

## 8.5 Ice Properties and Glacier Dynamics

CJ van der Veen, University of Kansas, Lawrence, KS, USA

© 2013 Elsevier Inc. All rights reserved.

8.5.1	Deformation of Glacier Ice	53
8.5.2	Force Balance	54
8.5.3	Modeling Glacier Flow	55
8.5.3.1	Perfect Plasticity	55
8.5.3.2	Lamellar Flow	56
8.5.3.3	Basal Sliding	56
8.5.3.4	Ice Temperature	56
8.5.3.5	Lateral Drag	57
8.5.3.6	Ice-Shelf Spreading	57
8.5.3.7	Continuity	57
8.5.4	Glacier Instability	58
8.5.4.1	Subglacial Hydrology	58
8.5.4.2	Release of Back Stress	58
8.5.4.3	Marine Instability	58
8.5.5	Concluding Remarks	59
References		59

### Glossary

**Basal plane** The plane perpendicular to the principal axis (*c*-axis) in ice crystals.

**Deviatoric stress** Full stress minus hydrostatic pressure.

**Equilibrium line** Altitude on a glacier at which annual snow accumulation equals annual melting.

**Grounding line** Transition zone from grounded to floating ice.

**Ice shelf** Floating peripheral extension of ice sheets nourished by drainage from the grounded parts, and basal accretion and surface accumulation.

**Laminar flow** Flow regime characterized by a balance between driving stress and drag at the glacier base, with

horizontal ice layers 'sliding' over one another similar to a deck of cards.

**Lithostatic stress** Stress at any depth induced by the weight of the overlying ice.

**Perfect plasticity** Property of ice to undergo irreversible deformation once a critical yield stress is reached.

**Resistive stress** Stress associated with restraint or resistance to glacier flow.

**Shape factor** Numerical factor introduced in the balance of forces to account for the effect of friction at lateral margins of glaciers.

**Surge** Short-lived speedup event resulting in rapid transfer of mass from higher elevations to the lower reaches of a glacier.

### Abstract

Glaciers and ice sheets gain material through accumulation of snow above the equilibrium line; they lose mass through ablation and meltwater runoff at lower elevations below the equilibrium line, and through iceberg calving where the terminus is in contact with the ocean or proglacial lakes. Additionally, mass may be gained or lost through freezing or melting at the glacier base; but for grounded ice masses, this component of the mass balance is generally small. The objective of modeling glaciers is to develop quantitative models to describe the transfer of mass from the accumulation zone to the ablation region.

van der Veen, C.J., 2013. Ice properties and glacier dynamics. In: Shroder, J. (Editor in Chief), Giardino, R., Harbor, J. (Eds.), *Treatise on Geomorphology*. Academic Press, San Diego, CA, vol. 8, *Glacial and Periglacial Geomorphology*, pp. 53–60.

### 8.5.1 Deformation of Glacier Ice

Glacier flow results from the deformation of individual crystals when subjected to stress. In an ice crystal, the molecules are arranged approximately in planes parallel to the basal planes. Crystal deformation involves the movement of dislocations or irregularities and results in these layers gliding over each other similar to the cards in a deck. The amount of crystal deformation depends on the component of shear stress parallel to its basal plane; crystals that are oriented optimally with respect to an imposed stress deform more rapidly than crystals for which the shear stress resolved on the basal plane is small. The common assumption is that ice is isotropic with the orientations of  $c$ -axes randomly distributed. In that case, an aggregate of many crystals will deform at a slower rate than a single crystal because many of the crystals are not oriented optimally for basal glide (Weertman, 1973; Schulson and Duval, 2009; Duval et al., 2010).

To model the glacier flow, it is not necessary to consider deformation of individual crystals. Rather, concepts of continuum mechanics can be applied. The basic premise is that ice is continuous, and that the response to applied stress can be described by a single constitutive relation, thus avoiding the need to model crystals individually. For glacier ice, the response to stress is viscoelastic, that is, the rate of deformation increases nonlinearly with applied stress. Provided the aggregate ice remains isotropic, with crystals oriented randomly, the commonly used flow law is that proposed by Glen (1955), which applies to situations in which the ice has undergone sufficient deformation to pass the initial stage of primary creep and reach the stage of secondary creep during which the rate of deformation remains constant.

Glen's flow law relates the rate of deformation, or strain rate, to applied stress. Because ice deformation is independent of hydrostatic pressure, deviatoric stresses are used, defined as the full stress minus the hydrostatic pressure. That is,

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} [\sigma_{xx} + \sigma_{yy} + \sigma_{zz}] \quad [1]$$

where the prime denotes the stress deviator and unprimed stresses are full stresses. Nye's generalization of Glen's flow law is then

$$\dot{\epsilon}_{ij} = A \tau_e^{n-1} \sigma'_{ij} \quad [2]$$

For stresses commonly occurring on glaciers ( $> 100$  kPa), theoretical arguments as well as laboratory and field studies suggest a value of  $n=3$  for the exponent (Budd and Jacka, 1989; Alley, 1992); evidence exists for a decrease in the exponent at low stresses (Schulson and Duval, 2009). The effective stress,  $\tau_e$ , is the second invariant of the deviatoric stress tensor,

$$2\tau_e^2 = \sigma'^2_{xx} + \sigma'^2_{yy} + \sigma'^2_{zz} + 2(\sigma'^2_{xy} + \sigma'^2_{xz} + \sigma'^2_{yz}) \quad [3]$$

The rate factor,  $A$ , is strongly temperature dependent (Hooke, 1981), reflecting that the rate of deformation for a given stress depends on the temperature of the ice. The warmer the ice, the easier deformation becomes. Consequently,

thermodynamics need to be included when modeling glacier flow and, in particular, the depth variation in ice temperature. Generally, deeper ice layers are warmer than layers closer to the surface and, combined with greatest shear stresses near the glacier base (except on floating glaciers and ice streams where basal drag is zero or very small), most vertical shear occurs in the lower ice layers.

Generally, the assumption is made that ice crystals are oriented randomly. However, one of the consequences of crystals deformation by basal glide is that the orientation of the basal plane changes as deformation progresses, such that the  $c$ -axis rotates toward the compressional axis and away from the tensile axis. Thus, if an aggregate is subjected to one particular stress regime for a sufficiently long time, all crystals become oriented such that their basal planes are near perpendicular to the principal compressive stress. In this geometry, no further crystal deformation occurs because the shear stress parallel to the basal plane vanishes. In reality, other processes become important, including displacement between crystals, crystal growth, boundary migration, and recrystallization, which result in more favorably oriented crystals, thereby permitting continued deformation (Van der Veen and Whillans, 1994; Duval et al., 2010). In some instances, the ice develops a pronounced fabric, with most  $c$ -axes aligned in one preferred direction, making the ice 'soft' with respect to some stress and 'hard' with respect to other stresses. For example, the basal ice layers of the Greenland ice sheet are characterized by  $c$ -axes clustering around the vertical direction, leading to greater rates of vertical shear than that would be found in isotropic ice (Shoji and Langway, 1988). The correct procedure for including this effect is to apply an anisotropic flow law. However, in practice, a more expedient procedure is to prescribe an enhancement factor to achieve greater shearing rates.

### 8.5.2 Force Balance

Glacier flow is sufficiently slow that accelerations may be neglected and Newton's second law reduces to balance of forces. The action force is the gravitational 'driving stress' responsible for making the ice flow in the direction of the surface slope (generally, there are small-scale exceptions, where the direction of flow does not coincide with the downslope direction). This action is opposed by resistive forces acting at the boundaries of an ice mass. These boundaries include the glacier bed ('basal drag'), the lateral margins where fast-moving ice is bounded by a rock or a slow-moving ice ('lateral drag'), and the up- and down-glacial ends ('gradients in longitudinal stress').

The balance of forces acting on a section of glacier has been discussed many times before, most recently in Hooke (1998: 191–195), Hughes (1998: 51–54), and Van der Veen (1999: 32–36). In most cases, the balance of forces is discussed in terms of stress deviators. Although deviatoric stresses are called for in the flow law for glacier ice, their use in discussing balance of forces unnecessarily complicates the interpretation because the longitudinal deviatoric stress in one direction depends on the full normal stresses in all three directions of a cartesian coordinate system, as can be seen from eqn [1]. It is more convenient to consider the stresses in a glacier as the

sum of the stress due to the weight of the ice above ('lithostatic stress') and special stresses due to the flow ('resistive stresses'). This partitioning makes a clear distinction between action and reaction in glacier mechanics (Van der Veen and Whillans, 1989).

Lithostatic stresses are associated with the gravitational force acting on the ice and horizontal gradients in this gravitational force give rise to the driving stress, the 'action' that makes the glacier flow. This lithostatic stress is a normal stress that increases linearly with depth (assuming constant ice density,  $\rho$ ):

$$L(z) = -\rho g(h - z) \quad [4]$$

in which  $h$  represents the elevation of the ice surface, and  $z$  the vertical direction (positive upward). By considering lithostatic stresses acting on a glacier column, it can be shown that the driving stress is given by (e.g., Van der Veen and Payne, 2003)

$$\tau_{dx} = -\rho g H \frac{\partial h}{\partial x} \quad [5]$$

where  $H$  represents thickness of the ice column and  $x$  the direction along flow. A similar expression can be derived for the driving stress corresponding to the other horizontal direction.

Glacier flow is driven by gravity and restrained by interaction against the bed, or the sides, or by compression or tension transmitted along the glacier. The effect of gravity is described by the driving stress (eqn [5]), whereas flow restraint is described by resistive stresses. Because the weight-induced lithostatic stress is accounted for through the driving stress, resistance to flow must originate from the remaining stresses, that is, from the differences between the full stresses and the lithostatic stress. These stress differences are referred to as resistive stresses, defined as

$$R_{ij} = \sigma_{ij} - \delta_{ij} L \quad [6]$$

in which  $\delta_{ij}$  represents the Kronecker delta ( $\delta_{ij} = 1$  if  $i=j$ , and zero otherwise).

Equations expressing balance of forces acting on an ice column can be derived by substituting the partitioning of full stress into lithostatic and resistive components into the momentum equations and integrating over the ice thickness (Van der Veen, 1999) or by using a more intuitive geometrical approach (Van der Veen and Payne, 2003). Both approaches yield the same balance equation, namely

$$\tau_{dx} = \tau_{bx} - \frac{\partial}{\partial x} (H \bar{R}_{xx}) - \frac{\partial}{\partial y} (H \bar{R}_{xy}) \quad [7]$$

In this equation, basal drag is defined to include all resistance at the glacier bed ( $z=b$ ),

$$\tau_{bx} = R_{xz}(b) - R_{xx}(b) \frac{\partial b}{\partial x} - R_{xy}(b) \frac{\partial b}{\partial y} \quad [8]$$

where the first term on the right-hand side represents skin friction, and the second and third terms form drag associated with flow over an irregular bed. A similar balance equation applies to the second horizontal direction.

According to the balance eqn [7], the gravitational action (left-hand side) is balanced by drag at the glacier bed (first term on the right-hand side), gradients in longitudinal stress (second term), and lateral drag (third term). Note that the gradients in longitudinal stress can resist flow or can act in concert with the driving stress to push or pull the ice forward.

For most modeling applications, force balance in the vertical direction need not be considered explicitly. Where the weight of the ice is supported by the bed, the vertical resistive stress,  $R_{zz}$ , is zero and the full vertical stress at any depth equals the lithostatic stress. On small horizontal scales, separation of the ice from the bed may occur and the weight of the ice may not be supported from below. In that case, local shear stress gradients will transfer the weight to the surrounding areas, where the ice is in contact with the bed, similar to a bridge whose span is not supported from below and the abutments carry the full weight of the bridge (Van der Veen and Whillans, 1989; Budd, 1970a, b; Kamb and Echelmeyer, 1986).

## 8.5.3 Modeling Glacier Flow

### 8.5.3.1 Perfect Plasticity

Models describing glacier flow can be ranked in order of increasing complexity as more physical processes are included. Perhaps the simplest model is the perfectly plasticity model. This model, which strictly speaking does not model glacier flow but which can be used to reconstruct the shape of glaciers and ice sheets, is based on the observation that deformation of glacier ice increases nonlinearly with applied stress. Thus, the flow law [2] can be approximated by the perfectly plastic criterion (Orowan, 1949; Nye, 1951; Reeh, 1982).

A material exhibits perfect plasticity if deformation is negligible when the applied stress is below a critical value (the yield stress), whereas for stresses exceeding this value the material deforms 'instantly' to relieve the applied stress. Consequently, the stress in the material never exceeds the yield stress. When applied to the glaciers, this means that basal drag equals the (constant) yield stress,  $\tau_0$ . Equating basal drag to the driving stress gives

$$-\rho g H \frac{\partial h}{\partial x} = \tau_0 \quad [9]$$

For a glacier resting on a horizontal bed, this expression can be integrated to yield the parabolic profile

$$H^2 = \frac{2\tau_0}{\rho g} (L - x) \quad [10]$$

with  $x=0$  at the ice divide and  $L$  the half-width of the glacier. For more realistic bed geometries, the profile can be reconstructed by integrating eqn [9] numerically, starting at the glacier margin (cf. Van der Veen, 1999: Section 6.2).

The most useful application of the perfectly plastic approximation is the reconstruction of former ice sheets based on geological observations of glacial landforms that indicate the maximum extent of the former ice sheet (e.g., Denton and Hughes, 1981; Hughes, 1985). However, it is not a dynamic model that considers glacier flow. To arrive at more realistic

dynamical models, the balance eqn [7] needs to be considered.

### 8.5.3.2 Lamellar Flow

It is possible to construct numerical ice-flow models that do not incorporate simplifying assumption and that solve the thermomechanical balance equations (e.g., Johnson and Staiger, 2007; Gagliardini and Zwinger, 2008; Durand et al., 2009). However, it is not computationally feasible to use such models to simulate the evolution of entire ice sheets over longer times (such as glacial cycles) and the common practice is to simplify the model by introducing certain assumptions that reduce the numerical complexity of the model. These assumptions are based on identification of the dynamical controls on glacier flow; that is, different resistive terms in the balance eqn [7] are retained, depending on the glacier being modeled.

The basic model is based on the assumption that the driving stress is balanced by drag at the glacier base. In that case, the only nonzero resistive stress is associated with vertical shear,  $R_{xz}$ , and this stress increases linearly with depth from zero at the surface to the maximum value at the glacier bed. The only nonzero strain rate is the vertical shear strain rate,  $\dot{\epsilon}_{xz}$ , related to the shear stress through the flow law [2]. Taking the rate factor,  $A$ , constant with depth, the vertical profile of the horizontal velocity is

$$u(z) = U_s \left[ 1 - \left( \frac{h-z}{H} \right)^{n+1} \right] \quad [11]$$

with the surface velocity given by

$$U_s = \frac{2AH}{n+1} \tau_{dx}^n \quad [12]$$

If two-dimensional flow is considered, a similar expression applies to the velocity component in the other horizontal direction.

### 8.5.3.3 Basal Sliding

The velocity profile [11] corresponds to internal deformation of the ice. This is typically a relatively slow process, with surface velocities ranging from a few meters per year to perhaps a few hundred meters per year. Where the basal ice is at the pressure-melting temperature and a lubricating water layer is present, basal sliding may result in velocities that can reach several kilometers per year. The form of the sliding relation has not been unambiguously established (cf. Fowler, 2010), but the most common form adopted in numerical models is the so-called Weertman sliding relation corrected for the effect of subglacial water pressure (e.g., Bindshadler, 1983). That is

$$U_b = A_s \frac{\tau_b^3}{P_e} \quad [13]$$

with the effective basal pressure,  $P_e$ , equal to the ice overburden pressure minus the subglacial water pressure:

$$P_e = \rho g H - P_w \quad [14]$$

To evaluate the water pressure at the glacier base, a hydrological model is needed, requiring some assumption about the nature of the subglacial drainage system. The recent study by Schoof (2010) shows that switches between drainage systems may result in acceleration or deceleration of a glacier.

### 8.5.3.4 Ice Temperature

The rate factor in the flow law [2] depends on the temperature of the ice, necessitating the need to add an equation for the temperature evolution at depth. This considerably adds to the complexity of any numerical model because of the need to explicitly calculate temperature at depth. It falls outside the scope of this review to discuss the numerical solution scheme (cf. Pattyn, 2003; Rutt et al., 2009), but some insight into the characteristics of the temperature profile can be obtained by considering the solution derived by Robin (1955).

To arrive at an analytical solution for the temperature profile, the assumption is made that horizontal diffusion and advection of heat is much smaller than vertical diffusion and advection so that the temperature depends on depth in the ice only. Furthermore, because strain heating is concentrated in the basal ice layers, heat generated by internal deformation can be taken into account by increasing the geothermal heat flux at the glacier base. Under these assumptions, the temperature equation becomes

$$\frac{\partial T}{\partial z} = K \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z} \quad [15]$$

where  $K$  represents the thermal diffusivity and  $w(z)$  the vertical velocity. Assuming that the ice sheet is in steady state, the vertical velocity can be approximated by

$$w(z) = -\frac{Mz}{H} \quad [16]$$

noting that  $z=0$  is at the glacier base, and  $M$  represents the surface mass balance.

To solve the temperature eqn [15], two boundary conditions are needed. At the surface, the temperature is prescribed to be equal to  $T_s$ , whereas at the base, the vertical temperature gradient must balance the geothermal heat flux,  $G$  (including heat generated by internal deformation):

$$\left( \frac{\partial T}{\partial z} \right)_b = -\frac{G}{k} \quad [17]$$

with  $k$  as the thermal conductivity. For positive values of  $M$ , the temperature profile is

$$T(z) = T_s - \frac{G\sqrt{\pi}}{2kq} [\operatorname{erf}(zq) - \operatorname{erf}(Hq)] \quad [18]$$

with

$$q^2 = \frac{M}{2KH} \quad [19]$$

and the error function is defined as

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-\bar{z}^2) d\bar{z} \quad [20]$$

This solution can be modified to include basal melting, the effect of horizontal advection, and strain heating at depth, but doing so does not significantly alter the characteristics of the solution. One of the important conclusions that can be drawn from the Robin temperature solution is that increasing the surface mass balance lowers the temperature of the basal ice.

### 8.5.3.5 Lateral Drag

The lamellar flow solution is based on the assumption that basal drag is the main resistance to flow. This is valid for interior regions of ice sheets but on valley glaciers, or fast-moving outlet glaciers and ice streams embedded within more slowly moving ice, lateral drag may provide additional flow resistance and basal drag must be less than the driving stress. To account for this effect, Nye (1965) introduced the shape factor,  $f$ , representing the fraction of driving stress that is supported by drag at the glacier base, and

$$\tau_b = f \tau_d \quad [21]$$

The shape factor can be estimated as that needed to produce the correct surface velocity at the glacier centerline if lamellar flow is assumed, or it can be calculated from force balance across the entire width of the glacier. Neither procedure yields a simple relation between the shape factor and ice velocity that can be readily implemented into a numerical model.

Integrated over the width of the glacier or ice stream, force balance is (e.g., Van der Veen, 1999: Section 5.5)

$$\bar{\tau}_d = \bar{\tau}_b + \frac{H_w \tau_s}{W} \quad [22]$$

where the overbar denotes width-averaged values.  $W$  represents the half-width of the glacier and  $H_w$  the thickness at the lateral margins. The shear stress at the margins,  $\tau_s$ , can be linked to the width-averaged velocity as (Van der Veen and Whillans, 1996)

$$\tau_s = B \left( \frac{5 \bar{U}}{2 W} \right)^{1/3} \quad [23]$$

where  $n=3$  in the flow law and the viscosity parameter,  $B$ , is related to the flow law parameter,  $A$ , as  $B=A^{-1/3}$ .

Finding an analytical solution for the velocity (comparable with the laminar flow solution) requires substituting eqn [23] into the balance eqn [22] and adopting some sliding relation to relate basal drag to glacier speed. For example, Van der Veen and Whillans (1996) adopt the sliding relation [13] and the width-averaged velocity can be estimated from

$$\bar{U}^{1/3} = \frac{\tau_d}{A_s P_e + \frac{BH}{W^{1/3}}} \quad [24]$$

### 8.5.3.6 Ice-Shelf Spreading

On floating ice shelves, basal drag is zero and the driving stress is balanced by lateral drag and gradients in longitudinal stress, and the balance equation integrated over the width of the ice shelf is

$$\tau_d = -\frac{\partial}{\partial x}(HR_{xx}) + \frac{H\tau_s}{W} \quad [25]$$

Again, lateral drag can be estimated from eqn [23], whereas the stretching stress  $R_{xx}$  can be related to the along flow gradient in ice velocity, to yield the following expression

$$\tau_{dx} = -\frac{\partial}{\partial x} \left[ 2B \left( \frac{\partial \bar{U}}{\partial x} \right)^{1/n} \right] + \frac{BH}{W} \left( \frac{5 \bar{U}}{2 W} \right)^{1/n} \quad [26]$$

This equation has to be solved iteratively to find the velocity along the ice shelf.

If the lateral drag is small, the second term on the right-hand side of eqn [26] may be neglected and, after invoking hydrostatic equilibrium to link surface elevation to ice thickness, the stretching rate is found to be (Weertman, 1957)

$$\frac{\partial U}{\partial x} = \left[ \frac{\rho g}{4B} \left( 1 - \frac{\rho}{\rho_w} \right) \right]^n H^n \quad [27]$$

This expression can be modified to allow for spreading in the other horizontal direction, but this only affects the constant factor and the stretching rate remains proportional to the  $n$ th power of ice thickness (Thomas, 1973; Van der Veen, 1999: Section 5.6).

Few, if any, ice shelves are floating freely and part of the driving stress is balanced by lateral drag. The effect of this is to reduce the stretching rate and

$$\frac{\partial U}{\partial x} = \left[ \frac{\rho g}{4B} \left( 1 - \frac{\rho}{\rho_w} \right) H - \frac{\sigma_b}{2B} \right]^n \quad [28]$$

with the back stress,  $\sigma_b$ , defined as the resistance to flow from lateral drag integrated from the calving front ( $x=L$ ) to some position  $x$  upglacier:

$$\sigma_b = \frac{1}{H} \int_x^L \frac{H\tau_s}{W} d\bar{x} \quad [29]$$

As a result of the back pressure, the stretching rate at the grounding line is reduced. Any lowering in back stress due to shortening or thinning of the ice shelf may lead to increased creep thinning at the grounding line, which could lead to irreversible collapse of the grounded glacier.

### 8.5.3.7 Continuity

The models discussed above allow the ice velocity to be estimated from glacier geometry. When modeling temporal changes in glacier geometry, the prognostic continuity equation needs to be considered. In essence, this equation states that no ice may be created or lost and thickness changes at any particular location must be entirely due to ice flow and local snowfall or melting. Because the density of ice may be



considered constant (ignoring firn compaction in the upper layers), conservation of mass corresponds to conservation of volume and the continuity equation becomes (e.g., Van der Veen, 1999: Section 6.3)

$$\frac{\partial H}{\partial t} = -\frac{\partial(HU)}{\partial x} + M \quad [30]$$

where  $M$  represents the surface mass balance expressed in meters of ice per unit time. To account for flow in the second horizontal direction, a term describing divergence of flow in that direction can be added to the right-hand side.

## 8.5.4 Glacier Instability

With increased observational capabilities from various remote sensing platforms at high spatial and temporal resolution, the conventional view that glaciers and ice sheets mostly respond sluggishly to environmental forcings has been challenged. It is now evident that ice streams and outlet glaciers can undergo dramatic changes, such as rapid speedup or significant thinning.

### 8.5.4.1 Subglacial Hydrology

A glacier surge is a brief period of fast motion following a prolonged period of quiescent flow. Surges occur periodically suggesting that the longitudinal profile of glaciers subject to surging is inherently unstable. It has long been recognized that surges involve a switch in subglacial hydrology, allowing a rapid transition from slow-to-fast flow. Fowler (1987) proposed a model based on transition for channelized drainage to a linked-cavity system. More recently, Schoof (2010) showed how such transitions may affect the seasonal velocity in Greenland.

Subglacial drainage can be through an arborescent system of channels or tunnels (Röthlisberger, 1972) or through a linked-cavity system (Kamb, 1987). The main difference between the two systems is that for channelized flow, the water pressure decreases as the water flux increases, whereas in a linked-cavity system, the water pressure increases with increasing water flux. Thus, a switch from tunnel drainage to linked cavities results in increased water pressure at the bed and, consequently, decreased effective pressure. This decrease in effective pressure allows the sliding speed to increase (eqn [13]). This transition from one drainage system to the other may result from changes in water supply to the glacier bed or from changes in the glacier speed. If the sliding speed is too small, dissipational heat generation becomes too small to maintain a linked-cavity system and the drainage system will collapse to tunnels. However, if the sliding speed becomes too large, a tunnel system will become unstable and a network of cavities connected by small channels will develop. This coupling between speed and stability of drainage systems allows a surge cycle to be described (Fowler, 1987).

During the quiescent phase, ice velocities are relatively low and drainage is through a tunnel system. As the glacier builds up and the driving stress increases, the velocity also increases until the threshold is reached, at which the tunnels become

unstable and the drainage system transitions to linked cavities. This transition results in a lower effective pressure, and the sliding velocity increases rapidly and the glacier enters the surge phase. During the surge, mass is transferred from upglacier to the terminus region and the glacier profile becomes flatter. The decrease in surface slope lowers the driving stress and thus the ice velocity decreases until sliding becomes too slow to maintain a linked-cavity system. A tunnel system then reforms and the glacier enters the slow-flow regime again.

Schoof (2010) used a model that captures the dynamic transition between tunnel and cavity drainage to show that when sufficient meltwater is provided to the glacier bed, a more efficient drainage system consisting of tunnels will develop, resulting in deceleration as has been observed recently on several glaciers in West Greenland (Sundal et al., 2011).

### 8.5.4.2 Release of Back Stress

Several studies have suggested that speedup of Greenland outlet glaciers and glaciers in the Antarctic Peninsula may have been caused by the weakening and breakup of peripheral floating ice shelves and ice tongues and associated release of back stress (e.g., Scambos et al., 2004; Howat et al., 2008; Joughin et al., 2008). This back stress, defined by eqn [29], represents resistance to flow offered by fjord walls or basal pinning points; and once this resistance vanishes, the stretching rate at the grounding line increases.

Acting by itself, it is doubtful that a lowering of back stress at the grounding line will lead to glacier instability. A lowering of back stress at the grounding line would result in an increase in resistive forces of 4–10 kPa immediately upstream of the grounding line (depending on the value of the coupling length). For most outlet glaciers, such an increase is small compared with the driving stress (typically a few hundred kilopascal) and cannot explain observed changes. In the absence of feedback processes that act to reinforce the glacier response, glacier response to the release of back stress should be similar to a kinematic wave perturbation with the perturbation amplitude rapidly decreasing with time and distance from the grounding line (Thomas, 2004; Van der Veen et al., 2011).

### 8.5.4.3 Marine Instability

Marine ice sheets are grounded on a bed that is well below sea level. Many studies have argued that the grounding line tends to be unstable if the depth of the bed below sea level is greater than some critical value and the sea floor slopes downward toward the ice-sheet interior (e.g., Weertman, 1974; Thomas, 1979). This bed geometry may provide a positive feedback that could initiate irreversible grounding-line retreat following the collapse of a peripheral ice shelf.

The basic premise of the marine instability hypothesis is that for a floating ice shelf the stretching rate is proportional to the  $n$ th power of ice thickness (eqn [27]). Breakup of the ice shelf and release of back pressure will cause an increase in stretching rate at the grounding line, leading to thinning and grounding-line retreat. Where the bed slopes downward toward the interior, grounding-line retreat actually increases the thickness at the grounding line and consequently, creep

thinning increases further. Thus, a positive feedback exists that amplifies the initial perturbation.

The model for studying grounding-line migration was developed by Thomas (1977) and applied to the Holocene retreat of the West Antarctic Ice Sheet by Thomas and Bentley (1978). A difficulty with these models is to properly account for changes in advection of ice from upglacier, which tends to stabilize the grounding line. Indeed, more recent modeling studies suggest that a downward sloping bed does not necessarily represent an unstable geometry (e.g., Nick et al., 2010).

### 8.5.5 Concluding Remarks

The brief overview of glacier flow presented in this section focuses on glacier dynamics. This is the first step toward glacier modeling. However, in many instances, a dynamic coupling between ice flow and the subglacial environment is needed. For example, fast flow on Whillans Ice Stream in West Antarctica is facilitated by a weak layer of water-saturated sediment maintained by melting at the base of the ice (Tulaczyk et al., 2000) and till deposition at the grounding line of this ice stream serves to stabilize the position of the grounding line (Anandakrishnan et al., 2007). Similarly, advance of calving tidewater glaciers through overdeepenings in fjords is facilitated by sediment transported to the glacier terminus in a conveyor belt fashion (Nick et al., 2007).

### References

- Alley, R.B., 1992. Flow-law hypotheses for ice-sheet modeling. *Journal of Glaciology* 38, 245–256.
- Anandakrishnan, S., Catania, G.A., Alley, R.B., Horgan, H.J., 2007. Discovery of till deposition at the grounding line of Whillans Ice stream. *Science* 315, 1835–1838.
- Bindschadler, R.A., 1983. The importance of pressurized subglacial water in separation and sliding at the glacier bed. *Journal of Glaciology* 29, 3–19.
- Budd, W.F., 1970a. The longitudinal stress and strain-rate gradients in ice masses. *Journal of Glaciology* 9, 19–27.
- Budd, W.F., 1970b. Ice flow over bedrock perturbations. *Journal of Glaciology* 9, 29–48.
- Budd, W.F., Jacka, T.H., 1989. A review of ice rheology for ice sheet modeling. *Cold Regions Science and Technology* 16, 107–144.
- Denton, G.H., Hughes, T.J. (Eds.), 1981. *The Last Great Ice Sheets*. John Wiley, New York, NY.
- Durand, G., Gagliardini, O., de Fleurian, B., Zwinger, T., Le Meur, E., 2009. Marine ice sheet dynamics: hysteresis and neutral equilibrium. *Journal of Geophysical Research* 114, F03009. <http://dx.doi.org/10.1029/2008JF001170>.
- Duval, P., Montagnat, M., Grennerat, F., Weiss, J., Meyssonier, J., Philip, A., 2010. Creep and plasticity of glacier ice: a material science perspective. *Journal of Glaciology* 56, 1059–1068.
- Fowler, A.F., 1987. A theory of glacier surges. *Journal of Geophysical Research* 92, 9111–9120.
- Fowler, A.C., 2010. Weertman, Llibouty and the development of sliding theory. *Journal of Glaciology* 56, 965–972.
- Gagliardini, O., Zwinger, T., 2008. The ISMIP-HOM benchmark experiments performed using the finite-element code Elmer. *The Cryosphere* 2, 67–76.
- Glen, J.W., 1955. The creep of polycrystalline ice. *Proceedings of the Royal Society of London Series A* 228, 519–538.
- Hooke, R.L.B., 1981. Flow law for polycrystalline ice in glaciers: comparison of theoretical predictions, laboratory data, and field measurements. *Reviews of Geophysics and Space Physics* 19, 664–672.
- Hooke, R.L.B., 1998. *Principles of Glacier Mechanics*. Prentice Hall, Upper Saddle River, NJ, 248 pp.
- Howat, I.M., Joughin, I., Fahnestock, M., Smith, B.E., Scambos, T., 2008. Synchronous retreat and acceleration of southeast Greenland outlet glaciers 2000–2006: ice dynamics and coupling to climate. *Journal of Glaciology* 54, 646–660.
- Hughes, T.J., 1985. The great Cenozoic ice sheet. *Palaeogeography, Palaeoclimatology, Palaeoecology* 50, 9–43.
- Hughes, T.J., 1998. *Ice Sheets*. Oxford University Press, Oxford, UK, 343 pp.
- Johnson, J.V., Staiger, K.W., 2007. Modeling long-term stability of the Ferrar Glacier, East Antarctica: implications for interpreting cosmogenic nuclide inheritance. *Journal of Geophysical Research* 112, F03S30. <http://dx.doi.org/10.1029/2006JF000599>.
- Joughin, I., Howat, I.M., Fahnestock, M., et al., 2008. Continued evolution of Jakobshavn Isbræ following its rapid speedup. *Journal of Geophysical Research* 113, F04006. <http://dx.doi.org/10.1029/2008JF001023>.
- Kamb, W., 1987. Glacier surge mechanism based on linked cavity configuration of the basal water conduit system. *Journal of Geophysical Research* 92, 9083–9100.
- Kamb, W., Echelmeyer, K., 1986. Stress-gradient coupling in glacier flow: IV. Effects of the 'T' term. *Journal of Glaciology* 32, 342–349.
- Nick, F.M., van der Veen, C.J., Oerlemans, J., 2007. Controls on advance of tidewater glaciers: results from numerical modeling applied to Columbia Glacier. *Journal of Geophysical Research* 112, F03S24. <http://dx.doi.org/10.1029/2006JF000551>.
- Nick, F.M., van der Veen, C.J., Vieli, A., Benn, D.I., 2010. A physically based calving model applied to marine outlet glaciers and implications for the glacier dynamics. *Journal of Glaciology* 56, 781–794.
- Nye, J.F., 1951. The flow of glaciers and ice-sheets as a problem in plasticity. *Proceedings of the Royal Society of London Series A* 207, 554–572.
- Nye, J.F., 1965. The flow of a glacier in a channel of rectangular, elliptic or parabolic cross-section. *Journal of Glaciology* 5, 661–690.
- Orowan, E., 1949. Remarks at joint meeting of the British Glaciological Society, the British Rheologists Club and the Institute of Metals. *Journal of Glaciology* 1, 231–236.
- Pattyn, F., 2003. A new three-dimensional higher-order thermomechanical ice sheet model: basic sensitivity, ice stream development, and ice flow across subglacial lakes. *Journal of Geophysical Research* 108, 2382. <http://dx.doi.org/10.1029/2002JB002329>.
- Reeh, N., 1982. A plasticity approach to the steady-state shape of a three-dimensional ice sheet. *Journal of Glaciology* 28, 431–455.
- Robin, G. deQ., 1955. Ice movement and temperature distribution in glaciers and ice sheets. *Journal of Glaciology* 2, 523–532.
- Röthlisberger, H., 1972. Water pressure in intra- and subglacial channels. *Journal of Glaciology* 11, 177–204.
- Rutt, I.C., Hagdorn, M., Hulton, N.R., Payne, A.J., 2009. The Glimmer community ice sheet model. *Journal of Geophysical Research* 114, F02004. <http://dx.doi.org/10.1029/2008JF001015>.
- Scambos, T.A., Bohlander, J.A., Shuman, C.A., Skvarca, P., 2004. Glacier acceleration and thinning after ice shelf collapse in the Larsen B embayment, Antarctica. *Geophysical Research Letters* 31, L18402. <http://dx.doi.org/10.1029/2004GL020670>.
- Schoof, C., 2010. Ice-sheet acceleration driven by melt supply variability. *Nature* 468, 803–806.
- Schulson, E.M., Duval, P., 2009. *Creep and Fracture of Ice*. Cambridge University Press, Cambridge, UK, 401 pp.
- Shoji, H., Langway, C.C., 1988. Flow-law parameters of the Dye 3, Greenland, deep ice core. *Annals of Glaciology* 10, 146–150.
- Sundal, A.V., Shepherd, A., Nienow, P., Hanna, E., Palmer, S., Huybrechts, P., 2011. Melt-induced speed-up of Greenland ice sheet offset by efficient subglacial drainage. *Nature* 469, 521–524.
- Thomas, R.H., 1973. The creep of ice shelves: theory. *Journal of Glaciology* 12, 45–53.
- Thomas, R.H., 1977. Calving bay dynamics and ice-sheet retreat up the St. Lawrence valley system. *Geographie et Physique Quaternaire* 31, 347–356.
- Thomas, R.H., 1979. The dynamics of marine ice sheets. *Journal of Glaciology* 24, 167–177.
- Thomas, R.H., 2004. Force-perturbation analysis of recent thinning and acceleration of Jakobshavn Isbræ, Greenland. *Journal of Glaciology* 24, 57–66.
- Thomas, R.H., Bentley, C.R., 1978. A model for the holocene retreat of the West Antarctic ice sheet. *Quaternary Research* 10, 150–170.
- Tulaczyk, S., Kamb, B., Engelhardt, H.F., 2000. Basal mechanics of ice stream B, West Antarctica. I. Till mechanics. *Journal of Geophysical Research* 105, 463–481.

- Van der Veen, C.J., 1999. *Fundamentals of Glacier Dynamics*. A.A. Balkema, Rotterdam, The Netherlands, 462 pp.
- Van der Veen, C.J., Payne, A.J., 2003. Modelling land-ice dynamics. In: Bamber, J.L., Payne, A.J. (Eds.), *Mass Balance of the Cryosphere: Observations and Modelling of Contemporary and Future Changes*. Cambridge University Press, Cambridge, UK, pp. 169–225.
- Van der Veen, C.J., Plummer, J.C., Stearns, L.A., 2011. Controls on the recent speed-up of Jakobshavn Isbræ, West Greenland. *Journal of Glaciology* 57, 770–782.
- Van der Veen, C.J., Whillans, I.M., 1989. Force budget: I. Theory and numerical methods. *Journal of Glaciology* 35, 53–60.
- Van der Veen, C.J., Whillans, I.M., 1994. Development of fabric in ice. *Cold Regions Science and Technology* 22, 171–195.
- Van der Veen, C.J., Whillans, I.M., 1996. Model experiments on the evolution and stability of ice streams. *Annals of Glaciology* 23, 129–137.
- Weertman, J., 1957. Deformation of floating ice shelves. *Journal of Glaciology* 3, 38–42.
- Weertman, J., 1973. Creep of ice. In: Jones, S.J., Gold, L.W. (Eds.), *Physics and Chemistry of Ice*. Royal Society of Canada, Ottawa, Canada, pp. 320–337.
- Weertman, J., 1974. Stability of the junction of an ice sheet and an ice shelf. *Journal of Glaciology* 13, 3–11.

## Biographical Sketch



Cornelis van der Veen is a glaciologist interested in the dynamics of fast-moving glaciers and ice streams. His research focuses on using measurements of ice velocity and geometry to identify mechanical controls on glaciers and how changes in these controls affect glacier flow and stability. After completing his PhD at the University of Utrecht in the Netherlands, he spent 20 years at The Ohio State University (1986–2006). He is currently a professor in the Geography Department at the University of Kansas.