Homework 2

NERS/ENGR 570 Fall 2020
October 12, 2020
DUE BY: October 22nd, 2020 at 11:59pm

Exercise 1 (20 pts) - Spectral Radius of Fixed Point Methods

In this exercise, calculate the spectral radius for the 1-D Poisson problem that was analyzed in the Jupyter Notebook here:

/gpfs/accounts/ners570f20_class_root/ners570f20_class/shared_data/lecture07_example/

A. Calculate the spectral radius from the numerical results--the plot of the norm of the residual--for Gauss-Siedel and Jacobi. Show your calculation.
B. Calculate the spectral radius from the theoretical expression given in Lecture 08. Describe your calculation in terms of the problem setup and what was calculated. It is recommended you use some library or tool to calculate the spectral radius.
C. Repeat for grid sizes of 100, 1000, 10,000.

Discuss your results.

- Do the numerical and theoretical calculations of the spectral radius agree?
- Which spectral radius is smaller: Jacobi or Gauss-Siedel?
- What was the effect of grid size on the condition number?

Exercise 2 (25 pts) - QR Factorization

In this exercise implement the Modified Gram-Schmidt algorithm to compute the QR factorization of a general dense square matrix of double precision type in either C, C++, or Fortran. Generate the QR factorization for the centered difference approximation of the 2D Laplacian with Dirichlet boundary conditions with 16 unknowns (a 4x4 grid).

1. Provide the coefficient matrix that you are factorizing
2. Provide Q and R
3. Have your code verify the implementation by checking that the column vectors of Q are orthonormal and that QR = A.
4. Provide the source code used to complete this assignment.
5. Provide a bash script or makefile to compile the source code.

Exercise 3 (15 pts) - UML Diagrams

For this exercise you'll get some practice in creating UML diagrams. Create the following UML diagrams:

A. Create a sequence diagram for PETSc's tutorial ex15 that was used in Lab 05. Be sure to include the major PETSc objects: vectors, matrices, solver, and preconditioner. See Ch. 4 of [Distilled UML]

B. Create a UML class diagram of the Abstract Factory Pattern for a "Linear Solver Factory" that would provide A, x, b, and a linear solver for dense and sparse linear systems.

C. Create a UML class diagram representing the composite pattern to describe a block-structured matrix. Here each entry in $\mathbf{A}$ can be thought of as another smaller matrix. Include attributes for the number of blocks for the rows (q in the figure below) and columns (s in the figure below). Try to develop a design that is "self-referential" or "recursive".

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1s} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{q1} & \mathbf{A}_{q2} & \cdots & \mathbf{A}_{qs} \end{bmatrix}$$

The provided reading materials/references from the lectures should be used as resources here. Having cross referenced this information here with Wikipedia--the latter also has an appropriate description.

Exercise 4 (40 pts) - Complete the Next Sprint from Lab 07

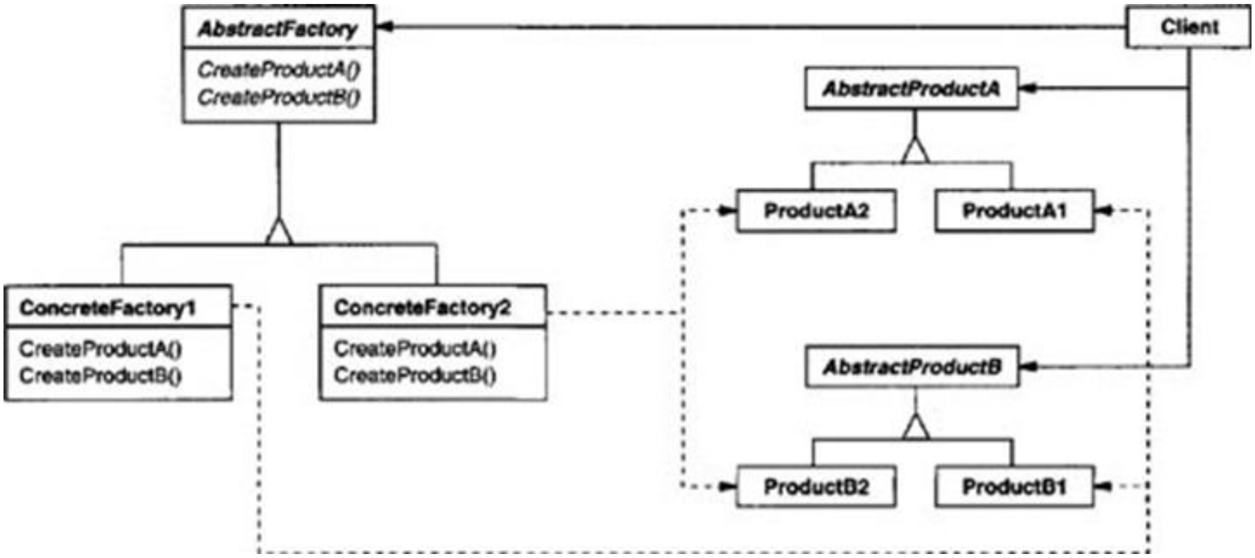
The scope of the work here is determined in Lab 07. This should include 4 issues. For this exercise provide the links to the issues you worked on/closed.

Deliverables and how to submit:

Please provide one pdf with the typed answers, source code, equations, figures, etc. for all exercises and submit this to Canvas.

Abstract Factory Pattern

[Design Patterns Ch. 3]
[Wikipedia]



Composite Pattern

[Design Patterns Ch. 4]
[Wikipedia]