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Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms: Fundamental relations

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ABSTRACT

The paper extends to three dimensions (3-D) the two-dimensional (2-D) Hilbert transform relations between potential field components. For the 3-D case, it is shown that the Hilbert transform is composed of two parts, with one part acting on the X component and one part on the Y component. As for the previously developed 2-D case, it is shown that in 3-D the vertical and horizontal derivatives are the Hilbert transforms of each other. The 2-D Cauchy-Riemann relations between a potential function and its Hilbert transform are generalized for the 3-D case. Finally, the previously developed concept of analytic signal in 2-D can be extended to 3-D as a first step toward the development of an automatic interpretation technique for potential field data.

INTRODUCTION

Whenever one attempts an interpretation of geophysical data, two major phases can be recognized:

(1) a qualitative-semiquantitative phase at the conclusion of which, out of a multitude of anomalies and possible solutions, a few are chosen for further study, and
(2) a quantitative phase, in which the solutions for the previously selected targets are refined and a complete geologic interpretation, including a drill targeting, is achieved.

The first phase is usually the most time-consuming, and great efforts have been made toward an automated interpretation of geophysical data in general and potential field data in particular. As a result, the early 1970s have seen a surge in papers dealing with the automatic computer processing of two-dimensional (2-D) potential field data (Hartman et al., 1971; O'Brien, 1971; Naudy, 1971; Nabighian, 1972 and 1974).

All of these techniques had various degrees of success when applied to real data as long as the main basic assumption, namely, two-dimensionality of data, was satisfied. As such, these techniques became applicable only for carefully selected profiles thus greatly limiting their general usefulness.

It is evident that the only way of achieving generality is by treating the data as representing truly three-dimensional (3-D) structures. Up to now most efforts in this area have been limited to developing iterative procedures for fitting the causative bodies with a distribution of rectangular blocks (Bhattacharyya, 1980). The main drawback of these techniques lies in their strong dependence upon the initial guess, i.e., the starting solution. In other words, one must practically know the expected answer before we can start a successful iteration process.

To achieve a completely general interpretation, with as few initial assumptions as possible, one must start from the fundamental properties of the potential field data and build the interpretation scheme with as few initial assumptions as possible. For the 2-D case, one of the simplest assumptions made about the causative body was that its cross-section can be represented by a polygon. One of the techniques developed to take advantage of this assumption was based on the fact that the horizontal and vertical derivatives of a potential function are the Hilbert transforms of each other (O'Brien, 1971; Nabighian, 1972, 1974.).

In the present paper we generalize to 3-D the previously published results for 2-D regarding the relationships between horizontal and vertical derivatives. As before, such relationships represent the first step toward developing procedures for automatic interpretation of potential field data in the most general cases.

Basic definitions

(1) One-dimensional Fourier transform

$$g(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx}\,dx,$$
$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(w)e^{iwx}\,dw, \tag{1}$$

or, in symbolic form

$$g(w) = \mathscr{F}[f(x)],$$

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$$f(x) = \mathscr{F}^{-1}[g(w)]. \tag{2}$$

(2) Two-dimensional Fourier transform

$$g(p, q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(px+qy)}\, dx\, dy,$$
$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(p, q) e^{i(px+qy)}\, dp\, dq, \tag{3}$$

or, in symbolic form

$$g(p, q) = \mathscr{F}[f(x, y)],$$
$$f(x, y) = \mathscr{F}^{-1}[g(p, q)]. \tag{4}$$

(3) Hilbert transform

$$g(\xi) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{\xi - x}\, dx,$$
$$f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\xi)}{\xi - x}\, d\xi, \tag{5}$$

or, in symbolic form[1]

$$g(\xi) = \mathscr{H}[f(x)],$$
$$f(x) = \mathscr{H}^{-1}[g(\xi)]. \tag{6}$$

Using the convolution theorem, one can easily write the following relation between the Fourier transform of a function and that of its Hilbert transform,

$$\mathscr{F}[g(\xi)] = -i \operatorname{sgn}(w) \mathscr{F}[f(x)] = H\mathscr{F}[f(x)], \tag{7}$$

where

$$\operatorname{sgn}(w) = \frac{w}{|w|} = \begin{cases} +1 \text{ for } w > 0, \\ 0 \text{ for } w = 0, \\ -1 \text{ for } w < 0, \end{cases}$$

and

$$H = -i \operatorname{sgn}(w)$$

is the one-dimensional Hilbert transform operator in the frequency domain.

(4) The following relations exist between the Fourier transform of a potential function *M* and of its horizontal and vertical derivatives.

(a) 2-D case

$$\mathscr{F}\left[\frac{\partial M}{\partial x}\right] = iw\, \mathscr{F}(M),$$
$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = |w|\, \mathscr{F}(M); \tag{8}$$

(b) 3-D case

$$\mathscr{F}\left[\frac{\partial M}{\partial x}\right] = ip\, \mathscr{F}[M],$$

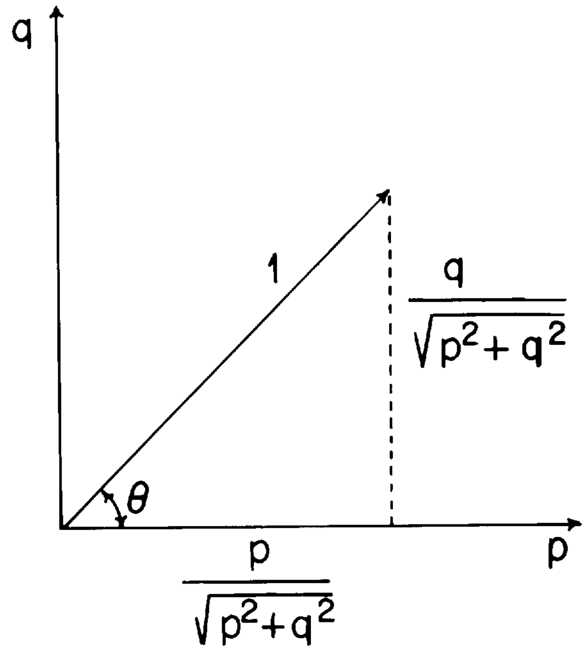

FIG. 1. Two-dimensional signum function representation.

$$\mathscr{F}\left[\frac{\partial M}{\partial y}\right] = iq\, \mathscr{F}[M], \tag{9}$$

and

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = \sqrt{p^2 + q^2}\, \mathscr{F}[M].$$

FUNDAMENTAL RELATIONS

Frequency-domain representations

The expressions that will be derived below represent generalizations to 3-D of previously published results for 2-D using Hilbert transforms and the concept of analytic signal (Nabighian, 1972)[2].

The starting point for the 2-D case was the proof that the vertical and horizontal derivatives of potential field data were the Hilbert transforms of each other. Using (N6), (N7), and (N8),

$$\frac{\partial M}{\partial z} = \mathscr{H}\left[\frac{\partial M}{\partial x}\right], \tag{10}$$

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = -i \operatorname{sgn}(w)\, \mathscr{F}\left[\frac{\partial M}{\partial x}\right], \tag{11}$$

and

[1]Space-domain transform operations (Fourier or Hilbert) are denoted with cursive capital letters.

[2]For brevity, all references to the 1972 paper will be preceded by the prefix N, i.e., (N7) will reference expression (7) from that paper. Also because of the alternate definition of Fourier transforms used in the present paper, I had to change i to $-i$ in all previous expressions.

$$\mathscr{F}\left[\frac{\partial M}{\partial x} + i\frac{\partial M}{\partial z}\right] = [1 + \operatorname{sgn}(w)]\,\mathscr{F}\left[\frac{\partial M}{\partial x}\right]. \qquad (12)$$

The generalization to 3-D can be accomplished in many ways, but with hindsight, it is easy to start from the following identity

$$\sqrt{p^2 + q^2} = ip\,\frac{-ip}{\sqrt{p^2 + q^2}} + iq\,\frac{-iq}{\sqrt{p^2 + q^2}}. \qquad (13)$$

Multiplying equation (13) by $\mathscr{F}(M)$ and using equation (9),

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = \frac{-ip}{\sqrt{p^2 + q^2}}\,\mathscr{F}\left[\frac{\partial M}{\partial x}\right] + \frac{-iq}{\sqrt{p^2 + q^2}}\,\mathscr{F}\left[\frac{\partial M}{\partial y}\right]. \qquad (14)$$

For $q = 0$ (2-D case),

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = -i\,\frac{p}{|p|}\,\mathscr{F}\left[\frac{\partial M}{\partial x}\right] = -i\operatorname{sgn}(p)\mathscr{F}\left[\frac{\partial M}{\partial x}\right],$$

which, per equation (11) above, represents the 2-D Hilbert transform relation between the vertical and horizontal derivatives of a potential function. To proceed further, I introduce a generalized signum function defined as a unit vector in the (p, q) plane (Figure 1) whose p and q components are given by $p/\sqrt{p^2 + q^2}$ and $q/\sqrt{p^2 + q^2}$, i.e.,

$$\operatorname{sgn}(p, q) = \frac{p}{\sqrt{p^2 + q^2}}\,\mathbf{e}_x + \frac{q}{\sqrt{p^2 + q^2}}\,\mathbf{e}_y,$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in the x and y directions, respectively.

The generalization of 2-D transform operator (7) can now be written for 3-D as

$$\mathbf{H} = -i\operatorname{sgn}(p, q) = H_1\mathbf{e}_x + H_2\mathbf{e}_y,$$

where

$$H_1 = -\frac{ip}{\sqrt{p^2 + q^2}},$$

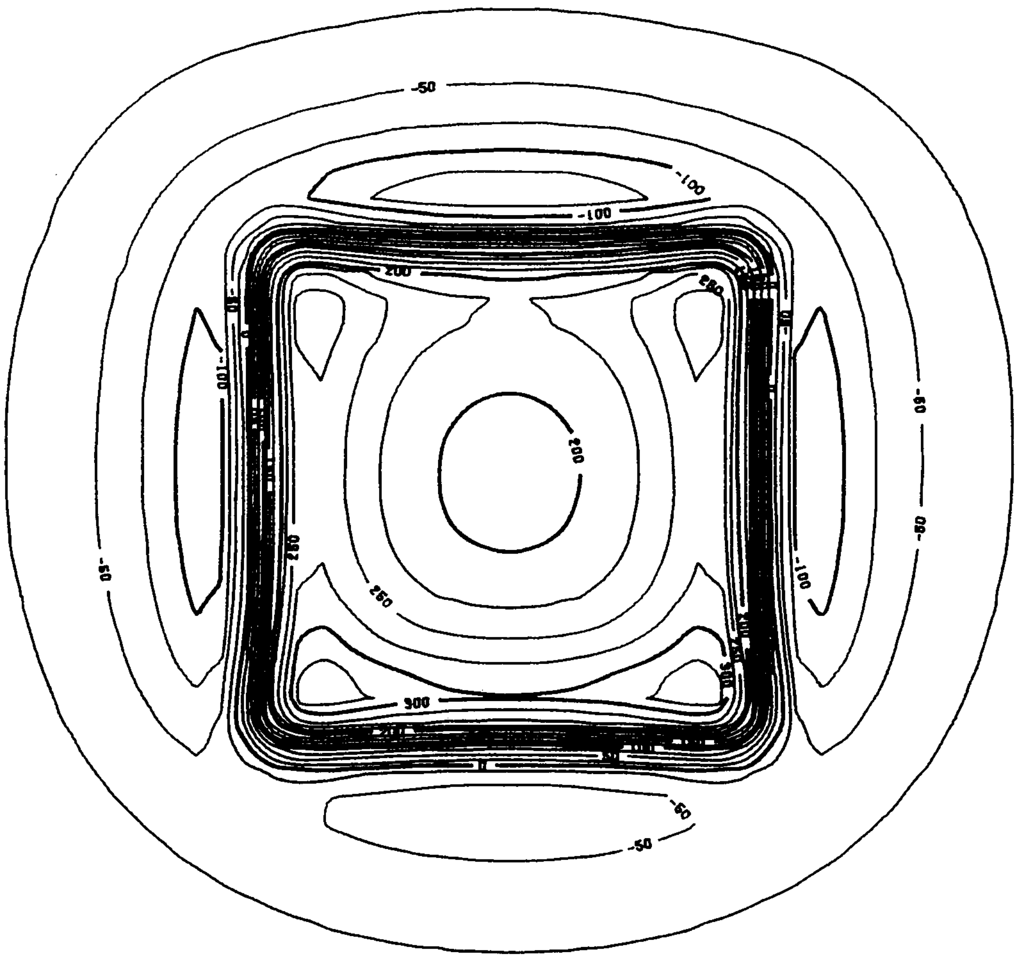

FIG. 2. Vertical derivative for a dipping prismatic body calculated by numerical differentiation. The 10 × 10 unit prism is buried at a depth of 1 unit and dips 45 degrees toward N.

and (15)

$$H_2 = -\frac{iq}{\sqrt{p^2 + q^2}}.$$

Letting

$$\nabla_h = \frac{\partial}{\partial x}\mathbf{e}_x + \frac{\partial}{\partial y}\mathbf{e}_y$$

be the horizontal gradient operator of a function, one can write equation (14) as the dot product between two vectors:

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = \mathbf{H} \cdot \mathscr{F}[\nabla_h M], \tag{16}$$

or, in longhand

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = H_1 \mathscr{F}\left[\frac{\partial M}{\partial x}\right] + H_2 \mathscr{F}\left[\frac{\partial M}{\partial y}\right]. \tag{17}$$

To the author's knowledge, the above expression represents a novel method of computing the first vertical derivative starting from the horizontal derivatives.

It is worth mentioning that the computations outlined in equation (17) are very stable since both H_1 and H_2 are well-behaved operators with values not exceeding ± 1. This becomes especially useful if the horizontal gradients are measured directly, as is currently done in some airborne systems.

Figure 2 shows the analytically calculated first vertical derivative over a prismatic body. The same derivative calculated by using expression (17) is presented in Figure 3, and, as expected, the agreement is very good despite the fact that the computations were carried out with few spatial points and the horizontal derivatives were computed by numerical differentiation.

By inspection and making use of relations (9), one can invert expression (17) to obtain

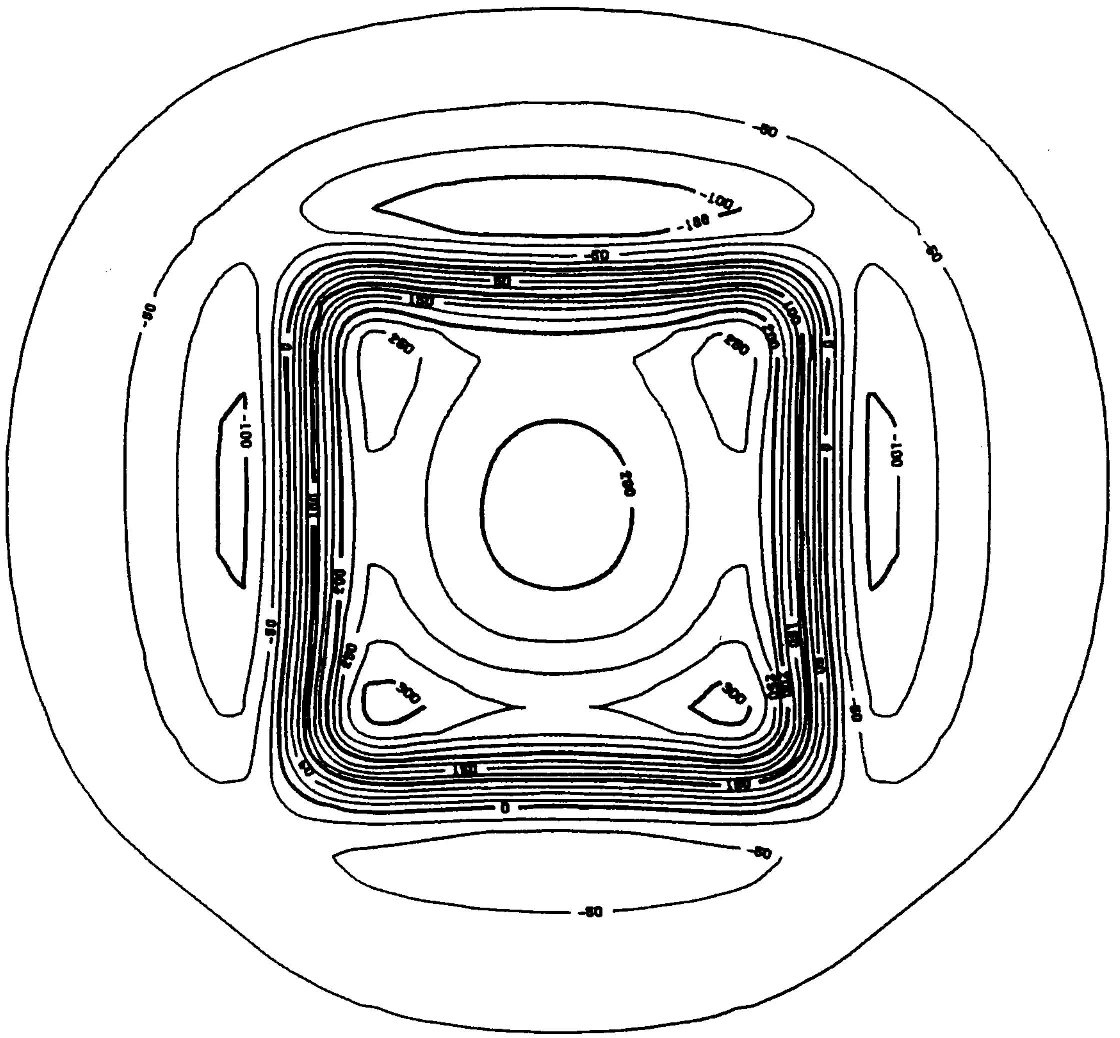

FIG. 3. Vertical derivative for the dipping prismatic body of Figure 2 calculated by using the generalized Hilbert transform relation (17). Computations were carried out using a coarse grid (40 × 40) with horizontal derivatives evaluated numerically.

$$\mathscr{F}\left[\frac{\partial M}{\partial x}\right] = -H_1\mathscr{F}\left[\frac{\partial M}{\partial z}\right],$$

and (18)

$$\mathscr{F}\left[\frac{\partial M}{\partial y}\right] = -H_2\mathscr{F}\left[\frac{\partial M}{\partial z}\right],$$

or

$$\mathscr{F}[\nabla_h M] = -\mathbf{H}\mathscr{F}\left[\frac{\partial M}{\partial z}\right]. \quad (18a)$$

By inspection,

$$H_1\mathscr{F}\left[\frac{\partial M}{\partial y}\right] = H_2\mathscr{F}\left[\frac{\partial M}{\partial x}\right]. \quad (19)$$

Operators H, H_1, and H_2 satisfy the following relations:

2-D case ($q = 0$, $H_1 = H$, $H_2 = 0$)

From equations (17) and (18), one can write in succession

$$\mathscr{F}\left(\frac{\partial M}{\partial z}\right) = H\mathscr{F}\left(\frac{\partial M}{\partial x}\right),$$

and

$$H\mathscr{F}\left(\frac{\partial M}{\partial z}\right) = H \cdot H\mathscr{F}\left(\frac{\partial M}{\partial x}\right) = -\mathscr{F}\left(\frac{\partial M}{\partial x}\right),$$

which immediately implies

$$H \cdot H = -1. \quad (20)$$

3-D case

From equations (17), (18) and (19), one can write in succession

$$\mathscr{F}\left(\frac{\partial M}{\partial z}\right) = H_1\mathscr{F}\left(\frac{\partial M}{\partial x}\right) + H_2\mathscr{F}\left(\frac{\partial M}{\partial y}\right),$$

and

$$H_1\mathscr{F}\left(\frac{\partial M}{\partial z}\right) = H_1 \cdot H_1\mathscr{F}\left(\frac{\partial M}{\partial x}\right) + H_2 H_1\mathscr{F}\left(\frac{\partial M}{\partial y}\right)$$
$$= (H_1 \cdot H_1 + H_2 \cdot H_2)\mathscr{F}\left(\frac{\partial M}{\partial x}\right) = -\mathscr{F}\left(\frac{\partial M}{\partial x}\right),$$

which immediately implies

$$H_1 \cdot H_1 + H_2 \cdot H_2 = -1. \quad (20a)$$

It is evident that equation (20a) represents the 3-D extension of relation (20). Finally, relation (12) which led to the development of the analytic signal concept in 2-D can now be generalized to 3-D as

$$\mathscr{F}\left[\frac{\partial M}{\partial x} + \frac{\partial M}{\partial y} + i\frac{\partial M}{\partial z}\right] = \left(1 + \frac{p}{\sqrt{p^2+q^2}}\right)\mathscr{F}\left(\frac{\partial M}{\partial x}\right) + \left(1 + \frac{q}{\sqrt{p^2+q^2}}\right)\mathscr{F}\left(\frac{\partial M}{\partial y}\right). \quad (21)$$

The simplicity of expression (21) as well as the obvious significance of the generalized signum function is indeed remarkable.

Space-domain representation

Since expressions (17) and (18) represent multiplications in the frequency domain, their space-domain counterparts are convolution integrals. To accomplish this, I use the following inverse transforms:

$$\mathscr{F}^{-1}\left(\frac{-ip}{\sqrt{p^2+q^2}}\right) = \frac{1}{2\pi}\frac{x}{(x^2+y^2)^{3/2}},$$

and

$$\mathscr{F}^{-1}\left(\frac{-iq}{\sqrt{p^2+q^2}}\right) = \frac{1}{2\pi}\frac{y}{(x^2+y^2)^{3/2}},$$

which are derived in Appendix A.

The space-domain equivalent of expressions (17) and (18) will represent the extension to 3-D of the 2-D Hilbert transform relations for potential field data. Thus

$$\frac{\partial M}{\partial z} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(x-\xi)\dfrac{\partial M}{\partial \xi} + (y-\eta)\dfrac{\partial M}{\partial \eta}}{R^3}\,d\xi\,d\eta,$$

$$\frac{\partial M}{\partial x} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(x-\xi)\dfrac{\partial M}{\partial z}}{R^3}\,d\xi\,d\eta, \quad (22)$$

and

$$\frac{\partial M}{\partial y} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(y-\eta)\dfrac{\partial M}{\partial z}}{R^3}\,d\xi\,d\eta,$$

where (ξ, η) is the integration point in the x, y plane and

$$R^2 = (x-\xi)^2 + (y-\eta)^2.$$

Following Fulton and Rainich (1932), expressions (22) can be written in a compact form for the total gradient

$$W = \frac{\partial M}{\partial x}\mathbf{i} + \frac{\partial M}{\partial y}\mathbf{j} + \frac{\partial M}{\partial z}\mathbf{k}.$$

Let α, β, γ be the direction cosines of the normal n to a surface, i.e.,

$$n = \mathbf{i}\alpha + \mathbf{j}\beta + \mathbf{k}\gamma$$

which for the x, y plane reduces to k. With the above notations, one can easily obtain

$$W(x, y, 0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(\xi, \eta, 0) \cdot n \cdot \nabla_h\left(\frac{1}{R}\right) d\xi\,d\eta,$$

where all above products are to be understood as quaternion products.[3] Letting

$$\nabla_h = \frac{\partial}{\partial \xi}\mathbf{e}_x + \frac{\partial}{\partial \eta}\mathbf{e}_y$$

be the horizontal gradient of a function at the integration point, one can write the Hilbert transform relations (22) in a more

[3]All theoretical developments in this paper can be derived by using quaternions. However, it was felt that the use of vector algebra would give a better insight into the problem.

compact form, i.e.,

$$\frac{\partial M}{\partial z} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nabla_h M \cdot \nabla_h \left(\frac{1}{R}\right) d\xi\, d\eta,$$

and (23)

$$\nabla_h M = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial M}{\partial z} \nabla_h \left(\frac{1}{R}\right) d\xi\, d\eta.$$

Letting $\partial M/\partial \eta = 0$ in equation (22), and taking into account that

$$\int_{-\infty}^{\infty} \frac{d\eta}{R^3} = \frac{2}{(x-\xi)^2},$$

one readily obtains the known 2-D Hilbert transform relations [see (N4), (N6), and (N7)].

The first expression (22) after integrating by parts can also be written as (Ackermann and Dix, 1955)

$$\begin{aligned}\frac{\partial M}{\partial z} &= -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\dfrac{\partial^2 M}{\partial \xi^2} + \dfrac{\partial^2 M}{\partial \eta^2}}{R}\, d\xi\, d\eta \\ &= -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\nabla_h^2 M}{R}\, d\xi\, d\eta, \end{aligned} \tag{24}$$

where

$$\nabla_h^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} = -\frac{\partial^2}{\partial \zeta^2}.$$

Similarly, integration of either of the last two integrals in (22) with respect to x or y leads to the well-known formula (Grant and West, p. 218)

$$M(x, y, 0) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\dfrac{\partial M}{\partial z}(\xi, \eta)}{R}\, d\xi\, d\eta. \tag{25}$$

In view of above, expressions (24) and (25) can be considered as inverse transforms of each other.

ADDITIONAL PROPERTIES OF 3-D POTENTIAL FUNCTIONS

Up to now, all developments have been made in parallel to the previously published 2-D case. The analysis can, however, be carried further as follows.

Expression (13) after multiplying by $\mathscr{F}(M)$ can also be written as

$$\begin{aligned}\sqrt{p^2+q^2}\, \mathscr{F}(M) = ip\left[\frac{-ip}{\sqrt{p^2+q^2}} \mathscr{F}(M)\right] \\ + iq\left[\frac{-iq}{\sqrt{p^2+q^2}} \mathscr{F}(M)\right]. \end{aligned} \tag{26}$$

The reader will recognize the terms in brackets as the Fourier transform of the x and y components of the Hilbert transform of M, in perfect analogy with (7). Thus

$$\mathscr{F}[\mathscr{H}_1(M)] = -\frac{ip}{\sqrt{p^2+q^2}} \mathscr{F}(M),$$

and (27)

$$\mathscr{F}[\mathscr{H}_2(M)] = -\frac{iq}{\sqrt{p^2+q^2}} \mathscr{F}(M).$$

Using relations (9) and (27), equation (26) can be written in the form

$$\mathscr{F}\left[\frac{\partial M}{\partial z}\right] = \mathscr{F}\left[\frac{\partial \mathscr{H}_1(M)}{\partial x}\right] + \mathscr{F}\left[\frac{\partial \mathscr{H}_2(M)}{\partial y}\right],$$

or (28)

$$\frac{\partial M}{\partial z} = \frac{\partial \mathscr{H}_1(M)}{\partial x} + \frac{\partial \mathscr{H}_2(M)}{\partial y} = \nabla_h \cdot \mathscr{H}(M),$$

where

$$\mathscr{H} = \mathscr{H}_1 \mathbf{e}_x + \mathscr{H}_2 \mathbf{e}_y.$$

Similarly expressions (18) lead to

$$\begin{aligned}\frac{\partial \mathscr{H}_1(M)}{\partial z} &= -\frac{\partial M}{\partial x}, \\ \frac{\partial \mathscr{H}_2(M)}{\partial z} &= -\frac{\partial M}{\partial y}, \end{aligned} \tag{29}$$

or

$$\frac{\partial \mathbf{H}(M)}{\partial z} = -\nabla_h M.$$

For the 2-D case ($q = 0$, $\mathscr{H}_1 = \mathscr{H}$, $\mathscr{H}_2 = 0$), expressions (28) and (29) reduce to [see also (N11)]

$$\frac{\partial M}{\partial x} = -\frac{\partial \mathscr{H}(M)}{\partial z},$$

and (30)

$$\frac{\partial M}{\partial z} = \frac{\partial \mathscr{H}(M)}{\partial x},$$

representing the Cauchy-Riemann conditions between a potential function and its Hilbert transform. Thus, relations (28) and (29) represent the 3-D extensions of the 2-D Cauchy-Riemann conditions.

Finally, using relations (29) in equation (22a), one obtains, after integrating with respect to z, still another relation between a potential function and its Hilbert transform:

$$M(x, y) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathscr{H}(M) \cdot \nabla_h \left(\frac{1}{R}\right) d\xi\, d\eta,$$

and (22b)

$$\mathscr{H}(M) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\xi, \eta) \nabla_h \left(\frac{1}{R}\right) d\xi\, d\eta.$$

All above expressions yielding the fundamental relations between potential field components were developed mainly for the case of magnetic data. Their extension to gravity data is straightforward. As outlined in the Introduction, they are essential in the development of a truly 3-D automatic interpretation technique, without unduly restrictive initial assumptions about the causative bodies.

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APPENDIX A

To evaluate the inverse Fourier transform of $\mathcal{H}_1 = ip/\sqrt{p^2 + q^2}$, start from the basic definition (3)

$$\mathcal{H}_1(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{-ip}{\sqrt{p^2 + q^2}} e^{i(px+qy)}\, dp\, dq$$

$$= -\frac{i}{4\pi^2} \int_0^{\infty} \int_0^{\infty} \frac{p}{\sqrt{p^2 + q^2}} (2i \sin px)(2 \cos qy)\, dp\, dq$$

$$= \frac{1}{\pi^2} \int_0^{\infty} \cos qy\, dy \int_0^{\infty} \frac{p}{\sqrt{p^2 + q^2}} \sin px\, dp.$$

The last integral can be solved by using expression 3.773.3, p. 429 from Gradshteyn and Ryzhik (1965) for $m = 0$, $n = 0$, $a = x$, and $\beta = q$. One then obtains

$$\mathcal{H}_1(x, y) = \frac{1}{\pi^2} \int_0^{\infty} q \cos qy K_1(xq)\, dq.$$

The above integral is solved by using expression 6.699.12, p. 749 from the above reference for $\mu = 1$, $a = x$, $b = y$, to yield the final result

$$\mathcal{H}_1(x, y) = \frac{1}{2\pi} \frac{x}{(x^2 + y^2)^{3/2}}.$$

In a similar fashion, it can be proven that

$$\mathcal{H}_2(x, y) = \frac{1}{2\pi} \frac{y}{(x^2 + y^2)^{3/2}}.$$