GRAPH THEORY

Presented by

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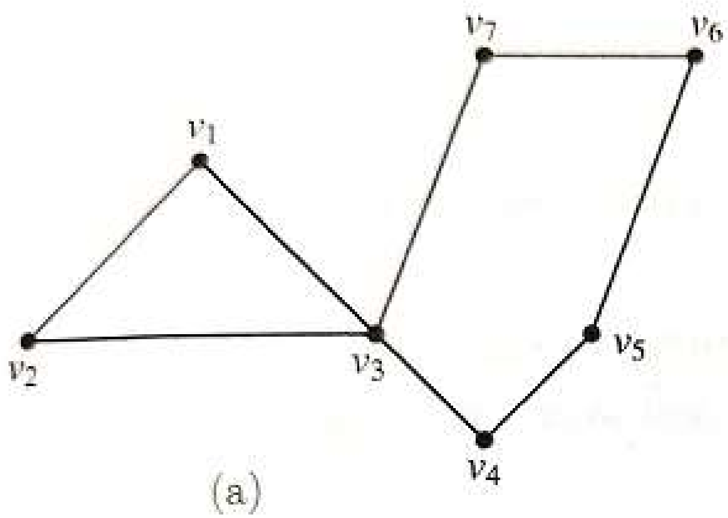
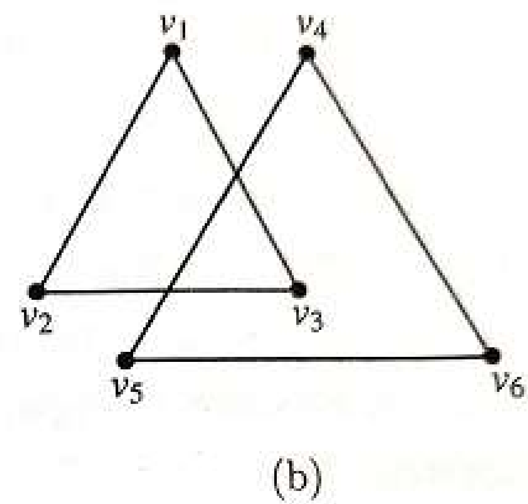
Connected and Disconnected Graphs

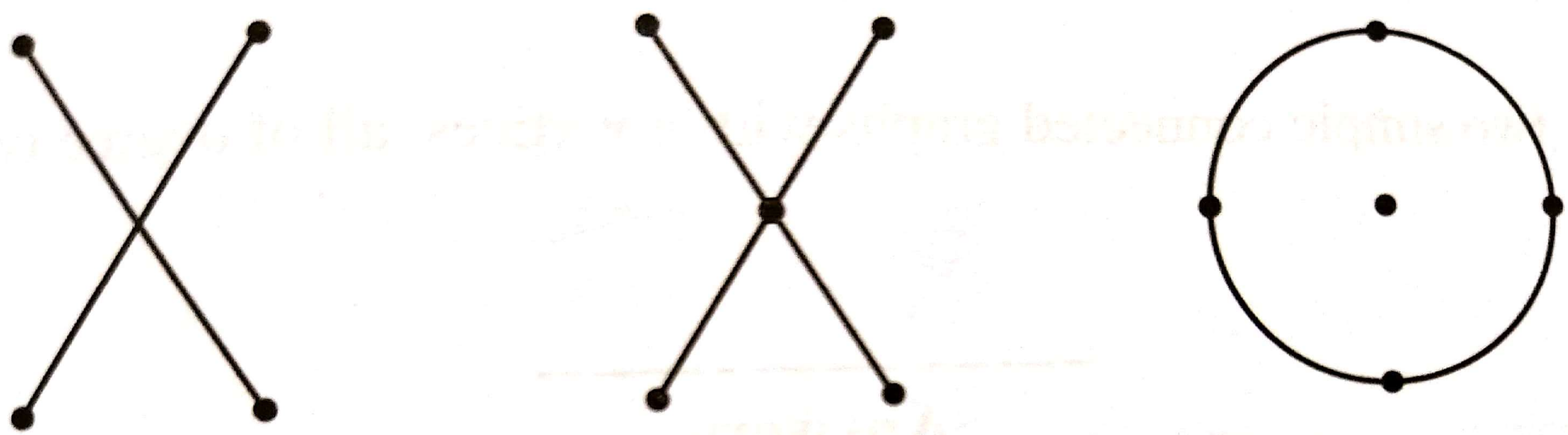
Def: Consider a graph G or order greater than or equal to two. Two vertices in G are said to be *CONNECTED* if there is at least one path from one vertex to the other.

Simply a graph G is a CONNECTED graph if every pair of distinct vertices in G are connected. Otherwise it is called as DISCONNECTED graph.





Indicate which of the following graphs are connected.



(a) (b) (c)

Connected and Disconnected Graphs

Note:

- In a graph G all walks and therefore, all trails, all circuits, all paths and all cycles are connected subgraphs of G.
- It is obvious that every graph G consists of one or more connected graphs. Each such connected graph is a subgraph of G and is called as COMPONENT of G.
- The number of components of graph G is denoted by K(G).
- How many components in connected graph?

Sol: Only one

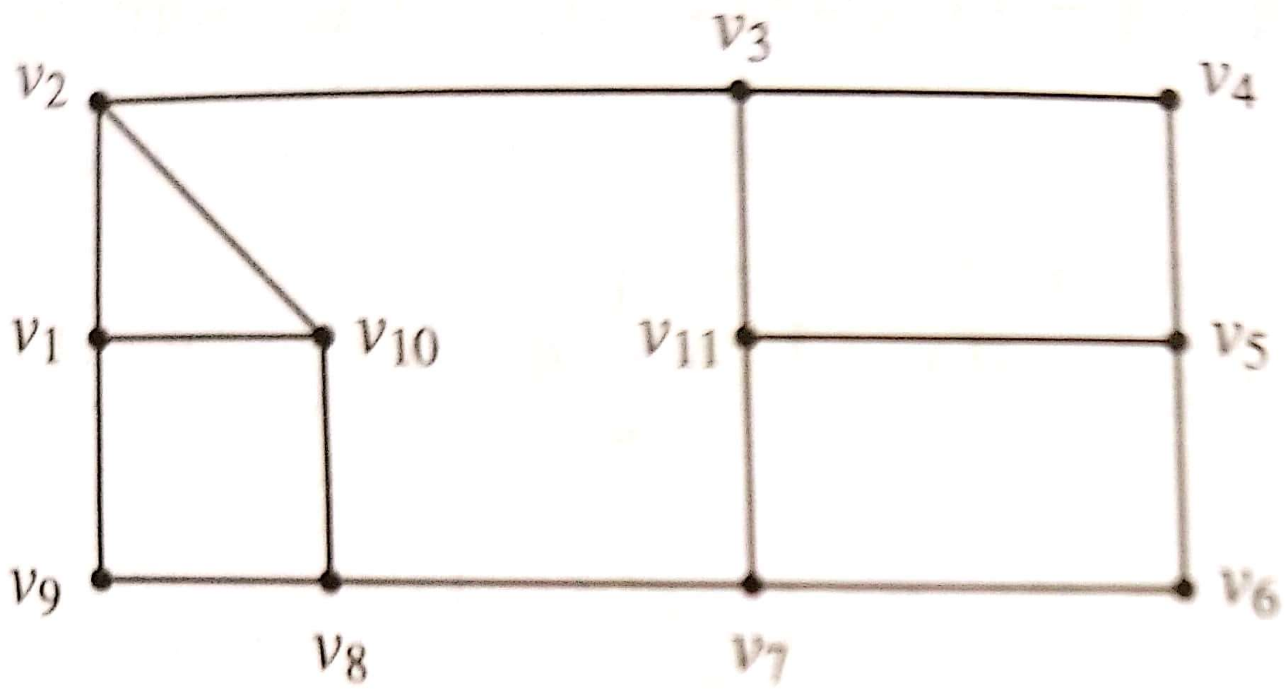
- How many components in Disconnected graph?

Sol: Two or More.

Connected and Disconnected Graphs

If *u* and *v* are two vertices in a connected graph G, then the length of the shortest path is called the ***DISTANCE*** between *u* and *v.*

Find the distance between the vertex v1 and the vertices v3, v5, v6 and v11 in the following graph.



Connected and Disconnected Graphs

PROPERTIES:

- If a graph has exactly two vertices of odd degree, then there must be a path connecting these vertices.
- A simple graph with n vertices and k components can have at most $\frac{1}{2}(n-k)(n-k+1)$ number of edges.
- If $m \geq \frac{1}{2}(n-1)(n-2)$ then a simple graph with n vertices and m edges is connected.
- A connected graph with n vertices has at least n-1 edges.
- A graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V1 and V2 such that there exists no edge in G whose one end vertex is in V1 and the other is in V2

PROBLEM

Prove that a connected graph G remains connected after removing an edge *e* from G if and only if *e* is a part of some cycle in G.

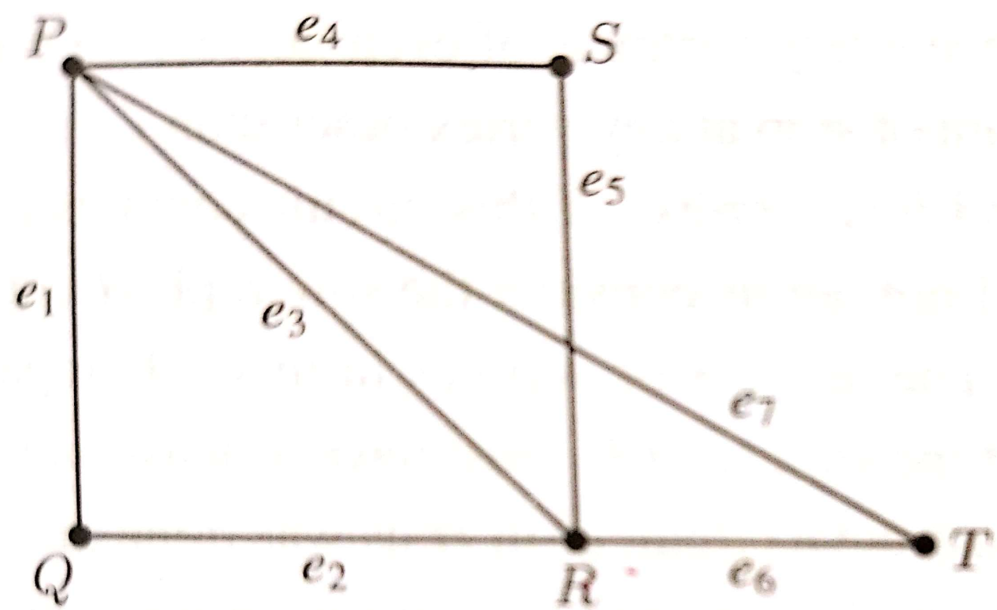
PROBLEM 2

Let G be a disconnected graph of even order n with two components each of which is complete. Prove that G has a minimum of n(n-2)/4 edges.

Euler Circuits and Euler Trails

Definition: Let G be a connected graph G. If there is a circuit in G that contains all the edges of G, then that circuit is called an ***EULER CIRCUIT*** in G.

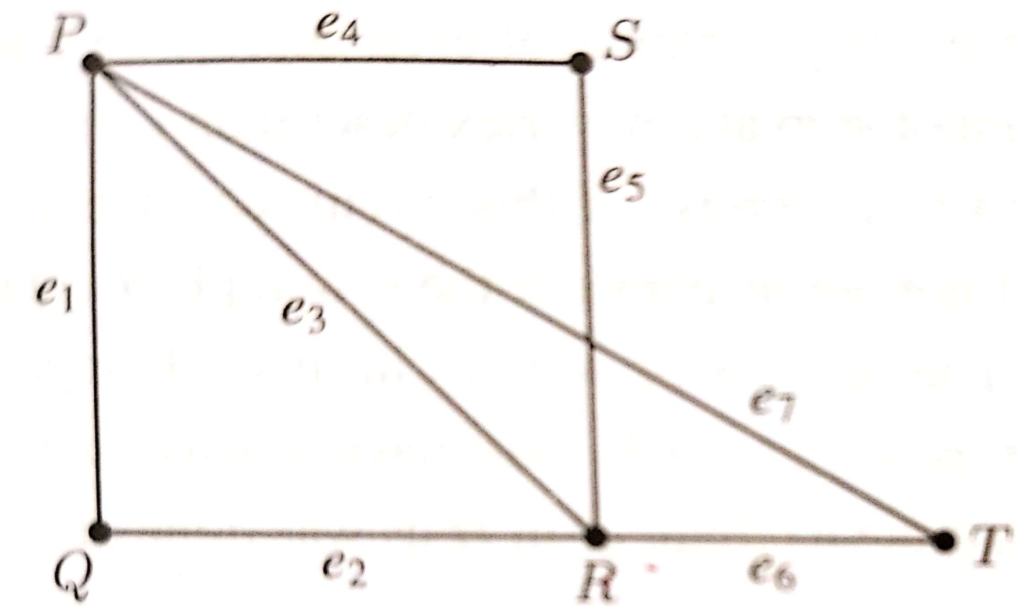
If there is a trail in G that contains all the edges of G, then that trail is called an ***EULER TRAIL*** in G



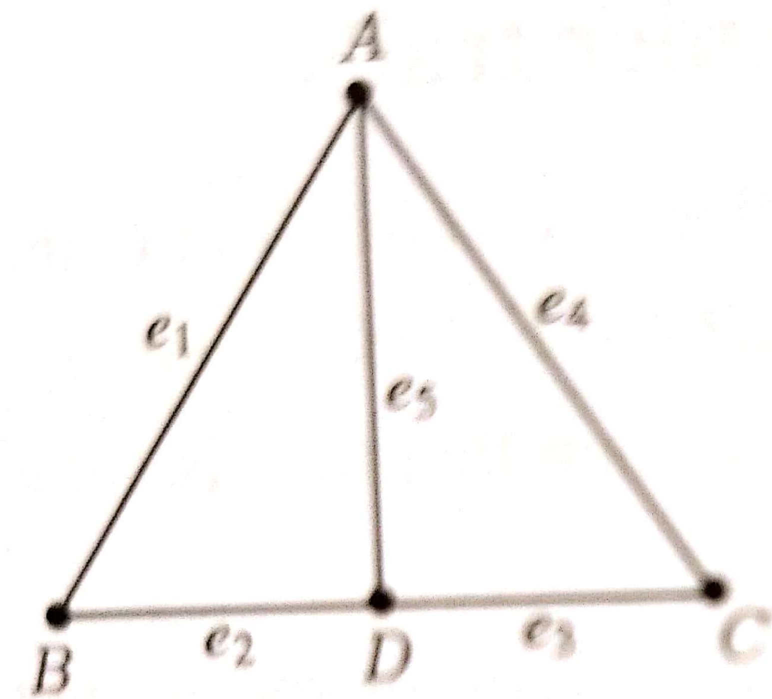
Euler and Semi-Euler Graph

Definition: A connected graph that contains an Euler circuit is called an EULER graph (or Eulerian graph).

A connected graph that contains an Euler trail is called a SEMI-EULER graph (or semi-Eulerian graph)



Find the following graphs are Euler or Semi-Euler graphs



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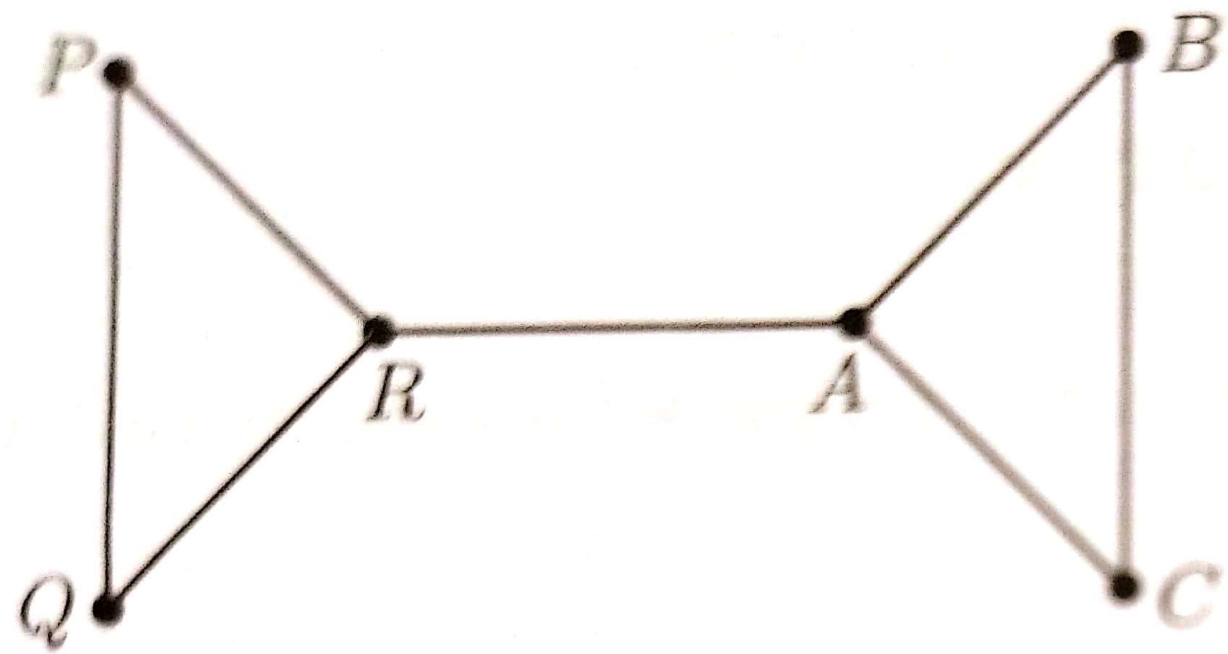

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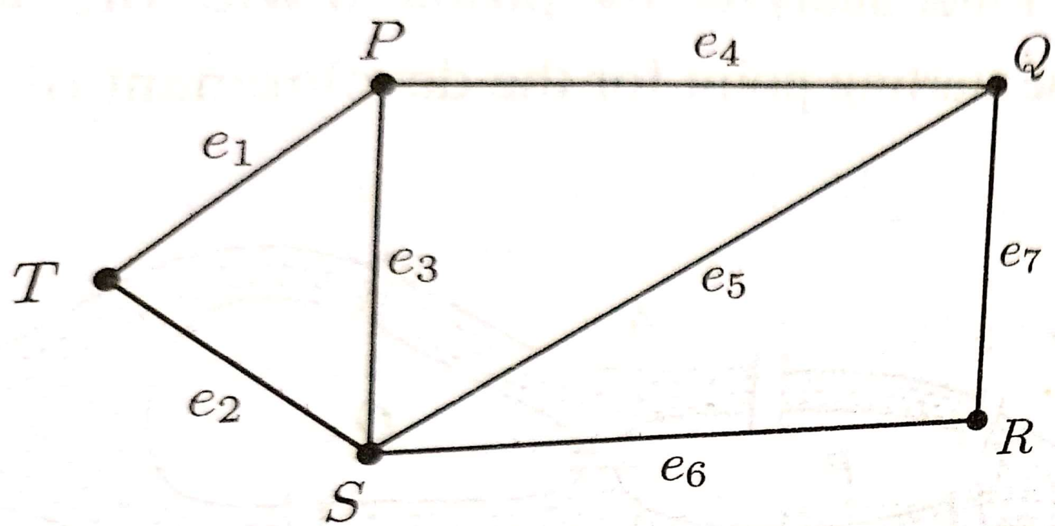


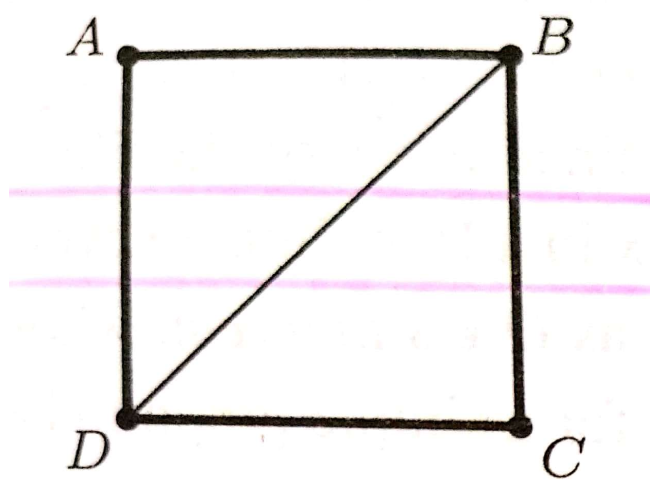
PROPERTIES

- A connected graph G has an Euler circuit if an only if all vertices of G are of even degree.
- A connected graph G has an Euler circuit if and only if G can be decomposed into edge-disjoint cycles.

Hamilton Cycles and Hamilton Paths

Definition: Let G be a connected graph. If there is a cycle (path) in G that contains all the vertices of G, then that cycle (path) is called a HAMILTON CYCLE (PATH) in G.

Definition: A graph that contains a Hamilton cycle is called a HAMILTON graph (or Hamiltonian graph)



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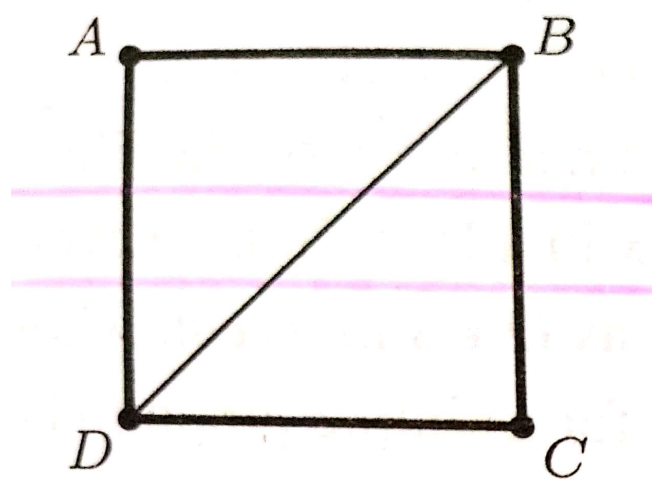
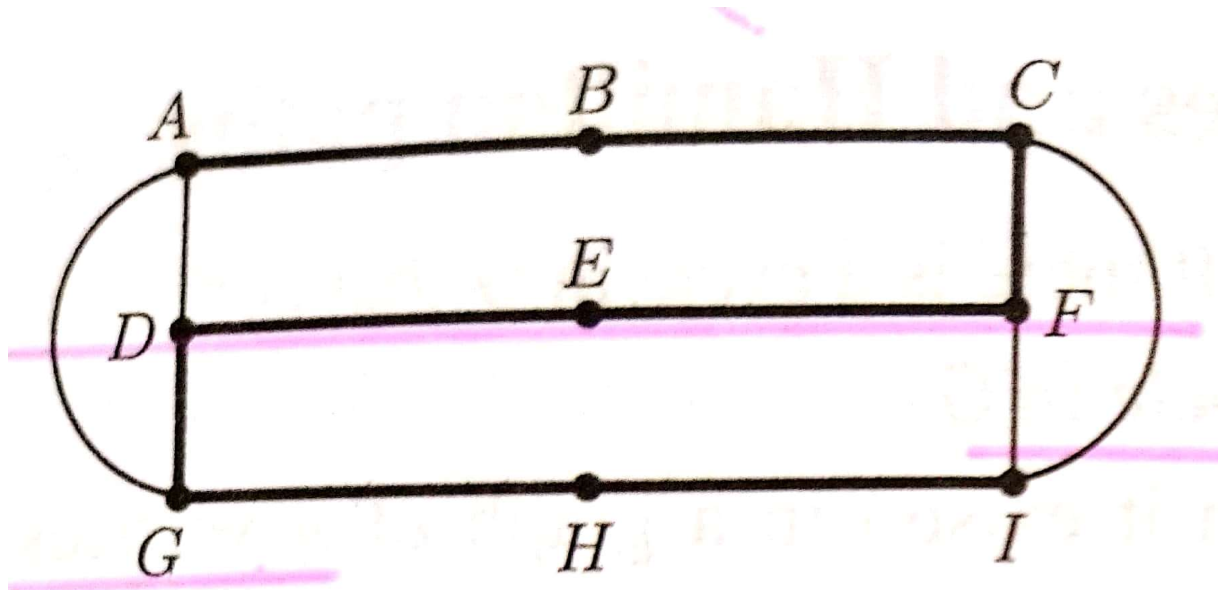


Verify the following are Hamiltonian graphs or Hamiltonian paths



Prove that the following graph is a Hamilton graph

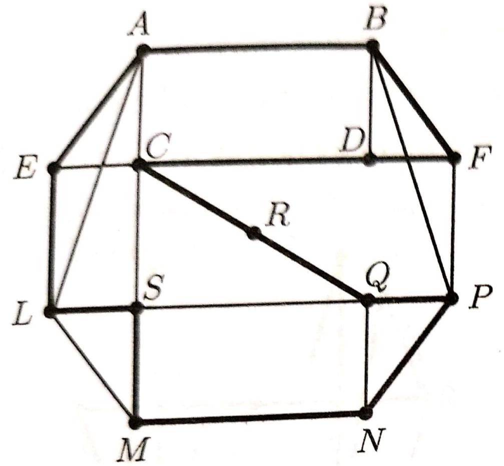
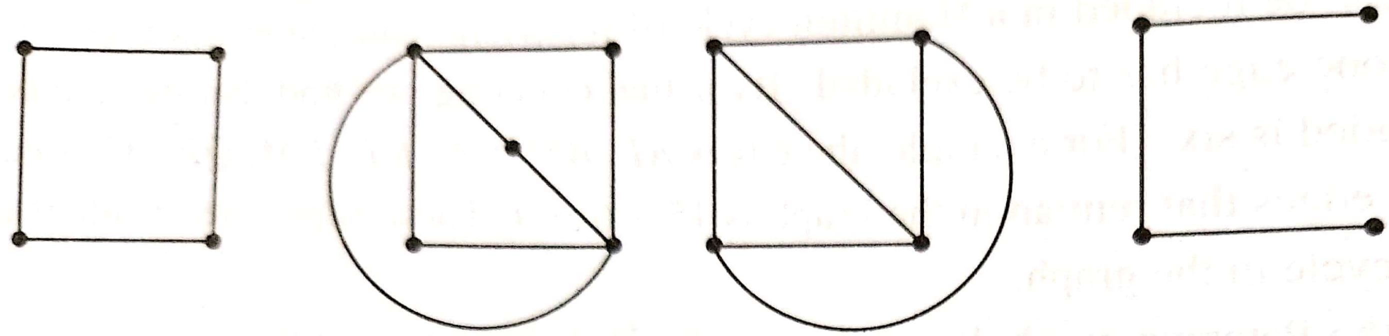


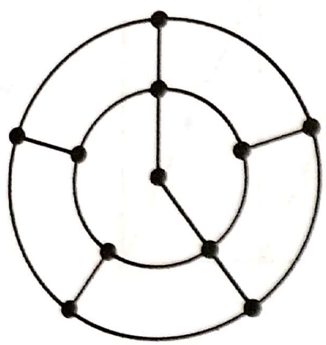
Exhibit the following:

- A graph which has both an Euler circuit and a Hamilton cycle.
- A graph which has an Euler circuit but no Hamilton cycle
- A graph which has a Hamilton cycle but no Euler circuit.
- A graph which has neither a Hamilton cycle nor an Euler circuit.

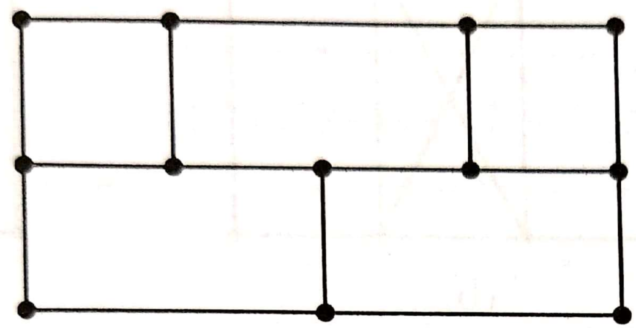


(a) (b) (c) (d)

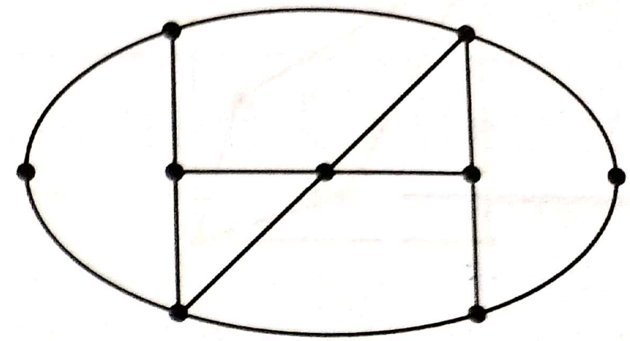
Which of the following are Hamiltonian graphs



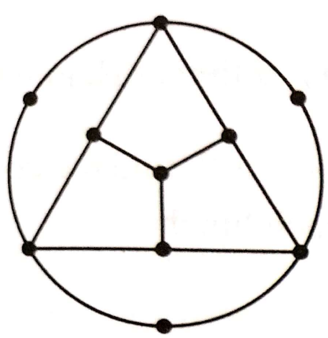
(a)



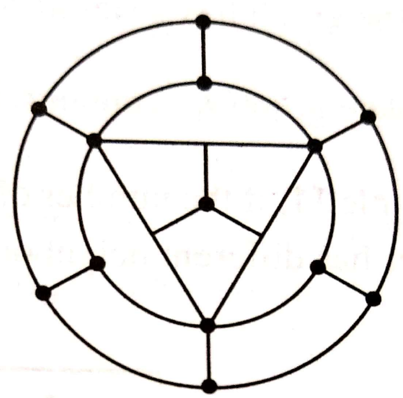
(b)



(c)

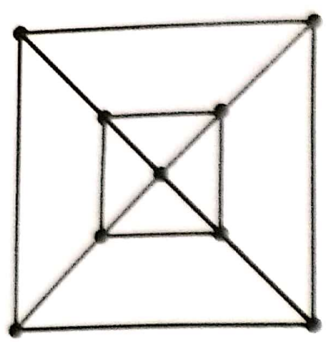


(a)

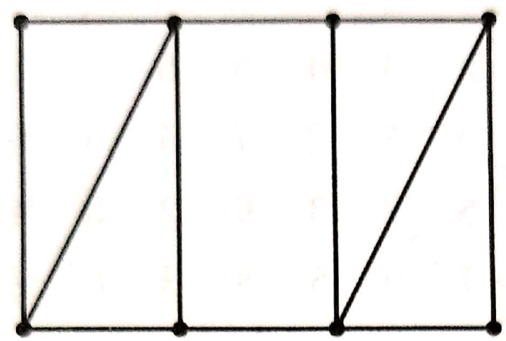


(b)

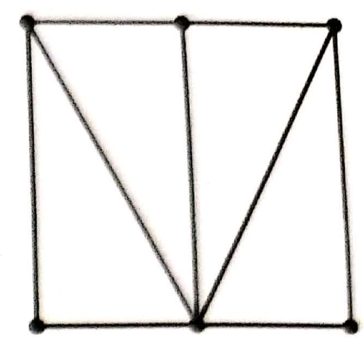
Prove that the following graphs are Hamiltonian but not Eulerian



(a)

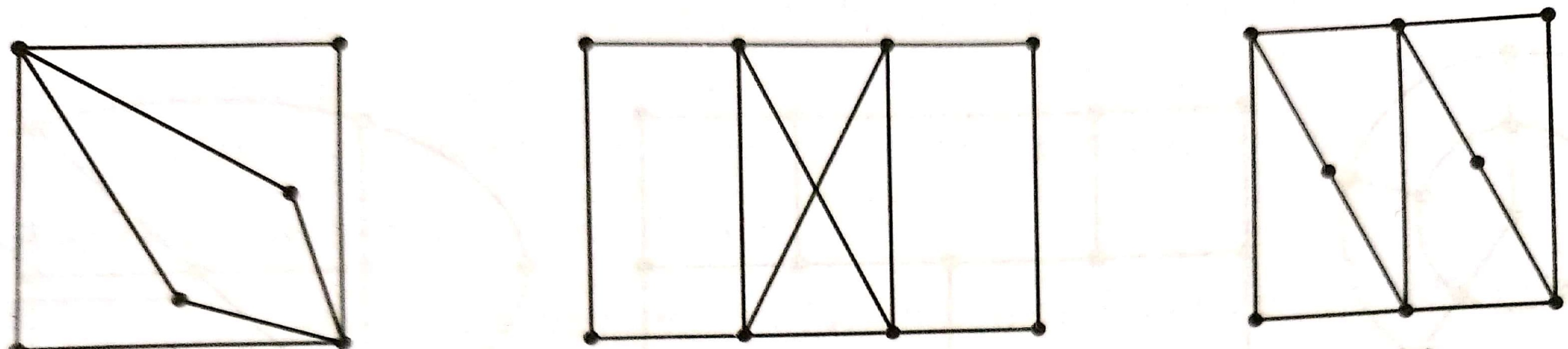


(b)



(c)

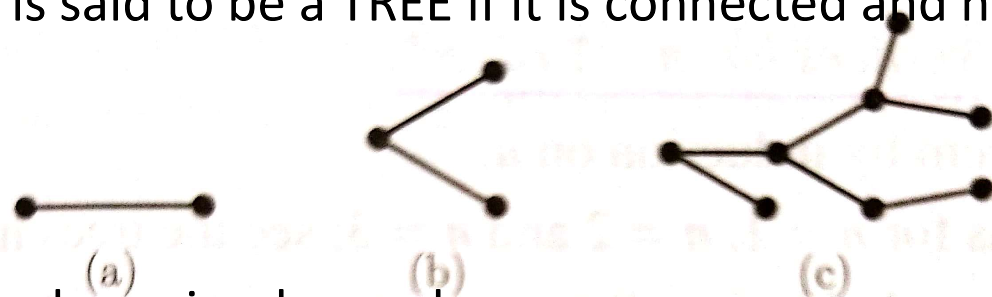
Which of the following graphs are Hamiltonian or Eulerian



(a) (b) (c)

TREES and THEIR BASIC PROPERTIES

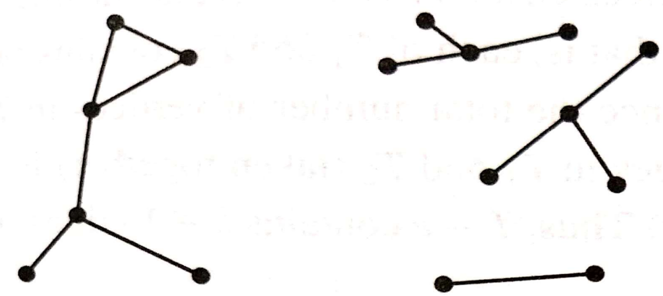
Definition: A graph G is said to be a TREE if it is connected and has no cycles.



Property: A tree has to be a simple graph

Def: Each tree possesses at least two pendant vertices. A pendant vertex of a tree is also called a LEAF.

Def: If each component of the disconnected graph is a tree, then that is called as a FOREST.



PROPERTIES

- A graph G is a tree if and only if there is one and only one path between every pair of vertices in G.
- A tree with n vertices has n-1 edges
- A graph with n vertices is a tree if and only of it is connected and has n-1 edges.
- A connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one cycle in G

SPANNING (SKELETON OR SCAFFOLDING) TREES

Definition: Let G be a connected graph. A subgraph T of G is called a SPANNING TREE of G if T is a tree which includes all vertices of G.

Since it includes all vertices of G it is also called as maximal subgraph of G.

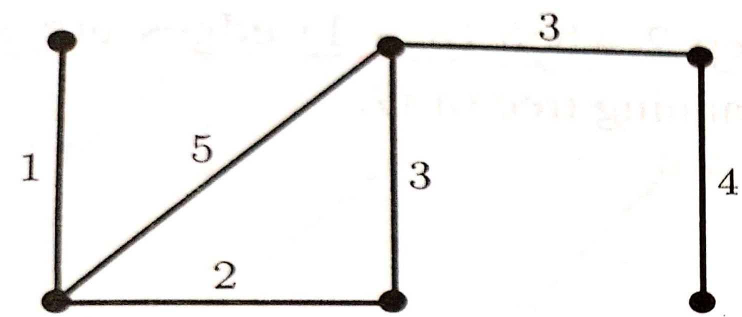
Every spanning tree is called as maximal tree.

The edges of a spanning tree are called its branches.

If T is a spanning tree of a graph G, then the edges of G which are not in T are called the chords of G with respect to T.

The set of all chords of G is called complement of T in G or chord-set of T in G which is denoted by $\overline{T}$

Therefore, $G = T \cup \overline{T}$

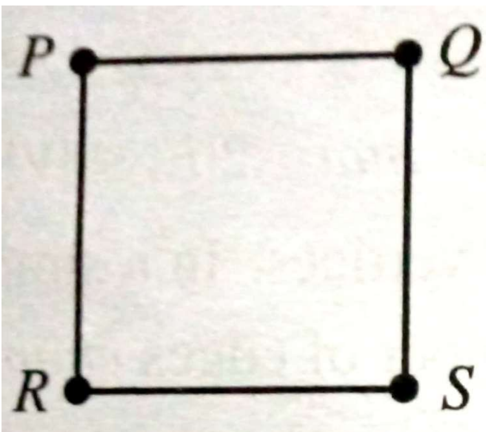


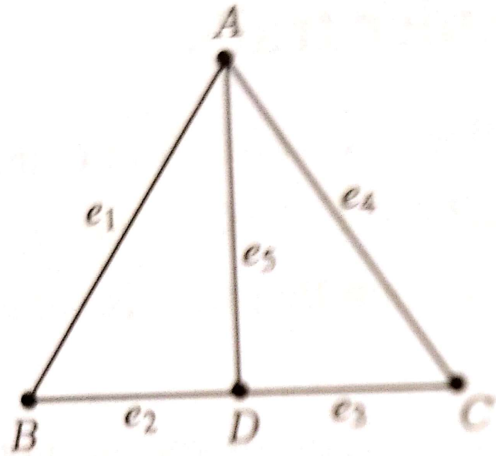
PROPERTIES

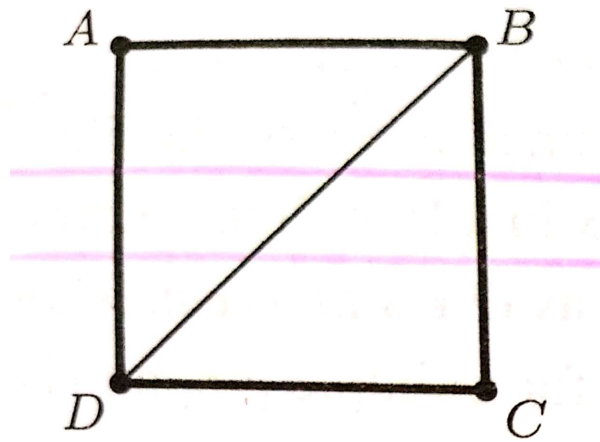
- A graph is connected if and only if it has a spanning tree.
- With respect to any of its spanning trees, a connected graph of n vertices and m edges has n-1 branches and m-n+1 chors

PROBLEM-1

Find all the spanning trees of the following graphs



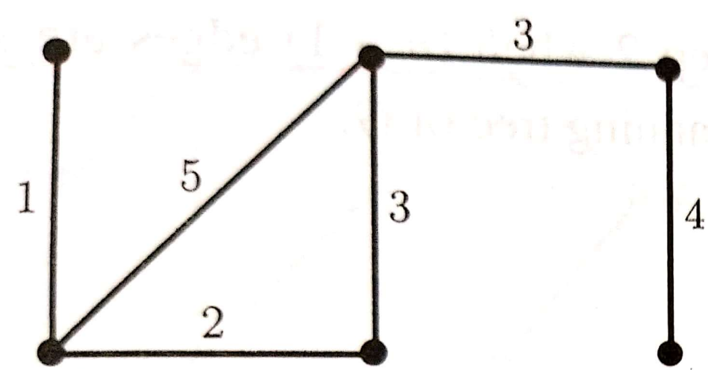




WEIGHTED GRAPH

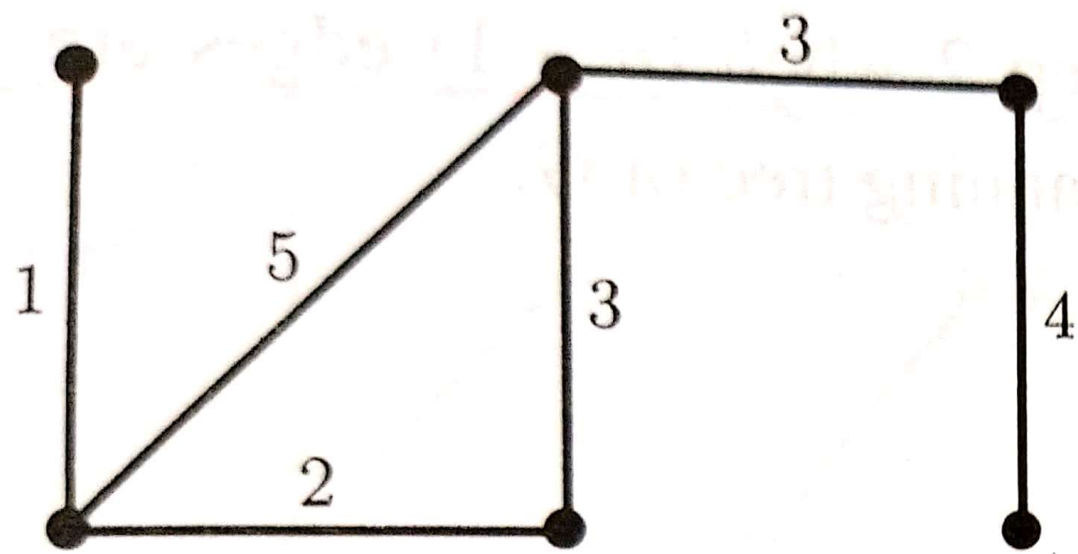
Definition: Let G be a graph and suppose there is a positive real number associated with each edge of G. Then G is called a WEIGHTED graph and the positive real number associated with an edge *e* is called the WEIGHT of the edge *e*.

If G is a connected, weighted graph, then the weight of an edge *e* of G is denoted by *wt(e)* and the weight of a spanning tree T of G is denoted by *wt(T)*.



MINIMAL SPANNING TREE

Def: A spanning tree whose weight is the least is called ***MINIMAL SPANNING TREE*** of the graph. This tree is not unique.



Algorithms for Minimal Spanning Tree

1) Kruskal’s Algorithm
2) Prim’s Algorithm

Algorithms for Minimal Spanning Tree

Kruskal's Algorithm: Steps involved in this are:

1) Given a connected, weighted graph G with n vertices, List the edges of G in the order of non-decreasing weiths.
2) Starting with a smallest weighted edge, proceed sequentially by selecting one edge at a time such that no cycle is formed.
3) Stop the process of Step 2 when (n-1) edges are selected.

These (n-1) edges constitute a MINIMAL SPANNING TREE of G.

Using Kruskal’s algorithm, find a minimal spanning tree of the following weighted graphs

Using Kruskal’s algorithm, find a minimal spanning tree of the following weighted graphs

Thank you