Rocket Project Report

This report seeks to provide justification for the variance from the given values in the parameters for step size, thrust, and initial propellant mass. It will also discuss the organization and interpretation of the results and underscore a few important limitations. Lastly, the usage of the Monte Carlo simulation will be summarized.

Stability testing began with a step size of 0.5, as advised. Testing proceeded with iterations corresponding to steps of 0.05 and 0.005. The difference between the outputs produced by the latter two iterations were minute and negligible for the purposes of this project (see the data set at the end of the next page). Therefore, it was unnecessary to decrease the step size beyond 0.005, given that such a reduction results in greater computational expense for the sake of negligible improvements in accuracy. Further, once an adequate step size is established, continuing to reduce it is inadvisable as it eventually introduces round off errors. The thrust curve used in this project is that of a single stage booster from the rocket Lambda-4s[1] (see footnote for more details). The csv file of thrust data was then interpolated to arrive at a continuous thrust function. The thrust was scaled by a factor of five to provide a more realistic flight trajectory. The initial propellant mass was modified because the given value of 1000 kgs was exhausted before the thrust curve reached completion. There were approximately 300 kgs of leftover fuel for a previous test value of 1500 kgs, so 1200 kgs was chosen as the best match for the thrust curve.

The notebook is organized in the following way. First, the physics model and RK4 methods are used to compute the two dimensional positions and velocities as well as the propellant mass. Then, the code computes the $\varepsilon(t)$ and $L(t)$ values at apogee, which are used to determine the orbit and escape conditions and compute apoapsis and periapsis. The code then provides a graph of the original thrust curve data overlaid with the interpolated thrust function, showing excellent correspondence to one another. The next graphs are $y(x)$, $x(t)$, $y(t)$, $v(t)$, and mprop(t). Lastly, Monte Carlo simulations are run to determine the likely variance in possible trajectories and apoapses.

These results were obtained using a constant launch angle, and they indicate a bound orbit around the center of mass, but one that intersects the surface of the earth in reality. The values for apoapsis and periapsis are calculated as measured from the earth's center of mass. In this case, periapsis occurred just a few thousand meters away. This indicates that the flight trajectory was very short considering the size of the earth: the rocket was very quickly overcome by gravity and intersected the surface. This is consistent with the distances described by the simulation. An important clarification is that apogee was chosen as the moment for the computation of $\varepsilon(t)$ and $L(t)$ because the motion there is at its slowest and is least affected by the atmosphere, which should correspond to the most accurate values from the RK4. It is also simply a convenient time to base multiple different calculations on given its consistency. Another point of interest is the second local maximum in the velocity function. After apogee, the rocket accelerates downward. However, while the gravitational acceleration remains practically constant, the density of the atmosphere exerts an increasingly powerful opposing force, causing

[1] Interpolated First Stage Thrust Curves

the speed of the rocket to begin decreasing again.

A few limitations of this simulation include the physical model, as well as the treatment of the rocket parameters. The drag and atmospheric density equations are simplified, and not extremely accurate at very high velocities and altitudes. The rotation of the earth is also not taken into account. This simulation uses one single stage booster with an 8 second burn to attempt an orbital flight path, which is effectively impossible in actual practice. The rocket is also launched at a 60 degree angle, which is retained throughout the trajectory. A more accurate representation would launch vertically and gimbal the engine later to produce the lateral thrust.

The Monte Carlo simulation coded here uses the method of quasi-random sampling via SciPy's LatinHypercube method, which creates a number of equal strata in an interval, and draws one random sample from each stratum. This method was used because of computational expense: it creates a fairly accurate distribution without requiring the very large number of runs that random sampling across the entire interval would necessitate. The number of runs used in the code considers the balance between computational expense and accuracy, finding that approximately 100 runs represents an acceptable middle ground. The results of the simulation are a graph displaying the various trajectories with the nominal trajectory in bold, as well as a histogram of the apoapses.

In short, while it has many significant limitations, this simulation can be summarized as a pedagogical introduction to flight trajectory modelling through applying fourth order Runge-Kutta methods to a basic physics model using Monte Carlo analysis.

**Stability Test:**

h = 0.5:
Specific orbital energy at apogee (epsf) = -62240481.71654878 $m^2/s^2$
Angular momentum at apogee (Lf) = 1781702725.4051788 $m^2/s$
Suborbital
Distance from earth's center of mass:
Apoapsis = 6399745.858626817 m
Periapsis = 3984.7843194449224 m

h = 0.05
Specific orbital energy at apogee (epsf) = -62235270.364953265 $m^2/s^2$
Angular momentum at apogee (Lf) = 1798051360.7016804 $m^2/s$
Suborbital
Distance from earth's center of mass:
Apoapsis = 6400208.5739085125 m
Periapsis = 4058.293781494277 m

h = 0.005

Specific orbital energy at apogee (epsf) = -62235255.94327908 $m^2/s^2$
Angular momentum at apogee (Lf) = 1798158388.578525 $m^2/s$
Suborbital
Distance from earth's center of mass:
Apoapsis = 6400209.574503668 m
Periapsis = 4058.7772366381787 m