Constrained inversion of geologic surfaces—pushing the boundaries

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During the past 10 years, there has been a shift in modelling for geophysical interpretation away from idealized geometrical bodies floating in air and toward fully three-dimensional representations. This transition has been driven by a number of factors, both technological and conceptual. The principal conceptual driver has been the growing recognition of the importance of integrated interpretations. Interpretation is a shared responsibility of geoscientists, the common goal being an Earth model consistent with all available information.

Inversion is a numerical process whereby an initial model is adjusted to improve the degree of agreement, or fit, between the measured geophysical data and the corresponding calculated data based on the model. In many circumstances, 3D geophysical inversion can be regarded as an extension of geologic modeling. Given that geologic models are generally constructed from surfaces, forms of inversion which manipulate geologic surfaces are especially well-suited for refining geologic models. Surface-based "geometry" inversion is therefore a strong driver for integrated geologic/geophysical interpretation.

Geologic surfaces define the three-dimensional distribution of geological events, and divide the subsurface into domains. The most common types of surfaces are lithological contacts and structural discontinuities. The domains are distinguished by rock type and quantitative attributes, such as grade and physical properties. Thus geologic models are categorical (domain-based) as well as numerical (property based).

In geometry inversion, the geologic surfaces themselves are manipulated to achieve an improved fit between observed and calculated data. The geologic significance of contacts is preserved, and after inversion the model is still fully recognizable as a geologic model. Physical property isosurfaces can be generated if desired, but they are not needed as surrogates for geologic boundaries. This is in contrast to conventional inversion of rock properties, after which subjective interpretation of the 3D property distribution is often required to define the position of a contact.

Not only do categorical/numerical models permit geometry inversion, they also permit greater flexibility in property inversion. In particular, property changes can easily be restricted to a particular geologic domain, by excluding all other rock types from inversion. In addition, remanent magnetization, which is specific to individual rock types, can be accommodated.

In greenfields exploration, geologic models may be either purely conceptual, or based on interpretations of geophysical data, e.g., seismic. Drilling may be limited or completely absent. Natural greenfields' roles for geometry inversion include depth to basement prediction, delineation of pods of mineralization, or (in petroleum exploration) definition of basal salt geometry.

In advanced exploration projects or near mines, the reliability of geologic surfaces is enhanced by drill core or downhole geophysical logs. Between the known points, the geologic surfaces are interpreted, from geophysical data or other characteristics. Hence, geologic surfaces are always to some degree subjective, and their reliability is variable. Geometry inversion honors the geologic observations, but allows the shape of the defining surfaces to vary between control points.

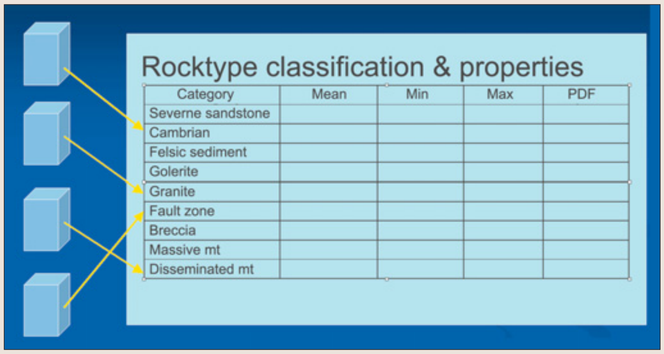

Figure 1. Parameterization of the rock property model. Each cell must belong to a rock type. Its physical properties are then assigned accordingly.

An inversion algorithm capable of accepting geologic surfaces as input, improving the fit to geophysical data, and outputting modified geologic surfaces (which honor fixed points) is desirable in itself. At the same time, the existence of geologic constraints enhances the effectiveness of geophysical inversion. This is especially true for potential field data which, when considered in isolation, will admit a wide variety of interpretations. Surface-based inversion therefore provides impetus for truly integrated interpretation of geologic and geophysical data. The refined model of the geology is then jointly owned, an interpretation shared by all geoscientists.

Although manipulation of geologic surfaces via inversion is the focus of this paper, physical properties cannot be divorced from geometry inversion. In general, the sensitivity of the data to the shape of a boundary is proportional to the property contrast; contacts associated with zero contrast will not move during geometry inversion. If knowledge of rock properties is sketchy, the inferred geometry will be correspondingly more uncertain. Characterization of the physical properties of the relevant geologic units is therefore crucial for geometry inversion as well as for property inversion. Ideally, the properties should be measured from core samples or with downhole probes. In petroleum and coal exploration, rock properties are recorded routinely in exploration holes, but in metalliferous and mineral exploration, rock property information is all too often very limited or completely absent.

Geometry inversion of gravity and magnetic data is illustrated below using a 3D potential fields inversion program, VPmg (Fullagar Geophysics), interfaced to a 3D geological modeling package, Gocad (Paradigm). Examples are drawn from a selection of exploration projects and mines.

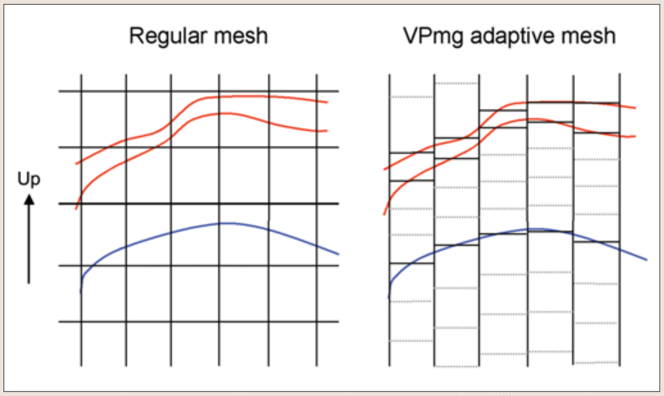

Figure 2. Schematic model sections illustrating the differences between a conventional fixed mesh (left) and the deforming mesh implemented in VPmg. Colored lines denote geological contacts; black lines are cell boundaries. Subcells (dotted boundaries) can differ in size from one rock type to another.

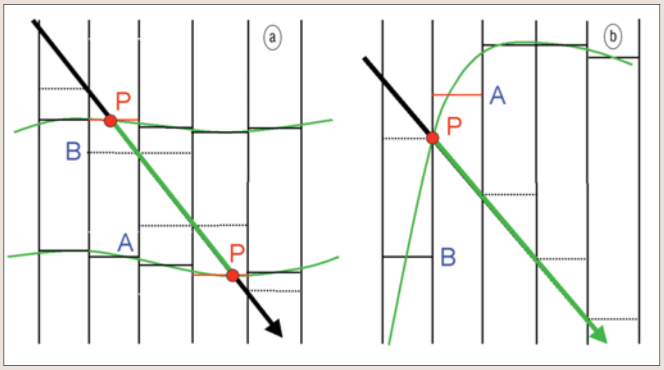

Figure 3. Schematic sections to illustrate the origin of bound constraints. Thick oblique line represents a drill hole. Fixed-cell boundaries are colored red; bound constraints are dotted. Pierce points, P, on the upper and lower contact of a particular unit (green) are marked with a red dot. (a) Dotted line at B marks the upper limit of travel for the interpreted contact at A. (b) Contact at A is deemed to be sufficiently close to P to remain fixed, while the dotted line at P marks the upper limit of travel for the interpreted contact at B.

Model parameterization. The goal of reshaping geologic boundaries via inversion imposes a fundamental requirement on the parameterization of the model. By definition, a geologic boundary separates one rock domain from another. Therefore, to manipulate and track bounding surfaces during inversion, each cell of the starting model must be assigned to a particular rock type. Thus the model must be categorical as well as numerical (Figure 1).

If a geologic unit is homogeneous, all its constituent cells share the same property value. On the other hand, if a geologic unit is heterogeneous, the property values of all its cells should collectively conform to the appropriate statistical distribution. Unfortunately, in mining applications the statistical variability of physical properties is often not well characterized.

Modification of geologic boundaries can be achieved either by reclassifying cells, with fixed cell boundaries, or by moving the cell boundaries, with cell rock types invariant. In the former case, the underlying model mesh is unchanging, and boundaries shift in discrete jumps of one or more cells. In the latter case, the mesh deforms, and arbitrarily small boundary adjustments are permitted. Some algorithms allow both styles of boundary modification. In this paper, we will explore a deforming mesh style of geometry inversion. The subsurface is discretized into close-packed vertical rectangular prisms. Prism tops honor surface topography, and internal horizontal contacts divide each prism into cells (Figure 2). Viewed in plan, the cells are uniform in size, but their vertical dimensions are arbitrary; thus a 0.2 m-thick cell can abut a 200 m-thick cell. This type of "adaptive mesh" has been implemented in the VPmg 3D inversion program. During geometry inversion, continuous movement of the horizontal cell boundaries is permitted, but vertical prism boundaries are fixed.

The VPmg adaptive mesh represents a compromise between generality and practicality. It offers several advantages over a conventional regular mesh:

1) details in geologic models, especially thin geologic units, can be retained in the inversion model;
2) all surfaces, including the ground topography, can be represented more accurately;
3) the adaptive mesh is more compact, i.e., fewer cells are required (especially for homogeneous units), so model files are smaller and run times are shorter.

Although geometry inversion is the focus of this paper, VPmg can adjust the densities or susceptibilities of the geologic units during inversion, as well as their bounding surfaces. The geologic units can be heterogeneous or homogeneous.

Constraints. Geometry inversion is always constrained to some extent, even in the absence of drill hole control points. In VPmg, the horizontal cell boundaries are prevented from erupting through the ground surface, or from passing through one another. In addition, the algorithm is conditioned to suppress large changes at shallow depths. Beyond these general constraints, it is possible to impose constraints on specific geologic surfaces. When drill holes exist, "ground truth" is captured and preserved in the VPmg model by means of both "hard" constraints (control points and bounds) and "soft" constraints (multiplicative weights).

Pierce point constraints. In VPmg, horizontal cell boundaries are held fixed, if pierced by a drill hole. If only a few vertical drill holes are involved, finding and tagging the fixed cell boundaries is straightforward. As the number of drill holes increases, especially if the holes are inclined away from vertical, and as the geology becomes more complex, tagging the fixed contacts is impracticable without suitable software, e.g., Gocad Mining Suite.

If a control point lies close to the center of a horizontal contact, there is no confusion as to which VPmg cell boundary should be fixed, especially if dips are gentle (Figure 3a). However, assigning interfaces as fixed or free becomes more subjective if the control point is close to a cell edge. Moreover, as dips increase a control point may be above or below the nearest model interfaces associated with the geological contact in question (Figure 3b). Therefore it is reasonable to invoke a "range of influence" around each control point, and to fix the interfaces which lie within range. In Figure 3b, contact A is deemed to be within range of control point P.

Bound constraints. Bound constraints restrict the travel of free interfaces during geometry inversion. They are most likely to arise when dips are steep or when drill holes are inclined from the vertical. In Figure 3a, the interpreted contact at A cannot move higher than B, since the green unit has been logged continuously between the two control points (P). Similarly, in Figure 3b the control point at P defines an upper limit for the model interface currently at

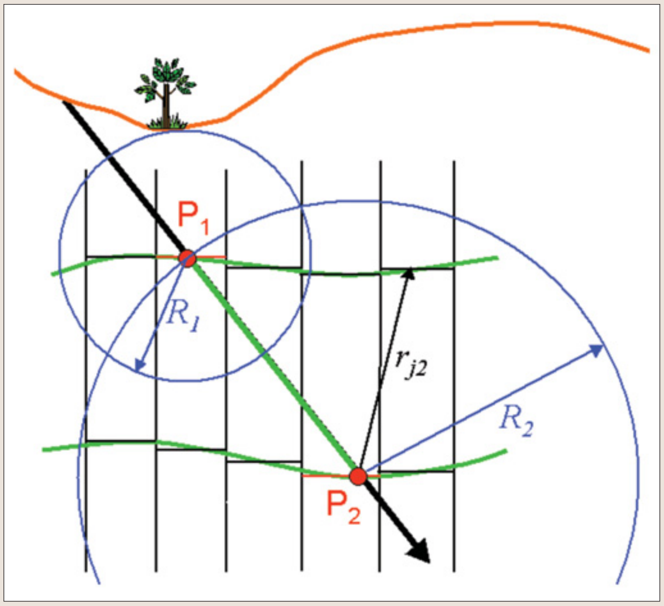

Figure 4. Schematic section showing radius of influence around control points, within which geometry changes are damped during inversion. In VPmg, R is defined as the lesser of depth and distance to nearest control point.

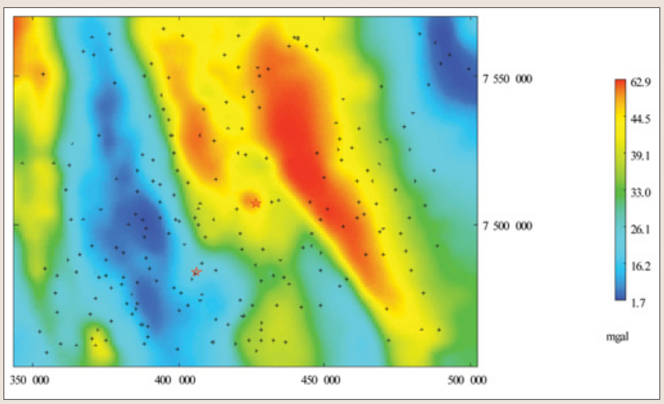

Figure 5. Free-air gravity image, Boulia 1:250 000 sheet, Queensland. Drill holes intersecting basement are marked as red stars. Basement depths were 1300 m (central) and 530 m (SW). Water borehole locations are marked with black crosses. Barrick's Osborne Cu-Au mine is in the NE corner. (Data courtesy Queensland Department of Natural Resources and Mines.)

B.

Upper bound constraints also arise when contacts are interpreted to lie below the reach of drilling. Similarly, a contact interpreted to occur within a percussion precollar can be bounded below, e.g., where core drilling commences.

Soft constraints. The manner in which a geologic surface is interpolated between known mapped or drilled points is subjective. This applies to computer programs as well as people. It is possible to distinguish two operations in the interpolation process: defining the neighborhood of influence for individual fixed points; and imposing a certain character on the contact surface. In VPmg, a simple distance weighting is applied. The radius of influence of a control point is defined as its depth or as distance to the nearest control point, whichever is smaller (Figure 4). The movement of each unconstrained ("free") interface within this radius is damped during inversion. The damping is achieved by applying

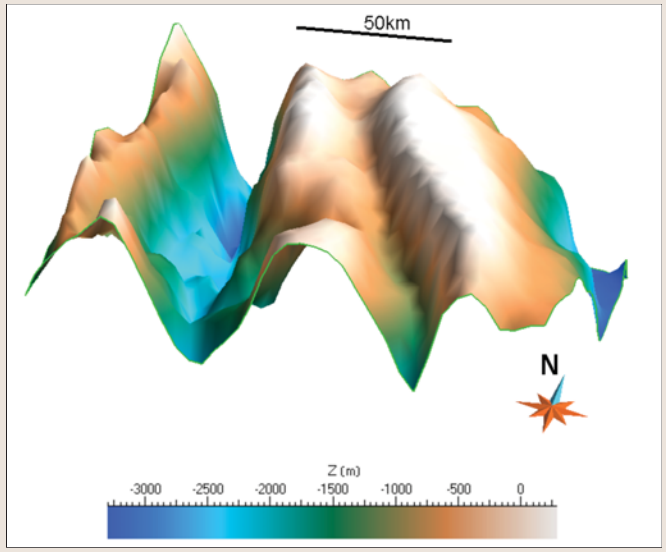

Figure 6. Perspective view of the basement surface after unconstrained geometry inversion. The color depicts elevation (m). Vertical exaggeration = 20.

weights to the derivatives associated with each free interface; the derivatives encapsulate the sensitivity of each data point to changes in elevation of a free interface. The weights are multiplicative. For the *j*th free interface lying distance r_{jk} from the *k*th control point, the weight w_j is updated by the factor r_{jk}/R_k, where R_k is the radius of influence of the *k*th control point. Thus

$$w_j = \prod_k \frac{r_{jk}}{R_k}$$

provided $r_{jk}<R_k$. Free interfaces which are far removed from any fixed points are assigned unit weight.

The net effect of the weighting is to desensitize the inverse problem to movement of free interfaces within the neighborhood of fixed interfaces; free interfaces far from fixed interfaces will be moved in preference. This approach is simple and effective, but is by no means the only way for the influence of drill hole control points to propagate through the model volume.

Examples of 3D geometry inversion.

Depth-to-basement inversion, Boulia, Queensland. Sedimentary cover thickness strongly influences area selection for exploration in Proterozoic and Archean terranes. Constructing a basement unconformity surface, consistent with potential field data, is a natural application for geometry inversion. Depth-to-basement prediction is illustrated here via inversion of gravity data over the Boulia 1:250 000 scale map sheet in western Queensland, Australia (Figure 5). The Boulia map sheet occupies an area where the Mount Isa Inlier plunges gently southwards beneath Palaeozoic sediments of the Georgina Basin. Topographic relief over the entire map sheet is very modest (less than 150 m).

Interpretation of gravity is fraught with ambiguity. The importance of a priori information during inversion is demonstrated here by inverting the gravity data twice, first without and then with constraints. For the first inversion it is simply assumed that the Proterozoic basement is denser than the overlying Palaeozoic cover. Both the basement (2.80 g/cc) and the sediments (2.42 g/cc) are assumed homogeneous in density. The basement contact is at a constant ele-

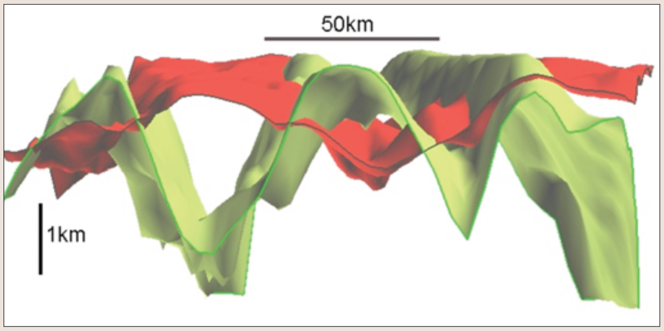

Figure 7. Comparison of inferred basement surface after constrained (red) and unconstrained (green) geometry inversion.

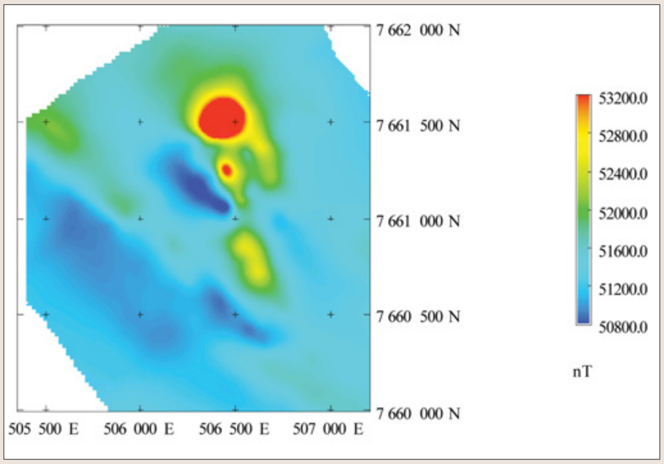

Figure 8. *Bull Creek total magnetic intensity image, based on ground readings. (Data courtesy Exco Resources.)*

vation initially (depth ~700 m).

The basement topography after inversion is depicted in Figure 6. Troughs have developed beneath gravity lows and ridges beneath gravity highs. The rms misfit was reduced from 12.86 mgal to 1.75 mgal. In the absence of additional information, this is a perfectly sensible hypothesis. The uncertainty associated with gravity interpretation can be minimized if all available information is exploited. For Boulia, depth-to-basement and Proterozoic geology interpretations based on aeromagnetics were available, as well as a compilation of density values. In addition, two drill holes were known to have intersected basement (Figure 5).

A 3D starting model was constructed, with basement elevation conforming with the drill hole pierce points and the aeromagnetic depth-to-source estimates. Density was assigned to interpreted basement domains in accordance with the density compilation. Basement geometry was again adjusted via inversion, with the basement elevation held fixed at the two drill hole control points this time. The basement surface was also bounded above by water bores which terminated in Paleozoic sediments (Figure 5). Basement density inversion was applied prior to geometry inversion to address some inconsistencies between interpreted basement lithology and the gravity data. Ultimately, a very close fit to the data was achieved (0.01 mgal rms).

The basement surfaces produced by constrained and unconstrained inversion are compared in Figure 7. The difference between the two is stark: the basement topography in the two models is almost reversed. Low gravity is actually due to the occurrence of felsics at shallow depth, not to thick cover. Constrained surface-based inversion has produced a 3D depth-to-basement model, consistent with all

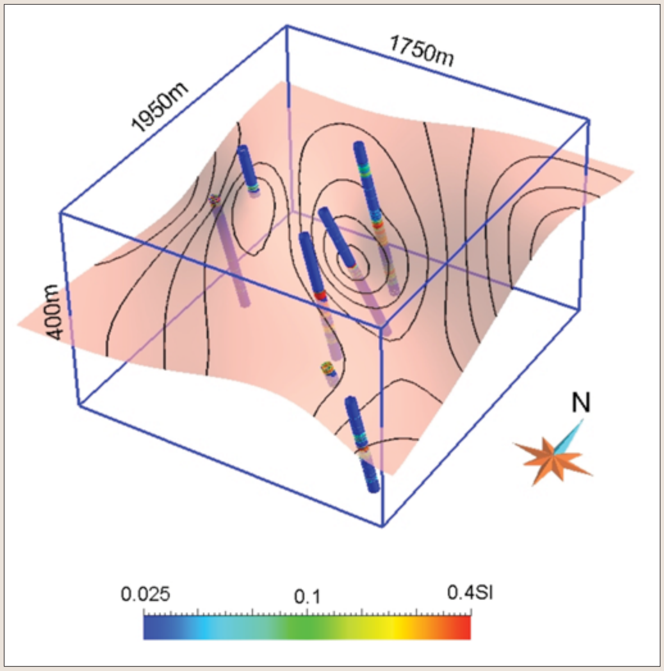

Figure 9. *Illustrates the position of the zero-thickness unit (layer) in relation to the drill hole susceptibilities. The colored susceptibilities are stretched logarithmically. Elevation contours on the starting model zero-thickness layer are displayed at 20-m intervals.*

information, which can assist area selection for exploration.

Growing a magnetic source "from nothing," Bull Creek, Queensland. Geometry inversion is performed on ground magnetics data acquired over the Bull Creek prospect near Cloncurry, Queensland, Australia. The magnetic response exceeds 4000 nT amplitude (Figure 8). The mineralization and alteration history of Bull Creek has been well documented previously. The Proterozoic basement is overlain unconformably by black shales. Six drill holes intersected magnetite-pyrrhotite mineralisation. Downhole logging defined susceptibilities between 0.1 and 0.8 SI in the mineralisation.

The starting model for inversion comprised a zero-thickness layer, shaped in Gocad so as to pass through the highest susceptibility intersection in each drill hole (Figure 9). The magnetic susceptibility assigned to the zero-thickness layer was 0.4 SI, a value consistent with downhole susceptibility logs. Geologically, this choice of starting model would be appropriate if all mineralization were hosted by a single, continuous magnetic horizon. During geometry inversion, the zero thickness layer was permitted to thicken where appropriate, to improve the fit to the magnetic data. Both top and bottom of the layer can move up or down together, so the center of the magnetic unit is not constrained to remain in its starting position. The initial rms data misfit was 479 nT; geometry inversion reduced this to 164 nT. From a fairly simple, rapidly constructed starting model, geometry inversion has generated a 3D magnetic source (Figure 10), broadly consistent with drill intersections, which reproduces the magnetic data fairly well. The approach is not limited to a single magnetic unit: multiple magnetic units could be allowed to develop, each with its own susceptibility.

Inferring kimberlite shape from magnetic data, Fort-á-la-Corne, Saskatchewan. The Fort-à-la-Corne kimberlite (FALC) field in northern Saskatchewan, Canada, is one of the largest in

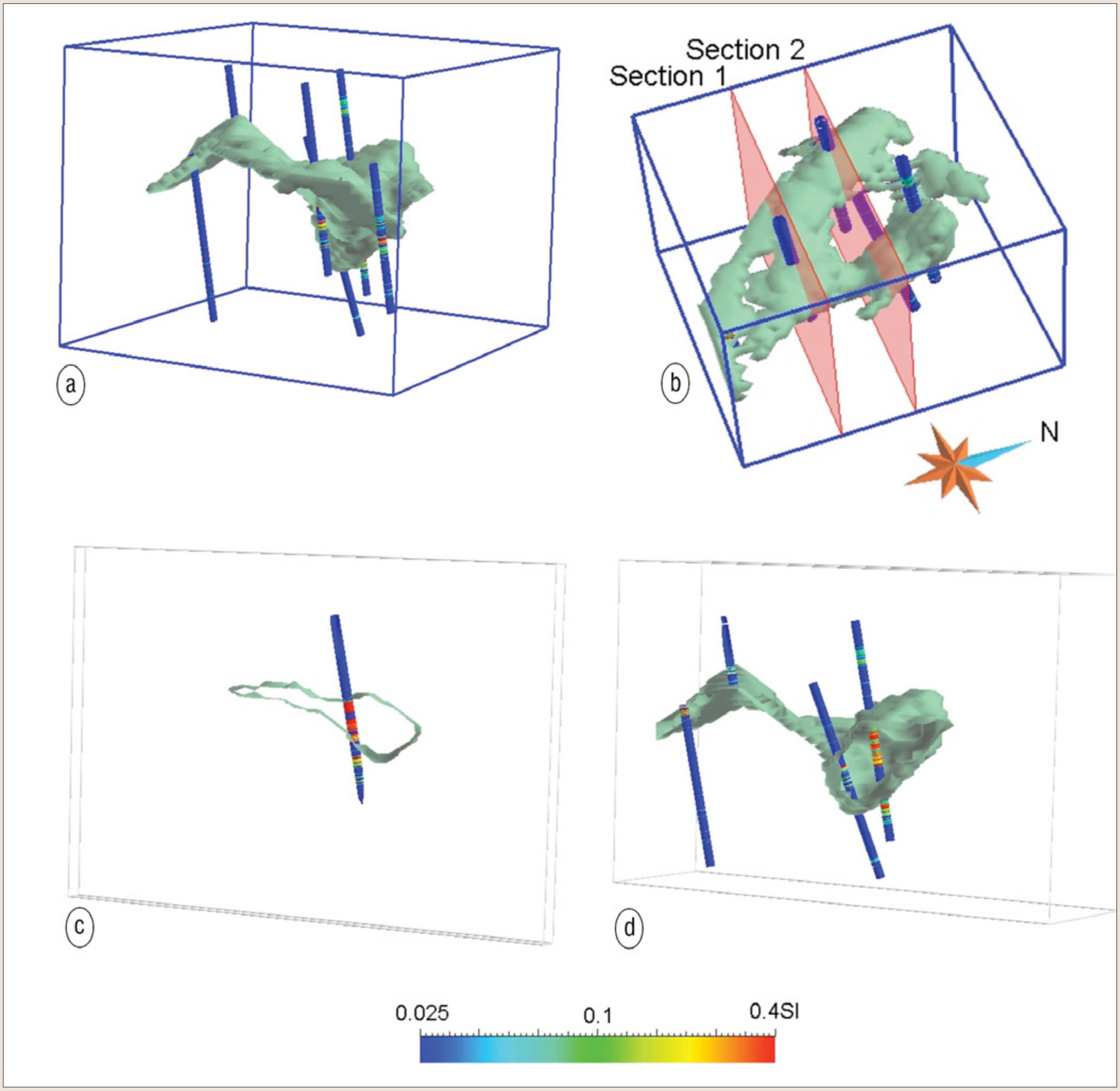

Figure 10. *3D magnetic source at Bull Creek, constructed via geometry inversion of a zero-thickness layer. Drill holes shown in all panels, colored according to susceptibility. Vertical exaggeration = 3. (a) View from the SSE. (b) View from ESE, with position of sections shown. (c) Section 1, viewed from SSE, shows peak drillhole susceptibility enclosed within the magnetic body. (d) "Thick" slice through the model, centered on Section 2.*

the world. In this example, geometry inversion is applied to aeromagnetic data to define the thickness and shape of two kimberlites. The aeromagnetic data are presented in Figure 11. Given a kimberlite starting model (conceptual or drill hole based), VPmg adjusts the model geometry (kimberlite shape) to improve the fit to the aeromagnetic data. Susceptibility within the kimberlite is assumed to be uniform here. Prior to geometry inversion the susceptibility of the kimberlite was optimized (from 0.06 SI to 0.047 SI) via VPmg "homogeneous unit" inversion.

The starting model for inversion comprised three units: kimberlite, overburden (till), and country rock (sediments). Surfaces were constructed representing the geologic boundaries between these units (Figure 12). The host geology is flat-lying and relatively weakly magnetic (~0.004 SI). The depths of overburden was known from drilling. An initial kimberlite shape was modelled on the basis of characteristics (shape, intensity, lateral extent) of the aeromagnetic anomalies (Figure 13, at left). Depth control on base of kimberlite was provided by four drill holes. During inversion, the basal kimberlite surface was pinned by the four control points, but was allowed to move elsewhere. All other geological interfaces were held fixed.

The basal kimberlite contact after inversion is shown in Figure 13 (at right). In this instance, follow up drilling permitted "ground truthing" of the results of the inversion. The correspondence between interpreted base of kimberlite and drill hole control points, before and after VPmg geometry inversion, is indicated in Figure 13. After inversion, the overall agreement with drilling is very good (white dots denote close correspondence). Thus inversion has generated a basal shape which more closely resembles true contact

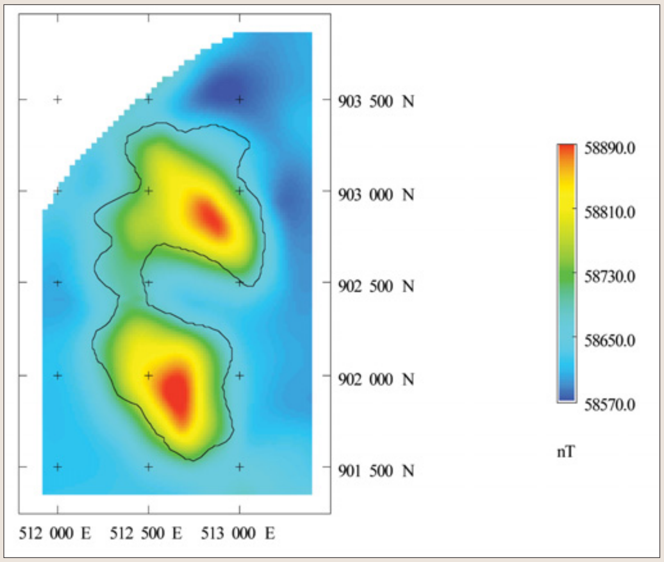

Figure 11. *Aeromagnetic image (nT) over kimberlite pipes 145/219, Fort á la Corne, Saskatchewan. Starting model kimberlite outline is shown in black. (Data courtesy De Beers, Canada.)*

geometry than the starting model. After surface-based inversion, volumetric analysis is straightforward, since the geological contacts are unambiguously defined within the rock property model. The kimberlite thickness distribution for the 145/219 pipes, as inferred from drilling and constrained inversion of magnetic data, is shown in Figure 14.

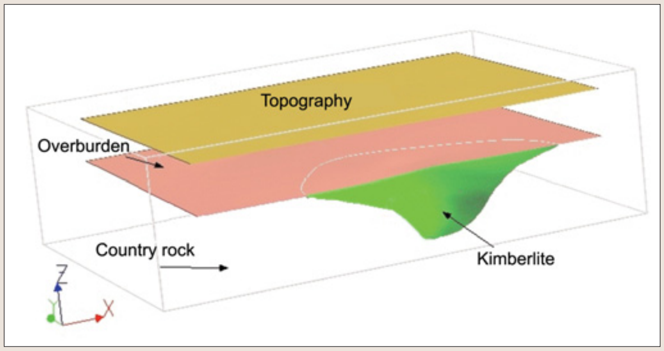

Figure 12. *Schematic illustration of the starting model configuration (not specifically depicting the 145/219 kimberlite).*

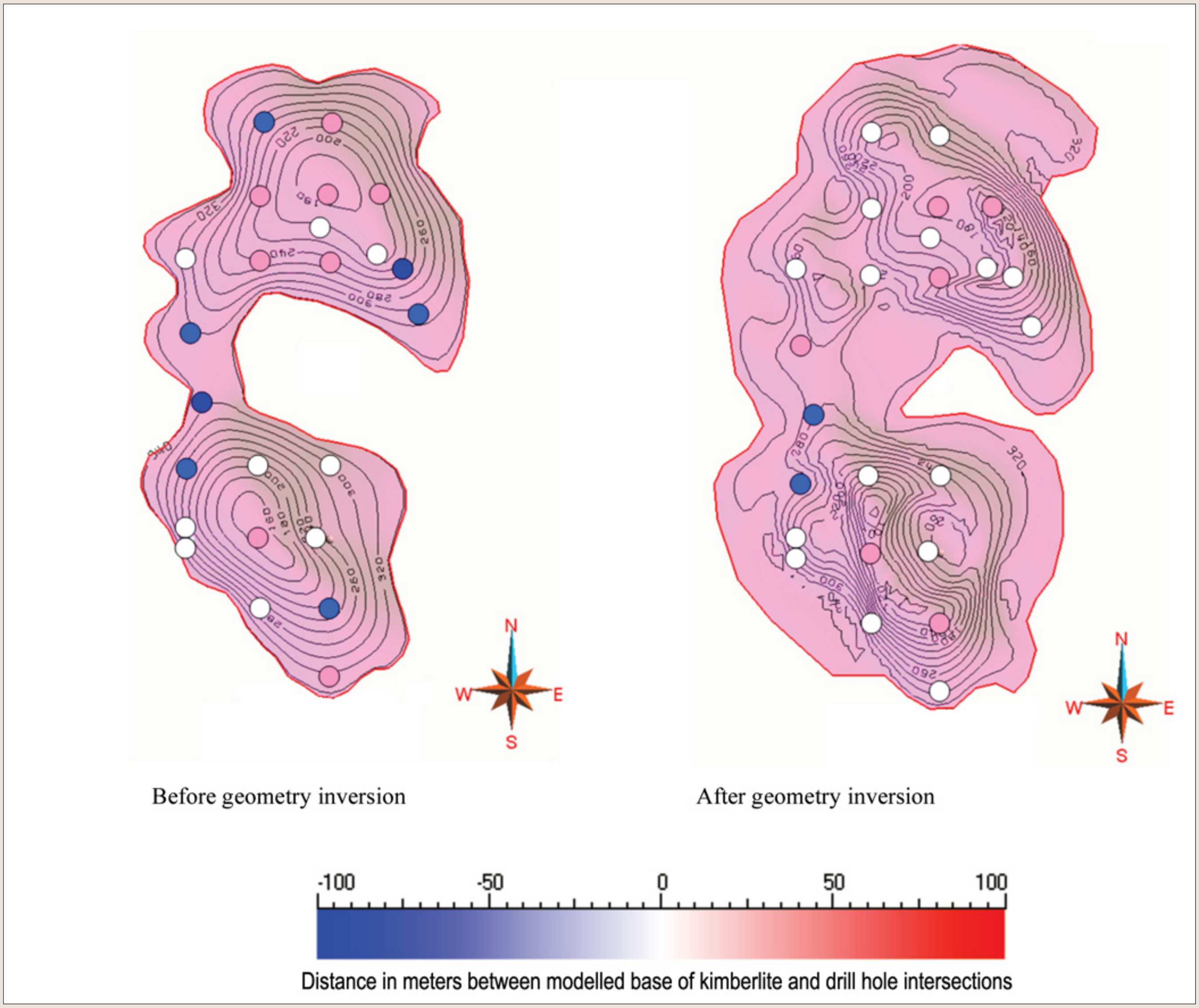

Figure 13. *Comparison between the interpreted base of kimberlite, before (left) and after (right) geometry inversion. All drill hole intersections (new and old) marked with dots. Colors indicate mismatch between predicted depth and drilled depth. Contour interval is 20 m.*

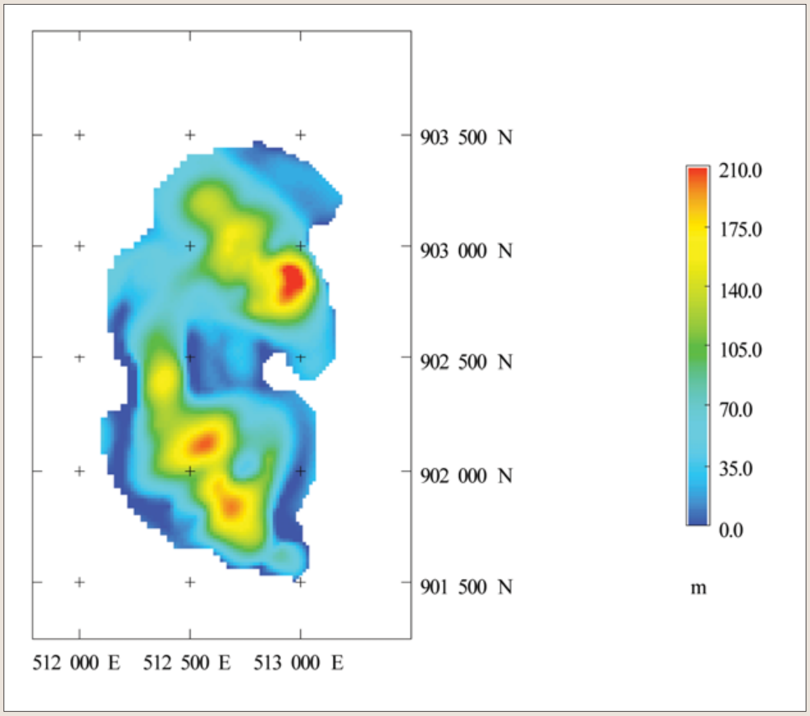

Figure 14. *Final kimberlite thickness (m) for 145/219, after successive phases of drilling, modeling, and geometry inversion.*

Conclusions. Given the nature of geologic models, a capability to manipulate geologic boundaries via inversion is very desirable. Geologic surfaces are constructed via 3D interpolation between control points, either drilled or inferred, e.g. from seismic. Geometry inversion can be regarded as a form of geological modeling, since it adjusts surfaces while honoring the control points. Attaching a surface to its control points is essential for geologic credibility; and is also highly advantageous because it reduces the inherent ambiguity of geophysical interpretation.

The fundamental prerequisite for geometry inversion is a geological starting model, i.e., one which is both categorical and numerical. In conventional property inversion, the model is purely numerical, with the result that explicit geologic boundaries and domains are lost. The application of 3D geometry inversion has been illustrated by applying VPmg (in conjunction with Gocad) to several gravity and magnetic data sets, at regional, prospect, and deposit scales.

Finally, all quantitative geophysical interpretation relies on physical property information. In mineral and metalliferous applications, knowledge of the physical properties of the local stratigraphy is often sketchy at best. In the absence of rock property data, pushing the boundaries can become an uphill battle.

Suggested reading. "Lithologic tomography: From plural geophysical data to lithology estimation" by Bosch (*Journal of Geophysical Research*, 1999). "Joint inversion of gravity and magnetic data under lithological constraints" by Bosch and McGaughey (*TLE*, 2001). "Drilling constrained 3D gravity inversion" by Fullagar et al. (*Exploration Geophysics*, 2000). "3D gravity and aeromagnetic inversion for MVT lead-zinc exploration at Pillara, Western Australia" by Fullagar et al. (*Exploration Geophysics*, 2004). "Constrained inversion of geological surfaces—pushing the boundaries" by Fullagar et al. (*Geoscience Australia Record*, 2006). "Towards geologically realistic inversion" by Fullagar and Pears (Fifth Dicennial International Conference on Mineral Exploration, 2007). "Comparison of airborne and ground TEM systems for a conductor beneath conductive cover—An example from North-West Queensland, Australia" by Hart and Lane (*ASEG 2001 Extended Abstracts*). "Physical property measurements on rock samples from the Mt. Isa Inlier, northwest Queensland" by Hone et al. (BMR Report 265, 1987). "3D inversion of magnetic data" by Li and Oldenburg (GEOPHYSICS, 1996). "3D inversion of gravity data" by Li and Oldenburg (GEOPHYSICS, 1998). "Boulia, interpreted geology" by Mackey et al. (AGSO, 1999). "Mapping Australian geology under cover: A case study applied to the Boulia and Springvale 1:250000 map sheets, Queensland" by Mackey et al. (*ASEG Preview*, 2000). "The common Earth model: A revolution in exploration data integration" by McGaughey and Morrison (Conference Proceedings, Mining Millenium International Convention and Technical Exhibition, 2000). "3D gravity modeling and interpretation for the 1:250,000 Boulia map sheet, Queensland" by Pears et al. (*ASEG 2001 Extended Abstracts*). TLE

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