GEOPHYSICS, VOL. 63, NO. 5 (SEPTEMBER-OCTOBER 1998); P. 1595–1603, 11 FIGS.

Weighted Euler deconvolution of gravity data

Pierre B. Keating*

ABSTRACT

Euler deconvolution is used for rapid interpretation of magnetic and gravity data. It is particularly good at delineating contacts and rapid depth estimation. The quality of the depth estimation depends mostly on the choice of the proper structural index and adequate sampling of the data. The structural index is a function of the geometry of the causative bodies. For gravity surveys, station distribution is in general irregular, and the gravity field is aliased. This results in erroneous depth estimates. By weighting the Euler equations by an error function proportional to station accuracies and the interstation distance, it is possible to reject solutions resulting from aliasing of the field and less accurate measurements. The technique is demonstrated on Bouguer anomaly data from the Charlevoix region in eastern Canada.

INTRODUCTION

Automated interpretation techniques are being used more as faster computers and commercial software are now widely available. One of these techniques, Euler deconvolution, is used increasingly in the interpretation of potential field data. In this technique, the field and its three orthogonal gradients (two horizontal and the vertical) are used to compute anomaly source locations. The field is sampled on a rectangular grid interpolated from the original data distribution. Usually, gradients are computed from the interpolated field, but measured gradients can be used when available. Reid et al. (1990) discuss the application of the technique to gravity data. They showed that a structural index of one corresponds to the finite step model. Marson and Klingelé (1993) found that, for noisy synthetic data, a value of 0.5 gives better results. Klingelé et al. (1991) show that, for gravity data, a point mass has a structural index of two. Marson and Klingelé (1993) advocate the use of the vertical gradient of the gravity field, rather than the gravity field itself, for Euler deconvolution. Their rationale is that the vertical gradient is more sensitive to horizontal discontinuities than the gravity field.

THEORY

Euler's homogeneity equation, when taking into account a base level for the background field, can be written as (Thompson, 1982)

$$(x - x_o)\frac{\partial T}{\partial x} + (y - y_o)\frac{\partial T}{\partial y} + (z - z_o)\frac{\partial T}{\partial z} = N(B - T), \tag{1}$$

where T is the observed field, x_o, y_o, z_o are source anomaly locations, B is the base level of the observed field, and N is a structural index. Since a regular grid is used, the equations can be solved over a small window. An $n \times n$ window results in an overdetermined system ($n > 2$) of equations from which the four unknowns (x_o, y_o, z_o, B) and their estimated errors are obtained. These estimated errors are often normalized by the estimated depth and expressed as percentages (Thompson, 1982). The technique, therefore, is to solve systems of equations over a moving window. For large grids, this can result in thousands of solutions. Good solutions are considered to be those that group well and have a small relative error.

When magnetic data are used, they have generally been acquired along regularly spaced lines; however, the field interpolated from this data set is aliased if the line spacing is too wide. Since gravity data are usually distributed irregularly, the grid calculated from such a station distribution can be aliased substantially, since interpolation techniques are unable to accurately represent the field. Because Euler deconvolution assumes that the gridded data accurately represent the true field, results, therefore, can be strongly affected by any aliasing problems present.

A solution to the aliasing problems produced by irregularly spaced data is to weight the individual Euler equations by a factor proportional to the data accuracy and the distance between the measured data and calculated grid points.

Presented at the 66th Annual International Meeting, Society of Exploration Geophysicists. Manuscript received by the Editor April 21, 1997; revised manuscript received February 17, 1998.
*Geological Survey of Canada, Geophysics Division, 615 Booth Street, Ottawa, Ontario K1A 0E9, Canada. E-mail: keating@gsc.nrcan.gc.ca.

WEIGHTED EULER DECONVOLUTION

Various schemes can be used to achieve this weighting. If the data all have the same accuracy, then simple inverse distance weights can be used. The distance is measured between the data nearest location and the grid point (x, y, z). If the data have different accuracies, an error map can be estimated from the known station accuracies (Keating, 1995) or, alternatively, if the grid is interpolated by kriging, the kriging interpolation error can be used.

Equation (1) can be written as

$$x_o \frac{\partial T}{\partial x} + y_o \frac{\partial T}{\partial y} + z_o \frac{\partial T}{\partial z} + NB = x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y} + z \frac{\partial T}{\partial z} + NT \tag{2}$$

or in matrix form as

$$\underset{\sim}{\mathbf{A}}\mathbf{u} = \mathbf{v}, \tag{3}$$

where $\mathbf{u}$ is the vector of m unknown parameters (x_o, y_o, z_o, B), $\underset{\sim}{\mathbf{A}}$ is the matrix of field gradients and structural index, and $\mathbf{v}$ is the vector of known parameters (x, y, z). The least squares estimate of $\mathbf{u}$ is given by

$$\mathbf{u} = (\underset{\sim}{\mathbf{A}}^t \underset{\sim}{\mathbf{A}})^{-1} \underset{\sim}{\mathbf{A}}^t \mathbf{v}, \tag{4}$$

and the estimated residual variance is

$$\hat{\sigma}^2 = \frac{\mathbf{v}^t \mathbf{v}}{(n-m)}. \tag{5}$$

The n data are weighted to give a relative degree of importance to each value. The weighting matrix $\underset{\sim}{\mathbf{W}}$ can be defined as

$$\underset{\sim}{\mathbf{W}} = \sigma^2 \underset{\sim}{\mathbf{S}}, \tag{6}$$

where $\underset{\sim}{\mathbf{S}}$ is the covariance matrix of the data. Assuming that the errors in the data are uncorrelated, the covariance matrix becomes a diagonal matrix with elements, σ_i^2, equal to the estimated data error (Inman, 1975). The weighted least squares estimate is then

$$\mathbf{u} = (\underset{\sim}{\mathbf{A}}^t \underset{\sim}{\mathbf{W}}^{-1} \underset{\sim}{\mathbf{A}})^{-1} \underset{\sim}{\mathbf{A}}^t \underset{\sim}{\mathbf{W}}^{-1} \mathbf{v}, \tag{7}$$

and the estimated residual variance is

$$\hat{\sigma}_w^2 = \frac{\mathbf{v}^t \underset{\sim}{\mathbf{W}}^{-1} \mathbf{v}}{(n-m)}. \tag{8}$$

SYNTHETIC EXAMPLE

The synthetic data set used here is the gravity anomaly due to two adjacent prisms of dimension 2 × 2 km buried at a depth of 200 m. Both prisms have a depth extent of 5 km and are located 1 km apart. The gravity grid has a 50-m spacing. Euler deconvolution is performed using a 10 × 10 data window with a structural index of zero. The calculated gravity anomaly and the resulting solutions are illustrated in Figure 1. The outlines of the causative bodies are well defined, and the solution depths, about 250 m, agree well with the upper edges of the initial model.

A more realistic approach is to compute the gravity field at locations similar to that encountered in a real station distribution. This data set is then gridded with the minimum curvature algorithm at a 50-m interval. The corresponding contour map is presented in Figure 2. Although the presence of the two prisms can be inferred from the map, the gravity field is poorly defined between them. The Euler deconvolution is calculated using the same parameters as previously. The calculated Euler solutions are plotted in Figure 3, superimposed on the gravity contour map. The two adjacent edges of the prisms are not identified by

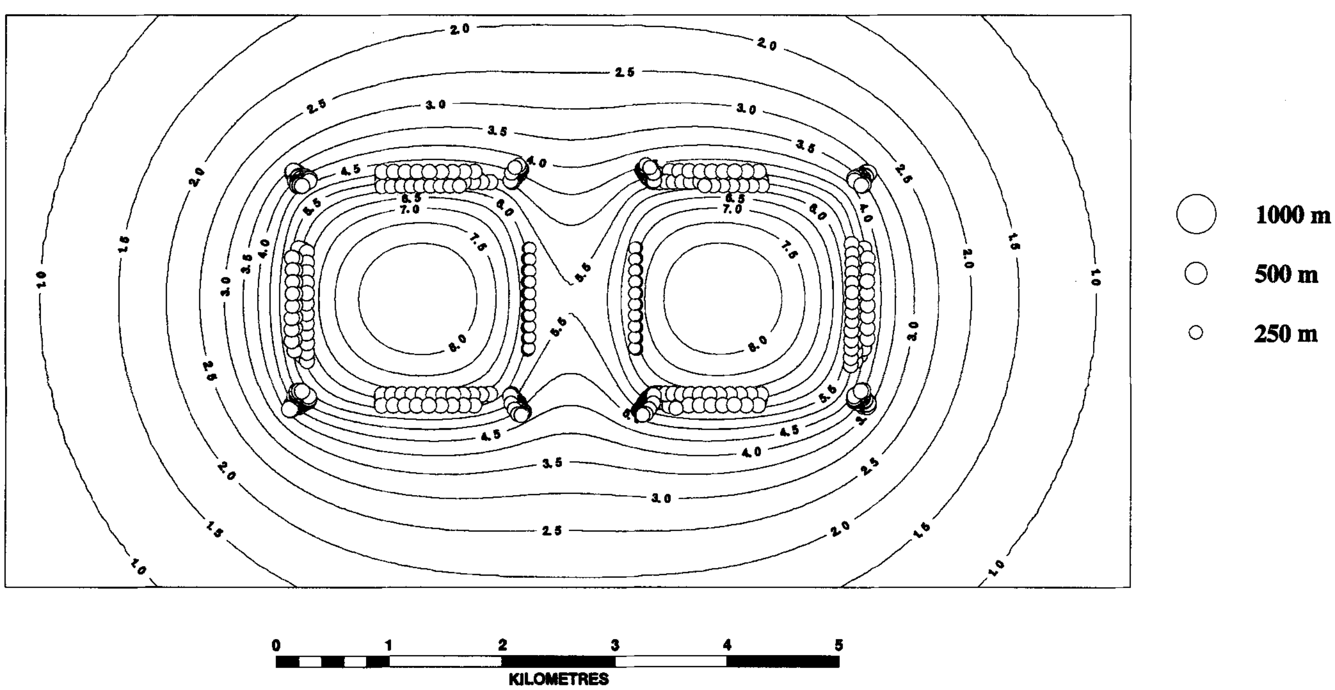

FIG. 1. Gravity anomaly due to two adjacent prisms of dimension 2 × 2 km buried at a depth of 200 m. Contour interval is 0.5 mGal. Circle diameter is proportional to the depth of the Euler solutions. A 10 × 10 window and a structural index of zero were used.

the Euler solutions, and deep, erroneous solutions now appear on the northern flank of the anomaly. Realistic station distributions have resulted in aliased gravity field data (Keating, 1993), and no interpolation technique can recover the missing information. Experienced interpreters could identify the deep solutions as possibly erroneous, but there is no way to quantify such an interpretation. In fact, these erroneous solutions are numerically as accurate as the valid solutions obtained from Euler deconvolution because standard Euler deconvolution assumes the gridded data is error free and perfectly represents the actual gravity field.

To compute the weighted Euler deconvolution, an error grid can be calculated using the method proposed by Keating (1995). In this technique, an error grid (Figure 4) is calculated

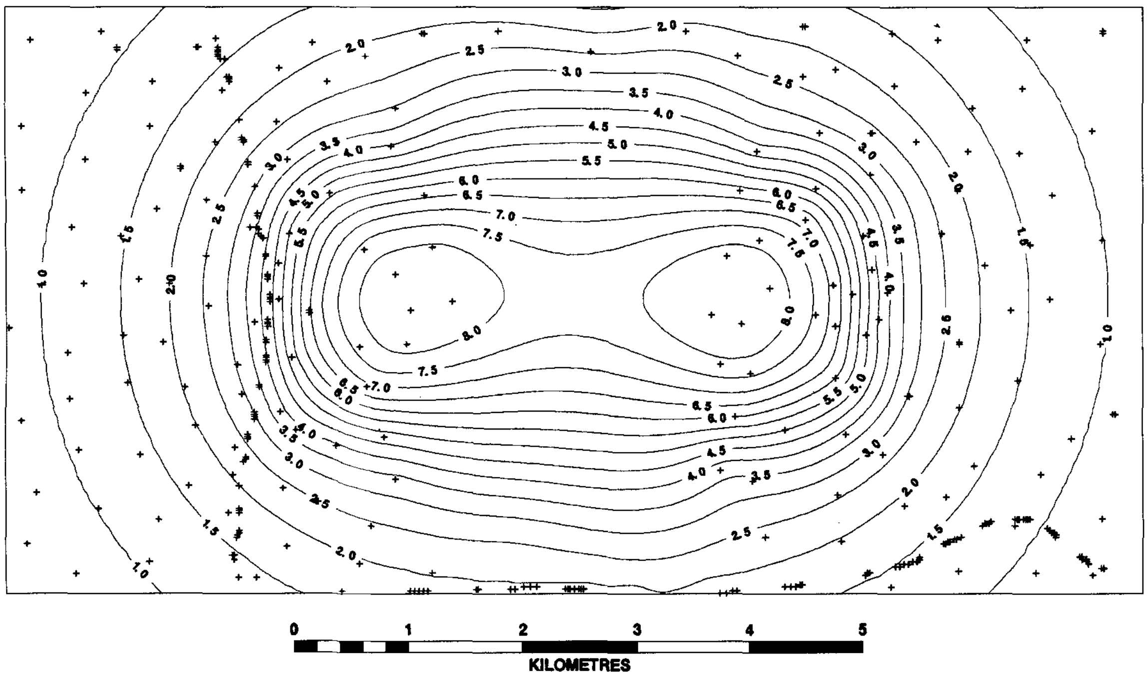

FIG. 2. Gravity anomaly gridded from gravity data obtained at the station locations indicated by crosses. Contour interval is 0.5 mGal.

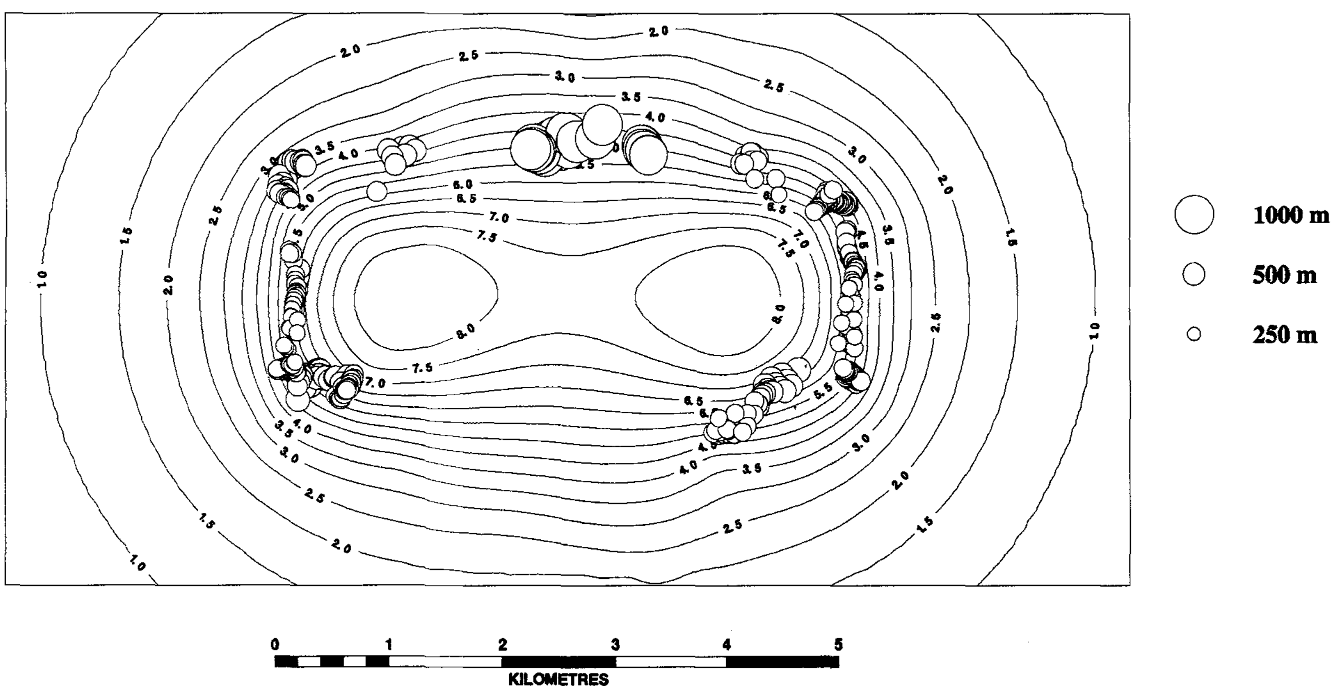

FIG. 3. Euler solutions calculated from the gravity anomaly shown in Figure 2. Solutions are superimposed on the gravity anomaly map. Contour interval is 0.5 mGal.

from the accuracy of the gravity stations. Deconvolution is still calculated with the same parameters as in the previous examples, and the weighted Euler solutions are presented in Figure 5. Weighting results in the rejection of solutions located in areas where the gravity field is aliased because these grid points are associated with large errors and, therefore, small weights. From equation (8), it can be seen that weighting results in a rescaling of the residual variance. For a given data set, grid points that are given larger weights because of their higher accuracies produce solutions that have smaller residual variances than those with small weights. This is why it is not possible to make a direct comparison between weighted and unweighted solutions based on the residual variances. It is better to compare the "n" best solutions obtained from both techniques. In

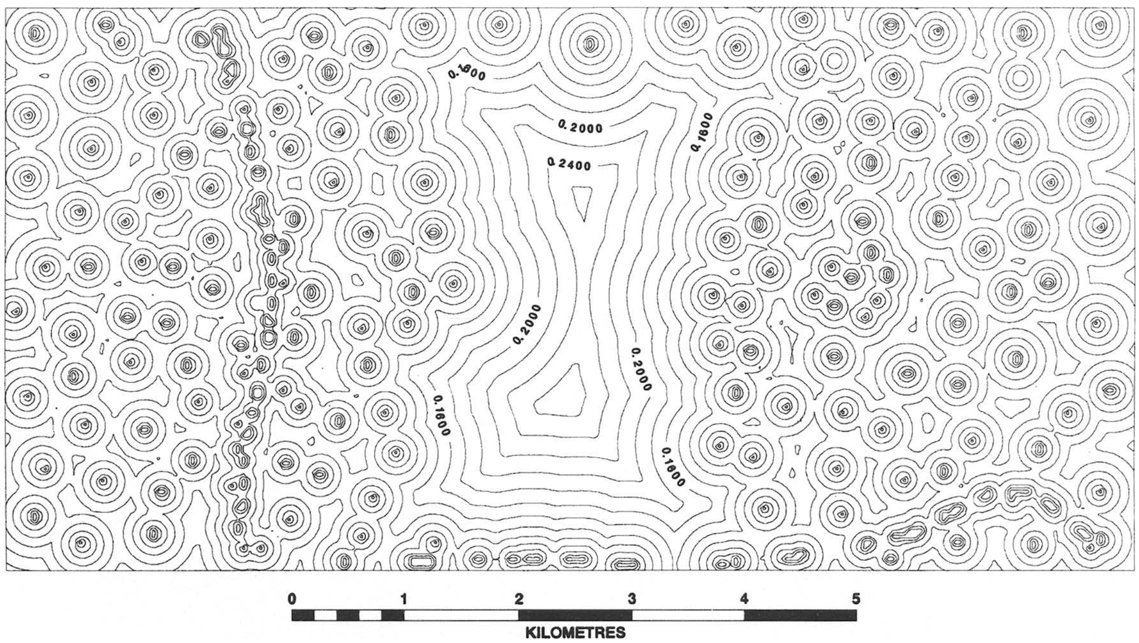

FIG. 4. Error map used to weight the Euler equations. Contour interval is 0.02 mGal.

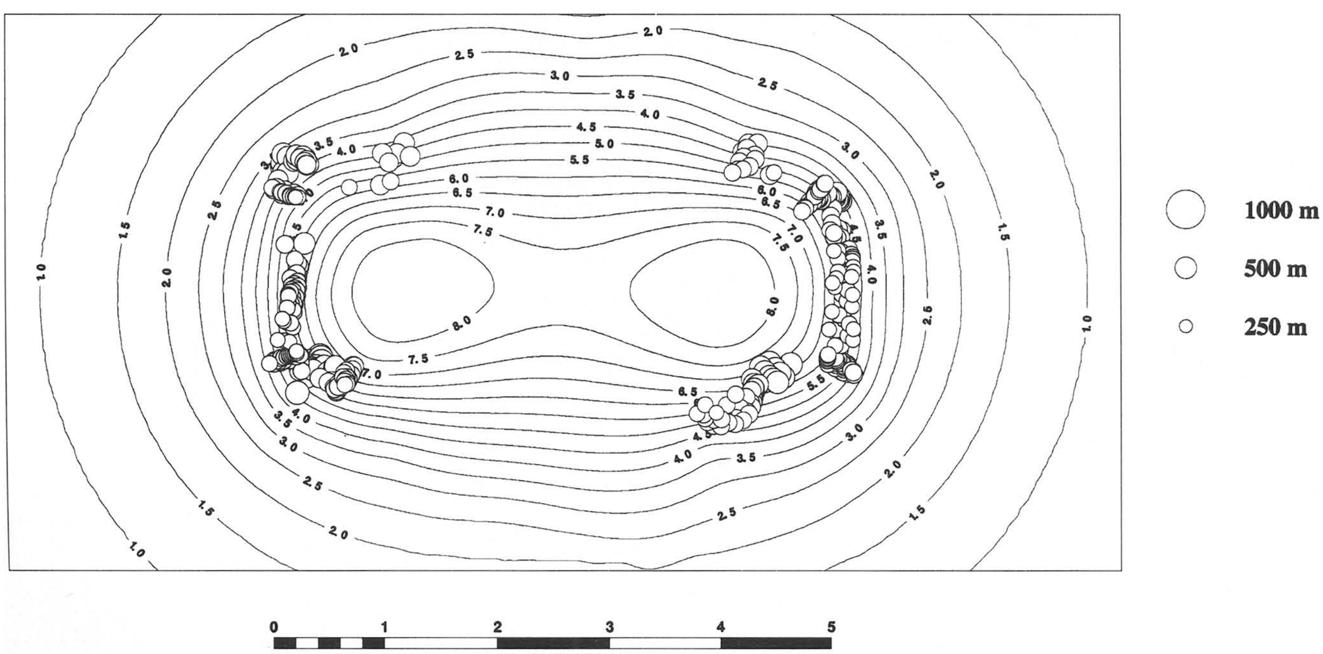

FIG. 5. Weighted Euler solutions calculated from the gravity anomaly shown in Figure 2. Weights were derived from Figure 4. The solutions are superimposed on the gravity anomaly. Contour interval is 0.5 mGal.

both previous examples, the best 415 solutions are presented. This corresponds to a maximum error of 27% for the standard technique and 9% for the weighted Euler deconvolution technique.

FIELD EXAMPLE

The technique is demonstrated on gravity data from the Charlevoix region in eastern Canada. Three main geological units make up the geology of the region: the Precambrian Shield of Grenvillian age, outcropping north of the St. Lawrence River; the Ordovician St. Lawrence platform sediments; and the Appalachian nappes. Four major structural events created the faults of the region: the Grenvillian collision, the rifting episode related to the opening of the Iapetus ocean, the reactivation of these faults at the closing of that ocean during the Taconian Orogeny and, finally, the Charlevoix impact crater of Devonian age. This region is seismically active (Lamontagne, 1987), and numerous studies have attempted to understand the structure of this area characterized by the presence of the impact crater that has a diameter of about 55 km (Rondot, 1971). It is unknown, however, if this impact crater is the direct cause of the seismic activity (Lamontagne and Ranelli, 1997). Gravity data have been acquired over many years in the area of interest. Stations are irregularly distributed and have various accuracies. There are three major data sets: (1) data acquired as part of the Canadian National Gravity Program at an average station spacing of about 10 km, (2) data from high-accuracy gravity surveys taken to investigate the impact structure, and (3) recent marine gravity data acquired over the St. Lawrence River to study in detail the seismically active zone located under the river. The Bouguer anomaly map and station locations are shown in Figure 6. The data are gridded at a 500-m interval with the minimum curvature algorithm.

Results from standard Euler deconvolution calculated for a structural index of 1 over a 10×10 moving window are shown in Figure 7. Reid et al. (1990) have shown that, for gravity data, the structural index of a finite step is unity. It is worthwhile to note that only a few simple geometries (dyke, fault, contact) satisfy Euler's homogeneity equation (Blakely, 1995). More complex geometries do not. In practice, however, the technique gives satisfactory results because, to a first approximation, bodies of complex geometries can often be approximated by an assemblage of bodies of simple geometry. It can be seen in Figure 7 that groups of deep solutions are obtained in areas where station spacing is large, especially in the northwest and southeast corners of the map. In these areas, the Bouguer anomaly map displays long wavelength anomalies that result from the wide station spacing. Although this is a reasonable interpolation, it causes deep Euler solutions with relatively small residual variances. The residual variance is a measure of the goodness of fit between the model, the Euler homogeneity equation, and the gridded data.

When the Euler equations are weighted by an error function estimated from the station accuracies (Figure 8), a very different pattern of solutions is obtained, as shown in Figure 9. A second set of error estimates based on inverse distance (Figure 10) were used to produced the solutions shown in Figure 11.

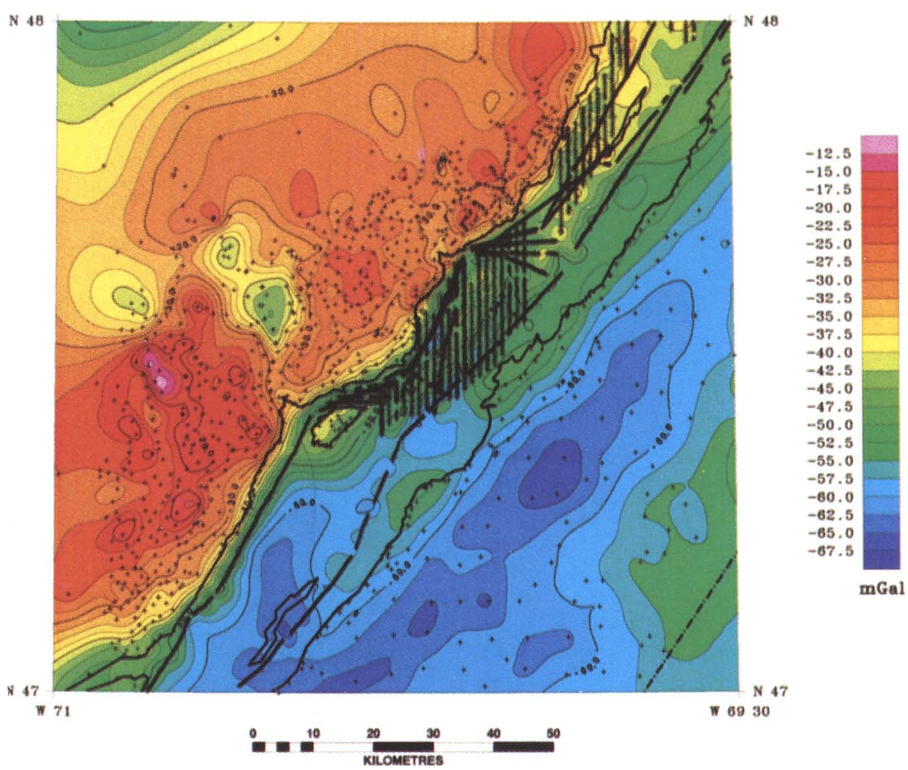

FIG. 6. Bouguer anomaly map of the Charlevoix region. Minimum contour interval is 2.5 mGal. Station locations are indicated by black dots. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

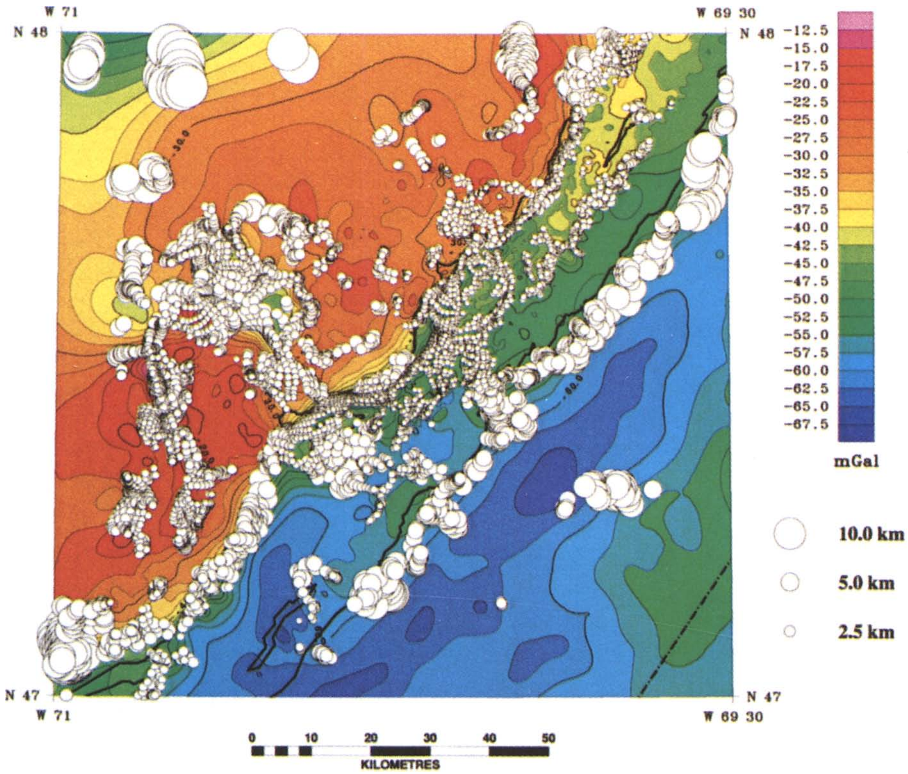

FIG. 7. Euler solutions of the Bouguer anomaly of the Charlevoix region. A 10 × 10 window and a structural index of 1 are used. The 9500 best solutions are shown. The maximum relative error is 5.4%. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

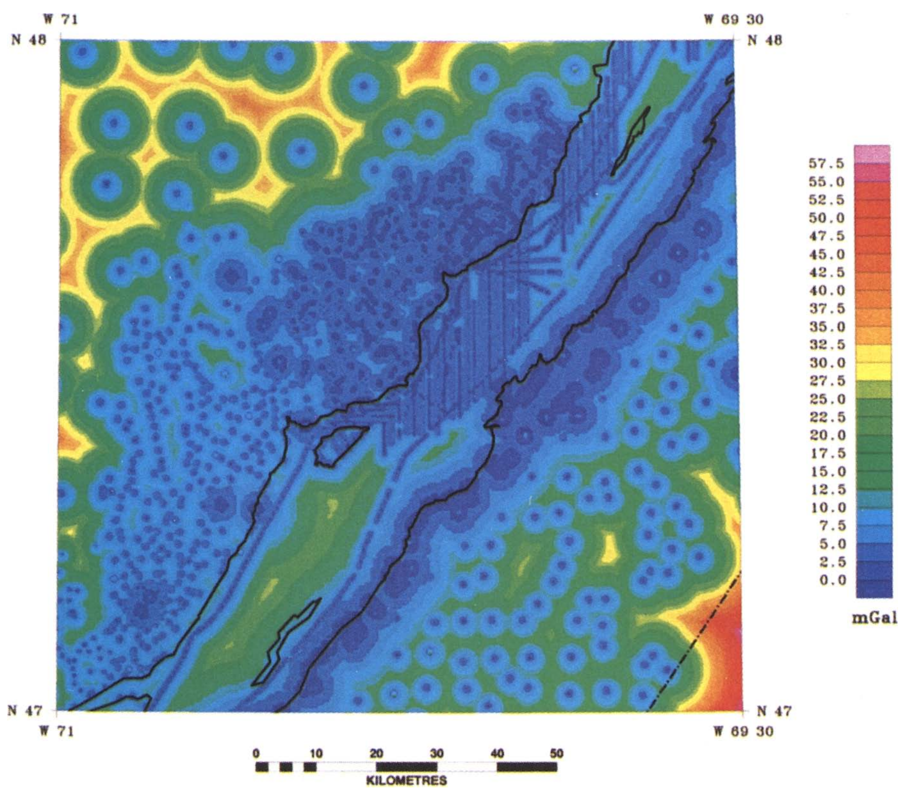

FIG. 8. Error map, based on gravity station accuracies, used to weight the Euler equations. Colour contour interval is 2.5 mGal. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

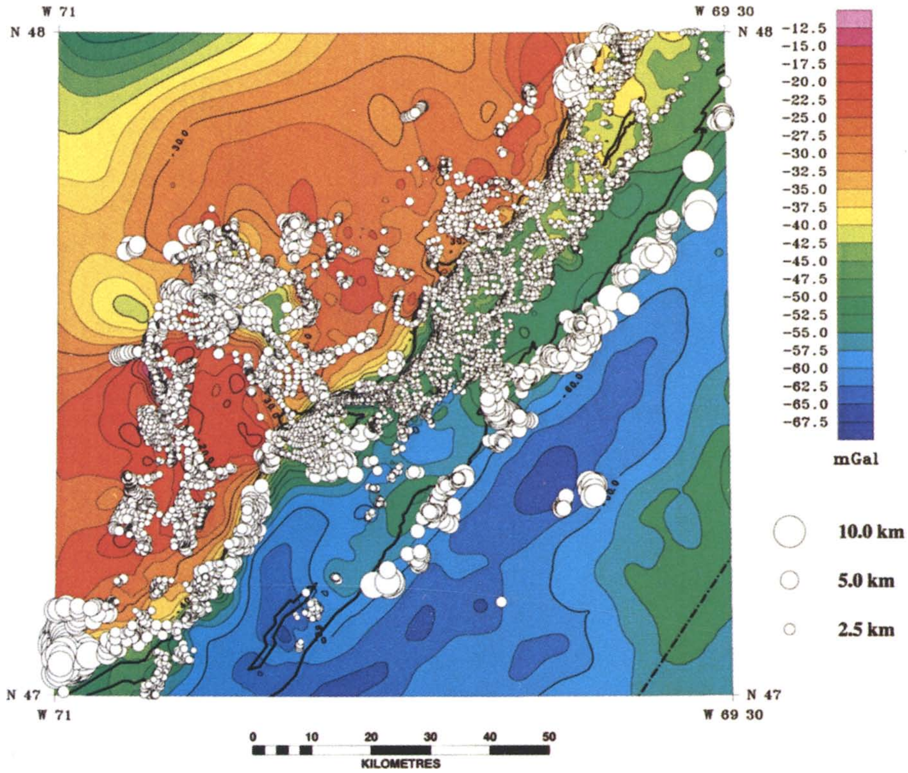

FIG. 9. Weighted Euler deconvolution of the Bouguer anomaly of the Charlevoix region. Weights are based on the error map of Figure 8. A 10×10 window and a structural index of 1 are used. The 9500 best solutions are shown. The maximum relative error is 11.3%. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

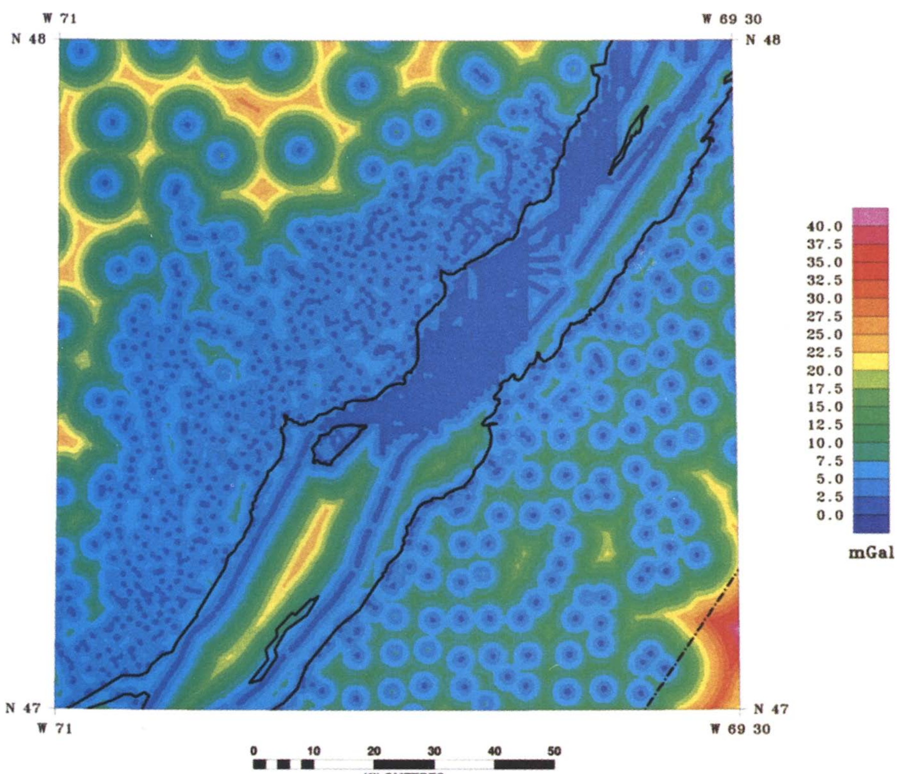

FIG. 10. Error map, based on an inverse distance criteria, used to weight the Euler equations. Colour contour interval is 2.5 mGal. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

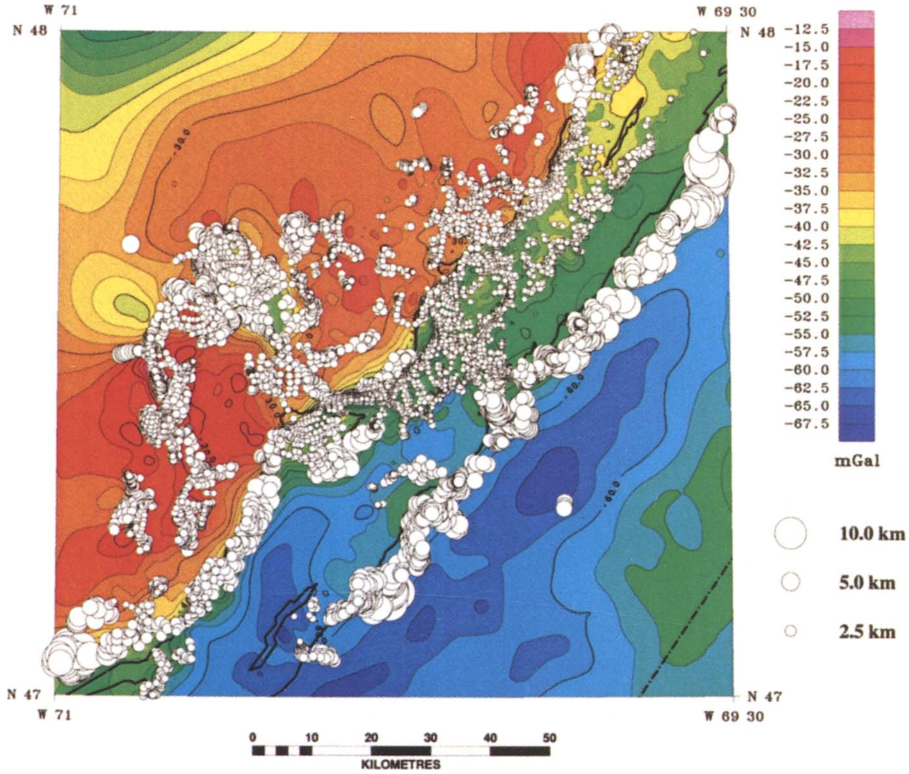

FIG. 11. Weighted Euler deconvolution of the Bouguer anomaly of the Charlevoix region. Weights are based on the error map shown in Figure 10. A 10 × 10 window and a structural index of 1 are used. The 9500 best solutions are shown. The maximum relative error is 10.5%. Shores of the St. Lawrence River are marked by thick black lines. The Canada-USA border is indicated by the line in the southeast corner of the map.

In both cases, solutions located in areas where data are sparse have high relative errors and are automatically rejected; minor discrepancies are observed along the south shore of the St. Lawrence River. Euler equations corresponding to grid points located far away from gravity observations are given smaller weights than grid points located near measurement points and, therefore, have a smaller influence on the solution. Deep solutions, previously observed in the northwest and southeast corners of the map, are rejected. The deep solutions located in the northwest corner of the map are about 40 km southwest of the epicenter of the 1988 Saguenay earthquake. This earthquake of magnitude 5.9 occurred at a depth of 29 km (Du Berger et al., 1990). It is unrelated to any known geological structure. It would be tempting to relate these deep solutions to the earthquake. However, these solutions are rejected by the weighted Euler deconvolution technique; the data do not support them. Solution patterns are similar where accurate and regularly spaced data exist, such as on the north shore of the St. Lawrence River where the center of the impact crater is located. Using an error grid based on station accuracies is more realistic than using one based on a simple inverse distance criteria. However, the later is easier to implement.

CONCLUSIONS

One of the major problems in using Euler deconvolution is the rejection of bad solutions. Assuming perfectly interpolated data can lead to gross interpretation errors. Long wavelength anomalies are generated by interpolation techniques in areas where data points are far apart. Closely spaced, inaccurate data can generate short wavelength anomalies. In both cases, the relative error of the calculated Euler solutions can be small, and these solutions cannot be distinguished from the "good" ones. Weighting the Euler equations by an error measure solves the problem and can improve the interpretation.

ACKNOWLEDGMENTS

I thank my colleague M. Pilkington for suggesting useful improvements to the manuscript and M. Lamontagne for allowing me to use his marine gravity data. I also thank R. O. Hansen, the Associate Editor, and the reviewers A. Reid and D. N. Ravat for suggesting improvements to the original manuscript.

This is Geological Survey of Canada publication 1997026.

REFERENCES

Blakely, R. J., 1995, Potential theory in gravity and magnetic applications: Cambridge Univ. Press.

Du Berger, R., Roy, D. W., Lamontagne, M., Woussen, R. G., North, R. G., and Wetmiller, R., 1990, The Saguenay (Quebec) earthquake of November 25, 1988: seismological data and geological setting: Tectonophysics, **186**, 59–74.

Inman, J. R., 1975, Resistivity inversion with ridge regression: Geophysics, **40**, 798–817.

Keating, P., 1993, The fractal dimension of gravity data sets and its implication for gridding: Geophys. Prosp., **41**, 983–993.

——— 1995, Error estimation and optimization of gravity surveys: Geophys. Prosp., **43**, 569–580.

Lamontagne, M., 1987, Seismic activity and structural features in the Charlevoix region, Quebec: Can. J. Earth Sci., **24**, 2118–2129.

Lamontagne, M., and Ranelli, G., 1997, Faults and spatial clustering of earthquakes near La Malbaie, Charlevoix Seismic Zone, Canada: Seis. Res. Lett., **68**, 337.

Klingelé, E. E., Marson, I., and Kahle, H. G., 1991, Automatic interpretation of gravity gradiometric data in two dimensions: Vertical gradient: Geophys. Prosp., **39**, 407–434.

Marson, I., and Klingelé, E. E., 1993, Advantages of using the vertical gradient of gravity for 3-D interpretation: Geophysics, **58**, 1588–1595.

Reid, A. B., Allsop, J. M., Granser, H., Millett, A. J., and Somerton, I. W., 1990, Magnetic interpretation in three dimensions using Euler deconvolution: Geophysics, **55**, 80–91.

Rondot, J., 1971, Impactite of the Charlevoix structure, Quebec, Canada: J. Geophys. Res., **76**, 5414–5423.

Thompson, D. T., 1982, EULDPH—A technique for making computer-assisted depth estimates from magnetic data: Geophysics, **47**, 31–37.