

# Efficient Simulation Of A Simple Evolutionary System

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# Outline

Background

Question 1: Distance between finite and infinite population

Question 2: Oscillation in finite population

Question 3: Oscillation in finite population under violation in mutation

Question 4: Oscillation in finite population under violation in crossover

Conclusion

# Terms

Population  $P$ : a collection of length  $\ell$  binary strings

Population vector  $\mathbf{p}$ :  $\mathbf{p}_j$  is the proportion of string  $j$  in the population

If  $P = 00, 01, 01, 10, 11, 11$ , then  $\mathbf{p}_3 = 2/6 = 1/3$

$\mathcal{R}$  denotes a set binary strings of length  $\ell$

Addition and multiplication of elements in  $\mathcal{R}$  are bitwise operations modulo 2

$$x = 1101, y = 1010$$

$$x + y = 1101 + 1010 = 0111$$

$$xy = 1101 \cdot 1010 = 1000$$

$$\bar{x} = 0010$$

# Crossover & Mutation

Crossover : Choose parents  $u$  and  $v$ , exchange bits using crossover mask  $m$ :

$$u' = um + v\bar{m}, v' = u\bar{m} + vm$$

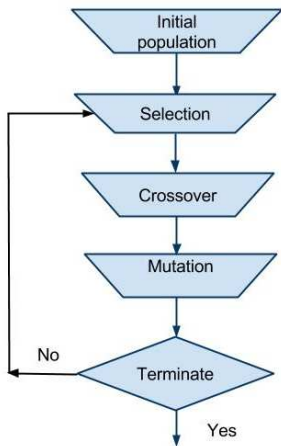
$$u = \mathbf{11001011}, v = 11011111, m = 11110000$$

$$\{\mathbf{11001011}, 11011111\} \rightarrow \{\mathbf{11000000} + 00001111, 0000\mathbf{1011} + 11010000\} \rightarrow \{\mathbf{11001111}, 1101\mathbf{1011}\}$$

Mutation: Flip bits using mutation mask:

$$x \rightarrow x + m$$

# Finite Population GA



Randomly select parents  $u$  and  $v$

Crossover  $u$  and  $v$  to produce  $u'$  and  $v'$

Keep one of  $u'$ ,  $v'$ , and mutate

Repeat above to form next generation

Repeat whole process until system stops to improve or threshold is reached

# Infinite Population Model

Population is modeled as a vector  $\mathbf{p}$

$\mathcal{G}$  maps  $\mathbf{p}$  to the next generation

$\mathcal{G}(\mathbf{p})_j$  = probability that string  $j$  occurs in the next generation

The infinite population model is the sequence

$$\mathbf{p} \rightarrow \mathcal{G}(\mathbf{p}) \rightarrow \mathcal{G}(\mathcal{G}(\mathbf{p})) \rightarrow \dots$$