

### Homework 4 Solutions

1. Problem 4.1: *Show the MIPS and VMIPS code for the Mr. Bayes likelihood table computation.*

#### MIPS Version

Assume: R2 = seq\_length and R3 = h.

```
1) LI R1, #0           // k = 0
2) Loop: SLT R0, R1, R2 // R0 = (k < seq_length) ? 1 : 0
3) BEQZ R0, Exit        // exit loop if k = seq_length
4) L.S F8, 0(RtiPL)     // load tiPL[0]
5) L.S F0, 0(RclL)      // load clL[0]
6) MUL.S F8, F8, F0     // F8 = tiPL[0]*clL[0]
7) L.S F9, 4(RtiPL)     // load tiPL[1]
8) L.S F1, 4(RclL)      // load clL[1]
9) MUL.S F9, F9, F1     // F9 = tiPL[1]*clL[1]
10) ADD.S F8, F8, F9     // F8 = intermediate sum
11) L.S F9, 8(RtiPL)    // load tiPL[2]
12) L.S F2, 8(RclL)     // load clL[2]
13) MUL.S F9, F9, F2    // F9 = tiPL[2]*clL[2]
14) ADD.S F8, F8, F9    // F8 = intermediate sum
15) L.S F9, 12(RtiPL)   // load tiPL[3]
16) L.S F3, 12(RclL)    // load clL[3]
17) MUL.S F9, F9, F3    // F9 = tiPL[3]*clL[3]
18) ADD.S F8, F8, F9    // F8 = left sum
19) L.S F10, 0(RtiPR)   // load tiPR[0]
20) L.S F4, 0(RclR)     // load clR[0]
21) MUL.S F10, F10, F4  // F10 = tiPR[0]*clR[0]
22) L.S F9, 4(RtiPR)    // load tiPR[1]
23) L.S F5, 4(RclR)     // load clR[1]
24) MUL.S F9, F9, F5    // F9 = tiPR[1]*clR[1]
25) ADD.S F10, F10, F9  // F10 = intermediate sum
26) L.S F9, 8(RtiPR)    // load tiPR[2]
27) L.S F6, 8(RclR)     // load clR[2]
28) MUL.S F9, F9, F6    // F9 = tiPR[2]*clR[2]
29) ADD.S F10, F10, F9  // F10 = intermediate sum
30) L.S F9, 12(RtiPR)   // load tiPR[3]
31) L.S F7, 12(RclR)    // load clR[3]
32) MUL.S F9, F9, F7    // F9 = tiPR[3]*clR[3]
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33) ADD.S F10, F10, F9	// F10 = right sum
34) MUL.S F9, F8, F10	// F9 = (left side)*(right side)
35) ADD R9, RclP, R3	// set offset of clP[h]
36) S.S F9, 0(R9)	// clP[h] = F9
37) ADDI R3, R3, #4	// h++
38) L.S F8, 16(RtiPL)	// load tiPL[4]
39) MUL.S F8, F8, F0	// F8 = tiPL[4]*clL[0]
40) L.S F9, 20(RtiPL)	// load tiPL[5]
41) MUL.S F9, F9, F1	// F9 = tiPL[5]*clL[1]
42) ADD.S F8, F8, F9	// F8 = intermediate sum
43) L.S F9, 24(RtiPL)	// load tiPL[6]
44) MUL.S F9, F9, F2	// F9 = tiPL[6]*clL[2]
45) ADD.S F8, F8, F9	// F8 = intermediate sum
46) L.S F9, 28(RtiPL)	// load tiPL[7]
47) MUL.S F9, F9, F3	// F9 = tiPL[7]*clL[3]
48) ADD.S F8, F8, F9	// F8 = left sum
49) L.S F10, 16(RtiPR)	// load tiPR[4]
50) MUL.S F10, F10, F4	// F10 = tiPR[4]*clR[0]
51) L.S F9, 20(RtiPR)	// load tiPR[5]
52) MUL.S F9, F9, F5	// F9 = tiPR[5]*clR[1]
53) ADD.S F10, F10, F9	// F10 = intermediate sum
54) L.S F9, 24(RtiPR)	// load tiPR[6]
55) MUL.S F9, F9, F6	// F9 = tiPR[6]*clR[2]
56) ADD.S F10, F10, F9	// F10 = intermediate sum
57) L.S F9, 28(RtiPR)	// load tiPR[7]
58) MUL.S F9, F9, F7	// F9 = tiPR[7]*clR[3]
59) ADD.S F10, F10, F9	// F10 = right sum
60) MUL.S F9, F8, F10	// F9 = (left side)*(right side)
61) ADD R9, RclP, R3	// set offset of clP[h]
62) S.S F9, 0(R9)	// clP[h] = F9
63) ADDI R3, R3, #4	// h++
64) L.S F8, 32(RtiPL)	// load tiPL[8]
65) MUL.S F8, F8, F0	// F8 = tiPL[8]*clL[0]
66) L.S F9, 36(RtiPL)	// load tiPL[9]
67) MUL.S F9, F9, F1	// F9 = tiPL[9]*clL[1]
68) ADD.S F8, F8, F9	// F8 = intermediate sum
69) L.S F9, 40(RtiPL)	// load tiPL[10]
70) MUL.S F9, F9, F2	// F9 = tiPL[10]*clL[2]
71) ADD.S F8, F8, F9	// F8 = intermediate sum
72) L.S F9, 44(RtiPL)	// load tiPL[11]
73) MUL.S F9, F9, F3	// F9 = tiPL[11]*clL[3]
74) ADD.S F8, F8, F9	// F8 = left sum

75) L.S F10, 32(RtiPR)	// load tiPR[8]
76) MUL.S F10, F10, F4	// F10 = tiPR[8]*clR[0]
77) L.S F9, 36(RtiPR)	// load tiPR[9]
78) MUL.S F9, F9, F5	// F9 = tiPR[9]*clR[1]
79) ADD.S F10, F10, F9	// F10 = intermediate sum
80) L.S F9, 40(RtiPR)	// load tiPR[10]
81) MUL.S F9, F9, F6	// F9 = tiPR[10]*clR[2]
82) ADD.S F10, F10, F9	// F10 = intermediate sum
83) L.S F9, 44(RtiPR)	// load tiPR[11]
84) MUL.S F9, F9, F7	// F9 = tiPR[11]*clR[3]
85) ADD.S F10, F10, F9	// F10 = right sum
86) MUL.S F9, F8, F10	// F9 = (left side)*(right side)
87) ADD R9, RclP, R3	// set offset of clP[h]
88) S.S F9, 0(R9)	// clP[h] = F9
89) ADDI R3, R3, #4	// h++
90) L.S F8, 48(RtiPL)	// load tiPL[12]
91) MUL.S F8, F8, F0	// F8 = tiPL[12]*clL[0]
92) L.S F9, 52(RtiPL)	// load tiPL[13]
93) MUL.S F9, F9, F1	// F9 = tiPL[13]*clL[1]
94) ADD.S F8, F8, F9	// F8 = intermediate sum
95) L.S F9, 56(RtiPL)	// load tiPL[14]
96) MUL.S F9, F9, F2	// F9 = tiPL[14]*clL[2]
97) ADD.S F8, F8, F9	// F8 = intermediate sum
98) L.S F9, 60(RtiPL)	// load tiPL[15]
99) MUL.S F9, F9, F3	// F9 = tiPL[15]*clL[3]
100) ADD.S F8, F8, F9	// F8 = left sum
101) L.S F10, 48(RtiPR)	// load tiPR[12]
102) MUL.S F10, F10, F4	// F10 = tiPR[12]*clR[0]
103) L.S F9, 52(RtiPR)	// load tiPR[13]
104) MUL.S F9, F9, F5	// F9 = tiPR[13]*clR[1]
105) ADD.S F10, F10, F9	// F10 = intermediate sum
106) L.S F9, 56(RtiPR)	// load tiPR[14]
107) MUL.S F9, F9, F6	// F9 = tiPR[14]*clR[2]
108) ADD.S F10, F10, F9	// F10 = intermediate sum
109) L.S F9, 60(RtiPR)	// load tiPR[15]
110) MUL.S F9, F9, F7	// F9 = tiPR[15]*clR[3]
111) ADD.S F10, F10, F9	// F10 = right sum
112) MUL.S F9, F8, F10	// F9 = (left side)*(right side)
113) ADD R9, RclP, R3	// set offset of clP[h]
114) S.S F9, 0(R9)	// clP[h] = F9
115) ADDI R3, R3, #4	// h++
116) ADDI RclL, RclL, #16	// clL += 4

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117) ADDI RclR, RclR, #16    // clR += 4
118) ADDI RtiPL, RtiPL, #64 // tiPL += 16
119) ADDI RtiPR, RtiPR, #64 // tiPR += 16
120) ADDI R1, R1, #1        // k++
121) J Loop                  // repeat loop
122) Exit:

```

### VMIPS Version

Assume: R2 = seq\_length and R3 = h.

```

1) LI R1, #0                // k = 0
2) LI VL, #4                // VL = 4
3) Loop: SLT R0, R1, R2      // R0 = (k < seq_length) ? 1 : 0
4) BEQZ R0, Exit            // exit loop if k = seq_length
5) LV V1, RtiPL              // load tiPL[0] – tiPL[3]
6) LV V2, RclL              // load clL[0] – clL[3]
7) MULVV.S V3, V1, V2       // V3[i] = tiPL[i]*clL[i]
8) SUMR.S F0, V3            // F0 = left sum
9) LV V4, RtiPR              // load tiPR[0] – tiPR[3]
10) LV V5, RclR              // load clR[0] – clR[3]
11) MULVV.S V3, V4, V5      // V3[i] = tiPR[i]*clR[i]
12) SUMR.S F1, V3           // F1 = right sum
13) MUL.S F2, F0, F1        // F2 = (left side)*(right side)
14) ADD R9, RclP, R3        // set offset of clP[h]
15) S.S F2, 0(R9)          // clP[h] = F2
16) ADDI R3, R3, #4         // h++
17) ADDI RtiPL, RtiPL, #16  // RtiPL += 4
18) ADDI RtiPR, RtiPR, #16  // RtiPR += 4
19) LV V1, RtiPL            // load tiPL[3] – tiPL[7]
20) MULVV.S V3, V1, V2      // V3[i] = tiPL[i]*clL[i]
21) SUMR.S F0, V3           // F0 = left sum
22) LV V4, RtiPR            // load tiPR[3] – tiPR[7]
23) MULVV.S V3, V4, V5      // V3[i] = tiPR[i]*clR[i]
24) SUMR.S F1, V3           // F1 = right sum
25) MUL.S F2, F0, F1        // F2 = (left side)*(right side)
26) ADD R9, RclP, R3        // set offset of clP[h]
27) S.S F2, 0(R9)          // clP[h] = F2
28) ADDI R3, R3, #4         // h++
29) ADDI RtiPL, RtiPL, #16  // RtiPL += 4
30) ADDI RtiPR, RtiPR, #16  // RtiPR += 4

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31) LV V1, RtiPL          // load tiPL[8] – tiPL[11]
32) MULVV.S V3, V1, V2    // V3[i] = tiPL[i]*clL[i]
33) SUMR.S F0, V3         // F0 = left sum
34) LV V4, RtiPR          // load tiPR[8] – tiPR[11]
35) MULVV.S V3, V4, V5    // V3[i] = tiPR[i]*clR[i]
36) SUMR.S F1, V3         // F1 = right sum
37) MUL.S F2, F0, F1      // F2 = (left side)*(right side)
38) ADD R9, RclP, R3      // set offset of clP[h]
39) S.S F2, 0(R9)         // clP[h] = F2
40) ADDI R3, R3, #4       // h++
41) ADDI RtiPL, RtiPL, #16 // RtiPL += 4
42) ADDI RtiPR, RtiPR, #16 // RtiPR += 4
43) LV V1, RtiPL          // load tiPL[12] – tiPL[15]
44) MULVV.S V3, V1, V2    // V3[i] = tiPL[i]*clL[i]
45) SUMR.S F0, V3         // F0 = left sum
46) LV V4, RtiPR          // load tiPR[12] – tiPR[15]
47) MULVV.S V3, V4, V5    // V3[i] = tiPR[i]*clR[i]
48) SUMR.S F1, V3         // F1 = right sum
49) MUL.S F2, F0, F1      // F2 = (left side)*(right side)
50) ADD R9, RclP, R3      // set offset of clP[h]
51) S.S F2, 0(R9)         // clP[h] = F2
52) ADDI R3, R3, #4       // h++
53) ADDI RtiPL, RtiPL, #16 // RtiPL += 4
54) ADDI RtiPR, RtiPR, #16 // RtiPR += 4
55) ADDI RclL, RclL, #16  // clL += 4
56) ADDI RclR, RclR, #16  // clR += 4
57) ADDI R1, R1, #1       // k++
58) J Loop                // repeat loop
59) Exit:

```

2. Problem 4.2: Assuming  $seq\_length = 500$ , what is the dynamic instruction count for both implementations?

#### MIPS Version

1 instruction for initialization, 120 instructions for single loop, 2 instructions to exit loop

$$\text{Dynamic instruction count} = 1 + (120)(500) + 2 = \mathbf{60,003}$$

#### VMIPS Version

2 instructions for initialization, 56 instructions for single loop, 2 instructions to exit loop

$$\text{Dynamic instruction count} = 2 + (56)(500) + 2 = \mathbf{28,004}$$

3. Problem 4.9: *Multiplication of vectors with single-precision complex values. Processor runs at 700 MHz and maximum vector length of 64.*

a. *What is the arithmetic intensity of this kernel? Justify your answer.*

Each loop has 6 single-precision memory accesses (i.e., 24 bytes total) and 6 FLOPs.

$$\text{Arithmetic intensity} = \frac{6}{24} = \mathbf{0.25}$$

b. *Convert this loop into VMIPS assembly code using strip mining.*

Strip mine remainder of loop:  $300 \bmod 64 = 44$

```
1) LI VL, #44           // first 44 operations
2) LI R1, #0            // i = 0
3) Loop: LV V1, a_re + R1 // load a_re
4) LV V3, b_re + R1     // load b_re
5) MULVV.S V5, V1, V3   // a_re*b_re
6) LV V2, a_im + R1     // load a_im
7) LV V4, b_im + R1     // load b_im
8) MULVV.S V6, V2, V4   // a_im*b_im
9) SUBVV.S V5, V5, V6   // top subtraction
10) SV V5, c_re + R1    // store c_re
11) MULVV.S V5, V1, V4   // a_re*b_im
12) MULVV.S V6, V2, V3   // a_im*b_re
13) ADDVV.S V5, V5, V6   // bottom addition
14) SV V5, c_im + R1    // store c_im
15) BNE R1, #0, Else    // Check if first iteration
16) ADDI R1, R1, #44     // first iteration, i += 44
17) J Loop              // repeat loop
18) Else: ADDI R1, R1, #256 // not first iteration, i += 256
19) Skip: BLT R1, #1200, Loop // Check if next iteration
```

4. Problem 4.11 (a): *Reduction and scalar expansion. Show how the C code will look for executing the second loop using recurrence doubling.*

```
for (int i = 64/2; i > 0; i /= 2)
    for (int j = 0; j < i; j++)
        dot[j] = dot[j] + dot[j + i];
```

5. Problem 4.14: *Analyze loop potential for parallelization.*

- a. *Does the following loop have a loop-carried dependency?*

Apply the greatest common divisor test to check for loop-carried dependency. In this example,  $a = 4$ ,  $b = 5$ ,  $c = 2$ , and  $d = 4$ , then  $\text{GCD}(2, 4) = 2$  and  $d - b = -1$ . Since 2 does not divide -1, no dependence is possible.

- b. *Find all the true dependences, output dependences, and antidependences. Eliminate the output dependences and antidependences by renaming.*

True dependences (RAW)

1.  $A[i]$  between S1 and S2
2.  $A[i]$  between S3 and S4

Output dependences (WAW)

1.  $A[i]$  between S1 and S3

Antidependences (WAR)

1.  $B[i]$  between S1 and S2
2.  $A[i]$  between S2 and S3
3.  $C[i]$  between S3 and S4

```
for (i = 0; i < 100; i++) {  
    A[i] = A[i] * B[i];  
    X[i] = A[i] + c;           // B to X, remove antidep. #1  
    Y[i] = C[i] * c;           // A to Y, remove output dep. #1 and antidep. #2  
    Z[i] = D[i] * Y[i];         // C to Z, remove antidep. #3  
}
```

6. Problem 4.16: Assume a hypothetical GPU with 1.5 GHz, 16 SIMD processors each with 16 SP FPU, and 100 GB/sec off-chip memory bandwidth. Without considering memory bandwidth, what is the peak single-precision floating-point throughput for this GPU in GFLOP/sec? Is this throughput sustainable given the memory bandwidth limitation?

$$\text{Peak FP throughput} = 1.5 \text{ G} \times 16 \times 16 = \mathbf{384 \text{ GFLOP/sec}}$$

The peak FP throughput performs 384 GFLOP/sec each requiring 2 SP FP values, or equivalently 8 bytes. The bandwidth required to sustain the computation needs to be at least,

$$384 \frac{\text{GFLOP}}{\text{sec}} \times 8 \text{ bytes} = 3,072 \text{ GB/sec}$$

Therefore, having only 100 GB/sec the computation becomes memory bound.