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Making Euler deconvolution applicable to small ground magnetic surveys

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Abstract

Euler deconvolution is a commonly employed magnetic interpretation method because it requires only a little a priori knowledge about the magnetic source geometry, and, more importantly, because it requires no information about the magnetization vector. As a result, it may be successfully applied in areas where the geology is poorly known. However, it requires a priori knowledge about the nature of an equivalent source producing a magnetic anomaly with the same falloff rate of the observed anomaly. This is a crucial limitation of the method, requiring that a parameter known as structural index (η) be determined. The customary application of the Euler method and the process of estimating η are benefited by having a large number of data and solutions preventing its application to ground magnetic surveys which may consist of a limited number of observations. We show that if the structural index is estimated by a new criterion, Euler deconvolution becomes a feasible technique to interpret anomalies defined by just a few observations. This new criterion is based on the correlation between the total-field anomaly h^o and the estimates of the base level b. These estimates are obtained for each position of the moving data window along the observed profile and for several tentative values for the structural index. However, differently from the customary method, instead of estimating the structural index as the tentative value producing the smallest solution dispersion, the best estimate of η is taken as the tentative value leading to the smallest correlation between h^o and the estimates of b. This criterion is deduced from the Euler's equation, so it does not depend on the inclination and declination of the geomagnetic field. The good results obtained with this new criterion in determining the correct structural index is illustrated in tests using synthetic data from different latitudes and the feasibility in using this criterion in applying Euler deconvolution to ground surveys is illustrated with a real magnetic anomaly defined by just 12 observations and produced by a basic/ultrabasic body at the emerald deposit of Socotó, Bahia, Brazil. The results of Euler deconvolution, combined with the geological information that the basic/ultrabasic body outcrops, show that the intrusive body may be approximated by an outcropping horizontal cylinder with a diameter of 68 m and center at a depth of 34 m, which is consistent with the geologic knowledge of the deposit.

Keywords: Ground magnetic survey; Euler deconvolution; Automatic aeromagnetic interpretations

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1. Introduction

A huge quantity of aeromagnetic data have been collected over the last 40 years, encouraging the development of automatic aeromagnetic interpretation methods during the 1970s and 1980s, as for example, CompuDepth (O'Brien, 1972), Werner deconvolution (Hartman et al., 1971), Naudy's (Naudy, 1971) method and Euler deconvolution (Thompson, 1982). In particular, Euler deconvolution has been applied successfully to interpret not only conventional aeromagnetic data, but also recent high-resolution data (Peirce et al., 1998; Johnson, 1998). The main reason for its success is that it does not require a priori knowledge about the anomalous source magnetization. However, it does require the knowledge of a parameter known as structural index (η), which specifies the kind of magnetic source which is implicitly employed by the method, that is, the interpretation model implicitly used.

Euler deconvolution (Thompson, 1982) is based on the application of Euler's homogeneity equation to a moving data window with η fixed at several tentative values. For each position of the moving data window, a linear system of equations is solved for the anomaly base level (b), and the horizontal (x_o) and vertical (z_o) positions of the equivalent source. As a result, a ''spray'' of possible solutions is produced, each solution being associated with a moving data window position. Thompson (1982) presented a criterion to reduce the number of possible solutions and to estimate, at the same time, the structural index. It consists in selecting (for a fixed tentative structural index), the solutions associated with the estimates of z_o with the smallest standard deviation, and estimating η as the value producing the smallest dispersion of the solutions in the subsurface. One disadvantage of this selection procedure to small surveys is that it requires a large number of possible solutions (each one associated with a position of the moving data window). These restrictions did not prevent the criterion of Thompson (1982) to be widely employed (Reid et al., 1990; Beasley and Golden, 1993), mainly because it has been the only existing criterion. Some applications incorporate slight modifications (Fairhead et al., 1994). Thompson's criterion, however, may sometimes be ambiguous.

Recently, Barbosa et al. (1999) developed a new criterion to determine the structural index which, differently from Thompson's criterion, rarely is ambiguous. This criterion is derived from Euler's equation itself and is based on the correlation between the total-field anomaly and the estimates of the base level using different tentative values for the structural index. The tentative value producing the smallest correlation is taken as the estimate of the true structural index.

The objective of this paper is to illustrate the feasibility of applying the Euler deconvolution to magnetic anomalies consisting of just a few observations. To this end, the criterion of Barbosa et al. (1999) is used to estimate the correct structural index. It is applied to a real magnetic anomaly defined by just 12 observations, which is produced by a basic/ultrabasic body at the emerald deposit of Socotó, Bahia, Brazil.

2. Methodology

The total-field anomaly $\Delta T \equiv \Delta T(x,\ z)$, known within an additive constant b and produced by a two-dimensional point (or line) source at (x_o, z_o) (referred to a right-hand Cartesian coordinate system), satisfies Euler's differential equation (Thompson, 1982):

$$x_o \frac{\partial}{\partial x} h^o(x,z) + z_o \frac{\partial}{\partial z} h^o(x,z) + \eta b = x \frac{\partial}{\partial x} h^o(x,z) + z \frac{\partial}{\partial z} h^o(x,z) + \eta h^o(x,z), \qquad (1)$$

where η, known as the structural index, is related to the nature of the source and $h_o(x, z)$ is the observed total field, given by $h^o(x, z) = \Delta T(x, z) + b$, where b is an unknown constant base level. In matrix notation, Eq. (1) can be written as:

$$\mathbf{G}\boldsymbol{p} = \boldsymbol{y}. \tag{2}$$

$\mathbf{G}$ is an $N \times 3$ matrix whose elements of the ith line are: $g_{i1} = (\partial/\partial x)h_i^o$, $g_{i2} = (\partial/\partial z)h_i^o$, and $g_{i3} = \eta$, $i = 1, \ldots, N$ where N is the number of observations in a data window, h_i^o is the total field observed at the ith observation point ($x = x_i$ and $z = z_i$) and $(\partial/\partial x)h_i^o$ and $(\partial/\partial z)h_i^o$, and the gradients of h_i^o evaluated at $x = x_i$ and $z = z_i$, respectively. The ith element of the N-dimensional vector $\boldsymbol{y}$ is $y_i = x_i(\partial/\partial x)h_{\mathrm{i}}^o + z_i = (\partial/\partial z)h_i^o + \eta h_i^o$ and the elements of the parameter vector $\boldsymbol{p}$ are x_o, z_o, and b.

Assigning several tentative values to η, we obtain, for each tentative value, various alternative solutions $\hat{\mathbf{p}}$ associated to different positions of the data window by solving Eq. (2) in the least-squares sense. These solutions are usually sprinkled on the x–z plane, producing in this way a ‘‘spray’’ of solutions.

We describe below the method presented by Barbosa et al. (1999). It consists of two independent procedures. The first one consists in estimating the structural index using the correlation between the observed anomaly and the estimated base level as a function of the data window position. This procedure does not require the knowledge of estimates of x_o and z_o. After estimating η, estimates of x_o and z_o are obtained by combining the criterion of Thompson (1982) with the additional criterion of Barbosa et al. (1999) that a solution $\hat{\mathbf{p}}$ must satisfy Eq. (2) in a quasi-exact way.

2.1. Estimating the structural index

Barbosa et al. (1999) proved analytically that z_o and η are linearly dependent and cannot, therefore, be simultaneously estimated. As a result, to compute $\hat{\mathbf{p}}$, a priori knowledge of η is required. In practice, this means that a priori information about the nature of the equivalent source to be used implicitly as interpretation model should be known, which rarely occurs. Thompson (1982) assumed the correct structural index was the one producing the smallest solution dispersion. However, this criterion to select the structural index depends on the estimated solutions and requires a large number of redundant solutions to perform a visual cluster analysis, restricting Euler deconvolution to be applied only to surveys with a large amount of data such as aeromagnetic surveys.

Barbosa et al. (1999) proposed the following objective criterion for determining the structural index. Assume that $z = 0$, that estimates $\hat{x}_o$, $\hat{z}_o$, and $\hat{b}$, are associated with the correct structural index η, and that $\hat{x}_{oi}$, $\hat{z}_{oi}$, and $\hat{b}_i$ are solutions of Eq. (2) in the least-squares sense using the observations inside a given moving data window at its ith position. Then we may write

$$\hat{x}_{o_i}\frac{\partial}{\partial x}h^o(x_j) + \hat{z}_{o_i}\frac{\partial}{\partial z}h^o(x_j) + \eta\hat{b}_i = x_j\frac{\partial}{\partial x}h^o(x_j) + \eta h^o(x_j) + \alpha_{i,j},\ \ i = 1,2,\ldots,M, \tag{3}$$

where $h^o(x_{\mathrm{j}})$, $(\partial/\partial x)h^o(x_j)$ and $(\partial/\partial z)h^o(x_j)$ are, respectively, the total-field anomaly and its gradients evaluated at any point $x = x_j$ inside the data window, α_{ij} is a residual, and M is the number of positions occupied by the moving data window. Assume also that, using a wrong structural index μ, we obtained estimates $\hat{x}'_{o_i}$, $\hat{z}'_{o_i}$, and $\hat{b}'_i$ as least-squares solution of Eq. (2). Then we may write after a minor re-arrangement:

$$\hat{x}'_{o_i}\frac{\partial}{\partial x}h^o(x_j) + \hat{z}'_{o_i}\frac{\partial}{\partial z}h^o(x_j) + \mu\hat{b}'_i = x_j\frac{\partial}{\partial x}h^o(x_j) + \eta h^o(x_j) + (\mu-\eta)h^o(x_j) + \beta_{ij},$$
$$i=1,\ldots,M, \tag{4}$$

where β_{ij} is a residual. Subtracting Eq. (3) from Eq. (4) we get

$$\left(\hat{x}'_{o_i} - \hat{x}_{o_i}\right)\frac{\partial}{\partial x}h^o(x_j) + \left(\hat{z}'_{o_i} - \hat{z}_{o_i}\right)\frac{\partial}{\partial z}h^o(x_j) + \mu\hat{b}'_i - \eta\hat{b}_i = (\mu-\eta)h^o(x_j) + \beta_{ij} - \alpha_{ij},$$
$$i=1,2,\ldots,M, \tag{5}$$

or

$$\hat{b}'_i = \frac{\eta}{\mu}\hat{b}_i + \frac{\hat{x}_{o_i} - \hat{x}'_{o_i}}{\mu}\frac{\partial}{\partial x}h^o(x_j) + \frac{\hat{z}_{o_i} - \hat{z}'_{o_i}}{\mu}\frac{\partial}{\partial z}h^o(x_j) + \frac{(\mu-\eta)}{\mu}h^o(x_j) + \frac{\beta_{ij}-\alpha_{ij}}{\mu},$$
$$i=1,2,\ldots,M. \tag{6}$$

Eq. (6) shows that the estimate $\hat{b}'_i$ as a function of the data window position may be correlated with the total-field anomaly. This correlation is positive for a tentative structural index greater than the correct one ($\mu > \eta$) and negative for a tentative structural index smaller than the correct one ($\mu < \eta$). The correct index produces a minimum correlation between the total-field anomaly and the estimate of its base level because the latter will be constant.

In practice, the criterion of Barbosa et al. (1999) can be implemented in the following way.

(1) Select the profile interval where the total-field anomaly exhibits a large horizontal gradient. In other words, take the values that make up the main portion of the anomaly under consideration. In the case of a limited number of observations, use the whole data set.

(2) Then, for each tentative index, μ, compute $\hat{\mathbf{p}}$ by solving Eq. (2) in the least-squares sense using the observations inside the moving data window. For M data window positions along the selected profile interval, compute the correlation coefficient r^{μ} by

$$r^{\mu} = \frac{\sum_{i=1}^{M}\hat{b}_{\mu i}h_i^o - \left(\sum_{i=1}^{M}\hat{b}_{\mu i}\sum_{i=1}^{M}h_i^o\right)/M}{\sqrt{\left\{\sum_{i=1}^{M}\hat{b}_{\mu i}^2 - \left[\left(\sum_{i=1}^{M}\hat{b}_{\mu i}\right)^2/M\right]\right\}\left\{\sum_{i=1}^{M}\hat{h}_i^{o^2} - \left[\left(\sum_{i=1}^{M}h_i^o\right)^2/M\right]\right\}}} \tag{7}$$

where $\hat{b}_{\mu i}$ is the estimated base level assuming a tentative structural index μ and using a given moving data window at its ith position, and h_i^o is the observed total-field anomaly coinciding with the central point of the data window at its ith position. All estimates of b in the selected profile interval are used to estimate the structural index.

(3) Identify the estimated structural index $\hat{\eta}$ as the value of μ producing the smallest $|r^{\mu}|$.

2.2. Barbosa et al.'s additional criterion for solution acceptance

Barbosa et al. (1999) proposed accepting only the solutions producing at the same time an error for the estimate $\hat{z}_o$ of z_o smaller than a threshold value, and a ‘‘best fit’’ to vector $\boldsymbol{y}$. These restrictions are applied to the solutions associated with the estimated structural index. The first restriction is the criterion of Thompson (1982) itself, that is, only solutions satisfying the inequality

$$\frac{\hat{z}_o}{\eta \sigma_{z_o}} > \epsilon \tag{8}$$

are accepted, where σ_{z_o} is the standard deviation of $\hat{z}_o$ and ϵ is a user-provided positive scalar (generally set to 20 for high-resolution data). The second restriction accepts only the solutions satisfying the inequality:

$$\left(\frac{\|\boldsymbol{y} - \mathbf{G}\hat{\boldsymbol{p}}\|^2}{(N-3)} \right)^{1/2} < \gamma \tag{9}$$

where γ is the smallest positive number that is able to produce coherent solutions. This restriction ensures that estimate $\hat{\mathbf{p}}$ satisfies Eq. (2) in a quasi-exact way.

By combining these two restrictions, the criterion of Barbosa et al. (1999) permits an efficient reduction of the ‘‘spray’’ of solutions. We stress that these criteria retain the most stable estimates of x_o and z_o, namely those produced by the data window containing the highest absolute value of the horizontal and vertical anomaly gradients (Barbosa et al., 1999).

3. Synthetic data application

In all applications presented in this paper, we used five tentative structural indices μ selected from the set: 0.001, 0.5, 1, 1.5, 2, and 3. The observation plane is $z = 0$. The horizontal and vertical gradients were obtained from the total-field anomaly using an equivalent source transformation (Emilia, 1973).

In this section, we compare the results of Euler deconvolution using the criteria of Barbosa et al. (1999) with those obtained with the criterion of Thompson (1982). Therefore, we use a synthetic anomaly with a large number of observations because Thompson’s criterion requires several alternative solutions to estimate the structural index. The results obtained with the criteria Barbosa et al. (1999), however, remain the same, regardless of using all observations or just a few observations centered at the anomaly maximum as will be shown graphically later.

3.1. Anomaly at the magnetic pole

Fig. 1a shows 100 equispaced noise-corrupted total-field observations and the horizontal and vertical gradients produced by a simulated two-dimensional vertical prism 2 km deep, infinitely extending in depth, 1 km wide, and centered at $x_o = 50$ km (Fig. 1b). The prism is uniformly magnetized in the vertical direction with an intensity of 1 A/m. This model simulates a magnetic dike and is associated with a structural index $\eta = 1$ (line of monopoles). The theoretical anomaly is contaminated with additive Gaussian pseudo-random noise with zero mean and standard deviation of 2 nT.

In applying Euler deconvolution to the data of Fig. 1a, we use a 7 point moving data window. Because in this test we compare the criteria of Thompson (1982) and Barbosa et al. (1999) for

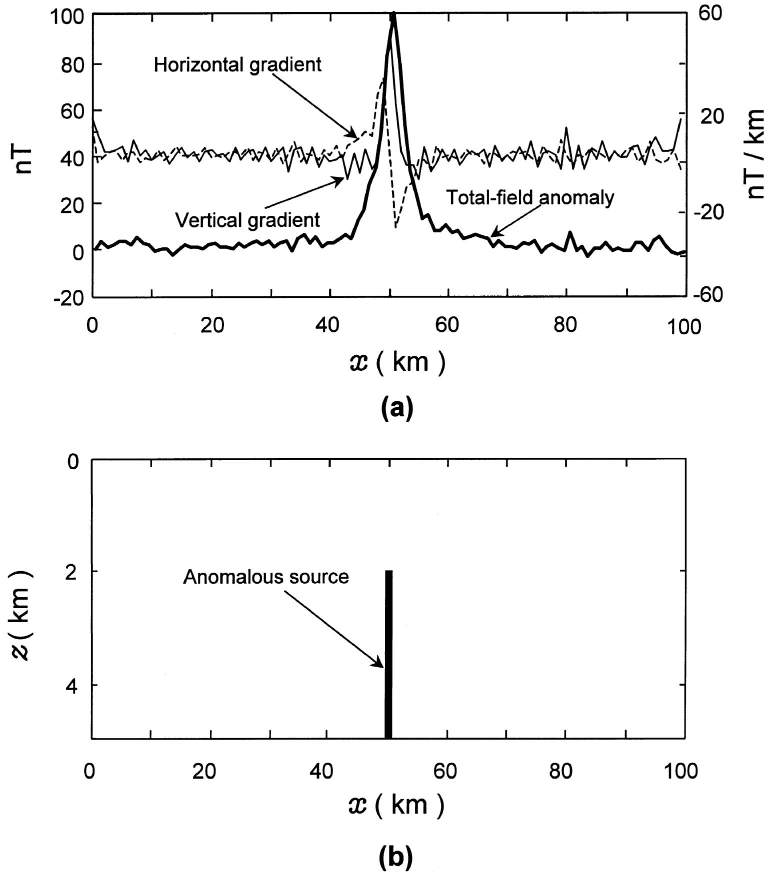

Fig. 1. Synthetic test. (a) Noise-corrupted total-field anomaly and its vertical and horizontal gradients, and (b) vertical, infinitely extending in depth prism uniformly magnetized in the vertical direction with a magnetization of 1.0 A/m producing the anomaly shown in (a). The ambient field is vertical.

solution selection, we analyze the complete set of solutions (associated with all values of μ) before estimating the structural index by the new criterion. However, we stress that the new structural index estimation and the new solution selection criterion are independent procedures. Fig. 2a shows, for all tentative indices, the solutions obtained using Thompson's criterion [inequality (8)] with $\epsilon = 20$. On the other hand, by applying the additional criterion of Barbosa et al. (1999) [combination of inequalities (8) and (9) with $\gamma = 18$ nT], we note that inequality (9) efficiently contributed in reducing the ''spray'' of solutions, as shown in Fig. 2b. By inspecting Fig. 2a, we note that the analysis of solution clustering to select the structural index (Thompson, 1982) leads to an ambiguity involving structural indices 1 and 1.5. These two indices might be equally accepted because they produce about the same (and smallest) dispersions of the estimated source position. This ambiguity produces rather different estimates of the source depth, whose averages are 1.95 km and 2.7 km, associated with structural indices 1 and 1.5, respectively.

To determine the structural index using the criterion of Barbosa et al. (1999) criterion, we compute the correlation coefficient r^{μ}, defined in the interval between $x = 10$ km and $x = 90$ km, for five tentative indices. The correlations are: $r^{0.5} = -0.83$, $r^{1} = -0.01$, $r^{1.5} = 0.73$, $r^{2} = 0.87$, and $r^{3} =$

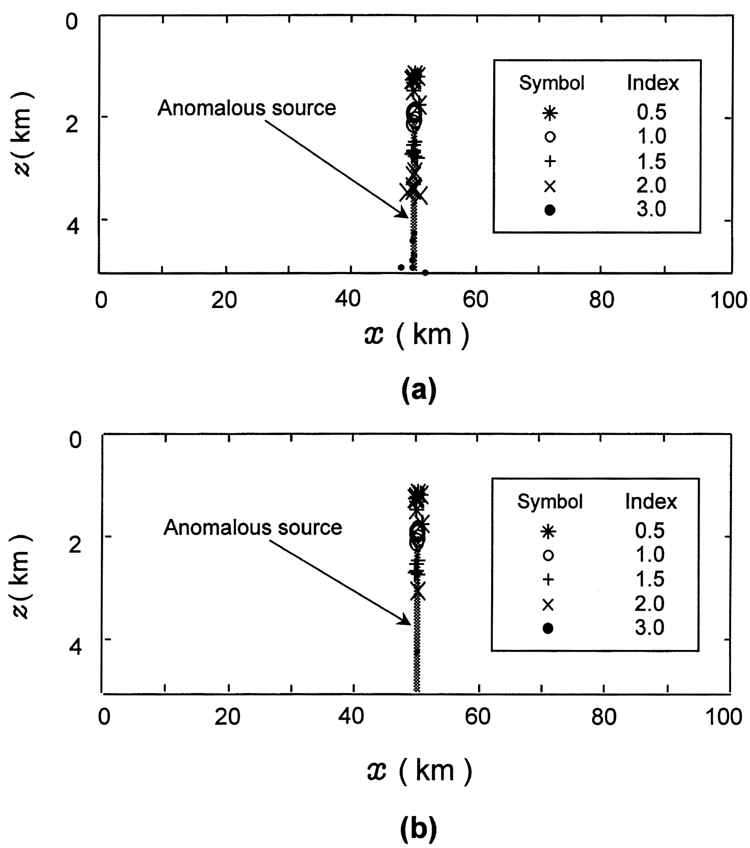

Fig. 2. Synthetic test: vertical prism at pole. Estimates of the source position using: (a) criterion of Thompson (1982) [inequality (8)]; (b) additional criterion of Barbosa et al. (1999), i.e., combining Thompson's criterion and the "best fit" criterion [inequalities (8) and (9), respectively] for each tentative structural index. Note that the additional criterion of Barbosa et al. (1999) reduced significantly the number of alternative solutions. For the correct structural index ($\eta = 1$) all alternative solutions map the top of the prism with good precision.

0.93, indicating that the correct index is 1 because it produces the minimum correlation. Graphically, this minimum correlation is illustrated in Fig. 3 where the estimated base level is plotted against the x-coordinate of the center of the moving data window for each tentative structural index.

Note that the solutions corresponding to $\eta = 1$ provide excellent estimates of the x- and z-coordinate of the prism top (Fig. 2b). Although this example uses a large number of data points, we emphasize that the approach of Barbosa et al. (1999) can be used in profiles with a reduced number of observations. Note that even if we had used just 21 data points in the interval $x \in [40 \text{ km}, 60 \text{ km}]$, we would have obtained the same minimum correlation. This can be graphically verified in Fig. 3, where, within the interval $x \in [40 \text{ km}, 60 \text{ km}]$, the minimum correlation is still associated with $\eta = 1$.

3.2. Anomaly at 30° inclination

The proposed criterion for estimating the structural index is based on Euler's equation, so it does not depend on the inclination and declination of the geomagnetic field. To illustrate this property, we

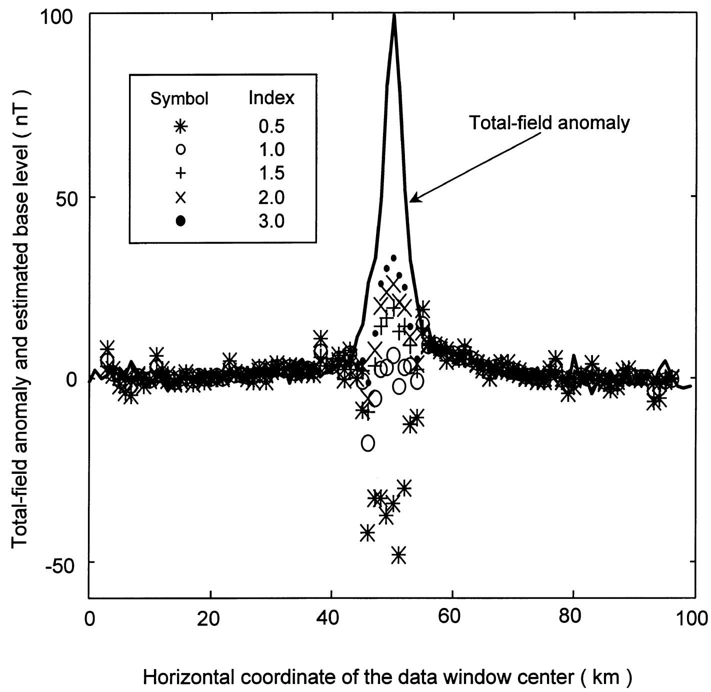

Fig. 3. Synthetic test: vertical prism at pole. Graphical illustration of the the minimum correlation criterion between the total-field anomaly and estimates $\hat{b}$. The assumption of the correct value for the structural index leads to a minimum correlation between the estimates of the baselevel (symbol ''o'') and the total field anomaly. Baselevel estimates obtained by assuming a structural index smaller than the true one produce negative correlations with the observed field (*) while an index greater than the true one produces a positive correlation ($+$, $\times$, and $\cdot$).

repeat the synthetic test presented in previous section using an inclination of 30° and a declination of 0° for both the geomagnetic field and the magnetization. The only additional differences are: standard deviation of the pseudo-random noise equal to 2.5 nT and $\gamma = 25$ nT. Fig. 4a shows the noise-corrupted total-field observations and the horizontal and vertical gradients.

To determine the structural index using the criterion of Barbosa et al. (1999), we compute the correlation coefficient r^{μ}, defined in the interval between $x = 0$ km and $x = 100$ km, for five tentative indices. The correlations are: $r^{0.001} = -0.72$, $r^{1} = 0.50$, $r^{1.5} = 0.71$, $r^{2} = 0.79$, and $r^{3} = 0.86$, indicating that the correct index is 1. To illustrate that the method is not sensitive to the inclusion of near null values at the tails of the anomaly, we computed the correlations in the interval between $x = 40$ km and $x = 60$ km and obtained: $r^{0.001} = -0.82$, $r^{1} = 0.23$, $r^{1.5} = 0.62$, $r^{2} = 0.77$, and $r^{3} = 0.86$, indicating again that the correct index is 1. Fig. 5a–e show graphically the correlation between the observed anomaly and the estimated base level for each tentative structural index employed.

By applying the additional criterion of Barbosa et al. (1999) [combination of inequalities (8) and (9) with $\gamma = 25$ nT], we obtained the solutions shown in Fig. 4b. Note that the solutions corresponding to $\eta = 1$ provide excellent estimates of the x- and z-coordinate of the prism top.

The tests with synthetic data presented in this section show that despite incorrect indices (either integer or not) produce oscillating parameter estimates (including $\hat{b}$) because of the variation of the

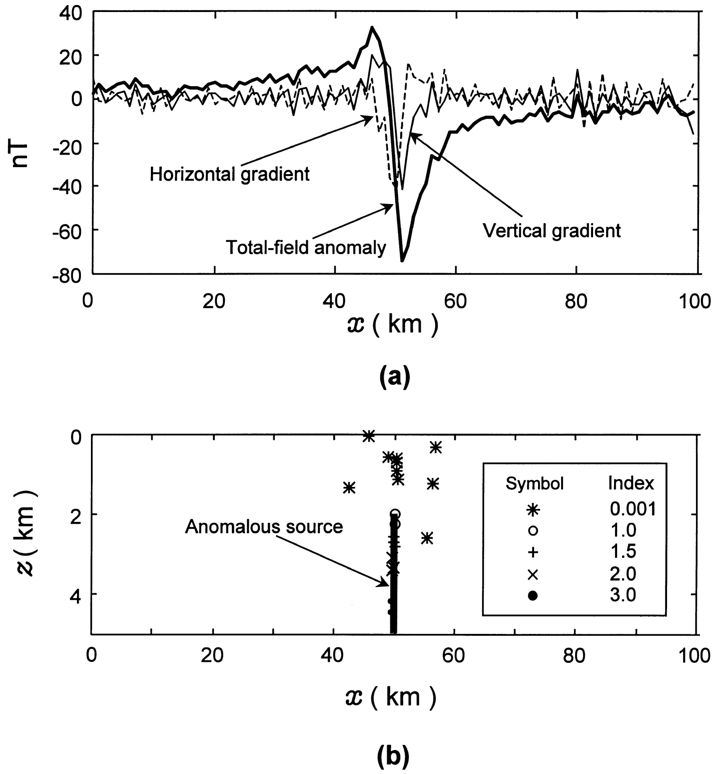

Fig. 4. Synthetic test: vertical prism at 30° inclination. (a) Noise-corrupted total-field anomaly and its vertical and horizontal gradients, and (b) Estimates of the source position using the additional criterion of Barbosa et al. (1999), i.e., combining Thompson's criterion and the ''best fit'' criterion [inequalities (8) and (9), respectively] for each tentative structural index. For the correct structural index ($\eta = 1$) all alternative solutions map the top of the prism with good precision, illustrating the applicability of the criterion of Barbosa et al. (1999) to magnetic anomalies at any magnetic inclination.

''structural index'' with the data window position (Ravat, 1996), the criterion of minimum correlation associated with the correct index still works.

4. Application to a real anomaly

In this section we demonstrate the feasibility of applying the criteria of Barbosa et al. (1999) to a ground magnetic survey having just 12 observations. The data were collected along an east-west profile perpendicular to the strike of a basic/ultrabasic intrusion known as the Socotó body, Bahia, Brazil (Collyer, 1990). This outcropping and intensely fractured intrusive body is a roof pendant whose origin is related to the Archean metamorphic–migmatitic complex and is emplaced in a granitic batholith of the Campo Formoso formation. This granite of semicircular shape is related to the end of the Transamazonian cycle and is intruded in the metasediments of the Jacobina group (Middle Proterozoic). This group, consisting of a volcano-sedimentary Proterozoic sequence, lies discordantly over an Archean metamorphic migmatitic and granulitic complex trending north-south. The economic importance of the Socotó body is the emerald mineralization at its contact with the granite. Therefore, by mapping the volume of this basic/ultrabasic body we can define the mineral potential of this emerald deposit.

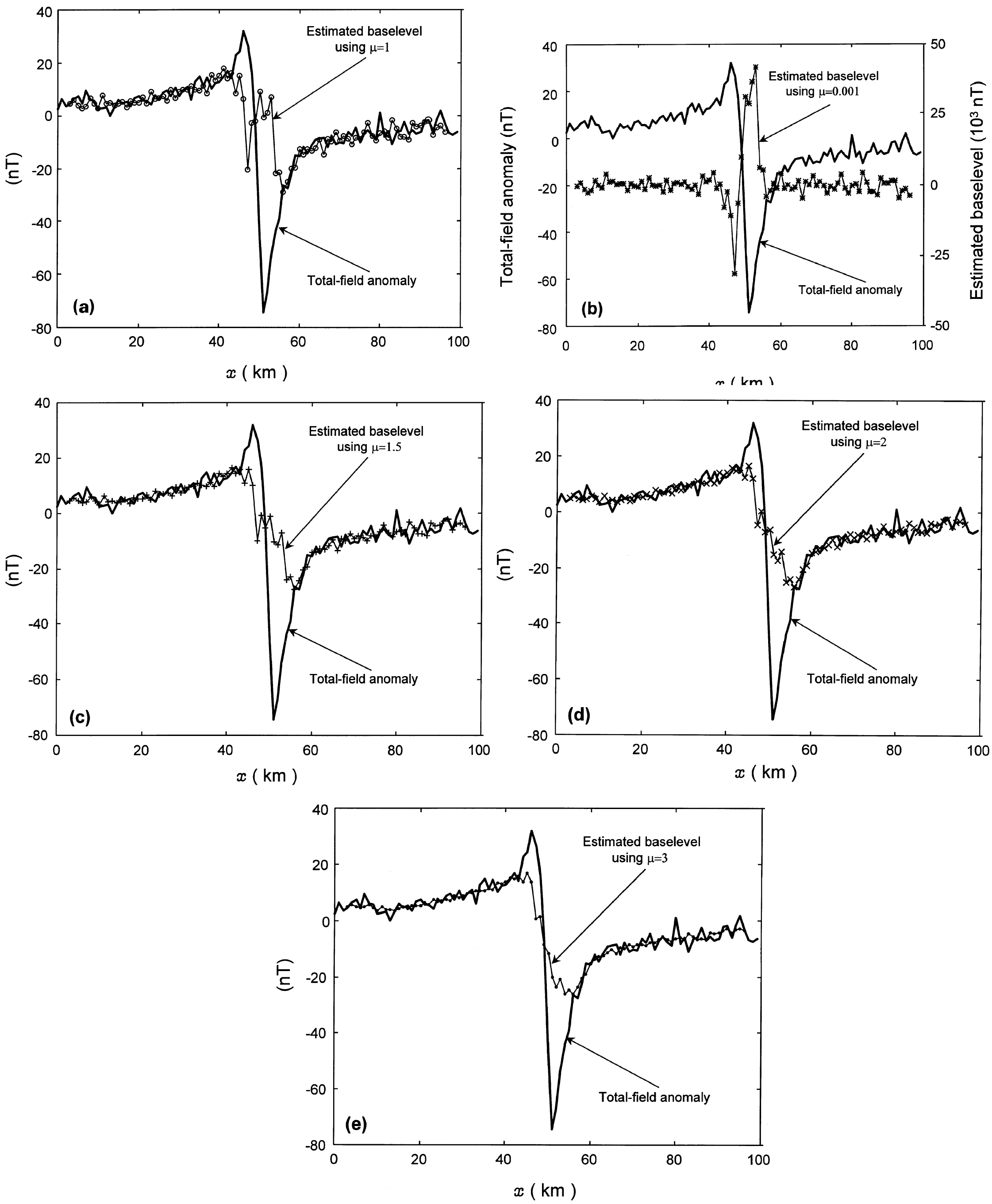

Fig. 5. Synthetic test: vertical prism at 30° inclination. Graphical illustration of the the minimum correlation criterion between the total-field anomaly and estimates $\hat{b}$. The assumption of the correct value for the structural index leads to a minimum correlation between the estimates of the baselevel and the total field anomaly (a). Baselevel estimates obtained by assuming a structural index smaller than the true one produce negative correlation with the observed field (b) while an index greater than the true one produces a positive correlation (c, d, and e).

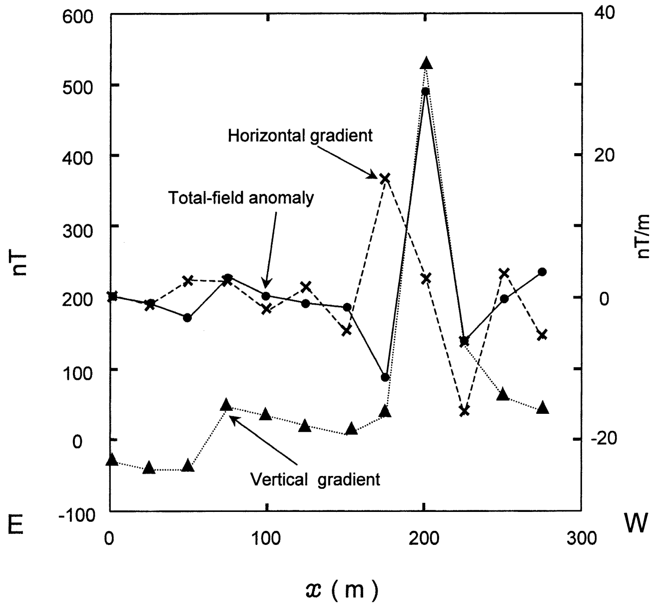

Fig. 6. Total-field anomaly and its vertical and horizontal gradients over the Socotó body, Bahia, Brazil. The dots mark the observations of the total-field anomaly. Crosses and triangles mark, respectively, the horizontal and vertical gradient values, computed by equivalent source transformation.

Fig. 6 shows the total-field anomaly over the Socotó body and the horizontal and vertical gradients obtained using an equivalent layer transformation. The use of an equivalent layer is fundamental in obtaining the gradients in this case because of the severe undersampling of the total-field anomaly. To apply Euler deconvolution to this anomaly, we selected the profile segment with the highest signal-to-noise ratio (the 8 points in the interval between [100 m, 275 m]. We employed a 3-point moving data window to estimate x_0, z_0, and b, producing six estimates for each parameter (a 3-point window produces two estimates less than the number of data in a profile). Fig. 7a shows, for all tentative indices, the solutions obtained using Thompson's criterion [inequality (8)] with $\epsilon = 200$. Thompson's criterion to determine the structural index indicates $\eta = 3$ and places the center of the equivalent source (dipole) at $x = 196.2$ m and $z = 45.3$ m. However, an estimated structural index equal to 3 is inconsistent with the geological information that the causative body is approximately two-dimensional. To determine the structural index employing the method of Barbosa et al. (1999), we computed, for the 6 estimates of b associated with the points in the interval $x \in [125$ km, 250 km$]$, the following correlation coefficients: $r^{0.5} = -0.78$, $r^1 = -0.64$, $r^{1.5} = -0.43$, $r^2 = -0.16$, and $r^3 = 0.32$, indicating that the correct index is 2. This result, in contrast with that using the criterion of Thompson (1982) criterion, defines the true source as equivalent to a line of dipoles (horizontal cylinder). To select the best solutions we used additionally the ''best fit'' criterion (Barbosa et al., 1999) with $\gamma = 0.00017$ nT in Eq. (9). The results are displayed in Fig. 7b, indicating that just one solution satisfies the criterion defined by Eq. (9). It corresponds to a horizontal cylinder with center located at $\hat{x}_o = 199.4$ m and $\hat{z}_o = 34$ m. Based on the geological information that the basic/ultrabasic body is outcropping and on the above estimate of the cylinder center, we consider that the anomalous source may be approximated by a horizontal cylinder with a radius of 34 m, extending to a depth of 68 m (dashed line in Fig. 4b). Our results are in agreement with measurements of the body width along its strike, averaging 100 m (Collyer, pers. comm.) and are consistent with the

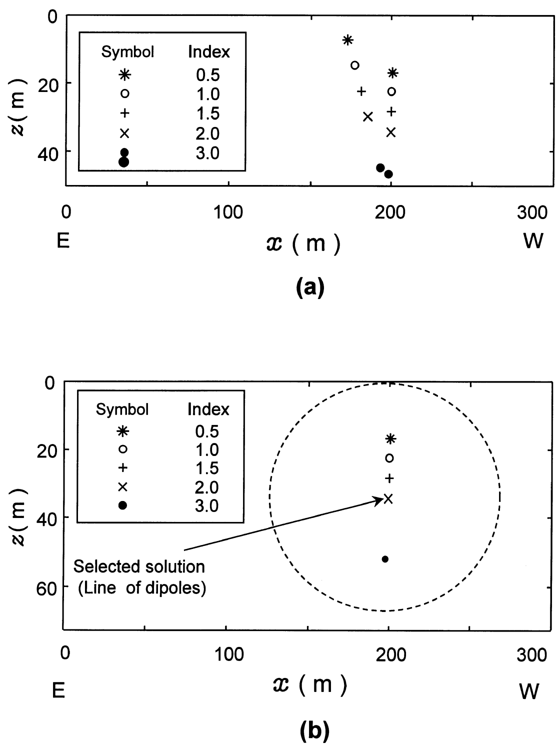

Fig. 7. Socotó body. Estimates of the source position using: (a) criterion of Thompson (1982) [inequality (8)]; (b) additional criterion of Barbosa et al. (1999), i.e., combining Thompson's criterion and the ''best fit'' criterion [inequalities (8) and (9), respectively] for each tentative structural index. The selected solution using criteria of Barbosa et al. (1999) corresponds to a line of dipoles, or, equivalently, to a horizontal cylinder. Because the intrusive Socotó body outcrops, one possible solution for its geometry is the cylinder shown as a dashed line.

geological information described by Collyer (1990), classifying this body as a roof pendant with an approximately lenticular shape.

To assess the significance of a correlation coefficient computed with a small number of points (6 in the above example), we analyzed numerically: (i) the probability of a random sequence to produce high correlation coefficient, and (ii) the probability that two correlated sequences produce small correlation coefficient because of their contamination with noise. In the first case we computed 10 000 correlation coefficients between two 6-point sequences randomly generated and following a Normal distribution with zero mean and unitary standard deviation. Fig. 8 shows the percentage of samples not exceeding (in modulus) a given value τ. Note that about 70% of all samples produce a correlation smaller (in modulus) than 0.5 and more than 90% produce a correlation smaller (in modulus) than 0.75.

In the second case, we generated two 6-point sequences from the equation

$$y_i = a x_i^2 + \delta_i, \quad i = 1,2,\ldots,6, \tag{10}$$

where a is equal to 1 and 0.5 for the first and second sequences, respectively, $x_i = -3, -2, \ldots, 2$ and δ_i is a realization of a random variable following a Normal distribution with zero mean. In this way, each sequence has a deterministic signal given by the quadratic law and a random noise. We computed 10 000 correlation coefficients (using different noise sequences) for each of several fixed

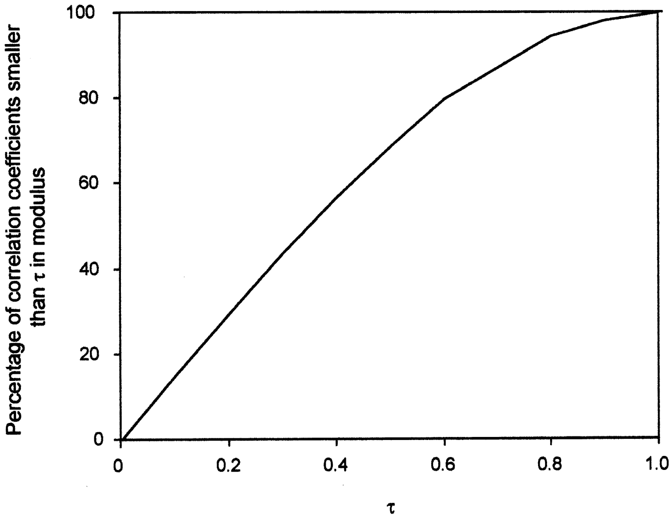

Fig. 8. Percentage of 10 000 correlation coefficients producing a value, in modulus, smaller than τ. The correlation is computed among pairs of 6-point random sequences.

standard deviations of the random variable in the interval [0, 30]. Fig. 9 shows the average of the 10 000 absolute correlation coefficient against the standard deviation. Note that the correlation coefficient starts to degrade only for values of the standard deviation greater about 45% of the deterministic signal amplitude of the second sequence, which is an intolerable percentage of noise for geophysical observations. From the above results, we conclude that a correlation coefficient computed with 6 data point is still likely to be significant.

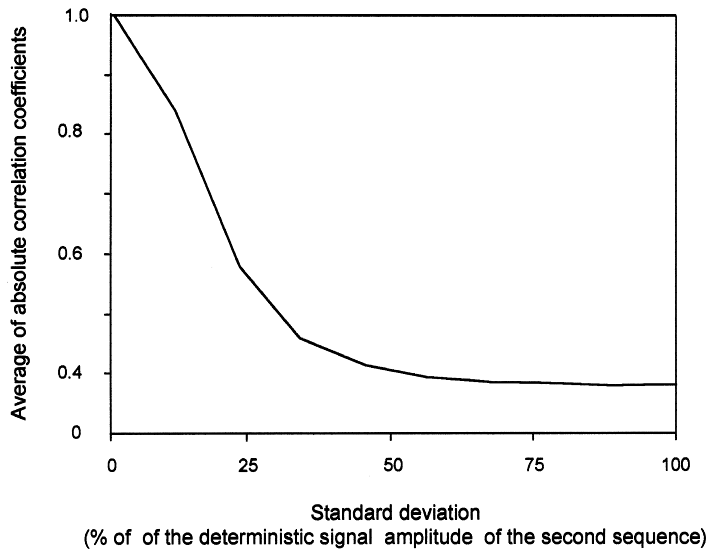

Fig. 9. Average of 10 000 absolute correlation coefficients between two 6-point sequences composed by a deterministic signal and a random variable as a function of the standard deviation expressed in percentage of the deterministic signal amplitude of the second sequence.

5. Conclusions

We have illustrated the feasibility of applying Euler deconvolution to surveys with a limited number of observations such as ground magnetic surveys. To this end, we employed the criterion of Barbosa et al. (1999) for estimating the structural index. This criterion which is different from the criterion of Thompson (1982) does not depend on a large number of solutions associated with different positions of a moving data window.

We applied the criterion of Barbosa et al. (1999) to a real ground magnetic profile consisting of just 12 observations across a basic/ultrabasic body at the emerald deposit of Socotó, Brazil. Only 8 out of the 12 observations were used to compute the set of alternative solutions, and only six estimates of the base level were used in computing the structural index. The results indicate that the intrusive body may be approximated by a horizontal cylinder with a diameter of 68 m and center at a depth of 34 m, which is consistent with the geological knowledge that describes this body as a roof pendant.

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