GRAVITY DISTURBANCES IN REGIONS OF NEGATIVE HEIGHTS: A REFERENCE QUASI-ELLIPSOID APPROACH

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ABSTRACT

Compilation of the bathymetrically and topographically corrected gravity disturbance, the so called BT disturbance, for the purpose of gravity interpretation/inversion, is investigated from the numerical point of view, with special emphasis on regions of negative heights. In regions of negative ellipsoidal (geodetic) heights, such as the Dead Sea region onshore or offshore areas of negative geoidal heights, two issues complicate the compilation and subsequently the inversion of the BT disturbance. The first is associated with the evaluation of normal gravity below the surface of the reference ellipsoid (RE). The latter is tied to the legitimacy of the harmonic continuation of the BT disturbance in these regions. These two issues are proposed to be resolved by the so called reference quasi-ellipsoid (RQE) approach. New bathymetric and topographic corrections are derived based on the RQE and the inverse problem is formulated based on the RQE. The RQE approach enables the computation of normal gravity by means of the international gravity formula, and makes the harmonic continuation in the regions of negative heights of gravity stations legitimate. The gravimetric inversion is then transformed from the RQE approach back to the RE approach, following the now legitimate harmonic upward continuation of the gravity data to stations on or above the RE. Stripping, the removal of an effect of a known density contrast, is considered in the context of the RQE approach. A numerical case study is presented for the RQE approach in a region of NW Canada.

Key words: BT gravity disturbance, RQE, normal gravity, harmonic continuation, inverse problem, interpretation

1. INTRODUCTION

This is a follow-up to the paper of *Vajda et al. (2008)*, where we showed by the decomposition of the Earth gravitational potential that the anomalous gravity quantity exactly equal to the attraction of the unknown anomalous density distribution, sought by interpreting gravity, globally enclosed onshore by the relief and offshore by sea bottom is but the bathymetrically and topographically corrected gravity disturbance, the so called 'BT disturbance', where both the topo-correction and the bathymetric correction are based on the reference ellipsoid (RE), instead of based on the sea level (geoid). It was also mentioned that the compilation of the BT gravity disturbance to be correctly interpreted poses some problems for observation points (stations) of negative (geodetic) height. The first problem is associated with the evaluation of normal gravity below the surface of the RE. Inside the RE the international gravity formula (IGF) is no longer valid, instead the internal normal gravity must be evaluated as the attraction of the model normal density distribution. Thus for each such a station the normal gravity must be computed by numerical evaluation of the Newton volume integral for attraction over the model normal density distribution in the RE, which is very inconvenient compared to the use of the IGF. The second problem is even more crucial. Although the direct inversion or forward modeling techniques for solving the gravimetric inverse problem (GIP) can be used at the natural (original) positions of the survey stations (points of observed gravity), they are typically formulated at a level (reference) surface to avoid dealing with the relief and topographical density. Such an approach, however, requires a continuation of the input gravity data ("observables"). We showed in the preceding paper that the harmonic continuation of the 'BT disturbance' in regions of negative ellipsoidal (geodetic) heights is not legitimate. This is a serious problem. This paper is devoted to overcoming this vital problem by means of the so called reference quasi-ellipsoid (RQE) approach, which makes use of a remove-restore technique for the upper layer of the model normal masses of the RE. We urge the reader to consider *Vajda et al. (2008)* as pre-requisite reading, since herein we build on it substantially, also using the same terminology, notation, and background (ibid, Section 2).

2. COMPILATION AND INVERSION OF THE BT GRAVITY DISTURBANCE

In the preceding paper (*Vajda et al., 2008*) we derived the exact match between the attraction of the unknown and sought anomalous masses and the "observable" gravity data

$$\delta g^{BT}\left(h,\Omega\right) = \delta A^{BT}\left(h,\Omega\right) . \tag{1}$$

On the left-hand side of Eq.(1) we have the "observables", the 'BT gravity disturbance'

$$\delta g^{BT}\left(h,\Omega\right)=g\left(h,\Omega\right)-\gamma\left(h,\Omega\right)-A^{ET}\left(h,\Omega\right)-\delta A^{EW}\left(h,\Omega\right), \tag{2}$$

where g is actual gravity observed at station, γ is normal gravity computed at station, $-A^{ET}$ is the topo-correction based on the RE given by *Vajda et al. (2008, Eq.(9) or (16))*, and $-\delta A^{EW}$ is the bathymetric correction based on the RE given by *Vajda et al. (2008, Eq.(10) or (13))*. On the right-hand side we have the attraction of the unknown anomalous density distribution $\delta\rho\left(h,\Omega\right)$ globally enclosed onshore (Ω_L) by the topographic relief $h_T\left(\Omega\right)$ and offshore (Ω_S) by the sea bottom $h_B\left(\Omega\right)$

$$\delta A^{BT}\left(h,\Omega\right)=G\int_{-R}^{h_T(\Omega')}\iint_{\Omega_L}\delta\rho\left(h',\Omega'\right)J\,d\vartheta'+G\int_{-R}^{h_B(\Omega')}\iint_{\Omega_S}\delta\rho\left(h',\Omega'\right)J\,d\vartheta' . \tag{3}$$

The J kernel of the volume integrals is the vertical derivative of the reciprocal Euclidean distance between computation and integration points (*Vajda et al., 2006*), R is the mean Earth radius. The $\delta\rho\left(h,\Omega\right)$ is defined relative to the reference (background) density distribution that consists of a constant average crustal density ρ_0 inside the solid topography, and of model normal density distribution $\rho_N\left(h,\Omega\right)$ inside the RE, cf. Table 1. While in geodesy the $\rho_N\left(h,\Omega\right)$ inside the RE, generating the external normal potential and normal gravity, is deemed unspecified (due to the non-uniqueness of the inverse problem for the external normal gravitational potential), it can be considered zero to some depth below the surface of the RE, making the (gravitational part of) normal potential harmonic there, enabling also the extension of the validity of the closed form or Taylor series expansion formulae for normal gravity (international gravity formula [IGF]) for this interval of negative ellipsoidal heights. On the other hand, in geophysics, we need a $\rho_N\left(h,\Omega\right)$ inside the RE that serves as the reference (background) density defining the

Table 1. Model normal density distributions in the RE and RQE approaches.

Region	Domain	Normal Density Re Approach	Normal Density Rqe Approach
above RE	$h \geq 0$	$\rho_N\left(h,\Omega\right)=0$	$\rho_N^*\left(h,\Omega\right)=0$
inbetween RE and RQE	$h\in\left(-h^*;0\right)$	$\rho_N\left(h,\Omega\right)=\rho_0$	$\rho_N^*\left(h,\Omega\right)=0$
inbetween RQE and deepest sea bottom	$h\in\left(h_B^{max};-h^*\right)$	$\rho_N\left(h,\Omega\right)=\rho_0$	$\rho_N^*\left(h,\Omega\right)=\rho_0$
below the level of deepest sea bottom	$h<h_B^{max}$	$\rho_N\left(h,\Omega\right)$	$\rho_N^*\left(h,\Omega\right)$ $=\rho_N\left(h,\Omega\right)$ $+\delta\rho_N^*\left(h,\Omega\right)$

unknown (sought by inversion) anomalous density $\delta\rho(h,\Omega)$ inside the RE. We assume here that an acceptable approximation to the geophysically meaningful $\rho_N(h,\Omega)$ can be found in terms of a 'model normal density distribution' $\rho_N(h,\Omega)$, which is an (confocal biaxial) ellipsoidally stratified distribution (with variable flattening of the strata) with a step-wise continuous mean-radial ("PREM-like" [PREM being an acronym of the Preliminary Reference Earth Model]) behavior matching that of the Earth (*Moritz, 1968*, *1973, 1990*; *Tscherning and Sünkel, 1981*). Thus the space inside the RE is filled with normal masses, and the normal gravity below the surface of the RE must be computed by evaluating the Newton integral for attraction over $\rho_N(h,\Omega)$, instead of by the IGF (inserting into it a negative ellipsoidal height).

Thus, when compiling the 'BT disturbance', Eq.(2), the normal gravity at stations (h,Ω) on or above the RE is computed using the IGF, but for stations located below the surface of the RE the internal normal gravity inside the RE must be computed at each station by evaluating the Newton volume integral for attraction over the $\rho_N(h,\Omega)$ of the RE, which is quite inconvenient.

Direct inversion and forward modeling techniques, based on Eq.(1), may be applied to interpreting the BT disturbance at the natural location of the stations (h,Ω) at which the gravity data are observed - the relief in the case of ground terrestrial survey, sea surface in the case of marine ship-borne survey, flight tracks in the case of air-borne surveys, eventually borehole and sea bottom observations, or an integration of all of the above - the stations being irregularly placed with respect to a vertical datum, or a level or reference surface. When inverting the BT disturbance at the natural locations of stations, no continuation of the BT gravity disturbance data is needed to any level or reference surface, but the topography with its (remaining anomalous) density and its relief must be included in the inversion or modeling. This has not become a common practice in gravimetric inversion yet. Instead, forward modeling software typically disregards the relief (topography), which gives rise to the need of continuing the 'observables', the BT disturbances, from the natural position of survey stations to a level (reference) surface. Also, the pattern recognition techniques and other techniques utilizing additional transformations of the observables, such as surface convolution integrals, inevitably require the observables be given on a level (reference) surface, in which case the continuation of the data is requisite. Then the legitimacy and numerical performance of a harmonic continuation of the observables becomes crucial.

The 'BT disturbance', in spherical approximation multiplied by geocentric distance $(R+h)$, is harmonic above the topo-surface while at the same time above the RE. In this space it can be harmonically continued using e.g., the Poisson integral (e.g., *Heiskanen and Moritz, 1967*; *Hofmann-Wellenhof and Moritz, 2006, p.247, Eq.(6–44)*) or methods of equivalent sources (e.g., *Ivan, 1994*). Below the topo-surface onshore the $(R+h)\delta g^{BT}(h,\Omega)$ is not harmonic due to the presence of anomalous masses there. In regions of the topo-surface being located below the RE (onshore the relief of negative geodetic heights, like the Dead Sea region, and offshore the sea surface [geoid] of negative geoidal heights) the $(R+h)\delta g^{BT}(h,\Omega)$ is not harmonic in the "free-air" space below the surface of the RE, due to the presence of normal masses $\rho_N(h,\Omega)$ there. In these regions the harmonic continuation of the BT disturbance is not legitimate. The RQE approach presented below remedies the two above discussed issues of compiling and inverting the BT disturbance in regions of negative ellipsoidal (geodetic) heights.

3. REFERENCE QUASI-ELLIPSOID (RQE) APPROACH

The compilation and inversion of BT disturbances at stations of negative heights may be encountered in ground and even air-borne surveys onshore in areas like the Dead Sea region, and offshore in marine ship-borne surveys in areas of negative geoidal heights, cf. Fig. 1.

Eq.(1), on which the direct inversion or forward modeling techniques of interpreting the BT gravity disturbances are based was derived by the decomposition of the actual gravitational potential of the Earth. This decomposition was based on the RE. We now introduce another reference surface, on which we will base the said decomposition, the so called 'reference quasi-ellipsoid' (RQE), cf. Fig. 2. This surface is defined as the surface the depth of which (h^*, reckoned along the ellipsoidal normal) below the surface of the RE is constant, $h(\Omega) = -h^*$. As such, it is a spheroidal, not an ellipsoidal surface, hence the name. The value of h^* is chosen so, that it is just greater than the maximum dip of the topo-surface below the RE elsewhere over the entire globe, e.g., as 500 m. We let the RQE serve as both the lower boundary of the "topographic masses" and the upper boundary of the "normal masses". This implies that we have to define a new model normal density distribution $\rho_N^*(h,\Omega)$ bound by the surface of the RQE, cf. Table 1, which generates the same normal potential and normal gravity in the exterior of the RE as

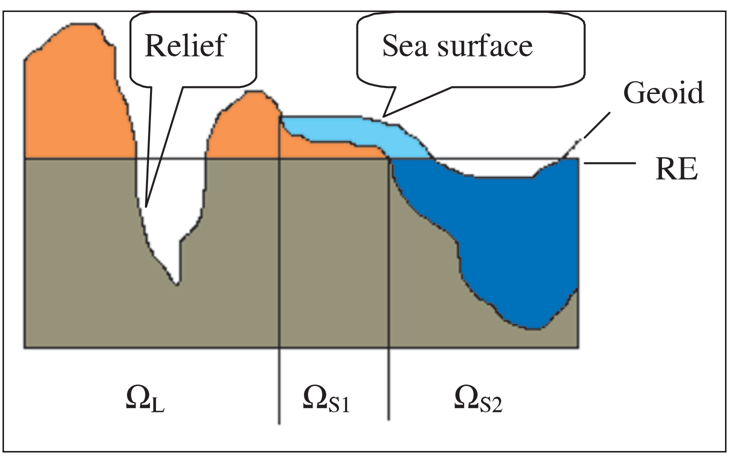

Fig. 1. The regions of negative ellipsoidal (geodetic) heights both onshore and offshore.

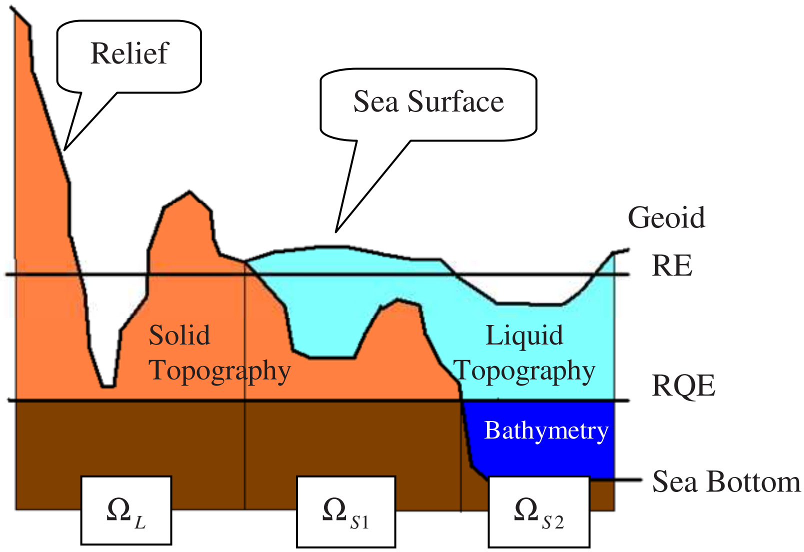

Fig. 2. Solid and liquid topography onshore and offshore based on the RQE.

is generated by $\rho_N(h,\Omega)$ bound by the surface of the RE. We do not require the $\rho_N^*(h,\Omega)$ to meet the geophysical constraints, and we even leave it unspecified. We only require that it generates the given normal gravity above the RE. In this approach the space between the surface of the RQE and the surface of the RE is void of normal masses.

In our approach it is vital that the top layer (stratum) of the model $\rho_N(h,\Omega)$ of the RE consists of constant density equal to ρ_0, cf. Table 1, which is a geophysically reasonable assumption. The normal masses from this (quasi-ellipsoidal) layer are now removed and are placed inside the RQE in the form of unspecified compensating surplus normal masses $\delta\rho_N^*(h,\Omega)$, for which we require that they reside deeper than the level of the deepest sea bottom h_B^{max} (about −11 km) on purposes associated with the bathymetric correction, to be revealed later on. Notice that these compensating surplus normal masses must generate, in the space above the RE, an attraction equal to the attraction of the quasi-ellipsoidal layer of constant density ρ_0 and constant thickness h^*, which in spherical approximation is above the RE (to a good approximation) constant (attraction of a Bouguer shell of constant density ρ_0 and thickness h^*).

The decomposition based on the RQE leads to the following exact link (match) between the observed gravity and the attraction of anomalous density (synthetic gravity)

$$\delta g^{BT*}(h,\Omega) = \delta A^{BT*}(h,\Omega) . \tag{4}$$

On the left-hand side we have the observables, the 'BT disturbance' based on the RQE instead of the RE (distinguished by the asterisk in the superscript)

$$\delta g^{BT*}(h,\Omega) = g(h,\Omega) - \gamma(h,\Omega) - A^{QET}(h,\Omega) - \delta A^{QEW}(h,\Omega) , \tag{5}$$

which is a bathymetrically and topographically corrected gravity disturbance, where the topo-correction is now based on the RQE (distinguished by the acronym "QET" standing for "Quasi-Ellipsoidal Topography, which indicates that the bottom interface of topo-masses is the RQE)

$$\begin{aligned} A^{QET}(h,\Omega) &= G\rho_0 \int_{-h^*}^{h_T(\Omega')} \iint_{\Omega_L} J\, d\vartheta' + G\rho_0 \int_{-h^*}^{h_B(\Omega')} \iint_{\Omega_{S1}} J\, d\vartheta' \\ &+ G\rho_W \int_{h_B(\Omega')}^{N(\Omega')} \iint_{\Omega_{S1}} J\, d\vartheta' + G\rho_W \int_{-h^*}^{N(\Omega')} \iint_{\Omega_{S2}} J\, d\vartheta' , \end{aligned} \tag{6}$$

and where the bathymetric correction is now also based on the RQE (distinguished by the acronym "QEW", "QE" standing for "Quasi-Ellipsoid" and "W" for "water")

$$\delta A^{QEW}(h,\Omega) = G\delta\rho_0 \int_{h_B(\Omega')}^{-h^*} \iint_{\Omega_{S2}} J\, d\vartheta' . \tag{7}$$

The upper boundary of the bathymetric contrast is the RQE while its lower boundary is the sea bottom globally.

On the right-hand side of Eq.(4) we have the attraction of the unknown anomalous masses, sought for in the gravimetric inversion,

$$\delta A^{BT*}(h,\Omega) = G \int_{-R}^{h_T(\Omega')} \iint_{\Omega_L} \delta\rho^*(h',\Omega') J\, d\vartheta' + G \int_{-R}^{h_B(\Omega')} \iint_{\Omega_S} \delta\rho^*(h',\Omega') J\, d\vartheta' . \tag{8}$$

The unknown anomalous density distribution (density contrast) $\delta\rho^*(h,\Omega)$, globally enclosed onshore by the relief and offshore by the sea bottom, is in the RQE approach defined anew, that is why the asterisk. It is defined inside the RQE relative to the (unspecified!) $\rho_N^*(h,\Omega)$, and above the RQE, within solid topography, relative to ρ_0, cf. Table 1. Obviously, to solve for a density contrast which is defined relative to an unspecified reference density makes no sense. We shall use the RQE approach only to remedy the two issues with the normal gravity and with the harmonic continuation of the BT disturbance, then we will switch the formulation of the gravimetric inverse problem back to the RE approach, where the unknown and sought anomalous masses are defined relative to the known reference (background) density $\rho_N(h,\Omega)$.

The great advantage of the RQE approach is that it makes the space between the surfaces of the RE and RQE void of normal masses. Consequently, the gravitational part of the normal potential becomes harmonic here, (1) extending the validity of the closed form or series expansion formulae for normal gravity (IGF) into the interval of negative ellipsoidal (geodetic) heights $h \in \left(0; -h^*\right)$; (2) the RQE based 'BT gravity disturbance' (in spherical approximation multiplied by geocentric distance $R + h$) becomes harmonic everywhere above the topo-surface, even in areas, where the topo-surface dips below the RE. The $(R+h)\delta g^{BT*}$ can be harmonically continued using either the Poisson integral (e.g., *Heiskanen and Moritz, 1967*; *Vaníček et al., 1996*; *Hofmann-Wellenhof and Moritz, 2006*) or the equivalent sources method (e.g., *Ivan, 1994*). The numerical aspects of this continuation are, however, considered beyond the scope of this paper, and are left for a separate work.

4. RESTORING THE INVERSION FROM RQE APPROACH BACK TO RE APPROACH

After upward harmonic continuation of the δg^{BT*} gravity data from regions of negative ellipsoidal heights of the stations to stations on or above the RE, the gravimetric inverse problem becomes formulated on or above the RE using the RQE based BT disturbance

$$\forall h \geq 0: \qquad \delta g^{BT*}(h,\Omega) = \delta A^{BT*}(h,\Omega). \tag{9}$$

Let us consider the attraction $A^{QELC}(h,\Omega)$ of a "Quasi-Ellipsoidal Layer of Constant thickness h^* and of constant density ρ_0" (acronym 'QELC' in the superscript), where this layer is bound by the surface of the RE from above and by the RQE from below. In spherical approximation, this attraction (of a Bouguer shell) is constant everywhere on or above the RE, disregarding a negligible height term (e.g., *Vaníček et al., 2001*)

$$\forall h \geq 0: \qquad A^{QELC}(h,\Omega) = \frac{4\pi G\rho_0}{3}\,\frac{R^3 - \left(R-h^*\right)^3}{(R+h)^2} \approx 4\pi G\rho_0 h^*. \tag{10}$$

For $h^* = 500$ m and $\rho_0 = 2.67$ g/cm^3, the $A^{QELC}(h, \Omega) = 112$ mGal and the neglected height term amounts to 35 µGal/km (e.g., *Meurers and Vajda, 2006*). Let us add this term (attraction) to both sides of Eq.(9)

$$\forall h \geq 0: \qquad \delta g^{BT*}(h,\Omega) + A^{QELC}(h,\Omega) = \delta A^{BT*}(h,\Omega) + A^{QELC}(h,\Omega). \tag{11}$$

Recalling what was said in Section 3 about the unspecified compensating surplus normal masses $\delta\rho_N^*(h,\Omega)$ and the condition they must obey, we have

$$\forall h \geq 0: \qquad A^{QELC}\left(h,\Omega\right) = G\rho_0 \int_{-h^*}^{0} \iint_{\Omega_0} J \; d\vartheta' = G \int_{-R}^{-h^*} \iint_{\Omega_0} \delta\rho_N^*\left(h',\Omega'\right) J \; d\vartheta' \; . \quad (12)$$

With the use of the above, the left-hand side of Eq.(11) becomes

$$\forall h \geq 0: \qquad \delta g^{BT*}\left(h,\Omega\right) + A^{QELC}\left(h,\Omega\right) = \delta g^{BT}\left(h,\Omega\right), \qquad (13)$$

i.e., the addition of the constant A^{QELC} term transforms at stations on or above the RE the BT disturbance based on the RQE to that based on the RE. Similarly, for the right-hand side of Eq.(11) we obtain

$$\forall h \geq 0: \qquad \delta A^{BT*}\left(h,\Omega\right) + A^{QELC}\left(h,\Omega\right) = \delta A^{BT}\left(h,\Omega\right), \qquad (14)$$

since the addition of the constant A^{QELC} term restores the removed quasi-ellipsoidal layer of normal masses that was compensated in the form of the surplus normal masses $\delta\rho_N^*\left(h,\Omega\right)$, cf. Section 3 and Table 1, transforms the $\rho_N^*\left(h,\Omega\right)$ of the RQE into the $\rho_N\left(h,\Omega\right)$ of the RE, and transforms the sought anomalous masses $\delta\rho^*\left(h,\Omega\right)$ into $\delta\rho\left(h,\Omega\right)$. Consequently,

$$\forall h \geq 0: \qquad \delta g^{BT}\left(h,\Omega\right) = \delta A^{BT}\left(h,\Omega\right). \qquad (15)$$

Thus the formulation of the inverse problem is restored back to the RE approach, this time for stations on or above the RE only.

To sum it up, in "free-air" regions below the surface of the RE (stations of negative geodetic heights) the preparation of the gravity data and their interpretation should follow the below steps:

(1) compilation of the RQE based BT gravity disturbances, where both the topographic and bathymetric corrections are computed based on the RQE;
(2) upward harmonic continuation of the RQE based BT gravity disturbances to a level surface above the RE;
(3) addition of the constant term (A^{QELC}), which transforms the data to become the RE based BT gravity disturbances;
(4) interpretation of the BT disturbances (the resulting anomalous masses are inside the RE relative to the model normal density distribution $\rho_N\left(h,\Omega\right)$).

5. ADDITIONAL STRIPPING STEPS

Stripping corrections can be computed in the RQE approach analogically to the stripping corrections based on the RE approach. In regions, where gravity stations are of negative heights, the RQE based 'stripped BT disturbance' can be harmonically upward continued onto or above the RE. However, it is simpler to upward continue the RQE based BT disturbance, convert the inverse problem to the RE approach, and then to correct the RE based BT disturbance by the stripping corrections (based on the RE) at the new stations residing now on or above the RE.

6. NUMERICAL CASE STUDY

To illustrate the global topographic and bathymetric corrections based on the RQE we evaluate them numerically in a region, where the corrections due to solid and liquid topography are expected to be significant. Our selected test area extends from 47° to 68° in northern latitudes and from 216° to 260° in eastern longitudes, covering thus the major part of the Pacific Coast Range including the Canadian Rocky Mountains. This region covers both rugged mountains and ocean. Orthometric heights there reach 5 959 m at Mt. Logan. Fig. 3 shows the mean $5' \times 5'$ relief onshore and sea bottom offshore, both referred in ellipsoidal (geodetic) heights (not in sea-level-based heights/depths) obtained by adding global geoidal (EGM'96) heights to the sea-level-based heights/depths. Fig. 4 shows the EGM'96 geoid (*Lemoine et al., 1998*) in our study region. The mean ($5' \times 5'$) gravity disturbances evaluated on the topo-surface are shown in Fig. 5. In the sequel we plot all the gravity data and corrections in mGal, 1 mGal = 10^{-5} m/s^2. These mean ground gravity disturbances range in our area from −166 to +331 mGal (with a statistical mean of −6.1 mGal). Note, that they are just disturbances, compiled as actual gravity on topo-surface minus normal gravity on topo-surface, no topographic correction yet.

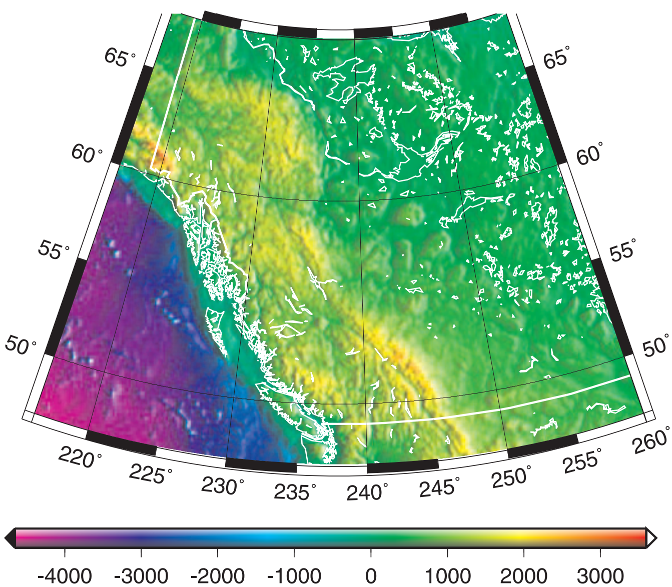

Fig. 3. The mean $5' \times 5'$ relief onshore and sea bottom offshore in our case study region. Both are given in ellipsoidal (geodetic) heights (m). Ellipsoidal heights are approximate, obtained by adding global geoidal heights (EGM'96) to the sea-level-based (orthometric) heights (and depths) of both the relief and sea bottom.

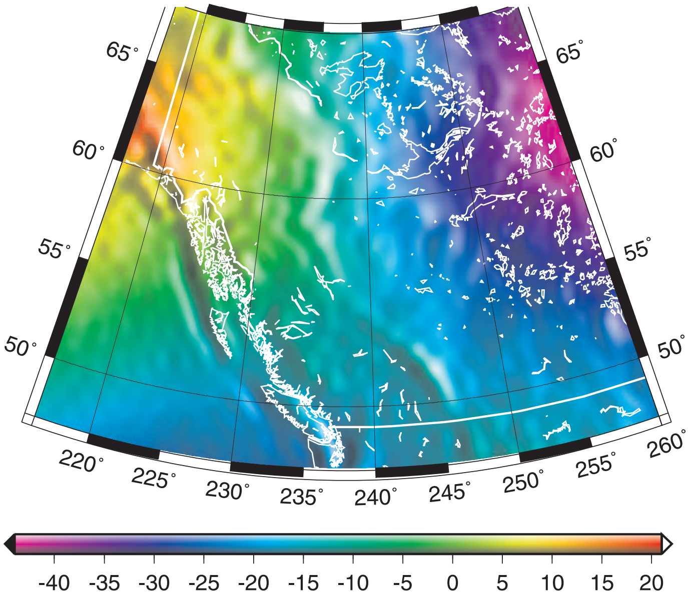

Fig. 4. The EGM'96 geoidal heights in our case study region.

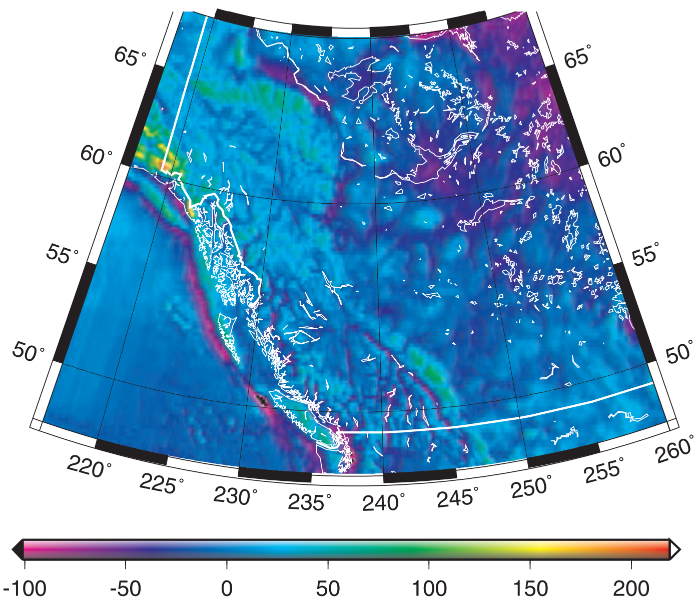

Fig. 5. The $5' \times 5'$ mean gravity disturbances given on the topo-surface (relief onshore, sea level offshore) in our study region (mGal). These ground gravity disturbances are just actual gravity on topo-surface minus normal gravity on topo-surface, no topographic correction yet.

Our RQE based corrections below adopt an RQE defined by $h^* = 500$ m. Next we shall consider the RQE based topo-correction given by Eq.(6). Its four individual terms are computed by numerical integration over the whole globe without truncating the integration domain to a spherical cap, which has not become in geophysical applications a standard practice yet (see also *Mikuška et al., 2006*). We do not split the volume integrals into a shell term and a terrain term. The J kernel of the volume integrals is evaluated in a spherical approximation as per Eq.(3) of *Vajda et al. (2008)*. The volume integrals here are evaluated in spherical approximation, but it is possible to evaluate them also exactly in ellipsoidal geometry (*Novák and Grafarend, 2005*; *Vajda et al., 2004*), which is not our objective here. The volume integrals are evaluated for computation points on the topographic surface on a $5' \times 5'$ grid. Gravity data and a $3'' \times 3''$ DTM of the Geodetic Survey Division of Natural Resources of Canada were used. The integration domain is split for each computation point into a rectangular $3° \times 3°$ near zone, in which $30'' \times 30''$ and $5' \times 5'$ mean DTM heights generated from the $3'' \times 3''$ DTM are used, and the far zone (the remainder to full globe), in which $30' \times 30'$ (obtained by averaging the $5' \times 5'$) ETOPO5 (of the National Geophysical Data Center of the NOAA) global topo heights and global sea depths are used. The J kernel changes slowly in the far zone, allowing the use of the $30' \times 30'$ integration grid there. The EGM96 (*Lemoine et al., 1998*) geoidal heights (global solution in spectral form of degree and order 360) were used for computing the GRS-80 related ellipsoidal (geodetic) heights of the relief and depths of the sea bottom. For the 'solid topography' the constant density of 2.67 g/cm^3 was assumed and for the 'liquid topography' that of 1.03 g/cm^3.

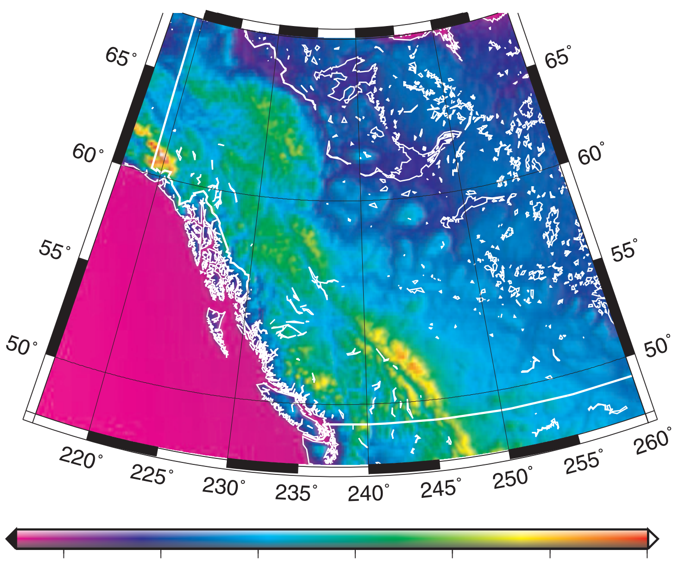

Fig. 6. The attraction A^{QET}, as per Eq.(6), of the global topography (both 'solid' and 'liquid'), reckoned from the RQE, in our study region (mGal). It is evaluated on the topo-surface.

The attraction of the 'solid' and 'liquid' global topography (RQE based), given by Eq.(6), computed on the topo-surface, is shown in Fig. 6. Within our study region it ranges from +75 to +515 mGal with a mean of +170 mGal. Fig. 7 shows the contribution of the 'liquid topography' to this attraction, which ranges from +13 to +37 mGal (with a mean of +18 mGal). The "shallow" waters (sea bottom above RQE), i.e., the third term on the right-hand side of Eq.(6), contribute only a fraction to this sum. The contribution of the 'solid topography' is not shown separately, as it represents the bulk of the attraction of the global topography, and quite closely resembles Fig. 6.

Fig. 8 shows the topo-corrected (RQE based) gravity disturbance given by the first three terms of Eq.(5) evaluated on the topo-surface in our test area. In other words it is a ground gravity disturbance from which the attraction (evaluated also on the topo-surface) of the global 'solid' and 'liquid' topography (reckoned from the RQE) was removed, using the reference (constant average crustal) density for the 'solid topo' and the real (constant) density of sea water for the 'liquid topo'. Within the study area it ranges from −339 to +67 mGal (with a mean of −176 mGal). Notice the pronounced low on the continent, which is due to two causes. The first one is the fact that the RQE based topo-correction removes more topo-masses than usual, as the lower boundary of the topo-masses is the surface of the RQE which is by h^* (in our case 500 m) deeper than the surface of the RE. The second cause is the isostatic roots of the mountains. Notice also the pronounced signature of the subduction zone of the oceanic plate. The signal caused by isostasy could be removed by an additional stripping correction, an isostatic correction, assuming some model of isostatic compensation (not performed here).

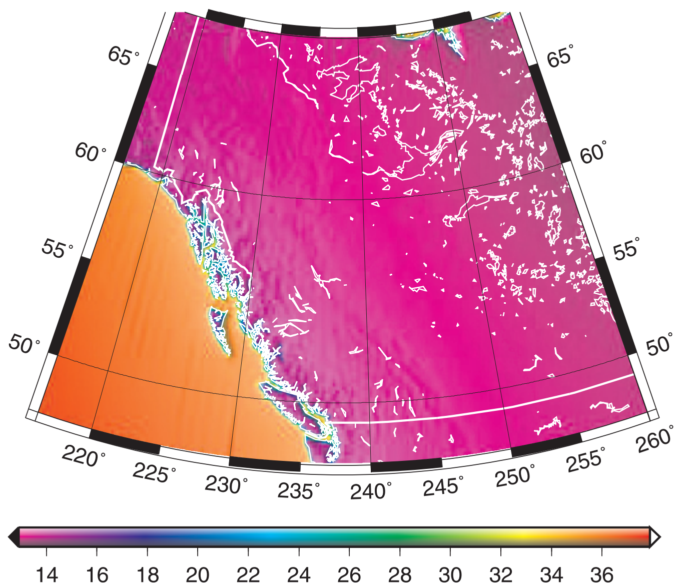

Fig. 7. The attraction of the global 'liquid topography', i.e., the last two terms of the right-hand-side of Eq.(6), in our study region (mGal). It is evaluated on the topo-surface.

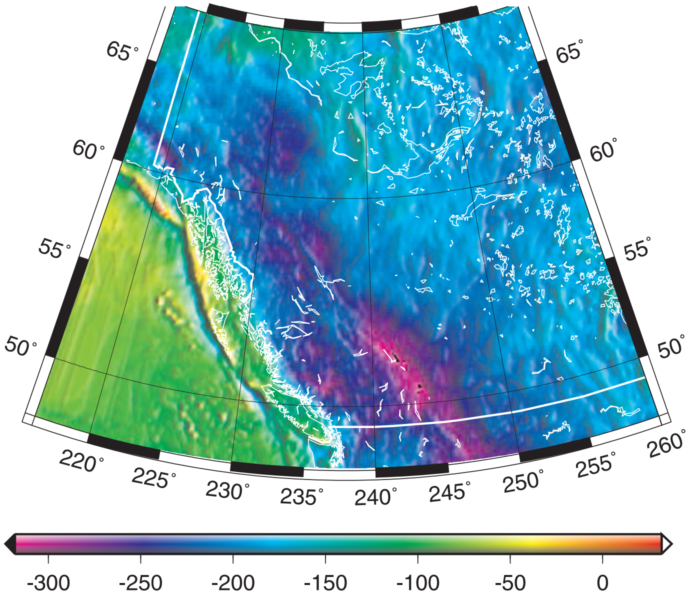

Fig. 8. The topo-corrected (RQE based) gravity disturbance evaluated on the topo-surface in our study region (mGal).

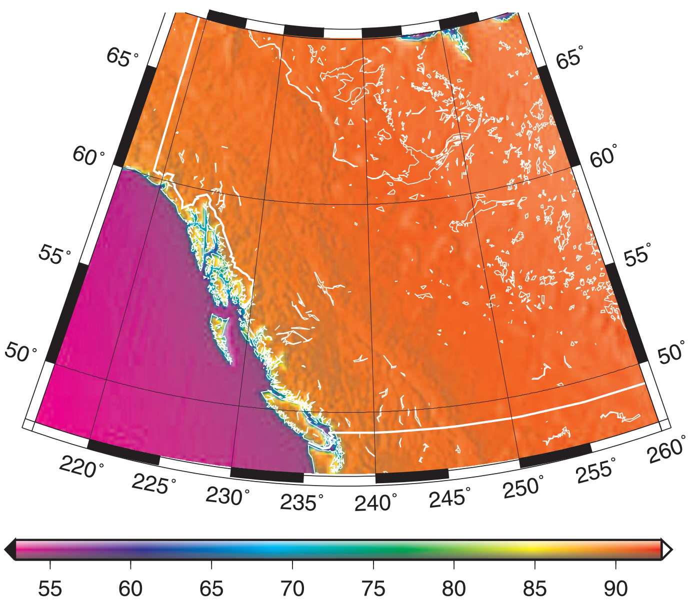

Fig. 9. The difference between the attraction of the conventional global topography (reckoned from sea level) and the attraction of our global 'solid' and 'liquid' topography reckoned from the RQE (cf. Fig. 6) evaluated on the topo-surface in our study region (mGal).

To compare the global "conventional" (classical, traditional) topographic correction, which adopts sea level (geoid) as the lower boundary of topographic masses (the relief onshore is referred in orthometric heights) and knows no 'liquid topography' offshore, with our RQE based topo-correction that considers the 'solid' and 'liquid' global topography reckoned from the RQE, we show the difference of the two in Fig. 9. Within our study region the difference ranges from +53 to +91 mGal (with a mean of +84 mGal).

Fig. 10 shows the bathymetric correction ($-\delta A^{QEW}$) given by Eq.(7), i.e., the (negative) attraction of the sea water density contrast down to the sea bottom, relative to the reference density inside the RQE. Down to some 11 km (the level of the deepest ocean) we assume the reference (background) density for the bathymetric density contrast to be constant and equal to 2.67 g/cm^3, cf. Table 1, yielding a constant bathymetric density contrast $\delta\rho_0 = -1.64$ g/cm^3. The bathymetric correction is evaluated on the topo-surface in our test area. Due to the bathymetric density contrast being negative, its attraction is negative, too, and the bathymetric correction is positive. In our study region the bathymetric correction attains a max of +380 mGal, a min of +70 mGal, and a mean of +107 mGal. Over the continental region the bathymetric correction is almost constant (~75 mGal). Its offshore features are highly correlated with the depths of the ocean (relief of the ocean bottom), as expected. It produces significant variations onshore close to the coastline.

Fig. 11 shows the 'bathymetric-topographic gravity disturbance', δg^{BT*}, the so called 'BT disturbance', based on the RQE, given by Eq.(5). The δg^{BT*} is compiled at the topo-surface. In our study region it attains a max of +301 mGal, a min of −263 mGal, and a mean of −69 mGal.

In our case study region there is a portion of the grid of computation points (stations) lying below the surface of the RE, namely a portion of the grid offshore where the geoidal heights are negative, cf. Fig. 2. In this portion of the grid, we should harmonically upward continue the δg^{BT*} onto the RE. We do not perform this step here, because we wish to leave the numerical aspects of the upward harmonic continuation for a separate work, comparing various numerical techniques. The next step would be adding a constant term of 112 mGal (the attraction, in spherical approximation, of a Bouguer shell of thickness $h^* = 500$ m and density $\rho_0 = 2.67$ g/cm^3) to the map of the δg^{BT*}. These latter two steps transform the RQE based 'BT disturbance' at stations on and above the RE, the δg^{BT*}, into the RE based 'BT disturbance', the δg^{BT}. The δg^{BT} is then used in the gravity data inversion or interpretation as described in Sections 4 and 2.

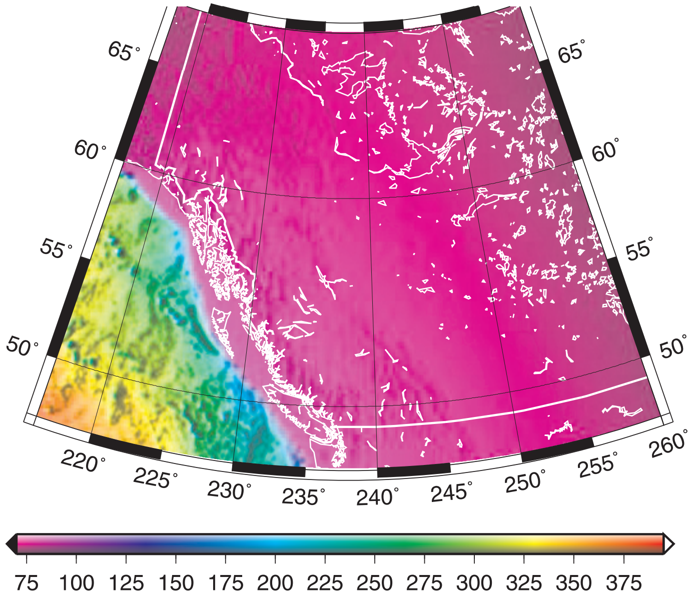

Fig. 10. The RQE based bathymetric correction ($-\delta A^{QEW}$) given by Eq.(7), evaluated on the topo-surface in our study region (mGal).

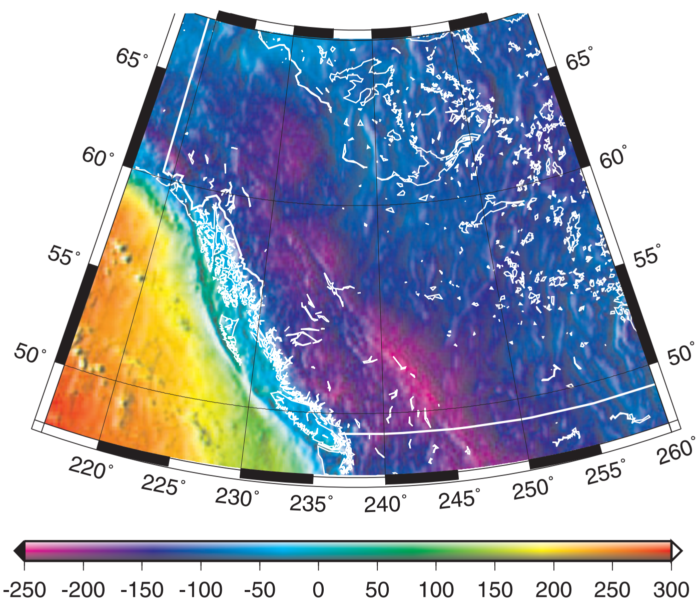

Fig. 11. The RQE based BT disturbance (δg^{BT*}) given by Eq.(5), evaluated on the topo-surface in our study region (mGal).

7. CONCLUSIONS

We have focused here at the occurrence of situations in interpreting observable anomalous gravity data, where the observation points (stations) (h, Ω), at which the matching (or direct inversion) takes place, have negative ellipsoidal heights, e.g., stations offshore at sea level of negative geoidal heights, or stations onshore in regions such as the Dead Sea. For stations of negative heights the normal gravity can no longer be evaluated using the international gravity formula (IGF). It has to be evaluated using the Newton integral over the $\rho_N(h,\Omega)$ inside the RE. Also the $(R+h)\delta g^{BT}(h,\Omega)$, i.e., the bathymetric-topographic gravity disturbance multiplied by geocentric distance (in spherical approximation), is no longer harmonic in the "free-air" regions below the surface of the RE due to the presence of normal masses inside the RE. So it cannot be harmonically continued in these regions. These two problems can be overcome by using the RQE approach described in this paper, which removes and later restores the normal masses between the RE and RQE surfaces.

The RQE approach by itself is not directly useful for gravity data inversion or interpretation, since the RQE based BT disturbance, δg^{BT*}, matches the attraction of anomalous masses $\delta\rho^*(h,\Omega)$ that are inside the RQE defined relative to an unspecified and geophysically meaningless normal density distribution $\rho_N^*(h,\Omega)$. However, the RQE approach helps us to overcome the harmonic continuation difficulty of the RE approach at stations of negative heights. If the compilation of gravity data and their inversion or interpretation takes place in regions of negative ellipsoidal heights, we compile the δg^{BT*} based on the RQE instead of the δg^{BT} based on the RE. The "pros" of the RQE approach are that now not only the normal gravity may be computed using the IGF even for stations below the RE (down to the RQE), but also that the $(R+h)\delta g^{BT}(h,\Omega)$ can now be (in spherical approximation) upward harmonically continued from "free-air" regions below the surface of the RE to stations on or above the RE. After continuing all the "observed" gravity data to points (stations) on or above the RE, we can transform the gravimetric inverse problem back to the RE approach, as described in Section 4, having the sought unknown anomalous masses (density contrast) defined relative to the known and geophysically meaningful model normal density distribution $\rho_N(h,\Omega)$ inside the RE.

Although the stripping corrections may be applied in the RQE approach as well as in the RE approach, as described in the preceding paper (*Vajda et al., 2008*), instead of upward continuing the RQE based 'stripped BT disturbances' it is simpler to first upward continue the RQE based 'BT disturbance' to new stations on or above the RE, transform the RQE based BT disturbances to the RE based ones, and only then apply the stripping corrections now based on the RE, as described in (ibid).

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