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A NEW METHOD FOR INTERPRETATION OF AEROMAGNETIC MAPS: PSEUDO-GRAVIMETRIC ANOMALIES*

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ABSTRACT

The purpose of this paper is to describe a method of interpretation based on a transformation of the total magnetic intensity anomalies into simpler anomalies. The result of this transformation is the elimination of the distortion due to the obliquity of the normal magnetic field, so that the resulting anomalies will be located on the vertical of the disturbing magnetized bodies.

The starting point of the theory is the well known relation between magnetic potential V and Newtonian potential U—relation which can be written:

$$\vec{\mathfrak{J}} \cdot \overrightarrow{\text{grad}}\ U = f\sigma \cdot V$$

with:

$\vec{\mathfrak{J}}$ = magnetization

$f = 66.7 \cdot 10^{-9}\ u \cdot CGS$

σ = assumed density of magnetized bodies

This relation may be considered as a partial differential equation. The boundary condition consists in the measured values of the total field $T(P)$ known at each point P at the datum plane. As $T(P)$ is the derivative of a harmonic function V, we can determine this function everywhere above the datum plane. Solving then the partial differential equation, we find the Newtonian potential U and its vertical derivatives $g, g', g'' \cdots$.

Finally, we obtain these quantities as functionals of the measured magnetic anomaly $T(P)$. For instance, one of the formulas is

$$g(M) = \frac{1}{2\pi} \iint H(M, P) T(P) dS_P$$

where $H(M, P)$ is the kernel of the transformation allowing the direct computation of $g(M)$—an anomalous field which will be called "pseudo-gravimetric anomaly."

Of course, the pseudo-gravimetric anomaly has all the usual properties of a gravimetric anomaly. The field of $g(M)$ presents no distortion and the interpretation becomes as easy as that of a Bouguer anomaly map.

To perform the actual routine calculations of this transformation, we start with the values of T taken on a rectangular or trigonal grid, as for the usual computation of the vertical derivatives. The use of punched-card equipment speeds up considerably these calculations.

I. INTRODUCTION

In applied geophysics, the interpretation of results assumes two different aspects, corresponding to two successive phases. The first phase is a detailed analysis of the data obtained by each of the methods applied. The second is the synthesis of both geophysical and geological data. This second stage most often concerns the geologist: the geophysicist only points out the possibilities and discards the inconsistent assumptions, but the first preliminary phase is the geophysicist's concern only.

In gravimetry, the main problem of the preliminary step is the elimination

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of the regional anomaly. Modern methods, computation of the vertical derivatives of the field, are justified only because of their objectivity. But, in magnetic work, the preliminary phase becomes very important.

As a matter of fact, the results of a magnetic survey are more difficult to interpret than those of a gravimetric survey. While, in gravimetry there is a simple connection between the residual anomalies and the tectonic features, in magnetism the anomalies do not take place at the apex of the disturbing bodies. The magnetic picture of the tectonic occurrences undergoes a distortion, due to the inclination of the magnetizing vector.

The difficulty is further increased in the case of an aeromagnetic survey. Then, instead of the vertical component of the field, the variations of the total magnetic field are recorded, and this again increases the distortion of the map: the effects of the obliquity of the recorded field are added to those of the inclination of the magnetization.

An additional difficulty arises at the final stage of the interpretation, when one tries to check the geological assumptions and to compare them, for instance, with a simple typical case. In gravimetry, the computation of the influence is quite immediate; in magnetism, the computation is much more complicated and a greater number of variable factors must be taken into account, such as the inclination of the normal field, the orientation of the structure with respect to the magnetic meridian, and so on.

These considerations show that, when faced with a problem of magnetic interpretation, it is advantageous, to begin with, to transform the isanomal map, in order to suppress the distortion, and to make the map as clear as a map of the Bouguer anomalies.

The purpose of this paper is to show that such a transformation is possible. It is not more difficult than the computation of vertical derivatives, and can be carried out as a routine job. The transformation is based on the fact that, in most practical cases, the magnetic potential V is connected to a gravimetric potential U. We shall first establish this relation.

II. FUNDAMENTALS OF THE METHOD

The gravimetric potential is a Newtonian potential, the general expression of which is

$$U = -f \iiint \sigma \frac{1}{r} \, dv$$

where f is the gravitational constant and σ the density, varying, in general, from point to point. The expression of the magnetic potential can be written as

$$V = \iiint \vec{\mathfrak{J}} \cdot \overrightarrow{\operatorname{grad}} \left(\frac{1}{r} \right) dv$$

where $\vec{\mathfrak{J}}$ is the magnetization vector, which may be variable too. The only assumption that we shall make is to suppose that the direction of $\vec{\mathfrak{J}}$ is the same in all magnetized masses. But we are not obliged to assume that this direction is the same as that of the normal terrestrial magnetic field.

Computing the scalar product of the magnetization $\vec{\mathfrak{J}}$ by the gradient of the gravimetric potential, we get the following relation:

$$\vec{\mathfrak{J}} \cdot \overrightarrow{\text{grad}}\, U = f\sigma V.$$

This formula seems to be known since Poisson's time. Let us recall that the founder of applied gravimetry, Eötvos, used Poisson's theorem to deduce two famous relations between the three components of the magnetic anomaly, and the values recorded on a torsion balance, but, now, the Eötvos' formulae are only of historical interest. However, the relation between the two potentials may be used to deduce more practical formulae, allowing direct transformation of the experimental magnetic anomalies into simpler anomalies, which we propose to call "pseudo-gravimetric" anomalies.

There are two reasons for this name: first we do not know the true density σ. Secondly, only the magnetized masses contribute to the potential U. The unmagnetized rocks do not show. Thus the anomalies deduced from U are not the true gravimetric anomalies: they are still magnetic anomalies, but computed on the assumption that the magnetization vector is vertical. To be perfectly clear, we call them "pseudo-gravimetric anomalies."

The essential fact is that anomalies deduced from U are as simple as Bouguer anomalies. They are located on the vertical of the magnetized masses, and do not depend on the inclination of the normal field, nor on the direction of the magnetization. Thus the interpretation and all the computations become very simple, as these factors no longer need to be taken in account.

Therefore, we shall select a conventional density, by writing:

$$f\sigma = |\mathfrak{J}|.$$

Then, calling $\vec{\nu}$ the unit vector representing the direction of $\mathfrak{J}$, we obtain:

$$\vec{\nu}\ \overrightarrow{\text{grad}}\, U = \frac{dU}{d\nu} = V \tag{2-1}$$

which means that the magnetic potential is the oblique derivative of the pseudo-gravimetric potential, in the direction of the magnetization.

III. TRANSFORMATION OF THE VERTICAL MAGNETIC COMPONENT

Let us now take the upward vertical as the Oz axis, and differentiate the relation (2-1) with respect to z. Thus:

$$\frac{dg}{d\nu} = Z \tag{3-1}$$

i.e., the vertical component Z of the magnetic anomaly is the derivative of the pseudo-gravimetric anomaly, that we shall call g, as in gravimetry, as there is no risk of confusion.

Before dealing with our main problem—the transformation of the anomalies of the total force—let us consider the equation (3-1), which we propose to integrate.

Let us consider (Fig. 1) a straight-line (D), with its origin at M and extending towards infinitv, either in the direction of the vector $\vec{\nu}$, or in the opposite direction,

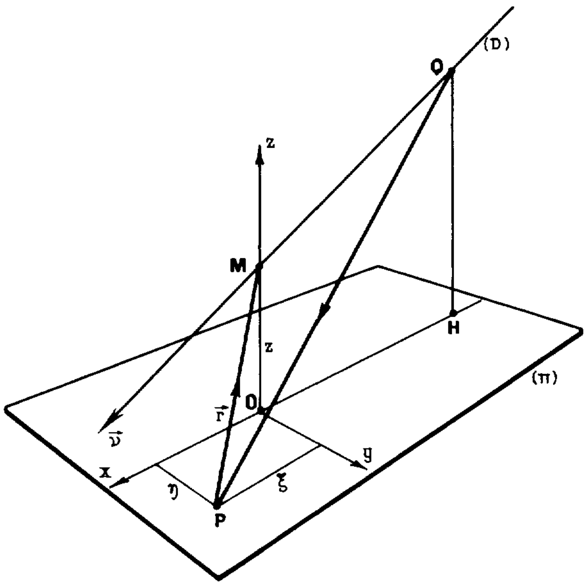

FIG. 1

but in such a manner that (D) will not cross zones where magnetized masses may have a chance to be found. More specifically, let us consider the northern hemisphere. The unit vector $\vec{\nu}$, then, will, in most cases, be directed downwards. We shall take for (D) the half-line with direction opposite to $\vec{\nu}$. It will be in totality above the earth's surface. Let us call Q any given point of this line.

To obtain the function $g(M)$ from (3-1), we need only to integrate this expression along the line (D). We get:

$$g(M) = \int_0^\infty Z(Q)ds$$

where s is the length of the variable sgement $\overline{MQ}$. This is an inverse operation with respect to the derivation in the direction of the vector $\vec{\nu}$. Of course, we do not know the value of the anomaly $Z(Q)$ on line (D). We only know the values of Z measured on the plane (π).

However, it is possible to compute Z at every point Q above the horizontal plane (π), using the well known upward continuation formula. If P is a point of the plane, $Z(Q)$ is given by the expression:

$$Z(Q) = -\frac{1}{2\pi}\iint_\pi \frac{\partial}{\partial z}\left(\frac{1}{\overline{QP}}\right) Z(P)dS_P$$

where dS_P is a surface element, the integration comprising the whole area of plane (π). Using this value of $Z(Q)$ and changing the order of integrations, we obtain:

$$g(M) = -\frac{1}{2\pi}\iint_\pi Z(P)dS_P \int_\pi^\infty \frac{\partial}{\partial z}\left(\frac{1}{\overline{QP}}\right) ds.$$

Let us put in evidence the function

$$K(M, P) = -\int_0^\infty \frac{\partial}{\partial z}\left(\frac{1}{\overline{QP}}\right) ds. \tag{3-2}$$

This is the kernel of the linear transformation

$$g(M) = \frac{1}{2\pi}\iint_\pi K(M, P)Z(P)dS_P \tag{3-3}$$

allowing direct passage from the anomaly of the vertical component Z to the pseudo-gravimetric anomaly $g(M)$ at any given point M above the plane (π).

The formula (3-2) suggests that the kernel K is the derivative, with respect to z, of an harmonic function that we shall denote by $u(M, P)$ and which will be useful hereafter. Thus we shall write

$$K(M, P) = \partial u(M, P)/\partial z \tag{3-4}$$

However, we cannot invert the two operations—integration and derivation with respect to z—because the integral dealing with the reciprocal of the distance $1/\overline{QP}$ is divergent. To remove this difficulty, we shall substitute to this function the following difference:

$$\frac{1}{\overline{QP}} - \frac{1}{s + s_0}$$

where s_0 is an adequate positive constant. If now we take the derivative with respect to this variable, the additional term, independent from z, disappears, while the integral

$$u(M, P) = \int_0^\infty \left(\frac{1}{s + s_0} - \frac{1}{\overline{PQ}}\right) ds$$

becomes absolutely convergent, as

$$\frac{1}{s+s_0} - \frac{1}{\overline{QP}} = O(s^{-2})$$

and the differentiation under the integral sign is justified.

The final integration is an elementary one. We easily obtain

$$u = \text{Log}\,(r + \vec{\nu}\vec{r}) - \text{Log}\,2s_0 \tag{3-5}$$

where $\vec{r} = \overrightarrow{MP}$ is the vector from M to P, and

$$r^2 = \xi^2 + \eta^2 + z^2.$$

We chose the origin of coordinates $Oxyz$ on the plane (π), so that point M, of elevation z, is on the vertical axis. The coordinates of point P are ξ, η, 0.

Last, to get K, we need only to take the derivative of u with respect to z. We have:

$$K(M, P) = \frac{1}{r}\,\frac{z + \mu r}{r + \vec{\nu}\vec{r}} \tag{3-6}$$

where μ is the cosine of the acute angle between the vector $\vec{\nu}$ and the axis Oz.

We can easily check that $u(M, P)$ are two functions the Laplacian of which is zero (except on D).

The formulae (3-3) and (3-6) completely solve the problem of the transformation of the anomalies of the vertical component into pseudo-gravimetric anomalies.

IV. TRANSFORMATION OF THE TOTAL FORCE

We shall now consider the anomalies of the total magnetic force T.

Let us call $\vec{\gamma}$ the unit vector in the direction of the normal terrestrial field. The anomaly T is the derivative, with the inverse sign, of the magnetic potential V, in the direction $\vec{\gamma}$

$$T = -\frac{dV}{d\gamma}\,\cdot$$

Thus, taking in account the fundamental relation (2-1) we have

$$T = -\frac{d^2U}{d\gamma d\nu}\,\cdot \tag{4-1}$$

Let us now compare this relation with the (3-1) formula. It can be noticed that, here, g is replaced by $-dU/d\gamma$ and Z by T. Thus, without repeating the whole reasoning, we can use the (3-3) formula and write

$$-\frac{dU}{d\gamma} = \frac{1}{2\pi}\iint K(M, P)T(P)dS_P.$$

After differentiating with respect to z, we get, with the usual conventions[1]

$$-\left(\frac{dg}{d\gamma}\right)_M = \frac{1}{2\pi}\iint \frac{\partial K(M, P)}{\partial z} T(P)dS_P.$$

To obtain g, of course, we must integrate again along the half-line, the direction of which is, now given by $\vec{\gamma}$ instead of $\vec{\nu}$. We shall get then

$$g(M) = \int_0^\infty \left(\frac{dg}{d\gamma}\right)_Q dS.$$

Let us use the expression of the derivative in the preceding formula, after substituting the variable point Q for M. If we invert the order of the integrations, we obtain:

$$g(M) = -\frac{1}{2\pi}\iint_\pi T(P)dS_P \int_0^\infty \frac{\partial K(Q, P)}{\partial z} ds.$$

We shall put

$$H(M, P) = \int_0^\infty \frac{\partial K(Q, P)}{dz} ds \tag{4-2}$$

$$g(M) = -\frac{1}{2\pi}\iint_\pi H(M, P)T(P)dS_P. \tag{4-3}$$

This latter formula makes it possible to pass from the anomaly of the total force to the pseudo-gravimetric anomaly. The kernel of the transformation $H(M, P)$ is given by (4-2) where the integral is extended along the half-line of direction $-\vec{\gamma}$, while K depends on $\vec{\nu}$.

It is known that in practice one can very often assume that the direction of the magnetization of the underground is that of the normal terrestrial field. Thus, we can assume that the unit vectors $\vec{\gamma}$ and $\vec{\nu}$ are identical. Our aim was to show that this simple assumption is not indispensable to carry out the transformation. However, hereafter, we shall put $\vec{\gamma}=\vec{\nu}$, to simplify the theory and to shorten the computations.

Before computing the kernel H, let us put

$$H(M, P) = \frac{\partial v(M, P)}{\partial z} \tag{4-4}$$

for the formula (4-2) suggests that H is the derivative with respect to z of a function that we call $v(M, P)$. But, before integrating with respect to s, we must subtract the principal part $\mu/(s+s_0)$ from $K(M, P)$, so that the kernel may be integrated. We get

$$v(M, P) = rK(M, P) - \mu u(M, P). \tag{4-5}$$

[1] The anomaly is considered as positive when it increases the recorded value.

The expression of the kernel $H(M, P)$ can be obtained in differentiating with respect to z as indicated in (4-4). However, we shall not need this general expression, as the only thing we are concerned with is the anomaly in plane $z=0$. We shall thus consider what become the kernels K and H at the limit, for $z=0$.

It will be convenient to choose the axis of coordinates so that the Ox axis will be directed towards the magnetic North, and to use polar coordinates, putting

$$\xi = \rho \cos \omega, \qquad \eta = \rho \sin \omega.$$

Then, if we call I the inclination of the normal field, we shall have $\mu = \sin I$. For practical purposes, we shall write $\lambda = \cos I$. We then obtain:

$$r + \vec{\nu}\cdot\vec{r} = \sqrt{\rho^2 + z^2} + \mu z + \lambda\rho \cos \omega$$

and therefore

$$u = \operatorname{Log}\left[\sqrt{\rho^2 + z^2} + \mu z + \lambda\rho \cos \omega\right]$$

and

$$K = \frac{1}{\sqrt{\rho^2 + z^2}} \; \frac{\mu\sqrt{\rho^2 + z^2} + z}{\sqrt{\rho^2 + z^2} + \mu z + \lambda\rho \cos \omega}.$$

For $z=0$, these two functions become

$$u = \operatorname{Log} \rho + \operatorname{Log}(1 + \lambda \cos \omega) \tag{4-6}$$

and

$$K = \frac{1}{\rho} \; \frac{\mu}{1 + \lambda \cos \omega}. \tag{4-7}$$

The kernel K is the product of the factor $1/\rho$ by the function of ω alone

$$L_1(\omega) = \frac{\mu}{1 + \lambda \cos \omega}. \tag{4-8}$$

The factor $1/\rho$ is infinite for $\rho=0$, thus K presents a singular point at the origin. However, as $dS = \rho \cdot d\rho \cdot d\omega$, this factor disappears under the sign sum (3-3) and the integral

$$g(0) = \frac{1}{2\pi} \iint Z(\rho, \omega) L_1(\omega) d\rho \cdot d\omega \tag{4-9}$$

giving the value of the pseudo-gravimetric anomaly at the origin of the coordinates includes no singular element.

Let us now consider the integral (4-3). Differentiating (4-5) with respect to z, and taking in account (3-4), we get

$$H = \frac{z}{r} K + r \frac{\partial^2 u}{\partial z^2} - \mu K. \tag{4-10}$$

The first term is null for $z=0$, the last one has been already computed. To compute the second derivative of u, we should remember that u is harmonic. Therefore,

$$\rho \frac{\partial^2 u}{\partial z^2} = -\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u}{\partial \rho}\right) - \frac{1}{\rho}\frac{\partial^2 u}{\partial \omega^2} = \frac{1}{\rho}\frac{\lambda(\lambda + \cos\omega)}{(1 + \lambda\cos\omega)^2}$$

which is obtained immediately from (4-6). At last, we have:

$$H(\rho, \omega) = \frac{1}{\rho}\cdot \Omega_2(\omega) \tag{4-11}$$

with

$$\Omega_2(\omega) = \frac{(\lambda^2 - \mu^2) + \lambda^3 \cos\omega}{(1 + \lambda\cos\omega)^2} \tag{4-12}$$

and hence

$$g(0) = -\frac{1}{2\pi}\iint T(\rho, \omega)\Omega_2(\omega)d\rho\cdot d\omega. \tag{4-13}$$

This integral enables the computation of the pseudo-gravimetric anomaly at the origin of the coordinates: it is a functional of the anomaly of the total force T.

V. DEVELOPED FORMULAE OF THE KERNELS

For the actual computation of the integrals, we shall be obliged to develop the functions K, H, L_1 and Ω_2 into Fourier's series.

Let us start from the well known series

$$\frac{1 - a^2}{1 + 2a\cos\omega + a^2} = 1 + 2\sum_{k=1}^{\infty}(-a)^k \cos k\omega$$

and write

$$a = \eta\frac{r - z}{\lambda} \quad \text{with} \quad \eta = \frac{1 - |\mu|}{\lambda}.$$

The development of (3-6) will be easily found:

$$K = \frac{1}{r} + \frac{2}{r}\sum_{k=1}^{\infty}(-\eta)^k\left(\frac{r - z}{\rho}\right)^k \cos k\omega. \tag{5-1}$$

Then, as

$$H = \frac{\partial}{\partial z}(rK) - \mu K$$

and

$$\frac{\partial}{\partial z}(rK) = -\frac{2}{r}\sum_{k=1}^{\infty}(-\eta)^k k\left(\frac{r-z}{\rho}\right)^k \cos k\omega$$

we shall get

$$H = -\frac{\mu}{r} - \frac{2}{r}\sum_{k=1}^{\infty}(-\eta)^k \frac{r-z^k}{\rho}(k+\mu)\cos k\omega. \qquad (5\text{-}2)$$

If we make $z=0$ and $r=\rho$ in (5-1) and (5-2), we shall have

$$L_1(\omega) = 1 + 2\sum_{k=1}^{\infty}(-\eta)^k \cos k\omega \qquad (5\text{-}3)$$

and

$$\Omega_2(\omega) = -\mu - 2\sum_{k=1}^{\infty}(-\eta)^k(k+\mu)\cos k\omega. \qquad (5\text{-}4)$$

These series are absolutely convergent as, in general, $\eta<1$. The case of the magnetic equator, where $\eta=1$, must be considered separately.

VI. COMPUTATION OF THE VERTICAL DERIVATIVES

Our next problem is the computation of the vertical derivatives g', g'', $\cdots$, directly from the recorded anomaly Z or T. It is well known that these derivatives are more convenient for the interpretation, as they more clearly point out the various anomalies, the separating power of the derivatives being greater. We shall see, further, that they are easier to compute, as the convergence of the integrals appearing in the computation is more rapid.

Let us first consider the meaning of the vertical pseudo-gravimetric gradient $g'(x, y)$ (the accent indicates the derivative with respect to z). The fundamental relation (3-1) shows that the vertical component Z is the oblique derivative of g. Let us assume that the geological structure, cause of the anomalies, is in the region of the magnetic pole. Then the vector $\vec{\nu}$ is vertical; the derivative with respect to ν is identical to $\partial g/\partial z$. On the other hand, at the pole, the component Z is identical to T, and, thus

$$\frac{\partial g}{\partial z} = T_{(\text{pole})}.$$

Thus the pseudo-gravimetric vertical gradient g' is nothing else than the *magnetic anomaly reduced to the pole.*

Let us suppose that the whole geological anomaly is carried over to the area of the magnetic pole, or, in other words, let us replace the actual magnetic field, which presents some inclination, by a vertical one. The results of measurements of the total field, performed under such conditions, would precisely give the pseudo-gravimetric vertical gradient.

When computing the vertical derivatives, one must keep in mind that the kernels are no longer continuous at the origin of the coordinates. This singular point, thus, should be isolated by a small circle of radius ϵ, to separate the plane π in two regions: the inside and the outside of this circle. Instead of (3-3), we shall write:

$$g = I_\epsilon(z) + \frac{1}{2\pi}\iint_{\rho>\epsilon} KZdS$$

$I_\epsilon(z)$ being the integral extended to the area $\rho<\epsilon$. The integral (4-3) will be similarly handled.

As to the field Z (or T) in the small circle, it is harmonic, thus analytic, and, accordingly, can be developed into series such as

$$Z = A + B\xi + C\eta + D\xi^2 + \cdots$$

or, in polar coordinates:

$$Z = A + (B \cos \omega + C \sin \omega)\rho + O(\rho^2).$$

We shall use this expression and that (5-1), of K, to compute $I_\epsilon(z)$. Integrating first with respect to ω, we get

$$\frac{1}{2\pi}\int_0^{2\pi} KZd\omega = \frac{A}{r} - \eta B \frac{r - z}{r} + O(\rho^3)$$

Integrating then, with respect to ρ, from 0 to ϵ

$$I_\epsilon(z) = \frac{1}{2\pi}\iint_{\rho>\epsilon} KZ\rho d\rho\cdot d\omega = A(r - z) - \eta B\left(\frac{\epsilon^2}{2} - zr + z^2\right)$$

where

$$r = \sqrt{(z^2 + \epsilon^2)}.$$

If we put, now, $z=0$, the value of $I_\epsilon(z)$ is $A \cdot \epsilon + O(\epsilon^2)$. It tends toward zero with ϵ. But, computing the derivatives, for $z=0$, we have

$$I_\epsilon'(0) = - A + \eta B\epsilon + O(\epsilon^2).$$

and

$$I_\epsilon''(0) = \frac{A}{\epsilon} - 2\eta B + O(\epsilon).$$

Thus:

$$g' = - A + \frac{1}{2\pi}\iint_{\rho>\epsilon} K'Z\rho d\rho\cdot d\omega$$

and

$$g'' = \frac{A}{\epsilon} - 2\eta B + \frac{1}{2\pi}\iint_{\rho>\epsilon} K''Z\rho d\rho \cdot d\omega.$$

The accents still indicate the derivatives with respect to z. We have the right to differentiate under the integral sign, after eliminating the singular element.

To compute the derivative K', we only need to take the second derivative of u, which we have already computed. We find:

$$u'' = K' = \frac{1}{\rho^2} L_2(\omega) \tag{6-1}$$

with

$$L_2(\omega) = \frac{\lambda(\lambda + \cos \omega)}{(1 + \lambda \cos \omega)^2} = 2\eta \cos \omega - 4\eta^2 \cos 2\omega + 6\eta^3 \cos 3\omega - \cdots. \tag{6-2}$$

The same method, applied to K, gives

$$K'' = \frac{1}{\rho^3} L_3(\omega) \tag{6-3}$$

with

$$L_3(\omega) = 1 + 6\eta^2 \cos 2\omega - 16\eta^3 \cos 3\omega + \cdots. \tag{6-4}$$

It is remarkable that the (6-2) series has no constant term. We can thus write the final formula as follows:

$$g' = -Z(0) + \frac{1}{2\pi}\iint Z(\rho, \omega) L_2(\omega) \frac{d\rho}{\rho} d\omega \tag{6-5}$$

and we know that the integral comprises no singular element.

As for the (6-4) series, the missing term is that in $\cos \omega$. Therefore

$$\frac{1}{2\pi}\int_0^{2\pi} L_3(\omega) \cos \omega d\omega = 0.$$

Let us now consider a constant anomaly, equal to $A = Z(0)$, and compute the integral for the whole area outside of the small circle. We have:

$$\frac{1}{2\pi}\iint_{\rho>\epsilon} L_3(\omega) \frac{d\rho}{\rho^2} d\omega = -\frac{A}{\epsilon}.$$

This value compensates the identical, but of opposite sign, term in $I''(0)$. Thus the singular element is completely eliminated and we can put $\epsilon = 0$, removing the circle which we excluded at first. We finally obtain the following formula:

$$g'' = -2\eta\left(\frac{\partial Z}{\partial x}\right)_0 + \frac{1}{2\pi}\iint [Z(\rho, \omega) - Z(0)]L_3(\omega)\frac{d\rho}{\rho^2}\,d\omega \tag{6-6}$$

for the second pseudo-gravimetric derivative.

The same method will permit to compute the derivatives g', g'', $\cdots$, as functionals of T. It is not necessary to repeat the reasoning; it suffices to observe that, when passing from K to H, the first term of the Fourier's series is multiplied by $-\mu$, the second one by $-(1+\mu)$. Thus the constant A must be replaced by $-\mu A$ and B by $-(1+\mu)B$. The final formulae are as follows:

$$g' = -\mu T(0) - \frac{1}{2\pi}\iint T(\rho, \omega)\Omega_3(\omega)\frac{d\rho}{\rho}\,d\omega \tag{6-7}$$

and

$$g'' = -2\lambda\left(\frac{\partial T}{\partial x}\right)_0 - \frac{1}{2\pi}\iint [T(\rho, \omega) - T(0)]\Omega_4(\omega)\frac{d\rho}{\rho^2}\,d\omega \tag{6-8}$$

with

$$\Omega_3(\omega) = 2\sum_{k=1}^{\infty}(-\eta)^k k(k+\mu)\cos k\omega \tag{6-9}$$

and

$$\Omega_4(\omega) = \mu - 2\sum_{k=1}^{\infty}(-\eta)^k(k^2-1)(k+\mu)\cos k\omega. \tag{6-10}$$

The transformations may be just as easily obtained for permitting one to form the derivative g''' directly from Z or T. We shall not elaborate, in order to save space.

VII. PRACTICAL FORMULAE FOR THE COMPUTATION OF THE PSEUDO-GRAVIMETRIC ANOMALIES—INTEGRATION WITH RESPECT TO ω

Expressions such as (4-13), (6-7), (6-8) are the theoretical formulae giving g, g', g'' as functionals of the magnetic anomaly T. Of course, it is necessary to find some simple, accurate and rapid means for the carrying out the practical computation of these expressions.

A very-large number of methods can be imagined. The most convenient seems to be the use of a "canonical grid." It is the method commonly used for the computation of the vertical derivatives. Its main advantage is that it does not entail numerous readings on the map, and lends itself easily to computation, through use of punched card machines. This is a well known method for geophysicists, so we do not need to go into details.

The usual square grid, or a trigonal grid—such as that illustrated in Figure 2—may be used equally. The method is the same, whichever be the data to be computed: g, g' or g''. We shall therefore consider only one of these formulae. For

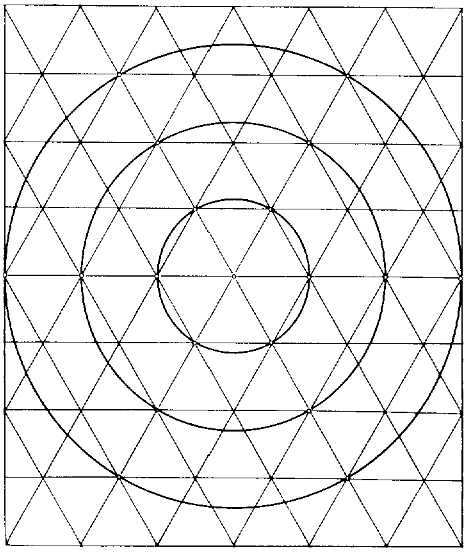

FIG. 2

instance, let us consider the expression (6-7), for the reduction of aeromagnetic anomalies to the pole. It first contains a constant term $-\mu T(0)$ which needs no computation (it is read directly on the map) and a double integral:

$$I = \frac{1}{2\pi} \iint T(\rho, \omega) \Omega_3(\omega) \frac{d\rho}{\rho} d\omega$$

covering the whole plane of measurements.

We shall first integrate with respect to ω, putting

$$\overline{T}(\rho) = \frac{1}{2\pi} \int_0^{2\pi} T(\rho, \omega) \Omega_3(\omega) d\omega. \tag{7-1}$$

Then, we shall integrate with respect to ρ

$$I = \int_0^{\infty} \overline{T}(\rho) \frac{d\rho}{\rho} \cdot \tag{7-2}$$

Note that this is not an absolute requirement. It would be possible, too, to integrate first with respect to ρ, this would require some precautions. In fact, it is the only possible method in the case of an irregular grid, for instance a rectangular or star-shaped one, which is sometimes more advantageous. But, in the case of a trigonal grid (Fig. 2) considered in our example, it is more convenient to integrate first with respect to ω.

The function $\Omega_3(\omega)$ appearing in (7-1) and acting as a weight function can be developed into Fourier's series (6-9). $T(\rho, \omega)$ is an experimental function known at six canonical points on the circle of fixed radius ρ. The six azimuths are equally spaced; their directions are determined by the angles

$$\omega = \frac{\pi\nu}{3} \quad \text{with} \quad \nu = 0, 1, 2, 3, 4, 5.$$

We shall represent these values read on the grid by the notation $T_\rho(\nu)$. It is obviously a periodic function; it would thus be natural to develop it, too, into a trigonometrical series. However, we propose to take a slightly different approach. Let us put:

$$T(\rho, \omega) = T\rho(0)\psi(\omega) + T\rho(1)\psi\left(\omega - \frac{\pi}{3}\right) + T\rho(2)\psi\left(\omega - \frac{2\pi}{3}\right) + \cdots + T\rho(5)\psi\left(\omega - \frac{5\pi}{3}\right).$$

$\psi(\omega)$ is an influence function of the point of zero azimuth, such azimuth being directed toward the magnetic North. For $\omega = 0$, its value is 1; it is zero in any other point. These conditions are satisfied for an infinite number of functions. To make a choice, we shall introduce a "simplicity condition." Like the anonaly T, the influence function must present as few higher harmonics as possible. This condi-

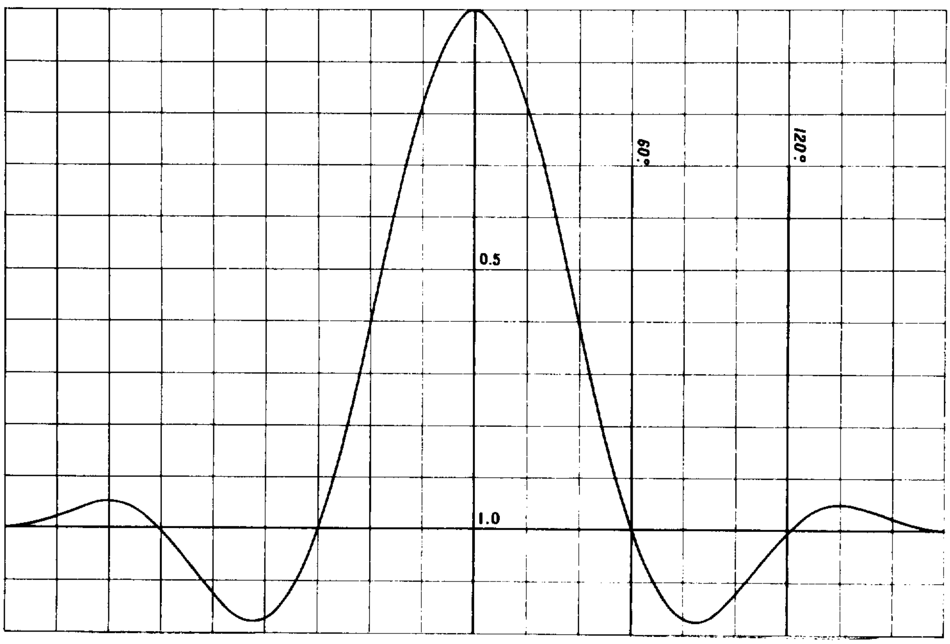

FIG. 3

tion, obviously, is in accordance with the filtering qualities of the canonical grid. We shall thus put:

$$\psi(\omega) = \frac{1}{6}(1 + \cos\omega)(2\cos\omega - 1)(2\cos\omega + 1)$$
$$= \frac{1}{6} + \frac{1}{3}\cos\omega + \frac{1}{3}\cos 2\omega + \frac{1}{6}\cos 3\omega \tag{7-3}$$

because this function is equal to zero for $\cos\omega = -1$, $+\frac{1}{2}$ and $-\frac{1}{2}$, while for $\omega = 0$, it has the value $\psi(0) = 1$. This is the simplest influence function satisfying to the foresaid conditions indicated above. Figure 3 shows the pattern of this function.

As to the other azimuths, we shall compute:

$$\psi\left(\omega - \frac{\nu\pi}{3}\right) = \frac{1}{6} + \frac{1}{3}\cos\frac{\nu\pi}{3}\cos\omega + \frac{1}{3}\cos\frac{2\nu\pi}{3}\cos 2\omega + \frac{1}{6}\cos\nu\pi\cos 3\omega$$
$$+ \text{ the odd terms in } \sin\omega, \sin 2\omega, \sin 3\omega$$

(we do not need to express these odd terms, as the function Ω_3 is an even one).

The Fourier's coefficients of these functions are tabulated hereafter:

ν	1	$\cos\omega$	$\cos 2\omega$	$\cos 3\omega$
0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$
2	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$
4	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$
5	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$

We can now develop the given function $T(\rho, \omega)$ into a Fourier's series on the circle of radius ρ. We obtain the following expression

$$\begin{aligned}
T(\rho, \omega) \\
&= \frac{1}{6}[T\rho(0) + T\rho(1) + T\rho(2) + T\rho(3) + T\rho(4) + T\rho(5)] \\
&+ \frac{1}{6}[2T\rho(0) + T\rho(1) - T\rho(2) - 2T\rho(3) - T\rho(4) + T\rho(5)]\cos\omega \\
&+ \frac{1}{6}[2T\rho(0) - T\rho(1) - T\rho(2) + 2T\rho(3) - T\rho(4) - T\rho(5)]\cos 2\omega \\
&+ \frac{1}{6}[T\rho(0) - T\rho(1) + T\rho(2) - T\rho(3) + T\rho(4) - T\rho(5)]\cos 3\omega \\
&+ \text{odd terms}
\end{aligned} \tag{7-4}$$

(it is useless to write these odd terms, since they disappear when integrating with respect to ω).

To compute the integral (7-1), we have to integrate the product of two functions T and Ω_3, given by their developments of the Fourier's series (5-9) and (7-4). To be more general, let us write the development of the weight function as follows:

$$\Omega(\omega) = \gamma_0 + 2\gamma_1 \cos \omega + 2\gamma_2 \cos 2\omega + \cdots.$$

For the function Ω_3, the values of the coefficients are:

$$\gamma_0 = 0, \qquad \gamma_1 = -\eta(1 + \mu)$$
$$\gamma_2 = 2\eta^2(2 + \mu)$$
$$\gamma_3 = -3\eta^3(3 + \mu)$$
$$\cdots\cdots\cdots$$

For the integration, we only need to sum up the products of the coefficients of (7-4) by the γs. We obtain an expression which can be conveniently rearranged according to the canonical values of the anomaly:

$$\overline{T}(\rho) = \alpha_0 T\rho(0) + \alpha_1 T\rho(1) + \cdots + \alpha_5 T\rho(5) \tag{7-5}$$

in order to show the numerical coefficients by which the readings from the grid must be multiplied. The expressions of these coefficients which depend on the inclination of the normal field are the following:

$$\alpha_0 = \frac{1}{6}(\gamma_0 + 2\gamma_1 + 2\gamma_2 + \gamma_3)$$

$$\alpha_1 = \alpha_5 = \frac{1}{6}(\gamma_0 + \gamma_1 - \gamma_2 - \gamma_3)$$

$$\alpha_2 = \alpha_4 = \frac{1}{6}(\gamma_0 - \gamma_1 - \gamma_2 + \gamma_3)$$

$$\alpha_3 = \frac{1}{6}(\gamma_0 + 2\gamma_1 + 2\gamma_2 - \gamma_3)$$

Thus the function $\overline{T}(\rho)$ may be easily computed for the values $\rho = 1, 2, 3, \cdots$, corresponding to the radii of the circles passing through the vertices of the grid illustrated on Figure 2.

VIII. INTEGRATION WITH RESPECT TO ρ

We must now compute the integral

$$I = \int_0^\infty \overline{T}(\rho) \frac{d\rho}{\rho} \tag{8-1}$$

where the function $\overline{T}(\rho)$, equal to zero at the origin, is known at points of abscissae $\rho=1, 2, 3, \cdots$. Numerous methods are available to effect the approximate quadratures. We shall outline here a method which, while perhaps not the most accurate, is very widely known, and the easiest to describe. The axis $O\rho$ will be divided into partial intervals:

$$(0, 1) - (1, 3) - (3, 5) \quad \text{and} \quad (5, \infty).$$

We shall admit that, in the first interval $(0, 1)$, the function $\overline{T}(\rho)/\rho$ may be represented by a parabola

$$\frac{\overline{T}(\rho)}{\rho} = A + B\rho + C\rho^2$$

passing through the points $\rho=1$, 2 and 3. We shall have:

$$A + \frac{1}{2} B + \frac{1}{3} C = \int_0^\infty \frac{\overline{T}}{\rho}\, d\rho$$

$$A + B + C = \overline{T}_1$$

$$A + 2B + 4C = \frac{1}{2} \overline{T}_2$$

$$A + 3B + 9C = \frac{1}{3} \overline{T}_3$$

where $\overline{T}_k = \overline{T}(k)$. Eliminating A, B, and C from these four equations, we get:

$$\int_0^1 \frac{T(\rho)}{\rho}\, d\rho = 2\overline{T}_1 - \frac{3}{4} \overline{T}_2 + \frac{1}{6} \overline{T}_3. \tag{8-2}$$

For the intervals $(1, 3)$ and $(3, 5)$, we shall apply Simpson's rule, which causes us to put:

$$\int_1^3 \overline{T}(\rho) \frac{d\rho}{\rho} = \frac{1}{3} \overline{T}_1 + \frac{1}{3} \overline{T}_2 + \frac{1}{9} \overline{T}_3 \tag{8-3}$$

$$\int_3^5 \overline{T}(\rho) \frac{d\rho}{\rho} = \frac{1}{9} \overline{T}_3 + \frac{2}{3} \overline{T}_4 + \frac{1}{15} \overline{T}_5. \tag{8-4}$$

Finally, the last interval $(5, \infty)$ corresponds to the "tail" of the curve $\overline{T}(\rho)$, where the function can be approximately expressed using the following formula:

$$\overline{T} = \frac{A}{\rho^2} + \frac{B}{\rho^3}.$$

In fact, the average weighted T anomaly decreases like the square or the cube of the distance. To compute the constants A and B we shall take into consideration the fact that the curve passes through the points $\rho=4$ and $\rho=5$. We shall then get three equations:

$$\frac{A}{50}+\frac{B}{375}=\int_5^\infty \overline{T}(\rho)\frac{d\rho}{\rho}$$

$$\frac{A}{16}+\frac{B}{64}=\overline{T}_4$$

$$\frac{A}{25}+\frac{B}{125}=\overline{T}_5$$

whence, eliminating A and B:

$$\int_5^\infty \overline{T}(\rho)\frac{d\rho}{\rho}=-\frac{32}{75}\overline{T}_4+\frac{7}{6}\overline{T}_5. \tag{8-5}$$

Summing the expressions (8-2), (8-3), (8-4) and (8-5), we get

$$\int_0^\infty \overline{T}(\rho)\frac{d\rho}{\rho}=\frac{7}{3}\overline{T}_1-\frac{1}{12}\overline{T}_2+\frac{7}{18}\overline{T}_3-\frac{7}{75}\overline{T}_4+\frac{37}{30}\overline{T}_5.$$

To check this formula, we chose a theoretical function

$$\overline{T}=\frac{a^2\rho}{(\rho^2+a^2)^{3/2}}\,. \tag{8-6}$$

The exact value of the integral is equal to unity.

The approximate values, computed with the (8-6) formula are shown in the table below:

a	I	Error
2	1.0906	9.1%
3	1.0227	2.3%
4	0.9950	−0.5%

For a small value of the constant a, the curve $\overline{T}$ is very sharp at the origin. With increasing a, the error rapidly decreases.

The integration may be made more accurate, through separate consideration of an additional partial interval (5, 7).

In any case, the result of the integration with respect to ρ is represented by a linear expression

$$\int_0^\infty \overline{T}(\rho)\frac{d\rho}{\rho}=\sum_{(k)}\beta_k\overline{T}_k \tag{8-7}$$

where the β_k are the numerical coefficients computed once for all. This formula should be combined with (7-5) to give the final result:

$$I = \sum_{\nu \cdot k} \alpha_\nu \beta_k T_k(\gamma). \tag{8-8}$$

Finally, we must not forget to add the constant term $-\mu T_0$ appearing in (6-7).

Figure 4 shows the values of the coefficients at their respective locations.

The process just described is well suited for dips $I > 30°$ of the normal magnetic field. For smaller dips, near the magnetic equator, the computation requires more care. In all cases, the computation practically involves the reading of the values of the field T on a canonical grid, followed by a cumulative multiplication

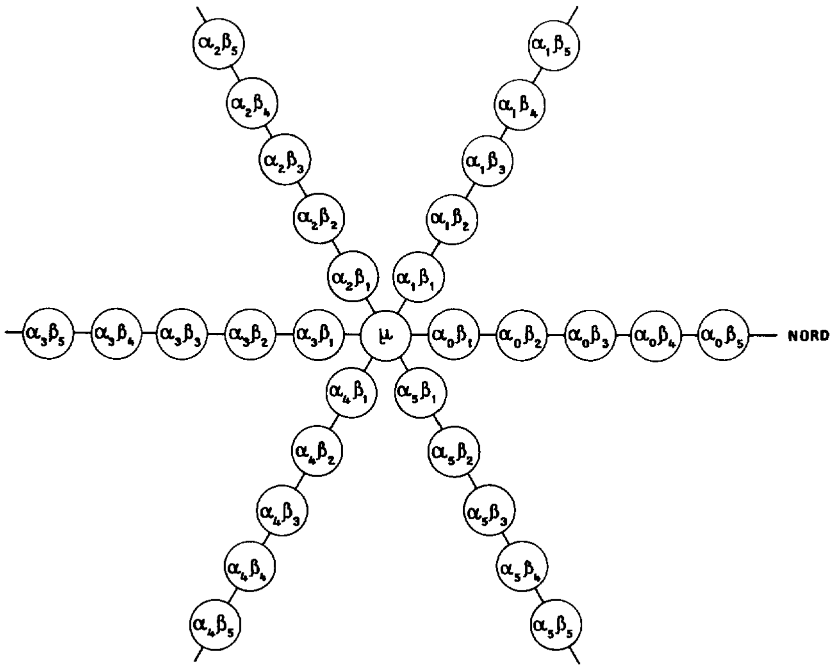

FIG. 4

of the readings by numerical coefficients computed once for all, and which depends only upon the mean average geomagnetic latitude of the area under consideration.

Of course, this computation is time consuming, if one uses ordinary computers. However, it can be considerably speeded up with the use of electronic computers operating with punched cards. The values of the field, read on a point of the canonical grid, and the coordinates of this point (oblique if the grid is trigonal) are plotted on "mark sensing" cards (one card for each point), transformed into normal cards by the reproducer.

A first list is then given by the tabulator, a list that is compared to the readings. This operation is designed to simultaneously check the readings, the plottings and the punching. The computation itself is carried out in the computer.

Finally, the cards pass again through the tabulator, which prepares the final list of the values of the transformed field.

Figure 5 gives an example of a grid with the numerical coefficients for the reduction to the pole of the field T in an area where the average dip is 60°.

FORMULA AND COEFFICIENTS TO COMPUTE THE PSEUDO--GRAVIMETRIC VERTICAL GRADIENT

$$-g' = -\sin I . T_0 + \frac{1}{2\pi} \iint T(\rho,\omega) \cdot \Omega_3(\omega) \frac{d\rho}{\rho} d\omega$$

$$\Omega_3 = \sin I \frac{3\cos I + \cos I(2+\cos^2 I)\cos\omega}{(1+\cos I \cdot \cos\omega)^3}$$

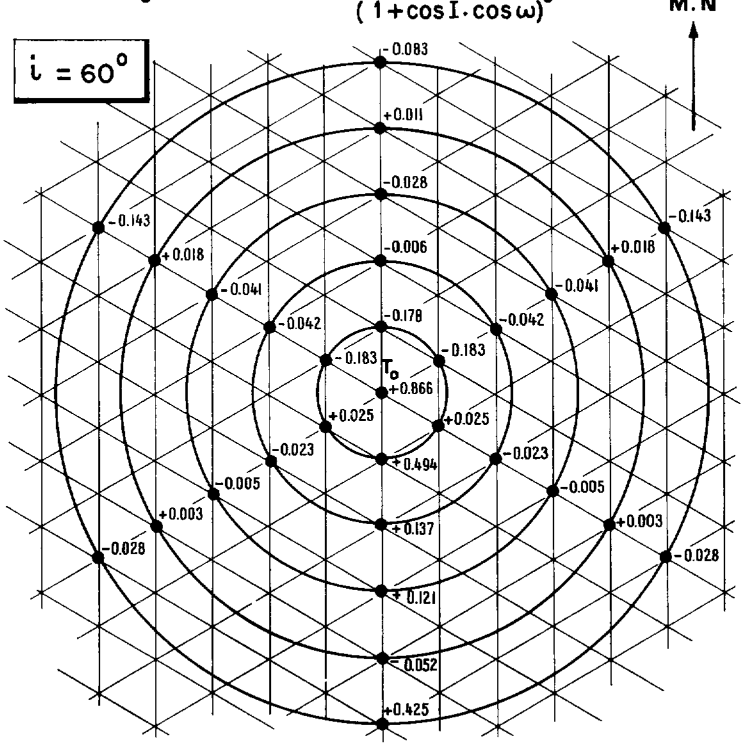

FIG. 5

Figure 6 shows the anomaly caused by a sphere buried at a depth h, shown on the map. On the left the contours of the total field anomaly are plotted. On the right side, the field of the computed pseudo-gravimetric vertical gradient is represented. The computed curves are almost perfect circles, and their common center on the apex above the sphere. The two external curves are very slightly affected by the unavoidable errors of the numerical computation.

Figure 7 shows the effect of a vertical prismatic body, of rectangular cross

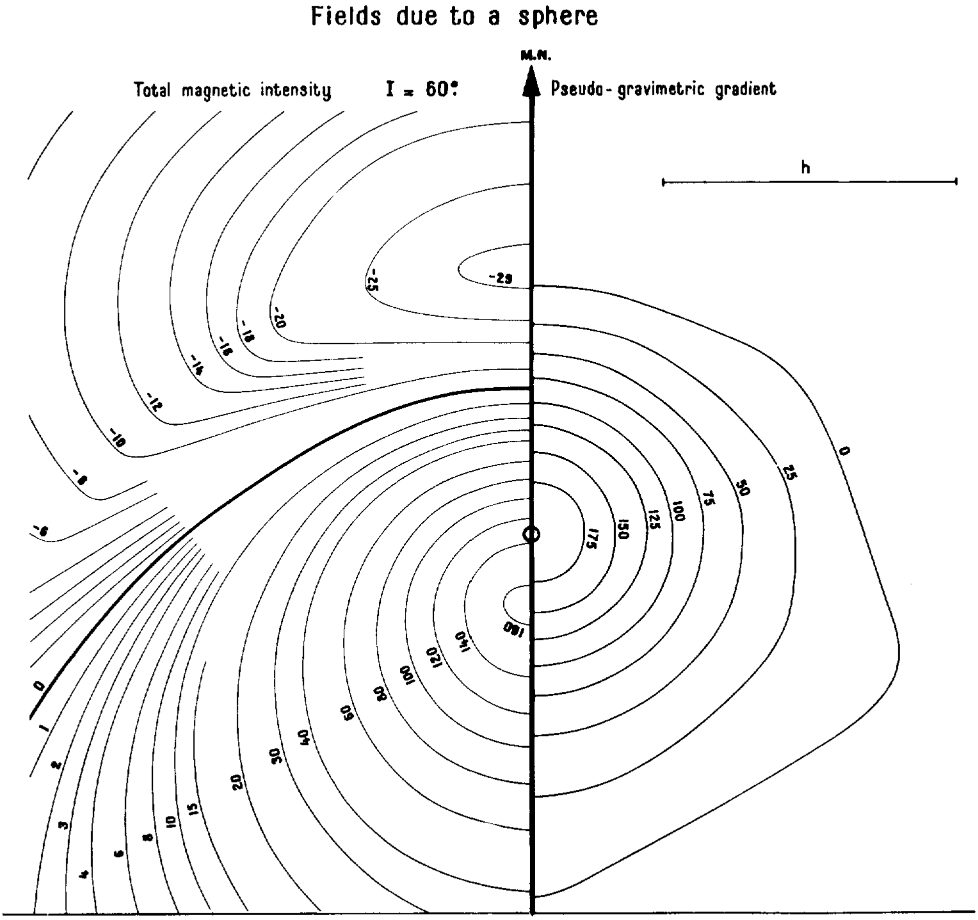

FIG. 6

section. The computed psuedo-gravimetric field, on the right of the figure, has a symmetric appearance, whereas the total field anomaly is shifted with respect to the magnetized mass.

The reduction of the field T to the pole seems to be a very advantageous transformation for the following reasons:

1) the qualitative interpretation of the transformed map is easier and more immediate, as the anomalies are not shifted as a result of the obliquity of the normal field;
2) the comparison of the results of a survey with a geological map is much easier. One does not need to take into consideration the shifting of the magnetic anomalies;
3) the transformed map can be compared to a gravimetric map, much better

than the original aeromagnetic map, especially if this latter too is transformed into a map of the first vertical derivatives. Indeed, the two quantities, then, are of the same nature;

4) the pseudo-gravimetric anomalies do not depend upon the inclination of the normal field, nor on the orientation of the tectonic structure. This simplifies the quantitative interpretation and the evaluation of depths. The number of typical cases, such as blocks, faults and so on, is consider-

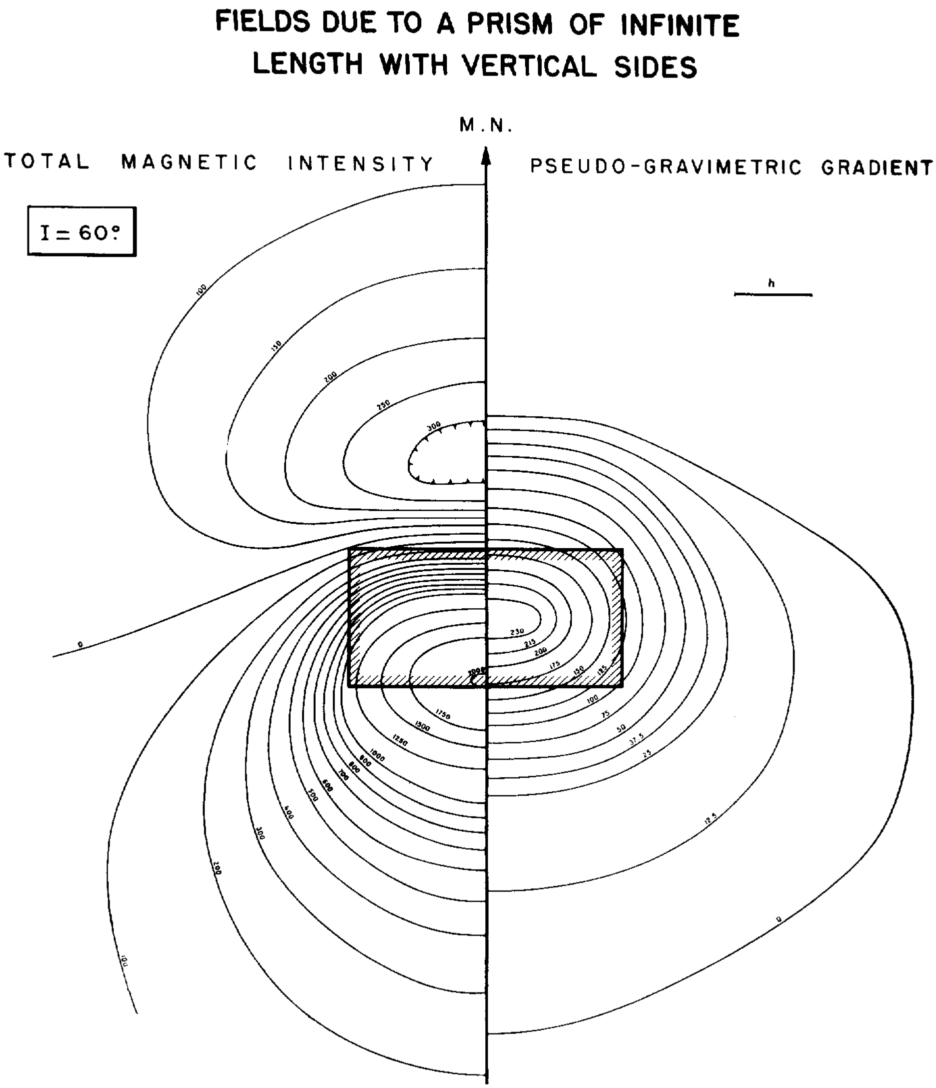

FIG. 7

ably reduced. Once computed, a single typical anomaly can be used for all dips and orientations;

5) the computation of the theoretical pseudo-gravimetric anomalies is simpler than that of the magnetic anomalies. For instance, let us consider a vertical block. In magnetism, the vertical walls of the compartment give rise to a very complex anomaly, which is eliminated if the structure is located at the pole. It is easy to draw a number of simple nomographs with which the computation of the pseudo-gravimetric anomalies becomes very rapid.

The approximate computation of integrals such as (6-7) and (6-8) has made some progress since 1955, so that a better method now exists than the one described in the paper. In particular, for the determination of the numerical coefficients of the transformation, it is no longer necessary to use a Fourier development of the kernels.

ACKNOWLEDGMENTS

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DISCUSSION OF V. BARANOV'S PAPER, "A NEW METHOD OF INTERPRETATION OF AEROMAGNETIC MAPS, PSEUDO-GRAVIMETRIC ANOMALIES"*

JAMES AFFLECK†

Baranov's pseudo-gravimetric anomaly method is based on well-known relationships and his development appears to be sound. Applications of his methods should be of very appreciable use in evaluating aeromagnetic surveys. The comments which follow should not be interpreted as criticisms. They are intended to emphasize the conditions and limitations under which Baranov's techniques might be utilized.

Conditions which must be recognized are these:

1. The direction of magnetization is not necessarily in the present direction of the earth's field. This is recognized by the author.
2. The direction of magnetization may be very much different for adjacent anomalies.

* Received by the Editor October 18, 1956.

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