# Noise Equivalent Bandwidth (NEB) for Second-Order ADPLL System

This document explains how to compute the noise equivalent bandwidth (NEB) for a second-order low-pass system, such as those used in analog and digital phase-locked loops (ADPLLs).

## System Transfer Function

For a second-order low-pass system with damping factor ζ and natural frequency ωₙ (in rad/s), the transfer function is given by:

H(s) = (2ζωₙs + ωₙ²) / (s² + 2ζωₙs + ωₙ²)

## Definition of Noise Equivalent Bandwidth

The NEB is defined as the bandwidth of an ideal rectangular filter that would pass the same total power (noise energy) as the actual filter. Mathematically, this is:

NEB = (1 / |H(0)|²) × ∫₀^∞ |H(jω)|² dω

For this second-order system, assuming the DC gain is 1 (|H(0)| = 1), the integral simplifies and results in the closed-form solution:

NEB = (π · ωₙ) / (2ζ)

## Conversion to Frequency in Hertz

Since ωₙ is in radians per second, and fₙ (the natural frequency) is in Hertz, we use the relation:

ωₙ = 2π · fₙ

Substituting this into the NEB equation gives:

NEB\_Hz = (π · 2π · fₙ) / (2ζ · 2π) = fₙ / (2ζ)

## Final Formula

NEB\_Hz = fₙ / (2ζ)

## Example

If the natural frequency is fₙ = 1 Hz and damping ζ = 1.0:

NEB\_Hz = 1 / (2 × 1) = 0.5 Hz

This result provides an intuitive understanding of how damping controls noise spreading in a second-order system.

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## Interpretation: Why is NEB Smaller than the 3 dB Bandwidth?

It may seem surprising that the noise equivalent bandwidth (NEB) is often smaller than the 3 dB bandwidth, especially since the 3 dB point is generally considered a defining measure of filter width. Here’s why this makes sense in the context of a second-order phase-locked loop (PLL) system:

1. The system's transfer function is not a simple low-pass filter, but rather a second-order loop filter with a peak (resonance) near the natural frequency ωₙ.

2. The 3 dB bandwidth measures the width of the response where the power drops by half from the peak, but it does not account for how concentrated the energy is within that range.

3. The NEB, by contrast, integrates the entire power response and compares it to an ideal rectangular filter. If the filter response has a sharp peak, then even though the 3 dB bandwidth is wide, the total energy passed is still small — resulting in a smaller NEB.

4. As damping increases and the response flattens, the NEB and 3 dB bandwidth values tend to get closer.

This distinction highlights that NEB and 3 dB bandwidth measure fundamentally different aspects of filter behavior:  
- 3 dB bandwidth indicates frequency span near the peak response.  
- NEB measures total energy passed across the frequency domain.  
Therefore, it is normal and expected that NEB < 3 dB bandwidth in resonant systems.

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