EE 122		**Spring 2016**
	HW1	
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1. If we order the edges as : $v_1v_2, v_1v_3, v_1v_4, v_1v_5, v_2v_3, v_3v_4, v_4v_5$, and keep the order of vertices, then the incidence matrix becomes:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

2. Running Dijkstra's Algorithm on the graph yields very similar results to just a nave BFS, except that node 6 is updated. Running Dijkstra's Algorithm, we end up with a table like:

Node	Shortest Distance	Previous Node
1	3	2
2	1	3
3	0	
4	8	3
5	7	1
6	9	5
7	6	1

Following the paths backwards to node 3, we can construct node 3's routing table:

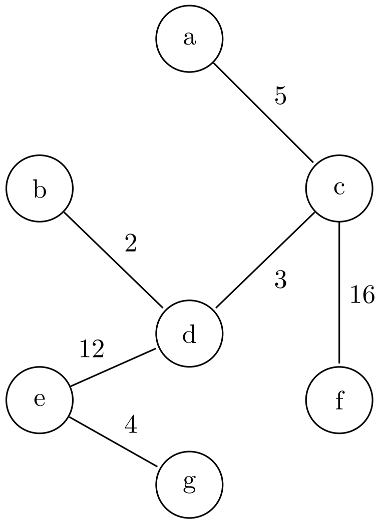
Destination	Next
1	2
2	2
3	
4	4
5	2
6	2
7	2

3. Spanning and Steiner Trees

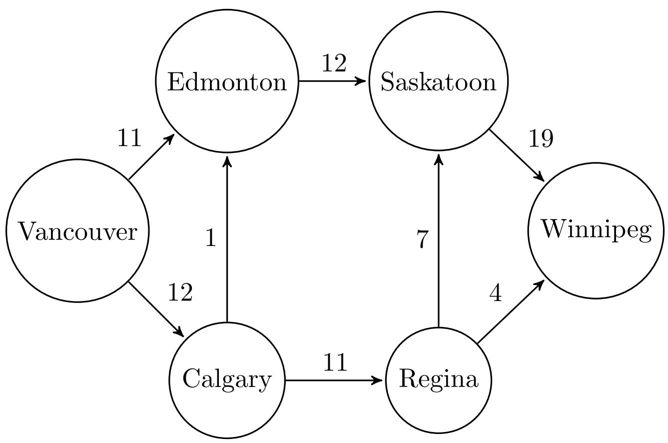
   3.1 A Spanning Tree is a subgraph of a graph that includes every node, but does not have any cycles (i.e. is a valid tree). A Minimum Spanning Tree is the Spanning Tree (since Spanning Trees are not unique) with the least total sum of the edge weights in the tree.

   3.2 The difference between a Steiner Tree and a Spanning Tree is that you may add extra edges and nodes to the original graph when constructing the Steiner Tree (in order to reduce the total cost of the resulting tree).

   3.3 The total cost of the minimum spanning tree (run using Kruskal's Algorithm) is **42**.



4. Using Ford-Fulkerson, we find the maximum flow in the graph is given by the following:



So we see that the maximum flow in the graph is **23**.

5. 5.1 The unreliability of the series is given by:

$$Q_S = 1 - R_{S1}R_{S2}$$

The reliability of the parallel components is given by:

$$R_P = R_{P1} + R_{P2} - R_{P1}R_{P2}$$

Running these two in series, we get:

$$Q_{\text{top}} = 1 - (R_{S1}R_{S2})(R_{P1} + R_{P2} - R_{P1}R_{P2})$$

Finally, putting this together with the bottom, we get:

$$Q_{\text{system}} = (1 - R_{\text{bottom}})(1 - (R_{S1}R_{S2})(R_{P1} + R_{P2} - R_{P1}R_{P2}))$$

5.2 With m=n=2 and a component reliability of 0.8, we just plug in the numbers to get:

$$Q_{\text{system}} = 0.06912$$

Which means about a 7% error.

5.3 When n becomes very large, Q_S approaches 1, which means Q_{top} approaches 1, which means failure of the system is dominated by Q_{bottom} at the bottom, and will get closer and closer to Q_{bottom}. Similarly, when m becomes very large, R_P approaches 0, and, again, Q_{top} approaches 1, and failure of the system is dominated by Q_{bottom}.