1 Membench

The *membench* programs are run to obtain the following plots in ?? to Fig. 3 for the effect of array length and stride length on type of memory access, and time to access.

In this plot, the solid vertical lines indicate the estimated memory sizes, and dashed vertical lines indicate the true memory sizes, with black lines indicating the cache line sizes, and blue lines indicating the total cache sizes. The green solid horizontal lines indicate the approximate cache access times.

In general, it is observed that for a given range of strides, and particularly in regions with plateaus of strides, say between 64B and 4KB, greater array lengths have greater access times. These parallel plateaus can be attributed to filling and then calling the lowest possible caches/accessing the lowest possible level of memory when there are cache misses in the even lower level of memory. This accessing is constant for the given lowest possible memory level.

Here, lowest possible memory level refers to the smallest cache at which the stride is less than the cache size and or line size. At this lowest possible level of memory, there will be the least number of cache misses, as it will be assumed that the array will fill this level of cache with as many full cache lines as possible. Larger arrays will fill more cache lines at that level, up to the level being full (and disregarding space being required for the code/instructions on top of space for the data). This filling also depends on whether a cache miss prompts a cache line to be overwritten, or a new cache line to be filled with data from a higher memory level, This depends on the specific association, inclusivity/exclusivity, hierarchy and relative sizes of the different cache levels.

So for larger arrays, the number of iterations through the array is obviously greater, and must take more time, and more cache misses will occur, regardless of the relative difference in stride size to cache line size. This greater number of misses (and corresponding new cache lines being filled from higher memory levels) is likely linear in the array size. This linear increase in cache misses explains the parallel plateaus at higher accessing times for greater array sizes, even if the time to access each element within a given cache size is roughly constant.

The access times will be measured where there are significant plateaus in the access times, and the *maximum* of these plateaus will be used as an estimate for the access times. The reason there are not sharp increases between plateaus is that as the stride increases, there are less cache hits and more misses as the indexing goes beyond the cache line size, however there are also less indexing calls with greater stride. So the increase up to the next plateau, which corresponds to the memory access time of the next greater memory level, is gradual, as there is a mix of hits in different memory levels.

The following script *calc.sh* in the appendix was used to get the cpu and memory statistics in Code. 1. The average processor core speed for the 36 processors on GreatLakes to be 3000 MHz, and the cache line sizes appear to be constant at 64 Bytes. The cpu core speed will be used to calculate the number of processor clock cycles required to access the various types of memory,

cycles per memory access = clock speed \times memory access time.

Code 1: CPU and Memory values.

```
Cache Info
Cache L1 Size: 32 kB
Cache L1 Line Size: 64 B

Cache L2 Size: 1024 kB
Cache L2 Line Size: 64 B

Cache L3 Size: 25344 kB
Cache L3 Line Size: 64 B
```

```
Cache L4 Size: 0 kB
Cache L4 Line Size: 0 B

Cache L4 Line Size: 0 B

CPU Info
CPU Cores: 36
CPU Speed: 2999.531 MHz
```

1.1 Processor Values

On the Greatlakes compute notes, there are $36 \times \text{Intel}(R) \text{ Xeon}(R)$ Gold 6140 CPU @ 2.30GHz. From repeated requests for the speed of each core, the average core speed over all cores and samples is 2.99 GHz.

1.2 L_1 Cache Values

For the L_1 cache line size, the true line size is 64B, and from the plots, particularly in Fig. 2, it can be seen that the access times do not start to initially increase until around strides of 64B, before then plateauing. This jump before plateauing indicates that strides greater than this value must start to involve more L_1 misses, and require accessing the L_2 memory.

$$L_1 \text{ Line } = 128B.$$

For the L_1 total cache size, although the true cache size is 32KB, from the plots, particularly in Fig. 2, it can be seen that the access times remain very constant, and at their minimum all way the up to 4KB for arrays with length \leq 32KB, suggesting the entire array, or at least half of the array can be loaded into the L_1 cache. In addition, the next largest 62KB array shows increased access times for up to 32KB strides, suggesting some L_1 cache misses possibly occur, and so the L_1 cache size is likely less than 32KB.

$$16KB \le L_1 \le 32KB$$
.

For the L_1 access time, given the quite constant access times up to 4KB strides for arrays with length \leq 32KB, the estimated access time is therefore the maximum of the 32KB length array curve:

$$T_1 = 0.57 \text{ns} = 2 \text{ cycles}.$$

1.3 L_2 Cache Values

For the L_2 cache line size, the true line size is 64B, and from the plots, particularly in Fig. 3, it can be seen that after the initial increase of access times, there is a slight plateau for strides ≤ 512 B, and arrays of sizes 64KB-8MB, indicating that possibly the array is being quickly indexed in the L_2 cache with a cache line of between 64B and 512B. There is then a jump in access times, but not a huge jump for arrays up to size 8MB, indicating there are possibly still cache hits in the L_2 cache, but at different lines.

$$64B \le L_2$$
 Line $\le 512B$.

For the L_2 total cache size, although the true cache size is 1MB, from the plots, particularly in Fig. 3, it can be seen that the access times remain very constant over a large range of array sizes between 64KM to 8MB, with strides between 1KB and 256KB. This suggests lots of these array sizes can be mostly loaded into the L_2 cache on several cache lines. This suggests the L_2 total cache size to be less than 256KB (and greater than the L_1 total cache size); the point where the access times for these array sizes drops dramatically when many less array elements are indexed. The difficulty at finding tighter bounds on the L_2 cache sizes is possibly due to the L_2 cache sometimes being shared by pairs of cores, affecting the timing, depending on which cores the array is being computed on.

$$16KB \le L_2 \le 256KB.$$

For the L_2 access time, given the quite constant access times for array sizes between 64KM to 8MB, with strides between 1KB and 256KB, the estimated access time is therefore the maximum of the 8MB size array curve along

this plateau:

$$T_2 = 3.83 \text{ns} = 12 \text{ cycles}.$$

1.4 L_3 Cache Values

For the L_3 cache line size, the true line size is 64B, however from the plots, it is difficult to tell where exactly there is a distinct plateau for array indexing with strides within the size of this larger cache's lines. This may be due to the L_3 cache being typically shared between all (36) cores, and so timings may be affected depending how the computations are distributed amongst the cores. However for array sizes of 16MB to 512MB, there is somewhat a plateau between 512B and 4KB, indicating a possible range for the L_3 cache line size. Here, there may be hits due to this larger cache being allowed to store more of these larger arrays, minimising cache misses. The lack of distinct plateau is also possibly attributed to the large arrays being far larger than the cache, and there being many hits and misses while the lines are being filled from the main memory.

$$64B < L_3$$
 Line $< 4KB$.

For the L_3 total cache size, although the true cache size is 24.75MB, from the plots, particularly in Fig. 4, it can be seen that the plateau between 512B and 4KB, for array sizes of 16MB to 512MB, rises, and then decreases gradually, before plateauing again for strides between 256KB and 8MB. This suggests that the time is not decreasing solely due to there being less elements indexed with greater stride, but there also possibly being effects of the elements still being in the faster L_3 cache compared to the main memory. There are still hits occurring in succession in a cache that are causing this plateau at non-zero access times. There is still though great uncertainty in the exact total size of this L_3 cache.

$$256KB \le L_3 \le 8MB.$$

For the L_3 access time, given the two different plateaus present in the access times for the larger arrays, the access time will be estimated as the maximum of these plateaus in Fig. 4.

$$T_3 = 10.36 \text{ns} = 32 \text{ cycles}.$$

1.5 Main Memory Values

Only the 1GB array sizes appear to be unable to be stored fully in any caches, and there are enough misses in the lower caches that the main memory must be accessed. It is assumed the upper plateau for the 1GB array are these memory hits, and the access time is assumed to be the maximum of this plateau. This is assumed to be a lower bound, if some of the array is in the L_3 cache, and there are some hits there, and some in the main memory.

$$T_{\text{mem}} \ge 13.69 \text{ns} = 42 \text{ cycles}.$$

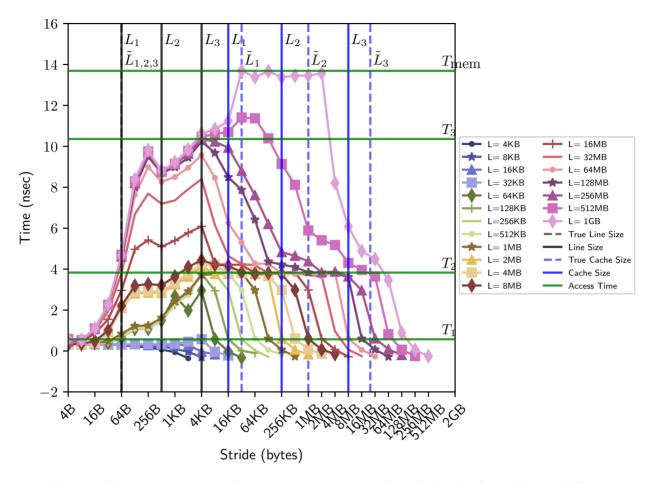


Figure 1: Memory access times for various array sizes, and stride lengths from 4B to 512MB.

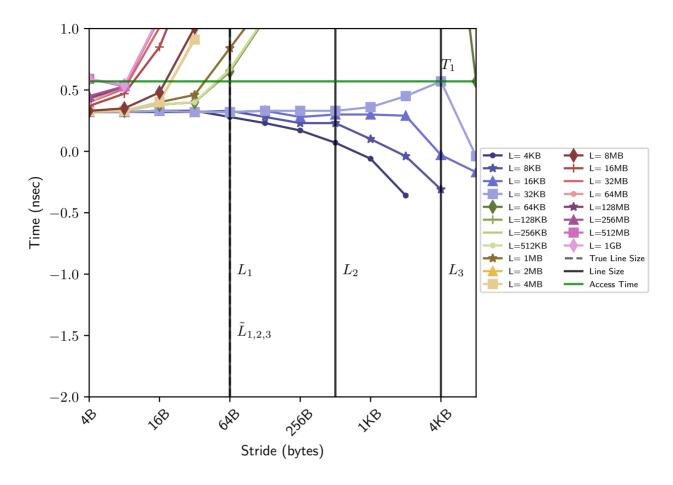


Figure 2: Memory access times for various array sizes, and stride lengths from 4B to 4KB.

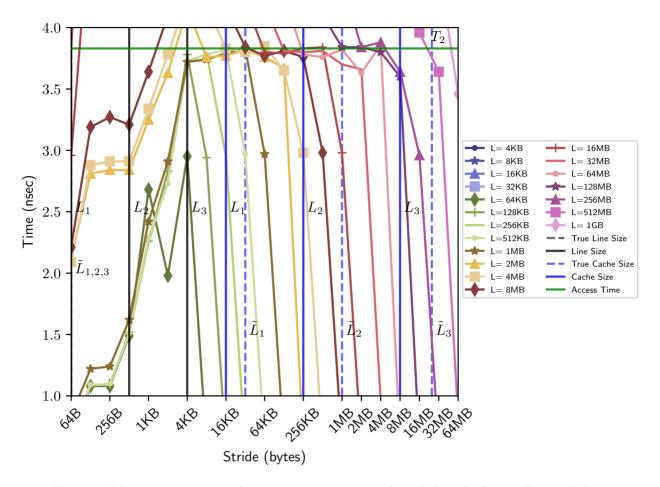


Figure 3: Memory access times for various array sizes, and stride lengths from 16B to 64MB.

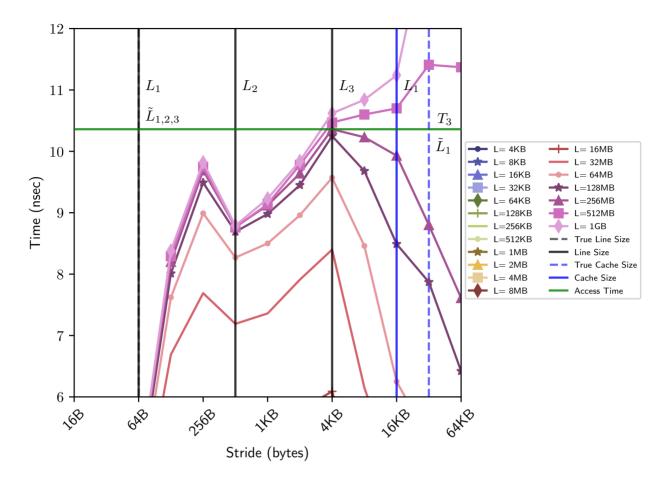


Figure 4: Memory access times for various array sizes, and stride lengths from 16B to 64KB.

2 SIMD Instructions

2.1 AVX Support

• The commands used to verify if the current machine/processor supports AVX are:

and if the current machine/processor supports AVX2 are:

These searches will show whether the supported AVX fields are in the flags section of the processor info from /proc/cpuinfo. This command will show all processors ($36 \times Intel(R) \times (R) \times (R) \times (R) \times (R) \times (R)$) Gold 6140 CPU @ 2.30GHz, for GreatLakes compute nodes).

• The commands used to verify if the current GNU compiler supports AVX or AVX2 by the constants/macros the compiler defines:

This command will show the boolean settings for the AVX constants:

• The commands used to verify if the current Intel compiler supports AVX or AVX2 by the constants/macros the compiler defines:

```
icc -march=native -dM -E - < /dev/null | grep "AVX" | sort
```

This command will show the boolean settings for the AVX constants:

```
#define __AVX__ 1
#define __AVX2__ 1
#define __AVX512BW__ 1
#define __AVX512CD__ 1
#define __AVX512DQ__ 1
#define __AVX512F__ 1
#define __AVX512VL__ 1
#define __AVX I 1
```

2.2 dgemm Assembly Instructions

The command to get the assembly instructions is as follows, where the function for the naive *dgemm.cpp* in Code. 6 in the appendix is translated into assembly code, with the avx2 and fast optimizations using the command:

```
g++ -S -Ofast -mavx2 dgemm.cpp
```

This command produces the following assembly code in Code. 2

Code 2: Matrix-Matrix multiply assembly with avx2 and fast optimization

```
"dgemm.cpp"
             .file
             .text
2
             .p2align 4,,15
             .globl
                            _Z5dgemmccjjjdPKPKdS2_dPPd
             .type
                           _Z5dgemmccjjjdPKPKdS2_dPPd, @function
5
     Z5dgemmccjjjdPKPKdS2_dPPd:
    .LFB1564:
             .cfi_startproc
                          8(%rsp), %r10
             leaq
9
             .cfi_def_cfa 10, 0
10
             andq
                          $-32, %rsp
11
                           %ecx, %ecx
             testl
                           -8(%r10)
             pushq
13
             pushq
                           %rbp
14
             .cfi_escape 0x10,0x6,0x2,0x76,0
15
                          %rsp, %rbp
             movq
16
                           %r15
             pushq
17
                           %r14
             pushq
18
                           %r13
             pushq
                           %r12
             pushq
20
                           %r10
             pushq
21
             .cfi_escape 0xf,0x3,0x76,0x58,0x6
22
             .cfi_escape 0x10,0xf,0x2,0x76,0x78
23
             .cfi_escape 0x10,0xe,0x2,0x76,0x70
24
             .cfi_escape 0x10,0xd,0x2,0x76,0x68
25
             .cfi_escape 0x10,0xc,0x2,0x76,0x60
26
                           %rbx
             pushq
```

```
.cfi_escape 0x10,0x3,0x2,0x76,0x50
28
                           %r8d, -52(%rbp)
29
             movl
                           (%r10), %r14
             movq
30
             movq
                           8(%r10), %rax
31
                         .L35
              jе
32
              testl
                            %edx, %edx
33
                         .L35
34
             jе
             subl
                           $1, %ecx
35
             leal
                           -1(\%rdx), \%r13d
             vbroadcastsd
                                    %xmm1, %ymm3
37
                           8(%rax,%rcx,8), %r15
             leaq
             movq
                           %rax, %rbx
39
40
             movl
                           %ecx, -56(%rbp)
              .p2align 4,,10
41
              .p2align 3
42
     .L9:
43
                           (%rbx), %r10
             movq
44
                           %r8d, %r8d
45
             xorl
             movq
                           %r10, %rcx
46
             shrq
                           $3, %rcx
47
             negq
                           %rcx
48
                           $3, %ecx
             andl
49
             leal
                           3(%rcx), %esi
50
                           %r13d, %esi
             cmpl
              ja
                         .L3
52
             testl
                            %ecx, %ecx
53
                         .L4
              jе
54
                              (%r10), %xmm1, %xmm2
55
              vmulsd
             cmpl
                           $1, %ecx
56
             movl
                           $1, %r8d
57
             vmovsd
                              %xmm2, (%r10)
58
             jе
                         .L4
59
                              8(%r10), %xmm1, %xmm2
             vmulsd
60
              cmpl
                           $2, %ecx
61
             movl
                           $2, %r8d
              vmovsd
                              %xmm2, 8(%r10)
63
                         .L4
             jе
64
              vmulsd
                              16(%r10), %xmm1, %xmm2
65
                           $3, %r8d
             movl
                              %xmm2, 16(%r10)
              vmovsd
67
     .L4:
68
                           %edx, %r12d
             movl
69
                           (%r10,%rcx,8), %rdi
             leaq
70
                           %esi, %esi
             xorl
71
72
             subl
                           %ecx, %r12d
             xorl
                           %ecx, %ecx
73
                           %r12d, %r11d
             movl
74
             shrl
                           $2, %r11d
75
              .p2align 4,,10
76
77
              .p2align 3
     L6:
78
                              (%rdi,%rcx), %ymm3, %ymm2
             vmulpd
79
                           $1, %esi
             addl
80
                               %ymm2, (%rdi,%rcx)
             vmovapd
                           $32, %rcx
             addq
82
              cmpl
                           %esi, %r11d
83
             ja
                         .L6
84
```

```
%r12d, %ecx
              movl
85
              andl
                             $-4, %ecx
86
               addl
                             %ecx, %r8d
87
               cmpl
                             %r12d, %ecx
88
               jе
                          .L7
89
     .L3:
90
                             %r8d, %ecx
91
              movl
              leag
                             (%r10,%rcx,8), %rcx
92
              vmulsd
                               (%rcx), %xmm1, %xmm2
               vmovsd
                               %xmm2, (%rcx)
94
              leal
                             1(%r8), %ecx
95
                             %ecx, %edx
               cmpl
96
               jbe
                            .L7
                             (%r10,%rcx,8), %rcx
              leaq
98
                               (%rcx), %xmm1, %xmm2
              vmulsd
99
                               %xmm2, (%rcx)
               vmovsd
100
                             2(%r8), %ecx
               leal
101
                             %edx, %ecx
102
               cmpl
               jnb
                            .L7
103
              leaq
                             (%r10,%rcx,8), %rcx
104
               vmulsd
                               (%rcx), %xmm1, %xmm2
105
               vmovsd
                               %xmm2, (%rcx)
106
              leal
                             3(%r8), %ecx
107
                            %ecx, %edx
               cmpl
108
               jbe
                            .L7
109
              leaq
                             (%r10,%rcx,8), %rcx
110
               vmulsd
                               (%rcx), %xmm1, %xmm2
111
                               %xmm2, (%rcx)
               vmovsd
112
                             4(\%r8), \%ecx
              leal
113
               cmpl
                             %ecx, %edx
114
               jbe
                            .L7
115
               leaq
                             (%r10,%rcx,8), %rcx
116
              addl
                             $5, %r8d
117
               cmpl
                             %r8d, %edx
118
               vmulsd
                               (%rcx), %xmm1, %xmm2
               vmovsd
                               %xmm2, (%rcx)
120
               jbe
                            .L7
121
                             (%r10,%r8,8), %rcx
              leaq
122
                               (%rcx), %xmm1, %xmm2
123
               vmulsd
               vmovsd
                               %xmm2, (%rcx)
124
     .L7:
125
                             $8, %rbx
               addq
126
                             %r15, %rbx
               cmpq
127
               jne
                            .L9
128
129
              movl
                             -52(\%rbp), %edx
              testl
                              %edx, %edx
130
              jе
                          .L37
131
                             -52(\%rbp), %ebx
              movl
132
              leaq
                             8(,%r13,8), %r13
133
                             %r14, %r15
              movq
              leal
                             -1(\%rbx), %edx
135
                             8(%r14,%rdx,8), %r12
              leaq
136
              movl
                             -56(\%\text{rbp}), \%\text{edx}
137
                             8(,%rdx,8), %r11
              leaq
138
               .p2align 4,,10
139
               .p2align 3
140
     .L12:
141
```

```
(\%r15), \%rbx
              movq
142
                            %r15, %r10
143
              movq
                            %edi, %edi
              xorl
144
              subq
                            %r14, %r10
145
              .p2align 4,,10
146
              .p2align 3
147
     .L11:
148
              leag
                            (%rbx, %rdi), %r8
149
              xorl
                            %edx, %edx
150
              .p2align 4,,10
151
              .p2align 3
152
     .L10:
153
                              (%r8), %xmm0, %xmm1
154
              vmulsd
              movq
                            (%r9,%rdx), %rsi
155
                            (%rax,%rdx), %rcx
              movq
156
                            $8, %rdx
              addq
157
                            %rdi, %rcx
              addq
158
                            %rdx, %r11
159
              cmpq
              vmulsd
                              (%rsi,%r10), %xmm1, %xmm1
160
              vaddsd
                              (%rcx), %xmm1, %xmm1
161
                              %xmm1, (%rcx)
              vmovsd
162
                           .L10
              jne
163
                            $8, %rdi
              addq
164
                            %r13, %rdi
              cmpq
              jne
                           .L11
166
                            $8, %r15
              addq
167
                            %r12, %r15
              cmpq
168
                           .L12
169
              jne
     .L37:
170
              vzeroupper
171
     .L35:
172
                            %rbx
              popq
173
                            %r10
174
              popq
              .cfi_def_cfa 10, 0
175
                            %r12
              popq
176
                            %r13
              popq
177
                            %r14
178
              popq
                            %r15
179
              popq
                            %rbp
180
              popq
              leaq
                            -8(%r10), %rsp
181
              .cfi_def_cfa 7, 8
182
              ret
183
              .cfi_endproc
184
     .LFE1564:
185
186
              .size
                             _Z5dgemmccjjjdPKPKdS2_dPPd, .-_Z5dgemmccjjjdPKPKdS2_dPPd
              .section
                                 .text.startup, "ax", Oprogbits
              .p2align 4,,15
188
                             _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd, @function
189
      _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd:
190
     .LFB2045:
191
              .cfi_startproc
192
                            _ZStL8__ioinit(%rip), %rdi
              leaq
193
194
              subq
                            $8, %rsp
              .cfi_def_cfa_offset 16
195
                            _ZNSt8ios_base4InitC1Ev@PLT
              call
196
                            _ZNSt8ios_base4InitD1Ev@GOTPCREL(%rip), %rdi
197
              movq
              leaq
                            __dso_handle(%rip), %rdx
198
```

```
_ZStL8__ioinit(%rip), %rsi
             leaq
199
                          $8, %rsp
             addq
200
              .cfi_def_cfa_offset 8
201
             jmp
                         __cxa_atexit@PLT
202
              .cfi_endproc
203
     .LFE2045:
204
                           _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd,
205
              .size
                .-_GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd
                               .init_array,"aw"
206
              .align 8
207
                           _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd
              .quad
208
                             _ZStL8__ioinit
              .local
209
                            _ZStL8__ioinit,1,1
              .comm
210
              .hidden
                              __dso_handle
211
                             "GCC: (Ubuntu 7.5.0-3ubuntu1~18.04) 7.5.0"
              .ident
212
                               .note.GNU-stack,"",@progbits
              .section
213
```

Code 3: memberch plotting script.

```
#!/usr/bin/env puthon
1
    import matplotlib
    import matplotlib.pyplot as plt
    import numpy as np
5
    matplotlib.rcParams['text.usetex'] = True
6
    # matplotlib.rcParams['text.latex.preamble'] = [r'\usepackaqe{raqqed2e']
    # Routine modified from:
      https://stackoverflow.com/questions/1094841/reusable-library-to-qet-human-readable-version-of-file-size
    def sizeof_fmt(num, suffix='B'):
10
        for unit in ['','K','M','G','T']:
11
             if abs(num) < 1024.0:
12
                 return '%3.0f%s%s' % (num, unit, suffix)
13
            num /= 1024.0
14
        return '%.1f%s%s' % (num, 'T', suffix)
15
16
    def fmt sizeof(fmt):
17
        bases={'B':2,'':10}
18
        units={k:v for k,v in zip(['','K','M','G','T'],[1,10,20,30,40])}
19
        try:
20
          base=fmt[-1]
21
          unit=fmt[-2]
22
          num = float(fmt[:-2])
23
        except ValueError:
24
          try:
             base=fmt[-1]
26
            unit=''
27
            num = float(fmt[:-1])
28
          except:
29
             try:
30
               base=''
31
               unit=''
32
               num = float(fmt)
33
             except:
34
               return fmt
35
        # print(num, (bases.get(base, 10) **units.get(unit, 1)))
36
        # print(num)
37
        num *= (bases.get(base,10)**units.get(unit,1))
38
        if int(num) == num:
39
          num = int(num)
        return num
41
42
    def indexer(array, sorter, value):
43
      i = np.where(sorter==value)[0]
44
      return array[i]
45
46
47
    file='membench_4'
48
    cpuspeed=3e9
49
50
    mbdata = np.genfromtxt('%s.out'%file,usecols=(1,3,5))
51
52
53
```

```
maxunit=10
54
    units=dict(zip(range(0,maxunit*4,maxunit),['','K','M','G']))
    sizes = \{\frac{b*}{0},\frac{b*}{0},\frac{b*}{0},\frac{b*}{0},\frac{b*}{0},\frac{b*}{0}\} for b,u in zip([2],['B']) for d in units for i
56
      in range(maxunit)}
57
58
    xtlabels=[*['4B','16B','64B','256B','1KB','4KB','16KB','64KB','256KB','1MB'],
59
               *['2MB','4MB','8MB','16MB','32MB','64MB','128MB','256MB','512MB','2GB']]
60
    xtvals = [sizes.get(1,fmt_sizeof(1)) for 1 in xtlabels]
62
64
66
    \# k, l, m = '1GB', '1MB', '512B'
67
    \# print((sizes.get(k,fmt\_sizeof(k)),indexer(indexer(mbdata[:,2],mbdata[:,0],sizes.get(l,|sizes)))))
      fmt_sizeof(l))),indexer(mbdata[:,1],mbdata[:,0],sizes.get(l,fmt_sizeof(l))),sizes.get(m, |
      fmt_sizeof(m)))[0]))
69
    lims={
70
            'all': [(xtlabels[0],xtlabels[-1]),(-2,16)],
71
            'lowerleft':[('4B','8KB'),(-2,1)],
72
            'middleupper': [('16B', '64KB'), (6,12)],
73
            'middle': [('64B','64MB'),(1,4)],
75
    heights={k:{1:-1 for 1 in ['h','v']}} for k in lims}
77
    heights['all']['v'] =15
    heights['lowerleft']['v'] = -1
79
    heights['middle']['v'] = 2.5
    heights['middleupper']['v'] = 11
81
    lines={
82
             'all':{'v': {
83
                           # **{(sizes.qet(k,fmt_sizeof(k))/2,height):r'$\tilde{L}_{1,2,1}
84
                             3}\\\textrm{%s}$'%k for i,k in
                             enumerate(['64B'])},
                           # **{(sizes.get(k,fmt_sizeof(k)),height):r'${L}_{%d}\\\textrm{%s}$'%(i+1, |
                             k) for i,k in
                             enumerate(['128B'])},
                           # # **{(sizes.qet(k,fmt_sizeof(k)),height):r'$\tilde{L}_{\}\d\}\f\'(i+1) for
86
                             i,k in enumerate(['128B', '4KB', '16KB'])},
                           # **{(sizes.get(k,fmt_sizeof(k)),
87
                             height):r'$\setminus tilde\{L\}_{%d}\setminus \det\{\%s\}$'\%(i+1,k) for i,k in
                             enumerate(['32KB','1MB','24.75MB'])},
                           **{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']-1):r'$\tilde{L}_{1,2,_|
89
                             3}$' for i,k in
                             enumerate(['64B'])},
                           **{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']):r'${L}_{%d}$'%(i+1)
90
                             for i,k in enumerate(['64B','512B','4KB'])},
                           # **{(sizes.get(k,fmt_sizeof(k)),
91
                             heights['all']['v']):r'\$\tilde\{L\}_{\d}$'\%(i+1) for i,k in
                             enumerate(['128B', '4KB', '16KB'])},
                           **{(sizes.get(k,fmt_sizeof(k)),
                             heights['all']['v']-1):r'$\tilde{L}_{\%d}$'\%(i+1) for i,k in
                             enumerate(['32KB','1MB','25MB'])},
```

```
**{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']):r'$L_{%d}$'%(i+1) for
93
                             i,k in enumerate(['16KB','256KB','8MB'])},
                           },
94
95
                    'h': {
                          # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0], |
97
                            sizes.qet(l,fmt\_sizeof(l)))):r'$T\_{%d}$''(i+1) for i,(k,l) in
                            enumerate(zip(['1GB'],['32KB']))},
                          **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
                            sizes.get(l, fmt_sizeof(l)))):r'$T_{{d}}''(i+1) if i<3 else
                            r'T_{\text{mem}} for i,(k,1) in
                            enumerate(zip(['1GB','1GB','1GB','1GB'],['32KB','1MB','256MB','1GB']))},
                          # **{(sizes.qet(k,fmt_sizeof(k)),indexer(indexer(mbdata[:,2],mbdata[:,0],
                            sizes.qet(l,fmt\_sizeof(l))), indexer(mbdata[:,1],mbdata[:,0],sizes.qet(l,,))
                            fmt\_sizeof(l))), sizes.get(m, fmt\_sizeof(m)))[0]):r'$T_{{d}}''(i+2) for
                            i, (k, l, m) in enumerate(zip(['1GB'], ['1MB'], ['512B']))},
                          }.
100
101
102
103
                   },
104
             'lowerleft':{'v': {
105
                                   \# **\{(sizes.qet(k,fmt\_sizeof(k)),height):r'\$\setminus tilde\{L\}_{\{1,2,\ldots\}}\}
106
                                     3}\\\textrm{%s}$'%k for i,k in
                                     enumerate(['64B'])},
                                   # **{(sizes.get(k,fmt_sizeof(k)),
107
                                     height):r'${L}_{%d}\\\textrm{%s}$'%(i+1,k) for i,k in
                                     enumerate(['128B'])},
108
                                   **{(sizes.get(k,fmt_sizeof(k)),
109
                                    \label{lowerleft'} heights['lowerleft']['v']-0.5):r'$\tilde{L}_{1,2,3}$' for i,k in
                                     enumerate(['64B'])},
                                   **{(sizes.get(k,fmt_sizeof(k)),
110
                                    heights['lowerleft']['v']):r'L_{\infty}(i+1) for i,k in
                                     enumerate(['64B','512B','4KB'])},
111
112
                                },
113
                          'h': {
                                 # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
115
                                  mbdata[:,0], sizes.qet(l,fmt\_sizeof(l))))):r'$T\_{%d}$'\%(i+1) for
                                   i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                          **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
116
                            sizes.get(1,fmt_sizeof(1)))):r'$T_{{d}}''(i+1) for i,(k,1) in
                            enumerate(zip(['4KB','4KB'],['32KB','1MB']))},
                                },
117
                          },
118
             'middle':{'v': {
119
                                   # **{(sizes.get(k,fmt_sizeof(k)),height):r'$\tilde{L}_{1,2,1}
120
                                     3}\\\textrm{%s}$'%k for i,k in
                                     enumerate(['64B'])},
                                   # **{(sizes.get(k,fmt_sizeof(k)),
121
                                     height):r'$\{L\}_{\d}\\\textrm{\s}$'\c (i+1,k) for i,k in
                                     enumerate(['128B'])},
122
```

```
**{(sizes.get(k,fmt_sizeof(k)), |
123
                                                                                                   heights['middle']['v']-0.5):r'$\tilde{L}_{1,2,3}$' for i,k in
                                                                                                   enumerate(['64B'])},
                                                                                              **{(sizes.get(k,fmt_sizeof(k)),
124
                                                                                                   heights['middle']['v']):r'L_{-{\text{d}}}''(i+1) for i,k in
                                                                                                   enumerate(['64B','512B','4KB'])},
                                                                                              **{(sizes.get(k,fmt_sizeof(k)),
125
                                                                                                   heights['middle']['v']-1):r'$\tilde{L}_{%d}$'\%(i+1) for i,k in
                                                                                                   enumerate(['32KB','1MB','25MB'])},
                                                                                                 **{(sizes.get(k,fmt_sizeof(k)),
126
                                                                                                      heights['middle']['v']):r'$L_{\%d}$'\%(i+1) for i,k in
                                                                                                      enumerate(['16KB','256KB','8MB'])},
                                                                                       },
                                                                       'h': {
128
                                                                                         # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
129
                                                                                             mbdata[:,0], sizes.get(l,fmt\_sizeof(l))))):r'$T_{{d}}$''(i+1) for
                                                                                              i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                                                                       **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
130
                                                                            sizes.get(1, \text{fmt\_sizeof}(1)))):r'T_{\text{d}}'%(i+1) for i,(k,1) in
                                                                            enumerate(zip(['16MB','16MB'],['32KB','1MB']))},
131
                                                                       },
132
                                     'middleupper':{'v': {
133
                                                                                              # **{(sizes.get(k,fmt_sizeof(k)),height):r'$\tilde{L}_{1,2,1}
                                                                                                   3}\\\textrm{%s}$'%k for i,k in
                                                                                                   enumerate(['64B'])},
                                                                                              # **{(sizes.get(k,fmt_sizeof(k)),
135
                                                                                                   height):r'$\{L\}_{\{d}\setminus \text{textrm}\{\%s\}$'\%(i+1,k) for i,k in
                                                                                                   enumerate(['128B'])},
136
                                                                                              **{(sizes.get(k,fmt_sizeof(k)), |
137
                                                                                                   \label{lem:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma
                                                                                                   in enumerate(['64B'])},
                                                                                              **{(sizes.get(k,fmt_sizeof(k)), |
138
                                                                                                   enumerate(['64B','512B','4KB'])},
                                                                                              **{(sizes.get(k,fmt_sizeof(k)), |
139
                                                                                                   heights['middleupper']['v']-1):r'$\dot{L}_{%d}$'%(i+1) for i,k
                                                                                                   in enumerate(['32KB','1MB','25MB'])},
                                                                                                 **{(sizes.get(k,fmt_sizeof(k)),
140
                                                                                                      heights['middleupper']['v']):r'L_{d}'%(i+1) for i,k in
                                                                                                      enumerate(['16KB','256KB','8MB'])},
                                                                                       },
141
                                                                       'h': {
142
143
                                                                                         # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
                                                                                             mbdata[:,0], sizes.get(l,fmt\_sizeof(l))))):r'$T_{{d}}$''(i+1) for
                                                                                              i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                                                                       **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
144
                                                                            sizes.get(1,fmt_sizeof(1)))):r'$T_{%d}$'%(i+1) for i,(k,1) in
                                                                            enumerate(zip(['16MB','16MB','32KB'],['32KB','1MB','256MB']))},
145
146
                                     \# \ 'middle': \{'v': \ \{**\{(sizes.get(k,fmt\_sizeof(k)),-4.5\}: 'L\%d \setminus LineSize: \setminus \%s'\%(i+1,k) \ for \
147
                                          i,k in enumerate(['4KB','16KB'])}}}
                              }
148
149
150
```

```
151
     print(lines)
153
154
155
     cmap = 'tab20b'
156
     lengths=np.array(list(sorted(set(mbdata[:,0]))))
157
158
     kwargs={}
     kwargs['marker'] = ['.','*','^','s','d','+','']
160
161
     kwargs['color'] = {k:v for k,v in zip(lengths,plt.get_cmap(cmap)(np.linspace(0, 1,
162
       len(lengths))).tolist())}
     kwargs['label'] = {k:'L=%s'%(sizeof_fmt(k)) for k in lengths }
163
164
     kwargs['marker'] = {k: kwargs['marker'][i%len(kwargs['marker'])] for i,k in enumerate(lengths)}
165
     kwargs['inds'] = {k: np.where(mbdata[:,0]==k)[0] for k in lengths}
166
167
168
169
     kwargs['lines'] = {k: {'linestyle':s,'color':c,'label':1,'alpha':a,'zorder':z} for k,1,c,s,a,z
170
       in zip(
                                                                                            ['tilde{L}_{1,2,|
171
                                                                                              3}',
                                                                                              'tilde{L}',
                                                                                              '{L}_','L_',
                                                                                              'T_','Mem',
                                                                                              None],
                                                                                            ['True Line
172
                                                                                              Size','True
                                                                                              Cache
                                                                                              Size','Line
                                                                                              Size','Cache
                                                                                              Size','Access
                                                                                              Time','Access
                                                                                              Time',''],
                                                                                            ['k','b','k',
173
                                                                                              'b','g','g',<sub>|</sub>
                                                                                             None],
                                                                                            ['--','--','-',
174
                                                                                              '-','-','-',<sub>|</sub>
                                                                                              None],
                                                                                            [0.6, 0.6, 0.8, 1]
175
                                                                                             0.8,0.8,0.8,
                                                                                              None],
                                                                                            [-1, -1, 10, 10, 10, 10, 10]
176
                                                                                              10, None],
                                                                                           )}
177
178
     for lim in lims:
179
180
181
       fig,ax = plt.subplots()
182
       # Plots
184
185
       #for i in range(9,27):
186
```

```
for L in lengths:
187
         #ax.plot(mbdata[istt:istt+i,1],mbdata[istt:istt+i,2],
188
         ax.plot(mbdata[kwargs['inds'][L],1],mbdata[kwargs['inds'][L],2],**{k:kwargs[k][L] for k in
180
           ['color','marker','label']})
         \#istt=istt+i+1
190
191
       ax.set_ylabel('Time (nsec)')
192
       ax.set_xlabel('Stride (bytes)')
193
       ax.set_xscale('log',base=2)
       ax.set_xticks(xtvals)
195
       ax.set_xticklabels(xtlabels,rotation=45)
196
       ax.set_ylim(*[l for l in lims[lim][1]])
197
       ax.set_xlim(*[sizes.get(1,fmt_sizeof(1)) for 1 in lims[lim][0]])
198
199
       # Lines
200
       plotlines={k:lambda line,k=k,i=i,**kwargs: getattr(ax,'ax%sline'%k)(line[i],**kwargs) for i,k
201
         in enumerate(['v','h'])}
       annotatelines={k:lambda line,text,k=k,i=i,**kwargs:
202
         getattr(plt, 'annotate')(text=text,xy=line,**kwargs) for i,k in enumerate(['v','h'])}
       for k in lines.get(lim,[]):
203
         plotline=plotlines[k]
204
         annotateline=annotatelines[k]
205
         for line in lines[lim][k]:
206
           text = lines[lim][k][line]
           _kwargs = kwargs['lines'][None]
208
           for x in kwargs['lines']:
209
             if str(x) in text:
210
                _kwargs = kwargs['lines'][x]
               break
212
           textline=list(line)
213
           if 0 and (_kwargs['color'] == 'k' and _kwargs['linestyle'] == '-'):
214
             textline[0] /=8
215
           elif (_kwargs['color'] == 'g') and (lim=='all'):
216
             textline[1] += 0.25
217
           elif (_kwargs['color'] == 'g') and (lim=='lowerleft'):
             textline[1] += 0.1
219
           elif (_kwargs['color'] == 'g') and (lim=='middle'):
220
             textline[0] *= 1.5
221
             textline[1] += 0.06
           elif (_kwargs['color'] == 'g') and (lim=='middleupper'):
223
             textline[0] *= 1.2
224
             textline[1] += 0.1
225
           elif lim not in ['lowerleft']:
226
             textline[0] *= 1.15
227
           else:
             textline[0] *= 1.15
229
           # if lim == 'lowerleft':
230
               textline[1] = -0.5
231
232
           plotline(line,**_kwargs)
           annotateline(textline,text)
234
235
       handles, labels = ax.get_legend_handles_labels()
236
       handles, labels = [h for i, (h,1) in enumerate(zip(handles, labels)) if l not in labels[:i]], [l
         for i,(h,1) in enumerate(zip(handles,labels)) if 1 not in labels[:i]]
       fig.legend(handles=handles,labels=labels,bbox_to_anchor=(0.7,0.5),loc='center
         left',ncol=2,prop = {"size": 6},)
```

```
fig.subplots_adjust(right=0.7,bottom=0.15)
fig.savefig('.../../figures/%s_%s.pdf'%('_'.join(file.split('_')[:-1]),lim))
```

Code 4: CPU Info script.

```
#!/bin/bash
    prog="/proc/cpuinfo"
    field="cpu MHz"
    pattern="s%.*: \([^ ]*\).*$%\1%"
    trials=5000
    file=tmp1234.tmp
    rm -f ${file}
10
11
    for i in $(seq 1 ${trials})
12
    do
13
            #echo Trial: $i
14
            grep "${field}" ${prog} | sed "${pattern}" >> ${file}
15
            sleep 0.0000001
16
    done
17
18
    N=$(wc -l ${file} | sed "s:\([^ ]*\).*:\1:")
19
    avg=$(paste -sd+ ${file} | bc)
    avg=$(echo "scale=3; $avg / $N" | bc)
21
    echo Avg\(N=${N},Trials=${trials}\) : ${avg}
23
24
    rm ${file}
25
```

Code 5: membench job script.

```
#!/bin/bash
2
    #SBATCH --account=ners570f20_class
3
    #SBATCH -- job-name=NERS570_Lab8
    \#SBATCH --partition=standard
    #SBATCH --mail-user=mduschen@umich.edu
6
    #SBATCH --mail-type=END
    #SBATCH --nodes=1
    #SBATCH --mem-per-cpu=8000m
    #SBATCH
                    --time=01:00:00
10
    #SBATCH
                    --ntasks-per-node=3
11
12
13
    ./run.sh
14
```

Code 6: Matrix-Matrix multiply

```
#include <cstdio>
     #include <iostream>
2
    void dgemm( char transa, char transb,
6
                    unsigned int m, unsigned int n, unsigned int k,
                    double alpha, const double * const * a,
                    const double * const * b,
                    double beta, double **c)
10
    {
11
         for(unsigned int i=0; i<n; i++){</pre>
12
                  for(unsigned int j=0; j<m; j++){</pre>
13
                      c[i][j] *= beta;
14
                      }
15
             };
         for(unsigned int l=0; l<k; l++){</pre>
17
                     for(unsigned int j=0; j<m; j++){</pre>
                           for(unsigned int i=0; i<n; i++){</pre>
19
                            c[i][j] += alpha*a[i][l]*b[l][j];
                               }
21
                      }
             };
23
24
25
    }
26
```