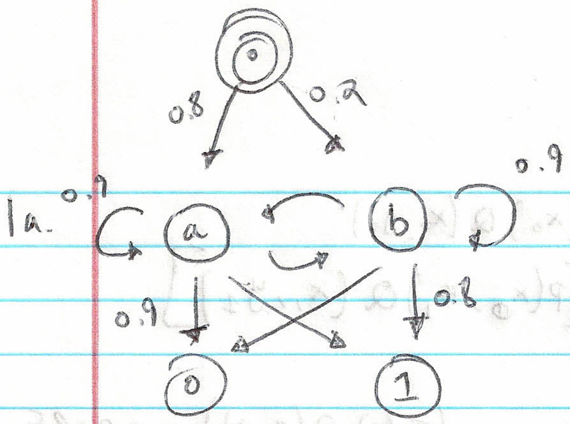


$\boxed{X_1 = a}$

$\boxed{X_2 = a}$

$\cancel{P(X_1 | Y_1, Y_2)} = \underset{X_1}{\operatorname{argmax}}\ P(X_1 | Y_1, Y_2)$

$= \underset{X_1}{\operatorname{argmax}}\ P(X_1) P(Y_1, Y_2 | X_1)$

$\{ P(X_1 = a) P(Y_1, Y_2 | X_1 = a),$
$P(X_1 = b) P(Y_1, Y_2 | X_1 = b) \}$

$= \{ 0.8 \cdot P(Y_1 | X_1 = a) P(Y_2 | X_1 = a, X_1),$
$0.2 \cdot$ " " $\}$

$= \{ 0.8 \cdot 0.9 \cdot (0.9 \cdot 0.1 + 0.1 \cdot 0.8),$
$0.2 \cdot 0.2 \, (0.9 \cdot 0.8 + 0.1 \cdot 0.1) \}$

$= \{ \underline{0.1229}, 0.0292 \}$

$\underset{X_2}{\operatorname{argmax}}\ P(X_2 | Y_1, Y_2)$

$= \underset{X_2}{\operatorname{argmax}}\ P(X_2) P(Y_1, Y_2 | X_2) = \{ P(X_2 = a) P(Y_1, Y_2 | X_2 = a), \ldots \}$

$= \{ (0.8 \cdot 0.9 + 0.2 \cdot 0.1) P(Y_2 | X_2 = a) P(Y_1 | Y_2, X_2),$
$0.26 \, P(Y_2 | X_2 = b) P(Y_1 | Y_2, X_2 = b)$

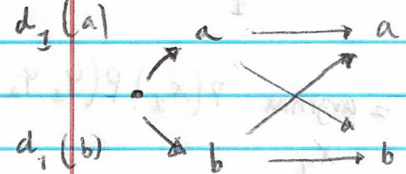
$= \{ 0.74 \cdot 0.1 \cdot (0.9 \cdot 0.9 + 0.1 \cdot 0.2),$
$0.26 \cdot 0.8 \cdot (0.1 \cdot 0.9 + 0.9 \cdot 0.2) \}$

$= \{ 0.06, 0.05 \}$

2,1

b. $\mathrm{MAP}\left[X^{(n)} = x^n \mid Y^n = y^n\right] = \log \pi_0(x_0)\, Q(x_0, y_0)$

$$+ \log\left[P(x_0, x_1)\, Q(x_1, y_1)\right]$$



~~$X_1 = a$~~
~~$X_2 = b$~~

$d_1(a) = -\ln\left(\pi_0(a)\, Q(a,0)\right) = 0.3285$

$d_1(b) = -\ln\left(\pi_0(b)\, Q(b,0)\right) = 3.2188$

$d_2(a,b) = -\ln\left(P(a \to b)\, Q(b,1)\right) = 2.525$

$d_2(b,a) = -\ln\left(P(b \to a)\, Q(a,1)\right) = 4.6051$

$d_2(a,a) = -\ln\left(P(a \to a)\, Q(a,1)\right) = 2.4079$

$d_2(b,b) = -\ln\left(P(b \to b)\, Q(b,1)\right) = 0.3285$

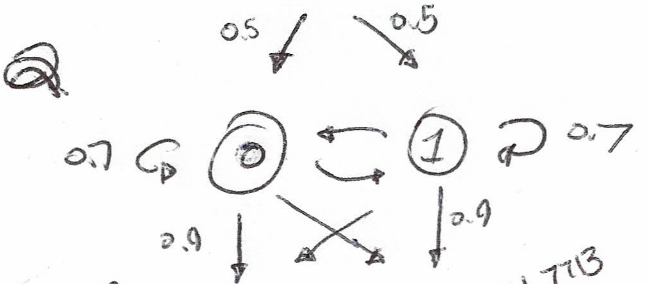
Fastest way through a_1 is to take a_2 : 2.4079

Fastest way through b_1 is to take b_2 : 0.3285

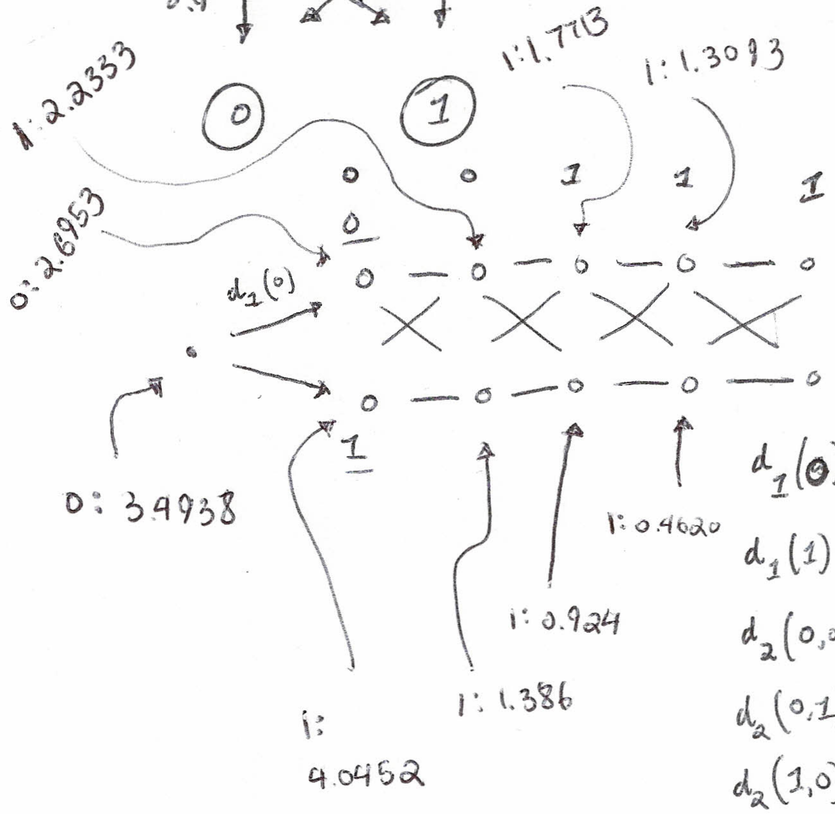
Fastest way is $\boxed{a_1, a_2}$: 2.7364

vs. b_1, b_2 : 3.5473

$$\boxed{\begin{matrix} X_1 = a \\ X_2 = a \end{matrix}}$$



$$Y = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad X = ?$$



$d_1(0) = -\ln(\pi_0(0)\,Q(0,0)) = 0.7985$

$d_1(1) = -\ln(\pi_0(1)\,Q(1,0)) = 2.9957$

$d_2(0,0) = -\ln(P(0\to 0)\,Q(0,0)) = 0.4620$

$d_2(0,1) = -\ln(P(0\to 1)\,Q(1,0)) = 3.5066$

$d_2(1,0) = -\ln(P(1\to 0)\,Q(0,0)) = 1.3093$

$d_2(1,1) = -\ln(P(1\to 1)\,Q(1,0)) = 2.6592$

$\cancel{d_3(0,0) = -\ln(P(0\to 0)\,Q(0,1))}$

$\cancel{d_3(x,y) = d_2(x,y)}$

$\cancel{d_4}$

$d_3(0,0) = -\ln(P(0\to 0)\,Q(0,1)) = 2.6592$

$d_3(0,1) = -\ln(P(0\to 1)\,Q(1,1)) = 1.3093$

$d_3(1,0) = -\ln(P(1\to 0)\,Q(0,1)) = 3.5066$

$d_3(1,1) = -\ln(P(1\to 1)\,Q(1,1)) = 0.4620$

$d_4(x,y) = d_3(x,y)$

$d_5(x,y) = d_3(x,y)$

Most likely sequence of X is $[0, 0, 1, 1, 1]$

3. We know that conditioned on Y, Z that X should be Gaussian.

With a slight modification to the derivation ~~x~~ from Walrand, we know that

$$E[X \mid Y, Z] = E[X] + \left(\operatorname{cov}(X,Y), \operatorname{cov}(X,Z)\right)\begin{pmatrix} \operatorname{var}(Y) & \operatorname{cov}(Y,Z) \\ \operatorname{cov}(Y,Z) & \operatorname{var}(Z)\end{pmatrix}^{-1}\begin{pmatrix} Y - E[Y] \\ Z - E[Z]\end{pmatrix}$$

$$= 0 + (3, 1)\begin{pmatrix} 9 & 3 \\ 3 & 2\end{pmatrix}^{-1}\begin{pmatrix} Y \\ Z\end{pmatrix}$$

$$= (3, 1)\begin{pmatrix} 2/9 & -1/3 \\ -1/3 & 1\end{pmatrix}\begin{pmatrix} Y \\ Z\end{pmatrix}$$

$$= \left(\tfrac{1}{3}, 0\right)\begin{pmatrix} Y \\ Z\end{pmatrix}$$

$$= \boxed{\tfrac{1}{3} Y}$$

which we know is Gaussian because it is equivalent to $(0, \tfrac{1}{3}, 1) \cdot (X, Y, Z)^T$

$\cancel{3B.\ E[X \mid Y,Z] = E[X-a}$

$U \sim N(0,2) \quad X-Z$

$Y \sim N(0,2) \quad X+Z$

4. $f_{U,Y} = f_{U|Y}(u|y)\, f_Y(y)$ $\quad (U,Y)^T \sim N\left(0, \begin{bmatrix} \cancel{var(U)}\ 2 & cov(Y,U) \\ cov(U,Y) & \cancel{var(Y)}\ 2 \end{bmatrix}\right)$

$cov(U,Y) = E[UY] - \cancel{E[U]}E[Y]$

$E\left[\frac{var(X+Y)}{4} Q - \frac{var(X-Y)}{4} R\right]$

$= \frac{1}{2} - \frac{1}{2} = 0$

a. $cov(X-Y, X+Y)$

$= E[X^2 - Y^2] - E[X-Y]E[X+Y]$

$= E[X^2] - E[Y^2]$

$= 1 - 1 = 0.$

So, $f_{U,Y} = f_U f_Y \sim N(0,2) \cdot N(0,2)$

$M_{U,Y}(s_1, s_2) = E\left[e^{s_1 U + s_2 Y}\right]$

$= E[$

$var(X) = E[X^2] - E[X]^2$

$E[X^2] = var(X) = 1$

b. $\hat{X}(Y) = E[X \mid Y]$

$= E[X \mid X+Z] = E[Z \mid X+Z]$

$E[X \mid X+Z] + E[Z \mid X+Z] = Y$

$\boxed{E[X \mid X+Z] = \frac{Y}{2}}$

c. $f_{E|Y} = P\left(X - \frac{Y}{2} = k \mid Y\right)$

$\sim \boxed{N\left(-\frac{Y}{2}, 0\right)}$

5 a. $E[X(0) \mid Y(0)] = E[\cancel{\ } X(0) \mid \beta X(0) + W(0)] = \dfrac{Y(0) - E[W]}{\beta}$

$= \boxed{\dfrac{Y(0)}{\beta}}$

b. $E[Y(n) \mid Y(0), \ldots, Y(n-1)]$

$= E[\beta X(n) + W(n) \mid Y(0), \ldots, Y(n-1)]$

$= E[\alpha\beta X(n-1) + \cancel{\ } \beta V(n) + W(n) \mid Y(0), \ldots, Y(n-1)]$

$= \alpha\beta E[X(n-1) \mid Y(0), \ldots Y(n-1)]$ (since V and W are 0-mean i.i.d)

$= \boxed{\alpha\beta\hat{X}(n-1)}$

$E[X(n) \mid Y(0), \ldots, Y(n-1)]$

$= E[\alpha X(n-1) + V(n) \mid Y(0), \ldots, Y(n-1)]$

$= \boxed{\alpha\hat{X}(n-1)}$