# Initial and Final Value Analysis using Laplace Transform

This document demonstrates how to compute the initial slope and final value of a system's response using the Laplace Transform. These techniques are useful in control systems and signal analysis to determine system behavior at the beginning and end of the response.

## System Description

Given Transfer Function:

H(s) = (2s + 1) / (s² + 2s + 1)

Input: Unit Step Function

U(s) = 1 / s

## Output in Laplace Domain

Y(s) = H(s) × U(s) = (2s + 1) / [s × (s² + 2s + 1)]

## Initial Slope (dy/dt at t = 0⁺)

Using the Initial Derivative Theorem:  
dy(t)/dt|ₜ₌₀⁺ = limₛ→∞ [s² × Y(s)] = limₛ→∞ [s²(2s + 1) / (s × (s² + 2s + 1))]

Simplifying:  
Numerator: s²(2s + 1) = 2s³ + s²  
Denominator: s(s² + 2s + 1) = s³ + 2s² + s  
As s → ∞, divide numerator and denominator by s³:  
limₛ→∞ [(2 + 1/s) / (1 + 2/s + 1/s²)] = 2

→ Initial slope at t = 0⁺ is \*\*2\*\*.

## Final Value (y(t) as t → ∞)

Using the Final Value Theorem:  
y(∞) = limₛ→₀ [s × Y(s)] = limₛ→₀ [s × (2s + 1) / (s × (s² + 2s + 1))]

Canceling s:  
limₛ→₀ (2s + 1) / (s² + 2s + 1) = 1 / 1 = 1

→ Final value as t → ∞ is \*\*1\*\*.

## Summary

• Initial Slope at t = 0⁺: 2

• Final Value as t → ∞: 1