GEOPHYSICS, VOL. 48, NO. 2 (FEBRUARY 1983); P. 234–237, 2 FIGS., 1 TABLE.

Short Note

Werner deconvolution of profile potential field data

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Werner (1953), in analyzing the magnetic fields of dipping, magnetized dikes, proposed a method of separating the field contributed by a particular dike under study from the interference of neighboring dikes. In addition to being a means of effecting a regional-residual separation, Werner's method of analysis also had the advantage of being easily programmed on a digital computer. This made it a convenient method for analyzing the large amounts of data from reconnaissance aeromagnetic surveys, and it became the basis of the automatic interpretation schemes of Hartmann et al (1971) and Jain (1976). The purpose of this note is to discuss some limitations of the Werner method of deconvolution and also to point out some possible extensions of the method to the general interpretation of potential field data.

THE DECONVOLUTION OF POTENTIAL FIELDS

Knowing the distribution of sources of a field is the same as knowing the field itself. For example, the magnetic scalar potential in the case of two-dimensional (2-D) symmetry is

$$\Psi_m = -\iint_\Omega \ln(|X' - X|)\nabla \cdot M(X')\,d^2X',$$

where $M(X')$ describes the distribution of magnetic dipoles in the source region, denoted by Ω, and $|X' - X|$ is the magnitude of the distance between a source point X' and a field point X (Jackson, 1975). Similarly, the gravitational potential is

$$\Psi_g = -G\iint_\Omega \ln(|X' - X|)\rho(X')\,d^2X',$$

where G is the gravitational constant and $\rho(X')$ is the anomalous distribution of mass (Grant and West, 1965). Both of these equations express the field as a convolution of the source distribution with the kernel function $\ln(X)$. The inverse problem, that of determining the distribution of mass $\rho(X')$ or magnetization $M(X')$ from the potential fields, is therefore a deconvolution.

Werner's approach to the inverse problem was to use simple, ideal forms for $M(X')$ that would result in a rational function form for the field. For example, a vertical, vertically magnetized dike buried at a depth $(y - y_0)$ may be represented by the form

$$M(x', y') = M_0\delta(x' - x_0)H(y - y_0)\mathbf{j},$$

where δ is the Dirac delta function, H is the Heaviside step function, and $\mathbf{j}$ is a unit vector in the positive y' direction. When used in the convolution integeral for the vertical component of the magnetic field, this results in

$$\begin{aligned} Y_a = -\frac{\partial\Psi}{\partial y} &= \iint_\Omega \frac{\dfrac{\partial M(x', y')}{\partial y'}(y' - y)}{(x' - x)^2 + (y' - y)^2}\,dx'dy' \\ &= M_o\iint_\Omega \frac{\delta(x' - x_0)\delta(y' - y_0)(y' - y)}{(x' - x)^2 + (y' - y)^2}\,dx'dy' \\ &= \frac{-M_0(y - y_0)}{(x - x_0)^2 + (y - y_0)^2}. \end{aligned}$$

An arbitrarily oriented and magnetized dike results in a form that is not much more complex:

$$Y_a = \frac{A(x - x_0) + B(y - y_0)}{(x - x_0)^2 + (y - y_0)^2},$$

where A and B are constants that depend upon the orientation and magnetization of the dike.

Werner assumed that the interference from a neighboring anomaly or a regional anomaly could be expressed as a simple polynomial added to the anomaly of interest. The quantity measured in the field is therefore

$$Y_{\text{field}} = Y_a + c_0 + c_1x + c_2x^2 + \ldots.$$

This expression may be rearranged to (Hartmann et al, 1971)

$$\begin{aligned} x^2Y_{\text{field}} = {} & a_0 + a_1x + b_0Y_{\text{field}} + b_1xY_{\text{field}} + k_0x^2 \\ & + k_1x^3 + \ldots \end{aligned}$$

which, when evaluated at a number of field points, results in a system of equations. Solving this system of equations results in estimates of the coefficients b_0, b_1, a_0, a_1, k_0, k_1, k_2 which, in turn, allow one to estimate the position and depth of the dike under study as well as the interference of neighboring anomalies.

It is apparent that Werner's method depends only on the ability to express the anomaly of interest as a rational function, for which

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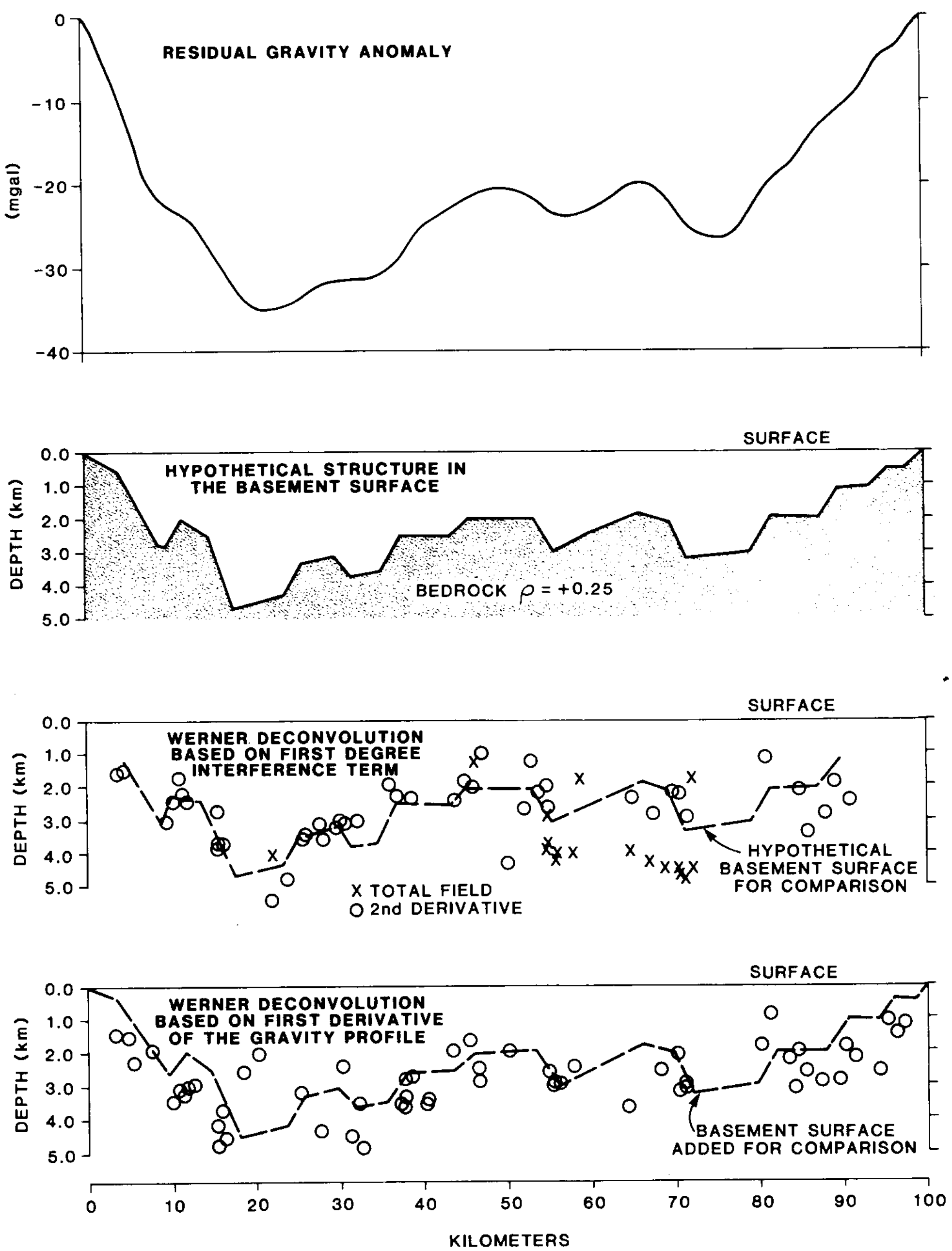

FIG. 1. Werner deconvolution of the gravity expression of a hypothetical basement structure. Vertical exaggeration in the figure is 4 times. Estimated depths have a large amount of vertical scatter, which makes an exact depth estimate impossible. However, the analysis does provide information on the maximum depth to basement because the deepest estimates always lie near or below the basement surface. In addition, the tendency of estimates to cluster at specific locations, for example at 12, 16, 38, 56, and 70 km, can be correlated from one profile to another, making a study of trends in the basement structure possible.

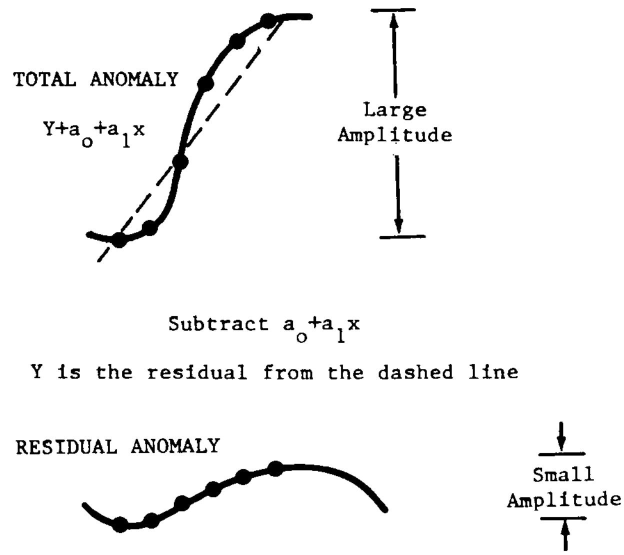

FIG. 2. Fitting a linear or higher order interference term to an anomaly reduces its amplitude substantially. This makes the analysis of the residual very susceptible to noise. Also, because the mass or susceptibility of the anomaly-causing body is only reflected in the amplitude of the anomaly, fitting an interference term removes the dependence of the solution on the actual mass or susceptibility of the body.

an example is

$$\frac{A(x - x_0) + B(y - y_0)}{(x - x_0)^2 + (y - y_0)^2}.$$

On this basis the method may be extended to models other than the magnetized dike. Hartmann et al (1971) and Jain (1976) discussed using a magnetic interface model which requires a Werner analysis not of the magnetic field, but of its horizontal derivative. Another suitable field is the vertical component of gravity caused by a horizontal cylinder which is

$$\frac{-GM_0(y - y_0)}{(x - x_0)^2 + (y - y_0)^2},$$

where M_0 is the excess mass of the cylinder per unit length. Dobrin (1975) suggested using such a model for topography on a basement surface. In an analogy with using the horizontal derivative of the magnetic field in Werner deconvolution, one may also use horizontal derivatives of the vertical component of gravity. For example, the second horizontal derivative of gravity over a vertical fault of small throw T and density contrast $\Delta\rho$ is approximately

$$\frac{4G\Delta\rho T(x - x_0)(y - y_0)}{(x - x_0)^2 + (y - y_0)^2},$$

which is once again a rational function of the proper form for a Werner deconvolution.

Figure 1 illustrates a Werner deconvolution of gravity data. The upper two panels in the figure show a hypothetical basement surface and the residual gravity anomaly that one would observe over it. The lower two panels show the Werner deconvolution estimates of the location of features on the basement surface based on the residual gravity itself and its first and second horizontal derivatives. There is a substantial amount of scatter in these estimates. However, the Werner deconvolution does provide an estimate of the maximum depth to basement because the deepest estimates lie near or below the basement surface. Also, the clusters of estimates at 12, 16, 38, 56, and 70 km can be correlated from profile to profile, making possible a study of trends in basement structure.

The method may also be extended to analysis of heat flow data by using the integral equation,

$$q(x) = -\iint_{\Omega} \frac{y'Q(x', y')}{(x - x')^2 + (y')^2}\, dx'dy',$$

which expresses the relationship between the vertical component of heat flow $q(x)$ and the distribution of temperature sources $Q(x', y')$. $Q(x', y')$ is the sum total of all temperature sources arising from convection, changes in thermal conductivity and contributions from radioactivity. Thus,

$$Q(x', y') = \frac{1}{4\pi}\left[\frac{A}{K} + \frac{VC\rho - \nabla K}{K} \cdot \nabla T\right],$$

where A is the distribution of radioactive sources, K is the thermal conductivity, ∇K is the thermal conductivity gradient, v is the Darcian velocity of groundwater, and $C\rho$ is the specific heat per unit volume of groundwater. Models such as groundwater flow in fractures and interfaces beween rock types result in proper forms for Werner deconvolution.

These few examples demonstrate that Werner deconvolution is a very general method for the interpretation of 2-D potential field data.

LIMITATIONS OF THE METHOD

In practice the accuracy and usefulness of the Werner deconvolution method is limited by three things. First, one is limited to an analysis in terms of simple 2-D models such as dipping dikes, interfaces, and horizontal cylinders and further limited by a minimum resolution criterion between neighboring bodies. Because Hartmann et al (1971), Jain (1976), and Godson (1978) discussed these limitations, I do not pursue them here. Second, the matrix of coefficients involved in the solution of the system of equations is generally ill-conditioned which makes its inversion very susceptible to noise. Third, the parameters k_0, k_1, and k_2, which are estimates of the interference, are correlated with the parameters a_0, a_1, b_0, and b_1, which are estimates of the depth, location, dip, and susceptibility of the dike, interface, horizontal cylinder, or other model being considered. Therefore, it is impossible to estimate and remove the effect of interference without altering the estimated shape and amplitude of the target anomaly. Let us consider these last two limitations in more detail.

The effect of noise

The effect of noise on the deconvolution process was determined by adding normally distributed random values to the theoretical magnetic anomaly of a single dipping dike, and then using Werner deconvolution to estimate the location and depth of the dike. Without any noise added, the deconvolution process located the top of the dike very successfully, although the results were always biased horizontally to one side of the dike depending upon the dip, magnetization, and degree of interference polynomial assumed.

When noise having a standard deviation equal to 1 percent of the peak amplitude of the anomaly was present, the resulting

Table 1. Correlation of parameters over the interval [0, x] assuming a first degree interference polynomial.

	Parameter					
a_0	1.00					
a_1	0.66	1.00				
b_0	**	**	1.00			
b_1	**	**	**	1.00		
k_0	0.75	0.97	**	**	1.00	
k_1	0.66	0.92	**	**	0.99	1.00
	a_0	a_1	b_0	b_1	k_0	k_1

**Indicates that the correlation depends upon the form of the anomaly

estimates of depth and position were scattered about the true values by about 20 percent of the true depth to the top of the dike. When the dike was vertical and vertically polarized, the scatter in the horizontal direction was approximately the same as in the vertical direction. The horizontal scatter tended to be larger than the vertical scatter when either the dike or its polarization was not vertical.

When noise having a standard deviation equal to 3 percent of the peak amplitude of the anomaly was present, the estimates of the depth and location of the dike were scattered about the true values by about 40 percent of the depth to top. If the standard deviation of the noise exceeded 3 percent of the peak amplitude of the anomaly, the method became incapable of finding any acceptable estimates.

Part of this sensitivity to noise is explained graphically in Figure 2. The upper part of the figure illustrates a total field anomaly of large amplitude and six points on the anomaly which are to be used in a hypothetical Werner deconvolution. By assuming a linear interference term, one effectively fits a straight line to these six values and uses the residual from this straight line to estimate the position of the causative body. The residual has a much smaller amplitude and is, therefore, much more susceptible to noise. Thus, the sensitivity to noise is dependent upon the type of interference term that one assumes.

The effect of correlation

The correlation that exists among the various parameters places some strict limitations on the certainty with which one may quote results of the deconvolution process. Because some of the parameters are highly correlated with one another, it is not the parameters themselves which are best determined in the deconvolution process, but rather some combination of the parameters.

Let $A(x_i)$ represent some quantity which is known through measurement at position x_i. In the case of Werner deconvolution of the magnetic field, $A(x_i) = H_i x_i^2$ where H_i is the total magnetic field. Furthermore, let $[k_n]$ be a set of parameters that determine the value of $A(x_i)$. According to Clifford (1973), the correlation coefficient between the k_α and k_β parameters is

$$r_{\alpha\beta} = \frac{\displaystyle\int \frac{\partial A}{\partial k_\alpha}\frac{\partial A}{\partial k_\beta}\,dx}{\left[\displaystyle\int \frac{\partial A^2}{\partial k_\beta}\,dx \int \frac{\partial A^2}{\partial k_\alpha}\,dx\right]^{1/2}}.$$

It is not possible to calculate all of the correlation coefficients explicitly because some depend upon the form of the anomaly. However, those which can be calculated explicitly are shown in Table 1. Note first that because the correlation among the estimates of interference, k_0, k_1 and the parameters b_0 and b_1 depends upon the anomaly, there is some probability of having a small correlation among these parameters somewhere over the anomaly. The parameters b_0 and b_1 together determine the location and depth of the dike. Second, the high correlation between a_1 and the interference terms indicates that it is impossible to obtain an accurate estimate of both a_1, which, together with a_0, determines the dip and susceptibility of the dike, and the interference. Godson (1978) expressed as much on the basis of his empirical studies.

An empirical examination of this correlation was performed using the theoretical gravity anomaly of a horizontal cylinder without adding any interference terms. When a Werner deconvolution was performed on this anomaly assuming no interference, the estimated depth and position of the center of the cylinder and the estimated excess mass were all within 0.2 percent of the true values. However, when the deconvolution was performed assuming a first- or second-order interference term, the depth and position of the center of the cylinder could still be estimated accurately, but over 99 percent of the excess mass estimate was removed in the interference terms. Again, this is clearly demonstrated in Figure 2 which shows that much of an anomaly's amplitude, which is the only part of the anomaly dependent upon the excess mass, is removed in the interference term.

CONCLUSIONS

The method of Werner deconvolution is a fairly general means of analyzing 2-D potential field data. Its main utility lies in the convenience with which it is programmed on a digital computer, making possible rapid analyses of reconnaissance aeromagnetic and gravity data, and in its automatic regional-residual separation. Its major drawbacks are sensitivity to geologic noise and noise in the field data and the rather large correlation among the parameters describing interference and those describing the desired characteristics of the anomaly-causing body. In general, unless one assumes that there is no interference, the result of this correlation is to make only the location and depth of the causative body known with any certainty. Moreover, the amount of scatter in the location and depth estimates makes Werner deconvolution less desirable than forward or inverse modeling when more detailed information is required.

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