**Math 113** **Fall 2015**

# Homework #12

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1. (a) Show that $x^3 + 2x + 1$ is irreducible in $\mathbb{Z}_3[x]$.

   We can just try out all of the possible roots:

   $$0^3 + 0x + 1 = 1 \neq 0$$

   $$1^3 + 2 * 1 + 1 = 1 \neq 0$$

   $$2^3 + 2 * 2 + 1 = 1 \neq 0$$

   None of the possible values are zeros of the polyomial, so it's irreducible in $\mathbb{Z}_3[x]$.

   (b) Let $\alpha$ be a zero of $x^3 + 2x + 1$ in an extension field of $\mathbb{Z}_3$. Show that the polyonmial factors into three linear factors in $\mathbb{Z}_3(\alpha)[x]$ by finding the factorization.

   $$x - \alpha \overline{\big) x^3 + 0x^2 + 2x + 1}^{\textstyle x^2 + \alpha x + (2 + \alpha^2)}$$

   (I couldn't format the division properly...but I've listed the remainders here)

   $$(x^3 + 0x^2) - (x^3 - \alpha x^2) = \alpha x^2$$

   $$(\alpha x^2 + 2x) - (\alpha x^2 - \alpha^2 x) = (2 + \alpha^2)x$$

   $$((2 + \alpha^2)x + 1) - ((2 + \alpha^2)x - (2\alpha + \alpha^3)) = 1 + 2\alpha + \alpha^3$$

   Because we know that our remainder is 0, since it's literally $\alpha$ plugged into our original polyonmial for which it's a solution, we have a clean division:

   $$(x - \alpha)(x^2 + \alpha x + (2 + \alpha^2)) = x^3 + 2x + 1$$

   I had a hectic week – couldn't finish the homework on time.