Table No.:	

NAME :
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SECTION :

Q1	
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Q3	
Total	

COLLEGE OF ENGINEERING
PUTRAJAYA CAMPUS
TEST 1 (SET 2)
SEMESTER 1, 2021/2022

PROGRAMME	: Bachelor of Electrical & Electronics Engineering (Honours) / Bachelor of Electrical Power Engineering (Honours)
SUBJECT CODE	: EEEB363/EEEB3024
SUBJECT	: Digital Signal Processing
DATE	: 14th October 2021
DURATION	: 2.5 hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **THREE (3)** questions.
2. **Answer all questions.**

DO NOT OPEN THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PRINTED PAGES INCLUDING THIS COVER PAGE

QUESTION 1 [40 MARKS]

(a) Calculate the conjugate anti-symmetric sequence of the complex signal $g[n]$ below.

$$g[n] = \{\ 1 - j1,\ \ -3 + j3,\ \ 5\ \}; \quad 0 \le n \le 2$$

[7 marks]

(b) Determine the linear convolution of the two signals given below:

$$x[n] = 8\,(0.5)^n\,(u[n] - u[n-4])$$

$$h[n] = 3\,\delta[n-1] + 4\,\delta[n-2]$$

where $u[n]$ is a digital unit step function, and $\delta[n]$ is a digital impulse function. Start by writing each signal in the form of a sequence of numbers.

[7 marks]

(c) Evaluate the normalized auto-correlation, ρ_{yy}, of the signal $y[n]$ in **Figure 1** below.

[7 marks]

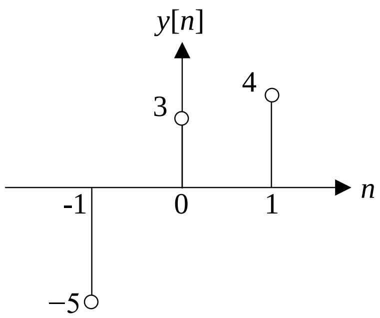

Figure 1: Signal *y*[*n*], for Question 1(c).

(d) Determine the frequency response, $H(e^{j\omega})$, from the linear time-invariant discrete-time system (LTI-DTS) described by the difference equation below where $x[n]$ and $y[n]$ are input and output of the system respectively:

$$y[n] - 0.3\,y[\,n-1\,] = x[n] + 1.1\,x[\,n-2\,] - 2.2\,x[\,n-3\,]$$

[7 marks]

(e) Let $Y(e^{j\omega})$ be the discrete-time Fourier Transform of the sequence

$$y[n] = \{ -1 , 3 , 2 \}; \quad 0 \leq n \leq 2$$

Determine $Y(e^{j\pi/2})$, the value of $Y(e^{j\omega})$ when $\omega = \pi/2$. Write the final answer in the form of a rectangular complex number.

[7 marks]

(f) Dr Zafri is supposed to sing the C note, which has a frequency of 261 Hz. But he sang too sharp at 271 Hz. You try countering his lack of talent by coming up with an autotune application. Your autotune app will correct the sharp note to become a perfect C note.

Using your knowledge of signal processing theorems, write the equation, $y[n]$, in terms of $x[n]$ that makes the conversion above. Include values in your equation. Use a sampling frequency of 30 kHz.

[5 marks]

<u>QUESTION 2 [30 MARKS]</u>

Consider the system in **Figure 2** with the input signal,

$$x(t) = 2 + \cos(100\pi t) + \frac{1}{2}\cos(200\pi t)$$

This input signal, $x(t)$ is sampled at every 1.0 ms, generating a sampled and digitized discrete-time sequence, $x[n]$.

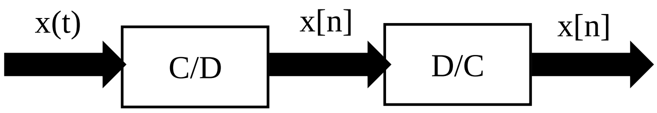

Figure 2: Input to Output signal conversion

(a) Evaluate and determine the frequency transform of $x(t)$ **AND** $x[n]$.

[6 marks]

(b) Sketch the $x(t)$ **AND** $x[n]$ signal spectrums.

[6 marks]

(c) Determine the time-domain equation of $x[n]$.

[4 marks]

(d) Determine aliasing condition of sampled signal, $x[n]$. Demonstrate and explain your answer.

[4 marks]

(e) The signal $x[n]$ is passed through an ideal lowpass filter with a cut-off frequency of $\omega_c = 0.18\pi$ to obtain an output signal, $y[n]$. Determine **AND** sketch the spectrum of output response, $Y(e^{j\omega})$.

[8 marks]

(f) What range of sampling frequency values would make the signal x[n] above be aliased?

[2 marks]

QUESTION 3 [30 MARKS]

(a) Consider a Discrete-Time Fourier transform (DTFT) of $x[n]$, $X(e^{j\omega})$ with real coefficients in the form of:

$$X(e^{j\omega}) = P_0 + P_1 e^{-j\omega} + \ldots + P_{M-1} e^{-j\omega(M-1)}$$

Let $X[K]$ denote the M-point DFT of the numerator coefficients $\{P_i\}$. **Derive** the exact expressions of the DTFT, $X(e^{j\omega})$ for $M = 4$ if the 4-point DFTs is given by

$$X[K] = \{-2,\ 1 + j, -2,\ 1 - j\};\quad 0 \leq K \leq 3$$

[10 marks]

(b) Compute the Discrete-Time Fourier Transform (DFT) of $x[n]$ using any Fast Fourier Transform (FFT) technique.

$$x[n] = \{1, 0, -2, 1\};\qquad 0 \leq n \leq 3$$

[15 marks]

(c) The five samples of the length-9 real sequence signal, $x[n]$ are given by $x[0] = 11,\ x[2] = 13,\ x[3] = 15,\ x[5] = 17$ and $x[8] = 19$. Determine the remaining four sample of $x[n]$.

[5 marks]

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Euler's relation: $e^{j\theta} = \cos\theta + j\sin\theta$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right), \sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

Convolution: $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

Autocorrelation: $r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l]$

Discrete Fourier Transform, DFT Pair:

$$X[k] = \sum_{n=0}^{N-1} x[n]\mathrm{W}_N^{kn}, \quad 0 \le k \le N-1$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\mathrm{W}_N^{-kn}, \quad 0 \le n \le N-1$$

$$\mathrm{W}_\mathrm{N} = e^{-j2\pi/N}$$

Sampling process equation,

$$X_p(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega - k\Omega_T))$$

Discrete-time Fourier Transform (DTFT) Pair:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

Symmetry Properties of DFT of complex sequence:

$$x[n] = x_{re}[n] + jx_{im}[n] \Leftrightarrow X[k] = X_{re}[k] + jX_{im}[k]$$

$$x^*[n] \Leftrightarrow X^*[<-k>_N]$$

$$x^*[<-k>_N] \Leftrightarrow X^*[k]$$

$$x_{re}[n] \Leftrightarrow X_{cs}[k] = \frac{1}{2}\left\{X[k] + X^*[<-k>_N]\right\}$$

$$jx_{im}[n] \Leftrightarrow X_{ca}[k] = \frac{1}{2}\left\{X[k] - X^*[<-k>_N]\right\}$$

$$x_{cs}[n] \Leftrightarrow X_{re}[k]$$

$$x_{ca}[n] \Leftrightarrow jX_{im}[k]$$

Table 1 Fourier Transform Pairs

	Sequence	Fourier Transform
1.	$\delta[n]$	1
2.	$\delta[n-n_0]$ (n_0 an integer)	$e^{-j\omega n_0}$
3.	$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4.	$a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{1-ae^{-j\omega}}$
5.	$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6.	$(n+1)a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
7.	$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p}u[n] \quad (\|r\| < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8.	$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \|\omega\| < \omega_c \\ 0, & \omega_c < \|\omega\| < \pi \end{cases}$
9.	$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10.	$e^{j\omega_0 n}$	$2\pi\sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + 2\pi k)$
11.	$\cos(\omega_0 n + \phi)$	$\pi\sum_{k=-\infty}^{\infty}\left[e^{j\varphi}\delta(\omega-\omega_0+2\pi k) + e^{-j\varphi}\delta(\omega+\omega_0+2\pi k)\right]$
12.	$\sin(\omega_0 n + \phi)$	$\frac{\pi}{j}\sum_{k=-\infty}^{\infty}\left[e^{j\varphi}\delta(\omega-\omega_0+2\pi k) - e^{-j\varphi}\delta(\omega+\omega_0+2\pi k)\right]$

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Table 2 Fourier Transform Theorems

	Sequence, $x[n]$, $y[n]$	Fourier Transform, $X(e^{j\omega})$, $Y(e^{j\omega})$
1.	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$
2.	$x[n-n_0]$ (n_0 an integer)	$e^{-j\omega n_0}X(e^{j\omega})$
3.	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
4.	$x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5.	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6.	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7.	$x[n]y[n]$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

Parseval's theorem

8. $\sum_{n=-\infty}^{\infty}|x[n]|^2=\dfrac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^2d\omega$ and $\sum_{n=-\infty}^{\infty}x[n]y^*[n]=\dfrac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})Y^*(e^{j\omega})d\omega$

Table 3 Some Common z-Transform Pairs

	Sequence	Transform	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert>1$
3.	$-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert<1$
4.	$\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
5.	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
6.	$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
7.	$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$
8.	$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert<\lvert a\rvert$
9.	$[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
10.	$[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
11.	$[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
12.	$[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
13.	$\begin{cases}a^n, & 0\le n\le N-1\\ 0, & \text{otherwise}\end{cases}$	$\dfrac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert>0$