Database Systems, CSCI 4380-01
Homework # 2 Practice Problem for Question 4

Question 4 [10 points]. Convert the following set of functional dependencies to minimal basis. Show all steps:

$$\mathcal{F} = \{ABCD \rightarrow E, E \rightarrow ABEFG, CD \rightarrow GH, A \rightarrow B, F \rightarrow E, CE \rightarrow D, G \rightarrow AG\}$$

See next pages for solution.

Step 1: convert to basis form (splitting rule)

$\mathcal{F} = \{A \to B,\ ABCD \to E,\ CD \to G,\ CD \to H,\ CE \to D,\ E \to A,\ E \to B,\ E \to E,\ E \to F,\ E \to G,\ F \to E,\ G \to A,\ G \to G\}$

Step 2: remove trivial functional dependencies ($E \to E$, $G \to G$)

$\mathcal{F} = \{$
$A \to B,\ ABCD \to E,\ CD \to G,\ CD \to H,\ CE \to D,$
$E \to A,\ E \to B,\ E \to F,\ E \to G,\ F \to E,\ G \to A$
$\}$

Step 3: remove redundant attributes

We only need to test the functional dependencies that have more than one attribute on the LHS.

1. Testing each attribute on the LHS of $ABCD \to E$:

   $\{BCD\}^+ = \{BCDGHAEF\}$, proves that A is not necessary (since BCD will determine A)

   $\{CD\}^+ = \{CDGHABEF\}$, proves that B is not necessary (since CD will determine AB)

   $\{D\}^+ = \{D\}$, so we cannot remove C

   $\{C\}^+ = \{C\}$, so we cannot remove D

2. Testing each attribute on the LHS of $CD \to G$:

   $\{D\}^+ = \{D\}$, so we cannot remove C

   $\{C\}^+ = \{C\}$, so we cannot remove D

3. Testing each attribute on the LHS of $CD \to H$:

   $\{D\}^+ = \{D\}$, so we cannot remove C

   $\{C\}^+ = \{C\}$, so we cannot remove D

4. Testing each attribute on the LHS of $CE \to D$:

   $\{E\}^+ = \{EABFG\}$, so we cannot remove C

   $\{C\}^+ = \{C\}$, so we cannot remove E

$\mathcal{F} = \{$
$A \to B,$
$CD \to E,$
$CD \to G,$
$CD \to H,$
$CE \to D,$
$E \to A,$
$E \to B,$
$E \to F,$
$E \to G,$
$F \to E,$
$G \to A$
$\}$

Step 4: remove redundant functional dependencies

$A \to B$
- $\{A\}^+$ over $F - \{A \to B\} = \{A\}$, does not include B so we cannot remove this dependency.

$CD \to E$
- $\{CD\}^+$ over $F - \{CD \to E\} = \{CDGHAB\}$, does not include E so we cannot remove this dependency.

$CD \to G$
- $\{CD\}^+$ over $F - \{CD \to G\} = \{CDEHABFG\}$, which includes G, so we **can remove this FD.**

$CD \to H$
- $\{CD\}^+$ over $F - \{CD \to H\} = \{CDEDABFG\}$, does not include H so we cannot remove this dependency.

$CE \to D$
- $\{CE\}^+$ over $F - \{CE \to D\} = \{CEABFG\}$, does not include D so we cannot remove this dependency.

$E \to A$
- $\{E\}^+$ over $F - \{E \to A\} = \{EBFGA\}$, includes A, so we **can remove this FD.**

$E \to B$
- $\{E\}^+$ over $F - \{E \to B\} = \{EFGAB\}$, includes B, so we **can remove this FD.**

$E \to F$
- $\{E\}^+$ over $F - \{E \to F\} = \{EGAB\}$, does not include F so we cannot remove this dependency.

$E \to G$
- $\{E\}^+$ over $F - \{E \to G\} = \{EF\}$, does not include G so we cannot remove this dependency.

$F \to E$
- $\{F\}^+$ over $F - \{F \to E\} = \{F\}$, does not include E so we cannot remove this dependency.

$G \to A$
- $\{G\}^+$ over $F - \{G \to A\} = \{G\}$, does not include A so we cannot remove this dependency.

$\mathcal{F} = \{$
$A \to B,$
$CD \to E,$
$CD \to H,$
$CE \to D,$
$E \to F,$
$E \to G,$
$F \to E,$
$G \to A$
$\}$

Minimal Basis:
$\mathcal{F} = \{A \to B,\ CD \to E,\ CD \to H,\ CE \to D,\ E \to F,\ E \to G,\ F \to E,\ G \to A\}$

Minimal Cover:
$\mathcal{F} = \{A \to B,\ CD \to EH,\ CE \to D,\ E \to FG,\ F \to E,\ G \to A\}$