# Bilinear Transform – Pre-warping Frequency Analysis

This document summarizes the interpretation of the formula:

ω\_ac = k · tan(Ω\_c / 2)

## What Does k Represent?

The factor k depends on how your analog frequency is defined:

### Case 1: Normalized Frequencies (Digital Filters, T = 1)

- Sampling rate f\_s = 1 Hz  
- Sampling period T = 1  
- Frequencies are in radians/sample  
  
Then:  
 k = 1 or 2 (depending on convention)  
So use:  
 ω\_ac = tan(Ω\_c / 2) or ω\_ac = 2 · tan(Ω\_c / 2)  
  
Note: This is used when working with normalized frequencies only. The scaling is absorbed in the filter design process, and units like rad/s or Hz are not used.

### Case 2: Physical Units (Analog System in rad/s or Hz)

- For real-world analog systems  
- Sampling period T ≠ 1  
  
Then:  
 k = 2 / T  
So use:  
 ω\_ac = (2 / T) · tan(Ω\_c / 2)  
  
This ensures the frequency in the analog design matches the desired cutoff when transformed to the digital domain using the bilinear transform.

## Clarifying the Confusion

The expression ω\_ac = k · tan(Ω\_c / 2) is always mathematically true with k = 2 / T.  
However, in purely normalized filter design (e.g., textbooks or DSP libraries), T is set to 1 and the analog frequency scale is redefined. In that context, k = 1 or 2 may be used to simplify expressions.  
  
✔ Always use k = 2 / T if you're mapping real analog specifications to digital.  
✔ Use k = 1 (or 2) only when working in normalized units with no physical time/frequency scales.

## Summary Table

|  |  |  |  |
| --- | --- | --- | --- |
| Context | T | k | Use Case Example |
| Normalized (digital world) | 1 | 1 or 2 | IIR filter design, normalized DSP libraries |
| Physical analog world | arbitrary | 2 / T | Mapping analog prototype to digital filter |

## Final Recommendation

- If converting digital filter specs into analog domain (pre-warping): use k = 2 / T  
- If working purely in normalized digital filter design: use k = 1 or 2 depending on your conventions

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