Overparameterization of Realistic Quantum Systems

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1. Problem

- Learn time-dependent *experimental parameters* for *precise* control of quantum systems Λ i.e) operator compilation $\Lambda \approx U$, or state preparation $\rho_\Lambda \approx \rho_U$ with trace overlap fidelities
- How do *realistic* effects (constrained parameters θ, noise γ) affect the optimal system?
- Are learning phenomena such as *overparameterization* [1] still observed in realistic settings?

2. Variables

N	Number of particles $\sim O(1)$
M	Number of time steps $\sim O(10^1 - 10^4)$
δ	Trotterization time step $\sim O(\mu\mathrm{s})$
λ	Evolution time $\leq M$, $M\delta \sim O(\mathrm{ms})$
J	Constant longitudinal coupling $\sim O(\mathrm{Hz})$
h	Constant longitudinal field $\sim O(\mathrm{kHz})$
θ	Variable transverse field $\sim O(\mathrm{MHz})$
γ	Noise probability $\sim O(10^{-12} - 10^{-1})$

3. Hamiltonian Ansatz

- Systems are represented as *channels* $\Lambda_{\theta\gamma} = \mathcal{N}_\gamma \circ \mathcal{U}_\theta$ with unitary evolution $\mathcal{U}_\theta$, and noise $\mathcal{N}_\gamma$
- Evolution generated by Hamiltonians with localized generators $\{G_\mu\}$ (Coloured in circuit $\longrightarrow$)

$$H_\theta^{(\lambda)} = \sum_\mu \theta_\mu^{(\lambda)} G_\mu \quad \to \quad U_\theta \approx \prod_\lambda^M U_\theta^{(\lambda)} \quad : \quad U_\theta^{(\lambda)} = e^{-i\delta H_\theta^{(\lambda)}} \approx \prod_\mu e^{-i\delta\theta_\mu^{(\lambda)} G_\mu} \tag{1}$$

i.e) *NMR* Hamiltonian with variable transverse fields and constant longitudinal fields [2]

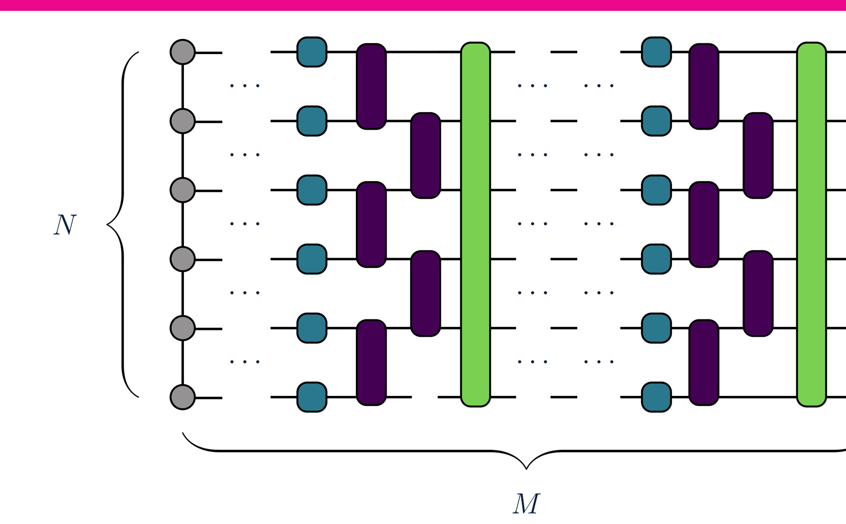
$$H_\theta^{(\lambda)} = \sum_i \theta_i^{x(\lambda)} X_i \; + \; \sum_i \theta_i^{y(\lambda)} Y_i \; + \; \sum_i h_i Z_i \; + \; \sum_{i<j} J_{ij} Z_i Z_j \tag{2}$$

- Noise generated by constant *Kraus* operators $\{\mathcal{K}_{\gamma_\alpha}\}$ i.e) Dephasing $\{\sqrt{1-\gamma}\, I, \; \sqrt{\gamma}\, Z\}$

$$\rho \; \to \; \rho_{\Lambda_{\theta\gamma}} = \prod_\lambda^M \mathcal{N}_\gamma \circ \mathcal{U}_\theta^{(\lambda)}(\rho) = \prod_\lambda^M \left[\sum_\alpha \mathcal{K}_{\gamma_\alpha} \; U_\theta^{(\lambda)} \; \rho \; U_\theta^{(\lambda)^\dagger} \; \mathcal{K}_{\gamma_\alpha}^\dagger \right] \tag{3}$$

- Native generators $\{G_\mu\}$ form a *dynamical Lie algebra* $\mathcal{G}$, with dimensionality $G = |\mathcal{G}|$, that determines the *expressibility* of an ansatz, depending if the circuit *depth* $M \lesseqgtr O(G)$ [3]

4. Variational Circuit

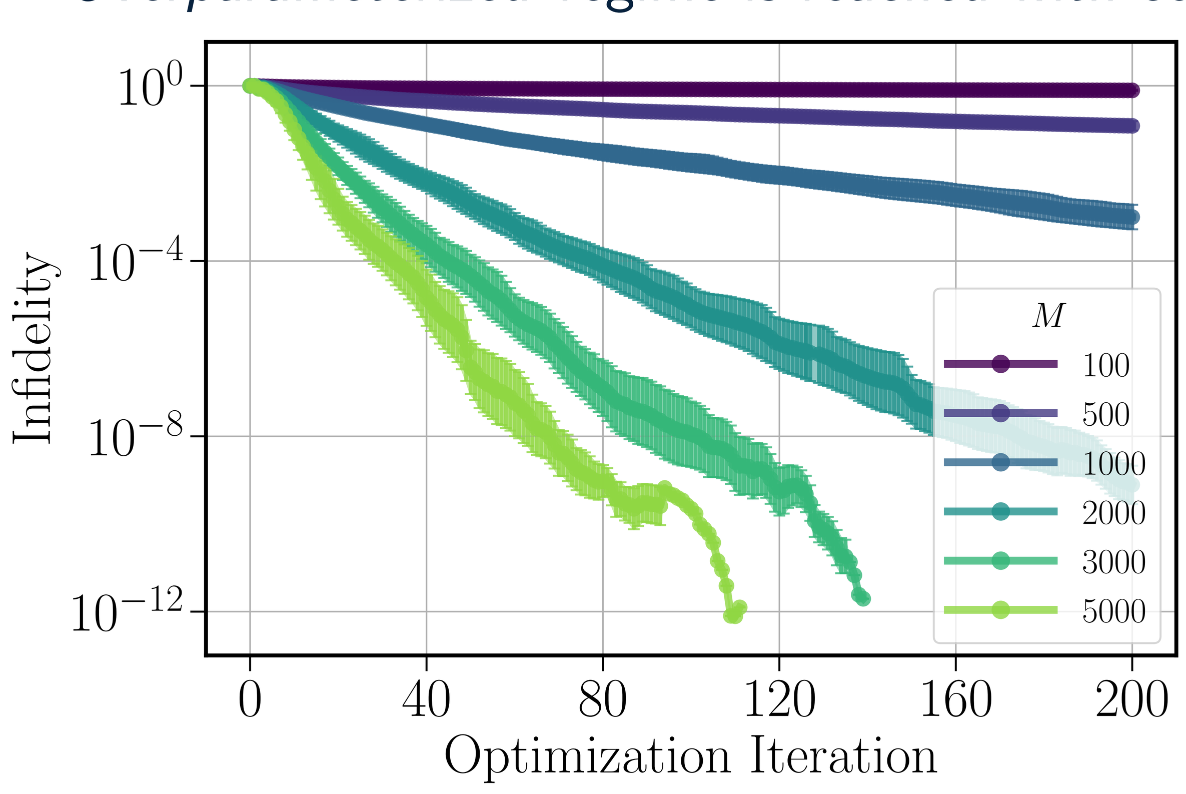


5. Methods

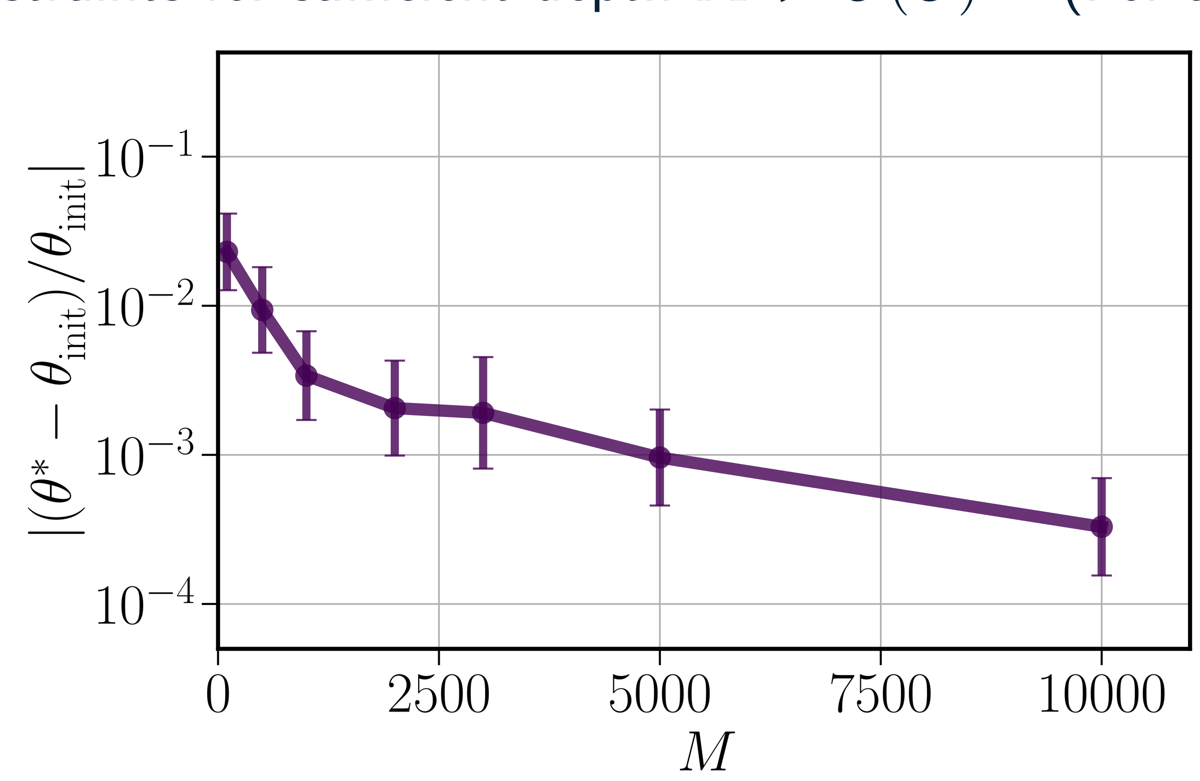
- Fast JIT compilation and gradient descent
- Analogous gradients of noisy/less channels

6. Constrained Optimization

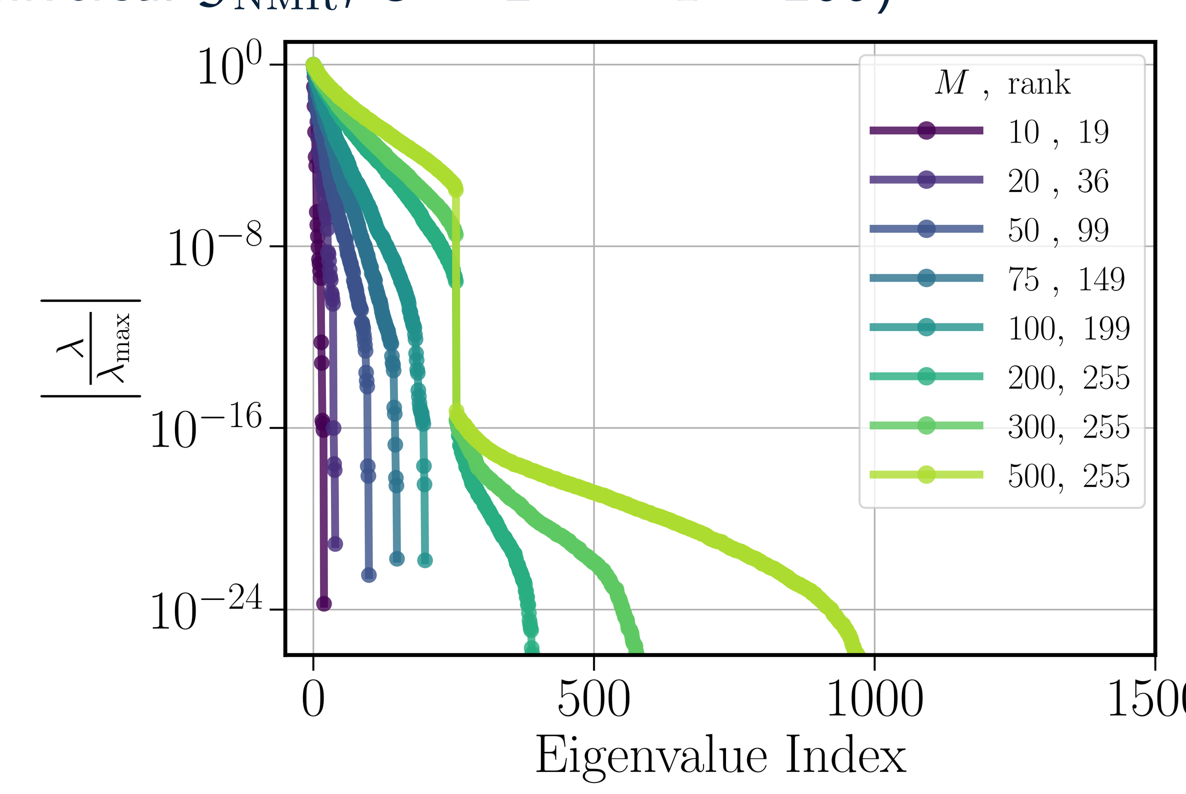
- Haar random unitary compilation for $N = 4$ qubits, with *bounded* fields *shared* across all qubits, and Dirichlet *boundary conditions*
- *Overparameterized* regime is reached with constraints for sufficient depth $M > O(G)$ (For universal $\mathcal{G}_{\mathrm{NMR}}$, $G = 2^{2N} - 1 = 255$)



(a) Exponential Convergence of Infidelity



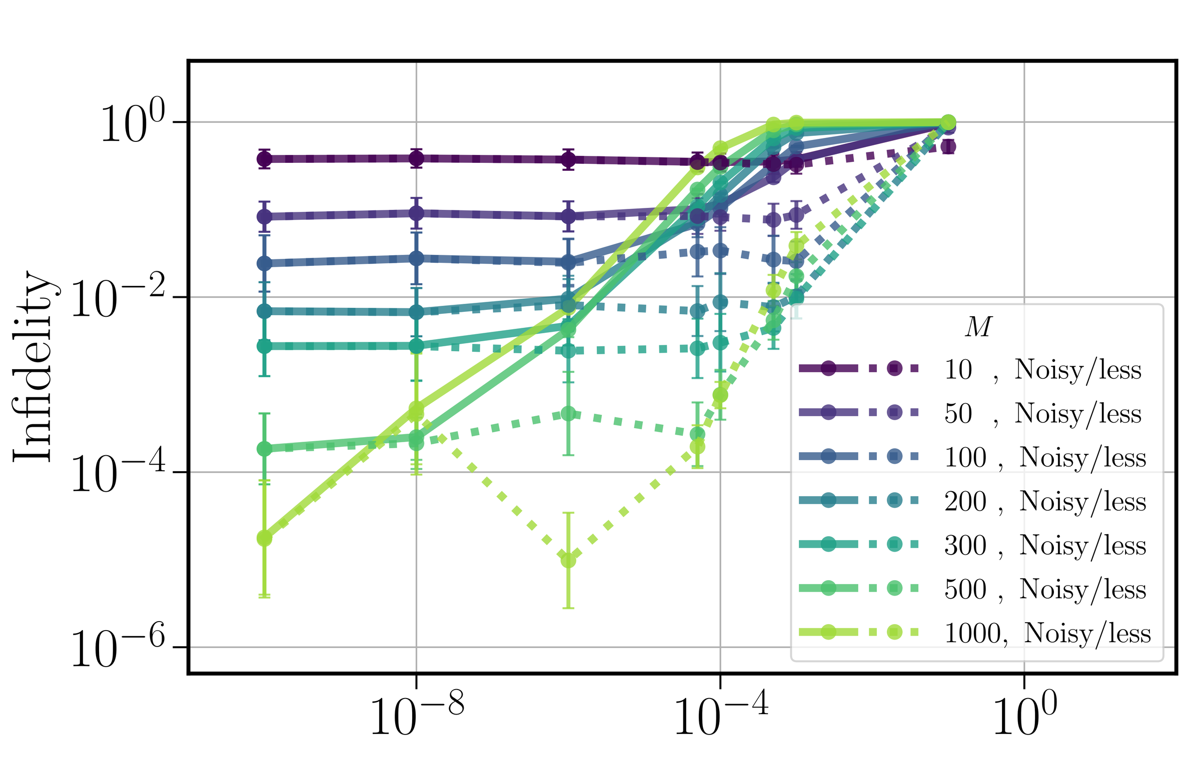
(b) Negligible Relative Change of Parameters



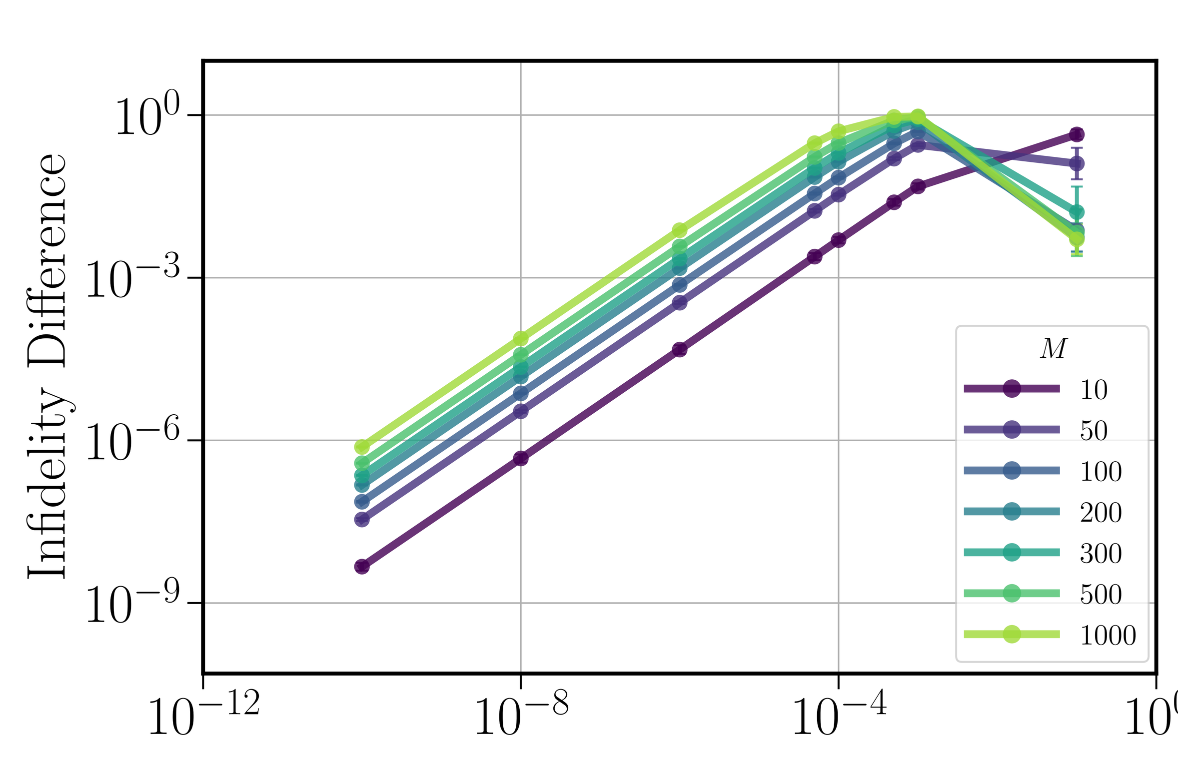
(c) Fisher Information Rank Saturation at G

7. Noisy Optimization

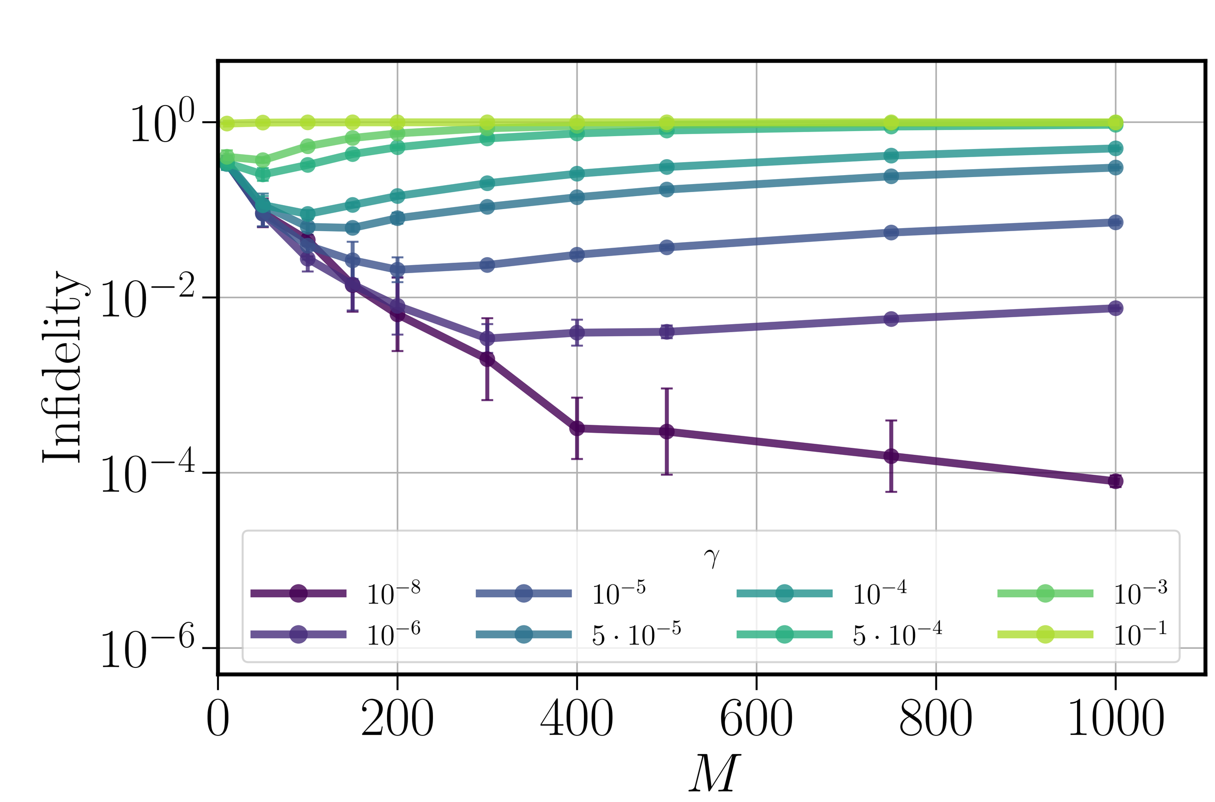
- Haar random state preparation for $N = 4$ qubits, with independent dephasing noise across qubits
- Training with noise learns about noiseless preparation, up to a critical $\gamma^* \sim O(M^{-1})$, where noise catastrophically accumulates



(d) Trained Noisy and Tested Noiseless Infidelity



(e) Trained Noisy and Tested Noiseless Infidelity Difference



(f) Critical Depth for Noisy Infidelity

8. Conclusions

- Overparameterization is robust to constraints, and requires $\sim O(N)$ greater evolution time
- Accumulation of noise induces a critical depth M_γ that prevents fidelity convergence
- Non-trivial compromises between numerical and experimental feasibility

9. References

[1] J. Liu, et al. Phys. Rev. X Quantum **3**, 3 (2020).

[2] J. Peterson, et al. Phys. Rev. Appl. **13**, 5 (2020).

[3] M. Larocca, et al. arXiv:2109.11676 [quant-ph] (2021).