

IIR First-Order Low-Pass Filter: 3 dB Bandwidth Analysis (via ChatGPT)

# Filter Equation:

$$y[n] = (1 - \alpha) * y[n - 1] + \alpha * x[n]$$
  
where  $0 < \alpha < 1$ 

### 3 dB Bandwidth Formula:

$$f_3dB = -ln(1 - \alpha) / (2\pi) * f_s$$

## Explanation:

- This defines the cutoff frequency (in Hz) where the output power falls by half (-3 dB).
- The frequency is expressed relative to the sampling rate f\_s (typically 1 Hz in time-based applications).
- As  $\alpha$  increases, the filter reacts faster and allows more bandwidth through.

#### Observations:

- Small  $\alpha$  → slow response → narrow bandwidth
- Large  $\alpha \rightarrow$  fast response  $\rightarrow$  wide bandwidth
- When  $\alpha \rightarrow 1$ , the cutoff frequency approaches Nyquist (0.5 \* f s)

## Use Cases:

- $\alpha$  is often tuned to match a desired cutoff frequency or smoothing behavior.
- This formula helps convert between intuitive "speed of reaction" and formal filter characteristics.

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