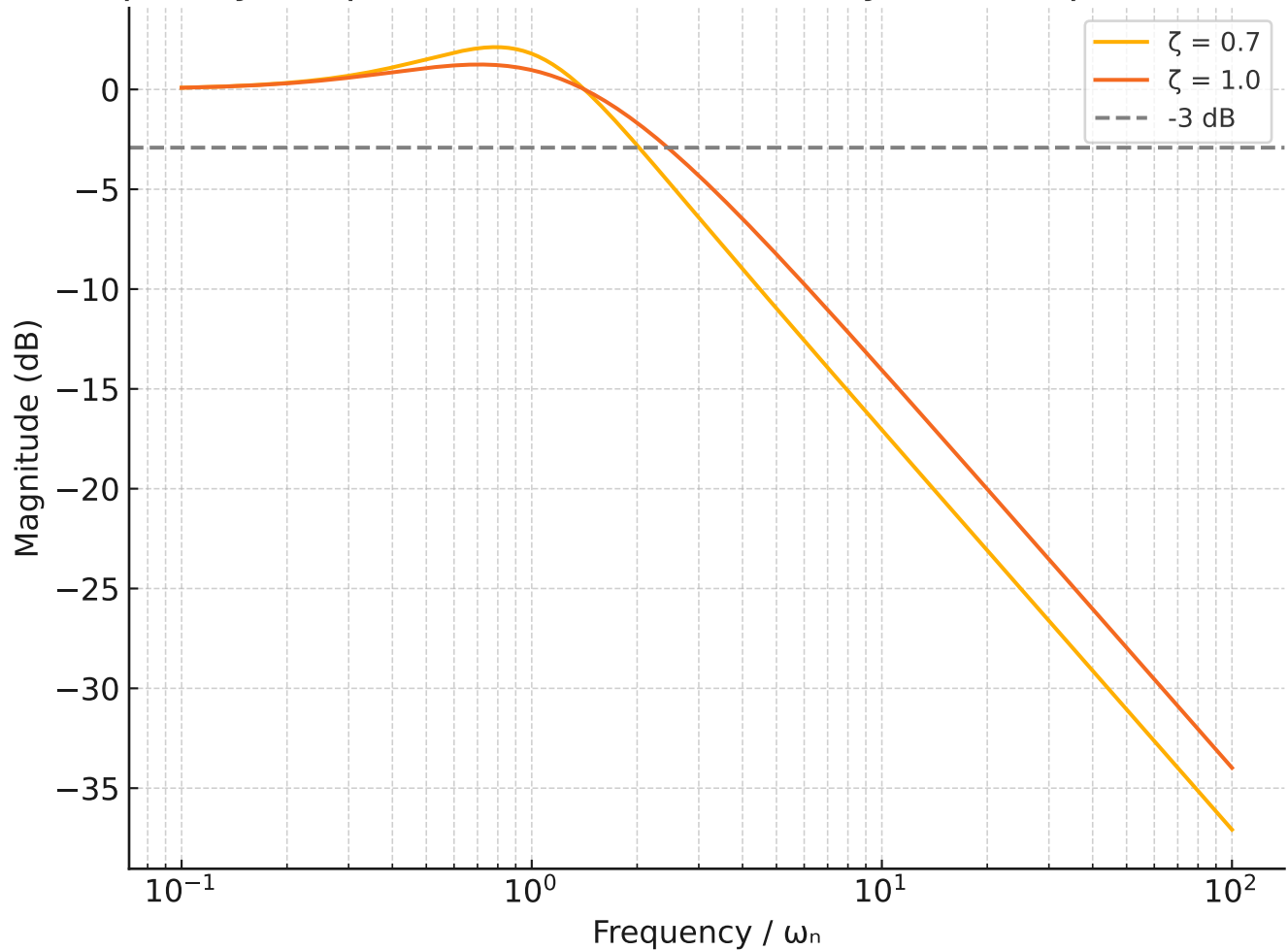


# Frequency Response of Second-Order System (Laplace Domain)



## ADPLL 3 dB Bandwidth Analysis (via ChatGPT)

Transfer Function (Laplace Domain):

$$H(s) = (2\zeta\omega_n s + \omega_n^2) / (s^2 + 2\zeta\omega_n s + \omega_n^2)$$

Magnitude squared:

$$|H(j\omega)|^2 = [(2\zeta\omega_n\omega)^2 + \omega_n^4] / [(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2]$$

To find the -3 dB bandwidth:

$$\text{Solve } |H(j\omega)| = |H(0)| / \sqrt{2} \text{ numerically}$$

Resulting Bandwidths (from analysis):

$$\text{For } \zeta = 0.7 \rightarrow f_{3\text{dB}} \approx 2.04 \times f_n$$

$$\text{For } \zeta = 1.0 \rightarrow f_{3\text{dB}} \approx 2.46 \times f_n$$

Approximate Formula (for quick estimation):

$$f_{3\text{dB}} / f_n \approx \sqrt{1 + 1 / (2\zeta^2)}$$

Comparison:

$$\zeta = 0.7 \rightarrow \text{Approx: } 1.42, \text{ Actual: } 2.04$$

$$\zeta = 1.0 \rightarrow \text{Approx: } 1.22, \text{ Actual: } 2.46$$

Conclusion:

- The approximation is useful for informal estimates
- For accurate results, refer to the frequency response graph
- This relationship is also valid for digital (Z-domain) systems

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