Fall 2014

# ECE471/571 – Homework #1 (Due 08/29)

Note: UG: 100+10, G: 100

**Problem 1:** The conditional density of class 1 in a one-dimensional feature space is Gaussian with mean 2 and variance 3; for class 2, the conditional density is also a Gaussian with mean 1 and variance 1. That is:

$$p(x \mid \omega_1) = \frac{1}{\sqrt{6\pi}} e^{\left[-\frac{(x-2)^2}{6}\right]}$$

$$p(x \mid \omega_2) = \frac{1}{\sqrt{2\pi}} e^{\left[-\frac{1}{2}(x-1)^2\right]}$$

(1) (10/10) Sketch the two density functions on the same figure using pencil and paper (i.e., without MATLAB or other software package). Assume equal prior probability, predict how many decision regions there would be. Use x=1 and x=2 as references, roughly describe where the intersection points would be.

(2) Assume equal prior probability, that is, $P(\omega_1) = P(\omega_2) = 0.5$ .

a. (10/5) If $x$=0, which class does $x$ belong to? Use the MAP method. Show detailed steps.

b. (10/5) Solve the same problem using likelihood ratio, assuming zero-one loss.

c. (10/5) Find the decision boundary using analytical methods instead of the sketch.

d. (10/5) Solve for the overall probability of error.

(3) Assume that $P(\omega_1) = 0.8, P(\omega_2) = 0.2$, and zero-one loss, i.e., $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 1, \lambda_{21} = 1$ .

a. (10/5) Use MATLAB to draw the posteriori probability.

b. (5/5) Redo question 2 (a)

c. (15/10) What kind of loss values would alter the decision making? Hint: Use likelihood ratio.

(4) (10/10) What would $\sigma_1$ need to be such that there are only two decision regions, i.e., one decision boundary?

(5) (10/10) Is it possible to have just one decision region?

**Problem 2:** The probability densities representing a two-class pattern are

$$p(y \mid \omega_1) = \begin{cases} \exp(y-2) \, when (y \leq 2) \\ 0 \, when (y > 2) \end{cases}$$

$$p(y \mid \omega_2) = \begin{cases} \exp(-(y-b)) \, when (y > b) \\ 0 \, otherwise \end{cases}$$

The prior probabilities are $P(\omega_1) = P(\omega_2) = 0.5$

(a) (+10/15) Sketch the two densities on the same figure for $b<2$. Show the regions corresponding to the decision rule that minimizes the probability of error.

(b) (0/10) What is $P(error \mid \omega_1)$ (conditional probability of error when we decide $\omega_2$ but actually it should be $\omega_1$) in terms of $b$? (Consider all values of $b$ from –inf to inf)

(c) (0/5) What is the value of $b$ that maximizes $P(error \mid \omega_1)$?