GRAPH THEORY

Presented by

Dr. B. MALLIKARJUNA

Assistant Professor

Department of Mathematics

Topics

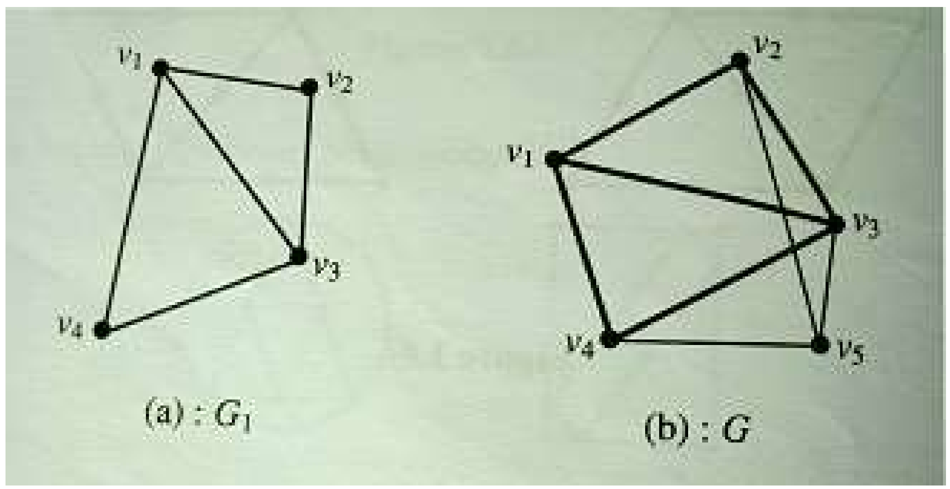
- Sub-Graphs
- Connected and Disconnected Graphs
- Eulerian Graph, Hamiltonian Graphs
- Trees: Spanning and minimal spanning tree: Kruskal’s and Prim’s Algorithms
- Shortest Path: Dijkstra’s algorithms

Subgraphs

Let G(V,E) and $G_1(V_1,E_1)$ be two graphs. The graph G_1 is said to be **SUBGRAPH** of G, if the following conditions hold

1) All the vertices and all the edges of G_1 are in G.
2) Each edge of G_1 has the same end vertices in G as in G_1

In general, a subgraph is a graph which is a part of another graph.



Subgraphs

Properties:

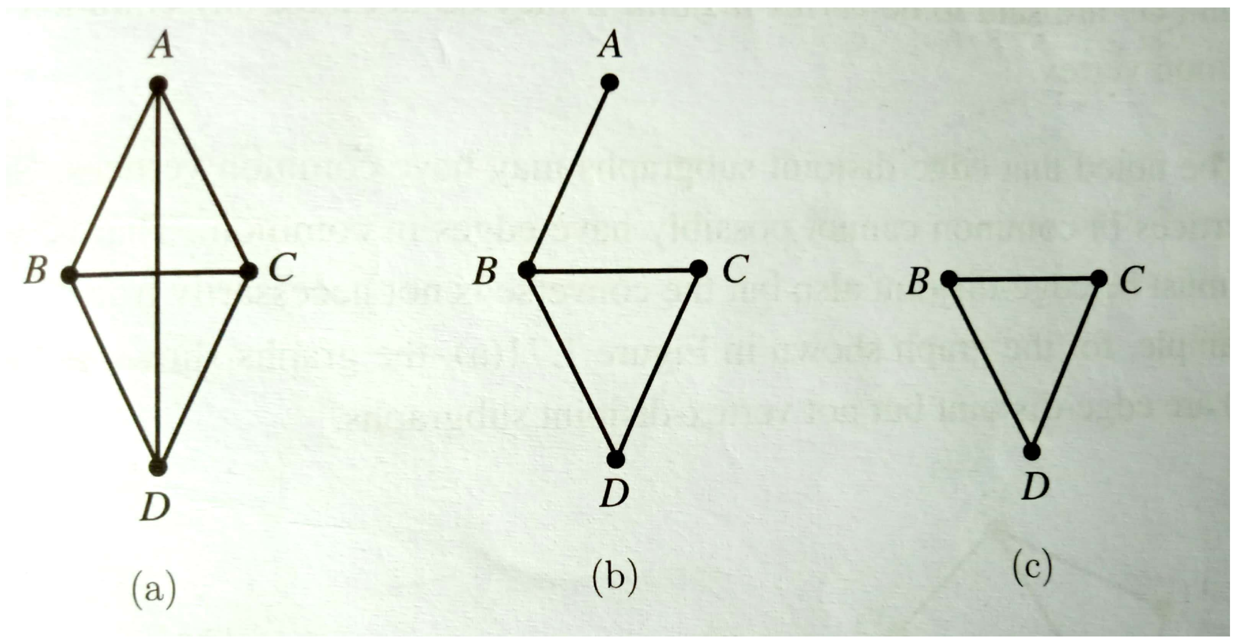
1) Any graph isomorphic to a subgraph of a graph G is also referred to as a subgraph of G.
2) Every graph is a subgraph of itself.
3) Every simple graph of n vertices is a subgraph of complete graph K_n
4) If G1 is a subgraph of a graph G2 and G2 is a subgraph of a graph G, then G1 is a subgraph of G.
5) A single vertex in a graph G is a subgraph of G.
6) A single edge in a graph G, together with its end vertices, is a subgraph of G.

Spanning Subgraph

Let G(V,E) be graph. If there is a subgraph $G_1(V_1,E_1)$ of G such that **V1=V**, then G1 is called a **SPANNING SUBGRAPH** of G.

In other words, a subgraph G1 of a graph G is a spanning subgraph of G whenever G1 contains all vertices of G.

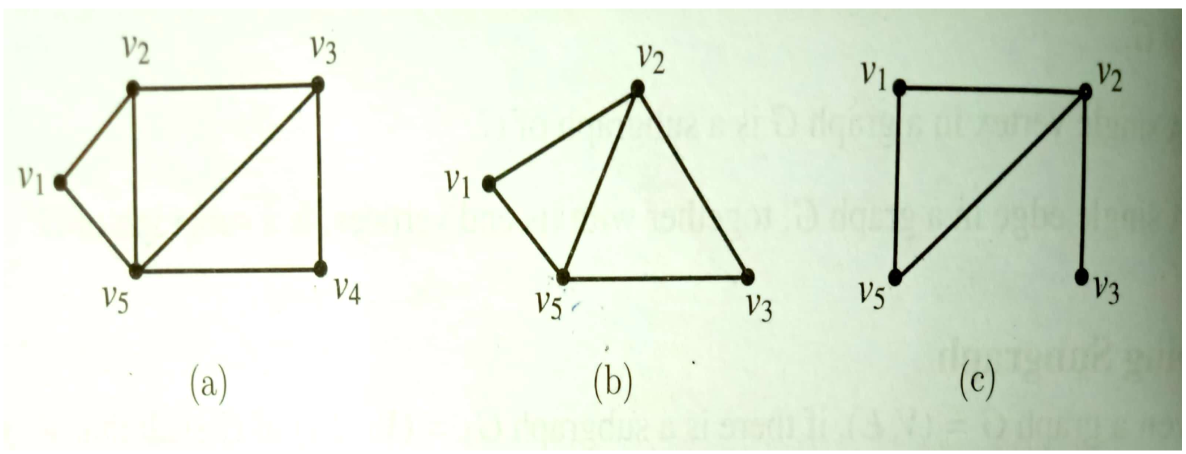
Note: Every graph is its own spanning subgraph.



Induced Subgraph

Let G(V,E) be graph. suppose there is a subgraph $G_1(V_1,E_1)$ of G such that every edge {A,B} of G, where A,B Є V1 is an edge of G1 also. Then G1 is called an **INDUCED SUBGRAPH** of G (induced by V1) and is denoted <V1>.

It follows that a subgraph G1 = (V1, E1) of a graph G=(V,E) is not an induced subgraph of G if for some A,B Є V1, there is an edge {A,B} which is in G but not in G1.



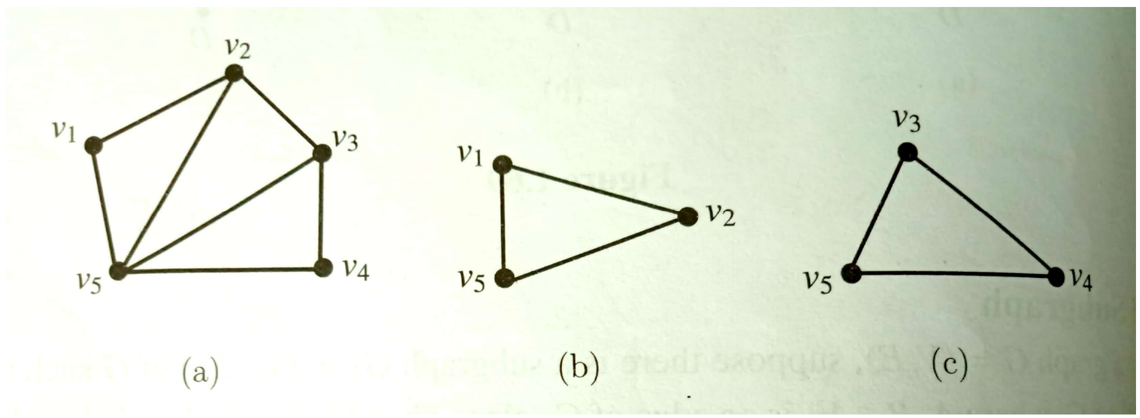
Edge-disjoint and Vertex-disjoint Subgraphs

Let G(V,E) be graph and G1 and G2 be two subgraphs of G. Then:

1) G1 and G2 are said to be **EDGE-DISJOINT** if they do not have any edge in common.
2) G1 and G2 are said to be **VERTEX-DISJOINT** if they do not have any common edge and any common vertex.

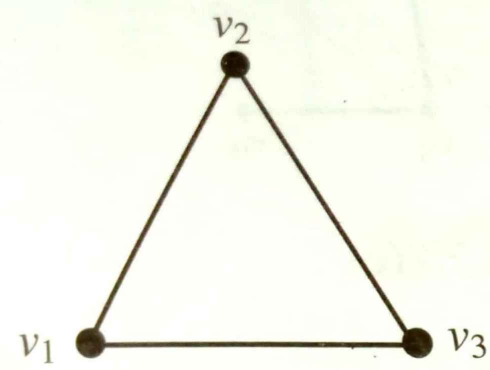
Note: 1) Edge-disjoint subgraphs may have common vertices.

2) Vertex disjoint subgraphs must be edge-disjoint also but converse need not be true.



PROBLEM-1

Given a graph G1, can there exist a graph G2 such that G1 is a subgraph of G2 but not a spanning subgraph of G2 and yet G1 and G2 have the same size?

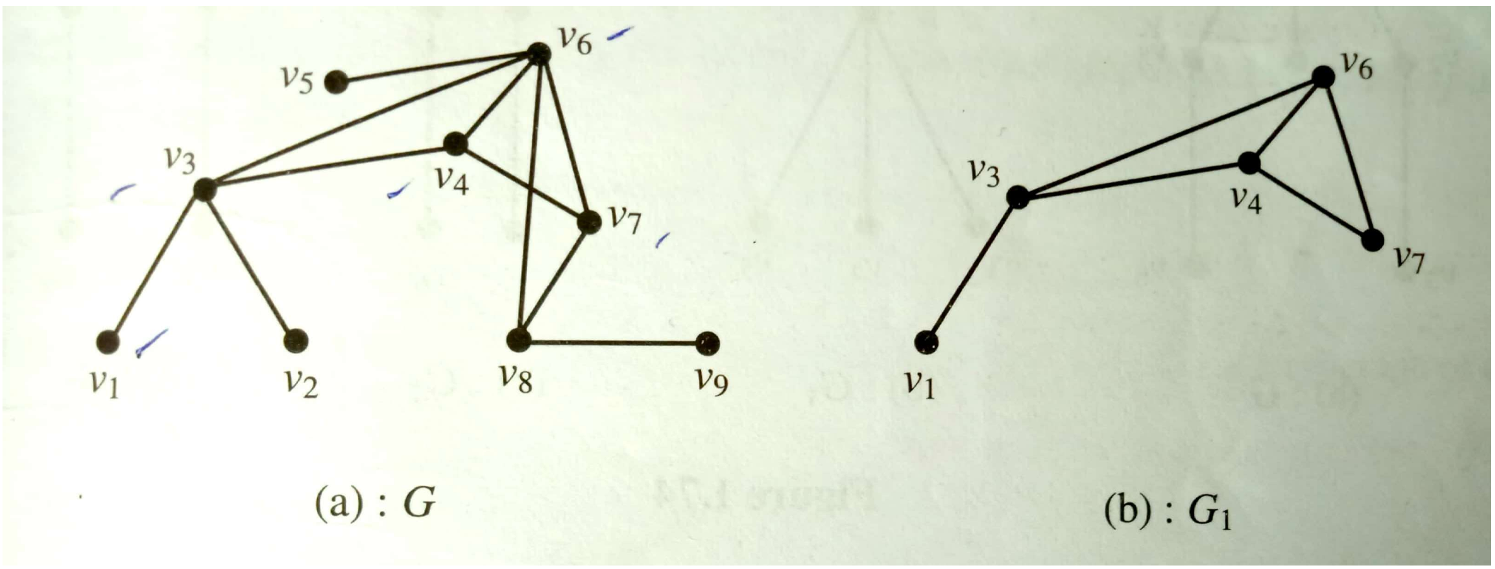


(a) : G_1

PROBLEM-2

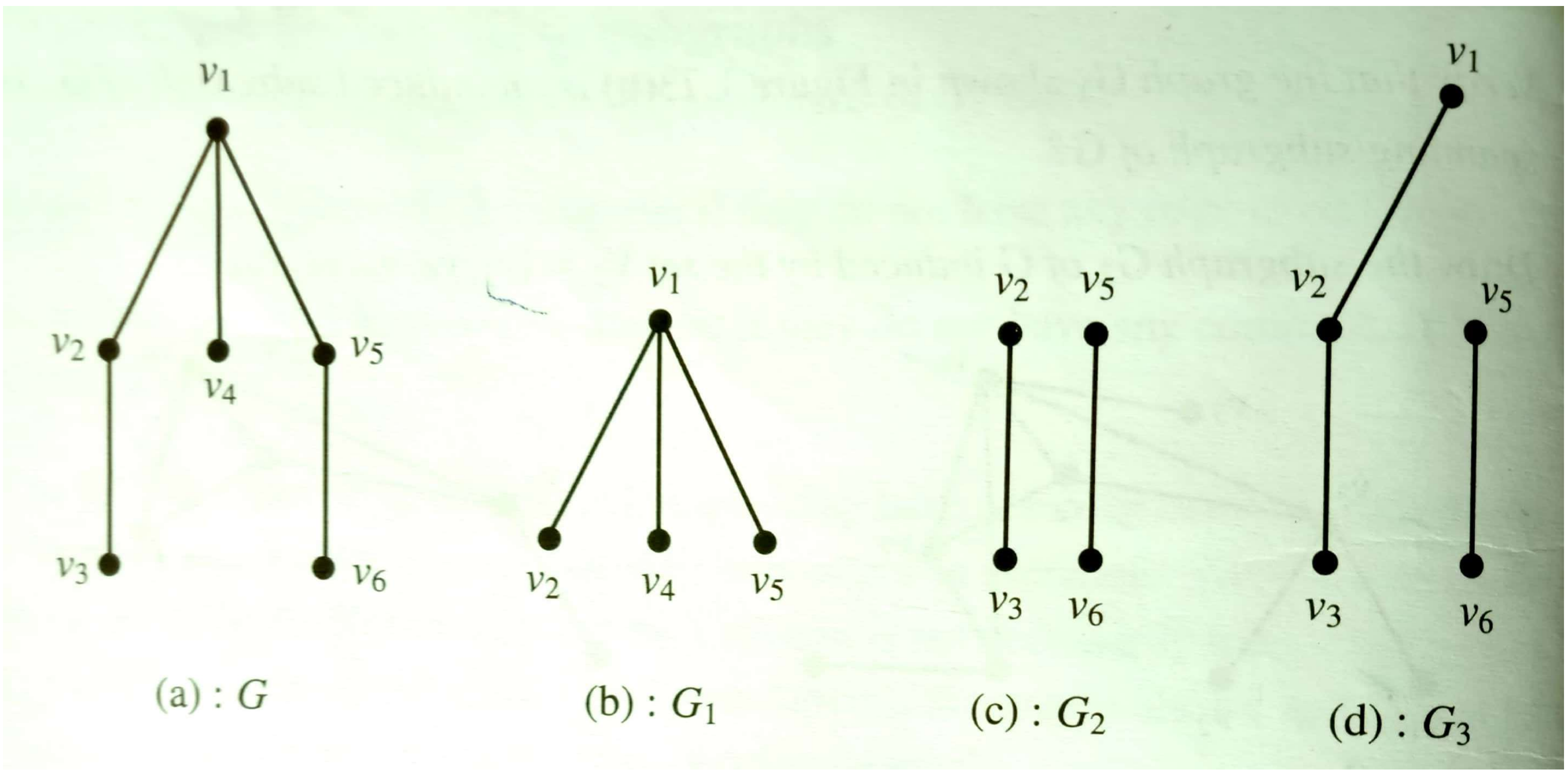
Consider the graph G shown in figure

1) Verify that the graph G1 is an induced subgraph of G. Is this a spanning subgraph of G?
2) Draw the subgraph G2 of G induced by the set V2={v3, v4, v6, v8,v9}.



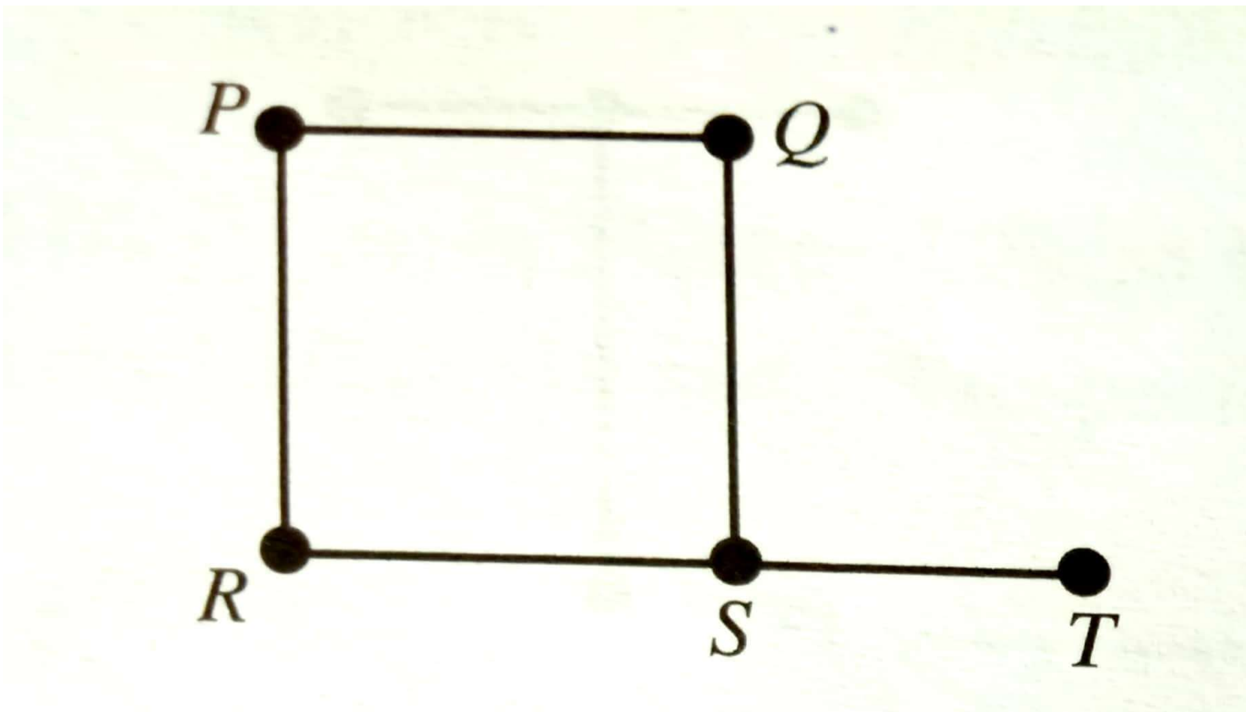
PROBLEM-3

Consider the graph G shown in figure. Verify that the graphs G1, G2 and G3 are induced subgraphs of G.



PROBLEM-4

Fog the given graph, find two edge-disjoint subgraphs and two vertex-disjoint subgraphs.



DEFINITIONS

1) Walk
2) Trail
3) Circuit
4) Path
5) Cycle

DEFINITIONS

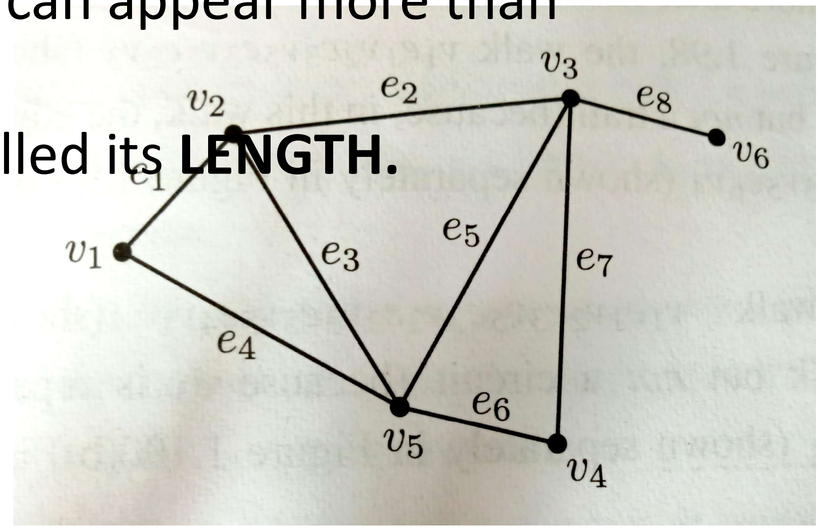
WALK: Let G be a graph having at least one edge. In G, consider a finite, alternating sequence of vertices and edges of the form

$$v_i\ e_j\ v_{i+1}\ e_{j+1}\ v_{i+2}, \ldots\ldots, e_k\ v_m$$

which begins and ends with vertices and which is such that each edge in the sequence is incident on the vertices preceding and following it in the sequence. Such a sequence is called a **WALK** in G.

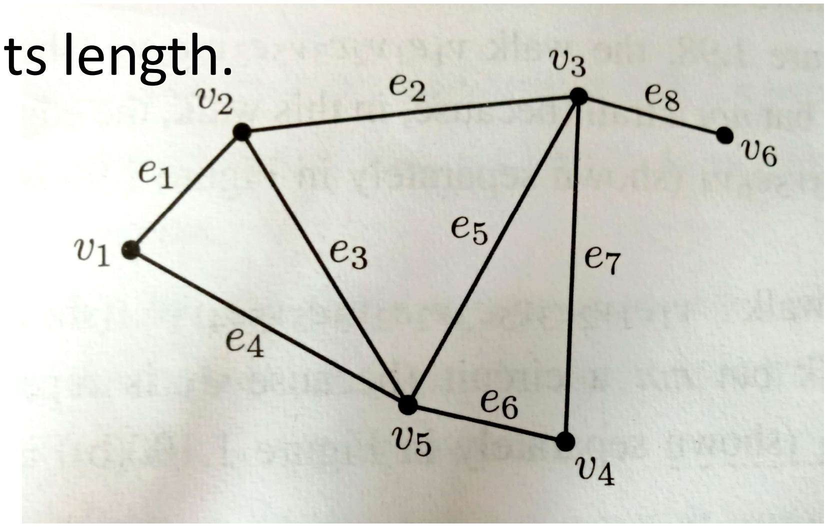
Note: In a walk, a vertex or an edge or both can appear more than once.

The number of edges present in a walk is called its **LENGTH.**



DEFINITIONS

Write different walks from graph and find its length.



DEFINITIONS

The vertex with which a walk begins is called the **initial vertex (or origin)** of the walk and the vertex with which a walk ends is called the **FINAL VERTEX (OR THE TERMINUS)** of the walk.

The initial and final vertex of a walk are together called its terminal vertices.

Note: Terminal vertices need not be distinct.

Nonterminal vertices of a walk are called its **INTERNAL** vertices.

Notation: A walk having u as the initial vertex and v as the final vertex is called a walk from u to v, or briefly a u-v walk.

OPEN AND CLOSED WALK

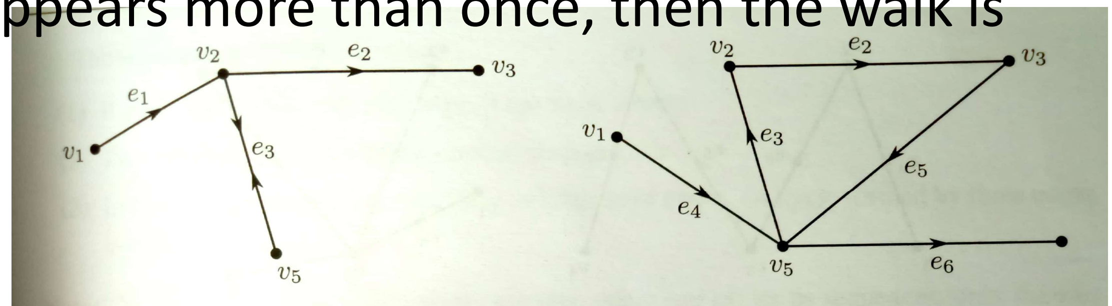
A walk that begins and ends at the same vertex is called a **CLOSED WALK**.

A walk which is not closed is called an **OPEN WALK.**

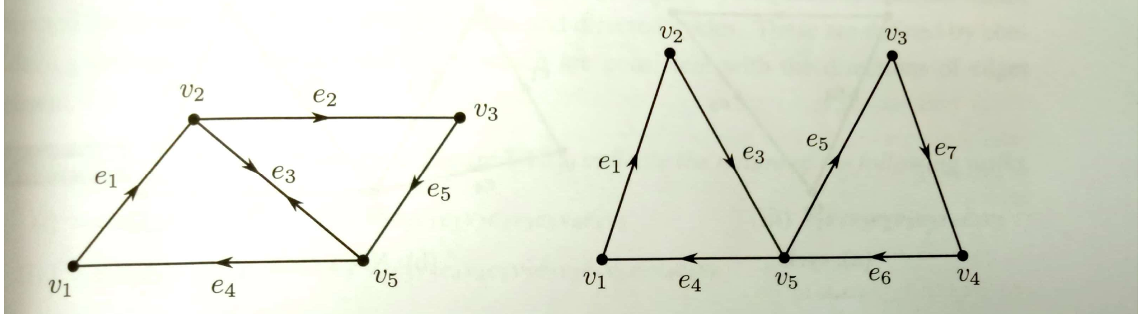
TRAIL AND CIRCUIT

If in an open walk no edge appears more than once, then the walk is called a **TRAIL.**

If in a closed walk no edge appears more than once, then the walk is called a **CIRCUIT.**



(a): Not a trail (b): Trail

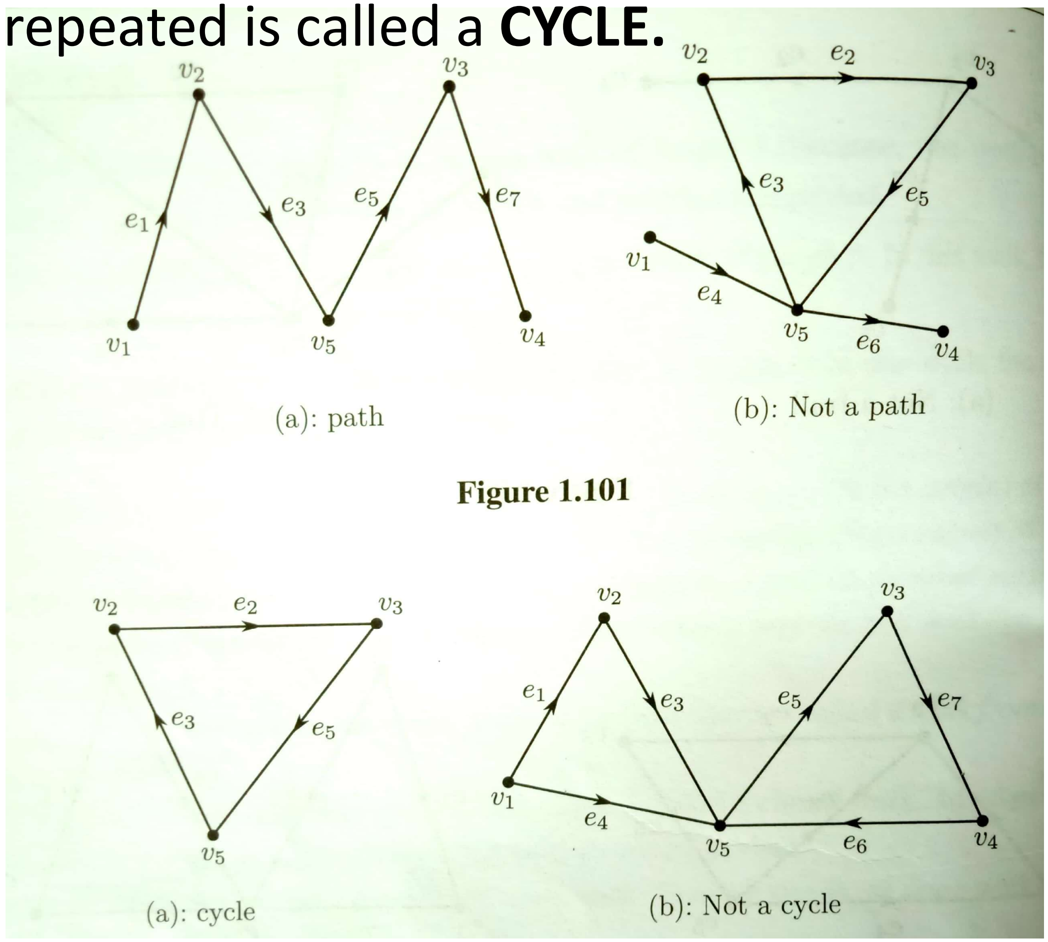


(a): Not a circuit (b): circuit

PATH AND CYCLE

A trail in which no vertex appears more than once is called a **PATH.**

A circuit in which the terminal vertex does not appear as an internal vertex and no internal vertex is repeated is called a **CYCLE.**



PROPERTIES

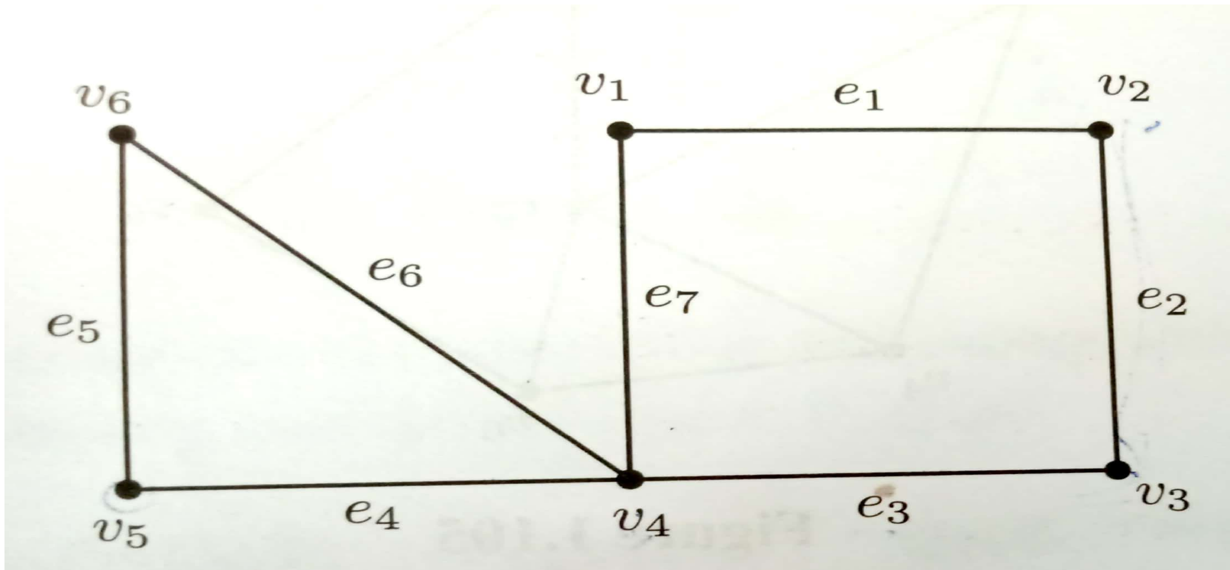
- A walk can be open or closed. In a walk (closed or open), a vertex and/or an edge can appear more than once.
- A trail (circuit) is an open (closed) walk in which a vertex can appear more than once but an edge cannot appear more than once.
- A path (Cycle) is an open (closed) walk in which neither a vertex nor an edge can appear more than once.
- Every path is a trail but a trail need not be a path
- Every cycle is a circuit but a circuit need not be a cycle.
- If a cycle contains only one edge, it has to be a loop.
- Two parallel edges (when they occur form a cycle.
- In a simple graph, a cycle must have at least three edges.
- A Cycle formed by three edges is called a **TRIANGLE.**

PROBLEM-1

For the graph shown in figure, indicate the nature of the following walks.

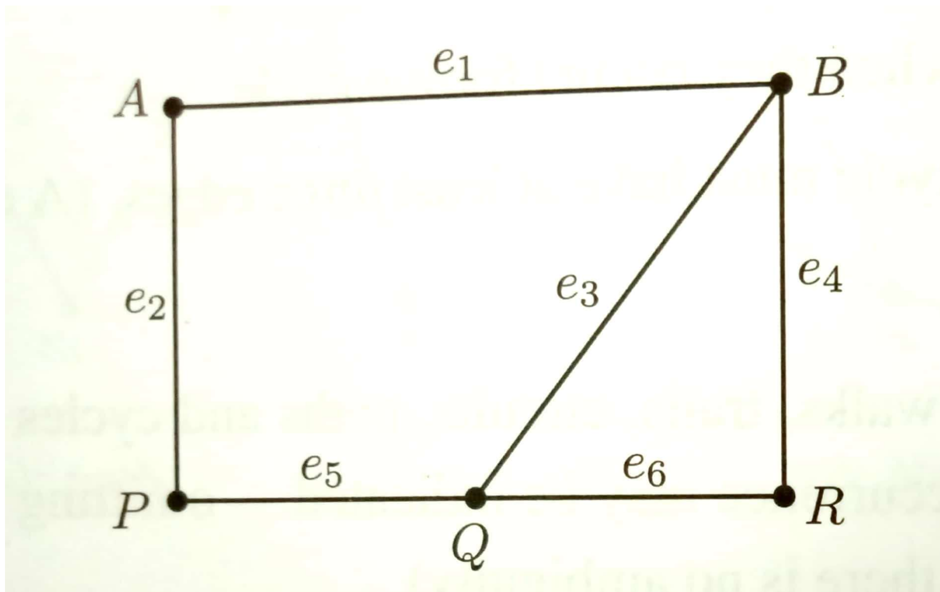
(i) v1e1v2e2v3e2v2 (ii) v4e7v1e1v2e2v3e3v4e4v5 (iii) v1e1v2e2v3e3v3e7v1

(iv) v1e1v2e2v3e3v4e4v5 (v) v6e5v5e4v4e3v3e2v2e1v1e7v4e6v6



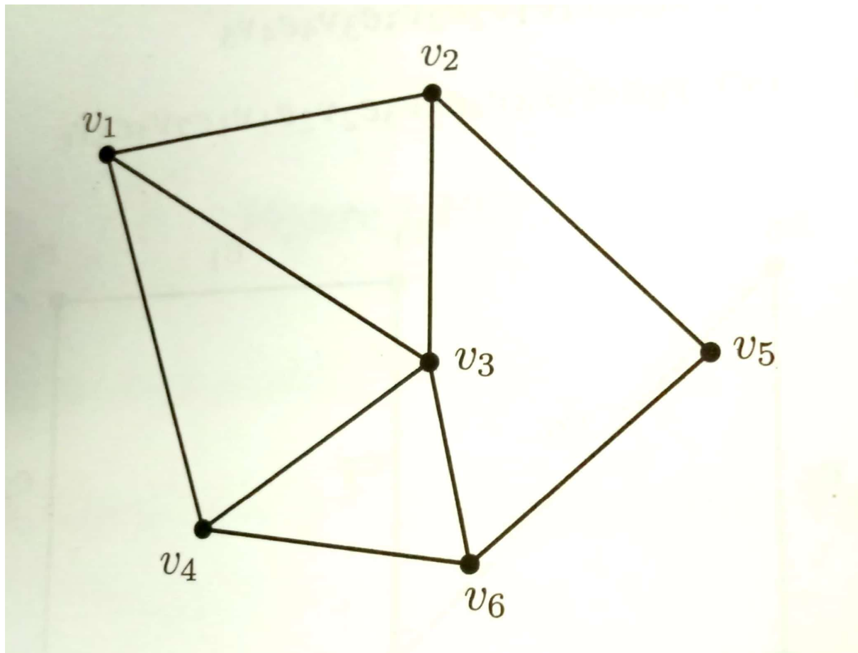
PROBLEM-2

From the graph find all paths from vertex A to vertex R. Also indicate their lengths?



PROBLEM-3

Determine the number of different paths of length 2 in the graph shown below?

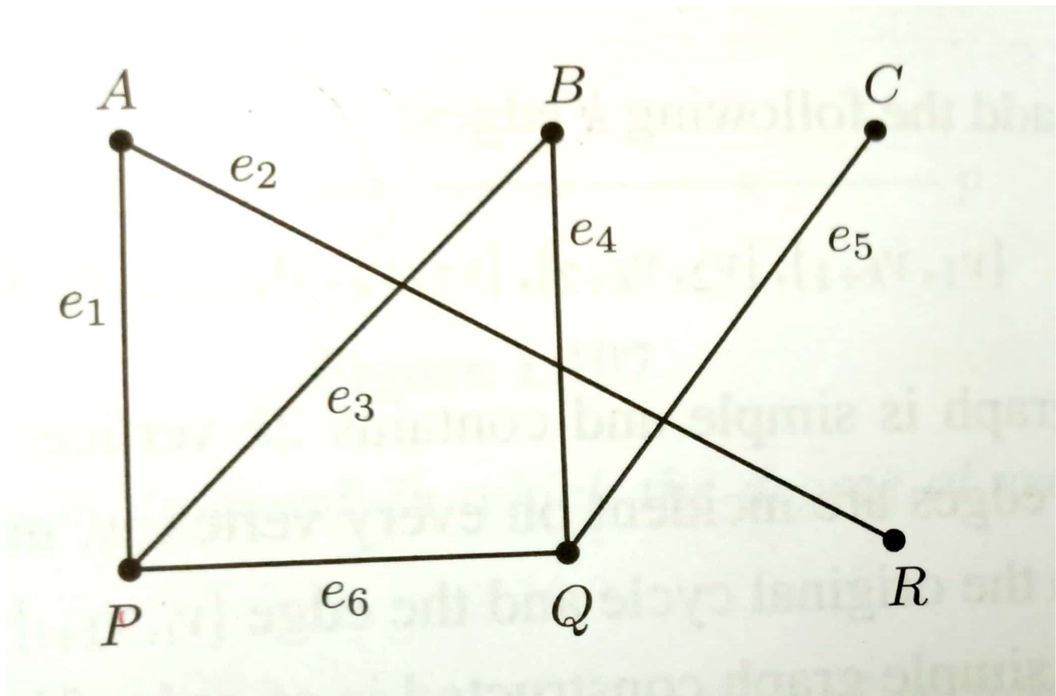


PROBLEM-4

If G is a simple graph in which every vertex has degree at least k, prove that G contains a path of length at least k.

PROBLEM-5

Find all the cycles present in the graph



PROBLEM-6

Prove the following:

(i) A path with n vertices is of length n-1

(ii) If a cycle has n vertices it has n edges

(iii) The degree of every vertex in a cycle is two.

Thank you