GEOPHYSICS, VOL. 42, NO. 1 (FEBRUARY 1977), P. 41–50, 6 FIGS., 4 TABLES

SPECTRAL ANALYSIS OF GRAVITY AND MAGNETIC ANOMALIES DUE TO RECTANGULAR PRISMATIC BODIES

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The spectra of gravity and magnetic anomalies due to a prismatic body can be expressed as sums of exponentials. The complex exponents of these exponentials are functions of frequency and locations of the corners of the body. An exponential approximation method is used for the analysis of the radial spectra of an anomaly and its first order moments for obtaining accurate estimates of the depths to the top and bottom of the body. A method has also been developed for determining approximately the location of the centroid of the body. When the location of the centroid and the depths to the top and bottom are known for the causative body, it is possible to calculate the horizontal dimensions with the help of the spectrum of the anomaly.

INTRODUCTION

In a recent paper, Bhattacharyya and Leu (1975a) described a method for spectral analysis of gravity and magnetic anomalies due to two-dimensional structures. The expressions for the spectra of these anomalies consist of, except for a factor, sums of exponentials. The exponents of these exponentials are functions of frequency and locations of the corners of the polygonal cross-section of the assumed structure. The locations of the corners, which completely define the structure, are determined with the help of a method called exponential approximation.

The present study deals with the extension of the above spectral approach to the analysis of anomalies due to those three-dimensional structures which can be reasonably represented by vertical, rectangular prismatic bodies. The expressions for the spectra of the anomaly and its moments due to a vertical prism consist of sums of exponentials, as in the case of two-dimensional bodies. The exponential approximation method can, therefore, be employed to determine the depths to the top and bottom of the prism from the complex spectrum of a given anomaly. A method also is described for computing the approximate location of the centroid of the causative body. A knowledge of the depths to the top and bottom and the centroid of the body is necessary for obtaining the horizontal dimensions from the spectrum of the anomaly. The accuracy and reliability of these methods have been tested with both gravity and magnetic anomalies due to various prismatic bodies. They have also been applied to two real anomalies shown on the filtered aeromagnetic data obtained over Yellowstone National Park.

SPECTRA OF GRAVITY AND MAGNETIC ANOMALIES

Let us consider a prismatic body with infinite vertical extent, as shown in Figure 1. The coordinates of the corners of the top surface are (α_1, β_1, h), (α_1, β_2, h), (α_2, β_1, h) and (α_2, β_2, h). The center of the top surface is located at (α_0, β_0, h). The horizontal dimensions of the body parallel to the x and y axes are a and b, respectively.

Gravity effect

The gravity effect at a point $(x,y,0)$, due to the volume V_0 occupied by the prismatic body with a uniform distribution of density ρ, is given by

$$\Delta g(x, y, 0) =$$

$$G\rho \int_{V_0} \frac{\gamma}{[(x-\alpha)^2 + (y-\beta)^2 + \gamma^2]^{3/2}}\, dv_0. \qquad (1)$$

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The Fourier transform of $\Delta g(x,y,0)$ is defined as

$$F_g(u,v) = \int_{-\infty}^{\infty}\int \Delta g(x,y,0)\, e^{-i(ux+vy)}\, dx\, dy, \quad (2)$$

where u and v are the angular frequencies in radians/unit spacings along x and y axes, respectively. A combination of equation (7) (p. 11) and equation (44) (p. 56) from Erdelyi et al (1954) yields the following:

$$2\pi e^{-\gamma s} = \iint_{-\infty}^{\infty} \frac{\gamma}{(x^2 + y^2 + \gamma^2)^{3/2}}\, e^{-i(ux+vy)}\, dx\, dy, \quad (3)$$

where

$$s^2 = u^2 + v^2.$$

Now, with the help of equations (1) to (3), we have

$$F_g(u,v) = 2\pi\, G\rho \int_{V_0} e^{-(s\gamma + iu\alpha + iv\beta)}\, dv_0. \quad (4)$$

For the given prismatic body, we can now express (4) in the following form:

$$F_g(u,v) = -\frac{2\pi G\rho}{uvs} \cdot e^{-sh} \cdot \Big[e^{-i(u\alpha_1 + v\beta_1)}$$

$$- e^{-i(u\alpha_1 + v\beta_2)} - e^{-i(u\alpha_2 + v\beta_1)} + e^{-i(u\alpha_2 + v\beta_2)} \Big]. \quad (5)$$

In the case of a prism with finite vertical extent, let us assume the depths to the top and the bottom of the body to be h_1 and h_2, respectively. Then, with the help of (5), we can write the Fourier transform $F_g'(u,v)$ of the gravity effect due to the body as

$$F_g'(u,v) = F_g(u,v)|_{h=h_1} - F_g(u,v)|_{h=h_2}. \quad (6)$$

Magnetic anomaly

Let us now consider the total field magnetic anomaly at a point $(x,y,0)$ due to a uniformly magnetized body of volume V_0 with a magnetization vector of intensity I_p and direction-cosines L, M, and N. It is assumed that the total magnetic field is measured in a direction given by the direction-cosines of the earth's field l, m, and n. Then the spectrum of the total-field anomaly $T(x,y,0)$ observed in a horizontal plane at an elevation h is given by (Bhattacharyya, 1966 and 1967):

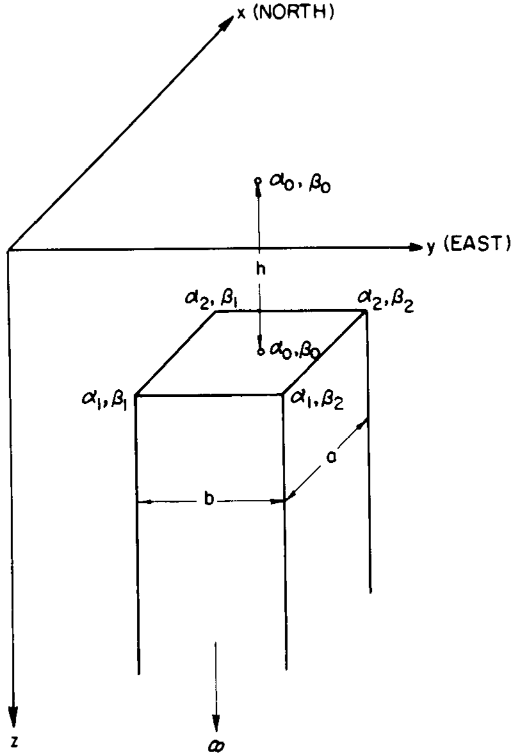

FIG. 1. A prismatic body with infinite vertical extent.

$$F_T(u,v) = \frac{2\pi I_p J}{s} \int_{V_0} e^{-(sh + iu\alpha + iv\beta)}\, dv_0, \quad (7)$$

where

$J = -lLu^2 - mMv^2 + nNs^2 - \alpha_{12}uv + is(\alpha_{13}u + \alpha_{23}v)$,

$\alpha_{13} = Ln + Nl$, $\alpha_{23} = Mn + Nm$ and $\alpha_{12} = Lm + Ml$.

For a prismatic body with infinite vertical extent (Figure 1), equation (7) yields the following expression:

$$F_T(u,v) = -\frac{2\pi I_p J}{uvs^2} \cdot e^{-sh} \big[e^{-i(u\alpha_1 + v\beta_1)}$$

$$- e^{-i(u\alpha_1 + v\beta_2)} - e^{-i(u\alpha_2 + v\beta_1)} + e^{-i(u\alpha_2 + v\beta_2)} \big]. \quad (8)$$

When the prism has a limited depth extent, we can write the expression for the spectrum of the magnetic anomaly as

$$F_T'(u,v) = F_T(u,v)|_{h=h_1} - F_T(u,v)|_{h=h_2}. \quad (9)$$

MOMENTS AND THEIR SPECTRA

The nth moment of a function of a single variable $f(x)$ is defined as $M_n = x^n f(x)$ (Bhattacharyya and Leu, 1975a). In a similar way, the nth x- and y- moments of $f(x,y)$ are described as

$$M_n^x(x,y) = x^n f(x,y), \tag{10}$$

and

$$M_n^y(x,y) = y^n f(x,y).$$

The Fourier transforms of M_n^x and M_n^y are given by

$$R_n^x(u, v) = \iint_{-\infty}^{\infty} x^n f(x, y) e^{-i(ux+vy)}\, dx\, dy,$$

and

$$R_n^y(u, v) = \iint_{-\infty}^{\infty} y^n f(x, y) e^{-i(ux+vy)}\, dx\, dy. \tag{11}$$

From (11), the iterative relations between transforms of moments of successive order may be written in the following form:

$$R_{n+1}^x(u, v) = i \frac{\partial}{\partial u} R_n^x(u, v),$$

and

$$R_{n+1}^y(u, v) = i \frac{\partial}{\partial v} R_n^y(u, v). \tag{12}$$

With the help of (5) and (12), we can write the spectra of the first order x- and y-moments of the gravity effect as

$$F_g^x(u, v) = -i \frac{2\pi G\rho}{uvs} e^{-sh} [A_1(u)\cdot\{e^{-iu\alpha_1 - iv\beta_1} - e^{-iu\alpha_1 - iv\beta_2}\} + A_2(u)\cdot\{e^{-iu\alpha_2 + iv\beta_2} - e^{-iu\alpha_2 - iv\beta_1}\}], \tag{13}$$

and

$$F_g^y(u, v) = -i \frac{2\pi G\rho}{uvs} e^{-sh}[B_1(u)\cdot\{e^{-iu\alpha_1 - iv\beta_1} - e^{-iu\alpha_2 + iv\beta_1}\} + B_2(u)\cdot\{e^{-iu\alpha_2 - iv\beta_2} - e^{-iu\alpha_1 + iv\beta_2}\}],$$

where

$$A_1(u) = \frac{uh}{s} + \frac{u}{s^2} + \frac{1}{u} + i\alpha_1,$$

$$A_2(u) = \frac{uh}{s} + \frac{u}{s^2} + \frac{1}{u} + i\alpha_2,$$

$$B_1(u) = \frac{vh}{s} + \frac{v}{s^2} + \frac{1}{v} + i\beta_1,$$

and

$$B_2(u) = \frac{vh}{s} + \frac{v}{s^2} + \frac{1}{v} + i\beta_2 \cdot$$

Similarly, from (8) and (12) the spectra of the first order x- and y-moments of the total magnetic field anomaly are given by

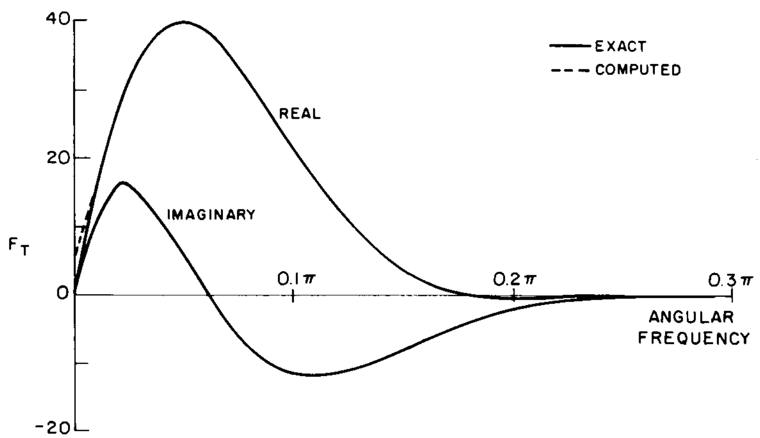

FIG. 2. Amplitude spectrum of the total-field-magnetic anomaly along a line at an angle of 45 degrees with the frequency axes.

$$F_T{}^x(u, v) = -i \frac{2\pi I_p J}{uvs^2} e^{-sh}[(G_1 + i\alpha_1)\{e^{-i(u\alpha_1 + v\beta_1)} - e^{-i(u\alpha_1+v\beta_2)}\} + (G_1 + i\alpha_2)\{e^{-i(u\alpha_2+v\beta_2)} - e^{-i(u\alpha_2+v\beta_1)}\}], \qquad (14)$$

and

$$F_T{}^y(u, v) = -i \frac{2\pi I_p J}{uvs^2} e^{-sh}[(K_1 + i\beta_1)e^{-iv\beta_1}\{e^{-iu\alpha_1} - e^{-iu\alpha_2}\} + (K_1 + i\beta_2)e^{-iv\beta_2}\{e^{-iu\alpha_2} - e^{-iu\alpha_1}\}],$$

where

$$G_1 = \frac{uh}{s} + \frac{2u}{s^2} + \frac{1}{u} - \frac{H}{J},$$

$$K_1 = \frac{vh}{s} + \frac{2v}{s^2} + \frac{1}{v} - \frac{I}{J},$$

$$H = -2u(lL - nN) - \alpha_{12}v + i \left(\alpha_{23}\frac{uv}{s} + \alpha_{13}\frac{u^2 + s^2}{s}\right),$$

and

$$I = -2v(mM - nN) - \alpha_{12}u + i \left(\alpha_{13}\frac{uv}{s} + \alpha_{23}\frac{s^2 + u^2}{s}\right).$$

Computation of the spectra of the moments

The paper by Bhattacharyya and Leu (1975a) presents a reliable and accurate method for evaluating the spectra of moments of data along a profile. This method has been extended to the case of two-dimensional observations. First, the fast Fourier transform algorithm is employed to obtain the discrete spectrum of the anomaly. Second, piecewise bicubic splines (Bhattacharyya, 1969) are fitted to the spectral values for computing the horizontal derivatives of the spectrum with respect to u and v. According to (12), these derivatives yield the spectra of the first order x- and y-moments of the anomaly.

In order to check the above method, the magnetic anomaly due to a prismatic body is calculated with the formula given by Bhattacharyya (1964). The dimensions of the body (Figure 1) are $a = 8$, $b = 4$, $\alpha_0 = 0$, $\beta_0 = 4$, $h_1 = 3$, and $h_2 = 6$. The body is assumed to be inductively magnetized in the geomagnetic field with inclination $I = 60$ degrees and $D = 0$ degrees. The intensity of magnetization I_p is taken to be unity. The total number of data points of the computed anomaly are 64×64 with a unit spacing between two successive points. The amplitude spectrum is obtained by calculating the two-dimensional Fourier transform of the anomaly values. For the prismatic body, the exact values of the spectrum can also be computed easily (Bhattacharyya, 1966).

Figure 2 shows the amplitude spectrum along a line at an angle of 45 degrees with the frequency axes. Along this line u is equal to v. The exact values are shown by solid lines and the calculated values by dashed lines. It is clear from the figure that, except at frequencies lower than the funda-

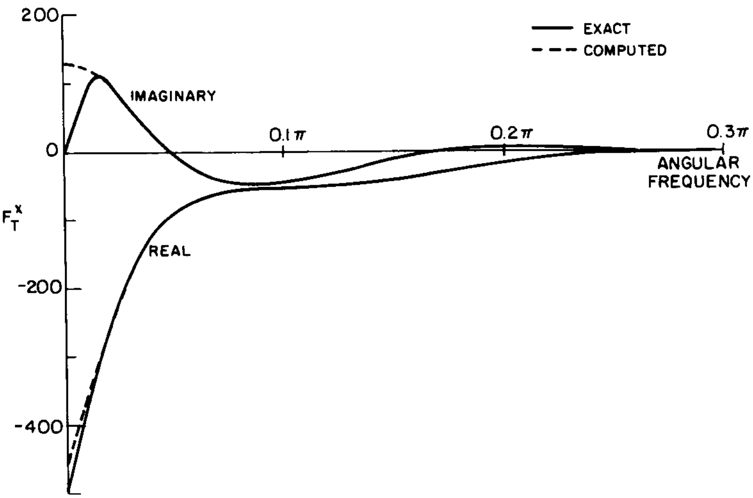

FIG. 3. Amplitude spectrum of the x-moment of the total-field-magnetic anomaly along a line at an angle of 45 degrees with the frequency axes.

mental frequency, the match between exact and computed values is excellent.

Bicubic splines are then fitted to the values of the two-dimensional discrete Fourier transform of the anomaly. The splines yield the horizontal u and v derivatives of the anomaly spectrum. Then these derivatives provide the spectra of the first x- and y-moments of the anomaly. In Figures 3 and 4, dashed and solid lines indicate, respectively, the calculated and exact spectra. Like the amplitude spectrum in Figure 2, these figures show clearly that the calculated spectra of the moments of the anomaly are sufficiently accurate and reliable, except at frequencies lower than the fundamental frequency.

CENTROID OF A CAUSATIVE BODY

Using equations (4) and (12), we can write the spectra of x- and y-moments of the gravity effect in the following way:

$$F_g^x(u,v) = 2\pi G\rho \int_{V0} (\alpha - iuh/s) \, e^{-sh + iu\alpha + iv\beta} dv_0,$$

and

$$F_g^y(u,v) = 2\pi G\rho \int_{V0} (\beta - ivh/s) \, e^{-sh + iu\alpha + iv\beta} dv_0. \qquad (15)$$

A comparison of (4) and (15) suggests the following relationships for approximate estimation of the location of the centroid of a three-dimensional causative body:

$$\frac{F_g^y(u,v)}{F_g(u,v)} \approx \beta_0 - \frac{ivh_0}{s}, \qquad (16)$$

and

$$\frac{F_g^x(u,v)}{F_g(u,v)} \approx \alpha_0 - \frac{iuh_0}{s},$$

where α_0, β_0, and h_0 are the coordinates of the centroid. The expressions in (16) are found to be accurate at low frequencies.

Similarly, a combination of equations (7) and (12) yields the following expressions for the spectra of the x- and y-moments of the total field anomaly $T(x,y,0)$:

$$F_T^x(u,v) = -\frac{i2\pi I_p J}{s} \int_{V0} \left[\frac{uh}{s} + \frac{u}{s^2} - \frac{H}{J} + i\alpha\right] \cdot e^{-sh + iu\alpha + iv\beta} dv_0, \qquad (17)$$

and

$$F_T^y(u,v) = -\frac{i2\pi I_p J}{s} \int_{V0} \left[\frac{vh}{s} + \frac{v}{s^2} - \frac{I}{J} + i\beta\right] \cdot e^{-sh + iu\alpha + iv\beta} dv_0.$$

From (7) and (17), we now obtain the following approximate relations:

$$\frac{F_T^x(u,v)}{F_T(u,v)} + i\left(\frac{u}{s^2} - \frac{H}{J}\right) \approx \alpha_0 - i\,\frac{uh_0}{s}, \qquad (18)$$

and

$$\frac{F_T^y(u,v)}{F_T(u,v)} + i\left(\frac{v}{s^2} - \frac{I}{J}\right) \approx \beta_0 - i\,\frac{vh_0}{s}.$$

Since for a prism with finite vertical extent $F_T(u,v)$ is equal to zero when $u = v = 0$, the relations given in (18) are not valid when u and v

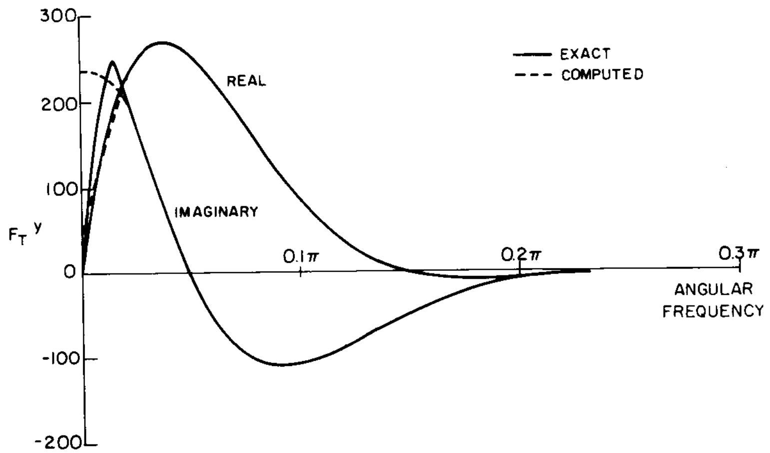

FIG. 4. Amplitude spectrum of the y-moment of the total-field-magnetic anomaly along a line at an angle of 45 degrees with the frequency axes.

are simultaneously equal to zero. It will be shown in the section on Results and Discussion that if equations (16) and (18) are used for frequencies higher than the fundamental, but not exceeding a few harmonics, accurate and reliable values can be obtained for the location of the centroid of the causative body. The accuracy of the values for the centroid can be improved by averaging several locations computed at many succeeding frequencies.

DEPTH ESTIMATION BY THE METHOD OF EXPONENTIAL APPROXIMATION

If we let $u = v$ in equations (5) and (8), we obtain the radial spectrum

$$\phi_0(u) = u^L F_0(u) = A \sum_{j=1}^{N} (-1)^{j-1} e^{-uc_j}, \quad (19)$$

where

$N = 8$, for a finite vertical prism;

$N = 4$, for an infinite vertical prism;

$c_1 = \sqrt{2}\, h + i\,(\alpha_1 + \beta_1)$;

$c_2 = \sqrt{2}\, h + i\,(\alpha_1 + \beta_2)$;

$c_3 = \sqrt{2}\, h + i\,(\alpha_2 + \beta_1)$;

$c_4 = \sqrt{2}\, h + i\,(\alpha_2\,, \beta_2)$, etc.

For the gravity effect, the constants in (19) are as follows:

$$A = \sqrt{2}\,\pi G\rho,\ L = 3 \text{ and } F_0(u) = F_g(u,v)|_{u=v}.$$

In the case of the total-magnetic-field anomaly, we have

$$L = 2,\ A = \pi\, I_p[-1L - mM + 2nN - \alpha_{12} + i\sqrt{2}(\alpha_{13} + \alpha_{23})],$$

and

$$F_0(u) = F_T(u,v)|_{u=v}.$$

Equation (19) is the expression for the radial spectrum along the line at 45 degrees with the u and v axes.

In order to consider the spectra of moments, we define

$$\phi_1(u) = \frac{\partial}{\partial u}\phi_0(u)\cdot \quad (20)$$

Now, using (19), we can write

$$\phi_1(u) = A \sum_{j=1}^{N} (-1)^j c_j e^{-uc_j}. \quad (21)$$

The complete expression for $\phi_1(u)$ in terms of the spectra of the anomaly and its moments is given by

$$\phi_1(u) = 3u^2F_0(u) - iu^3F_1(u), \text{ for gravity effect:} \quad (22)$$

and

$$\phi_1(u) = -iu^2F_1(u) + 2uF_0(u), \text{ for magnetic anomaly:} \quad (23)$$

where

$$F_1(u) = i\,\frac{\partial F_0(u)}{\partial u}.$$

It is interesting to note that $F_1(u)$ may also be expressed in the following forms:

$$F_1(u) = [F_g{}^x(u,v) + F_g{}^y(u,v)]_{u=v},$$
for gravity effect; (24)

and

$$F_1(u) = [F_T{}^x(u,v) + F_T{}^y(u,v)]_{u=v},$$
for magnetic anomaly.

Thus, we note in equation (24) that $F_1(u)$ can be computed with the help of the spectra of the x- and y-moments of either the gravity effect or the magnetic anomaly.

Both equations (19) and (21) are expressed as sums of exponentials. Each exponential term in the sum contains in the exponent the location of one edge of the prismatic body. As shown by Bhattacharyya and Leu (1975a), the unknown exponents can be determined by utilizing the exponential approximation method (Hildebrand, 1956) when the value of $\phi_0(u)$ or $\phi_1(u)$ are available. However, with $\phi_0(u)$ unrealistic results are obtained for the bottom edges of the causative body, particularly in the case of a large vertical extent or thickness of the body. The spectrum $\phi_1(u)$, on the other hand, enhances the effect of the bottom edges of the causative body and, thus, increases the resolution of the spectrum in the low frequency region.

As indicated before, the complex exponents c_j provide the information on the locations of the edges of a prismatic body. The real parts of the exponents contain the depths to the edges and the imaginary parts are determined by the sum of α and β. So with a given exponent, the horizontal coordinates α and β of the edges cannot be separately computed. For this reason, the exponential approximation method is used in this paper only for obtaining the depths to the top and bottom of a rectangular prism.

HORIZONTAL DIMENSIONS

Expression (5) for the spectrum of the gravity effect and expression (8) for that of the magnetic anomaly can be written in the following form:

$$F(u, v) = \frac{A(u, v)}{uvs} \cdot \sin\left(\frac{ua}{2}\right) \cdot \sin\left(\frac{vb}{2}\right) \cdot e^{-sh + iu\alpha_0 + iv\beta_0}, \quad (25)$$

where a and b are the horizontal dimensions of the prismatic body along x and y directions, respectively;

$$A(u,v) = 8\pi G\rho, \text{ for gravity effect,}$$
$$= 8\pi I_p J/s, \text{ for magnetic anomaly.}$$

Now, if we consider two pairs of frequencies (u_1,v_1) and $(u_1,2v_1)$, we can write with the help of (25):

$$\frac{F(u_1, 2v_1)}{F(u_1, v_1)} = \frac{1}{2} \cdot \frac{A(u_1, 2v_1)}{A(u_1, v_1)} \cdot \frac{\sin(v_1 b)}{\sin(v_1 b/2)} \cdot e^{-(s_2-s_1)h - iv_1\beta_0}, \quad (26)$$

where

$$s_1 = (u_1^2 + v_1^2)^{1/2}, s_2 = (u_1^2 + 4v_1^2)^{1/2},$$

and

$$\frac{A(u_1, 2v_1)}{A(u_1, v_1)} = 1, \text{ for gravity effect;}$$
$$= \frac{J(u_1, 2v_1)}{J(u_1, v_1)} \cdot \frac{s_1}{s_2}, \text{ for magnetic anomaly.}$$

If h and β_0 are calculated by the methods described before, it is straightforward to determine b with the help of equation (26). In a similar fashion, by considering pairs of frequencies (u_1, v_1) and $(2u_1,v_1)$, we can obtain the other horizontal dimension a.

It should be noted here that the method described for the computation of the horizontal dimensions is also valid for prisms with finite vertical extent.

RESULTS AND DISCUSSION

The methods discussed in the previous sections have been tested with both gravity and magnetic anomalies due to various prismatic bodies. In the

Table 1. Depth to the tops and bottoms of magnetic models.

No.	h	Computed depths 1	2	3	4	Mean	Exact	Remarks n	N	l	S	Spectrum used
1	h_1	2.993	3.008	2.854	−0.432*	2.95	3.0	10	1	1	1	$\phi_0(u)$
	h_2	6.602	6.708	15.703*	−8.836*	6.67	6.0					
2	h_1	3.175	3.106	3.005	2.998	3.07	3.0	10	1	1	1	$\phi_0(u)$
	h_2	5.071	5.900	6.186	13.059*	5.613	6.0					
3	h_1	3.920	4.008	3.822	1.422*	3.950	4.0	10	1	1	1	$\phi_1(u)$
	h_2	10.617	7.998	19.630*	2.850*	9.308	10.0					
4	h_1	5.053	5.131	4.706	5.629	5.131	5.0	10	2	2	2	$\phi_0(u)$
	h_2	9.211	10.506	−0.975*	−1.317*	9.858	10.0					
5	h_1	3.117	3.079	−3.621*	−5.383*	3.098	3.0	10	1	1	2	$\phi_1(u)$
	h_2	5.967	6.021	18.926*	14.007*	5.989	6.0					
6	h_1	4.203	3.915	3.844	0.259*	3.987	4.0	10	2	1	2	$\phi_1(u)$
	h_2	14.206	14.525	17.969*	20.032*	14.365	14.0					
7	h_1	5.708	5.987	5.975	5.936	5.901	6.0	10	1	1	2	$\phi_1(u)$
	h_2	17.877	17.885	20.670	24.054	21.626	20.0					

* Unrealistic depths

Table 2. Depth to the tops and bottoms of gravitational models.

No.	h	Computed depths 1	2	3	4	Mean	Exact	Remarks n	N	l	S	Spectrum used
1	h_1 h_2	3.119 6.301	3.240 4.804	−0.763* 7.679	−0.808* −5.823*	3.178 6.261	3.0 6.0	15	1	1	1	$\phi_0(u)$
2	h_1 h_2	2.941 10.948	2.881 7.669	3.523 1.223*	1.556* 0.631*	3.115 9.309	3.0 10.0	10	1	1	1	$\phi_1(u)$
3	h_1 h_2	4.716 11.609	4.076 15.954	4.338 21.648*	0.766* 0.236*	4.376 13.781	5.0 14.0	10	1	1	2	$\phi_1(u)$
4	h_1 h_2	2.986 12.722	2.929 11.084	2.611 17.152*	−2.937* 4.294*	2.842 11.903	3.0 12.0	10	1	1	2	$\phi_1(u)$
5	h_1 h_2	4.706 18.264	3.954 10.935	3.751 −0.224*	0.382* −6.433*	4.173 14.599	4.0 15.0	10	1	1	1	$\phi_1(u)$
6	h_1 h_2	5.775 18.441	5.746 14.929	6.710 21.734	0.729* −6.055*	6.077 18.688	6.0 20.0	9	2	1	2	$\phi_1(u)$
7	h_1 h_2	5.171 11.031	5.048 12.075	5.107 −3.395*	2.252* −2.758*	5.108 11.556	5.0 10.0	8	2	1	2	$\phi_0(u)$

* Unrealistic depths

case of magnetic anomalies, it is assumed that the causative bodies are all inductively magnetized in the presence of a geomagnetic field with an inclination of 60 degrees and a declination of 0 degrees. The anomalies are computed on a 64 × 64 equispaced array of points spaced one grid unit apart. The computed values are then used to obtain the spectra of the anomalies and their moments.

Equations (16) and (18) are used to determine the location (α_0, β_0, h_0) of the centroid of the body for gravity and magnetic anomalies, respectively. In order to compute the depths to the top and the bottom of the body, the exponential approximation method is applied to the radial spectrum $\phi_0(u)$ or $\phi_1(u)$. For a finite vertical prism, there are eight values of depth corresponding to the eight corners of the body. The mean of four of these values provides the depth to the top and the mean of the remaining four gives the depth to the bottom. These values are presented in Tables 1 and 2. In most cases, one or two unrealistic depths marked with the asterisk symbol are obtained. By comparing the four values obtained for the depth

Table. 3. Parameters of gravitational models.

No. of cases	Parameters	h_0	α_0	β_0	h_1	h_2	a	b
1	Exact Computed	4.5 4.69	3.5 3.49	3.5 3.49	3.0 3.18	6.0 6.26	5.0 5.03	3.0 3.04
2	Exact Computed	6.5 6.28	0.0 0.00	4.0 3.99	3.0 3.12	10.0 9.31	8.0 8.04	4.0 4.07
3	Exact Computed	9.5 8.40	5.0 4.99	5.0 4.99	5.0 4.38	14.0 13.78	6.0 6.09	4.0 4.11
4	Exact Computed	7.5 7.22	0.0 0.00	0.0 0.00	3.0 2.84	12.0 11.90	8.0 8.05	8.0 8.04
5	Exact Computed	9.5 7.97	3.0 2.99	1.5 1.50	4.0 4.17	15.0 14.60	4.0 4.11	7.0 7.08
6	Exact Computed	13.0 11.12	0.0 0.00	5.5 4.91	6.0 6.08	20.0 18.69	12.0 12.10	7.0 7.14
7	Exact Computed	7.5 7.86	1.0 1.00	5.5 4.93	5.0 5.02	10.0 11.56	10.0 10.05	15.0 15.03

Table 4. Parameters of magnetic models.

No. of cases	Parameters	h_0	α_0	β_0	h_1	h_2	a	b
1	Exact	4.5	2.5	2.5	3.0	6.0	3.0	3.0
	Computed	4.49	2.49	2.46	2.95	6.67	2.89	2.89
2	Exact	4.5	0.0	0.0	3.0	6.0	8.0	8.0
	Computed	5.06	0.00	0.00	3.07	5.61	7.99	7.99
3	Exact	7.0	−3.0	−1.0	4.0	10.0	8.0	4.0
	Computed	7.01	−2.97	−1.06	3.95	9.31	7.95	3.99
4	Exact	7.0	5.0	5.0	5.0	9.0	6.0	4.0
	Computed	7.18	4.98	4.93	5.13	9.86	5.51	4.25
5	Exact	4.5	0.0	4.0	3.0	6.0	8.0	4.0
	Computed	5.07	0.00	3.94	3.10	5.99	7.55	3.66
6	Exact	9.0	6.0	9.0	4.0	14.0	4.0	6.0
	Computed	7.73	5.97	8.89	3.99	14.37	3.28	5.52
7	Exact	13.0	−2.0	1.0	6.0	20.0	8.0	12.0
	Computed	11.82	−1.98	0.94	5.90	21.63	8.98	12.586

to either the top or the bottom of a prism, we can easily find the erroneous values; if any. In this process, the location of the centroid is also of considerable help. The values which are considered reliable are then averaged to obtain a mean value for the depth either to the top or to the bottom of the body.

The symbols in the "Remarks" column indicate different parameters used in the exponential approximation method, as described by Bhattacharyya and Leu (1975a). In this method, as noted before, very low frequencies cannot be considered for analysis, because of inaccuracies in the calculated spectrum at such frequencies. Consequently, the frequencies from zero to the Nth harmonic of the fundamental frequency are often discarded. If the fundamental angular frequency is $d\omega$, then a band of discrete, equispaced frequencies, spaced $\Delta\omega$ apart, is considered for obtaining an equation relating spectra at all these frequencies. The ratio of $\Delta\omega$ and $d\omega$ is denoted by l. Then this band of frequencies is shifted in the frequency scale by an amount $S\,d\omega$ to obtain another equation. The number of such equations generated for determining the depths is given by n.

With the help of the computed location of the centroid and depths to the top and bottom of the prism, the horizontal dimensions are determined by using (26) for "b" and a similar equation for "a". Several values of a and b are normally obtained for different pairs of frequencies and the averages of these values are found to be accurate and reliable. Tables 3 and 4 present the calculated and exact values for the parameters of various models used in generating gravity and magnetic fields. The tables show the high accuracy of the computed dimensions.

ANALYSIS OF REAL ANOMALIES

The method as presented in this paper for spectral analysis of gravity and magnetic anomalies due to three-dimensional bodies was applied to the filtered aeromagnetic data over Yellowstone National Park for mapping Curie-point isothermal surface (Bhattacharyya and Leu, 1975b). Two anomalies have been selected from this data for

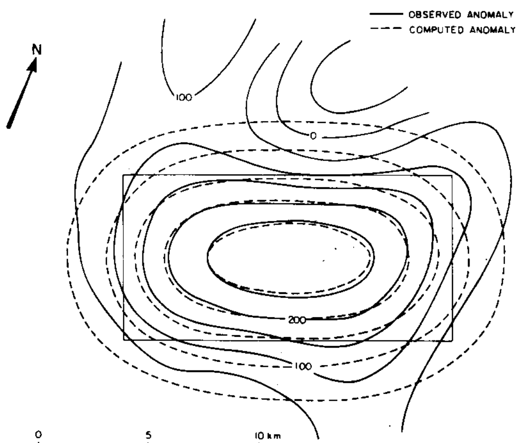

FIG. 5. Interpretation of filtered total-field-magnetic anomaly shown in solid lines. Dashed contour lines show the anomaly due to a rectangular prismatic body. The horizontal cross-section of the body is shown by a thin line. Contour interval is 50 gammas.

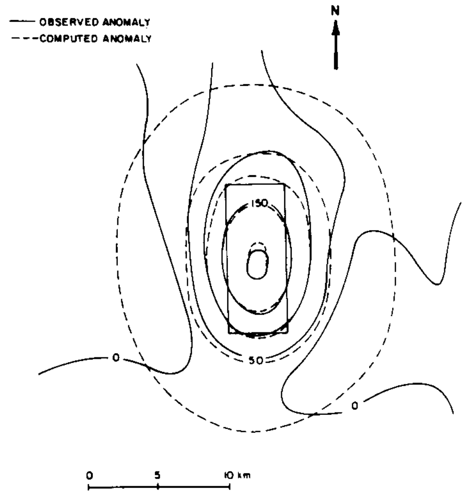

FIG. 6. Interpretation of filtered total-field-magnetic anomaly shown in solid lines. Dashed contour lines show the anomaly due to a rectangular prismatic body. The horizontal cross-section of the body is shown by a thin line. Contour interval is 50 gammas.

detailed analysis. Figures 5 and 6 present these anomalies in solid lines.

The digitized data for these anomalies are spaced 2.08 km apart in NS and EW directions. From the data bicubic spline interpolation is used to obtain 64 × 64 values for each of the anomalies. Then the fast Fourier transform algorithm is applied to the new sets of values for obtaining the spectra of the anomalies.

It is assumed that the causative bodies were vertically dipping rectangular prisms. The original data were reduced to the pole, on the assumption of induced magnetization of all the bodies. Due to lack of knowledge about remanent magnetization of these bodies, the inclination and declination of both the magnetization vector and the geomagnetic field are supposed to be 90 degrees and 0 degrees, respectively.

For the anomaly in Figure 5, the dimensions of the vertical prism are calculated to be a = 5.6 km, b = 12.5 km, h_1 = 4.5 km, h_2 = 11.0 km, and the susceptibility χ = 0.00365 cgs emu. The depths h_1 and h_2 are determined with respect to the level of observation of the original aeromagnetic data which was 3.66 km above sea level. The horizontal cross-section of the body is shown in Figure 5. The dashed contour lines in the figure represent the theoretical anomaly due to the prismatic body. There is a good correspondence between the theoretical and observed anomalies near their peaks.

The dimensions of the body as calculated for the anomaly in Figure 6 are a = 4.2 km, b = 10.4 km, h_1 = 4.2 km, h_2 = 12.5 km, and χ = 0.00224 cgs emu. In this case also, the theoretical anomaly due to the body of the above dimensions are in good agreement with the given anomaly near the central portion (Figure 6).

CONCLUDING REMARKS

This paper has presented a method for spectral analysis of gravity and magnetic anomalies due to prismatic bodies. The spectrum of an anomaly contains a predominantly large effect from the upper edges of the prism relative to that of the deeper edges. In order to enhance the effect of the bottom edges, it is necessary to transform the anomaly in the space domain and then use the spectrum of the transformed anomaly for analysis. The moments of the anomaly contain the larger effects of the bottom edges, and so their spectra are useful for obtaining accurate estimates of the depths to the bottoms of the causative bodies.

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