V14 MARKO DOKIN

1.4 0036524589

$$D = \begin{bmatrix} (9.5, -0.7, -2.8), \\ (8.8, -0.8, -3.2), \\ (6.5, -0.2, -0.8), \\ (2.3, 0.3, 1.2), \\ (2.2, 0, 0), \\ (3.6, 0.3, 1.2) \end{bmatrix}$$

a) $\hat{\mu}_{MLE} = \frac{1}{N} \sum x^{(i)} = [5.48, -0.18, -0.733]$

$\hat{\Sigma}_{MLE} = \frac{1}{N} \sum (x^i - \mu_{MLE}) \cdot (x^i - \mu_{MLE})^T$

$$= \begin{bmatrix} 5.48 & -0.1833 & -0.73 \\ 8.77 & -1.198 & -4.79 \\ -4.79 & 0.765 & 3.062 \end{bmatrix}$$

gustoća nije dobro definirana jer moramo dijeliti s nulom

b) determinanta Σ matrice je 0. a fja za gustoću je:

$\ast \; p(x|\mu, \Sigma) = \frac{1}{(2\pi)^{n/2} \cdot |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$

c) $\rho_{12} = \frac{-0.183}{\sqrt{5.48 \cdot -1.198}} = -0.92$

$\rho_{13} = -0.92 \quad \rho_{23} = 1$

Varijable 2 i 3 su savršeno ZAVISNE.

Izbacujemo varijablu 3

Sada nam primjer x^i imaju 2 značajke 1 i 2.

$\hat{\mu}_{MLE} = [5.48, -0.183]$

$$\hat{\Sigma}_{MLE} = \begin{bmatrix} 8.77 & -1.19 \\ -1.19 & 0.19 \end{bmatrix}$$

$|\Sigma| \neq 0$

$\ast : p([-2, 1]) = 0.0083$

1.5

a) $\theta_{MAP} = \arg\max_{\theta} p(\theta|D) = \arg\max_{\theta} p(D|\theta) \cdot p(\theta)$

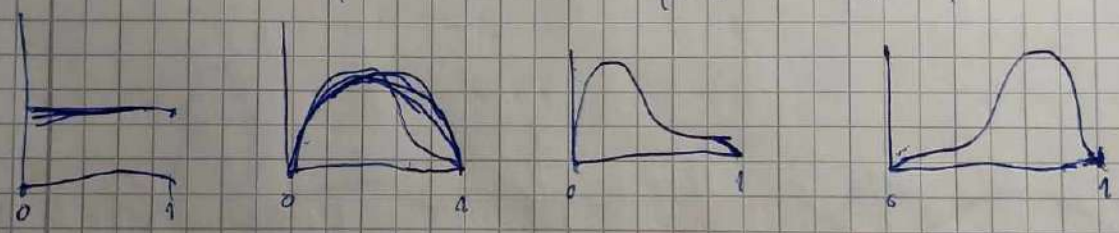
MAP je bolji od MLE jer koristimo apriorno znanje i informacije iz skupa podataka.

b) 1) Konjugatne distribucije su one koje pripadaju istoj familiji.

2) Konjugatna apriorna distribucija je ona distribucija, koja pomnožena sa distribucijom podataka, daje istu vrstu distribucije kao posterior.

c) Bitno nam je jer možemo raditi "online" učenje.

$\alpha = \beta = 1$ $\alpha = 2, \beta = 2$ $\alpha = 2, \beta = 4$ $\alpha = 4, \beta = 2$



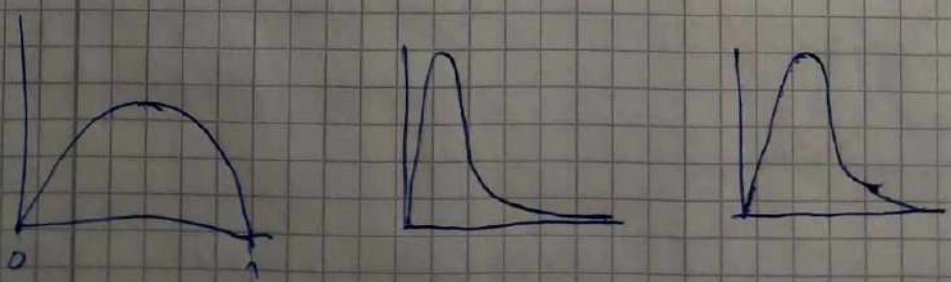
d) $p(\mu|D, \alpha, \beta) = \mu^m (1-\mu)^{N-m} \cdot \frac{1}{B(\alpha,\beta)} \cdot \mu^{\alpha-1} (1-\mu)^{\beta-1} \cdot \frac{1}{p(D)}$

$= \mu^{m+\alpha-1} \cdot (1-\mu)^{N-m+\beta-1} \cdot \frac{1}{B(\alpha,\beta) \cdot p(D)}$

$= \mu^{\alpha'-1} \cdot (1-\mu)^{\beta'-1} \cdot \frac{1}{B(\alpha',\beta')}$

e)

APRIORI PODACI UMNOŽAK



MARKO ĐUKIN 0036524583

1) $\hat{\mu}_{MAP} = \frac{1+2-1}{2+10+2-2} = \frac{2}{12} = 0.167 = \frac{m+\alpha-1}{\alpha+N+\beta-2}$

$\hat{\mu}_{MLE} = \frac{1}{10} = 0.1$ NI : razlika između $\hat{\mu}_{MAP}$ i $\hat{\mu}_{MLE}$ bi se smanjivala

g) Ako odaberem $\alpha = \beta = 2$

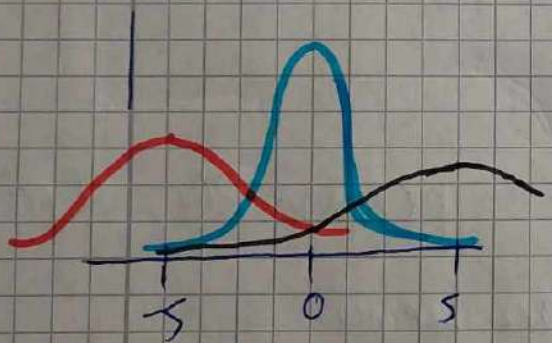
$\hat{\mu}_{MAP} = \frac{m+\alpha-1}{\alpha+N+\beta-2} = \frac{m+1}{N+2}$ ⇒ Laplace-ov procjenitelj

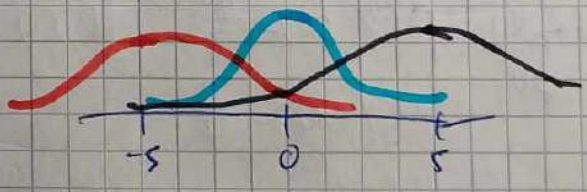
---

VIS, 1.3

$p(x|y)$
- $p(x|y=1)$
- $p(x|y=2)$
- $p(x|y=3)$

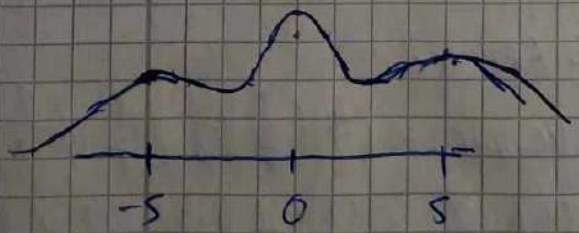
$p(x,y)$
- $p(x|y=1) \cdot p(y=1)$
- $p(x|y=2) \cdot p(y=2)$
- $p(x|y=3) \cdot p(y=3)$

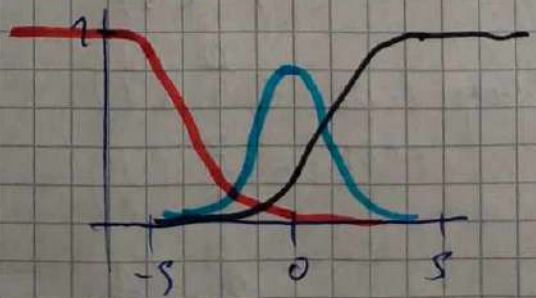




$p(x) = \sum p(x|y=i) \cdot p(y=i)$

- $p(y=1|x)$
- $p(y=2|x)$
- $p(y=3|x)$





MARKO ĐUKIN 0036524585

MARKO ĐUKIĆ
0073524587

V15.

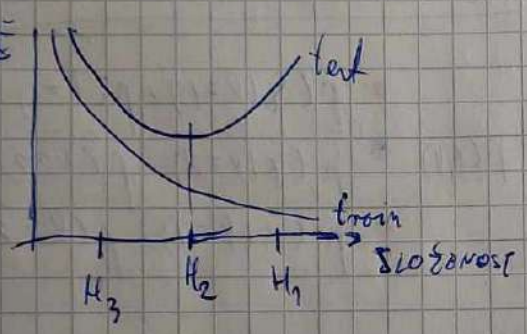
1.5) a) H_1: $\frac{n}{2}(n+1) + k \cdot n + k - 1 = 34$

H_2: $n + k \cdot n + k - 1 = 19$

H_3: $1 + k \cdot n + k - 1 = 14$

b) H_2 jer nije blizu preraspodjeli kao H_1 ili podraspodjeli kao H_3.

c)



d) Unakrsnom provjerom

V16.

1.2a) $p(Y=DA) = \frac{4}{7}$ U C) Laplac

b) $p(Y=NE) = \frac{3}{7}$

$p(Y|X) = P(Y) \cdot \prod p(X_i|Y)$

BEZ LAPLACA:

$P(DA|X) = \left(\frac{4}{7}\right)\left(\frac{2}{4}\right)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{0}{4}\right)\cdot\left(\frac{0}{4}\right) = 0$

$P(NE|X) = \left(\frac{3}{7}\right)\left(\frac{0}{3}\right)(1)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = 0$

Ne možemo raditi klasifikaciju

S LAPLACOM

$P(DA|X) = \left(\frac{4}{7}\right)\left(\frac{3}{7}\right)\left(\frac{1}{3}\right)\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)$

$P(NE|X) = \left(\frac{3}{7}\right)\left(\frac{1}{6}\right)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

DA: $1.66 \cdot (0^{-3})$

NE: 0,014

$|X_1| = 3$: $p(X_1 = kuća | Y=DA) = \frac{2}{4}\left(\frac{3}{7}\right)$, $p(X_1 = kuća | Y=NE) = \frac{0}{3}\left(\frac{1}{6}\right)$

$|X_2| = 2$: $p(X_2 = Ne | Y=DA) = \frac{1}{4}\left(\frac{1}{3}\right)$, $p(X_2 = Ne | Y=NE) = \frac{3}{3}\left(\frac{4}{5}\right)$

$|X_3| = 3$: $p(X_3 = kompl | Y=DA) = \frac{0}{4}\left(\frac{1}{7}\right)$, $p(X_3 = kompl | Y=NE) = \frac{2}{3}\left(\frac{1}{2}\right)$

$|X_4| = 3$: $p(X_4 = kuća | Y=DA) = \frac{0}{4}\left(\frac{1}{7}\right)$, $p(X_4 = kuća | Y=NE) = \frac{2}{3}\left(\frac{1}{2}\right)$