Lecture 06 – Algorithms for Linear Algebra

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NERS/ENGR 570 - Methods and Practice of Scientific Computing (F20)

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Outline

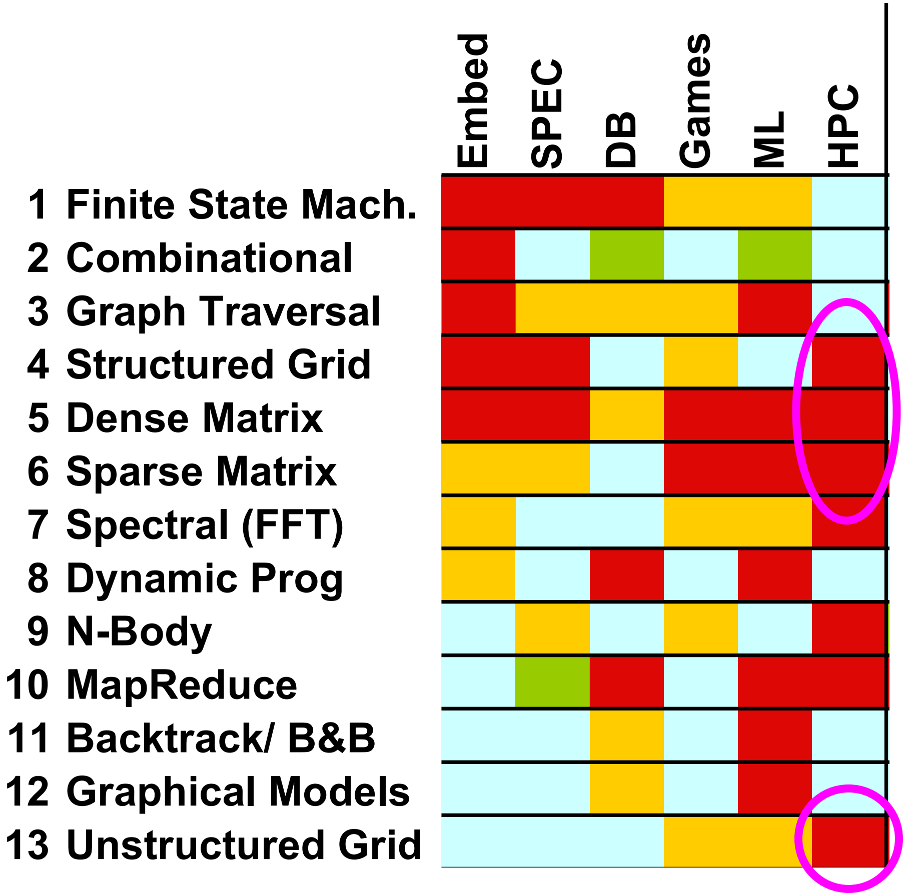
- Resume class exercise with Git
- Linear Algebra/Linear System Fundamentals
- Solution of Linear Systems

Learning Objectives: By the end of Today's Lecture you should be able to

- *(Skill)* Perform Conflict Resolution (but just in git)
- *(Knowledge)* Interpretate meaning of some vector norms
- (Value/Knowledge) explain how to think about programming equations in linear algebra
- *(Knowledge)* know when to use direct vs. iterative solution algorithms
- (*Knowledge*) implement LU factorization

Overview

- What types of problems are solved with Linear Algebra?
  - Linear systems of equations
  - Eigenvalue problems
  - matrix factorization
  - Overdetermined system of equations
    - (Data fitting)



Linear System Fundamentals

Basics of Linear Systems

The Residual and Norms

Norms of Vectors

Going from Chalkboard to Terminal

Types of Linear Algebra Operations

Solving Linear Systems

How do we solve linear systems?

Overview of Solution Methods

Direct Solution Methods

aka Matrix Factorizations

Types of Matrices

LU Factorization

LU: Forward Elimination

LU: Backward Substitution

Classical Iterative Methods

aka Fixed Point Iteration Schemes

Classical Iteration Schemes

Jacobi

Gauss-Siedel

Do they converge?

- Fixed point iteration

$$\mathbf{x}^{(\ell+1)} = \mathbf{F}\mathbf{x}^{(\ell)} + \mathbf{c}$$

- Express iterate as combination of exact solution and error

$$\mathbf{x} + \boldsymbol{\varepsilon}^{(\ell+1)} = \mathbf{F}\left(\mathbf{x} + \boldsymbol{\varepsilon}^{(\ell)}\right) + \mathbf{c}$$

- If the method converges then:

$$\lim_{\ell \to \infty} \boldsymbol{\varepsilon}^{(\ell)} = 0$$