

Yes! From analyzing the image, I can extract the key information regarding the **transfer function transformation** from the **Laplace domain** ($H(s)$) to the **Z-domain** ($H(Z)$) for a **Digital PLL (DPLL)**.

Key Points from the Image:

1. Laplace-Domain Representation:

- The continuous-time transfer function is given as:

$$H(s) = \frac{2\eta\omega_n s + \omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2}$$

- This corresponds to an **analog second-order system**.

2. Discretization Using the Z-Transform:

- The discrete-time **Z-domain transfer function** is:

$$H(Z) = \frac{C_2(Z - 1) + C_1}{(Z - 1)^2 + C_2(Z - 1) + C_1}$$

- Here, Z replaces s as the complex frequency variable.

3. Mapping from Continuous to Discrete:

- The transformation from $H(s)$ to $H(Z)$ is done assuming **high sampling frequency** ($f_s \gg f_n$), which allows the approximations:

$$C_2 = 2\eta\omega_n T, \quad C_1 = \frac{C_2^2}{4\eta^2}$$

- Where:
 - $T = \frac{1}{f_s}$ (sampling period),
 - $\omega_n = 2\pi f_n$ (natural frequency),
 - η is the damping factor.

4. Relation to the Unit Circle:

- In discrete-time systems, the Z-transform is expressed as:

$$Z = e^{j\Omega}$$

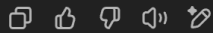
- This means that the digital filter operates in the **unit circle** of the complex plane, where:
 - Ω is the discrete-time angular frequency,
 - The poles and zeros define system stability and frequency response.

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Interpretation:

- The analog transfer function ($H(s)$) describes a continuous second-order PLL.
- The discrete transfer function ($H(Z)$) is obtained using a Z-transform, mapping the system into a digital filter.
- The substitution $Z = e^{j\Omega}$ places the analysis on the unit circle, where digital frequency responses are evaluated.

Would you like me to generate the frequency response of $H(Z)$ using the given parameters? 🚀



I have no problem with the transformation but I am not sure for the meaning of the variable ω_n relative to T . Maybe at this point you can have some insight that are missing in the paper?

Good question! Let's clarify the meaning of ω_n (natural frequency) relative to the sampling period T .

Understanding ω_n in Continuous and Discrete Time

1. Continuous-Time Interpretation (s -Domain)

- The natural frequency of the system is given by:

$$\omega_n = 2\pi f_n$$

- Where:
 - f_n is the natural frequency in Hz.
 - ω_n is the natural angular frequency in rad/s.
 - This applies to the analog domain.

2. Discrete-Time Interpretation (Z -Domain)

- The system is sampled with a period:

$$T = \frac{1}{f_s}$$

- The sampling frequency is related to the discrete-time variable Ω , which represents **digital angular frequency** in the unit circle:

$$\Omega = \omega_n T$$

- In digital systems:
 - Ω is in **radians per sample**.
 - It is the discrete-time equivalent of ω_n .
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Key Relationship:

- In continuous time, ω_n is given in **radians per second**.
 - In discrete time, the equivalent frequency is $\Omega = \omega_n T$, which is in **radians per sample**.
 - Since T is the sampling interval, the **discrete-time system operates in a normalized frequency space**.
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Possible Missing Insights in the Paper

1. Interpretation of ω_n in Discrete-Time Analysis

- The paper assumes $T \ll 1$, meaning the system is sampled at a high rate.
- However, it does not explicitly mention the transformation:
$$\Omega = \omega_n T$$
- This is important because it **scales** the frequency response in the digital domain.

2. Normalized Frequency in Digital Systems

- The paper defines $\omega_n = 2\pi f_n$, but it does not explicitly relate it to the **Nyquist frequency**.
- In digital filtering, **frequencies are often normalized**:

$$\hat{\Omega} = \frac{\omega_n}{f_s} = \frac{\Omega}{2\pi}$$

- This normalization ensures that digital frequencies are **bounded** between:

$$-\pi \leq \Omega \leq \pi$$

- This might be missing in the paper's explanation.

Conclusion

- The missing step is the **transition from continuous-time frequency (ω_n) to discrete-time frequency ($\Omega = \omega_n T$)**.
- If the system response is analyzed in digital form, we must consider the effect of **sampling** and interpret Ω on the **unit circle**.

Would you like me to illustrate this relationship with a graph? 🇮🇹

