## 1 Membench

The *membench* programs are run to obtain the following plots, from Code. 5, in Fig. 1 to Fig. 3 for the effect of array length and stride length on type of memory access, and time to access.

In this plot, the solid vertical lines indicate the estimated memory sizes, and dashed vertical lines indicate the true memory sizes, with black lines indicating the cache line sizes, and blue lines indicating the total cache sizes. The green solid horizontal lines indicate the approximate cache access times.

In general, it is observed that for a given range of strides, and particularly in regions with plateaus of strides, say between 64B and 4KB, greater array lengths have greater access times. These parallel plateaus can be attributed to filling and then calling the lowest possible caches/accessing the lowest possible level of memory when there are cache misses in the even lower level of memory. This accessing is constant for the given lowest possible memory level.

Here, lowest possible memory level refers to the smallest cache at which the stride is less than the cache size and or line size. At this lowest possible level of memory, there will be the least number of cache misses, as it will be assumed that the array will fill this level of cache with as many full cache lines as possible. Larger arrays will fill more cache lines at that level, up to the level being full (and disregarding space being required for the code/instructions on top of space for the data). This filling also depends on whether a cache miss prompts a cache line to be overwritten, or a new cache line to be filled with data from a higher memory level, This depends on the specific association, inclusivity/exclusivity, hierarchy and relative sizes of the different cache levels.

So for larger arrays, the number of iterations through the array is obviously greater, and must take more time, and more cache misses will occur, regardless of the relative difference in stride size to cache line size. This greater number of misses (and corresponding new cache lines being filled from higher memory levels) is likely linear in the array size. This linear increase in cache misses explains the parallel plateaus at higher accessing times for greater array sizes, even if the time to access each element within a given cache size is roughly constant.

The access times will be measured where there are significant plateaus in the access times, and the *maximum* of these plateaus will be used as an estimate for the access times. The reason there are not sharp increases between plateaus is that as the stride increases, there are less cache hits and more misses as the indexing goes beyond the cache line size, however there are also less indexing calls with greater stride. So the increase up to the next plateau, which corresponds to the memory access time of the next greater memory level, is gradual, as there is a mix of hits in different memory levels.

The following script *calc.sh* in the Code. 6 in the appendix was used to get the cpu and memory statistics in Code. 1. The average processor core speed for the 36 processors on GreatLakes to be 3000 MHz, and the cache line sizes appear to be constant at 64 Bytes. The cpu core speed will be used to calculate the number of processor clock cycles required to access the various types of memory,

# cycles per memory access = clock speed  $\times$  memory access time.

Code 1: CPU and Memory values.

```
Cache Info
Cache L1 Size: 32 kB
Cache L1 Line Size: 64 B

Cache L2 Size: 1024 kB
Cache L2 Line Size: 64 B

Cache L3 Size: 25344 kB
Cache L3 Line Size: 64 B
```

```
Cache L4 Size: 0 kB
Cache L4 Line Size: 0 B

Cache L4 Line Size: 0 B

CPU Info
CPU Cores: 36
CPU Speed: 2999.531 MHz
```

# 1.1 Processor Values

On the Greatlakes compute notes, there are  $36 \times \text{Intel}(R) \text{ Xeon}(R)$  Gold 6140 CPU @ 2.30GHz. From repeated requests for the speed of each core, the average core speed over all cores and samples is 2.99 GHz.

#### 1.2 $L_1$ Cache Values

For the  $L_1$  cache line size, the true line size is 64B, and from the plots, particularly in Fig. 2, it can be seen that the access times do not start to initially increase until around strides of 64B, before then plateauing. This jump before plateauing indicates that strides greater than this value must start to involve more  $L_1$  misses, and require accessing the  $L_2$  memory.

$$L_1 \text{ Line } = 128B.$$

For the  $L_1$  total cache size, although the true cache size is 32KB, from the plots, particularly in Fig. 2, it can be seen that the access times remain very constant, and at their minimum all way the up to 4KB for arrays with length  $\leq$  32KB, suggesting the entire array, or at least half of the array can be loaded into the  $L_1$  cache. In addition, the next largest 62KB array shows increased access times for up to 32KB strides, suggesting some  $L_1$  cache misses possibly occur, and so the  $L_1$  cache size is likely less than 32KB.

$$16KB \le L_1 \le 32KB$$
.

For the  $L_1$  access time, given the quite constant access times up to 4KB strides for arrays with length  $\leq$  32KB, the estimated access time is therefore the maximum of the 32KB length array curve:

$$T_1 = 0.57 \text{ns} = 2 \text{ cycles}.$$

### 1.3 $L_2$ Cache Values

For the  $L_2$  cache line size, the true line size is 64B, and from the plots, particularly in Fig. 3, it can be seen that after the initial increase of access times, there is a slight plateau for strides  $\leq 512$ B, and arrays of sizes 64KB-8MB, indicating that possibly the array is being quickly indexed in the  $L_2$  cache with a cache line of between 64B and 512B. There is then a jump in access times, but not a huge jump for arrays up to size 8MB, indicating there are possibly still cache hits in the  $L_2$  cache, but at different lines.

$$64B \le L_2$$
 Line  $\le 512B$ .

For the  $L_2$  total cache size, although the true cache size is 1MB, from the plots, particularly in Fig. 3, it can be seen that the access times remain very constant over a large range of array sizes between 64KM to 8MB, with strides between 1KB and 256KB. This suggests lots of these array sizes can be mostly loaded into the  $L_2$  cache on several cache lines. This suggests the  $L_2$  total cache size to be less than 256KB (and greater than the  $L_1$  total cache size); the point where the access times for these array sizes drops dramatically when many less array elements are indexed. The difficulty at finding tighter bounds on the  $L_2$  cache sizes is possibly due to the  $L_2$  cache sometimes being shared by pairs of cores, affecting the timing, depending on which cores the array is being computed on.

$$16KB \le L_2 \le 256KB.$$

For the  $L_2$  access time, given the quite constant access times for array sizes between 64KM to 8MB, with strides between 1KB and 256KB, the estimated access time is therefore the maximum of the 8MB size array curve along

this plateau:

$$T_2 = 3.83 \text{ns} = 12 \text{ cycles}.$$

# 1.4 $L_3$ Cache Values

For the  $L_3$  cache line size, the true line size is 64B, however from the plots, it is difficult to tell where exactly there is a distinct plateau for array indexing with strides within the size of this larger cache's lines. This may be due to the  $L_3$  cache being typically shared between all (36) cores, and so timings may be affected depending how the computations are distributed amongst the cores. However for array sizes of 16MB to 512MB, there is somewhat a plateau between 512B and 4KB, indicating a possible range for the  $L_3$  cache line size. Here, there may be hits due to this larger cache being allowed to store more of these larger arrays, minimising cache misses. The lack of distinct plateau is also possibly attributed to the large arrays being far larger than the cache, and there being many hits and misses while the lines are being filled from the main memory.

$$64B < L_3$$
 Line  $< 4KB$ .

For the  $L_3$  total cache size, although the true cache size is 24.75MB, from the plots, particularly in Fig. 4, it can be seen that the plateau between 512B and 4KB, for array sizes of 16MB to 512MB, rises, and then decreases gradually, before plateauing again for strides between 256KB and 8MB. This suggests that the time is not decreasing solely due to there being less elements indexed with greater stride, but there also possibly being effects of the elements still being in the faster  $L_3$  cache compared to the main memory. There are still hits occurring in succession in a cache that are causing this plateau at non-zero access times. There is still though great uncertainty in the exact total size of this  $L_3$  cache.

$$256KB \le L_3 \le 8MB.$$

For the  $L_3$  access time, given the two different plateaus present in the access times for the larger arrays, the access time will be estimated as the maximum of these plateaus in Fig. 4.

$$T_3 = 10.36 \text{ns} = 32 \text{ cycles}.$$

#### 1.5 Main Memory Values

Only the 1GB array sizes appear to be unable to be stored fully in any caches, and there are enough misses in the lower caches that the main memory must be accessed. It is assumed the upper plateau for the 1GB array are these memory hits, and the access time is assumed to be the maximum of this plateau. This is assumed to be a lower bound, if some of the array is in the  $L_3$  cache, and there are some hits there, and some in the main memory.

$$T_{\text{mem}} \ge 13.69 \text{ns} = 42 \text{ cycles}.$$

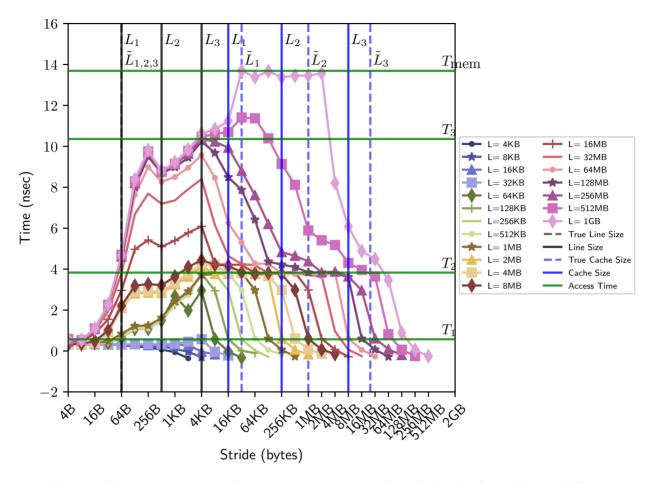


Figure 1: Memory access times for various array sizes, and stride lengths from 4B to 512MB.

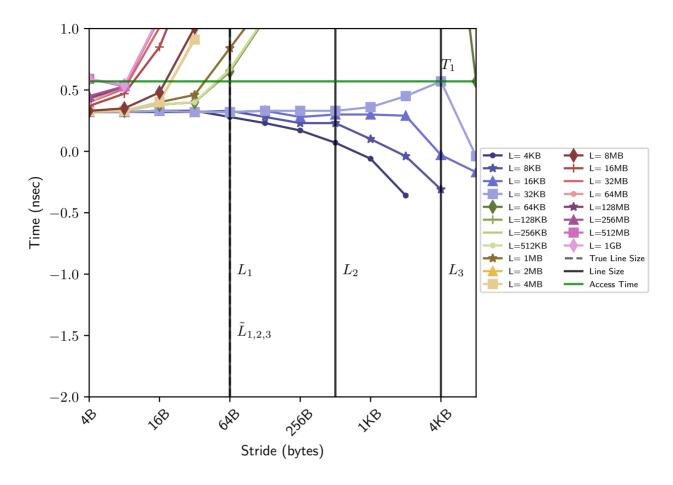


Figure 2: Memory access times for various array sizes, and stride lengths from 4B to 4KB.

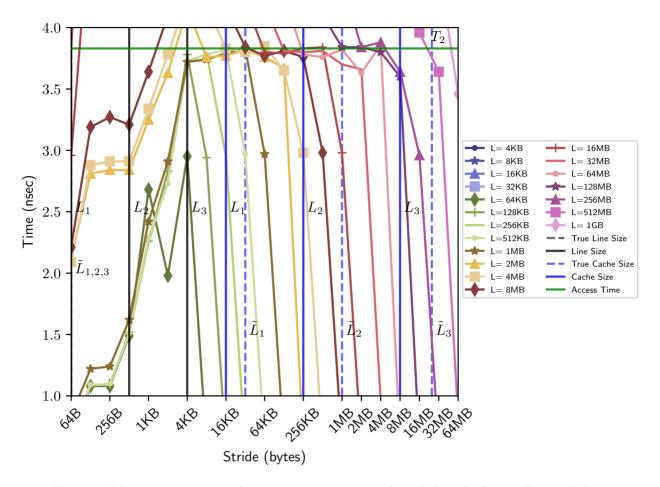


Figure 3: Memory access times for various array sizes, and stride lengths from 16B to 64MB.

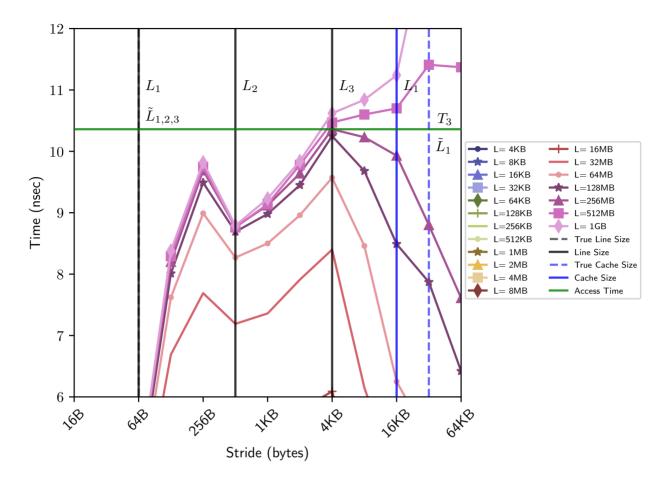


Figure 4: Memory access times for various array sizes, and stride lengths from 16B to 64KB.

# 2 SIMD Instructions

# 2.1 AVX Support

• The commands used to verify if the current machine/processor supports AVX are:

and if the current machine/processor supports AVX2 are:

These searches will show whether the supported AVX fields are in the flags section of the processor info from /proc/cpuinfo. This command will show all processors ( $36 \times Intel(R) \times (R) \times (R) \times (R) \times (R) \times (R)$ ) Gold 6140 CPU @ 2.30GHz, for GreatLakes compute nodes).

• The commands used to verify if the current GNU compiler supports AVX or AVX2 by the constants/macros the compiler defines:

```
gcc -mavx2 -dM -E - < /dev/null | grep "AVX" | sort
```

This command will show the boolean settings for the AVX constants:

```
#define \_\_AVX\_\_1
#define \_\_AVX2\_\_1
```

• The commands used to verify if the current Intel compiler supports AVX or AVX2 by the constants/macros the compiler defines:

This command will show the boolean settings for the AVX constants:

```
#define __AVX__ 1
#define __AVX2__ 1
#define __AVX512BW__ 1
#define __AVX512CD__ 1
#define __AVX512DQ__ 1
#define __AVX512F__ 1
#define __AVX512VL__ 1
#define __AVX I 1
```

# 2.2 dgemm Assembly Instructions

The command to get the assembly instructions is as follows, where the function for the naive *dgemm.cpp* in Code. 8 in the appendix is translated into assembly code, with the avx2 and fast optimizations using the commands for gnu and intel compilers:

This command (with the gnu compiler) produces the following assembly code in Code. 3. Here it can be seen in lines 81-120, there is the assembly code for the three loops (add, add,compute, cmp, jump, jne, as well as the vectorized multiply vmulsd, add add, and vectorized move vmovsd. The specific SIMD fused multiply-add commands are vfmadd132sd.

The assembly instructions are known to contain AVX2 instructions because they are writing to the AVX specific register keywords ymm, and AVX2 is confirmed, because fused-multiply-add commands (such as vfmadd132sd) are

in the assembly instructions.

Considering the correct compiler options and hardware compatibility for AVX2 instructions has appeared to be verified, no modifications of the original matrix-matrix multiplication source code were necessary. The mydgemm, compiled with the above avx2 and fma options, as well as the fast optimizations, does not appear on GreatLakes to run faster, at least for matrix sizes up to 2000.

Code 2: Matrix-matrix multiplication loop assembly code

```
testl
                              %r8d, %r8d
81
               jе
                          .L29
82
                             16(%rbp), %r12
              movq
               leal
                             -1(%r8), %eax
                             8(, %r14,8), %rbx
              leaq
85
                             %r8d, %r8d
              xorl
86
                             8(%r12,%rax,8), %r13
               leaq
               .p2align 4,,10
88
               .p2align 3
     .L10:
90
                             (%r12), %r14
              movq
91
               xorl
                             %esi, %esi
92
               .p2align 4,,10
93
               .p2align 3
94
     .L9:
95
                             (%r14,%rsi), %rdi
              leaq
96
               xorl
                             %eax, %eax
97
                            .L8
               jmp
               .p2align 4,,10
99
               .p2align 3
100
     .L13:
101
              movq
                             %rdx, %rax
102
     .L8:
103
                             (%r11,%rax,8), %rdx
104
              movq
              movq
                             (%r9,%rax,8), %rcx
105
                               (%rdi), %xmm0, %xmm1
               vmulsd
106
               addq
                             %rsi, %rdx
107
               vmovsd
                               (%rdx), %xmm4
108
               vfmadd132sd
                                     (%rcx,%r8), %xmm4, %xmm1
109
               vmovsd
                               %xmm1, (%rdx)
110
                             1(%rax), %rdx
              leaq
111
               cmpq
                             %rax, %r10
112
                            .L13
               jne
113
                             $8, %rsi
               addq
114
                             %rsi, %rbx
115
               cmpq
               jne
                            .L9
116
                             $8, %r12
               addq
117
               addq
                             $8, %r8
118
                             %r12, %r13
               cmpq
119
                            .L10
               jne
120
```

Code 3: Matrix-Matrix multiply assembly with avx2 and fast optimization

```
_Z5dgemmccjjjdPKPKdS2_dPPd:
6
    .LFB1538:
             .cfi_startproc
8
             pushq
                            %rbp
             .cfi_def_cfa_offset 16
10
             .cfi_offset 6, -16
11
             movq
                           %rsp, %rbp
12
             .cfi_def_cfa_register 6
13
                            %r14
             pushq
                           24(%rbp), %r11
             movq
15
                            %r13
             pushq
                            %r12
             pushq
17
                            %rbx
18
             pushq
             .cfi_offset 14, -24
19
             .cfi_offset 13, -32
20
             .cfi_offset 12, -40
21
             .cfi_offset 3, -48
22
             testl
                            %ecx, %ecx
23
             jе
                         .L27
24
             testl
                            %edx, %edx
25
             jе
                         .L27
26
                           %edx, %r12d
             movl
27
             leal
                           -1(\%rcx), \%eax
28
                           %edx, %ebx
             movl
             movq
                           %r11, %rdi
30
                           $2, %r12d
             shrl
31
                           %rax, %r10
             movq
32
                           8(%r11,%rax,8), %r13
             leaq
33
                           $-4, %ebx
             andl
34
             leal
                           -1(\%rdx), \%r14d
35
             salq
                           $5, %r12
36
                                    %xmm1, %ymm3
             vbroadcastsd
37
             .p2align 4,,10
38
             .p2align 3
39
    .L7:
40
             movq
                           (%rdi), %rsi
41
                           $2, %r14d
             cmpl
42
             jbe
                          .L12
43
                           %rsi, %rax
             movq
             leaq
                           (%r12,%rsi), %rcx
45
             .p2align 4,,10
46
             .p2align 3
47
    .L4:
48
                              (%rax), %xmm5
             vmovupd
49
50
             vinsertf128
                                   $0x1, 16(%rax), %ymm5, %ymm2
             addq
                           $32, %rax
51
                             %ymm3, %ymm2, %ymm2
             vmulpd
52
                              %xmm2, -32(%rax)
             vmovups
53
             vextractf128
                                    $0x1, %ymm2, -16(%rax)
54
                           %rax, %rcx
             cmpq
             jne
                          .L4
56
                           %ebx, %eax
             movl
57
             cmpl
                           %ebx, %edx
58
                         .L5
             jе
    .L3:
60
             movl
                           %eax, %ecx
61
             leaq
                           (%rsi,%rcx,8), %rcx
62
```

```
(%rcx), %xmm1, %xmm2
              vmulsd
63
                              %xmm2, (%rcx)
              vmovsd
64
              leal
                            1(%rax), %ecx
65
              cmpl
                            %ecx, %edx
66
              jbe
                           .L5
67
                            (%rsi,%rcx,8), %rcx
              leaq
68
              addl
                            $2, %eax
69
              vmulsd
                               (%rcx), %xmm1, %xmm2
70
              vmovsd
                              %xmm2, (%rcx)
              cmpl
                            %eax, %edx
72
                           .L5
              jbe
73
                            (%rsi,%rax,8), %rax
              leaq
74
                               (%rax), %xmm1, %xmm2
75
              vmulsd
              vmovsd
                              %xmm2, (%rax)
76
     .L5:
77
                            $8, %rdi
              addq
78
              cmpq
                            %r13, %rdi
79
                           .L7
80
              jne
              testl
                             %r8d, %r8d
81
              jе
                          .L29
82
                            16(%rbp), %r12
              movq
83
                            -1(\%r8), \%eax
              leal
              leag
                            8(,%r14,8), %rbx
85
                            %r8d, %r8d
              xorl
              leaq
                            8(%r12,%rax,8), %r13
87
              .p2align 4,,10
88
              .p2align 3
89
     .L10:
90
              movq
                            (%r12), %r14
91
              xorl
                            %esi, %esi
92
              .p2align 4,,10
93
              .p2align 3
94
     .L9:
95
              leaq
                            (%r14,%rsi), %rdi
96
              xorl
                            %eax, %eax
97
              jmp
98
              .p2align 4,,10
99
              .p2align 3
100
     .L13:
101
              movq
                            %rdx, %rax
102
     .L8:
103
                            (\%r11,\%rax,8), \%rdx
              movq
104
                            (\%r9,\%rax,8),\%rcx
              movq
105
                               (%rdi), %xmm0, %xmm1
              vmulsd
106
107
              addq
                            %rsi, %rdx
              vmovsd
                               (%rdx), %xmm4
108
                                    (%rcx,%r8), %xmm4, %xmm1
              vfmadd132sd
109
              vmovsd
                              %xmm1, (%rdx)
110
              leaq
                            1(%rax), %rdx
111
                            %rax, %r10
              cmpq
              jne
                           .L13
113
                            $8, %rsi
              addq
114
                            %rsi, %rbx
              cmpq
115
              jne
                           .L9
116
                            $8, %r12
              addq
117
              addq
                            $8, %r8
118
                            %r12, %r13
              cmpq
119
```

```
.L10
              jne
120
     .L29:
121
              vzeroupper
122
     .L27:
123
                           %rbx
124
              popq
                           %r12
              popq
125
                           %r13
126
              popq
              popq
                           %r14
127
                           %rbp
              popq
              .cfi_remember_state
129
              .cfi_def_cfa 7, 8
130
              ret
131
     .L12:
132
              .cfi_restore_state
133
                           %eax, %eax
              xorl
134
                           .L3
              jmp
135
              .cfi_endproc
136
     .LFE1538:
137
              .size
                             _Z5dgemmccjjjdPKPKdS2_dPPd, .-_Z5dgemmccjjjdPKPKdS2_dPPd
138
                                .text.startup, "ax", @progbits
              .section
139
              .p2align 4,,15
140
                            _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd, @function
141
              .type
     _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd:
     .LFB2019:
143
              .cfi_startproc
144
              subq
                           $8, %rsp
145
              .cfi_def_cfa_offset 16
146
              movl
                           $_ZStL8__ioinit, %edi
147
              call
                           _ZNSt8ios_base4InitC1Ev
148
              movl
                           $__dso_handle, %edx
149
                           $_ZStL8__ioinit, %esi
              movl
150
              movl
                           $_ZNSt8ios_base4InitD1Ev, %edi
151
              addq
                           $8, %rsp
152
              .cfi_def_cfa_offset 8
153
              jmp
                          __cxa_atexit
154
155
              .cfi_endproc
     .LFE2019:
156
                             _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd,
              .size
157
                .-_GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd
                                .init_array, "aw"
              .section
158
              .align 8
159
                             _GLOBAL__sub_I__Z5dgemmccjjjdPKPKdS2_dPPd
              .quad
160
              .local
                              _ZStL8__ioinit
161
              .comm
                             _ZStL8__ioinit,1,1
162
              .hidden
                               __dso_handle
163
                              "GCC: (GNU) 8.2.0"
              .ident
164
                                .note.GNU-stack,"", @progbits
              .section
```

# 3 In-line Assembly

With help from the internet, the following matrix-vector multiplication y = Ax, in-line assembly, as well as conventional c code, is shown in Code. 9 in the appendix. The fused-multiply adds assembly was not quite able to be implemented, however the procedure of writing the fused-multiply adds with avx instructions (in the assembly syntax, as opposed to the intel syntax) was attempted. There are no compiler errors, however the issue seems to be that the y value is not being assigned the computed multiplied value in the set register in  $\_mm\_storel\_pd(y, \_mm256\_castpd256\_pd128(Ax))$ .

The assembly registers for multiplies and adds currently only allow for operating on  $4 \times 4$  matrices if double precision is used. So, blocks of the matrix and vector in these sizes must be passed to the assembly function  $\_matvec\_avx(A,x,y)$  in a loop. This loop can then be unrolled, to assign each part of y separately.

The program is compiled with the following command, and assembly snippet is shown in Code. 4.

 ${\it gcc}$ -Ofast -mavx2 -mfma -funroll-all-loops matvec.c

Code 4: Matrix-Vector multiplication inline assembly code

```
void _matvec_avx(const double* A,const double* x,double* y){
 202
                                        asm volatile ("# avx code begin"); // looking at assembly with qcc -S
203
                                        _{m256d xrow = mm256_loadu_pd(x);}
204
205
                                        __m256d a = _mm256_mul_pd(_mm256_loadu_pd(A), xrow);
206
                                        _{m256d} b = _{mm256} (_{mm256} (_
207
                                        _{m256d} c = _{mm256} (_{mm256} (_{mm256} (_{nm256} (_
208
                                        _{\rm m256d} d = _{\rm mm256\_mul\_pd(_{\rm mm256\_loadu\_pd(A+12)}, xrow);}
209
210
                                        // our task now is to get {sum(a), sum(b), sum(c), sum(d)}
211
                                        // This is tricky because there is no hadd instruction for avx
212
213
                                        // \{a[0]+a[1], b[0]+b[1], a[2]+a[3], b[2]+b[3]\}
214
                                        _{\rm m256d\ sumab} = _{\rm mm256\_hadd\_pd(a, b)};
215
216
                                        // \{c[0]+c[1], d[0]+d[1], c[2]+c[3], d[2]+d[3]\}
217
                                        _{m256d} = _{mm256\_hadd\_pd(c, d)};
218
219
                                        // \{a[0]+a[1], b[0]+b[1], c[2]+c[3], d[2]+d[3]\}
                                        _{\rm m256d} blend = _{\rm mm256\_blend\_pd(sumab, sumcd, 0b1100)};
221
222
                                        // \{a[2]+a[3], b[2]+b[3], c[0]+c[1], d[0]+d[1]\}
223
                                        __m256d perm = _mm256_permute2f128_pd(sumab, sumcd, 0x21);
225
                                        // {sum(a), sum(b), sum(c), sum(d)}
226
                                        _{\rm m256d} Ax = _{\rm mm256\_add\_pd(perm, blend)};
227
                                        // printf("yinit = %f",*y);
                                        _mm_storel_pd(y, _mm256_castpd256_pd128(Ax));
229
                                        // printf("yfinal = %f",*y);
230
                                        // _mm_storel_pd(y, Ax);
231
                                        asm volatile ("# avx code end");
232
233
                     };
234
```

# 4 Appendix

Code 5: memberch plotting script.

```
#!/usr/bin/env python
    import matplotlib
2
    import matplotlib.pyplot as plt
    import numpy as np
4
    matplotlib.rcParams['text.usetex'] = True
6
    # matplotlib.rcParams['text.latex.preamble'] = [r'\usepackage{ragged2e']
    # Routine modified from:
      https://stackoverflow.com/questions/1094841/reusable-library-to-get-human-readable-version-of-file-size
    def sizeof_fmt(num, suffix='B'):
10
        for unit in ['','K','M','G','T']:
11
            if abs(num) < 1024.0:
12
                 return '%3.0f%s%s' % (num, unit, suffix)
            num /= 1024.0
14
        return '%.1f%s%s' % (num, 'T', suffix)
15
16
    def fmt_sizeof(fmt):
17
        bases={'B':2,'':10}
18
        units={k:v for k,v in zip(['','K','M','G','T'],[1,10,20,30,40])}
20
          base=fmt[-1]
          unit=fmt[-2]
22
          num = float(fmt[:-2])
23
        except ValueError:
24
          try:
25
            base=fmt[-1]
26
            unit=''
27
            num = float(fmt[:-1])
          except:
29
            try:
30
              base=''
31
              unit=''
32
              num = float(fmt)
33
            except:
              return fmt
35
        # print(num, (bases.get(base, 10) **units.get(unit, 1)))
        # print(num)
37
        num *= (bases.get(base,10)**units.get(unit,1))
38
        if int(num) == num:
39
          num = int(num)
40
        return num
41
42
    def indexer(array,sorter,value):
      i = np.where(sorter==value)[0]
44
      return array[i]
45
46
    file='membench_4'
48
    cpuspeed=3e9
49
50
    mbdata = np.genfromtxt('%s.out'%file,usecols=(1,3,5))
52
```

```
53
       maxunit=10
        units=dict(zip(range(0,maxunit*4,maxunit),['','K','M','G']))
55
        sizes = \{'\%d\%s\%s'\%(b**(i),units[d],u):b**(i+d) \text{ for b,u in } zip([2],['B']) \text{ for d in units for i}
56
            in range(maxunit)}
57
58
        xtlabels=[*['4B','16B','64B','256B','1KB','4KB','16KB','64KB','256KB','1MB'],
59
                            *['2MB','4MB','8MB','16MB','32MB','64MB','128MB','256MB','512MB','2GB']]
        xtvals = [sizes.get(l,fmt_sizeof(l)) for l in xtlabels]
61
63
65
66
        \# k, l, m = '1GB', '1MB', '512B'
67
        # print((sizes.qet(k, fmt_sizeof(k)), indexer(indexer(mbdata[:,2], mbdata[:,0], sizes.qet(l, |
68
            fmt\_sizeof(l)), indexer(mbdata[:,1], mbdata[:,0], sizes.get(l,fmt\_sizeof(l))), sizes.get(m, [:, 0], sizes.ge
            fmt_sizeof(m)))[0]))
69
        lims={
70
                       'all': [(xtlabels[0],xtlabels[-1]),(-2,16)],
71
                       'lowerleft': [('4B', '8KB'), (-2,1)],
72
                       'middleupper': [('16B', '64KB'), (6,12)],
                       'middle': [('64B','64MB'),(1,4)],
74
                      }
76
        heights={k:{1:-1 for 1 in ['h','v']} for k in lims}
        heights['all']['v'] =15
78
        heights['lowerleft']['v'] = -1
        heights['middle']['v'] = 2.5
80
        heights['middleupper']['v'] = 11
81
        lines={
82
                         'all':{'v': {
83
                                                    # **{(sizes.get(k,fmt_sizeof(k))/2,height):r'$\tilde{L}_{1,2,1}
                                                       3}\\\textrm{%s}$'%k for i,k in
                                                        enumerate(['64B'])},
                                                    # **{(sizes.qet(k,fmt_sizeof(k)),height):r'${L}_{%d}\\\textrm{%s}$'\(i+1, \
85
                                                       k) for i,k in
                                                        enumerate(['128B'])},
                                                   # # **{(sizes.get(k,fmt_sizeof(k)),height):r'$\tilde{L}_{\%d}$'\%(i+1) for
                                                        i,k in enumerate(['128B', '4KB', '16KB'])},
                                                    # **{(sizes.get(k,fmt_sizeof(k)),
87
                                                       height):r'\$\tilde\{L\}_{\d}\\\ for i.k in
                                                        enumerate(['32KB','1MB','24.75MB'])},
88
                                                   **{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']-1):r'$\tilde{L}_{1,2,_|
89
                                                       3}$' for i,k in
                                                       enumerate(['64B'])},
                                                   **{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']):r'${L}_{%d}$'%(i+1)
                                                       for i,k in enumerate(['64B','512B','4KB'])},
                                                    # **{(sizes.get(k,fmt_sizeof(k)),
91
                                                       heights['all']['v']):r'$\tilde{L}_{{d}}$'%(i+1) for i,k in
                                                        enumerate(['128B', '4KB', '16KB'])},
                                                   **{(sizes.get(k,fmt_sizeof(k)),
92
                                                       heights['all']['v']-1):r'$\tilde{L}_{\%d}$'\%(i+1) for i,k in
                                                       enumerate(['32KB','1MB','25MB'])},
```

```
**{(sizes.get(k,fmt_sizeof(k)),heights['all']['v']):r'$L_{%d}$'%(i+1) for
93
                             i,k in enumerate(['16KB','256KB','8MB'])},
                           },
94
95
                    'h': {
                          # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0], |
97
                            sizes.qet(l,fmt\_sizeof(l)))):r'$T\_{%d}$''(i+1) for i,(k,l) in
                            enumerate(zip(['1GB'],['32KB']))},
                          **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
                            sizes.get(l,fmt_sizeof(l)))):r'$T_{{d}}''(i+1) if i<3 else
                            r'T_{\text{mem}} for i,(k,1) in
                            enumerate(zip(['1GB','1GB','1GB','1GB'],['32KB','1MB','256MB','1GB']))},
                          # **{(sizes.qet(k,fmt_sizeof(k)),indexer(indexer(mbdata[:,2],mbdata[:,0],
                            sizes.qet(l,fmt\_sizeof(l))), indexer(mbdata[:,1],mbdata[:,0],sizes.qet(l,,))
                            fmt\_sizeof(l))), sizes.get(m, fmt\_sizeof(m)))[0]):r'$T_{{d}}''(i+2) for
                            i, (k, l, m) in enumerate(zip(['1GB'], ['1MB'], ['512B']))},
                          }.
100
101
102
103
                   },
104
             'lowerleft':{'v': {
105
                                   \# **\{(sizes.qet(k,fmt\_sizeof(k)),height):r'\$\setminus tilde\{L\}_{\{1,2,\ldots\}}\}
106
                                     3}\\\textrm{%s}$'%k for i,k in
                                     enumerate(['64B'])},
                                   # **{(sizes.get(k,fmt_sizeof(k)),
107
                                     height):r'${L}_{%d}\\\textrm{%s}$'%(i+1,k) for i,k in
                                     enumerate(['128B'])},
108
                                   **{(sizes.get(k,fmt_sizeof(k)),
109
                                    \label{lowerleft'} heights['lowerleft']['v']-0.5):r'$\tilde{L}_{1,2,3}$' for i,k in
                                     enumerate(['64B'])},
                                   **{(sizes.get(k,fmt_sizeof(k)),
110
                                    heights['lowerleft']['v']):r'L_{\infty}(i+1) for i,k in
                                     enumerate(['64B','512B','4KB'])},
111
112
                                },
113
                          'h': {
                                 # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
115
                                  mbdata[:,0], sizes.qet(l,fmt\_sizeof(l))))):r'$T\_{%d}$'\%(i+1) for
                                   i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                          **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
116
                            sizes.get(1,fmt_sizeof(1)))):r'$T_{{d}}''(i+1) for i,(k,1) in
                            enumerate(zip(['4KB','4KB'],['32KB','1MB']))},
                                },
117
                          },
118
             'middle':{'v': {
119
                                   # **{(sizes.get(k,fmt_sizeof(k)),height):r'$\tilde{L}_{1,2,1}
120
                                     3}\\\textrm{%s}$'%k for i,k in
                                     enumerate(['64B'])},
                                   # **{(sizes.get(k,fmt_sizeof(k)),
121
                                     height):r'$\{L\}_{\d}\\\textrm{\s}$'\c (i+1,k) for i,k in
                                     enumerate(['128B'])},
122
```

```
**{(sizes.get(k,fmt_sizeof(k)), |
123
                                                                   heights['middle']['v']-0.5):r'$\tilde{L}_{1,2,3}$' for i,k in
                                                                   enumerate(['64B'])},
                                                                **{(sizes.get(k,fmt_sizeof(k)),
124
                                                                   heights['middle']['v']):r'L_{-{\text{d}}}''(i+1) for i,k in
                                                                   enumerate(['64B','512B','4KB'])},
                                                                **{(sizes.get(k,fmt_sizeof(k)),
125
                                                                   heights['middle']['v']-1):r'$\dot{L}_{k}^{(i+1)} for i,k in
                                                                   enumerate(['32KB','1MB','25MB'])},
                                                                  **{(sizes.get(k,fmt_sizeof(k)),
126
                                                                     heights['middle']['v']):r'$L_{\dd}$'\%(i+1) for i,k in
                                                                      enumerate(['16KB','256KB','8MB'])},
                                                           },
                                                'h': {
128
                                                            # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
129
                                                               mbdata[:,0], sizes.get(l,fmt\_sizeof(l))))):r'$T_{{d}}$''(i+1) for
                                                                i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                                                **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
130
                                                    sizes.get(1, \text{fmt\_sizeof}(1)))):r'T_{\frac{d}{t}}'%(i+1) for i,(k,1) in
                                                    enumerate(zip(['16MB','16MB'],['32KB','1MB']))},
131
                                                },
132
                         'middleupper':{'v': {
133
                                                                # **{(sizes.get(k,fmt_sizeof(k)),height):r'$\tilde{L}_{1,2,1}
                                                                    3}\\\textrm{%s}$'%k for i,k in
                                                                    enumerate(['64B'])},
                                                                # **{(sizes.get(k,fmt_sizeof(k)),
135
                                                                    height):r'$\{L\}_{\{d}\setminus \text{textrm}\{\%s\}$'\%(i+1,k) for i,k in
                                                                    enumerate(['128B'])},
136
                                                                **{(sizes.get(k,fmt_sizeof(k)), |
137
                                                                   \label{lem:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma:lemma
                                                                   in enumerate(['64B'])},
                                                                **{(sizes.get(k,fmt_sizeof(k)), |
138
                                                                   enumerate(['64B','512B','4KB'])},
                                                                **{(sizes.get(k,fmt_sizeof(k)), |
139
                                                                   heights['middleupper']['v']-1):r'$\dot{L}_{%d}$'%(i+1) for i,k
                                                                   in enumerate(['32KB','1MB','25MB'])},
                                                                  **{(sizes.get(k,fmt_sizeof(k)),
140
                                                                     heights['middleupper']['v']):r'L_{d}'%(i+1) for i,k in
                                                                      enumerate(['16KB','256KB','8MB'])},
                                                           },
141
                                                'h': {
142
143
                                                            # **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],
                                                               mbdata[:,0], sizes.get(l,fmt\_sizeof(l))))):r'$T_{{d}}$''(i+1) for
                                                                i, (k, l) in enumerate(zip(['256B'], ['32KB']))),
                                                **{(sizes.get(k,fmt_sizeof(k)),np.max(indexer(mbdata[:,2],mbdata[:,0],
144
                                                    sizes.get(1,fmt_sizeof(1)))):r'$T_{%d}$'%(i+1) for i,(k,1) in
                                                    enumerate(zip(['16MB','16MB','32KB'],['32KB','1MB','256MB']))},
145
146
                         \# 'middle':\{'v': \{**\{(sizes.qet(k,fmt\_sizeof(k)),-4.5\}: 'L\%d\LineSize:\\\%s'\%(i+1,k) for \}\}
147
                            i, k in enumerate(['4KB', '16KB'])}}}
                    }
148
149
150
```

```
151
     print(lines)
153
154
155
     cmap = 'tab20b'
156
     lengths=np.array(list(sorted(set(mbdata[:,0]))))
157
158
     kwargs={}
     kwargs['marker'] = ['.','*','^','s','d','+','']
160
161
     kwargs['color'] = {k:v for k,v in zip(lengths,plt.get_cmap(cmap)(np.linspace(0, 1,
162
       len(lengths))).tolist())}
     kwargs['label'] = {k:'L=%s'%(sizeof_fmt(k)) for k in lengths }
163
164
     kwargs['marker'] = {k: kwargs['marker'][i%len(kwargs['marker'])] for i,k in enumerate(lengths)}
165
     kwargs['inds'] = {k: np.where(mbdata[:,0]==k)[0] for k in lengths}
166
167
168
169
     kwargs['lines'] = {k: {'linestyle':s,'color':c,'label':1,'alpha':a,'zorder':z} for k,1,c,s,a,z
170
       in zip(
                                                                                            ['tilde{L}_{1,2,|
171
                                                                                              3}',
                                                                                              'tilde{L}',
                                                                                              '{L}_','L_',
                                                                                              'T_','Mem',
                                                                                              None],
                                                                                            ['True Line
172
                                                                                              Size','True
                                                                                              Cache
                                                                                              Size','Line
                                                                                              Size','Cache
                                                                                              Size','Access
                                                                                              Time','Access
                                                                                              Time',''],
                                                                                            ['k','b','k',
173
                                                                                              'b','g','g',<sub>|</sub>
                                                                                             None],
                                                                                            ['--','--','-',
174
                                                                                              '-','-','-',<sub>|</sub>
                                                                                              None],
                                                                                            [0.6, 0.6, 0.8, 1]
175
                                                                                             0.8,0.8,0.8,
                                                                                              None],
                                                                                            [-1, -1, 10, 10, 10, 10, 10]
176
                                                                                              10, None],
                                                                                           )}
177
178
     for lim in lims:
179
180
181
       fig,ax = plt.subplots()
182
       # Plots
184
185
       #for i in range(9,27):
186
```

```
for L in lengths:
187
         #ax.plot(mbdata[istt:istt+i,1],mbdata[istt:istt+i,2],
188
         ax.plot(mbdata[kwargs['inds'][L],1],mbdata[kwargs['inds'][L],2],**{k:kwargs[k][L] for k in
180
           ['color','marker','label']})
         \#istt=istt+i+1
190
191
       ax.set_ylabel('Time (nsec)')
192
       ax.set_xlabel('Stride (bytes)')
193
       ax.set_xscale('log',base=2)
       ax.set_xticks(xtvals)
195
       ax.set_xticklabels(xtlabels,rotation=45)
196
       ax.set_ylim(*[l for l in lims[lim][1]])
197
       ax.set_xlim(*[sizes.get(1,fmt_sizeof(1)) for 1 in lims[lim][0]])
198
199
       # Lines
200
       plotlines={k:lambda line,k=k,i=i,**kwargs: getattr(ax,'ax%sline'%k)(line[i],**kwargs) for i,k
201
         in enumerate(['v','h'])}
       annotatelines={k:lambda line,text,k=k,i=i,**kwargs:
202
         getattr(plt, 'annotate')(text=text,xy=line,**kwargs) for i,k in enumerate(['v','h'])}
       for k in lines.get(lim,[]):
203
         plotline=plotlines[k]
204
         annotateline=annotatelines[k]
205
         for line in lines[lim][k]:
206
           text = lines[lim][k][line]
           _kwargs = kwargs['lines'][None]
208
           for x in kwargs['lines']:
209
             if str(x) in text:
210
                _kwargs = kwargs['lines'][x]
               break
212
           textline=list(line)
213
           if 0 and (_kwargs['color'] == 'k' and _kwargs['linestyle'] == '-'):
214
             textline[0] /=8
215
           elif (_kwargs['color'] == 'g') and (lim=='all'):
216
             textline[1] += 0.25
217
           elif (_kwargs['color'] == 'g') and (lim=='lowerleft'):
             textline[1] += 0.1
219
           elif (_kwargs['color'] == 'g') and (lim=='middle'):
220
             textline[0] *= 1.5
221
             textline[1] += 0.06
           elif (_kwargs['color'] == 'g') and (lim=='middleupper'):
223
             textline[0] *= 1.2
224
             textline[1] += 0.1
225
           elif lim not in ['lowerleft']:
226
             textline[0] *= 1.15
227
           else:
             textline[0] *= 1.15
229
           # if lim == 'lowerleft':
230
               textline[1] = -0.5
231
232
           plotline(line,**_kwargs)
           annotateline(textline,text)
234
235
       handles, labels = ax.get_legend_handles_labels()
236
       handles, labels = [h for i, (h,1) in enumerate(zip(handles, labels)) if l not in labels[:i]], [l
         for i,(h,1) in enumerate(zip(handles,labels)) if 1 not in labels[:i]]
       fig.legend(handles=handles,labels=labels,bbox_to_anchor=(0.7,0.5),loc='center
         left',ncol=2,prop = {"size": 6},)
```

```
fig.subplots_adjust(right=0.7,bottom=0.15)
fig.savefig('../../figures/%s_%s.pdf'%('_'.join(file.split('_')[:-1]),lim))
```

### Code 6: CPU Info script.

```
#!/bin/bash
    prog="/proc/cpuinfo"
    field="cpu MHz"
    pattern="s%.*: \([^ ]*\).*$%\1%"
    trials=5000
    file=tmp1234.tmp
    rm -f ${file}
10
11
    for i in $(seq 1 ${trials})
12
    do
13
            #echo Trial: $i
14
            grep "${field}" ${prog} | sed "${pattern}" >> ${file}
15
            sleep 0.0000001
16
    done
17
18
    N=$(wc -l ${file} | sed "s:\([^ ]*\).*:\1:")
19
    avg=$(paste -sd+ ${file} | bc)
    avg=$(echo "scale=3; $avg / $N" | bc)
21
    echo Avg\(N=${N},Trials=${trials}\) : ${avg}
23
24
    rm ${file}
25
```

# Code 7: membench job script.

```
#!/bin/bash
2
    #SBATCH --account=ners570f20_class
3
    #SBATCH -- job-name=NERS570_Lab8
    \#SBATCH --partition=standard
    #SBATCH --mail-user=mduschen@umich.edu
6
    #SBATCH --mail-type=END
    #SBATCH --nodes=1
    #SBATCH --mem-per-cpu=8000m
    #SBATCH
                    --time=01:00:00
10
    #SBATCH
                    --ntasks-per-node=3
11
12
13
    ./run.sh
14
```

#### Code 8: Matrix-Matrix multiply

```
#include <cstdio>
     #include <iostream>
2
    void dgemm( char transa, char transb,
6
                    unsigned int m, unsigned int n, unsigned int k,
                    double alpha, const double * const * a,
                    const double * const * b,
                    double beta, double **c)
10
    {
11
         for(unsigned int i=0; i<n; i++){</pre>
12
                  for(unsigned int j=0; j<m; j++){</pre>
13
                      c[i][j] *= beta;
14
                      }
15
             };
         for(unsigned int l=0; l<k; l++){</pre>
17
                     for(unsigned int j=0; j<m; j++){</pre>
                           for(unsigned int i=0; i<n; i++){</pre>
19
                            c[i][j] += alpha*a[i][l]*b[l][j];
                               }
21
                      }
             };
23
24
25
    }
26
```

#### Code 9: Matrix-Vector multiply

```
#include <immintrin.h>
    #include <stdio.h>
    #include <stdlib.h>
    #include <string.h>
    #include <math.h>
5
    #define ALIGN 16
    #define SIMD_BLOCK (ALIGN/sizeof(double))
    #define base 0
10
11
12
     // Coords (i_{0},...,i_{dim-1}) from linear index (Naive operations)
13
    void position(int z, const int dim, const int * shape,int * indices){
14
             int i,j,L;
15
             for (i=0;i<dim;i++){</pre>
16
                      L = 1;
                      for (j=i+1; j < dim; j++){}
18
                               L *= shape[j];
19
                      };
20
                      indices[i] = (z/L)%shape[i];
21
             };
22
23
             return;
24
    };
25
26
27
    // Linear order of indices (i_{0}, ..., i_{dim-1}),
28
    // with shape(N_{\{0\}},...,N_{\{dim-1\}})
29
    int lorder(const int dim,const int * indices,const int * shape){
30
31
             int z = 0;
32
             int w = 1;
33
             int i,j;
34
             for (i=0;i<dim;i++){</pre>
35
                      w = 1;
36
                      for (j=i+1; j < dim; j++){
37
                               w *= shape[j];
38
                      }
39
                      z += indices[i]*w;
40
             }
             return z;
42
    };
44
45
46
    // z order of indices (i_{0},...,i_{dim-1}) (Bitwise operations)
47
    int zorder(const int dim, const int * indices,const int *shape){
48
             const int zsize=8*sizeof(int);
49
             int i,j;
50
             int z = 0;
51
             int x;
52
             for (i=0;i<zsize;i++){</pre>
53
                      for(j=0;j<dim;j++){</pre>
```

```
x = (indices[dim-1-j] & (1 << i));
55
                                z = (x << (i+j));
56
                                };
5.7
                       };
58
              return z;
59
     };
60
61
     // z order of indices (i_{1}, 0), ..., i_{n} dim-1) for (2d) array (Bitwise operations)
62
     void z_order(const int N, const int dim, double * arr){
              int i,j,z;
64
              int indices[dim];
              int shape[dim];
66
              int size = 1;
              for (i=0;i<dim;i++){}
68
                       shape[i] = N;
69
                       size *= N;
70
              };
71
              double tmp[size];
72
              for(i=0;i<size;i++){
73
                       position(i,dim,shape,indices);
                       // printf("(%d, %d) \rightarrow %d n", indices[0], indices[1], i);
75
                       z = zorder(dim,indices,shape)+base;
76
                       tmp[i] = arr[z];
77
              };
              for(i=0;i<size;i++){</pre>
79
                       arr[i] = tmp[i];
              };
81
              return;
     };
83
84
     int size(const int dim,const int * shape){
85
              int size=1;
86
              for(unsigned int i=0;i<dim;i++){</pre>
87
                       size *= shape[i];
88
              };
              return size;
90
     };
91
92
     void arr_init(const int dim,const int * shape, double * arr,const char * type){
              int N=size(dim,shape);
94
95
              for(unsigned int i=0;i<N;i++){</pre>
96
                       if (strcmp(type, "RANDOM") == 0) {
                                arr[i] = rand();
98
                       }
                       else if (strcmp(type,"LINEAR")==0){
100
                                arr[i] = i;
101
                       }
102
                       else if (strcmp(type, "ONES") == 0){
103
                                arr[i] = 1;
105
                       else if (strcmp(type,"ZEROS")==0){
106
                                arr[i] = 0;
107
                       }
                       else {
109
                                arr[i] = 0;
110
                       };
111
```

```
};
112
              return;
113
     };
114
115
     void arr_pad(const int n,const int m,const int N, const int M, const double * A, double *
116
       A_,double padding){
              unsigned int i,j,z,w;
117
              for(i=0;i<N;i++){
118
                       for(j=0;j<M;j++){
                                if ((i < n) \&\& (j < m)){
120
                                         z = i*M+j;
121
                                         w=i*m+j;
122
                                         A_[z] = A[w];
123
                                }
124
                                else{
125
                                         z = i*M+j;
126
                                         A_[z] = padding;
127
                                };
128
                       };
129
              };
130
     };
131
132
133
     // Print out array of shape (N,M)
     void printa(const int N, const int M, double const * arr, char const * label){
135
              const int dim = 2;
136
              int indices[dim];
137
              int shape[] = {N,M};
138
              int i,j;
139
              int a,z;
140
              // int label_size = strlen(label);
141
              // printf("label size = %d", label_size);
142
              // if(label_size>1){
143
              //
                          char spacing[label_size-1];
144
              //
                          for (unsigned int i=0;i<label_size-1;i++){
              //
                                   spacing[i]=" ";
146
              //
                          };
147
              // }
148
              // else{
              //
                          char spacing[]="";
150
              // };
151
              char spacing[]="";
152
              printf("%s = [",label);
153
              for (i = 0; i < N; i++) {
154
155
                         for (j = 0; j < M; j++){
                                  indices[0] = i;
156
                                  indices[1] = j;
157
                                  z = lorder(dim,indices,shape);
158
                                  a = arr[z];
159
                                  if (j==0 \&\& i>0){
                                           printf("%s",spacing);
161
                                           printf("
                                                           ");
162
                                  }
163
                                  else if (j==0 \&\& i == 0){
                                           printf(" ");
165
                                  };
166
                           printf("%d",a);
167
```

```
if (j== (M-1) \&\& i==(N-1)){
168
                                                                                                                                                      printf(" ]\n");
 169
170
                                                                                                                        else if (j == (M-1)) {
171
                                                                                                                                               printf("\n");
172
                                                                                                                       }
173
                                                                                                else if (a < 10){
174
                                                                                                                               printf("
                                                                                                                                                                               ");
175
                                                                                                }
                                                                                                else if (a < 100){
177
                                                                                                                                                                          ");
                                                                                                                               printf("
 178
                                                                                                }
179
                                                                                                else if (a < 1000){
                                                                                                                               printf(" ");
181
                                                                                                }
182
                                                                                                else{
183
                                                                                                                               printf(" ");
184
                                                                                               };
185
                                                                                        };
186
                                                 };
 187
                                                 printf("\n");
188
                                                 return;
 189
                   };
190
 191
                   void matvec(const int n,const int m,const double * A,const double *x, double *y){
192
                                                 for(unsigned int i=0;i<n;i++){</pre>
193
                                                                                for(unsigned int j=0;j<m;j++){</pre>
194
                                                                                                                 // std::cout << i << " " << j << " :: " << y[i] << " = " << A[i*m+j] << " | |
195
                                                                                                                        " * " << x[j] << std::endl;
                                                                                                                y[i] += A[i*m +j]*x[j];
196
                                                                                }
197
                                                 };
198
                   };
199
200
                   void _matvec_avx(const double* A,const double* x,double* y){
202
                                  asm volatile ("# avx code begin"); // looking at assembly with gcc -S
203
                                  _{m256d xrow = mm256_loadu_pd(x);}
204
                                  _{m256d} = _{mm256_{mul_pd}(_{mm256_{loadu_pd}(A), xrow)};
206
                                  _{m256d} b = _{mm256} (_{mm256} loadu_{pd}(A+4), xrow);
207
                                  _{m256d} c = _{mm256} (_{mm256} (_
208
                                  _{m256d} d = _{mm256} d = _{m
209
210
211
                                  // our task now is to get {sum(a), sum(b), sum(c), sum(d)}
                                  // This is tricky because there is no hadd instruction for avx
212
213
                                  // {a[0]+a[1], b[0]+b[1], a[2]+a[3], b[2]+b[3]}
214
                                  _{m256d sumab} = _{mm256\_hadd\_pd(a, b)};
215
216
                                  // {c[0]+c[1], d[0]+d[1], c[2]+c[3], d[2]+d[3]}
217
                                  _{m256d} sumcd = _{mm256}_hadd_pd(c, d);
218
219
                                  // \{a[0]+a[1], b[0]+b[1], c[2]+c[3], d[2]+d[3]\}
                                  _{\rm m256d} blend = _{\rm mm256\_blend\_pd(sumab, sumcd, 0b1100)};
221
222
                                  // \{a[2]+a[3], b[2]+b[3], c[0]+c[1], d[0]+d[1]\}
223
```

```
__m256d perm = _mm256_permute2f128_pd(sumab, sumcd, 0x21);
224
225
         // {sum(a), sum(b), sum(c), sum(d)}
226
         _{\rm m256d} Ax = _{\rm mm256\_add\_pd(perm, blend)};
227
         // printf("yinit = %f",*y);
228
         _mm_storel_pd(y, _mm256_castpd256_pd128(Ax));
229
         // printf("yfinal = %f",*y);
230
         // _{mm\_storel\_pd(y, Ax)};
231
         asm volatile ("# avx code end");
233
     };
234
235
236
237
     void matvec_avx(const int n,const int m, const double* A,const double* x,double* y){
238
              // Break up multiplication into 4x4 blocks for avx instructions
239
              //Pad A,x,y with zeros to make multiple of 4 for SIMD
240
              int N = n, M=m;
241
              if (n%SIMD_BLOCK != 0){
242
                       N = SIMD_BLOCK*(n/SIMD_BLOCK+1);
243
              };
244
              if (m%SIMD_BLOCK != 0){
245
                       M = SIMD_BLOCK*(m/SIMD_BLOCK+1);
246
              };
              if (N < M){
248
                       N = M;
249
              }
250
              else if(N>M){
                       M = N:
252
              };
253
              const int L = N*M;
254
              const int S = N/SIMD_BLOCK;
255
              double A_[L],x_[M],y_[N];
256
              double y__;
257
              arr_pad(n,m,N,M,A,A_,0.0);
259
              arr_pad(m,1,M,1,x,x_,0.0);
260
              arr_pad(n, 1, N, 1, y, y_0, 0.0);
261
              // Use z-index ordering for blocks of 4
263
              // printa(N, M, A_, "A");
264
              // z_order(N,2,A_);
265
              // printa(N, M, A, "A");
266
267
              printf("Padded Arrays\n");
268
              printa(N,M,A_,"A");
269
              printa(M,1,x_,"x");
270
              printa(N,1,y_,"y");
271
272
              _matvec_avx(A_,x_,y_);
274
275
276
              // unsigned int i,j,z;
              // for(z=0;z<L;z+=SIMD_BLOCK){
278
              //
                          i=L/M;
              //
                          j=L\%M;
280
```

```
//
                           printf("z=%d, i=%d, j=%d \setminus n", z, i, j);
281
              //
                           _{matvec\_avx(A\_+z,x\_+j,\&y\_\_);}
              //
                           y[i] += y_{-};
283
              // };
284
              return;
285
     };
286
287
288
290
     int main(int argc, char** argv){
291
              //Shape of matrix
292
              const int dim = 2;
              int shape[dim];
294
              for (int i=dim-1;i>-1;i--){
295
                       shape[i]=0;
296
                        if (argc > (i+1)){
297
                                 for(unsigned int j=0;j<dim;j++){</pre>
298
                                          if (j<=i){
299
                                                   shape[j] = atoi(argv[j+1]);
300
                                          }
301
                                          else{
302
                                                   shape[j] = atoi(argv[i+1]);
303
                                          };
                                 };
305
                                 break;
306
                       };
307
              };
309
              const int N=size(dim,shape);
310
311
              double A[N];
312
              arr_init(dim,shape,A,"LINEAR");
313
314
316
              int dim_x=1,dim_y=1;
317
              int shape_x[1]={shape[1]};
318
              int shape_y[1]={shape[0]};
320
              double x[shape[1]],y[shape[0]];
321
              arr_init(dim_x,shape_x,x,"ONES");
322
              arr_init(dim_y,shape_y,y,"ZEROS");
323
324
325
              printf("Init Arrays\n");
326
              printa(shape[0],shape[1],A,"A");
327
              printa(shape[1],1,x,"x");
328
              printa(shape[0],1,y,"y");
329
              // for(unsigned\ int\ i=0;i< m;i++){}
331
              //
                           x[i] = i;
332
              // };
333
              // for(unsigned\ int\ i=0;i< n;i++){
                           y[i] = 0.0;
335
              // };
336
              printf("Naive Mat-Vec Arrays\n");
337
```

```
matvec(shape[0],shape[1],A,x,y);
338
339
             printf("Naive Result\n");
340
             printa(shape[0],1,y,"y");
341
342
343
             printf("ASM Mat-Vec Arrays\n");
344
              arr_init(dim_y,shape_y,y,"ZEROS");
345
             matvec_avx(shape[0],shape[1],A,x,y);
347
             printf("ASM Result\n");
             printa(shape[0],1,y,"y");
349
              // for(unsigned\ int\ i=0;i< n;i++){
351
                         std::cout << i << " " << y[i] << std::endl;
352
              // };
353
354
355
356
             return 0;
     };
357
```