Two-step processing for 3D magnetic source locations and structural indices using extended Euler or analytic signal methods

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Summary

In situations where a physical surface defines an upper bound on acceptable magnetic source locations, a two-step inversion process can be used to correct initial solutions that fall above the reference surface. The first step involves solving for source locations using an assumed structural index, usually zero. The second step involves moving solutions above the reference surface down to the surface and either solving for a new horizontal location and structural index (Euler method) or adjusting the structural index using a pre-determined formula (analytic signal method). If the initial structural index is zero, solutions falling below the reference surface will represent the shallowest possible distribution of sub-surface magnetic sources. For both methods, the number and reliability of solutions is greatly increased by analyzing generalized Hilbert transform components in addition to the observed magnetic field.

Introduction

In traditional magnetic source-location methods, the structural index of the source is specified by the interpreter prior to the analysis. In the Euler method (Reid and others, 1990), the specified index may range continuously from zero (a contact) to three (a dipole). In the analytic signal method (Roest and others, 1992; Phillips, 2000), the index is usually chosen to be either zero (a contact) or one (a sheet edge). When the chosen structural index is too small, the estimated source locations will be too shallow; conversely, when the chosen index is too large, the estimated source locations will be too deep.

Often there is a physical surface representing an upper bound to the acceptable source locations. For aeromagnetic data this might be the topographic surface, the seafloor, or the seismic basement. This paper outlines how this known surface can be used to constrain the source locations and lead to estimates of structural index for sources lying directly on the surface.

For both Euler and analytic signal methods, an increased number of solutions can result from the use of generalized Hilbert transform components (Nabighian, 1984; Nabighian and Hansen, 2001). Combinations of these components with the observed magnetic field provide additional Euler equations, and permit up to seven Euler solutions within each small data window.

The Two-step Method

The basic two-step approach involves first solving for source locations assuming a specified structural index (usually zero) and retaining those solutions that lie on or below the reference surface. For a structural index of zero, these retained solutions represent the shallowest possible sub-surface sources. Solutions lying above the reference surface are moved down to the reference surface, and a second solution is attempted for the horizontal location and structural index on the reference surface (Euler method), or a new structural index is assigned using a pre-determined formula (analytic signal method).

Applied to Extended Euler Deconvolution

For the extended Euler method, the first step involves solution of one or more of the equations:

$$
\begin{aligned}
&x_0\frac{\partial T}{\partial x}+y_0\frac{\partial T}{\partial y}+z_0\frac{\partial T}{\partial z}+\alpha \\
&\qquad\qquad = x\frac{\partial T}{\partial x}+y\frac{\partial T}{\partial y}+z\frac{\partial T}{\partial z}+nT, \\
&x_0\frac{\partial H_x}{\partial x}+y_0\frac{\partial H_x}{\partial y}+z_0\frac{\partial H_x}{\partial z}+\alpha_x \\
&\qquad\qquad = x\frac{\partial H_x}{\partial x}+y\frac{\partial H_x}{\partial y}+z\frac{\partial H_x}{\partial z}+nH_x, \\
&x_0\frac{\partial H_y}{\partial x}+y_0\frac{\partial H_y}{\partial y}+z_0\frac{\partial H_y}{\partial z}+\alpha_y \\
&\qquad\qquad = x\frac{\partial H_y}{\partial x}+y\frac{\partial H_y}{\partial y}+z\frac{\partial H_y}{\partial z}+nH_y
\end{aligned}
\tag{1}
$$

The left-hand sides contain the unknown source location (x_0, y_0, z_0), and unknown constants α, α_x, α_y. The right-hand sides contain the known observation location (x, y, z), the observed field T, the calculated Hilbert transform components (H_x, H_y), and the assumed structural index n. Although it may appear that n could be treated as a fifth unknown, Barbosa and others (1999) have shown that this approach leads to instabilities. Normally the Hilbert transform components and the derivatives are computed assuming that the observations T are on a flat surface z=0. This same surface should be used to solve for the unknowns, with z_0 adjusted later to reflect the true observation elevation.

Within a small data window of size M by M, there are seven possible combinations of equations (1), leading to seven different solutions (Table 1).

Component	Equations	Unknowns
T only	M^2	4
H_x only	M^2	4
H_y only	M^2	4
T and H_x	$2M^2$	5
T and H_y	$2M^2$	5
H_x and H_y	$2M^2$	5
T, H_x, and H_y	$3M^2$	6

Table 1 – The seven extended Euler solutions.

One strategy is to attempt all seven solutions, then average the successful solutions. Solutions are successful if the system of equations can be solved, if the error in z_0 (either absolute or percent) is below a specified maximum, if z_0 is above a specified maximum depth, and if (x_0, y_0) is within a specified radius of the center of the window.

Successful solutions that lie on or below the reference surface are retained. Successful solutions that lie above the reference surface are moved down to the reference surface elevation z_1 and one or more of the following equations are solved for x_0, y_0, α, α_x, α_y, and n:

$$x_0\frac{\partial T}{\partial x}+y_0\frac{\partial T}{\partial y}+\alpha-nT=x\frac{\partial T}{\partial x}+y\frac{\partial T}{\partial y}+(z-z_1)\frac{\partial T}{\partial z},$$
$$x_0\frac{\partial H_x}{\partial x}+y_0\frac{\partial H_x}{\partial y}+\alpha_x-nH_x =x\frac{\partial H_x}{\partial x}+y\frac{\partial H_x}{\partial y}+(z-z_1)\frac{\partial H_x}{\partial z}, \qquad (2)$$
$$x_0\frac{\partial H_y}{\partial x}+y_0\frac{\partial H_y}{\partial y}+\alpha_y-nH_y =x\frac{\partial H_y}{\partial x}+y\frac{\partial H_y}{\partial y}+(z-z_1)\frac{\partial H_y}{\partial z}$$

Again all seven possible solutions are attempted, and the successful solutions are averaged. Solutions are successful if the system of equations can be solved, if (x_0, y_0) is within a specified radius of the center of the window, and if n is between zero and three. The average absolute error in z_0 is left unchanged from the solution of the first step, although the percent error in z_0 (if used) must be reduced to reflect the new and lower source elevation z_1.

At the end of this two-step process, there is either (1) no solution in the window, or (2) one solution containing the average source location, the average depth error, the average structural index, and the number of components contributing to the average. Solutions produced from a large number of components are more reliable than those produced from a small number of components, as are solutions with smaller average error in depth. These two parameters can be combined in an "Euler information index",

$$EII=\frac{number\ of\ components}{average\ error\ in\ depth} \qquad (3)$$

This index is high for more reliable solutions and low for less reliable solutions. Less reliable solutions should not necessarily be rejected, because many real sources are only detected in one or two of the seven components. However the index is useful for determining which solutions are the most repeatable and have the lowest error.

Applied to the Analytic Signal Method

In the analytic signal method, the horizontal location (x_0, y_0) of a contact source (structural index zero) is estimated from the crest of the analytic signal amplitude function:

$$|A(x,y)|=\sqrt{\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2} \qquad (4)$$

The depth coordinate z_0 is estimated from the maximum curvature of $|A(x,y)|^2$ at the crest (Phillips, 2000):

$$z_0{}^2=\frac{|A(x,y)|^2[(x-x_0)\cos\phi+(y-y_0)\sin\phi]}{|A(x_0,y_0)|^2-|A(x,y)|^2} \qquad (5)$$

where ϕ is the strike of the crest.

If the depth solution z_0 lies above the reference surface, the source can be moved down to the reference surface z_1 by increasing the structural index n using the formula

$$n=\left(\frac{z_1}{z_0}\right)^2-1 \qquad (6)$$

which was determined by considering theoretical sources having integer structural indices. As with the extended Euler method, additional solutions can be found by substituting the Hilbert transform components H_x and H_y for T in equation (3).

Example

Aeromagnetic data over two buried lava flows in New Mexico (USGS and Sander Geophysics, Ltd, 1998) are used to illustrate the method (Fig. 1a). The longer lava flow, which extends along the diagonal of the area, is buried at least 150 meters below the ground surface. The shorter lava flow, which is parallel and to the southwest, lies approximately 15 meters below the ground surface (Grauch, 2001). Both flows appear to originate from the same volcanic center in the northwest corner of the area. In addition to the flows, there are several faults trending north and northeast within the sedimentary section (Grauch, 2001). The flight surface is approximately 130 m above the topography.

The Hilbert transform components (Fig. 1b,c) computed using the method of Nabighian (1984) should be equivalent data sets to the observed magnetic field for purposes of interpreting source locations (Nabighian and Hansen, 2001). To illustrate this, the result of two-step Euler analysis on the observed field only (Fig. 2a,d) is compared to the results of separate and joint analyses on the Hilbert transform components (Fig. 2b,e). By averaging results from all seven combinations of these three data sets, many additional solutions emerge (Fig. 2c,f).

The large number of solutions resulting from the combined analysis is useful in mapping the depth distribution of sources, but it also can be an impediment to the interpretation of individual faults and contacts. Fortunately there are several measures that highlight the more significant solutions. These include the reciprocal of the error in the depth estimate (Fig. 3a,d), which is a measure of the reliability of each solution, the number of solutions contributing to the average in each window (Fig. 3b,e), which is a measure of the repeatability of each result, and the *EII*, or ratio of the number of solutions to the depth error (Fig. 3c,f), which is a measure of both.

Conclusions

A two-step solution process can be used to correct initial magnetic source locations lying above a known reference surface. The approach can be applied to Euler or analytic signal solutions, and it yields structural indices for all solutions corrected to the reference surface. The use of generalized Hilbert transform components adds significant information to the solutions.

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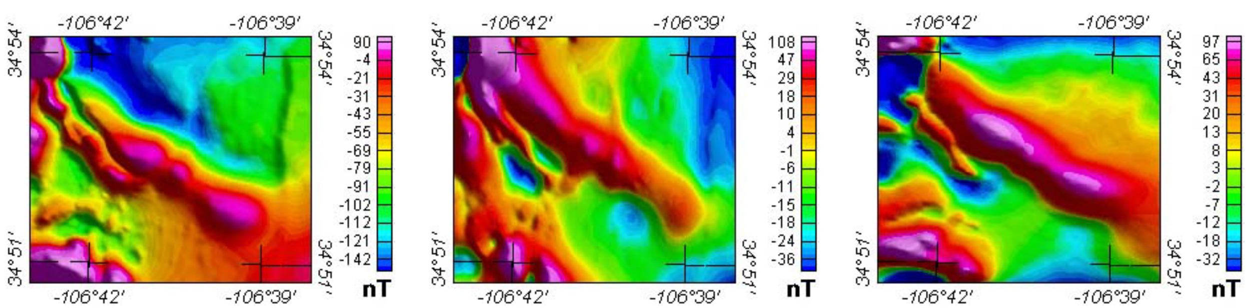

Figure 1. Observed magnetic field at 130 m above terrain (a), and generalized Hilbert transform components H_x (b) and H_y (c).

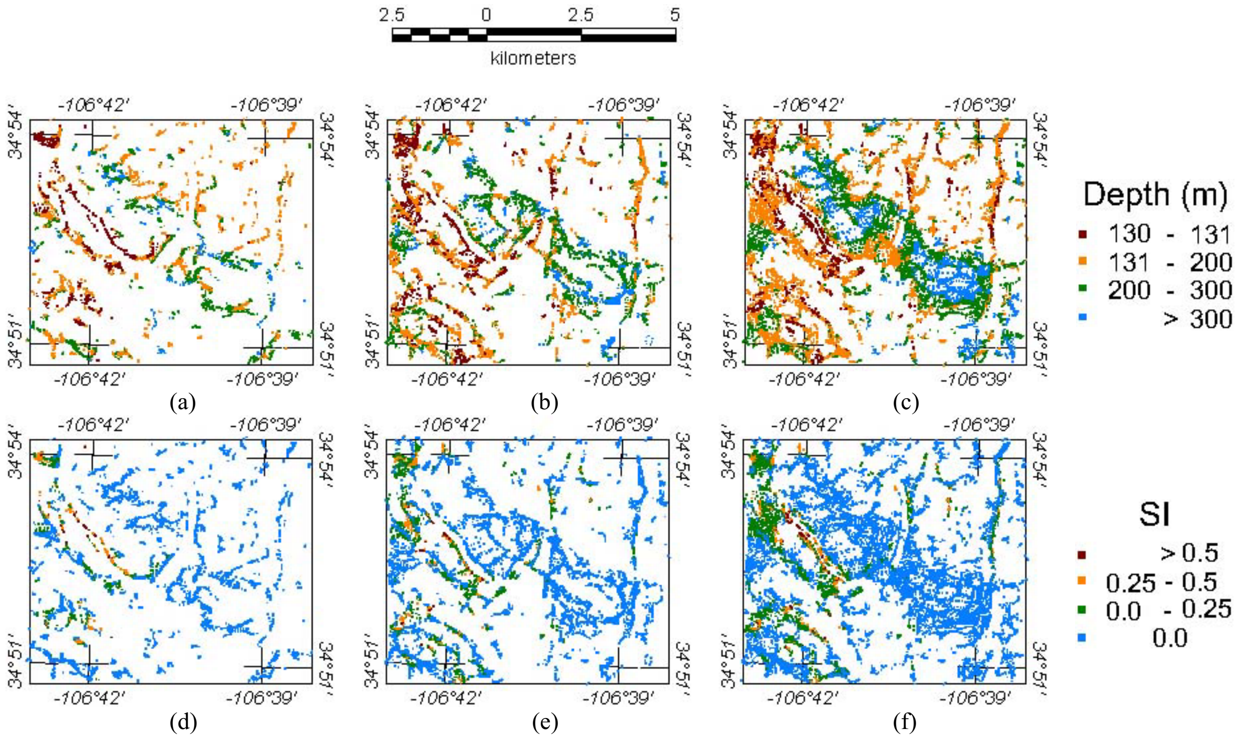

Figure 2. Two-step Euler depth solutions, zero initial structural index, 7x7 window, 10% maximum depth error. (a) Solutions from only the observed magnetic field. (b) Averaged joint and separate solutions from H_x and H_y. (c) Averaged solutions from all combinations of components. (d) Structural indices for (a). (e) Structural indices for (b). (f) Structural indices for (c).

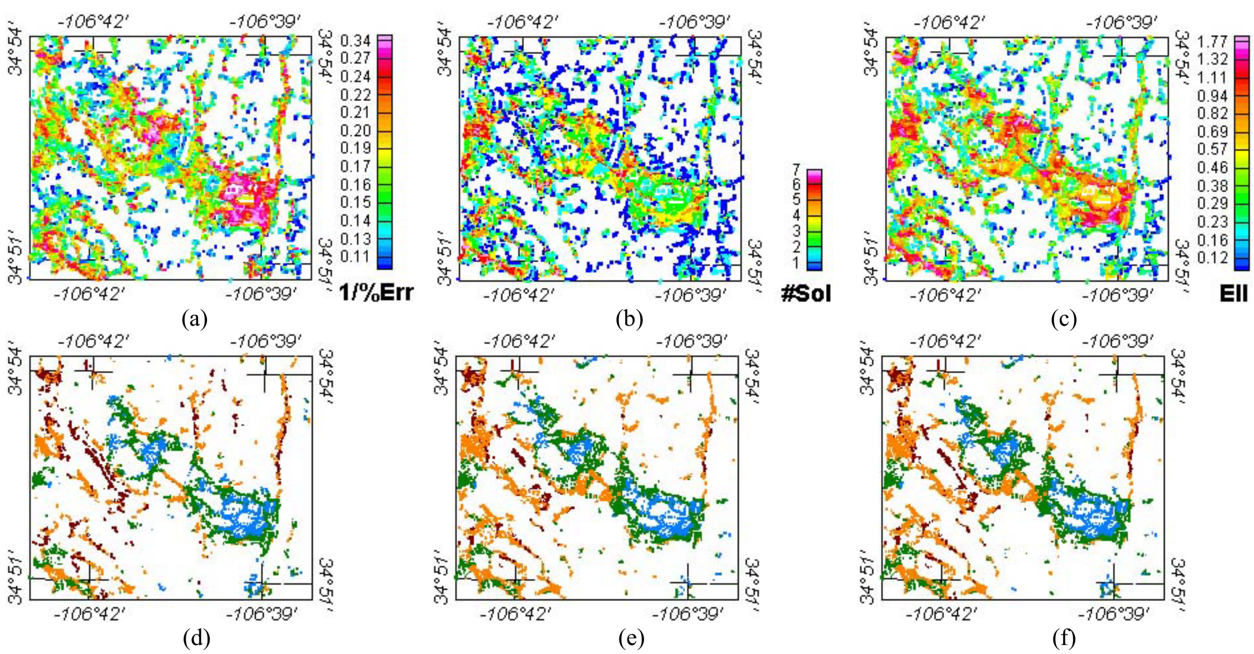

Figure 3. Three measures of reliability and repeatability for the averaged Euler solutions of Fig. 2c. (a) The reciprocal of the percent error on depth. (b) The number of solutions contributing to the average. (c) Euler information index, the product of (a) and (b). (d) Depth solutions with 1/%Err > 0.18. (e) Depth solutions with #Sol >2. (f) Depth solutions with *EII* > 0.4.