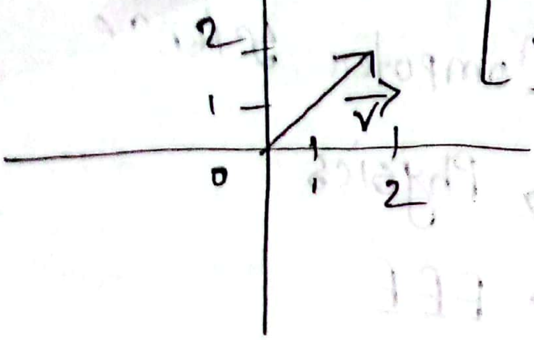


$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

→ There are 3 distinct but related ideas about vectors.

Phy stu perspective: Vectors are arrows pointing in space.

CS stu perspective: Vectors are ordered list of numbers.

ex: Say that I am doing some analytics about house prices, and the only features I cared about were square footage and price. I might model each house with a pair of numbers: the first indicating square footage and the second indicating price.

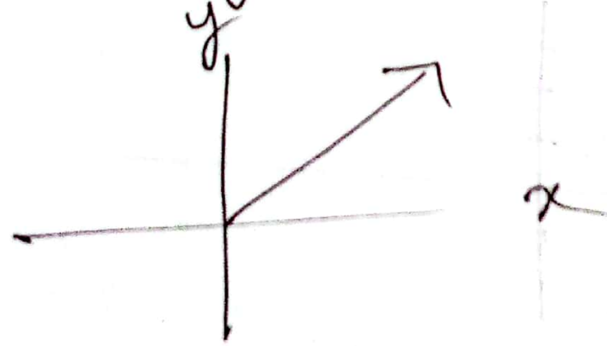
$$\begin{bmatrix} 2600\ ft^2 \\ \$\,300{,}000 \end{bmatrix} \neq \begin{bmatrix} 300000\ ft^2 \\ \$\,2600 \end{bmatrix}$$

The order matters here

In the lingo, you'd be modeling houses as 2D vectors. In this context, what makes it 2D is the fact that the length of that list is 2.

$$\left.\begin{bmatrix} 2600\ ft^2 \\ \$300{,}000 \end{bmatrix}\right\} 2D$$

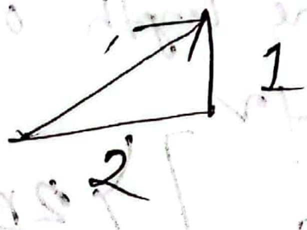
* Whenever a new topic involving vector is introduced, I should think about an arrow — and specifically think about that arrow inside a coordinate system, like the x-y plane, with its tail sitting at the origin.



In linear algebra it's almost always the case that ~~my~~ your vector will be rooted at the origin.

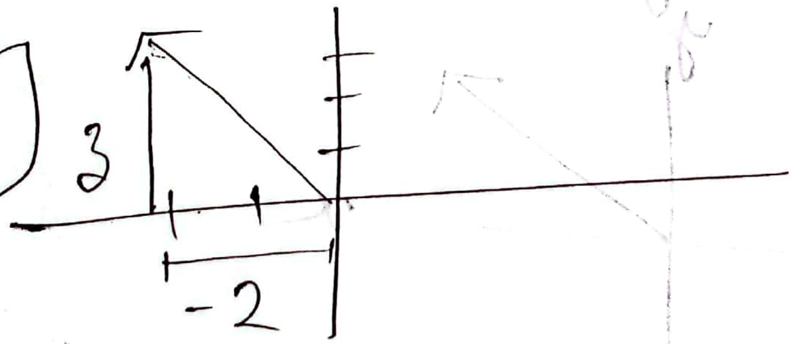
After considering this, we'll translate it over to the list-of-numbers point-of-view, which we can do by considering the coordinates of the vector

$\begin{bmatrix}2\\1\end{bmatrix} \Longleftrightarrow$



* The coordinates of a vector is a pair of numbers that basically give instructions ~~for~~ how to get from the tail ~~to~~ of that vector — at the origin — to its tip

$\begin{bmatrix}-2\\3\end{bmatrix}$

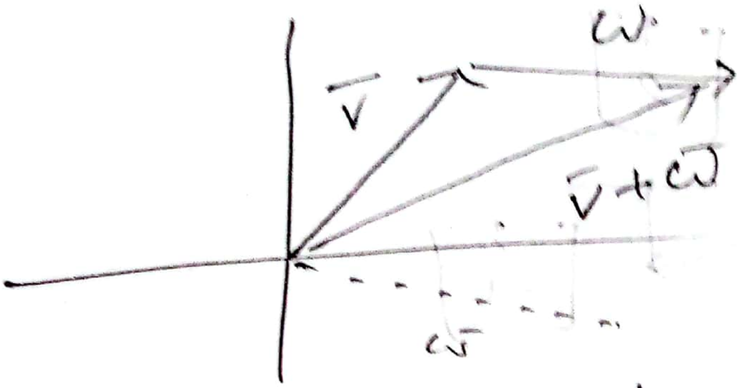


* In 3-D

$\begin{bmatrix}x\\y\\z\end{bmatrix}$, x represents how far to move along the x axis
, y represents how far to move parallel to the y axis
z represents how far to move parallel to the z axis

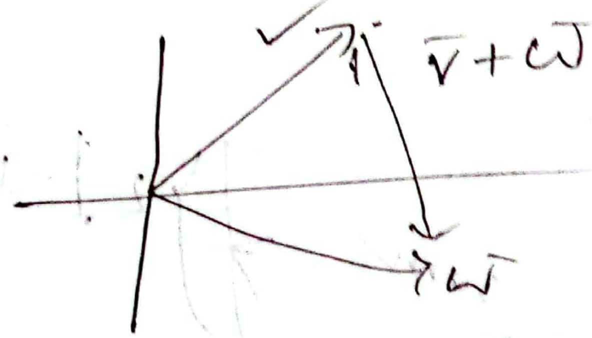
* Every topic in linear algebra is going to center around these two operations.

*** <u>Vector addition:-</u>



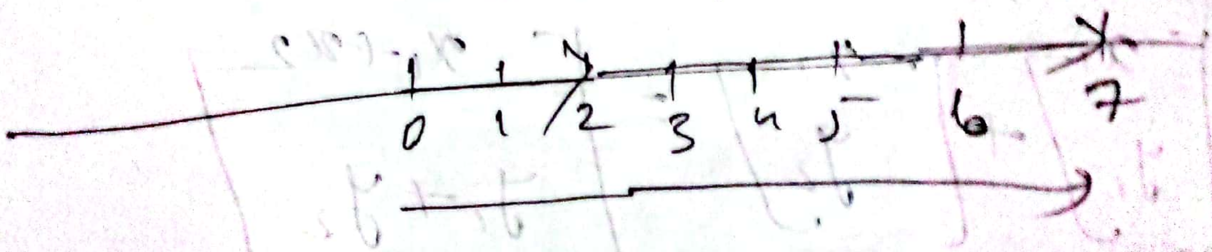
this is the only time when we let vector stay away from its origin.

• But why does this method work and why doesn't ~~the~~ the others? like . .



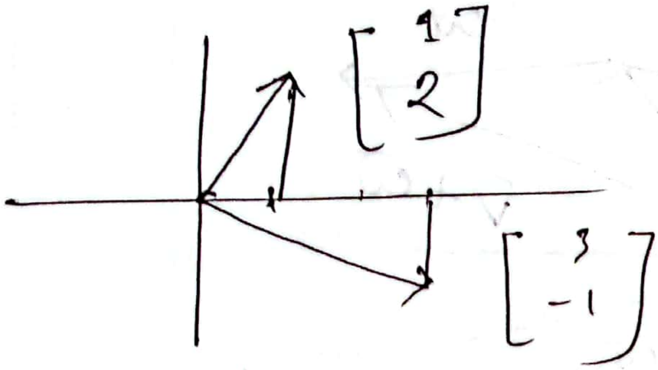
→ Each vector represent a certain movement. this is the same as,

$$2+5=7$$



* vector addition numerically :-

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$



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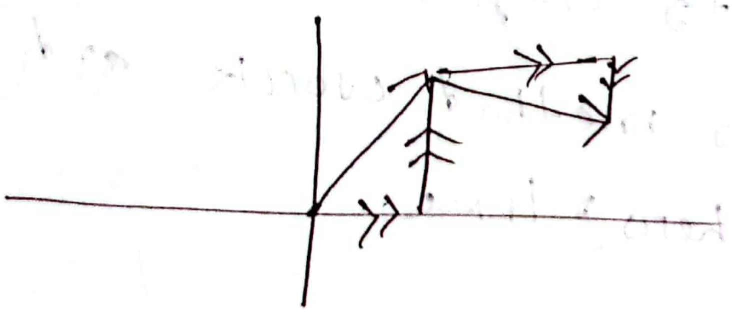
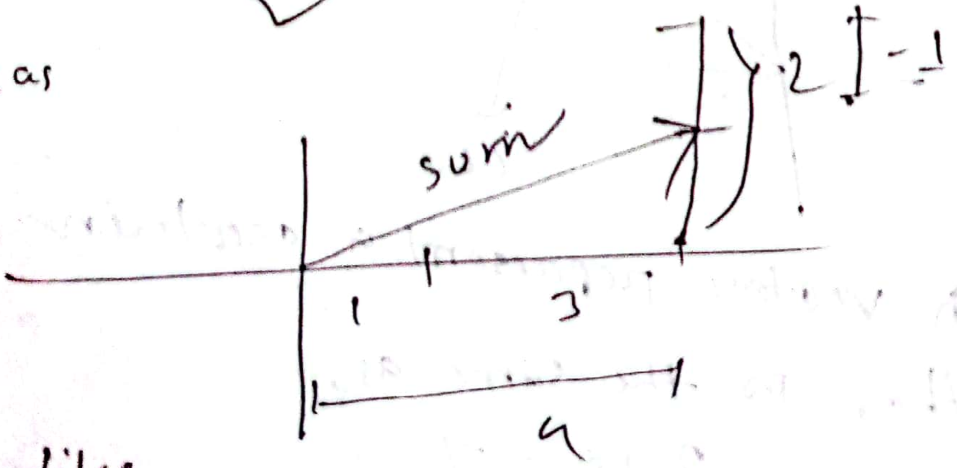


||

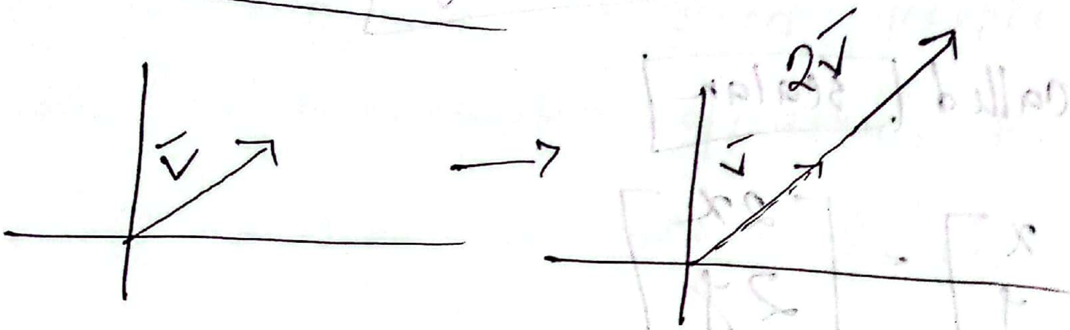
same as



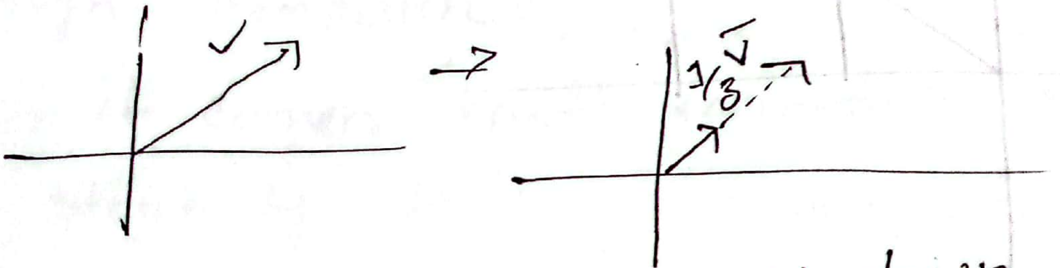
It's like,

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$$

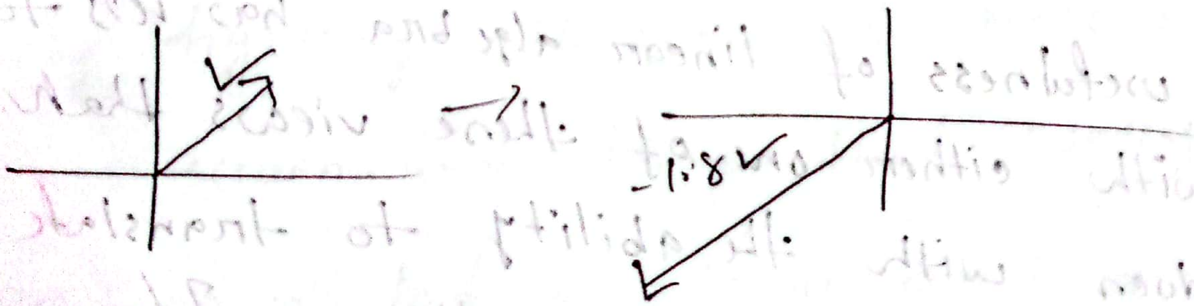
* multiplication by a number:



$2\bar{v}$ means stretching out the vector so that it's 2 times as long as when I started



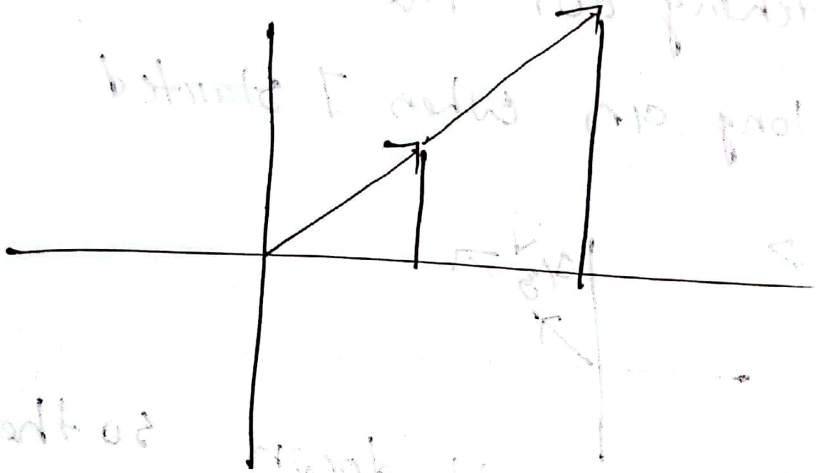
$\frac{1}{3}\bar{v}$ means squishing it down so that it's $1/3$ of the original length

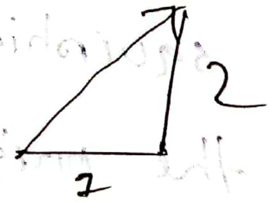


First it ~~flipped~~ gets flipped around and then streched out by that factor of 1.8

This is called $\boxed{\text{scaling}}$ & the num is called $\boxed{\text{scalar}}$

$$2\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2x\\2y\end{bmatrix}$$



*** $\begin{bmatrix}1\\2\end{bmatrix} \Longleftrightarrow$ 

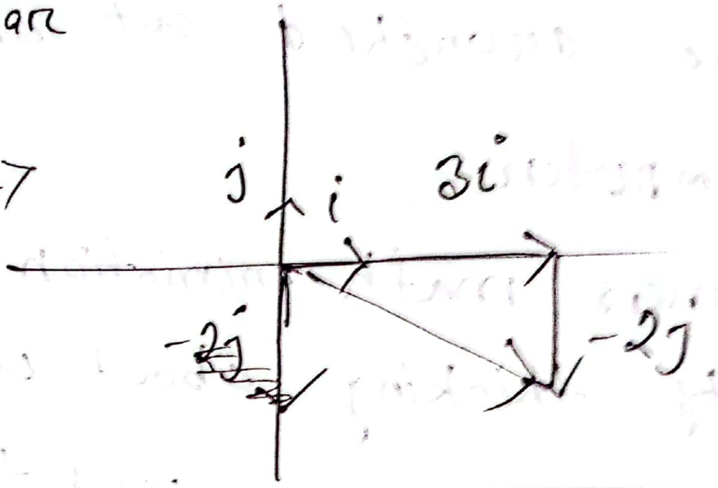
The usefulness of linear algebra has less to do with either one of these views than it does with the ability to translate back & forth between them. It gives the data analyst a nice way to conceptualize many lists of numbers in a visual way, which can seriously clarify patterns in data, and give a global view of what certain operations do. and on the flip side, it gives people like physicists and computer graphics programmers a language to describe space & the manipulation of space using numbers that can be crunched ~~out~~ and run through computers.

when it comes math animation, for ex one start by thinking about what's actually going on in space, and then get the computer to represent things numerically, thereby figuring out where to place the pixels on the screen. and doing that usually relies on a lot of linear algebra understanding

* There's another way to think about these coordinates that is pretty central to linear algebra.

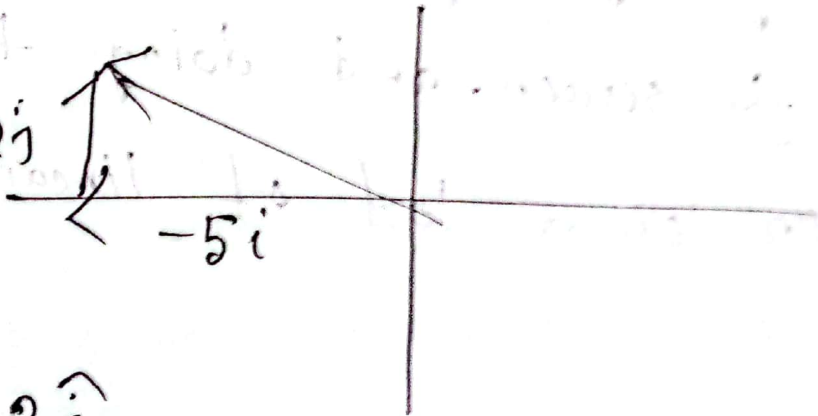
→ Think of each coordinate as a scalar

$\begin{bmatrix} 3 \\ -2 \end{bmatrix} \rightarrow$



$\hat{i}$ & $\hat{j}$ are the $\boxed{\text{basis vectors}}$ of the xy coordinate system.

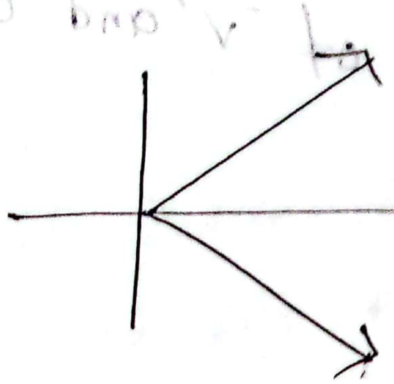
$\begin{bmatrix} -5 \\ 2 \end{bmatrix} \rightarrow$



$(-5\hat{i}) + 2\hat{j}$

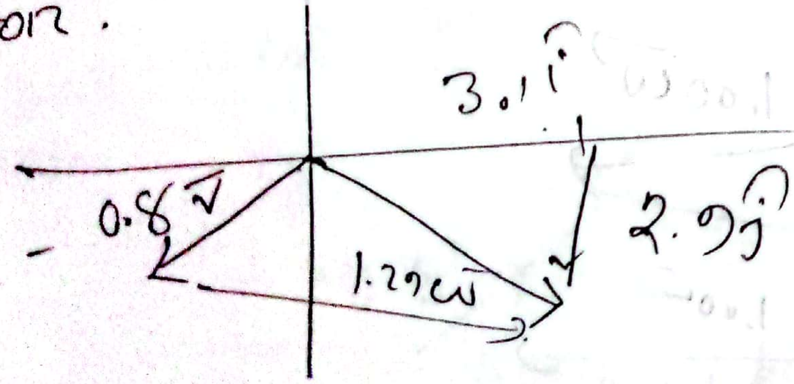
* what if we chose different basis vectors?

→ for ex, take some vector pointing up and to the right, along with some other vector pointing down ~~and~~ and to the right, in some way.



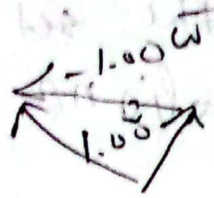
Take a moment to think about all the different vectors that you can get by choosing two scalars, using each one to scale one of the vectors, then adding together ~~what~~ of the vectors,

Then by altering the choices of scalars we ~~will get~~ can reach every possible 2D vector.



$\begin{bmatrix} -0.80 \\ 1.10 \end{bmatrix} +$

$\begin{bmatrix} 3.1 \\ 2.9 \end{bmatrix}$

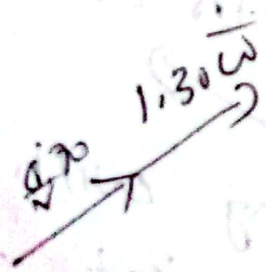


then the tip of the resultant vetor drawn a straight line.

(ii) Now if you let both scalars range freely, and consider every possible vector that you can get, there will happen 2 things:

— for most of vectors you'll be able to reach every possible point in the plane.

— In the unlucky case where your two original vectors happen to line up, the tip of the resulting vector is limited to just this sigle line passing through the origin



— both your vector could be zero in which case you'd just be stuck at the origin.

The 'span' of $\vec{v}$ and $\vec{w}$ is the set of all their linear combinations,

$$a\vec{v} + b\vec{w}$$

let a & b vary over all real numbers.

** the span of 2 vectors is basically a way of asking what are all the possible vectors you can reach using only these 2 fundamental operations, vector addition & scalar multiplication?

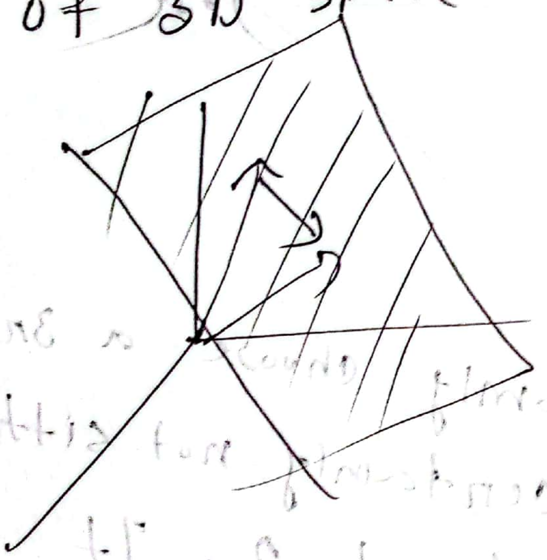
Vectors vs points:

→ In general, if you're thinking about a vector on its own, think of it as an arrow .

→ If you're dealing with a collection of vectors, it's convenient to think of them all as points

② What does the span of two 3d vectors look like?

→ It indicates all the possible vectors if we take a flat sheet, cutting through the origin of 3D space



~~of~~ ~~of~~

linear combination of $\bar{v}, \bar{w}, \bar{u}$:

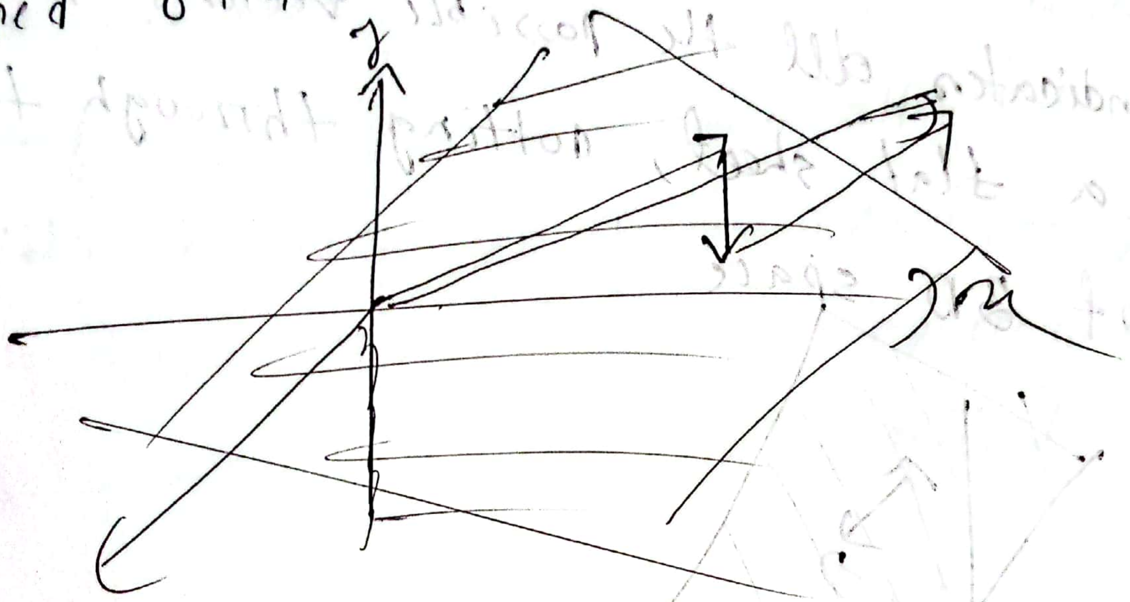
$$a\bar{v} + b\bar{w} + c\bar{u}$$

scalars

for span, lets these vary

Two different things could happen here,

— If 3rd vector happens to be sitting on the span of the first 2 then the span doesn't change, you're short of trapped on the same flat sheet



— If you just randomly choose a 3rd vector, it's almost certainly not sitting on the span of the first 2, It unlocks access to every possible 3D vector.

another way to think that the 3rd vector moves around that span sheet of the ~~flat~~ first 2, sweeping it through all of space.

— In the case where the 3rd vector was already sitting on the span of the 1st two on the case where 2 vectors happen to line up we want some terminology to describe the fact that at least of these vectors is redundant.

The relevant terminology is to say that they are linearly dependent,

"Linearly dependent"

$$\vec{u} = a\vec{v} + b\vec{w}$$

for some values of a & b

[one of vectors can be expressed as a linear combination of the others since its already in the span of others]

→ on the other hand if each vector really does add another dimension to the span, "Linearly independent"

$\vec{w} \neq a\vec{v} + b\vec{w}$

for all values of a

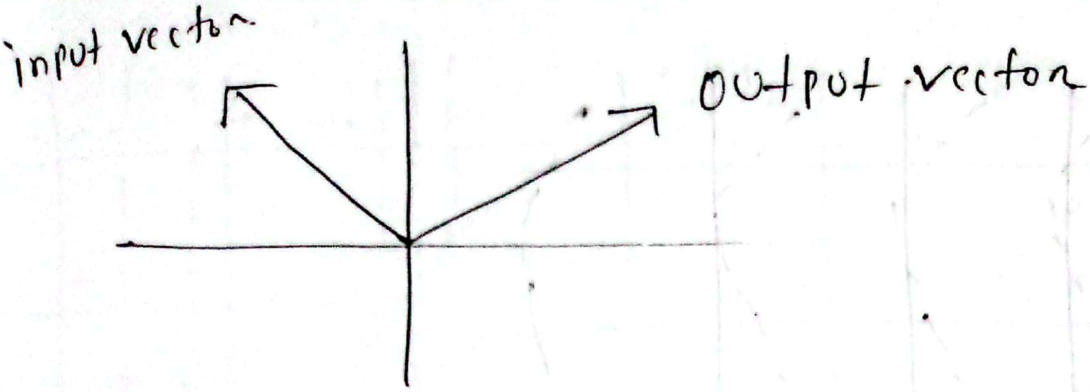
Technical definition of basis: The basis of a vector space is a set of linearly independent vectors that span the full space.

Linear Transformation:

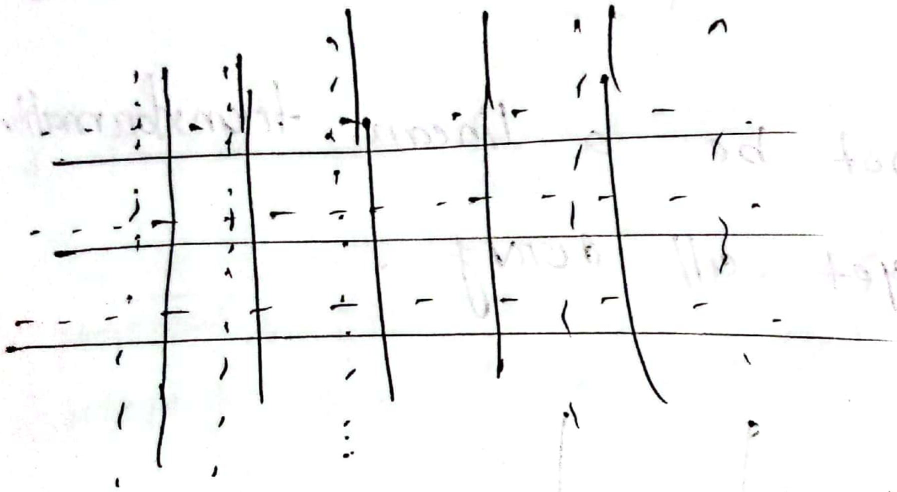
→ transformation is a fancy word for function

→ why we use this terminology instead of function?

→ the word "transformation" suggests that you think using movement



here, the input vector just moving over to the output vector.

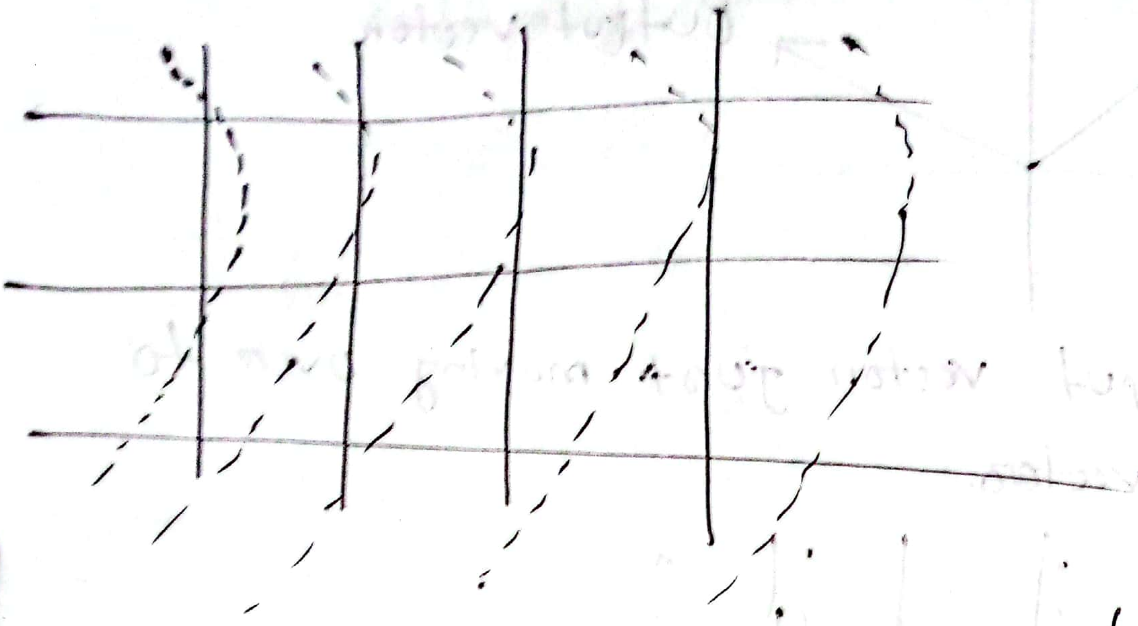


here, the vectors showing us some movement.

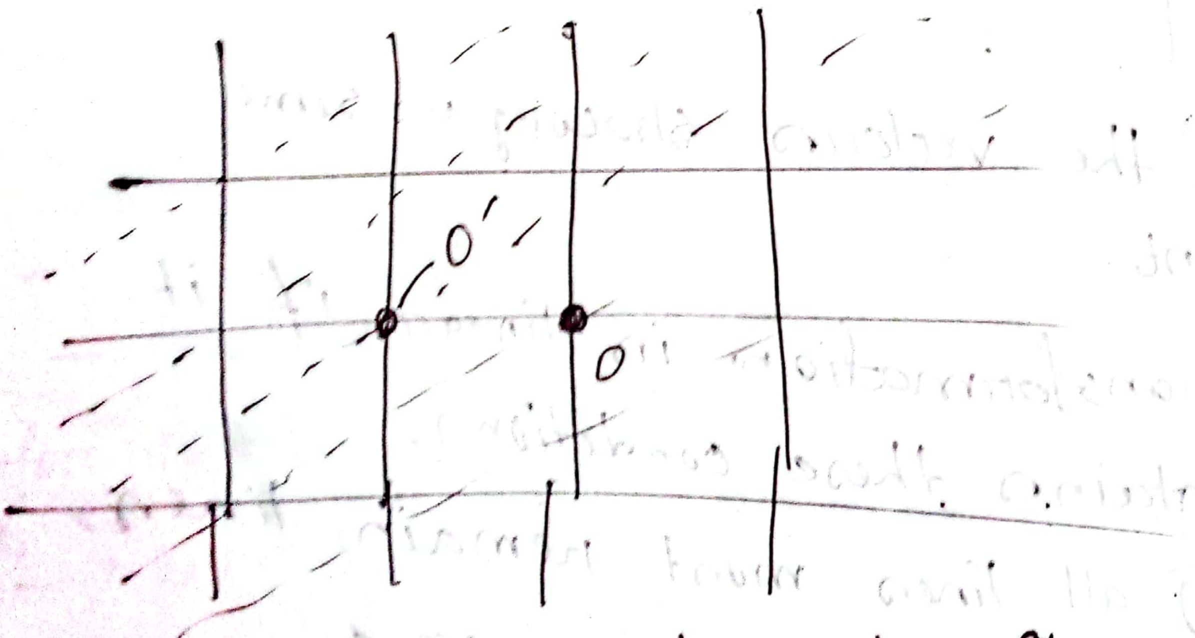
→ A transformation is linear if it maintains these conditions.

① all lines must remain lines, without getting curved

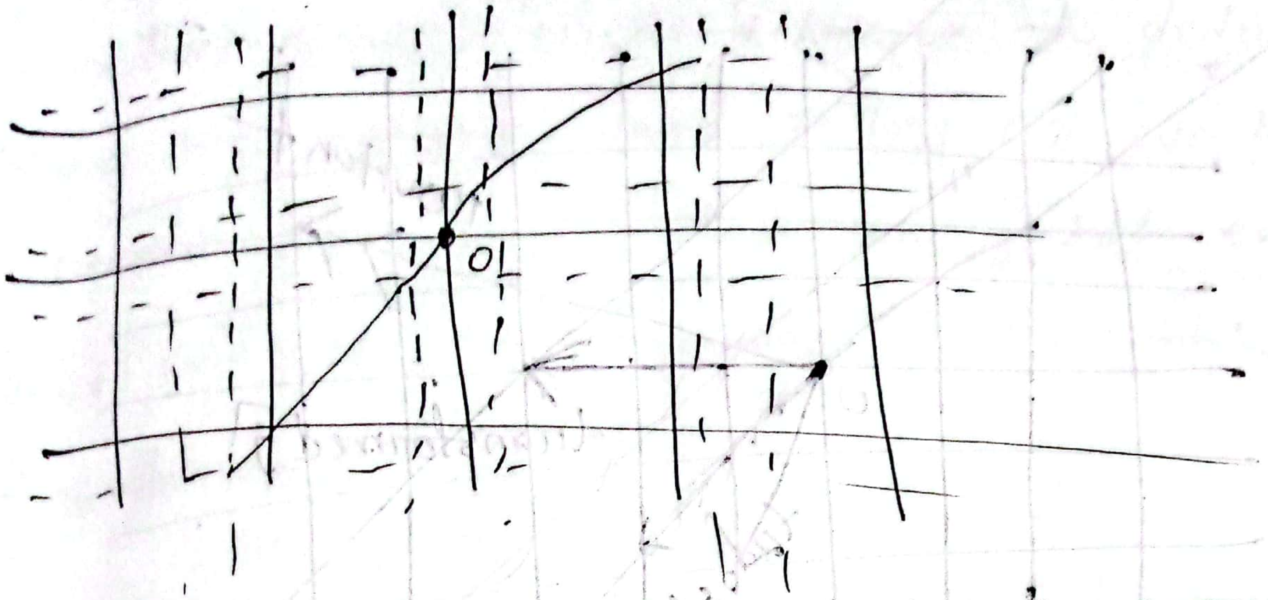
② origin must remain fixed in place



This would not be a linear transformation since the lines get all curvy.



these one is also not a linear transformation because it moves the origin.



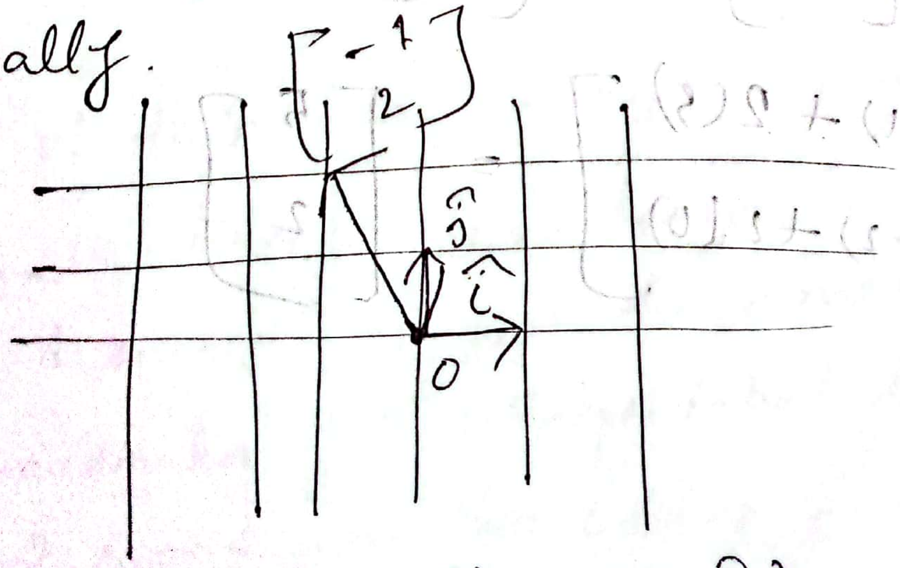
not linear as the diagonal gets curved.

In general, you should think of linear transformation as keeping grid lines parallel and evenly spaced.

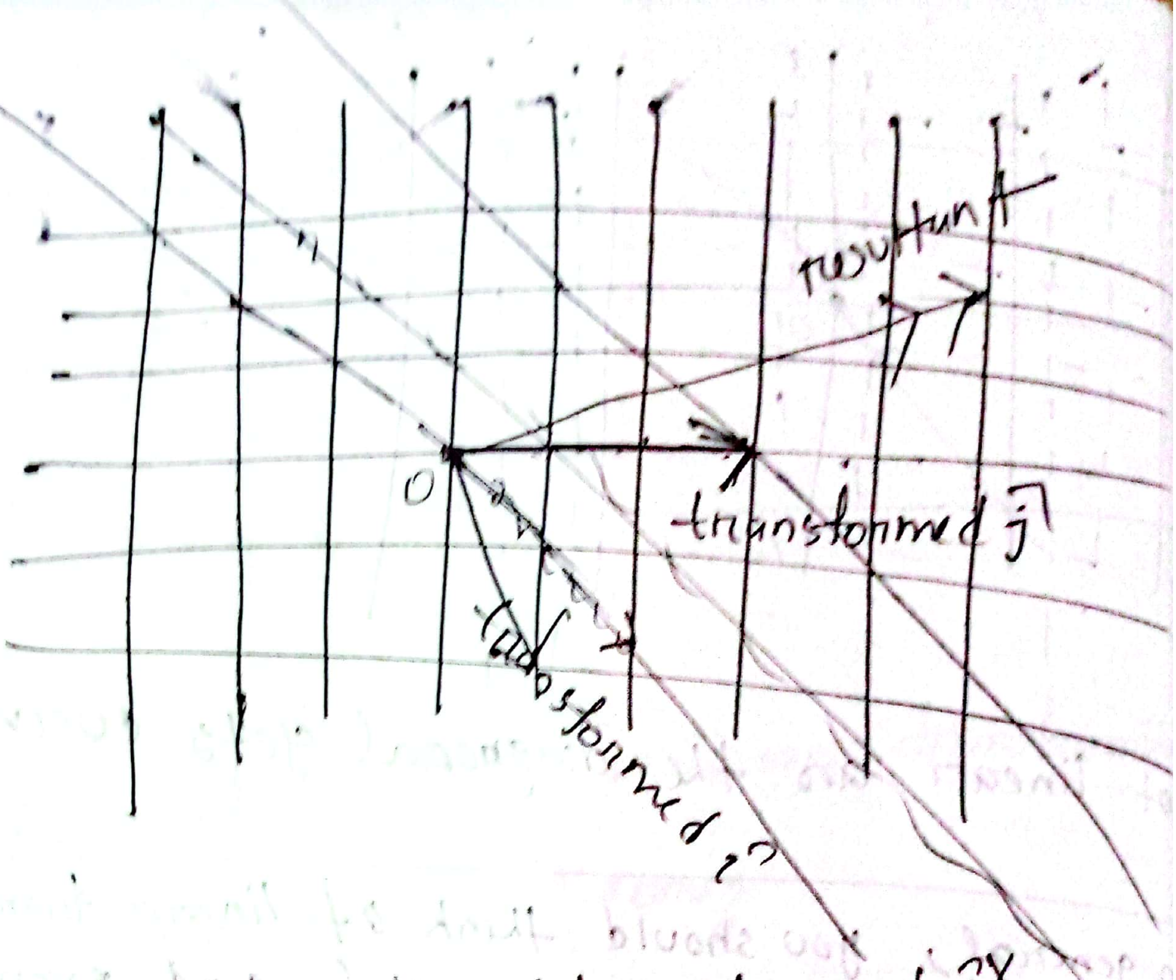
~~some are~~

* describing this transformation numerically.

→



$$\vec{v} = -1(\hat{i}) + 2(\hat{j})$$



$\therefore$ transformed $\bar{v} = -1(\text{transformed } \hat{i}) + 2(\text{transformed } \hat{j})$

$$= -1\begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}3\\0\end{bmatrix}$$

$$= \begin{bmatrix}-1(1)+2(3)\\-1(-2)+2(0)\end{bmatrix} = \begin{bmatrix}5\\2\end{bmatrix}$$

→ This gives us a technique to deduce where any vectors land so long as we have a record of where i-hat and j-hat each land without needing to watch the transformation itself

$$\hat{i} \rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \hat{j} \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x \begin{bmatrix} 1 \\ -2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$

So all of this is saying is that a 2 dimensional linear transformation is completely described by just four numbers: the 2 co-ordinates for where i-hat lands, and the 2-coordinates for where j-hat lands.

It's common to package these coordinates into a ~~two~~ two-by-two grid of numbers.

$$\hat{j} \to \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \hat{i} \to \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$: (2×2 matrix)

→ where $\hat{j}$ lands.

where $\hat{i}$ lands

If you're given a two-by-two matrix describing a linear transformation and some specific vector & you want to know where that linear transformation takes that vector. $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$$5 \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix} + 7 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

In most general case,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Place where 1st basis vector lands

Place where 2nd basis vector lands.

Applying this transformation to some vector $\begin{bmatrix} x \\ y \end{bmatrix}$ we get,

intuition ✓

~~vecto~~ pair

$$x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Q) If we rotate all of space 90° counterclockwise, $i \to [0, 1]$, $j \to [-1, 0]$.

figure out what happens to any vector after 90 rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= x \begin{bmatrix} 0 \\ 1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

A fun transformation with a special name, called a "shear"

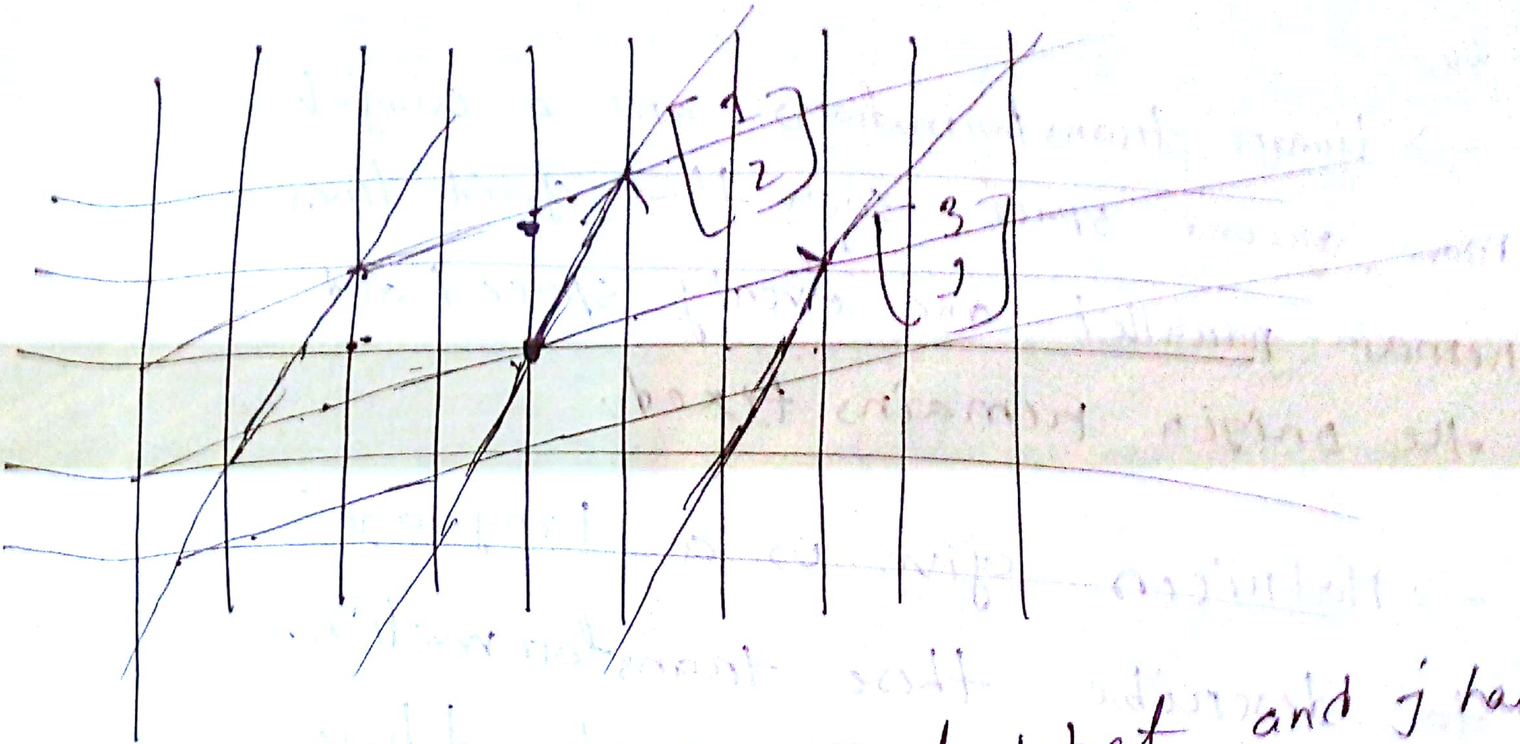
where, transformed $\hat{i} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ [fixed]

$\hat{j} \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

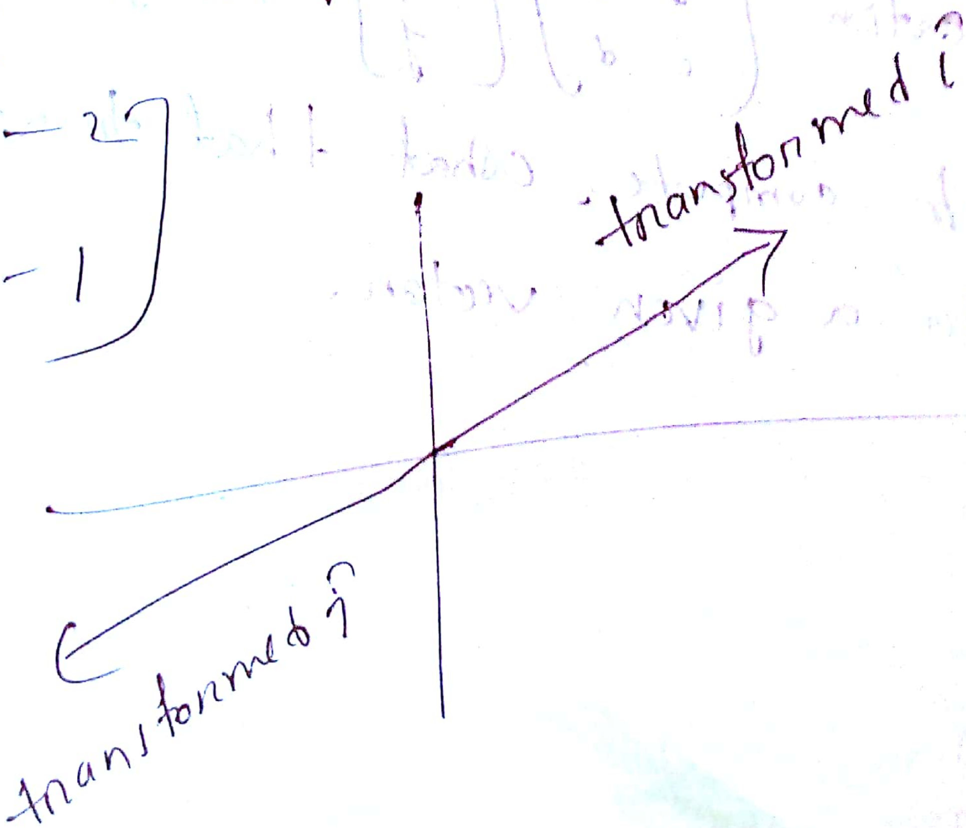
Q) given a coloomn $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$.

What its transformation look like.



* If the vectors that i-hat and j hat land on linearly dependent.

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$



so,

→ linear transformations are a way to move around space such that grid lines remain parallel and evenly spaced and the origin remains fixed.

→ Matrices give us a language to describe these transformations where the columns represents those coordinates. and matrix vector multiplication $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ is just a way to compute what that transformation does to a given vector.

Matrix multiplication as composition:

→ matrix multiplication has the geometric meaning of applying one transformation then another.

$i \to (1,0)$ (initially)

$j \to \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{shear}} \left(\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{rotation}} \begin{bmatrix} x \\ y \end{bmatrix} \right) = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{composition}} \begin{bmatrix} x \\ y \end{bmatrix}$$

(first rotation then shear)

← Read right to left

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{rotation}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{composition}}$$

You first apply the transformation represented by the matrix on the right, then you apply the transformation represented by the matrix on the left.

This is same as, $f(g(x))$

Step 1

$$\overbrace{\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}}^{M_2} \overbrace{\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}}^{M_1} =$$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = -2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}^{M_2}\overbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}^{M_1} =$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ g \end{bmatrix} = e\begin{bmatrix} a \\ c \end{bmatrix} + g\begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \begin{bmatrix} ae + bg \\ ce + dg \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} f \\ h \end{bmatrix} = f\begin{bmatrix} a \\ c \end{bmatrix} + h\begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \begin{bmatrix} af + bh \\ cf + dh \end{bmatrix}$$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

** $M_1 M_2 \neq M_2 M_1$

$M_2 \rightarrow 90°$ rotation

$M_1 \rightarrow$ shear

* Multiplication in associative:

$$(AB)C = A(BC)$$

→ this can be proved by the precedence of applying transformation. ~~It~~

It doesn't matter who are in the parenthesis but in both time first apply C, then B, then A

3D Linear transformation:

In 3D,

$$\hat{i} \rightarrow \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad \hat{k} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{j} \rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix}}_{\text{2nd transformation}} \underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{\text{1st transformation}}$$

→ It is useful in computer graphics & robotics

The determinant:

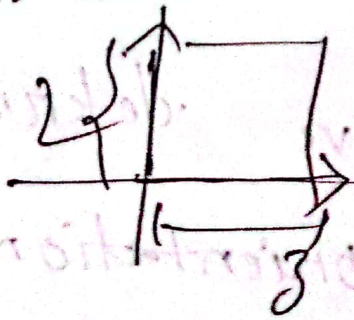
“The purpose of computation is insight, not numbers”

→ Exactly how much are things being stretched or squished.

→ to measure the factor by which the given region increases or decreases.

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow$$ before transformation its area was = 1

then,



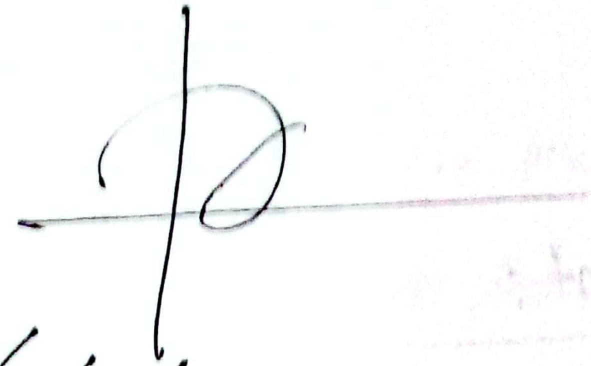
area = 6

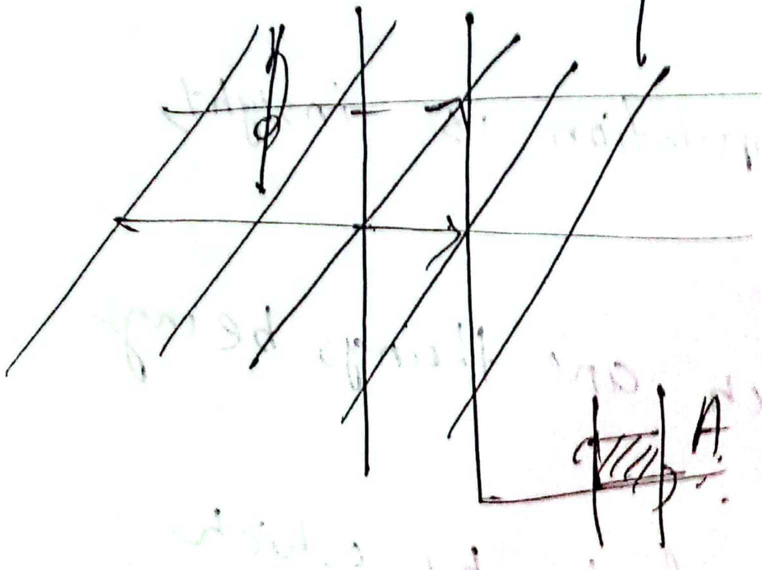
= 6·1

6 → Determinant

If we take a shear,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$





area before = 1

area after = 1×1
= 1

∴ Determinant = 1

$$\det\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = 0 \cdot A$$

↳ squishes everything into a smaller dimension

*** Negative determinant means inverting the orientation of space where the factor remain positive but we put negative sign in order to indicate flipping scene.

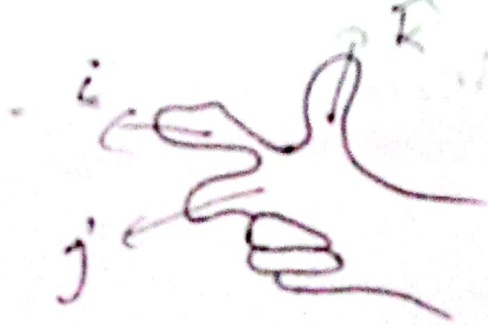
$$\det\left(\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}\right) = -3.0$$

The space just flipped over & areas are scaled by a factor of 3.

→ In 3D determinant tells how much volume gets scaled

* A determinant of 0 would mean that, all of space is squished onto something with 0 volume meaning either a flat plane, a line or, in the most extreme case onto a single point

* negative det in 3D:-



If I can still do that the orientation hasn't changed and det is +ve

Otherwise if after the transformation it only makes to do that by using left hand then det is -ve

Q) $\det(M_1 M_2) = \det(M_1) \times \det(M_2)$?

→ think of an example

"To ask the right question is harder than to answer it" — Georg Cantor

Linear Algebra lets us solve certain system of equations.

$$\begin{aligned} 2x+5y+3z &= -3 \\ 4x+0y+8z &= 0 \\ 1x+3y+0z &= 2 \end{aligned} \longrightarrow \overset{A}{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{\vec{v}}{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}$$

$$A\vec{x} = \vec{v}$$

transformation

This means we will apply some linear transformation A on vector $\vec{x}$ which will transform into $\vec{v}$

Let's start by 2D

$$\begin{aligned} 2x+2y &= -4 \\ 1x+3y &= -1 \end{aligned}$$

$$\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

$$A \quad \vec{x} = \vec{v}$$

If $\det(A) \neq 0$ then.

then there will be only one vector which will land on $\vec{v}$

Transformation $\underbrace{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}}_{A}$ → Inverse Trans $\underbrace{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}^{-1}}_{A^{-1}}$

If,

Transformation $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then, Inverse trans $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

90° Counterclockwise　　　90° clockwise

If

Transformation $\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Rightward shear}}$ then Inverse trans $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\underbrace{A^{-1}A}$

The transformation that does nothing

Once you have this : A^{-1} , then you can solve this by multiplying

$$A^{-1} A \vec{x} = A^{-1} \vec{v}$$
$$\Rightarrow \vec{x} = A^{-1} \vec{v}$$

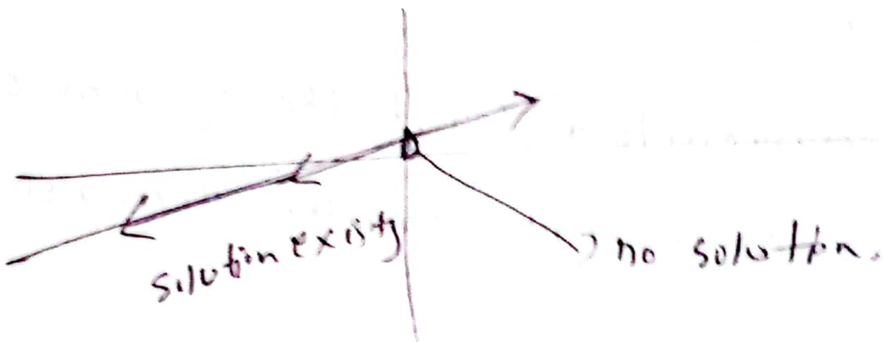
$$\det(A) \neq 0$$
$$\downarrow$$
$$A^{-1} \text{ exists}$$

When, $\det(A) = 0$,
Transformation associated with the system of equation squishes spaces into smaller dimension. There is no inverse. That is,
You can't unsquish a line to a 2D plane.
That could require transforming each individual vector into a while full line of vectors. But func can take only single input & give single output does not a function

plane

Similarly, in 3D there will be no inverse if the corresponding transformation squishes 3-D space into the plane or squishes into a line or to the point.

* Solutions can still exist when $\det(A) = 0$
You have to be lucky enough for the vector $\vec{v}$ exist somewhere in the line



Rank 1: when the output of a transformation is a line meaning its 1-D

Rank-2: If the output lands on a 2-D plane.

Rank-3: If the output lands on a 3-D plane

Rank = Num of dimensions in the output

In 2-2 matrix, Rank 2 is the best it can be

* set of all possible outputs $A\vec{v}$ $\longleftrightarrow$ "column space" of A

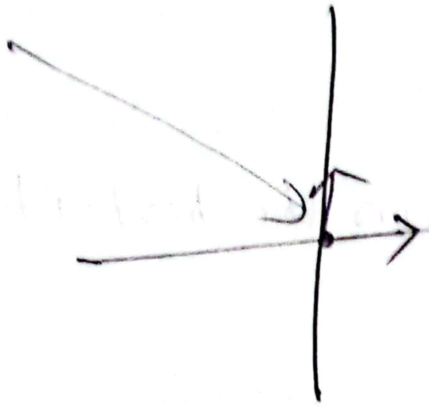
$$\underbrace{\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}}$$

Span of columns
$\Updownarrow$
"column space"

more precise def of rank : num of Dimension in the column space.

when the rank is highest it can be it means ~~the~~ it is equal the num of columns we call it Full rank

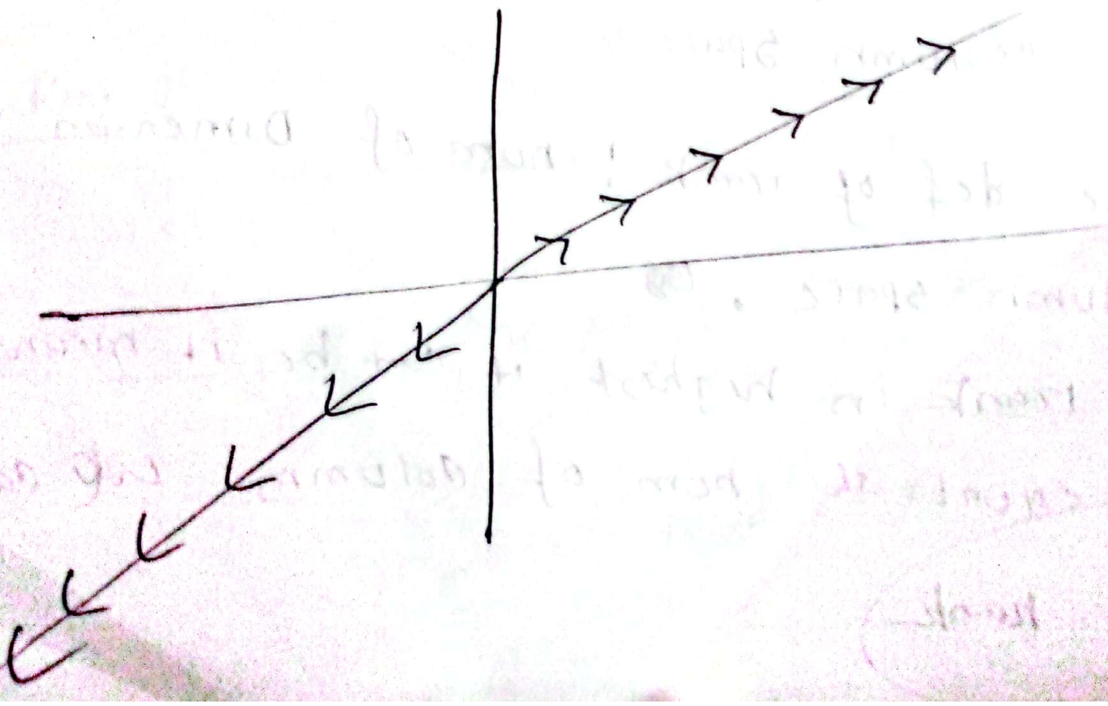
$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is always in the column space



For a full rank transformation,

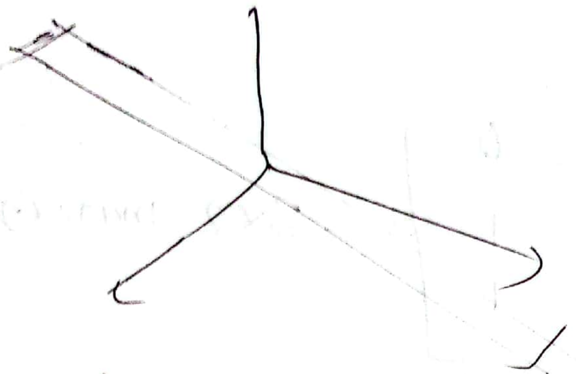
only $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ lands on $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

But the matrices that aren't in full rank which squished into a smaller dimension you can have a whole bunch of vectors which land on zero.



If a 3-D transformation squishe into a plane then there is also a full line of vectors lands on origin.

If a 3-D transformation squishes all of space into a line then there is whole plane land on origin



* These set of vectors that lands on the origin is called the $\boxed{\text{null space or kernel}}$ of your matrices. ~~This is~~ It is the space of all vectors that become null in the sense that they land on zero vector.

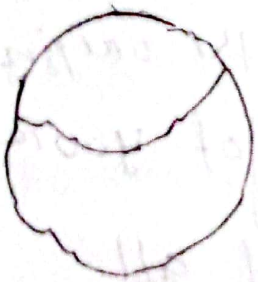
If $A\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

then null space gives all possible solutions for the equations.

Non-square matrix:

$$\underbrace{\begin{bmatrix} 2 \\ 7 \end{bmatrix}}_{\text{2d input}} \longrightarrow L(\vec{v}) \longrightarrow \overbrace{\begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}}^{\text{3d output}}$$

$$\underbrace{\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ -2 & 1 \end{bmatrix}}_{\text{span of columns}} \quad 3\times 2 \text{ matrix}$$



span of columns

$\Updownarrow$

Column space

$\downarrow$

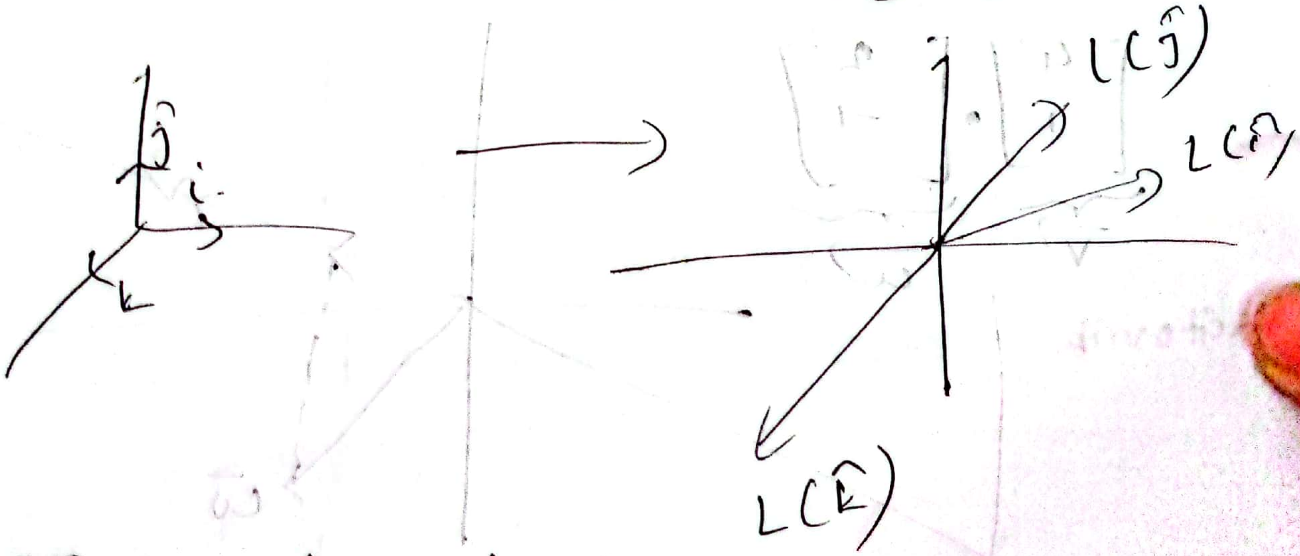
here in the 2D plane slicing to the origin of 3D Plane space

* $\begin{bmatrix} 3 & 1 \\ 4 & 7 \\ 5 & 9 \end{bmatrix}$, it has a geometric interpretation of mapping 2-D into 3-D

Two columns indicate that input space has 2 basis vectors & 3-rows indicate the landing spot for each of the basis vectors, is described with 3-separate co-ordinates

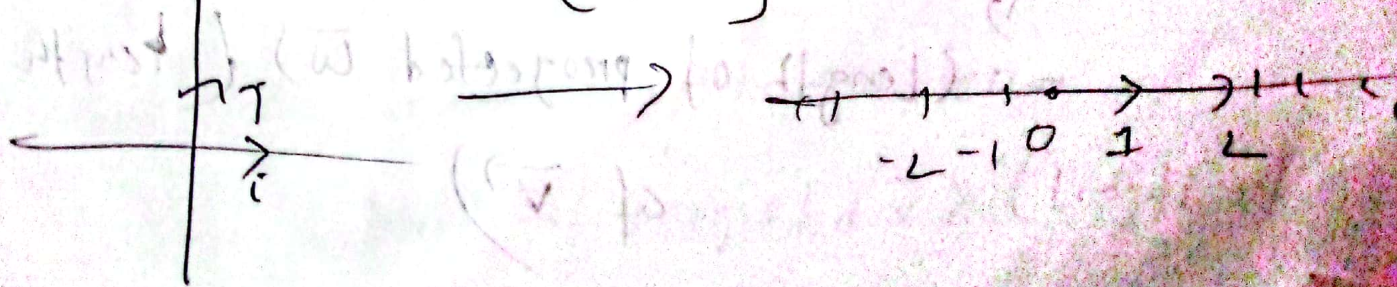
$$\overbrace{\begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \end{bmatrix}}^{\text{3 basis vectors}} \Big\} \text{ 2x3 matrix} \rightarrow \text{2 coordinates for each landing spot}$$

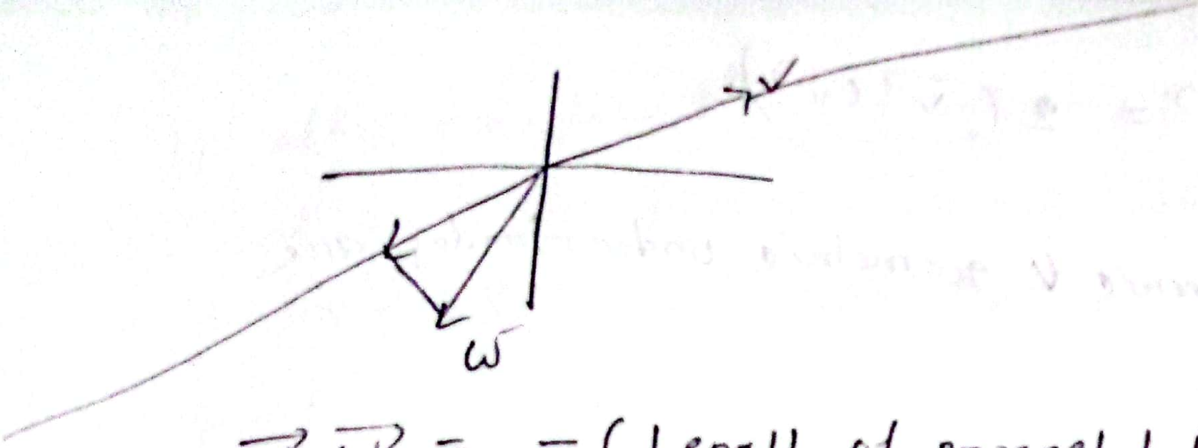
3D input ⟶ output in 2D



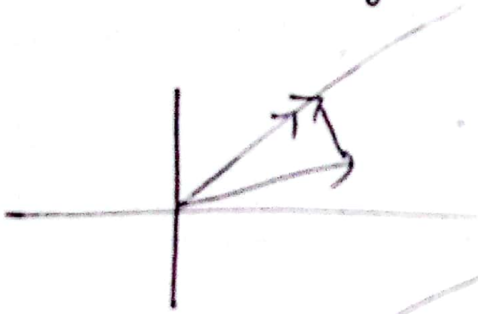
* 1D space (number line)

[1 2]



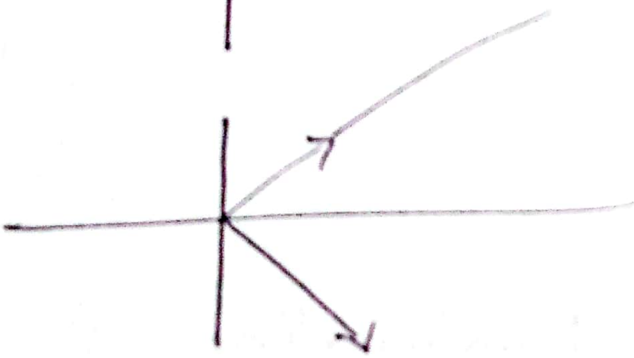


$\vec{v} \cdot \vec{w} = -$(Length of projected $\vec{w}$)(length of $\vec{v}$)



$\vec{v} \cdot \vec{w} > 0$

similar direction

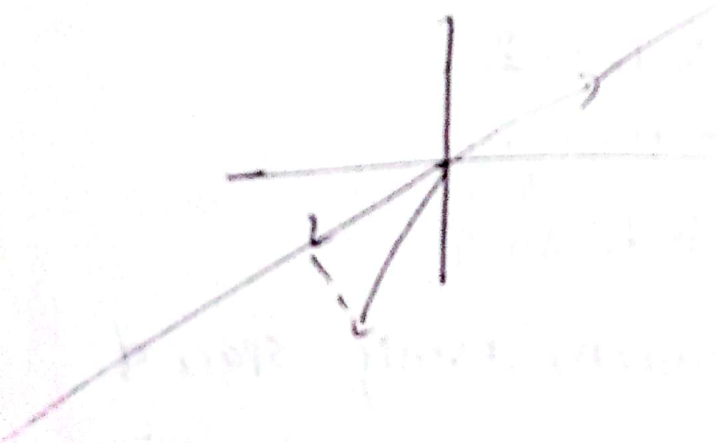


$\vec{v} \cdot \vec{w} = 0$

perpendicular

$\vec{v} \cdot \vec{w} < 0$

opposing direction



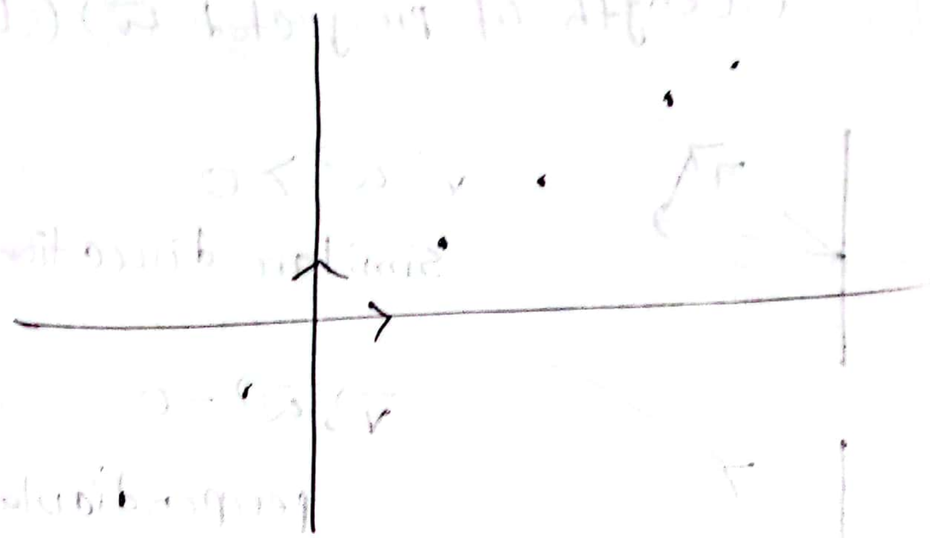
* Here the order doesn't matter.

$\bar{v} \cdot \bar{w} =$ (length of projected $\bar{v}$). (length of $\bar{w}$).

$(2\vec{v}) \cdot \vec{w} = 2(\vec{v} \cdot \vec{w})$

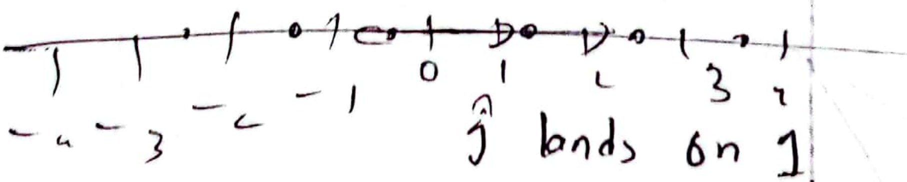
(3) Why numeric & geometric understanding are connected?

→



After transformation,

$\hat{i}$ lands on 2



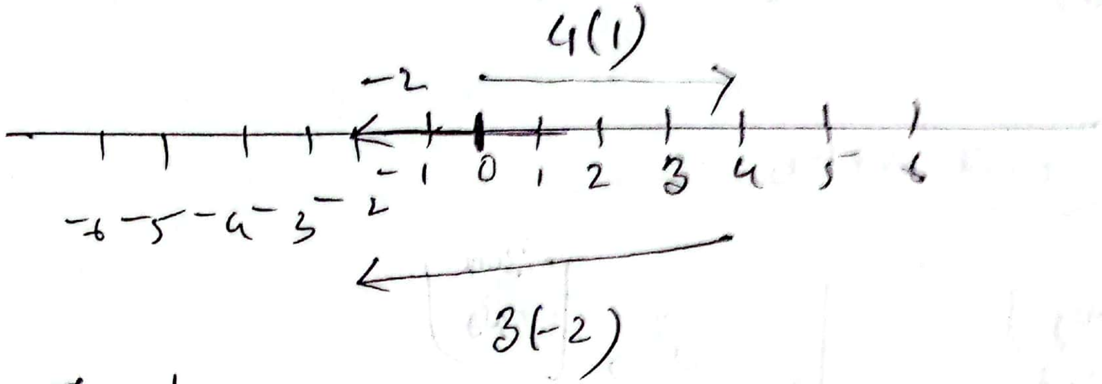
$\hat{j}$ lands on 1

Line of dots remains evenly spaced

Transformation matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$

If after transformation, $[1 \quad -2]$
$\hat{i}$ lands on 1 & $\hat{j}$ lands on -2, then,

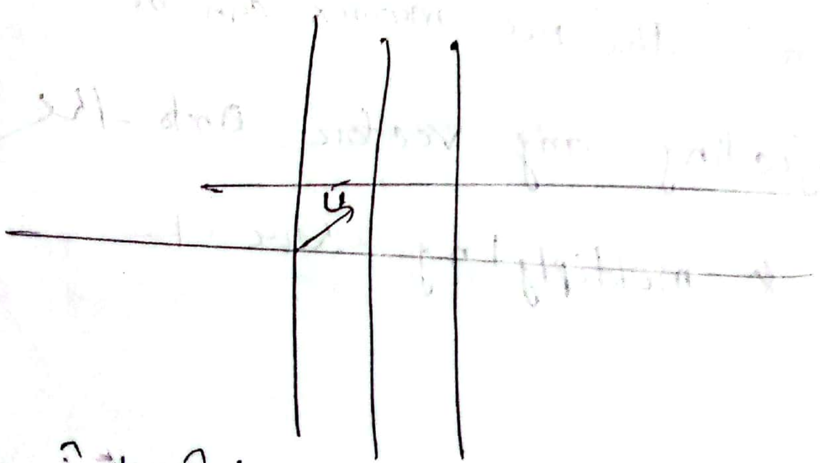
$\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ will land on,



Transform

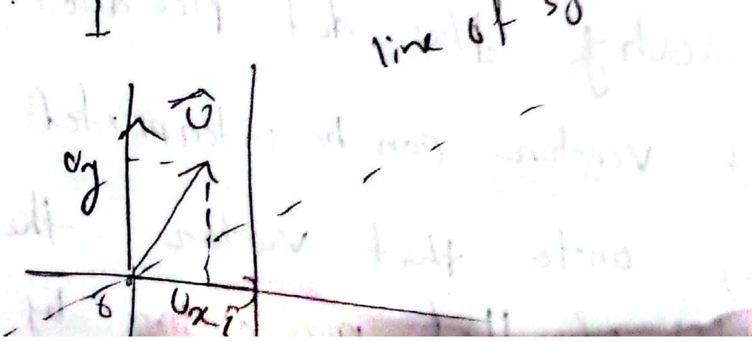
$$[1 \quad -2]\begin{bmatrix} 4 \\ 3 \end{bmatrix} = 4(1) + 3(-2) = -2$$

1×2 matrices $\longleftrightarrow$ 2d ~~vec~~ vectors



Where do $\hat{i}$ & $\hat{j}$ land?

$[u_x \quad u_y]$



Matrix vector product $\left[u_x \;\; u_y\right]\begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y$

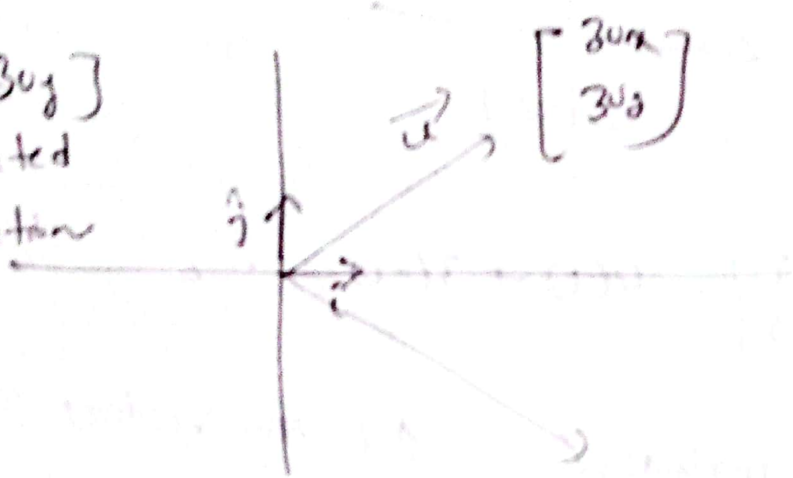
$\Downarrow$

Dot product $\begin{bmatrix} u_x \\ u_y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y$

for non-unit vectors

$\left[3u_x \;\; 3u_y\right]$

Associated transformation



Since this is linear the new matrix can be interpreted as projecting any vector onto the number line copy & multiplying where it lands by 3

This is why the dot product with a non-unit vector can be interpreted as first projecting onto that vector then scaling up the length of that projection by the length of the vector

So the main theme is

We had a linear transformation from 2D space to the number line which was not defined in terms of numerical vectors on numerical dot products. It was just defined by projecting space onto a diagonal copy of the number line

As the transformation is linear, it was necessarily described by some 1x2 matrix and since multiplying 1x2 matrix by a 2D vector is the same as turning that matrix on its side & taking a dot product.

The lesson here, is that anytime you have one of these linear transformations (like [4, 1]) whose output space is the number line no matter how it was defined there's going to be some unique vector v.

corresponding same to that transformation

In the sense that applying the transformation

is the same thing as taking a dot product

with that vector.

Duality $\Longleftrightarrow$ Natural but surprising correspondance.

For the LA case:

you'd say that "dual" of a vector is

the linear transformation, that it exactly

be the dual of a linear transformation from

space to one dimension in a certain

vector in that space.

* *
the dot product is very useful geometric tool for understanding projections & for testing whether or not vectors tend to point in the same direction.

bot at deeper level, dotting two vectors together is a way to translate one of them into the world of transformations.

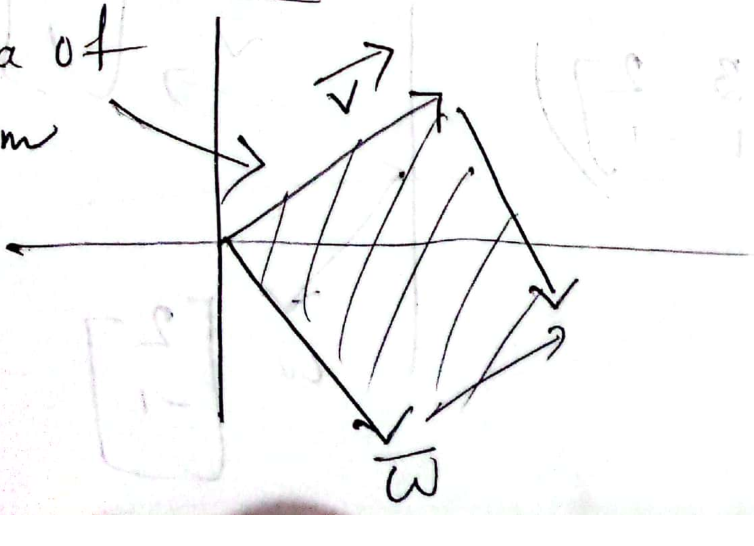
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\Downarrow$$

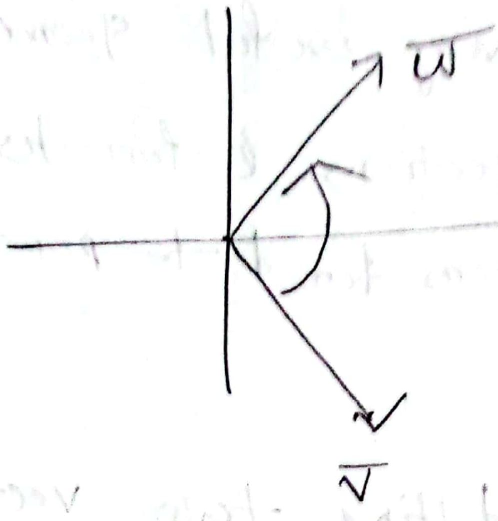
$$\begin{bmatrix} x_1 & y_1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Cross product;

$\vec{v} \times \vec{w}$ = Area of parallelogram

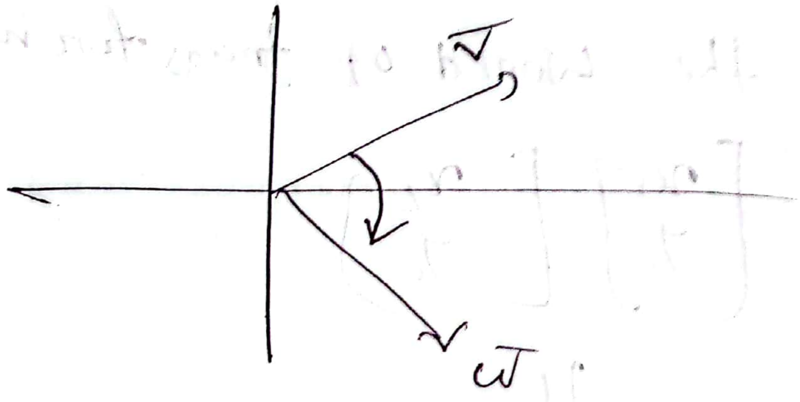


If,



~~Area~~ $\bar{v} \times \bar{w} = +$ Area of parallelogram

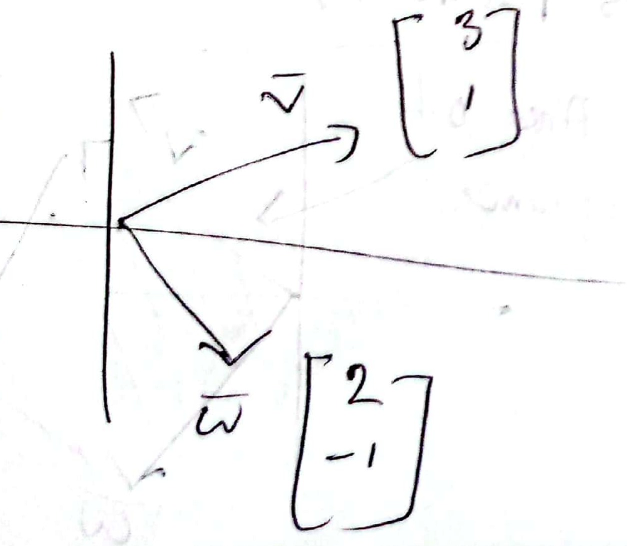
if,



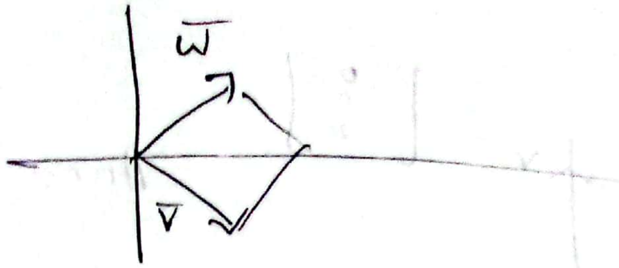
$\bar{v} \times \bar{w} = -$ Area of parallelogram

$\therefore \; \bar{v} \times \bar{w} = - \bar{w} \times \bar{v}$

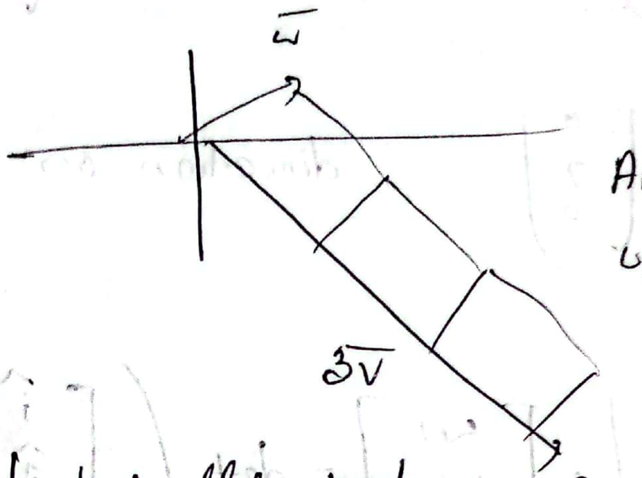
$\bar{v} \times \bar{w} = \det\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\right)$



$\vec{v} \times \vec{w}$



$(3\bar{v} \times \bar{w})$
$= 3(\bar{v} \times \bar{w})$



Area also scaled up a factor by 3

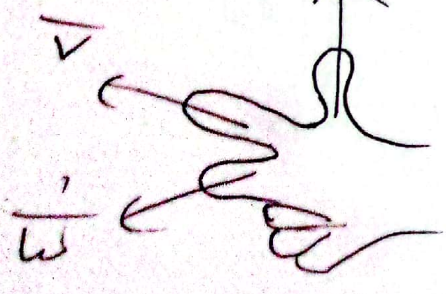
But that was technically not the cross product.

True cross product is something that combines 2 diff 3D vectors to get a new 3D vector. Just as before

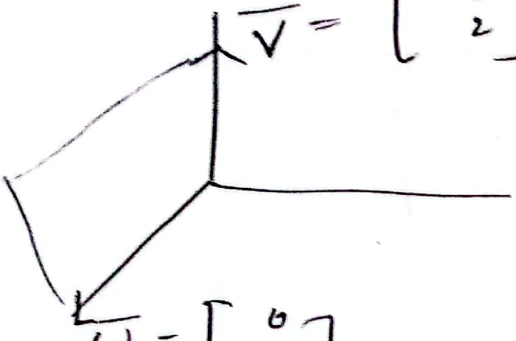
$$\bar{v} \times \bar{w} = \underset{\sim}{\bar{p}}$$

length will be equal to the area of parallelogram

& the direction ~~wi~~ is perpendicular to the parallelogram



For ex:



$\bar{v} = \begin{bmatrix}0\\0\\2\end{bmatrix}$

Area = 4

$\bar{v} \times \bar{w} = \begin{bmatrix}-4\\0\\0\end{bmatrix}$

$\bar{w} = \begin{bmatrix}0\\2\\0\end{bmatrix}$

direction on $-\hat{i}$

$$\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \times \begin{bmatrix}w_1\\w_2\\w_3\end{bmatrix} = \det\left(\begin{bmatrix}\hat{i} & v_1 & w_1\\ \hat{j} & v_2 & w_2\\ \hat{k} & v_3 & w_3\end{bmatrix}\right)$$

Eigenvalues & Eigenvectors

* Linear transformation এর পরে যে vector গুলো যদি তার span এর মাঝেই থাকে সেটি eigenvector.

~~প্রতি Eigen.~~
Each eigenvector has a value associated with it by which it is stretched or squished.

$$A\vec{v} = \lambda \vec{v}$$

$\downarrow$
Eigenvalue

$$\Rightarrow A\vec{v} = \lambda I \vec{v}$$

$$\Rightarrow (A - \lambda I)\vec{v} = \vec{0}$$