GEOPHYSICS, VOL. 39, NO. 4 (AUGUST 1974), P. 526-536, 6 FIGS., 1 TABLE

THE INVERSION AND INTERPRETATION OF GRAVITY ANOMALIES

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A rearrangement of the formula used for the rapid calculation of the gravitational anomaly caused by a two-dimensional uneven layer of material (Parker, 1972) leads to an iterative procedure for calculating the shape of the perturbing body given the anomaly. The method readily handles large numbers of model points, and it is found empirically that convergence of the iteration can be assured by application of a low-pass filter. The nonuniqueness of the inversion can be characterized by two free parameters: the assumed density contrast between the two media, and the level at which the inverted topography is calculated. Additional geophysical knowledge is required to reduce this ambiguity. The inversion of a gravity profile perpendicular to a continental margin to find the location of the Moho is offered as a practical example of this method.

INTRODUCTION

Gravity profiles are often characterized by a smooth regional trend superimposed on higher frequency information which is usually referred to as the "gravitational anomaly." It is of interest to "invert" these data to determine the nature of the mass distribution that would give rise to such an anomalous field. Unfortunately, as with other problems in potential theory, the solution is nonunique, i.e., many different distributions of mass can produce the same gravitational field, and even the very strong assumption that the gravity anomaly is caused by a single two-dimensional disturbing mass of uniform density does not lead to an unambiguous interpretation (Skeels, 1947). Nevertheless, a meaningful interpretation may be obtained if seismic data, and perhaps geologic data from boreholes or wells, are used to constrain sufficiently the range of possible models.

The problem we shall study is the following: Given a single profile of a gravitational anomaly, what is the shape of a two-dimensional mass of constant density which will produce this anomaly? Original inversion attempts were based on trial-and-error methods where the shape of an initial starting structure was perturbed until its gravitational attraction, calculated by means of graticules, matched the observed field (Skeels, 1947).

More recent attempts have centered about automating the perturbation scheme (Bott, 1960; Corbató, 1965; Tanner, 1967; Negi and Garde, 1969). The perturbing body is usually approximated by a set of rectangular prisms of constant density, and its gravitational field is then calculated. The residuals between the calculated and observed fields are used to adjust the heights of these rectangles; the amount of the adjustments is calculated by solving a linearized set of normal equations (Corbató, 1965).

Disadvantages to this technique are numerous. The final model consists of a number of rectangular prisms rather than a smooth curve which would be more reasonable geologically; and two types of iteration schemes, depending upon whether the upper or lower surface of the perturbing body is assumed known and fixed, are required to insure probable stability and convergence (Tanner, 1967). Published examples using this method are characterized by slow convergence and instability of the iteration scheme when the number of parameters, i.e., the number of rectangles, is increased. For buried bodies convergence of the iteration method also requires that

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the depth of burial of the rectangular prism be greater than its width (Tanner, 1967).

Dyrelius and Vogel (1972) increased stability and rate of convergence by using an alternate method of calculating some of the linearized quantities in the normal equations and preventing the iteration scheme from making large adjustments to the heights of the rectangles by requiring that the center of mass and the total mass of the model be the same as those calculated from the observed anomaly by Gauss' formula. Their method, however, was applied only when the upper surface of the rectangular prisms coincided with the observational plane, and the effectiveness of their method is not known when the perturbing body is buried at depth.

The purpose of this paper is to present an inversion technique which can accommodate easily a large number of model points (more than a hundred) and still maintain the desired numerical properties of stability and relatively rapid convergence. The inversion is accomplished by an iterative scheme based on the rearrangement of the forward algorithm given by Parker (1973). The scheme calculates the Fourier transform of the gravitational anomaly as the sum of the Fourier transforms of powers of the perturbing topography. Because Fourier transforms can be computed rapidly (IEEE, 1967), this method is computationally much more efficient than calculating the gravitational field by breaking up the model into a set of prisms whose contributions are calculated separately and summed. Indeed, it is this speed with which the forward algorithm is executed that allows us to present the following inversion method as a practical one.

THEORY

We consider Parker's method of calculating the gravitational attraction of a two-dimensional, uneven layer of material of constant density. In an $x-z$ Cartesian coordinate system the gravitational anomaly is given by $\Delta g(x)$, and the lower and upper boundaries of the perturbing layer are denoted by $z=0$ and $z=h(x)$, respectively. The entire mass of the perturbing layer must lie below the horizontal line on which the observations are specified. Since our profile is of only finite length and in order to avoid problems of convergence, we assume that the layer vanishes outside some finite domain D, i.e., $h(x)=0$ if $x \not\subset D$. In practice $h(x)$ is measured relative to some reference level a distance z_0 below the surface.

We define the one-dimensional Fourier transform of a function $h(x)$ by

$$F[h(x)] = \int_{-\infty}^{\infty} h(x) e^{ikx} dx, \tag{1}$$

where k is the wavenumber of the transformed function. The Fourier transform of the gravitational anomaly is obtained by reducing Parker's two-dimensional formula to the one-dimensional form required here. Hence,

$$F[\Delta g(x)] = -2\pi G \rho e^{-|k|z_0} \sum_{n=1}^{\infty} \frac{|k|^{n-1}}{n!} F[h^n(x)], \tag{2}$$

where ρ is the density contrast between the two media and G is Newton's gravitational constant. Transposing the $n=1$ term from the infinite sum and rearranging, we obtain

$$F[h(x)] = -\frac{F[\Delta g(x)] e^{|k|z_0}}{2\pi G \rho} - \sum_{n=2}^{\infty} \frac{|k|^{n-1}}{n!} F[h^n(x)]. \tag{3}$$

When ρ and z_0 are known (or assumed), this equation may be used iteratively to calculate $h(x)$ in the following manner: The most recent determination of $h(x)$ [for the first iteration a guessed solution or $h(x) \equiv 0$ is satisfactory] is used to evaluate the right-hand side of equation (3); the inverse Fourier transform of this quantity then gives an updated value for the topography. The iterative procedure is continued until some convergence criterion is met or a maximum number of iterations has been completed. It is important to note that the calculation of $h(x)$ by equation (3) involves approximately the same number of computations as solving the forward algorithm; hence, each iteration can be done quickly, and total computation time for the inversion is therefore governed by the number of iterations required before the convergence criterion is satisfied.

CONVERGENCE AND UNIQUENESS

Tests for convergence must be applied in evaluating the infinite sum of Fourier transforms in equation (3) and also in the iteration technique used to find the topography $h(x)$. Before convergence properties can be considered, reasonable restrictions must be imposed upon $h(x)$. In a manner similar to that given by Parker (1972), we require that $h(x)$ be bounded and integrable and that it vanishes outside some finite domain D, i.e., $h(x)=0$ for $x \not\subset D$. Then,

$$F[h^n(x)] = \int_D h^n(x) e^{ikx} dx$$

$$\leq \int_D | h^n(x) | \, dx \leq LH^n,$$

where L is the length of D and $H = \max | h(x) |$. The sum

$$\sum_{n=2}^{\infty} \frac{|k|^{n-1}}{n!} F[h^n(x)]$$

$$\leq \frac{L}{|k|} \sum_{n=2}^{\infty} \frac{|k|^n}{n!} H^n \qquad (4)$$

$$= \frac{L}{|k|} (e^{|k|H} - 1 - |k| H).$$

The right-hand side of (4) is bounded for any finite value of k; hence, from properties of the exponential function (Whittaker and Watson, 1962, p. 581), the sum of Fourier transforms is absolutely and uniformly convergent in any bounded domain of the k-plane. In principle then, no problem exists with the convergence of the infinite sum although, in practice, large relief may require many terms (say 20 or 30) of the sum to be evaluated before an appropriate convergence criterion is met. Let

$$S_n = \max_{\substack{\text{over all} \\ k}} \left| \frac{|k|^{n-1}}{n!} F[h^n(x)] \right| .$$

The convergence criterion chosen requires that successive terms in the sum are computed until $S_n/S_2 < E$, where E is some sufficiently small number. The term $e^{-|k|z_0}$ in equation (2) allows a stronger statement to be made about the convergence of the direct problem. Parker has shown that for $z_0 > 0$

$$e^{-|k|z_0} \sum_{n=1}^{\infty} \frac{|k|^{n-1}}{n!} F[h^n(x)]$$

$$\leq \frac{L}{|k|} \sum_{n=1}^{\infty} e^{-|k|z_0} \frac{|k|^n H^n}{n!} \qquad (5)$$

$$< \frac{L}{|k|} \sum_{n=1}^{\infty} \left(\frac{H}{z_0} \right)^n ,$$

and hence that the series is uniformly convergent, independent of the value of k, when $H/z_0 < 1$. The rate of convergence of $\sum (H/z_0)^n$ is maximized when H/z_0 is a minimum, thus the obvious requirement that the topography be measured relative to a level z_0 which is the median of the largest and smallest values of $h(x)$. Numerical experiments carried out by Parker have shown that this choice of z_0 falls very close to the optimum one. Unfortunately, if the material giving rise to the gravitational anomaly comes into contact with the observation plane, then $H/z_0 = 1$ and $\sum (H/z_0)^n$ is not convergent. However, the first inequality in equation (5) remains valid thereby ensuring the convergence of the sum for finite k, but in practice this rate of convergence is slowed as the perturbing layer approaches the observational surface. Convergence of the forward algorithm is monitored by

$$R_n = \max_{\substack{\text{over all} \\ k}} \left| e^{-|k|z_0} \frac{|k|^{n-1}}{n!} F[h^n(x)] \right| .$$

A sufficient number of terms has been evaluated when $R_n/R_1 < \delta$, where δ is some arbitrarily chosen small number.

The convergence of the infinite sum in equation (3) has therefore been assured; but, even assuming that this sum has been computed exactly, we still have no control over convergence of the iterative procedure to find $h(x)$. The question of this convergence is complicated by the fact that the right-hand side of equation (3) is a nonlinear functional of the model $h(x)$.

There is an additional problem for the term $F[\Delta g(x)] e^{|k|z_0}$ is numerically equivalent to the downward continuation of the gravity data by an amount z_0. We do not mean to imply that this term is the true Fourier transform of the gravitational field at the level $z=0$, for this would mean that we are downward continuing through the source material, but only that such a term may lead to fallacious results because, as in downward continuation, short wavelength components, usually associated with "noise" in the data or truncation errors in the Fourier transforms, are multiplied by large exponential factors.

As a result of these complications we have not been able to make analytical statements concerning the convergence of the iterative procedure but, instead, have been forced to resort to empirical results. We have used as our convergence criterion the requirement that the root-mean-square difference between two successive approximations of $h(x)$ be less than some arbitrarily chosen value. It has been found that the straightforward application of the iterative procedure usually resulted in a divergent solution characterized by high wavenumber oscillations. Only in special cases, when the observed gravitational anomaly could be accounted for by an $h(x)$ with sufficiently small relief, was convergence obtained.

However, problems involving downward continuation of potential field data usually provide meaningful results only when the downward continued data are smoothed (Bullard and Cooper, 1948). In gravitational problems such smoothing may be physically realistic since short wavelength anomalies are more easily generated by structures near the surface than those at depth. If it is required to invert a gravitational anomaly the bulk of whose source is believed to be at considerable depth, then we may justifiably filter out much of the short wavelength information which is probably caused by near-surface structure. These high-frequency[1] oscillations can be eliminated by multiplying the right-hand side of equation (3) by a suitable low pass filter $B(k)$ which passes all frequencies up to WH and passes none above a cutoff frequency SH.

If the gravitational anomaly to be inverted is given at N equally spaced points, then the model (i.e., the shape of the topographic surface causing the anomaly) is determined by specifying the N complex amplitudes of the equally spaced frequency components between $-f_N$ and f_N, where f_N is the Nyquist frequency. The effect of the filter $B(k)$ is to reduce the number of free parameters of the model by approximately the factor of SH/f_N, since for $SH<f_N$ the frequencies between SH and f_N, and $-SH$ and $-f_N$, are omitted from the composition of $h(x)$. The omission of this high-frequency information results in a loss of structural detail that can be seen in the topography. Thus, as SH is decreased in value, the number of complex amplitudes to be determined is reduced, and we must be satisfied with a smoother $h(x)$.

In practice it is found that for any gravitational profile the filter parameters WH and SH can be adjusted so that the iterative process converges; however, because of the application of this arbitrary low-pass filter, we can no longer be assured that the gravitational anomaly calculated from the inverted topography will agree with the original observations. In such cases our filtering has been too severe. If, by adjusting the filter parameters, we obtain only divergence of the iterative procedure or convergence to a topography which has a gravitational field not satisfying the observed data, then either the assumed density contrast ρ or the assumed level z_0 in equation (3) must be adjusted. Indeed, if the assumed density contrast is made sufficiently large, then $F[h(x)] = -F[\Delta g(x)]e^{|k|z_0}/2\pi G\rho$ and the difference between the field calculated from $h(x)$ and the observed

$$B(k) = \begin{cases} 1 & |k/2\pi| < WH \\ \dfrac{1}{2}\left[1 + \cos\left(\dfrac{k - 2\pi WH}{2(SH - WH)}\right)\right] & WH \le |k/2\pi| \le SH \\ 0 & |k/2\pi| > SH. \end{cases}$$

This form of filter is only one of many that could have been chosen. In particular, it was selected over the filter obtained by setting $B(k)=0$ for $|k/2\pi| > WH$ to reduce the effect of Gibbs phenomenon in the untransformed domain.

[1] Editors note: Throughout this report the author uses the term frequency for the quantity $k/2\pi$.

field can be arbitrarily small. This is because a thin layer of material with high density can yield the same gravitational attraction as a thicker layer with lower density, and the choice of a large ρ so diminishes the magnitude of $h(x)$ that only the first term in the infinite sum in equation (2) is important. This term in fact calculates the gravitational attraction of a surface mass distribution obtained by condensing the matter in the vertical column from $z=0$ to $z=h(x)$ down into the $z=0$ plane. Although there is no upper bound to the value of the density which can give rise to the anomaly, there is a minimum value.

Maximum depth rules (Grant and West, 1965; Parker, 1974) indicate that for an assumed density contrast there is a maximum depth above which some portion of the perturbing body must lie in order to cause the anomaly; alternatively, if the anomaly is caused by a density contrast at some specified depth, then the magnitude of this contrast must be greater than some $\rho_{\min}$. Therefore, if, in the inversion scheme, the assumed value of ρ is too small and/or the assumed value of z_0 is too large, no filter exists which will force the iteration procedure to converge to a topography satisfying the observations.

The nonuniqueness of the gravity inversion problem results from the existence of two free parameters, ρ and z_0. Skeels (1947) appears to have given one of the first published examples of how the change in level is responsible for an ambiguity in interpretation. After fixing the density contrast between the basement rocks and the overlying sediments, he varied the depth to the undisturbed basement and obtained different configurations of the basement and overburden whose resultant gravitational attraction fitted the original observations to within 0.1 mgal. Skeels also pointed out that gravity data assumed to arise from a density contrast of a single layer at depth could be inverted uniquely if the value of the density contrast and either the depth to the surface of density contrast at one point or the maximum relief of the structure were known.

Parker (1973) has shown that a formula analogous to equation (2) can be used for the rapid calculation of a magnetic anomaly arising from a two-dimensional magnetized layer. If the thickness and location of this layer are known, then the rearrangement of this formula (Parker and Huestis, 1974) has been shown to lead to an iterative procedure for calculating a linear distribution of magnetization $m(x)$ which could cause the anomaly. It is interesting to note the difference in the roles that the choice of z_0 (i.e., the level relative to which the topography is measured) plays in the magnetic and gravitational inversion procedures. For the magnetic problem the value of z_0 affects the rate of convergence of the iterative procedure but not the resulting $m(x)$. In contrast, for the gravitational problem a different $h(x)$ is obtained for each value of z_0, hence this parameter characterizes an entire family of possible topographies which can cause the observed anomaly.

The inversion of any gravitational profile therefore involves a set of three parameters, ρ, z_0, and $B(k)$. To provide some insight as to the effects of these parameters, the nonuniqueness of gravitational interpretation, and the rapidity of convergence of the iterative process, a numerical example is given.

NUMERICAL EXAMPLE

We consider the gravitational anomaly caused by the "bump" on the subsurface stratum in Figure 1. The bump is 20 km wide at the base; it has a height of 4 km and an assumed density contrast of 1.0 gm/cm^3. The gravitational attraction of this bump has been calculated at 128 equidistant points (1 km apart) by using equation (2). Evaluation of only the first nine terms of the infinite sum resulted in $R_9/R_1 < 10^{-6}$.

This gravity anomaly has been inverted for different assumed values of ρ and z_0. The results, shown in Table 1, typify the interrelation between z_0, ρ, and the low-pass filter for such an inversion. Three different low-pass filters were applied, and the iterative procedure was terminated when the rms difference between successive values of $h(x)$ was less than 0.5 m or when a maximum of 10 iterations had been completed. The infinite sum. which had to be evaluated at each iteration, was said to have converged when $S_n/S_2 < 5\times 10^{-3}$.

For each inversion, Table 1 shows the rms difference in $h(x)$ at the last iteration (ERR), the total number of iterations taken (ITER), and the absolute value of the maximum error (in mgals) between the observed gravity and the gravity calculated from the inverted topography (M.E.). Also indicated is whether the iterative process had diverged (DIV) or was still converging (S.C.) There is some difficulty in concisely presenting the effects of the three different parameters on the inversion procedure. Nevertheless, from Table 1 we can draw the following general conclusions which have been empirically shown to be valid for the inversion of many different profiles.

We shall first consider the effects of varying ρ

and z_0 for a particular low-pass filter. If z_0 is fixed, then a decrease in the assumed value of ρ requires that more iterations be performed before convergence is obtained. Indeed, there exists ρ_{min}, a minimum value of ρ, such that for $\rho < \rho_{min}$ the iterative procedure no longer converges. Similarly, if ρ is fixed, an increase in the assumed value of z_0 requires that more iterations be performed before convergence is obtained. There also exists z_{max}, a maximum value of z_0, such that for $z_0 > z_{max}$ the iterative procedure no longer converges. If we now fix z_0 and ρ and allow the filter to vary, we find that a decrease in the value of the filter parameters WH and SH results in faster convergence.

It must be emphasized that even though convergence can be rapidly attained by this filtering, there is no guarantee that the final topography will give rise to a gravitational anomaly satisfying the observations. In some instances convergence is attained without filtering; however, if the value $\Delta f = SH - WH$ is assumed fixed, then in most cases there appears to exist an upper limit to the value of SH, called SH_{max}, such that for $SH > SH_{max}$ the iterative procedure no longer converges. As examples, the filter $(WH, SH) = (.15, .20)$ enabled an acceptable $h(x)$ to be found for $z_0 = 3.0$ km but not for $z_0 = 5.0$ km or below.

For the filter (.075, .150) convergence was obtained for $z_0 = 5.0$ km and $\rho = 1.25$ gm/cm^3 or $\rho = 1.0$ gm/cm^3, but the iterative procedure diverged for $\rho = .75$ gm/cm^3 (5 more iterations were required to make ERR<.5 m for $\rho = 1.0$). The application of the filter (.025, .075) was an example of "over-filtering"; all inversions converged but resulted in an $h(x)$ which gave a gravitational anomaly that did not satisfy the observations.

The nonuniqueness of gravity inversion is further illustrated in Figure 2 where each of the six structures plotted produced a gravity anomaly that differed from the observed by less than 0.1 mgal. The filter used for each inversion is shown on the topographic profile. A density contrast of $\rho = 1.0$ gm/cm^3 has been used throughout. Since the topographic relief varies with the assumed value of z_0, the knowledge of either the value of this relief or the depth to any one point on the structure greatly constrain the range of possible models satisfying the observations. The speed of

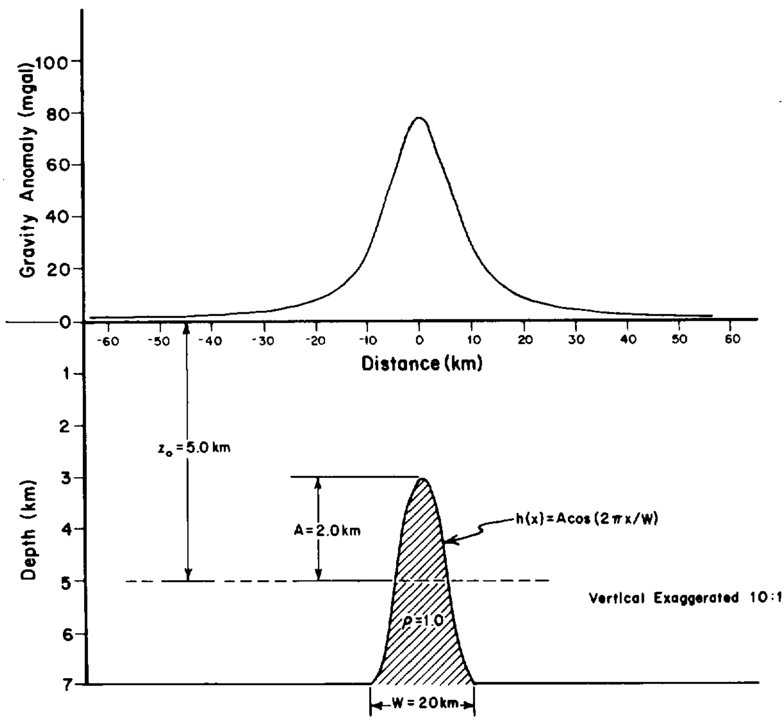

FIG. 1. Gravitational anomaly from a "bump" on a flat substratum.

Table 1. The effects of the density contrast ρ, the level Z_o, and the low-pass filter (WH, SH), on the inversion of a gravitational profile.[1]

Z_o \ ρ	1.25	1.0	.75
filter: (.15, .20) ≡ (WH, SH)			
3.0	ERR = .3 m ITER = 4 M.E. = 0.7	ERR = .3 m ITER = 6 M.E. = .09	ERR = .3 m ITER = 10 (S.C.) M.E. = .87
5.0	ERR = 36 m ITER = 2 (DIV) M.E. = 5.9	ERR = 70 m ITER = 2 (DIV) M.E. = 7.9	ERR = 200 m ITER = 2 (DIV)
7.0	All inversions diverged		
filter: (.075, .150)			
3.0	ERR = .13 m ITER = 4 M.E. = .05	ERR = .27 m ITER = 4 M.E. = .04	ERR = .38 m ITER = 5 M.E. = .10
5.0	ERR = .44 m ITER = 7 M.E. = .07	ERR = 1.2 m ITER = 10 (S.C.) M.E. = .29	ERR = 37 m ITER = 3 (DIV) M.E. = 9.95
7.0	All inversions diverged		
filter: (.025, .075)			
3.0	ERR = .29 m ITER = 3 M.E. = 7.2	ERR = .12 m ITER = 4 M.E. = 6.9	ERR = .33 m ITER = 4 M.E. = 6.4
5.0	ERR = .38 m ITER = 3 M.E. = 7.1	ERR = .17 m ITER = 4 M.E. = 6.8	ERR = .47 m ITER = 4 M.E. = 6.3
7.0	ERR = .12 m ITER = 4 M.E. = 7.0	ERR = .26 m ITER = 4 M.E. = 6.7	ERR = .22 m ITER = 5 M.E. = 6.0

[1]ITER is the number of interations completed; ERR is the rms difference between the last two iterations of $h(x)$; and M.E. is the absolute value of the maximum error (in mgals) between the observed gravitational anomaly and that calculated from the inverted topography. S.C. beside ITER means the iteration procedure was still converging, while DIV means it was diverging, i.e., the rms error between successive iterations had begun to increase.

the inversion scheme is illustrated by the fact that the six inverted profiles in Figure 2 were calculated in approximately 1.2 sec on a CDC 7600.

GEOPHYSICAL EXAMPLE

Unfortunately, the assumptions that an observed gravitational anomaly is caused by a single surface between two constant density media and that the perturbing body is two-dimensional are sufficiently strong that they are never exactly met in nature. In many cases though, enough is known about the near-surface structure from seismology or from boreholes that its effect can be stripped off leaving a reduced gravitational anomaly which may approximately fulfill our assumptions. An example of geophysical interest is the determination of the depth to the interface between the earth's crust and mantle, i.e., the Mohorovičić discontinuity or Moho.

Seismological work indicates that the Moho is at a depth of 11 km beneath most ocean surfaces (Hart, 1969, p. 228) and that it plunges under the thick continental crust. We applied this inversion technique to see if gravitational observations can be used to delineate this discontinuity along a traverse perpendicular to a continental margin. Since continental margins are approximately linear features over lengths of a few hundred kilometers, the error introduced by the assumption of two-dimensionality of the various layers is probably small.

We used as gravity observations the 22 pendulum measurements for the Mt. Desert Section profile given by Worzel (1965) and redrawn in Figure 3. We interpolated these points with a spline routine and extended the lengths of the water and sediment profiles in order to produce a total profile length of 1024 km.

The reasons for this "bordering" are two-fold. The computation of a finite transform assumes that the data series is infinitely replicated. Achievement of this replication with no discontinuities requires that the beginning and end points of the series should have approximately the same value. It is also desirable to extrapolate the observed layering of water and sediments so that the calculated gravitational attraction from these layers will be approximately correct even close to the edges of the unbordered profile. In the present case the water and sediments were extrapolated as horizontal layers for a distance of 150 km, then both rose gently to zero in the next 150 km and remained at zero until 1024 km. All structures were then digitized at intervals of 8 km yielding series 128 points long. In spite of this bordering, it must be remembered that we are interested in a model fitting the observations only in the first 600 km, where real data exist.

Assuming the sediment and water distributions as given in Figure 3, we calculated their gravita-

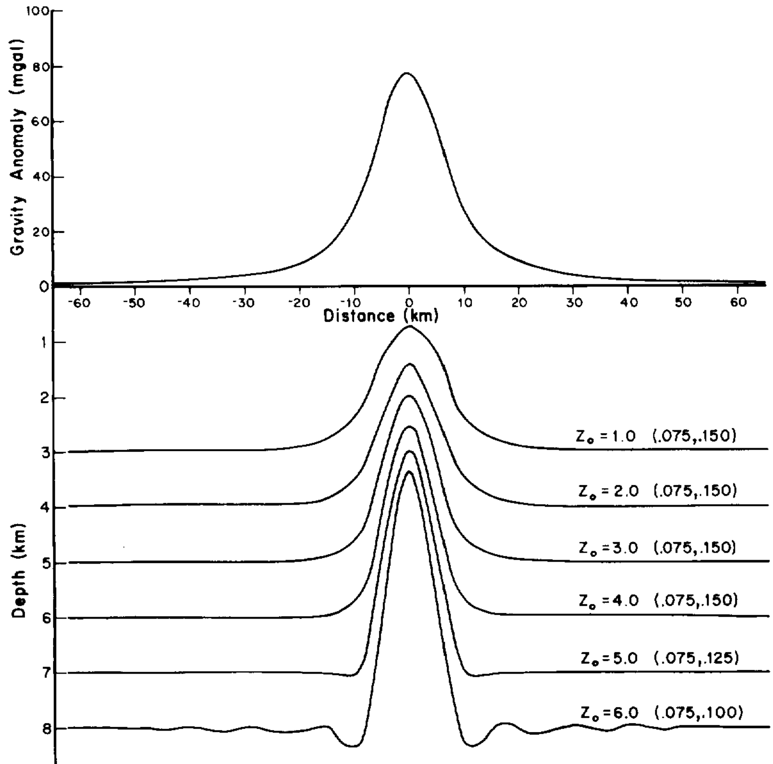

FIG. 2. Ambiguity in gravity interpretation caused by the choice of level. The "observations" are those from Figure 1, and each of the six reliefs shown gives rise to a gravitational anomaly differing from the observed anomaly by less than 0.1 mgal.

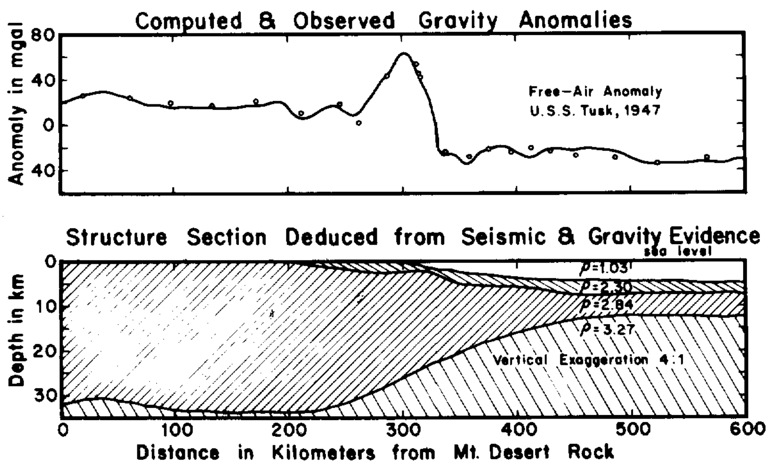

FIG. 3. The 22 pendulum measurements for the Mt. Desert section and the locations of the water and sedimentary layers (redrawn from Worzel, 1965). Also shown is the agreement between the observations and the gravitational attraction calculated from the stratigraphic section.

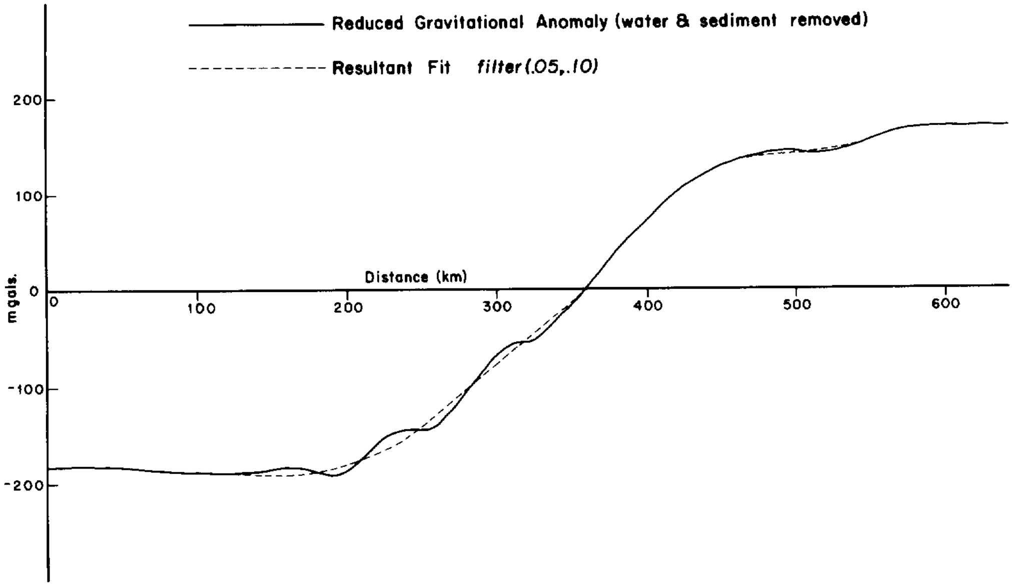

FIG. 4. The observed gravitational anomaly with water and sediments removed and the fit obtained from inversion with the filter (.05, .10).

tional contribution to obtain the reduced gravitational profile shown in Figure 4. The densities used were those given by Worzel (1965): ρ(water) $=1.03$ gm/cm^3 and ρ(sediment)$=2.30$ gm/cm^3. Because the calculation of an unambiguous structure requires additional information, we assumed a constant density contrast between the crust and mantle of 0.43 gm/cm^3, i.e., ρ(crust)$=2.84$ gm/cm^3 and ρ(mantle)$=3.27$ gm/cm^3. We also assumed that the depth of the Moho at $x=600$ km is about 11 km beneath the ocean surface. With these added constraints a value of $z_0=22$ km was found. The results of inverting the gravity profile of Figure 4, using three different filters, are shown in Figure 5. Different amounts of high frequency data are present in each of the curves, but the general shape of the Moho is clearly outlined.

The depth of the Moho beneath the continent (0 to 150 km) is approximately 33–34 km, a very reasonable value. The Moho then rises at an overall rate of about .1 km/km (slope $\sim$6 degrees) in the region 200 to 450 km, remains flat between 450 to 525 km, then rises once more, and finally levels off at a depth of approximately 11 km beneath the ocean surface. Although each of the three topographies gives rise to nearly the same gravitational effect, one feels intuitively that the undulations seen in the two less filtered profiles are artificial and do not represent plausible geologic models. The smooth profile corresponding to the filter (.050, .100) seems more likely to represent the true structure.

The agreement between the reduced gravity observations and the anomaly calculated from the topography obtained by inversion for filter (.050, .100) is shown in Figure 4. The dashed line represents a smoothing of the oscillations occurring between $x=128$ and $x=350$ km; with the exception of the points near $x=192$, 230, and 260 km, the agreement along the whole profile is quite good.

The short wavelength anomalies probably arise from errors in defining the water-sediment boundary or the sediment-crust boundary, but the non-two-dimensionality of these surfaces, and the effect of possible additional shallow geologic features, may also contribute significantly to these anomalies. Indeed, application of the results of the 2-body gravity problem (Parker, 1974) shows that the depth of a disturbing body, with density contrast $\rho=.43$ gm/cm^3, giving rise to the observations at $x=192$, 208, and 224 km must be shallower than 15 km. For comparison purposes, the original 22 observations and the anomaly calculated from the inverted topography are shown in Figure 6.

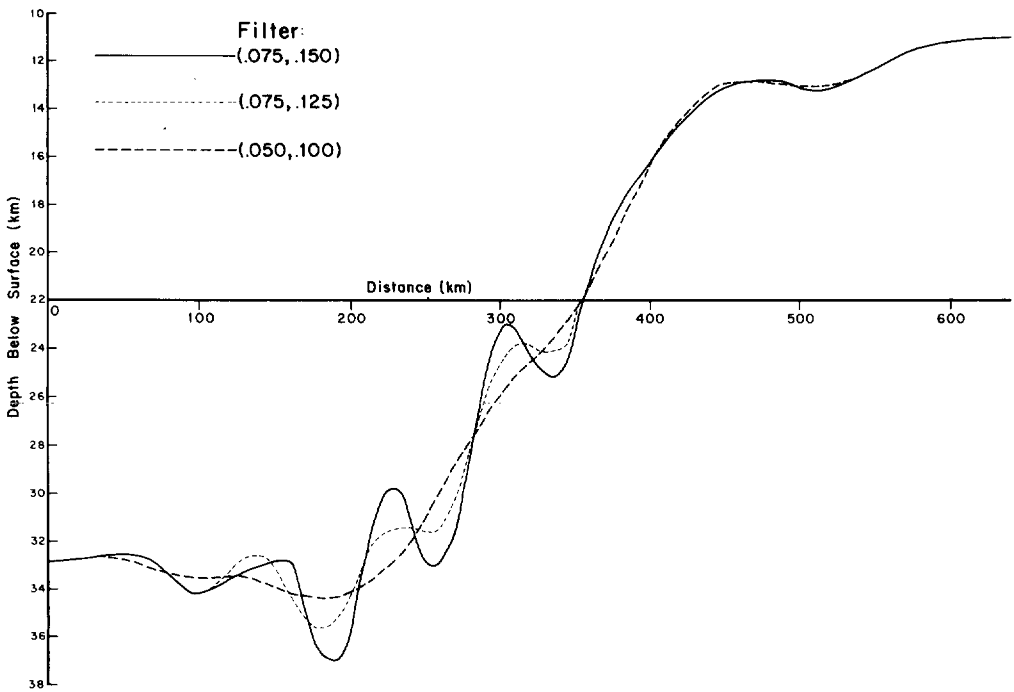

FIG. 5. Location of the Moho obtained from the inversion of the reduced gravitational anomaly in Figure 4. Three different low pass filters have been used to produce Moho reliefs with varying amounts of structural detail.

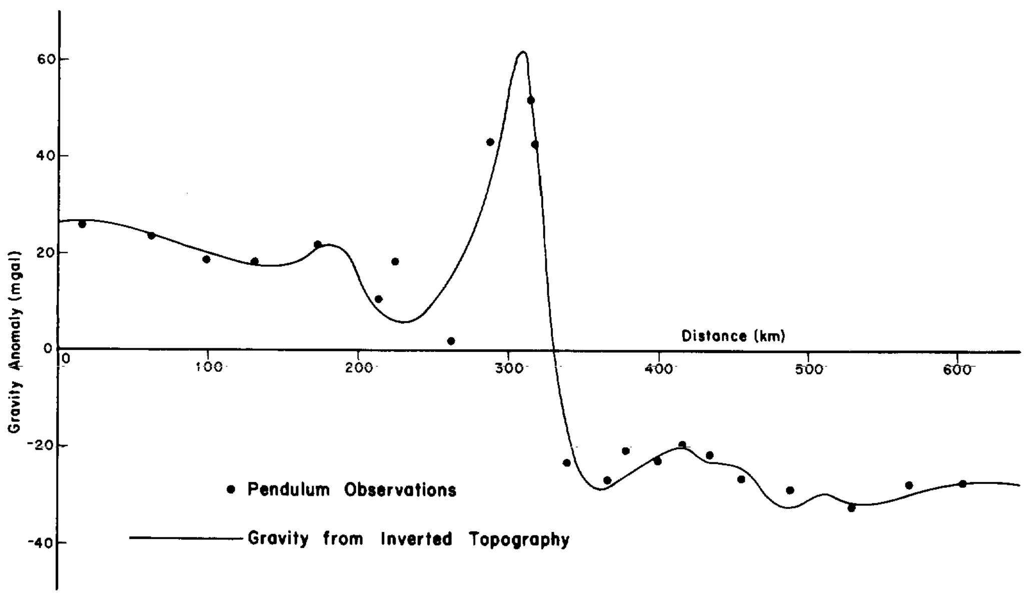

FIG. 6. Agreement between the 22 original gravity observations and the anomaly calculated from the Moho topography as shown in Figure 5 [filter: (.010, .100)]. Effects of the water and sediments have been included.

CONCLUSIONS

The formula developed by Parker (1973) for the rapid calculation of potential anomalies has been rearranged to yield an iterative procedure for inverting one-dimensional gravity profiles. Because computations in the inversion scheme involve primarily the evaluation of Fourier transforms which can be computed very quickly, this method is capable of handling large numbers of model points without requiring much time.

The nonuniqueness of the gravity inversion, as shown by explicit examples in the text, is characterized by two free parameters: ρ, the density contrast between the perturbing layer and the surrounding medium, and z_0, the level at which the inversion is made. Without additional information constraining these two parameters, the ambiguity in the gravity interpretation cannot be reduced. A low-pass filter has been applied to improve convergence of the iteration process but, in trials where the assumed density was too small or z_0 too large, no topography could be found which would give rise to an anomaly agreeing with the initial observations.

As a real example, a gravity profile perpendicular to a continental margin was inverted to determine the location of the Moho. The ability of this inversion scheme to handle large numbers of model points without greatly decreasing the numerical stability or greatly increasing the computation time makes it particularly attractive. Also, since the original formula for the direct problem calculated the gravitational effect from a three-dimensional topography, an extension of this inversion scheme to invert a two-dimensional gravitational anomaly measured over a plane is possible.

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