

# Lecture 06 – Algorithms for Linear Algebra

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NERS/ENGR 570 - Methods and Practice of Scientific Computing (F20)



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# Outline

- Resume class exercise with Git
- Linear Algebra/Linear System Fundamentals
- Solution of Linear Systems

# Learning Objectives: By the end of Today's Lecture you should be able to

- *(Skill)* Perform Conflict Resolution (but just in git)
- *(Knowledge)* Interpretate meaning of some vector norms
- *(Value/Knowledge)* explain how to think about programming equations in linear algebra
- *(Knowledge)* know when to use direct vs. iterative solution algorithms
- *(Knowledge)* implement LU factorization

# Overview

- What types of problems are solved with Linear Algebra?
  - Linear systems of equations
  - Eigenvalue problems
  - matrix factorization
  - Overdetermined system of equations
    - (Data fitting)

	Embed	SPEC	DB	Games	ML	HPC
1 Finite State Mach.	Red	Red	Red	Yellow	Yellow	Light Blue
2 Combinational	Red	Light Blue	Green	Light Blue	Green	Light Blue
3 Graph Traversal	Red	Yellow	Yellow	Yellow	Red	Light Blue
4 Structured Grid	Red	Red	Light Blue	Yellow	Light Blue	Red
5 Dense Matrix	Red	Red	Yellow	Red	Red	Red
6 Sparse Matrix	Yellow	Yellow	Light Blue	Red	Red	Red
7 Spectral (FFT)	Yellow	Light Blue	Light Blue	Yellow	Yellow	Red
8 Dynamic Prog	Yellow	Light Blue	Red	Light Blue	Red	Light Blue
9 N-Body	Light Blue	Yellow	Light Blue	Yellow	Light Blue	Red
10 MapReduce	Light Blue	Green	Red	Light Blue	Red	Red
11 Backtrack/ B&B	Light Blue	Light Blue	Yellow	Light Blue	Red	Light Blue
12 Graphical Models	Light Blue	Light Blue	Yellow	Light Blue	Red	Light Blue
13 Unstructured Grid	Light Blue	Light Blue	Light Blue	Yellow	Yellow	Red



# Linear System Fundamentals



# Basics of Linear Systems

# The Residual and Norms





# Norms of Vectors





# Going from Chalkboard to Terminal



# Types of Linear Algebra Operations



# Solving Linear Systems



# How do we solve linear systems?



# Overview of Solution Methods



# Direct Solution Methods

aka Matrix Factorizations



# Types of Matrices



# LU Factorization





# LU: Forward Elimination



# LU: Backward Substitution



# Classical Iterative Methods

aka Fixed Point Iteration Schemes



# Classical Iteration Schemes

**Jacobi**

**Gauss-Siedel**

# Do they converge?

- Fixed point iteration

$$\mathbf{x}^{(\ell+1)} = \mathbf{F}\mathbf{x}^{(\ell)} + \mathbf{c}$$

- Express iterate as combination of exact solution and error

$$\mathbf{x} + \boldsymbol{\varepsilon}^{(\ell+1)} = \mathbf{F}(\mathbf{x} + \boldsymbol{\varepsilon}^{(\ell)}) + \mathbf{c}$$

- If the method converges then:

$$\lim_{\ell \rightarrow \infty} \boldsymbol{\varepsilon}^{(\ell)} = 0$$