GEOPHYSICS, VOL. 35, NO. 2 (APRIL 1970), P. 293-302, 9 FIGS.

STATISTICAL MODELS FOR INTERPRETING AEROMAGNETIC DATA†

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A mathematical basis for the application of power spectrum analysis to aeromagnetic map interpretation is developed. An ensemble of blocks of varying depth, width, thickness, and magnetization is considered as a statistical model. With the use of the fundamental postulate of statistical mechanics, a formula which can be used to analyze the power spectrum of an aeromagnetic map is developed. The influences of horizontal size, depth, thickness, and depth extent of the blocks on the shape of the power spectrum are assessed. Examples which include power spectra of maps from Canada and Central America demonstrate the application of the approach. In the cases studied a double ensemble of blocks appears to best explain the observed power spectrum characteristics.

INTRODUCTION

It has become one of the cliches of contemporary scientific writing that many a concept which yesterday seemed abstruse is today accepted with an enthusiasm that seems almost inexplicable. Geophysics has many examples to show of such shifts in the winds of convention, and one of these undoubtedly has been in the attitude taken toward Fourier analysis as a tool for interpreting gravity and magnetic field data. Within recent years, spectrum analysis has quite suddenly become a technique that is not only acceptable but, in some places, almost *de rigeur* in the processing of aeromagnetic maps. Nowadays the two-dimensional power spectrum within this context has become a term of common technical use.

We do not, of course, claim any special priority for the idea of using power spectra in the analysis of aeromagnetic data. Suggestions of this kind can be traced back for at least 10 years (see, for example, Horton, Hempkins and Hoffman, 1964) and perhaps even farther than that. We have, however, actively been engaged in the application of Fourier techniques during at least half of this period (Spector and Bhattacharyya, 1966, and Spector, 1968), and we have developed certain concepts which we believe have proven to be useful, particularly in aeromagnetic interpretation. One of these has been the use of statistical models and ensemble averaging. In this paper we would like to elaborate on the role of this particular concept.

AN INTERPRETATION MODEL

Let us start with the basic model that is often used by geophysicists to simulate the magnetic effects of various geological units. This is the rectangular, vertical-sided parallelepiped (Figure 1). The model has enjoyed wide and continued popularity over many years as a device for estimating depths to individual magnetic bodies (or zones) whenever their anomaly patterns are easily separated from neighboring influences. The success that this model has achieved probably can be explained by the observation that aeromagnetic anomaly patterns are shaped very largely by the depths and volumes of the sources (and also, of course, by the directions of their magnetizations). They are shaped relatively little by the details of their boundaries. In any event, success

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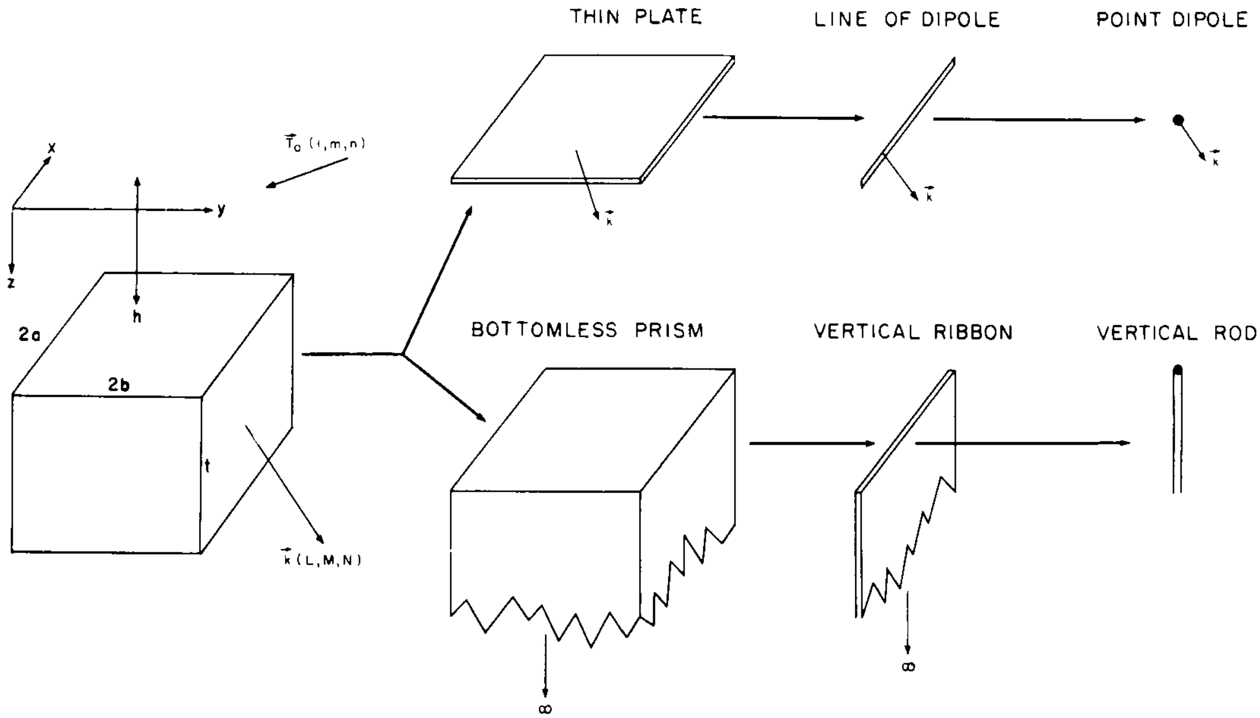

FIG. 1. The rectangular, vertical-sided parallelepiped: an interpretation model.

does give one quite a high degree of confidence in the model.

If the rectangular parallelepiped works satisfactorily in the case of individual anomalies, it ought to work even better as the unit of an ensemble, where wide variances are tolerated in all of the parameters. Whatever deficiencies there may be in the model will be minimized in the process of ensemble averaging and smoothing. We are thus led to make the following hypothesis: For the purpose of analyzing aeromagnetic maps, the ground is assumed to consist of a number of independent *ensembles* of rectangular, vertical-sided parallelepipeds, and each ensemble is characterized by a joint frequency distribution for the depth h, width a, length b, depth extent t, and direction cosines of magnetization L, M, N (see Figure 1).

According to this hypothesis, the map of the magnetic field intensity over an area, after the removal of the main geomagnetic component, is assumed to consist of the superposition of a large number of individual anomalies, most of them overlapping, which are caused by several ensembles of blocks having various dimensions and magnetizations. It is our intention to try to effect a separation of the magnetic effects of the different ensembles in the aeromagnetic map. Notice, incidentally, that when the size parameters of the block model a and b are allowed to vary, the model is capable of taking on a very wide variety of forms such as the vertical rod, vertical ribbon or plate, horizontal lamina, horizontal rod, and the bottomless prism. These models adequately represent many common geological forms such as pipes, veins, dikes, sills, and stocks.

Let us now consider the power spectrum (i.e. the square of the Fourier amplitude spectrum) of the total magnetic field intensity anomaly over a single rectangular block. The expression, which was first given by Bhattacharyya (1966), is here transcribed into polar wavenumber coordinates in the u, v frequency plane. Thus if

$$r = (u^2 + v^2)^{1/2} \quad \text{and} \quad \theta = \tan^{-1} u/v,$$

$$E(r, \theta) = |F(\Delta T)|^2$$
$$= 4\pi^2 k^2 e^{-2hr}(1 - e^{-tr})^2 S^2(r, \theta) R_T^2(\theta) R_k^2(\theta), \quad (1)$$

where $k/4ab$ is the magnetic moment/unit volume of the body (k is a magnetic moment/unit depth),

$$S(r, \theta) = \frac{\sin(ar\cos\theta)}{ar\cos\theta} \cdot \frac{\sin(br\cos\theta)}{br\cos\theta},$$

$$R_T^2(\theta) = [n^2 + (l\cos\theta + m\sin\theta)^2],$$

$$R_k^2(\theta) = [N^2 + (L\cos\theta + M\sin\theta)^2];$$

l, m, n are direction cosines of the geomagnetic

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field vector $\mathbf{T}_o$ and L, M, N are direction cosines of the magnetic moment vector $\mathbf{k}$. The symbols h, t, a, and b are explained in Figure 1.

The simplicity of equation (1) is striking when we compare it with the exceedingly cumbersome formula for the magnetic field anomaly itself (see Bhattacharyya, 1964). Moreover, the parameters which define the model separate from one another in the power spectrum in quite a remarkable way.

AN ENSEMBLE OF BLOCKS

We now use the fundamental postulate of statistical mechanics to obtain from equation (1) a formula which we can apply directly to the power spectra of aeromagnetic maps. The postulate states that the mathematical expectation of the value of the power density function is equal to the *ensemble average* of E. It is strictly valid only for large samples, but in actual fact works very well even if the number of bodies present is as small as 5 or 6. Thus we may write

$$\langle E \rangle = \int \cdots \int E \cdot \Phi(a, b, t, h, I, D, k) dV, \quad (2)$$

where Φ is the ensemble joint frequency distribution for the parameters a, b, etc. and I and D are respectively the inclination and declination of the magnetic moment vector $\mathbf{k}$. Integration is over the entire 7-dimensional parameter space.

The question that now arises is what form to use for Φ. We assume that the parameters vary independently of one another, so that we may write $\Phi(a, b, \cdots) = \Phi(a) \cdot \Phi(b) \cdots$, but these Φ's are prior frequency distributions in the sense that there is no way in which we can deduce them from the observations. Accordingly, we assume that each Φ is rectangular, this being the simplest form to deal with analytically. If we assume that the width, a, is uniformly distributed in $(0, 2\bar{a})$, t in $(0, 2\bar{t})$, h in $\bar{h} \pm \Delta h$, I in $\bar{I} \pm \Delta I$, and D in $\bar{D} \pm \Delta D$, equation (2) becomes

$$\langle E(r, \theta) \rangle = \frac{1}{V} \int \cdots \int E(r, \theta) \, da\, db\, dt\, dh\, dI\, dD\, dk, \quad (3)$$

where V is the 7-dimensional parallelepiped which is defined by the limits of the frequency distribution functions.

Putting the expression (1) under the integral sign in (3), we obtain

$$\langle E(r, \theta) \rangle = 4\pi^2 \bar{k}^2 \, R_T^2(\theta) \cdot \langle R_k^2(\theta) \rangle \langle e^{-2hr} \rangle \langle (1 - e^{-tr})^2 \rangle \langle S^2(r, \theta) \rangle. \quad (4)$$

If we make the further assumption that for a moderately large number of bodies the average values of I and D will not differ appreciably from the inclination and declination, respectively, of the geomagnetic field, as long as ΔI and ΔD are not too large (say $\leq 20°$), we may put $R_k^2 = R_T^2$ in expression (4). We can then define a new form for the power spectrum, which we write as

$$\langle \tilde{E}(r, \theta) \rangle = \frac{\langle E(r, \theta) \rangle}{R^4_T(\theta)} = 4\pi^2 \bar{k}^2 \langle e^{-2hr} \rangle \langle (1 - e^{-tr})^2 \rangle \langle S^2(r, \theta) \rangle. \quad (5)$$

This is called "reduction (of the power spectrum) to the north magnetic pole." What the phrase means is that if all the bodies were magnetized in the direction of $\mathbf{T}_o$ and if observations had been taken close to the north magnetic pole, the power spectrum should have an ensemble average given by (5). There is no difficulty in theory about reducing the power spectrum to the north magnetic pole because $R_T(\theta)$ is always known. Care must be taken in low magnetic latitudes, however, and close to the geomagnetic equator, R_T becomes so exceedingly asymmetrical that special numerical procedures must be used. The validity of the assumption that $\bar{I}$ and $\bar{D}$ are equal to the inclination and declination of the geomagnetic field can only be judged according to whether or not the majority of the anomaly patterns look "normal" for the given magnetic latitude.

PROPERTIES OF THE POWER SPECTRUM

It is useful as a general rule to look at power spectra in one-dimensional or profile form rather than in two-dimensional or map form. This is mainly because the quantity $\langle S^2 \rangle$ is a somewhat bumpy function of θ when $\bar{a}$ is moderately large, and the bumpiness imparts a certain irregularity to the contours. Accordingly, taking the average of (5) with respect to θ gives

$$\langle E(r) \rangle = 4\pi^2 \bar{k}^2 \langle e^{-2hr} \rangle \langle (1 - e^{-tr})^2 \rangle \langle S^2(r) \rangle, \quad (6)$$

where

$$\langle S^2(r) \rangle = \frac{1}{\pi} \int_0^{\pi} \langle S^2(r, \theta) \rangle d\theta.$$

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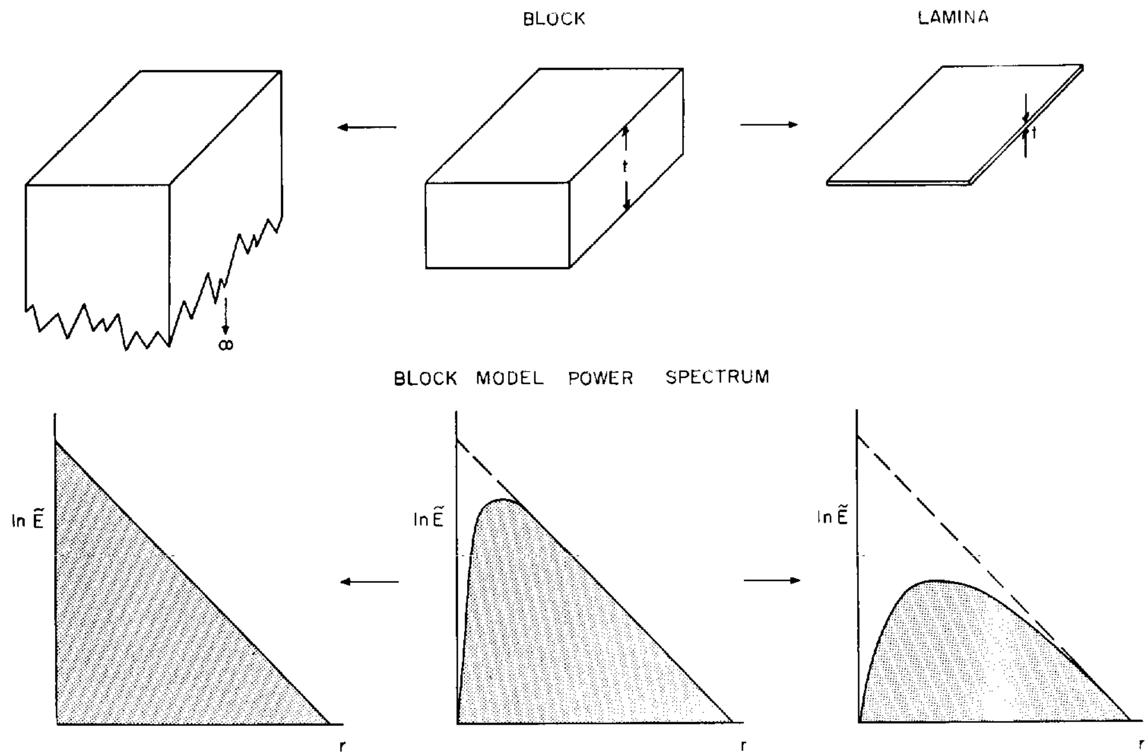

FIG. 2. Effect of finite thickness on the shape of the spectrum.

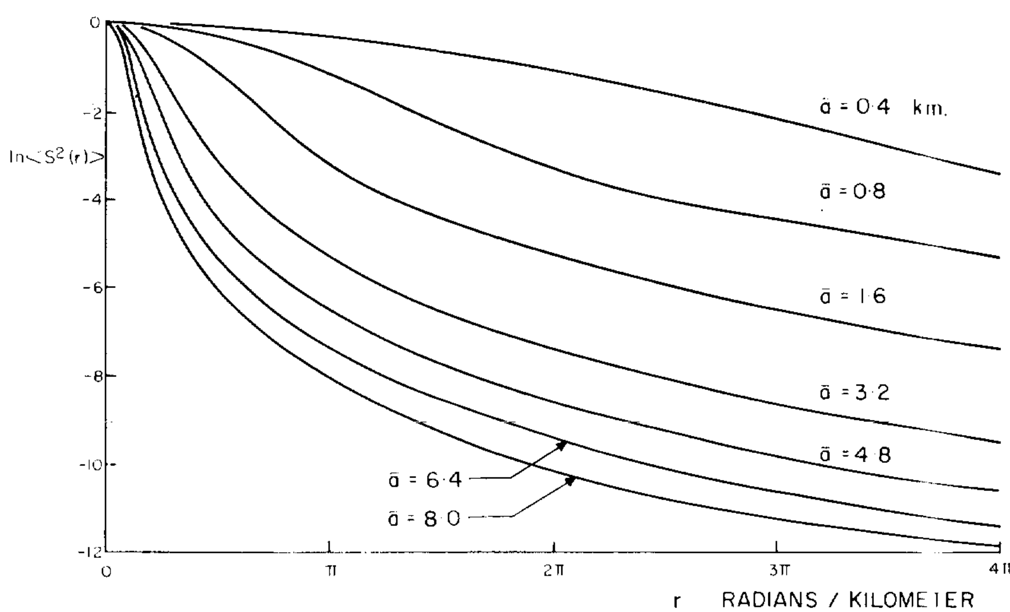

FIG. 3. Finite horizontal size effect.

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Notice that the ensemble parameters $\bar{k}$, $\bar{h}$, $\bar{t}$, and $\bar{a}$ are completely factored in this expression. In the logarithmic form of (6), therefore, their influences will simply add. The additive property is one of the chief reasons logarithmic spectra are preferred for analysis.

Effect of depth

The ensemble average depth $\bar{h}$ enters only into the factor $\langle e^{-2hr}\rangle = e^{-2\bar{h}r} \sinh (2r\Delta h)/4r\Delta h$. Normally one would set Δh at a value not greater than $0.5\bar{h}$; otherwise $\bar{h}$ itself has no interpretational value. For values of r which are $<1/\bar{h}$, therefore, $\langle e^{-2\bar{h}r}\rangle = e^{-2\bar{h}r}$ and the logarithm of this factor approximates a straight line whose slope is $-2\bar{h}$. The $e^{-2\bar{h}r}$ term is invariably the dominating factor in the power spectrum. Map spectra are usually declining functions of r whose rate of decay is largely determined by the mean depth of the bodies.

Effect of depth extent

The mean depth extent of the sources enters only into the factor

$$\begin{aligned}\langle C^2(r)\rangle &= \langle (1 - e^{-tr})^2\rangle \\ &= 1 - (3 - e^{-2\bar{t}r})(1 - e^{-2\bar{t}r})/4\bar{t}r.\end{aligned}$$

The parameter $\bar{t}$ plays a rather interesting role in shaping the power spectrum. When combined with the depth factor $e^{-2\bar{h}r}$ (for not too large values of r), the effect of $\langle C^2(r)\rangle$ is to introduce a peak into the spectrum whose position shifts toward smaller wavenumbers with increasing values of $\bar{t}$ (Figure 2). If the majority of the bodies in the area extend to such depths that their bottoms cannot be discerned clearly through the map window (a phrase which we shall explain presently), $\bar{t}$ becomes so large that the spectrum peak cannot be seen at all. In that case we get the spectrum for an ensemble of bottomless prisms, a spectrum which has its maximum value at $r=0$.

Whether the sources appear to be depth-limited or not will depend very much upon the size of the map. Since we must work with maps of finite area, we cannot, of course, calculate their Fourier transforms at wavenumber intervals smaller than $1/2L$, where L is the maximum length of the sample. If there were no restrictions upon either the size of the map or the size of the computer available to us, then presumably we could see right down to the Curie point isotherm; but to see to that depth would require a map at least 200 km$\times$200 km and would involve upwards of 40,000 T-values digitized on a 1.0 km square grid.

Effect of Size

The mean size of the bodies enters into the factor

$$\langle S^2(r)\rangle = \frac{1}{\pi}\int_0^{\pi} \langle S^2(r,\theta)\rangle d\theta$$

and has the effect of tapering the spectrum toward the higher wavenumbers, i.e. of speeding its decay. The magnitude of the effect is indicated in Figure 3, which shows $ln\langle S^2(r)\rangle$ versus r for several values of $\bar{a}$. If we do not allow for the $\langle S^2(r)\rangle$ factor (by assuming the sources to be poles or dipoles), estimates of $\bar{h}$ which are based upon the rate of decay of the power spectrum will be too large. The important property of these curves to note is the degree to which they tend to become straight lines (signifying exponential decay) toward the larger wavenumbers.

DOUBLE ENSEMBLE CASE

To make use of the theory at this stage, one would have to be presented with a map whose power spectrum clearly indicates the presence of only a single ensemble of sources. Such cases do exist, but they are rare. Even more rare, however, are situations in which three or more ensembles can clearly be discerned. By long odds, the most common occurrence is where just two sets of sources are discernible in the spectrum. They are easily recognized, as later examples will show, by the marked change that takes place in the spectral decay rate. Thus the average depth of burial $\bar{h}$ is the most influential parameter of the ensemble, and a substantial change in the value of this parameter will be reflected unmistakably in the power spectrum by a distinct change in the decay rate.

Let us suppose, therefore, that in accordance with observation there are two ensembles of sources. Each ensemble fills a well-defined rectangular volume of parameter space. Ensemble averaging accordingly requires us to integrate over the two regions, and we get

$$\langle E\rangle = \langle E\rangle_I + \langle E\rangle_{II}.$$

Suppose that region I is characterized by parameters $\bar{K}$, $\bar{H}$, $\bar{T}$, and $\bar{A}$, and region II by $\bar{k}$, $\bar{h}$, $\bar{t}$,

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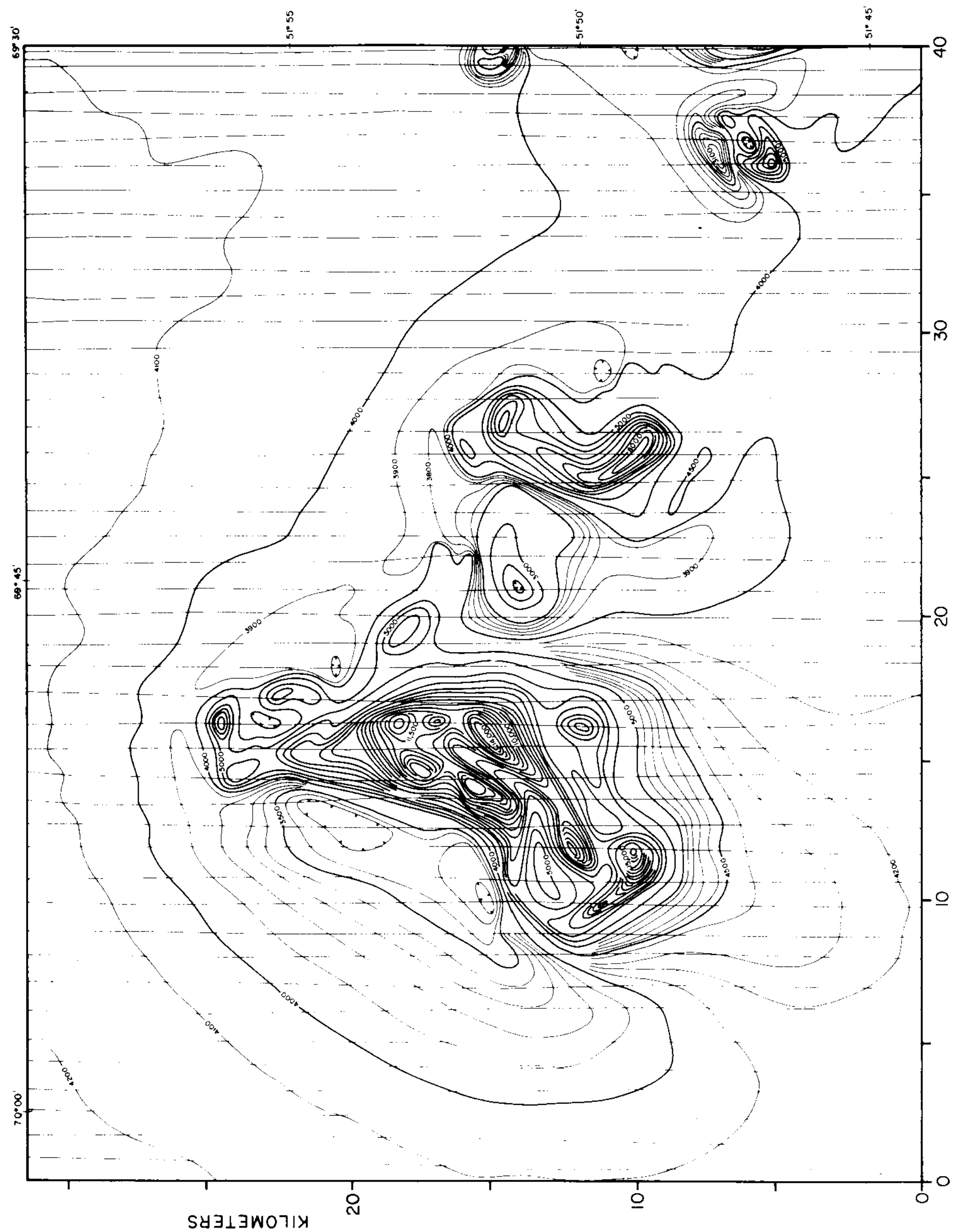

FIG. 4a. Total field intensity map of Mantonipi, Quebec. Minimum contour interval is 100 gammas.

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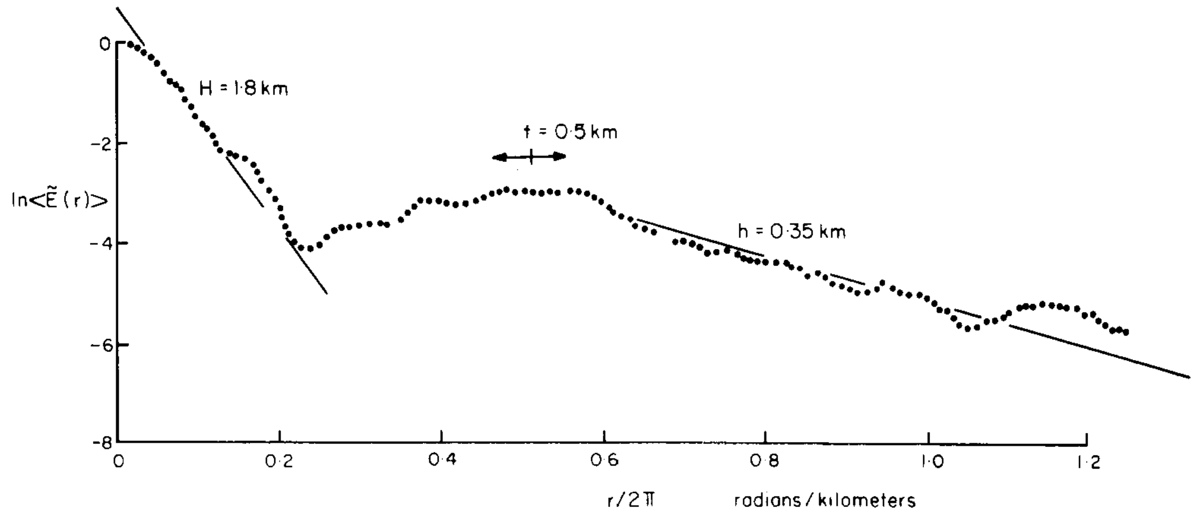

FIG. 4b. Power spectrum analysis of an aeromagnetic map of Figure 4a.

and $\bar{a}$. We assume that both ensembles have their mean direction of magnetization more or less along the geomagnetic vector. Reducing the spectrum to the north magnetic pole, therefore, gives the following formula:

$$\langle \bar{E}(r) \rangle = 4\pi^2 \overline{K}^2 e^{-2\bar{H}r} \langle S^2(r, A) \rangle + 4\pi^2 \bar{k}^2 \langle C^2(r, \bar{t}) \rangle e^{-2\bar{h}r} \langle S^2(r, \bar{a}) \rangle. \quad (7)$$

It is assumed that $\overline{T}$ is sufficiently large in relation to the map window L that the bottom surfaces of the deeper ensemble cannot be detected. Such is almost always the case in practice.

The power spectrum of the double ensemble thus consists of two parts. The first, which relates to the deeper sources, is relatively strong at the small wavenumbers and decays away rapidly. The second, which arises from the shallower ensemble of sources, dominates the short wavelength end of the spectrum.

EXAMPLES

The examples shown in Figures 4, 5, and 6 are three among many we have examined which illustrate the fact that a formula of the type (7) may be fitted to the majority of aeromagnetic power spectra. Actually, we have not yet encountered an instance in which it has proved impossible.

Example 1: Matonipi Lake area, Province of Quebec, Canada, (Figure 4)

This is an area of Precambrian rock which has been subjected to intense regional metamorphism. Iron formation outcrops in various places within this area. The power spectrum indicates that a major contribution to the aeromagnetic map is from bodies which lie at depths of about 2 km and that the outcropping iron formation has very limited depth extent (about 150 m).

Example 2: Central America (Figure 5)

The entire area is overlain with a thick veneer of Tertiary volcanic lava. The aeromagnetic map is extremely complex. The power spectrum indicates that the lavas have an average thickness of about 1.5 km, and that below them there is a deeper set of sources which are rather weakly magnetic as compared with the volcanics. A sequence of nonmagnetic material, probably sediments, which have an estimated thickness of 2 km, appears to overlay the deeper magnetic basement.

Example 3: Petry area, Province of Ontario, Canada (Figure 6)

This is again Precambrian country with numerous outcroppings of steeply-dipping gneiss and schist. The area is immediately north of Lake Superior. The power spectrum indicates that some rather strong source(s) lies at considerable depths within this region and that the near-surface bodies have quite a large depth extent. Deeper bodies are thought to be metavolcanic remnants.

If it is possible to fit aeromagnetic power spectra with formulas of the type (7), the same formula may be used as a basis for designing numerical filters to remove one part or the other from the spectrum.

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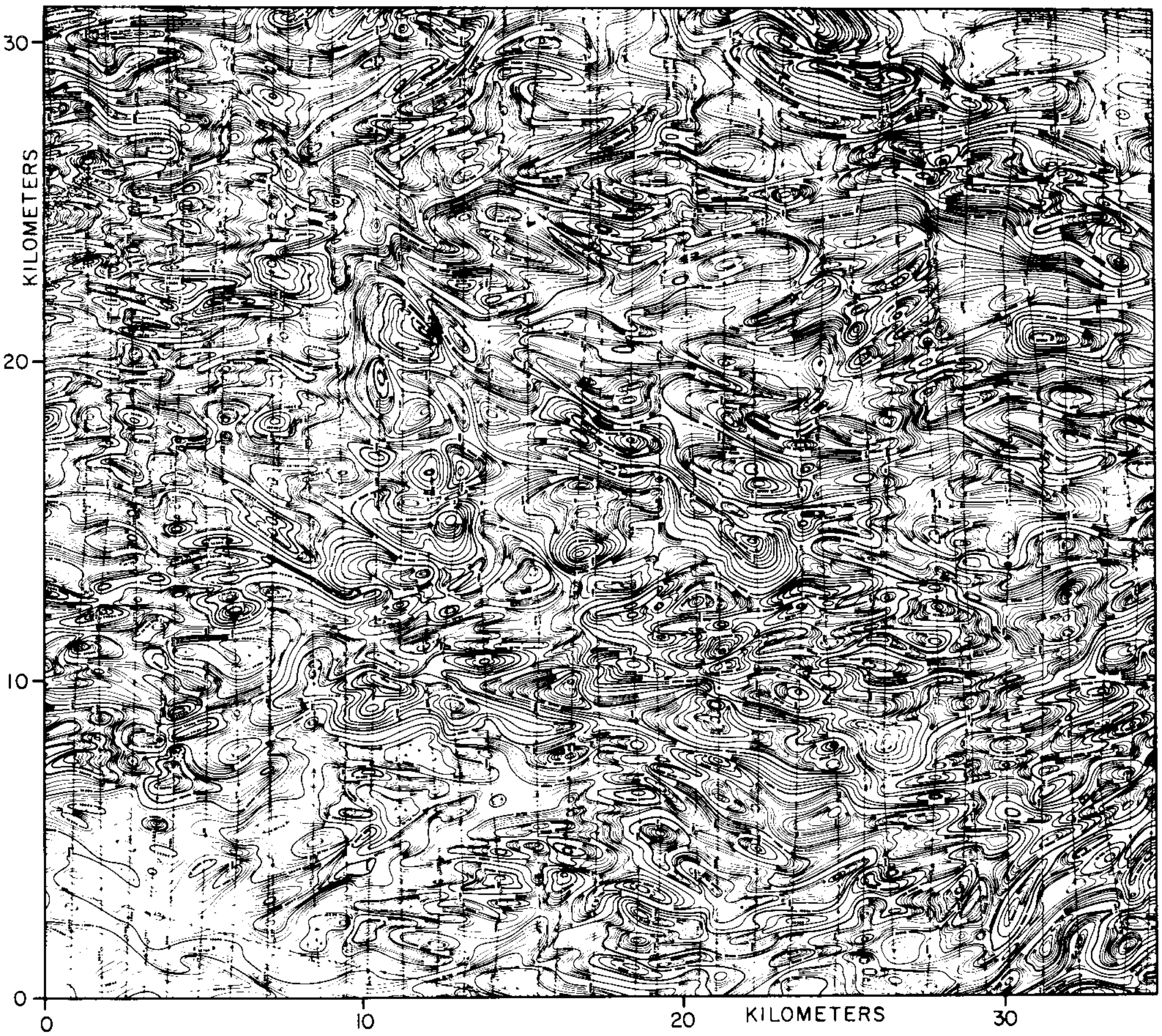
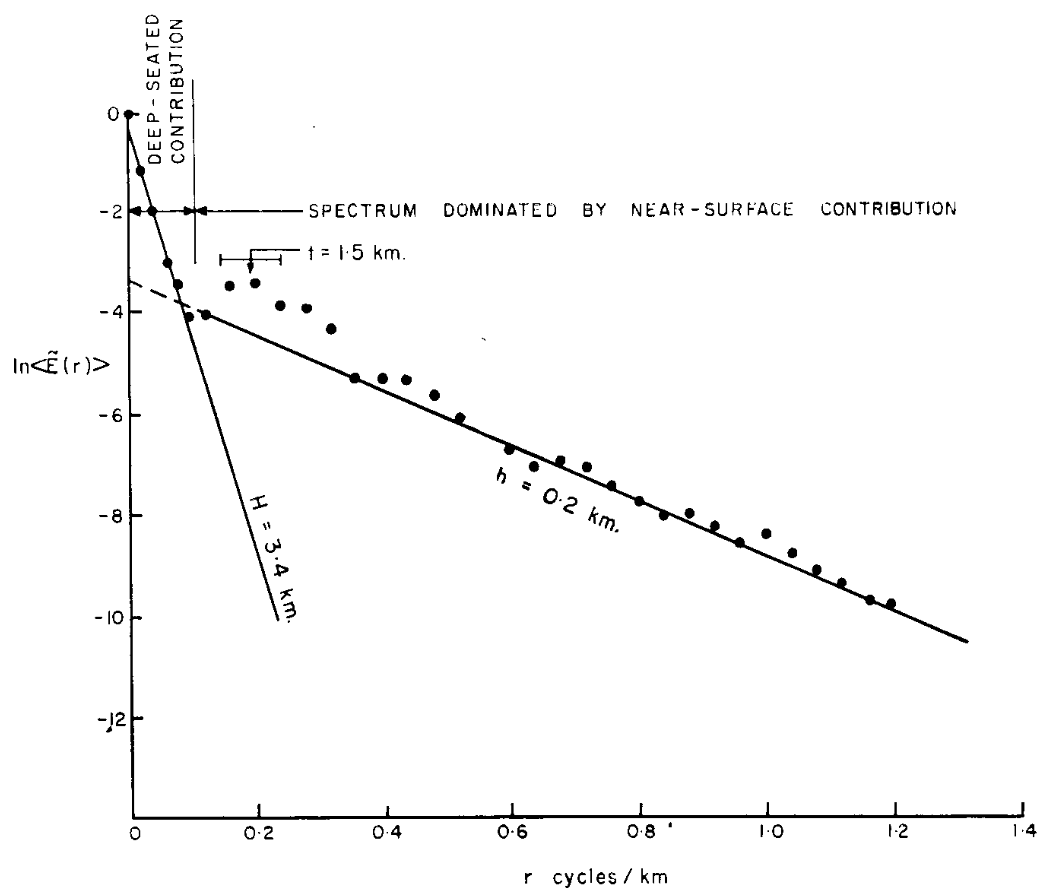


FIG. 5a. Central America aeromagnetic map with contour intervals at 20 gammas.



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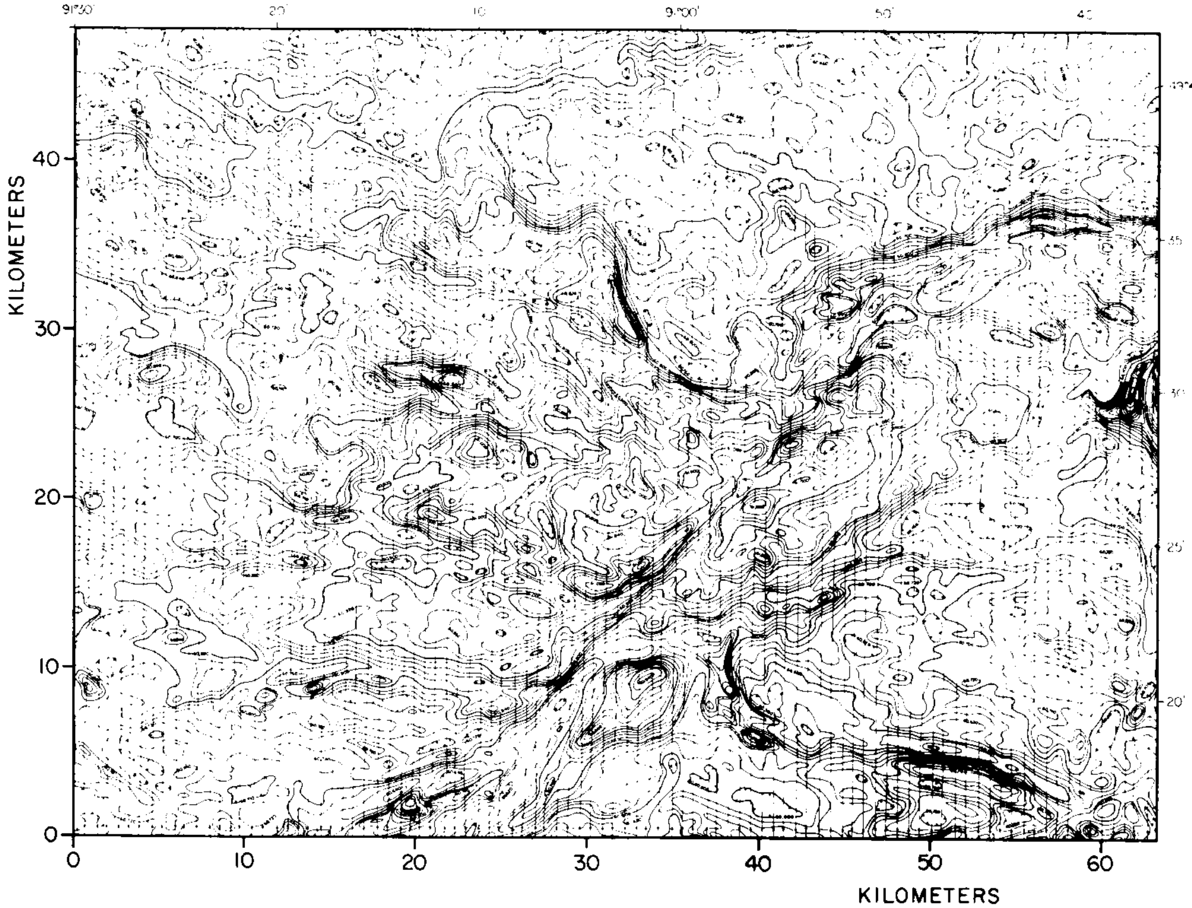

FIG. 6a. A total field intensity map of Petry, Ontario, with contour intervals at 20 gammas.

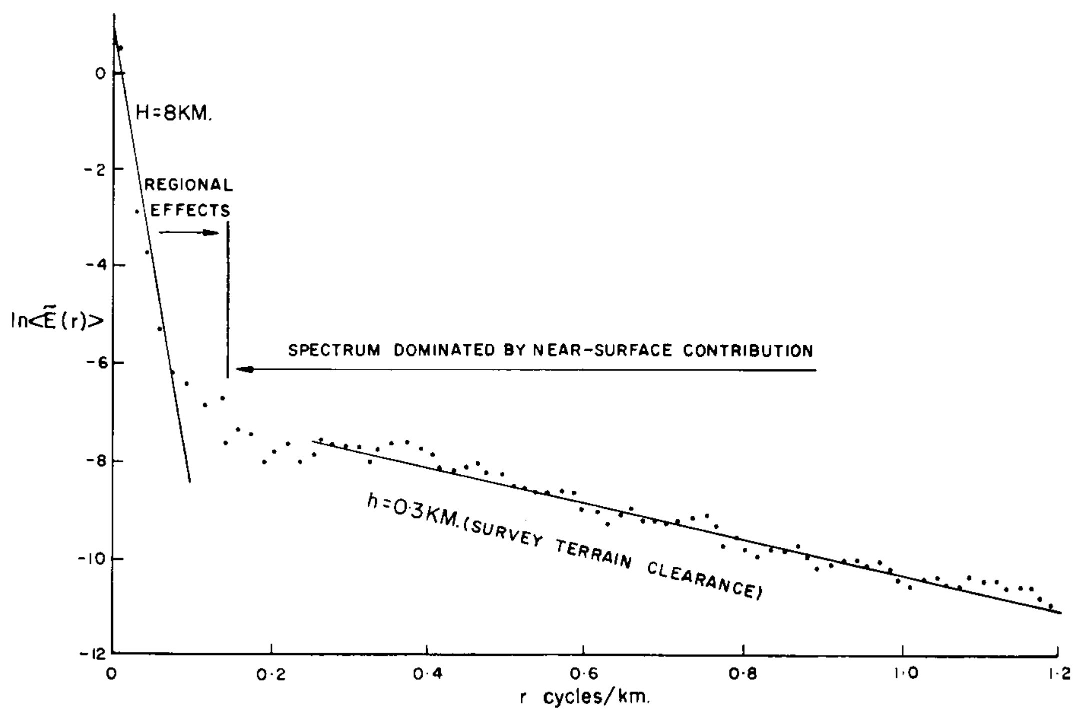

FIG. 6b. A power spectrum analysis of an aeromagnetic map of Figure 6a.

FIG. 5b. A power spectrum analysis of an aeromagnetic map of Figure 5a.

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