# Initial and Final Value Analysis using Laplace and Z-Transform

## Laplace Domain Analysis

Given H(s) = (2s + 1)/(s² + 2s + 1), input: U(s) = 1/s

Initial slope (t → 0⁺): lim\_{s→∞} s² Y(s) = 2

Final value (t → ∞): lim\_{s→0} s Y(s) = 1

## Z-Domain Analysis

Initial value: lim\_{z→∞} Y(z)

Final value: lim\_{z→1} (1 - z⁻¹) Y(z)

Nyquist point z = -1 is for frequency response, not time-domain slope analysis.

# Analysis of Final Slope Behavior

While initial and final values can be directly extracted from Laplace and Z-domain transforms, the slope at the final value (i.e., the derivative as time approaches infinity) cannot be directly extracted.

## Laplace Domain Perspective

The Final Value Theorem provides:  
 lim\_{t→∞} y(t) = lim\_{s→0} s × Y(s)  
To compute the slope at t → ∞ (i.e., dy/dt as t → ∞), one might consider:  
 lim\_{s→0} s² × Y(s)  
However, this expression actually yields the initial slope (as t → 0⁺), not the final slope. Laplace transforms emphasize initial behavior due to the high-frequency weighting of s.

## Z-Domain Perspective

Similarly, in the Z-domain, initial and final values are computed using limits at z → ∞ and z → 1 respectively. There is no direct method to extract the final slope (i.e., the difference y[n] - y[n−1] as n → ∞) from Y(z).

## Interpretation for Stable Systems

For most stable systems, the output y(t) (or y[n]) converges to a steady value as time increases. This implies that the slope (rate of change) at large time approaches zero:

lim\_{t→∞} dy(t)/dt = 0

This is typically the case for causal, asymptotically stable systems. If a system exhibits exponential decay like y(t) ≈ A·e^(−αt) near the end, then the final slope also decays exponentially to zero.

## Conclusion

While Laplace and Z-transforms provide valuable tools for analyzing system dynamics, extracting the slope at final time directly from these domains is not possible. Such information typically requires knowing the explicit time-domain form of y(t) or its long-term decay behavior.