Applied Geomatics (2021) 13:945–951
https://doi.org/10.1007/s12518-021-00402-2

ORIGINAL PAPER

Leveling airborne and surface gravity surveys

Xiaopeng Li[1]

Received: 7 June 2021 / Accepted: 28 September 2021 / Published online: 8 November 2021

Abstract

For both airborne gravimetric and airborne magnetic surveys, systematic residual errors usually remain in the data after standard data processing. In the literature, the mismatches at the crossover points are used to either adjust the processing procedures or directly adjust the results. This is usually called crossover adjustment. Due to the limited crossover points, it has a risk of leaking the random errors into a systematic distortion of the entire data. In aeromagnetic surveys, the low-wavenumber information is used to construct a smooth field to level the survey data without the need for tie lines. However, this method relies on the long-wavelength component of the flight line data accurately sampling the regional field. Here, an alternative approach is developed to model the physical field using radial basis functions (RBF) and to parameterize the systematic errors at the same time, which avoids all of the aforementioned problems. Numerical results show that the new method provides more stable results than the classical approach. The method is also tested on terrestrial gravity surveys (where it is challenging to directly apply the crossover analysis). Here also, it yields promising results.

Keywords Gravity · Airborne Gravimetry · Survey

Introduction

Airborne gravimetry (Bell et al 1999; Olesen 2003) and aeromagnetic (Luyendyk 1997) surveys are very useful in local geoid modeling and geophysical explorations, respectively. In most of these surveys, a part of the flight trajectories are usually designed to "meet" with the other parts to generate a certain amount of the so-called crossover points. However, these "intersection" points between two different flights actually do not always intersect each other in 3D space due to the height differences between two flight lines. By neglecting the time variation of the physical field (either the gravity field or the magnetic field), these crossover points provide duplicated measurements of the same physical quantities at the same location after considering the gradient effect at different heights. The differences between these values at the crossover points often serve as quality indicators of the entire survey.

Most of the time, researchers also use these mismatches to adjust the entire data, which is called "crossover adjustment" in geodesy (Hwang et al 2006), and "leveling" in aeromagnetic surveys (Luyendyk 1997). However, this procedure opens a possibility of leaking the random errors at the limited crossover points into systematic distortions of the entire survey (Olesen 2003; Li 2011a). White and Beamish (2015) use the low wavenumber information to generate a smooth field proportional to the flight trajectories to estimate the line biases without directly tying the flights at the crossover points. However, the methodology relies on the long-wavelength component of the flight line data accurately sampling the regional field (White and Beamish 2015). Moreover, the modeling of the physical field and the estimating of the errors are still done in two steps, which lacks theoretical elegance and is cumbersome during implementation. In this study, a new model based on radial basis functions (RBF) and random constant biases is developed to model the gravity field and to estimate the flight line biases in the same step. The method is then extended into surface gravity surveys to avoid the rather time-consuming path searches as described by Saleh et al. (2013).

The rest of the paper is organized as follows: "The mathematical model" section gives a short description of the mathematical model that is used in this investigation. Some numerical experiments are given in "Case study in airborne survey scenarios" section. The method is then extended into

✉ Xiaopeng Li
Xiaopeng.Li@noaa.gov

[1] Geoscience Research Division, National Geodetic Survey, 1315 East-West Highway, Silver Spring, MD, USA

surface gravity surveys in "Case study in terrestrial survey scenarios" section. Finally, some conclusions are given in "Conclusions" section.

The mathematical model

The Earth's gravity field is one of the main research subjects in Earth sciences such as geodesy and geophysics; please see Li and Götze 2001 for their differences and common grounds. The gravity field is usually solved as a boundary value problem (Heiskanen and Mortiz, 1967), whose solution can be represented by a series of spherical harmonics (Thomson and Tait 1867, i.e., the famous "T and T" in mathematics). Despite its elegance in theory, the solution is suffering from numerical problems, especially in the high degrees and orders, due to the limitation in global data coverages (Jekeli 2005) and sometime poor data qualities. Vast amount of airborne gravimetric surveys either the traditional scale one (Li 2013) or the vector one based on inertial measurement units (Li and Jekeli 2008; Li 2011b among others) is carried out to fill in these gaps or to provide updated measurements in the areas where significant geological events happen. In particular, the National Geodetic Survey of the USA is collecting airborne gravity data in the US territories with about 200 km penetrations to its neighboring countries and open ocean areas (Smith 2007). Cleaned airborne gravity data is a key pillar to the success of these missions. However, no matter what kind of data processing algorithms are used, there are always some systematic/inharmonic errors left over in the airborne gravity data due to various reasons. After removing the long-wavelength information, provided by very accurate satellite missions, the characteristics of these errors are fortunately quite different from the true physical field. Thus, they are quite easy to be parameterized in the case of modeling the residual gravity field with RBFs.

The mathematical model leads to Eq. (1).

$$\delta g\left(r_i, \phi_i, \lambda_i\right) = -\frac{\partial}{\partial r} T\left(r_i, \phi_i, \lambda_i\right) = \frac{GM}{R^2} \sum_{k=1}^{\infty} \alpha_k \sum_{n=N_{min}}^{N_{max}} b_n \left(\frac{R}{r_i}\right)^{n+2} (n+1)\sqrt{(2n+1)}\overline{P}_n(cos\psi) + \mu_l, l = 1,2,3,\dots L \tag{1}$$

where μ_l and α_k are the unknown bias along flight line l and the scaling coefficient that need to be determined from the data, [N_{min}, and N_{max}] are the bandwidth of the data usually available during the airborne data processing, ψ is the spherical distance between the computation point $\boldsymbol{r}_i$ and the location of the RBF (Schmidt et al 2007, and Klees et al 2008), $\overline{P}_n(cos\psi)$ is the normalized Legendre function (Heiskanen and Mortiz 1967), and b_n is used to control the spectrum contents (if a Shannon kernel is used,$b_n = 1$). GM = 3.986004415 × 1014 m3/s2, and R is the mean radius of the Earth under spherical approximation.

After centering the data (removing the mean differences over the entire survey area) with respect to a global reference model, the sum of the line biases should be zero. This leads to the extra constraint as shown in Eq. (2) that can avoid the selection of a fixed line in the normal crossover adjustment or leveling procedure.

$$\sum_{l=1}^{L} \mu_l = 0 \tag{2}$$

Please note that removing the entire block mean bias with respect to the reference field is different from removing each line bias with respect to the same reference field. The former one is an area bias while the latter ones are more subjected to sample aliasing. Equation (1) and Eq. (2) is the so-called Gauss-Markov model with constraints, which can be solved via least squares technique.

Case study in airborne survey scenarios

First, simulation tests are carried out to verify the proposed method. A high degree and order coefficient model, GOCE and Egm2008 Combination (GECO Gilardoni et al 2016), published by the International Centre for Global Earth Models (ICGEM), is used to generate a set of synthetic data along real flight trajectories of a typical GRAV-D campaign conducted by the National Geodetic Survey (NGS) over the state of Iowa. It is well known that the airborne gravity data is essentially band-limited (Schwarz and Li 1997; Jekeli 2016). Previous studies (Huang et al., 2017 and Li et al., 2021) on the NGS airborne gravity data show that the main power is concentrated in the band between degree 200 and 1,000. Thus, the GECO simulated gravity disturbances in the band of airborne spectrum (degree/order 200 to 1,000) are shown in Fig. 1.

Majority of the flight lines are in the east–west direction. Only a few cross tracks are in the north–south direction, which only gives a few crossover points. It does not make too much sense to use these sparsely distributed crossovers to adjust tens of thousands of observations. To apply RBFs, first, a Reuter grid (Eicker 2008) is generated over the target area under 50 km of the local topography with spatial resolution to match the maximum degree 1,000. Note that the grid area is normally smaller than the data area to avoid edge effects. Researchers usually use trial and error to find the proper borders of the model zone. However, a rule of thumb is given by Naeimi (2013); i.e., the buffers between the data zone and the model zone are about a half of the minimum spatial resolution of the RBFs, which leads to a total number of 980 grid points. One can also use a priori information

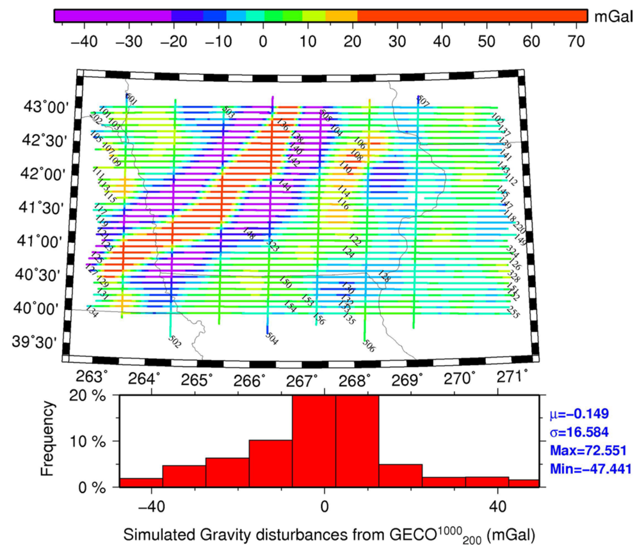

Fig. 1 Simulated gravity disturbances at the real flight trajectories of the GRAV-D flights over the state of Iowa (flight numbers are annotated at the beginning of each line)

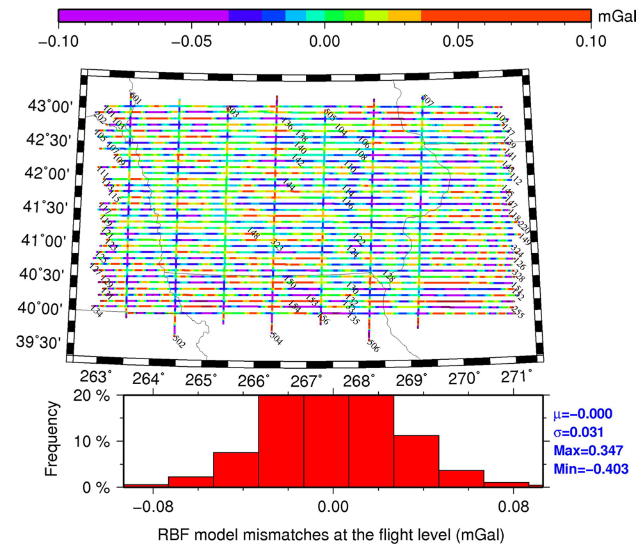

Fig. 2 The RBF model mismatches of the simulated data in Fig. 1

outside the data area to maximize the model area if losing data in the edge becomes a critical issue. Then, a Shannon band-pass, $N_{min} = 200$, and $N_{max} = 1000$ in Eq. (1), kernel-based RBF is generated at each of the grid point. The observation at each GRAV-D flight point is used as the input to the model.

The sampling rate of the GRAV-D flights is about 100 m. So, there are usually tens of thousands observations for each GRAV-D block. This is much more than the number of the unknown parameters $\{\alpha_\bullet\}$. Since the observation number is much bigger than 3 times of the unknowns (a rule of thumb provided by Klees and Wittwer (2007) to determine the stableness of the system), the model can be solved by using standard least squares adjustment. Note that to speed up the computation, one can precompute the common terms in the design matrix, and recursively compute $\left(\frac{R}{r_i}\right)^n$. The computation is fairly fast in nowadays multithread shared memory computation systems. Figure 2 shows the model mismatches at the flight level. The residuals of the model are generally small: the standard deviation is well below sub-mGal level with extreme values not exceeding half mGal.

To test if the Reuter grid automatically gives a reasonable number of RBFs, the relationship (Bentel et al 2013) between its level and N_{max} of the model is broken. Instead of using this default number of RBFs, i.e., 980, a series of other numbers starting from 500 to 1500 are tested. Figure 3 shows the plot of the model errors for all of these extra tests. From this figure, it is clear that the errors are too big if we use less numbers of RBFs and the model does not improve after using more than 1000 of RBFs. Thus, the default number of the Reuter grid is a good choice to avoid under-fitting or over-fitting problems.

Some extra tests on the band width are also carried out while fixing the number of the RBFs. Table 1 shows the RMS errors of the model residuals under several alternative setups of the kernel. From the table, we see that though the theoretical default values of the pass-band do not give the numerical minimum error, it is very close to the numerical best, 0.03mGal to 0.02mGal. Altering too much of the pass band gives worse results. For instance, if the model (plan D in Table 1) only contains partial of the information in the data, it produces a very poor (> 10mGal) result. Thus, it is important to have certain a priori information on the spectrum of the data, which is not a very unreasonable requirement for the airborne data that is usually band-limited due to filtering. Even though the data itself is not band-limited, such as the surface data, we can make it band-limited by removing the topographic effects; please see the example in the next section.

The above simulation tests demonstrated the performance of RBFs in zero noise circumstances. However, as it is well-known, the real data normally contain an unprecedented amount of noise. To test that scenario before directly dive into real data, 66 random constants are generated and added to the simulated data in Fig. 1 along with a 0.5mGal random noise assumed across over all the flights. These biased and noisy data are then used as the input values to test the proposed model. Figure 4 shows both the generated artifacts and the model estimated biases for all of these 66 lines over the target area, which clearly shows that the proposed method works and provides very close estimates to the true values for almost all of the flights. The classical crossover adjustment method is also used to these simulated data. The

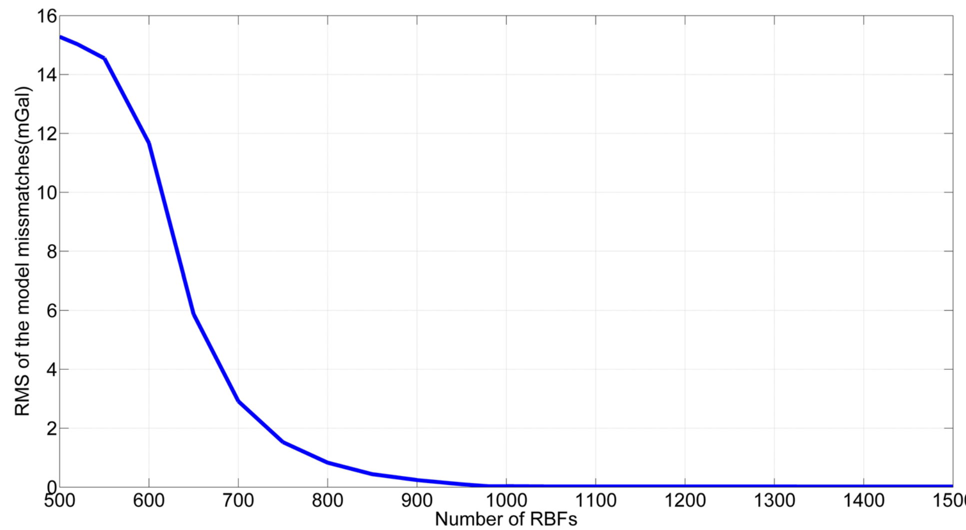

Fig. 3 The model mismatches by using different number of RBFs

Table 1 The band width effects on model performance

	N_{min}	N_{max}	RMS (mGal)
(Theoretical) Default	200	1000	0.03
Plan A	2	1000	0.02
Plan B	200	1500	1.75
Plan C	2	1500	1.72
Plan D	400	800	10.44

corresponding results are also given in Fig. 4 for comparison purpose. The proposed new method provides roughly a magnitude better result than the traditional crossover adjustment as demonstrated in Fig. 4.

After all of these simulation tests, we are confident in the algorithm and its associated computational codes. At this stage, the verified method is applied to the real GRAV-D flight data over the target area, whose crossover values are

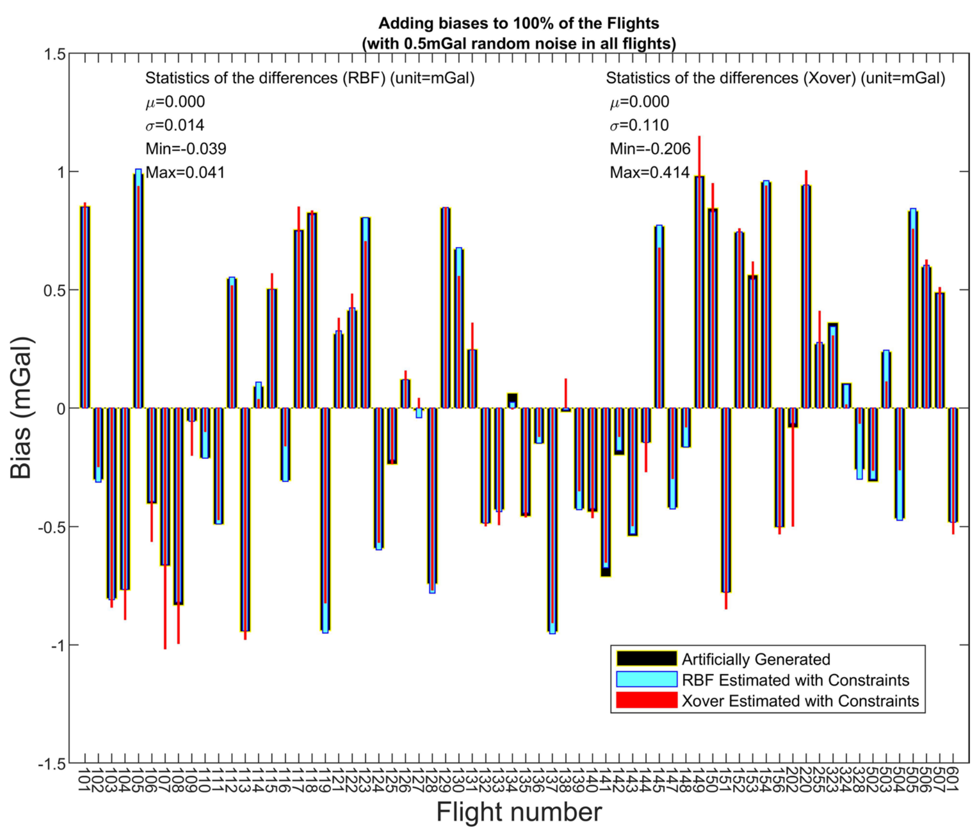

Fig. 4 The simulated line biases and their estimates from both the proposed method and the crossover adjustment

shown in Fig. 5. The standard deviation of the crossover is well above 2mGal. From Fig. 5, we see that the crossovers do not really have a random distribution in space. For instance, line 503 is apparently biased with respect to other lines. A blind crossover adjustment will distort all other lines while trying to absorb the problems in several particular lines, which will generate large artifacts that are not desirable for many applications. Figure 6 shows the estimated biases along each of the flight trajectories after removing the block mean bias. Figure 7 shows the crossover of the processed data after removing both the mean bias and the individual line biases estimated by the new method. From Fig. 7, we see that the standard deviation of the crossover is under half mGal, which is much better than before.

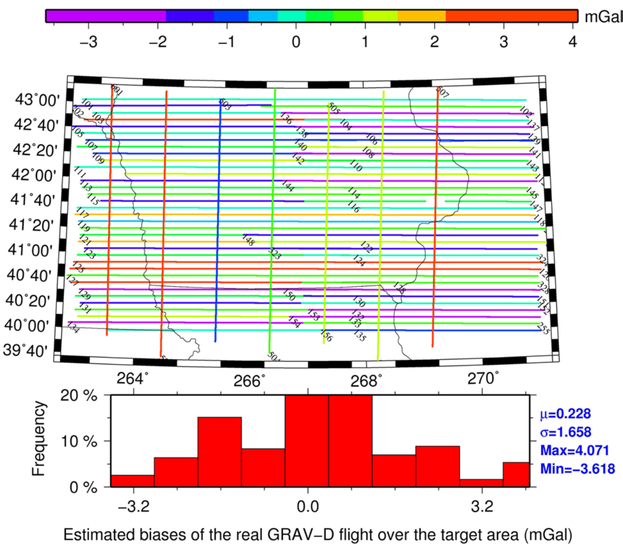

Fig. 6 The estimated biases in the real GRAV-D flights over the target area

Case study in terrestrial survey scenarios

The proposed method is also applied to the surface data over the target area, i.e., the state of Iowa. The surface gravity surveys have much more complicated spatial patterns than the data from a typical airborne campaign, of course. All of the data are then decomposed into sub surveys according to the available NGS metadata. The same step by step simulations as described in the previous section are carried out to the surface data. Similar results are obtained. To save space, they are not all listed here. There is only two extra things need to be mentioned. The first one is to change the term (n + 1) in Eq. (1) into (n − 1) because now we are using the gravity anomalies instead of the gravity disturbances (Heiskanen and Mortiz 1967). The second one is to remove the residual terrain effects so to make the data bandlimited. Figure 8 shows the estimated biases of in these NGS gravity surveys over the target area. The values are ranging from − 3mGal to about 2mGal.

Conclusions

A new one-step leveling method is introduced to model the physical gravity field and estimate the systematic biases without the need for tie lines or requiring that the long-wavelength component of the flight line accurately model the regional field. Synthetic data without any added noise

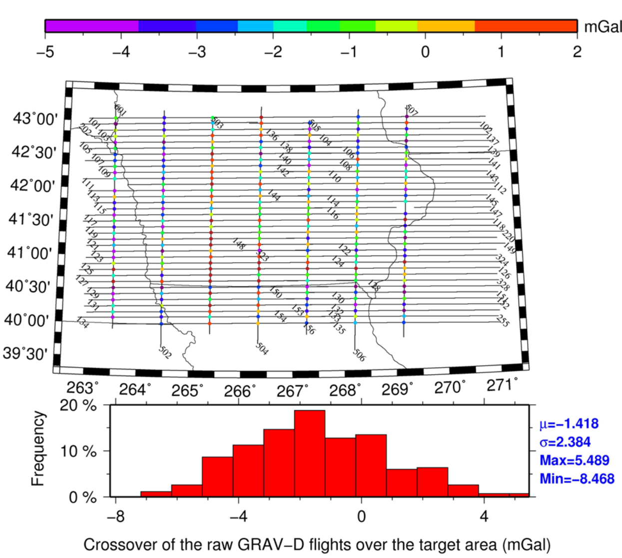

Fig. 5 The crossover the raw GRAV-D data in the target area

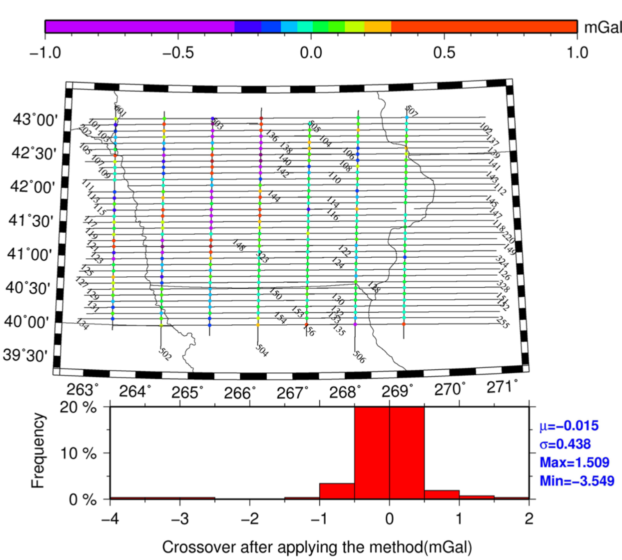

Fig. 7 Crossover of the processed GRAV-D flights over the target area

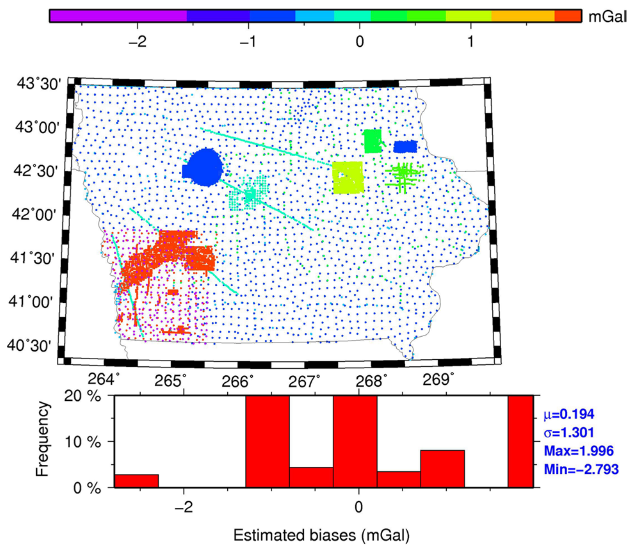

Fig. 8 The estimated biases of the real NGS survey gravity surveys over the target area

are used as benchmarks to verify the algorithm and its associated implementation source codes. Both random biases and random noise are then added to the simulated data to test the robustness of the newly developed procedure. The results show that the novel methodology is much more stable than the tie line (crossover) method, especially in the event of limited crossover points with large random noise values. The suggested method not only provides a fairly accurate bias estimate but also tells where the bias comes from. Without fixing any lines, the method successfully provides good line bias estimates for a typical GRAV-D flight campaign. The process is then further extended into surface gravity surveys, where the random spatial distribution of the data makes it difficult to directly apply a conventional crossover adjustment. Encouraging results are also obtained from the software package here, unless the biases are significantly small relative to the random noise. Micro scaled leveling and adjusting procedures may be still required to resolve the case of relatively small biases.

Acknowledgements The author sincerely thanks the useful comments from the reviewer.

References

Bell RE, Childers VA, Arko RA, Blankenship DD, Brozena JM (1999) Airborne gravity and precise positioning for geologic applications. J Geophys Res 104(B7):15281–15292

Bentel K, Schmidt M, Gerlach C (2013) Different radial basis functions and their applicability for regional gravity field representation on sphere. GEM-International Journal on Geomathematics 4:67–96

Eicker A (2008) Gravity field refinement by radial basis functions from in-situ satellite data. Ph.D. Thesis University of Bonn, Bonn

Gilardoni M, Reguzzoni M, Sampietro D (2016) GECO: a global gravity model by locally combining GOCE data and EGM2008. Stud Geophys Geod 60(2):228–247

Heiskanen WA, Mortiz H (1967) Physical Geodesy. Freeman, San Francisco

Huang, J., S. A. Holmes, D. Zhong, M. Véronneau, Y. Wang, J. W. Crowley, X. Li, and R. Forsberg, 2017, Analysis of the GRAV-D airborne gravity data for geoid modelling. In: Vergos G., Pail R., Barzaghi R. (eds) International Symposium on Gravity, Geoid and Height Systems 2016. International Association of Geodesy Symposia, vol 148. Springer, Cham. https://doi.org/10.1007/1345_2017_23

Hwang C, Hsiao YS, Shih HC (2006) Data reduction in scalar airborne gravimetry: theory, software and case study in Taiwan. Computer & Geosciences 32:1573–1584

Jekeli C (2005) Spline representation of functions on a sphere for geopotential modeling, Technical report 475. Department of Geodetic Science, the Ohio State University

Jekeli C (2016) Theoretical fundamentals of airborne gravimetry, NGS summer school. https://www.ngs.noaa.gov/GRAV-D/2016SummerSchool/presentations/day-1/2Jekeli_Theoretical_Fundamentals.pdf. Accessed 5 Oct 2021

Klees R, Wittwer T (2007) Local gravity field modelling with multipole wavelets. In: Tregoning P, Rizos C (eds) Dynamic planet—monitoring and understanding a dynamic planet with geodetic and oceanographic tools. In: International association of geodesy symposia, vol 130. Springer, Berlin, pp 303–308

Klees R, Tenzer R, Prutkin I, Wittwer T (2008) A data-driven approach to local gravity field modeling using spherical radial basis functions. J Geodesy 82:457–471

Li, X. and C. Jekeli, 2008, Ground-Vehicle INS/GPS Gravimetry, Geop., 73, 2, I1-I10. Li X., 2011a, Strapdown INS/DGPS airborne gravimetry tests in the Gulf of Mexico, J Geod., 85, 9, 597–605, https://doi.org/10.1007/s00190-011-0462-2

Li X (2011a) An exact formula for the tilt correction in scalar airborne gravimetry. Journal of Applied Geodesy 5(2):81–85

Li X (2011b) Strapdown INS/DGPS airborne gravimetry tests in the Gulf of Mexico. J Geod 85(9):597–605. https://doi.org/10.1007/s00190-011-0462-2

Li X (2013) Examination of two major approximations used in the scalar airborne gravimetric system — a case study based on the LCR system. Journal of Geodetic Science 3(1):32–39. https://doi.org/10.2478/jogs-2013-0004

Li X, Götze HJ (2001) Tutorial ellipsoid, geoid, gravity, geodesy, and geophysics. Geophysics 66(6):1660–1668

Li X, Huang J, Klees R, Forsberg R, Willberg M, Slobbe C, Hwang C, Pail R (2021) Characterization and stabilization of the downward continuation problem for airborne gravity, in the reviewing process of Journal of Geodesy

Luyendyk APJ (1997) Processing of airborne magnetic data. Journal of Australian Geology & Geophysics 17(2):31–38

Naeimi M (2013) Inversion of satellite gravity data using spherical radial base functions. PhD thesis, Institute of Geodesy, University of Hanover, Germany

Olesen AV (2002) Improved airborne scalar gravimetry for regional gravity field mapping and geoid determination. Ph.d. dissertation, National Survey and Cadastre of Denmark Technical Report 24:123

Saleh J, Li X, Wang YM, Roman DR, Smith DA (2013) Error analysis of the NGS surface gravity database. J Geodesy 87(3):203–221

Schmidt M, Fengler M, Mayer-Guerr T, Eicker A, Kusche J, Sanchez L, Han SC (2007) Regional gravity field modeling in terms of spherical base functions. J Geod 81(1):17–38

Schwarz, K.P., and Z. Li, 1997, An introduction to airborne gravimetry and its boundary value problems. In: Sansó F., Rummel R. (eds) Geodetic Boundary Value Problems in View of the One Centimeter Geoid. Lecture Notes in Earth Sciences, vol 65. Springer, Berlin, Heidelberg. https://doi.org/10.1007/BFb0011709

Smith DA (2007) The GRAV-D project: Gravity for the Redefinition of the American Vertical Datum, http://www.noaa.gov/GRAV-D/pubs/GRAVDv20071219.pdf, accessed 20 March 2017

Thomson W, Tait PG (1867) Treatise on Natural Philosophy. Oxford University Press

White JC, Beamish D (2015) Levelling aeromagnetic survey data without the need for tie-lines. Geophys Prospect 63:451–460