Needed Simulations: corrected (Jan. 18, 2016)

$G = 8, H = 100$

Set # 1a:

Chiefs don't contribute, don't punish, don't take taxes, don't evolve.

Leaders: don't punish; but y evolves; initial $y = U01$.

Commoners: initial $x = rnd(2)$.

Strategy updating: $(v_1, v_2, v_3) = (0.01, 0.24, 0)$ and $(0, 0.01, 0.24)$.

10 runs for each parameter combination; $T = 1500$; compute averages over the last 500 steps.

Set $c_x = c_y = 1$.

Graphs of x, y, P in one graph and π_c and π_l in another graph as functions of $b = 1, 2, \ldots, 10$ for $X_0 = 2, 4, 8$ and $n = 4, 8, 16$. Use different colors for different X_0 and different thickness for different n (see the example)

Notice that we will use X_0 in the equation for P rather than nx_0!

Arrange the graphs in two 9×9 sets of graphs:

- "inside" parameters x, y, P (or π_c, π_l) (vertical) and $\theta = 0, 0.1, 0.2$) (horizontal).

- outside parameters $e = 1, 2, 4$ (vertical) and $y_0 = 1/16, 1/8, 1/4$ (horizontal) (see the 2×2 example).

Set # 2a:

Chiefs don't punish, but z evolves; initial $z = U01$; $c_z = 1$.

Everything else is the same as in Set #1.

Graphs of x, y, P, Q in one graph and $\pi_c, \pi_l, \pi_{chief}$ in another graph.

$Y_0 = \theta_{max} \times \eta_{max} \times n_{max} \times b_{\max}/2 = 0.2 * 0.2 * 16 * 10/2 = 3.2$

Additional parameters: $\eta = 0, 0.1, 0.2; E = 1, 2, 4; z_0 = 1/(2G), 1/G, 2/G$. This gives $3 \times 3 \times 3 = 27$ different combinations of parameters.

One set: chiefs keep everything (i.e., $\theta_d = \eta_d = 1$).

Another set: equal taxes up and down (i.e., $\theta_u = \theta_d = \theta, \eta_u = \eta_d = \eta$).

Needed code modifications: Dec.26, 2016

- Different update rates for commoners, leaders, and chiefs (to look at, say, optimization for commoners and selective learning for leaders).
- QRE approach for optimization with several candidate strategies.

Needed Simulations: Cooperation in 3-level systems without punishment (Dec.12/26, 2016)

Set # 1:

Chiefs don't contribute, don't punish, don't take taxes, don't evolve.

Leaders: don't punish; but y evolves; initial $y = U01$.

Commoners: initial $x = rnd(2)$.

Strategy updating: $(v_1, v_2, v_3) = (0.01, 0.24, 0)$ and $(0, 0, 0.25)$.

10 runs for each parameter combination; $T = 1500$; compute averages over the last 500 steps.

Set $c_x = c_y = 1$.

Graphs of x, y, P in one graph and π_c and π_l in another graph as functions of $b = 1, 2, \ldots, 20$ for $X_0 = 2, 4, 8, 16$ and $n = 4, 8, 16$. Use different colors for different X_0 and different thickness for different n (see the example)

Notice that we will use X_0 in the equation for P rather than nx_0!

Arrange the graphs in two 9×9 sets of graphs:

"inside" parameters x, y, P (or π_c, π_l) (vertical) and $\theta = 0, .1..2)$ (horizontal.

outside parameters $e = 1, 2, 4$ (vertical) and $y_0 = 1/8, 1/4, 1/2$ (horizontal) (see the 2×2 example).

We will likely be using the same graph structure for the propaganda project!

Set # 2:

Chiefs don't punish, but z evolves; initial $z = U01$; $c_z = 1$.

Everything else is the same as in Set #1.

Additional parameters: $\eta = 0, 0.1, 0.2$; $E = 1, 2, 4$; $z_0 = 1/G, 2/G, 4/G$. This gives $3 \times 3 \times 3 = 27$ different combinations of parameters.

One set: taxes go only up (i.e., $\theta_d = \eta_d = 0$).

Another set: equal taxes up and down (i.e., $\theta_u = \theta_d = \theta, \eta_u = \eta_d = \eta$).

Multilevel model with punishment (10/22/16)

There are three hierarchical levels: individuals, groups, and polities (i.e., groups of groups). Within each group one individual is a "leader"; other n individuals are "commoners". G groups form a polity; their leaders are subordinate to a "chief". There are H polities in the system.

Efforts

Each commoner makes a (binary) production effort $x = 0$ or $x = 1$.

Each leader makes a coordination effort y and ~~punishment~~ monitoring effort p. The taxes/rewards rates for commoners are θ_u (up) and θ_d (down), respectively ($y \geq 0, 0 \leq p, \theta_u, \theta_d \leq 1$).

Each chief makes a coordination effort z and ~~punishment~~ monitoring effort q. The taxes/rewards for leaders are η_u (up) and η_d (down), respectively ($z \geq 0, 0 \leq q, \eta_u, \eta_d \leq 1$).

12/3/16:That is, efforts evolve; taxes are constant.

Group's CA

Groups are engaged in "us vs. nature" collective actions. The group produces value $(bn)P$ with

$$P = \frac{\sum x}{\sum x + X_0/r} = \frac{\sum x}{\sum x + nx_0/r}, \text{ where } r = 1 + e\frac{y}{ny_0 + y} \quad (2)$$

is the leader's effect on production; y is the leader's coordination effort; e, x_0 and y_0 are parameters.

Punishment of commoners

Each free-rider (i.e. commoners with $x = 0$) is punished by the leader with probability p (that is, punishment of a commoner is a stochastic event). The punishment means the free-rider looses κ from his payoff. The act of punishment is costly: the leader looses δ from his payoff for each act of punishment.

12/3/16: It looks we don't have any cost of monitoring here, only that of punishing (?)

Group's spoils

The leader takes share θ_u of the group production $(bn)P$; the rest is shared equally by commoners with each getting $(1 - \theta_u)bP$. The chief takes share η_u from each leader leaving each leader $(1 - \eta_u)\theta_u(bn)P$ with a corresponding P value.

Polity's CA

The group's production reaching the chief from a leader is $Y = \eta_u\theta_u(bn)P$. Each chief coordinates productive efforts of the groups in its polity in producing value $(BnG)Q$ in an "us vs. nature" game with

$$Q = \frac{\sum Y}{\sum Y + GY_0/R}, \text{where } R = 1 + E\frac{z}{Gz_0 + z}, \quad (3)$$

with z being the chief's coordination effort and E and z_0 being parameters.

Punishment of leaders

Each of L least effective leaders (i.e. with smallest Y's) is punished by the chief with probability q (that is, punishment of a leader is a stochastic event). The punishment means the leader looses

K from his payoff. The act of punishment is costly: the chief looses Δ from his payoff for each act of punishment.

Polity's spoils

The chiefs keeps $\eta_d(BnG)Q$ of the polity production and sends $(1-\eta_d)(Bn)Q$ to each leader.

Each leaders keeps $\theta_d(1-\eta_d)(Bn)Q$ of the polity production and sends $(1-\theta_d)(1-\eta_d)BQ$ to each villager.

Payoffs

Commoners' payoff is

$$\pi_0 = 1 + (1-\theta_u)bP - c_x x - \text{cost of punishment if free-riding} + (1-\theta_d)(1-\eta_d)BQ, \tag{4}$$

where c_x is the commoner's cost coefficient.

The leader's payoff is

$$\begin{aligned}\pi_1 =& (1-\eta_u)\theta_u(bn)P - c_y y - \text{cost of punishing free-riders} \\ &+ \theta_d(1-\eta_d)(Bn)Q - \text{cost of punishment by the chief},\end{aligned} \tag{5}$$

where c_y is the leader's cost of coordination effort.

The chief's payoff is

$$\pi_2 = \eta_d(BnG)Q - c_z z - \text{cost of punishing leaders}, \tag{6}$$

where c_z is the chief's cost of coordination effort.

Dynamic variables: x, y, p, z and q.

Three strategy updating protocols with probabilities (v_1, v_2, v_3):

Random mutation (v_1): normal deviation $N(0,\sigma)$; same σ for each type of individuals

Selective copying (v_2): randomly choose another individual of the same type and copy his strategies if he has higher payoff. Commoners: choose from the same group with probability $1-m$ and from a different group in the same polity with probability m. Leaders: choose from the same polity with probability $1-m$ and from a different polity with probability m. Chiefs: choose from a different polity. Parameter m is the probability of migration (e.g. $m = 0.1$).

New parameter!

Best response (v_3): commoners consider only interactions with their leader (not possible payoffs from the chief); leaders consider interactions both with commoners and the chief; chiefs consider only interactions with leaders (not their effects on commoners).

Constant parameters:

taxes $\theta_u, \theta_d, \eta_u, \eta_u$

half-efforts and efficiency: x_0, y_0, z_0, Y_0, e, E

benefits and costs: b, B, c_x, c_y, c_z

punishment: $\delta, \kappa, \Delta, K, L$

system size: n, G, H

Strategy update rates: v_1, v_2, v_3