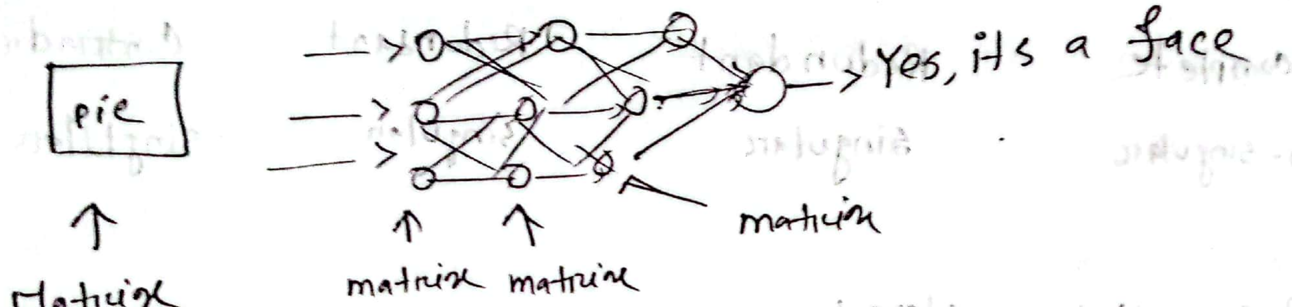
# Linear Algebra

Neural networks – Matrix operations




Neural networks – image recognition

systems of sentences,

| system 1 | system 2 | system 3 |
|---|---|---|
| The dog is black | The dog is black | The dog is black |
| The cat is orange | The dog is black | The dog is white |
| Complete | Redundant | Contradictory |
| Non-singular | singular | Singular |

| System 1 | System 2 | System 3 | System 4 |
|---|---|---|---|
| The dog is black | The dog is black | The dog is black | The dog is black |
| The cat is orange | The dog is black | " | The dog is white |
| The bird is red | The bird is red | " | The bird is red |
| complete | Redundant | Redundant | Contradictory |
| Non-singular | singular | singular | singUlar |

Q Give this system:

- Between the dog, the cat, & the bird, one is red
- " " " " " , one is orange
- The dog is black

→ The bird is obviously red & the system is non-singular

Sentences $\longrightarrow$ Equations

Sentences with numbers $\longrightarrow$ Equations

The price of an apple and a banana is \$10 $\longrightarrow$ $a+b=10$

(8) You bought an apple & a banana & they cost \$10

You bought ~~two~~ one apples & two banana they cost \$12

$\longrightarrow$

$$\left.\begin{array}{l} a+b=10 \\ a+2b=12 \end{array}\right\} \text{non-singular system}$$

$b=\$2, \; a=\$8$

(9) You go 2 days in a row & collect this info

Day 1: bought 1 apple + 1 banana they cost \$10

Day 2: " 2 " + 2 " " " \$20

$\longrightarrow$ No solution, Redundant system

Infinite solutions singular

① You go two days in a row & collect this info.

Day 1: Bought an apple + a banana cost \$10

Day 2: " 2 " + 2 " " \$24

⟶

Contradiactory, singular

No solution

**Linear Eq**

$a+b=10$

$2a+3b=15$

$3.4a-48.99b+$

$2c=122.5$

All numbers

**Non-linear**

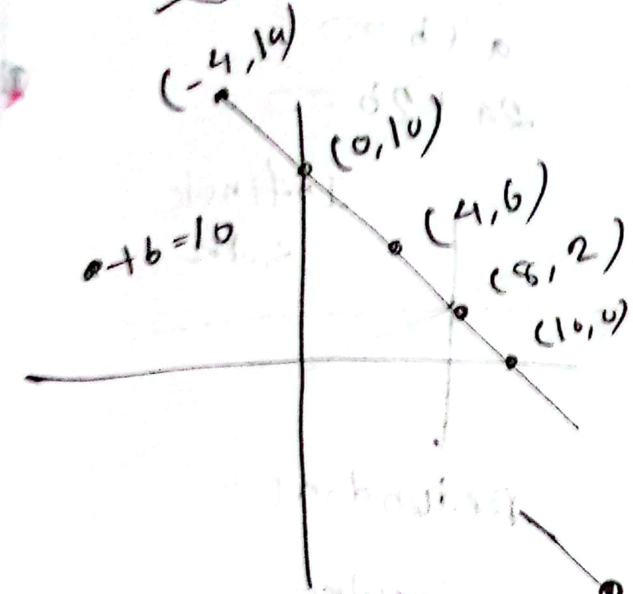
$a^2+b^2=10$

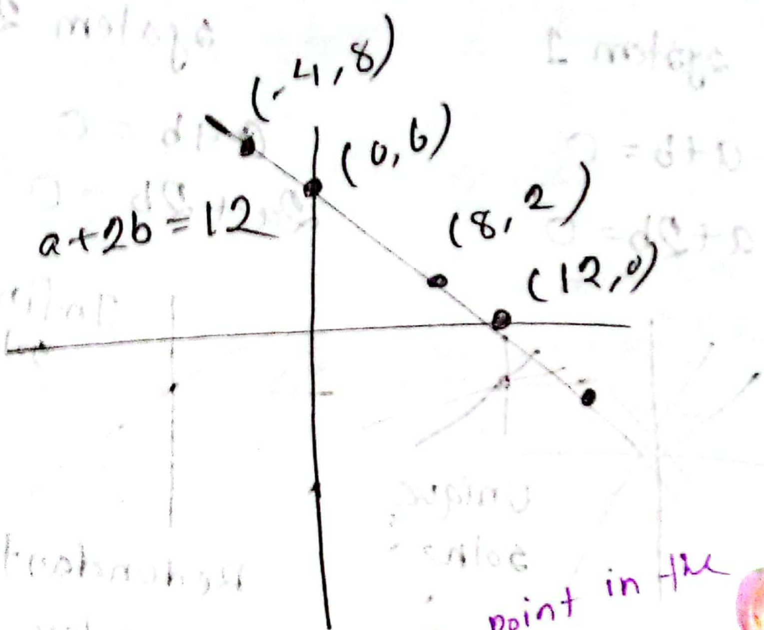
$\sin(a)+b^5=15$

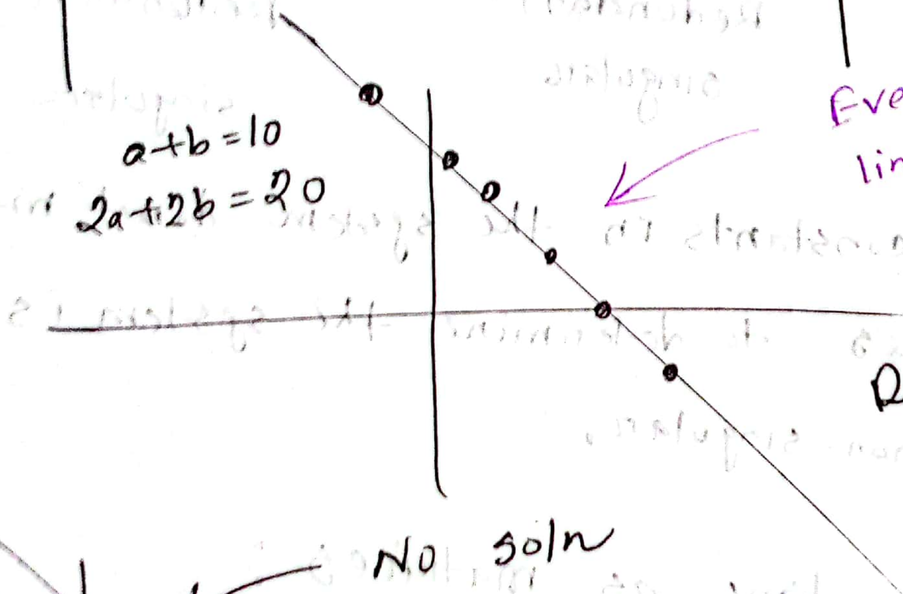
$2^a-3^b=0$

$ab^2+\frac{b}{a}-\frac{3}{b}-\log(c)=$

Linear Equation → Line



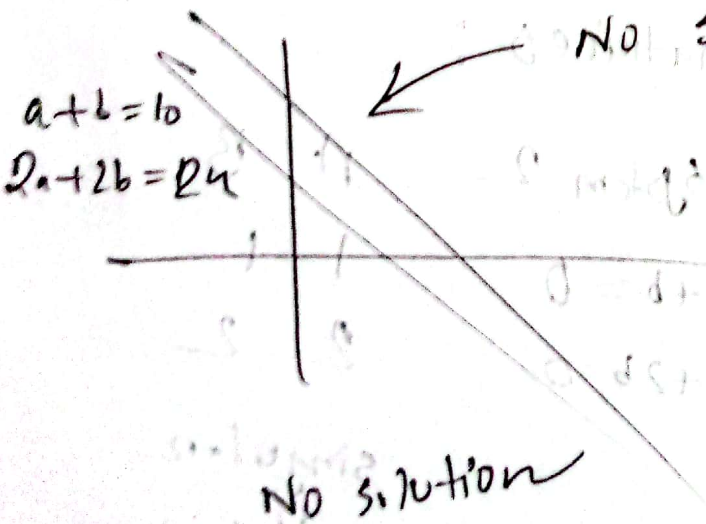




Every point in the line is a soln

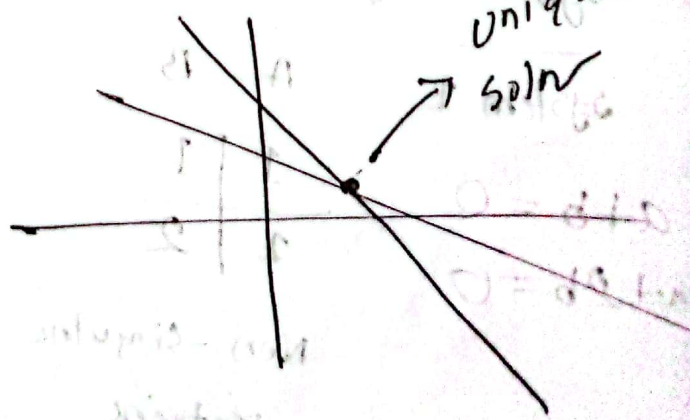
Redundant
singular

No soln




No solution
Contradictory
singular

Unique soln

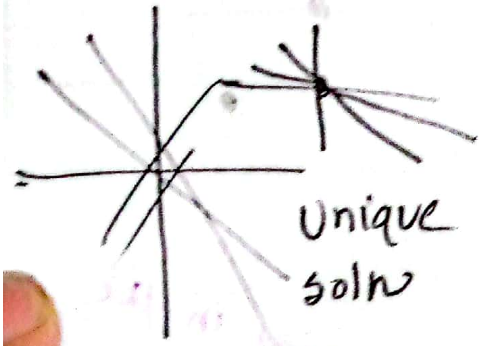


Complete
Non-singular

system 1

$a+b=0$

$a+2b=0$



Unique soln

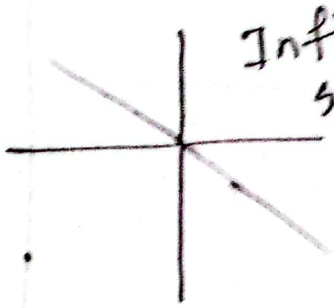
Non-singular

system 2

$a+b=0$

$2a+2b=0$

Infinite soln



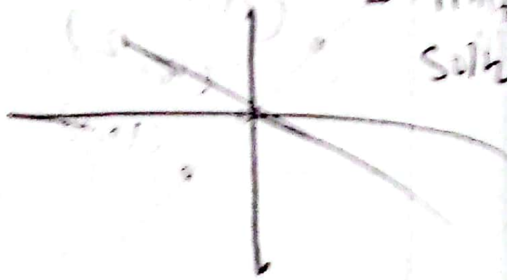
Redundant singular

system 3

$a+b=0$

$2a+2b=0$

Infinite soln



Redundant singular

note : The constants in the system don't matter when it comes to determine the system is singular or non-singular.

## System of equations as matrices :-

system 1

$a+b=0$

$a+2b=0$

$$\begin{array}{cc} A & B \\ 1 & 1 \\ 1 & 2 \end{array}$$

Non-singular Matrix

(unique soln)

System 2

$a+b=0$

$2a+2b=0$

$$\begin{array}{cc} A & B \\ 1 & 1 \\ 2 & 2 \end{array}$$

singular Matrix

(Infinite soln)

Linear dependence between rows

Non-singular

| | a | b |
|---|---|---|
| $a+b=0$ | 1 | 1 |
| $a+2b=0$ | 1 | 2 |

No eqn is a multiple of the other one

No row is a multiple of the other one

Rows are linearly independent

Singular system

| | a | b |
|---|---|---|
| $a+b=0$ | 1 | 1 |
| $2a+2b=0$ | 2 | 2 |

2nd eqn is a multiple of the first one

2nd row is a multiple of the first row

Rows are linearly dependent

Determinant

- If $\det(A)=0$ then matrix is singular
- If $\det(A)\neq 0$ " " " non-singular

(8)

$$2b+3m=15$$
$$2b+4m=16$$

$-m=-1$ $m=1$

$b=6$

# system of equations (3x3)

## system 1

$a+b+c=10$

$a+b+2c=15$

$a+b+3c=20$

Infinite soln

$c=5$

$a+b=5$

$(0,5,5), (1,4,5), (2,3,5), \ldots$ Redundant singular

## system 2

$a+b+c=10$

$a+b+2c=15$

$a+b+3c=18$

No soln

from (i) & (ii)

$c=5$

from (ii) & (iii)

$c=3$

contradictory singular

## system 3

$a+b+c=10$

$2a+2b+2c=20$

$3a+3b+3c=30$

Infinite soln

Any 3 nums to add to 10 works

$(0,0,10), (2,7,1), \ldots$

Redundant singular

## system 4

$a+b+c=10$

$a+2b+c=15$

$a+b+2c=12$

Unique soln

complete non-singular

Constants don't matter for singularity

| S-1 | S-2 | S-3 | S-4 |
|---|---|---|---|
| $a+b+c=0$ | $a+b+c=0$ | $a+b+c=0$ | $a+b+c=0$ |
| $a+2b+c=0$ | $a+b+2c=0$ | $a+b+2c=0$ | $2a+2b+2c=$ |
| $a+b+2c=0$ | $a+b+3c=0$ | $a+b+3c=0$ | $3a+3b+3c=$ |

unique

$a=0$
$b=0$
$c=0$

Infinite soln

$c=0$
$a+b=0$
(i.e, $a=-b$)

Infinite soln

$a+b+c=0$
(i.e, $c=-a-b$)

complete singular

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

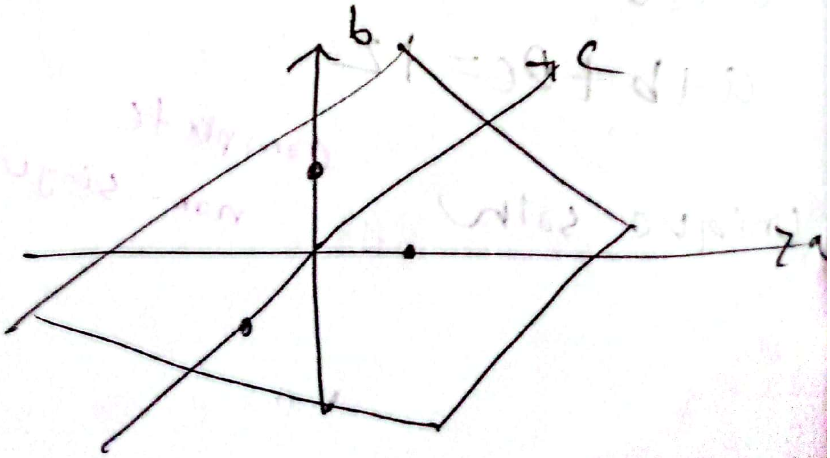
Linear equation in 3 variables → Plane

$a+b+c=1$

$1+0+0=1$

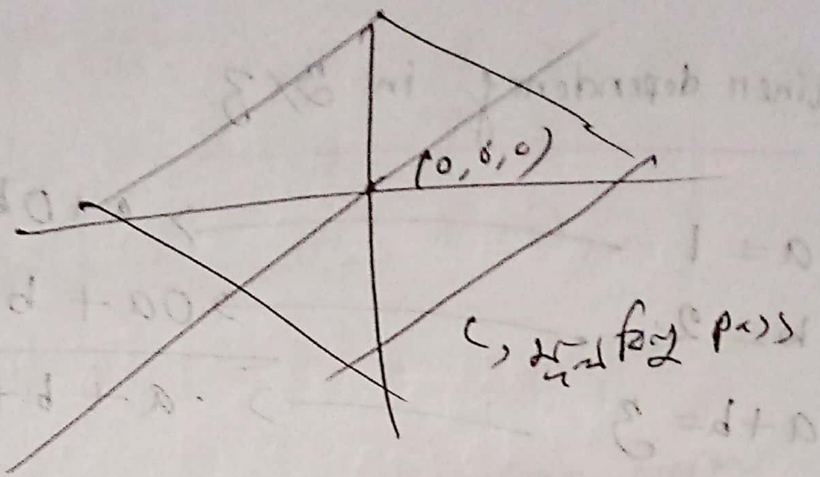
$0+1+0=1$

$0+0+1=1$



$3a-5b+2c=0$

$3(0)-5(0)+2(0)=0$




$a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$

The 3 planes intersects into a point (0,0,0) That's why it has unique soln

$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

All 3 plane go through the same line. so the set of soln is not just a point, it's an entire line

$a+b+c=0$
$2a+2b+2c=0$
$3a+3b+3c=0$

Since all are multiple of one another, they refers to same plane. Therefore the set of solutions to the system is every single point in the plane, multiple soln, singular.

Linear dependency in 3×3

$a = 1$ → $a + 0b + 0c = 1$

$b = 2$ → $0a + b + 0c = 2$

$a + b = 3$ → $a + b + 0c = 3$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{matrix}$$

$R1 + R2 = R3$

R-3 depends on R-1 & R-2

Rows are linearly dependent

$a + b + c = 0$ → $a + b + c = 0$

$2a + 2b + 2c = 0$ → $2a + 2b + 2c = 0$

$3a + 3b + 3c = 0$ → $3a + 3b + 3c = 0$

$R1 + R2 = R3$

Rows are linearly dependent

$a + b + c = 0$ → $a + b + c = 0$

$a + b + 2c = 0$

$a + b + 3c = 0$ → $+ a + b + 3c = 0$

$2a + 2b + 4c = 0$

$\div 2$

→ $a + b + 2c = 0$

Avg of R1 & R3 is R2

Hence, R-2 depends on R1 & R3

Rows are linearly dependent

$a+b+c=0$
$a+2b+c=0$ → No relations between eqn
$a+b+2c=0$

1 1 1
1 2 1 → No relations between rows
1 1 2

Rows are linearly independent

(Q)
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix}$$
Dependent

→ $R_3 - 2R_1 = R_2$

(Q)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

→ $R_1 = R_2 + R_3$

dependent

(8) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$\rightarrow$ Independent

(8) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$

$\rightarrow$

~~2 7 8~~

1 5 3

$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix} \rightarrow$ dependent

$\rightarrow$

$r_3 - 2r_1 \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ non zero rows = 2

$2 < 3$, then dependent

(8) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}$

$\rightarrow$ det $= 1(3-0) + 1(0-3)$

$= 3 - 3 =$

(9) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

$\xrightarrow{\mathcal{D}}$ $1(-1-0) - 1(-1-0) + 1(0-0)$

$= -1 + 1$

# Week-2

Neural networks — Sound recognition

Acoustic monitoring : Monitoring ecosystems through sounds

Sound recognition : tracking species through sound to preserve bio-habitats

If contradictory,

$a+b=10$ $\quad a+b=10$ $\quad$ no solution

$2a+2b=24$ $\quad a+b=12$

$0=2$

(3)

$5a+b=11$ $\quad \cancel{a+\frac{1}{5}b=11}$ $\quad$ let, $b=m$

$10a+2b=22$ $\quad a=\frac{11-m}{5}$

$10a+2b=22$

$10a+2b=22$

$0=0$

Elimination method

$a+b+2c=12$

$3a-3b-c=3$

$2a-b+6c=24$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 3 & -3 & -1 & 3 \\ 2 & -1 & +6 & 24 \end{array}\right]$$

$r_3 \to 2r_1 - r_3 \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 3 & -3 & -1 & 3 \\ 0 & 3 & -2 & 0 \end{array}\right]$

$r_2 \to 3r_1 - r_2 \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ & & & \\ & & & \end{array}\right]$

$r_3 \to r_1 + r_3 \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 3 & -3 & -1 & 3 \\ 3 & 0 & 8 & 30 \end{array}\right]$

$r_3 \to r_2 - r_3 \to \left[\begin{array}{ccc} 1 & 1 & 2 \\ 3 & -3 & -1 \\ & & \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 3 & -3 & -1 & 3 \\ 2 & -1 & 6 & 24 \end{array}\right]$$

$$R_3 \rightarrow 2R_1 - R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 3 & -3 & -1 & 3 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$$R_2 \rightarrow 3R_1 - R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 0 & 6 & 7 & 33 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$$R_3 \rightarrow R_2 - 2R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 0 & 6 & 7 & 33 \\ 0 & 0 & 11 & 33 \end{array}\right]$$

$$\begin{array}{l} R_2 \rightarrow 1/6 R_2 \rightarrow \\ R_3 \rightarrow 1/11 R_3 \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 0 & 1 & 7/6 & 33/6 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$a + b + 2c = 12$$

$$b + \frac{7}{6}c = 33/6$$

$$c = 3$$

$$b = \frac{[illegible]}{[illegible]} = \frac{[illegible]}{6} = \frac{12}{6} = 2$$

$$a = 12 - b - 2c = 12 - 2 - 6 = 4$$

## Matrix row reduction

| Original system | Intermediate system | Solved Sys |
|---|---|---|
| $5a + b = 17$ | $a + 0.2b = 3.4$ | $a = 3$ |
| $4a - 3b = 6$ | $b = 2$ | $b = 2$ |

| Orig matrix | upper diagonal ~~form~~ matrix | Digonal |
|---|---|---|
| $\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |
| | Row ~~ehe~~ echelon form | Reduced row echelon |

# Row echelon form

$$\begin{matrix} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

~~**~~ If you apply row operation to solve the systems ~~you~~ like for a singular matrix you get singular, for a non-singular matrix you get non singular.

The determinant stays the same after ~~back~~ the operations.

[ Adding a nonzero fixed value entry of the row would change the value of the determinant ] x+

(1) $x+y=4$

$x-0.33y=-2.67$

$1.33y=6.67$

$\Rightarrow y=5.02$

$\therefore x=-1.02$

(2) $Det(M)=-32+21\neq 0$

(3) $Det(A)=-3(4+6)+8(4-5)+1(12+10)$

$\Rightarrow -30+8+22$

(4) $\begin{bmatrix} a & b & c \\ d & e & f \\ 2a & 2b & 2c \end{bmatrix} \to \begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$

(5) $ac=ab$

(7) Lois went yesterday to the bank to find out the interest rate of 3 diff financial instruments. He recieved the following info.

| Financial instrument | Savings acc | CD(s) | Bonds |
| --- | --- | --- | --- |
| Annual Interest | 2% | 3% | 4% |

He wanna invest his USD 10000 savings in these accounts. By doing so, he knows that after a year he would recieve a total of US 260 $ in interest if he put twice as much money in the savings account as in the CDs & 2 money in bonds.

What is the value of 2?

Rank:

Application: Compressing images.

The pixeled images are matrices & the rank of matrix is related to the amnt of space that is needed to store the corresponding image

Rank also refers how much info ~~that~~ matrix or its corresponding system linear eqn is carrying.

* There's a very powerful technique on singular value decomposition (SVD) which is ~~quite~~ big can reduce the rank of a matrix while changing it as little as possible.

## System 1

The dog is black
The cat is orange

2 piece of info

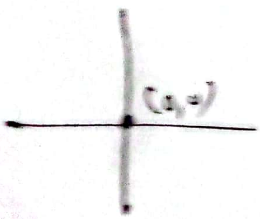
Rank = 2

$a+b=0$
$a+2b=0$

Rank = 2

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Dimension of soln space = 0




Non singular

## System 2

The dog is black
"

One piece of info

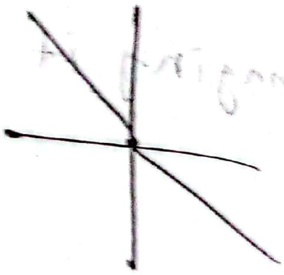
Rank = 1

$a+b=0$
$2a+2b=0$

Rank = 1

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Dimension of soln space = 1



singular

## System 3

The dog
The dog

Zero piece of info

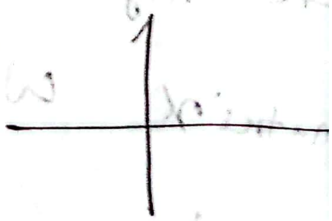
Rank = 0

$0a+0b=0$
$0a+0b=0$

Rank = 0

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Dimension of soln space = 2



Singular

Rank = 2 − (Dimension of soln space)

* A system is non-singular if it has full rank, namely if the rank is equal to the number of rows. This is the same as saying that a system of equations is non-singular if it carries as many pieces of info as equations it has, meaning carrying max num of info

| | | |
|---|---|---|
| $a+b+c=0$ ✓ | $a+b+c=0$ ✓ | $a+b+c=0$ ✓ |
| $a+2b+c=0$ ✓ | $a+b+2c=0$ ✗ | $2a+2b+2c=0$ ✗ |
| $a+b+2c=0$ ✓ | $a+b+3c=0$ ✓ | $3a+3b+3c=0$ ✗ |
| Rank = 3 | Rank = 2 | Rank = 1 |

$0a+0b+0c=0$ ✗

$0a+0b+0c=0$ ✗

$0a+0b+0c=0$

Rank = 0

* Rank is num of 1 in the diagonal in the row echelon ~~form~~ / num of non-zero rows.

Assignment:—

$$\left[\begin{matrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{matrix}\right] \begin{matrix} \cancel{120} \\ \cancel{70} \\ \cancel{20} \end{matrix}$$

$$\left[\begin{matrix} 1 & 5/7 & 3/7 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{matrix}\right]$$

Neural Networks — AI generated images

- AI generated human face
- Generative learning: Generating realistic looking images

→ Text to image & image to text generation

Text

↓

Frozen text Encoder

↓ Text Embedding

Text-to-image Diffusion model

↓ 64x64 image

Super Resolution Diffusion Model

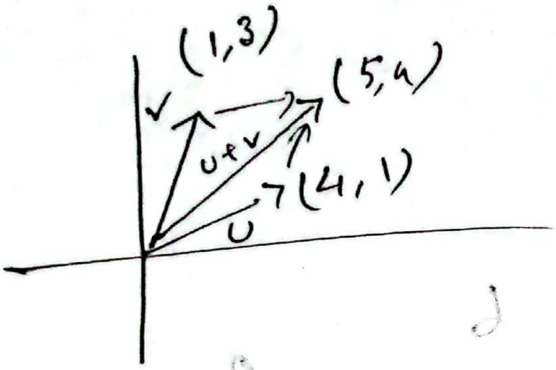
↓ 256x256 image

Super Resolution Diffusion Model

↓

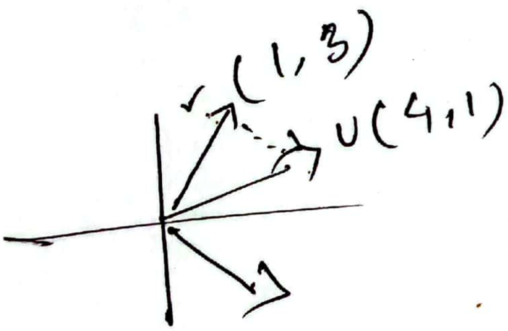
1024x1024 image

## Vectors & their properties




$u+v = (4+1, 1+3)$
$= (5,4)$

In machine learning, its very useful to know distances between vectors bcz many times you want to calculate diff similarities between data points, & these measures are useful.




$u-v = (4-1, 1-3) = (3,-2)$

(-) also refers the distances

L-1 dist, $|u-v|_1 = |5| + |-3| = 8$

L-2 " , $|u-v|_2 = \sqrt{5^2+3^2} = 5.83$

# The dot product

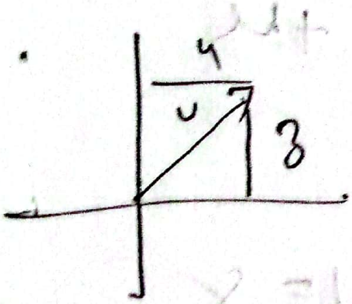
2 apples — apples : $\$3$

4 bananas — bananas : $\$5$

1 cherry — cherries : $\$2$

$$\begin{matrix} 2 & a & 3 & — & 6 \\ 4 & a & 5 & — & 20 \\ 1 & a & 2 & — & 2 \end{matrix} = 28$$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

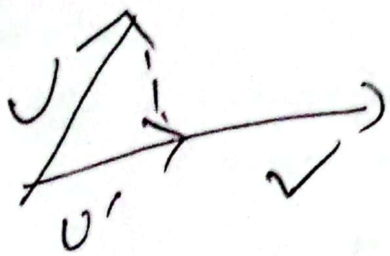



$\sqrt{4^2+3^2} = 5$

$4 \; 3 \; \begin{matrix} 4 \\ 3 \end{matrix} = 25$

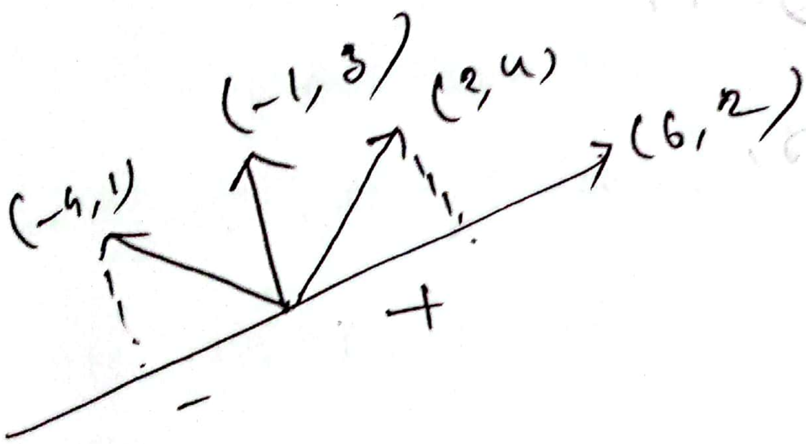
$L_2$ norm $= \sqrt{\text{dot product}(u,u)}$

$$\|u\|_2 = \sqrt{(u,u)}$$




$\langle u, v \rangle = |u'| \cdot |v|$

$= |u||v| \cos\theta$




Linear Transformation

Using row reduction

Some elementary operations will be needed.

1. Multiply any row by a non-zero num
2. Add two rows and exchange one of the original rows with the result of the addition
3. Swap rows.
4. Unify matrix A & B horizontally.

```
A_system = np.hstack((A, b.reshape(3,1)))

def MultiplyRow(M, row_num, row_num_multiple):

    M_new = M.copy()
    M_new[row_num] = M_new[row_num] * row_num_multiple

    return M_new
```

# multiply row_num_1 by row_num multiple & add it to the row_num_2

```
def AddRows(M, row_num_1, row_num_2, row_num_1_multiple):

    M_new = M.copy()
    M_new[row_num_2] = row_num_1_multiple * M_new[row_num_1] + M_new[row_num_2]

    return M_new
```

# exchange row_num_1 & row_num_2

```
def SwapRows(M, row_num_1, row_num_2):
    M_new = M.copy()
#swapping M_new[[row_num_1, row_num_2]] = M_new[[row_num_2, row_num_1]]
    return M_new
```

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & -10 \\ 2 & 1 & 3 & 0 \\ -1 & 2 & -5 & 17 \end{array}\right]$$

$$r_3 \leftrightarrow r_1 \longrightarrow \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 2 & 1 & 3 & 0 \\ 4 & -3 & 1 & -10 \end{array}\right]$$

$-4 \quad 8 \quad -20 \quad 68$

$$r_3' \rightarrow 4r_1 + r_3 \longrightarrow \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 2 & 1 & 3 & 0 \\ 0 & 5 & -19 & 58 \end{array}\right]$$

$-2 \quad 4 \quad -10 \quad 34$

$$r_2' \rightarrow 2r_1 + r_2 \longrightarrow \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 5 & -7 & 34 \\ 0 & 5 & -19 & 58 \end{array}\right]$$

$$r_3' - 2r_2 + r_3 \longrightarrow \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 5 & -7 & 34 \\ 0 & 0 & 12 & -24 \end{array}\right] \quad \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 5 & -7 & 34 \\ 0 & 0 & -12 & 24 \end{array}\right]$$

$$r_3' \rightarrow \frac{1}{-12} r_3 \longrightarrow \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 5 & -7 & 34 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 5 & -7 & 34 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$R_2' \to \frac{1}{5} R_2 \to \left[\begin{array}{ccc|c} -1 & 2 & -5 & 17 \\ 0 & 1 & -7/5 & 34/5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\therefore x_3 = -2$$

$$-x_1 + 2x_2 - 5x_3 = 17$$

$$x_2 - \frac{7}{5} x_3 = 34/5$$

$$x_2 = \frac{34}{5} + \frac{7}{5} \times x_3$$

x_3 = -2

x_2 = A_ref [1,3] - A_ref[1,2] * x_3)

x_1 = (A_ref [0,3] - A_ref [0,2] * x_3 - A_ref[0,1] * x_2) / A_ref [0,0]

## Matrix multiplication

→ corresponds to combining two linear transformation into a 3rd one.

→ apply one transformation then another

## Identity Matrix

main diagonal → all 1

Identity matrix × vector = no change of the vector

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(0, 0) = (0, 0)

(1, 0) = (1, 0)

(0, 1) = (0, 1)

(1, 1) = (1, 1)

CamScanner

# Inverse matrix

Transformation $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then Inverse trans.

90° counterclockwise

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

90° clockwise

Ex: $\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $|D| = 10 - 2 = 8$

$D_{11} = 2$ $D_{12} = -1$, $D_{21} = -2$, $D_{22} = 5$

$\begin{bmatrix} 2 & -1 \\ -2 & 5 \end{bmatrix}$

$\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ $\frac{1}{8} \rightarrow \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{8} & \frac{5}{8} \end{bmatrix}$

* singular matrix are non invertible

$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, Det $= 0$

Neural networks & matrices :

Quiz: NLP

Imagine u have a spam dataset, & the spam dataset you pinpoint 2 words that are quite deterministic for spam, which are lottery & win. Of course their appearance doesn't guarantee that the email is spam. So you have counted the num of appearances got this table,

| spam | lottery | win |
|---|---|---|
| y | 1 | 1 |
| y | 2 | 1 |
| N | 0 | 0 |
| y | 0 | 2 |
| N | 0 | 1 |
| N | 1 | 0 |
| y | 2 | 2 |
| y | 2 | 0 |
| y | 1 | 2 |

Goal: spam filter

You assign a score to the word lottery, & a score to the word win, then calculate the score of sentence.

ex:

lottery : 3 points

win : 2 points

"win, win the lottery" : 7 points

If the num of points of the sentence is bigger than some amnt called a thresh, then the email is classified as spam.

Goal: find the best points & threshold

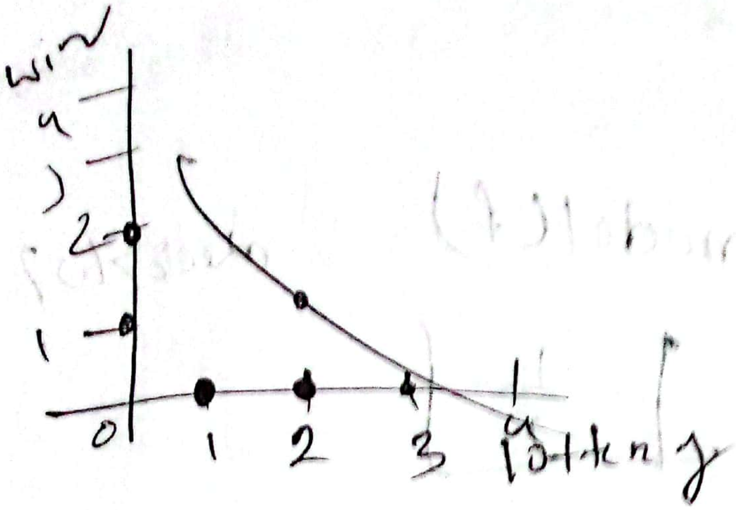
lottery: $\frac{1}{1}$ point

win: $\frac{1}{}$ point

Threshold: $\frac{1.5}{}$ point

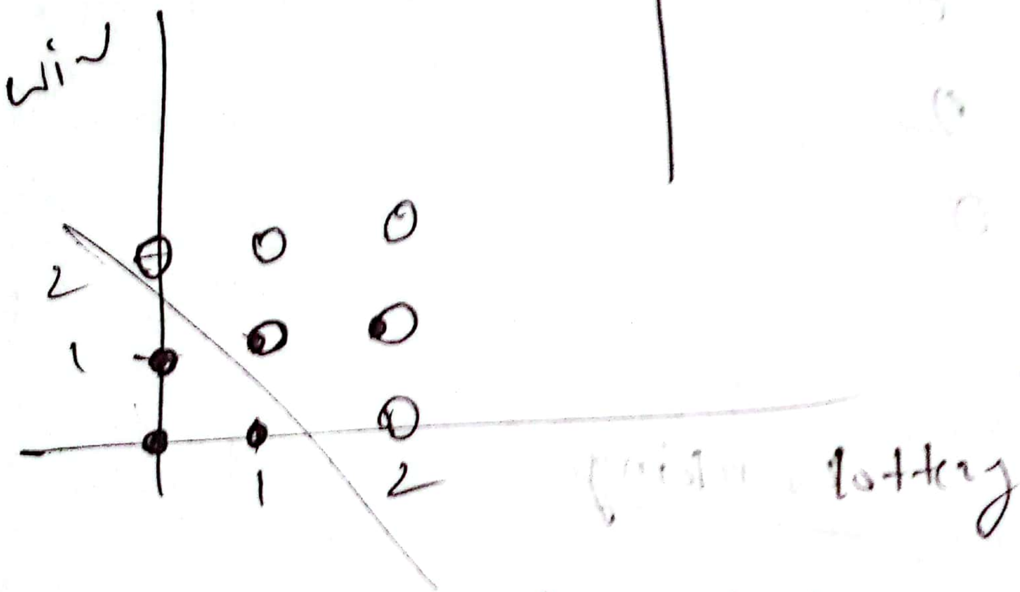
| score | |
|---|---|
| 2 | y |
| 3 | y |
| 0 | N |
| 2 | y |
| 1 | N |
| 1 | N |
| 4 | y |
| 2 | y |
| 3 | y |

"This is called NLP as the input was language, it was words, & it used these words to make prediction."




→ linear classifier

→ Neural network with one layer




1. win + 1. lottery = 1.5

+ve zone: 1. win + 1. lottery > 1.5

"A 1 layer neural network can be seen as a matrix product followed by a threshold check"

| spam | lottery | win |
|---|---|---|
| y | 1 | 1 |
| y | 2 | 1 |
| N | 0 | 0 |
| y | 0 | 2 |
| N | 0 | 1 |
| N | 1 | 0 |
| y | 2 | 2 |
| y | 2 | 0 |
| y | 1 | 2 |

· model(f)

clicks(

working

Model

[0] [1] $\boxed{\phantom{x}}$ = 3

clicks?

✓

spam

efficient way,

— is to take the product of the matrix & vector spam lottery win

| modes | | pred | | check |
|---|---|---|---|---|
| 1 | = | 2 | | y |
| 1 | | 3 | | y |
| 1 | | 0 | check | N |
| | | 2 | | y |
| | | 1 | | N |
| | | 1 | | N |
| | | 4 | | y |
| | | 2 | | y |
| | | 3 | | y |

OR,

spam lot win

check

1.win + 1.lottery > 1.5 → T

1.win + 1.lottery − 1.5 > 0

bias

model

1

1

−1.5

bias

check > 0 ?

* Sometimes you'll see ~~a~~ classifiers with a bias & sometimes with threshold. For more complicated neural networks the bias tends to be more common.

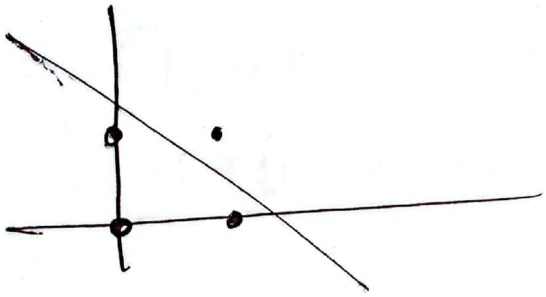
The And operation "Why bring And?"

| And | $x$ | $y$ | model | | Out Pred | | checks |
|---|---|---|---|---|---|---|---|
| N | 0 | 0 | | | 0 | → | |
| N | 0 | 1 | 1 | = | 1 | | N |
| N | | | 1 | | 1 | | N |
| y | | | | | 2 | | N |
| | | | | | | | y |

~~az~~ if model

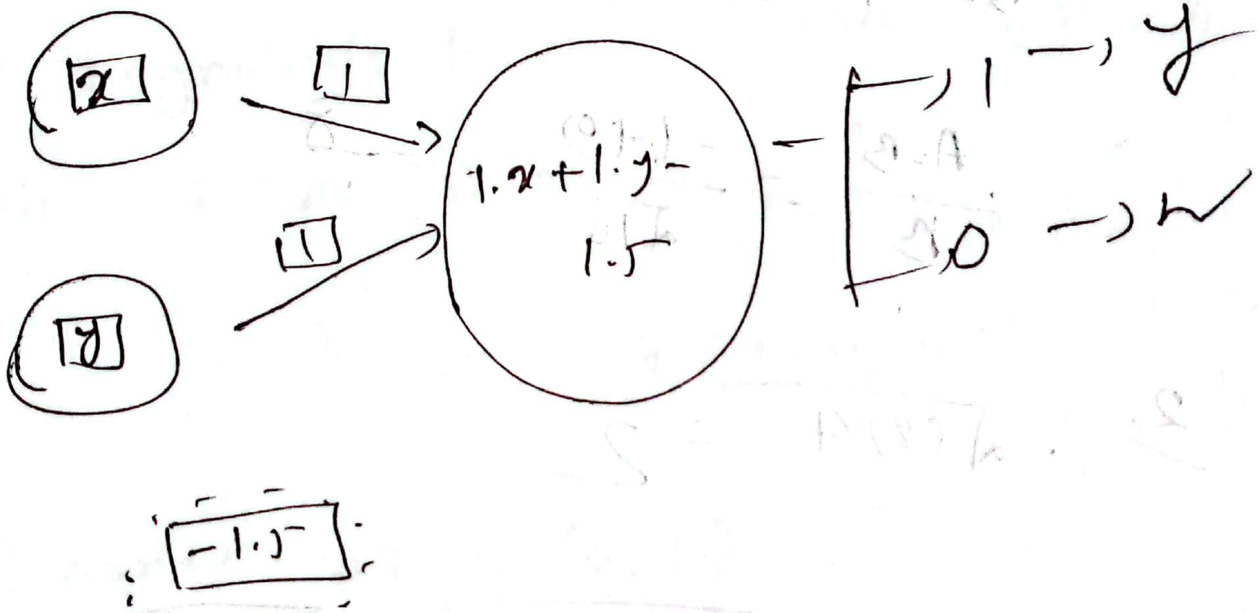
→ az if you consider this as a small dataset then you can model it with a neural network, in fact exact the same one for spam.

so the and dataset can be modeled as a perceptron, as a 1 layer neural network.

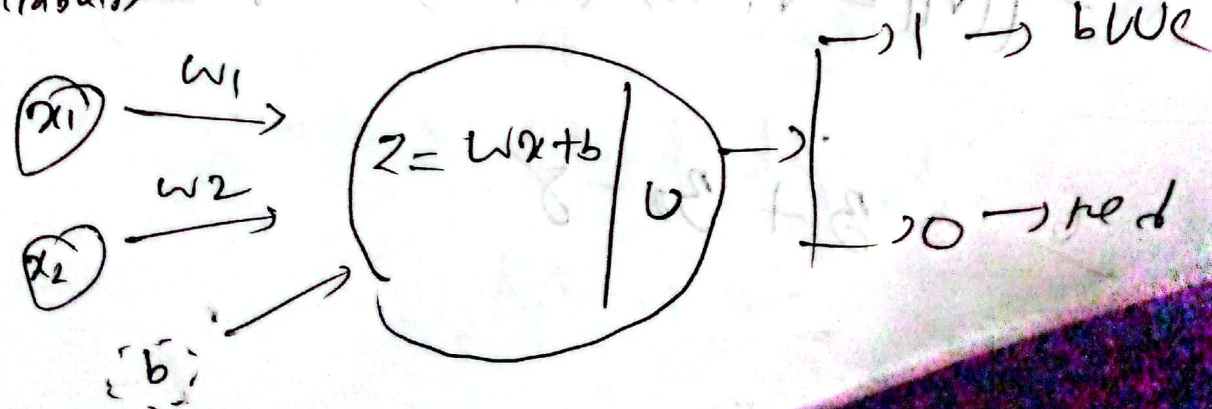


$1.x + 1.y = 1.5$

## The perceptron




Using variables,



distance of 2 vectors, $\bar{u} = \hat{i} + 0\hat{j} + 7\hat{k}$

1. $d = \sqrt{(1-0)^2 + (0+1)^2 + (7-2)^2}$ $\bar{v} = 0\hat{i} - \hat{j} + 2\hat{k}$

$= \sqrt{1+1+25}$

$= \sqrt{27}$

2.

$A = \hat{i} + 0 - 3\hat{k}$ $|A| = \sqrt{1+9} = \sqrt{10}$

$B = -\hat{i} - 3\hat{k}$ $|B| = \sqrt{10}$

$\frac{A \cdot B}{\cdot B} = \frac{-1+9}{\sqrt{10}} = \frac{8}{}$

2. $\sqrt{(2)^2 +} = 2$

4. $||v|| = \sqrt{1+25+4+9} = \sqrt{39}$

6. $3 + 3\theta - 8$

$=$

$$M_1 M_2 = \begin{bmatrix} 2 & -1 \\ 3 & -3 \end{bmatrix} \times \begin{bmatrix} 5 & -2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10+0 & -4-1 \\ 15+0 & -6-3 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ 15 & -9 \end{bmatrix}$$

8   $27+5$

* magnitude from vector $P(x_1, y_1)$ to $Q(x_2, y_2)$

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

* norm of $\vec{v}(0,1) = \sqrt{0^2 + 1^2}$

CamScanner

Application of L.T — Computer Graphics

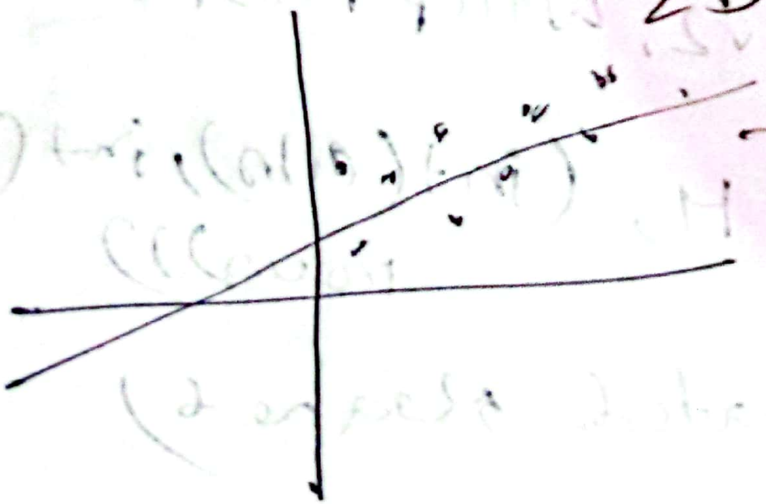
- L.T are often used to generate comple[x] shapes
- GPU is designed to handle huge calculation

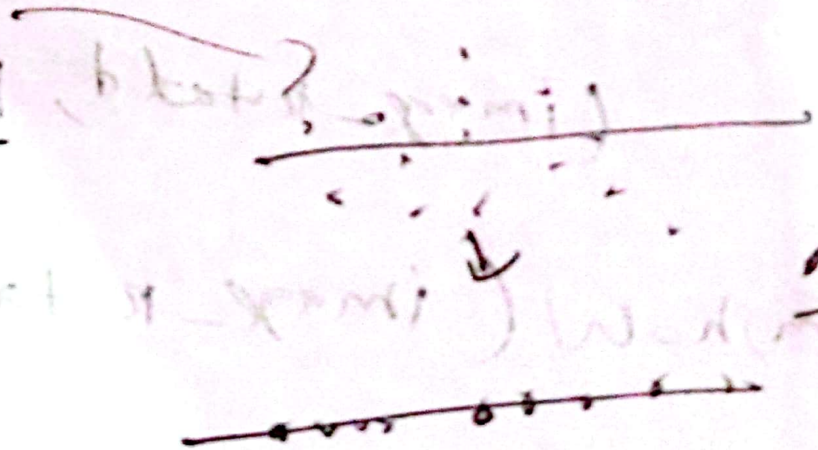
# Principle Component Analysis

high Dimensional dataset ⟶ reduced dimension dataset

(without losing at least info)

2D





1D

→ dimensionality reduction Algorithm

## Singularity & rank of linear transformation

non-singular ↓ can span the whole space

singular
1 1
2 2
" only a line "
2D → line
3D → plane/line/point

singular
0 0
0 0

Rank = Num of dimension in the output

Det (A B) = det (A) × det (B)

**Determinant**

**Quiz :** the product of a singular & a non-singular matrix is

— singular

— Non-singular

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

det = 5 det = 0

2nd matrix being singular means it send everything into a part of line. And that means that when you combine them & them it sends the fundamental basis into some segments. so it has this: det 0 as this segment has area = 0

0.15625 − 0.03125

## Det of inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5 det = 0.2

$5^{-1} = 0.2$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

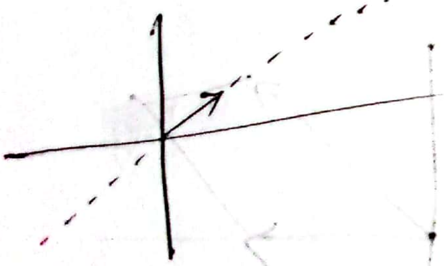
$\det = 8$ $\qquad\qquad$ $\det = 0.125$

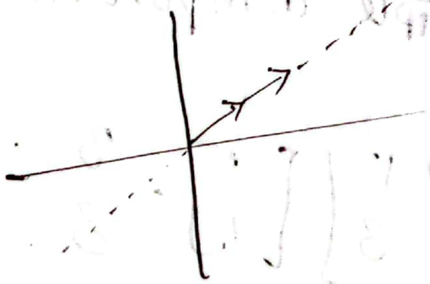
$$8^{-1} = 0.125$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

## Basis & span

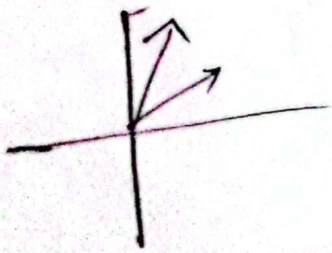
- A basis is a minimal spanning set



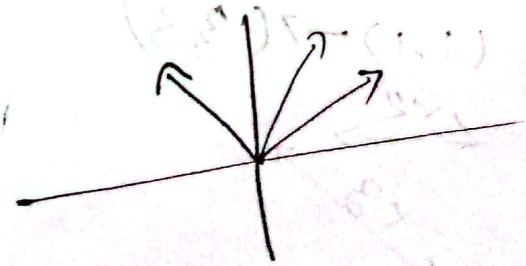


Not a basis (too many)

basis

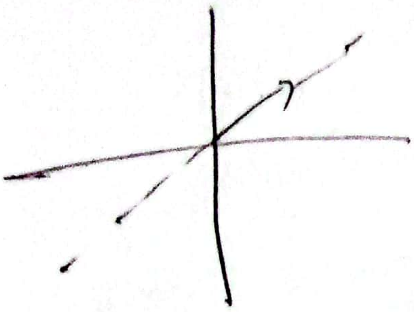


Basis

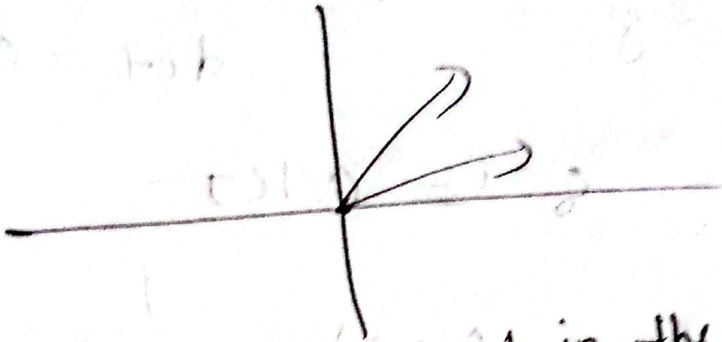


Not basis (any 2 can be basis & 3rd one is redundant)

* The basis is the space of the dimension of the plane.



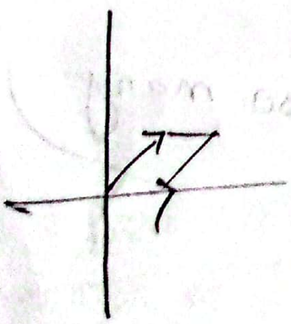
1 element
1 dimension



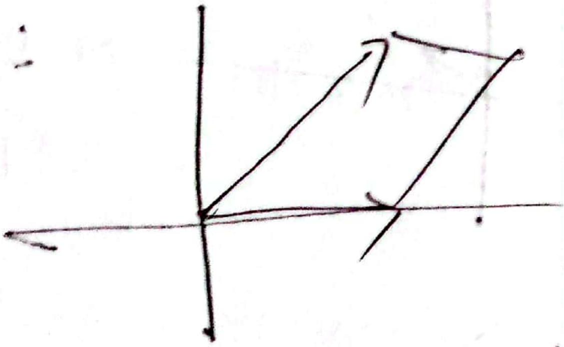
2 element in the basis
Dimension-2

## Eigenbasis

→ for principle component analysis

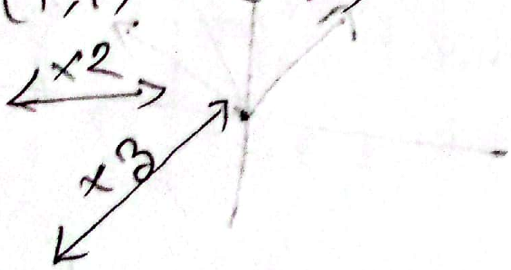


$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{matrix}3\\3\end{matrix}$$



(1,0) → (2,0)

(1,1) → (3,3)




"Eigen basis"

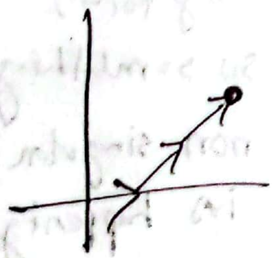
why this is useful?

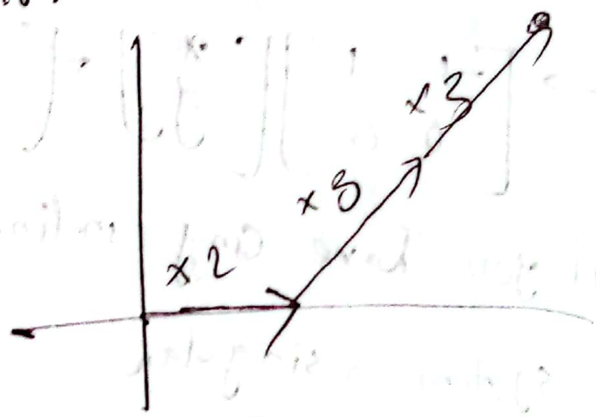
→ you want to find the image of the point (3, 2)

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

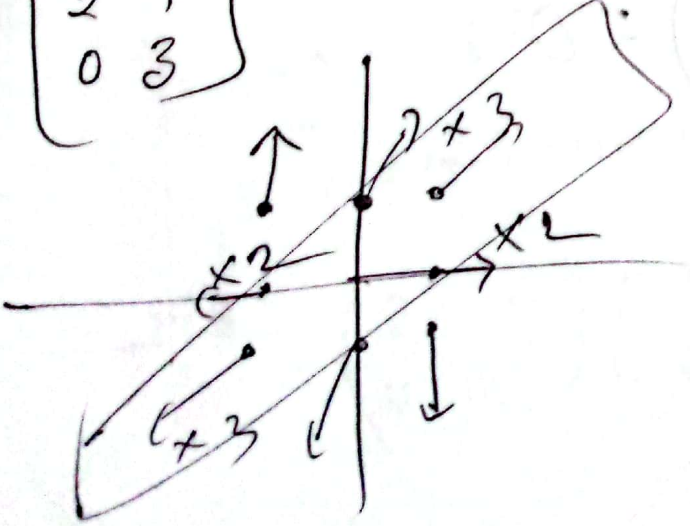
$(3, 2) \to (8, 6)$

It can be expressed by as a combination of elements in the basis.

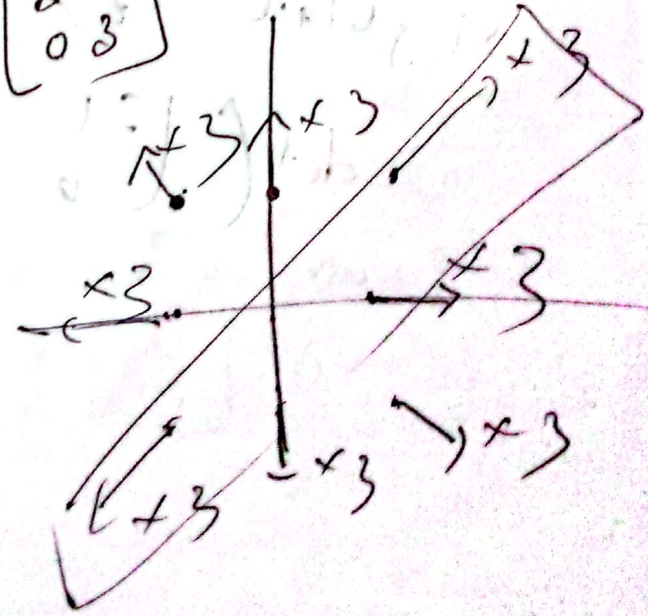





$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

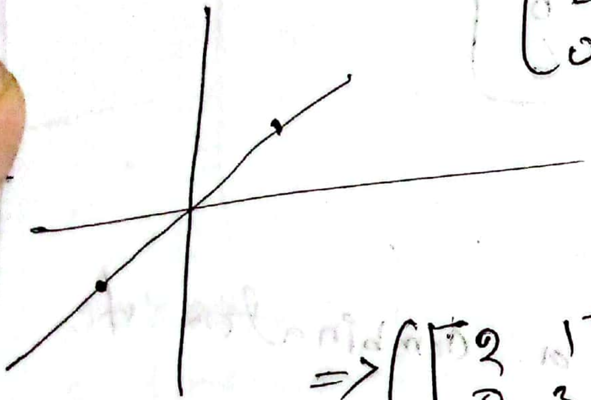


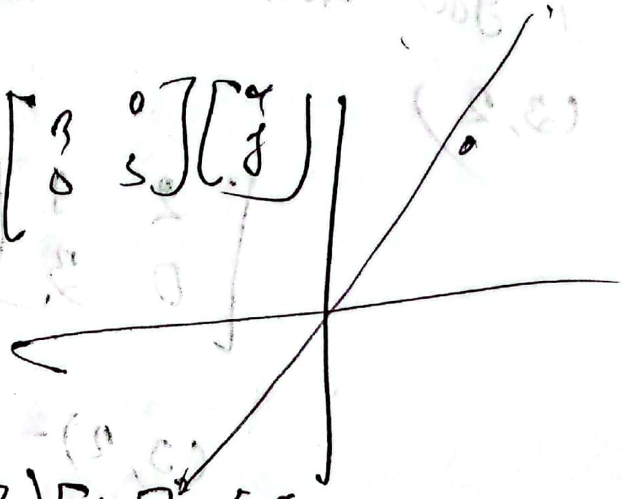

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$



they act exact same way for infinitely many points all the points in the line

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$





$$\Rightarrow \left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

→ as they are same for many points. so something non singular is happening

so, if you have any infinity many solve system → singular

singular (2,

$$\det\left(\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\right) = 0$$

If $\lambda$ is an eigenvalue:

transformation, given by our matrix,

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

transformation given by scaling the plane entirely by $\lambda$ are equal for infinitely many vectors $x, y$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\right) = 0$$

characteristic polynomial,

$$(2-\lambda)(3-\lambda) - 1.0 = 0$$

eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$

$2x + y = 2x$

$0.x + 3y = 2y$

$x = 1$

$y = 0$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 3\begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{matrix} 2x+y=3x \\ 0x+3y=3y \end{matrix}$$

$x=1$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$y=1$

(3) $\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow$

characteristic polynomial,

$(9-\lambda)(3-\lambda) - 16 = 0$

$\lambda = 11,$ $(9-11)(3-11)$ | $\lambda = 2$

✓ $-2 \times -8$ | $(9-2)(3-2)$

16 | $7 \times 1 = 7$

$\lambda = 1,$ $8 \times 2$ $\quad$ $\lambda = 0,$ $9 \times 3$

" Eigenvalue of identity
matrix = 1 "

$\lambda$ eigenvalue of $\longrightarrow$ nonsingular $A$

$\lambda^{-1} \longrightarrow A^{-1}$

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Reflection over y axis

→ multiply $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} x \text{ shear} \\ y \text{ shear} \end{bmatrix}$

Rotation

**Application of Eigenvalues**

→ Navigating webpages