CHAPTER 5

Numerical Calculation of Terrain Correction Within the Bouguer Anomaly Evaluation (Program Toposk)

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5.1 INTRODUCTION

The new software Toposk was developed as part of the research project "Bouguer anomalies of new generation and the gravimetrical model of Western Carpathians" funded by the Slovak Research and Development Agency. One of the main tasks of this project was the unification and recalculation of the Slovak gravimetric database, which today contains approximately 320,000 points. Although our new program was designed primarily for the territory of Slovakia, it was later modified for a wider use, e.g., various coordinate systems and arbitrary subzone divisions were incorporated. There is also a possibility of using an arbitrary local orthogonal coordinate system for the inner zones calculation, which enables it for a global usage inside the continental areas. This version does not calculate bathymetric effects, as they relate to the territories outside the standard distance of 166.7 km in the case of inland countries like Slovakia. At the present time, we are developing a universal modification for worldwide calculations, which will also incorporate bathymetric corrections.

The program enables one to keep the real position of the calculated (measured) point with regard to the topographic surface, i.e., above, on or below it. It is relevant for example in the case of correcting the measured vertical gradient of gravity (VGG). There is also an option for using a density grid instead of a constant density of the topographic masses.

We have performed several numerical tests in order to check the quality of the computing algorithm. We have used simple synthetic

Understanding the Bouguer Anomaly. DOI: http://dx.doi.org/10.1016/B978-0-12-812913-5.00004-X

topography models represented e.g., by a cone, paraboloid, and others within these tests.

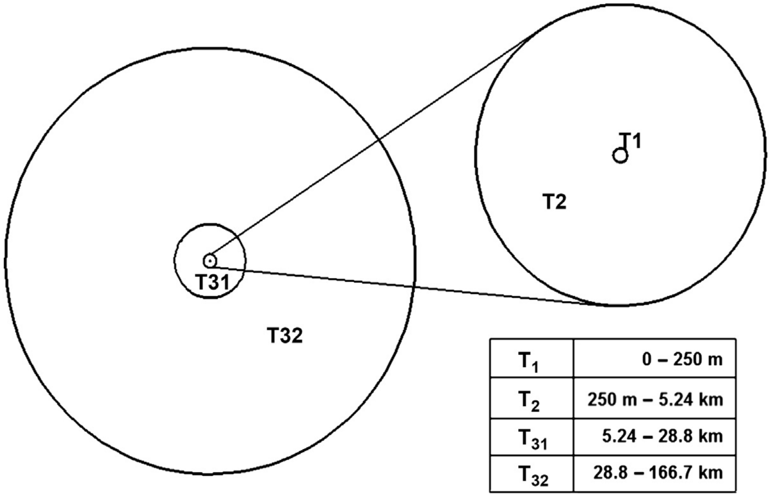
The software was programmed in C++ for 32 or 64 bit Windows applications.

5.2 MAIN FEATURES OF THE NEW SOFTWARE TOPOSK

The main idea is the straightforward calculation of the gravitational effect of the topographic masses, which we call the topographic effect (sometimes, it is called the "mass correction," e.g., Hammer, 1974 or Meurers et al., 2001b). The terrain correction is then derived from the topographic effect by its subtraction from the gravitational effect of the truncated spherical layer (or vertical cylinder in the planar approach within the inner zones). The relationship between the masses considered in terrain correction and topographic effect evaluation, respectively, is clearly demonstrated in Fig. 5.1.

We perform the calculation up to the standard angular distance 1°29′58″ (approximately 166.7 km), which is the outer limit of zone O of the Hayford–Bowie system (Hayford and Bowie, 1912). The dividing of this calculated area into several zones follows the traditional approach originated in former Czechoslovakia, where Pick et al. (1960) defined inner or local zone up to 5.24 km (the outer limit of the zone H of the Hayford–Bowie system), which was later (Bližkovský et al., 1976) divided into the zones T1 (square 500 × 500 m; today, we use circular zone with radius 250 m instead of that square area) and T2 (up to 5240 m). Outer zone was later modified to the zones T31 (5240–28,800 m) and T32 (28.8–166.7 km) after Mikuška and Grand (1989). The radius 28,800 m corresponds to the outer limit of the zone L of the Hayford–Bowie system. Different digital elevation models (DEMs), with increasing resolution toward the calculation point, are used within particular zones. This system (Fig. 5.2) was proven as a

Figure 5.1 Relation between the masses considered in terrain correction (left) and topographic effect (right) calculation at point P.



T_1	0 – 250 m
T_2	250 m – 5.24 km
T_{31}	5.24 – 28.8 km
T_{32}	28.8 – 166.7 km

Figure 5.2 Standard system of terrain correction zone divisions used in Slovakia, after Grand et al. (2004).

suitable approach for the terrain corrections recalculation of the Slovak regional gravity database (Grand et al., 2001, 2004), except the inner zone T1, where the DEM used in that work was insufficient. Therefore, we have focused our attention particularly on this zone regarding the more detailed elevation models, as well as the deeper study of the concept of the interpolated heights of the calculation points (Zahorec, 2015). The inner (T1) and intermediate (T2) zones are treated in planar approach (this yields a small, in most cases, negligible error), and DEMs in local orthogonal coordinates are used. Outer zones are treated using a spherical approach; the calculation is based on the DEMs in ellipsoidal coordinates (e.g., ETRS89). The program enables the transformation between known local and global coordinate systems used in the area of Slovakia (Slovak national systems JTSK, S-1942, global systems UTM 33/34 and ETRS89/WGS84). We use a system of normal heights—Kronstad base system Baltic after adjustment (Bpv). Transformation between direct measured ellipsoidal heights from GNSS (in ETRS89, ellipsoid GRS80) to this local height system Bpv is made using Slovak local quasigeoid DVRM (Klobušiak et al., 2005) related to the same ellipsoid GRS80. The mentioned subdivision and the limits of the individual zones can be changed by the user.

The calculation can be performed utilizing a set of scattered points of known elevation (acquired, e.g., by in situ geodetic measurements; e.g., Lyman et al., 1997; LaFehr et al., 1988) instead of a standard DEM grid, using a set of triangular facets created by means of the triangulation method of Joe (1991). This option was introduced because of the possible imperfection of gridding processes in the case of irregularly distributed elevation (measured) data. However, this approach is more time consuming than the standard calculation with a DEM grid. There is a check box *Or use toposk database* just under the selection box *Elevation grid* in the T1 and T2 tabsheet for this option (Fig. 5.6).

Although various numerical approaches can be used for the calculation of the topographic effect within particular zones, the formula of Pohánka (1988) for a 3D polyhedral body is preferred. This formula makes the calculation of the topographic effect possible in an arbitrary point, so also inside the topographic masses. This property is very useful for the calculation of the effect of topography related to VGG measurements, underground gravity measurements, and others. It enables us also to calculate the topographic effect exactly in the gravity meter sensor position, which is required for the case of precise geodetic absolute gravity measurements.

The calculation is performed by default for a given constant density of the topographic masses (e.g., 2.67 g/cm^3). But there is also an option to make the calculation for variable densities defined by a density grid, if it is available. In this case, the topographical effect of each segment (prism) within the particular zones is calculated using the supplied density. The total computational time for all zones in today's common PC (CPU 3.4 GHz) is less than one second per point.

5.3 INNER ZONE T1

Our standard radius for the inner zone T1 is 250 m. The topography within this zone is approximated by one 3D polyhedral body (see an example in Fig. 5.3), the gravitational effect of which is calculated using the formula of Pohánka (1988). The upper surface of the polyhedral body is created by a triangular net constructed from the elevation data, which are usually in a grid format. The transition between the rectangular grid and the circular boundary of the zone is realized by

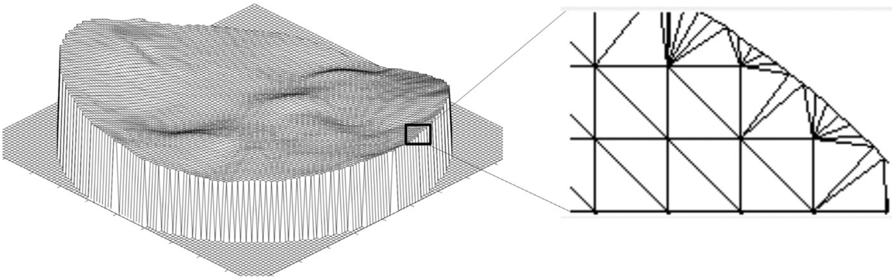


Figure 5.3 Example of 3D polyhedral body approximation of the topography within the inner zone T1 and a detailed sketch of the transition area along its boundary.

additional 360 points interpolated from the grid along the zone boundary (Fig. 5.3). The interpolation method is at choice (see next section), a bilinear interpolation as default.

The terrain correction is obtained as the difference between the gravitational effect of the vertical cylinder (with the same radius and the height equal to the calculation point elevation) and the calculated topographic effect. During the recalculation of the regional gravimetrical database, we used a high resolution DEM with grid cell size of 10 m (DMR-3, Topographic Institute, 2012), which is considerably better than the previous models. However, detailed or microgravity surveys need even greater detail for the DEM (perhaps 1×1 m, which typically can only be obtained from mentioned in-field surveying).

The interpolated heights of the calculation points can be used instead of the measured heights for the inner zone terrain corrections. This approach reduces the errors resulting from the discordance between the real (measured) and the model heights of the calculation points. Those height differences can achieve 100 m or even more in mountainous areas, which leads to errors of several mGal within terrain corrections for zone T1. We have made a detailed study of the optimal distance for use of interpolated heights. The tests with real data as well as synthetic topography models suggest that the optimal distance depends on the quality of the DEM. In most cases, it could be less than 250 m (Zahorec, 2015). Anyway, by using interpolated heights of calculation points to a distance of 250 m, we observe lower errors than when using measured heights.

5.4 INTERMEDIATE ZONE T2

We use the standard extent for this zone 250–5240 m from the point of calculation. The topography within this zone can be approximated by two methods: by a set of triangular prisms or by a set of segments of a vertical cylinder. The first method is the analogy of the polyhedral body method applied within the inner zone T1, where the gravitational effect of the prism is calculated by Pohánka's formula. For the second method, we can designate using the classic or template approach, as can be seen in Fig. 5.4. Both methods give similar results in the case of detailed DEM but the classic method is considerably faster.

The heights of particular segments (prisms) are estimated by one of the standard interpolation methods, e.g., bilinear interpolation, average, bicubic spline, and others. We leave the choice of interpolation method to user experience and standardly use a bilinear interpolation because it is simple and sufficient in the case of detailed elevation grids. The segment size should be approximately the same as the grid cell size of the DEM. Today, we typically use elevation grids with cell sizes of 30×30 m. The terrain correction T2 is obtained as the difference between the gravitational effect of the hollow vertical cylinder (with the corresponding inner and outer radius and with height equal

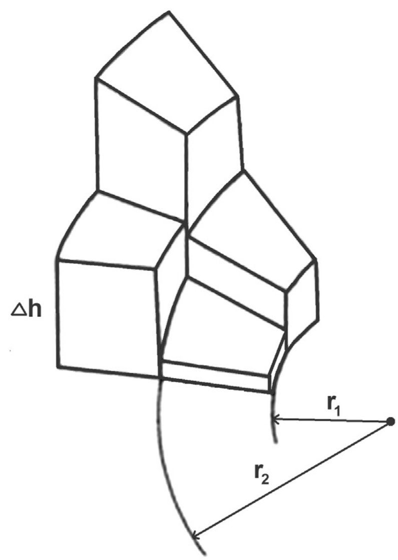


Figure 5.4 "Classic" approximation of topography using vertical cylinder segments, r1 and r2 are inner and outer radii of the particular segment, Δh is the mean height of the segment

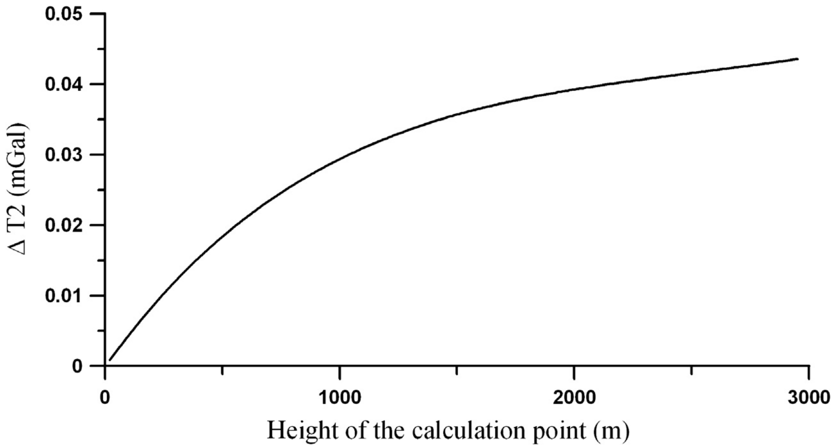


Figure 5.5 Difference between the spherical and planar concepts of the terrain corrections T2 (2.67 g/cm^3).

to the calculation point elevation) and the calculated topographic effect. We have estimated the error resulting from the planar approximation within the intermediate zone. We compared the planar and spherical approaches to the calculation of the terrain correction T2 on a set of synthetic calculation points covering the elevation range that occurs in Slovakia. An elevation model with constant zero level was used within this test to obtain maximal T2 values. The maximum expected error is in the range of a few tens of μGal for a density 2.67 g/cm^3, see Fig. 5.5.

5.5 OUTER ZONES T31 AND T32

The standard distances which we use for these zones are 5240–28,800 m and 28,880–166,730 m, respectively. The topography can also be approximated by two methods: by the set of triangular prisms, this is similar to the zone T2 but here in a spherical modification (geographic coordinates are transformed to Cartesian, and the position and shape of prisms is adapted to the sphere). The second method is represented by the set of segments of the spherical layer, where their effects are calculated by the formula of Mikuška et al. (2006). Both methods give virtually the same results, but the latter method is faster. The elevation grids based on ellipsoidal coordinates WGS84 or ETRS89 are used, e.g., from SRTM data (Jarvis et al., 2008). Today, we use grids $3 \times 3''$ for T31 and $30 \times 30''$ for T32 calculations. The terrain correction within these zones is obtained as the difference between the gravitational effect of the truncated spherical layer (with the corresponding inner and outer radii and the height equal to the calculation point elevation) and the sum of the gravitational effects of all segments in the given zone.

The calculation of the gravitational effect of each segment is made for a given constant density, or for a density interpolated from the density model (grid). Such option is available for all zones.

5.6 USER INTERFACE

The user interface is shown in Fig. 5.6. The upper section is designed for the choice of input file, coordinates type, and the density. The program enables to check the correctness and range of known coordinates types.

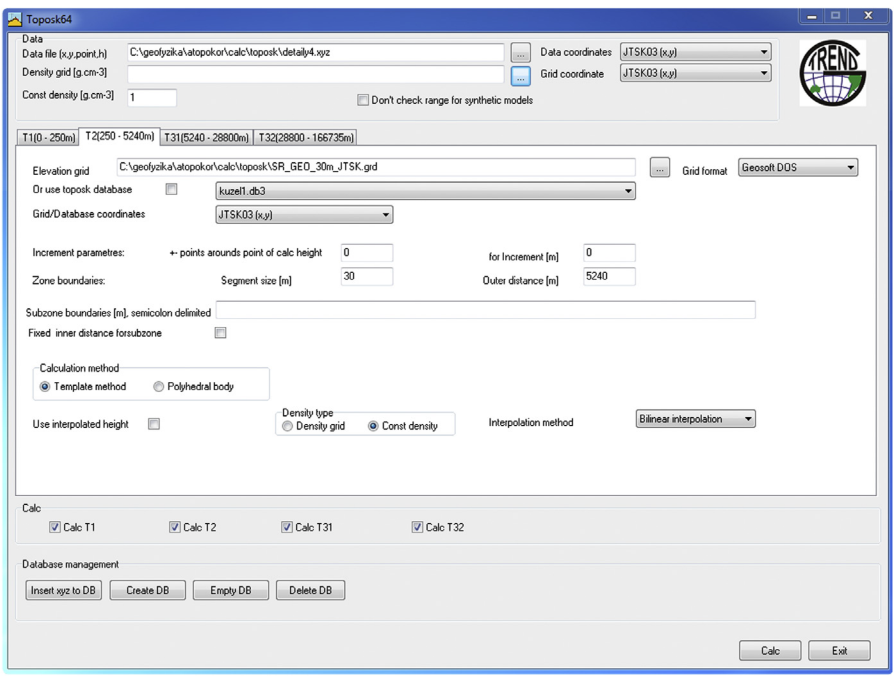


Figure 5.6 User interface of the software Toposk with active T2 tabsheet as an example. There are pop-up tooltips connected to all check boxes.

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------------------------------------------------------------------------------
Data name.............: C:\geofyzika\atopokor\calc\toposk\database_points.xyz
Data coordinate.......: JTSK03 (x,y)
Increment [m].........: 0.000000
Use interpolate height: NO
Grid name.............: C:\geofyzika\atopokor\calc\toposk\SR_GEO_30m_JTSK.grd
Grid coordinate.......: JTSK03
Grid type.............: Surfer 6 binary
Interpolation method..: Bilinear interpolation
Inner distance [m]....: 250.000000
Outer distance [m]....: 5240.000000
Calculation method....: Template method
------------------------------------------------------------------------------
          X          Y      point        H       H-Hi       NTE2        T2
  -372082.5 -1181817.9      501   1353.179      3.122   35.98968   4.01673
  -368073.6 -1187666.5      502    956.366      2.416   25.93893   1.39551
  -340764.5 -1186926.9      503   1675.012     -1.325   45.89653   3.67710
  -346379.2 -1186669.2      504   1505.837     -0.613   40.93490   3.69174
  -359324.1 -1187170.4      505    929.189      1.276   25.42678   1.00804
  -335894.6 -1186792.1      506   1306.620     -3.726   35.96564   2.60247
  -344292.3 -1177741.4      507   1026.553     -1.276   26.72140   2.91932
  -332727.1 -1174225.0      508    919.482      1.200   25.52640   0.58624
```

Figure 5.7 Example of output file from the T2 zone calculation.

The main section contains four tabsheets, which control the parameters of the calculation for each zone: the elevation model, the zone boundaries, the segment size, the calculation method, the interpolation method, and others. In the bottom section, the user can manage the elevation-data databases (SQLite system) in the case of nongrid calculation.

The input data file must contain the coordinates and elevations of the calculation points. An example of the output file for the zone T2 calculation is shown in Fig. 5.7. There are the calculated values of topographic effect (NTE2—"near topographic effect") and terrain correction T2 for the given density (in mGal), as well as the parameter $H-H_i$ for each point, which is the difference between measured elevation and the elevation interpolated from the DEM. This parameter gives useful information about the quality of the elevation data (DEM) used for the respective terrain correction calculation.

5.7 PROGRAM TESTING ON SYNTHETIC DATA

We have performed several tests to check the quality of the numerical algorithms implemented in the Toposk approach. We focused on simple synthetic models of the topography, the analytical gravitational effects of which are possible to calculate by means of closed formulas (the cone, paraboloid, planar, or spherical layer) and compare them

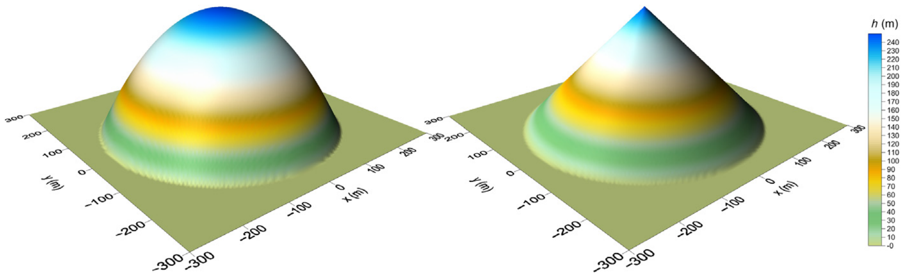

Figure 5.8 Simple topography models approximated by the paraboloid and cone.

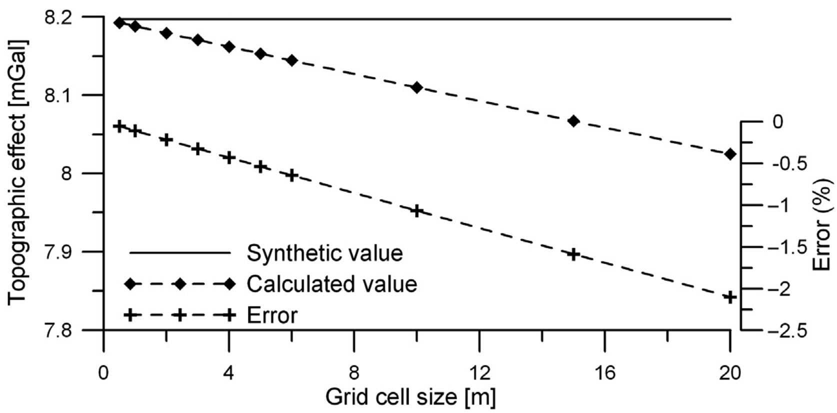

Figure 5.9 Test on conus-like topography showing the calculation error dependence on the model resolution. The calculation point is situated on the cone apex.

with the calculated topographic effects, see Fig. 5.8. We have used common formulas found in the literature for the analytical calculations of the gravitational effect at the cone (paraboloid) apex (e.g., Helmert, 1884; Hammer, 1939; Válek, 1969).

The tests have confirmed that the calculation error is purely a function of the resolution of the model (grid). Highly detailed models (e.g., grid cell size 1×1 m) lead to errors only of few μGal, see Fig. 5.9. The important question was to confirm the accuracy of the calculation in the case when the calculation point is above or below the surface (i.e., inside the topographic masses, e.g., in the case of underground gravity measurements). Programs previously in use in Slovakia did not allow such situations. In Fig. 5.10, we show the results of a comparison of analytical and calculated gravitational (topographic) effect along the

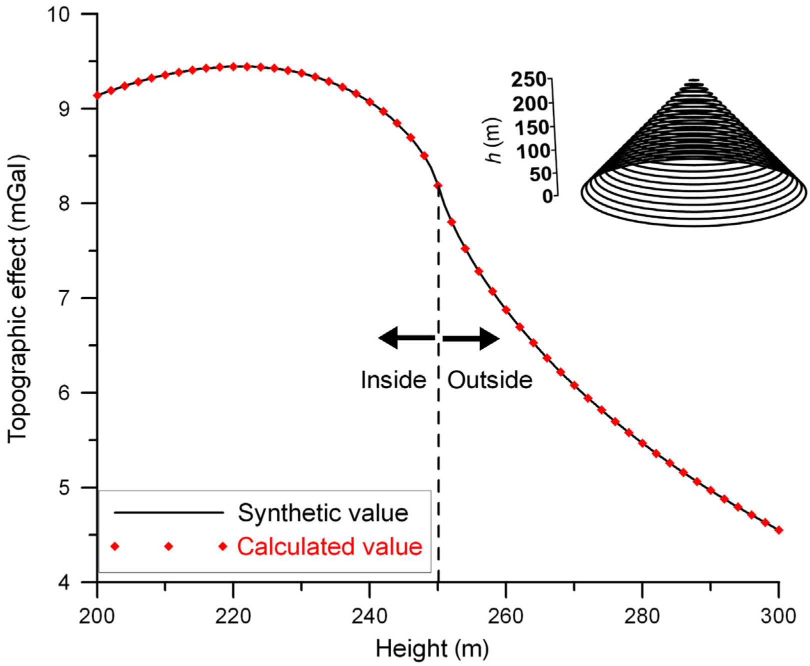

Figure 5.10 Comparison of the analytical value and program Toposk-calculated effects of a cone along its axis. The dashed line shows the calculation point with height of 250 m, lying directly on the cone apex.

cone axis. The analytical values were calculated by the formula published in Meurers (2001a). The results confirm the perfect coincidence of the calculated values in the case of very detailed topography model (e.g., 1×1 m).

5.8 REAL DATA CALCULATIONS

We have proven the program by calculations also on the basis of real measured data. This is particularly important for measurements of VGG (or rather tower VGG, as we determine it by means of the gravity measurements at different height levels) because of the sensitivity to near-station topography (e.g., Zahorec et al., 2014). The measured values of tower VGG can deviate considerably from the "expected" or "normal" value (-0.3086 mGal/m) especially in mountainous regions, as is shown in Fig. 5.11 (black symbols). After allowing for the topographic effect, the corrected values (blue crosses in Fig. 5.11) are considerably closer to the normal gradient value.

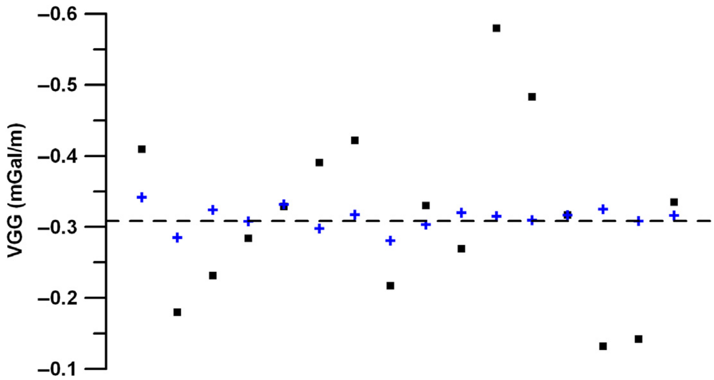

Figure 5.11 Set of field VGG measurements in Slovakia. The black squares represent measured values (Min = −0.580, Max = −0.132 mGal/m), whereas the blue (gray in print versions) crosses represent values after the correction for topography effect (Min = −0.342, Max = −0.281 mGal/m). Dashed line represents the normal VGG (−0.3086 mGal/m).

5.9 CONCLUSIONS

As the most important attribute of the new software Toposk, we have shown the correct calculation of the topographic effect and terrain corrections for the arbitrary point location with regard to the real Earth surface (above or below it). The detailed approximation of topography within the inner zone by 3D polyhedral body has been applied. Alternative computing methods, namely the "classic" segment-based methods and the vertical prism method, are available within the intermediate and the outer zones. Several tests on synthetic topography models proved the correctness of the implemented algorithms. The optional concept of using interpolated heights for the calculation points within the nearest zone is recommended. We have also estimated the maximum expected error resulting from use of a planar approach within zone T2 (up to 5240 m) to a few tens of μGal (for density 2.67 g/cm^3). The user has a choice to either use the default zone division (to four zones with radii 250, 5240, 28,800, and 166,730 m) or define custom zones. The possibility of using various local coordinate systems leads to more universal usage of the software. The use of scattered elevation data instead of an elevation grid is also available, however, this option is more time consuming.

This software was recently used during the recalculation of the complete Slovak gravimetrical database (see Chapter 7, National gravimetric database of the Slovak Republic in this book). The concept of interpolated heights of calculation points was applied within the nearest zone T1 (0–250 m).

ACKNOWLEDGMENTS

The authors are grateful to the Slovak Research and Development Agency APVV (grants No. APVV-0194-10, APVV-0827-12, and APVV-0724-11) and the Slovak Grant Agency VEGA (grants No. 2/0067/12 and 1/0095/12) for the support of the software development. We thank to Allen Cogbill, John Bain, and one anonymous reviewer for improving the manuscript.

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