# Estimating PLL Parameters from Unit Step Response

This document explains how to estimate the parameters of a second-order system or phase-locked loop (PLL) based on its response to a unit step input. This method is useful for validating filter performance or estimating internal settings from external observations.

## Applicable System

The method applies to systems with the following transfer function:  
  
 H(s) = (2ζωₙs + ωₙ²) / (s² + 2ζωₙs + ωₙ²)  
  
This represents the output of a PLL just after the integrator. It is the 'velocity-type' output of a second-order system that can track changing input related to speed.

## Step Response Features to Measure

You can extract the following key features from the unit step response:

* • Peak Value (y\_max)
* • Final Value (y\_inf)
* • Overshoot = (y\_max - y\_inf) / y\_inf
* • Time to Peak (T\_p): when the peak occurs

## Formulas to Estimate Parameters

Estimate the damping factor (ζ) from overshoot:

ζ ≈ -ln(M\_p) / sqrt(π² + ln²(M\_p))

Estimate the natural frequency (ωₙ) from time to peak:

ωₙ ≈ π / (T\_p \* sqrt(1 - ζ²))

If needed, convert ωₙ to frequency (Hz):

fₙ = ωₙ / (2π)

## Example

Measured response:  
• Overshoot ≈ 16%  
• Time to Peak ≈ 6 seconds  
  
→ ζ ≈ 0.503  
→ ωₙ ≈ 0.53 rad/s  
→ fₙ ≈ 0.085 Hz

## Usage in Code

You can automate this analysis in VB.NET by:  
• Collecting step response data  
• Computing overshoot and time to peak  
• Applying the above formulas  
  
This is useful for validating PLL behavior in live systems or during development.

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