprise Corporation, 100 Enterprise Way, Scotts Valley, CA.), which is based on the Pascal language. GeoGrid can take up to 1000 gravity points and grid them to a 100×100 grid, i.e. 10,000 grid points. A fast (≥ 300 MHz) Pentium II Processor and 64 MB of Ram or better are recommended for its usage. GeoGrid allows the display of the original data points, the gridded data points, a contour map produced from the gridded points, the equivalent layer

itself and the misfit between the equivalent layer and the original data points. All plots are displayed with data values colour coded. The user can set the colours assigned to the minimum, maximum and mean values and all intermediate colours are produced by linear interpolation in RGB colour space. Hardcopy output to all Windows supported devices is provided. The gridded data can be saved to disk in either ASCII format or as a Surfer.grd file (Golden Software, 1996. 809 14th street, Golden, CO).

GeoGrid is available free via ftp from ftp.cs.wits.ac.za in directory /pub/general/geophys, or from ftp.iapg.org.

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