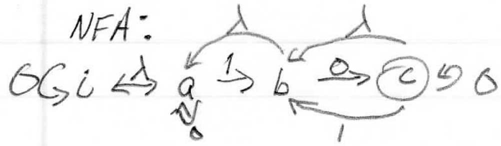
Part 1 Karen Matney

NFA:

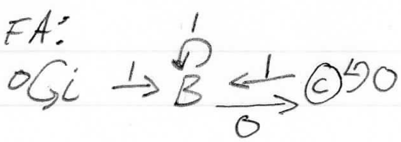


Intial State: $\{i,a\}$

$\{i,a\}0 \rightarrow \{i,a\}$	$0i \rightarrow 0\underline{a} \rightarrow 0\underline{a} \rightarrow 0\underline{i}$
	$0i \rightarrow 0\underline{a} \rightarrow 0\underline{i} \rightarrow 0i$
$\{i,a\}1 \rightarrow \{i,a,b\}$	$1\underline{i} \rightarrow 1\underline{a} \rightarrow 1\underline{b}$
	$1\underline{i} \rightarrow 1\underline{a} \rightarrow 1\underline{i} \rightarrow 1\underline{a} \rightarrow 1\underline{b}$
$\{i,a,b\}0 \rightarrow \{i,a,b,c\}$	$0b \rightarrow 0\underline{c} \rightarrow 0\underline{b} \rightarrow 0\,\underline{a} \rightarrow 0\,\underline{i} \rightarrow 0\underline{i} \rightarrow 0a$
$\{i,a,b\}1 \rightarrow \{i,a,b\}$	$1b \rightarrow 1\underline{a} \rightarrow 1\underline{i} \rightarrow 1\underline{a} \rightarrow 1\underline{b}$
$\{i,a,b,c\}0 \rightarrow \{i,a,b,c\}$	$0c \rightarrow 0\underline{b} \rightarrow 0\underline{a} \rightarrow 0\underline{i} \rightarrow 0\underline{a}$
	$0c \rightarrow 0\underline{c} \rightarrow 0\underline{b} \rightarrow 0\underline{a} \rightarrow 0\underline{i} \rightarrow 0\underline{a}$
$\{i,a,b,c\}1 \rightarrow \{i,a,b\}$	$1c \rightarrow 1\underline{b} \rightarrow 1\underline{a} \rightarrow 1\underline{i} \rightarrow 1\underline{a} \rightarrow 1\underline{b}$

$\{i,a\} = i$; $\{i,a,b\} = B$; $\{i,a,b,c\} = C$

DFA:



$R_i = 0R_i + 1R_b + 0R_c$

$R_b = 0R_c + 1R_b$

$R_c = 0R_c + 1R_b + \lambda$

$R_b = 1R_b + 0R_c \Rightarrow 1^*0R_c$

$R_b = 1^*0(1^*0)^*$

$R_c = 0R_c + 1(1^*0R_c) + \lambda$

$= (0 + 11^*0)R_c + \lambda = (1^*0 + 0)R_c + \lambda = (1^*0)R_c + \lambda \Rightarrow (1^*0)^*\lambda = (1^*0)^*$

$R_i = 0R_i + 1(1^*0)(1^*0)^* + 0(1^*0)^*$

$= 0R_i + (11^*0 + 0)(1^*0)^*$

$= 0R_i + (1^*0)(1^*0)^* \Rightarrow \boxed{0^*1^*0(1^*0)^*}$ = Regular Expression

$r = 0^*(1^*0)(1^*0)^*$

$D_\lambda r = r = 0^*1^*0(1^*0)^*$

$D_0 r = D_0((0^*1^*0)(1^*0)^*)$

$= (D_0(0^*1^*0))(1^*0)^* + \delta(0^*1^*0)\, D_0(1^*0)^*$

$= ((D_0(0^*))1^*0 + \delta(0^*)\, D_0(1^*0))(1^*0)^*$

$= ((D_0(0))0^*1^*0 + \lambda)(1^*0)^*$

$= (\lambda 0^*1^*0 + \lambda)(1^*0)^* = (\lambda + \lambda 0^*1^*0)(1^*0)^*$

Accepting state $= \lambda 0^*1^*0(1^*0)^* = 0^*1^*0(1^*0)^* = r$

$D_1 r = D_1((0^*1^*0)(1^*0)^*)$

$= (D_1 0^*1^*0)(1^*0)^* + \delta(0^*1^*0)\, D_1(1^*0)^*$

$= ((D_1 0^*)(1^*0) + \delta(0^*)\, D_1(1^*0))(1^*0)^*$

$= (\cancel{(D_1 0)0^*1^*0} + \delta(0^*)(D_1(1^*)0 + \cancel{\delta(1^*)D_1 0}))(1^*0)^*$

$= (\lambda(D_1(1)1^*0))(1^*0)^*$

$= (\lambda\lambda 1^*0)(1^*0)^* = (1^*0)(1^*0)^*$

$D_{10} = D_1(D_0 r)$

$= D_1(r)$

$= D_1 r$

$D_{11} = D_1(D_1 r)$

$= D_1((1^*0)(1^*0)^*)$

$= (D_1 1^*0)(1^*0)^* + \delta(1^*0)\, D_1(1^*0)^*$

$= ((D_1 1^*)0 + \delta(1^*)D_1 0)(1^*0)^*$

$= ((D_1 1)1^*0)(1^*0)^*$

$= \lambda 1^*0(1^*0)^* = (1^*0)(1^*0)^* = D_1 r$

Part (2)

Simplification Rules

(2) $(\alpha\beta)\gamma = \alpha(\beta\gamma)$

$\mathcal{L}(\alpha)\mathcal{L}(\beta)\gamma = \alpha\mathcal{L}(\beta)\mathcal{L}(\gamma)$

$\mathcal{L}(\alpha)\mathcal{L}(\beta)\mathcal{L}(\gamma) = \mathcal{L}(\alpha)\mathcal{L}(\beta)\mathcal{L}(\gamma)$

(5) $(\alpha+\beta)(\gamma+\delta)$; if $(\gamma+\delta) = e$

$\stackrel{?}{=} (\alpha+\beta)e = \alpha e + \beta e = \alpha(\gamma+\delta) + \beta(\gamma+\delta)$

$= \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta$

(12) $\alpha^{+} = \alpha\alpha^{*}$

$\alpha^{n} = \alpha^{1}\alpha^{y} = \alpha^{(1+y)}$

Since bases are the same can set exponents equal to each other

$n = 1+y$ $\quad n \geq 1$; $y \geq 0$

$1 = 1+0 = 1$ base

α^{+} means there is at least 1 α, even with no second α there is always one α.

(1) $\alpha = \lambda\alpha = \alpha\lambda$

For $\alpha = \alpha\lambda$

$\alpha^{0} = \lambda$

$\alpha^{0+1} = \alpha^{1} = \alpha^{1}\alpha^{0}$

$= \alpha\lambda$

For $\alpha = \lambda\alpha$

$\alpha^{0}\alpha^{1} = \alpha^{0+1} = \alpha^{1}$

$= \lambda\alpha = \alpha$

(18) $(\alpha+\beta)^{*}\alpha = (\alpha^{*}\beta)^{*}\alpha^{+}$

From rule 17: $(\alpha+\beta)^{*}\alpha = (\alpha^{*}\beta)^{*}\alpha^{*}$

$(\alpha+\beta)^{*}\alpha = ((\alpha^{*}\beta)^{*}\alpha^{*})\alpha$

$= ((\alpha^{*}\beta)^{*})(\alpha^{*}\alpha)$ From Rule 12: $\alpha^{+} = \alpha^{*}\alpha$

$= (\alpha^{*}\beta)^{*}\alpha^{+}$