EE 122	**Spring 2016**

HW1

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1. 1.1 Just writing out the probabilities in order, it's simply:

$$\begin{pmatrix} 0 & 1-p & p \\ 1 & 0 & 0 \\ 0 & \alpha & 1-\alpha \end{pmatrix}$$

   1.2 We have paths to all points from all points (it is irreducible) as long as both $p > 0$ and $\alpha > 0$. And it is also aperiodic under the same conditions.

   1.3

$$\begin{pmatrix} 0 & 1-p & p \\ 1 & 0 & 0 \\ 0 & \alpha & 1-\alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

   Separating it out as a system of equations we get:

$$(1-p)y + pz = x$$

$$x = y$$

$$\alpha y + (1-\alpha)z = z$$

   Solving it, we find that $x = y = z$. So:

$$\pi = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

2. 2.1 The states of the system are $\{0, 1, \cdots, k\}$.

   2.2

3. We can calculate this with Little's Result. First we need to find the average occupancy. The packets coming in on average have $\frac{4000+400}{2} = 2200$ bits. And the average Mbps is $0.8 * 4 + 0.2 * 1 = 3.4$ Mbps. So the average occupancy should be $2200 * \frac{1}{3.4} * \frac{1}{10^6} = .000647059$ seconds. So then, average delay is $\frac{.000647059}{100} = .000006471$ seconds, or .006471 ms.

4. 4.1 It can be modeled with a CTMC because, if we're at any number of lines, call it k, with probability λ, we could get another call at any point in time, but with probability μ, they could finish a call at any point in time. And because of the "memoryless" property of the Poisson process and the Exponential distribution, the amount of time we've already waited for a new call to come in, or for a call to finish does not change the probability of when the call is going to finish or when a new call is going to come in. Given all of these Markov properties, it can be modeled with a CTMC.

   4.2 With $\lambda = \frac{30}{60} = 1/2$ call/min, and $\mu = 4$ min, the matrix would look like:

$$Q = \begin{pmatrix} -1/2 & 1/2 & 0 & 0 \\ 4 & -9/2 & 1/2 & 0 \\ 0 & 4 & -9/2 & 1/2 \\ 0 & 0 & 4 & -9/2 \end{pmatrix}$$

   4.3 We can show it by Little's Result.

4.4 From the lecture slides, we know $\pi(n) = (1-\rho)^n \rho^n$. So we get:

$$\pi = \begin{pmatrix} 1-\rho \\ \rho-\rho^2 \\ \rho^2-\rho^3 \\ \rho^3-\rho^4 \end{pmatrix} = \begin{pmatrix} 7/8 \\ 7/64 \\ 7/512 \\ 7/4096 \end{pmatrix}$$

And so the blocking probability is $\frac{7}{4096}$.

4.5 Looking at the invariant distribution, they should subscribe to at least two phone lines, since the blocking probability at just one phone line is slightly over 10%.

5. 5.1 Again, using the ρ formula above, it's the probability of $\pi_1(2)\pi_2(3)$. This is just $(1-\frac{\lambda_1}{\mu_1})^n(\frac{\lambda_1}{\mu_1})^n * (1-\frac{\lambda_2}{\mu_2})^n(\frac{\lambda_2}{\mu_2})^n$, with $\lambda_1 = \lambda_2 = \frac{1}{6}$ customers per minute, $\mu_1 = 2$ minutes, and $\mu_2 = 4$ minutes.

5.2 Using Little's Law again, we know the average occupancy is $2+4 = 6$ minutes. And the arrival rate is 10 customers per hour, or $\frac{1}{6}$ customers per minute. So then the average delay is $6 * 6 = 36$ minutes.

5.3 We can treat this problem as an M/M/1 of arrival rate $\frac{1}{6}$ customers per minute, and average service time of 6 minutes. Then, $\pi(4) = (1-\frac{1}{36})^n(\frac{1}{36})^n = .000000532$

6. 6.1 Yes, we could still possibly transmit high-fidelity radio. If we increase the power of the signal, we may be able to achieve a high enough SNR.

6.2 Under Shannon's Theorem: $C = Wlog_2(1+\frac{S}{N})$. Plugging in, we get:

$$270880 = 200000log_2(1+SNR)$$

$$SNR = 1.5569$$

With an SNR of 10, we get:

$$C = 200000log_2(1+10) = 691.88 \text{ Kbps}$$