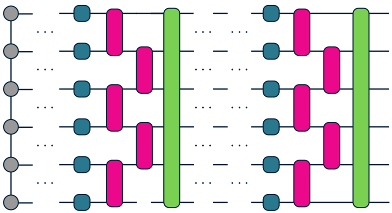
How Can We Use Quantum Systems To Learn?

Matthew Duschenes
University of Waterloo, Institute for Quantum Computing, Perimeter Institute & Vector Institute



June 27, 2024

PSI Start Seminar

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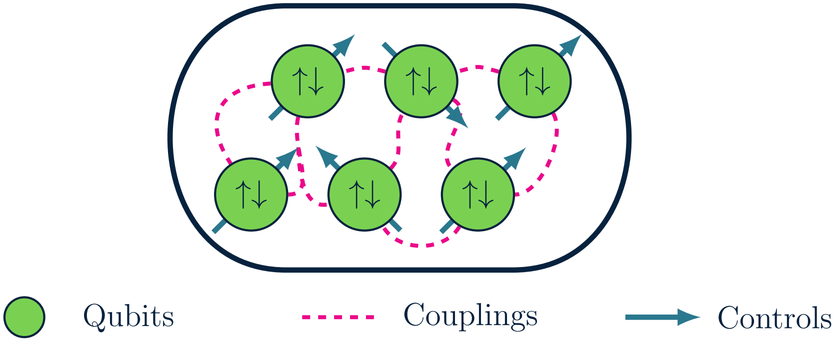
Quantum Systems



Quantum Tasks Of Interest

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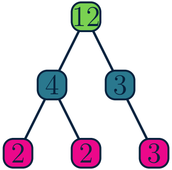

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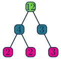
  - *Quantum algorithms* i.e) Factoring numbers 

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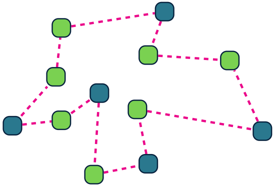

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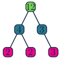
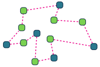
    - *Quantum algorithms* i.e) Factoring numbers 

    - *Optimization* problems i.e) Travelling Salesman Problem 
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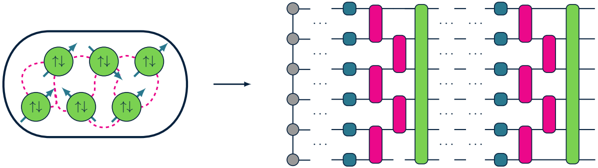

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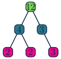
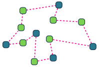
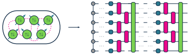
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
  - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
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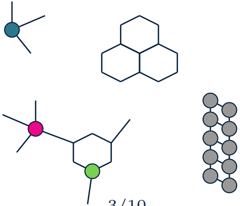

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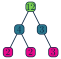
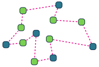
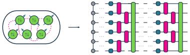
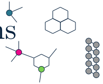
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
  - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
  - *Simulate* quantum systems i.e) Complicated molecules and chemical reactions 
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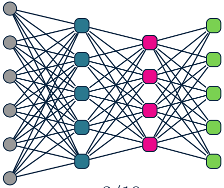

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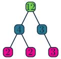
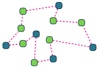
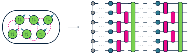
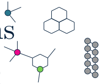
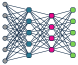
    - *Quantum algorithms* i.e) Factoring numbers 

    - *Optimization* problems i.e) Travelling Salesman Problem 
    - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
    - *Simulate* quantum systems i.e) Complicated molecules and chemical reactions 
    - *Machine learning* functions i.e) Classification, Regression, Generative 
- What makes quantum systems *potentially better* than classical systems?

What Is Quantum Advantage?

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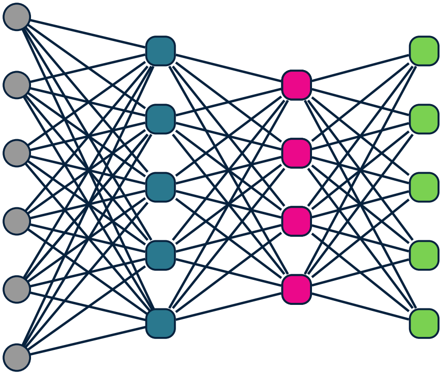
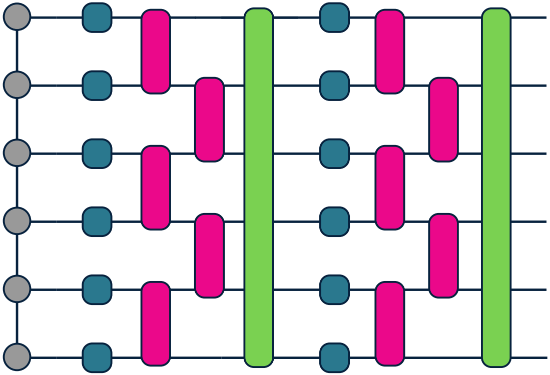
- It remains up for *debate* on the quantum-classical *complexity hierarchy*

Classical versus Quantum Algorithms



Classical Neural Network

Classical versus Quantum Algorithms



Quantum Unitary Circuit

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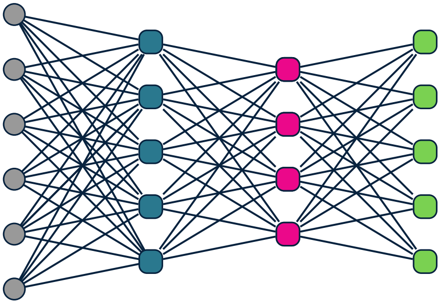
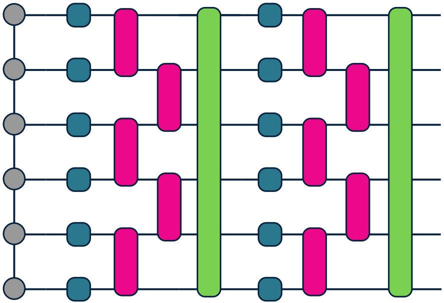


Classical Neural Network
$f_\theta(x)$



Quantum Unitary Circuit
$U_\theta |\psi(x)\rangle$

Classical versus Quantum Algorithms

Quantum
Encode State
$x \to |\psi(x)\rangle$

$\longrightarrow$

Quantum
Envolve State
$|\psi_\theta(x)\rangle = U_\theta |\psi(x)\rangle$

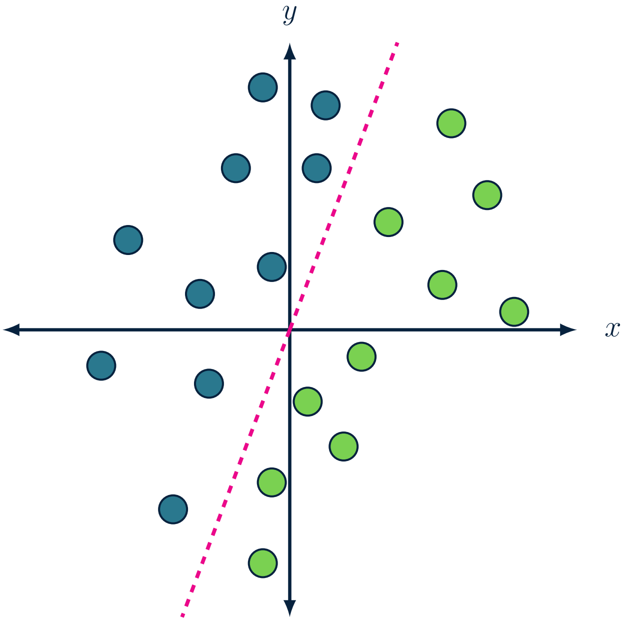
$\longrightarrow$

Quantum
Statistics
$\mathcal{L}_\theta \sim \langle O \rangle_{\psi_\theta(x)}$

$\longrightarrow$

Classical
Optimization
$\theta \to \theta - \eta \partial_\theta \mathcal{L}_\theta$

Variational Quantum Algorithms



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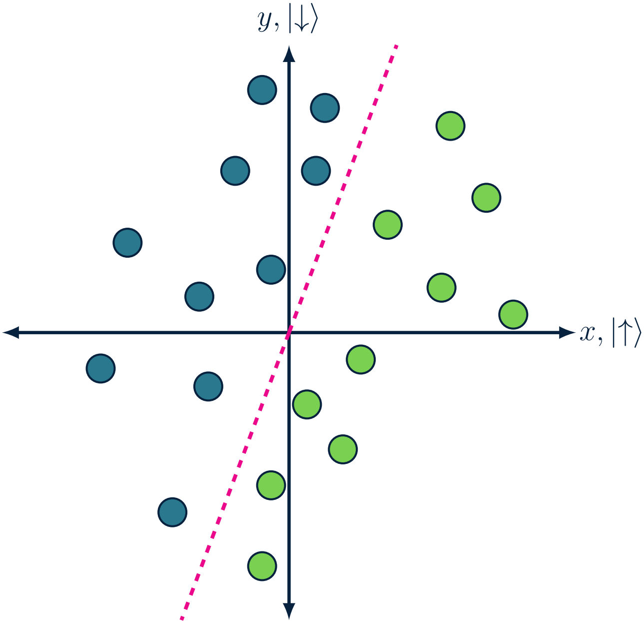

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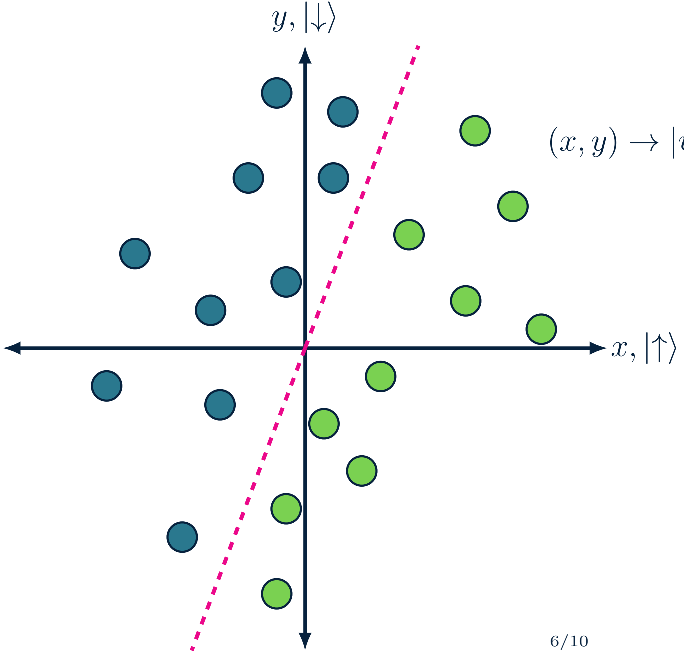

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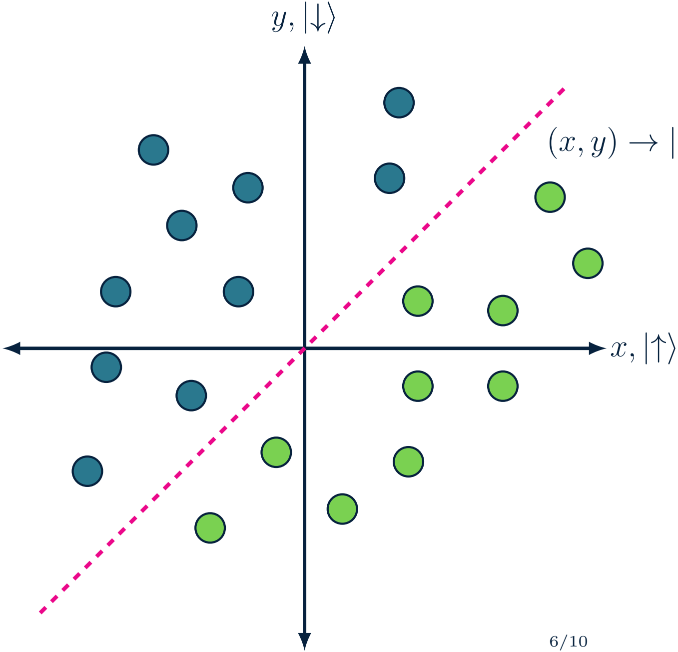

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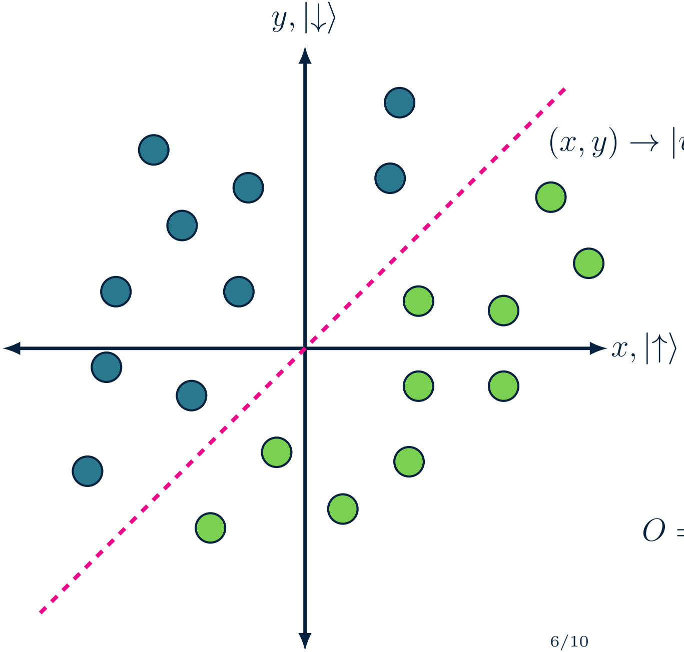

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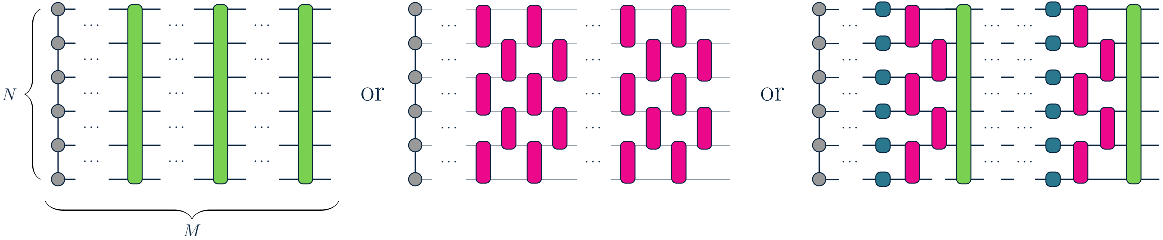
Quantum Architecture Selection

- *Basis*: How do we choose our set of parameterized operators $\{U_\theta^{(i)}\}_i^M$?



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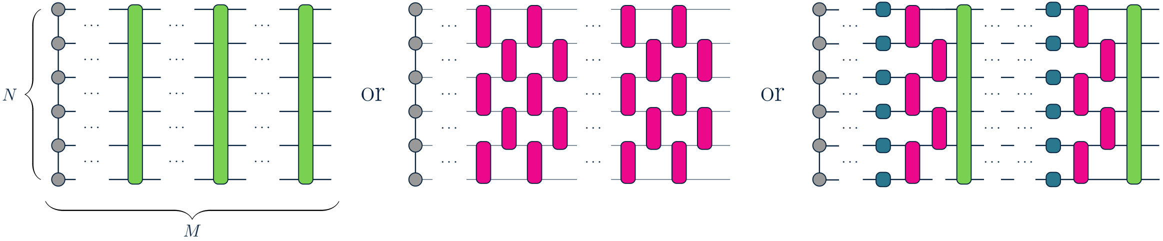

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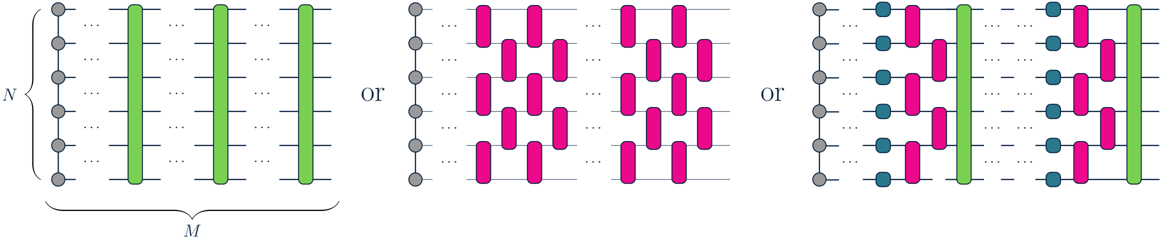
- *Structure*: How do we incorporate *patterns* in the data and objectives?



Classical Simulation and Operator Compilation

- Translate or *compile* operators into a form that suits native device operators

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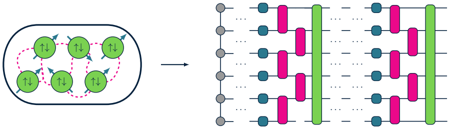

(3)

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- Develop *quantum-inspired* classical models

What About Noise?

What if we are are unable to *experimentally implement* purely unitary operators, but *noisy* operators?

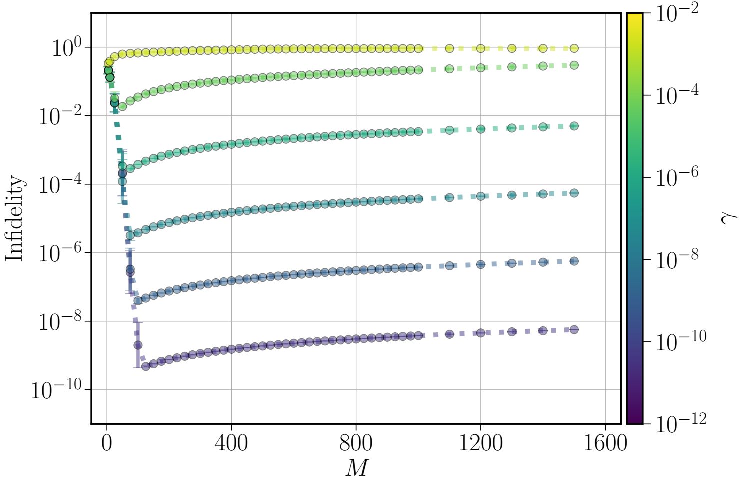

Figure 1: Entropy-Increasing Noise: Unital Dephasing

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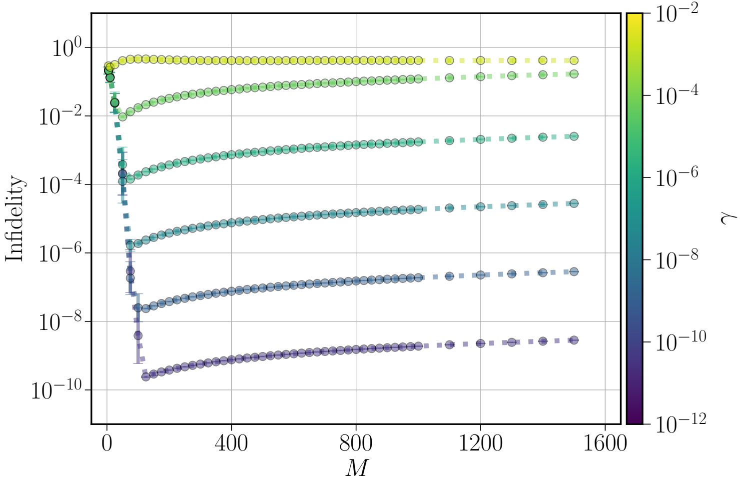

Figure 2: Entropy-Decreasing Noise: Non-Unital Amplitude Damping

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- Useful Reviews:

1. Cerezo, M. *et al.* , Variational quantum algorithms. Nature Reviews Physics, 3(9), 625–644. (2021).
2. Schuld, M. *et al.* , Is Quantum Advantage the Right Goal for Quantum Machine Learning? PRX Quantum, 3(3), 030101. (2022).
3. Bharti, K. *et al.* , Noisy intermediate-scale quantum algorithms. Reviews of Modern Physics, 94(1), 015004. (2022).
4. Duschenes, M. *et al.* , Characterization of overparametrization in the simulation of realistic quantum systems. Phys. Rev. A, 109, 062607. (2024).