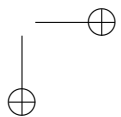
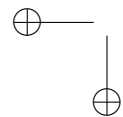
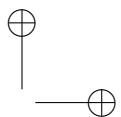
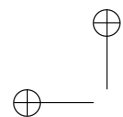


*In memory of Christian Freksa*

*26.06.1950–12.11.2020*



# PREFACE

## *Who should read this book*

---

This book is intended for readers from any background interested in the issues that arise when computing technology meets spatial information—in other words, in GIS! You do not need to be a specialist computing scientist already; the text develops the necessary background in specialist areas, such as databases, system architecture, and AI, as it progresses. Nevertheless, some knowledge of, and interest in the basic components and functionality of computers is essential for understanding the importance of certain key issues in GIS. Where some aspect of general computing bears a direct relevance to our development, the background is given in the text.

This book can be used as a teaching text, taking readers through the main concepts by means of definitions, explications, and examples. However, the more advanced reader is not neglected, and the book attempts to highlight the threads and references that can be used to follow up on particular research topics.

## *Changes to the third edition*

---

In a highly technology-led area, such as GIS, the pace of technological change sometimes feels like shifting sand under one’s feet. Returning to write the third edition, it was again encouraging to see that the spine of the first edition—databases and spatial data, structures, algorithms, and indexes (Chapters 2, 3, 5, 6)—continues to stand the test of time.

While that core is still sound, the third edition has of course been extensively revised and updated, and complemented by significant new material, especially in those other areas where the field has moved more rapidly. Graph databases have now been added to the core as a major new topic. The material on time has been significantly reorganized and extended, reflecting a tighter integration of space and time in the field more generally. The fundamental material on models and modeling has also been significantly revised and reorganized to strengthen the clarity and messaging. Major new material reflecting important advances in the technology has been added in connection with architectures (Chapter 7, including cloud computing, stream computing, and sensor networks). The material on cartography and visualization (Chapter 8) has been extensively rewritten and redeveloped with largely new content. Finally, the introduction of an entirely new chapter on AI and GIS (Chapter 9, including ontologies, spatial reasoning, machine learning and spatial analysis, and deep learning) is a reflection of the rapid advances in this area over the past decade.

In addition to the changes in content, we have striven to make further improvements to the format to produce a more attractive and readable book. The format for the book has been completely revised in full color, in particular with all figures either new or completely regenerated. The text has likewise been completely rewritten with the continuing aim of making the book more accessible to an ever-wider audience. Every chapter begins with a new summary, outlining the major ideas and learning objectives in that chapter. At its close, every chapter ends with some more personal reflections and perspectives on the topic from the authors.

Before sitting down to write a third edition, it was essential for us to move the book towards open access. We are very grateful to the Taylor & Francis team for working with us to enable five chapters—half the book—to be made freely available online and open access. Another important change, which may not be immediately obvious, is the conscious effort to rebalance the list of references. The cited references in the second edition were overwhelmingly to male authors—of more than 300 references in the second edition, over 86% were authored exclusively by men. In writing the third edition, we have attempted to address this stubborn bias by actively seeking out the abundance of excellent research by women GI scientists. It has been a joy to read and reread these works of women researchers, who continue to be systematically under-cited and under-recognized as they have been for the entire history of the field.<sup>1</sup>

<sup>1</sup> Tracking gender in bibliographies is not straightforward, but we estimate over 50% of cited references in this third edition are by female authors or include female co-authors. Nevertheless, the book remains a long way from gender parity (i.e., in terms of the total proportion of male authors, as opposed to the proportion of references with exclusively male authors) and still offers no visibility for the contributions of non-binary GI scientists.

Finally, following the pattern Mike Worboys began in 2004 by inviting Matt Duckham to become a co-author of the second edition, Matt and Mike welcome Qian (Chayn) Sun on to this third edition as a third author. With a background in geography and GIS, Chayn has brought her own fresh perspective and distinct expertise to the third edition, in particular, in web mapping, cloud computing, critical geography, and machine learning with GIS. We hope you will agree that this third edition has succeeded in maintaining the high standards set by *GIS: A Computing Perspective* back in 1994!

### *Formatting used in this book*

---

Several formatting conventions, continued from the second edition, have been used in this book. Material that is relevant to the main themes in the text, but not essential to the reader, is clearly separated out in purple inset boxes, usually at the top of a page. Boxes typically contain interesting asides, more challenging material, or background to a topic, as well as references and links that readers may wish to pursue. With over 60 such boxes, a complete list is included in the front matter to the book for ease of reference. A list of insets is included in the book's front matter.

Throughout this book, we have used margin text to allow rapid reference to important terms. When an important term is first defined or introduced, that term will appear in the margin. A corresponding entry can be found in



the index, with the page reference in bold typeface. This enables the reader to use the index as an extensive glossary of more than 1200 terms used in this book. Each index term has at most one bold typeface page reference, and a term can be rapidly located within a page by finding the corresponding margin entry. In addition to normal- and bold-typeface index entries, those index entries that appear in italics refer to terms that appear within an inset box.

### Structure of this book

Figure 1 indicates the overall structure of interdependencies between chapters. Readers may find it helpful to refer to Figure 1 to tailor their use of this book to their own particular interests and prior background.

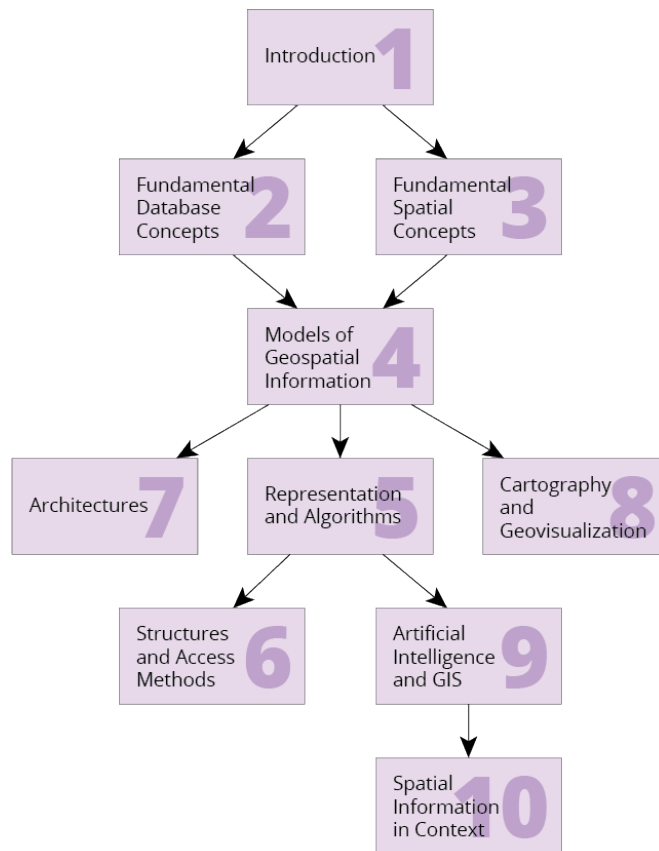


Figure 1: Relationships between chapters

*Chapter 1* motivation and introduction to GIS; preparatory material on general computing; and an overview of what makes “spatial special”;

*Chapters 2–3* relational and graph databases; conceptual data modeling; foundations and formalisms for spatial concepts;

*Chapter 4* high-level modeling of space and time; object- and field-based models; time in GIS;

*Chapters 5–6* exposition of the core material; spatial algorithms and data structures; spatial indexes and access methods;

*Chapter 7* GIS system architecture and distributed systems; web mapping; stream computing and sensor networks;

*Chapter 8* graphic design and cartography; GIS interface and interaction design; visualization of geospatial data;

*Chapter 9* ontology engineering and spatial reasoning; machine learning and deep learning; “GeoAI”; and

*Chapter 10* uncertainty and imperfection in spatial information; location privacy; critical GIS.

### *Online resources*

---

The website that accompanies this book can be found at:

<http://gisacp.duckham.org>

The resources at this site are constantly under development, but they include resources such as sample exercises, lecture slides and notes, open-source computer code, sample material, useful links, errata, and contact information. We welcome suggestions from readers as to resources that we should include on the website, or indeed any feedback or comments on the book itself. Matt can be contacted on Mastodon on @mduckham@mastodon.au or Twitter at @geospatial\_md; other up-to-date contact information can be found on the website.

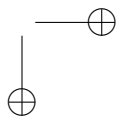
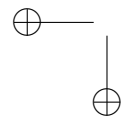
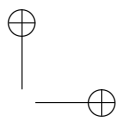
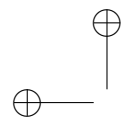
# ACKNOWLEDGMENTS

Matt and Chayn are very grateful for the support of RMIT University friends and colleagues in Melbourne, including members of the Geospatial Sciences team, the STEM College, and the Research and Innovation teams led by Swee Mak and Calum Drummond.

Special thanks are due to Sarah Rigbye and Prag Sapkota who assisted with the conversion of second edition chapters and the development of some of the figures. The original characters adapted for use in many figures are from a series by Nadia Snopek. The authors are grateful to Mark Newman for permission to use his cartogram on page 320. We would also like to thank the Taylor and Francis production and editorial team for their support, and in particular Irma Britton for her patience and persistence over more than a decade that this edition was in the making.

Finally, Matt will be forever grateful to his family in Melbourne—Laurie, Nola, Tess, Sophie, Eloise, James, Russell, Josh, Frankie, Nina, Allison, Andy, Aidan, and Anja—for their love, patience, and encouragement, and most especially to Ingrid for sharing her joyful laughter, wise counsel, and inexhaustible kindness.

Matt Duckham, Chayn Sun, and Mike Worboys, January 2023

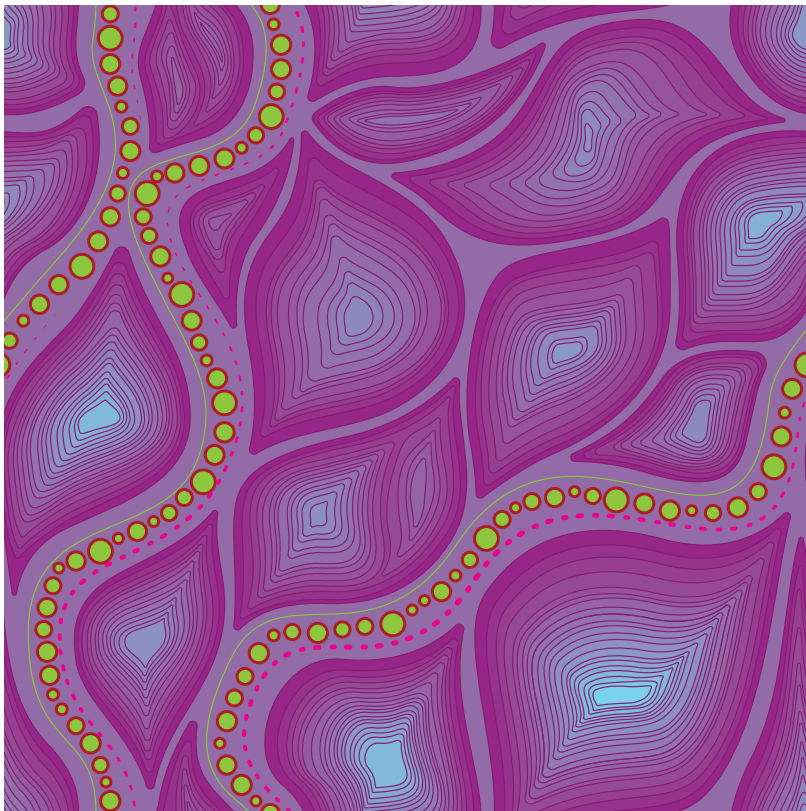


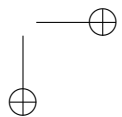
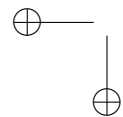
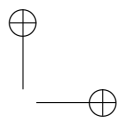
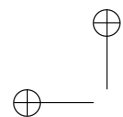
# ARTWORK

I am a part Indigenous Australian artist originally from Grafton, Northern NSW, identifying as Bundjalung for the land I was born and grew up on, as well as Anaiwan and Kamilaroi—the nations of my ancestors. I draw my artistic inspiration from nature and my everyday surroundings, highlighting the similarities in our life’s journeys with that of the universe and everything living within it.

This artwork reflects the geographical lines and pathways of the lands we walk upon every day. What once were carved by the oceans currents and tides, are now the hills and valleys we travel through. Much like the paths we take with every decision we make, the winds and waters travel scientifically the same.

Lou Bloomer, June 2023





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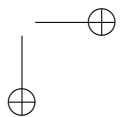
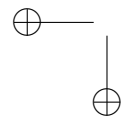
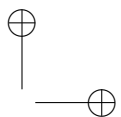
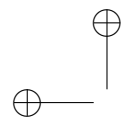
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# APPENDIX: NUTTY NUGGETS RELATIONAL DATABASE



*Nutty Nuggets relational database example*

---

*NUGGETS database schema*

ADDRESS (ADDRESS\_ID, STREET\_AD, ZIP, CITY, STATE)  
 CONTENT (OID, SID, WEIGHTG)  
 CUSTOMER (TEL, HID, TITLE, GNAME, FNAME)  
 ORDERS (ORDER\_NO, CUS\_TEL, AID, MEAL, SIZE)  
 STOCK (INGREDIENT, QUANT, UNITS)

(See page 57.)

*ADDRESS relation*

ADDRESS_ID	STREET_AD	ZIP	CITY	STATE
10012-BS01	177a Bleecker St	10012	New York	NY
10560-GL01	1407 Graymalkin Lane	10560	North Salem	NY
11050-PA01	98 Pleasant Avenue	11050	Fort Washington	NY
11232-4201	738 42nd Street	11232	Brooklyn	NY
19104-CS01	85 Chestnut Street	19104	Philadelphia	PA
20002-VRD2	3501 Valley Road Drive	20002	Washington	DC
74012-RA01	554 Railroad Avenue	74012	Broxton	OK
94110-MS01	1128 Mission St	94110	San Francisco	CA

*CUSTOMER relation*

TEL	GNAME	FNAME	HID	TITLE
202-555-0125	Joanna	Cargill	20002-VRD2	Ms
213-555-0506	Roberto	Da Costa	94110-MS01	Mr
610-555-0195	Lorna	Dane	10560-GL01	Ms
757-555-0112	Bobby	Drake	11050-PA01	Mr
785-555-0189	Loki	Laufeyson	10012-BS01	Mx
939-555-0177	Jane	Foster	74012-RA01	Dr

*ORDERS relation*

ORDER_NO	CUS_TEL	AID	MEAL	SIZE
M066-22-06	610-555-0195	19104-CS01	Regular	2
M066-22-07	610-555-0195	19104-CS01	Regular	1
M066-22-08	610-555-0195	19104-CS01	Regular	2
M113-22-09	939-555-0177	74012-RA01	Vegan	1
M315-22-06	213-555-0506	94110-MS01	Lo carb	4
M315-22-07	213-555-0506	94110-MS01	Lo carb	4

*CONTENT relation*

OID	SID	WEIGHTG
M066-22-06	Chickpea	400
M066-22-06	Chili	10
M066-22-06	Rice	400
M066-22-06	Tomato	400
M066-22-07	Cheese	150
M066-22-07	Garlic	20
M066-22-07	Mushroom	400
M066-22-08	Carrot	300
M066-22-08	Corn	200
M066-22-08	Potato	300
M066-22-08	Vegetable stock	500
M113-22-09	Chickpea	400
M113-22-09	Garlic	20
M113-22-09	Lemon	200
M113-22-09	Mushroom	600
M315-22-06	Chickpea	800
M315-22-06	Chili	20
M315-22-06	Corn	600
M315-22-06	Tomato	600
M315-22-07	Cheese	350
M315-22-07	Garlic	30
M315-22-07	Mushroom	900

*STOCK relation*

INGREDIENT	QUANT	UNITS
Carrot	61	kg
Cheese	17	kg
Chickpeas	18	cans
Chili	750	g
Corn	15	kg
Garlic	8	kg
Lemon	8	kg
Mushroom	30	kg
Oil	42	l
Onion	25	kg
Potato	70	kg
Rice	64	kg
Tomato	104	cans
Vegetable stock	48	l

# APPENDIX: DISCRETE MATHEMATICS PRIMER

## B

THIS appendix provides a brief primer on the discrete mathematics structures and syntax used in the book, developed from an earlier primer that first appeared in Duckham (2013).

### Sets

Sets are used throughout the book but introduced in some depth in Section 3.2. In summary, a *set* is a collection of “things,” each called a *member* or an *element* of a set.<sup>1</sup> Some common sets are also identified with their own symbol, including:

- $\emptyset$ , the *empty set* containing no elements, also written  $\{\}$ ;
- $\mathbb{B}$ , the Boolean set of two values (e.g., with members true/false, 1/0, on/off);
- $\mathbb{Z}$ , the set of positive and negative integer numbers, including zero;
- $\mathbb{Q}$ , the set of rationals—ratios of two integers; and
- $\mathbb{R}$ , the set of reals—measurements on the number line.

The order of elements in a set is not significant.<sup>2</sup> A set that contains all the elements of another set, and perhaps some additional elements, is termed a *superset*. We write  $X \subseteq Y$  to indicate  $Y$  is a superset of  $X$ , or equivalently that  $X$  is a *subset* of  $Y$ .<sup>3</sup> The empty set is a subset of every set, e.g.,  $\emptyset \subseteq X$ . A set  $X$  is said to be a *proper subset* of  $Y$ , written  $X \subset Y$ , if  $X$  is a subset of  $Y$  and  $X \neq Y$ .

A set can be specified explicitly by itemizing each of its members. Alternatively, it is often easier to specify sets by *intension*, essential for sets with a large or infinite number of elements such as  $\mathbb{Z}$  and  $\mathbb{R}$ . For example, another special set  $\mathbb{N}$  (the set of natural numbers—positive integers from 1 to infinity) can be defined as  $\mathbb{N} = \{x \in \mathbb{Z} | x > 0\}$  (read “the set containing numbers  $x$  in  $\mathbb{Z}$  such that  $x$  is greater than 0”).<sup>4</sup>

We often specify by intension subsets of the real numbers as *intervals*:

- $[a, b]$  is the set of reals between  $a$  and  $b$  including  $a$  and  $b$ , termed a *closed interval*;
- $]a, b[$  is the set of reals between  $a$  and  $b$  excluding  $a$  and  $b$ , termed an *open interval*; and

<sup>1</sup> For example,  $A = \{a, b, c\}$  defines the set  $A$  with three members  $a$ ,  $b$ , and  $c$ . If we wish to assert that  $a$  is an element of set  $A$ , we may write  $a \in A$ .

<sup>2</sup> So, set  $A = \{a, b, c\} = \{b, c, a\} = \{c, a, b\} = \{c, b, a\} = \dots$

<sup>3</sup> If  $A = \{a, b, c\}$  and  $B = \{a, b, c, d\}$  then  $A \subseteq B$ . It is also true that  $A \subseteq A$ , but  $A$  is not a proper subset of itself.

<sup>4</sup> The set  $C = \{2, 3, 4, 5, 6\}$  can be specified as  $C = \{x \in \mathbb{N} | 1 < x < 7\}$ .

- $[a, b]$  is the set of reals between  $a$  and  $b$  including  $a$  but excluding  $b$ , termed a *semi-open* interval.

<sup>5</sup> With reference to the sets above,  $A \cap \{a, b, d\} = \{a, b\}$ ,  $A \cup \{b, d\} = B$ , and  $A \cap C = \emptyset$ .

<sup>6</sup> For example,  $A \setminus B = \{b\}$ .

<sup>7</sup> For the sets in the margins above,  $|A| = 3$ ,  $|B| = 4$ ,  $|C| = 5$ .

<sup>8</sup> For example,  $\{a, c\} \in \mathcal{P}(A)$ .

<sup>9</sup> The set  $\{\{a, c\}, \{b\}, \{d\}\}$  is a partition of  $B$ .

<sup>10</sup> Pairs are sometimes written without parentheses or commas, e.g.,  $xy$ .

<sup>11</sup> Thus, for example,  $(b, d) \in A \times B$ .

<sup>12</sup> For example,  $(b, c, 3) \in A \times B \times C$ .

<sup>13</sup> For example,  $L = \{(d, a), (b, a), (c, b), (a, b)\}$  is a relation from  $B$  to  $A$ .

<sup>14</sup> Thus, for example,  $(15.2, -1.9) \in \mathbb{R}^2$ .

<sup>15</sup> For example,  $N = \{(1, 2), (2, 1), (2, 5), (5, 2)\}$ , a (binary) relation on  $\mathbb{Z}$ , is not transitive; is symmetric (and hence not antisymmetric); and is irreflexive (and hence not reflexive).

The *intersection* of two sets  $X$  and  $Y$ , written  $X \cap Y$  is the set of elements in  $X$  and in  $Y$ . The *union* of  $X$  and  $Y$ , written  $X \cup Y$  is the set of elements in  $X$  or in  $Y$ .<sup>5</sup> The set *difference*,  $X \setminus Y$ , is the set of elements in  $X$  but not in  $Y$ .<sup>6</sup>

The *cardinality* of a set  $X$  is the number of elements in that set, written  $|X|$ .<sup>7</sup> The *complement* of a set  $A$  is the set of elements *not* in that set, written  $A'$ . Set complement makes implicit reference to a *universal* set of all elements under consideration. The *power set* of a set  $X$  is the set of all subsets of that set (including the empty set), written  $\mathcal{P}(X)$  or sometimes  $2^X$ . Note that the power set of a set is still a set: the elements of a set may themselves be sets.<sup>8</sup> A *partition* of a set is a set of subsets such that each and every member of the original set appears in exactly one subset in the partition.<sup>9</sup> Therefore, a partition of a set is a subset of the powerset of that set.

### Products and relations

The *product* of two sets  $X$  and  $Y$ , written  $X \times Y$ , is the set of all distinct pairs with the first element  $x \in X$  and the second element  $y \in Y$ . A pair is usually written with enclosing parentheses, for example,  $(x, y)$ .<sup>10</sup> Unlike the elements of a set, a pair is ordered, so  $(x, y) \neq (y, x)$ . Note that the product of two sets is, again, a set—albeit a set containing pairs as its elements.<sup>11</sup> The product of two sets can be extended to an  $n$ -ary product (product of  $n$  sets) in a straightforward way.<sup>12</sup>

A *relation* between two sets is simply a subset of the product of those sets.<sup>13</sup> Many important relations are from a set to itself, termed a *binary relation*. The set  $\mathbb{R} \times \mathbb{R}$ , often written  $\mathbb{R}^2$ , is the familiar points in the Cartesian plane (i.e., planar coordinates).<sup>14</sup>

A binary relation  $R$  on  $X$  is said to be *reflexive* if for every  $x \in X$ ,  $(x, x) \in R$ . A relation  $R$  is *irreflexive* if for every  $x \in X$ ,  $(x, x) \notin R$ . A relation  $R$  on  $X$  is said to be *symmetric* if whenever  $(x, y) \in R$  it is also the case that  $(y, x) \in R$ . A relation is *antisymmetric* relation if whenever  $(x, y) \in R$ , then  $(y, x) \notin R$ . Finally, a relation  $R$  on  $X$  is said to be *transitive* if whenever  $(x, y)$  and  $(y, z) \in R$ ,  $(x, z) \in R$ .<sup>15</sup>

### Functions

A *function* is a special case of a relation between two sets,  $X$  and  $Y$ , where every element in  $X$  is related to a unique element in  $Y$ . The first set,  $X$ , is the set of all permitted inputs to the function, termed the *domain*. The second set,  $Y$ , is the set of all possible outputs of the function, termed the *codomain*.

Although functions are a special case of relations, they have their own special formal syntax to highlight their particular constraints. A function  $f$  with domain  $X$  and codomain  $Y$  is specified as  $f : X \rightarrow Y$  (read “ $f$  is a function from  $X$  to  $Y$ ”). We can indicate that the function  $f$  relates a particular input  $x \in X$  to a particular result  $y \in Y$  by writing  $y = f(x)$  or equivalently



$f(x) \mapsto y$  (read “ $f$  of  $x$  maps to  $y$ ”). To highlight the correspondence with relations, if  $f$  were defined as a relation (rather than as a function) we might instead choose to write  $(x, y) \in f$ . Both are correct, the difference is a matter of style and convention.

A function where every element in the domain maps to a distinct (different) element in the codomain is termed an *injection*. A function where every element in the codomain has some element from the domain that maps to it is termed a *surjection*. A function that is both an injection and a surjection is termed a *bijection*.<sup>16</sup>

The *image* of a function  $f$ , denoted by  $f_*$ , is the set of elements that a function maps to.<sup>17</sup> Thus, the image of a function is necessarily a subset of the codomain for that function. Any injection has the property that there exists another function that maps back to elements in the domain from elements in the image of that function, termed an *inverse function*. A bijection has an inverse that maps back to elements in the domain from elements in the codomain of that function (since the image of a bijection is its codomain).

## Graphs

A *graph* is a pair  $G = (V, E)$ , where:

1.  $V$  is some set of elements, called *vertices* or *nodes*; and
2.  $E$  is a binary relation on  $V$ ,  $E \subseteq V \times V$ , each element of which is called an *edge*.

If  $E$  is symmetric, then the graph  $G$  is termed *undirected* or *bidirected*; otherwise,  $G$  is a *directed* graph. For an undirected graph  $G = (V, E)$ , we may sometimes use the shorthand  $\{x, y\} \in E$  instead of writing both  $(x, y) \in E$  and  $(y, x) \in E$  (since in an undirected graph  $E$  is symmetric, so if  $(x, y) \in E$ , then necessarily  $(y, x) \in E$ , and the ordering does not matter).

Graphs (and relations) have a natural graphical representation, with vertices depicted as dots and edges depicted as lines connecting two vertices. Directed graphs may be similarly depicted, with the addition of arrows to indicate the direction(s) of edges (as in Figure B.1).

Two nodes that are connected by an edge are said to be *adjacent*. An edge is said to be *incident on* the nodes it connects; likewise, a node is said to be *incident with* an edge that connects to it. The *degree* of a node is its number of incident edges. In a directed graph, we may distinguish between the number of incoming edges (a node’s *in-degree*) and outgoing edges (a node’s *out-degree*). The *order* of a graph is the number of nodes it contains.

A *path* is a sequence of adjacent nodes.<sup>18</sup> Two nodes are said to be *connected* if there exists a path between them. A graph is said to be *connected* if every pair of nodes in the graph is connected.

A *tree* is a (connected) graph where there exists exactly one path between any pair of nodes. A *rooted tree* is a tree with one node designated as the *root*.

A *planar* graph is a graph that can be drawn in the plane with edges only intersecting at nodes (i.e., no edges crossing). A graph that *can* be drawn in

<sup>16</sup> For instance, function  $g : A \rightarrow B$  where  $g(a) = a$ ,  $g(b) = b$ ,  $g(c) = c$  is an injection, but not a surjection (and so not a bijection).

<sup>17</sup> For example, for the function  $f : X \rightarrow Y$ , the image of the function  $f_* = \{y \in Y \mid y = f(x) \text{ for some } x \in X\}$ .

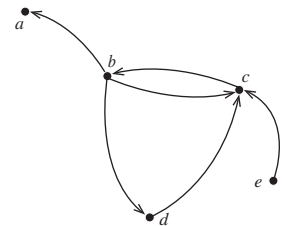


Figure B.1: Example directed graph, from Figure 2.14

<sup>18</sup> For example,  $bdc$  is a path in  $G$  Figure B.1;  $bcd$  is not.

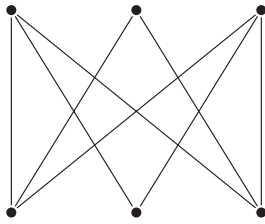


Figure B.2: Example non-planar graph, from Figure 3.42

the plane with no crossing edges is called planar; a graph that is drawn in the plane with no crossing edges is called *plane*. Thus all plane graphs are also planar; and all non-planar graphs (such as Figure B.2) can never be depicted as a plane graph.

### Logic

Logical expressions in mathematics and computer science can be assigned a *truth value*. In the most basic (two-valued) logic, expressions have a truth value of either true or false, but not both. For example, the truth value of the expression “ $5 < 10$ ” is true; the truth value of the expression “ $1 = 2$ ” is false. We often use the formal symbols  $\top$  (true) and  $\perp$  (false).

The *conjunction* of two logical expressions  $p \wedge q$  is true if *both*  $p$  and  $q$  are true; it is false otherwise. The *disjunction* of two logical expressions  $p \vee q$  is true if *either*  $p$  or  $q$  are true; it is false if both  $p$  and  $q$  are false.<sup>19</sup> Other connectives include implication  $\Rightarrow$ , equivalence  $\Leftrightarrow$  (if and only if), and negation  $\neg$ . The logical structure of connectives can be captured in truth tables, such as those in Figure B.3.

<sup>19</sup> For example, the truth value of “ $a \in A \wedge a \in C$ ” is false; the truth value of “ $5 < 10 \vee 1 = 2$ ” is true.

Figure B.3: Truth tables for predicate logic connectives

$\wedge$	$\top$	$\perp$	$\vee$	$\top$	$\perp$	$\neg$	$\top$
$\top$	$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\perp$
$\perp$	$\perp$	$\perp$	$\perp$	$\top$	$\perp$	$\perp$	$\top$

$\Rightarrow$	$\top$	$\perp$	$\Leftrightarrow$	$\top$	$\perp$
$\top$	$\top$	$\perp$	$\top$	$\top$	$\perp$
$\perp$	$\perp$	$\top$	$\perp$	$\perp$	$\top$

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