EE 126 **Spring 2016**

HW7

Nikhil Unni

1.

2. The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

   (a) Find $MLE[X|Y]$.

   MLE should just be $\arg\max_x P(X = x|Y = y) = \arg\max_x P(Y = y|X = x)$, since all priors are equall likely. So we have:

   $$\arg\max_x xe^{-xy}$$

   =

   $$\arg\max_x \ln(x) - xy$$

   Taking the partial derivative we get:

   $$\frac{\partial}{\partial x} \ln(x) - xy = \frac{1}{x} - y$$

   Settng it to 0 and solving for x, we have:

   $$\frac{1}{x} - y = 0 \implies x = \frac{1}{y}$$

   (b) Find $MAP[X|Y]$.

   Again, we have $\arg\max_x P(X = x|Y = y) = \arg\max_x P(Y = y|X = x)P(X = x)$. Plugging in the actual distributions, we get:

   $$= \arg\max_x e^{-x}(xe^{-xy})$$

   $$= \arg\max_x \ln(x) - x(y + 1)$$

   Taking the partial derivative and setting to 0, we have:

   $$\frac{\partial}{\partial x} \ln(x) - x(y + 1) = 0$$

   $$\frac{1}{x} - y - 1 = 0$$

   $$x = \frac{1}{y + 1}$$

3. The stochastic block model (SBM) as defined in Lab 9 is a random graph $G(n, p, q)$ consisting of two communities of size $\frac{n}{2}$ each such that the probability an edge exists between two nodes of the same community is p and the probability an edge exists between two nodes in different communities is q, where $p > q$. The goal of the problem is to exactly determine the two communities given only the graph. Show that the MAP-decision rule is equivalent to finding the min-bisection of the graph.

4. In this problem, we use similar settings which were considered in HW2. Consider a random bipartite graph, G_1, with K left nodes, and M right nodes. Each of the KM possible edges of this graph is connected with probability p independently. In the following problems, we consider the situations when M and K are large and Mp and Kp are constants. Hint : Use the Poisson distribution to approximate binomial distribution and apply law of large numbers.

   (a) A singleton is a right node of degree one. As M and K get large, how many left nodes are connected to right nodes which are singletons?

   (b) A doubleton is a right node of degree two. As M and K get large, how many doubletons do we have?

   (c) We call 2 doubletons distinct, if they are not connected to the same 2 left nodes. As k and M get large, what is the probability that two doubletons are distinct?