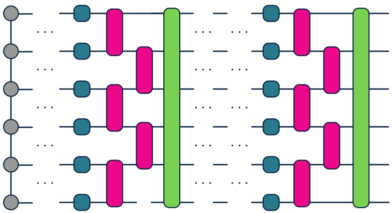
How Can We Use Quantum Systems To Learn?

Matthew Duschenes
University of Waterloo, Institute for Quantum Computing, & Vector Institute

November 2, 2023



Quetzal Quantum Computing Career Accelerator

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PERIMETER INSTITUTE

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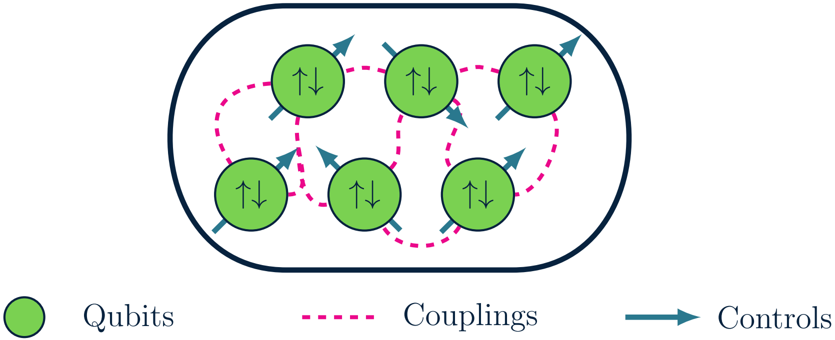
Outline

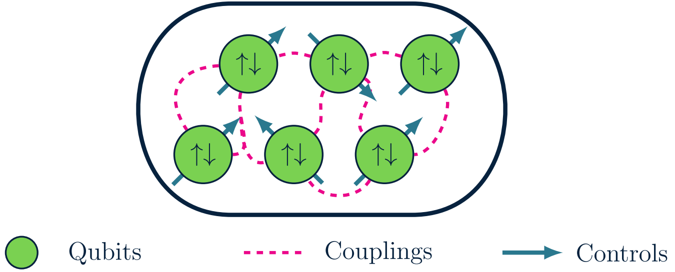
What Are Quantum Computers (Useful For)?

Quantum Systems



Quantum Systems

- Quantum systems comprise of particles with properties that *encode* information: *qubits* $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle \quad : \quad |\alpha|^2 + |\beta|^2 = 1$



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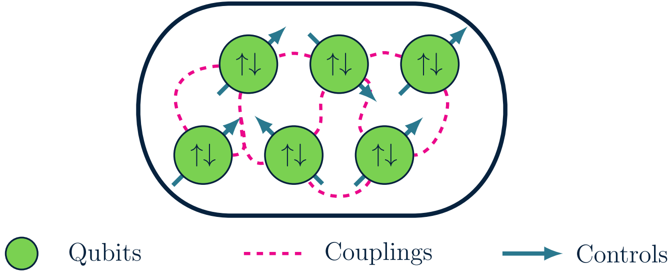

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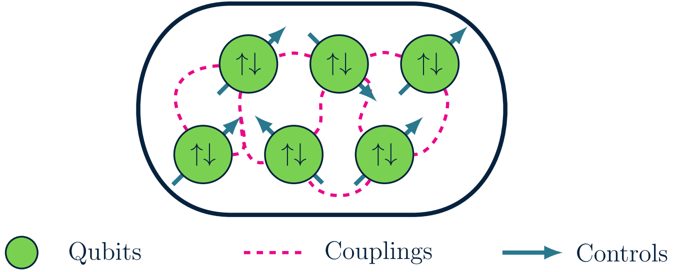

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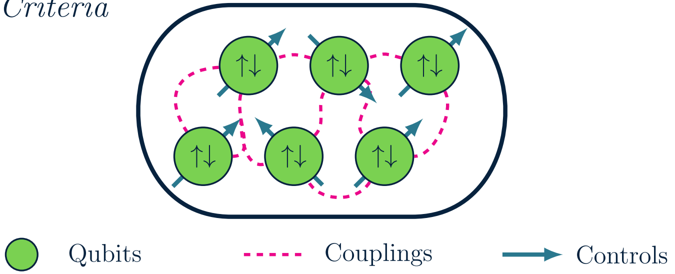
i.e) Nuclear Magnetic Resonance, Trapped Ions, Neutral Atoms:

DiVincenzo Criteria



Quantum Tasks Of Interest

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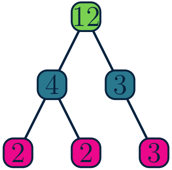

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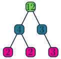
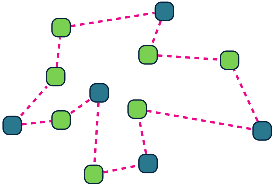
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem



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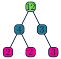
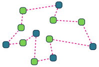
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
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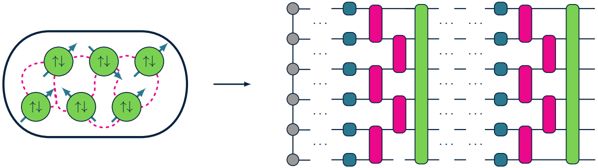

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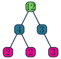
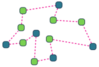
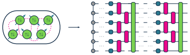
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
  - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
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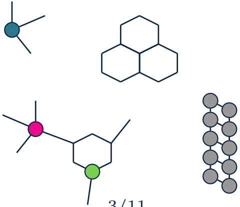

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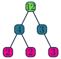
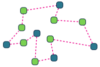
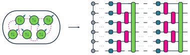
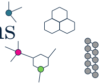
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
  - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
  - *Simulate* quantum systems i.e) Complicated molecules and chemical reactions 
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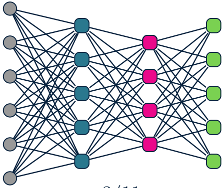

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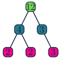
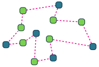
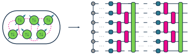
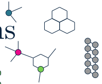
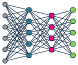
  - *Quantum algorithms* i.e) Factoring numbers 

  - *Optimization* problems i.e) Travelling Salesman Problem 
  - *Compilation* tasks i.e) Form operators U given native gates $\{V\}$ 
  - *Simulate* quantum systems i.e) Complicated molecules and chemical reactions 
  - *Machine learning* functions i.e) Classification, Regression, Generative 

- What makes quantum systems *potentially better* than classical systems?

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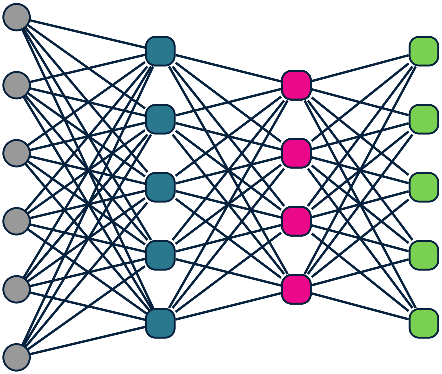
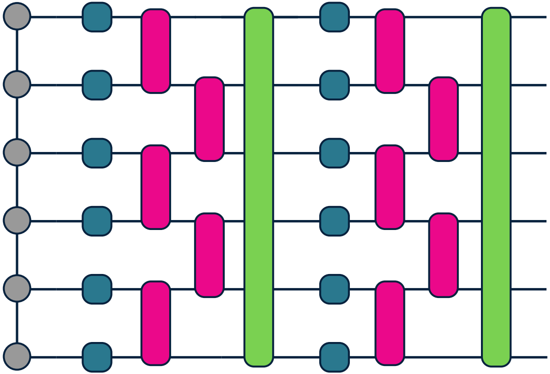
- Back and forth between *state-of-the-art* classical and quantum methods
- It remains up for *debate* on the quantum-classical *complexity hierarchy*

Classical versus Quantum Algorithms



Classical Neural Network

Classical versus Quantum Algorithms



Quantum Unitary Circuit

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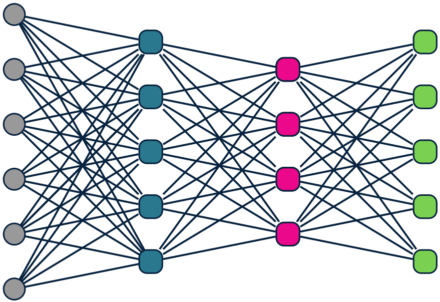
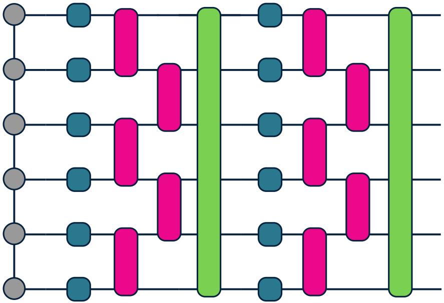


Classical Neural Network

$f_\theta(x)$



Quantum Unitary Circuit

$U_\theta \left|\psi(x)\right\rangle$

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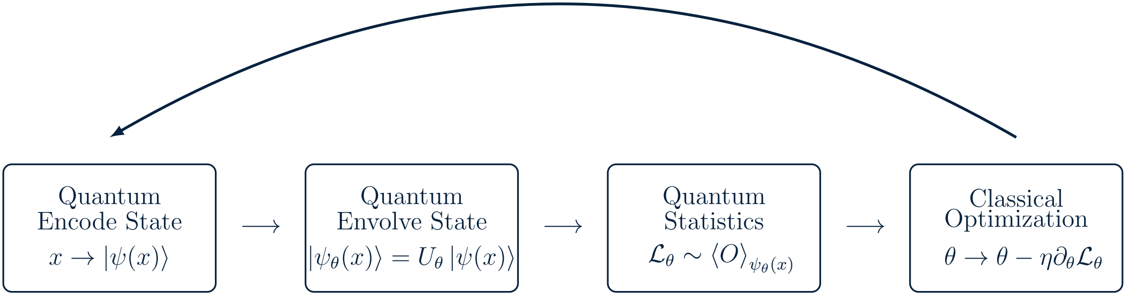

Hybrid Classical-Quantum Approaches

Variational Quantum Algorithms

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     i.e) Phases: $U_\theta(\alpha |\uparrow\rangle + \beta |\downarrow\rangle) = \alpha |\uparrow\rangle + e^{i\theta}\beta |\downarrow\rangle$
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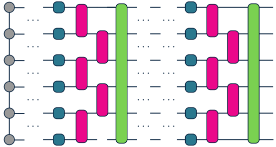

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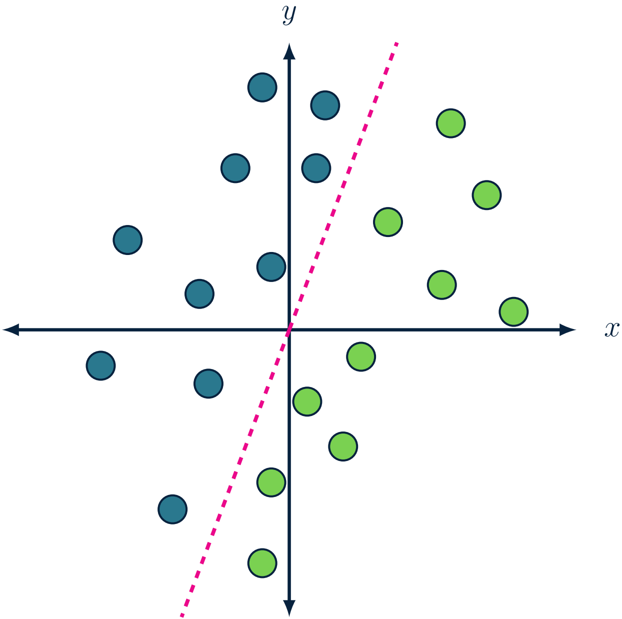
- Series of *local* operators form *circuits*

Quantum Machine Learning



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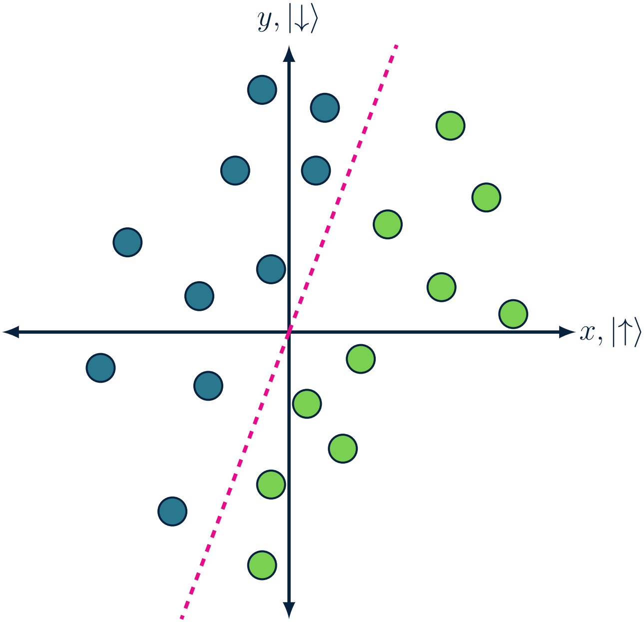

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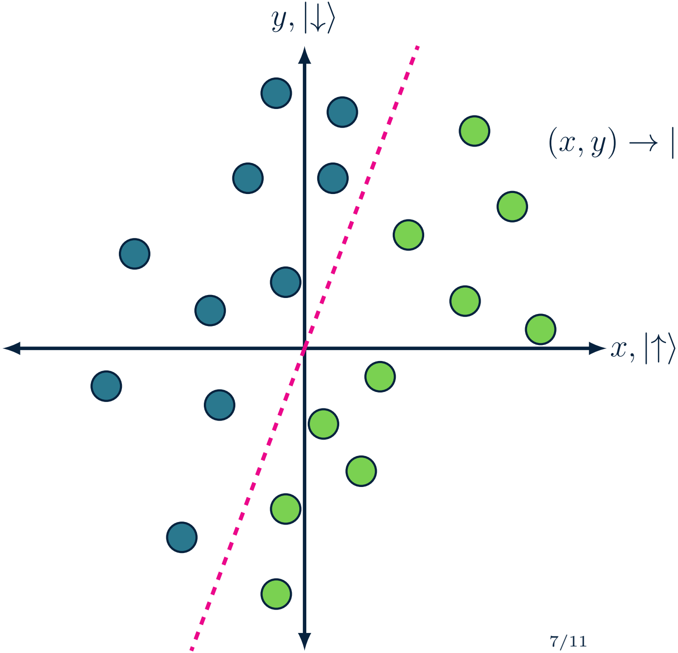
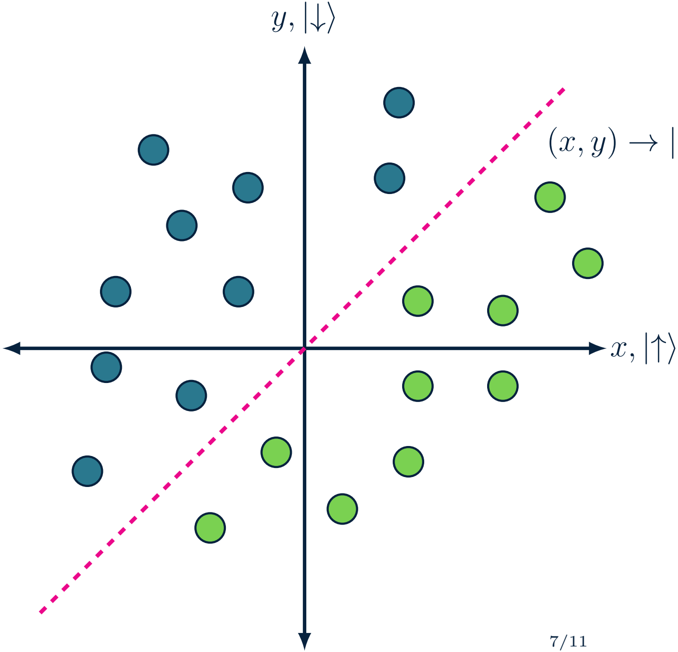


$$(x, y) \to |\psi(x, y)\rangle = \frac{x}{\sqrt{x^2+y^2}} |\uparrow\rangle + \frac{y}{\sqrt{x^2+y^2}} |\downarrow\rangle$$

Quantum Machine Learning



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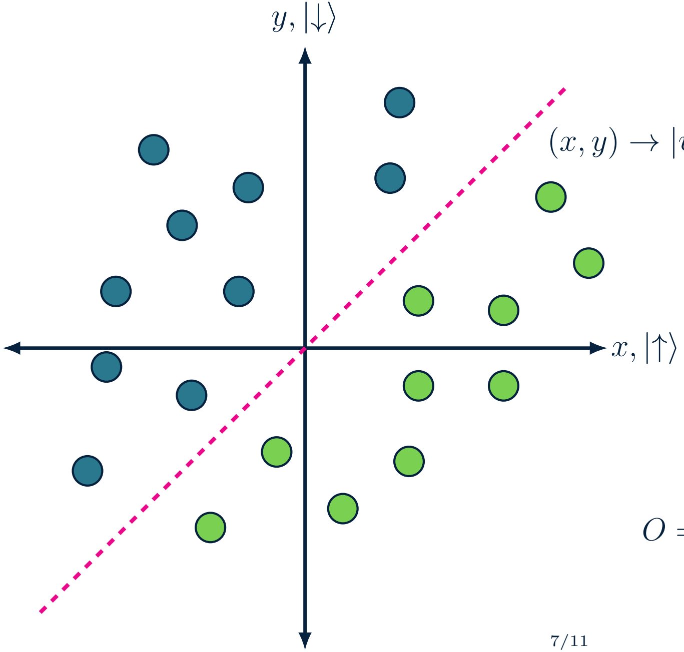

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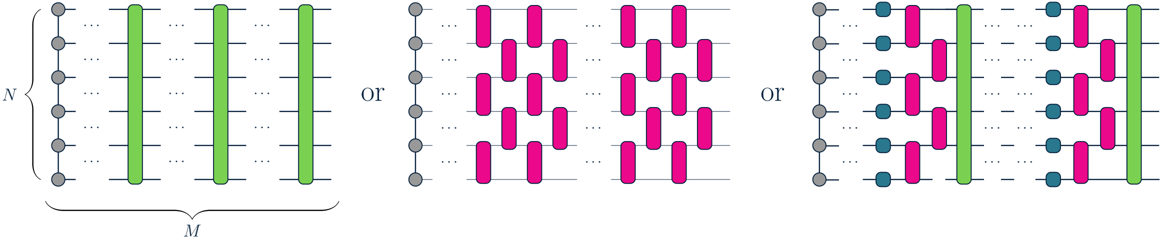
$$O = |\uparrow\rangle \langle\uparrow| - |\downarrow\rangle \langle\downarrow|$$

Quantum Architecture Selection

- *Basis*: How do we choose our set of parameterized operators $\{U_\theta^{(i)}\}_i^M$?



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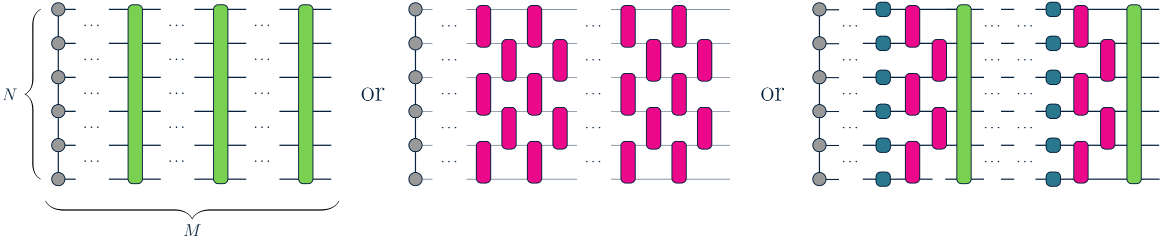

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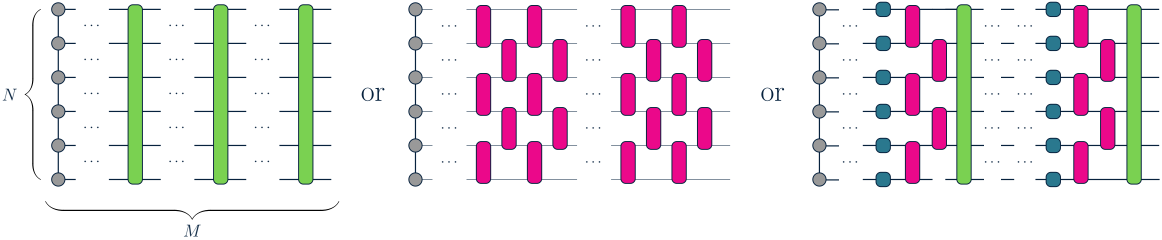
- *Basis*: How do we choose our set of parameterized operators $\{U_\theta^{(i)}\}_i^M$?
- *Depth*: Many models are *periodic layers*, repeated M times
- *Structure*: How do we incorporate *patterns* in the data and objectives?



Abilities of Realistic Quantum Systems

Classical Simulation and Operator Compilation

- Translate or *compile* operators into a form that suits native device operators

$$U_\theta \approx U \;\; : \;\; \mathcal{L}_\theta^U \sim \text{Fidelity}(U, U_\theta) \tag{3}$$

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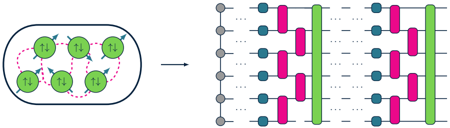

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- Discretize continuously evolving systems into a discrete product of operators

$$U_\theta(t) = e^{-i \int_0^t d\tau \;\; H_\theta(\tau)} \rightarrow U_\theta = \prod_i^M e^{-iH_\theta^{(i)}} \tag{4}$$

Classical Simulation and Operator Compilation

- Translate or *compile* operators into a form that suits native device operators

$$U_\theta \approx U \;:\; \mathcal{L}_\theta^U \sim \text{Fidelity}(U, U_\theta) \tag{3}$$

- Discretize continuously evolving systems into a discrete product of operators

$$U_\theta(t) = e^{-i\int_0^t d\tau \;\; H_\theta(\tau)} \to U_\theta = \prod_i^M e^{-iH_\theta^{(i)}} \tag{4}$$

- The span of $\{U_\theta^{(i)}\}_i^M$ determines how *expressive* our model is and how many $O(2^{2N})$ possible operators or states it can *represent*

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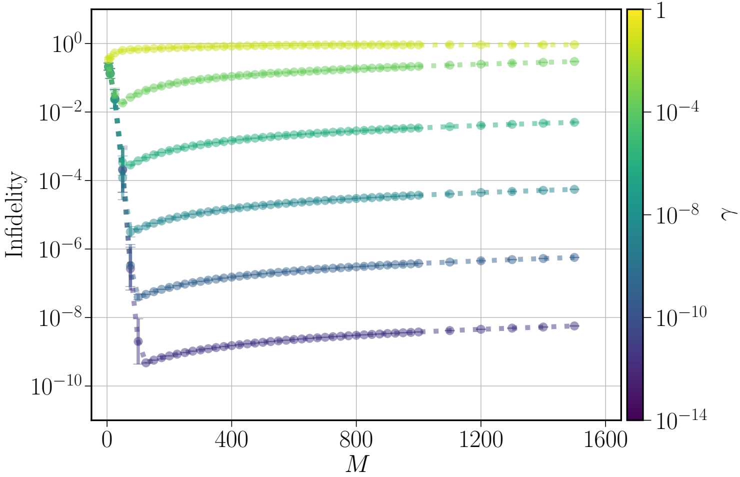
- Develop *quantum-inspired* classical models

What About Noise?

What if we are are unable to *experimentally implement* purely unitary operators, but *noisy* operators?



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- Useful Reviews:

1. Schuld, M. *et al.* , An introduction to quantum machine learning. Contemporary Physics, 56(2), 172–185. (2015).
2. Cerezo, M. *et al.* , Variational quantum algorithms. Nature Reviews Physics, 3(9), 625–644. (2021).
3. Schuld, M. *et al.* , Is Quantum Advantage the Right Goal for Quantum Machine Learning? PRX Quantum, 3(3), 030101. (2022).
4. Bharti, K. *et al.* , Noisy intermediate-scale quantum algorithms. Reviews of Modern Physics, 94(1), 015004. (2022).